Computational illumination optics

Wiskundige modellen voor optisch ontwerp

WND-conferentie, Noordwijkerhout, 15 december 2023

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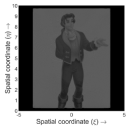
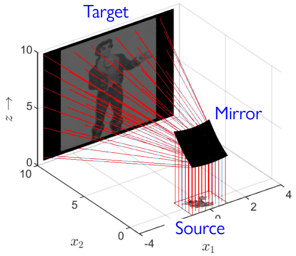
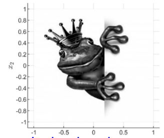
TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Can we turn a frog into a prince?



Light distribution target





Light distribution source

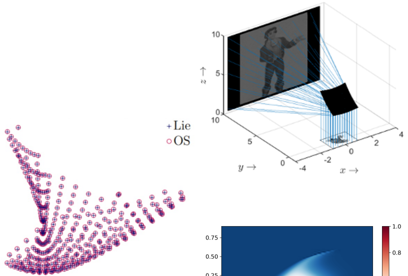
- *"In the original Grimm version of the story, the frog's spell was broken when the princess threw the frog against the wall, at which he transformed back into a prince, while in modern versions the transformation is triggered by the princess kissing the frog."*

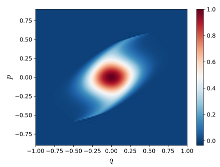
  https://en.wikipedia.org/wiki/The_Frog_Prince
- Can we do it with light?

Romijn, L. B. (2021)
Generated Jacobian Equations in Freeform Optical Design: Mathematical Theory and Numerics
PhD thesis, Eindhoven University of Technology

Outline

- Computational illumination optics at TU/e
- Nonimaging freeform optics
  - Sixteen basic systems
  - 1: Standard Monge-Ampère equation
  - 2: Generalized Monge-Ampère equation
  - 3: Generated Jacobian Equation
  - Iterative least-squares solver for 3D systems
  - Numerical results
- Imaging optics
- Improved direct methods
- Conclusions and future work





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Computational Illumination Optics Group at TU/e

Martijn Anthonissen | Wilbert IJzerman *Signify Research* | Lisa Kusch | Koondi Mitra | Jan ten Thije Boonkkamp

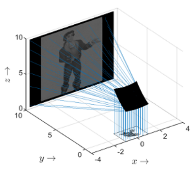
Antonio Barion | Pieter Braam | Roel Hacking | Willem Jansen | Vi Kronberg | Teun van Roosmalen | Sanjana Verma

Maikel Bertens | Carmela Filosa | Corien Prins | Bart van Lith | Lotte Romijn | Robert van Gestel | Nitin Yadav

OPTIC
AWAVE
Math & CS
OTP

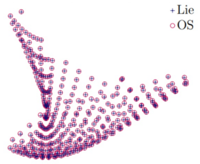
Lines of research

Line A
Nonimaging freeform optics



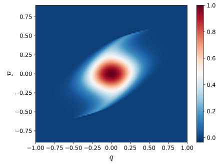
- Compute optical surfaces that convert given source into desired target distribution
- Freeform surfaces
- Fully nonlinear PDE of Monge-Ampère type

Line B
Imaging optics



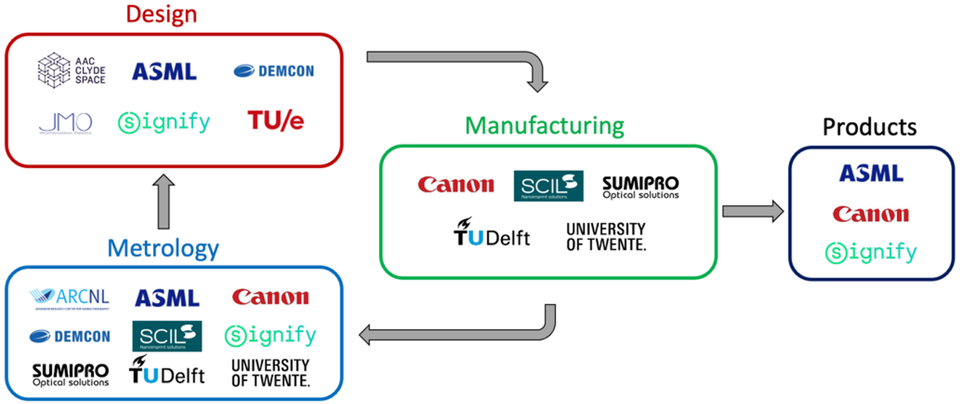
- Make a very precise image of an object, minimizing aberrations
- Description with Lie transformations

Line C
Improved direct methods



- Ray tracing: iterative procedure to compute final design. Slow convergence
- Backward ray mapping
- Hamiltonian structure of equations and advanced numerical schemes for PDEs

Academic cooperation and industrial embedding



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Nonimaging freeform optics

Design of optical systems for illumination purposes

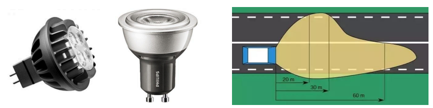
- LED lighting
- Road lights
- Car headlights

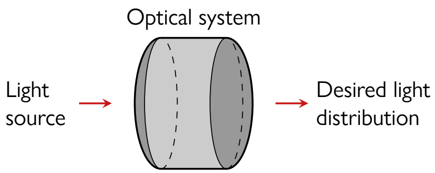
Industry standard: ray tracing

- Easy to implement
- Slow convergence
- Design, ray trace, change design, ray trace, …

