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Developing Techno-mathematical Literacies in higher technical professional education

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Developing Techno-mathematical Literacies in higher technical professional education

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(met een samenvatting in het Nederlands)

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1 Introduction

1.1 Research topic

This dissertation is the result of a research project with the aim to design, implement, and evaluate a sustainable educational innovation and to gain theoretical insights into how such can be achieved. The project originated from the wish to improve the mathematics curriculum in the School of Life Sciences and Environmental Technology at Avans University of Applied Sciences where the author works as a lecturer in mathematics and chemistry. The dissertation addresses several aspects of this process: the preliminary inventory of the use of mathematics in technical workplaces, the design of an innovative course in applied mathematics for technical domains, the challenges that arose whilst implementing this new course in the curriculum, and the testing of the effectiveness of the course in relation to the learning of students.

Over the past decades, the use of ICT, digital technology, and computer-driven equipment in the workplace has changed the professional practices of engineers worldwide (Advisory Committee on Mathematics Education, 2011). Calculations are performed by computers and therefore, mathematics is often hidden (Hoyles, Noss, Kent, & Bakker, 2010). Because of these changes in professional practices, a new repertoire of skills, which is labelled Techno-mathematical Literacies (TmL), is necessary. Bakker, Hoyles, Kent, and Noss (2006) defined TmL as functional mathematical knowledge mediated by tools and grounded in the context of specific work situations (p. 343). TmL integrate mathematical, workplace and ICT knowledge, communicative skills (Kent, Bakker, Hoyles, & Noss, 2005), the ability to interpret abstract data (Kent, Noss, Guile, Hoyles, & Bakker, 2007), having a sense of number and having a sense of error (Hoyles et al., 2010). It is gradually being acknowledged that employees often lack these skills. Because TmL are rarely picked up on the job, it is essential that these skills are developed explicitly (Hoyles, Noss, Kent, Bakker, & Bhinder, 2007).

In the Netherlands, there is an ongoing discussion about mathematics curricula for higher technical professional education (Van Asselt & Boudri, 2013). At present, mathematics courses are mainly abstract with very few context- and workplace-related examples, and with limited use of software. Classes are generally based on transmission teaching and working on abstract practice questions. This kind of teaching seems to create inert knowledge which cannot be transferred to solving problems in practice (Brown, Collins, & Duguid, 1989).

It is important to bridge the gap between what is required in workplaces and what is addressed in education and to implement TmL as learning goals in mathematics curricula (e.g., Bakker, 2014). Which specific Techno-mathematical Literacies engineers use in their practices, however, was still unknown before the start of this study, yet it is relevant to know, so aspiring engineers and students can be better prepared for their future tasks. Hoyles et al. (2010) researched how employees could develop TmL in workplaces, yet how students in higher professional technical education can acquire these skills in mathematics courses is also unknown. The main research question that guided this project was:

> *How can an innovative course in applied mathematics in the technical domain of higher professional education help students acquire the TmL necessary for their future workplaces?*

This main question was addressed by studying four questions of an inventory, design-oriented, design implementation based and evaluative nature.

1.2 Research and design approach

A suitable approach in cases where it is unclear how particular educational goals can be achieved is design research. Because TmL as learning goals are not yet part of the mathematics curricula of higher technical professional education, a new course had to be designed. The research approach we used in this study is (educational) design research (Bakker, 2018; McKenney & Reeves, 2012; Plomp, 2013; Gravemeijer & Cobb, 2006), and more specifically, Design-Based Implementation Research (DBIR) (Fishman & Penuel, 2018). The strategy of DBIR consists of a process of theory-and practice-informed cycles of design, testing, evaluating and adjustment of a module. This approach yields scientific knowledge about breaking down the traditional research–practice barriers, by bringing about systemic change that makes it more likely that lecturers can adapt innovations productively (Fishman, Penuel, Allen, Cheng, & Sabelli, 2013). In this study, three cycles and a preliminary pilot were conducted in the School of Life Sciences and Environmental Technology in the years 2015–2019.

In the new mathematics course, two parallel learning tracks were developed. The first track consists of abstract pre-calculus mathematics in ALEKS™[1], an

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[1] Assessment and LEarning in Knowledge Spaces is a Web-based, artificially intelligent assessment and learning system. ALEKS uses adaptive questioning to quickly and accurately determine exactly what a student knows and does not know in a course. ALEKS then instructs the student on the topics (s)he is

electronic learning environment. This track functions as a prerequisite for the second track, in which students collaboratively work on context-based cases with a focus on TmL on roughly the same mathematical topics. The learning of TmL was stimulated by the design of the cases but also during the classroom discussions in the so-called feedback hours, in which the lecturer used several teaching strategies for this purpose. One research instrument used is called Hypothetical Learning Trajectory (HLT) (Simon & Tzur, 2004). With this instrument, predictions and conjectures, for example regarding the learning of TmL in our case, can be empirically tested, so an effective learning strategy can be developed. Based on the results of the interviews and the input of literature research, such an HLT, including conjectures about the learning of TmL, was composed and was adjusted after every design cycle.

1.3 Research overview

This dissertation contains four papers in Chapters 2 to 5, which form its core and address the specific research questions and goals. The content of each chapter is specified below. Figure 1 shows an overview of the series of studies.

Employees often do not recognise the mathematics in their work, although researchers think otherwise, based on their observation of these employees at work (cf. Duchhardt & Vollstedt, 2016; Noss & Hoyles, 1996). Therefore, to identity which TmL engineers use in their daily practices, a task-based interview study was conducted (Goldin, 1997). This study is described in Chapter 2 and provides an answer to research question 1:

RQ1: Which TmL do engineers use in different professional practices and what are their opinions and ideas regarding their own and future mathematics education?

The identified TmL function as learning goals for an innovative mathematics course for engineering education. Chapter 3 focuses on a new course in applied mathematics in higher technical professional education to foster acquiring TmL for future engineers. It describes the design of the new course with the use of context-based cases, and the strategies that the lecturer uses to stimulate TmL learning in classroom discussions. This chapter answers research question 2:

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most ready to learn. As a student works through a course, ALEKS periodically reassesses the student to ensure that topics learned are also retained.

RQ2: During the discussions of context-based cases, what teaching strategies did the lecturer use to foster TmL?

Chapter 4 presents the challenges and successes of the implementation of the course within a curriculum of the School of Life Sciences and Environmental Technology with the approach of Design-based Implementation Research. As professional development and support of the teaching lecturers was provided, their experiences, opinions, dilemmas, feelings, and beliefs regarding teaching the new course were investigated through an interview study. The goal of the chapter is to answer research question 3:

RQ3: Which challenges may arise in the relation between teachers' agency and the aim of professional development, and the goal of a sustainable implementation of an innovation?

Because there were no tests available that measure TmL, we had to design such a test ourselves. During the third cycle, starting September 2018, a pre- and posttest on TmL were conducted and analysed.

Chapter 5 reports the assessment of the learning effect of the course and the design and validation of the two tests that were administered as pre- and posttest to answer research question 4 and 5:

RQ4: What is the learning effect of a course in applied mathematics on students' development of Techno-mathematical Literacies?

RQ5: What are possible explanations for the unexpected results? Chapter 6 provides a summary of the conclusions and main findings of the study. In this final chapter, the study's contribution to design research theory is discussed, as well as limitations and directions for future research.

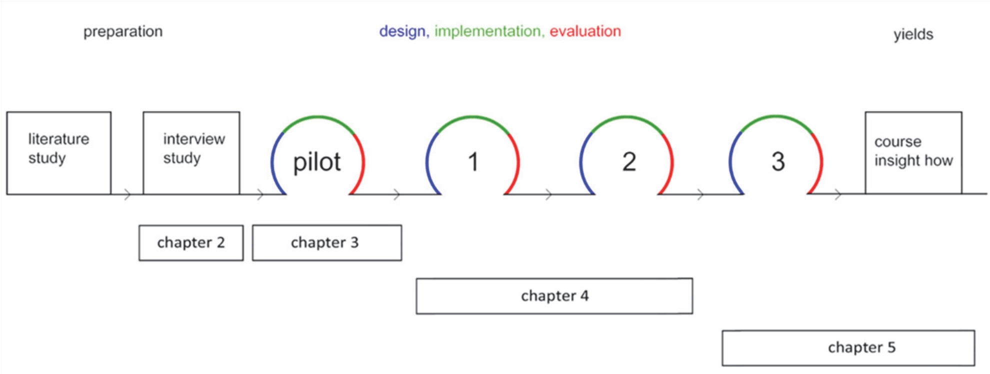

Figure 1. Overview of the phases of the design study and the chapters of the dissertation

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2 Which Techno-mathematical Literacies Are Essential for Future Engineers?

Van der Wal, N.J., Bakker, A., & Drijvers, P. (2017). Which Techno-mathematical Literacies Are Essential for Future Engineers? *International Journal of Science and Mathematics Education, 15*, 87–104. https://doi.org/10.1007/s10763-017-9810-x

Abstract Due to increased use of technology, the workplace practices of engineers have changed. So-called Techno-mathematical Literacies (TmL) are necessary for engineers of the 21st century. Because it is still unknown which TmL engineers actually use in their professional practices, the purpose of this study was to identify these TmL. Fourteen semi-structured interviews were conducted with engineers with a background in different educational tracks in higher professional education (e.g. civil, chemical, biotechnical and mechanical engineering). As a result of the data analysis, 7 commonly used TmL are identified: data literacy, technical software skills, technical communication skills, sense of error, sense of number, technical creativity and technical drawing skills. Engineers also noted a discrepancy between their education and workplace needs; they characterized mathematics in their education as an island with limited relevance. These findings lead to recommendations for the future of science, technology, engineering and mathematics (STEM) in higher technical professional education that can help students learn STEM for the future.

Keywords Engineering education ● Mathematics education ● STEM education ● Workplace competencies ● Workplace skills

2.1 Introduction

The practices of science, technology, engineering and mathematics (STEM) change, and so should STEM education. But what is this STEM for the future, and what should the future of STEM education look like? This paper is concerned with one aspect of this question, namely which mathematical skills engineers use in practice. Through identifying these skills, the future of mathematics curricula for engineering education can be reshaped in line with the future of STEM.

Over the past few decades, the use of ICT, digital technology and computer-driven equipment in technical workplaces has changed professional practices (Advisory Committee on Mathematics Education, 2011). Calculations are predominantly performed by computers, and therefore mathematics often remains hidden (Hoyles, Noss, Kent, & Bakker, 2010; Williams & Wake, 2007a). Results of these calculations are not transparent and can be unexpected (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; Williams & Wake, 2007b).

The need for mathematical skills that transcend mathematical knowledge and content has been recognised for many years. In her book *What counts as mathematics*, FitzSimons (2002) described a wide spectrum of terms that are used to identify these skills, such as “specific and generic numeracies” (Buckingham, 1997). Broader definitions are also mentioned, for example, “technical competence”, which includes mathematical competence and reflective knowledge (Wedege, 2000). In our study, we specifically focused on the technology-mediated nature of using mathematics in the workplace. Kent, Bakker, Hoyles, and Noss (2005) stated:

> *Individuals need to be able to understand and use mathematics as a language that will increasingly pervade the workplace through IT-based control and administration systems as much as conventional literacy (reading and writing) has pervaded working life for the last century. (p. 1)*

The ubiquitous use of ICT in all sectors changes the nature of the mathematical skills that are required, but does not reduce the need for mathematics (Hoyles et al., 2002). These new mathematical skills have been labelled Techno-mathematical Literacies (TmL) by Kent et al. (2005). TmL integrate mathematical, workplace and ICT knowledge, and communicative skills (Kent et al., 2005), for example the ability to interpret abstract data (Kent, Noss, Guile, Hoyles, & Bakker, 2007), having a sense of number and a sense of error (Hoyles et al., 2010). It is gradually being acknowledged that employees often lack these skills. Because TmL are rarely learned on the job, it is essential that these skills are developed explicitly (Hoyles, Noss, Kent, Bakker, & Bhinder, 2007).

In higher technical professional education in the Netherlands, there is an ongoing discussion about the content and relevance of mathematics curricula (Van Asselt & Boudri, 2013). Mathematics courses are mostly theoretical, with very few context- and workplace-related examples. Estimation and use of software are rarely a part of these courses, although they are a key part of TmL required in workplaces (Hoyles et al., 2010). Furthermore, students have limited motivation for mathematics because of its perceived lack of relevance. The Dutch situation thus seems similar to international observations (Roth, 2014; Wedege, 1999).

Because of the discrepancy between school and work mathematics (Bakker & Akkerman, 2014), students are insufficiently prepared for their future jobs. Although mathematics education, even at the bachelor level, does not only prepare for future work, it is still important to bridge this gap between what is required in workplaces and addressed in education and to implement TmL as learning goals in mathematics curricula.

In engineering research, mathematical skills of engineers have been identified (e.g., Gainsburg, 2007; Kent & Noss, 2000, 2001, 2002). In this study, we deploy the concept of Techno-mathematical Literacies that engineers with a background in higher technical professional education use in their practices. This use in practice is still unknown, yet relevant to know, so as to be able to better prepare students for their future tasks, not only at work but also in future study and as citizens. Hoyles et al. (2010) researched how employees could develop TmL in their workplaces, but how students in higher professional technical education can acquire these skills in mathematics courses is also unknown.

The global research question in the overall study is how mathematics curricula in the technical domain of higher professional education can be developed to help engineers acquire the necessary TmL for their future workplaces. The questions addressed in this article are as follows: (1) which TmL do engineers use in different professional practices and (2) what are their opinions and ideas regarding their own and future mathematics education?

2.2 Techno-mathematical Literacies in Engineers' Workplaces

Over the last 50 years, there has been a shift in the Western world from an industrial economy to an information-based economy. Education in the twenty-first century no longer prepares for lifetime employment. Rapid technological change and globalisation ask for individuals who have a broad general education, good communication skills, adaptability and who are committed to lifelong learning (Millar & Osborne, 1998). On all levels, there is a shift from routine to non-routine tasks, and this asks for specific skills in employees. Voogt and Pareja Roblin (2010) identified these as twenty-first-century skills, encompassing problem-solving, creativity, technology skills, critical thinking and complex communication skills.

2.3 Mathematics in Technical Workplaces

Because computers perform most calculations, decisions based upon output are therefore prone to serious errors, if the user does not understand the underlying mathematics (Gravemeijer, 2012). On the one hand, it seems that less mathematical knowledge is needed as computers take over a growing number of mathematical tasks. On the other hand, for "mathematics consumers", there is an increased need to be able to handle and understand quantitative information (Gravemeijer, 2013; Levy & Murnane, 2007).

In this study, we consider the whole spectrum of technical domains at the bachelor level. In engineering education research, Kent and Noss (2002) studied the workplaces of civil and structural engineers. Although mathematics plays a central role in engineering, they found that in their own perception, engineers merely use simple mathematics. They practise division of labour by delegating computational aspects of the work to software and by outsourcing to mathematical experts. In the pre-computer era, engineers developed understanding through daily practice of hand calculation. Because of computers, mathematics is more accessible, and engineers now learn by "understanding through use". To understand and use this technology adequately, mathematical literacy is necessary, analogous to language literacy (Kent & Noss, 2001). James (1995) defined the term "literacy" as the ability that enables the process of good communication and that requires a range of experience.

Bergsten, Engelbrecht, and Kågesten (2015) studied the view of engineers towards the relevance and role of "procedural" and "conceptual" mathematical skills. Kent and Noss (2002) distinguish those two views as "doing" and "understanding" mathematics. Gainsburg (2007) also studied the workplaces of structural engineers and recognised the use of "engineering judgement" which encompasses engineering expertise with mathematical and other capabilities and introduced the term "sceptical reverence"—a critical attitude towards mathematics. Williams, Wake and Boreham (2001) mention "mathematical competence" as the ability to use mathematics needed in workplaces.

To define mathematical needs that transcend mathematical content, a variety of terms is used. For mathematics in technology-based workplace practices, Kent et al. (2005) introduced the term Techno-mathematical Literacies. They wrote: "We use the term as a way of thinking about mathematics as it exists as part of modern, increasingly IT-based workplace practices" (p. 1). In 2007, they elaborated on the choice of this term by explaining that they found "mathematical literacy" a term of more relevance than traditional notions of numerical skills and competences and added the prefix "techno" to stress that mathematics is mediated by technology and literacies – plural – as a wide variety of knowledge, necessary in workplace practices (Kent et al., 2007).

These complex skills go well beyond basic numeracy because they are grounded in data and the context of specific work situations (Bakker, Hoyles, Kent, & Noss, 2006; Hoyles et al., 2002). For example, graphs are often misinterpreted, even by professional scientists, when they do not originate in their own specific domain (Roth, 2003). Interpreting graphs, charts and outcomes with weak connections to data or underlying mathematical relationships is defined as pseudo-mathematics by Hoyles, Noss, Kent, and Bakker (2013). Context and activity in which mathematical understanding takes place are therefore essential, and employees must make mathematical sense of situations that are quite different from their formal mathematical education (Bakker et al., 2006).

Kent et al. (2005) state that dealing with models and taking decisions based on abstract information have always been tasks of highly trained employees, but due to technology, an increasing number of people engage in these systems, which brings a new complexity to the workplace. Furthermore, whereas visible mathematics is mostly associated with routine tasks and less visible mathematics with non-routine tasks, information technology brings an extra layer of invisibility to the processes behind the screen or printout (Hoyles et al., 2013).

2.4 Mathematics in Technical Education

To prepare students for their future jobs, a significant change in education is essential. Schools focus on conventional skills, facts and procedures, whilst learning for the twenty-first century is about using and integrating knowledge in a problem-oriented interactive curriculum (Fadel, Honey, & Pasnik, 2007). Wagner (2008) mentions an achievement gap between curricula in school and what students need to succeed in a global economy. Students do not learn the skills that matter in today's and tomorrow's world. He adds that our system and methods of education were created in a previous century with other needs.

There is a growing interest in implementing twenty-first-century skills as educational goals. For example, the OECD Programme for International Student Assessment is currently undergoing significant changes. Last year, the assessment of 15-year-olds was expanded with more skill-related questions, involving for instance collaborative problem-solving.

STEM education, in general, has received increasing attention in recent years (Kuenzi, 2008), and there is a widespread call for improvement in higher education. Research on STEM learning in the past decades has provided a substantial knowledge base of effective pedagogies and instructional strategies on how students learn (Singer, Nielsen, & Schweingruber, 2012). However, the implementation rates, adoption and scale-up of this knowledge remain low. One reason for this is that STEM educators in higher education are not used to letting educational change processes be informed by theory or literature (Borrego & Henderson, 2014).

As for mathematics education, Garfunkel (2011) states that different sets of mathematical skills are useful for different careers. He also emphasises the importance of learning mathematics in the context of science and providing both useable knowledge and abstract skills. He identifies the mathematics that is needed as "quantitative literacy" and "mathematical modelling".

The use of mathematics in workplaces is rather different from conventional mathematics education. According to Steen (2003), school mathematics is complex, but used in simple problems, whereas workplace mathematics is simple, but is used in complex problems. Alpers (2010) states that mathematics in engineering education has two major goals: "It should enable students to understand, set up and use the mathematical concepts, models, and procedures that are used in the application subjects" and "to provide students with a sound mathematical basis for their future professional life" (p. 2). Whilst content is still essential in engineering education, Cardella (2008) states the importance of "mathematical thinking". The curriculum documents of the SEFI Mathematics Working Group are changing "from contents to outcomes to competencies". In the latest edition of 2013 (Alpers et al.), the main message is that "although contents are still important, they should be embedded in a broader view of mathematical competencies". The document follows the mathematical categories identified by Niss (2003), comprising thinking mathematically, posing and solving mathematical problems, modelling mathematically, reasoning mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating about mathematics and making use of aids and tools. Kent and Noss (2000) add that, in this computer era, the conventional approach of content first and application later in school and work, where the engineer and student apply tools made by others, is no longer adequate.

A gradual shift from mathematical content to a broader definition of mathematical needs can be seen. An example of an initiative regarding mathematics in context and use of technology is the design of a software tool to help students learning laboratory techniques in secondary vocational education develop a better understanding of proportional reasoning in dilution (Bakker, Groenveld, Wijers, Akkerman, & Gravemeijer, 2014). In engineering education, model-eliciting activities (MEAs) are increasingly used at the introductory course level. MEAs are case study problems that small groups solve over one or two class periods and that form a bridge between mathematics and engineering (Hamilton, Lesh, Lester, & Brilleslyper, 2008). In this article, we focus on TmL used by engineers in their workplace practices to specify and elaborate STEM skills to be addressed in STEM education of the future.

2.5 Method

The answers to the two research questions were addressed by means of a task-oriented interview study. In 14 semi-structured interviews, the engineers' tasks and practices were used as prompts to reveal the TmL for the first research question. Additionally, the ideas and opinions of the engineers about their previous education and mathematics education, in general, were asked to answer the second research question.

This method was chosen for the following reasons. When asked what mathematics they use, employees often reply "nothing", even if researchers, whilst observing these employees at work, think otherwise (cf. Duchhardt & Vollstedt, 2016; Noss & Hoyles, 1996). Because employees often do not recognise the mathematics in their work, using questionnaires was not an adequate method as they would not provide valid information about the TmL used by engineers, which is the first research question in this study. Also, ethnographic observation was not an efficient choice because this method would take too much time to cover all technical domains of higher professional education. The interviews, each taking 1.5-2 h, were conducted in the authentic workplaces of engineers in the technical domains of applied science, ICT, built environment, and mechanical and electrical engineering. All participants (eight male and six female) had a background in higher technical professional education (HBO) and originated in the whole spectrum of technical domains. The participants, presented in Table 1, were recruited on a voluntary basis via LinkedIn and other available professional networks and had working experience varying from 2 to 40 years.

Because the goal of these interviews was to make an inventory of TmL in use, the interview scheme of the task-based interviews was inspired by the sparsely available literature on this topic (e.g. Hoyles et al., 2013). It consisted of sets of questions of the following categories: the tasks of the engineer; the engineer was asked to explain and show all of his/her tasks, including software and computer-driven equipment. As a result, in the data analysis, the data could be searched for technological and mathematical elements. Furthermore, there were questions about what mathematics skills were used in their opinion, communication with colleagues, management and customers, and division of labour. Finally, to answer the second research question, questions about the engineers' previous education and ideas about how to improve mathematics education at HBO were also addressed.

For the data collection, audio was recorded and screenshots from software usage were taken and the interviews were transcribed verbatim. The goal of the analysis was to identify and define TmL categories and to be exhaustive for these categories, but we did not consider them to be exclusive in advance, and they did, in fact, turn out to overlap somewhat. We used Atlas TI™ to support open, axial and finally selective coding (Boeije, 2005).

Table 1. Educational background and workplace settings of the participants

Participant	Education	Workplace setting	Gender
1	Construction	Sales engineer	F
2	Chemical engineering	Quality assurance engineer	F
3	Environmental science	Permit/license advisor	M
4	Mechanical engineering	Sales engineer	M
5	Civil engineering	Department manager	M
6	Mechanical engineering	Calculator	F
7	Chemistry	Research assistant	M
8	Mechatronics	Quality and process manager	M
9	Computer science	Technical consultant	F
10	Biology and applied medical laboratory technology	Senior research assistant	M
11	Electrical engineering	Application lead	M
12	Electrical engineering	Technical writer	M
13	Business mathematics	Web analyst	F
14	Business engineering	Marketing and communication officer	F

Based on the TmL definitions and examples provided in the literature (Bakker et al., 2006; Hoyles et al., 2002; Hoyles et al., 2013), to establish a list of TmL, the data were analysed in cycles, bottom-up and top-down, for mathematics and mathematical skills, technical skills, technical communication and software usage. Codes and sub-codes, with a unit of analysis of fragments with one topic, were assigned and grouped in families. First, technological and mathematical elements were given provisional codes. After consulting experts, combinations of these sub-codes were grouped into

TmL categories, which together formed a code family. For example, provisional codes such as “analysing information”, “drawing conclusions” and “searching information” were assembled in the TmL category *data literacy*. Another family consisted of the engineers’ opinions about their previous mathematics education and ideas for mathematics education in general. Together with a family of miscellaneous codes, this resulted in a codebook of three families. Two of the 14 interviews were also coded by an external coder, who was provided with the codebook and the full transcription of the interviews and asked to assign TmL categories to the transcripts. The interrater reliability, Cohen’s kappa, was .84, suggesting that the coding procedure was reliable.

Additionally, the frequencies were determined. These frequencies do not show the use of TmL categories of a representative sample of the population of engineers but provide an indication of the TmL use of a set of engineers from various domains. We do, however, think that the frequencies provide a sense of which TmL may be more common across engineering fields (generating rather than confirming hypotheses).

2.6 Results

2.6.1 TmL Used by the Engineers

In answer to the first research question on the TmL used in engineers’ practices, the data analysis led to the identification of seven main categories of Techno-mathematical Literacies used by the engineers, presented in Table 2. Because the tasks of engineers are complex, we often observed the use of multiple TmL categories in a task and also some overlap between the categories.

To illustrate and substantiate the TmL categories used by engineers, we now shall present each TmL category with an exemplary excerpt, followed by a few examples of combinations of TmL categories.

2.6.2 TmL 1: Data Literacy

Data literacy concerns the ability to handle textual, numerical and graphical data sensibly. The engineer has to analyse, interpret, draw conclusions and take action accordingly. One of the tasks of the marketing and communication officer (P14) was to gain as many clicks as possible on advertisements in her company’s campaigns in Google AdWords™.

> I: So, you watch these numbers [of clicks on advertisements]. And then what do you think or do?

> P14: I think: All right, this is not going well. This one here, which is very low, must be removed. But I can also leave it as it is. I watch the numbers and think: OK, this one is very low, but do I mind that? No, because the fact that they show is enough. They don't have to click it [click on the advertisement].

She analysed the numbers (which numbers are low and whether that is a problem) and drew conclusions (the fact that they show is enough), which are important elements of *data literacy*. Working with both textual or numerical data and with graphical representations is a part of data literacy. For example, a technical writer, whose job was to write manuals for machines that produce digital chips (P12), emphasised that his skills to produce insightful graphics are very important.

Table 2. TmL categories with descriptions and frequencies

	TmL category	Description	Frequency
1	Data literacy	The ability to analyse and interpret technical data and graphical representations, draw conclusions and take action accordingly	29
2	Technical software skills	The ability to use professional software, e.g. Excel™, as a calculation tool	80
3	Technical communication skills	The ability to communicate technical information with colleagues, customers, supervisors and other parties	32
4	Sense of error	The ability to check and verify data and detect errors	27
5	Sense of number	The ability to handle and interpret numbers sensibly	14
6	Technical creativity	The ability to produce creative solutions to puzzles and problems (by using, e.g. cleverness or experience)	20
7	Technical drawing skills	The ability to understand and produce technical drawings (by using, e.g. spatial insight)	12

2.6.3 TmL2: Technical Software Skills

The second TmL category, *technical software skills*, is the most frequently observed category in this study and relates to the ability to use technical software. It includes working not only with general (e.g. Excel™) but also domain-specific technical company software. A relevant issue regarding technical software skills is the level of transparency. One can distinguish three levels of transparency, called white box, grey box or black box. In the case of a white box, the user knows exactly which calculations and mathematics are behind the interface of the software and understands these. When software is perceived as a grey box, the user understands only a part of the processes, and in the case of a black box, none (Kent & Noss, 2002, Williams & Wake, 2007a). We have observed these three levels of transparency in the tasks of the engineers in the interviews and shall present some examples. Furthermore, we found that Excel™ as a calculation and planning tool, in particular, was employed in every technical domain, and some engineers also programmed in this application.

An example of programming in Excel™ was presented by the sales engineer (P4), who sells stabilisation fins for yachts. To calculate the price of a fin for the customers, he created an Excel™ calculation tool to save time, thus emphasising the potential gain in efficiency through software tools:

> At one point, I looked at how to make the calculations easier. Because every time, they searched for lists, what did we get last time, what were the costs from the beginning all over again? I considered that insensible, so at one point I made something [an Excel™ calculation tool for price calculation], with a purpose to make things easier for my colleague and myself.

For all three levels of transparency, an example is now given. For a department manager in the field of civil engineering (P5), the calculations in his Excel™ calculation tool that converts rain in millilitres to cubic metres, were completely transparent to him (white box) because he considered them “basic knowledge”. For a chemical engineer who works at a plant for potato products (P2), the software she uses is sometimes less transparent (grey box). She could understand the integrals behind a software tool she uses to calculate the amount of reducing sugar in the potato product with infrared spectrometry, but she does not have to. However, she does have to know what the numbers in the outcome mean. The research assistant (P7) in chemistry perceives his software for the spectrometer as a black box, which is not a problem because of alternative checks. When measuring the spectrum of an enzyme, he knows what the shape of the spectrum generally should look like, then checks with a small ruler and draws conclusions by reasoning back.

2.6.4 TmL 3: Technical Communication Skills

The third category of TmL, *technical communication skills*, includes communication with various parties. One can distinguish horizontal communication (with colleagues and other departments) and vertical communication (with management, customers and employees). A broad variety of these skills is important, as explained by the quality and process manager (P8). He stated that his educational background, mechatronics, supports these skills: “I can talk with technical people as well as management”. Sometimes it is important that simple language is used. The license advisor in environmental engineering (P3) asks for plain “Miffy” language to facilitate mutual understanding (Miffy is a famous Dutch rabbit figure for toddlers made by Dick Bruna, with very simple language use).

Interaction with customers requires more advanced technical communication skills. The sales engineer (P1) who designs climate ceilings that can both cool and heat the room explained how she asks her customer's specific additional questions. For the interviewer, she used an analogue (cars and colours) to vividly elaborate these questions:

> When we calculate the ceiling, we have to deal with the design. The customer often says: this [climate] ceiling should feel like such and such and good luck.
>
> And then we ask: You want a white car, but do you want a Fiat or a BMW? That is a big difference. And then we mostly succeed and know which other party we have to involve. In the end, the customer says: I want the BMW.

To communicate effectively with customers, one has to consider what representations to use. The web analyst (P13) was aware that she cannot use tables for her customers. She argued: "I think presenting a lot of tables is very adequate, but for most people, that is not the case. People cannot work with those". She recognises that her level of *data literacy* does not always correspond with that of a client, so she adapts her *technical communication*.

Division of labour and collaboration are evidenced in multiple references to asking each other questions, interacting with colleagues from different disciplines and seeking advice. Of course, a junior often asks for advice from a senior, but the other way around is also important, according to our senior application lead (P11): "I can still read C+ [a programming language], but of course my knowledge is outdated, and then I ask one of the youngsters".

2.6.5 TmL4: Sense of Error

As a fourth TmL category, we distinguished having a *sense of error*, which encompasses the ability to detect errors in all sorts of data. Because errors – also small ones – can have a large effect, this is an important skill. The license advisor (P3) often scrutinises the reports he receives: "We have to do something with that when we see something conspicuous in such a report. Maybe it is a sloppy mistake; we have to detect that and pass it on". When errors in numbers are detected, this TmL category has some overlap with the next category, *sense of number*. We have often observed the combination of these two TmL categories in the engineers' tasks. In the next section, a combination of TmL categories, we will provide an example of this.

2.6.6 TmL5: Sense of Number

Handling numbers sensibly is very important for engineers. It is essential to understand what a certain number means and how to interpret such numbers. When using software, for example, the input of the right numbers and interpreting the output numbers correctly are crucial. The technical writer (P12), with a background in electrical engineering, speaks about a kind of "numeracy" that is needed, which includes handling units, for example, to know the difference between m (milli) and µ (micro). *Sense of number* not only often combines with *sense of error* but also with *technical software skills*. In the combined TmL section, we discuss this.

