What types of mathematical reasoning are evaluated in school mathematics?
- An analysis of Swedish high school exams

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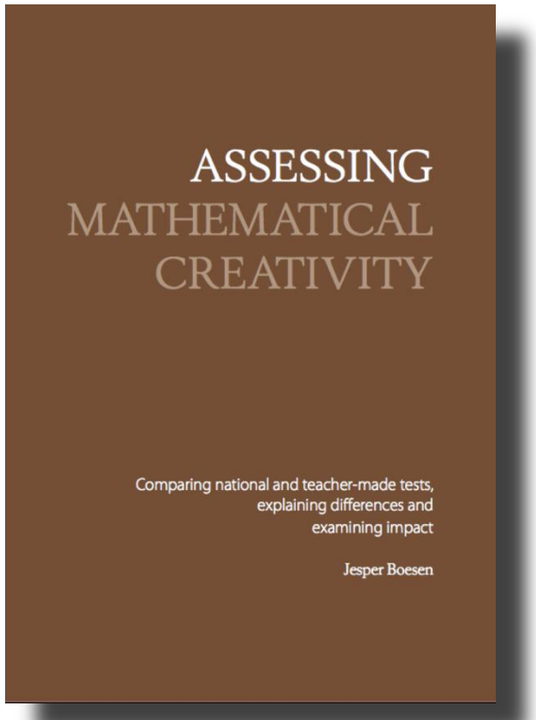


Paper I Palm, T., Boesen, J., & Lithner, J. (2005). *The Requirements of Mathematical Reasoning in Upper secondary Level Assessments* Research Reports in Mathematics Education (No. 5). Umeå: Mathematics.

Paper II Boesen, J., Lithner, J., & Palm, T. (2005). *The relation between test task requirements and the reasoning used by students – An analysis of an authentic national test situation* Research Reports in Mathematics Education (No. 4). Umeå: Mathematics.

Paper III Boesen, J. (2006). *Why emphasise imitative reasoning? – Teacher made tests* Research Reports in Mathematics Education (No. 3). Umeå: Mathematics.

Paper IV Boesen, J. (2006). *The National Course Tests' Impact on Teachers' Written Classroom Assessment* Research Reports in Mathematics Education (No. 4). Umeå: Mathematics.

Tenets:

'Mathematically superficial aspirations towards algorithmic reasoning are common and often dominant'

‘What you test is what you get’

Research aims:

Expand knowledge about students' learning difficulties. What types of math skills are required to solve problems on math tests?

Expand knowledge about the national examination system. Is teacher's test construction affected?

Theories or conceptual frameworks:

Mathematical reasoning:

Johan Lithners

Mathematical competencies:

Palm et. al. (Niss o Jensen, NCTMs, Kilpatric et. Al.)

