THE ROYAL SOCIETY

A new approach to mathematics and data education

A discussion paper from the Mathematical Futures Board of The Royal Society's Advisory Committee on Mathematics Education (ACME).

KEY MESSAGES

1. Mathematical and data sciences are everywhere, and their influence is growing. Our education system must adapt.
2. The nature of the mathematical education that is needed is changing from 'mathematics' to a fusion of mathematics, statistics, data science and computer science; what we have called mathematics and data education (MDE).
3. England has a system that serves some students well but fails too many. MDE will improve outcomes for everybody.
4. MDE is for everybody up to age 18.
5. MDE is not just a matter for mathematics teachers and departments.
6. MDE will require change across the whole system including:
   a. a new National Curriculum for mathematics;
   b. particular reform of pathways and qualifications from the age of 14;
   c. assessments that provide accurate information about what pupils know and can do; and
   d. better use of digital technologies.
7. The changes will take 10 – 15 years to implement fully and will need investment and careful planning. The involvement and support of teachers will be critical to success.

The Mathematical Futures Programme

The Mathematical Futures Programme was established by RS ACME in 2020. With a 20-year time horizon, it set out to address two questions:

- What mathematical competencies will be needed for citizens and society to thrive in the future?
- How should education systems develop these mathematical competencies?

We have engaged with a wide range of stakeholders from industry, policy, education, mathematics and related disciplines and we have commissioned a number of research reports, which are referred to in this paper and can be viewed online at: **royalsociety.org/mathematical-futures**

About this paper

This paper takes a high-level and long-term view. It is not the final report of the programme and, by design, it does not include detailed recommendations for the future. It does, however, describe a vision which we hope readers will find compelling. We hope they will find the supporting arguments convincing and that they will wish to contribute to its further development.

The publication of this discussion paper is the start of a process of consultation with the many communities that have a stake in the future of mathematics and data education. There will be a number of engagement events and meetings during Autumn 2023.

The purpose of the consultation is to air the underlying principles and possibilities, to gauge the appetite for change, and to explore how we could move from strategic vision to implementation. We invite readers to respond to any or all of the following questions via the online form at: **royalsociety.org/mathematical-futures**

1. Do you support our vision for the future of mathematics and data education?
2. Is this vision appropriate for all students?
3. Are there areas of our vision that need further development?
4. What are the first steps needed to begin the process of change?

Our report, informed by this consultation, will be published in 2024.

Opening statement

Mathematical and data sciences are everywhere. They increasingly support thinking and decisions in government, in industry, finance and business, and in academic disciplines. They influence the day-to-day lives of individuals as employees, citizens and consumers of information. The massive increase in the use and availability of data through digital technologies means that this influence can only grow.

Our education system must adapt to this rapid change and in this paper we set out the case for a new approach to mathematics education. We believe this new approach, which we are calling mathematics and data education, can help to equip future citizens with the capabilities, skills, adaptability and resilience they need to lead fulfilled lives in a fast-changing, data-rich world where mathematics and data play increasingly important roles in everyone's lives.

Our aim is to provide a better mathematics education for everyone, from the everyday needs of citizens to the brilliant mathematicians of the future. We believe our new approach will result in many more people wanting to continue with the study of mathematics and data science and will lead to a more mathematically skilled labour force. The improvement in the skills of future generations would make a transformative contribution to the UK's economy and to our preparedness for the future. By contrast, continuing with our present arrangements condemns the UK to life in the slow lane.

While the prize is great, so are the challenges. The reforms we seek cannot be developed by limited short-term measures; they would take 10 – 15 years to implement fully. They would need investment, careful planning, design, implementation, and evaluation. They would require collaboration between the stakeholders involved, cross-party support and determination to stay the course. We are encouraged by the recent publication of a 15-year strategy for the NHS, which we think exemplifies the kind of long-term thinking that is needed.

The reforms we are suggesting are radical, but we believe that the rewards will repay the effort many times over, and that the country cannot afford not to embrace them.
The pace of change is fast and is accelerating.

Now is the time to act.

Sir Martin Taylor FRS
Chair, Mathematical Futures Programme

PART 1

Why a new approach to mathematics education is needed

The changing role of data

In the 21st century the reach and use of mathematics has expanded dramatically. This is due in part to the massive increase in the availability and power of computing and statistical analyses. Powerful analyses that were once the preserve of rare supercomputers are now routinely available to all on low-cost laptops. Data already plays a central role in most areas of employment – from healthcare to financial services and manufacturing – and is now a fundamental feature of everyday life. The Covid pandemic showed how mathematical and data skills were fundamental to understanding what is going on in a complex world (see Box 1). This is a trend that is certain to grow in the future, as data-driven intelligence and analyses provide goods and multiple app-driven services with unprecedented efficiency and convenience.

The combination of capabilities known broadly as data science "… is effecting a revolution in the way we do business, access knowledge, communicate, and understand the world"[1]. This revolution is underpinned by mathematics.

BOX 1

Data science transforming human health

"Analysing 'big data' and getting insights from it can be immensely powerful, and one of the most striking areas of application is human health. Big data analytics have helped enormously with understanding better the viruses that cause Covid-19, Ebola and HIV/AIDS.

