

shadow

and

depth

SHADOW AND DEPTH

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You may draw, write and colour in this book.

Make sure you have:

- a sharp pencil
- a ruler
- scissors and glue
- a rubber.

Many problems should be worked on together. Take a look at what the others in your group are drawing and writing. Tell them why you've done it differently!

This booklet was devised by Aad Goddijn of the Wiskivon department of the IOWO, Utrecht, the Netherlands (1980).

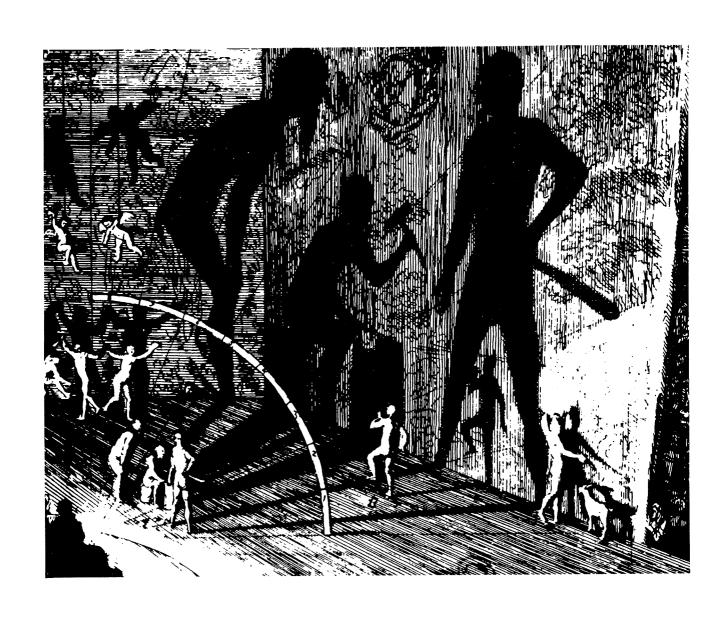
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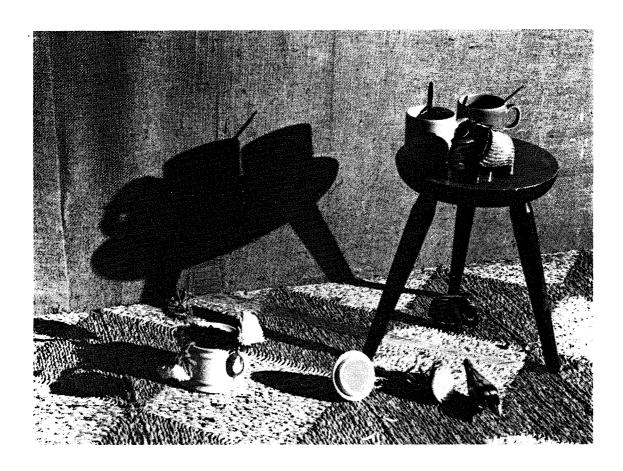
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lighting the shadows



- ➤ 1. Without light you don't see anything. From which side does the light fall in this photo? How can you tell?
- 2. Check all the shadows in the photo. Maybe something needs to be added or taken away.
- ≫ 3. Take a look at the picture on page 1 of this book. Why are some shadows so big and others so small?
- 4. Take a good look at your shadow outside on a sunny day. What are your shadow-feet attached to? Can your shadow be longer than you are? When?

How must you stand in order to make the slimmest possible shadow?

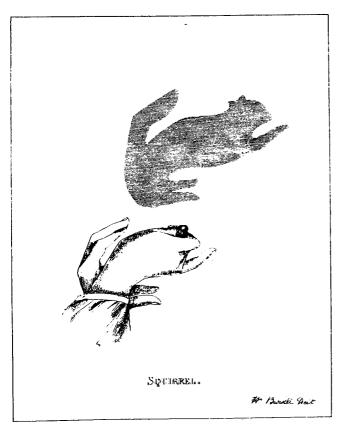
Which part of your shadow is the sharpest?

Which part is more blurred?

Stick out your hand so that you have five shadow-fingers. What do you have to do to make a nice sharp shadow?

SHADOWS CAST ON THE WALL

Using a bright lamp you can cast all sorts of shadows on the wall. Here is an example.



≫ 5. How is the squirrel's eye
made?

Which fingers form the tail?

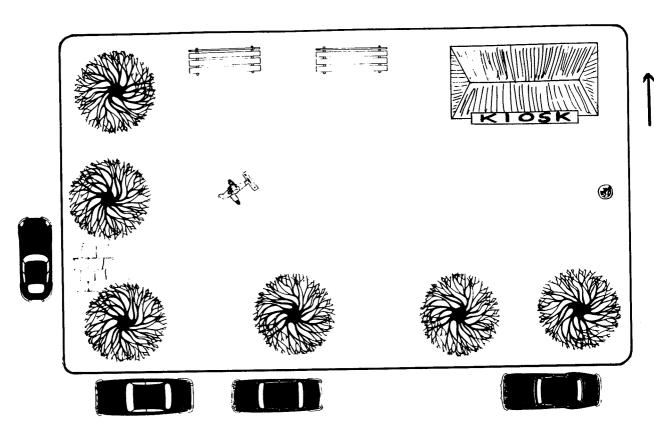
And how are the ears made?

> 6. Try making some shadow-pictures
on a wall at home and then show
them in class.
Write down what sorts of

pictures you made.

DIRECTION

Note: > 7. Here is a top-view of a square. The direction of North is indicated by an arrow. Draw arrows pointing north in all four corners of the square.

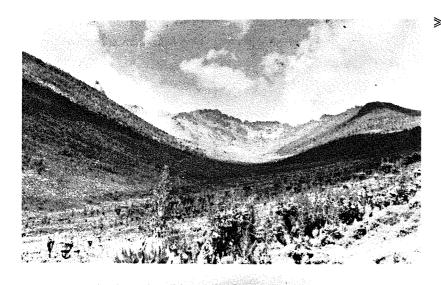


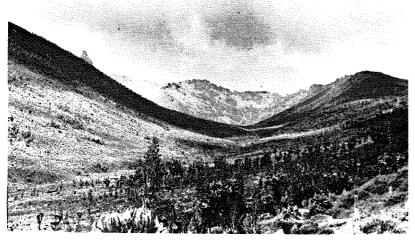
- ≫ 8. The sun is shining from the south-west. Draw red arrows pointing to
 the sun in all four corners.
- \gg 9. Show the dark side of the trees, benches, etc. by shading them with a pencil.
- ≫ 10. Imagine you're walking outside with the sun on your left.

 Where is your shadow then? And if you walk towards the sun?
- \geqslant 11. And now in general: where is the shadow always found in relation to the light source?

OTHER SHADOWS

- ≥ 12. If you watch football on television, you may sometimes see that each player has four shadows. Why is that?
- ≥ 13. In what kind of weather won't you see any shadow of yourself at all?





And where do the shadows come from?

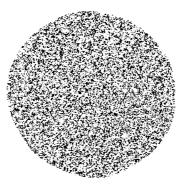
they stay in one place?

- ▶ 15. Night is a shadow. Can you explain that?
- ≥ 16. Tell in your own words what exactly a shadow is.

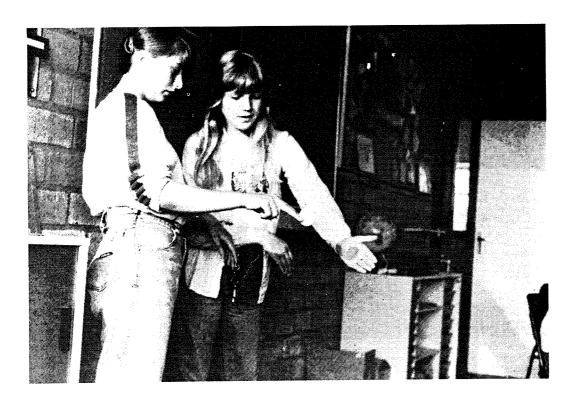
CATCHING SHADOWS

Now you can try things out yourself with light and shadow. It would be nice if the sun came out, but if it won't then you can make do with a bright lamp which casts nice sharp shadows.

➤ 17. Tape a small paper circle onto a window pane. About so big:



If the sun isn't shining, then someone should hold the paper circle a couple of meters from the lamp.



Catch the shadow of the circle in the palm of your hand, close to the window. Lay the shadow very carefully on the floor; be careful that the shadow doesn't roll off your hand!

What kind of figure did your hand make as it moved through the air?

≥ 18. Make a drawing here which shows how your hand moved.

Draw the sun (or lamp) and the paper circle as well.