Methods

Classification of tasks
Math reasoning, Competencies

Article I, II, IV

Analyses of students´solutions

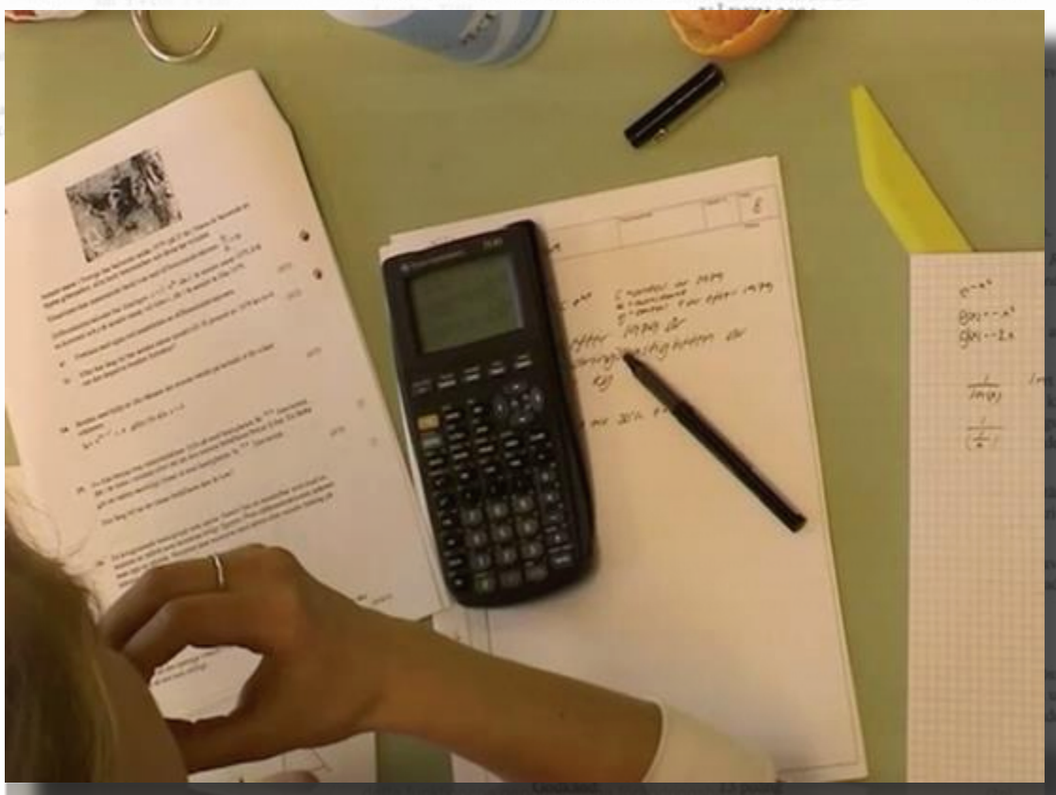
Math reasoning

Article II

Teacher interviews:

Math reasoning, Competencies, connected to teachers own tests

Article III, IV



P.S.

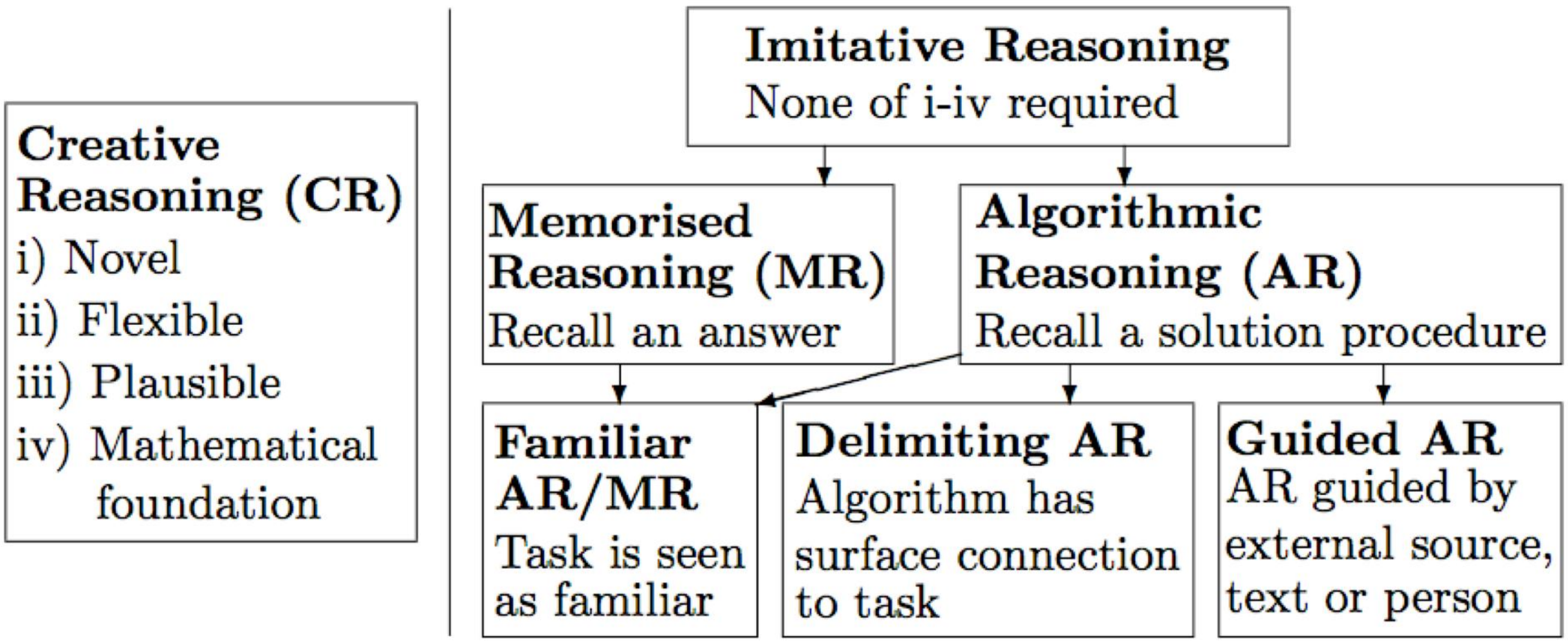
Exakt svar!

(Tipstack till Rudolf Teichter, Gbg!)

