Let's play Darts

Final assignment for the

Alympiade 2006

Garderen,
March 17 and 18, 2006

GUIDELINES FINAL MATH A-LYMPIADE 2006

BEFORE YOU START:

❑ First read the whole text of the assignment so that you know what you have to do this weekend!
❑ The parts A, B and C do not have to be done in this order. Part B requires each team member to throw 100 darts. Do not forget to put your team on the entry list.
❑ Divide the tasks where possible and confer when needed.
❑ It is important that, for the final assignment, your team fully describes your children's board and provides supporting arguments for your choices.

PLANNING:

- On Friday afternoon (after lunch until 'happy hour') there will be opportunity to throw darts in the central area (assignment B). Do not forget to sign up for this assignment.
- Keep an eye on the time while working on the various assignments.
- Make sure to have enough time to prepare your poster presentation on Saturday!
- Your report must be handed in on Saturday afternoon, before two o'clock.

HAND IN
The full answers to all parts. Be sure to include all information on the poster in your report as well!

The jury will be given copies of your work. To make sure these copies are legible, write using a black pen and only print on A4 paper. Any drawings included in your work must be clear enough for copying, so please do not use a pencil.

JUDGING
The jury will look, among other things, at:
- Whether all parts of the assignment were answered in full.
- Use of mathematics
- The reasoning used and justification of choices
- In-depth solutions to the different assignments
- Presentation: form, legibility, illustrations etc.
- Originality and creativity.
The final assignment (part D) will count heaviest towards the overall result; this includes the connection with the results from assignments A, B and C.

Let's play Darts!

JELLE KLAASSEN NEW DARTS KING

For a year, Jelle Klaassen may call himself darts world champion (BDO – British Darts

Organisation). In an exciting finale the 21 year old defeated favourite and title defender Raymond van Barneveld 7-5.

During the whole tournament Klaassen displayed no nerves, and even in the final he appeared on top of the situation. The man from Alphen hit his stride straightaway and won the first two sets.

'Barney', who knew his idol, five times winner Eric Bristow was watching, did not get his cool back until the third set. The title defender did not want to give up without a fight and won the next seven legs.

The sets that followed developed into a nerve-wracking match, in which every leg was disputed to the last dart. The match went evenly as far as 5-5, but Klaassen managed to win the eleventh set, which had been started by Van Barneveld, and suddenly found himself in an excellent position.

With a 2-1 leg lead in hand, the 'Young Matador' struck at the first opportunity. He took the title with a 100+ finish to become the youngest winner ever.

"I hadn't expected this", the winner said afterwards. "Barney missed some double, and I profited from that. He is still number one for me."

Source: http://www.nos.nl/nosstudiosport

A sensation at the Lakeside darts tournament: Jelle Klaassen, a 21 year old debutant, beats the best of the world! During this tournament, which is seen as the world championship for professional darts players, a large part of the world (and certainly the Netherlands) was enthralled by darts.
High time for a mathematical reflection on the game.

The Assignment

The topic of the 2005/2006 math Alympiade final is darts. The assignment consists of four parts:

Part A: the game and the rules
> In which you look at how a game may go using the existing rules

Part B: the throw
> In which you look at whether the pattern of throws can be a measure for the quality of the player, and in which you determine your own level of playing

Part C: the numbers on the board
> in which you find out who invented the distribution of the numbers on the board and for what reasons

Part D: final assignment
> in which you design your own "children's board"

Part A: the game and the rules

There are several variants of darts. The one that is used most in competition is called 501. The rules of variant 501 are largely as follows:

- Two players play each other.
- Both players start with the same number of points: 501.
- The players take turns to throw three darts, with the intention of getting as high a score as possible; after each dart is thrown, the points scored are subtracted from the total (501 at the start).
- After three darts, the other player gets to throw.
- The first to reach zero points is the winner.