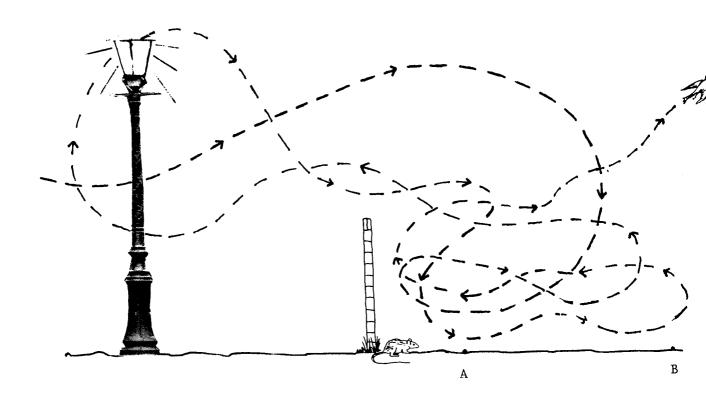
- ≥ 19. Put your hand in the light and look at the shadow on the wall. When
 you move your hand, the shadow slides along the wall. At least, most
 of the time, because you can also move your hand in such a way that
 the same bit of wall remains in the shadow. How do you do that?
- ≥ 20. And if you now move your hand farther and farther away from its shadow, where will you end up?
- ≥ 21. Do this together with at least two others: All shake hands together, except without touching! In other words, shadow hands, because the shadows do touch each other. How must you hold the three hands in the light in order to do this?

STREET SHADOWS

≥ 22. Street-lamp, bird, rat, wall and street. A side-view. The rat is
walking away from the wall. Is he in the light of the lamp at A?

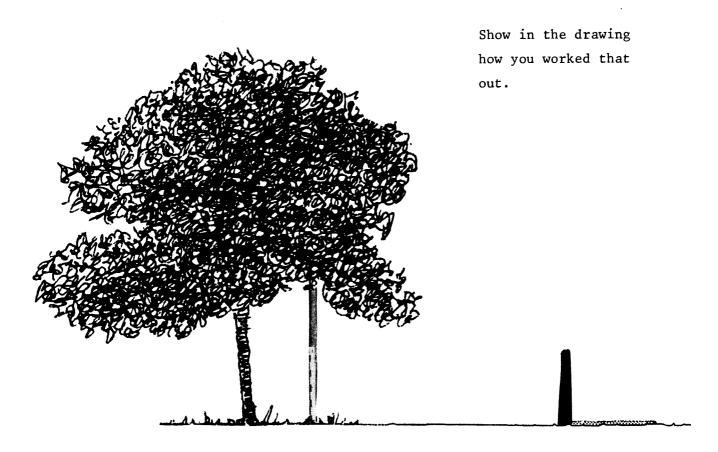
And at B?

Show exactly where the rat first comes into the light. Draw the first ray of light which the rat meets.

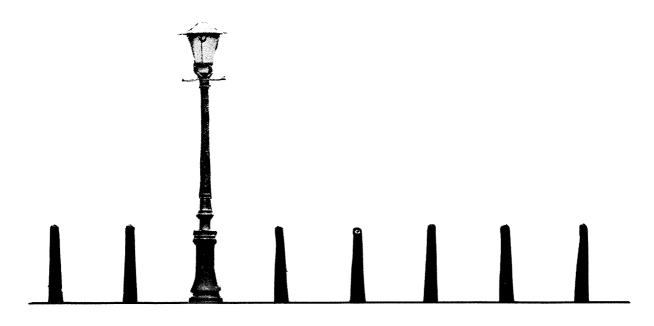


 \gg 23. The bird flies along the dotted line. Show exactly the areas where it is flying in the shadow.

≥ 24. There's a street-lamp behind this tree. You can just see the bottom of it. The small post casts a shadow as the lamp is on.
Can you tell the height of the street-lamp?



 \gg 25. A side-view of a street-lamp with a row of small posts. The lamp is on. Draw the shadow of the posts in blue on the ground.



≫ 26. You're walking down the street in the dark. Your shadow is following you. At the street-lamp it catches up and goes on in front of you. Then the shadow disappears. But at the next street-lamp it catches up with you again.

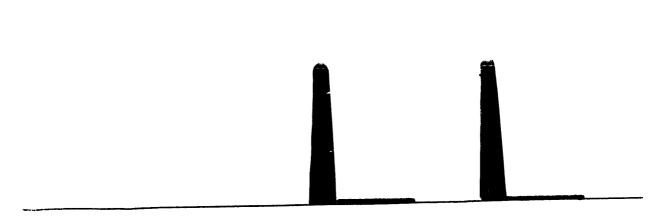
As you approach a street-lamp does your shadow get bigger or smaller?

And as you walk away from the lamp?

Why is it that your shadow goes faster than you do?

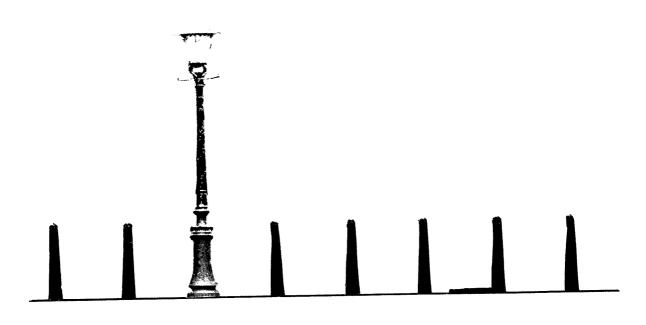
 \gg 27. Here are two posts with their shadows drawn in. Work out exactly where the lamp is.

 \gg 28. Nearly the same problem. But make sure to do it very precisely with a long ruler.



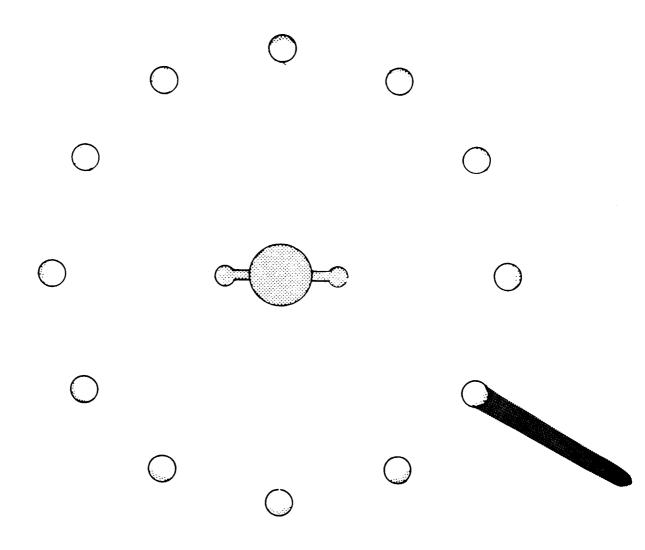
You've noticed that this is different from promblem 27. Where does the light come from in this case?

 \gg 29. The same drawing as a moment ago. But now the sun is shining. One shadow has already been drawn in. Do the rest as well. Draw the shadow of the street-lamp too.

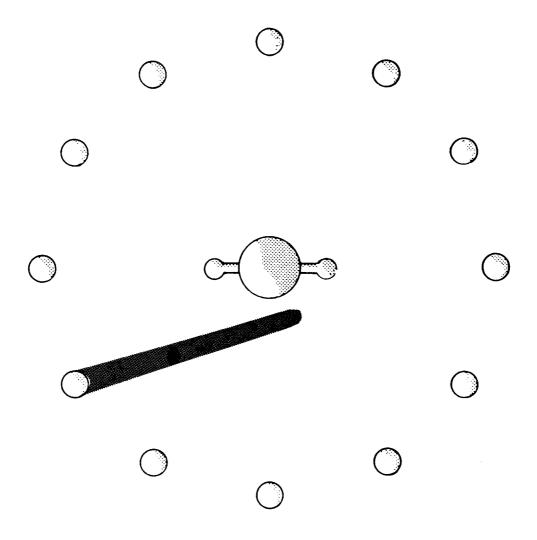


≼ 30. You're walking along the beach in the sun. Who's going faster now,
you or your shadow?

≫ 31. Here's a top-view of a street-lamp and some posts. The lamp is on.
One shadow has already been drawn in. Draw the other shadows as well. Do it as exactly as possible!



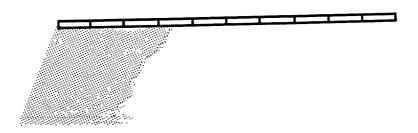
 \gg 32. The same situation, only now the street-lamp is out and the sun is shining. Draw the rest of the shadows.



≥ 33. Here you have a top-view of a street-lamp shining on a low wall,
one meter high.

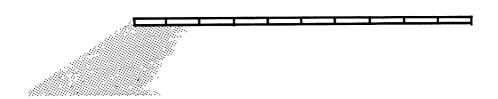
Finish drawing in the shadow.