2.6.7 TmL 6: Technical Creativity

The sixth TmL category, *technical creativity*, is a particular skill which encompasses cleverness, experience and puzzle-solving abilities, especially for engineers who design. The mechanical engineer of large cooling systems (P6) solves a lot of puzzles in her work: "We are specialised in solving puzzles. It is puzzling and boggling and calculating. Is it correct, is this all?" When programming software, technical creativity plays an important role, she explained: "My colleague, for example, designed a small programme in Excel™ to calculate the costs of pipe lengths. Considering all the variables, it is not so difficult in a mathematical sense; it is just puzzling with formulas".

2.6.8 TmL 7: Technical Drawing Skills

The seventh and last TmL category entails *technical drawing skills*. These skills include understanding and interpreting technical drawings and, for some engineers, producing them as well. An important component of these skills is spatial insight. Although this skill is merely domain-specific and applies, for example, more to the domain of built environment (e.g. construction) than to the domain of applied science (e.g. chemistry), the permit advisor with a background in environmental engineering (P3) also needs this skill when he interprets technical drawings.

> I need spatial insight so that I understand what this thing looks like from the top, side, and front. Because I know that when there is a driveway on the right in the drawing, it is on the other side of the section drawing. You have to be able to think in planes and think spatially.

2.6.9 Combination of TmL Categories

As mentioned before, because the tasks of engineers are often complex, multiple TmL categories were assigned to many fragments. The TmL category *sense of error*, for example, often combines with a *sense of number*. The sales engineer (P4) who sells stabilisation fins for yachts explained how he checks the numerical data of the yachts (e.g., sizes) for errors because "one-tenth or two-tenth can have a big influence on the outcome". The research assistant (P7) states the importance of context for a *sense of number* with the analogue of the speed of a car to detect an error (*sense of error*).

> When you calculate the speed of a car and the outcome is 6000 km/h, then you know there must be a mistake somewhere in the calculation. When it's about bare numbers, it is more complicated to detect errors.

Having *data literacy* does not mean the engineer always understands all the details, as explained by the technical writer (P12) of manuals for machines that produce digital chips. He combines *data literacy* with a *sense of error* when receiving input and explains that, as a technical writer, he knows a little about a lot. Looking at graphs and formulas of polynomials, for example, he often does not know which formula corresponds with a certain graph. However, he learns by doing; when seeing something strange (*sense of error*), he puzzles and searches until he understands (*data literacy*).

The sales engineer (P4) of the stabilisation fins for yachts integrates TmL categories *data literacy*, *technical software skills* and *technical communication skills* in his task of calculating power versus fin size. His Excel™ calculation tool, with endless formulas, embodies years of experience. When another company offers a fin of 3.5 m^2, where his company can offer only 2.94 m^2, he concludes that the competition is wrong or has a better solution. He then starts thinking and sparring with his colleagues (*technical communication skills*): "How can the competition offer this size for this amount of power (*data literacy* and *technical software skills*)?"

The permit advisor, with a background in environmental engineering (P3), whilst interpreting technical drawings also uses *data literacy* and *sense of number*. He explained: "In supporting reports, we have to be able to interpret the drawings. All these parameters, for example, 50 m^3, how much water is that? All these pipes, we have to understand how it works." To interpret the technical drawings, he needs *data literacy* (how pipes work) and *sense of number* (how much water is 50 m^3).

2.7 Engineers' Ideas and Opinions

With respect to the second research question on the ideas and opinions of the engineers, we discuss the findings in subsections regarding their previous mathematics education, the benefits of mathematics in general and the future of mathematics.

2.7.1 *Mathematics: Island or Mainland*

All engineers' previous education had been theoretical, mostly without context, professional tasks or products. They perceived their mathematics courses as islands, with no relation to the rest of their education. They asked themselves (and their teachers): "Why do we have to learn this? What has this to do with my major?" This had a significant influence on the motivation for mathematics, as the sales engineer (P4) explained that he perceived mathematics as something that just had to be done. It was not his favourite subject; he found other, applied, courses much more interesting.

Almost all interviewees thought mathematics should be taught in context to enhance student motivation and to recognise its purpose (only two interviewees did not spontaneously say this). They stated that their mathematics was completely separated from the rest of the courses and a real obstacle for many students. After all, higher technical professional education is about application! The engineer with a background in mechatronics (P8) once had a teacher who made a drawing of a car crash with a tree as an example to illustrate movement and to explain why the driver hit the steering wheel, and then he thought: this is useful!

Besides the advantage of context in mathematics for motivation, the sales engineer with a background in construction (P1) emphasised the importance of context for better understanding.

> I experience technical mathematics as hard because, at university, we had plain exercises in a book. It would have been better if there had been practice examples because I understand *water* × *air* much better than $a \times b$. But I still have the problem that when we calculate a tennis court, I do not recognise the same rectangle in a soccer field.

The technical writer with a background in electrical engineering (P12) also wished he had more knowledge of applied mathematics. He engaged in advanced mathematics during his education, but one time, during a presentation when someone demonstrated calculated forces in a forklift truck, he did not understand the calculations and wished he had been trained in applications "back then".

2.7.2 *Mathematics: Useful for Cognitive Skills*

Although engineers say they do not use advanced mathematics, the engineers who were former high achievers in mathematics emphasised the general importance of mathematics for analytic and logical thinking, learning and interpretation abilities. The business engineer (P14) recognised an indirect profit of her mathematics education. She said that she had not solved any equation since university, but her mathematical background guaranteed a certain level in the way of looking at things.

2.7.3 Mathematics: Even More Technology in the Future

For the future, the interviewees think that the skills they need will be the same as now, but that there will be even more computer tools and technology. They state that mathematics will always be very important because technical developments rely on it completely. The chemical engineer (P2) thinks one has to know and understand the basics in mathematics education, but the focus should be on comprehension of the software and not on performing complex calculations by hand. The sales engineer (P4) added:

> I have to say, calculus and such, I have never used it. Most of the time it is hidden in the software, and it would be nonsense to let someone calculate for a whole day what a computer can do in a minute.

The technical writer of electrical engineering (P12) took it a step further and provided an example of trial and error with a PID controller instead of understanding the software or calculating by hand.

> A proportional-integral-derivative controller (PID-controller) calculates with derivative and integral terms. There are an entry and an exit, and you try to set the exit in a way that the controller does what you want, based on the entry. But you do not have to calculate PID areas! There are just some variables you can set, and then you watch what happens. Trial and error. Nobody builds these controllers; you just buy them.

What implications do these results bear for the education of engineers with a background in higher technical professional education?

2.8 Conclusion and Discussion

The first research question concerned the identification of the TmL used in the professional practices of engineers with a background in higher professional education. In reply, seven main categories of TmL, summarised in Table 2, were identified. This inventory is in line with the generic categories of Bakker et al. (2006), Hoyles et al. (2002) and Hoyles et al. (2013), who found that the ability to interpret data and graphs (TmL1, *data literacy*), the ability of calculation and estimation, having a sense of error (TmL4, *sense of error*, and TmL5, *sense of number*) and the ability to communicate (e.g. technical information; TmL3, *technical communication skills*) are common TmL in workplaces within the technical and economic domains.

The earlier finding that mathematical skills are almost always mediated by technology is confirmed in this study (Hoyles et al., 2010). The category with the highest frequency is the second TmL, *technical software skills*, and it is assumed to be the most important TmL. Hoyles et al. (2013) identified TmL in the financial sector. A similar TmL appreciates the existence of a mathematical model underlying computer output. The software is then experienced as a white box (an element of TmL2, *technical software skills*). In engineers' workplaces, this is often, but not always, the case. Sometimes, the calculation tool is a black box, which is not always a problem, as we saw in the example of the Research Assistant and his spectrometer software and the example of the PID controller.

Some TmL mentioned in the literature were not found in our current study on engineers. For example, the TmL of model making is hardly used by engineers with a background in higher professional education, but is used at the academic level. On the lower end of the educational levels, Hoyles et al. (2002) found the precise entry of data as a TmL in the manufacturing sector. This was not found in the workplace of engineers either. We attribute this to the level of the interviewees' positions and educational backgrounds: They had attended higher professional education (bachelor's level), but were not academically trained (master's level; cf. Frejd & Bergsten, 2016). Furthermore, even within the same educational level, TmL are not merely generic skills. This study also points out that specific TmL are required for particular domains. *Technical creativity* and *technical drawing skills* are TmL that are specifically found in the technical domain and that, to our knowledge, have not been described in TmL literature before. In answer to the second research question on the opinions and ideas of engineers about their previous education and mathematics education in general, we found foremost that the engineers thought that mathematics should be taught in the context of professional tasks. This is not only important for student motivation but also to gain experience with, and to better understand, applied mathematics as it is used in the workplace.

For the future, they still see a core role for the TmL skills as identified above, but using, even more, calculation tools, software and technology. Most engineers pursue extra education for specific work areas such as statistics, software and project management and say they will continue to need to do so.

Several engineers advise that mathematics education should focus on using and understanding technical software. Students have to know the basics, but should not perform complex calculations that computers can do instantly by hand. To understand underlying software and to uncover the black box, it can be useful to use mechanical tools as an intermediate step to digital technology (Bakker, Wijers, Jonker, & Akkerman, 2011). The Advisory Committee on Mathematics Education (2011) states that employers think employees should study mathematics at a higher level than they use in their practices to provide them with confidence and versatility to use mathematics in new situations at work.

Although the engineers state that they do not use the advanced mathematics of their education, they emphasise the general importance of mathematics for analytic and logical thinking and also recognise technology to rely completely on mathematics. Gainsburg (2007) advocates a realistic view of mathematics. Bialik and Kabbach (2014) agree with this view and state that it is most likely that higher-order thinking skills support mathematical skills and not the reverse. However, Rajagukguk and Simanjuntak (2015) found improvement in students' critical thinking ability using integrated problem-based mathematics teaching kits implemented with ICT. Also, Huang, Ricci, & Mnatsakanian (2016) recommend "thinking through math" with meaningful mathematical experiences to enhance critical thinking.

The identified TmL and the information obtained from the remarks about the engineer's previous education and their ideas about mathematics education in general lead to some recommendations for mathematics curricula in higher technical professional education. In our view, mathematics education should be rooted in professional tasks and products, and TmL should be amongst the main learning goals because today's and tomorrow's engineers need these new skills in the modern workplace which is equipped with more and more advanced technology. Therefore, subsequent (design) research of applied mathematics courses in engineering education that aims to foster these TmL is necessary.

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Appendix 1

Table 1. Interview scheme

Subject	Question	Subquestion
Job	What is your job description?	
	What are your tasks?	
	Do you have collaborations and/or division of labour in your job?	
	What mathematical skills do you use in your job?	Do you use diagrams and/or tables?
	What software and equipment do you use?	What is the purpose of your software and equipment usage?
	Can you give examples of your tasks and demonstrate them, including software usage?	
Communication	With which parties do you communicate?	How is this communication?
	Do you ask for advice or aid?	From whom?
Previous education	What mathematical skills did you learn?	What are the most important skills that you have learned?
	What software and equipment did you use?	
	What are your experiences regarding your own mathematics education?	Did you use software and/or pen and paper?
	What are your opinions and ideas about mathematics education for higher technical professional education in general?	
Education at work	What mathematical and technical skills did you learn on the job?	Did you use software and/or pen and paper?
Future	Which skills do you need in 5 years' time?	
	What are your ideas about your lifelong learning?	

3 Teaching Strategies to Foster Techno-mathematical Literacies in an Innovative Mathematics Course for Future Engineers

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Abstract The workplace practices of engineers have changed due to the ubiquity of digital technology. So-called Techno-mathematical Literacies (TmL), seen as a domain specification of 21st-century skills, are essential for future engineers. How these TmL can be fostered in their education, however, is still unclear. To address this issue, we conducted a design study in which we developed a course in applied mathematics for higher technical professional education with TmL as central learning goals. This paper describes the design and implementation of the course in a first design cycle with 59 chemistry students. We focus on the teaching strategies that the lecturer used to stimulate the development of students' TmL. In classroom discussions, in the so-called feedback hours on which students' collaborative work on TmL was centred, context-based cases were discussed. Results include didactical, process, and inquiry-based learning strategies used by the lecturer.

Keywords Techno-mathematical literacies ● Mathematics education ● Engineering education ● Design research ● Inquiry-based learning

3.1 Introduction

Technical practices have changed significantly due to the ubiquity of information and communication technology (ICT), digital technology, and computer-driven equipment (Advisory Committee on Mathematics Education, 2011). Because of these changes, the development of 21st-century skills is becoming increasingly important for future work life (Voogt & Roblin, 2012). Science, technology, engineering, and mathematics (STEM) education plays an essential role in supporting the development of these skills, integrated with other competencies and content knowledge (Schleicher, 2012). Implementing and assessing 21st-century skills is known to be challenging (Ananiadou & Claro, 2009); in particular, there is a need to specify and promote these in STEM domains.

As specification of 21st-century skills for the domain of mathematics, the term Techno-mathematical Literacies (TmL) was introduced by Kent, Bakker, Hoyles, & Noss (2005). Bakker, Hoyles, Kent, and Noss (2006) defined TmL as functional mathematical knowledge mediated by tools and grounded in the context of specific work situations (p. 343). They consist of mathematical, workplace and software knowledge, multi-step calculation and estimation (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002), and the ability to interpret abstract data and communicative skills (Hoyles, Noss, Kent, & Bakker, 2013).

Mathematics is often hidden in the interface of the software and is regularly experienced as a black box, so, numerical or graphical output can be unexpected or ill-understood (Kent, Noss, Guile, Hoyle, & Bakker, 2007; Williams & Wake, 2007). TmL are rarely learned on the job and need to be developed explicitly (Hoyles, Noss, Kent, Bakker, & Bhinder, 2007). Therefore, they should be considered important learning goals in engineering education.

Ridgway (2000) researched the mathematical needs of engineering apprentices and concluded that mathematics in their work differs in important aspects from mathematics education, and stated that mathematics curricula should support the development of a broad range of skills, practised in contexts. Bakker (2014) investigated the implications of technology for what students need to know about statistics; is the required knowledge more, less or different? His answer is all three: because technology does the computations for the user, less knowledge about the exact formulas (e.g., standard deviation, SD) may be needed, but more advanced techniques are faced in workplaces. Moreover, knowledge of technology is needed to do the relevant statistics (e.g., knowing which of the different SD options in Excel™ should be used in a particular case).

TmL are intertwined with abstract mathematical knowledge and skills: communication about numerical or graphical data, for example, cannot exist without mathematical expertise in these data. Therefore, when TmL are central learning goals, what does this mean for mathematical content and pedagogy? Table 1 shows which TmL engineers with a background in higher professional education use in their technical practices (Van der Wal, Bakker, & Drijvers, 2017, see Chapter 2 of this dissertation). In spite of their importance, TmL are not explicitly addressed as learning goals in higher technical professional education in the Netherlands. Mathematics education, in particular, is still mainly theoretical with little workplace-related contexts. We notice a gradual introduction of software in these courses, but it is important that TmL also be introduced. How these TmL can be supported in mathematics education within technical higher professional education, however, remains unknown.

To address this gap, we conducted a design study to investigate how emphasis on technology use and communication about authentic cases, along with teaching basic mathematics, can promote students' TmL. Our longer-term goal is to design a sustainable course within the prerequisites of the program, which can be taught by multiple lecturers to hundreds of students (Roesken-Winter, Hoyles, & Blömeke, 2015). For this goal, we need proofs of principle, means of scaling-up, and an evaluation of the course's effectiveness. In previous research on TmL, the focus was merely on tools and tasks (e.g., Bakker & Akkerman, 2014; Hoyles, Noss, Kent, & Bakker, 2010). What strategies teachers use in their classes to foster the development of TmL in students, however, was yet to be investigated. Because we needed this information for our large-scale implementation of the new course, we analysed what teaching strategies the lecturer used to stimulate TmL reasoning in discussing context-based cases in the first design cycle (Bakker, 2018) of an innovative mathematics course for first-year students in life sciences. The research question of the first cycle was as follows: During the discussions of context-based cases, what teaching strategies did the lecturer use to foster TmL?

3.2 Background

In a rapidly changing world, influenced by informatisation, automatisation, digitalisation, and globalisation, knowledge is changing and expanding at high speed. Information sharing, teamwork, and innovation are key, and manual and routine work have become less important because computers and machines accomplish those tasks. New standards of what students should be able to do, instead of basic knowledge and skills of the past, are identified as 21st-century skills. These include critical and creative thinking, flexible problem solving, ICT literacy, and collaboration and communication skills (Binkley et al., 2012). Development of 21st-century skills will require systematic instruction and additional resources, over and above what is common in current practice (National Research Council, 2013).

Because mathematics is at the core of what computers do, its role increases together with technology. Within STEM education, mathematics appears to be a particularly good fit for the purpose of supporting 21st-century skills, and therefore, its content and pedagogy need to change (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017). TmL categories can be linked to general 21st-century skills. Data literacy, sense of error and number are considered specifications of critical thinking. Creative use of software skills and technical drawing skills, and technical communication, reflect essential skills for communication with supervisors, customers, colleagues, etc.

In this study, we focus on engineering education. Therefore, in preliminary research, we administered an interview protocol (N = 14) on the use of TmL by engineers in their daily practice. We identified seven TmL categories, listed in Table 1 (Van der Wal et al., 2017, see Chapter 2).

Table 1. The seven TmL categories that engineers use in their work

	TmL category	Description
1	Data literacy	The engineer is able to analyse and interpret technical data and graphical representations, draw conclusions and take action accordingly
2	Software skills	The engineer is able to use professional software, e.g., Excel™ as calculation tools
3	Technical communication skills	The engineer is able to communicate technical information with colleagues, customers, supervisors and other parties
4	Sense of error	The engineer is able to check and verify data and detect errors
5	Sense of number	The engineer is able to handle and interpret numbers sensibly
6	Technical creativity	The engineer is able to produce creative solutions to puzzles and problems (by using, e.g., cleverness or experience)
7	Technical drawings skills	The engineer is able to understand and produce technical drawings (by using, e.g., spatial insight)

To foster the development of TmL, we chose to use an inquiry-based learning (IBL) approach. Inquiry, the use of open questions, is playing an important and growing role in science education, as science is a question-driven process. In this way, inquiry learning mimics as closely as possible the actual pattern of inquiry in science practice. Inquiry in authentic activities motivates students to acquire, understand, and apply science concepts (Linn, Songer, & Eylon, 1996; Edelson, Gordin, & Pea, 1999). As the National Research Council (1996) stated: “Scientific inquiry refers to the diverse ways in which scientists study the natural world and propose explanations based on the evidence derived from their work” (p. 23). In this way, inquiry learning reflects the nature of science inquiry. Chu and colleagues (2017) defined IBL as follows: “A learner-centered approach focussing on questioning, critical thinking and problem-solving. The learner is actively involved in formulating the question/naming a problem” (p. 7). It is a learning process in which students are engaged to stimulate an inquiry mindset and develop understanding and skills (Anderson, 2002), supported by the teaching strategy of using process-focussed questions. According to the NSES (1996), inquiry refers to a learning process in which students are engaged. It is said to be an active learning process – “something that students do, not something that is done to them” (p. 2).

The idea of inquiry is not new (Barrow, 2006). Dewey (1910), a former science teacher, advocated the need for active science practice in science learning rather than the passive reception of knowledge. In this model, there is a central role for the student and for the teacher as a facilitator and guide. Current support for inquiry-based learning comes from cognitive science, which emphasises the importance of authentic contexts for learning (Collins, Greeno, Resnick, Berliner, & Calfee, 1992).

Many studies of pre-college instruction have shown improved student learning as a result of inquiry approaches (Savelsbergh et al., 2016). At college-level, findings are mixed on whether inquiry can significantly change student learning or attitudes toward science (Gormally, Brickman, Hallar, & Armstrong, 2009). The American Association for the Advancement of Science (1993), however, stated that inquiry-based teaching methods provide students the opportunity to discuss science and are the best path to scientific literacy. For the designed course in our study, IBL methods were used in the classroom discussions to stimulate creativity, communication, and deep understanding of problem- and context-based learning.

3.3 Design rationale

In the design study described here, we used the methodological orientation of design-based research (Hoadley, 2005; Bakker, 2018) or educational design research (Plomp, 2013), which evolved to bridge the gap between research and practice. It involves developing both theoretical insights and practical solutions in the real world for complex educational problems. The process consists of flexible, iterative cycles of problem analysis and design of the intervention, conducting the intervention, and reflection to produce design principles and adjustment of the design (Cobb, Confrey, Lehrer, & Schauble, 2003). The design process is based on theory and practical experiences, and typically requires teamwork (McKenney & Reeves, 2012).

3.3.1 Background of the study

In higher technical professional education in the Netherlands, there are ongoing discussions regarding mathematics education. There is variation both in the expectations of employers and in topics and level of curricula (Van Asselt & Boudri, 2013). What all courses have in common, however, is that they are mainly theoretical with very few contexts and workplace-related examples. Classes are taught mainly based on a transmission model, which means transmission of knowledge from an external source to the learners (Vermunt & Verloop, 1999). This is the situation also at Avans University of Applied Sciences, where the study took place. This kind of teaching has come under pressure, because it is assumed to lead to inert knowledge, meaning that students may not be able to use this knowledge to solve problems in practice (Brown, Collins, & Duguid, 1989). It may also have a strong negative association with students' mathematics dispositions (Pampaka & Williams, 2016).

At the School for Life Sciences and Technology, part of Avans University of Applied Sciences, the introductory mathematics course previously also consisted of abstract mathematics without context, followed by a written test. A few years previously, pilot studies had been conducted with word problems in the mathematics course. However, students experienced difficulties with recognising and interpreting the mathematics in the text, performance remained low, and the initiative was terminated. After returning to abstract mathematics, the lecturers experienced continuing low student motivation and performance. Furthermore, students did not understand why they have to learn mathematics, and, in the experience of the lecturers, they often did not recognise the mathematics when it was used or needed in other courses.

We suspected that mathematics anxiety played a large role in this phenomenon. In general, roughly 20% of students appear to suffer from high mathematics anxiety (Ashcraft & Ridley, 2005), which is defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). Although chemistry students are expected to suffer less mathematics anxiety on average, this appeared not to be the case in the observation of the lecturers. Therefore, a significant change in the introductory mathematics course was imperative.

3.3.2 Design team

The course intervention was designed by the researcher, in collaboration with an interdisciplinary design team of three lecturers from the School for Life Sciences and Technology, the School of Built Environment and Infrastructure (both part of Avans University of Applied Sciences), and the Institute for Engineering and Design (HU University of Applied Sciences Utrecht). These lecturers helped in creating support for innovations, contributed expertise from different technical domains, and provided the technical contexts. The design team met every 3 weeks over approximately 8 months. The tasks of the members consisted of attending the meetings, reading given literature, providing input for the design, developing cases in their specific technical domains, and reviewing each other's cases. We elaborate on these cases in the next section of this paper.

3.3.3 Design premises

The design of the course was based on several premises, provided by the input from the literature, the interview study at engineers' workplaces (Van der Wal et al. 2017, see Chapter 2), and the professional experience of the team members. The learning goals were based on the TmL categories of the interview study and the engineers' opinions and ideas regarding their previous mathematics education. They all experienced it as an island with limited relevance and claimed that it would have been better if mathematics had been taught in context with professional tasks, and related to the other subjects of their major. Furthermore, they assumed this change would make students both more motivated and better prepared for the workplace, where mathematics is always encountered and used in context. This is in line with the results of several studies. Herrington (2006) claimed that more student-centred, problem-based, and technology-enriched higher education learning environments can engage students, and may enhance learning and retention.

The course should contain a firm base of pure, abstract mathematics of sufficient level because TmL and mathematical knowledge go hand in hand. We implemented a slow progression in mathematical difficulty and the production of calculation tools. Because the course is designed as an introductory mathematics course for every technical domain, such as electrical engineering, computer science, and chemical engineering, the course should cover pre-calculus, with optional adjustments for specific needs in different domains (see Appendix 1 for the mathematical topics).

In some of these domains, the curriculum consisted of just one mathematics course, and it was, therefore, necessary to ensure the course would be sufficient for these students' mathematical needs. In close dialogue with those involved, we chose to add a basis of calculus with a focus on qualitative understanding, rather than computational rules and calculations. For other technical domains, this new course with basic mathematics and qualitative understanding of derivatives and integration was assumed to provide a basis for subsequent courses in calculus.

A research technique often used in design studies is the development of a hypothetical learning trajectory (HLT). Simon (1995) first defined HLT as being comprised of the goal for student learning, the mathematical tasks that will be used to promote student learning, and hypotheses about the process of student learning. The hypotheses and the tasks are interdependent; tasks are chosen based on the hypotheses and vice versa. With this instrument, predictions can be empirically tested, so an effective learning strategy can be developed (Clements & Sarama, 2004). In later work, Simon added a framework for thinking about the learning process, the selection of mathematical tasks, and the role of these tasks in the learning process (Simon & Tzur, 2004). The HLT that the design team developed, described the aforementioned premises, the starting situation with the pre- requisite mathematical knowledge of the students, the learning goals, the conjectures about students' learning, teaching instructions, the structure of the course, general practicalities and the rationale of the mathematical and pedagogical choices, all of which we elaborate in the following sections of this paper. Because of the uncertain nature of the hypotheses, the HLT could be adjusted at any given moment. Parts of the HLT are presented as examples in Appendix 2.

3.3.4 Course structure

The new course included two parallel learning tracks. In the first track, students practised pre-calculus topics in ALEKS™, an electronic learning environment, to build a strong mathematical base for the TmL of the second track. For each technical domain, the lecturer has the option to add specific topics, for instance, trigonometry. Students can work individually with this software, outside the classroom, but can ask questions during class hours. Because mathematical self-concept and self-efficacy have been found to have a negative association with mathematics anxiety (e.g., Meece, Wigfield, & Eccles, 1990; Pajares & Miller, 1994), we assumed that this system could contribute to reducing mathematics anxiety, for students can work at their own level and their own pace, creating experiences of success.

A screenshot of a representative exercise in ALEKS™ of the first objective (first and second week) is presented in Fig. 1. The ability to use the quadratic formula is necessary for the first case of the second learning track, in which the proton concentration of weak acids is modelled.

The second track, aiming for TmL, consists of working on complex, guided cases in groups of two or three students during class hours, on roughly the same pre-calculus topics as in the ALEKS™ track. Students can work on each case for 2 weeks (6 weeks in total) and the seventh and final week is dedicated to completion. The first track focusses on individual learning and the second on collaborative learning. There is no prescribed textbook, because, in our experience, students merely use books for exercises and rarely read the textual explanations. Therefore, we decided that the short texts in the cases would suffice. For each case, a rubric is available for the students and the lecturers (see Appendix 3 for an example). In the method section, we discuss and elaborate upon the design of these cases.

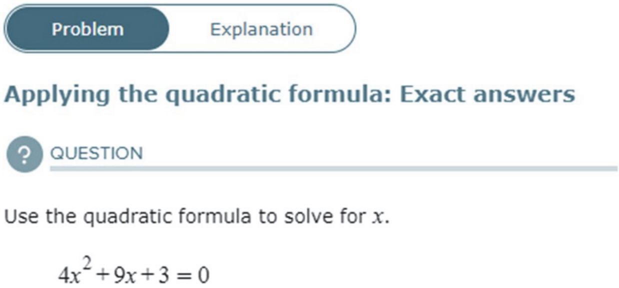

Figure 1. Screenshot of a representative exercise in ALEKS™ regarding topics of the first and second week of the course

The study load for the course equals two ECTS (European Credit Transfer and Accumulation System), one for each educational track, which is equal to two tracks of 28 h each. To pass the course, students have to master 90% of the topics of the ALEKS™ track. Additionally, they submit their documents for all cases. The summative grade follows from calculating the average grade of the three rubrics, where in each rubric, 0, 1 or 2 points can be scored for every question, and the sum is divided by the total score to get the grade. If students fail the review of the cases, they have to pass an oral exam. As for pedagogical choices, we decided to take an inquiry-based, rather than the aforementioned transmission approach. With collaborative learning and classroom discussions, we administered student-centred activities (Teo, Chai, Hung, & Lee, 2008), which are increasingly used in education in general (Howard, McGee, Schwartz, & Purcell, 2000).

The course is built on weekly 4-h classes for 7 weeks in total. Table 2 depicts the class structure. In every first hour, the case of that particular fortnight is presented by the lecturer, with a short mathematical instruction added. In every second and third hour, students work on the case in their group, without the lecturer being present. In every fourth hour, the feedback hour, three groups of three students present their work and are queried by the lecturer who is seated among the students in the classroom. Helping each other is the motto of this hour, and creating an open atmosphere is imperative for students to feel safe while presenting, thereby reducing mathematics anxiety.

For complex skills, such as TmL, classroom discussions can stimulate understanding and competence. In these feedback hours, the focus is on assessment for learning, which should have priority over assessment of learning according to Schuwirth and Van der Vleuten (2011). The feedback hours also have the function of testing progress, which has a positive influence on student behaviour by discouraging binge learning (learning in a limited period of time, usually just before the test) and enhances future competence and retention of knowledge (Schuwirth & Van der Vleuten, 2012). The choice to assign three clusters of topics in so-called objectives of 2 weeks each in the educational track of ALEKS™ has the same goal.

Table 2. Weekly class hours

1st hour	Introduction/questions with lecturer
2nd hour	Collaborative work without lecturer
3rd hour	Collaborative work without lecturer
4th hour	Feedback hour with lecturer

As mentioned before, we focussed on inquiry-based learning in the tasks of the cases and in the feedback hours to foster and develop inquiring minds and to help students discover knowledge for themselves. The feedback (how I did it), feed up (where am I going), and feed forward (what is the next step), we used in the feedback hours were assumed to be more effective on the level of process and self-regulation than on the level of tasks (Hattie & Timperley, 2007). Therefore, students were asked how they approached the problems, rather than what their answer was. The IBL question strategies that were used in the feedback hours can be found in Appendix 4 (Doorman, Jonker, & Wijers, 2016).

The students in the classroom were expected to contribute actively to the presentations given by their peers. Firstly, presenting groups were asked to show what went well, in order to stimulate an experience of success before addressing the problems they were experiencing. During every feedback (fourth) hour, students were asked to fill in a ‘feedback form’. This form is assumed to stimulate student awareness of feedback and activation of its use. In this form, students can elaborate on the feedback they received and how they processed it, as well as the feedback given to other groups, and how they could use that for themselves. Finally, students could describe the contribution to the case by each team member. The feedback form was submitted, along with the cases.

In this course, we chose case-review as the form of assessment, because this form has several advantages. We expect students to get engaged in the feedback hours to improve their documents and to learn from each other, whereas written tests can cause too much focus on learning for the test, and also cause much stress for students with a fear of failing. Another important goal of the classroom discussions in the feedback hours is to bridge the two educational tracks of the course, the abstract track of ALEKS™, which is needed to be able to work on the cases, and developing TmL and the applied mathematics of the cases. Where students use x and y in ALEKS™, they use different symbols in the cases, but on the same mathematical topics. In the HLT, specific suggestions are made for lecturers on how and where they can support this bridging.

