Can we understand the tippee top?

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Abstract
Tippee tops have long been a fascination, yet many explanations go beyond the reach of the average reader. Here I attempt to give an insight into the workings in a way that most will be able to understand and open the door to more investigation and wonder.

Tippee tops have been around for many years and are often portrayed as being mysterious or magical by those trying to sell these delightful items. These little tops invert themselves, standing on their ends after spinning for a while. They are indeed mesmerizing to those who encounter them for the first time. Bohr and Pauli were fascinated by this small top when they first came across it at Lund University[1]; however, a more contemporary photograph is shown in figure 1. The tops can still amaze.

So, is the physics behind these spinning tops, which turn upside down and reverse their direction of spin, that complicated, or can anyone with some basic physics appreciate what lies behind their motion? It has been said that ‘tops are the playthings of children and adults with PhDs in mathematics’. I hope to show that the reference to PhDs is unnecessary and that we can all have a good basic understanding of these wonderful tops. In this article I will therefore take the reader on a step-by-step journey describing the motion and exploring explanations without drowning ourselves in mathematical equations.

First, let us describe the motion of a tippee in some detail. The tippee top is a fairly small round bottomed top, shaped so that its centre of mass is lower than the centre of curvature of the rounded bottom. It has a short stem with a rounded top, so that when inverted the centre of mass of the top lies above the centre of curvature of the end of the stem (figure 2).

When spun in a clockwise sense (from above) the top begins to lean over away from the vertical and bodily rotates clockwise. (This is normal top precession, described elsewhere [1].) As this leaning increases, the rotational spin speed of the top about its own axis slows, the bodily rotation increases and eventually the top of the stem touches the surface. At this point in time the top quickly stands up on its stem, i.e. upside down from its initial spin position, and is now seen to be spinning (from the top’s point of view) in the opposite direction, having reversed its direction. Viewed from above, however, the top is still spinning in a clockwise direction.

Much has been written about this motion, which quickly seems to move beyond the understanding of many in schools, hence the reference to higher degrees. However, this article will hopefully stay within everyone’s reach. Understanding physics is in many ways like peeling an onion—we can peel deeper and deeper, with each layer offering a new description, with greater insight. The skill of the teacher is to peel sufficiently to challenge students without going too far and disengaging their interest. It is also important that we understand that these descriptions are often simply applications of

[1] There is a photograph of Niels Bohr and Wolfgang Pauli studying a tippee top, taken by Erik Gustafson at the opening of the new institute of physics at the University of Lund 31 May 1951 and held in the Emilio Segre Visual Archives, Margrethe Bohr collection, www.aip.org/history/esva.

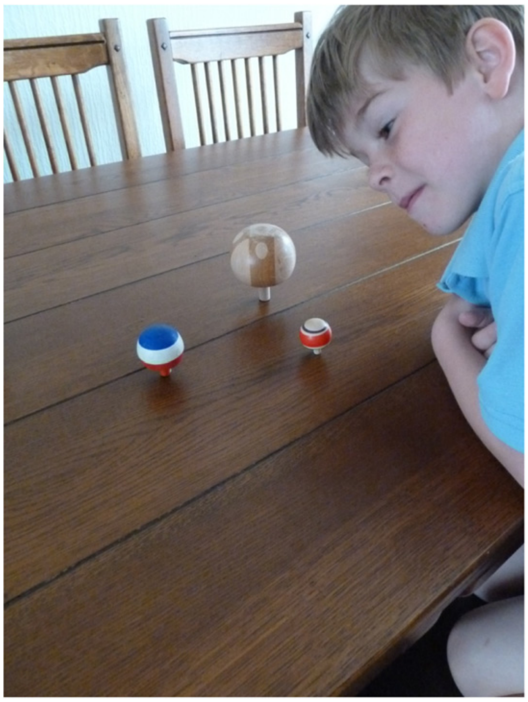

Figure 1. Young child fascinated by inverted spinning tippee tops.

Figure 2. Picture of tippee tops.

general principles, rather than being the ultimate explanation of why things are as they are.

Beginning with any spinning top, we need to appreciate the fundamental difference between a top with a pointed bottom and one with a small rounded bottom. A sharp pointed top will noticeably in time precess about the vertical, gradually lowering itself towards the surface on which the top rests. (This happens with a gyroscope too, which has a supported point.) This precessional motion is a result of the top's spin and the gravitational couple trying to pull it over. The same thing will happen with a spinning round bottomed top, i.e. it precesses and eventually

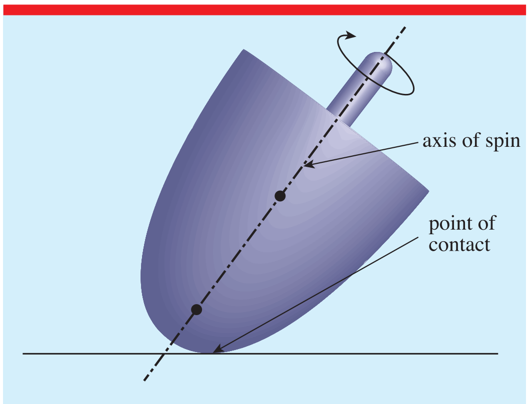

Figure 3. Round bottomed top showing the axis of spin and point of contact.

falls over. This precession is adequately described elsewhere, with varying degrees of mathematical complexity. An excellent explanation using simply the concept of momentum is to be found in Rod Cross's article on spinning tops [2].

