Teaching standard STEM topics with a key competence approach

STEMkey Higher Education Module 3

Measurement

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Summary

This module deals with the mathematical standard topic of measurement, quantities and their dimensions. Measurement provides answers about sizes of objects or phenomena and applies to basic physical quantities like length, area, volume, weight, time, speed, force and energy. The aim of the module is to provide a rich learning experience for future teachers on this topic and demonstrate the relevance of being able to apply measurement concepts and use them to solve various situations in their personal and professional lives.

Innovative teaching methods are used for a shift from basic skills, that often compete with what technology can do, towards a mathematical key competence. This is the ability to solve (measurement) problems in a range of everyday situations. This involves mathematical thinking as well as a meaningful and critical use of mathematical representations, language and technology in measurement. Authentic and cross-disciplinary contexts are used to provide opportunities for students to learn and further develop measuring skills. Inquiry-based learning is used to help to organize the use of these contexts in classroom practices.

Digital technology will be integrated since many measuring instruments are nowadays available on smartphones. Moreover, augmented reality technology enhances the environment with information instruments like compasses, temperature and rulers. Future teachers need to understand the potential of using modern technology in their classroom, and they need to learn how they can instruct their students at school how to use them to act competently in today's society.

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Subject Introduction

This module deals with the mathematical standard topic of measurement, quantities and their dimensions. Measurement provides answers about sizes of objects or phenomena and applies to basic physical quantities like length, area, volume, weight, time, speed, force and energy (Van den Heuvel-Panhuizen & Buys, 2008). These quantities can be primary (e.g. length and time) or compound quantities (e.g. speed).
We will address dimensions and units for a variety of quantities and pay attention to the development of mathematical representations and language like micro, kilo and tetra. Measurement is a standard STEM topic across Europe. We will address this in combination with innovative teaching methods. In particular in measurement such methods are needed for a shift away from basic skills that compete with what technology can do towards the mathematical (key) competence in which mathematical thinking as a tool for solving everyday problems is the central theme. Mathematical competence also involves a meaningful and critical use of technology (Gravemeijer et al., 2017). Authentic contexts are used to provide opportunities for students to learn (basic) measuring skills and to develop transversal skills like critical thinking (Doorman, 2019; Wijers & Jonker, 2017). Inquiry-based learning is used to help to organize the use of these contexts in classroom practices by intentionally eliciting students' intuitive reasoning about measurement and connecting these ideas and language to conventional notations and strategies (Cairns & Areepattamannil, 2019).
One of the pitfalls in measurement is the use of diagrams of the metric system (often shaped as a staircase) which allow mechanical strategies based on moving the decimal comma (a transformation from cm to m requires a move of the comma two steps to the left: 18,5 cm = 0,185 m) and resulting in isolated and not flexible strategies (Ballering, 2012). In addition, different calculation procedures in different STEM disciplines make it difficult for students to become fluent in measurement (ibid, 2012). A flexible understanding of measurement is part of the mathematical (key) competence and is important for being confident and skilled in daily life, workplace and in all STEM domains.

The primary target group for the module is teaching staff (educators) in ITE; an important second target audience are future teachers. We will include illustrative classroom scenarios, tasks and students' answers. The tasks can also be used by teachers and their students in daily practice.

This module shows the importance and the teaching of measurement for the digital society of the future with cross-disciplinary problem solving, authentic contexts, IBL and digital technology.

The connection with authentic contexts and personal reference measures supports future teachers in developing an understanding of the importance of measurement, measuring skills, and a positive mind-set towards its relevancy for daily life, work and further study. Future teachers will have developed the abilities to build on students' measuring reasoning, to recognize and use the potential of innovative technology and inquiry-based learning (IBL) to guide their students towards the intended measurement knowledge, measuring skills and positive attitudes towards the usefulness of measurement. Their students will be able to use their personal innovative technology for daily life or work-related measuring problems, to identify unrealistic results, and or make realistic estimates.

Student teachers need to become aware of the general need for a shared understanding of measurements to share and compare information. In particular in science it is fundamental to be able to develop a critical attitude towards measuring tools, number of measurements needed and precision achieved. Student-teachers need to engage in this (scientific) attitude, and develop skills of knowing that, why and how to use ‘objectively’ retrieved information to create evidence-based arguments for a certain conclusion or in decision making.

The module uses contexts from other disciplines such as physics, chemistry and geography. Moreover, the measuring strategies will have a close connection with applications in sectors like agriculture and health care by connecting to their instruments and strategies (e.g. rules of thumb for the size of a heap of soil when digging out a cube meter, or for creating 1% chemical solutions).

For this module we identified four different topics in relation to measurement, each with its own specific learning outcomes. A topic includes activities to support future teachers in reaching the learning outcomes, as well as activities oriented on how to address the topic in the classroom in an innovative way, using authentic contexts, inquiry-based learning strategies an technology.