3.3.5 The cases

The guided cases consist of short pieces of text, pictures, and questions, and are focussed on TmL. Although it is difficult to pinpoint specific behaviours relevant to TmL, especially regarding Tml *technical creativity*, the design team tried to create questions that address specific aspects of TmL. TmL *software skills* are practised in all cases using Excel™ to build calculation tools. TmL *data literacy* takes up a central position in the cases because of questions focused on analysing, interpreting, and searching data. TmL *technical communication skills* are practised in exercises in which students are asked to explain and elaborate, and during the classroom discussions in the feedback hours (as explained later in this section). As for the TmL of *sense of numbers*, we expanded the definition of this TmL category (Van der Wal et al. 2017, see Chapter 2) to number sense, thus including symbols and formulas. The TmL *sense of error* and *sense of number* are addressed by questions of trial and error, interpreting numbers, symbols and formulas, searching for errors, and estimation. TmL *technical creativity* plays a role in questions in which students are asked to create a solution strategy. The last aspect of TmL, *technical drawing skills* are not used in the cases of the first cycle because this TmL category is not common in the domain of Chemistry.

The titles of the two cases that the design team developed for the School of Life Sciences and Environmental Technology are *Solutions and acids*, and *Bacterial growth*. The third case, *Change*, was provided by the School of Built Environment and was adjusted for the domain of life sciences. The cases have several links to other courses in the curriculum. In the section on solutions in the first case, the mathematical topics are linear functions and equations, and in the section on acids, quadratic functions and equations are addressed. This case starts with a worked example because for novice learners it is more effective to add an example before the problem-solving phase (Van Gog, Kester, & Paas, 2011). The second case discusses bacterial growth by means of exponential, logarithmic, and rational functions and equations. The last case addresses the subject of change through a qualitative approach to calculus. In the first part of this case, distance, time, and speed of a runner are chosen, because this context stimulates an intuitive understanding of change. In the last part, the design team added a chemical topic. All cases consist of short pieces of text with pictures and contain 15–40 questions, and the third case also uses apps in Geogebra™.

In the cases, students are stimulated to develop TmL with other types of tasks that go beyond mere calculations. For example, TmL *sense of error* was practised in the last question of the first case on solutions and weak acids. In this section, the concentration of the protons (H^+) in weak acids is modelled as a function of the acid concentration m. The variables K_a, the acid constant, and r, the added proton concentration, are fixed in cells B1 and B3. Students are asked to find the error in one of the two solutions of the quadratic formula in linear form for the calculation of the proton concentration. In Appendix 5, the Excel™ screenshot is given. Can you find the error?

$$= (-(\$B\$3 + \$B\$1) + SQRT((\$B\$3 + \$B\$1)^2 - A7 * \$B\$1))/2$$

In the electronical supplementary material, the integral version of this case is given, with the answers, the rationale on TmL, and parts of the HLT.

3.4 Method

3.4.1 Participants

The implementation of the first cycle with 59 first-year students (26 female, 33 male) of the Chemistry major, aged 17–21 years old, divided over three groups, was conducted by the researcher and a member of the design team from the School for Life Sciences and Technology. The new course was named Applied Mathematics and implemented in the curriculum of the Chemistry major in the second half of the second semester. Because of the different learning goals, a control group would not have yielded a fair comparison.

3.4.2 Data collection

Data collection concerning the teaching strategies consisted of video-recordings of all feedback hours, with their TmL-focussed class discussions. These meetings were filmed every week for the two classes that were taught by the researcher, resulting in 12 lessons of 45 min of video. The camera with a built-in microphone was set up in the middle of the classroom, facing the digital board and the presenting students in the front. Because the lecturer was ill in the fifth week, there were no classes that week, and therefore the video data consist of 12 instead of 14 lessons. The students of these two classes were asked to sign consent forms for using this material for research purposes; we used the data of those 30 students, out of a total of 35, who signed for approval.

3.4.3 Data analysis

Data analysis was performed using a version of the constant comparative method (CCM). Watching all video recording chronologically, and, with the HLT and research question as theoretical orientations, initial conjectures were generated, and when necessary were revised during further analysis of subsequent episodes (cf. Cobb & Whitenack, 1996). In this way, teaching strategies of the lecturer relevant to conducting feedback hours and to stimulating TmL learning were identified in the first two recorded feedback hours and tested in the successive feedback hours. The analysis led to a list with occurrences of teaching strategies used during every student presentation (each lasting approximately 15 min). These occurrences are meant as a descriptive overview of teaching strategies used, and not as a generalisation.

In each second and third hour of the 4-hour schedule (see Table 2), the students worked collaboratively on the cases. Often, they did not finish the assigned work in the given time. During the feedback hours, when students presented their work, the lecturer continuously promoted discussion by using particular IBL strategies, until some intended TmL reasoning among the group was observed and most students expressed understanding (as judged during teaching). Hence, we concluded that the intended TmL reasoning was realised. In the analysis, we distinguished pedagogical and process strategies that we considered conditional for conducting each feedback hour, from specific IBL strategies that stimulated elicitation of TmL. In this paper, we focus on the latter. We investigated and listed how the lecturer used IBL questions in the classroom interactions, and looked for moments in which the lecturer used certain teaching strategies, such as asking students to elaborate on what they did, or how they solved certain problems (*technical communication skills*), to find an error (*sense of error*), leading discussions on the several expressions of a formula (*sense of numbers* and *technical creativity*) or discussing the interpretation of data (*data literacy*). In case 2, for example, an expression of an equation had to be transformed (*sense of numbers*). After discussing how the students handled this problem and listing the several answers students had given, the lecturer encouraged the class to formulate a general approach to these kinds of problems. Another member of the design team, who was not involved in teaching the course, was provided with the list of teaching strategies that the researcher had identified and asked to analyse independently 1.5 h of the video data, to compare findings and to test agreement on interpretation (peer examination). He identified several teaching strategies from the provided list, but also some others. Most of these other strategies appeared to be similar to the ones already identified but were formulated differently or were considered part of another strategy. For example, we agreed that "asking a student how a particular calculation changes with different numbers" (as formulated by the second coder) could be seen as a part of the strategy of "asking deeper questions about data, tables, formulas, and figures" as formulated by the first coder. Two identified teaching strategies were explicitly new and were added to the list.

3.5 Results

3.5.1 Pedagogical and process strategies

The pedagogical and process strategies that were frequently used by the lecturer are provided in the first two rows of Table 3 with the occurrences in brackets. Firstly, we saw the deliberate use of expectation and process management, because the structure and way of assessment of the course were more complex than, and different from, most other courses of the curriculum. Every week, the lecturer set out the program of the feedback hour, chose groups of students to present, and often reminded the students to open the feedback form on their laptops, and she repeated how to use it. Secondly, she often elaborated on the learning goals of the course. For instance, she mentioned that being able to explain your work (TmL *technical communication*) is an important skill for future work life, which requires ample practice.

Furthermore, we saw a constant effort to achieve a good and safe atmosphere in the class, including using positive phrasing, emphasising the aim of helping one another, stimulating applause for one another, and using humour. Students started their presentation of their work with a part that went well to stimulate a success experience. Later on, they were asked where they got stuck so that the whole class could help. Finally, the students chose group names, and we had the impression that their choices, which were often playful, added positivity to the atmosphere in the classroom. We heard, for example, The Algebro's, SyntaxError, and The MathCrew, but also nonmathematical names.

Table 3. Teaching strategies used by the lecturer during the feedback hours, with numbers of occurrences in brackets

Pedagogical strategies
The lecturer stimulates that this hour is meant to help each other; Acknowledges the difficultness of the subject; Stimulates pride for their work in students; Formulates in a positive way and uses humour; Stimulates applauding for each other; Encourages feedback from students on the cases to further improve them for future students; Addresses possible feelings of frustration at students.

Process strategies
Stimulates working together more and emphasises not to divide sections between group members; Stimulates to simmer on the problem after class; Checks the progress of the groups and adjusts accordingly; Stimulates to write down the feedback after presentation; Stimulates to write down feedback of others that they can use themselves; Asks the contribution of each student to the product; Explains the rationale of the assignment; Asks what students need to continue before ending the class.

General IBL strategies
Asks to show something that went well (stimulating success experience); Asks where students got stuck; Structures answers and theory and recapitulates what students say; Asks what the thought process was; Starts problem-solving on whiteboard and asks students to finish; Asks how students will proceed with this problem; Gives a tip; Compares used heuristics in groups; Explains connection between math and other courses or future profession; Discusses the way one can work on these cases, advantages and disadvantages of strategies.

TmL specific IBL strategies
For all TmL: Asks deeper questions about data, tables, formulas, and figures. *For TmL technical communication:* Stimulates to rephrase in own words; Stimulates to take a helicopter view in elaborating; Stimulates explaining to each other; Asks a student to elaborate on the answer; Asks class to formulate a general strategy. *For TmL Sense of number and sense of error:* Stimulates to use numbers that are realistic or easy; Asks class to spot an error; Let students discover their mistake by stimulating thinking about the logical answer. *For TmL technical software skills:* Asks a student to show and explain their Excel™ calculation tool live.

3.5.2 IBL strategies

Relating to fostering the development of TmL, we saw a variety of IBL strategies used, both general and TmL-specific. When students presented their work, the lecturer constantly structured the process with comments and recapitulated the explanations and answers of the students. Then further questions were asked to deepen the thought processes, and classroom discussions on the topic were encouraged. For example, students were asked to spot an error, to formulate a general approach to a certain problem, or to elaborate on the general subject of the case.

3.5.3 Challenges

The lecturer also faced some challenges. In classroom activities, it proved not always possible to engage every student. In the videos, we saw a few students who were distracted by their phones or were yawning and almost sleeping. In late classes, students could be tired, or perhaps some experienced the assignment as too simple. A few students complained about the difficult phrasing of some questions. The lecturer then asked how they would formulate them. The students often concluded that they had not read the questions properly. Furthermore, after working in their groups in the 2 h without the lecturer, not every group had finished the same amount of work. The lecturer had to consider these differences and choose presenting groups carefully.

3.5.4 Classroom dialogues

To illustrate findings, we present two examples of classroom dialogues and discussions, in which the lecturer used IBL strategies to support the learning of TmL. In the first week, TmL *technical communication skills* were practised in the feedback hour with the whole class in the first case. This TmL category was continuously stimulated in every feedback hour, but in the excerpt below, we focus explicitly on *how to explain something*. Students often struggled with this skill; they tended to start with details and often could not see the whole picture. The lecturer (L) tried to practise how explaining something could be structured with the presenting students (PS) and the other students in the classroom (S), to generate a template for this aspect.

L Can you explain what the example at the start of this case is about?

PS1 Uh…we fill in variables in the equation

L That's right. But let's take some steps back. If you look at the worked example, which we started with, which we build upon this whole case, what was it about?

PS1 Uh…it is about the H+ concentration where you add a certain volume

L Yes, that is a part of it, indeed. OK, the goal of this is to learn to explain things, what you do. That can be difficult, and that is why we practise it.

Student 2 [in class], can you try to explain the example?

S2 There is a solution with a pH of 1.5, and by adjusting the formula and by adding variables, you can calculate other concentrations

L That is what we do after the example, indeed, but let's look at the example again, what do we start with?

S3 [in class]. You start with 20 mL of 1 mol per litre HCl, then we add a certain volume of 0.25 mol per litre HCl, so you get a solution….with pH 1.5

L Indeed. So, what is important when you have to explain such a thing, is that you start with what you have, and then explain where you want to go

In the next example from the feedback hour of the third lesson, regarding the case about bacterial growth, the lecturer tried to focus on the TmL of *sense of error, software skills,* and some *technical creativity* by asking specific IBL questions. First, she asked students to show the formula in Excel™, to stimulate the students in developing *software skills*:

L How did you implement a generation time of 0.5?

PS4 We used the same formula but made another timetable, 0, 0.5, 1…etc.

L Can you show the formula?

The lecturer now focussed on the formula and asked questions about what to expect for the number of bacteria, to give the students the opportunity to discover that something was wrong (*sense of error)*.

L Ok, suppose we start with one bacterium, can you change that? [S4 adjusts the cell in Excel™ with the aid of other students (technical software skills)]. So, when we start with one bacterium and the generation time is 1 h, how many bacteria do you have after 1 h?

PS5 2

L 2, indeed. And when the generation time is 0.5, how many bacteria do you have after 1 h?

PS5 4

The students saw that they did not have this amount. Then the lecturer tried to stimulate *technical creativity* by asking for another solution to this problem. She mentioned that the students used a timetable from 0, 1, 2….and changed that into 0, 0.5, 1, 1.5, 2.5…, and acknowledged that this solution was a smart way to adjust the generation time, but then asked whether there might be an alternative way. If the timetable was kept as in the beginning, how could the formula be adjusted? The lecturer helped the students by suggesting the solution was in the adjustment of the formula. She could have chosen not to disclose this point, to challenge the students even more. A student in the class then replied with "dividing by a half". Subsequently, the lecturer practised *sense of error*, by asking whether one has to divide by half or multiply by a half to obtain four bacteria after an hour.

3.6 Conclusion and discussion

In this study, we addressed the question of what teaching strategies the lecturer used to foster techno-mathematical literacies in an innovative mathematics course. Through collaboration within an interdisciplinary STEM design team, we designed an innovative mathematics course with a central role for TmL. Firstly, TmL learning is stimulated by using context-based cases, adjusted to specific technical domains. The guiding questions in the cases are designed to stimulate TmL by not only focussing on producing calculations but also, for example, detecting errors (TmL *sense of error*), elaborating (TmL *technical communication*) and stimulating creating alternative solutions (TmL *technical creativity*). The feedback hours, with their classroom discussions, and usage of IBL questions, seem to contribute to the learning of TmL. The lecturer used a variety of teaching strategies for this purpose, including prerequisite strategies such as process management. IBL strategies are used to enhance learning in general, but also to specifically address TmL. Conducting the feedback hours appears to ask a lot from the lecturer, who has to take multiple roles, as a teacher, coach, discussion leader, and organiser.

The results of this first cycle function as a proof of principle, and the premises of the course, described in this paper, are an example of how the development of TmL can, in principle, be fostered. By choosing TmL as learning goals in mathematics education, the development of these skills is made explicit (Hoyles et al. 2007). In combining those learning goals with IBL teaching strategies in context-rich materials, we hope to contribute to well-prepared students who can transfer and apply knowledge to solve problems in their working future (Brown et al., 1989; Voogt & Roblin, 2012; Ridgway, 2000). In this paper, we did not address the collaborative work of the students in the hours without the lecturer present, nor did we report on the revision of the course after the first cycle. Because the administering of this course with its process and expectation management, and IBL approach, is demanding for the lecturer, we recommend training to prepare lecturers for this sort of TmL-focussed mathematics education. We also would like to create more open instead of guided cases, to stimulate increased development of TmL. With all the changes students and lecturers already face with this innovation, however, we will postpone this next step to a later stage.

For the second cycle, a large implementation for all majors of the School of Life Science has been administered, and because of the complex character of all the challenges involved, we decided to add a study on how lecturers can be effectively supported, in which we would build on very different literature, namely on teacher professional development. The second and third cycles are, therefore, the topic of a future report. A fourth design cycle will be devoted to measuring the effectiveness of the course by means of pre- and posttests. We hope that the design, premises, and implementation of this new course will be an example and inspiration of how mathematics education can contribute to the development of 21st-century skills.

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Appendix 1

Mathematical topics in the applied mathematics course

1 Linear functions, equations, and graphs.
2 Solving systems of linear equations
3 Quadratic functions, equations, and graphs.
4 Rational functions, equations, graphs, and computational rules.
5 Exponential functions, equations, graphs, and computational rules.
6 (Natural) logarithmic functions, equations, graphs, and computational rules.
7 Qualitative approach to derivatives and integration.
8 Optional: trigonometry, vectors, and matrices.

Appendix 2

Parts of the HLT with teaching instructions and TmL in case-questions.

TmL as learning goals

The applied mathematics course has a focus on mathematics in workplace contexts and a central role in fostering TmL. The TmL we use in this course are *data literacy*, *software skills*, *technical communication skills*, *sense of number*, *sense of error*, and *technical creativity*. In the cases, students use Excel™ to develop a calculation tool, as part of TmL *software skills*.

A teaching instruction for the feedback hour

In the feedback hour, two groups are invited to present their case. Firstly, ask the groups to present a part that went well and of which they are proud, to create an experience of success. Then continue to the part where students encounter difficulties. Try to help the students and activate the rest of the class by using inquiry-based questions which focus on the process, rather than the answers. What do you know here? What is the next step? How did you approach this? Is there another way to solve this? Explain your method to the class, etc. In the IBL form [Appendix 4] you can find more process related questions you can use.

Instructions for bridging abstract and applied mathematics

[In question 13 of the second case, students are asked to derive one formula from another with calculation rules]

During the feedback hour, after the formula is derived, ask the presenting group what kind of function this is (linear), what the main formula is of such a function ($y = ax + b$), and which variable represents x, y, a, and b, to support recognition of abstract mathematics in applied formulas.

Appendix 3

Part of the rubric of the first case

Question	2 points	1 point	0 points
1	Correct formula, incorrect layout and Excel™ formula, table and graph (including titles) correct	Formula correct OR Excel™ correct	Both incorrect
3	Equation formed and solved correctly with unity, and graph correct	Partly incorrect	Incorrect
7	Calculation posed and solved correctly and indicated which solution for *x* is right.	Partly incorrect	Both incorrect
8	Function/formula correctly posed and simplified	Partly incorrect	Both incorrect
9	Tables, graphs (including titles) and trend lines correct	Partly incorrect	Incorrect
13	Equilibrium equation and answer to the negative a correct	One of two correct	Both incorrect
14	Derive to quadratic equation and solving correctly	One of two correct	Both incorrect
15	Debugging correct		Debugging incorrect

Appendix 4

IBL questions for the feedback hour, Doorman et al. 2016, p. 42

IBL teaching strategies	Suggested questions
Allow students time to understand the problem and engage with it	Take your time, don't rush What do you know? What are you trying to do?
Discourage students from rushing in too quickly or from asking you to help too soon	What is fixed? What can be changed? Don't ask for help too quickly—try to think it out between you.
Offer strategic rather than technical hints	How could you get started on this problem?
Avoid simplifying problems for students by breaking it down into steps	What have you tried so far? Can you try a specific example? How can you be systematic here? Can you think of a helpful representation?
Encourage students to consider alternative methods and approaches	Is there another way of doing this? Describe your method to the rest of the group
Encourage students to compare their own methods	Which of these two methods do you prefer and why?
Encourage explanation	Can you explain your method?
Make students do the reasoning and encourage them to explain to one another	Can you explain that again differently? Can you put what Sarah just said into your own words? Can you write that down?
Model thinking and powerful methods	Now I'm going to try this problem myself, thinking aloud
When students have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed	I might make some mistakes here—try to spot them for me This is one way of improving the solution

Appendix 5

The Excel™ input provided to detect the error in the formula.

AANTAL | =(-($B$3+$B$1)+WORTEL(($B$3+$B$1)^2+A7*$B$1))/2

	A	B	C	D	E	F	G	H	I
1	Ka	6,30E-05							
2									
3	r	0,010							
4									
5	m	$[H^+]_1$	$[H^+]_2$	pH					
6									
7	0,00	=(-($B$3+B	-1,01E-02						
8	0,02	3,12E-05	-1,01E-02	4,51					
9	0,04	6,22E-05	-1,01E-02	4,21					
10	0,06	9,30E-05	-1,02E-02	4,03					
11	0,08	1,24E-04	-1,02E-02	3,91					
12	0,10	1,54E-04	-1,02E-02	3,81					
13	0,12	1,84E-04	-1,02E-02	3,73					
14	0,14	2,15E-04	-1,03E-02	3,67					
15	0,16	2,44E-04	-1,03E-02	3,61					
16	0,18	2,74E-04	-1,03E-02	3,56					
17	0,20	3,04E-04	-1,04E-02	3,52					
18	0,22	3,33E-04	-1,04E-02	3,48					
19	0,24	3,63E-04	-1,04E-02	3,44					
20	0,26	3,92E-04	-1,05E-02	3,41					
21	0,28	4,21E-04	-1,05E-02	3,38					
22	0,30	4,49E-04	-1,05E-02	3,35					

Appendix 6

Modelling with solutions and acids

In this case, students will work with linear and quadratic functions and create models. The case has been divided into 11 questions that each contain a topic with sub questions. Teacher guidance is shown in red, elaborations/answers in blue, and feedback hour questions in green. These questions are meant to stimulate students to think deeper and understand, and encourage discussion within the group or the classroom. This will focus on the TmL technical communication. The TmL involved are listed for each part. This first case does not include the TmL technical creativity. That will not be addressed until cases 2 and 3. The case will start with two preparatory questions, that do not need to be written down; they are intended as a warm-up for the students.

Elaborate the case in an Excel document. Put all equations in the correct layout using the equation editor and keep an eye on units used. The case will start with an introduction, an example and a few warm-up questions. Take your time and read carefully!

A. Concentrations and mixing solutions

An example of a linear function is:

$$y = 3x + 4$$

or in a different notation:

$$f(x) = 3x + 4$$

The graph of this function is a straight line (see figure 1). We say that y is a function of x.

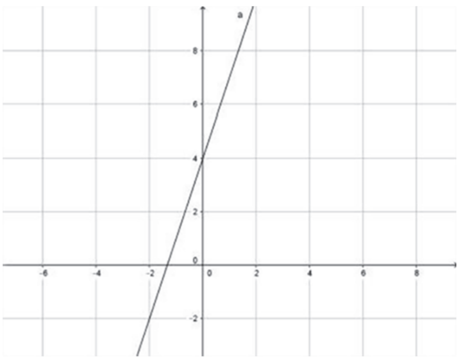


Figure 1. Graph of function y = 3x + 4

In the equation $3x + 4 = 6$ you set the function equal to a specific number, in this case 6. In fact, you are calculating for which x this function y is equal to 6.

Preparatory question: Calculate this. For which x is y equal to 6? Check in the illustration above whether your answer can be correct. Explain this.

$$3x + 4 = 6$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Mathematical equations of the form

$3x + 4 = 6$ or $2x + 6 = x + 8$

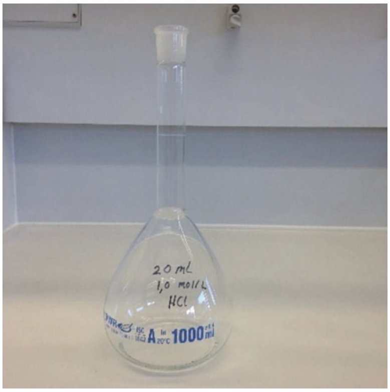
are linear equations and these can be solved easily with a bit of algebra. In a chemical laboratory we will often encounter problems that look slightly more complex, but which in fact come down to solve equations that are just as simple as the ones shown above. It may be hard at times to recognise the simple mathematics in a real problem. Look at the following example closely:

Example: A 1 L volumetric flask contains 20 mL 1.0 mol/L HCl. How many mL of a 0.25 M HCl-solution must be added to make 1.0 L of a solution with pH = 1.50?

Goal: A solution with pH = 1.50. This has $[H^+]$ = $10^{-1.50}$ = 0.032 mol/L.

We will now elaborate the example step by step.

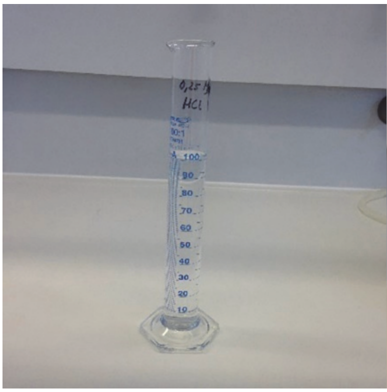
A 1 L volumetric flask contains 20 mL 1.0 mol/L HCl:

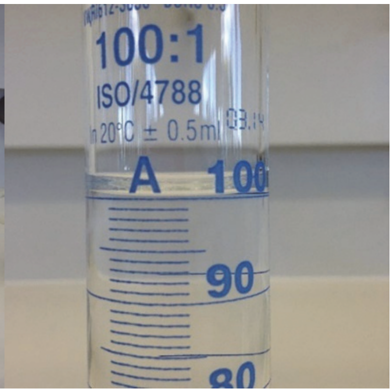


20 mL of 1.0 mol/L HCl contains 0.020 × 1.0 = 0.020 mol H^+.

(amount mol = concentration x volume)

We have 100 mL of a 0.25 M HCl solution of which a part has to be added to the volumetric flask to obtain a pH of 1.50.





A volume V (in L) of the 0.25 mol/L HCl-solution is added. So, the amount of HCl (in mol) that is added is:

$$0.25 \times V$$

The function for total amount (in mol) HCl in the volumetric flask is:

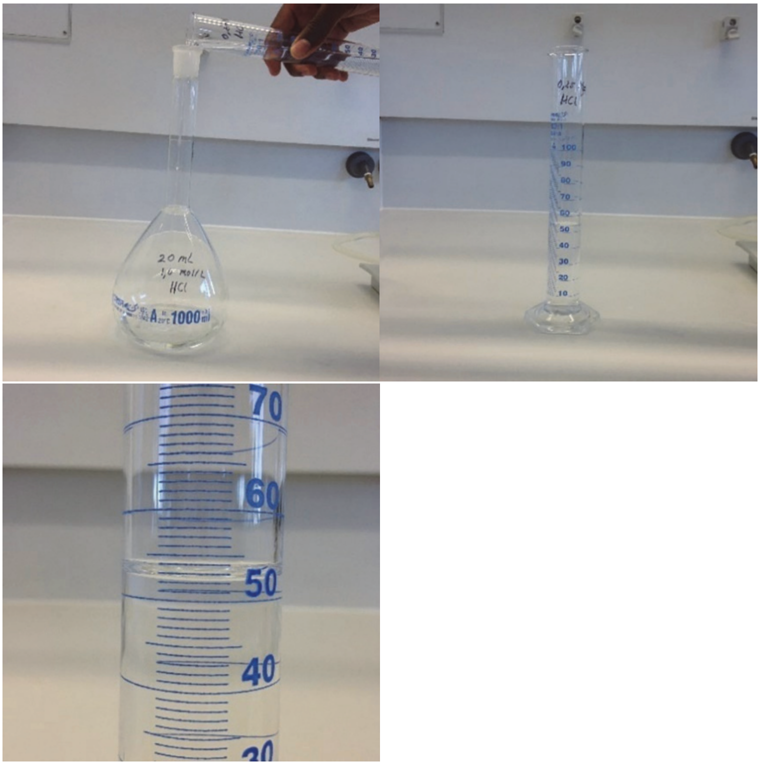
$$n(H^{+}) = 0.020 + 0.25 \times V$$

Because the volume of the volumetric flask is 1 L, the function for $[H^{+}]$ is also

$$[H^{+}] = 0.020 + 0.25 \times V$$

So $[H^{+}]$ is a function of the added volume V.

V is poured into the volumetric flask:



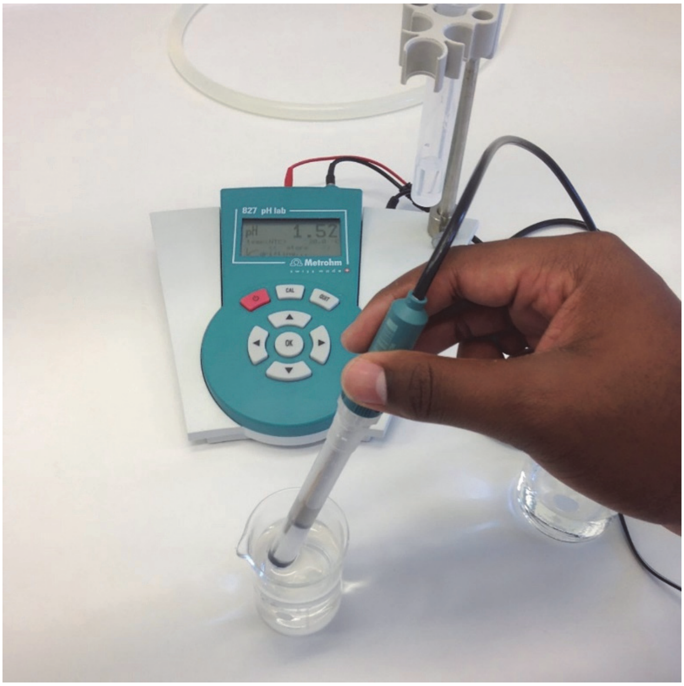
Preparatory question: how many mL has been added? How many mL is left? Read this off in the photo.

How many mL must be added to end up with an $[H^+]$ of 0.032 mol/L in the volumetric flask?

$$0.25 \times V + 0.020 = 0.032$$

From this, it follows that $V = 0.048$ L. So, 48 mL of the 0.25 mol/L HCl solution has to be added, and finally, the liquid must be topped up to 1.0 L with purified water. We will then have made a solution with pH = 1.50.

Preparatory question: We pour some of the solution in a beaker and measure the pH to check. Why isn't it exactly 1.50?



We will now perform the same steps as in the example, but with different values.

In this question the students will go to work themselves after a worked example. The question calls on TmL data literacy (in this case, interpreting the example and making tables and graphs), TmL software skills and TmL sense of number (symbols, numbers and equations)

1. A 2 L volumetric flask contains 10 mL 0.80 mol/L HCl. We have a solution of 0.50 mol/L HCl to add.

- Give $[H^+]$ as a function of the added volume V, shown in regular layout, using the equation editor.

- Calculate the amount of added volume V that is needed for a pH of 1.50. Explain your calculations.

- In Excel, make tables with $[H^+]$, the pH and the added volume V.

- Make a graph of $[H^+]$ as a function of the added volume V. This means that the $[H^+]$ will be on the vertical axis and the added volume V on the horizontal axis. Remember to add a suitable name for the graph and for the axes. Do the same for a graph of the pH and the added volume V.

A 2 L volumetric flask contains 10 mL 0.80 mol/L HCl

$$10\ mL \times 0.8\ \frac{mol}{L} = 0.0080\ mol$$

We now add a volume V (in L) of a 0.50 mol/L HCl-solution. So, we add $0.5 \times V$ mol HCL-solution.

$$n(HCl) = 0.5V + 0.008\ mol$$

HCl is a strong acid which separates completely into H^+ and Cl^-. The volumetric flask is 2 L. The concentration $[H^+]$ as a function of the added volume V in the 2.0 L solution is

$$[H^+] = \frac{0.5V + 0.008}{2}\ mol/L$$

$$[H^+] = 0.25V + 0.004\ mol/L$$

$$\text{pH} = -\log(0.25V + 0.004)$$

See Excel

How much of the 0.50 mol/L solution must be added to end up with a pH of 1.50:

$0.25V + 0.004 = 0.032\ or\ 0.03162 \ldots$ (concentration) or $0.5V + 0.008 = 0.064$ (amounts mol)

$V = 0.110 - 0.112\ L\ depending\ on\ rounding\ off = 110 - 112\ mL$ and with 2 significant numbers $1.1\ \times 10^3\ mL$)

Feedback hour: Check the solution of the equation in your Excel. How do you explain a possible small deviation? Rounding off in Excel. (TmL sense of error, sense of number)

For the next question the starting volume is changed. TmL sense of number, software skills.

1. Assume that the starting volume is not 10 mL HCl in the volumetric flask, but 30 mL.

- Give $[H^+]$ as a function of the added volume V and simplify your answer as far as possible.

-Calculate again how much added volume is needed for a pH of 1.50.