How to score

Below you see the standard dart board, which is also used in competitions:
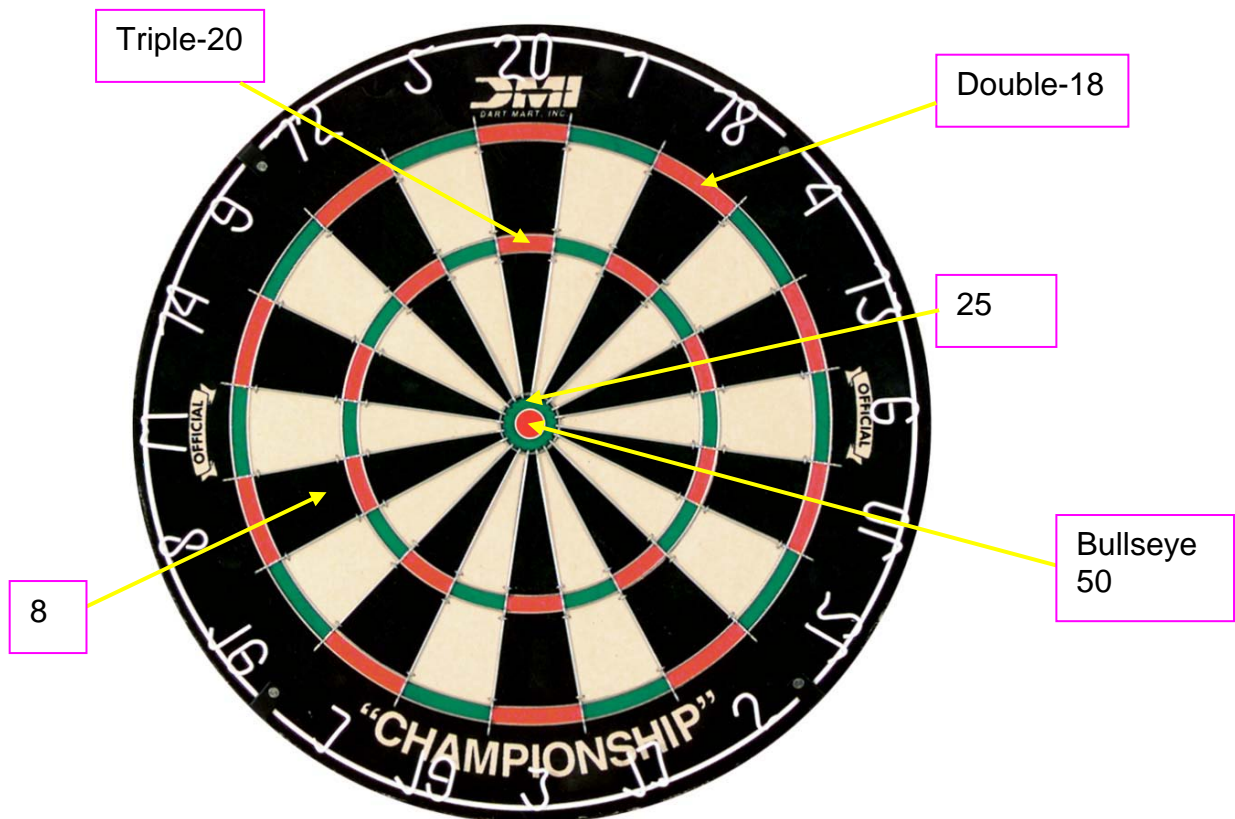
- If you hit the white or black compartments in a number's "slice of pie" on the board, your score is that number, for instance 8.
- A dart in the outer green-red ring ('double' ring), counts double. For instance: a dart in the outer ring for 18 counts as 2 x 18 = 36 points.
- If you hit the inner green-red ring ('triple' ring), the number is multiplied by 3. For instance: a dart in the inner ring for 20 counts as 3 x 20 = 60 points.

- The centre of the board, the small red circle, is called 'bullseye', and is worth 50 points.
- The narrow green ring just outside the bullseye is called the 25-ring. It is worth 25 points.

Scoring and the end of the game

The number of points for each dart is subtracted from 501 straightaway, resulting in a new point standing.

The game can only end by hitting a 'double' or the bullseye. So, a player who has a score of 40, can end the game by throwing a double 20. If he misses the double and hits a single 20 with his first throw, he can still win by throwing a double 10 with his second dart. Should he miss this one as well, his last chance to win in this turn is throwing double 5 with his third dart.

If a player throws higher than the required double, his points standing remains the same and the other player gets his turn. So if a player is at 10 points and throws a double 12, his turn is over and his score remains at 10.

The same happens if a player would end up with 1 point: for instance, if one has a score of 40 and throws triple 13, the score remains at 40 and the other player takes his turn.