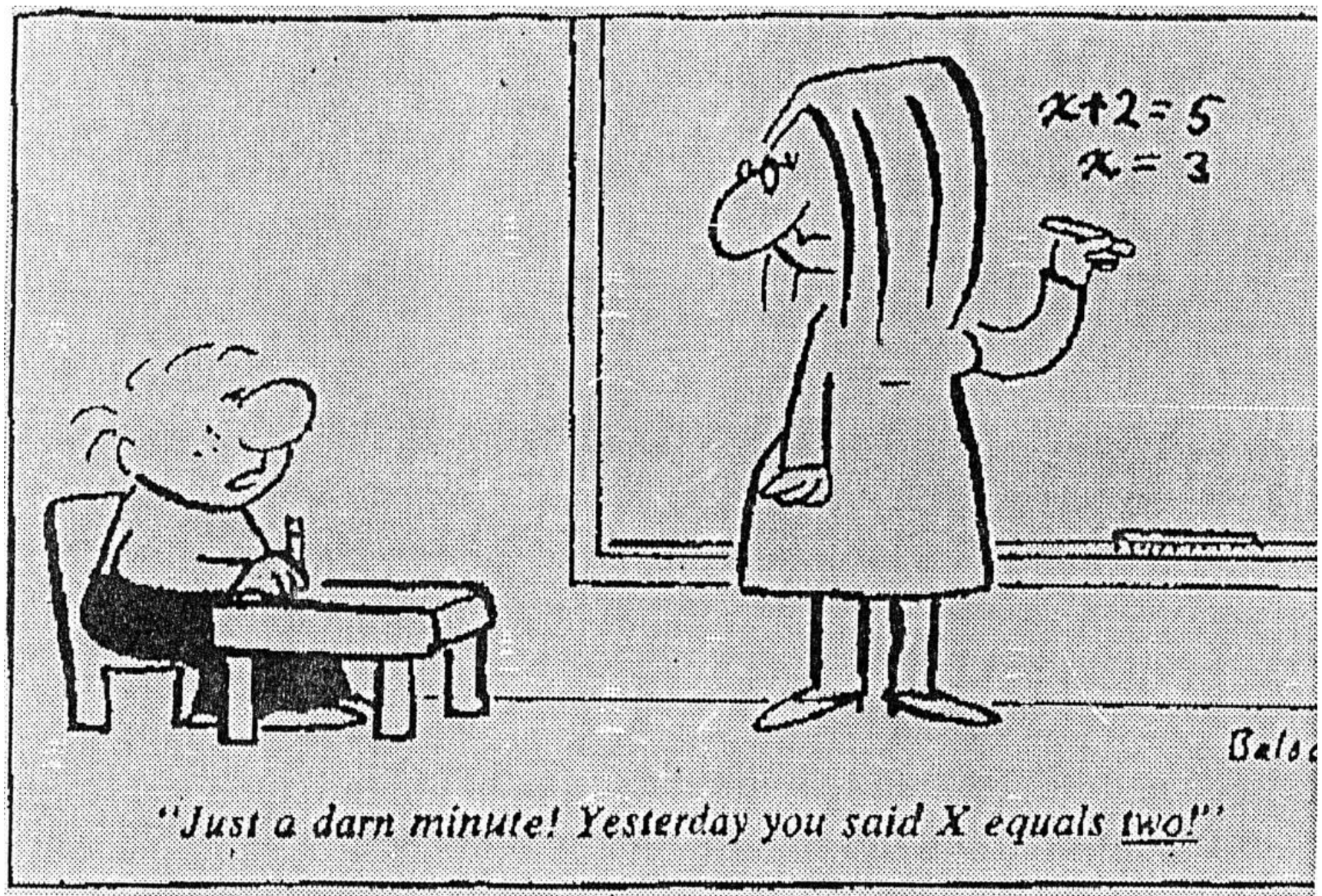
Mathematical reasoning

Reasoning in this thesis is the line of thought, the way of thinking, adopted to produce assertions and reach conclusions. It is not necessarily based on formal deductive logic and may even be incorrect as long as there are some kind of sensible (to the reasoner) reasons that guides the thinking.

Types of reasoning:



Example MR



Consistent:

At Konsum, a large soft drink costs 15 SEK. At ICA the soft drink costs 2 SEK more.
How much does soft drink cost at ICA?

15+2=17

Inconsistent:

At Konsum, a large soft drink costs SEK 15 SEK. That is 2 SEK less than what the soft drink costs at ICA. How much does soft drink cost at ICA?

15-2=13

Example IR

Solved example from textbook:

Of the 80 students who left Vallaskolan, 16 had applied to the natural science program in the upper secondary school. What percentage of the students were there?