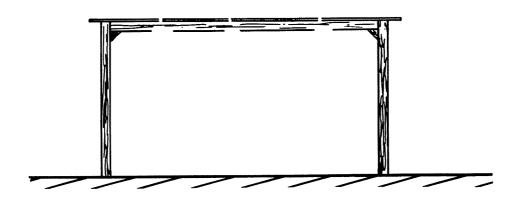
 \gg 34. The same, only the street-lamp is out and the sun is shining again. Starting at the X, draw an arrow pointing towards the sun. Now finish the shadow.





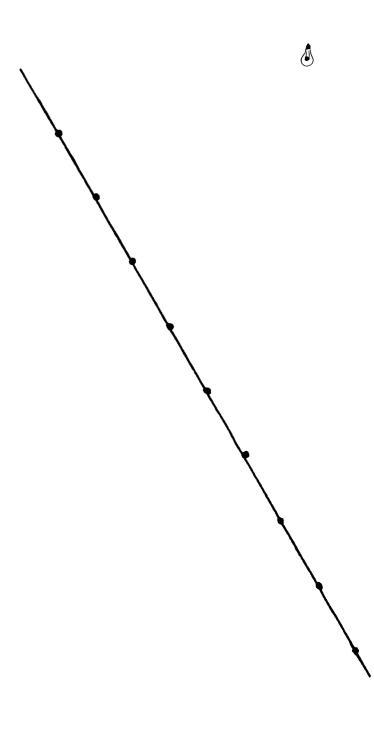
≫ 35. A light bulb is hanging above the table. In the table-top are three holes, through which the light shines. Where on the floor do you find the spots of light?





 \gg 36. What happens to the spots of light if the lamp is raised? Make a drawing of it.

≫ 37. Here is a knotted rope and a light bulb. The knots are all
equidistant. Draw the shadows of the knots on the floor under them.
Are the shadows of the knots also equidistant?



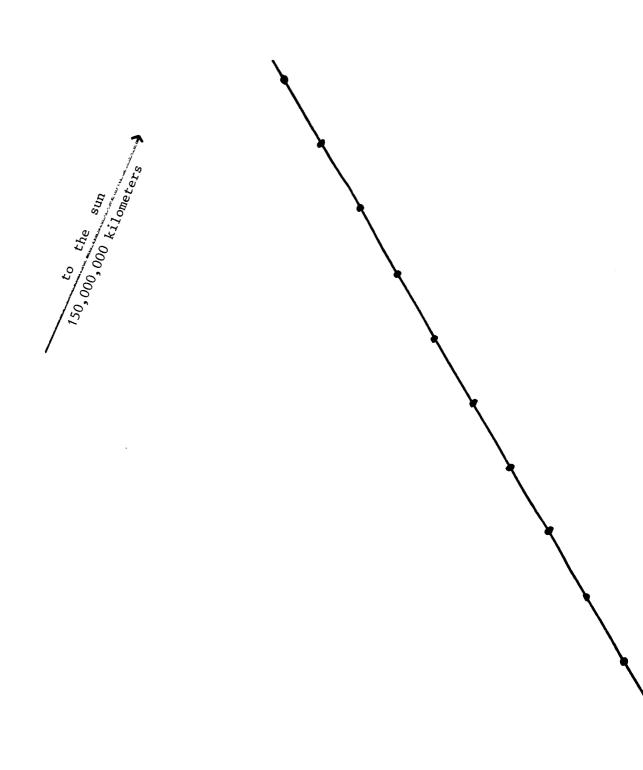
≫ 38. Here the shadows of the knots have already been drawn. Cut off the knotted rope from the edge of the page and try to place it in such a way that it will produce the shadows shown.

How must the rope be held in order to produce equidistant shadows?

(

 \gg 39. The same drawing as in problem 37, only here there is no light bulb. However, the direction of the sun is indicated.

(The sun itself is 150 million kilometers away and so won't fit in this book.) Draw the shadows of the knots here as well. What is the difference between these shadows and the ones in problem 37?



 \gg 42. What is the longest shadow you can make?

.... AND FROM A CUBE

≫ 43. Find or make a cube.

What kind of shadows can you make with it?

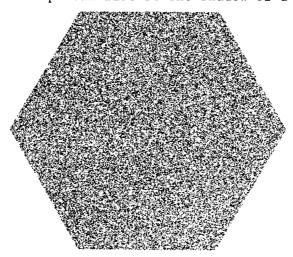
≥ 44. Can you make a square shadow with your cube on a piece of paper?

Don't forget that the paper can move too!

How must you hold the cube and the paper in relation to the sun or lamp?

Make a drawing of how you did this as well.

 \geqslant 45. See if this shape can also be the shadow of a cube.



Explain how you must hold the cube to do this.

≫ 46. Try to make a shadow in the form of a regular pentagon from your
cube. Can you do it? Explain.

IN SIGHT OF THE LIGHT

 \gg 47. Hold the cube in the sun. Hold it still and count how many faces are in the light at once.

And how many faces can you see at once?

 \gg 48. Try it with other things as well: tetrahedron, dodecahedron, pyramid, etc.

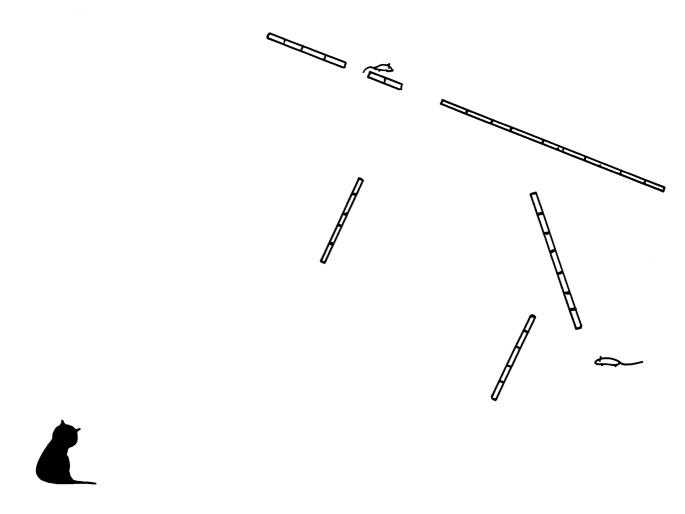
Fill this table in and make it longer.

	visible at once	in the light at once
cube	faces	faces
dodecahedron	11	"

 \geq 49. What do you notice? Explain!

≫ 50. As you know, cats love mice. But not the other way round.

Here you have a top-view of some low walls of a ruin. The cat is on the prowl. Colour the ground red where the mice had better not show their faces.



≫ 51. Now put on a heat-proof space suit, take a quick drink of cold water and fly to the sun. From the sun let's look at the earth through a very powerful telescope. Can you make out any shadows? Explain why. ≥ 52.

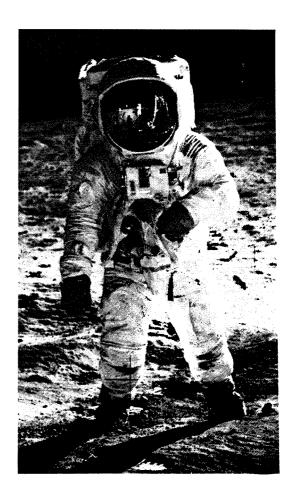


Was this photo taken with or against the light?

≫ 53. The white queen
on her own. Why
do you see a
thin line of
light on her
left side?



≫ 54. Here is another photo taken against the light. It was made on the moon in 1967. What do you suppose is the source of light?



At the front of the classroom we'll let a bright lamp shine on a ball. The lamp is quite a way away, to the left of the ball.

The ball represents the moon, the lamp the sun.



- \gg 55. What percentage of the ball is illuminated by the lamp?
- ≫ 56. Draw the ball as precisely as possible from where you are sitting.
 Pay particular attention to the border between light and dark.
 Will everyone have made the same drawing? Explain.
- \gg 57. From where does the ball look like this?