Inverse methods

- Directly compute required optical system
- Need solving PDE of Monge-Ampère type
- Avoid iterations and manual optimization





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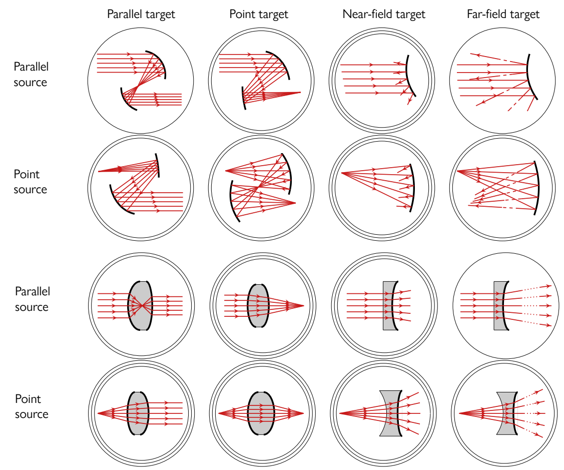
Sixteen basic systems

SMA GMA GJE

Reflector systems

Lens systems



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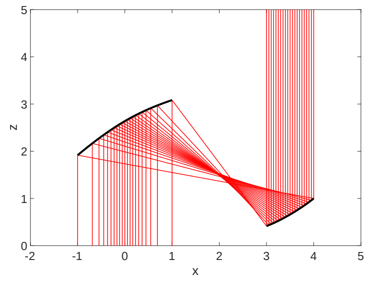
Parallel-to-parallel reflector 2D



- Source: parallel beam with Gaussian light distribution
- Desired at target: parallel beam with uniform distribution
- Find the two freeform reflector surfaces

Mathematical model: optimal transport formulation

- Path of a ray
  - Leaves source S at $P = (x, 0)$
  - Hits first reflector at $A = (x, u(x))$
  - Hits second reflector at $B = (y, L - w(y))$
  - Arrives at target T at $Q = (y, L)$
- **Optical path length**

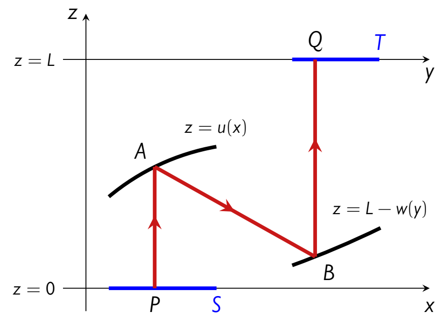
$$V = u(x) + d(A, B) + w(y)$$

- Let $\beta := V - L$. Then

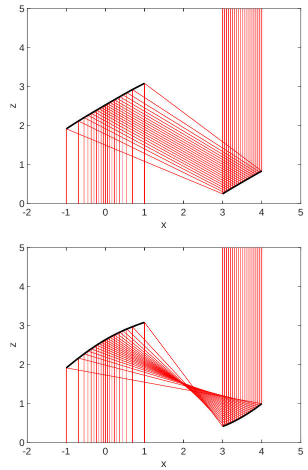
$$u(x) + w(y) = \tfrac{1}{2}\beta + L - \frac{(y-x)^2}{2\beta} =: c(x,y)$$

$c(x,y)$ is called cost function

- Differentiate to x: **optical mapping** $y = m(x) = x + \beta u'(x)$



Energy conservation



- $S = (-1, 1)$, $T = (3, 4)$
  Source light distribution: $E = E(x)$, $x \in S$, Gaussian
  Target light distribution: $G = G(y)$, $y \in T$, uniform
- Energy conservation:

$$\int_{-1}^{x} E(\xi)\, d\xi = \pm \int_{m(-1)}^{m(x)} G(y)\, dy = \pm \int_{-1}^{x} G(m(\xi))\, m'(\xi)\, d\xi$$

- Differentiate to x:

$$m'(x) = \pm \frac{E(x)}{G(m(x))}$$

Solve ODE for m

- Differentiate $u(x) + w(y) = c(x,y)$ to x, substitute $y = m(x)$:

$$u'(x) = \frac{\partial c}{\partial x}(x, m(x))$$

Solve ODE for u

- Second reflector: $w(m(x)) = c(x, m(x)) - u(x)$

Parallel-to-parallel reflector, 2D and 3D

2D

- Optical mapping:

$$y = m(x) = x + \beta u'(x) = \phi'(x)$$

- Optimal transport formulation:

$$u(x) + w(y) = c(x,y)$$
$$c(x,y) = \tfrac{1}{2}\beta + L - \frac{(y-x)^2}{2\beta}$$

- Energy conservation:

$$m'(x) = \phi''(x) = \pm \frac{E(x)}{G(m(x))}$$

3D: $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$

- Optical mapping:

$$\mathbf{y} = \mathbf{m}(\mathbf{x}) = \mathbf{x} + \beta \nabla u(\mathbf{x}) = \nabla \phi(\mathbf{x})$$

- Optimal transport formulation:

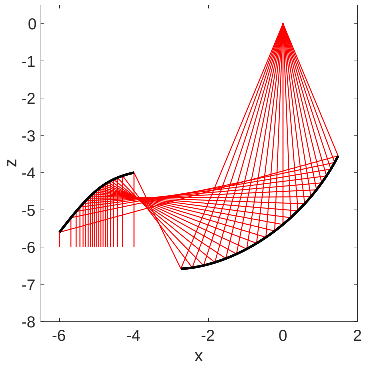
$$u(\mathbf{x}) + w(\mathbf{y}) = c(\mathbf{x}, \mathbf{y})$$
$$c(\mathbf{x}, \mathbf{y}) = \tfrac{1}{2}\beta + L - \frac{\|\mathbf{y} - \mathbf{x}\|^2}{2\beta}$$

- Energy conservation:

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \frac{\partial^2 \phi}{\partial x_1^2}\frac{\partial^2 \phi}{\partial x_2^2} - \left(\frac{\partial^2 \phi}{\partial x_1 \partial x_2}\right)^2 = \pm \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))}$$

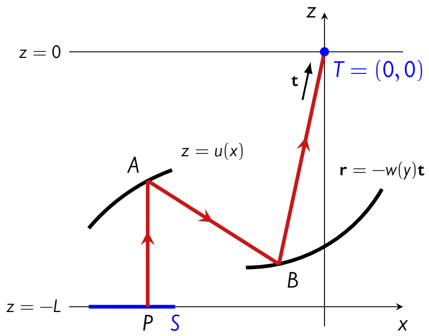
Standard Monge-Ampère equation
Second-order nonlinear PDE

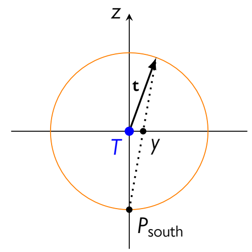
Parallel-to-point reflector 2D



- Source: parallel beam with Gaussian light distribution
- Point target with uniform distribution
- Find the two freeform reflector surfaces

Stereographic projection from the south pole





- Circle has radius 1, so

$$\mathbf{t} = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \qquad t_1^2 + t_2^2 = 1$$

- South pole: $P_{\text{south}} = (0, -1)$
- One can show that

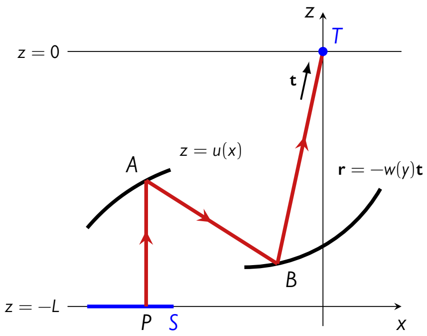
$$t_1 = \frac{2y}{1+y^2} \qquad t_2 = \frac{1-y^2}{1+y^2} \qquad y = \frac{t_1}{1+t_2}$$

Mathematical model: optimal transport formulation

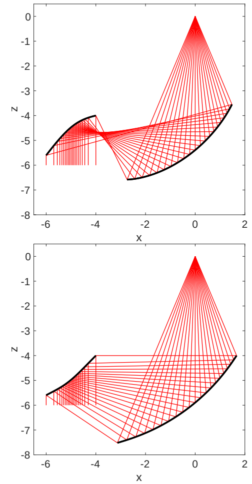
- Path of a ray
  - Leaves source S at $P = (x, -L)$
  - Hits first reflector at $A = (x, -L + u(x))$
  - Hits second reflector at $B = (-w(y)t_1, -w(y)t_2)$
  - Arrives at target $T = (0, 0)$
- Optical path length: $V = u(x) + d(A, B) + w(y)$
- Geometric relation

$$\left(-\frac{u}{\beta} - \frac{x^2}{2\beta^2} + \frac{V+L}{2\beta}\right) \cdot \left(\frac{\beta}{w}(1+y^2) - 2y^2\right) = \left(1 + \frac{xy}{\beta}\right)^2$$

- Take logarithms: $u_1(x) + u_2(y) = c(x, y)$
  Cost function is non-quadratic



Numerical method



- Energy conservation:

$$m'(x) = \pm \frac{E(x)}{G(m(x))} \frac{1 + (m(x))^2}{2}$$

  Solve ODE for m
- Differentiate $u_1(x) + u_2(y) = c(x, y)$ to x, substitute $y = m(x)$:

$$u_1'(x) = \frac{\partial c}{\partial x}(x, m(x))$$

  Solve ODE for u_1
  From u_1 compute u
- Second reflector:

$$u_2(m(x)) = c(x, m(x)) - u_1(x)$$

  From u_2 compute w

Parallel-to-point reflector, 2D and 3D

GMA

2D

- Energy conservation:

$$m'(x) = \pm \frac{E(x)}{G(m(x))} \frac{1+(m(x))^2}{2}$$

- Optimal transport formulation:

$$u_1(x) + u_2(y) = c(x,y)$$

$$c(x,y) = \log\left(\left(1+\frac{xy}{\beta}\right)^2\right)$$

3D: $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $\quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ $\quad \mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$

- Energy conservation:

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \pm \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))} \frac{\left(1+\|\mathbf{m}(\mathbf{x})\|^2\right)^2}{4}$$

- Optimal transport formulation:

$$u_1(\mathbf{x}) + u_2(\mathbf{y}) = c(\mathbf{x},\mathbf{y})$$

$$c(\mathbf{x},\mathbf{y}) = \log\left(\left(1+\frac{\mathbf{x}\cdot\mathbf{y}}{\beta}\right)^2\right)$$