Percentage of applicants: 16/80 = 0.20 = 20%

Answer: 20% of the students applied for the science program.

Task from the same textbook:

During a police check outside a school, it was found that 84 cars out of 400 were driving too fast. What percentage were speeding?

CR:

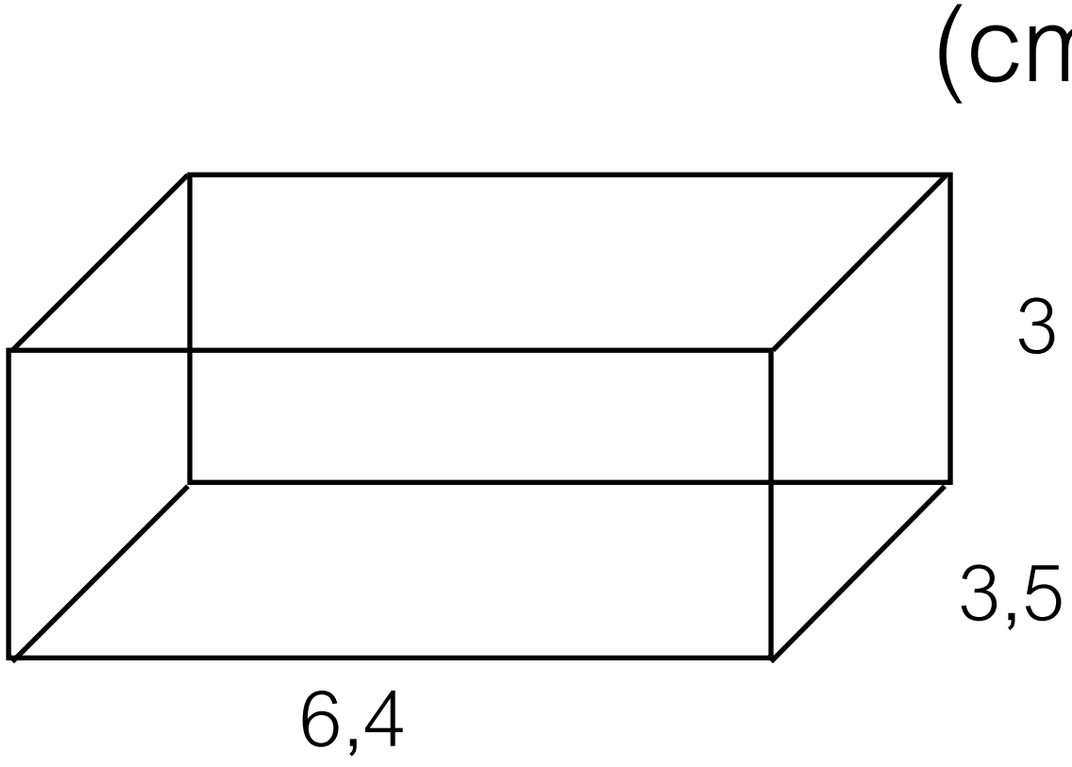
$$x^{5} \cdot x^{3} = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{8}$$

AR:

$$x^{5} \cdot x^{3} = x^{5+3}$$

Example AR

Determine the volume of the rectangle.



Example (global)

You will build a glass aquarium of approx. 160 litres.

Suggest suitable measurements.

Describe how you arrived at these measurements and draw a sketch of the aquarium with the measurements indicated.

Investigate how isosceles triangles that have an angle of 70° can look like.

Determine the other angles in the triangles you find.

Example CR Local

A car's value, y SEK, can be described with the formula

$$y=120000\cdot 0{,}85^{x}$$

where x is the time in years after the purchase.

a) How much did the car cost when it was new?

b) How much is the car worth after 3 years?