This means it is important, especially for a player with some experience, to know the different possibilities to end a game quickly with a double or a bullseye.

Check-out

He who first manages to collect 501 points by the rules above, and ends up at exactly zero, is the winner. From a certain score downwards, it becomes possible to end the game in one turn. Remember: you have three throws per turn. This situation is called a 'check-out'.

task 1

The quickest game possible requires 9 darts for the winner. Give all possible combinations of throws for the existing rules with 9 darts and 501 points.

task 2

Jelle Klaassen finished his final match with a 100+ check-out (that is a check-out of more than 100 points)

List all possible 100+ check-outs, and ways to throw them. Use your answer for assignment 1 here.

task 3

There are lists of all possible check-outs. It sometimes seems as if experienced darters know them by heart. But even an experienced darter may hit the wrong space…

Below you see a part of a check-out-list. The list gives a two-dart check-out so that you can switch to another option if the first throw goes wrong.

107	Triple 19	Bullseye		90	Triple 20	Double 15
104	Triple 18	Bullseye		89	Triple 19	Double 16
101	Triple 17	Bullseye		88	Triple 20	Double 14
100	Triple 20	Double 20		87	Triple 17	Double 18
98	Triple 20	Double 19		86	Triple 18	Double 16
97	Triple 19	Double 20		85	Triple 15	Double 20
96	Triple 20	Double 18		84	Triple 20	Double 12
95	Triple 19	Double 19		83	Triple 17	Double 16
94	Triple 18	Double 20		82	Bullseye	Double 16
93	Triple 19	Double 18		81	Triple 15	Double 18
92	Triple 20	Double 16		80	Triple 20	Double 10
91	Triple 17	Double 20				

This list is said to contain the optimal choices to continue in case your first dart misses its intended target.
Examine the criteria used to determine these 'optimal choices'. Make your own list, based on the criteria you found, for the check-outs from 79 to 60.

Part B: the throw

Of course a good darter usually hits what he wants to. Most likely, the dart game and the board have been designed assuming that wrong throws are distributed in a 'normal' way around the intended target.
You may expect experienced, good players to throw closer to their intended target more often. But what about the distribution of the throws? Perhaps every darter has a deviation to above or below? Left-handed players to the left, right-handed players to the right, or the other way around?

Good aim

Two people have been aiming for the same point (for example the bullseye). In the illustration you can see where the darts ended up.
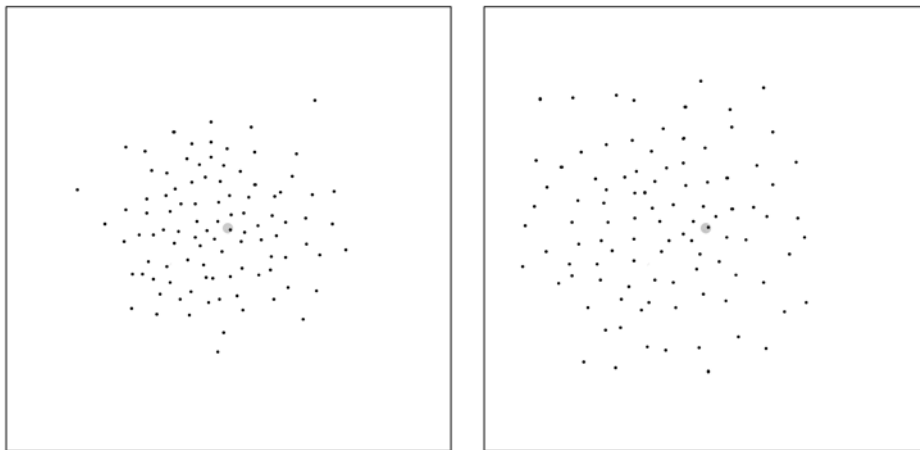
There are many questions you can ask: who is the better player? And are there deviations in the players' throwing patterns?

In the following assignment you will look for a method to determine, on the basis of this kind of figure:

- 'target orientation': how 'good' a player is at hitting his target
and
- the 'deviation': does the player have a deviation, what is it and how large is it?

task 4

Design a way to derive 'target orientation' and 'deviation' from such pattern of dots. Represent both in a number, and include a scale to interpret the numbers.
On the worksheets there are four patterns you can use (extra copies of the sheets are available).
Describe your method in such a way that everyone can use it to determine their own 'target orientation' and 'deviation'.