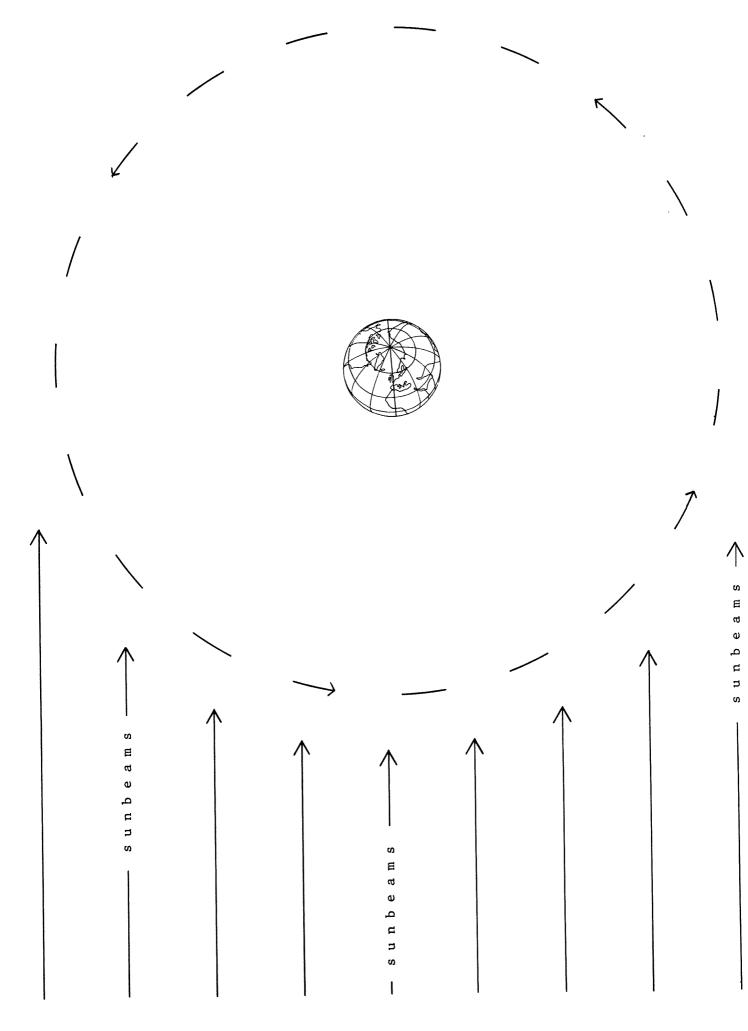


 \gg 58. And from where like this?

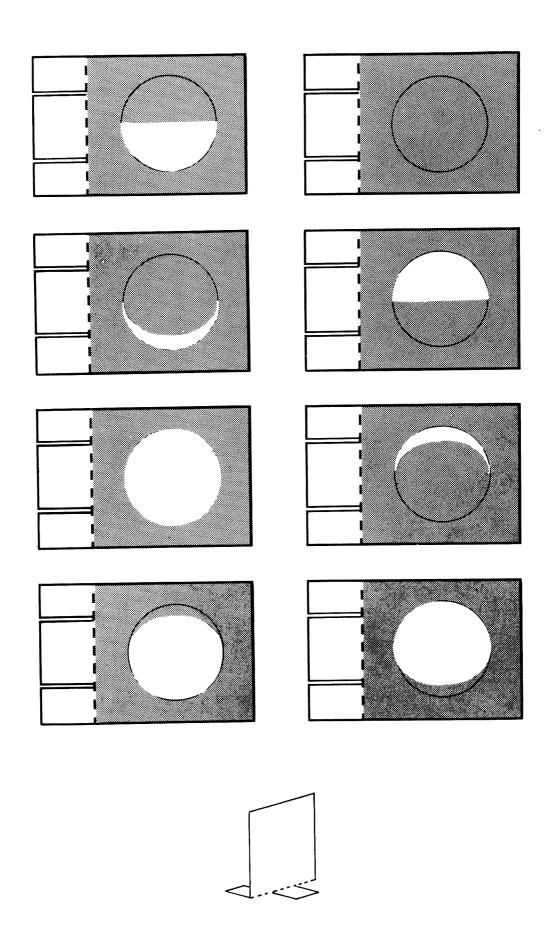


≫ 59. On the following page you see the earth from above. The short
arrows indicate the journey which the moon covers every 29 days.
Cut out the eight faces of the moon from the next page and fold
them as shown in the drawing. Pick a spot in the moon's journey
and place there the correct moon-face as seen from the earth.

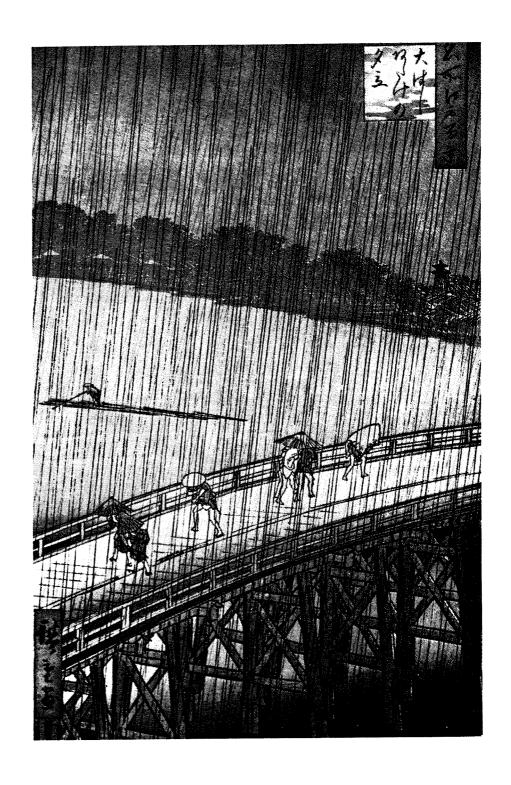
Do this carefully for all eight moon-faces. Show it to the
teacher and then glue them in place.



- 26 -



- 28 -



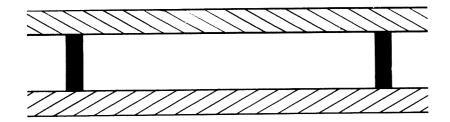
paralle1s

≥ 60. Holland seen from the air.



The ditches run straight on for a long way beyond the photo. Will they at some point get closer together?

 \gg 61. Two long beams, kept apart by two short supports. Imagine that the beams continued on to the left and right. On which side would they meet each other?

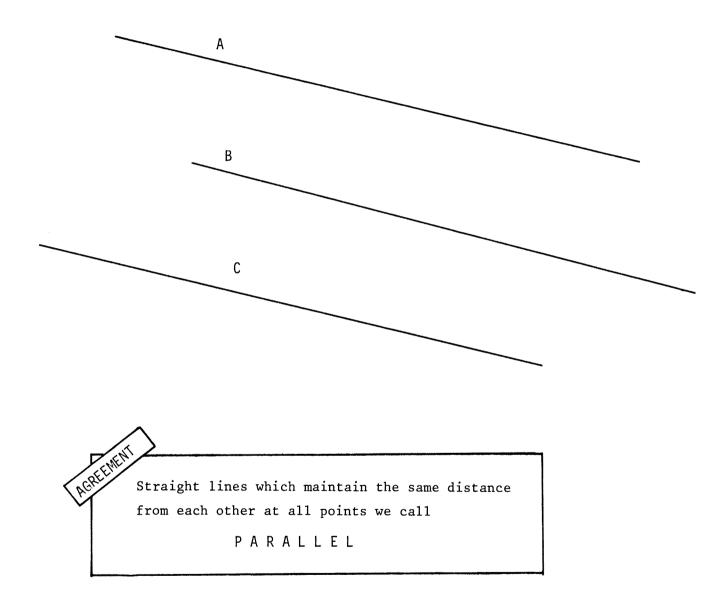


Don't be fooled! Take the measurements. What can you best measure to make sure?

≥ 62 . Look at the top and bottom of the blackboard. Imagine the lines continuing straight on to the right. What things will they have to pass through?

How far will they be from each other after 3 kilometers?

≥ 63. Here you see three perfectly straight lines. Two are running in the same direction. One diverges slightly. Which one is that?



≫ 64. Which two lines in problem 63 are parallel according to this
agreement?

PARALLELS IN THE CLASSRO

>	65.	Find three lines in the classroom which run parallel with each other.
≫	66.	Is the top edge of the blackboard parallel to any of the edges of the classroom? To which ones?
≫	67.	Imagine a straight line running diagonally across the front wall of the classroom, from lower left to upper right. Which other line in the classroom runs parallel with it?
>	68.	Look at the upper left edge of the classroom. Which other edges run parallel with it?
>	69.	Make a drawing of your classroom in this space. Give parallel lines the same colour.

How many colours do you need?

≫ 70. Can you divide the lines of this building into the same kind of groups? Make up names for the various groups which you find.



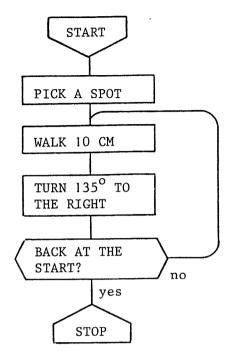
 \gg 71. Can you think of a building which has even more groups of parallel lines?

 \gg 72. Here's a robot.

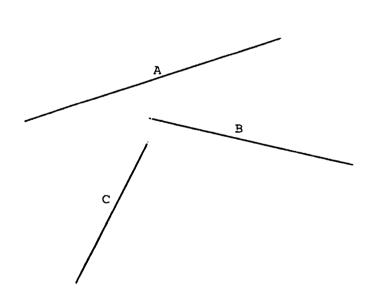
He moves according to this flow chart.

Draw very carefully the figure which the robot makes. Use a separate piece of paper for this.

