In the sun and the wind in your back

- The realisation of Ambergreen -

Preliminary round assignment for the 28th Mathematics Alympiad - November 18 2016

Colophon

The Mathematics Alympiad (Wiskunde Alympiade) is an initiative of the Freudenthal Institute, Utrecht University. The Alympiad committee is responsible for the organisation of the Alympiad and for producing the assignment.

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CASIO, Amstelveen
Texas Instruments, Utrecht

Guide for the preliminary round assignment for the 2016/2017 Mathematics Alympiad

This Mathematics Alympiad assignment consists of five initial assignments and a final assignment in two parts. The five initial assignments are a run-up to the final assignment: all knowledge and insights gained in these assignments are necessary to successfully complete the final assignment.

General advice for working on this assignment:

- First read the full text of the assignment so you will know what you have to do.
- Keep an eye on the time you spend on the initial assignments; take plenty of time for the final assignment, at least three hours.
  If you divide up tasks within your team, discuss the results with each other after every assignment.
- If you adapt certain approaches, methods or procedures while working on the assignments, describe your adaptations in your report and include why you made them.
- It may be a good idea to use Excel or another spreadsheet program for this assignment.

Handing in:

- The final assignment (take care, this consists of two parts, a folder and a report)
- The initial assignments 1 to 5 as attachment(s)

The jury will receive a digital copy of your work. If you have any appendices with your work, hand in everything in a zipped folder. Include the name of your school *and* your own names in the file name.

Judging:

These are some of the points that will be considered:

- Legibility and clarity of the final assignment;
- How complete the work is;
- The use of maths;
- The argumentation used and justifying choices that have been made;
- The depth to which the various assignments have been answered;
- Presentation: form, coherence, legibility, illustrations and use of appendices;
- (Mathematical) creativity in your elaboration of the assignments.

Have fun and good luck!

In the sun and the wind in your back - The realisation of Ambergreen

Amberhavn has been growing a lot in recent years, and therefore a new residential area is being built. It will be a modern and sustainable area of which the name is already known: Ambergreen. The location is also known already: in the south, on the eastern edge of the city (See the map on the first page). Amberhavn wants to realise renewable energy production and is researching whether it is possible to have the area be energy neutral. The city has decided to perform an experiment: the new inhabitants of the 175 houses to be built are communally responsible for producing renewable energy. The inhabitants will decide together whether and how solar and wind energy will be used. The costs are divided fairly over the inhabitants. The inhabitants do not want to research everything themselves and together with the Amberhavn city council they arrange independent expert advice..

Initial assignments

In the initial assignments you will research the average cost for a household for different energy sources, and how the yield for each source depends on the weather. The data you will use for your calculations are often simplified, but are based on reality.
An important unit you will be using is kWh, which stands for kilowatt-hour. Kilowatt-hour is a unit for energy. This unit is mostly used for electrical energy. If you operate an appliance with a capacity of 1kW (that is 1 kiloWatt = 1000 Watt) for one hour, you will have used 1kWh of energy

Initial assignment 1: Energy costs

Determine, using the data below, the difference in cost per household between energy use that is completely based on fossil energy (gas and coal-fired power plants) and energy use where electricity is drawn from solar panels. Give as clear an overview of your calculations as possible.

Data:

- The average energy use per household per year is 3500 kWh of electricity and 1500 m^3 of gas.
- The cost of electricity from a coal-fired power plant (fossil energy) for a household is on average 17 cents per kWh (including fixed charges, taxes and tax credits).
- Gas costs 28 cents per m^3, with another 26 cents of energy tax per m^3 gas added on. In addition, fixed charges come to an average of over €195 per year (including VAT).
- You do not pay to the electricity net for solar energy – but you do of course pay for the purchase. Solar panels cost €300 per m^2, and 1 m^2 of solar panels provides on average 150 kWh per year. Solar panels last 20 years.