How good are you?

How good is your score, according to your own method?
Every team member determines their own 'target orientation' and 'deviation' –
darts measurements – based on experimental data (your own throws).

task 5

Go to the central area and throw 100 times, using the target on the sheet of
paper you are given.
Mark the holes with a thick felt tip pen and make digital photos for your report
(if needed, the organisers can make these photos for you).

Adapt your method if needed. If you do so, clearly explain how and why you
made these changes!

Part C: the numbers on the board

Why is the dart board the way it is? Who invented the order of the numbers? The following article gives you some background information.

WHY ARE THE NUMBERS ON A DARTBOARD IN THE ORDER THEY ARE?

This is probably the most asked question about the origins of the modern game. Who was the devious person who structured the segments of the dartboard in such a frustrating manner?

The man who is credited with the 'invention' of the numbering sequence of the modern standard dartboard is BRIAN GAMLIN. Gamlin was a carpenter from Bury in the County of Lancashire, England and came up with the infuriating sequence in 1896, at the age of 44. He died in 1903 before he could patent the idea.

In those days many working men – and in particular those with carpentry skills – manufactured dartboards out of elm or poplar wood as a sideline. This cottage industry was later prevalent across the North of England, the Midlands and the South East as darts grew in popularity from the mid-1920s onwards. The reason for producing dartboards at home, or more properly in the garden shed, was to sell the boards to local pubs, thereby supplementing the family income. However, more often than not, this income never found its way home at all. Dartboards were exchanged for credit in the local pub or money earned would finds its way back over the bar.

The numbering of a standard dartboard is designed in such a way as to cut down the incidence of 'lucky shots' and reduce the element of chance. The numbers are placed in such a way as to encourage accuracy. That's it. Pure and simple. The placing of small numbers either side of large numbers e.g. 1 and 5 either side of 20, 3 and 2 either side of 17, 4 and 1 either side of 18, punishes inaccuracy. Thus, if you shoot for the 20 segment, the penalty for lack of accuracy or concentration is to land in either a 1 or a 5.

There are 2,432,902,008,176,640,000 different possible arrangements of the 20 segments on a standard dartboard so it is perhaps a little surprising that Gamlin's arrangement of the numbers is almost perfect.

Gamlin himself is an enigma. Like the lost court records in the case of William 'Bigfoot' Annakin, there is a vital piece of information in the Gamlin story that is missing. Despite the most thorough of searches no record can be found of Gamlin's death in 1903. Looking three years either side, for both counties of Lancashire and Suffolk reveal no one of that name terminating at that time. However, the answer may be that this is because Gamlin was on the move.

The Daily Mirror in 1992 was asked the question "Who decided the numbers on a dartboard should be so jumbled and why?" The reply read:

"Brian Gamlin of Bury, Lancs, introduced the odd numbering system in our fairgrounds in 1896, boasting "No Skill Required". Drunks had no chance, as a test of sobriety, the darts game 'round the clock' (in which players have to score with darts in numerical order) became a great success".

So this is why his death cannot be traced. If Gamlin was a showman then, sure, he would be on the road for at least six months of the year. It makes a lot of sense for the idea to have come from within the fairground community. They were the primary cause of the importation of so many 'French darts' which have, over the years become known as 'fairground darts'. Darts has been a feature of fairground sidestuff from the mid-19th century onwards, so who better than a showman – always looking for new ways of attracting punters – to come up with this devious numbering arrangement?

Note: For those new to the game of darts the left-hand side of the dartboard is recommended as there are proportionately more high numbers grouped there, i.e. 16, 8, 11, 14, 9 and 12. No huge scores can be guaranteed with this tactic, but at least you'll never hit 5's or 1's. (At least that's the theory!) This side of the dartboard is known as the 'married man's side' because married men always play safe!

A fascinating sentence in this article is:

There are 2,432,902,008,176,640,000 different possible arrangements of the 20 segments on a standard dartboard so it is perhaps a little surprising that Gamlin's arrangement of the numbers is **almost** perfect.

task 6

Examine, using a calculation, whether that figure of 2,432,902,008,176,640,000 is correct.