Take the example of COVID-19. In 2020, Oxford physician and data scientist, Martin Landray, proposed studies of the effects of certain common and inexpensive drugs on severe Covid sufferers. Studies like this are careful, large-scale experiments, designed to be analysed by applying data science rigorously. The result with one drug, Dexamethasone, was a big success. It is estimated that treatment with Dexamethasone helped save hundreds of thousands of lives worldwide." *The Economist*[2]

Navigating our data-rich world

We need a society in which everyone has the knowledge and skills to thrive in this fast-moving world; to strengthen the national economy and to tackle major global challenges. In all areas, and at all levels of employment, we will need mathematically educated people who are equipped to deal with the opportunities and challenges of varied and ever-changing work roles, who know how to ask the right questions and to play their role in finding the answers. Similarly, and no less importantly, all citizens, regardless of their employment, need to be able to manage their daily lives, to play their role in civic society, and to increase and embrace the opportunities available to them.

An education system for all

The increasing digitalisation of work and communications and the ever-growing importance of data and computing require fundamental changes to current mathematics education. Children entering primary school today will leave school to take up roles and occupations that do not yet exist. Their daily lives will be different and will continue to change over their lifetimes. And as the content and goals of mathematics education change so must the process, to reflect the digital and technological environment into which children will grow. Our objective must be to create an education system that is fit for twenty years hence and that is agile enough to respond to what will be a continual process of change.

1. Royal Society 2019 Dynamics of data science skills, p.4: Royal Society. See https://royalsociety.org/-/media/policy/projects/dynamics-of-data-science/dynamics-of-data-science-skills-report.pdf (accessed 11 July 2023).
2. The Economist 2021, How British science came to the rescue. See https://www.economist.com/britain/2021/02/27/how-british-science-came-to-the-rescue (accessed 1 September 2023).

A new approach to mathematical education

The nature of the mathematical education that is needed is changing from 'mathematics' to a fusion of mathematics, statistics, data science and computer science; what we have called mathematics and data education (MDE). MDE is built on a secure grasp of the basic toolbox of essential mathematics (broadly speaking the content of foundation level GCSE mathematics). It comprises three core elements that have complementary goals. Across each of these core elements, the use of appropriate digital tools is commonplace.

The first is an evolution of the current mathematics curriculum, with a greater emphasis on data and computing.

1. **Foundational and advanced mathematics:** Foundational mathematics establishes the essentials for life and further learning. Advanced mathematics builds capacity for more demanding focused study and the application of mathematics and data science in subsequent learning. Together they incorporate the understanding of fundamental mathematical ideas, the development of more complex mathematical and data techniques, and skilled use of computational, data-analytic and other tools at levels appropriate to the age of the learner. Examples include basic arithmetic, proportional reasoning and probability, which underpin mathematics and data analysis. At a more advanced level, algebra and trigonometry, followed by calculus and mechanics, are needed for a huge range of mathematical applications, while sampling theory is essential for the study of statistical inference.

The second addresses the growing and currently unmet need for all students to confidently apply their mathematical and data skills to the common, real-world, quantitative problems they are likely to face, and in a range of educational, employment and everyday contexts.

2. **General Quantitative Literacy (GQL)** is the ability to use and apply mathematical concepts and use digital tools to solve real-world quantitative problems. It is developed throughout education and is essential for operating effectively in daily life and work. Confidence and fluency in general arithmetic and proportional reasoning are its foundations, together with an appreciation of presenting and interpreting data. GQL underpins the ability to take a critical view of arguments that are based on mathematics and data. Examples of GQL include the ability to calculate the affordability of a payday loan or mortgage, or to critique quantitative claims made in advertising or the media.

The third recognises that mathematics and data skills are increasingly used outside of the mathematics classroom in a job or domain-specific context.

3. **Domain-specific competencies (DSCs)** enable learners to use and apply mathematics and data skills in a range of other subjects and disciplines. They are developed increasingly through the secondary years and beyond, using general digital technologies as well as those designed for applied contexts. DSCs may originate in mathematics lessons but are as likely to be acquired within the disciplines (for example in non-mathematics A levels and in technical and vocational qualifications such as T Levels). Examples include estimations in construction, procedures used by technicians in batch monitoring and the calculations used to plan sampling in biological fieldwork.

Mathematics and data education for all

All citizens need mathematical and data competence, and they need it for many reasons. Every day, most individuals will need to draw on more than one of the three core elements to meet their needs.

All citizens

All citizens need foundational mathematics skills and general quantitative literacy for daily life, including personal finance and general employment. General quantitative literacy will enable them to critique quantitative claims with confidence in a variety of contexts, for example political and scientific claims in the mass media.

Vocational and technical occupations

People in vocational and technical occupations (eg electricians, technicians, nurses and other health care professionals) will often need domain-specific competencies particular to their occupations. These may be of a very high order. They will be built on foundational or advanced mathematics and general quantitative literacy.