In the module we use the following vocabulary:

Quantity	Dimension(s)	Unit
Length	L	meter
Area	Length and Width	m^2
Weight (mass)	M	kilogram
Time	T	second
Volume	L	liter
Velocity	Distance and Time	m/s or km/h

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Key Competence Approach

The main focus of this module is the development of the mathematical key competence, defined by the EU as:

> *Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentations (formulas, models, constructs, graphs and charts*).

The module also relates to other key competences such as critical thinking and scientific and digital competences. All topics in this module contribute to the mathematical competence by addressing activities that can be used to develop the competence and activities to teach the competence in classroom practices.

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Learning Outcomes

The first expected learning outcome of this module is that future teachers understand the importance of measurement, extend their own measuring competences, and acquire a positive mind-set towards its relevancy for daily life, work and further study.

The second learning outcome is that future teachers have developed abilities to build on students' measuring intuitions, to recognize and use the potential of innovative technology and inquiry-based learning (IBL) to guide their students towards the intended measurement knowledge, measuring skills and positive attitudes towards the usefulness of measurement. They learn how to use their personal innovative technology for daily life or work-related measuring problems in their teaching.

An overview of measurement learning outcomes aimed at in the module with respect to the successive chapters:

Topics/Learning outcomes	knowledge	skills	attitude
Generating and using personal reference measures	Units for everyday quantities, sizes of familiar objects as reference measures	Using a reference strategy for estimations and ability to find new reference points	Willingness and confidence in using familiar reference points for measurement
Schematizing structures	Relationships between units, use of prefixes, models for calculations and conversions	Ability to relate quantities, schematize situations with structure models, perform calculations	Confidence in (re)constructing models that support reasoning in measurement
Identifying quantities and using measuring tools	Knowledge of various quantities, measuring tools, scales and accuracy	Ability to select and use appropriate measuring tools	Confidence and interest in using tools and apps available for measurement
Using language to communicate about measurement	Daily and academic names of quantities, units and prefixes	Ability to use language for measurement appropriately	Confident in communicating about measurement in and outside the classroom

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HE Module plan

The structure of the module plan is as follows:

0. Introduction into measurement: 20 min
1. Topic: Generating and using personal reference measures: 60 min
2. Topic: Schematizing structures: 60 min
3. Topic: Identifying quantities and using measuring tools: 30 min
4. Topic: Using language to communicate about measurement: 30 min

For each task/activity/session a table with the following information should be presented.
Template for single activities/sessions/tasks:

0 Introduction into measurement	
Introduction	
Whole class	20 min
We suggest to start with two introductory activities to have student teachers become aware of the use of personal reference points and the importance of schematizing in measurement. The second activity can also be used to highlight the two kinds of activities in this module: for developing your own competence, and for learning to teach the competence.	
Description of the activity Ask your students the two questions below and let them think and share briefly their results. Plenary discuss these results as an introduction to the module. 1 Estimate the height of the room you are in. Explain how you estimated. 2 How many floor tiles? The floor area of your room is 4 x 5m. You want to cover it with tiles of 30 x 60cm. (own competence) How many tiles do you need to order? Find at least two strategies to solve the problem. (teaching the competence) How can this problem be used to promote students' understanding and reasoning about area?	

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1 Generating and using personal reference measures

Generating and using personal reference measures refers to the ability to develop and apply personal reference points for measurement in order to solve a range of (estimation) problems in (everyday) situations involving quantities, dimensions and units of measurement. The focus on estimation problems also relates to promoting the competence of modeling real phenomena and situations which includes the estimation of quantities. Estimation and the use of personal reference measures are key facilitators that enable students to tackle complex situations in all STEM disciplines (Ärlebäck and Albarracín, 2019).

Working arrangement in class: whole class, pairs, alone (use icons as given in STEMkey sync-share-folder)	60 min

Knowledge

Necessary knowledge involves

- knowledge of (metric) measurement units and its names for everyday quantities such as length/distance (cm, m, km, …), liquid volume (ml, cl, dl, l), weight (mg, g, kg) , time (s, min, h, …), speed (km/h, m/s, ..)
- knowledge of everyday objects that correspond with formal units of measurement: 1 m is one big step, 1 kg is a bag of sugar, 1 minute is counting slowly to 60, etc.
- knowledge of measurements (dimensions and units) of some real 'world objects' like knowing one's own height, one's own speed when walking or biking, the volume of a soda can, the duration of a lesson, etc.

Skills

- Being able to estimate and measure using a reference strategy.
- Being able to find/make new reference points for 'new' units.

Attitude

- Willing to look for and use personal reference points for measurement. Having confidence in personal knowledge.

Activities – your personal reference points

In activities 1.1 and 1.2 the aim is for students to develop and apply personal reference points for measurement in order to solve a range of (estimation) problems in (everyday) situations involving quantities, dimensions and units of measurement.