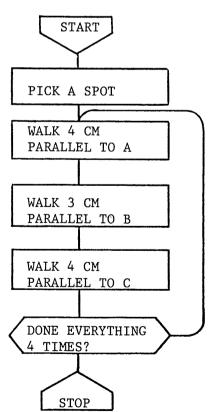
Give parallel lines the same colour.



≫ 73. Trace lines A, B and C onto another piece of paper. Then follow
this flow chart.



Did everyone get the same figure? How many different turns does the robot make?



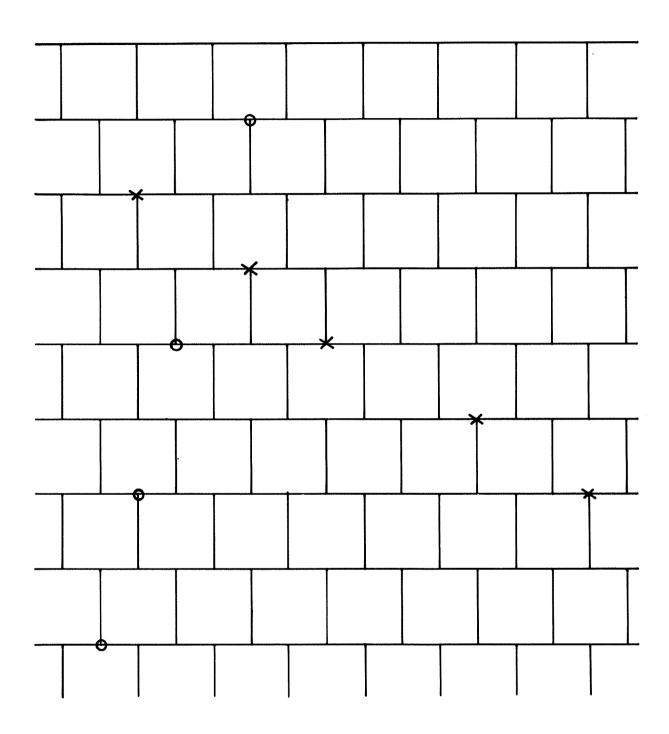
 \gg 74. This is how the robot moved.



Now you make a flow chart which tells how the robot should continue after C.

Use the word 'parallel' in your flow chart.

 \gg 75. Make another drawing and flow chart of your own.

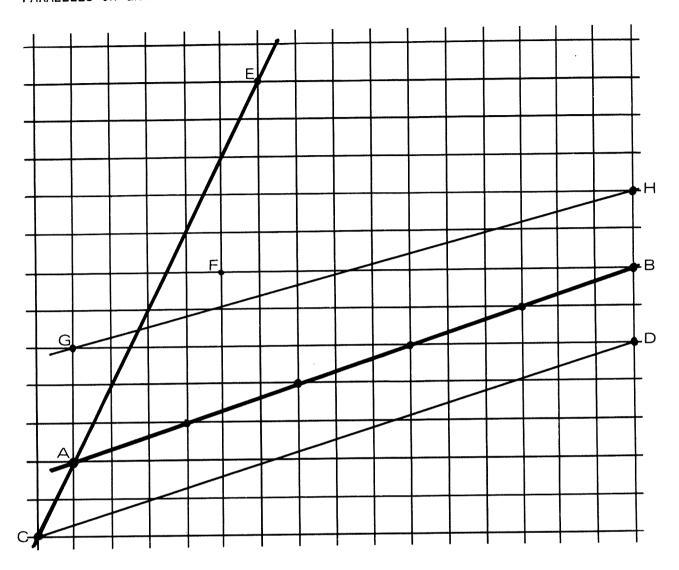


- ≫ 76. Do all the small circles lie on one line?

 Can you see that without using a ruler? How?

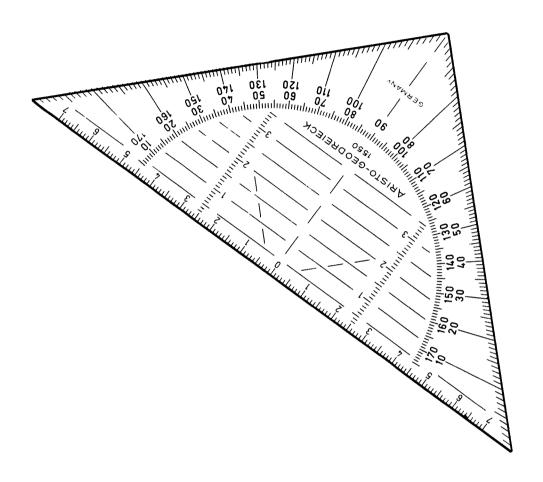
 Cross out the circle which doesn't belong and draw three more lines, parallel to the first one. Make use of the squares!
- \gg 77. Do the same with the X's.

PARALLELS ON GRAPH-PAPER



- \gg 78. Draw a line through F which is parallel to AB. And one parallel to EC.
- \gg 79. Are AB and CD parallel? How can you tell?
- \gg 80. AB and GH do not run parallel. Write down where they will meet each other.

 \geq 81. You can find parallel lines on this protractor. Where?



How can you use it to draw parallel lines yourself? You can now use the protractor to do problem 63. How? ≥ 82.

23

21

20

19

18

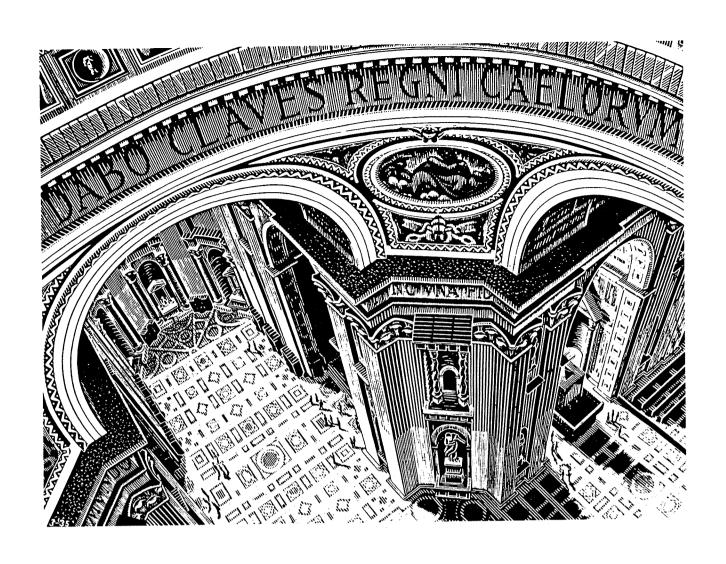
Which lines run parallel with 8-23?

How many lines run parallel with 1-2?

And with 1-3?

 \gg 83. How many lines are there altogether?

- 40 -



eyeing the depths

- 42 -

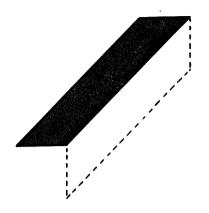
FLAT OR NOT

≫ 84. Cut the grey figure out of cardboard.

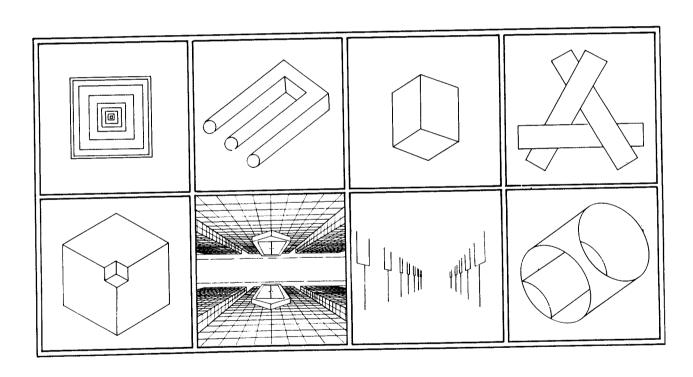
Draw around it on a piece of paper and then flip it over as shown by the dotted lines. Draw and flip five more times.

You've now made a drawing where you can see depth.

What sort of thing does it look like to you?

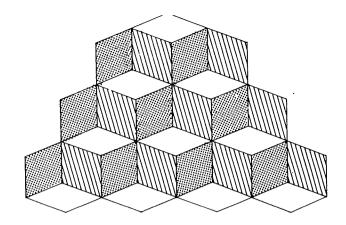


≫ 85. What kinds of spatial things do you see in the following flat drawings?



≫ 86. You see, the paper stays flat but you can imagine spatial objects.
Did everyone see the same things?

- ≫ 87. A stack of cubes.
 How many are there?
- Now count again How is that possible?



▶ 89. Colour the left-most cube red.
Turn the book upside-down and colour the right-most cube blue.
Now do you know the answer to the question in 88?