Mathematical Competencies

Problem solving Competence

Algorithmic Comptence

Concept Competence

Modelling Competence

Reasoning Competence

Communication Competence

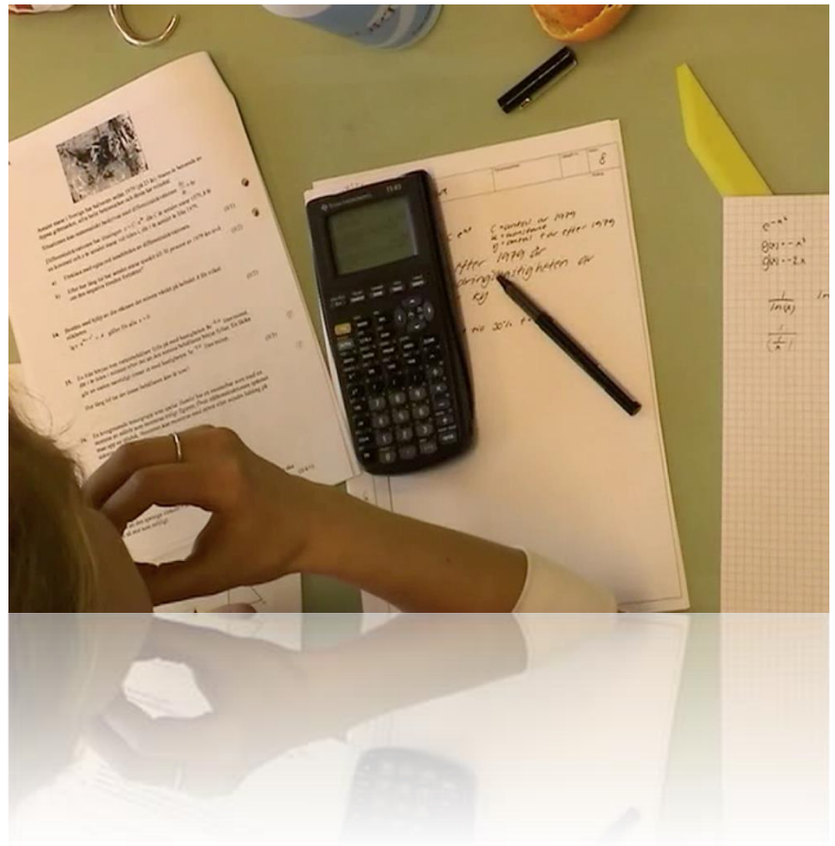
Explicit connection to curricula

Results Article I:

	HP A	SP A	SP B	SP TOT	NV A	NV B	NV C	NP D	NV TOT	NP A	NP B	NP C	NP D	NP TOT
MR/A R/IS	70	88	74	80	78	73	60	58	67	27	42	49	26	36
LCR	14	6	12	9	13	15	16	22	16	19	11	7	18	14
GCR	16	7	15	11	8	12	24	20	16	54	47	44	56	50
CR (L+G)	30	12	26	20	22	27	40	42	33	73	58	51	74	64

Results Article II:

High consistency between requirements and reasoning used.



Higher proportion of both requirements and used CR in the A course compared to the other courses.

		Reasoning demands			
		MR	AR	LCR	GCR
Course	A	2,7%	18,9%	18,9%	59,5%
	B	4,5%	50,0%	9,1%	36,4%
	C	3,8%	42,3%	15,4%	38,5%
	D	4,5%	45,5%	18,2%	31,8%