Traditionally non-quantitative roles

Those in roles that have traditionally been thought of as non-quantitative, but now require increased mathematical and data skills (eg administrators, lawyers, agricultural and land management professionals, journalists, civil servants, politicians), will have moderate to advanced general quantitative literacy and, as appropriate, domain-specific competencies.

Professions with high quantitative demands

People requiring mathematical and data skills as a core component of their work (eg analysts, engineers, financial professionals, scientists, social scientists) need highly developed domain-specific competencies and strong advanced mathematics and in many instances will also need excellent general quantitative literacy to navigate the quantitative demands of their particular occupation.

Professional mathematical scientists

Professional mathematical scientists (eg academic mathematicians, teachers of advanced mathematics) will excel in advanced mathematics and might also have advanced general quantitative literacy.

The present position

Other countries have recognised the need for change and have been evolving their approaches to mathematical and data education. Our commissioned research provides many examples of these new approaches[3, 4, 5]. The overall picture is one of intense activity and repositioning for the future in nearly all mathematically high performing countries.

Within the UK there have been different approaches to, and prioritisation of, the three elements of MDE outlined in part 1. Wales has developed a GCSE in numeracy alongside GCSE mathematics, and Scotland has a Higher in application of mathematics and National Progression Awards in data science. These qualifications seek to address the MDE needs of learners following different pathways to different kinds of futures, at different ages. They are interesting developments, from which we have learned. That said, no UK models have completely satisfied their multiple stakeholder demands. In this report we focus on England, simply because it is the largest of the UK's education systems, with >85% of the UK's learners.

England has a system that serves some students well, especially the highest attaining post-16 learners. Mathematics A level is well regarded, both as a preparation for mathematically demanding courses in higher education (mathematics, science, engineering etc) and as a course of study in its own right. Mathematics is the most popular A level subject. In 2023, almost 91,000 students, about 14% of 18-year-olds, took A level mathematics[6]. Disproportionately more were boys than girls. Around 15,000 of these students took further mathematics[7]. However, there are important respects in which mathematics education in England does not do as well.

Low levels of numeracy

First, many young people are turned off mathematics at an early age. To quote a 2023 Ofsted report: "Pupils who are learning mathematics more slowly than their peers frequently receive a mathematics education that does not meet their needs .... Many of these pupils develop a negative view of mathematics[8]".

Government data show that 17 million working-age people in England, equivalent to 49% of the labour force, have a level of numeracy equivalent to that of children when they leave primary school[9]. This level of numeracy is significantly below other 'developed' countries and has created an entrenched culture in which it is somehow socially acceptable to be bad at mathematics. This is despite evidence that individuals with low levels of numeracy are likely to have reduced earning potential and poorer health, with knock-on negative impacts on social wellbeing and economic prosperity. These disadvantages are all too often perpetuated down the generations.

GCSE 'failure' rates

Second, each year around 30% of 16-year-old students attending state-funded schools in England fail to attain a standard or strong pass in GCSE mathematics (grades 4 – 9)[10]. These students are required to resit GCSE mathematics examinations until they attain at least a grade 4, or else pursue a 'pass' in an equivalent functional skills qualification. Two thirds of these students do not make the grade by age 19, while others may resit multiple times before achieving it. Such experiences are humiliating and demoralising. The burden falls particularly on students from disadvantaged backgrounds.

3. Adams G and Boylan, M 2023 Landscaping mathematics education policy: horizon scanning of international policy initiatives. Sheffield Hallam University. See https://royalsociety.org/~/media/policy/projects/maths-futures/landscaping-international-mathematics-education-policy.pdf (accessed 1 September 2023).
4. Smith C, Kathotia V, Ward-Penny R, Howson O and Wermelinger M 2023 Mathematical and data literacy. Competencies and curriculum implications at the intersection of mathematics, data science, statistics and computing. The Open University. See https://royalsociety.org/~/media/policy/projects/maths-futures/intersection-mathematics-data-statistics-computing.pdf (accessed 1 September 2023).
5. Firetail 2022 An evidence review on the changing nature and importance of mathematics in the 21st century. See https://royalsociety.org/~/media/policy/projects/maths-futures/changing-nature-of-mathematics.pdf (accessed 1 September 2023).
6. Office for National Statistics 2022, Estimates of the population for the UK, England, Wales, Scotland and Northern Ireland https://www.ons.gov.uk/peoplepopulationandcommunity/populationandmigration/populationestimates/datasets/populationestimatesforukenglandandwalesscotlandandnorthernireland (accessed 1 September 2023).
7. Ofqual 2023, Provisional entries for GCSE, AS and A level: summer 2023 exam series 7. See https://www.gov.uk/government/statistics/provisional-entries-for-gcse-as-and-a-level-summer-2023-exam-series/provisional-entries-for-gcseas-and-a-level-summer-2023-exam-series (accessed 30 August 2023).
8. Ofsted 2023, Coordinating mathematical success: the mathematics subject report. See https://www.gov.uk/government/publications/subject-report-series-maths/coordinating-mathematical-success-the-mathematics-subject-report (accessed 24 July 2023).
9. KPMG and National Numeracy 2017 The essentials of numeracy. A new approach to making the UK numerate. National Numeracy. See https://www.nationalnumeracy.org.uk/sites/default/files/documents/nn124_essentials_numeracyreport_for_web.pdf (accessed 11 July 2023).
10. Department for Education 2023 Post-16 maths participation for pupils ending KS4 in 2018/19. See https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1132372/Post_16_maths_participation_ad_hoc_statistics.pdf (accessed 1 August 2023).