-Add this situation to your Excel-tool.

$$30\,mL \times 0.8\,\frac{mol}{L} = 0.024\,mol$$

$$n(HCl) = 0.5V + 0.024\,mol$$

$$[H^+] = \frac{0.5V + 0.024}{2}\,mol/L$$

$$[H^+] = 0.25V + 0.012\,mol/L$$

The equation to be solved then becomes

$$0.25V + 0.012 = 0.03162 \ldots.$$

With as the solution

$$0.25V = 0.01962\,mL$$

$$V = 0.7848\,L = 78\,mL$$

Or an equation with amounts of mol.

Feedback hour question: Why is there less added volume necessary than in the situation with 10 mL at the start? TmL data literacy; interpretation and understanding, sense of number

We will now generalise some values by using letters. This is called modelling. The advantage of modelling is that you can calculate different situations with only one equation.

Modelling. TmL sense of number (symbols and formulas), software skills and also data literacy (tables and graphs)

2. Assume that the starting volume isn't 10 mL or 30 mL, but a (mL or L, choose which and indicate which one you have chosen) HCl.

- Give $[H^+]$ as s function of the added volume V and simplify your answer as far as possible.

-Calculate again how much added volume is needed for a pH of 1.50. Pay attention: you will now have an expression with a in it.

-Add this situation to your Excel-tool, so that you can fill in various values for a and change them in one place. Use a cell reference for this.

$$[H^+] = \frac{0.5V + \left(\frac{a}{1000}\right) \times 0.8}{2} \; mol/L$$

This is when a is in mL, when it is not: without dividing by 1000.

$$[H^+] = 0.25V + \frac{a}{1000} \times 0.4 \text{ mol/L}$$

$$[H^+] = 0.25V + 0.0004a \text{ mol/L}$$

Check for $a = 10 \; mL$:

$$[H^+] = 0.25V + 0.004 \; mol/L$$

$$0.25V + 0.0004a = 0.032$$

$$V = 0.128 - 0.0016a$$

This can also be done with an equation of amounts mol, everything times 2 L, gives the same answer.

Feedback hour question: How could you check whether your answer is correct? By filling in a specific value of a, and calculate the answer "by hand" and compare with what Excel calculated. TmL sense of error, sense of number.

With choosing your own units: TmL sense of number, data literacy (tables and graphs)

3. We will model a further step. Assume that there was a (mL or L) of b mol/L HCl present.

- Give $[H^+]$ as a function of the added volume V and simplify your answer as far as possible.

- Calculate again how much added volume is needed for a pH of 1.50. Pay attention: you will now have an expression with a in b it.

- Add this situation to your Excel-tool, so that you can fill in various values for a and b and change them in one place. Use a cell reference for this.

$$[H^+] = \frac{0.5V + (\frac{a}{1000}) \times b}{2}\ mol/L$$

This is when a is in mL, when it is not: without dividing by 1000.

$$[H^+] = \frac{0.5V + (\frac{ab}{1000})}{2}\ mol/L$$

$$[H^+] = 0.25V + 0.0005ab\ mol/L$$

$$0.25V + 0.0005ab = 0.032$$

$$0.25V = 0.032 - 0.0005ab$$

$$V = 0.128 - 0.002ab$$

This can also be done with an equation of amounts mol, everything times 2 L, gives the same answer.

Bonus

4. We can model two more values.

What is the function of $[H^+]$ of a (mL or L) of a b mol/L HCl solution in a volumetric flask of c (mL or L), to which V mL of a d mol/L HCl solution is added, after which the volumetric flask is filled up to the grade mark.

$$[H^+] = \frac{d \times V + \frac{a}{1000} \times b}{\frac{c}{1000}} = \frac{1000d}{c}V + \frac{ab}{c}$$

With a and c in mL, if they are not: without dividing by 1000

B. Weak acids

In this part we will look at quadratic equations using calculations on weak acids. We will be modelling again.

Hydrochloric acid, which is mentioned above, is a strong acid that completely dissociates its H^+ (single arrow).

$$HCl \rightarrow H^+ + Cl^-$$

As well as strong acids, there are weak acids, like benzoic acid, C_6H_5COOH. A weak acid does not completely dissociate its H^+, but will reach an equilibrium (double arrows)

$$C_6H_5COOH \leftrightarrows H^+ + C_6H_5COO^-$$

The location of that equilibrium is described by the acid dissociation constant K_a

$$K_a = \frac{[H^+][C_6H_5COO^-]}{[C_6H_5COOH]}$$

For benzoic acid at 25 °C, $K_a = 6.3 \times 10^{-5}$.

We can use this to determine the pH of a 0.30 mol/L benzoic acid solution. The method you can use for this is a table with starting, difference and final concentrations.

$C_6H_5COOH \leftrightarrows H^+ + C_6H_5COO^-$

Starting	0.30	0	0
Difference	$-x$	$+x$	$+x$
Equilibrium	$0.30 - x$	x	x

Then the equilibrium equation is:

$$K_a = \frac{x^2}{0.30 - x} = 6.3 \times 10^{-5}$$

Here the unknown x stands for $[H^+]$ which in this situation is equal to the concentration $C_6H_5COO^-$. This equation can be rewritten to a quadratic equation:

$$6.3 \times 10^{-5}(0.30 - x) = x^2$$

$$6.3 \times 10^{-5} \times 0.30 - 6.3 \times 10^{-5}x = x^2$$

$$x^2 + 6.3 \times 10^{-5}x - 1.89 \times 10^{-5} = 0$$

In general, quadratic equations of the form

$$ax^2 + bx + c = 0$$

have the following general solution (the quadratic formula):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TmL data literacy (interpreting and processing of information above) and sense of number

5. Calculate $[H^+]$ and the pH of a 0.30 mol/L solution of benzoic acid using the quadratic formula.

$$x = [H^+] = \frac{-6.3 \times 10^{-5} \pm \sqrt{(6.3 \times 10^{-5})^2 - 4 \times 1 \times (-1.89 \times 10^{-5})}}{2 \times 1}$$

$$= 4.3 \times 10^{-3}\ or - 4.4 \times 10^{-3}\ mol/L$$

$$pH = -\log(4.3 \times 10^{-3}) = 2.37$$

Feedback hour question: Which of the two concentrations is the right one? And what if the result had been 0,1 and 0,4 mol/L, two positive numbers? Answer: look at the starting concentration of benzoic acid. The $[H^+]$ concentration can never be higher than that. TmL sense of error, sense of number

Just as in part A we will be modelling again by replacing numbers with letters, so that we can calculate different situations.

TmL software skills, data literacy (graphs and tables)

6. We will now replace the benzoic acid concentration of 0.30 mol/L by a concentration m.

- Again, calculate $[H^+]$. Pay attention: your answer now is not a number, but a function: $[H^+]$ as a function of the concentration of benzoic acid m.

- In Excel, make a table of $[H^+]$ in a benzoic acid-solution and the concentration benzoic acid m. Choose a series of concentrations m, including the concentration 0.30 mol/L and check using your answer to question 7 whether your function is correct.

- Make a graph of $[H^+]$ as a function of the concentration benzoic acid m.

- In your table, add the pH and make a graph of the pH as a function of the concentration benzoic acid m. Do not include the point with 0 mol/L in your graph.

$$x = [H^+] = \frac{-6.3 \times 10^{-5} \pm \sqrt{(6.3 \times 10^{-5})^2 - 4 \times 1 \times (-m \times 6.3 \times 10^{-5})}}{2 \times 1}$$

$$x = [H^+] = \frac{-6.3 \times 10^{-5} \pm \sqrt{(6.3 \times 10^{-5})^2 + 4 \times m \times 6.3 \times 10^{-5}}}{2}$$

Feedback hour question: Why is this not a linear line? Explain mathematically and chemically. Mathematical: it is a root function. Chemical: it is a weak acid, with a rising starting concentration relatively less $[H^+]$ will be released. TmL data literacy (interpretation and understanding) and sense of number (square root function)

Feedback hour question: Why can you not include the point 0 mol/L? The log of 0 does not exist.

In the next question again TmL technical software skills, sense of number and also sense of error.

7. We will now continue modelling, so that we can also use our Excel-tool for other weak acids. Therefore, instead of $K_a = 6.3 \times 10^{-5}$ we will use a general K_a.

- Again, calculate $[H^+]$. Pay attention: your answer is now $[H^+]$ as a function of the concentration of benzoic acid m and K_a

- Change your Excel-tool so that you can fill in different K_a-values by using a cell reference.

- Check the equation in your Excel-tool by choosing a weak acid and a solution at random and calculating the answer "by hand" and comparing it to the answer in your Excel-tool. Explain the result.

$$x = [H^+] = \frac{-K_a \pm \sqrt{(K_a)^2 - 4 \times 1 \times (-m \times K_a)}}{2 \times 1} =$$

or:

$$x = [H^+] = \frac{-K_a \pm \sqrt{(K_a)^2 + 4 \times m \times K_a)}}{2} =$$

If we add 0.30 mol benzoic acid to a 1,0 L solution where a concentration r mol/L H^+ is already present, the table will change:

$C_6H_5COOH \leftrightarrows H^+ + C_6H_5COO^-$

Start	0.30	r	0
Difference	$-x$	$+x$	$+x$
Equilibrium	$0.30 - x$	$r + x$	x

8. Work out the following steps for this situation

- Formulate the equilibrium equation.

- Elaborate the equilibrium equation to a quadratic equation of the form $ax^2 + bx + c = 0$.

- Formulate the equation for $[H^+]$ at 0.30 mol benzoic acid, where a concentration r mol/L H^+ is already present. Pay attention: you will now have $[H^+]$ as a function of r.

- Add this situation to your Excel-tool so that you can fill in different r-values by using a cell reference.

Feedback hour question: Experiment with different values for r. Can you explain the changes in the graphs? TmL data literacy (interpretation of graphs)

$$K_a = \frac{x(r+x)}{0.30-x} = 6.3 \times 10^{-5}$$

$$(0.30-x)K_a = x(r+x)$$

$$(0.30-x)K_a = rx + x^2$$

$$0.30K_a - xK_a = rx + x^2$$

$$x^2 + rx + K_a x - 0.30K_a = 0$$

$$x^2 + (r+K_a)x - 0.30K_a = 0$$

and using K_a for benzoic acid:

$$x^2 + (r + 6.3 \times 10^{-5})x - 0.30 \times 6.3 \times 10^{-5} = 0$$

$$x^2 + (r + 6.3 \times 10^{-5})x - 1.89 \times 10^{-5} = 0$$

$$x = \frac{-(r + 6.3 \times 10^{-5}) \pm \sqrt{(r + 6.3 \times 10^{-5})^2 - 4 \times 1 \times -1.89 \times 10^{-5})}}{2 \times 1}$$

or:

$$= \frac{-(r + 6.3 \times 10^{-5}) \pm \sqrt{(r + 6.3 \times 10^{-5})^2 - 7.56 \times 10^{-5})}}{2}$$

$$[H^+] = r + x$$

The final question is purely aimed at finding an error: sense of error

9. In the Excel-file for this case you will find an elaboration for a weak acid with a random acid dissociation constant (K_a), a random starting concentration H^+ (r) and a random concentration of the weak acid (m), to calculate the pH with. However, there is an error somewhere! Track it down. ☺

$$x = \frac{-(r + K_a) \pm \sqrt{(r + K_a)^2 + 4 \times m \times K_a)}}{2}$$

$$[H^+] = r + x$$

In the Excel-tool the 4 in this equation is missing.

4 Fostering Techno-mathematical Literacies in Higher Technical Professional Education: Reflections on Challenges and Successes of DBIR

Van der Wal, N.J., Bakker, A., Moes, A., & Drijvers, P. (Accepted). Fostering techno-mathematical literacies in higher technical professional education: Reflections on challenges and successes of DBIR. In Z.A. Philippakos, J.W. Pellegrino, & E. Howell, (Eds.), *Design Based Research in Education: Theory and Applications.* New York, NY: Guilford Press.

In this chapter, we describe the challenges and successes in the implementation phase of a design study using a Design-Based Implementation Research (DBIR) approach. Implementations of research-based innovations in education have proven to be difficult and often unsuccessful (Fishman & Penuel, 2018). This gap between research and practice is under-represented in educational literature, where scalability and sustainability of innovations are scarcely addressed and typically left to practitioners (e.g., Akkerman, Bronkhorst, & Zitter, 2013; Farley-Ripple, May, Karpyn, Tilley, & McDonough, 2018; Snow, 2016). Furthermore, innovation is a process rather than a single act, and many stakeholders, such as teachers, teacher educators, school management, and policymakers, are involved. Because teaching contexts differ between schools, regions, and countries, it is crucial to understand how innovations can be adapted to local situations (Maass, Cobb, Krainer, & Potari, 2019).

A bridge between "*what works*" in design research and "*what works where, when and for whom*" (Means & Penuel, 2005, p. 181, emphasis original) is provided by design-based implementation research (DBIR). Where design research focuses on design, DBIR can be seen as an expansion supporting usability and effectiveness of the sustainable implementation of educational innovations (Penuel & Fishman, 2012). To this end, DBIR aims to break down barriers between disciplines of educational research and reconfigures the roles of researchers and practitioners to bring about systemic change (Fishman, Penuel, Allen, Cheng, & Sabelli, 2013). Four key principles of DBIR (p. 332) are:

1. A focus on persistent problems of practice from multiple stakeholders' perspectives
2. A commitment to iterative, collaborative design
3. A concern with developing theory and knowledge related to both classroom learning and implementation through systematic inquiry
4. A concern with developing capacity for sustaining change in systems.

Faced with this aspiration of DBIR, Cobb et al. (2013, p. 342) noted:

> *Given DBIR's current status as an emerging methodology and the limited guidance for instructional improvement at scale provided by current research, it is reasonable to expect that the systematic inquiry to which Penuel et al. (2011) refer will be a bootstrapping process.*

As the word bootstrapping refers to achieving something without necessary external input, Cobb et al. emphasize here that DBIR requires large effort with limited guidance from literature. Furthermore, DBIR needs multiple theoretical frameworks to work from (DiSessa & Cobb, 2004), and these theories need to be networked, that is, coordinated and integrated (e.g. Alberto, Bakker, Walker-van Aalst, Boon, & Drijvers, 2019; Bikner-Ahsbahs, Bakker, Haspekian, & Maracci, 2018).

It is challenging for the teaching community to benefit from research insights. Literature, for example, can be less accessible because of jargon, high level of abstraction, paywalls, or lack of actionable knowledge Therefore, the gap between educational research and practice can be seen as a barrier for education reform. Also, teachers' voices are not often part of research design or implementation.

To create alignment between academic research communities and professional teaching practices, teacher-researchers, as so-called *brokers,* can cross boundaries between those two worlds (Wenger, 1998). Such brokers face many challenges, occupying an ambiguous position operating in the very different cultures of school and academia; each culture bearing different expectations (Bakx, Bakker, Koopman, & Beijaard, 2016).

In this chapter, we describe our DBIR project as a case study, in which a teacher-researcher (the first author), in the role of broker, gained insights into how theory and practice can be brought together by both designing and implementing an educational innovation while being assisted by a team of lecturers. Many challenges and successes were experienced, and this chapter presents a variety of factors and mechanisms that play a role in such an adventure.

4.1 Professional Development and Agency

Because all organizations need to innovate to keep up with social and technological developments, change is a requirement in teachers' professional lives. However, transformations in practices and teachers' identities are hard to achieve (Vähäsantanen, 2015). Success in educational innovations highly depends on the teachers who shape them (Lieberman & Pointer Mace, 2008). Visnovska et al. (2012) and Goodchild (2014) stated it might be too ambitious to expect teachers to engage in the design process from the start, but Hargreaves (2003) argued that innovations that are only based on volunteers are less sustainable. Teachers benefit from involvement in innovation in terms of professional learning, but they also make sure that aspects such as practicality and authenticity are not neglected (Wake, Swan, & Foster, 2015).

Professional development (PD) is widely seen as the centre of innovation (Heck, Plumley, Stylianou, Smith, & Moffett, 2019), and evidence is growing that quality PD improves student learning (Desimone 2009). Not all PD formats, however, are equally efficient (Kennedy, 2016), and they are not always based on how teachers learn. Learning outcomes that teachers report in terms of changes in knowledge and beliefs are attained mostly by experimenting and reflection on their own teaching practices and less by literature or input from colleagues. However, there are substantial differences between teachers, and some experience friction, struggling not to revert to old ways and avoiding learning (Bakkenes, Vermunt, & Wubbels, 2010). Little (2007) advised that teachers exchange experiences with one another for personal growth.

Teacher beliefs and values influence teaching practices (e.g., Phillip, 2007) but also drive professional agency (Meirink, Imants, Meijer, & Verloop, 2009). Agency is defined as the opportunity to influence one's work, to have the power to act, think for oneself, to make decisions and choices, and take stances (Ketelaar, Beijaard, Boshuizen, & den Brok, 2012; M. Priestley, Edwards, & A. Priestley, 2012). In agency, people express their personal power, and it can be developed (Eteläpelto, Vähäsantanen, Hökkä, & Paloniemi, 2013; Holland, Lachicotte, Skinner, & Cain, 2001).

There is an emerging tendency to acknowledge the importance of teachers' agency (Biesta, Priestley, & Robinson, 2015). Agentic teachers perceive themselves as active learners, as pedagogical experts, and are able to implement and develop their own expertise (Toom, Pyhältö, & Rust, 2015). When opportunities for participation and influence are absent, teachers' agency is strongly reduced (Vähäsantanen, 2015). Therefore, agency can manifest itself in line with an innovation but also in criticism and resistance, and it is constructed in the middle of dilemmas and pedagogical uncertainties (Pyhalto, Pietarinen, & Soini, 2012).

In this chapter, we address the research question:

Which challenges may arise in the relation between teachers' agency and the aim of professional development, and the goal of a sustainable implementation of an innovation?

4.2 Project Context: Implementation of a New Mathematics Course for Future Engineers

The implementation of a new course in applied mathematics was part of a larger design study to improve mathematics education in the technical domain of higher technical professional education in the Netherlands. Mathematics curricula are a topic of ongoing discussion, because of variation in topics and level, and employer expectations (Van Asselt & Boudri, 2013). This is the situation also at Avans University of Applied Sciences, where the study took place. At the School of Life

Sciences and Environmental Technology and other technical schools, low success rates and motivation of students were encountered, and a wide variety in students' mathematical background levels provided additional problems. Mathematics curricula focused on abstract mathematics, and most teaching was transmission-based. So, our project started from the multiple issues (low student motivation and performance, diverse entrance levels of students) we faced in mathematics education, and therefore aligns with the first principle of DBIR.

Because one of our goals was to improve the connection between mathematics education and workplace practices, we decided to introduce the development of *Techno-mathematical Literacies* (TmL) as central learning goals in the course. The term TmL was introduced by Kent, Bakker, Hoyles, & Noss (2005). Bakker, Hoyles, Kent, & Noss (2006) defined TmL as "functional mathematical knowledge mediated by tools and grounded in the context of specific work situations" (p. 343). TmL include mathematical, workplace and software knowledge, multi-step calculation and estimation, and the ability to interpret abstract data and communicative skills (Hoyles, Noss, Kent, & Bakker, 2010; Hoyles, Noss, Kent, & Bakker, 2013).

To identify which TmL engineers use in their daily work practices, we conducted an interview study in a range of technical domains (Van der Wal, Bakker, & Drijvers, 2017, see Chapter 2). With these TmL categories as central learning goals, a new course in applied mathematics for technical schools was developed by the lecturer-researcher in collaboration with an interdisciplinary design team of three lecturers from the School for Life Sciences and Environmental Technology, the School of Built Environment and Infrastructure (both part of Avans University of Applied Sciences), and the Institute for Engineering and Design (HU University of Applied Sciences Utrecht). These lecturers helped create support for innovations, contributed expertise from different technical domains, and provided the technical contexts.

By designing and implementing a new course in applied mathematics in higher technical education, we aimed to gain insights into how to better align mathematics education with workplace practices. This involved creating learning materials, adjusting domain-specific pedagogy, and designing professional development of lecturers, as well as formulating prerequisite conditions for the course. Designing the course materials was labour-intensive but without insuperable difficulties. To promote the development of TmL, we had reasonably clear learning goals in mind, based on interviews with engineers (Van der Wal, et al., 2017, see Chapter 2). However, involving lecturers who had not taken part in the design process proved the most substantial challenge. We, therefore, broadened our initial focus on the design of the course to include the support of all lecturers who would teach the course. We added evaluation interviews after the first and second cycle of the implementation, in which we investigated the lecturers' experiences, opinions, dilemmas, feelings, and beliefs regarding teaching the new course.

Our experiences underpin the importance of Engeström's (2011) critique of most interventionist research, namely that it adopts a linear model with assumptions about control and prediction that do not hold in a setting where people have agency and sometimes are resistant to change. In this work, we focus on the lecturers and the support we provided while student learning outcomes are a topic for a future report.

4.3 Course specifics

In the designed course, we focused on the application of mathematics by using context-based cases, to improve recognition of mathematics in technical practices, and enhance motivation in students. Because basic, abstract knowledge of mathematics is essential for successfully working on these cases, we added a learning track using the ALEKS™[2] software. Furthermore, we changed our pedagogical approach; transmission teaching was limited, and Inquiry-based Learning (IBL) was introduced during so-called feedback hours. The choice for IBL was based on the premise that TmL are inquiry-based by nature. An extended description of the course can be found in van der Wal, Bakker & Drijvers (2019), see Chapter 3.

The new mathematics course was implemented for all students of the School of Life Sciences and Environmental Technology at Avans University of Applied Sciences in the Netherlands. After a pilot in the spring of 2016 with 59 chemistry students, three iterative cycles (second principle of DBIR) were administered, monitored, evaluated, and adjusted in the years following. We started with eight lecturers and 400+ students in the fall of 2016 and proceeded with nine lecturers and 500+ students in the 2017 cohort.

4.3.1 Lecturers

Our design team's setup resembles a Teacher Design Team (TDT), a type of emerging professional development in which a group of teachers focuses on the

[2] Assessment and LEarning in Knowledge Spaces is a Web-based, artificially intelligent assessment and learning system. ALEKS uses adaptive questioning to quickly and accurately determine exactly what a student knows and does not know in a course. ALEKS then instructs the student on the topics (s)he is most ready to learn. As a student works through a course, ALEKS periodically reassesses the student to ensure that topics learned are also retained.

design of educational materials. Because participation in such a team stimulates ownership, TDT can be the basis of a successful implementation. Enthusiasm is key; teachers in TDTs are often motivated and ambitious (Binkhorst, Handelzalts, Poortman, & van Joolingen, 2015). To support sustainability of the innovation, it is advised to involve all teachers (Hargreaves, 2003).

In our case, however, for multiple reasons, most lecturers involved in teaching the course could not be a member of the design team. We did not precisely know which and how many lecturers would be teaching. Over the last few years, the student population had grown; therefore, more lecturers were required each year. Also, some lecturers left because of other tasks and priorities, and others joined later; thus, we did not have a steady teaching team during the project. Also, we introduced many innovations in the design. This would not have been possible had too many people been involved.

Teachers are the most important agents in educational change and many innovations have failed due to lack of teacher learning (Borko 2004; Guskey, 2002). “Top-down” approaches from policymakers or researchers to teachers do not work (Altrichter, Feldman, Posch, & Somekh, 2008; Tirosh & Graeber, 2003). Therefore, in our sub-optimal circumstances, not being able to involve all lecturers in the design process was a significant challenge. Although the lecturers in the design team were very motivated and ambitious, not all teaching lecturers were enthusiastic about starting the new course with different content and pedagogy. They were asked by the management to participate, but most of them were used to teaching “old-school” mathematics for years. The design team was conscious of this situation and tried to provide as much support as possible to assist with the necessary changes.

4.3.2 Support Kit for Teaching Lecturers

To support the teaching lecturers, the design team developed a support toolkit for them. The choice of tools was based on the experiences of the two lecturers, the researcher-lecturer and the design-lecturer from the School of Life Sciences and Environmental Technology, in the pilot of the new course. We provided materials such as manuals and written information about IBL and TmL and sent weekly e-mails with instructions, suggestions, and tips to the teaching lecturers. Support via phone, e-mail, or live with the lecturer-researcher was available. We also provided the lecturers with a reflection form to monitor their experiences. Most importantly, we organized lecturer meetings three times each in the first and second year, with a seventh, final meeting in the third year, moderated by a coach-lecturer from the School of Life Sciences and Environmental Technology. These meetings, designed to prepare and support the lecturers, addressed a range of topics, presented in Table 1.

By conducting iterative design cycles, we developed knowledge about factors that influence an implementation's success (third principle of DBIR). In applying changes to the innovation's design to support sustainable implementation in the curriculum, our project aligns with the fourth principle of DBIR.

Table 1. Lecturers' meetings in three years of implementation

	Content of meeting
Beginning of year 1, meeting 1	Working on case and reflect. Presentation of theory on IBL and TmL, rationale behind the course, results engineer-field study. Explanation of course details and video examples of feedback hours from pilot study.
Half-way year 1, meeting 2	Working on another case and reflect. Discussion of factors that influence the feedback hours. More video-examples for motivation and support.
End of year 1, meeting 3	Evaluation. Discussion of first round of interviews. Inventory of ideas and wishes for adjustments.
Beginning of year 2, meeting 1	Discussion of changes in the course, based on the first round of interviews and the evaluation meeting 3 of previous year.
Half-way year 2, meeting 2	Experiencing TmL and IBL by working on assignment in pairs.
End of year 2, meeting 3	Evaluation. Sharing students' success rates and other facts. Inventory of ideas and wishes for adjustments.
End of Year 3, meeting 1	Evaluation.

4.3.3 Interviews

The first cycle of implementing the new course led us to discover that the teaching lecturers were the most critical factor in successful implementation. We also realized that our support toolkit did not suffice. We needed to deeply understand the lecturers' experiences, opinions, dilemmas, feelings, and beliefs to apply necessary changes in the design. Therefore, as mentioned above, in the spring of the first cycle, interviews with the six teaching lecturers were conducted by the member of the design team from HU University of Applied Sciences Utrecht (third author of this chapter). This choice was based on the teaching lecturers and the design lecturer not being acquainted, which provided a certain distance in which lecturers would be able to speak more freely. The interview questions originated from DBIR principle two, three, and four, by gaining insights for the design (both materials and support) and in lecturers. The interview questions are listed in Table 2. The approximately 1-hour long semi-structured interviews were audio-recorded, transcribed, and coded according to the method of Boeije (2005) by the design lecturer. The assigned codes were guided by the interview questions, but also emerged from the material and were grouped by theme. A total of 321 fragments in 2017 and 93 in 2018 was coded. Frequent discussions took place with the lecturer-researcher regarding interpretation of findings.

Although we originally planned to conduct these interviews once, we decided to add a second round, to find out the lecturers' experiences with the design changes (see Table 5) after the first implementation cycle. This illustrates a need for flexibility in conducting this kind of research that goes beyond the four key principles of DBIR. These informal, short interviews with the six teaching lecturers from the previous year and two new lecturers were conducted by the lecturer-researcher in the spring of the second cycle. The lecturer-researcher asked for experiences, opinions, and feelings about teaching the course that year. These interviews were not audio-recorded but were summarized on-the-spot on a laptop. The notes were coded and interpreted by the design lecturer using the same approach as in the first round.

Table 2. The interview questions for the teaching lecturers

Theme	Questions
Classes	You have taught the course Applied Mathematics. I'm curious as to how classes went. Can you tell me a bit about that? What were the students mainly doing? What were you mainly doing? What went well? What went less well? Were you able to teach the classes in a way that suits you? Can you explain that? If not, what would suit you better? Were you able to teach the classes as intended by the design team? Can you explain that?
The design of the course	How would you explain to colleagues what this course is about? What is new or different about this course?
Support for the lecturers	How did you prepare for the lessons? Did you work out any problems in ALEKS™? Have you worked out the cases yourself? Did you do any other things to prepare yourself? Support has been offered in several ways. I have them printed on these cards. For you, some of those ways will have been more

	effective than others. Would you please lay out the cards, in order of effectiveness? Can you explain this order? [cards: lecturer meetings, feedback form, asking questions via e-mail, asking questions via telephone, weekly e-mail] For your colleagues and yourself, what would be effective ways to be supported in teaching these classes in the future?
Lecturers input in the redesign	(Only for lecturers that have taught the course before) You have taught the course for the second time. What have you done differently? Next year the course Applied Mathematics will be taught again. In the next months, the design of the course will be reviewed and adjusted. What suggestions do you have for improvement of the course? Would you change anything about the content of the course? Would you change anything about the teaching methods? Would you change anything about the cases? Would you change anything about testing and assessment? What else could help to make this course a success?

Table 3. The interview questions for the director

Theme	Questions
Goals of the course	What purpose did you have in mind with this course? Why is that important? When would you call it a success?
Role in the implementation of the course	What role did you have in the implementation process? How would you describe that yourself? By what means can you influence this process?
Supporting lecturers	What does it require of a lecturer to be able to teach this course? How can you manage this? What kind of support can you offer? What kind of support can you facilitate? Which conditions are necessary?

The former director of the School of Life Science was involved at the beginning of this research project and was supportive and facilitating during its process. Thus, we decided to organize a semi-structured interview with him to identify his role in stimulating a successful implementation. This interview was also conducted by the designer-researcher, audio-recorded, and transcribed, but because of the different goals of this interview, we decided not to code it, but interpret and draw conclusions directly from the transcription. The interview questions for the director can be found in Table 3. After the second cycle of implementation, the director retired, and we were pleased that the new director continued the support for our project.

4.4 Insights into challenges and successes gained from the interviews

4.4.1 The former director

The former director of the School of Life Sciences and Environmental Technology turned out to be quite valuable in the implementation of the new course and was an important stakeholder (first principle of DBIR). He had a clear vision and ideas on needed improvements to the mathematics curriculum. He addressed the problems with the differences in the level of mathematical skills between students and the fact that abstract mathematics does not stimulate students' and lecturers' motivation. At the beginning of the project, he suggested that we should investigate what the mathematical needs of the workplace are to improve student motivation, design the course accordingly, add technology to support differences in students, and use appropriate pedagogical tools.

Indicators of success were, according to him, satisfaction in lecturers and students, and a strong connection between the course, the rest of the curriculum, and the workplace. He hoped lecturers would be happy with a new approach, and motivation would improve. He also suggested that, after this intervention, other courses would be adjusted accordingly.

Although he did not share his vision extensively with all lecturers, he recruited lecturers to participate and to conform to the prerequisites of the course, allocated task hours, and provided support to the researcher-lecturer. He also facilitated the professional development of the teaching lecturers to support the learning of the required pedagogy.

The most striking aspect of the interview was his conjecture that the lecturers, especially the younger ones, were reflective professionals who wanted to develop professionally, who would be motivated by the domain-specific context in mathematics and thus would be able to motivate students. He also hypothesized that younger lecturers, more recently educated in teaching, would be more enthusiastic for new courses with new content and pedagogy.

Table 4. Percentages of fragmented codes (experiences, opinions, dilemmas, feelings, and beliefs of the lecturers) in the first and second round of interviews (percentages above ten are highlighted)

	First round spring 2017	Second round spring 2018
Learning outcomes	15%	8%
Cases	8%	4%
ALEKS	6%	9%
First hour: instruction	5%	3%
Second and Third hour, collaborative work	10%	7%
Fourth hour: feedback hour	13%	9%
Assessment	3%	14%
Lecturer preparation	6%	
Support toolkit	12%	
Teacher roles	7%	13%
Students	11%	5%
Feedback hour Menu		22%
Miscellaneous	4%	8%
Total	100%	100%

The first round of lecturer interviews

The purpose of the lecturer interviews was to identify their experiences, opinions, dilemmas, feelings, and beliefs and therefore gain insight in factors that play a role in the success of the implementation. We shall discuss the most distinctive, surprising, and influential themes and illustrate these with excerpts of the interviews. The themes emerged from the data, as we combined open codes to selective codes (Boeije, 2005). Some themes are only discussed in one round, but most are addressed in both. Table 4 shows the percentage breakdown. In the first round, most codes and most comments by the lecturers concerned learning outcomes, the feedback hour, the support toolkit, and the students.