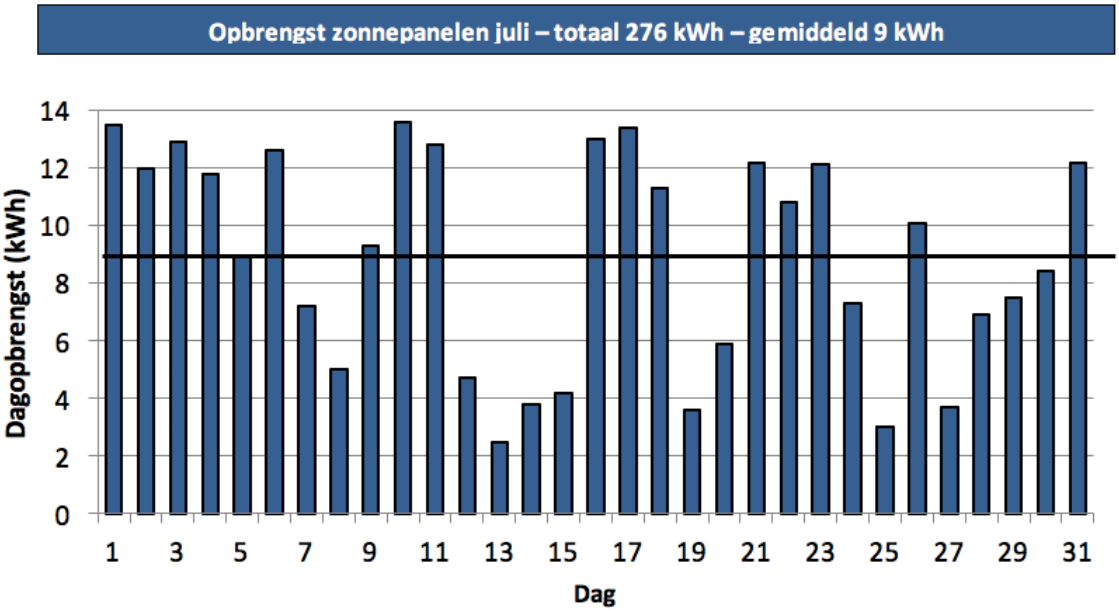
Initial assignment 2: Heating with gas or electricity

If you want to heat your house and your water (washing up, showering, etc.) using electricity instead of gas you will use 18.500 kWh extra electricity.
Calculate the amount you would spend per year on heating and warm water if you do this in one of the following three ways:

1 gas
2 electricity from fossil fuel sources
3 electricity from solar panels

Initial assignment 3: Solar panels

Obviously the real situation is a lot more complicated than the one in the previous assignment. This is mostly caused by the variation in solar power and energy use.



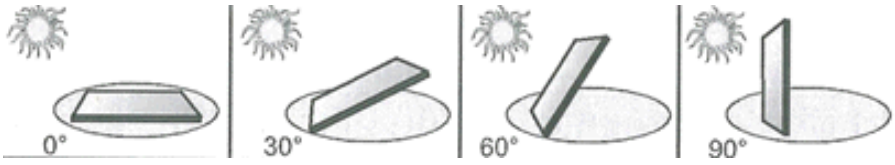
In the chart above, the household with solar panels uses on average 8 kWh per day in electricity. Since they generated an average of 8,9 kWh per day (see the horizontal line in the chart) that should be enough. Yet there were many days this month that this household did not have enough electricity.

Calculate the total electricity shortfall for this household in this month..

Initial assignment 4: Installation of solar panels

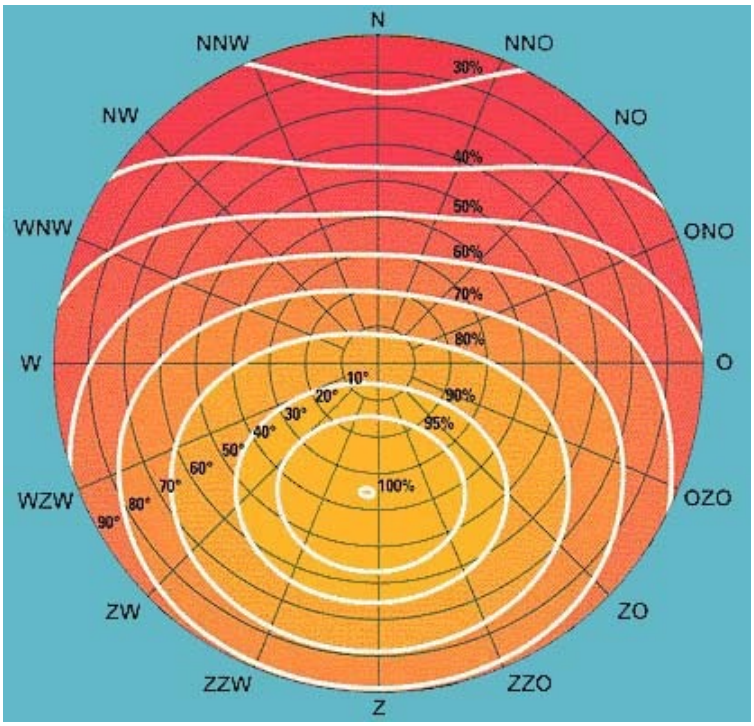
In addition to variation in solar power the yield of solar panels also depends on the placement of the panels. Two points play a role here:

1 The **angle of placement** is the angle the solar panel makes with the ground. See the figure below.

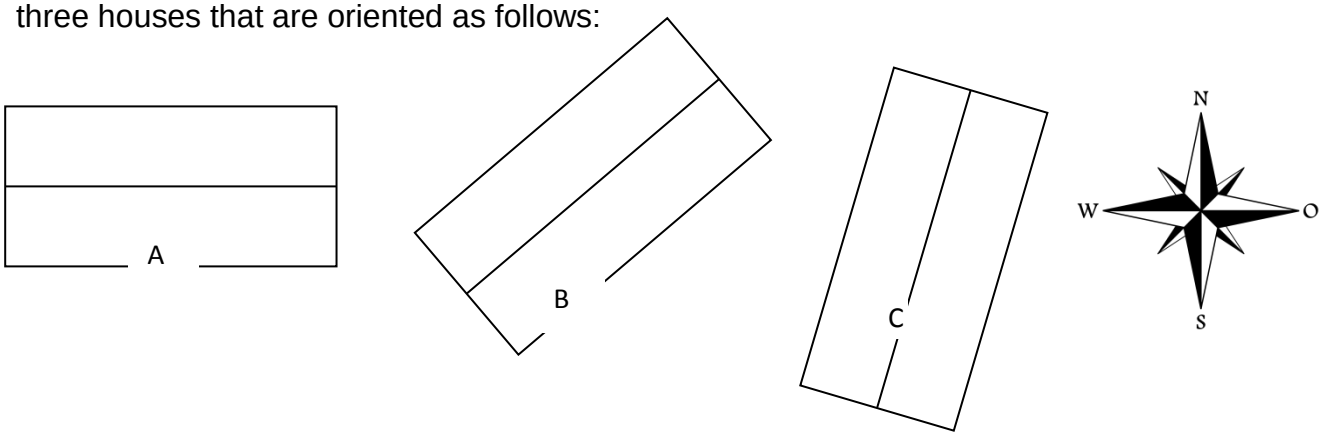


2 The **(wind)orientation** of the house: A household that can orient its solar panels on the south has more of a profit from them than a household where the panels can only be installed due east or west.

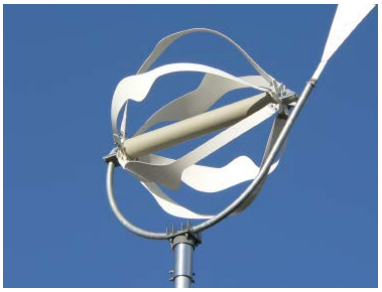
The diagram below (**Please note: appendix C has a larger version of the diagram**) provides an overview of the various yields of solar panels, depending on the (wind)orientation and the angle of placement of the solar panel.



Determine the maximum yield and the ideal angle of placement of solar panels for the three houses that are oriented as follows:

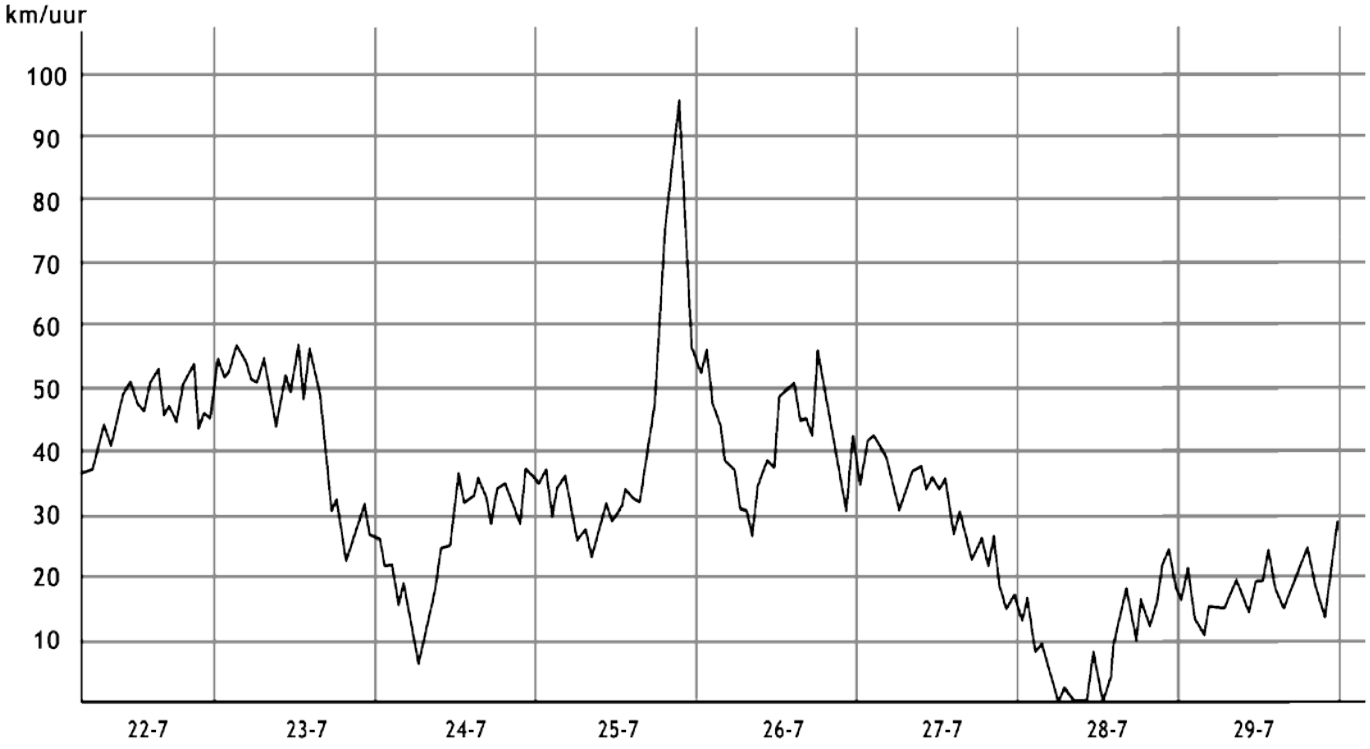


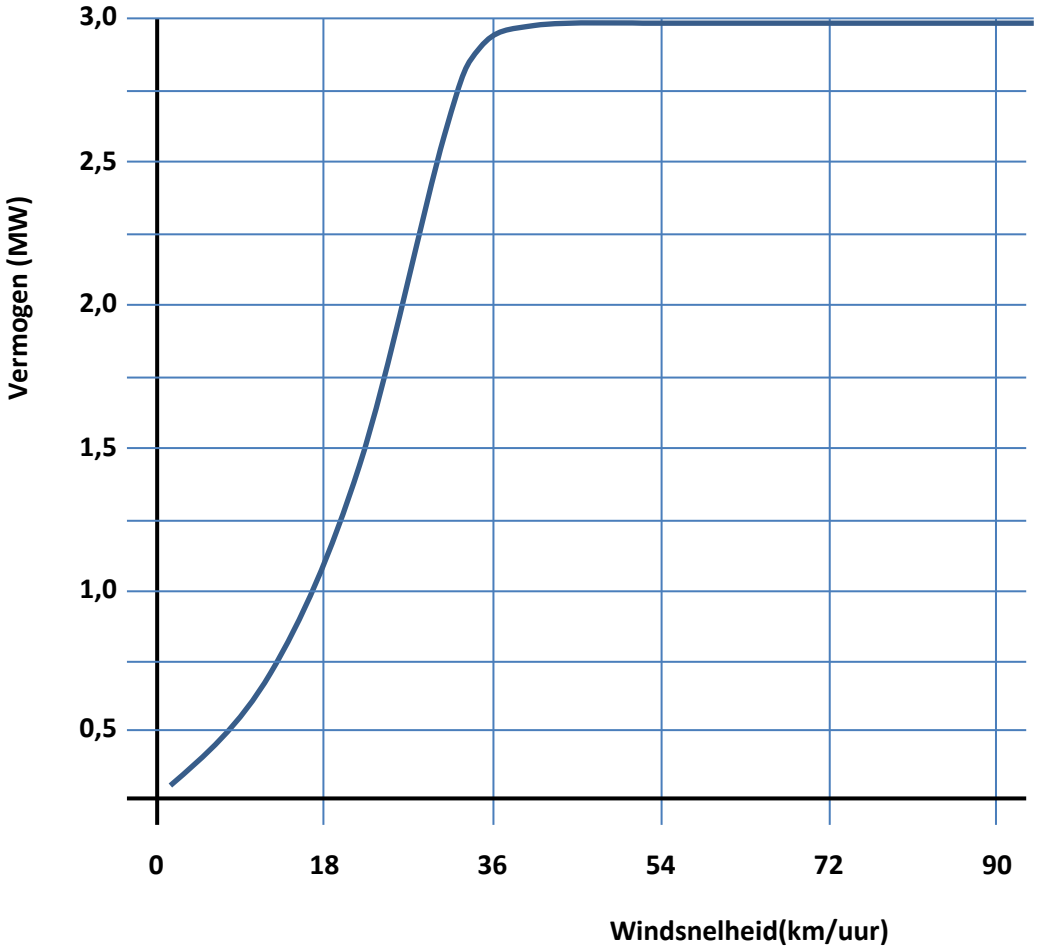
Initial assignment 5: Wind



Up to now we only looked at solar power. Another renewable source of energy is wind. A wind mill (aka a wind turbine) converts wind energy into electricity. How much electricity a wind mill provides depends among other things on wind speed, also called wind strength. Of course the size of a wind mill is also a factor. There are very large wind mills with a capacity of 3 MW (that is 3 Megawatt = 3000 kW), but there are also small wind mills that you can for example install on the roof of a house, with a capacity of 0,5 kW (like in the photo).

There is a similar process for wind mills as there is for solar panels. Wind speed varies both daily and for the time of day. Where the production of solar energy has a strong seasonal aspect (and no production at night), wind strength is not as obviously periodical. In the chart below you can see the development of wind strength in a specific place during a particular week in Amberhavn. In the chart after that you can see the capacity that is produced at a specific wind speed.



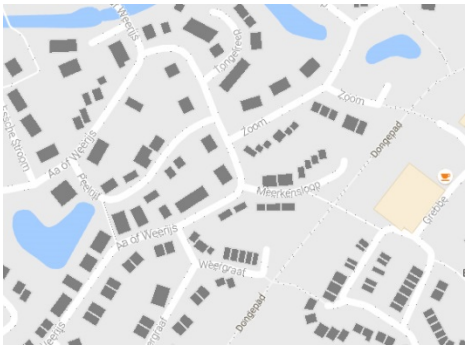


As mentioned before, Kilowatt hour (kWh) is a unit for energy.
Example:
If you let a machine with a 2 MW (that is 2000 kiloWatt) capacity run for ten hours, you have used (2000 times 10 is) 20.000 kWh of energy.

Determine as well as you can how much energy this wind mill produces in the week from 23 up to and including July 29.

Final assignment

Ambergreen wants to switch entirely to green energy for its electricity production. It's not yet possible in 2016, the capacity per m^2 of solar panels is still too small. But the expectation is that that capacity will double in a few years, and then it will be possible. A household can opt for a large solar panel, a small solar panel and a wind mill or a windmill. There are three types of windmill to choose from. It is of course best if solar panels can be aimed at the south. However, not all houses are oriented on the south, as you can see from the planned street view of Ambergreen (**Please note: there is a larger version in appendix D**).



Ambergreen aims to have all inhabitants produce their electricity through green energy, throughout the year, so every month. Unfortunately it is not possible at the moment to return surplus energy to the power grid and battery storage is not yet effective.
Electricity use may vary per household, but an average household uses around 3600 kWh of electricity per year. Gas is still used for heating purposes, and we may assume that there is no monthly variation in the need for electricity.
You can find all data about solar panels and wind mills in appendices A and B.