Low levels of participation post-16

Third, while mathematics A levels work well for some, they are not for everybody. For most students there are few other mathematics options on offer post-GCSE, so compared with other countries few students study advanced mathematics after the age of 16. A 2012 report from the Nuffield Foundation showed that in England, Wales and Northern Ireland fewer than one in five students studied any mathematics after the age of 16[11]. This compares with participation rates of over 80% in most of the 24 countries in the study, (and 100% in 8 of them). The GCSE resits policy has increased the number of students studying maths post-16, but most of this is at lower, indeed remedial, levels.

It is widely accepted that there are more than 200,000 students each year with GCSE grade 4 or above who could, but do not, enter for a maths qualification after the age of 16[12]. Many of these students would benefit from continuing their MDE to 18, not least because it will be of great value to them as they progress to Higher or Further Education and employment.

In 2014, a new set of qualifications with the title 'Core Maths' was introduced. In terms of demand on the student they equate, more or less, to AS levels. Core Maths qualifications vary considerably but include topics such as statistical analysis, mathematical modelling, financial mathematics, and critical analysis of data. They focus on developing problem-solving and decision-making skills, as well as improving mathematical fluency and general QL. Core Maths courses are intended to be more practical and applied than A level. They use real-world data and explore mathematical concepts in authentic contexts that are relevant to everyday life, business, or social issues. Core Maths has many points of similarity with the kinds of courses developed in other countries and we believe it is a useful post-16 option for the future. However, the number of students taking the qualification has been modest, with around 12,000 students (less than 2% of 18-year-olds) taking the qualification in 2023[13]. Understanding how this number could be significantly increased is an important issue[14].

Digital technologies

Finally, as a recent JMC report[15] shows and research commissioned by the programme confirms, England is struggling to embed digital technology effectively into teaching and learning. Developments have been piecemeal and sporadic, and there has been no central drive or long-term planning. There has been a lack of development of the kind of computational thinking within mathematics that is evident in many more forward-looking countries[16].

Summary

These shortcomings Illustrate a problem that has been evident for many years, one which governments of both sides have identified and tried to solve. Recently the Prime Minister announced a new initiative aimed at involving all students in the study of mathematics to the age of 18[17]. We welcome the intent, but we believe achieving it will require a bold strategy. Certainly, the shortcomings of the post-16 curriculum need to be addressed, but improvement requires action at all stages of education, from early years onwards.

11. Hodgen J, Pepper D, Sturman L and Ruddock G 2012 Is the UK an outlier? An international comparison of upper secondary mathematics education. Nuffield Foundation. See https://www.nuffieldfoundation.org/sites/default/files/files/Is%20the%20UK%20an%20Outlier_Nuffield%20Foundation_v_FINAL.pdf (accessed 11 July 2023).
12. Thomson D 2023 Could there be demand for more post-16 maths? See Could there be demand for more post-16 maths? – FFT Education Datalab, https://ffteducationdatalab.org.uk/2023/01/could-there-be-demand-for-more-post-16-maths (accessed 13 July 2023).
13. MEI 2023 Summary of Core Maths entries and results August 2023 (UK). See https://mei.org.uk/summary-of-core-maths-entries-and-results-august-2023-uk (accessed 30 August 2023).
14. Royal Society and British Academy 2022 Core Maths qualifications are 'practical and valuable' to students of all disciplines, say British Academy and Royal Society. See https://royalsociety.org/news/2022/01/core-maths-qualifications (accessed 11 July 2023).
15. Noyes A, Clark-Wilson A, Hodgen J and Button T (eds) 2023 Mathematics education and digital technology. A report from a working group of the Joint Mathematical Council of the United Kingdom. See https://www.jmc.org.uk/wordpress-cms/wp-content/uploads/2023/07/JMC-Digitech-Report-July-2023-1.pdf (accessed 12 July 2023).
16. UCLC Consultants 2023 Educational technologies in mathematics education. UCL Institute of Education. See https://royalsociety.org/~/media/policy/projects/maths-futures/educational-technology-mathematics-education.pdf (accessed 1 September 2023).
17. Gov.uk 2023 Prime Minister outlines his vision for Maths to 18. See https://www.gov.uk/government/news/prime-minister-outlines-his-vision-for-maths-to-18 (accessed 13 July 2023).

What mathematical and data education is needed?

The Mathematical Futures Programme has identified seven core requirements for mathematical and data education (MDE), which we discuss below.

In summary, MDE needs:

1. to be based on clear principles.
2. to be based on a sound understanding of foundational mathematics.
3. to make full use of digital technologies.
4. to have general quantitative literacy for all at its core.
5. to value mathematics as a subject in its own right and encourage the study of advanced mathematics.
6. to provide a series of pathways that meet the different needs of learners.
7. to provide an opportunity to develop mathematics across all subject domains.