The theme Feedback hour menu is only discussed in the second round of interviews, and is addressed in the section: the second round of lecturer interviews.

4.4.2 Inquiry-based Learning

Lecturers experienced many difficulties with using IBL during the feedback hours. In general, most lecturers in higher professional education have worked in industry before switching to teaching, and receive only a course of 6 ECTS on pedagogy. They tend to teach in the way they were taught in their own school days with a focus on sending information and explaining procedures. Conducting feedback hours with IBL questions, addressing TmL, managing classroom discussions, and creating a safe atmosphere in the class are complex tasks. They require taking on multiple teacher roles, something most lecturers are not used to, and that some do not prefer, as they explained in the lecturer meetings. Although all lecturers received the same support toolkit, we noticed their interpretations of, for example, the approach of IBL were very different. All lecturers tried to administer the classes as they thought they were meant to, but some lecturers assumed instruction and explaining were not allowed anymore, while others used all teacher roles freely. In the following example, we see a lecturer (L1) struggling with the desire of students to know whether an answer is correct and her effort to focus on the approach rather than the answer.

> L1: And then they asked: Is it correct? I got asked that question a lot. And then I said: Does everyone agree that it has to be solved like this? Then everyone said: Yes, that must be correct. Then I said: If everyone agrees, then it must be correct. They did not like that.

4.4.3 Content and Pedagogy

In the School of Life Sciences and Environmental Technology, mathematics courses are not taught by mathematicians or mathematics teachers, but by biologists, chemists and chemical engineers who have an affinity with mathematics or statistics. We expected their background would turn out to be an advantage for the new applied mathematics course, as contexts provide meaning for these professionals. As mentioned above, the School's director shared this expectation.

However, we did not see this conjecture confirmed. It is very challenging for many mathematics teachers to change from instructor teaching procedures for solving specific tasks to a more facilitating role, supporting both conceptual understanding and procedural fluency (Swan, 2007). Some lecturers preferred to teach abstract mathematics and we suspect that context-based cases are actually mathematically more difficult. According to Heck et al. (2019), mathematics is often seen as a static body of knowledge of rules and procedures, while an inquiry-oriented approach stimulates a more dynamic view and actively engages students to construct knowledge. This aligns with Freudenthal (1973), who perceives teaching mathematics as a human activity in his Realistic Mathematics Education principle. Pozzi, Noss, and Hoyles (1998) state that the key idea is mathematization, which they define as a complex set of relations (including mathematical relations) between resources, activities, and settings as they are operationalized to achieve a particular goal at work.

In the next example, we present a lecturer (L2) who experiences a lack of mathematical content and misses teaching his specialty in basic mathematics *(students study basic mathematics in ALEKS™ outside class).* His drive to help students is challenged by the shifted content and pedagogy of the course:

> L2: For myself, I miss the content component very much and I wonder: Do you need me, or should you use someone who is better in process coaching and perhaps less mathematically educated? That would work for this class too. I cannot teach my specialty, [which is] basic mathematics. It [the new approach] is something I struggle with, and I would like to teach that. To share my knowledge, this is how you can approach this, and you can use these methods to solve this problem. I want to be able to use and teach my mathematical expertise. Those mathematical skills [in students] are not yet developed. I can contribute to that development, [and offer] a significant contribution. I would like to have a possibility for that. To provide mathematical support to students, which is not asked of me. Students don't ask for it, but it would be beneficial to them.

Because the teaching load of lecturers is generally high, most had limited preparation time for teaching the new course. From informal conversations with the lectures, it appeared that almost all lecturers in both cycles did not work out the cases by themselves and did not study or even open the trajectory of abstract mathematics in ALEKS™. We suspect this lack of preparation played a significant role in lecturers feeling less confident with the content of the course. We also noticed that some lecturers tended to turn to abstract mathematics and calculation rules. In this example, the lecturer is more comfortable with basic differentiation rules than the applied mathematics content:

> L1: It's getting accustomed for me. You [usually] just say "x square is two x" and go for it. Not this kind of teaching. This takes getting used to, this content.

4.4.4 Feedback hour

All lecturers struggled to engage students during the feedback hour. A lecturer describes the challenges with the different levels of understanding and some lack of motivation among students:

> L3: During the feedback hour, it is challenging to keep everyone engaged. That is hard work. Some students want to listen carefully to what is going on, because they don't understand it very well.

> However, there are also some who are noisy, and then you have to ask for quiet continuously. They are obligated to be there and then motivation is not always very high. They are noisy because they already get it. When we talk more about a specific topic because some students struggle with it and some don't, then the engagement is gone. I try to keep them engaged, but it costs a lot of energy.

4.4.5 Cases and TmL

We considered the lecturer meetings as an essential tool in the support kit, hoping to educate the lecturers in TmL and IBL. We saw, however, that lecturers were not focused on these meetings, but appreciated the more practical tools of the kit. Short-term support in the form of informal phone calls and weekly e-mails was experienced as most helpful. Although lecturers reported positive experiences with students learning to work with Excel™ and estimations, which are TmL, the concept of TmL appeared to be challenging for the lecturers and we saw limited engagement with it. Here, we have an example of a lecturer who focuses on mathematical content rather than on TmL, but then realizes TmL was in fact part of it:

> L4: The lecturers' version of the cases gave theory: learning goals and the skills students work on. I did not read that well. There were all kinds of abbreviations and difficult concepts I could not understand directly. Nathalie [first author] did explain that in the lecturer meetings, however. Something with numbers and getting a sense of numbers, but it was all in English. Something with a letter…. [Interviewer says TmL]. Uh, yes, TmL. Those were present in the lecturers' version of the cases, but in class we didn't do anything with that. That would be too vague for the students too. That is how I dealt with it, I focused on the content, the math, the Excel™, the lesson itself. The thoughts behind, I did not occupy myself with that… Perhaps then I am not right if I say that the name of the course [applied mathematics] is misleading because that TmL is part of it. But when I say mathematics, then it is about topics, and how much they come up.

There were also some doubts about desired learning outcomes in using these cases, but most lecturers thought they were useful and suitable tasks for students to work on mathematical applications in the technical domain. One lecturer explains how students have to really think instead of applying procedural rules.

> L1: This is interesting because you let them really think about something instead of giving them some rules to learn by heart.

> Because they forget those, you see that with me too. But anyway, do you become a good biologist by this? I think you can be one without, but on the other hand, I do think it is important that they learn to think a certain way, and I think you can accomplish that with such cases.

4.4.6 Revision of the design

Not many changes were made to the original design of the course in the iterative cycles of the implementation. There were small adjustments in the cases on the level of text or added pictures, the construct of two learning trajectories comprising the use of ALEKS™, and the cases were maintained, because the purposes and learning goals of the course appeared to match. The only substantial change we implemented was the form of assessment. In the initial design of the first cycle, cases were submitted by the students and reviewed by the lecturers. Because we experienced some cases of cheating by students and reviewing the cases appeared to be too time-consuming for the lecturers, we decided to introduce a digital assessment after the first year, consisting of TmL questions regarding the cases. Lecturers thought this was a considerable improvement. In general, lecturers at the School of Life Sciences and Environmental Technology are used to written assessments and value this form as the most reliable to test students' knowledge.

We learned from the interviews and the evaluation meeting at the end of the first cycle that the whole-class feedback hours with students presenting their work were the most substantial challenge. The amount of work students had finished during the collaborative second and third hour varied enormously, which raised the difficulty of administering this hour. We, therefore, introduced a menu with different options for the feedback hour, see Table 5. This local adjustment in the design was in line with formative interventions (Engeström, 2011) but also fitted the second principle of DBIR. For more advanced students, alternative and extra assignments were developed. We also suggested to the lecturers to vary the length of the collaborative second and third hour as they saw fit.

Table 5. Menu for organizing a feedback hour

	Option
1	Two or three groups of presenting students and whole-class discussion using IBL
2	Visit all groups and discuss using IBL
3	Alternate options 1 and 2 per week
4	Peer review per 2 groups and proceed with whole class reflection
5	Option 2 in weeks 1–6 and with short presentations of products in week 7

4.4.7 The second round of lecturer interviews

In the second cycle, during the spring of 2018, informal interviews with the researcher-lecturer were conducted. Table 4 shows a shift toward codes regarding assessment form, teacher roles, and the menu of feedback. Lecturers were positive toward the feedback-hour menu. Adding more options in teaching allowed for more autonomy and professional agency. We chose not to audio-record these interviews but to take notes, because we did not want to possibly disturb the collegial relationship between the researcher-lecturer and the teaching lecturers. This interview round provided less data, as the interviews were shorter and because there was no audio-recording, we cannot present excerpts.

In the support toolkit for the teaching lecturers, a reflection form was provided to be completed after every class. The aim was to provide support and to gain insight into the experiences, thoughts, and feelings of lecturers. Only one lecturer used this form and reported that it helped to improve their teaching in successive classes. This lecturer also held the most positive opinions about the new course and reported extensive content preparation.

We considered whether changing both mathematical content and pedagogy by introducing a new course was too ambitious in a college where lecturers are used to more traditional ways of teaching. Most lecturers appeared to be ill-equipped for the pedagogical choices in the design. However, content and pedagogy are often intertwined. Because TmL are inquiry-based by nature, IBL seemed to be the preeminent choice for learning these skills. Therefore, although administering IBL in the classroom is complex, we did not want to abolish this approach in the course. It became apparent that additional professional development for the lecturers was inevitable to achieve a sustainable and successful implementation.

In the third cycle (2018–2019), two lecturers left the team, and two others joined. There was no official professional development available anymore, but new lecturers were supported and trained individually by the researcher-lecturer. We tried to recruit new lecturers who fit the profile of a reflective practitioner. We also continued with conducting an evaluation meeting at the end of each year. We saw that, after two cycles, the lecturers were getting more used to the course, and although some lecturers deviated from the design, we also saw more confidence in lecturers and fresh ideas and successes in newcomers. It became more and more apparent that sustainable implementation required a long-term effort.

4.5 Conclusion and Discussion

While design studies are developed in a specific local context, researchers should try to create general knowledge that can be of value to other researchers and contributes to theory (Anderson & Shattuck 2012; McKenney & Reeves 2013). In this chapter, we shared the lessons learned in a DBIR project and illustrate why this kind of research is complicated and challenging since there is not just one theoretical frame to work from. However, we also shared successes and have implemented a new course in the curriculum, which can last many years. While our first efforts focused on the design of the course, over time, we encountered the influence of many practical constraints, and the importance of continuous professional development for lecturers became apparent.

We conjectured that lecturers would appreciate applied mathematics in their domain in contrast to abstract mathematics. Students are focused on answers and ask confirmation explicitly. It seems that both lecturers and students have a belief in mathematics containing problems that have one correct solution. TmL provides a perspective of mathematics as an activity with multiple solutions and with less certainty. We wanted to keep our focus on these skills because of their importance in the workplace. However, we do think this contrast might be part of why many obstacles in the implementation occurred.

Design components were considered successful if they worked as conjectured and intended, or could be made to work effectively after small adjustments, and were positively received by the lecturers. The most substantial success we can point at in the implementation is the menu for the feedback hour. The factors we could not predict and struggled with or could not find an immediate solution for during the process, we consider as challenges. We saw that the act of interviewing added to the positive experience of being seen and heard. Reflection on theory, which we did not accomplish during the lecturer meetings, was stimulated by the interviews. Therefore, listening and trying to adjust the design, but to avoid lethal mutations (Brown & Campione, 1996), was both our most significant success and challenge regarding the lecturers. Therefore, a design should have a certain amount of robustness so that it can survive small changes (Roschelle, Tatar, Shechtman, & Knudsen, 2008).

Although we knew from literature that it is advisable to involve all lecturers in the design process, this was not practically possible. Even with this knowledge, we were surprised by the enormous impact of lecturers' beliefs regarding mathematics content, learning outcomes, and pedagogy on the success of the implementation. Our support toolkit did not provide enough basis for successful implementation, and we did not expect the limited effect of the lecturers' meetings, which were appreciated less than the more practical tools in the support kit. Can we accommodate lecturers by operating more closely in agreement with their beliefs and preferences? Cole and Engeström (2007) state, in line with Vygotsky and followers, that practice is essential for testing and improving theory, and that activity should be the unit of analysis. Although DBIR includes practice by using iterative cycles of implementation, we do think that we should move towards formative interventions, in which Vygotsky's principle of a second stimulation fosters learner's agency (Engeström, 2011). Alternatively, perhaps, designs should be more *half-baked*, as Kynigos (2007) defines his microworlds, which are pieces of software, to be built upon and changed by all its users. This lets a design be both robust and flexible.

In this chapter, we did not report on students' experiences and learning outcomes. In student evaluations, we saw a direct correlation between confident and content lecturers and content students. As for learning outcomes, we administered a pre- and posttest, and results will be the topic of a future report.

Based on the experiences in this project, the lecturers and the researcher-lecturer recently formulated a plan for the coming years. Long-time management support for implementation is imperative, and fortunately, a green light was given for more official, external professional development and peer feedback in the coming years and to recruit more lecturers with an innovative mindset. We hope these actions will contribute to more success and a sustainable future for the course and will have a positive influence on other courses.

Within our design study, it appeared that the implementation phase was the most challenging, and we needed to focus on professional development of the lecturers. On one side, we have a course and ideas for pedagogy, but on the other, we have some teaching lecturers who are not comfortable and need agency and autonomy. Although Luckner et al. (2018) provided design research advice, every project has different challenges, and the authors emphasized the unpredictability of "research in the wild" (p. 818). Research is always directed backward, but life is forwards, and one has to consider how much leeway there is. Our project aligns with the four principles of the theory of DBIR (Fishman et al., 2013). Yet, Engeström and Sannino (2010) warned that researchers should not expect "nicely linear results from their efforts." And even with the DBIR principles in mind, implementation of innovation requires, most of all, flexibility of the researchers; one cannot blindly follow abstract guidelines. In the non-ideal circumstances of real practice, we should expect the unexpected.

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5 Assessing the Development of Techno-mathematical Literacies in an Innovative Mathematics Course for Future Engineers

Van der Wal, N.J., Bakker, A., & Drijvers, P. (2020). Assessing the development of Techno-mathematical Literacies in an innovative mathematics course for future engineers. Submitted for publication.

Abstract Techno-mathematical Literacies, which are defined as a combination of mathematical, workplace and ICT knowledge, and communicative skills, are acknowledged as important learning goals in STEM education. Still, much remains unknown about ways to address them in teaching and in to assess their development. To investigate this, we designed and implemented an innovative course in applied mathematics with a focus on Techno-mathematical Literacies for 1st-year engineering students, and we set out to measure the learning effect of the course. Because assessing TmL is an uncharted terrain, we designed tests that could serve as pre- or posttests. To prevent a test learning effect, we aimed designing two different but equally difficult tests A and B. These were assigned randomly to 68 chemistry students, as a pretest, the other one serving as posttest after the course. A significant development in TmL was found in the B-pre group, but not in the A-pre group. Therefore, as a follow-up analysis we investigated whether the two tests were equally difficult and searched for possible explanations. We found that students who were assigned B (pre) were previously higher achieving than A (pre), and a sound mastery level of basic skills that ground the higher-order TmL seemed necessary. Furthermore, as TmL are very heterogenous by nature, some of them are easier learned and tested than others. Based on the results, we propose improved ways of testing TmL, that should be validated in future research.

Keywords: Techno-mathematical literacies ● Mathematics education ● Engineering education ● STEM education ● Design research ● Validation ● Assessment

5.1 Introduction

The professional practices in which science, technology, engineering, and mathematics (STEM) are used have changed over the last few decades because of tremendous changes in available knowledge and digital technology (Duderstadt, 2010; Kent & Noss, 2001). Nowadays, most calculations are performed by computers, and mathematics behind the interfaces are therefore less visible and transparent (Williams & Wake, 2007; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002). In general, mathematical application skills beyond pure mathematical knowledge have been recognised to be increasingly important (FitzSimmons, 2002). In this study, therefore, we focus on Techno-mathematical Literacies (TmL), which are defined as a combination of mathematical, workplace and ICT knowledge, and communicative skills (Kent, Bakker, Hoyles, & Noss, 2005). Examples of TmL are the ability to interpret abstract data (Kent, Noss, Guile, Hoyles, & Bakker, 2007), and having a sense of number and a sense of error (Hoyles, Noss, Kent, & Bakker, 2010). TmL are not automatically developed in the workplace and need to be learned explicitly (Hoyles, Noss, Kent, Bakker, & Bhinder, 2007).

To prepare for these new practices for the case of future engineers, engineering education curricula should align with the need for a broader range of mathematical skills, practised and applied in future professional contexts (Ridgway, 2000; Kent & Noss, 2000, 2001, 2002; Williams & Wake, 2001). The implementation of TmL in mathematics education for engineering students can contribute to an improved preparation for the workplace. Therefore, we carried out a design study in fostering these students' TmL. As TmL are domain-specific, we identified in an earlier study seven commonly used TmL categories in engineering practices: data literacy, software skills, technical communication skills, sense of error, sense of number, technical creativity, and technical drawings skills (Van der Wal, Bakker, & Drijvers, 2017, see Chapter 2).

To address these TmL as central learning goals, we designed an innovative course in applied mathematics for 1st-year engineering students through a design research approach (Bakker, 2018; Plomp, 2013). This course, which included group work on applied cases as a core element, was implemented in the curriculum of all majors of the School of Life Sciences and Environmental Technology of Avans University of Applied Sciences and taught by 11 lecturers to over 1400 students in three years (Van der Wal, Bakker, & Drijvers, 2019, see Chapter 3). To evaluate the learning effect of the designed TmL-rich course, the first step involved the measurement of TmL. In this final study of the project, therefore, we investigated the learning effect of the course by means of a pre- and posttest. The initial research question that guided this study was: What is the learning effect of a course in applied mathematics on students' development of Techno-mathematical Literacies?

Because there were no tests available that measure TmL, we faced the challenge of designing such tests. A key choice was to develop two tests which were sufficiently different to avoid a test learning effect – performing better on a posttest because of experience with a pretest – but were also equally difficult. For the pre-test, the tests A and B were randomly assigned to the 68 participating students. The two tests were assigned vice versa as posttest to the 62 present students. The analysis of the development from pre- to posttest showed contrasting results: scores from students on test B (pre)–A (post), on the one hand, did increase significantly, but scores from students on test A (pre)–B (post), on the other, did not. In search for explanations, it appeared that test B was more difficult than test A, although we – as content experts – had not recognised this beforehand. In trying to explain these results, therefore, we further investigated the tests on validation and equivalence in the second stage of the study and formulated a follow-up research question: What are possible explanations for the unexpected results? Data analysis, therefore, includes two phases, a first phase to answer the initial research question and a second phase addressing the follow-up research question.

5.2 Theoretical Background

5.2.1 Design-based Implementation Research

The methodological approach of design research aims to develop theoretical insight and at the same time provide practical solutions to complex educational problems. It has emerged to bridge the gap between theory and practice (Bakker, 2018; Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). It includes iterative, flexible cycles of designing, monitoring, evaluating and adjusting an intervention, mostly conducted in a team (McKenney & Reeves, 2012). As the implementation of a design is often the most challenging part, and as an expansion of DBR, Design-based Implementation Research (DBIR) has been developed (Fishman & Penuel, 2018). DBIR provides guidelines to support for usability and sustainability of educational interventions. It transcends barriers between the educational disciplines to provide systemic change (Fishman, Penuel, Allen, Cheng, & Sabelli, 2013). In our design study, we followed these guidelines to support the implementation of the innovative course in applied mathematics (Van der Wal, Bakker, Moes, & Drijvers, submitted, see Chapter 4).

5.2.2 Techno-mathematical Literacies

The rapid changes in the world due to globalisation, digitalisation and automatisation have caused knowledge to expand at high speed over the last decades. Computers and technology-driven machines have taken over calculations from handwork. Because input has to be monitored with great scrutiny and output has to be interpreted sensibly, there is an increased need to be able to understand quantitative data (Gravemeijer, 2013; Levy & Murnane, 2007). Although mathematics plays a central role in engineering, engineers often perceive themselves as using only simple mathematics (Kent & Noss, 2002). Handling and interpreting abstract information has always been a task for highly trained employees, but because of technology, an increasing number of people engages in these challenges (Kent et al., 2005). Furthermore, because work tasks are nowadays far more complex than they have been in the past, division of labour is practiced, and computations are often outsourced to computers and to expert mathematicians and statisticians. In non-routine tasks, the use of ICT can add a certain mathematical invisibility behind the screen or the print-out, and this can be perceived as a black box (Hoyles, Noss, Kent, & Bakker, 2013; Van der Wal et al., 2017, see Chapter 2). Engineers learn technology mostly by use, and therefore, mathematical literacy, analogous to language literacy, is necessary (Kent & Noss, 2001).

The mathematical understanding or literacy needed in professional contexts differs significantly from what is typically taught in formal mathematics education (Bakker, Hoyles, Kent, & Noss, 2006). While the latter addresses merely conventional skills, facts and procedures, learning in the 21st century should integrate knowledge in a problem-oriented interactive curriculum (Fadel, Honey, & Pasnik, 2007). Garfunkel (2011) emphasises the importance of mathematics learned in the context of science and states that a different set of mathematical skills are necessary, which he identifies as mathematical modelling and quantitative literacy.

As for mathematical skills that go beyond mathematical knowledge, several definitions have been introduced, and because of the technology-driven nature of engineering, we chose to focus on Techno-mathematical Literacies (TmL) in our study. TmL are complex skills that are context specific and based on data and go far beyond numeracy and calculations (Bakker et al., 2006). Even for professional scientists, for example, graphs that originate in other technical domains are often misinterpreted (Roth, 2003). As shown in Table 1, our interview study with fourteen engineers from several technical domains led to the identification of seven TmL categories, that these engineers often use in combination (Van der Wal et al., 2017, see Chapter 2).

With the aim of using TmL as a central learning goal in an applied mathematics course, we decided to use the approach of inquiry-based learning (IBL). Inquiry is playing an increasing role in science education, as it mimics the patterns of science practices. It stimulates students to acquire and apply science concepts (Linn, Songer, & Eylon, 1996). IBL is defined as a student-centred approach to stimulate critical thinking, problem-solving and developing an investigating mindset (Anderson, 2002; Chu, Reynolds, Tavares, Notari, & Lee, 2017). Teaching according to this approach involves process-focused questions, while students are engaged and learn actively.

Table 1. Seven TmL categories that engineers reported to use in their work (from Van der Wal et al., 2017)

	TmL category	Description
1	Data literacy	The ability to analyse and interpret technical data and graphical representations, draw conclusions and take action accordingly
2	Technical software skills	The ability to use professional software, e.g. Excel™, as a calculation tool
3	Technical communication skills	The ability to communicate technical information with colleagues, customers, supervisors and other parties
4	Sense of error	The ability to check and verify data and detect errors
5	Sense of number	The ability to handle and interpret numbers sensibly
6	Technical creativity	The ability to produce creative solutions to puzzles and problems (by using, e.g. cleverness or experience)
7	Technical drawing skills	The ability to understand and produce technical drawings (by using, e.g. spatial insight)

5.3 Course design

After the identification of the TmL used by engineers, a new course, *Applied Mathematics*, was designed with these TmL as central learning goals. The course was collaboratively developed by an interdisciplinary design team of four lecturers consisting of a mathematician (the first author), a chemical engineer, an electrical engineer and a built environment engineer. These lecturers provided technical contexts and their professional experiences. The premises of the course were determined based on the input of literature and the interviews with the engineers from the previous study (Van der Wal et al., 2017, see Chapter 2). As the engineers stressed the importance of application, and TmL need context by nature, we decided to build a problem-based and technology-enriched course of applied mathematics.

Because TmL need a strong base of mathematical knowledge, the course consisted of two learning tracks. For the track of abstract mathematics, the ALEKS™ software[3] was used, on which students could work individually, outside class, and at their own level and pace. In the second track with TmL as goal, students worked collaboratively in groups of 2 or 3 on guided cases during classes. The topics were aligned; for example, linear and quadratic functions were addressed in the first two weeks of the course in both tracks. Because the TmL category of *technical drawing skills* is not used in the technical domains of chemistry and biology, this TmL category was not a learning goal in these cases.

Table 2. The weekly class hours schedule

1st hour	Introduction/questions with lecturer
2nd hour	Collaborative work without lecturer
3rd hour	Collaborative work without lecturer
4th hour	Feedback hour with lecturer

The class schedule consisted of 4 hours of classes each week, see Table 2. The first hour was dedicated to introduction of the cases and some instruction. Subsequently, the students worked on the cases for two hours, without the lecturer being present. The fourth hour was dedicated to feedback. Students presented their work to the whole class, or the lecturer visited each group separately. The lecturer adopted the philosophy of IBL to discuss the solutions of the students to the case-problems. In focusing on the approach of problems, rather than solutions, the lecturer tried to foster inquiry-based thinking and help students obtaining knowledge for themselves. Classroom or group discussions can stimulate understanding and competence for complex skills (Nathan & Kim, 2009), such as TmL.

Because the new course was to be implemented in the curriculum of the School of Life Sciences and Environmental Technology, we designed the cases specifically for the realms of chemistry and biology. The cases were developed to foster the use of the six TmL categories. The course contained seven weekly classes, in which every case was worked on for a fortnight. A total of three cases was designed, with the seventh, and last class, dedicated to questions. The first case

[3] Assessment and LEarning in Knowledge Spaces is a Web-based, artificially intelligent assessment and learning system. ALEKS uses adaptive questioning to quickly and accurately determine exactly what a student knows and doesn't know in a course. ALEKS then instructs the student on the topics (s)he is most ready to learn. As a student works through a course, ALEKS periodically reassesses the student to ensure that topics learned are also retained.

addressed the topic of pH in solutions and weak acids. In the second case, students worked on bacterial growth and the third case consisted of derivatives and antiderivatives, mainly focusing on qualitative understanding. An extended description of the course can be found in Van der Wal et al. (2019), see Chapter 3.

5.4 Methods

This study involves two phases, a phase 1 in which two tests A and B were developed, validated and administered with the course as treatment to answer the first research question. In phase 2, we zoom in on the mixed results, to find explanations for them and address the follow-up research question through evaluating the tests. To elaborate on the two phases, we carried out a series of research activities (RA). An overview of these activities is shown in Table 3. Activities 1-11 refer to the first phase, addressing the initial research question of the learning effects. Activities 12-16 concern the follow-up research question in the study's second phase. These research activities are explained in more detail in the next subsections.

Table 3. Overview of research activities

	Research activity	Description	Gain insight into
1	Test design	Designing test items in co-design with students	feasibility
2	Validation	Discussing test items with TmL experts	concept validity
3	Redesign	Adjusting test items and assigning to two tests A and B	content validity
4	Validation	"Thinking-aloud" session with 4th-year chemistry student	construct validity and feasibility
5	Redesign	Adjusting test items regarding language, errors and number of items	content validity
6	Administering pre- and posttests	Conducting pretest with 68 and posttest with 62 students in 30 minutes	validity and feasibility
7	Grading	Grading tests by the researcher	test scores
8	Data analysis	Checking for internal consistency of the items with measures of classical test theory.	criterion validity
9		Performing *t* tests to compare pre- and posttest results	content validity
10		Performing one-way ANCOVA to check influence of lecturers.	construct validity
11		Compare P values to investigate development for different TmL categories	content validity

12	Follow up analysis	Performing *t* test to compare marks from course chemical calculation with assignment test A or B as pretest	construct validity
13		Performing *t* tests to compare test A and B	construct validity
14		Performing *t* tests to compare scores on posttest with marks for summative test.	construct validity
15	Redesign proposal	Mixing items within tests to spread missing values	
16		Mixing matching items between test A and B and detect discriminating items to standardise difficulty	

5.4.1 Design and validation of TmL tests

To assess the learning effect of the new course in applied mathematics, we developed in research activity one (RA1) two tests that both could serve as pre- or posttest of Techno-mathematical Literacies. As the course was part of the regular curriculum, a control group was not possible; moreover because of the new learning goals comparison with results from previous years would not make sense. Because testing of TmL has not been done before, there was no material available to build on, so we had to design such a test ourselves. A co-design team consisting of the lecturer-researcher (first author) and students in Electrical Engineering at Fontys University of Applied Sciences was formed to align more closely to the lifeworld of students and assure feasibility. They were asked to get acquainted with relevant literature and create test items for extracurricular study credits. They developed these items using as many TmL categories as possible, and to implement these in non-related, non-chemical contexts to avoid treatment-inherency, because with TmL, we aim for transfer beyond the exact tasks used in the course (Cheung & Slavin, 2016). For example, one test item involved an ant that can carry it's a multiple of its own body weight. The TmL category *sense of number* is the TmL which most resembles the "standard" way of mathematics test items, and although we intended not to measure this TmL too often, we see that it is required in many items, as can be seen in Table 8 and 9.

Subsequently, in RA2, the pool of potential test items was discussed with two TmL experts, with the aim to validate whether the items indeed measured TmL (concept validity). After further adjustments based on their feedback (RA3), a voice-recorded "thinking aloud" session with a 4th-year Chemistry student of Avans University of Applied Sciences was conducted (RA4). Some possible language confusions were detected, and it appeared that for a half-hour test, we needed to decrease the number of test items. In RA5, the test items were divided in two sets, A (Appendix 1) and B (Appendix 2), by the lecturer-researcher with approximately the same distribution of topics and TmL categories. Subsequently, a grading scheme for both tests was designed by the lecturer-researcher. Points per test item were based on the number of steps a student had to take to come to a solution. Test A yielded a total of 33 points and test B 28 points, both normalised to percentages (0–100). We estimated that 13 items for test A versus 12 items for test B would take approximately the same amount of time for students.

5.4.2 Participants and procedure

The tests were administered to all 1st-year Chemistry students (N = 68, 38 female and 30 male, aged 17-24) between April and June of 2019, during the third cycle of the implementation (RA6). In the pretest, half of the 68 participating students were randomly assigned to test A, and half of them to test B. To prevent the previous mentioned test learning effect, we assigned the 62 students that were present during the posttest to the other test.

The pretest was conducted at the start of the first class of the course in applied mathematics. All students signed a consent form and were given exactly 30 minutes to perform the test. Because the seventh and last class was not obligatory and functioned as a question session, we decided to conduct the posttest during the sixth week. As participating in this test relied on the goodwill of the students, we had to schedule it at the beginning of the sixth class rather than at the end, so the students would not be tired already. The mathematical content of the sixth class, therefore, was not covered yet, and this could have influenced scores on the posttest, which we discuss further in the results section. The tests were conducted with pen, paper and calculator. The students were asked to fill in how much time of the half-hour was left when they had finished, to gain insight into how much time students need for doing the two tests (feasibility). The mark that students received for the course in applied mathematics was unrelated to the pre- and posttest scores. To pass the course, they had to master 90% of the assigned topics in the learning track of ALEKS™ as a prerequisite to receive a mark for their summative digital test, two weeks after the posttest. With TmL questions in the contexts of the cases, the students worked on during the course, summative test items were more familiar to them than the pre- and posttest. As a measure of item difficulty, we use the P value, the average correct score of all students (not to be confused with p value to estimate statistical significance).