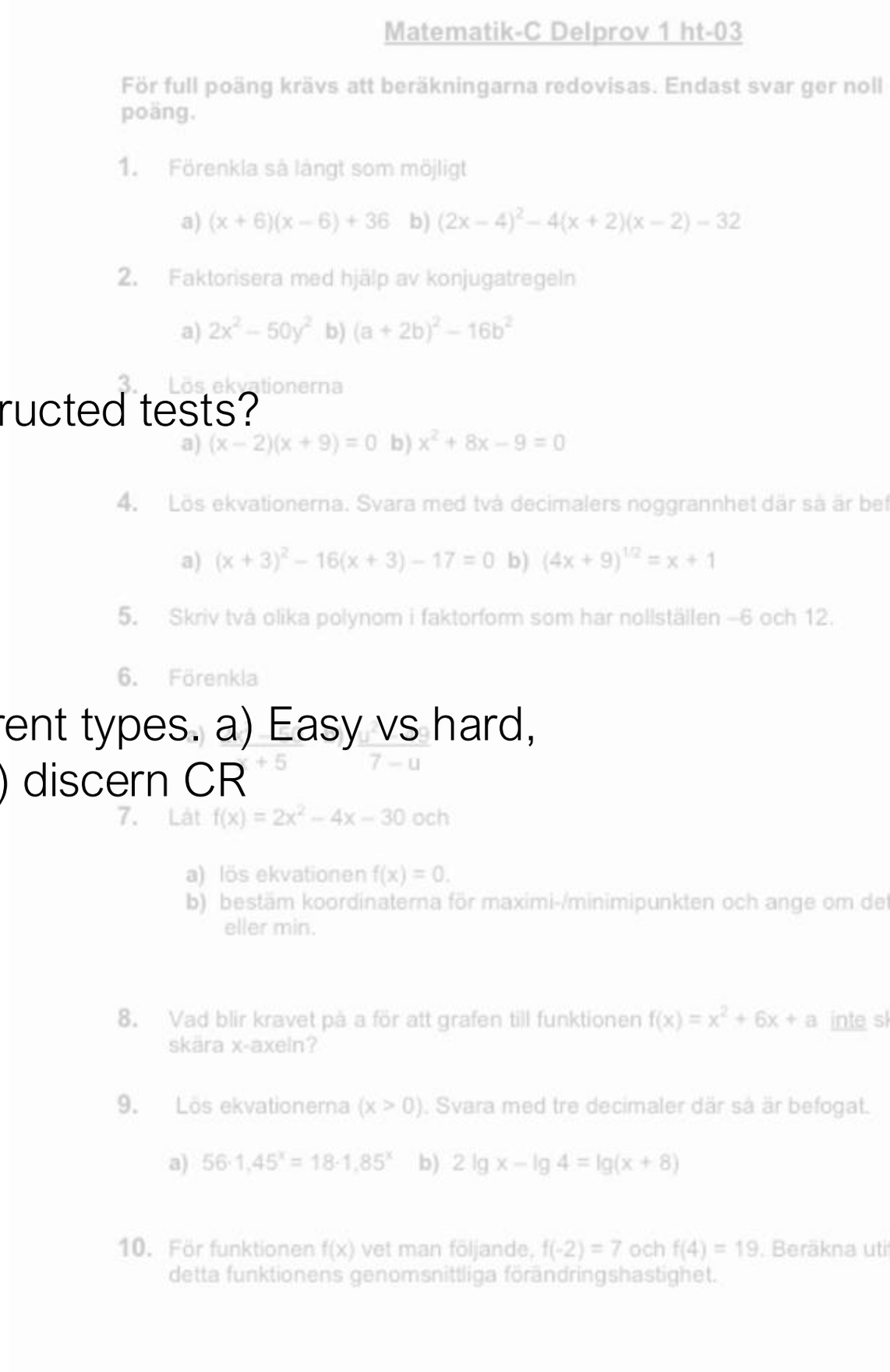
Results Article III:

Why such a high focus on MR/AR in teacher-constructed tests?

Low awareness of CR.
Different types of perceptions of AR/CR. Four different types. a) Easy vs hard, b) Geometry vs algebra, c) “Something special”, d) discern CR

'Not all students can learn CR.’ Takes time to develop CR tasks, difficult to do so,

The desire to get as many students as possible approved.


Matematik-C Delprov 1 ht-03

För full poäng krävs att beräkningarna redovisas. Endast svar ger noll poäng.

1. Förenkla så långt som möjligt

   a) $(x + 6)(x – 6) + 36$ b) $(2x – 4)^2 – 4(x + 2)(x – 2) – 32$

2. Faktorisera med hjälp av konjugatregeln

   a) $2x^2 – 50y^2$ b) $(a + 2b)^2 – 16b^2$

3. Lös ekvationerna

   a) $(x – 2)(x + 9) = 0$ b) $x^2 + 8x – 9 = 0$

4. Lös ekvationerna. Svara med två decimalers noggrannhet där så är bef

   a) $(x + 3)^2 – 16(x + 3) – 17 = 0$ b) $(4x + 9)^{1/2} = x + 1$

5. Skriv två olika polynom i faktorform som har nollställen –6 och 12.

6. Förenkla

   $x + 5$ $7 – u$

7. Låt $f(x) = 2x^2 – 4x – 30$ och

   a) lös ekvationen $f(x) = 0$.
   b) bestäm koordinaterna för maximi-/minimipunkten och ange om det eller min.

8. Vad blir kravet på a för att grafen till funktionen $f(x) = x^2 + 6x + a$ inte sk skära x-axeln?

9. Lös ekvationerna ($x > 0$). Svara med tre decimaler där så är befogat.

   a) $56 \cdot 1{,}45^x = 18 \cdot 1{,}85^x$ b) $2 \lg x – \lg 4 = \lg(x + 8)$

10. För funktionen f(x) vet man följande, $f(-2) = 7$ och $f(4) = 19$. Beräkna uti detta funktionens genomsnittliga förändringshastighet.