1. Principles

The mathematics and data education (MDE) of the future should be underpinned by a number of principles. These principles are not contentious, but need to be stated firmly because they are evident only weakly in our present system, when they should be at its heart.

The first is that any new approach should be inclusive. It should cater for all students, acknowledging different start and end points. Each student should have the opportunity to achieve the best they can. It should be challenging, with appropriate levels of demand to ensure development and progress.

Second, the curriculum and qualifications must be relevant, so that everyone, whatever their interests and aspirations, leaves school confident that what they have learned is of use and value to them. They should be able to see how the mathematical and data competencies they have acquired will play a central part in their future lives, as citizens and in their education and employment.

Third, MDE should be engaging, so that students at all stages and at all levels of ability experience intellectual stimulus and satisfaction, encouraging them to continue their study of mathematics or data science or both. MDE should therefore be connected; both within itself and with other subjects. It should be flexible – MDE should not restrict the ambitions for individual students because of their starting point or level of maturity.

Fourth, MDE should be agile and up to date, linking to contemporary developments in data science, IT and computing. It should make the best use of technology to enhance MDE thinking. If it is to achieve this, it must be well resourced.

And finally, every aspect of it – from pedagogy, to professional learning, to policy implementation – should be informed by evidence, constantly improving through a strong underpinning body of research.

2. Solid foundations

A key component must be the foundational mathematical and data competencies that everyone should have acquired by the age of 14, and in many cases by the end of primary schooling (with appropriate arrangements for those with special educational needs and disabilities). The important word here is “everyone”. Just as no education system contemplates students leaving school unable to read or write, so the expectation must be that every student leaves with these foundational competencies. This should be tested and certified in such a way that students, teachers and ultimately employers know with confidence what the student understands and can do. There are various ways in which this might be achieved, and different views about when the right time would be to make these judgements. We do not believe that the present GCSEs and Functional Skills qualifications in mathematics serve the purpose, especially for those who are awarded lower grades, where the grade gives an overall score, but no indication of what the student can or cannot do.

3. Digital technologies

Digital technologies of varying sophistication are rapidly transforming our daily lives and workplaces. Their impact on the teaching of mathematics in schools has been less obvious. To quote a recent report from the Joint Mathematical Council: “There has been considerable change in the use and application of digital technologies in all aspects of the education system, but when it comes to mathematics education this has been rather organic, patchy, inequitable and with little evidence of widespread impact. That said, in HEIs [Higher Education Institutions], there has been a rapid shift in the use of digital technologies and genuine efforts to use the tools that professional mathematicians and users of mathematics will use in their employment futures”[18]. The last point is particularly important, as schools must prepare many of their students for higher education.

Programming, in particular, has strong links with mathematics. In recent initiatives to include programming in national curricula and standards, some jurisdictions (eg Ontario, Spain and Norway) have added new programming requirements to revised mathematics curricula. In England, the national curriculum programmes of study in computing are well established for Key Stages 1 – 4, with increasing number of pupils electing to take the GCSE in computer science at 16. MDE should strengthen its links with programming, drawing on pupils’ coding skills developed through the computing curriculum to solve problems in MDE, and providing an authentic context to apply the coding and computational thinking they have learnt in computing[19].

Recently, large language model generative AI, such as ChatGPT, have had a great deal of professional and media attention. These systems are able to solve a wide range of MDE problems, including through writing and running code. Even with such systems, a solid foundation of MDE knowledge is needed to:

- Frame problems effectively.
- Make sense of the solution offered.
- Discern between correct and incorrect solutions and explanations.

Large language model chatbots might offer much as adaptive tutors for those learning MDE, as early initiatives suggest[20]. Generative AI is in its early stages, and it will become clearer over the next year or two how both the technology and the regulation of it will evolve. As the technology matures its advantages and applications for education in general, and mathematics in particular, will become more evident.