- Differentiate, substitute $\mathbf{y} = \mathbf{m}(\mathbf{x})$, differentiate

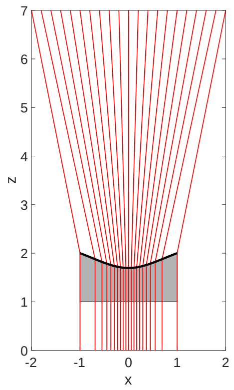
$$\underbrace{\mathrm{D}_{\mathbf{xy}}c(\mathbf{x},\mathbf{m}(\mathbf{x}))}_{\mathbf{C}}\ \mathrm{D}\mathbf{m}(\mathbf{x}) = \underbrace{\mathrm{D}^2u_1(\mathbf{x}) - \mathrm{D}_{\mathbf{xx}}c(\mathbf{x},\mathbf{m}(\mathbf{x}))}_{\mathbf{P}}$$

- **Generalized Monge-Ampère equation**

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \frac{\det(\mathbf{P})}{\det(\mathbf{C})} = \pm \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))} \frac{\left(1+\|\mathbf{m}(\mathbf{x})\|^2\right)^2}{4}$$

Parallel-to-near-field lens

GJE



- Source: parallel beam with Gaussian light distribution
- Near-field target with uniform distribution
- Find the single freeform lens surface

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Mathematical model

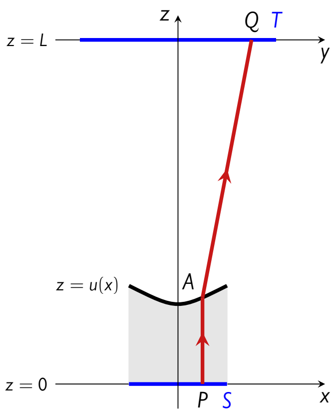
GJE

- Path of a ray
  - Leaves source S at $P = (x, 0)$
  - Hits freeform lens surface at $A = (x, u(x))$
  - Arrives at target T at $Q = (y, L)$
- Optical path length:

$$V(y) = n \cdot u(x) + d(A,Q)$$

$$= n \cdot u(x) + \sqrt{(y-x)^2 + (L-u(x))^2}$$

Cannot be formulated as optimal transport problem



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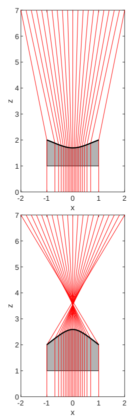
Numerical method

GJE



- Energy conservation:

$$m'(x) = \pm \frac{E(x)}{G(m(x))}$$

Solve ODE for m

- Geometric relation:

$$V(y) = n \cdot u(x) + \sqrt{(y-x)^2 + (L-u(x))^2}$$

differentiate to x, substitute $y = m(x)$, solve for $u'(x)$:

$$u'(x) = \frac{m(x) - x}{n \cdot \sqrt{(m(x)-x)^2 + (L-u(x))^2} + u(x) - L}$$

Solve ODE for u

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Parallel-to-near-field lens, 2D and 3D

GJE

2D

- Energy conservation:

$$m'(x) = \pm \frac{E(x)}{G(m(x))}$$

- Geometric relation:

$$V(y) = n \cdot u(x) + \sqrt{(y-x)^2 + (L-u(x))^2}$$

3D

- Energy conservation:

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \pm \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))}$$

- Geometric relation:

$$V(\mathbf{y}) = n \cdot u(\mathbf{x}) + \sqrt{\|\mathbf{y}-\mathbf{x}\|^2 + (L-u(\mathbf{x}))^2} =: H(\mathbf{x}, \mathbf{y}, u(\mathbf{x}))$$

- Differentiate to $\mathbf{x}$: $\nabla_{\mathbf{x}} \big(H(\mathbf{x}, \mathbf{y}, u(\mathbf{x})) \big) = \mathbf{0}$
- Let $\widetilde{H}(\mathbf{x}, \mathbf{y}) := H(\mathbf{x}, \mathbf{y}, u(\mathbf{x}))$, substitute $\mathbf{y} = \mathbf{m}(\mathbf{x})$, differentiate to $\mathbf{x}$:

$$\mathrm{D}_{\mathbf{xx}}\widetilde{H}(\mathbf{x}, \mathbf{m}(\mathbf{x})) + \underbrace{\mathrm{D}_{\mathbf{xy}}\widetilde{H}(\mathbf{x}, \mathbf{m}(\mathbf{x}))}_{\mathbf{C}} (\mathrm{D}\mathbf{m})(\mathbf{x}) = \mathbf{O}$$

- **Generated Jacobian equation**

$$\det\big(\mathrm{D}_{\mathbf{xx}}\widetilde{H}(\mathbf{x}, \mathbf{m}(\mathbf{x}))\big) = \pm \det(\mathbf{C}) \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))}$$

Iterative least-squares solver for 3D systems

SMA GMA GJE

- Find mapping $\mathbf{m} : S \to T$ such that

$$\det(\mathrm{D}\mathbf{m}) = \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))}$$
$$\mathbf{m}(\partial S) = \partial T$$

- Break down in substeps
  We compute $\mathbf{P}$, $\mathbf{b}$, $\mathbf{m}$ such that

$$\mathbf{P} = \mathbf{C}\,\mathrm{D}\mathbf{m}$$
$$\det(\mathbf{P}) = \det(\mathbf{C}) \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))}$$
$$\mathbf{b}(\mathbf{x}) = \mathbf{m}(\mathbf{x}) \qquad \mathbf{x} \in \partial S$$