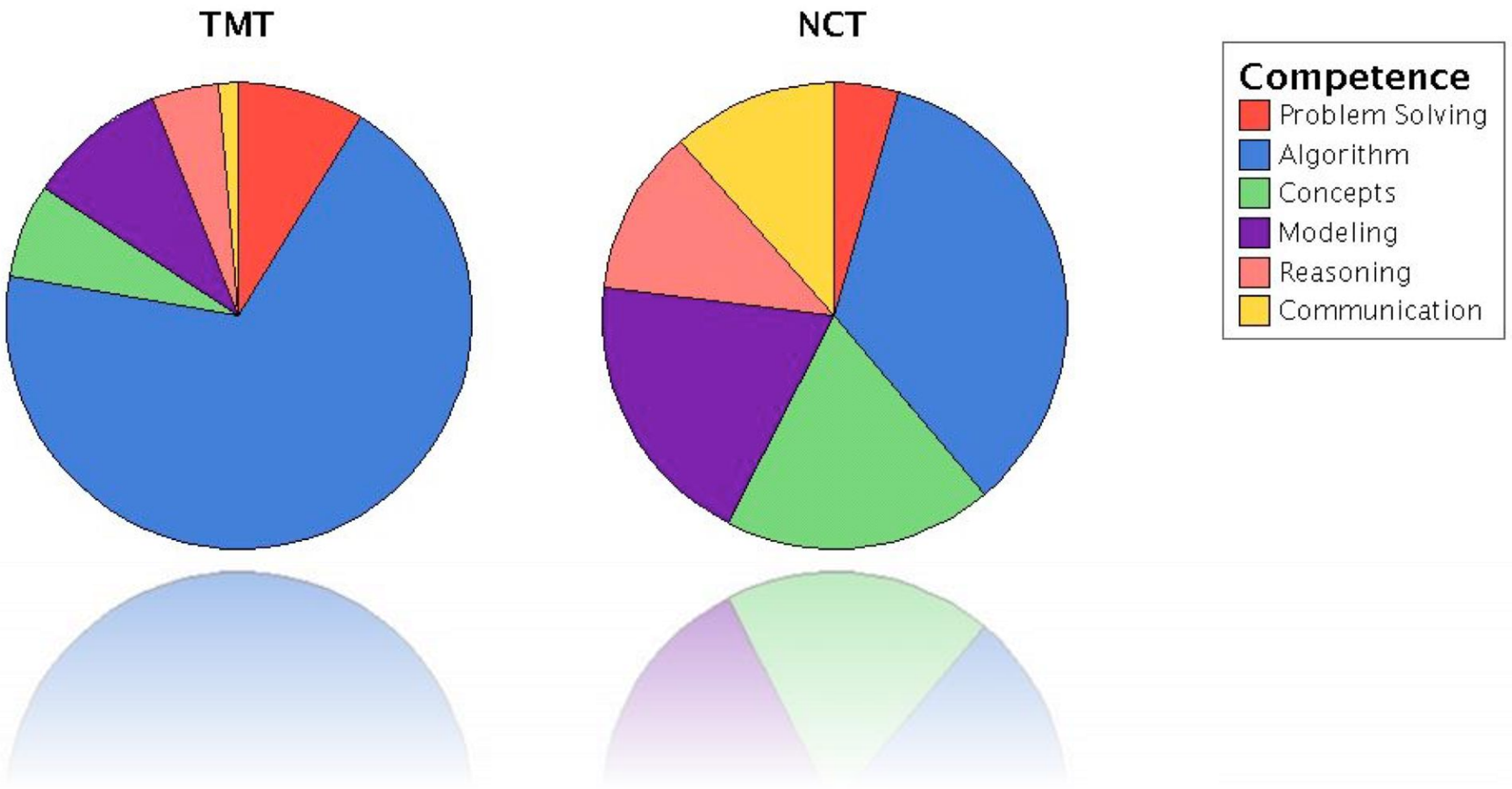
Results article IV:

Impact?