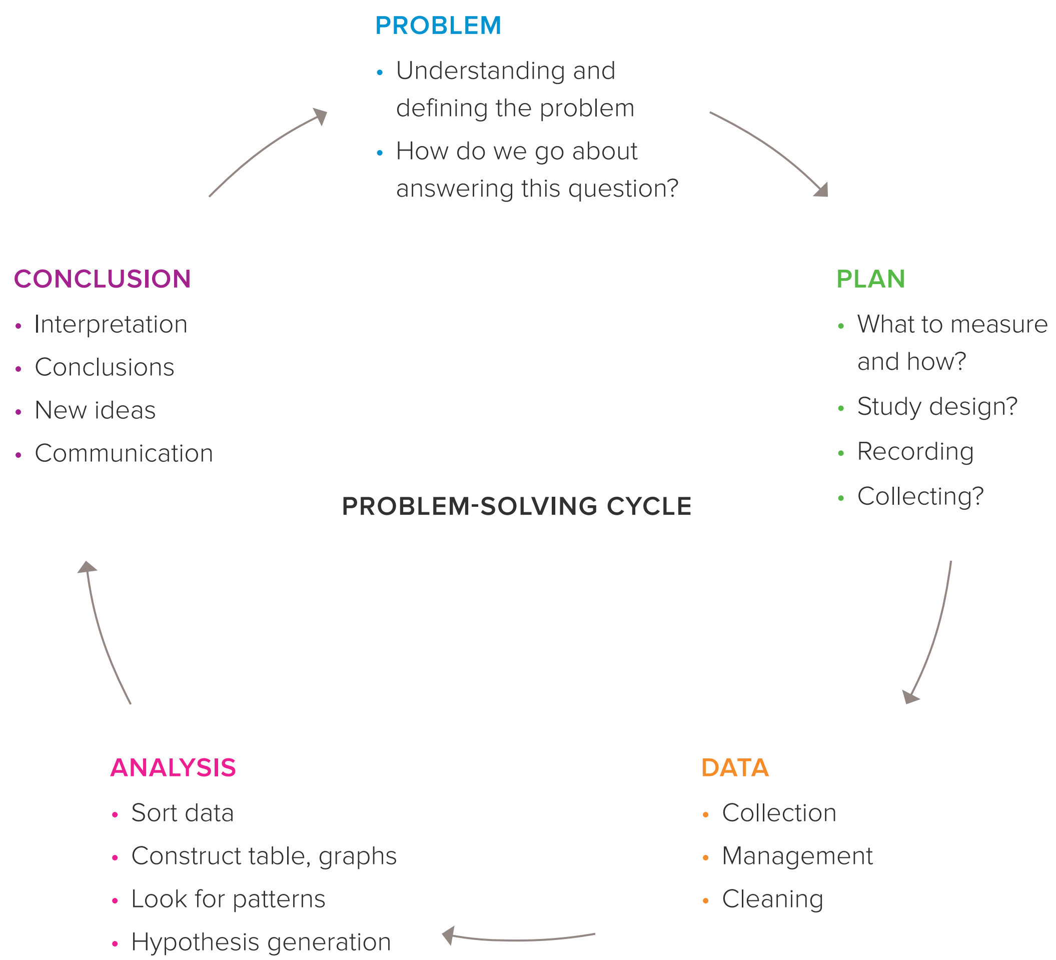
The use of digital technology in assessment offers particular opportunities. While it is not a panacea, and raises its own issues, the technologies should be explored and developed, albeit with a properly critical eye. It is likely that the use of digital methods of assessment will of itself be an important driver for the wider adoption of technology in the classroom[21].

A new MDE will seize the opportunities these technological developments present for the learning of mathematics. It must prepare citizens for personal, civic and employment futures where mathematics, data science and computing are blended and evolving quickly. The growth of digital technology demands a shift of focus away from traditional drill in mathematical techniques, in favour of teaching computational thinking. These powerful tools demand rethinking of the ways in which learners use the data/problem cycle to address relevant and interesting problems (see Figure 1).

18. Noyes A, Clark-Wilson A, Hodgen J and Button T (eds) 2023 Mathematics education and digital technology. A report from a working group of the Joint Mathematical Council of the United Kingdom. London: JMC. See https://www.jmc.org.uk/wordpress-cms/wp-content/uploads/2023/07/JMC-Digitech-Report-July-2023-1.pdf (accessed 12 July 2023).
19. Bocconi S, Chioccariello A, Kampylis P, Dagienė V, Wastiau P, Engelhardt K and Stupurienė G 2022 Reviewing computational thinking in compulsory education: State of play and practices from computing education. See https://publications.jrc.ec.europa.eu/repository/handle/JRC128347 (accessed 1 September 2023).
20. Khan Academy 2023 Harnessing GPT-4 so that all students benefit. A nonprofit approach for equal access. See https://blog.khanacademy.org/harnessing-ai-so-that-all-students-benefit-a-nonprofit-approach-for-equal-access/ (accessed 24 July 2023).
21. UCLC Consultants 2023 Educational technologies in mathematics education. UCL Institute of Education. See https://royalsociety.org/~/media/policy/projects/maths-futures/educational-technology-mathematics-education.pdf (accessed 1 September 2023).

FIGURE 1

Problem-solving cycle, going from problem, plan, data, analysis to conclusion and communication, and starting again on another cycle, based on the New Zealand Census at School project[22].



22. David Spiegelhalter 2020 The Art of Statistics. See https://www.penguin.co.uk/books/294857/the-art-of-statistics-by-spiegelhalter-david/9780241258767 (accessed 30 August 2023).

4. Improving general quantitative literacy

As we have said, the mathematics education that is needed is evolving into a fusion of mathematics, statistics, data science and computer science. MDE has a stronger focus on QL for all learners, at every stage and level. As mathematics and data become increasingly embedded in society, so citizens and employees need the capabilities to read their quantitatively saturated worlds and workplaces. For this reason the major international comparison of mathematics education (OECD:PISA) foregrounds mathematical literacy (see Box 2). Improving QL extends much further than 'basic skills' and now, more than ever, the capacity to navigate the world with mathematics depends on a capacity to use computational tools.