Bordering differences

Gamlin's intention was to put wrong throws at as much of a disadvantage as possible, which is why high values (such as 20) are always next to low ones (in this case 5 and 1). The 'bordering differences' are always as big as possible. The perfect board, according to this criterion, would have maximum sums for all bordering differences. The Gamlin board is a 198 board (i.e. a board where the sum of all bordering differences is 198), which makes it an *almost* perfect board. The maximum is *not* 198.

task 7

Find out what the maximum is for the sum of the bordering differences. How many really different perfect boards can you make? Provide clear arguments for your result.

task 8

Not really nice for Gamlin to be so hard on wrong throws! Also design a board with the minimum sum for all bordering differences and explain how you found it.

PART D: final assignment

As said before, Gamlin used bordering differences as the criterion for designing his board. Of course there are other criteria you can use for designing an optimal board.
In this final assignment you will design a special board for children: it contains only the numbers from 1 to 10.

Use all the results from what you have researched and discovered in the earlier tasks.
First formulate your criteria, before you start on designing the accompanying board. The result does not have to look anything like a Gamlin board.

Of course there are rules for your board. Formulate them and make a thorough analysis of possible games on your board, including lists of check-outs. How about chances for better and worse throwers?

You will deliver a children's board, with a motivation for your design; rules and possible games, and everything else you think is relevant for players and judges.
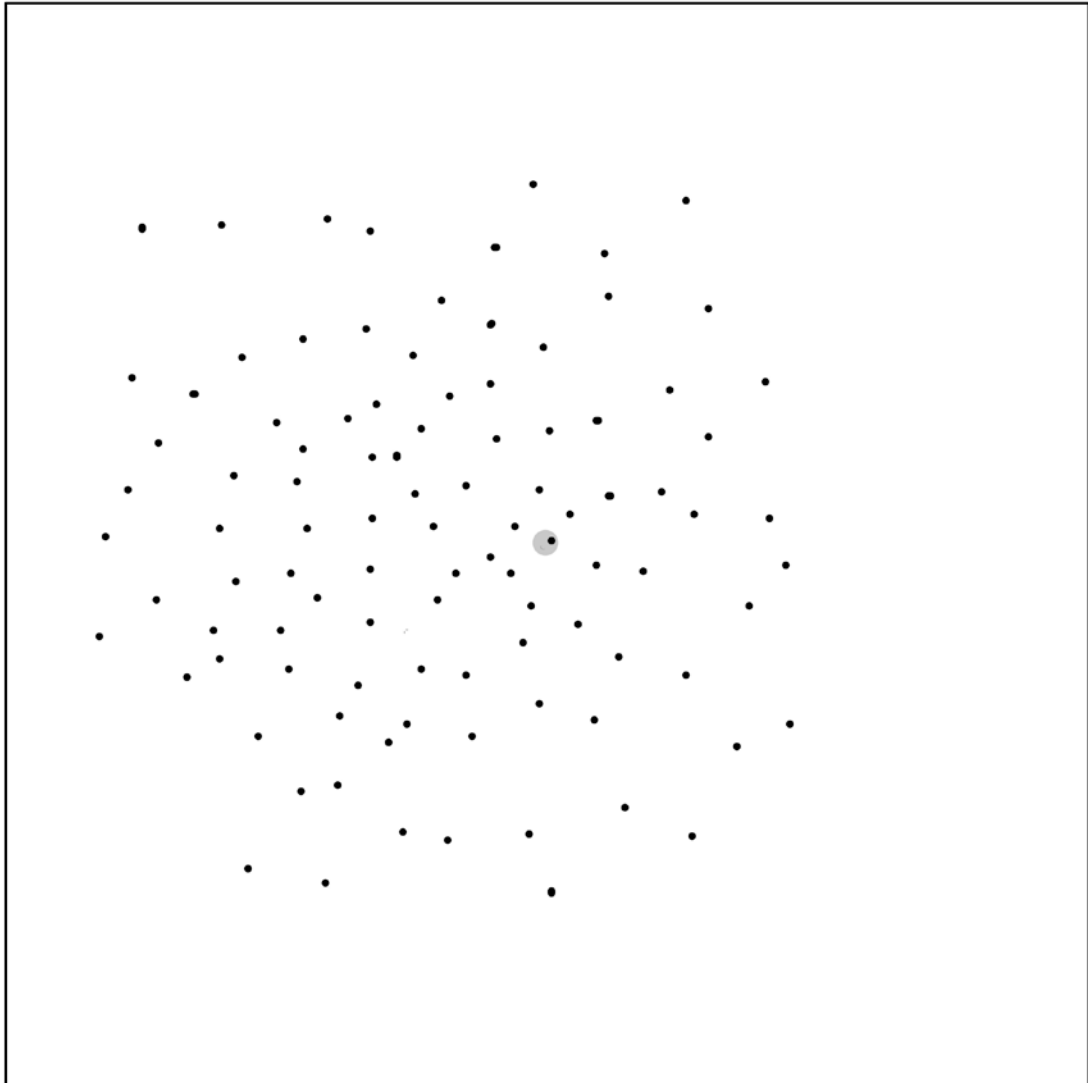
presentation