On the other hand, if a round bottomed top (like, for example, a traditional whipping top) is spinning fast enough, it will, if leaning over to one side (and precessing), right itself into the vertical position again. This is most clearly seen, for example, just after being rewhipped. The top leans over very noticeably but soon is standing vertically again. This erection happens because of a frictional couple produced at the point of contact. The key is that the point of contact is at a distance from the rotational axis and centre of mass of the top (figure 3). The spinning round bottomed top presents an additional frictional force, and hence couple, to produce the righting behaviour due to the rounded surface at its base and the position of the point of contact. A couple about the centre of mass is therefore produced. This frictional couple leads to the additional resultant couple on the round bottomed top, which re-erects the spinning top.

This is a first level of explanation which helps us to understand the tippee top's unusual behaviour. When a round bottomed top is tilting, a couple acts which rights a small radius round bottomed top (because the centre of mass is on one side of the point of contact), but further topples a large radius round bottomed top (which has its centre of mass on the opposite side) (figure 4). This righting effect is emphasized

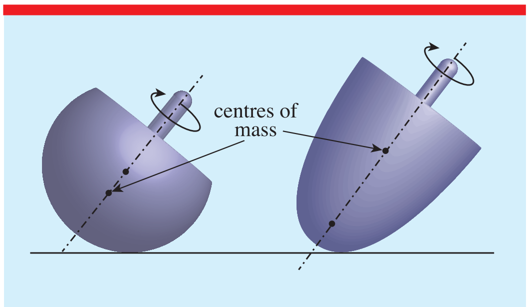

Figure 4. Tippee top, and small radius tops showing positions of centres of mass.

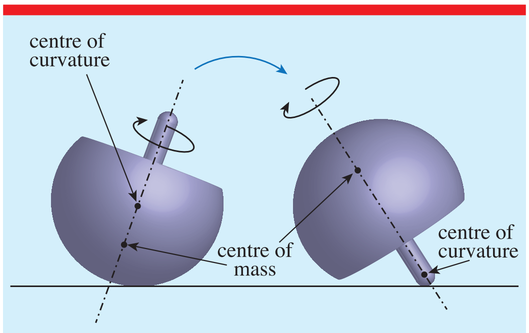

Figure 5. Tippee top showing progression from a large bottom (round base) to a small one (stem).

when a particularly small radius is used, as with a small ball bearing embedded in the bottom of the top. This is because of the relative positions of the centres of mass of the tops. Here, we have a simple rule of explanation of the motion of a tippee top. If moving fast enough, a small radius round bottomed top rights itself, whereas a larger radius round bottomed top tilts further and further.

Therefore, once the tippee top—with its large radius bottom—begins to tilt, it continues to tilt further until the smaller radius shaft reaches the surface (figure 5). Once that contact is made, the smaller radius of the shaft means that the top erects itself on the shaft. The tippee changes from being a large radius round bottomed top (which tips itself further) to a small radius round bottomed top (which erects itself).

Angular momentum has to be conserved. Hence, when upside down, the top must spin in the same sense, viewed from above, as it was spinning initially, hence it must have reversed its direction as it turned upside down. This means that at some point the top itself must stop spinning, all the angular momentum being within the bodily rotation of the top. The upside down top does rotate at a slightly slower angular speed due its increase in potential energy (the centre of mass having been raised), and the loss of angular momentum is accounted for because of the frictional couple which causes the erection.

It must be recognized that this is not really an explanation at all, but simply a description of two alternatives (tipping or erecting) which describe what can happen with the additional frictional couple of spinning tops with round bottoms. Almost all bottoms of tops are rounded. The compromise is a gyroscope which has a pointed base which rotates within the frame. The gyroscope spins but the frame does not slip over the surface. The gyroscope precesses but does not re-erect once it begins to tilt over because it does not have the sliding frictional couple to do it.

To understand the motion further we need to look more carefully at the couples acting and the directions of these couples. A small radius top has its centre of mass above the centre of curvature of the base, whilst a large radius top may have its centre of mass below the centre of curvature. This means that the centres of mass are on opposite sides of the point of contact, leading in turn to couples in opposite senses.

Let us ask now why the turning occurs at all. This level of explanation lies in the way that vectors at right angles combine. Students of physics are familiar with the principle, through the infamous Fleming's rule in electricity. A current and a magnetic field at right angles combine to produce a force at right angles to the two. Therefore, we ask where are the vectors to combine with the top?