QL should be central at primary level and continue throughout lower secondary, where new contexts become increasingly important as the student encounters new subjects and important issues. Thereafter further development of QL complements both foundational and advanced mathematics and domain-specific competencies.

The core objective of QL is empowering all learners to be able to use and apply foundational mathematical ideas confidently and competently in a variety of contexts of varied complexity. For many, developing QL will be more important than learning more complex mathematical techniques that they will be unlikely to use.

Emphasis needs to be given to learning how to use and apply these essential mathematical and data skills in solving relevant and real-life problems. This may mean reducing existing curriculum content, to create time for developing QL.

BOX 2

OECD definition of mathematical literacy[23]

"...Mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens."

5. Mathematics as a subject in its own right

For all its economic and social importance, the place of mathematics in a young person's education is not simply one of instrumental value. Students should learn to appreciate that mathematics is one of the greatest intellectual achievements of mankind. They should understand that mathematical modelling, in the sense of quantitative descriptions of the world, is one of the key means by which humanity can understand the world. They should learn that mathematics is underpinned by a set of ideas of extraordinary power and beauty such as prime numbers, the concept of a variable, the idea of a proof, and the key concepts of space and geometry. For some students the encounter with these ideas will be a gateway to pursuing advanced mathematics and a lifetime of interest and enjoyment in mathematics for its own sake. Not all students will follow this path, of course, but we believe that all students should have the opportunity.

23. OECD 2018 PISA 2018 Mathematics Framework. See https://www.oecd-ilibrary.org/sites/13c8a22c-en/index.html?itemId=/content/component/13c8a22c-en (accessed 13 July 2023).

6. Pathways for mathematics and data education

Across the UK, learners have for many years followed different routes through secondary mathematics education (from age 14). This was typically understood, in England at least, as how far each learner progressed on a 'ladder' (eg basic, foundation, higher, and advanced mathematics).

Recently there has been focus on different mathematics pathways. The key recommendation of the 2004 report, *Making mathematics count*, by the now Royal Society President, Sir Adrian Smith, describes a vision for mathematics education that we entirely subscribe to[24]. A new mathematical and data education (MDE) will need a similar structure of holistic and coherent pathways with multiple threads, including data science and computing.

> **" We wish to see a highly flexible set of interlinking pathways that provide motivation, challenge and worthwhile attainment across the whole spectrum of abilities and motivations but avoid the danger of returning to the O-level/CSE 'sheep and goats' divide."**
>
> Sir Adrian Smith, *Making mathematics count*, 2004.

All learners should continue to develop general quantitative literacy from ages 14 to 18. Qualifications such as Core Maths offer a means of doing this, albeit currently only at advanced level. Many learners should continue to develop foundational and advanced mathematics skills and/or domain-specific competencies.

There are well established standalone qualifications for advanced mathematics skills (eg A level mathematics and further mathematics, A level statistics). These should offer better support to the data science and computing aspects of MDE.

Post-16 learners in England now have a variety of routes to developing domain-specific competencies as they are signposted in other subjects, for example discipline-specific mathematics in some A levels. This is a welcome development although it is relatively new, and how much it is affecting actual practice is uncertain.

Tests and examinations that enable all to succeed at some level, and to demonstrate what they can do, will be critical to a future MDE in which foundational and advanced mathematics, general quantitative literacy and domain-specific competencies are developed along flexible pathways. Current forms of norm-referenced high-stakes assessment, particularly at key grade thresholds (eg GCSE grade 4), do not helpfully inform anyone of what learners know and can do. New approaches to competence-based assessment might address this, particularly for the essential QL strand. Tests such as the driving test and graded music examinations are good examples from which we should learn.

24. Smith A 2004 Making mathematics count. The report of Professor Adrian Smith's inquiry into post-14 mathematics education. See http://www.mathsinquiry.org.uk/report/MathsInquiryFinalReport.pdf (accessed 13 July 2023).

7. Mathematics and data education across the curriculum

MDE is not just a matter for mathematics teachers and departments. Over time it should involve teachers of all subjects, in the same way that all teachers have a responsibility for developing the literacy of their pupils.

There have been some moves in this direction. As discussed above, a number of recent A level specifications include explicit statements of the mathematical content that students are expected to have mastered (domain-specific competencies), and the examinations are designed to test this knowledge. Similarly, general mathematical competencies (GMCs) frame the mathematical requirements of T Levels[25]. Writing such aspirations into curriculum specifications is a good start although it will take time for good teaching of these domain-specific competencies to become the norm.

This development should be extended to Key Stages 3 and 4. The aim should be to ensure that pupils encounter mathematical and data ideas in ways that are consistent across subjects, where possible using common vocabulary. This is not to argue for uniformity: different disciplines use quantitative methods in different ways, but understanding those differences is itself an important goal of learning.

25. Royal Society 2019 Mathematics for the T Level qualifications: a rationale for General Mathematical Competences (GMCs). See https://royalsociety.org/~/media/policy/topics/education-skills/Maths/Mathematics%20for%20the%20T%20Level%20Qualifications%20-%20a%20rationale%20for%20GMCs.pdf?la=en-GB (accessed 13 July 2023).