The results are presented on Saturday afternoon.
For this presentation, you need to make a poster which depicts the children's board you designed, the rules, and the most important aspects of the design.
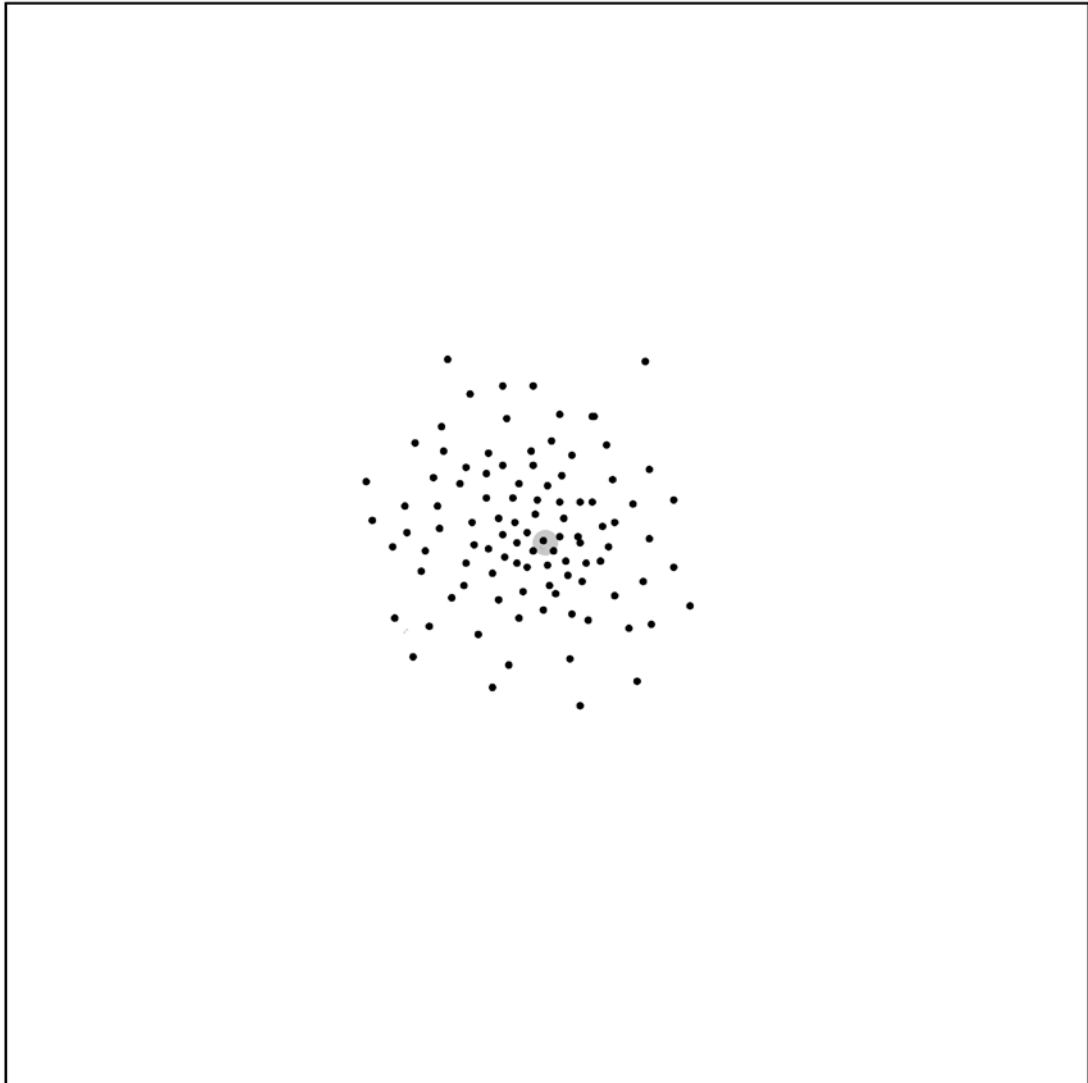The text on the poster has to be in English.