$\mathbf{b}$ maps ∂S to ∂T

- Iterative procedure:
  1. Choose an initial guess $\mathbf{m}^0$
     Let n = 0
  2. Let $J_I(\mathbf{m}, \mathbf{P}) = \frac{1}{2} \iint_S \|\mathbf{C}\,\mathrm{D}\mathbf{m} - \mathbf{P}\|^2 \, d\mathbf{x}$

     $\mathbf{P}^{n+1} = \underset{\mathbf{P}}{\operatorname{argmin}}\ J_I(\mathbf{m}^n, \mathbf{P})$

     Constrained minimization problem
  3. Let $J_B(\mathbf{m}, \mathbf{b}) = \frac{1}{2} \int_{\partial S} \|\mathbf{m} - \mathbf{b}\|^2 \, ds$

     $\mathbf{b}^{n+1} = \underset{\mathbf{b}}{\operatorname{argmin}}\ J_B(\mathbf{m}^n, \mathbf{b})$

     Projection on boundary ∂T
  4. Let $J(\mathbf{m}, \mathbf{P}, \mathbf{b}) = \alpha J_I(\mathbf{m}, \mathbf{P}) + (1-\alpha) J_B(\mathbf{m}, \mathbf{b})$

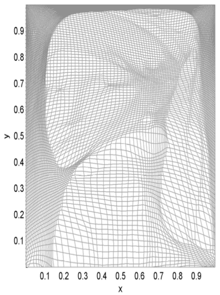
     $\mathbf{m}^{n+1} = \underset{\mathbf{m}}{\operatorname{argmin}}\ J(\mathbf{m}, \mathbf{P}^{n+1}, \mathbf{b}^{n+1})$

     Elliptic PDEs for m_1 and m_2 — FVM
  5. Let $n := n + 1$, go to Step 2

Girl with a pearl earring (parallel-to-far-field reflector 3D)

- Parallel source
- Uniform source distribution
- Far-field target
- Single reflector

Target



Mapping

Ray trace

Prins, C. R., ten Thije Boonkkamp, J. H. M., van Roosmalen, J., IJzerman, W. L., Tukker, T. W. (2014)
A Monge-Ampère-solver for freeform reflector design
SIAM Journal on Scientific Computing, 36(3), B640-B660
https://doi.org/10.1137/130938876

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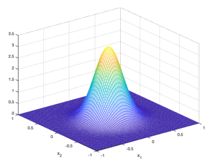
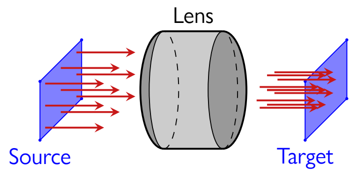
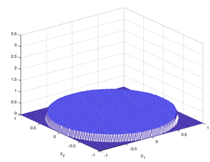
Laser beam shaping (parallel-to-parallel lens 3D)







Source distribution:
Gaussian profile
Source emits parallel light rays

Optical system:
one lens with two freeform surfaces

Desired target distribution:
circular top hat profile
Output beam: parallel light rays

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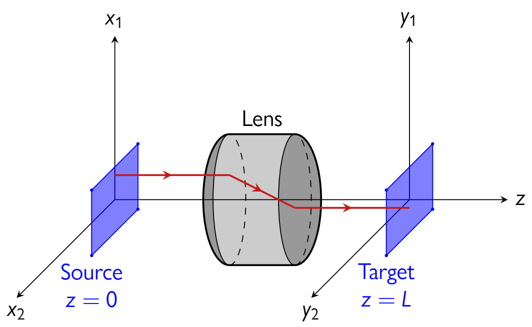
Laser beam shaping



- Light rays travel from left to right
- Source plane: $z = 0$

  Cartesian coordinates $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

  Source S emits parallel light rays

  Exitance: $f(\mathbf{x})$, $\mathbf{x} \in S$
- Target plane: $z = L$

  Cartesian coordinates $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

  Desired irradiance: $g(\mathbf{y})$, $\mathbf{y} \in T$
- First lens surface: $z = u(\mathbf{x})$, $\mathbf{x} \in S$

  Second lens surface: $L - z = w(\mathbf{y})$, $\mathbf{y} \in T$

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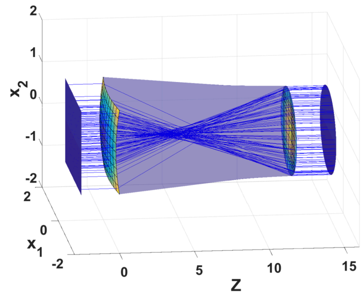
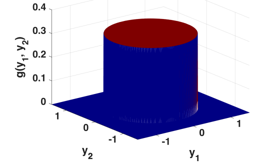
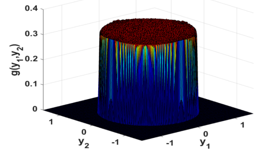
Numerical results for the laser beam shaping problem