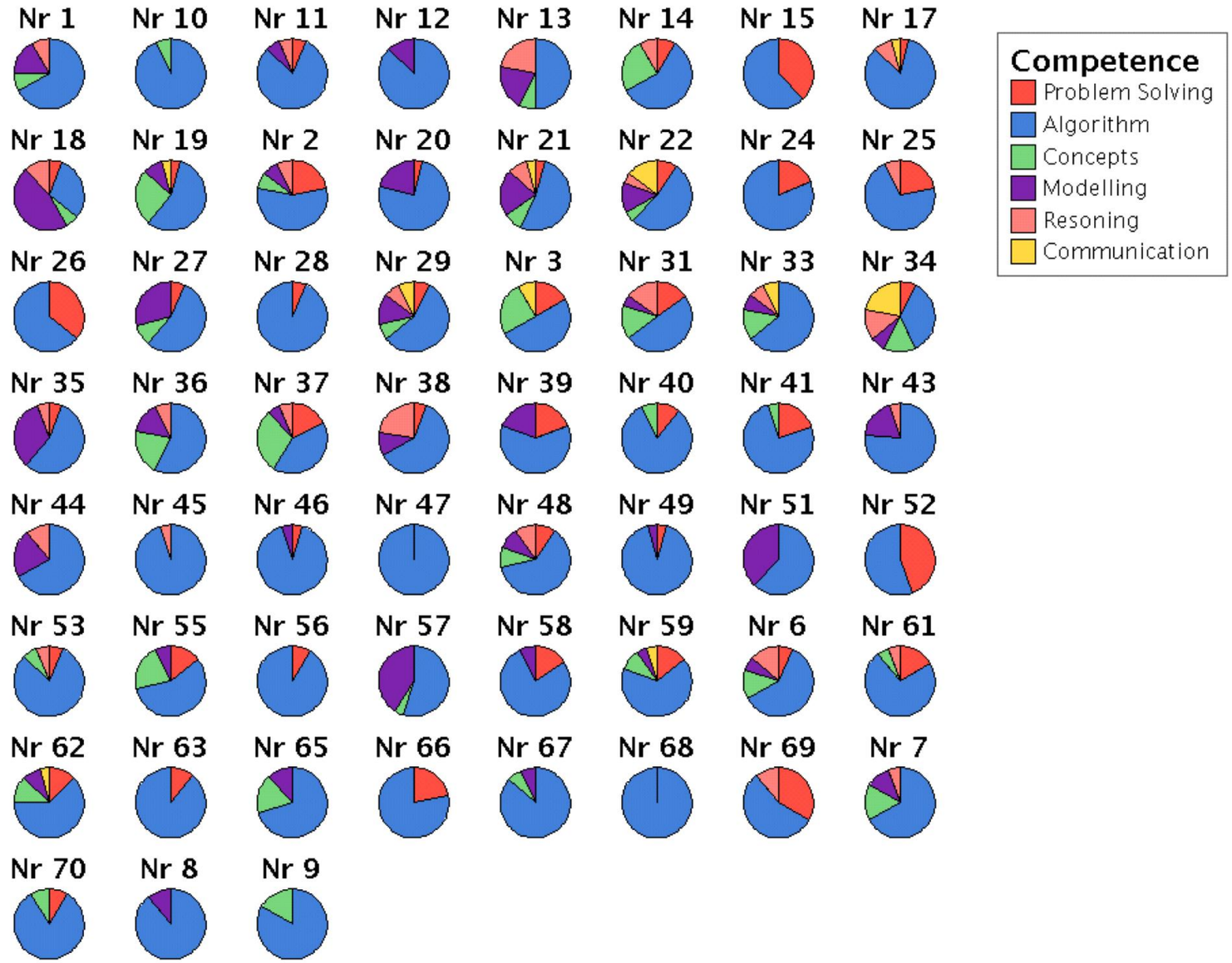
NCT → TMT?

Regarding tested competencies

Results article IV:



Results article IV:



Results article IV:

Impact?

NCTs → TMTs

Interviews with teachers. Two groups.
Teachers with tests simalar to the NCTs,
Teachers with tests not simalar to NCTs.

Both groups had similar views of the NCTs
Delibarate choices to mimics the NCTs

Conclusions:

Teacher-made tests mainly focus on tasks that can be solved without having to consider intrinsic mathematical properties.

National tests focus to much higher degree on tasks which cannot be solved without having to consider intrinsic mathematical properties.

In the case where a test requires creative mathematically founded reasoning, as in the national tests, students also use non-imitative solution strategies to a much higher extent.

A very content-dense curriculum may hamper the development of creative mathematically founded reasoning and conceptual understanding.

Conclusions:

The focus on tasks possible to solve by imitative reasoning in teacher-made tests seems explainable by teachers limited awareness about creative mathematically founded reasoning and their wish to get as many students as possible to reach the grade level Passed as possible, which was believed difficult to do when requiring high quality reasoning from weaker students.

Impact of the national course tests on teachers' development of tests, in terms of tested competences, seem to be fairly modest.

The impact of the national course tests has however been proved to exist and partly shown how to operate, on some teachers test development. I.e. a causal relationship has been shown (cf. request from Mehrens (2002)).

Conclusions:

The frameworks and analysis procedures used throughout this thesis have shown to be useful as research tools and can be used to further evaluate and develop tests.

Discussion

Generally, in Sweden, there´s a big gap between test results and grades, how to explain that?

Whats the nature of your own tests?

Are you, influenced by your National Tests?

What is a good distrubution of AR(IR) and CR? In learnings situations? Why?

末端

[The end]