Assignments

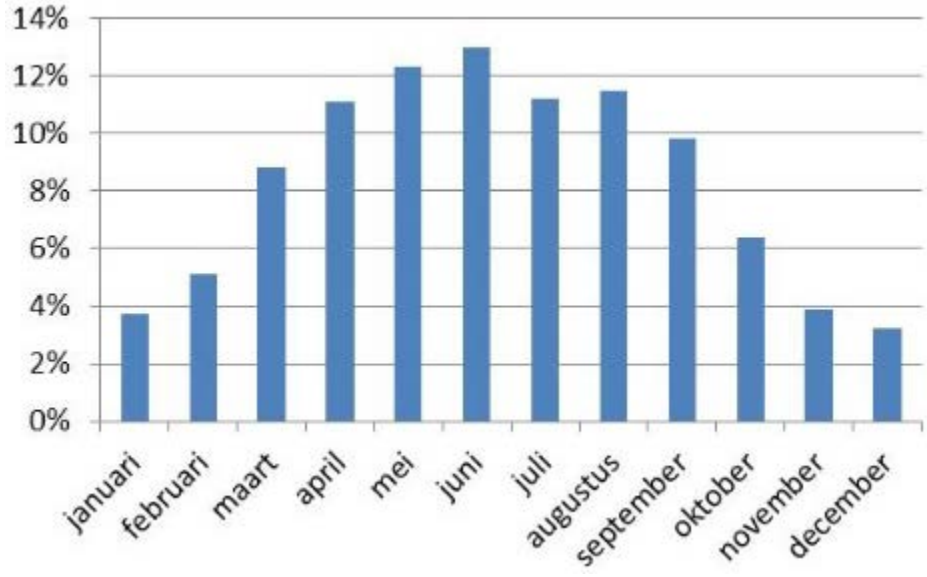
Every house in Ambergreen will need to receive individual advice about the purchase of solar panels or a wind mill, depending on the position of the house. And if possible with alternatives, in case there are people who definitely do not want a wind mill. Or solar panels.

1. The borough council of Ambergreen is giving you the task of designing an A4 flyer, with which every inhabitant of Ambergreen can determine for his own situation what the optimal choice is between solar panels and/or which windmill.
2. The borough council doesn't want to give preferential treatment to either the companies that produce solar panels or the ones that produce wind mills, but wants people to really make the choice that is best for them. That is why you must substantiate your advice in a report for the borough council.

Appendix A

Solar panels

- Price large set of solar panels (30m2): € 9000,-
- Price small set of solar panels (20m2): € 6000,-
- Solar panels have a 20 year lifespan.
- Solar panels must be installed with a $40°$ angle.
- The yield for the orientation of the house can be found in the diagram in Appendix C.
- The maximum annual yield (in a north-south direction): 330 kWh per year per m^2. Those 330 kWh are divided over the months as follows:



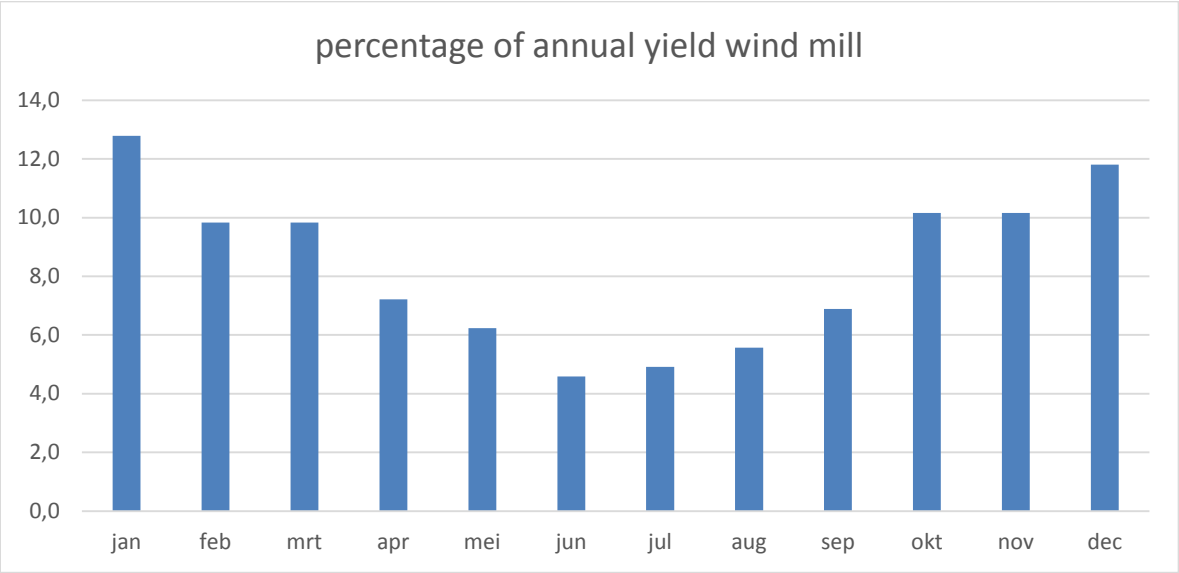
Appendix B

Wind mills

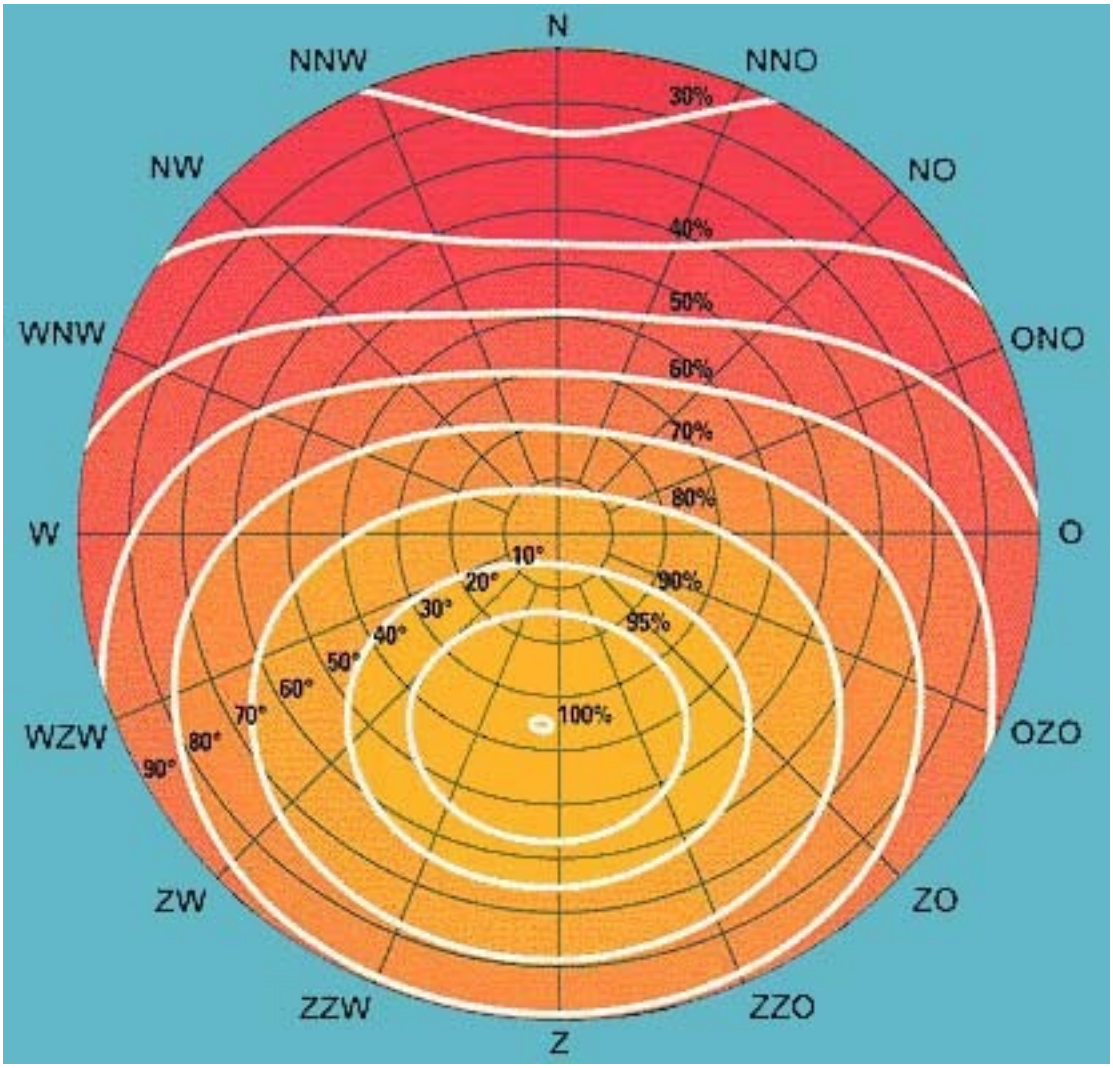
There are three wind mills to choose from:

	price	Max energy per year
Matthew	**€ 14000**	**12000 kWh**
Kathrina	**€ 10000**	**8000 kWh**
Sandy	**€ 3500**	**1100 kWh**