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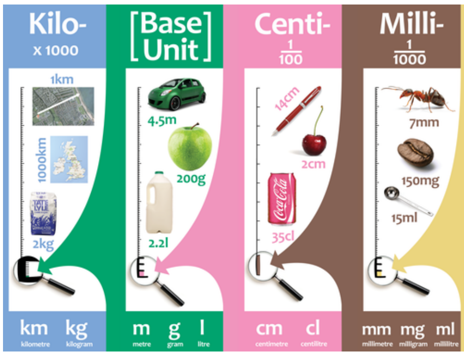

Figure: Some sense of measurement is indispensable in daily life

Activity 1.1 Reference points

In this activity students first individually solve estimation problems for various dimensions using personal reference points (**Worksheet 1.1A**). In small groups they compare their reference points and estimation strategies. In the whole group you collect the findings and reflect on reference points for length and speed. In the discussion also involve reference points for other quantities, such as weight, liquid volumes, time.

As homework students make their personal reference guide (**Worksheet 1.1C**) and bring it to the next session.

You can also insert an optional activity before the homework (see **Worksheet 1.1B**) in which teams of students solve Fermi problems, for which they need reference points and schematizations for several quantities in the estimation tasks. The schematizations can be used for or referred to in activity 2.

Activity 1.2 references for area

In this activity students reflect on reference measures for area. In **Worksheet 1.2A** a newspaper heading is used as a start. You can replace this with a local or recent example. A football field is used as a reference. why? And is this a good reference for everyone? In **Worksheet 1.2B** students relate measures, units and reference for length to those for area.

Activities – teaching personal reference points

The aim of these activities is to have students in ITE explore and reflect on ways to teach their students about estimation and the use of personal reference points.

One of the ways to teach this is making use of open estimation questions (see activity 1.1B). Such questions relate to Fermi problems (Ärlebäck and Albarracín, 2019).

> *Fermi problems (FPs) are open questions offering little or no specific information for the problem solvers to direct them in the solution process. Examples of FPs are: estimate the number parking places needed for an apartment building and estimate the area that can be covered by all leaves of a tree. The most prominent characteristic of FPs is the use of assumptions based on personal knowledge and 'simple' chains of reasoning.*

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The student teachers learn that the task of the teacher is to share and compare personal reference measures and solution strategies in order to value and reflect on their own list. This activity, also focusing on how to talk about and reflect on the use of personal reference points in measurement, supports the development of a critical attitude towards measuring and estimation skills.

Activity 1.3 lesson plans for estimation tasks and reference points
Worksheet 1.3A lets student teachers design a lesson plan and discuss how to organize estimation tasks and how to ensure classroom understanding of lessons learnt. **Worksheet 1.3B** lets student teachers reflect on the work of students on a task about reference points and design a follow up activity to improve this task. **Worksheet 1.3C** is an example of how to use the news as a resource for teaching about measurement.

2 Schematizing structures

Schematizing structures focuses on the ability to model, structure and visualize relations between quantities, dimensions and units of measurement in order to solve problems. This focus shifts attention from calculation models (e.g. the metric ladder, staircase models) towards structure models (e.g. a structured ruler or structured volume cubes) that help to measure, to choose the unit of measurement and to perform calculations for unit conversions (e.g. from m^3 to liters).
Let student teachers critically reflect on the use of staircase-models for supporting students to perform calculations. These visuals work when you understand the model, while students are in the middle of a learning process. We suggest more attention for structure models in teaching measurement.

Working arrangement in class: whole class, pairs, alone (use icons as given in STEMkey sync-share-folder)	Duration; write time expected for this activity (plus use icon)--- 60 min

Knowledge

- knowledge of (common) relationships between units of measurement
- knowledge of the (most commonly used) pre-fixes in the metric system and their meaning
- knowledge of models like the (double) number line, bar, area-model and ratio-table
- knowledge of metric systems (and knowledge of limitations of teaching a metric system and of possible alternatives for unit conversions via 'known' units (e.g. centimetres, meters, kilometres).

Skills

- being able to relate measurement units for the same quantity to each other
- being able to relate units between quantities e.g. 1 dm^3 = 1 litre

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- Reconstruct relations between measures by visualisation e.g. in a sketch of a cubic meter, indicate centimetres in each direction (l, w, h) to find the factor 1 million (100 x 100 x 100).
- Being able to analyse the affordances and limitations of schematisations such as the staircase for the metric system.

Attitude

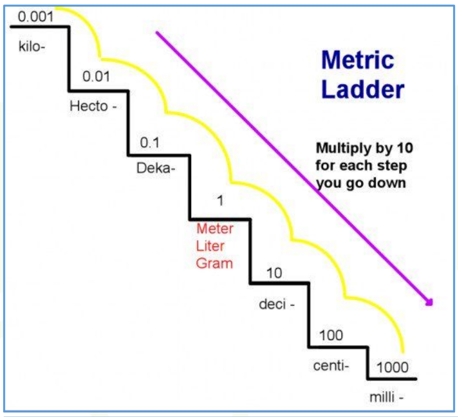
- Trusting one's own knowledge and ability to use drawings for calculations and conversions. Confidence in ability to reconstruct instead of remembering.