Desired target distribution

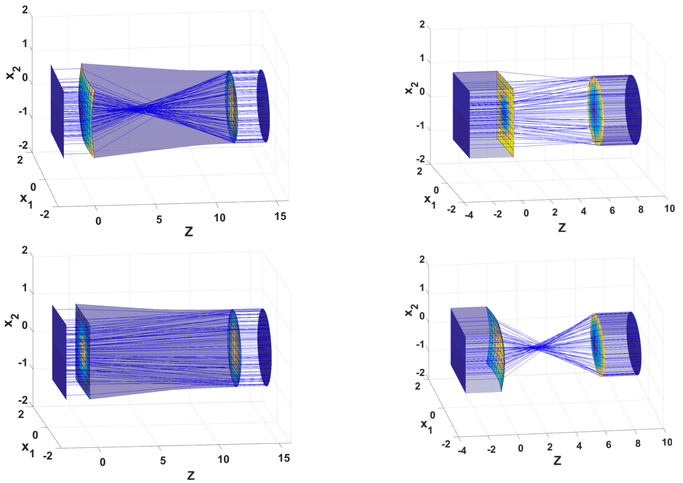


Achieved



Yadav, N. K., ten Thije Boonkkamp, J. H. M., IJzerman, W. L. (2019)
Computation of double freeform optical surfaces using a Monge–Ampère solver: Application to beam shaping
Optics Communications, 439, 251-259
https://doi.org/10.1016/j.optcom.2019.01.069

Alternative optical systems



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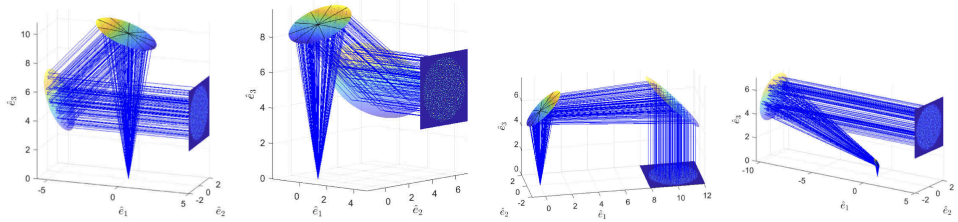
Point-to-parallel reflector system 3D



Van Roosmalen, A. H., Anthonissen, M. J. H., IJzerman, W. L., ten Thije Boonkkamp, J. H. M. (2021)
Design of a freeform two-reflector system to collimate and shape a point source distribution
Optics Express, 29(16), 25605-25625
https://doi.org/10.1364/OE.425289

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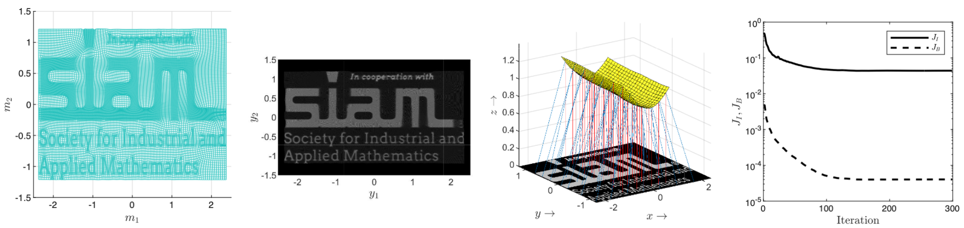
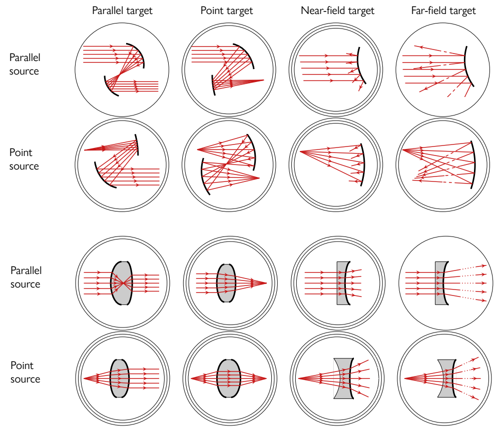
Parallel-to-near-field reflector 3D



Mapping Ray trace Surface Convergence

Romijn, L. B., Anthonissen, M. J. H., ten Thije Boonkkamp, J. H. M., IJzerman, W. L. (2021)
An iterative least-squares method for generated Jacobian equations in freeform optical design
SIAM Journal on Scientific Computing, 43(2), B298-B322
https://doi.org/10.1137/20M1338940

Sixteen basic systems

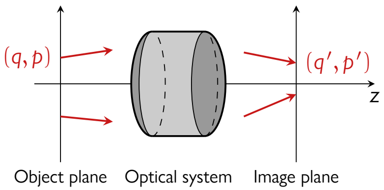


- **Model 1: Standard Monge-Ampère equation (SMA)**
  Optimal transport formulation with quadratic cost function
- **Model 2: Generalized Monge-Ampère equation (GMA)**
  Optimal transport formulation with non-quadratic cost function
- **Model 3: Generated Jacobian equation (GJE)**
  No optimal transport formulation

Anthonissen, M. J. H., Romijn, L. B., ten Thije Boonkkamp, J. H. M., IJzerman, W. L. (2021)
Unified mathematical framework for a class of fundamental freeform optical systems, Optics Express, 29(20), 31650-31664, https://doi.org/10.1364/OE.438920

Imaging optics

- Consider imaging optical system



- Each ray is defined by position q and direction p
- Hamiltonian optics:

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}, \quad H = -\sqrt{n^2 - p^2}$$

- Optical map:

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = M \begin{pmatrix} q \\ p \end{pmatrix}$$

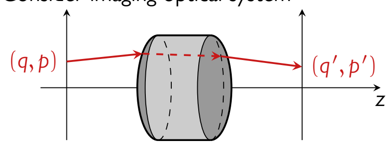
Ideal system:

$$M = M_{\text{linear}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

- In practice M is nonlinear
  Goal: Mathematical description of nonlinearities, called **aberrations**