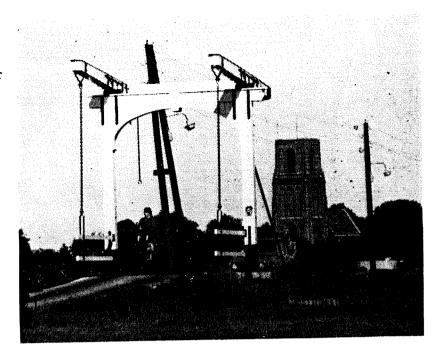
HOW REAL IS A PHOTO?

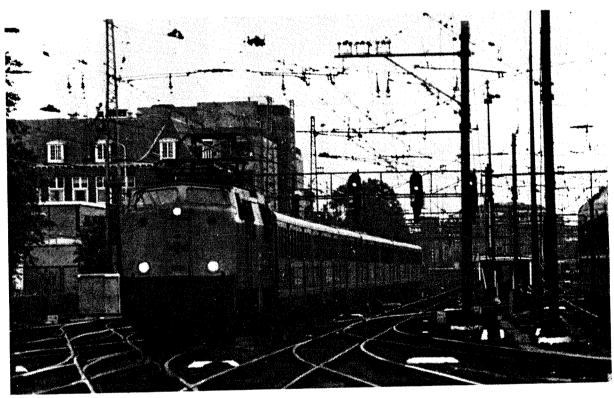


≫ 90. Almost everything is circular on this table. Trace a plate onto
a piece of paper. What do you notice?

≫ 91. Do you suppose that the tower is really shorter than the bridge?

> Then why does it seem that way in this photo?





 \gg 92. If the locomotive is 3 meters wide, then how long is the train according to the photo? And is that really so?

 \gg 93. Photos show everything just as you see it. Is that true?



≫ 95. How can you tell?
What things can you find which don't fit?

FROM THE MIDDLE AGES



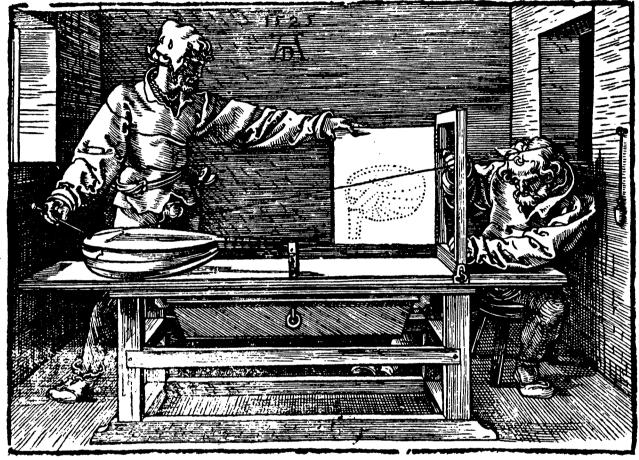
- ≥ 96. It is not so
 easy to draw
 something
 exactly as
 you see it.

 Take a look
 for example at
 this mosaic from
 the Middle Ages.

 How was the
 chapel on the
 right drawn:
 looking down
 from above or up
 from below?
- ≫ 97. And now the building on the left behind the angel. Are you looking at it from above or from below?

- \gg 98. Could you look at these buildings in reality both from above and below at the same time?
- \gg 99. Could this have happened in a photo?

≥ 100. Take a close look at this picture. It is found in an old book about drawing in depth. The two man are drawing the lute which is lying on the left end of the table. The drawing is fixed on hinges to the window. They are using a piece of string.



Which part of the lute are they doing now?

What is the weight on the string for which you see at the far right?

The sitting man is looking for a particular spot in the window. He's holding a piece of chalk there. In what way does he use the stretched string to find this spot?

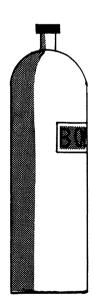
Why is the standing man rather than the sitting one swinging the drawing towards the window?

When the drawing has been swung shut, the sitting man will make a mark on it, exactly where he's holding his chalk. What does this mark indicate?

≥ 101. After a while they will have drawn the whole lute in this way, exactly as you would see it from one particular spot in the room. Where is that? Explain.

Now we'll do the same thing in class. Instead of the hinged drawing-screen, we'll use the swinging segment of the blackboard. Instead of the lute we'll use a chair. Write down afterwards what happened and make a drawing of it.

 \geqslant 103. Trace this bottle onto cardboard and cut it out.



seen from the side. Put the bottle on the table at A and draw the string running to the bottom and top On the following page you'll find a table, a drawing-screen and an eyelet for the piece of string, of the bottle.

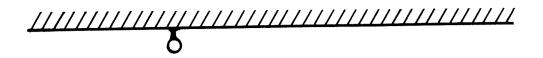
How tall is the bottle drawn on the screen? And how high is the top edge of the label?

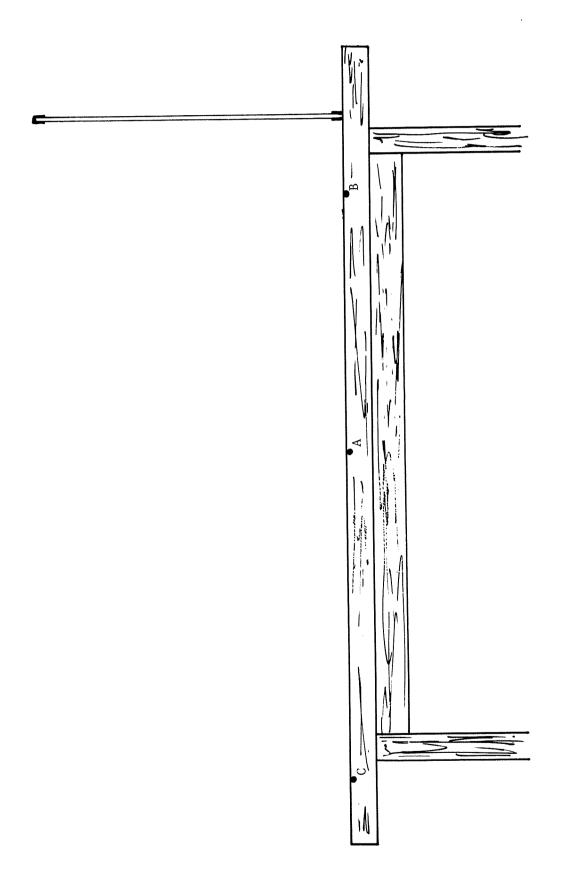
≥ 104. Now put the bottle at B. Draw blue lines. Check the label again.

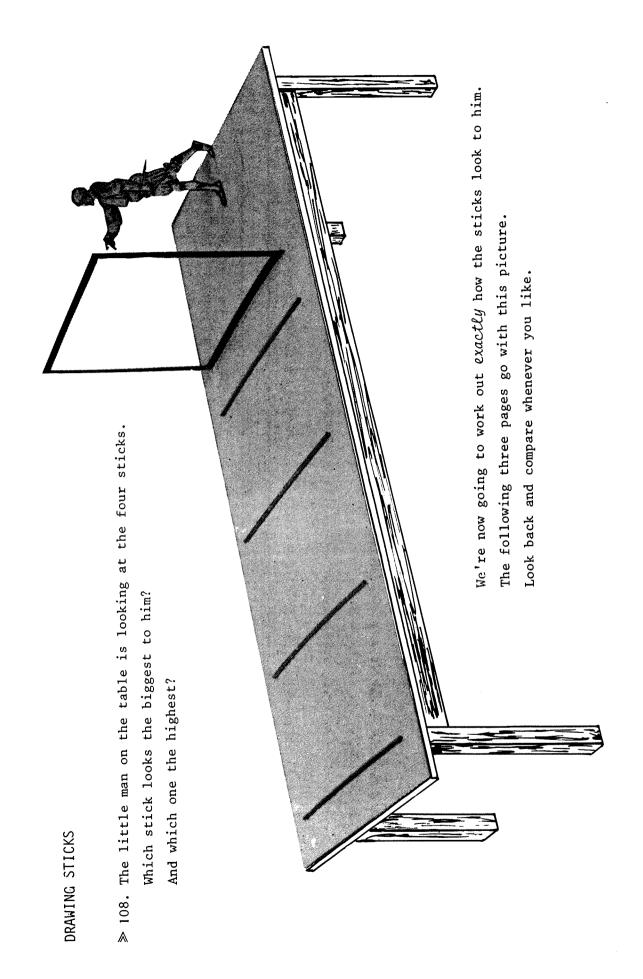
≥ 105. And finally at C. Draw red lines.

≥ 106. In problems 103, 104 and 105 you've in fact drawn the bottle three times on the screen. These bottles are not all the same size. Why is that?