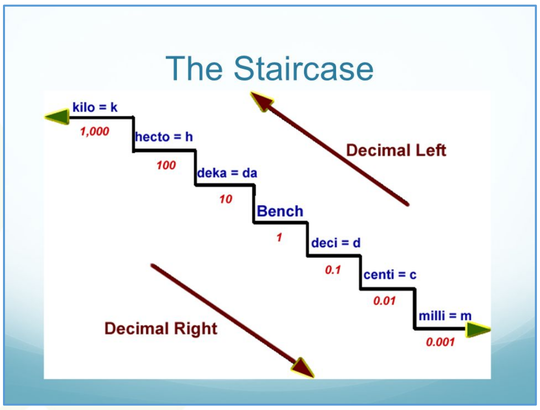
Activity 2.1 Explore calculation strategies and the use of sketches/representations

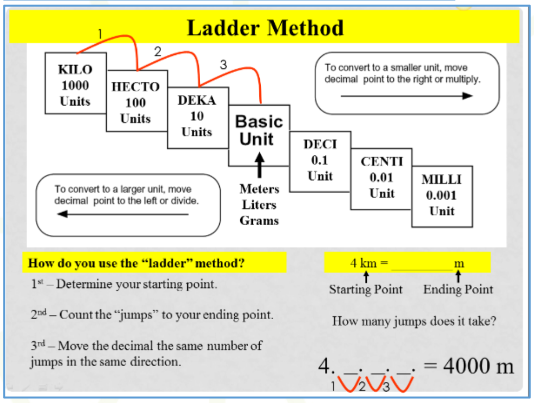
This activity is an introduction on calculations that include conversions of units or dimensions. Let the students calculate the volume in liters of a swimming pool with length, width and height given in meters (maybe in connection with time needed to fill the pool in liters per second). Next, let them reflect on knowledge and skills needed to get the answer (we expect that most students will not draw a pool to support reasoning, but some might make a sketch).

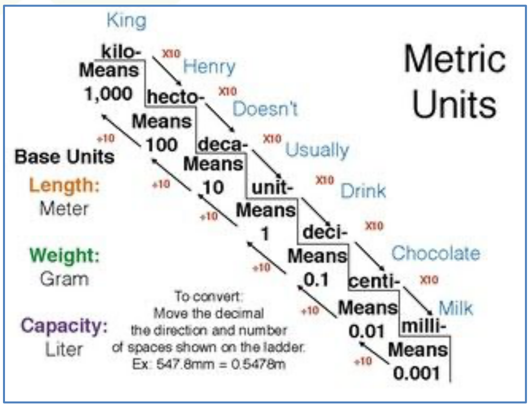
Activity 2.2 Discuss staircase models

In many situations you need to convert units (e.g. from kilo… to centi…) or dimensions (from liters to m^3). Traditionally, 'staircase models' dominate the teaching of these conversions. Discuss with your future teachers reasons for and risks of using 'staircase models' for conversions between units of measurement:



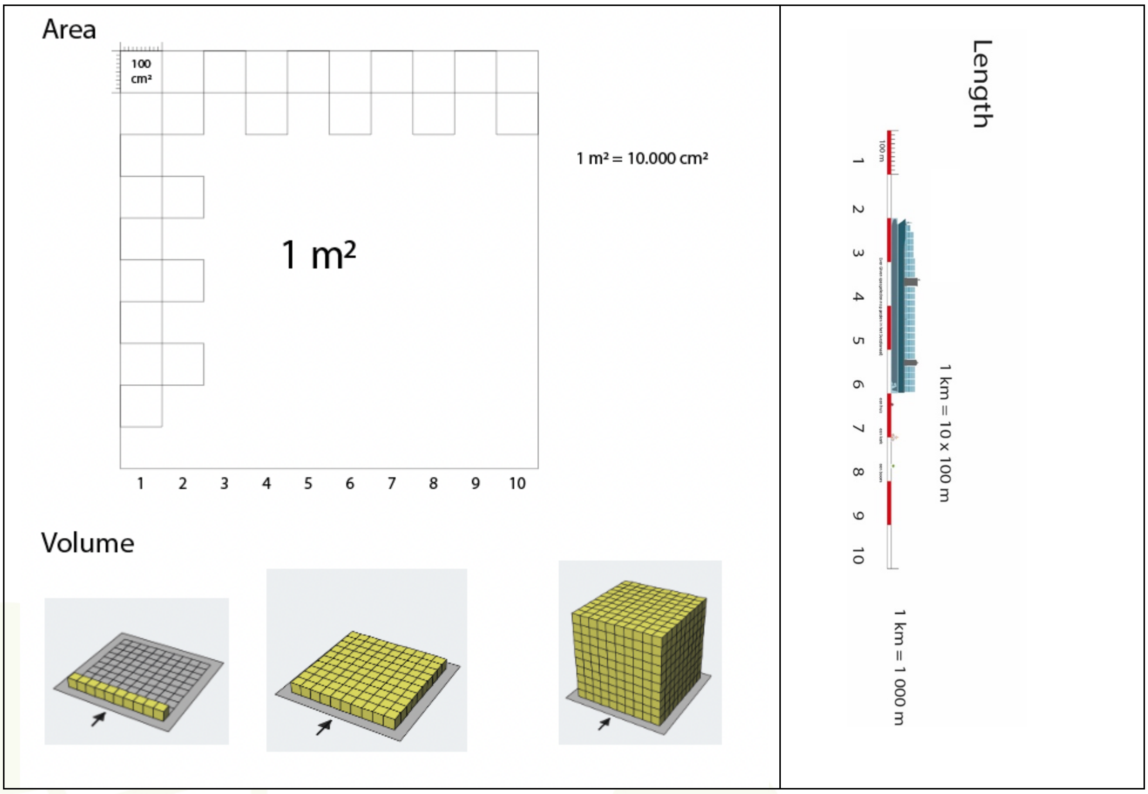






Activity 2.3 Discuss alternatives for the metric ladder

Why don't we use the metric ladder for conversions in time (e.g. from seconds to hours)? How could you visualize time and support conversion calculations?
Visual representations that directly connect to the situation or dimension being measured can provide an alternative for the metric ladder, and conceptually support what calculations to perform for converting between units.
The visual representations in the figure below help to develop and reconstruct relations between measures. Explain how a sketch of a cubic meter can be used to explain the factor of 1 million when converting from cubic meters to cubic centimeters.



Activity 2.4 Progressive schematizing in a learning trajectory

Provide the student teachers with one or more research papers on learning trajectories for measurement. Analyze the role of schematizing and tool use in learning about measurement, e.g. for the concept of a standard measuring unit: from using feet or hand spans for pacing distances to the use of measuring strips (Gravemeijer, 2004) and exploiting fairness while playing traditional games (Wijaya e.a., 2011).
Summarize knowledge needed for a schematizing approach in measurement (liter equals dm^3, meter to dm, volume equals length x width x height), skills, and attitude. Ask your student teachers to comment. Let them find similar tasks in textbooks and provide a similar competence oriented analysis.
Discuss the instructional sequence and design sample activities for (some of) the rows in the paper by Gravemeijer: https://doi:10.1207/s15327833mtl0602_3 .

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TABLE 1
Overview of the Proposed Role of Tools in the Instructional Sequence

Tool	*Imagery*	*Activity/T-a-s Interests*	*Potential Mathematical Discourse Topics*
Feet (heel to toe)		Measuring	
Masking tape	Record of activity of pacing	Reasoning about activity of pacing	Focus on covering distance
Footstrip	Record of pacing (builds on masking tape) (Form/function shift: using a record of pacing as a tool for measuring)	Measuring with a "big step" of five = measuring by iterating a collection of paces	Measuring as divorced from activity of measuring Structuring distance in collections of 5s and 1s
Smurf cans	Stack of Unifix cubes signifies result of iterating	Measuring by creating a stack of Unifix cubes	Builds on measuring divorced from activity of iterating
Smurf bar	Signifies result of iterating	Measuring by iterating a collection of 10 Unifix cubes Structuring distance into measures of 10s and 1s	Accumulation of distances Coordinating measuring with 10s with measuring by 1s
10-strip	Signifies measuring 10s and 1s with the Smurf bar	Measuring by iterating the 10-strip, and using the strip as a ruler for the 1s	Accumulation of distances Coordinating 10s & 1s
Measurement strip	Signifies measuring with 10 strip/ Starts to signify result of measuring (Form/function shift: inscription developed for measuring is used for scaffolding and communicating)	(1) Measuring: strip alongside item; counting by 10s and 1s => reading of endpoint (2) Reasoning about spatial extensions (results of measuring have become entities in and of themselves)	Distance seen as already partitioned; extension already has a measure Part-whole reasoning/quantifying the gaps between two or more lengths Shift in focus: focus on number relations; developing and using emergent framework of number relations
Empty number line	Signifies reasoning with measurement strip	Means of scaffolding & means of communicating about reasoning about number relations	Numbers as mathematical entities (numbers derive their meaning from a framework of number relations) Various arithmetical strategies

(Gravemeijer, 2004)

3 Identifying quantities and using measuring tools

This topic focuses on the selection of the relevant quantity, the selection of an appropriate measuring tool or instrument, reading and interpreting the result from the tool. Reading and interpreting results from a measuring tool includes interpreting a measuring scale and may involve a change of unit or rounding.

Working arrangement in class: whole class, pairs, alone (use icons as given in STEMkey sync-share-folder)	Duration; write time expected for this activity (plus use icon)--- 30 min

Knowledge

- Knowledge of the difference between measuring qualitative and quantitative dimensions.
- Knowledge that (quantitative) measuring means 'fitting to a unit'.
- Knowledge that for measuring one can use measurement tools and instruments and know some examples of these.
- Knowledge of the way measurement scales 'work' and can be read (both digital and analogue)
- Knowledge of accuracy, precision, significant figures, and rounding: when reading a number from a device that represents a measure you know that some/all figures are significant, depending on the context, that rounding might be needed and that this impacts further calculations

Skills

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- Being able to connect qualitative and quantitative measurements for a quantity
- Being able to select and use an appropriate measurement tool/instrument
- Reading different scales, interpolating and/or rounding if necessary
- Converting between units if necessary
- Interpreting results (readings, sense of significance related to context)
- Experiment systematically to find out how a measurement instrument/tool works

Attitude

- Becoming a confident user of modern technologies
- Being able to critically reflect on your own measuring activity (what is precision, and feeling the need to measure more than once)
- Critically evaluate measurements presented by others (numbers are seldomly objective)
- Awareness of measuring cultures in other disciplines or professions and willingness to share and engage in interdisciplinary activities related to measurement.