Finding aberrations

- Consider imaging optical system



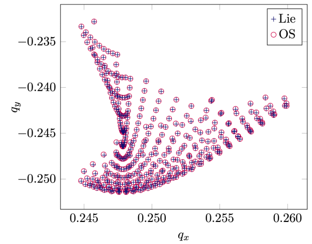
- Break down in steps: P—R—P—R—P
  P: propagation
  R: refraction
- Write each step as a **Lie transformation**

$$M_i = \exp\left([f_i, \cdot]\right)$$

$$[f_i, \cdot] = \frac{\partial f_i}{\partial q}\frac{\partial}{\partial p} - \frac{\partial f_i}{\partial p}\frac{\partial}{\partial q}$$

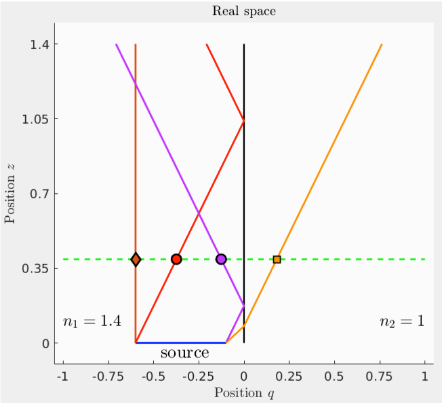
- Optical map is product of M_i

- Spot diagram off-axis object



Barion, A., Anthonissen, M. J. H., ten Thije Boonkkamp, J. H. M., IJzerman, W. L. (2023)
Computing aberration coefficients for plane-symmetric reflective systems: A Lie algebraic approach
Journal of the Optical Society of America A, Optics, Image Science and Vision, 40(6), 1215-1224
https://doi.org/10.1364/JOSAA.487343

Improved direct methods



- New methods work in **phase space**
- $q < 0$: Water, refractive index $n_1 = 1.4$
  $q > 0$: Air, $n_2 = 1$
  Light source at $z = 0$
- Each ray can be defined by position $q(z)$ and direction $p(z)$
- Four rays:
  - ◆ $p(0) = 0 \longrightarrow q(z)$ constant
  - ● $p(0) > p_{\text{crit}} \longrightarrow$ ray is reflected
  - ● $p(0) > p_{\text{crit}} \longrightarrow$ ray is reflected
  - ■ $0 < p(0) < p_{\text{crit}} \longrightarrow$ ray is refracted

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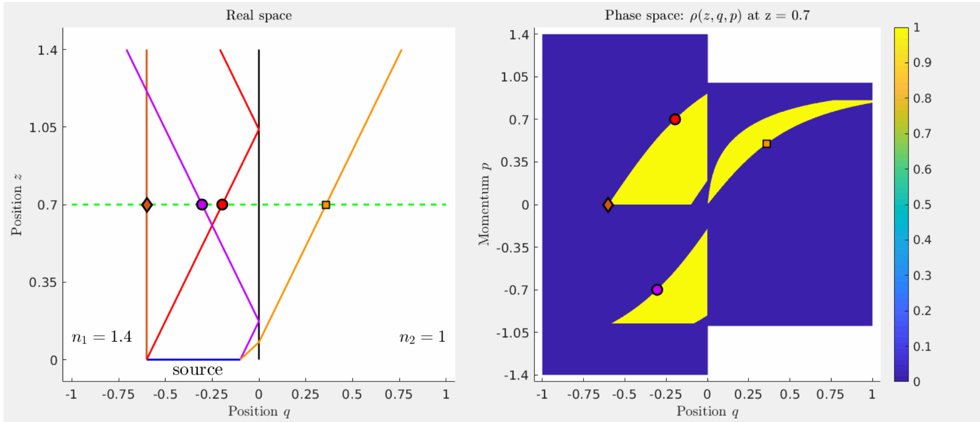
Phase space



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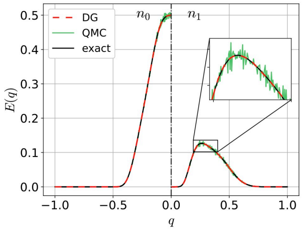
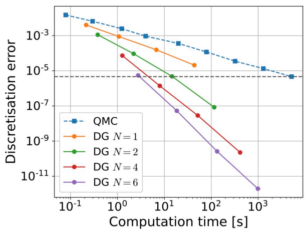
Liouville's equation





- Conservation of luminance $\rho = \rho(z, q, p)$

$$\frac{\partial \rho}{\partial z} + \nabla \cdot (\rho \mathbf{u}) = 0, \qquad \mathbf{u} = \frac{1}{\sqrt{n^2 - p^2}} \begin{pmatrix} p \\ n\frac{\partial n}{\partial q} \end{pmatrix}$$

Liouville's equation

- Discontinuous Galerkin spectral element method (DG)
- Compared to quasi Monte Carlo ray tracing (QMC)

van Gestel, R. A. M., Anthonissen, M. J. H.,
ten Thije Boonkkamp, J. H. M., IJzerman, W. L. (2023)
An ADER discontinuous Galerkin method on moving meshes for Liouville's equation of geometrical optics
Journal of Computational Physics, 488, 112208
https://doi.org/10.1016/j.jcp.2023.112208