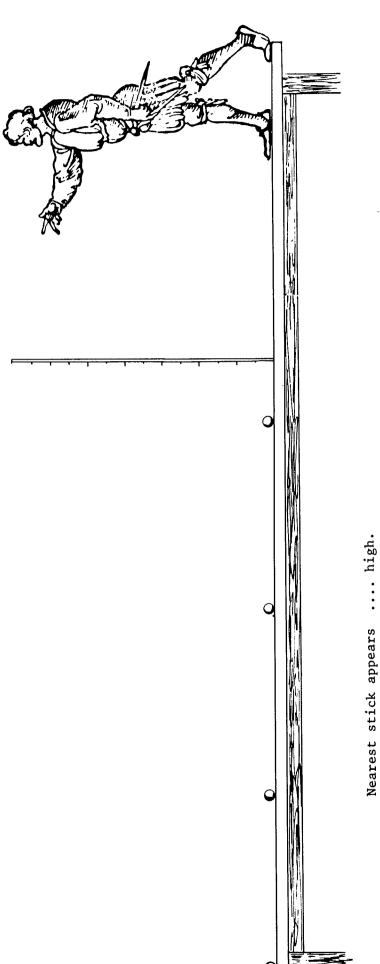
≥ 107. What did you notice in drawing the label three times on the screen? Why is that?







≥ 109. Here is a side-view. How high above the table should the sticks be drawn on the screen? Draw the lines you need to find this out.



.... high. Second stick appears

.... high. Third stick appears Furthest stick appears high.

≥ 110. A top-view of the same thing. Find just where the man's eye is.

How long do the sticks appear on the screen?

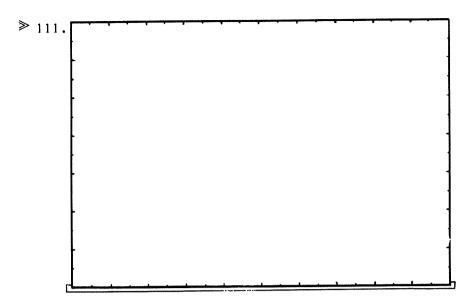
.... long.

Third stick appears Furthest stick appears

.... long.

Nearest stick appears: Second stick appears:

- 54 -



Using the heights and lengths which you found in the last two problems you can now draw exactly what you see through the screen.

Here is the screen as it stands in front of the little man.

Draw the sticks on the screen. Does it look convincing?

STEPS WITH STICKS

≥ 112. Work on this problem together with someone else.

Lay a stick one meter long crossways on the floor in front of you. Hold a ruler in your hand, stretch your arm out in front of you and close one eye.

How many centimeters on the ruler does it take to just cover the stick? Write that down.

Now the other person can move the stick a step further away. You stay in one place and look again how many centimeters it takes to cover the stick.

Always keep one eye closed and your arm stretched out in front. Make a note of the measurements each time the stick is moved further away.

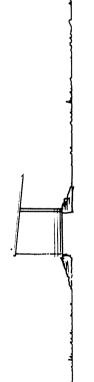
distance from you to the stick	how many cm needed to cover the stick
1 step	cm
2 steps	cm
etc.	etc.

≥ 113. If you went on like this for a million more steps, what kind of numbers would you get in the righthand column?

And what would happen to your outstretched arm? (besides getting)

tired of course!)

You're standing at point C. Which one looks higher, the tower or the bridge? \gg 114. A side-view of the tower and the bridge from problem 91. Does this agree with the photo?



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From where would the photo have been taken?

 \gg 115. A top-view of a row of poles and the screen on which to draw them.

- Where do poles 1 to 5 appear on the screen?

Draw the lines you need.

- Which pole belongs with point A?

- Does point B belong with any of these poles?

Ω

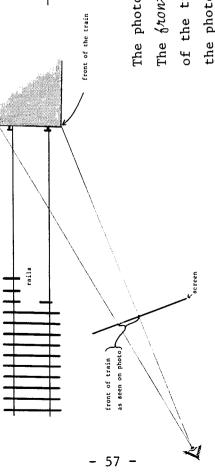
Draw on the screen where pole 1,000,000 would appear

≥ 116. On the screen poles 1 and 2 are mm apart
1 and 10 are mm apart

10 and 1,000,000 mm apart.

> 117. Remember the foreshortened train from problem 92?

Here you have a top-view: train, rails, drawing screen.



The $\ell \pi ont$ of the train is already shown on the screen. Now draw the $\delta \dot{\lambda} de$ of the train on the screen, being sure to use the same proportions as in the photo from problem 92. You can now see where the end of the train The photo from problem 92 is pasted on the screen. appears on the screen.

Where does the end of the train appear on the hails?

≥ 118. The door of the classroom is nearly one meter wide. Even so, from somewhere in the room the door looks like just a thin line.

Where is that?

And what has that got to do with the train photo?

≥ 119. Read this comic strip about Brer Rabbit. What has this got to do
with problem 100?



≥ 120.



At the back of this book you'll find a piece of yellow paper.

Detach it and cut out the innermost section.

Closing one eye, let's look at all sorts of things through this picture frame.

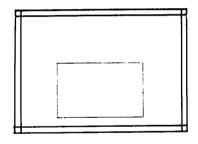
Look through the frame at one of your classmates and imagine that there's a photo in the frame instead of a person standing about one meter behind it.

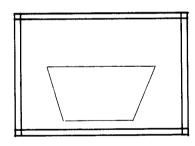
≫ 121. Stand directly in front of your table, about one meter away. Look
through the frame at the table-top. How do you see it,

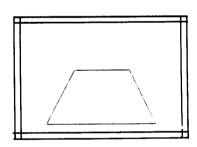
like this,

this,

or this?



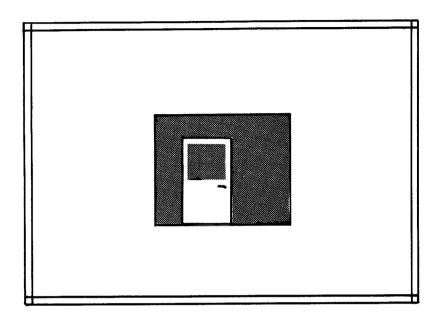




≥ 122. What seems shorter: the front or the back of the table-top?

In which problem did something like this happen?

≥ 123. Stand with your frame in a long straight corridor. The far end of the corridor is the grey bit in this drawing. (Maybe yours isn't exactly the same, but that doesn't matter.)

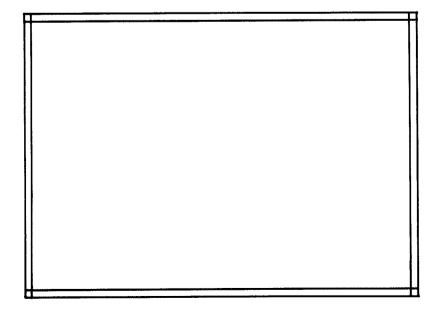


Imagine that there's a piece of glass in the frame.

Trace on the glass with your finger the main lines of the corridor.

In other words, the lines which move away from you. Draw these on the above picture as you saw them through the frame.

≥ 124. Imagine that the corridor was much much longer. What would the far end of the corridor then look like as seen through the frame? Make a drawing of this as well.



≥ 125.



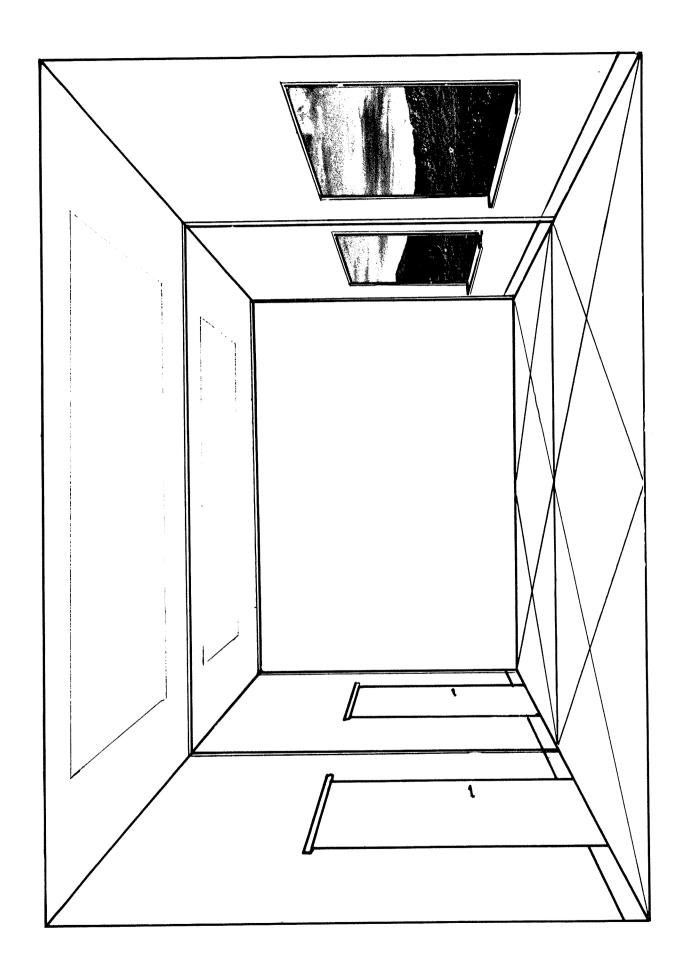
Railway tracks have to run parallel.
Why is that?