In this activity students first individually collect and compare analogous and digital measuring instruments and discuss what can be measured with what scale (activity 3.1). In small groups they can do the activities. Whole group discussions are needed to collect the findings and reflect on concepts like significance, outliers and tool use.
As homework students … (worksheet 3.2) and bring it to the next session.

Activity 3.1 measuring tools

Measuring instruments change. Below you see a set of analogous and a set of digital measuring instruments. What do you notice? What do they measure? How do you measure? What is the impact on teaching and learning to measure?

A concrete activity could be to let three fellow students measure your length with a measuring tool (maybe a measuring strip, an electronic device, a smartphone, a 15x15 cm tile if it is at hand). Do you get every time the same result? How precise is the tool? Can you take the average? How many significant numbers do you get?
Homework could link this activity to critical thinking: let your students collect newspaper items that include measurements that can be questioned (precision, sampling, …).

Analogous instruments	Digital instruments
	

Activity 3.2 measuring with your mobile phone

What can you measure with your mobile phone? Explore measuring apps. Make a list of what each app measures, give an example how it works, what kind of quantities, what is the unit of measurement? How precise?

Compare issues of precision and rounding when reading results from a speedometer in your car and from a radar gun used by the police:

List at least three situations in which reading results from a measuring tool involves rounding (e.g. measuring your body for clothing sizes versus measuring wooden planks for constructing furniture).

Activity 3.3 qualitative and quantitative measurements

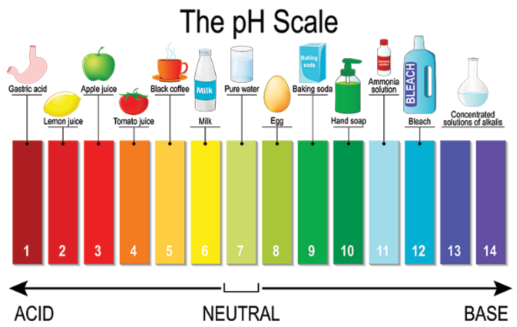
A characteristic of most of the measuring instruments is that they use some sort of quantitative scale. This scale relates in a specific way to a more qualitative understanding of the quantity being measured. For instance, we call something heavy when it is not easy to lift by hand (qualitative), and this object might weigh more than 20 kg (quantitative). Some quantities are often measured qualitatively (e.g. acid with soluble or paper indicator, or wind strength by watching phenomena outside), while they also have quantitative scales (respectively pH value by titration and Beaufort scale with a anemometer or Windy.app). This activity makes students aware of these two perspectives on measurement. In **Worksheet 3.3** a full table for wind speed is provided.

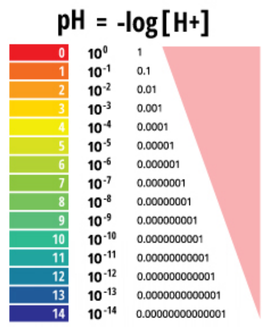
Let your students describe a quantity that can be measured qualitatively and quantitatively (including dimensions and units). Also discuss the number of significance digits you can measure with the a quantitative measuring tool in a specific situation.

Beau-fort	Description	Wind speed	Wave height	Sea conditions	Land conditions
0	Calm	< 2 km/h	0 m	Sea like a mirror	Smoke rises vertically.
1	Light air	2–5 km/h	0–0.3 m	Ripples with appearance of scales are formed	Direction shown by smoke drift but not by wind vanes.

3	Gentle breeze	12–19 km/h	0.6–1.2 m	Large wavelets; crests begin to break; foam of glassy appearance; perhaps scattered white horses	Leaves and small twigs in constant motion; light flags extended.

This activity can be connected to the importance of interdisciplinary work when addressing measurement and scale conversions. For instance by exploring the scales for the PH-value.





Depending on your target audience you can also include an activity on measuring in vocational contexts (Bakker e.a., 2011). For example: create a measurement lesson based on the information about fishes and fishery (http://www.fao.org/3/F0752E/F0752E03.htm) or on hear beats (check your pulse or use a pressure machine, fitness tracker or a smartphone).