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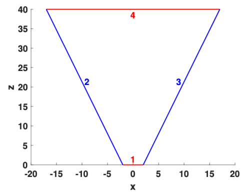
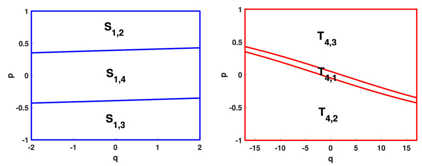
Backward ray mapping

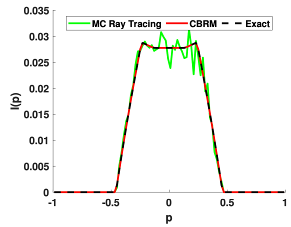
- Two-faceted cup



- Phase spaces source and target



- Intensity at target



Jansen, W. G. T., Anthonissen, M. J. H.,
ten Thije Boonkkamp, J. H. M., IJzerman, W. L. (2023)
Concatenated Backward Ray Mapping on the Compound Parabolic Concentrator
Journal of Mathematics in Industry. Submitted

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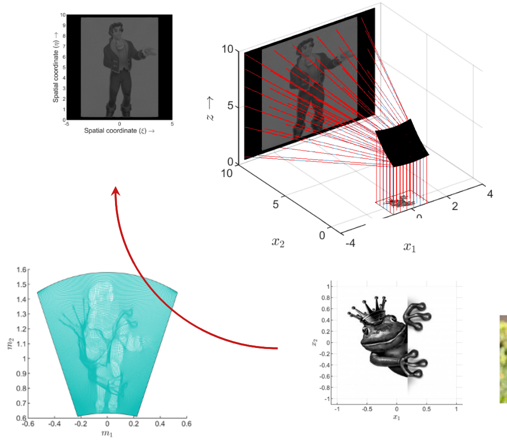
Conclusions



Optical mapping

- Yes we can transform a frog into a prince with a mirror!
- And we can model and simulate another fifteen basic optical systems

Romijn, L. B. (2021)
Generated Jacobian Equations in Freeform Optical Design: Mathematical Theory and Numerics
PhD thesis, Eindhoven University of Technology

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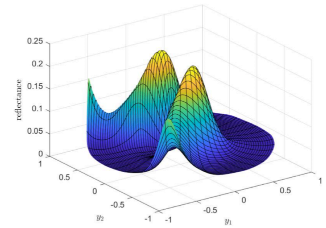
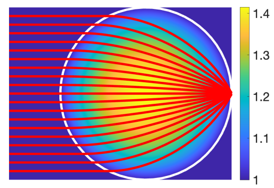
Extensions and future work

- Include Fresnel reflection and scattering
- Design lens arrays with smooth surfaces
- Use machine learning to speed up or replace the least-squares solver
- Model finite sources
- Model GRIN optics
- Move beyond the sixteen basic systems
- Include phase information



Reflectance on a freeform lens for street illumination



Luneburg lens

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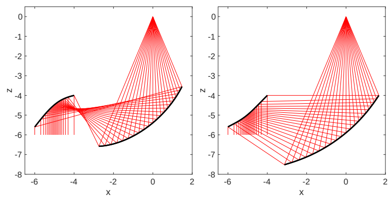
Concave, convex and saddle surfaces

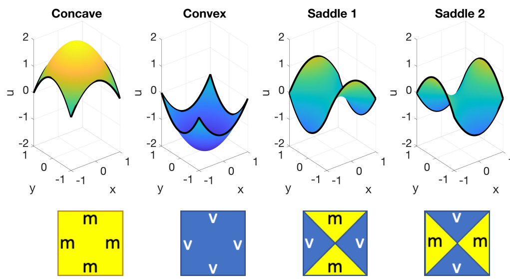
2D

$$m'(x) = \pm \frac{E(x)}{G(m(x))} \frac{1 + (m(x))^2}{2}$$

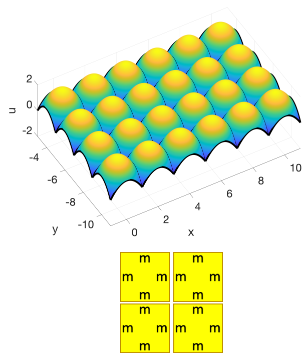


3D

$$\det(\mathrm{D}\mathbf{m}(\mathbf{x})) = \pm \frac{E(\mathbf{x})}{G(\mathbf{m}(\mathbf{x}))} \frac{\left(1 + \|\mathbf{m}(\mathbf{x})\|^2\right)^2}{4}$$

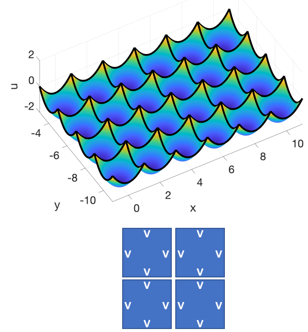


Lens arrays



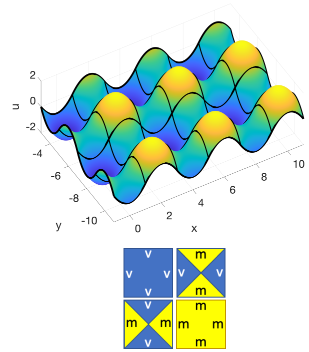
Concave elements only

Not smooth at interfaces



Convex elements only

Not smooth at interfaces



Concave, convex and saddle

Smooth everywhere

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