Yet you certainly wouldn't say so from this photo!
Are the sleepers all the same length in reality?

- ≥ 126. In reality there are lots more lines which are parallel with the railway tracks. Draw these lines on the photo further into the mist. What do you notice in particular about all these lines?
- ≥ 127. On the next page you'll find a drawing of the beginning of a corridor.

 Two equal segments have been completed but the corridor is not yet finished.

Draw the next two segments yourself. Start with the most important lines. Take care to continue the floor pattern into the following corridor segments as well.

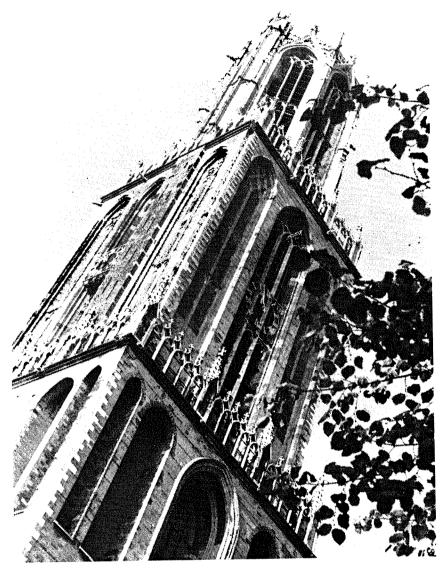


➤ 128. The Santa Maria Maggiore is a church in Rome which is 1500 years old. It is built in the form of a rectangle. Find the lines in the church which in reality are parallel and move away from you.
Draw them further on the photo where they would pass through the far wall. Where do all these lines go?



If you've drawn them carefully, all the lines will pass through one point. This is called a VANISHING POINT.

≥ 129.

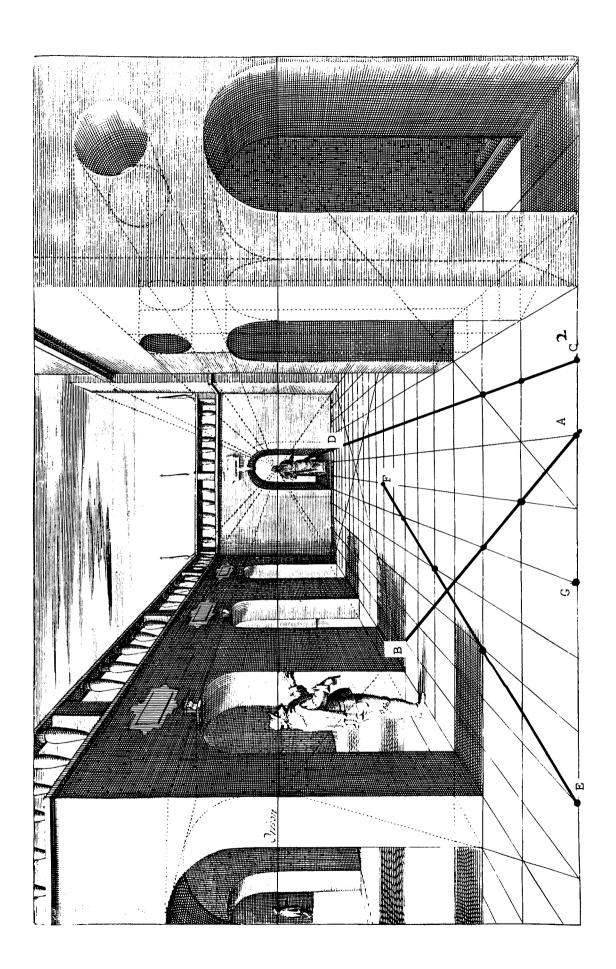


This high church tower in Utrecht,
Holland also contains plenty of parallel lines.
Draw these where they would continue outside the photo, using an old newspaper to go further.
Where do you find a vanishing point now?

- ≥ 130. In the picture on the following page you see a tiled floor.

 Draw a few lines on the floor in the picture which would be parallel with AB on the real tiled floor. Start for example at E and again at G. Find the vanishing point for this group of three lines.
- \gg 131. Do the same for EF. Start for example at G and at A. And one again for CD.
- ≥ 132. You've now found 3 vanishing points on the drawing. What does the
 artist call the line where these points are found?

And what do you notice about the eyes of the people in the drawing?





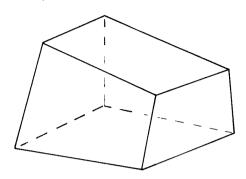
- ≥ 133. A block of flats viewed from another one.

 Which floor do you look down on and which do you look up at?

 In other words, on which floor was the photographer standing?
- ≥ 134. Draw the horizontal lines of the building further. How high lies
 the vanishing point of these lines?
- ≥ 135. Look at the photo of the church in problem 128: was the photographer standing on the ground or was the photo taken from higher up?

DRAWING A CUBE

 \gg 136. This isn't a very good drawing of a cube.



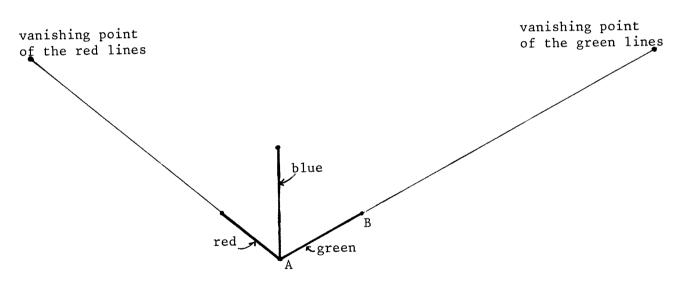
Make the lines which should run parallel with each other the same colour: towards left back - red
towards right back - green
from top to bottem - blue

≫ 137. Now we'll draw the cube well. The beginning has been done for you.

AB is one edge. Make sure that the blue lines from top to bottom

run exactly parallel. Use the vanishing points of the red and green

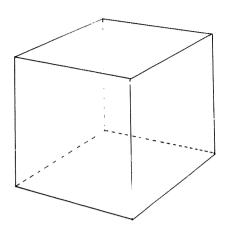
lines.



 \gg 138. Are the red lines parallel in reality? And in the drawing? Complete the following:

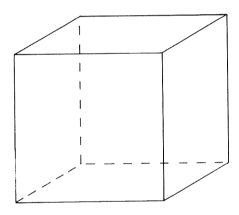
•	in reality	in the drawing
red	parallel	not parallel
green		
blue		•••••

 \gg 139. Another cube. Find the vanishing points.



 \geq 140. The last cube.

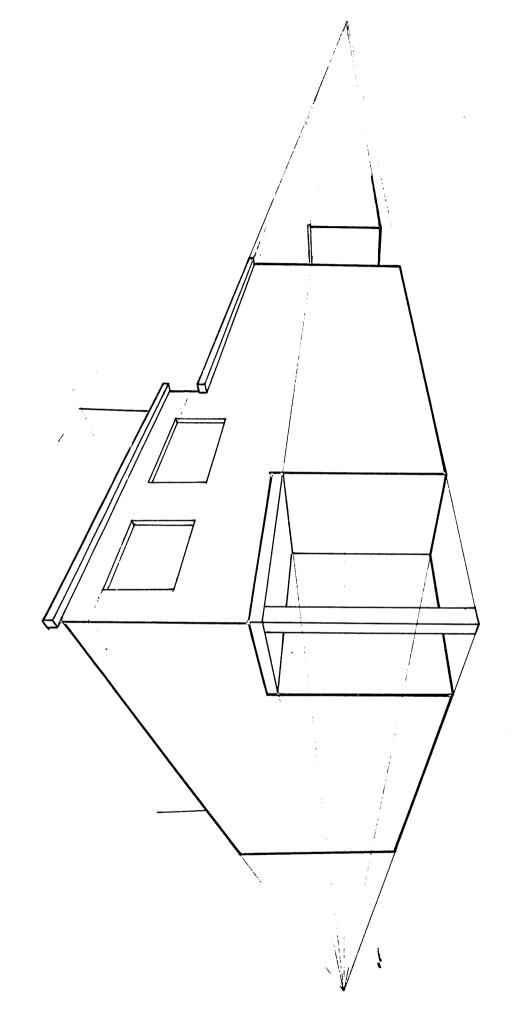
This is the way a cube is usually drawn.



Colour the lines red, green and blue as in problems 136 and 137. Are the red lines in the drawing parallel?

Make a table like the one in problem 138.

Is there a difference between this and the cube in problem 137? And the one in problem 139?



≫ 141. Complete this drawing: garage, more windows, door, t.v. antenna, chimney, etc. Use the vanishing points!

≫ 142. These clouds are parallel in reality. See if you can find their vanishing point. Where does it lie approximately?