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Table 3 Examples of competence descriptions related to measurement randomly chosen from each sector

Ex.	Competence description (search terms in bold)	Qualification file and sector	Level
1	He samples and takes **measurements** for product and process control in skilled and accurate ways taking into account the vulnerability and decay of food	Assistant food and environment Food technology	1
2	The carpenter interprets the construction drawing and understands the structure and shape of the product. He chooses and manipulates the equipment for **measuring** and assembling the various parts correctly	Carpenter Economics	2
3	The lab assistant uses equipment for analysis and uses chemicals according to regulations. He treats them in a proper and careful way to ensure a good condition of the equipment and to produce reliable **measurement** results	Lab assistant Care	2
4	The metal worker accurately assembles different parts of the product in order to finalize the production process according to quality requirements in terms of **measure** (length, width, height, design), number, and delivery time	Metal worker Technology	3
5	The First mate/marine engineer of small fishing boats professionally recognizes the fish based on **size** and species and sorts them into the correct bins. Discards are thrown back into the water and in case of doubt he will use a **measuring** board	First mate/marine engineer small fishing boats Fisheries and food technology	3
6	The sport and activity coach collects sufficient relevant data about the performance of the SA-participant, for instance through observations and **measurements** and judges, based on these data, whether the SA-participant meets the expected standards, allowing the (social) skill level of the SA-participant to be determined correctly and any skill qualifications can be issued appropriately	Sport and activity coach Care	4
7	The aircraft technician examines possible causes of malfunction, verifies them, analyzes the results of the **measurements**, and draws logical conclusions on the fault so as to locate and analyze the problem adequately	Aircraft technician Technology	4
8	The senior logistics timber selects the **measurement** tools suitable for identifying stocks. He controls the quality standards during inventory of the appropriate wooden materials and makes efficient use of the **measurement** tools	Senior logistics manager for timber Economics	4

Activity 3.4 repeated measurements

When not addressed in activity 3.1: Let your students measure in groups their average height and width with arms spread using what they have at hand (e.g. books, pencils) and to create graphs of their measurements.
Discuss how and why graphs can be different. The role of outliers. Possible measuring mistakes. The need for splitting data, for instance for different sexes. Connect to measuring in science: you sometimes need repeated measurements to prevent mistakes, outliers, be more precise.
Contact a colleague from Physics or Chemistry, for instance a (student) teacher, and ask how precision of measurements is involved in their teaching, in particular ask for the concept of ‘significant numbers’. Reflect on how, or whether, that concept is addressed in mathematics education and how the two disciplines can support each other.

Activity 3.5 from foot to phone

This activity involves students in the use of mobile apps for measurement and to learn about unit transformation, in this case from steps to (kilo)meter. When you like historical stories you can connect the step or foot as a measure to early distance measures. This activity can also easily be extended or more challenging for higher grades by adding properties of the routes or shapes to be traced (e.g. a circular path with a length of 1 km).

Source: https://www.maa.org/press/periodicals/convergence/the-right-and-lawful-rood

Your mobile phone probably has an activity meter that measures your number of steps during the day and the distance covered. The two screen shots in this figure are from the mobile phone of the same person. How do you think that the phone calculates distance from the number of steps?

Is a walk around your school more or less than 1 km? Use two ways to determine the length of this walk: using maps (with a clever route using street names), and with a activity meter from a mobile phone. Compare the results and decide which of the two is more precise. Let the student teachers design a lesson for this activity.

4 Using mathematical language for measurement

The importance of language in mathematics learning has been widely acknowledged (e.g. Smit e.a., 2016). Also in measurement language is prominent for naming quantities and performing measurements. Words like weight, volume and liters are related, but also have different meaning or use, depending on the context and the language used in academic, school or daily life situations. The following activities illustrate how awareness for the role of language can be developed and how language-oriented support in learning about measurement can be provided by using language promoting activities.

Working arrangement in class: whole class, pairs, alone (use icons as given in STEMkey sync-share-folder)	Duration; write time expected for this activity (plus use icon)--- 30 min

Knowledge

- Knowledge of names (and meaning) of quantities, units and of prefixes
- Knowledge of common expressions used in the context of measurement, this involves daily language as well as academic language and being able to connect between these language systems.

Skills

- Use of specific language of measuring in an appropriate way.
- Connecting representations

Attitude

- Confident in engaging measuring discussions
- Awareness of role of language in measurement and willingness to support students with language-scaffolding activities

Activity 4.1

Have students look back at the previous activities they did and from these find or think of and write down a few lines in which they show how they talk: for example when explaining a strategy for estimating the height of the room (activity 1) or when discussing an alternative for the metric 'staircases' activity etc. and write (when solving a task or reporting on findings) about measurement and measuring.

Activity 4.2

Have your students in pairs make a wordspin of mathematical language used for measurement.

Combine two pairs into a group of 4 and have them order the words in categories (related terms). They have to make these categories themselves.

In the whole group discuss the 'category systems' of the groups

- What language elements are used in making the categories?

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Next discuss that mathematical language is not used a bout words: it is about expressions (like '*height* is measured in *centimeters*, using a tape measure) and relations. Don't forget: symbols are also part of the language of mathematics.

Make sure that student teachers also focus on one specific category: the prefixes (milli, micro, kilo, mega, tera, …) that are used for various dimensions/units (meter, gram, byte, …).