Couples are defined by a vector at right angles to the plane of the couple. Angular momentum is defined with a vector along the axis of rotation. A couple acting on a rotating body with angular momentum produces a resultant couple at right angles to both the angular momentum vector and the applied couple vector.

The relative positions of the centre of mass of the top and the frictional couple will determine whether the resultant couple erects or inverts the top. It is the couple due to gravity that leads to precession, but the frictional couple that leads to erection or inversion.

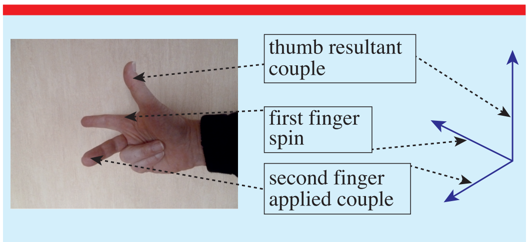

Figure 6. Vector diagram used to describe precession which can be applied to erection or toppling.

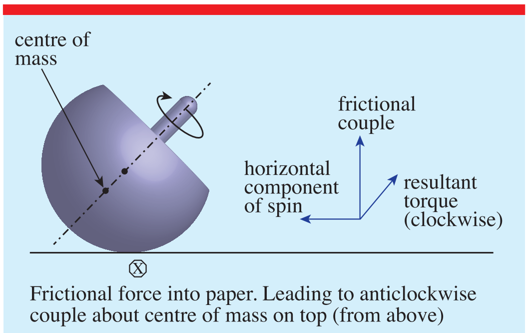

Figure 7. Diagram showing the vectors for a large radius round bottomed top.

The rule we use to describe the erection and inversion is the same as that often used to explain precession (figure 6). It is a right-hand rule with the fingers allocated as follows.

The first finger points in the direction of spin.

The second finger represents the vector of the applied couple.

The resultant couple or turning is in the sense of the thumb vector.

Now we apply this to the round bottomed top.

Working through the vectors of a clockwise spinning large radius round bottomed top (see figure 7), the clockwise spin leads to an inwards force at the point of contact (because of the outward motion of the top at this point). This frictional force produces an anticlockwise couple about the centre of mass. Combining this couple with the horizontal spin vector leads to a resultant clockwise couple on the top which tips it further.

For a small radius round bottomed top (see figure 8), the spin once more leads to an inwards frictional force at the point of contact, which this time, because of the higher centre of mass, leads

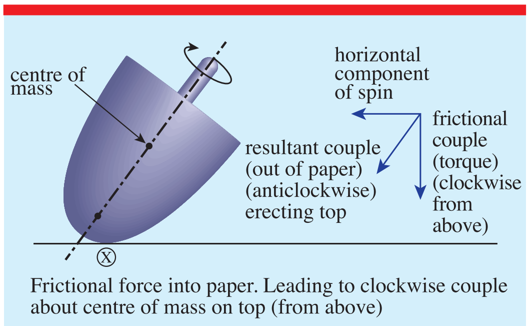

Figure 8. Diagram showing the vectors for a small radius round bottomed top.

Figure 9. Cream eggs: spinning erect and before rotation.

to a clockwise couple about the centre of mass. Combining this couple with the spin this time leads to a resultant anticlockwise couple which erects the top.

Thus, we have the next level of explanation, by looking at the vector addition. However, we are simply applying the rule for vector addition to the vectors of the top. This is for many the best place to stop. There are many other articles that give a more mathematical approach in explanation [3].

I have tried to give simple levels of explanation of the tippee top. Each, as the reader will have seen, is a description in more general terms, rather than a full explanation. Nevertheless, this is largely how science works, and most of us will find satisfaction in one of the levels of description given here, and be able to make further predictions from it. Ultimately we have to apply the principle ‘that’s the way things are’.

Note: a chocolate cream egg or hard boiled egg can also behave in a similar way, if spun correctly (figure 9). However, the actual shape of the egg and position of its contents is critical. Some egg shapes certainly perform better than others. The chances of a chicken's egg erecting are greatly improved by hard-boiling it in a vertical position with the small end at the top.

Spin a cream egg with its axis horizontal. We can assume that the centre of mass is on the horizontal axis of symmetry but nearer to the end of the egg with greater radius of curvature. Any wobble that takes the egg towards a position with the less rounded end down will turn the egg in the opposite direction, whilst moving towards the more pointed end raises the centre of mass, so that the spinning egg stands upright. Further details can be found in Rod Cross's articles [4].

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David Featonby 'retired' from school physics teaching after 35 years in the classroom, and worked until 2011 for the Institute of Physics as a Teacher Network Coordinator in north-east England, UK. He represented the UK at Science on Stage in Geneva 2005, Grenoble 2007, Berlin 2008, Copenhagen 2011 and Slubice 2013 and now works voluntarily with the International Science on Stage (Europe) Committee as the UK representative and a member of its European Executive Board. David is author of several 'hands on' articles in *Physics Education*, and the European journal *Science in School* and has led workshops at many conferences throughout the UK and Europe. He is particularly interested in showing the physics in everyday things to the public, whatever their age.