PART 4

How do we get there?

What is needed?

The new approach to mathematical and data education will:

- encourage more people, at every stage, to develop foundational and advanced mathematical skills.
- by improving general quantitative literacy, be inclusive of the needs of a diverse population and help develop better equality and opportunity for all.
- lead to a more mathematically skilled labour force, better equipped with the domain-specific competencies needed, with huge benefits to the economy.
- resolve the waste and injustice of a system where the failure of a third of students is inbuilt.
- address the indifference and anxiety many people feel towards mathematics.

Our vision is of a virtuous cycle in which more learners see MDE as an intrinsic element of their education; interesting, engaging and relevant to their daily lives. Mathematical skills and general quantitative literacy become a core component of all learners' experience, as part of a modern education in which the connections between subjects are more evident. With recognition for the level of mathematical skill and QL they have attained, more will want and be able to study at advanced levels. These changes in turn will improve the supply of teachers (and not just teachers of MDE) at all levels able to teach these competencies to future generations.

This improvement in the skills of the workforce would make a substantial contribution to the UK's economy and preparedness for the future. Continuing with our present arrangements condemns the UK to life in the slow lane.

A new MDE requires these changes:

- recognition across the system that MDE would be for everybody up to age 18;
- changes to the National Curriculum for mathematics to accommodate MDE;
- reform of pathways and qualifications from the age of 14, to enable students to make choices based on their abilities and aspirations;
- assessments that provide accurate information about what pupils know and can do, to provide a reliable overview for students, their teachers and, in due course, employers;
- a sustained and well-funded programme for teachers' professional learning and an entitlement to this learning and development for all teachers;
- a new approach to teacher education, so that new teachers at all levels (and especially at primary level) are prepared for MDE; and
- recognition by subject associations, learned societies and others of the implications for the teaching of all subjects, not just mathematics.

And as the Royal Society has long argued, all of the above would fit much better in a broad and balanced curriculum[26].

26. Royal Society 2014 Vision for science and mathematics education. See https://royalsociety.org/topics-policy/projects/vision (accessed 11 July 2023).

What next?

The UK education system is complex and has become increasingly so. One of the first pieces of research commissioned by the Mathematical Futures Programme was a study of the recent history of education policy in the UK, so that we could learn what does and does not lead to effective long-term change[27].

The message of the commissioned report is clear: while some large-scale initiatives have been successful (eg, the National Literacy and Numeracy strategies) there is a long list of incomplete or unsuccessful changes. Many of these have failed because they have focussed on a single issue and have not considered sufficiently the interrelationships with other parts of the complex educational jigsaw. Cross-party support is important: over the years a number of promising initiatives have been abandoned as a consequence of changes of government. Above all, initiatives which do not give enough attention to the need to recruit, train and support teachers are bound to fail.

Our reform agenda is ambitious. It will take 10 – 15 years to implement fully. It needs careful planning, design, implementation, and evaluation. It will need an agreed plan so that schools and teachers are able to refine the vision and help to design its implementation. All this will require collaboration between the stakeholders, cross-party support and determination to stay the course.

The purpose of this paper, and the consultation that accompanies its publication, is not to suggest detailed plans for the future, but to engage the many communities that have a stake in MDE; to discuss the underlying principles and possibilities, to gauge the appetite for change, and to explore how we could refine the strategic vision and consider implementation. Our report, informed by this consultation, will be published in 2024.

While the challenge is great so is the prize. We are encouraged by the recent publication of a 15-year strategy for the NHS. While the substance is very different there are clear points of similarity, in particular the need to prepare for a future that will look very different from the present, and the recognition that success depends on the people who are doing the job on the ground. We believe this is the kind of long-term thinking that is needed in mathematical education. This kind of planning can only be done by government. We believe the changes we are suggesting are necessary and that the rewards will repay the effort many times over. The world is changing quickly and the cost of inaction would be significant. The time to act is now.

27. Boylan M, Adams G and Birkhead A 2022 Landscaping national mathematics education policy. Sheffield Hallam University. See https://royalsociety.org/~/media/policy/projects/maths-futures/landscaping-national-mathematics-education-policy.pdf (accessed 1 September 2023).

The Royal Society Advisory Committee on Mathematics Education (RS ACME)

The committee oversees the delivery of activities aligned with the Society's vision for science, mathematics and computing education and is regarded as a trusted voice on mathematical and quantitative education in UK schools and colleges. It comprises eight experts with an interest in mathematical and quantitative education from the Royal Society Fellowship, classroom, research and industry. Currently chaired by Sir Martin Taylor FRS, the Committee works closely with the Joint Mathematical Council of the UK, the Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society.

The Mathematical Futures Programme

Launched in February 2020, the programme seeks to answer two core questions:

- What mathematical competencies will be needed for citizens and society to thrive in the future?
- How should education systems develop these mathematical competencies?

The programme is overseen by the Mathematical Futures Board and the Royal Society's Advisory Committee on Mathematics Education. It is supported through donor funding by industry partners: Arm, GSK and Google, and ongoing support from the Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society.

Members of the Mathematical Futures Programme Board (July 2023)