Activity 4.3
Some background on Language sensitive teaching
Language is an important tool for 'thinking' and thus for understanding.
Give examples of words and sentences for measurement to that are everyday language, general academic language, and specific mathematical language
As an example you can have your students analyze the Laundry powder task with the following questions:
1. What kind of language is used in the problem of the laundry powder? Which daily language, which mathematical language (words) and symbols are used, and which general academic language?

2. Look at the problem (language and pictures) through the eyes of a student: what might be difficult for the student?

Suppose you use a dosage of 100 ml soap for each wash. How many washes can you do with one pack of 1 kg washing powder?

Dosage			
4-5kg	soiling		
water hardness			
soft	60ml	75ml	100ml
medium	75ml	100ml	115ml
hard	100ml	115ml	140ml
handwash: dissolve 50ml of powder per 10L of water.			
for 6-7kg machines: add an additional 30ml / kg extra to the recommended dosage.			
prewash: add an extra 50ml of powder.			
100 ml = 68 g			
*The standard number of washes (100) is based on the dosage for normal soiling in medium water hardness and average content.			

Activity 4.4 Scaffolding language
Watch the video 'Oom Kees' Zie: https://www.youtube.com/watch?v=H3gYgreiAdw
The topic of the activity is (the language) of graphs, but you may also have noticed measurement related language. Discuss this
-> how does the teacher support the development of the mathematical language of the pupils? What techniques does she use?

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Materials and resources

List attached materials and give one-sentence-description. you can also include icons Note: Table lines shall not be visible.

Examples of materials:

Presentation 1 (pptx). Title + One-sentence-description

Readings. Title + One-sentence-description

Presentation 2 (pptx). Title + One-sentence-description

Worksheets. Title + One-sentence-description

Digital media, e.g.youtube video, social media…

Online repository

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Evaluation

Personal reference measures
What are personal reference measures? (include example)
Why are personal reference measures important in measurement?
How can your future students be stimulated to develop and use personal reference measures? (design a lesson plan for an activity)
Evaluation goal: Knowledge on measurement education and teaching competence related to lesson preparation for measurement.

The fictitious dialogue
In this assignment, student teams of 2-3 are asked to write and perform a dialogue about the role of measurement in mathematics and one or two other disciplines. For the dialogue, students are asked to imagine that they are a representative of a specific discipline and invent a dialogue about the role of measurement in their discipline, its use and what is important to know and to master. Each group prepares a dialogue, with an emphasis on the disciplinary grounding of their arguments and perform it in class. The personalized arrangement and speaking in the 'I'-form intensify the experience.
Evaluation goal: Interdisciplinary awareness in measurement and engagement with different perspectives (e.g. mathematics focuses on dimensions and conversions, while science subjects focus on its role in experiments such as using tools, precision, representativity, errors). The rubrics for interdisciplinary skills can be used to evaluate the students´ work.
https://cat-database.sites.uu.nl/knowledge_item/how-do-you-assess-interdisciplinary-skills/

Critically reflect on measurements
Provide a list of measurements with a few outliers and with a variation in precision. Ask for the measuring context or question whether this list gives enough information to make a decision.
Evaluation goal: to show awareness of possible measurement errors and the ability to critically reflect on and compare the numbers (in relation with the measuring context and tools).
Evaluation criteria: give feedback referring to levels varying from immediately saying "yes, this enough" or "don't understand what to do", to ignoring some outliers, to doing some extra measures and triangulating more available data.

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ICON (follows)

Transdisciplinarity

Interdisciplinary elements concern the attention for qualitative and quantitative aspects of measuring in other disciplines (e.g. pH in chemistry and wind speed in geography), and attention for the role of measurement in (critical) scientific work (e.g. significance of the number of digits when measuring a quantity with a specific tool and repeated measurement for preventing/identifying measuring errors). This relates in particular to physics and chemistry.

Furthermore, we have a link with the human anatomy module (e.g. BMI and Leonardo's body measures and in particular Activity 3.5 "from foot to phone" on step size) and with the material-cycle module on the carbon cycle: how to measure the amount of carbon in soil? E.g. https://www.agric.wa.gov.au/soil-carbon/measuring-and-reporting-soil-organic-carbon?page=0%2C0#smartpaging_toc_p0_s0_h2 . A bulk density (BD) estimate is required to calculate soil organic carbon stocks in tons of carbon per hectare. Bulk density is the dry weight of a known volume of soil. It can be taken using a core, exhaust tube or pipe hammered into soil for a given depth. Bulk density (BD) estimate example: An exhaust tube 7cm in diameter (3.5cm radius) and banged in to a depth of 10cm has a volume of: 3.14 x (3.5 x 3.5) x 10 = 385cm^3. If dry soil weight was 500g then BD = 500/385 = 1.3g/cm^3 = 1 300 000kg/ha …

Finally, the link with water shortage / water saving concerns questions like: how to measure water use during showering? What difference can you make in a year when decreasing shower time?

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One reference one row. Insert rows if needed. APA style
In this section please put literature used for production of this IO

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