Worksheet 1
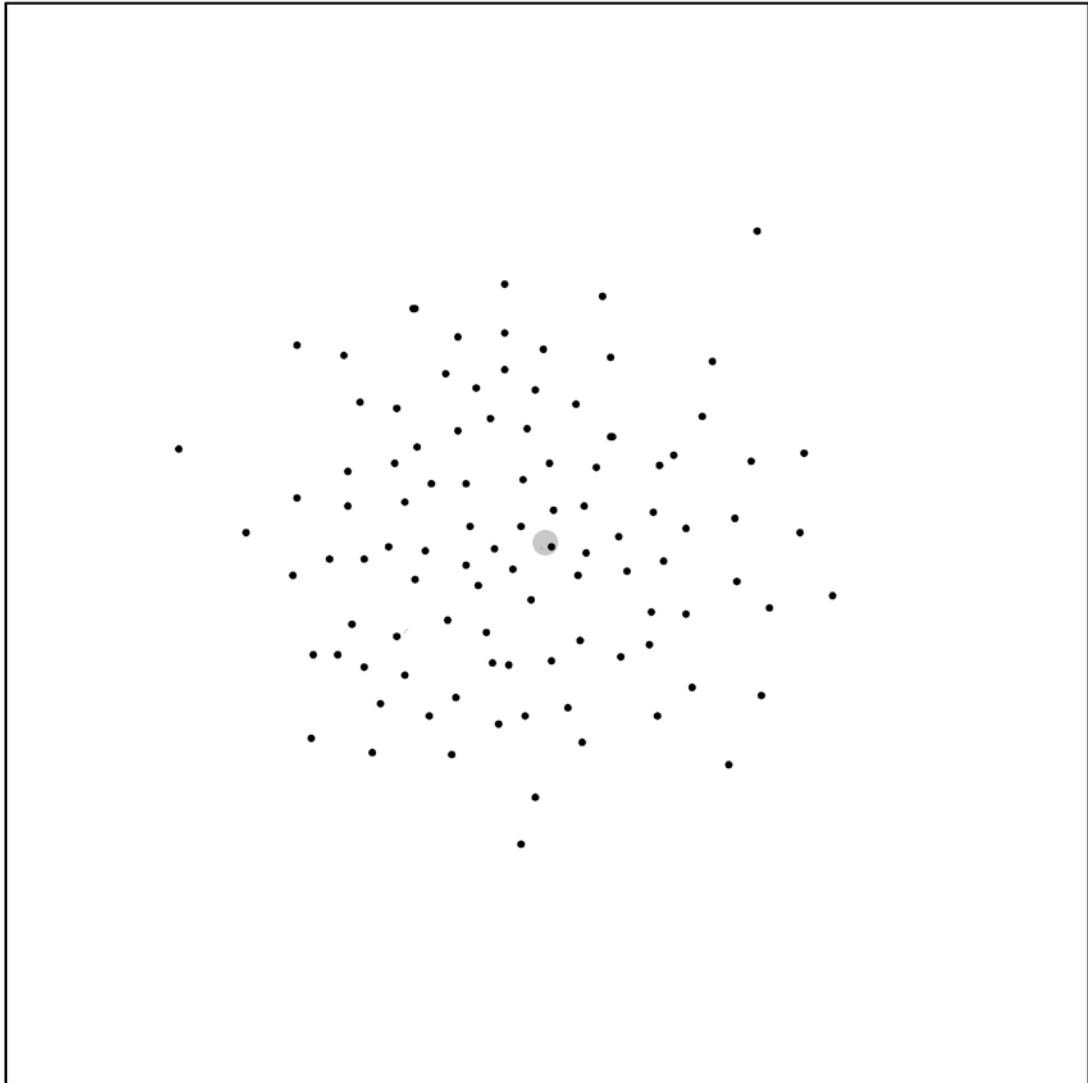
Worksheet 2

Worksheet 3

Worksheet 4