5.4.3 Scoring

After conducting both pre- and posttests, the tests were divided and scored by the lecturer-researcher and the other teaching-lecturer. Because the test was paper based, all information about the students and which test they were assigned to was visible for the lecturers. Based on the grading scheme, both lecturers graded the tests with frequent peer consultation. After intensive discussion of the grading procedure with the other lecturer, the lecturer-researcher did all the grading to ensure consistent application of the grading procedure (RA7). To be able to judge whether items were too difficult or whether there were too many items (feasibility criterion), we distinguished the assignment of a code of zero points or a code 999. The code 999 was used for missing data; when nothing was filled in, when a question mark or an “I don’t know” was filled in, or when the question was rewritten but no answer was given. A score of zero points was used when something was tried, but the answer was wrong. Zero points were also assigned when something was tried wrongly, but crossed out; this was not considered as missing data.

5.4.4 Data collection

The scores on the items of the two tests were the main data collected for this study in phase 1. However, after the unexpected contrasting results of the tests to measure the development of TmL from pre- to posttest, and further research was necessary, extra data was collected for phase 2. First, we wanted to test whether the random assignment of test A and test B among the students led to groups with about the same average level in mathematics. Therefore, for RA12, we collected the marks of the students of a course on chemical calculation, that the students followed earlier that academic year and compared these marks, normalised to percentages (0−100) with the students’ assignment to test A or B as pretest. Secondly, for RA14, we collected the marks of the summative digital test, also normalised to percentages (0−100) and compared these marks with the scores on the tests A and B as posttest, to find out whether test A and B have the same level of difficulty.

5.5 Results

In sections 5.5.1− 5.5.5, we present the results regarding phase 1, answering the initial research question on the learning effect concerning TmL from pre- to posttest, as described in research activities 8−11. Sections 5.5.6 – 5.5.8 contain the results on the follow-up research question on explanations for the test results, corresponding to research activities 12−14 (phase 2).

5.5.1 Internal consistency

Validity is not a singular concept, but in essence boils down to measuring what one intends to measure (Borsboom, Mellenbergh, & Van Heerden, 2004; Hoogland, Pepin, Bakker, de Koning, & Gravemeijer, 2016). The Standards of Educational and Psychological Testing define validity as “the degree to which evidence and theory support the interpretation of test scores for proposed uses of tests” (AERA, APA, &

NCME, 2014, p. 11). Gardner (1995) emphasises the fact that in a rating scale, when scores are summed up, it is important that all items reflect the same construct and according to Taber (2018), a high value of alpha is not always a good thing, for it cannot ensure that an instrument or scale is unidimensional and a high alpha may even indicate a use of an inefficient level of redundant items. Therefore, interpretation of this value is not unambiguous.

In our context, we focused on the feasibility for students to do the test in time as well as concept and construct validity with the proposed use as pre- and posttest to be able to assess development in the TmL developed by engineering students. Although we did not consider TmL to be a homogenous construct and although we were aware of the discussion among experts about internal consistency, we studied the internal consistencies of the tests to learn about the heterogeneity of the skills we purported to measure.

Although, we conjectured low alphas because of the heterogenous nature of TmL, we see in Table 4, relative normal alpha's for test A, but lower values for test B.

Table 4. Cronbach's alphas for test A (pre), test A (post), test B (pre) and test B (post)

Test	Cronbach's alpha
A (pre)	.61
A (post)	.64
B (pre)	.47
B (post)	.51

5.5.2 P values and Rit and Rir values

In RA8 a classical test item analysis is performed. As for test A used as pretest, Table 5 shows an average P value of .63, which means students did well on this test. In test A (post), which was assigned to the other half of the student population, we see an average P value of .65. As for test B (post), with an average P value of .59, we have fewer high outliers than in test A (post). This suggests that test B was more difficult than test A. Concluding, we see a small increasing P value, and therefore a small learning effect from pre to post in both tests.

Table 5. P values and pre-post differences for tests A and B

	test A (pre)	test A (post)	
average	.63	.65	
item	p-score	p-score	difference
la	1.00	.97	-.03
lb	.60	.57	-.03
le	.40	.41	.01
2a	.97	1.00	.03
2b	.49	.68	.19
3a	.53	.40	-.13
3b	.61	.71	.10
4a	.52	.65	.13
4b	.63	.51	-.12
5	.70	.77	.07
6	.24	.43	.19
7a	.86	.58	-.28
7b	.70	.82	.12

	test B (pre)	test B (post)	
average	.57	.59	
item	p-score	p-score	difference
1a	.76	.78	.02
1b	.36	.29	-.07
2	.52	.38	-.14
3a	.80	.58	-.22
3b	.67	.67	.01
3c	.35	.55	.20
4a	.48	.59	.11
4b	.20	.55	.35
5	.50	.50	.00
6	.50	.73	.23
7a	.98	.78	-.19
7b	.75	.61	-.14

For a short 30-minute test consisting of respectively 13 and 12 items, we cannot expect Rit- and Rir-values to be very high. The Rit and Rir values in Table 6 show a common pattern. In test A (post) we see somewhat lower Rit and Rir values and as a whole, and just slightly higher values than test A (pre). The boxes for item 1a in test A (pre) and 2a in test A (post) in Table 6 are empty, because all students scored the maximum 2 points for that item. Test B (pre) shows a normal pattern, but in test B (post), there are less negative outliers than in test A (post). In general, the results of the P values, and the Rir and Rit values suggest that test B was more difficult than test A.

In RA8 we also calculated the percentages of missing values. Although we shortened the tests after the aloud session with the test student, the percentage of items that were reviewed as missing value increased towards the end of the tests – see Table 7 – which suggests that the tests were too long to finish in time for many students. Test B shows more missing values in general, which could indicate that this test was more difficult for the students. Furthermore, item 4b in test A and item 1b and 2 in test B, show high values, which are probably more difficult items.

Table 6. Rit and Rir values for test A (pre), test A (post), test B (pre) and test B (post)

test A (pre)	item	1a	1b	1c	2a	2b	3a	3b	4a	4b	5	6	7a	7b
	rit		0.48	0.39	0.11	0.55	0.44	0.30	0.33	0.40	0.44	0.46	0.18	0.30
	rir		0.26	0.14	0.05	0.32	0.28	0.15	0.15	0.17	0.21	0.31	0.10	0.15

test A (post)	item	1a	1b	1c	2a	2b	3a	3b	4a	4b	5	6	7a	7b
	rit	0.36	0.61	0.42		0.34	0.32	-0.09	0.51	0.23	0.49	0.68	-0.05	0.44
	rir	0.318	0.31	0.14		0.08	0.14	-0.26	0.37	-0.14	0.25	0.48	-0.21	0.29

test B (pre)	item	1a	1b	2	3a	3b	3c	4a	4b	5	6	7a	7b
	rit	0.35	0.18	0.33	0.45	0.60	-0.09	0.29	0.38	0.48	0.58	0.34	0.52
	rir	0.222	0.04	0.02	0.29	0.49	-0.24	0.09	0.22	0.31	0.31	0.29	0.33

test B (post)	item	1a	1b	2	3a	3b	3c	4a	4b	5	6	7a	7b
	rit	0.45	-0.08	0.52	0.19	0.36	0.11	0.51	0.51	0.36	0.68	0.49	0.72
	rir	0.335	-0.17	0.28	-0.03	0.21	-0.05	0.31	0.31	0.04	0.49	0.33	0.61

Table 7. Percentages of missing values in tests A (pre), A (post), B (pre) and B (post)

A	Percentage missing values	B	Percentage missing values
1a	1.6 %	1a	1.5 %
1b	4.7 %	1b	26.2 %
1c	3.1 %	2	35.4 %
2a	1.6 %	3a	12.3 %
2b	9.4 %	3b	1.5 %
3a	3.1 %	3c	20.0 %
3b	9.4 %	4a	4.6 %
4a	9.4 %	4b	6.2 %
4b	29.7 %	5	40.0 %
5	9.4 %	6	36.9 %
6	53.1 %	7a	35.4 %
7a	37.5 %	7b	43.1 %
7b	43.8 %		

5.5.3 Pre- and posttest compared

In comparing the results on the pre- and posttest in RA9, we performed three paired *t* tests. The *t* test on the total scores for pretest (M = 58.38*,* SD = 18.17) and posttest (M = 61.41*,* SD = 19.11) did show improvement, but not a significant increase, $t(60) = -1.059$, $p = .294$, 95%CI[-8.760, 2.694], $d = 0.16$. Scores on test A (pre) (M = 62.69*,* SD = 17.61) and test B (post) (M = 58.63*,* SD = 20.14) showed a decrease in results, but not significant, $t(31) = 1.003$, $p = .324$, 95%CI[-4.202, 12.327]. However, scores on test B (pre) (M = 53.62*,* SD = 17.87), and test A (post) (M = 64.48*,* SD = 17.74) did improve significantly, $t(28) = -3.047$, $p = .005$, 95%CI[-18.165, -3.559], d = 0.61. Wilcoxon signed rank tests showed similar results. Concluding, we see an overall small, not significant, development in learning TmL, because the results of B (pre) – A (post) are contrasting with A (pre) – B (post). These results indicate that test B was most likely more difficult than test A.

As mentioned before, the posttest was scheduled in the sixth week before all the mathematical content was provided to the students. The topic of anti-derivatives was not covered yet, and this topic was addressed in item 3 of both tests. We therefore excluded this item and performed the three paired *t* tests again, but this did not show a different result. Because of this result, we decided to include this item in further analysis.

5.5.4 Lecturers compared

Because the course was taught by two lecturers to three classes and to make sure that a difference in score pre- to posttest is unlikely due to lecturers' influence, a one-way ANCOVA was conducted in RA10 to determine a difference between lecturer 0 ($M = 65.12$, $SD = 20.59$) and lecturer 1 ($M = 60.32$, $SD = 23.09$) on the scores of the students on the posttest controlling for the pretest. This difference proved not to be significant ($F(1, 58) = .604$, $p = .440$).

5.5.5 Different TmL compared

As to see how P values differ when we match them with their corresponding TmL categories 1–6, we grouped the items in RA11 for both tests A and B as shown in Table 8 and 9. This distribution shows that test items often include multiple TmL categories.

As we saw earlier, students who were assigned test B (pre) and test A (post) were higher achieving students than students with test B (pre) and test A (post). This might explain why we see a development in four TmL categories in test A and in only two in test B. The largest progress, however, is found in the two TmL *technical software skills* and *sense of error* in test B, which can be explained by the extensive and explicit presence of these TmL categories in the course. The TmL category *sense of error* is the only TmL that increased in both tests. TmL category *technical communication skills* was only addressed in item 3b in test A and in item 3a in test B, part of items that deal with theory students had not received yet. The decrease of .21 in test B, versus an increase of .09 in test A, could be attributed to the fact that test B (post) was assigned to the lower achieving students. We see that some TmL categories, such as *sense of error* and *software skills* are quite concrete, demarcated, and probably more easily learned and measured, whilst others, for example *data literacy* and *technical creativity* are broader concepts and more difficult to be learned and measured.

Table 8. Distribution of test items over TmL categories in test A and B

Test A	TmL	Item												
	1	1a			2a		3a	3b	4a	4b	5		7a	7b
	2								4a	4b				
	3							3b						
	4					2b			4a	4b				
	5		1b	1c	2a	2b	3a		4a	4b	5	6	7a	7b
	6	1a					3a	3b		4b	5			

Test B	TmL	Item											
	1	1a	1b		3a	3b	3c			5		7a	
	2							4a	4b				
	3				3a								
	4							4a	4b		6		
	5	1a	1b	2		3b		4a	4b	5	6	7a	7b
	6			2			3c			5		7a	

Table 9. P values and differences of TmL categories in test A and B

		A			B		
TmL		pre	post	difference	pre	post	difference
1	Data literacy	.72	.71	-.02	.55	.52	-.03
2	Technical Software skills	.57	.56	-.01	.33	.57	.24
3	Technical communication skills	.61	.70	.09	.80	.58	-.21
4	Sense of error	.54	.61	.06	.38	.62	.24
5	Sense of number	.55	.62	.07	.57	.54	-.03
6	Technical creativity	.58	.66	.09	.59	.55	-.04

5.5.6 Marks on course in chemical calculation compared with test A or B as pretest

In comparing the marks of the students on the course in chemical calculation with being assigned to test A or B as pretest (RA12), we found, in an unpaired *t* test, a difference between test A (M = 71.94*,* SD = 14.83) and test B (M = 78.35*,* SD = 13.82), although marginally not significant, $t(63) = -1.779$, $p = .077$, 95%CI[-6.414, 3.565]. It seems that students who performed test B as a pretest achieved higher on the course chemical calculation than the students who were assigned to test A as pretest. This could be an explanation as to why the B (pre) – A (post) students showed TmL development, and group A (pre) – B (post) did not. Because test B (pre) students were higher achievers, and the difference between the scores in test A (post) and in test B (pre) is significant, that we can also assume that these students have learned more in this course. For higher-order skills, such as TmL, the learning effect often depends on the academic level.

5.5.7 Tests A and B compared

In RA13 we compared test A and B with independent *t* tests to investigate whether they indeed differ in difficulty, as was suspected by the results on the development from pre- to posttest in section, as shown in section 5.5.1. In comparing test A (pre) (M = 62.46*,* SD = 17.46) with test B (pre) (M = 53.97*,* SD = 18.05), we see that students on test A scored higher, although marginally statistical not significant, $t(66) = 1.971$, $p = .053$, 95%CI[8.487, 4.306]. We found the same in the posttest; although scores on test A (post) (M = 64.48*,* SD = 17.74) are higher than on test B (post) (M = 58.63*,* SD = 20.14), it is not a significant difference, $t(59) = 1.200$, $p = .235$, 95%CI[-3.910, 15.626]. Three Mann-Whitney tests confirm these results. Boxplots of the scores on both pretests and posttests are presented in Figure 4. Altogether, there is reason to suspect that test B is more difficult than test A.

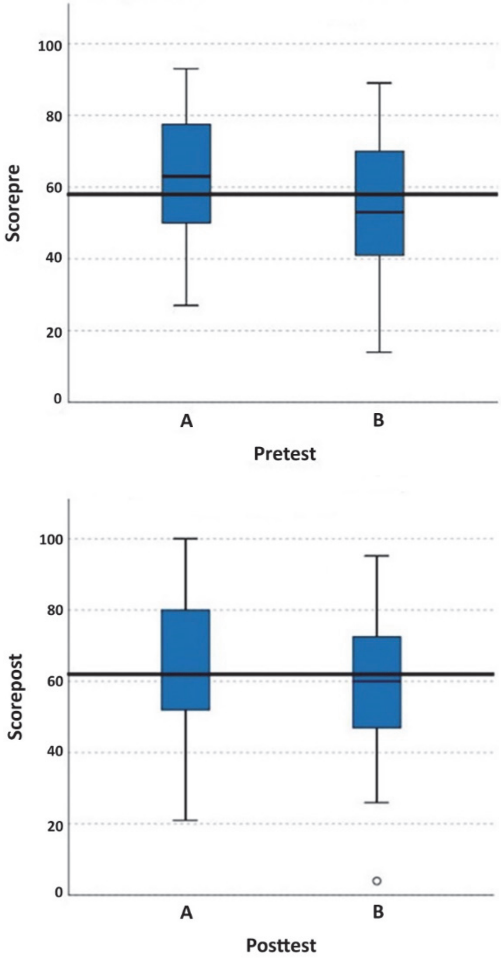

Figure 1. Boxplots of the scores of test A and test B in pre- and posttest (normalised scores ranging from 0 – 100)

5.5.8 Posttests scores and summative digital test marks compared

To investigate whether test B was indeed more difficult than test A, the marks of the students' summative test are compared with the scores on the posttest in RA14. A paired *t* test, comparing the test A (post) results (M = 64.48, SD = 18.39) with the summative test marks (M = 71.48, SD = 16.81), showed not to be significant, $t(26) = -1.723$, $p = .097$, 95%CI[-15.352, 1.352]. The paired *t* test, comparing test B (post) results (M = 58.63, SD = 20.14) with the summative test marks (M = 75.56, SD = 12.78), did show to be significant, $t(31) = -4.200$, $p < .001$, 95%CI[-25.163, -8.172], d = 1.00, which indicates that test B is indeed more difficult than test A.

As content experts, it is not clear to us why test B is perceived as more difficult. Especially item 7b of both tests, in which a growing factor is asked, seems equally difficult. However, item 7b of test A (Figure 2) showed an increase in P value of .12, but test B (Figure 3) a decrease of .14. Therefore, we presented both items to a non-participating colleague in the School of Life Sciences and Environmental Technology to judge these two items. He stated that he could not see any difference in difficulty in both items. Moreover, he conjectured that the item in test A could be perceived as more difficult because of the use of more formal mathematical language. An explanation for this could be, again, the fact that the higher achieving students were assigned to test A as posttest. It is also often the case that last items of a difficult test score rather poorly, because students get tired, lack time, or give up.

Saving for a home

Dirk is saving to be able to buy a home. wants to take out the lowest possible mortgage and therefore decides to save as long as possible. The level of his income hardly rises and the savings thus proceed almost linearly as a function of time. However, house prices are rising exponentially. The type of house he has in mind costs € 180,000,- when he starts saving. This price increases by 8% per year. The house price as a function of time in years can be shown with the equation below. What is the value of g?

$$House\ price\ (t) = 180{,}000 \cdot g^t$$

$g = 1.08$

Figure 2. Item 7b from test A (see also Appendix 1)

Salary increase

First-year student Yara from Chemistry Den Bosch got a very good side job last year in which she earns 600 euros per month. Every year she continues to work there she receives a salary increase of 2%. The equation for her salary is shown below. What is the value of g?

$$Salary\ (t) = 600 \cdot g^t$$

$g = 1.02$

Figure 3. Item 7b from test B (see also Appendix 2)

5.6 Conclusion and Discussion

In this paper, we first addressed the research question of what the learning effect is of a course in applied mathematics on students' development of Techno-mathematical Literacies. Unfortunately, we can only provide a mixed answer to this question. On the one hand, we do see progress from pre- to posttest scores, which suggests a positive learning effect on TmL skills. On the other hand, the answer is preliminary, as the progress is not large, and the picture is somewhat distorted by differences between the two test that were used and the slight initial differences between the two test condition groups.

The tests A and B were randomly assigned to the 68 participating students. The two tests were assigned vice versa as posttest to the 62 present students as to prevent a test learning effect. In analysis we compared pre- and posttest scores and found a mixed result; the scores of the students in test B (pre) and test A (post) increased significantly, but the scores from students in test A (pre) and test B (post) did not. Furthermore, by performing a one-way ANCOVA, we made sure that a difference in score due to lecturers' influence is unlikely.

We did, however, report a progression on scores in certain TmL categories. TmL *sense of error* showed a large increase in scores between pre and post in both tests. For TmL *software skills* we see a large improvement in test B. These TmL categories are rather concrete and demarcated, and we think they are easier to learn and measure than a TmL category such as *technical creativity* or *data literacy*, which are more complex. With this, we mention the discussion about "measure what you find important" versus "finding important what you measure" and the challenge not to focus on just teaching what you can measure (Collins, 2017). Can TmL be measured per se, or can TmL development be better assessed via qualitative methods? Our experience with oral assessments, not reported here, was promising in that respect, although practically too time-consuming and calibration among colleagues proved to be challenging. Furthermore, one can wonder whether complex TmL can be developed in just one course or might need more time to develop. Can our course be improved to foster their development? These are all questions for further research.

From these results, we suspected that test B was more difficult than test A. Therefore, we investigated the follow-up question of what possible explanations there might be for the contrasting results. To answer this question, we first investigated if the random assignment of the two tests as pretest had led to equivalent groups of students. To this end, the scores of a previous course in chemical calculation that students followed earlier that year was compared with the assignment to test A or B as pretest. We found that students who were assigned to test B as pretest were higher achieving than the students with test A as pretest, although marginally statistically insignificant. This could be an explanation as to why the higher achieving students showed more development from pre- to posttest, but also indicated that test B could be more difficult than test A.

Subsequently, we compared test A (pre) to test B (pre) and test A (post) to test B (post). Although an unpaired *t* test did not show a significant difference between test A and B, and the lecturers could not recognise a difference in difficulty, we decided to compare posttest scores with the summative test marks of the course, and with these paired *t* tests, it was proved that test B was indeed more difficult than test A. We cannot provide an explanation for this fact, it was unexpected and as content experts, we do not recognise this difference. The test items distribution over test A and test B, therefore, should be re-evaluated, and discriminating items should be identified, so as a next step, the tests A and B should be equivalated with respect to difficulty.

Before we discuss these conclusions, we first have to point out some of the study's limitations. On the practical side, we had to deal with a limited sample size and number of test items, because we could not spend more than half an hour to perform the test, as class-time is limited, and students participated voluntarily in this study. However, this did not lead to very low Cronbach's alphas in the tests, although lower for test B than for test A. So, although we argued that TmL are complex and heterogenous constructs, we were surprised to find a medium internal consistency.

Reflecting on these conclusions, we notice that, in measuring TmL, we entered new territory, as this has not been done before and no material was available. In general, designing test items for complex skills is not an easy task (Drijvers, Kodde-Buitenhuis, & Doorman, 2019). The measurability of TmL, however, is yet completely unclear and this study aims to take a first step and contribute to the knowledge of testing TmL. In the process of designing, validating and conducting this test we faced several challenges and mixed results, which we will discuss.

Designing adequate test items for TmL proved to be very challenging, results are not always straightforward, and sometimes, certain effects cannot be explained. As a next step, we plan to randomise test items both within (RA15) and between the two tests A and B (RA16). This will ensure that the highest percentages of missing values are not concentrated at the end of each test, and more importantly, as mentioned above, by mixing matching items from both tests and detecting discriminating items, we can equivalate the tests in difficulty. With this study we laid a first foundation for testing the development of Techno-mathematical Literacies.

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Appendix 1

TEST A

1. Tunnel drilling

In Switzerland, a new tunnel is being drilled for a new highway through a mountain. The speed of drilling in this tunnel is almost linear. The total length of the tunnel is 3.8 km.

2 tunnel drills are used:

- The tunnel drill that starts at 0 meters in the starting station is called "tunnel drill_there" and drills towards the end station at 3.0 meters per day
- The tunnel drill that starts at 3800 meters in the end station is called "tunnel drill_back" and drills towards the starting station at 4.2 meters per day.

a. Make a sketch of the situation. (Sketching means not drawing very accurately or neatly) *TmL 1,6*

This can be a picture of a line one way and one the other way, can be anything. A graph with linear function (straight line) is also good.
2p

The following equation for the distance in meters as a function of time in days for tunnel drill_there:

$$S_{td\_there}(t) = 3.0t$$

This equation is a form of the standard linear equation: $y = ax + b$ with $a = 3.0$ and $b = 0$.

b. Make the equation for the distance (s) in meters as a function of time (t) in days for the above-mentioned tunnel drill_back. Use the standard linear equation for this:
$y = ax + b$ and determine a and b. *TmL 5*

$s(t) = -4.2t + 3800$
Initial value is 3800 meters.
Speed is 4.2 meters per day back to 0. Direction coefficient must therefore be negative.
4p

---

c. After how many days is the tunnel expected to be ready? *TmL 5*

Make equal and solve:
$3.0t = -4.2t + 3800$

$7.2t = 3800$
$t = \frac{3800}{7.2} = 527.9\ or\ 528\ days$
3p

2. Losing weight

In the TV show My 600-lb life, people from the U.S. with morbid overweight who are given a stomach reduction by Dr. Nowzaradan are followed. After they have lost a lot of weight, often part of their now too large skin surface is removed.

With the Dubois equation you can calculate for a given body weight (G) in kg, height (L) in cm and the skin surface (H) in m^2. The equation reads:

$$H = 0.0072 \cdot G^{0.425} \cdot L^{0.725}$$

A man in this show weighs 296 kg and is 185 cm tall.

a. Show with a calculation that this man has 3.56 m^2 of skin. *TmL 1,5*

$H = 0.0072 \cdot G^{0.425} \cdot L^{0.725}$
$H = 0.0072 \cdot 296^{0.425} \cdot 185^{0.725} = 3.56\ \text{m}^2$
2p

After his surgery with Dr. Now, he loses a lot in two years. He now has no less than 0.63 m^2 of skin too much.

b. The calculation of how much weight he has lost is shown below. However, there is 1 mistake, track it down and explain. The error has not been calculated. The final answer is therefore good. *TmL 4,5*

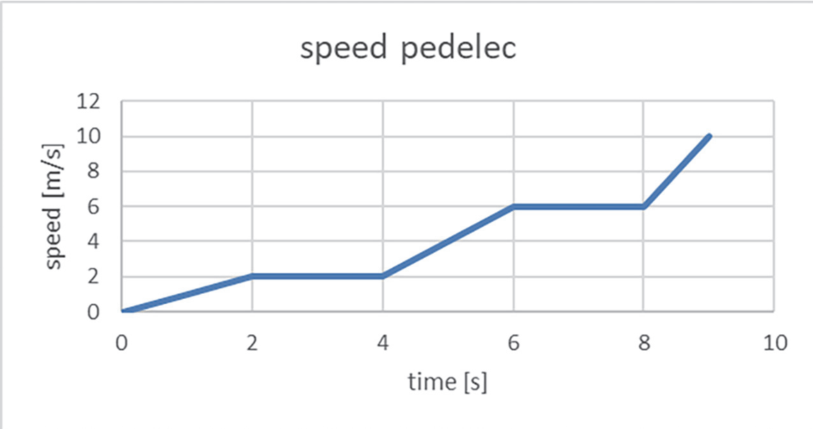
$H = 0.0072 \cdot G^{0.425} \cdot L^{0.725}$ (1)
$3.56 - 0.63 = 0.0072 \cdot G^{0.425} \cdot 185^{0.725}$ (2)
$G^{0.425} = \frac{0.0072 \cdot 2.93}{185^{0.725}}$ (3)
$G^{0.425} = 9.2435$ (4)
$G = 9.2435^{1/0.425} = 187\ \text{kg}$ (5)

The 0.0072 is in the numerator at equation 3
and should be in the denominator.
3p

So he lost 296 - 187 = 109 kg. What a guy!

3. Speed pedelec

Because Mees lives 28 km from school and is tired of all train delays, he is considering purchasing a speed pedelec. This is an e-bike with a maximum speed of 45 km/h. Since Mees has not had any math this year, he makes a simplified graph of the speed of the first nine seconds of an accelerating speed pedelec.



a. The distance covered by the speed pedelec can be determined by calculating the area under the speed-time graph. What is the approximate distance travelled in the interval 4 to 8 seconds? *TmL 1,5,6*

Several possibilities. For example, large square - small triangle

$Large\ square\ = (8-4)\cdot(6-0) = 24$ m

$Small\ triangle\ = 0.5\cdot\ (6-4)\cdot(6-2) = 4$ m

$Distance\ traveled\ = 24-4 = 20$ m

2p

---

b. Describe how the graph of the speed pedelec actually works. *TmL 1,3,6*

Something like no angular points in the graph, but more smoothly, perhaps no straight lines at the part where speed increases. Or as a linear line (intuitive). Or as a root function. Exponential line is not correct.

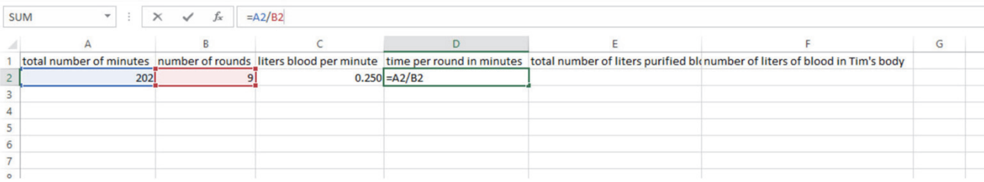
2p

4. Kidney dialysis

The kidneys purify the blood of all kinds of waste products. If the kidneys function very poorly, the blood is not properly purified, and it must be purified artificially. This artificial purification is called renal dialysis. Tim will start kidney dialysis every Monday, Wednesday and Friday. On the site www.nieren.nl he finds the following about the haemodialysis machine:

> *Blood repeatedly passes through the artificial kidney.*
>
> *1 liter of blood passes through the artificial kidney every 3 to 5 minutes. An adult human has about 5 liters of blood. This means that after 15 to 25 minutes all your blood has passed through the artificial kidney. Hemodialysis treatment takes an average of 4 hours. The blood passes through the artificial kidney several times during 1 dialysis treatment. That improves the filtering of waste products and superfluous substances from the blood.*

As a curious chemistry student, Tim would like to learn more. With the data from his first dialysis, he wants to calculate exactly how much blood he has in his body. He sees that the machine is set to 250 mL of blood per minute and has counted 9 rounds. He prepares an Excel file:



SUM =A2/B2

	A	B	C	D	E	F	G
1	total number of minutes	number of rounds	liters blood per minute	time per round in minutes	total number of liters purified bl	number of liters of blood in Tim's body	
2	202	9	0.250	=A2/B2			
3							
4							
5							
6							
7							

In the formula in cell D2, Tim has chosen to use cell references A2 and B2 instead of numbers 202 and 9, so he can also use this tool for other patients by simply adjusting the numbers in A2, B2 and C2.

a. What formula did Tim use in cell E2? *TmL 1,2,4,5*

=A2*C2
2p

b. Tim arrives at 5.61 L of blood in his body in cell F2 with the formula "=E2/B2". This is possible with two other formulas as well. Give one. *TmL 1,2,4,5,6*

=D2*C2 or =A2*C2/B2 or similar
3p

5. Virus infection

On November 1, the number of 1st-year students of Chemistry in Den Bosch with the flu increased according to the quadratic function $N = -d^2 + 4d$, where N is the number of sick students and d the number of days. After 4 days everyone was better again. On which day was the number of sick students greatest? Tip: sketch a graph. *TmL 1,5,6*

This can be solved in 3 ways:

- by making the derivative equal to 0: $N'(d) = 0$, $N'(d) = -2d + 4$, $-2d + 4 = 0$, $d = 2$.
- the equation for the top of a parabola: $d_{\text{top}} = \frac{-b}{2a} = \frac{-4}{2 \cdot -1} = 2$
- The equation is a parabola that intersects the x axis in d is 0 and 4. Half-way is the top, so that is at $d = 2$.

3p

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6. Strong ants

Ants can lift a weight that is greater than their own. The weight (G) an ant can lift is represented by the following equation: $6mG = G^2 + 9m^2$. Below is the start of describing this equation in G as a function of m. Finish it. *TmL 5*

Tip: use $(a - b)^2 = a^2 + b^2 - 2ab$ or the quadratic-equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$6mG = G^2 + 9m^2$
$G^2 - 6mG + 9m^2 = 0$
$(G - 3m)^2 = 0$
$G = 3m$
3p

7. Saving for a home

Dirk is saving to be able to buy a home. wants to take out the lowest possible mortgage and therefore decides to save as long as possible. The level of his income hardly rises and the savings thus proceed almost linearly as a function of time. However, house prices are rising exponentially. Dirk can save an average of € 1,000 per month. When he starts saving, he has € 11,000 in his account. Dirk wants to calculate a few things. First, he establishes the function of his savings.

a. Draw up the savings as a function of time (in years). Use the standard linear equation for this: $y(x) = ax + b$ *TmL 1,5*

$Savings\ (t) = 12000t + 11000$
2p

b. The type of house he has in mind costs € 180,000,- when he starts saving. This price increases by 8% per year. The house price as a function of time in years can be shown with the equation below. What is the value of g? *TmL 1,5*

$House\ price\ (t) = 180{,}000 \cdot g^t$

$g = 1.02$
2p

Appendix 2

TEST B

1. Mortgage

Ilona wants to buy a house and needs a mortgage for this. She wants to collect information about the monthly costs for a linear mortgage.