- Wind mills have a 20 year lifespan.
- The amount of wind is not consistent throughout the year. The annual yield has been divided according to the diagram below:



Appendix C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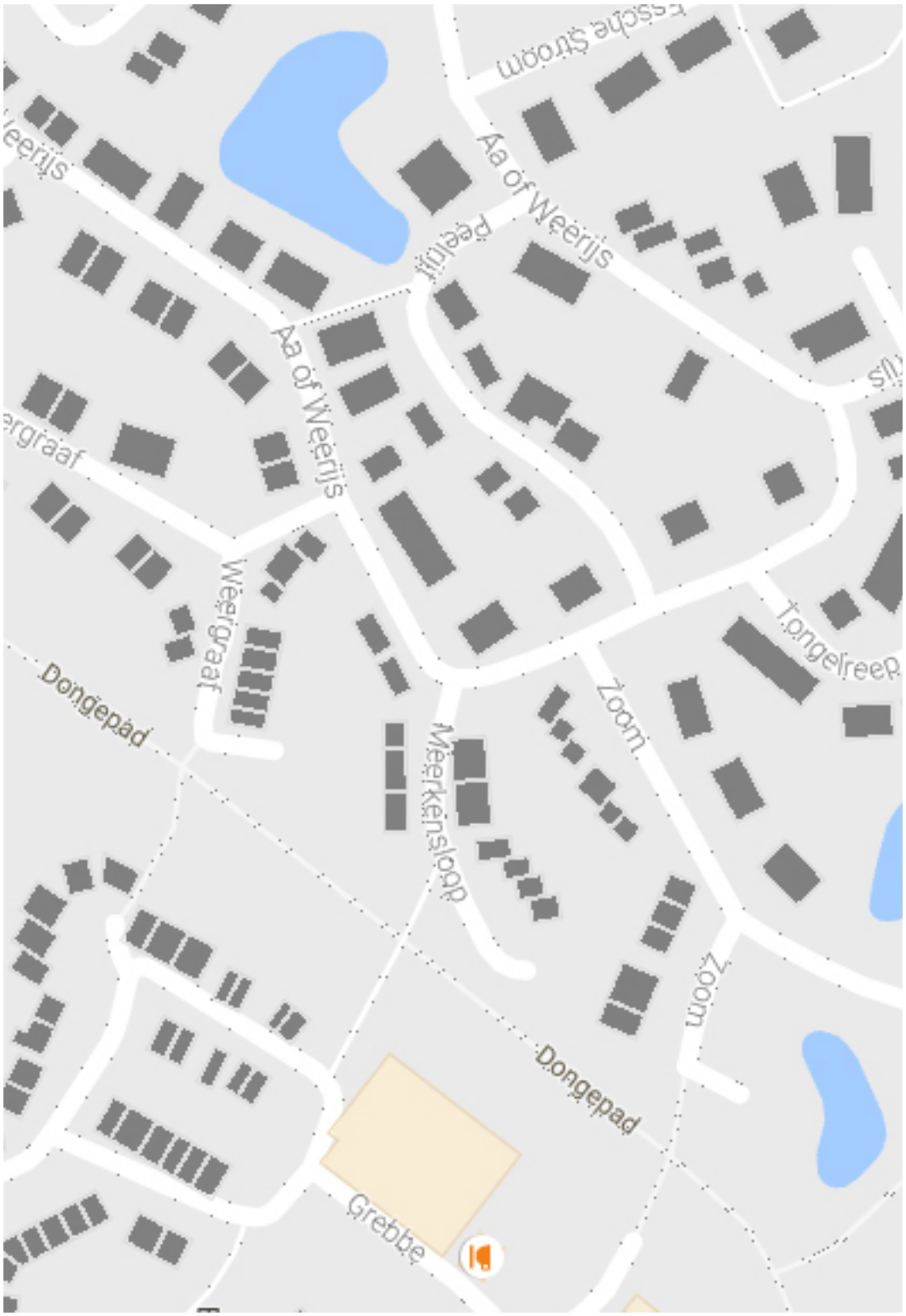
Appendix D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