Figure 1 shows the concept of a linear mortgage.

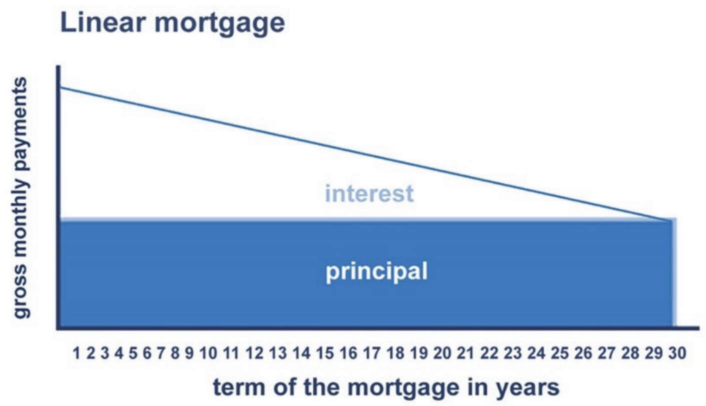


Figure 1. Linear mortgage

Ilona needs a mortgage of € 288,000. The term for the mortgage is 30 years and the annual mortgage interest is 2%. This interest applies to the remaining principal and is therefore initially € 288,000.

a. Calculate the interest for the first month. *TmL 1,5*

$$interest = \frac{Mortgage \cdot interest}{12\ months}$$

$$= €288{,}000 \cdot \frac{0.02}{12\ months} = €480$$

2p

The monthly repayment is calculated by dividing the mortgage by the term and is therefore equal to:

$$monthly\ payment = \frac{mortgage}{term\ in\ years \cdot 12\ months} = \frac{€288{,}000}{30 \cdot 12} = €800$$

The gross monthly charge equals the monthly repayment, which is constant, plus the monthly interest that gradually falls, so:

$$gross\ monthly\ charge\ = monthly\ repayment + interest$$

b. The gross monthly charge is a linear decreasing line ($y = -ax + b$), see the figure. Draw up the equation for this line. So, calculate a and b. Tip: b is not equal to the monthly payment! *TmL 1,5*

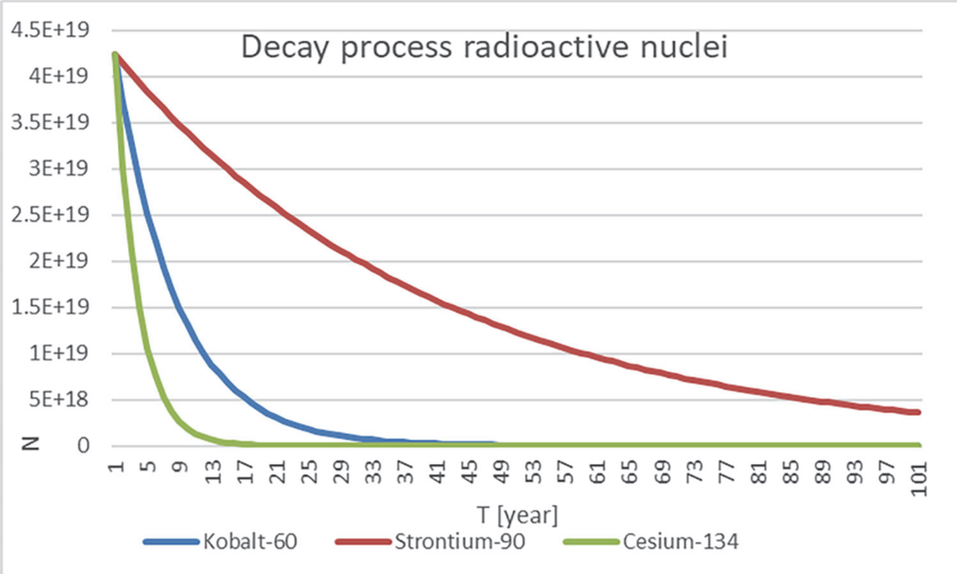
$General\ formula\ for\ linear\ relationships$:
$y = ax + b$
$Equation\ monthly\ interest = \frac{\Delta interest}{\Delta months}$
$= 0 - \frac{480}{360} = 0 - \frac{4}{3} \rightarrow a = -\frac{4}{3}$
$calculate\ b \rightarrow fill\ in\ y\ with\ a\ given\ value\ x$:
$1280(0) = -\frac{4}{3} \cdot 0 + b \rightarrow 1280 = 0 + b \rightarrow b = 1280$
$fill\ in\ a\ and\ b\ in\ general\ formula$: $y = -\frac{4 \cdot t}{3} + 1280$
Or fill in 2 points (first month and last month)
3p

2. Half-life

Sharon conducts research into the decay process of radioactive materials. This decay process depends, among other things, on the half-life. The half-life $t_{0.5}$ is the time after which exactly half of an original amount of substance remains. The number of nuclei N is a function of the time in years and the half-life $t_{0.5}$. $N_0$is the number of nuclei at $t = 0$. *TmL 5,6*

$$N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{t_{0.5}}}$$

The decay process of a number of radioactive nuclei is shown below.



The number of nuclei at $t = 0$ is $4.24 \cdot 10^{19}$. Below, the half-life for the nucleus Radium-226 is calculated. The decay process of this nucleus is not shown in the graph. After 3200 years, there are still $1.06 \cdot 10^{19}$ nuclei. Complete the half-life calculation.

$$N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{t_{0.5}}}$$

$$1.06 \cdot 10^{19} = 4.24 \cdot 10^{19} \cdot \left(\frac{1}{2}\right)^{\frac{3200}{t_{0.5}}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{3200}{t_{0.5}}}$$

$$\frac{1}{2}^{2} = \left(\frac{1}{2}\right)^{\frac{3200}{t_{0.5}}}$$

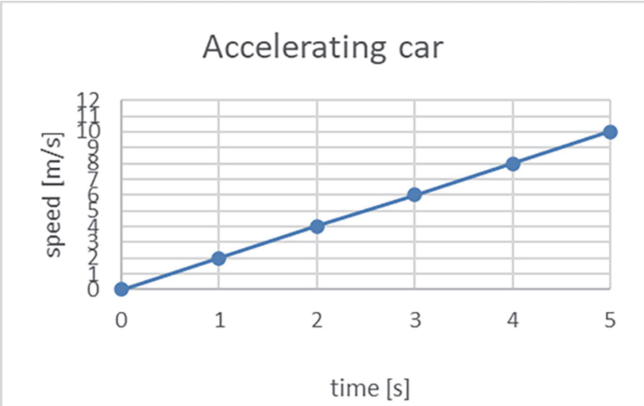
$$2 = \frac{3200}{t_{0,5}}$$

$$t_{0,5} = 1600\ years$$

(Or creatively: After 3200 years there is still a quarter of cores left. So, after 1600 years half 😊)
3p

3. An accelerating car

A car accelerates. It has a speed of 10 m / s after 5 seconds. Look at the following chart.



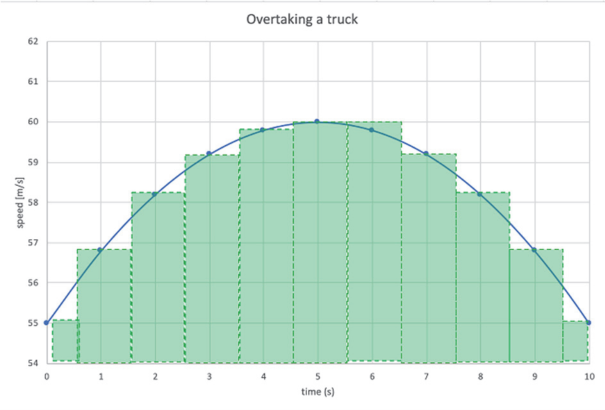
a. Explain why the area under the graph is equal to the distance traveled by the car. *TmL 1,3*

You multiply time times speed, which is equal to distance: $s = v \cdot \mathrm{t}$
2p

b. How much distance has the car travelled? *TmL 1,5*

Area under the graph: $0.5 \cdot 5 \cdot 10 = 25$ m
2p

With a non-linear line, the area is more difficult to calculate. We then use partial areas that we add up. The car now drives on and overtakes a truck, for which it needs to accelerate. After overtaking, the driver takes his foot off the accelerator and his speed drops. The speed-time graph is shown below.



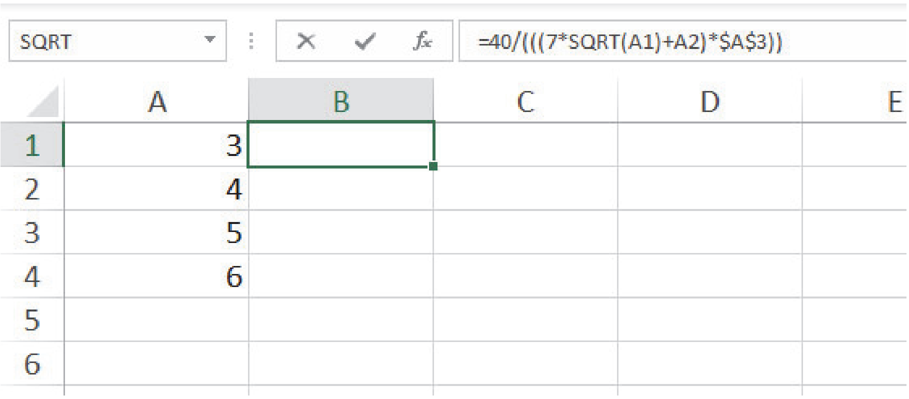
We approximate the distance travelled by adding the areas of the rectangles.

c. How could we determine the distance travelled even more precisely? Just indicate HOW, you don't have to calculate anything! *TmL 1,6*

By taking more and narrower bars. Or another creative explanation such as making triangles or something like that.
2p

4. Excel formula

Look at the image below. The formula in the formula bar belongs to cell B1.



a. What value will Excel automatically fill in in cell B1? *TmL 2,5*

$\frac{40}{(7\cdot\sqrt{3}+4)\cdot 5} = 0.4961$ or with rounding off
2p

b. The formula in cell B1 is extended to cell B2 by means of the green block. What value will be in cell B2? *TmL 2,5*

$\frac{40}{(7\cdot\sqrt{4}+5)\cdot 5} = 0.42105$ or with rounding off
2p

5. Golf course

A new golf course is being constructed near 's-Hertogenbosch. The available surface is rectangular and comes out at 1 km^2. The length of the golf course is 400 meters longer than the width. Determine the length and width of the golf course.

Tip: Use symbols for the golf course, for example L for length and B for width, where $L = B + 400$. Make a drawing. The quadratic equation: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

TmL 1,5,6

The area of a rectangle is $L \cdot B$. 1 km^2 is equal to 1000 m · 1000 m = 10^6 m^2
$10^6 = (B + 400)B$
$10^6 = B^2 + 400B$
$B^2 + 400B - 10^6 = 0$ (use quadratic equation)
$B = 819.8$ m and $B = -1219.8$ m
B = 819.80 m, L = 1219.8 m or with rounding off
3p

6. Capacitor

A capacitor is an electrical component that stores electrical charge. During charging, the electric current I decreases exponentially as a function of time according to the equation:

$$I = \frac{V}{R} e^{\frac{-t}{RC}} \qquad (1)$$

V, R, C are variables. We convert this equation into t as a function of I. However, one error has crept in here. Detect and explain this error. The error was used for further calculations. *TmL 5,6*

$$I = \frac{V}{R} e^{\frac{-t}{RC}} \qquad (1)$$

$$\frac{IV}{R} = e^{\frac{-t}{RC}} \qquad (2)$$

$$ln\left(\frac{IV}{R}\right) = \frac{-t}{RC} \qquad (3)$$

$-t = RC \cdot ln\left(\frac{IV}{R}\right)$ (4)

$t = -RC \cdot ln\left(\frac{IV}{R}\right)$ (5)

The error is in equation 2 (it should be IR/V)
3p

7. Salary increase

First-year student Yara, who studies chemistry in Den Bosch got a very good side job last year in which she earns 600 euros per month. Every year she continues to work there she receives a salary increase of 2%.

a. After how many years will she earn more than 630 euros per month? *TmL 1,5,6*

This can be done by calculating a number of years
After 1 year: $600 \cdot 1.02 = 612$ or $600 + 0.02 \cdot 600 = 612$
After 2 years: $612 \cdot 1.02 = 624.24$
After 3 years: $624.24 \cdot 1.02 = 636.72$
So, after 3 years she earns more than 630 euros a month.
Or via solving the equation: $600 \cdot 1.02^t = 630$
2p

The equation for her salary is shown below. What is the value of g? *TmL 5*

$Salary\ (t) = 600 \cdot g^t$

$g = 1.02$
2p

6 Conclusions and reflections

6.1 Introduction

The aim of this study is to contribute to a knowledge base on the teaching and learning of Techno-mathematical Literacies in higher technical professional education, through developing and implementing an innovative and sustainable course in applied mathematics for 1st-year students in the School of Life Sciences and Environmental Technology at Avans University of Applied Sciences. In this final chapter, we summarise and reflect upon the major results and conclusions of the study. The research question which guided this project, as formulated in Chapter 1, is:

How can an innovative course in applied mathematics in the technical domain of higher professional education help students acquire the TmL necessary for their future workplaces?

The main research question is divided into four subquestions that are addressed in the Chapters 2–5:

RQ1: Which TmL do engineers use in different professional practices and what are their opinions and ideas regarding their own and future mathematics education?

RQ2: During the discussions of context-based cases, what teaching strategies did the lecturer use to foster TmL?

RQ3: Which challenges may arise in the relation between teachers' agency and the aim of professional development, and the goal of a sustainable implementation of an innovation?

RQ4: What is the learning effect of a course in applied mathematics on students' development of Techno–mathematical Literacies?

RQ5: What are possible explanations for the unexpected results?

To answer the first question, an interview study was conducted with fourteen engineers in various technical domains. With the results of this study as input, a new course of applied mathematics was designed for higher technical professional education in the Netherlands. After a pilot study of this innovative course, in which teacher strategies were analysed, the course was implemented in the curriculum of the School of Life Sciences and Environmental Technology for all 1st-year students by 11 lecturers, and monitored, evaluated, and adjusted in three year-long cycles. To address the challenges in teaching the course, an interview study was conducted. Lastly, the effect of the course on the learning of TmL was studied by means of a pre- and posttest study in the third year of the implementation.

In the following sections, the results and conclusions of the studies are described and linked to the body of literature. Furthermore, limitations of the project are addressed, and ideas and directions for future research are shared.

6.2 Techno-mathematical Literacies in the workplace of engineers

The interview study with fourteen engineers was carried out to address the first research question:

> *RQ1: Which TmL do engineers use in different professional practices and what are their opinions and ideas regarding their own and future mathematics education?*

In answering the first part of this research question, data analysis led to the identification of seven TmL categories that these engineers use in their daily practices, as presented in Table 1.

It was quite striking that most engineers claimed to hardly use any mathematics in their work, whereas observation of their use of software, for example, painted a completely different picture. In general, the perception that the engineers have of mathematics is quite small. For the engineers, it appeared that even logarithms were not considered to be mathematics at first sight, let alone the equations in their calculation tools. Most mathematics in engineers' practice, however, remains hidden behind the interfaces of software. Therefore, it may seem that less mathematical knowledge is needed. On the contrary, there is an increased need to understand and interpret quantitative data (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002). This was also found in this study, as engineers showed to practice a broad range of TmL, which matches examples of TmL in literature (e.g., Hoyles, Noss, Kent, & Bakker, 2013; Bakker, Hoyles, Kent, & Noss, 2006). The TmL categories *technical creativity* and *technical drawing skills* proved to be specific for the technical domain and were not included before.

As for the second part of the first research question, engineers explained how they have experienced mathematics in their own education as an island in the curriculum. Mathematics was mostly abstract to them and without context. This often caused a decrease in motivation and difficulties in recognising mathematics in other domains. They suggested, therefore, that mathematics should be taught in the context of professional tasks and situations, not just to increase student motivation, but also to better prepare students for the workplace in which mathematics is always used in application, with real data, not ideal 'easy' numbers, as is also advised in literature (e.g., Herrington, 2006).

Table 1. TmL categories identified in interviews with in-service engineers

	TmL category	Description
1	Data literacy	The ability to analyse and interpret technical data and graphical representations, draw conclusions and take action accordingly
2	Technical software skills	The ability to use professional software, e.g. Excel™, as a calculation tool
3	Technical communication skills	The ability to communicate technical information with colleagues, customers, supervisors and other parties
4	Sense of error	The ability to check and verify data and detect errors
5	Sense of number	The ability to handle and interpret numbers sensibly
6	Technical creativity	The ability to produce creative solutions to puzzles and problems (by using, e.g. cleverness or experience)
7	Technical drawing skills	The ability to understand and produce technical drawings (by using, e.g. spatial insight)

The identified TmL in this study and the engineers' views on mathematics education result in recommendations for higher technical professional education that are in line with our view that mathematics needs to be based on professional tasks with TmL as important learning goals, because students' future workplaces will be more and more immerged by advanced technology.

6.3 An innovative course in applied mathematics

With the identified TmL in the workplaces of engineers as central learning goals and the interviewed engineers' advice to teach mathematics in context, a new course in applied mathematics was designed by an interdisciplinary team of lecturers from Avans University of Applied Sciences and HU University of Applied Sciences Utrecht through the approach of (educational) design research (Bakker, 2018; Plomp, 2013). The course consisted of two complementary learning tracks in parallel. The basic skills were addressed in a learning track in which students practiced abstract mathematics individually through the use of an electronic learning environment called ALEKS™. In this way, they built a robust foundation of basic skills for the second track. This second track focused on complex cases in applied contexts, on which students worked collaboratively. We developed three cases for the domain of life sciences, to be studied in a six-week period. The mathematical topics that were needed in the applied cases matched with the mathematical foundations treated in the abstract track. Linear and quadratic functions were practiced in the first case, on chemical solutions and weak acids. The second case, on bacterial growth, made use of exponential, (natural) logarithmic and rational functions. The third case was dedicated to the principles of derivatives and anti-derivatives, first in the context of distance, time and speed of a runner, then about acid-base titrations.

Classroom or group discussions can be essential in fostering understanding and competence for complex skills (Nathan & Kim, 2009), such as TmL. Therefore, so-called feedback hours were held as most important part of the classes, in which Inquiry-based Learning (IBL) was used to focus on the process, as inquiry is at the base of science practice (Anderson, 2002; Chu, Reynolds, Tavares, Notari, & Lee, 2017). During the feedback hours, the lecturers used a variety of teacher strategies to foster TmL learning by their students, ranging from pedagogical strategies, general IBL strategies and TmL-specific IBL strategies. Table 2 elaborates these strategies. The analysis of the video-recordings of feedback hours of the pilot with 59 chemistry students indicated that TmL learning was stimulated during the classroom discussions. The second study tried to answer the following research question:

RQ2: During the discussions of context-based cases, what teaching strategies did the lecturer use to foster TmL?

Conducting feedback hours proved to be quite challenging for the lecturer (the author), as she had to take on multiple roles as an instructor, coach, moderator and socialiser. Engaging all students was not always possible, as distractions, such as phones and the whole wide world on a laptop are always present. Some students had difficulties with the formal language of the cases, although we tried to minimize mathematical jargon as much as possible. Process management proved to be extremely important, and the lecturer extensively explained the course content and new didactical approach with the students, because almost everything in this course was different from what they were used to. After the pilot, a questionnaire was distributed, and nine short interviews with participating students were conducted by a research-assistant. Students stated that they had to get used to the different didactical approaches and mathematical language in the cases, but that they were content in general and more motivated because they recognised the practical use of this kind of mathematics in their education.

Table 2. Teaching strategies used by the lecturer during the feedback hours

Pedagogical strategies
The lecturer stimulates that this hour is meant to help each other; Acknowledges the difficultness of the subject; Stimulates pride for their work in students; Formulates in a positive way and uses humour; Stimulates applauding for each other; Encourages feedback from students on the cases to further improve them for future students; Addresses possible feelings of frustration at students.

Process strategies
Stimulates working together more and emphasises not to divide sections between group members; Stimulates to simmer on the problem after class; Checks the progress of the groups and adjusts accordingly; Stimulates to write down the feedback after presentation; Stimulates to write down feedback of others that they can use themselves; Asks the contribution of each student to the product; Explains the rationale of the assignment; Asks what students need to continue before ending the class.

General IBL strategies
Asks to show something that went well (stimulating success experience); Asks where students got stuck; Structures answers and theory and recapitulates what students say; Asks what the thought process was; Starts problem-solving on whiteboard and asks students to finish; Asks how students will proceed with this problem; Gives a tip; Compares used heuristics in groups; Explains connection between math and other courses or future profession; Discusses the way one can work on these cases, advantages and disadvantages of strategies.

TmL specific IBL strategies
For all TmL: Asks deeper questions about data, tables, formulas, and figures. *For TmL technical communication:* Stimulates to rephrase in own words; Stimulates to take a helicopter view in elaborating; Stimulates explaining to each other; Asks a student to elaborate on the answer; Asks class to formulate a general strategy. *For TmL Sense of number and sense of error:* Stimulates to use numbers that are realistic or easy; Asks class to spot an error; Let students discover their mistake by stimulating thinking about the logical answer. *For TmL technical software skills:* Asks a student to show and explain their Excel™ calculation tool live.

In general, because of the changing nature of the workplace, it is important for a professional to be flexible and resilient. Jobs are not for life anymore, and continuous development is increasingly important. Education needs to follow in becoming more flexible in content, level and tempo, and teacher roles need to change alongside.

Therefore, there is a growing awareness in higher education of the importance to foster the development of a professional identity in both students and lecturers, which involve social and emotional skills, such as a growth mindset, self-efficacy, self-confidence and resilience for success in study and work (Yeager & Dweck, 2012). In our technical education, this is even more necessary, because we, as engineering lecturers and students, do not excel in these skills by default. In the School of Life Sciences and Environmental Technology, several projects have been started over the last few years to promote the development of professional identity in both lecturers and students.

When education changes and courses innovate, we often ask more independent behaviour of students, as we did in this new course in applied mathematics. As we try to spread the key aspects of the new course into educational curricula inside and outside Avans University, we hope that students will be more equipped for innovative courses in the coming years. We know, however, that educational changes, in general, take a lot of time (Hargreaves, Lieberman, Fullan, & Hopkins, 2010).

As for the learning of TmL, we ask ourselves whether we could further develop the use of these skills in mathematics education in general and in our course in particular. In designing context-based cases, it is challenging to develop truly different questions rather than the usual ones in which students are asked to solve something. The usual type of questions depends mostly on the TmL *sense of numbers* and was too dominantly present in our cases. As for future plans, we are also researching whether we can adjust cases to be more open and less guided. This is something for the medium-term future, as lecturers build more confidence and experience in teaching this course, and as students develop skills to handle such cases.

6.4 Implementing the new course in applied mathematics

After the pilot, the course was revised and implemented in the curriculum of the School of Life Sciences for all majors and taught by 11 lecturers to over 1400 first-year students during the years 2016–2019. We anticipated that teaching a new course with both different content and didactics would need extensive support and training for the lecturers. Most of them had not been involved in the design process, because this was not possible due to mostly practical circumstances. From literature, we learn that teachers are the most important agents, and many educational innovations have failed due to a lack of teacher learning (Borko, 2004). The approach of Design-based Implementation Research (DBIR) provides four key principles to support the transfer from design to implementation (Fishman, Penuel, Allen, Cheng, & Sabelli, 2013, p. 332):

1. A focus on persistent problems of practice from multiple stakeholders' perspectives
2. A commitment to iterative, collaborative design
3. A concern with developing theory and knowledge related to both classroom learning and implementation through systematic inquiry
4. A concern with developing capacity for sustaining change in systems.

The research question we therefore formulated for this study was:

RQ3: Which challenges may arise in the relation between teachers' agency and the aim of professional development, and the goal of a sustainable implementation of an innovation?

To answer this question, we conducted two rounds of semi-structured interviews with the lecturers, to deeply understand their experiences, opinions, dilemmas, feelings, and beliefs in teaching the new course. Data analysis was performed using a version of the constant comparative method (CCM) in which initial conjectures were generated and revised during subsequent episodes. From this analysis, we learned that conducting the feedback hour with an IBL approach was the most challenging for the lecturers for the following reasons. In the School of Life Sciences and Environmental Technology, mathematics courses are not taught by mathematicians, but by chemists, biologists and chemical engineers, with exception of the author. Although we expected that teaching applied mathematics would be better aligned and more motivating for them than abstract mathematics, this was not the case. We think that context-based cases are quite complex and mathematically more difficult, and even more so with the use of TmL. Furthermore, the lecturers did not always recognise the mathematics in the cases, like the engineers on the workplace described in the first study. It also seemed that the lecturers, like their students, believe that mathematical problems have only one correct solution, whereas TmL involve a mathematical activity with more possible solutions and less certainty. Finally, it appeared that although all lecturers received the same support and theory, they all had very different interpretations of IBL, for example, the idea that they were no longer allowed to explain things, and we noticed that this caused difficulties for some.

The support kit that we designed for the lecturers proved to be insufficient. It consisted of materials such as manuals and written information about IBL and TmL, a reflection form, weekly e-mails with instructions, suggestions, and tips, all-time support via phone, e-mail, or live with the lecturer-researcher and most importantly, three lecturer meetings each year. Lecturers appeared to be more focused on the practical tools of the kit than on the underlying theory of IBL and TmL we shared in the lecturer meetings. Provided literature about the theory was scarcely read, because these professionals were not used to reading social science literature. Due to the generally high workload, lecturers were prepared to teach the course only to a limited extent. Most of them had not worked out the cases by themselves beforehand and some had not even opened the ALEKS™ software. We suspect that this caused less confidence in the content and stimulated the tendency to focus on abstract mathematics with easy standard/routine calculation rules. We think that, ideally, lecturers should fit the profile of the so-called reflective practitioner, as just one of them did. She was the only lecturer that used the reflection form, a tool from the support kit in which experiences, thoughts and feelings could be noted after each lesson. She explained that this helped her improving her lessons. In addition to extensive content preparations, this lecturer also reported the most positive opinions about the new course.

During the three subsequent years of implementation, only a few adjustments were made to the design of the course by the lecturer-researcher, as the initial idea of the two learning tracks and the use of context-based cases seemed to work out rather well. However, we did try to help the lecturers by providing a menu with several options to conduct the feedback hours, to expand their autonomy and agency. In the three years, some lecturers continued, some lecturers stopped, and new lecturers joined. We were happy to see lecturers building experience and their confidence growing. We have planned to conduct guided peer-feedback sessions and an expert workshop on IBL for continuous lecturers' training, and to recruit new lecturers who fit the reflective professional's profile and who can offer fresh ideas. We have also made sure that new lecturers receive more time to prepare and provide them with easy and accessible literature.

The largest challenge we faced in the implementation was the enormous impact of lecturers' opinions and beliefs about mathematics, in both content and didactical respect, on their feelings about teaching the course. Most lecturers in higher technical professional education in the Netherlands previously worked in industry and have received limited didactical education. We think that the fact that mathematics courses in the School of Life Sciences and Environmental Technology, mathematics courses are not taught by mathematicians, but by chemists, biologists and chemical engineers has advantages and disadvantages. We face the challenge to both listen to the lecturers and keep the original ideas and innovations of the design alive. Changes and adjustments are necessary and feasible, but it is important to try to avoid *lethal mutations* of the course (Brown & Campione, 1996). Perhaps we need to switch to formative interventions, in which a design is not fully developed, but with open aspects to be filled in by the lecturers. In that way, professional agency of the lecturer is stimulated, and more autonomy is possible (Engeström, 2011). Foremost, we see that challenging opinions and beliefs about didactics and mathematics in lecturers and students is the most important task to achieve successful change, and this will be our continuing mission.

6.5 Assessing the development of TmL in an applied mathematics course

Chapter 5 reports the results of a learning effect study of the new course in applied mathematics to answer research questions 5 and 6:

RQ4: What is the learning effect of a course in applied mathematics on students' development of Techno-mathematical Literacies?

RQ5: What are possible explanations for the unexpected results?

An overview of these activities undertaken in design and validation are summarised in Table 3 and explained in the subsequent text. Data analysis in the study is divided in two phases, the first phase to address the initial research question and a second phase for the follow-up question.

Table 3 Overview of research activities

	Research activity	Description	Gain insight into
1	Test design	Designing test items in co-design with students	feasibility
2	Validation	Discussing test items with TmL experts	concept validity
3	Redesign	Adjusting test items and assigning to two tests A and B	content validity
4	Validation	"Thinking-aloud" session with 4th-year chemistry student	construct validity and feasibility
5	Redesign	Adjusting test items regarding language, errors and number of items	content validity
6	Administering pre- and posttests	Conducting pretest with 68 and posttest with 62 students in 30 minutes	validity and feasibility
7	Grading	Grading tests by the researcher	test scores
8	Data analysis	Checking for internal consistency of the items with measures of classical test theory.	criterion validity
9		Performing *t* tests to compare pre- and posttest results	content validity
10		Performing one-way ANCOVA to check influence of lecturers.	construct validity

11		Compare P values to investigate development for different TmL categories	content validity
12	Follow up analysis	Performing *t* test to compare marks from course chemical calculation with assignment test A or B as pretest	construct validity
13		Performing *t* tests to compare test A and B	construct validity
14		Performing *t* tests to compare scores on posttest with marks for summative test.	construct validity
15	Redesign proposal	Mixing items within tests to spread missing values	
16		Mixing matching items between test A and B and detect discriminating items to standardise difficulty	

As a last study in this project, we designed (research activity 1) and validated (2) two tests which were administered as pre- and posttest on TmL during the third year of the implementation with 68 1st-year chemistry students. The test items with non-chemical contexts were developed in co-design with students of electrical engineering at Fontys University of Applied Sciences in which they implemented TmL. As a first step in content-validation, the test items were discussed with two TmL experts. After adjustments, the items were distributed over two tests, A and B (3), and tested in an "thinking-aloud" session with a 4th-year chemistry student (4), to check for mistakes or unclear language and investigate construct validity and feasibility, and adjusted accordingly (5). The two tests were then randomly assigned to the students as pretests at the start of the first class of the course. In the sixth week, the two tests were switched among the students and assigned as posttests (6).

After grading of the tests (7), we determined the internal consistency with measures of the classical item response theory for criterion validity (8). Then, performing *t* tests (9), we found that a significant development in TmL was found in the pre B – post A group, but not in the pre A – post B group. By performing a one-way ANCOVA (10), we made sure that a difference in score due to lecturers' influence is unlikely. When we investigated the improvement of individual TmL (11), we found that scores on more concrete TmL such as *sense of error* and *technical software skills* increased, and we think that these kind of TmL categories are easier to measure, and perhaps learned, than complex TmL such as *technical creativity* and *data literacy* – which seem to be much broader, more context-dependent categories.

Because of these mixed results, we formulated a follow-up question and continued data analysis. First, the grades of a previous course in chemical calculations was used to compare with the scores on the pretest using a paired *t* test (12), to test whether the random assignments to test A and B had indeed led to equal groups of students regarding their starting level. It appeared that the students with test B as pretest were higher achieving students than the test A students. Furthermore, we discovered, comparing test A and B with an independent *t* test, that scores on test B were lower, although not significantly, than on test A (13). We conjectured that test B was probably more difficult than test A. Although we, as researchers and designers, could not discover any differences in difficulty, this conjecture was confirmed by comparing posttest A and B results with the official summative end test of the course with paired *t* tests (14), and it became clear that only test B had significantly different results, which indicates that it was indeed more difficult than test A.

We assume that these two findings – pretest B students were higher achievers, and test B was more difficult than test A – caused the identified contrasting results in the paired *t* tests that higher achieving students (pre B – post A) showed significant improvement, but the lower achieving students (pre A – post B) did not. We ask ourselves how TmL development can be measured better, perhaps in a more qualitative manner? Our experiment with oral assessments was promising in that respect, although practically too time-consuming and calibration among colleagues proved to be challenging. We do want to keep these TmL as learning goals, though, because there is a danger in only teaching what is measurable and not to measure wat you find important (Collins 2017).

Designing test items for complex skills is a creative and challenging task. Using TmL questions instead of "standard" questions or calculations is not straightforward, and we noticed that the TmL of *sense of number*, which resembles calculation questions the most, is required often in our tests. It is for future research to find out how complex skills in general, and TmL in particular can be developed in a valid and reliable manner. As a next step, we propose to randomise test items within (15) and between (16) the two tests A and B; by detecting discriminating items, and by mixing matching items from both tests, they will be equivalated in difficulty.

6.6 Conclusion

As a result of this PhD study, we have an innovative and sustainable course in applied mathematics for 1st-year students in higher technical professional education, which is implemented in the curricula of all majors of the School of Life Sciences and Environmental Technology at Avans University of Applied Sciences, and has already inspired other universities of applied sciences to adopt several features of the course.

We also contributed to knowledge about TmL and design research, and most importantly, implementation. Our overall research question was:

> *How can an innovative course in applied mathematics in the technical domain of higher professional education help students acquire the TmL necessary for their future workplaces?*

To answer this question, we first identified seven TmL categories that engineers use in their daily workplace practices: *data literacy*, *technical software skills*, *technical communication skills*, *sense of error*, *sense of number*, *technical creativity*, and *technical drawing skills*. With these TmL as central learning goals, we developed an innovative course in applied mathematics with the approach of design research and implemented, monitored, analysed, evaluated and adjusted it during three subsequent years. We analysed what teaching strategies the lecturer-researcher used in the feedback hours of her class to foster students' acquisition of TmL. Finally, we investigated the learning effect of the course. It appeared that conducting the feedback hour, using the approach of IBL knows many challenges and we learned that a variety of teaching strategies is necessary for success. Implementing a new course with colleague lecturers who could not all be involved in the design process turned out to be rather difficult, because teachers need autonomy and agency. We also found that it takes a long time to sustainably implement an educational innovation and continuous professional development of the lecturers is imperative. To measure the effectiveness of the course in students' development of TmL, we developed and conducted a pre- and posttest. For skills such as TmL, this is a very creative and complex task. We were confronted with the unforeseen fact that the two variants of the test were not equally difficult, which caused a more complicated analysis.

As mentioned before, implementing an educational innovation with lecturers who cannot be involved in the design process, has been the most challenging part of this study. We think that, perhaps, designing a more *half-baked* innovation, which has to be built upon and can be changed by its users, can provide both flexibility and robustness (Kynigos, 2007). Further research on design implementation in practical, non-ideal circumstances is therefore imperative. We have also seen how difficult it is to develop test items for measuring students' improvement in TmL. How can one create items in neutral contexts, and, for example, make sure the level of difficulty is comparable in test-variants? Furthermore, how can schools and universities help and guide their lecturers in educational innovations? Lecturers are already so often confronted with obligated changes in their work, that we need to focus on intrinsic motivation and stimulate their professional agency. For this, I think that we need to inform our lecturers on lessons learned in educational research and share best practices. We need to do this in an accessible manner; not by providing scientific literature in educational sciences, but in a practical and non-time-consuming way. On a more specific note, we need to address the ideas, opinions and convictions about what mathematics entails in both lecturers and students. Too often, as for mathematical skills, this view is limited to calculations and procedures. As for content, we see that mathematics is often not recognised when it does not involve complicated formulas and, for example, some integral signs. In conclusion, these are all challenges and topic for more future research.

For me as a researcher, the project has been tremendously interesting and challenging in many respects, especially working as a lecturer-researcher in a "broker"-role with colleagues who were not all interested in change. In addition to being a researcher in collecting data, analysing, reading and writing, I faced many practical tasks in this role. First, in collaboration with school management, I had to align the wishes of all parties involved. Then I had to form a design team and manage the process of developing a course and setting up premises. The implementation of the course in the School of Life Sciences and Environmental Technology required developing extensive people skills to collaborate with the colleagues involved, but also providing them with support and training. According to McKenney and Brand-Gruwel (2015), communicative and collaborative skills and the ability to handle insecurity are abilities a design researcher should have, with which I fully agree. I am very glad to be able to continue conducting educational research in higher technical education and to contribute to developing and evaluating innovative mathematics education that helps to provide technical solutions to world-wide challenges.

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Nederlandse samenvatting

In het Nederlandse hoger technisch beroepsonderwijs is het wiskundeonderwijs in veel gevallen geen geïntegreerd onderdeel van de opleiding. Het is meestal abstract, maakt weinig gebruik van beroepscontexten en software wordt schaars ingezet. In onze ervaring is bovendien de motivatie voor wiskunde bij studenten in technische domeinen die minder op wiskunde leunen, zoals chemie, biologie en bouwkunde, beperkter. Deze studenten twijfelen vaak aan het nut van het vak voor hun opleiding en herkennen de wiskunde in de andere vakken van het curriculum meestal niet.

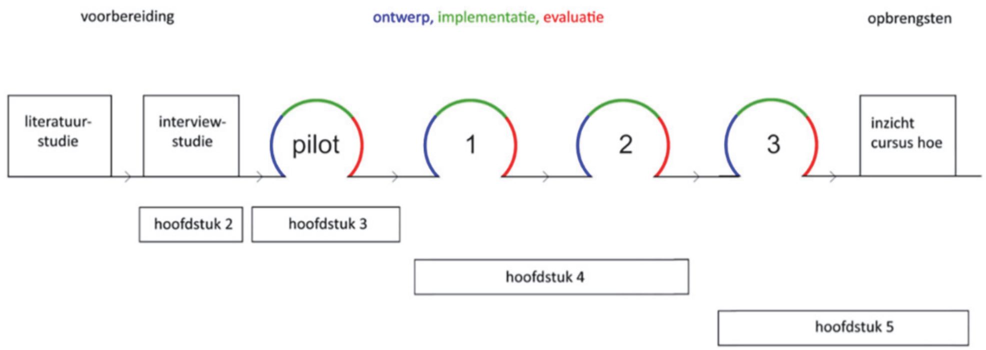
In de technische beroepspraktijk, waar deze aankomende ingenieurs voor opgeleid worden, hebben de laatste decennia grote veranderingen plaatsgevonden als gevolg van de enorme groei van het gebruik van computers en digitale technologie. Berekeningen worden vooral uitgevoerd door software en de wiskunde erachter blijft vaak verborgen voor de gebruikers van de software. Hierdoor hebben toekomstige ingenieurs nieuwe en andere, maar niet minder, wiskundige vaardigheden nodig. Welke vaardigheden dat zijn is een vraag die ook internationaal gesteld wordt. Cardella (2008) stelt dan ook dat wiskundige inhoud voor technische opleidingen nog altijd belangrijk is, maar dat deze ingebed zou moeten worden in bredere wiskundige competenties.

Voor deze nieuwe wiskundige vaardigheden is de term *Techno-mathematical Literacies* (TmL) geïntroduceerd, als onderdeel van de zogenaamde *21st-century skills* (Kent, Bakker, Hoyles, & Noss, 2005). TmL combineren wiskundige, technische, ict- en communicatieve vaardigheden en omvatten onder andere de interpretatie van abstracte data, softwarevaardigheden en een gevoel voor getallen, nauwkeurigheid en foutenmarges. Het voorvoegsel *Techno* benadrukt het feit dat het gebruik van wiskunde mogelijk wordt gemaakt door technologie. TmL gaan veel verder dan *basic numeracy* omdat ze specifiek zijn voor beroepscontexten: verhoudingsgewijs redeneren kan voor chemici anders ogen dan voor verpleegkundigen, ook als de wiskundige kern gelijk is (Hoyles, Noss, & Pozzi, 2001; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002). Deze vaardigheden worden niet vanzelfsprekend in de praktijk geleerd en dienen daarom expliciet via de opleiding aangeboden te worden (Hoyles, Noss, Kent, Bakker, & Bhinder, 2007).

Deze dissertatie beschrijft het ontwerp- en onderzoeksproces rond een wiskundecursus voor eerstejaarsstudenten van het technisch hbo waarbinnen deze TmL expliciete en centrale leerdoelen vormen. Het project had enerzijds het praktische doel om een duurzame eigentijdse wiskundecursus te ontwikkelen en te implementeren waarin TmL centraal staan, en anderzijds een theoretisch doel om een bijdrage te leveren aan kennisontwikkeling over (de implementatie van) TmL in de technische beroepspraktijk, en factoren die daarin een rol spelen. De centrale onderzoeksvraag voor het gehele project luidt:

> *Hoe kan een innovatieve cursus toegepaste wiskunde in het technisch hoger beroepsonderwijs studenten helpen de TmL te verwerven die nodig zijn in hun toekomstige beroepspraktijk?*

Figuur 1 geeft een overzicht van de gehele ontwerpstudie:



Figuur 1. Overzicht van de verschillende fases van de ontwerpstudie en de hoofdstukken van de dissertatie

Hoofdstuk 2: Techno-mathematical Literacies in de beroepspraktijk van ingenieurs

De centrale onderzoeksvraag wordt in vier deelvragen in de afzonderlijke hoofdstukken beantwoord. Als eerste stap in het onderzoek was het noodzakelijk te onderzoeken in welke richting het wiskundeonderwijs zou moeten veranderen. Daarom is gestart met een veldstudie onder ingenieurs met als onderzoeksvraag:

> *Onderzoeksvraag 1: Welke TmL gebruiken ingenieurs in hun professionele beroepspraktijk en wat zijn hun meningen over het eigen onderwijs en hun ideeën voor het toekomstige wiskundeonderwijs voor ingenieurs?*

Hoofdstuk 2 beschrijft het onderzoek naar de TmL die ingenieurs in hun dagelijkse beroepspraktijk gebruiken. In deze studie zijn 14 semigestructureerde interviews gehouden met ingenieurs van diverse technische domeinen. De interviews van 1,5–2 uur lang werden gehouden op de werkplek van de ingenieur. Alle deelnemers (zes vrouwen en acht mannen) hadden een hbo-achtergrond en hadden 2–40 jaar werkervaring. Het interviewschema was taak-gebaseerd; het bevatte vragen over hun taken, de software en de apparatuur waarmee ze dagelijks werkten en er werd ook gevraagd deze te demonstreren. Verder waren er vragen over hun gebruik van wiskunde, communicatie met diverse partijen, hoe zij zelf dachten over wiskundeonderwijs in het algemeen en het door henzelf genoten wiskundeonderwijs in het bijzonder. Van de interviews werden geluidsopnamen gemaakt, en deze werden met behulp van Atlas TI™ geanalyseerd en gecodeerd met open, axiale en ten slotte selectieve codes (Boeije, 2005). Twee van de interviews werden door een tweede, externe codeur gecodeerd en dit leverde een Cohen's kappa op van .84, wat op een goede betrouwbaarheid duidt.

Hoewel wiskunde een centrale rol speelt in techniek, gaven de ingenieurs zelf aan voornamelijk eenvoudige wiskunde te gebruiken, vooral vanwege het uitbesteden van taken aan computers. Uit de analyse van de interviewgegevens zijn zeven veel gebruikte TmL-categorieën naar voren gekomen, gepresenteerd in tabel 1. De laatste twee categorieën bleken specifiek bij technische domeinen te horen en zijn niet eerder in de literatuur beschreven. De TmL categorie s*ense of number* is belangrijk voor ingenieurs. Wanneer bijvoorbeeld software gebruikt wordt, is niet alleen de juiste *input* van getallen belangrijk, maar bovenal de juiste interpretatie van de *output.* In deze interpretatie is de TmL-categorie *sense of error* minstens zo belangrijk, om te bepalen of de uitkomsten kunnen kloppen.

Tabel 1. TmL-categorieën die door ingenieurs worden gebruikt

	TmL-categorie	Beschrijving
1	Data literacy	De ingenieur is in staat technische en grafische data te interpreteren, conclusies te trekken en overeenkomstige acties te nemen.
2	Technical software skills	De ingenieurs is in staat professionele software, zoals bijvoorbeeld Excel™, als rekentool te gebruiken.
3	Technical communication skills	De ingenieur is in staat om met collega's, klanten, leidinggevenden en anderen over technische informatie te communiceren.
4	Sense of error	De ingenieur is in staat om data te controleren, te verifiëren en fouten te detecteren.
5	Sense of number	De ingenieur is in staat om met getallen om te gaan en deze te kunnen interpreteren.
6	Technical creativity	De ingenieur is in staat creatieve oplossingen te bedenken voor technische puzzels en problemen (door slimheid of ervaring).
7	Technical drawings skills	De ingenieur is in staat technische tekeningen te gebruiken en begrijpen (door bijvoorbeeld ruimtelijk inzicht).

Wat betreft het tweede deel van de onderzoeksvraag, gaven bijna alle ingenieurs aan dat zij hun wiskundeonderwijs vroeger hadden ervaren als geïsoleerd van en met beperkte relevantie voor de rest van de opleiding. Het was voornamelijk theoretisch, zonder context of relatie met de rest van het curriculum. Omdat technisch hoger beroepsonderwijs om de praktijk draait, vinden zij dat wiskunde in context aangeboden zou moeten worden om motivatie voor en begrip van de stof te verhogen, iets wat zij vroeger gemist hebben. Ze verwachten dat wiskunde in de toekomstige ingenieurspraktijk belangrijk blijft omdat technologische hulpmiddelen daarin een steeds grotere plaats innemen en deze volledig gebaseerd zijn op wiskunde. Over de vraag of het nodig is de wiskunde "onder de motorkap" te kunnen begrijpen werd verschillend gedacht. Het lijkt erop dat bij de steeds meer toenemende complexiteit van de technologie dit een ondoenlijke opgave wordt, en andere wiskundige vaardigheden nodig zijn.

Hoewel de ingenieurs zelf geen geavanceerde wiskunde gebruiken, vinden de meeste ingenieurs dat wiskunde leren in het algemeen logisch en analytisch denken stimuleert, wat een essentiële vaardigheid is in hun beroep. Ook in de literatuur wordt bevestigd dat betekenisvolle wiskundige ervaringen kritisch denken stimuleren (Huang, Ricci, & Mnatsakanian, 2016; Rajagukguk & Simanjuntak, 2015). Bialik and Kabbach (2014) denken daarentegen dat het andersom werkt en dat hogere-orde denkvaardigheden juist wiskundige vaardigheden stimuleren. De vastgestelde TmL in deze studie en de ideeën van de ingenieurs over wiskundeonderwijs geven handvatten voor het ontwikkelen van nieuw wiskundeonderwijs. Naar onze mening zouden professionele taken en producten in het wiskundeonderwijs centraal moeten staan en TmL een belangrijke rol spelen.

Hoofdstuk 3: Onderwijsstrategieën om TmL te bevorderen

Na de interviewstudie in het beroepenveld is door een ontwerpteam vervolgens een nieuwe cursus toegepaste wiskunde ontwikkeld voor het technisch hbo met de geïnventariseerde TmL als centrale leerdoelen. Hoofdstuk 3 behandelt het ontwerp van de nieuwe cursus toegepaste wiskunde en de leerstrategieën die de docent gebruikt om TmL te bevorderen tijdens de klassendiscussies in de zogeheten feedbackuren. De nieuwe cursus is door een interdisciplinair team van diverse hogescholen ontworpen en tijdens een pilot met 59 eerstejaarsstudenten (26 vrouwen, 33 mannen) van de opleiding Chemie van Avans Hogeschool in het voorjaar van 2016 getest. Voor het ontwerp werd gebruikgemaakt van een hypothetisch leertraject (HLT), waarin de leerdoelen worden beschreven, als ook de cruciale wiskundige taken en de onderliggende hypotheses over het leerproces van de studenten (Simon, 1995). In het HLT werden ook de gebruikte TmL opgenomen en gespecificeerd. Het resultaat, de ontworpen cursus, bestaat uit twee leerlijnen: een leerlijn in basiswiskunde met de software ALEKS™ waarin studenten individueel werken en een leerlijn met toegepaste, contextrijke casussen met een focus op TmL, die groepsgewijs uitgewerkt worden.

Het rooster van de zeven wekelijkse werkcolleges bestond uit vier uren; het eerste uur was gewijd aan instructie, dan volgden twee uren groepswerk door groepjes van twee of drie studenten in afwezigheid van de docent en in het vierde uur kwam de docent terug en werd het feedbackuur gehouden. In deze feedbackuren werd klassikaal of in kleinere groepen de eigen uitwerking van de casussen door de studenten besproken en de abstracte en toegepaste wiskunde door de docent nadrukkelijk gekoppeld om *transfer* te stimuleren. In deze besprekingen werd gebruikgemaakt van *inquiry-based learning* (IBL), waarin onderzoekend leren centraal staat, omdat hierin het vraaggericht/gestuurde karakter van wetenschap geïmiteerd wordt (Chu, Reynolds, Tavares, Notari, & Lee, 2017). Het is een proces waarin studenten worden gestimuleerd om een onderzoekgerichte houding, begrip en vaardigheden te ontwikkelen, ondersteund door procesgerichte vragen van de docent (Anderson, 2002). De onderzoeksvraag die in hoofdstuk 3 centraal staat luidt:

Onderzoeksvraag 2: Welke onderwijsstrategieën gebruikte de docent tijdens de discussies over de contextrijke casussen om het leren van TmL te stimuleren?

Om de leerstrategieën vast te stellen die de docent toepaste, werden video-opnames gemaakt van de feedbackuren van de pilot van de werkcolleges bij de docent-onderzoeker (auteur). Van de zeven werkcolleges bij twee klassen werden 12 maal 45 minuten video gemaakt. Voor de analyse in het programma NVivo™ werd gebruikgemaakt van een versie van de constant comparatieve methode, waarbij met de input van de HLT en de onderzoeksvraag chronologisch de video's werden bestudeerd, initiële vermoedens werden vastgesteld en vervolgens aangepast waar nodig bij volgende fragmenten (cf. Cobb & Whitenack, 1996). Vervolgens werd de lijst met onderwijsstrategieën en 1,5 uur aan video-materiaal voorgelegd aan een docent van het eerdere ontwerpteam om bevindingen te vergelijken. Hij vond veel strategieën uit de lijst maar identificeerde ook nog twee nieuwe. Uit de analyse werden vier categorieën vastgesteld:

- Pedagogische strategieën, waarin de nadruk lag op het stimuleren van een goede, positieve sfeer en een leergericht klimaat in de klas.
- Processtrategieën, die gericht waren op het reguleren en activeren van het leerproces.

- Algemene IBL-strategieën, zoals het structureren en recapituleren van antwoorden van studenten en het vragen naar de verschillende oplossingsstrategieën van een probleem.
- TmL-specifieke strategieën, zoals het dieper doorvragen naar aanleiding van data, formules, tabellen en grafieken, het gezamenlijk opstellen van een oplossingsstrategie, vragen een fout op te sporen, of het laten uitleggen van een concept.

De eerste twee categorieën waren vooral voorwaardelijk voor een goed verloop van het feedbackuur. De docent gebruikte diverse IBL-strategieën. Wanneer studenten hun werk presenteerden, structureerde ze het proces met opmerkingen, recapituleerde antwoorden van studenten, en stelde vervolgvragen om een klassendiscussie te stimuleren. Verder stimuleerde ze TmL door te vragen naar het opsporen van een fout in een uitwerking, het toe te lichten of een generieke aanpak voor het probleem te laten formuleren door de klas.

Het uitvoeren van het feedbackuur was een complexe opdracht voor de docenten, waarin ze verschillende rollen op zich namen, zoals die van coach, gespreksleider, entertainer en instructeur. Natuurlijk lukte het niet altijd om alle studenten geïnteresseerd en actief te houden. Een frequente afleider was de telefoon, maar ook het tijdstip van de les, het weer en diverse andere factoren hadden invloed. Sommige studenten moesten wennen aan het taalgebruik in de casussen of hadden moeite met tekst in het algemeen.

De resultaten uit dit onderzoek functioneren als een *proof of principle*. De premisses van deze cursus met het samenwerken aan context- en TmL-rijke casussen en de klassendiscussies zijn een voorbeeld hoe het leren van TmL gestimuleerd kan worden.

Hoofdstuk 4: Implementatie van de nieuwe cursus

Na de pilot is de cursus aangepast en geïmplementeerd in het curriculum van de Academie voor de Technologie van Gezondheid en Milieu van Avans Hogeschool. Gedurende 2016–2019 is de cursus met meer dan 1400 eerstejaarsstudenten en 11 docenten onderwezen, gemonitord, geëvalueerd en aangepast. De uitvoering door de docenten, waarvan de meesten niet bij de ontwerpfase betrokken konden worden, bleek de grootste uitdaging te vormen. Dit werd duidelijk uit gesprekken met de docenten en evaluaties onder studenten. Daarom is besloten een extra studie aan het project toe te voegen met als onderzoeksvraag:

Onderzoeksvraag 3: Welke uitdagingen ontstonden in de relatie tussen de professionele ontwikkeling en autonomie van docenten en het doel van een duurzame implementatie van een innovatie?

Hoofdstuk 4 beschrijft de successen en uitdagingen van de implementatiefase in het ontwerponderzoek, wat de moeilijkste fase in het project is geweest. Waar ontwerponderzoek focust op ontwerp, tracht DBIR een brug te slaan naar toepassing en een duurzame implementatie van een educatieve innovatie (Penuel & Fishman, 2012). Veel innovaties mislukken door een “top-down” benadering en een gebrek aan docenttraining (Borko, 2004; Tirosh & Graeber, 2003). Toch was het, om diverse en praktische redenen, helaas niet mogelijk om alle docenten bij het ontwerpproces te betrekken. Daarom ontwikkelde het ontwerpteam een uitgebreide *support kit* om docenten te ondersteunen bij de veranderingen. Deze *support kit* bevatte directe, praktische hulp en ondersteuning, maar ook trainingen om de theorie van IBL en TmL te ervaren en te doorgronden. Toch bleek deze ondersteuning niet voldoende, en rezen er naast successen ook vele uitdagingen in de eerste drie jaar dat de cursus gegeven werd.

We besloten daarom om in de lente van het tweede jaar een interviewstudie te houden met de zes docenten die niet bij het ontwerp betrokken waren geweest om hun ervaringen, meningen, dilemma’s, gevoelens en overtuigingen te onderzoeken. De semigestructureerde interviews van ongeveer een uur werden door het ontwerpteamlid van de Hogeschool Utrecht gehouden, zodat voldoende afstand tussen interviewer en geïnterviewde gewaarborgd was. Van de interviews werden geluidsopnames gemaakt en vervolgens geanalyseerd in NVivo™ met de methode van Boeije (2005). De codes waren deels gebaseerd op de ideeën achter de interviewvragen, maar ook op basis van de resulterende antwoorden zijn ze vervolgens gegroepeerd naar thema. Frequent overleg vond plaats met de docent-onderzoeker (auteur) over de interpretatie.

Het originele plan was om deze interviews één maal te houden, maar we besloten het jaar erop nogmaals een korte ronde te organiseren, om de veranderingen te evalueren die we na de eerste ronde hadden doorgevoerd. Deze interviews met de zes docenten van het vorige jaar en twee nieuwe docenten van dat jaar, duurden slechts een kwartier en werden door de docent-onderzoeker gehouden. Zij vroeg naar de ervaringen, gevoelens en meningen over de cursus van dat jaar en een samenvatting werd ter plekke genoteerd op een laptop. Deze notities werden door het ontwerpteamlid op dezelfde manier verwerkt als in de eerste ronde.

In de Academie voor de Technologie van Gezondheid en Milieu, waar de nieuwe wiskundecursus in het curriculum is opgenomen en het project werd uitgevoerd, wordt het wiskundeonderwijs verzorgd door chemici, chemisch technologen en biologen. De verwachting was dat een contextrijke toegepaste wiskundecursus beter zou aansluiten bij de expertise en voorkeuren van deze docenten. Dit bleek niet het geval. Het vermoeden ontstond dat de wiskunde in toegepaste casussen vaak niet (h)erkend werd en dat het werken hiermee, en met name de focus op TmL, moeilijker is dan het onderwijzen van meer eenduidige, abstracte wiskunde met een focus op rekenregels en procedures. Bij het implementeren van onderwijskundige innovaties is het essentieel dat een docent bij het profiel van een reflectieve professional aansluit of zich hiernaar ontwikkelt en wij herkenden dit slechts in één van de deelnemende docenten die plezier, succes, maar ook een grondige voorbereiding rapporteerde. Voor de toekomst hebben we afspraken met het management gemaakt dat nieuwe docenten meer voorbereidingstijd krijgen en dat we de professionele ontwikkeling van docenten blijven stimuleren door middel van diverse soorten training en intervisie.

Dit hoofdstuk deelt geleerde lessen in DBIR en laat zien waarom dit soort onderzoek gecompliceerd is. De grootste uitdaging in de duurzame implementatie van een innovatie blijkt te liggen in het luisteren naar en aansluiten bij docenten, het ontwerp daarop aan te passen, en *lethal mutations* (dodelijke afwijkingen) te vermijden (Brown & Campione, 1996). Het is daarom te adviseren een verschuiving naar zogeheten *formative interventions* te overwegen (Engeström, 2011), waarin autonomie en eigen invloed (*agency*) van docenten een grotere plaats heeft, of wellicht een opener ontwerp te ontwikkelen, wat deels nog afgemaakt, ingevuld of veranderd kan worden door zijn gebruikers (Kynigos, 2007). We vragen ons echter af of dit voor de Nederlandse situatie beter zou werken, gezien de voorkeur van veel docenten voor *readymade* cursussen en de focus op de praktische kanten van de door ons geboden *support kit*.

Hoofdstuk 5: Het meten van de ontwikkeling van TmL in de cursus toegepaste wiskunde

Als laatste onderdeel van het onderzoekstraject werden twee toetsen ontwikkeld die fungeerden als voor- en natoetsen. Hoofdstuk 5 rapporteert over de resultaten van het ontwerp en de validatie van deze twee toetsen en hun items om het leereffect van de nieuwe cursus te testen. Deze studie werd tijdens het derde jaar van de implementatie uitgevoerd en probeert een antwoord te geven op de onderzoeksvraag:

Onderzoeksvraag 4: Wat is het leereffect van de cursus toegepaste wiskunde?

Onderzoeksvraag 5: Hoe kunnen onverwachte resultaten verklaard worden?

Het meten van TmL is nog nooit eerder gebeurd en er was dan ook geen materiaal beschikbaar waarop kon worden voortgebouwd. We hebben daarom een eerste stap gezet in het testen van deze vaardigheden door het zelf ontwikkelen van twee toetsen. Het ontwikkelen van TmL-vragen in aansprekende contexten in plaats van gebruikelijke procedurele vragen vergt ontwerpcreativiteit. De toetsitems werden in samenwerking met studenten van de opleiding elektrotechniek van Fontys Hogeschool ontwikkeld, ondersteund door literatuur. Omdat we met TmL *far transfer* beogen, werd door het gebruik van niet-chemische contexten een zekere afstand tot de stof van de cursus gewaarborgd.

De toetsitems werden vervolgens inhoudelijk gevalideerd door middel van overleg met twee TmL-experts en verdeeld over twee toetsen A en B van ongeveer dezelfde lengte en met gelijke wiskundige onderwerpen. Daarop volgde een test met een ouderejaars chemiestudent in een zogenaamde hardop-denksessie voor constructvalidatie en om de haalbaarheid te testen. Vervolgens is bij alle 68 eerstejaars studenten chemie in het voorjaar van 2019 aan het begin van het eerste werkcollege een voortoets afgenomen en, aan het begin van het zesde en laatste werkcollege, bij de aanwezige 62 studenten, een natoets. In de voortoets kreeg (willekeurig) de helft van de studenten toets A, en de andere helft toets B. Bij de natoets werd dit omgedraaid.

Als eerste stap in de analyse werd de ontwikkeling in TmL van voor- naar natoets getest. Hieruit kwam een gemengd resultaat tevoorschijn: De scores van studenten met voortoets B en natoets A lieten een significante vooruitgang zien, maar bij de studenten die voortoets A en natoets B hadden gemaakt niet. Over het geheel werd een kleine, niet-significante vooruitgang gemeten. Verder werd vastgesteld dat het statistisch onwaarschijnlijk was dat de docent invloed heeft gehad op dit resultaat. Bij het indelen van de toetsitems naar TmL zagen we in toets A vooruitgang in de meeste TmL-categorieën. In toets B toonden de TmL-categorieën *technical software skills* en *sense of error* de grootste vooruitgang. TmL *sense of error* was in beide toetsen de grootste stijger. We zagen dan ook dat deze TmL's heel expliciet in de cursus aan bod kwamen maar ook dat deze TmL concreter zijn, in tegenstelling tot de meer complexe en heterogene TmL zoals *data literacy* en *technical creativity*.

Door de gemengde resultaten werd besloten om een additionele onderzoeksvraag te formuleren en daartoe extra data-analyse uit te voeren. Als eerste stap in de analyse werd een vergelijking gemaakt met de cursus Chemisch Rekenen, die studenten eerder dat jaar gevolgd hadden, om het startniveau van de studenten te bepalen. Hieruit bleek dat de studenten die als voortoets B hadden, bij toeval, ondanks de willekeurige toewijzing, een hoger wiskundig niveau hadden. Vervolgens werden de toetsen zelf vergeleken, maar hieruit kwam geen significant verschil in moeilijkheid. Wij, als docenten, herkenden ook geen verschil. Een niet-betrokken collega werd geraadpleegd om een zeer vergelijkbaar toetsitem uit beide testen te beoordelen. Ook hij gaf aan geen verschil te zien in moeilijkheidsgraad, en een item uit toets A wellicht zelfs iets moeilijker te vinden omdat het meer gebruikmaakt van wiskundig jargon dan het vergelijkbare item uit toets B. Na het vergelijken van de scores op de natoetsen met de summatieve eindtoets van de cursus, werd bevestigd dat toets B inderdaad significant moeilijker was dan toets A.

Deze twee bevindingen – toets B is moeilijker dan toets A en studenten met voortoets B hadden een hoger starniveau – kunnen een verklaring zijn voor de tegengestelde resultaten op de meting van TmL ontwikkeling van voor- naar natoets. We vragen ons af hoe goed deze bredere TmL meetbaar zijn, hoe belangrijk we ze ook vinden. Dit betekent niet dat we ze niet meer als leerdoel willen voorstellen. Er is immers een gevaar van alleen datgene te onderwijzen wat meetbaar is (Collins, 2017), dus waarderen wat gemeten kan worden (Biesta, 2012).

Het is duidelijk dat het ontwikkelen van toetsitems voor complexe vaardigheden veel creativiteit vergt en geen makkelijk proces is (cf. Drijvers, Kodde-Buitenhuis, & Doorman, 2019). We vermoeden ook dat, in het algemeen, studenten een hoger startniveau nodig hebben voor en ook meer leren in cursussen die zich richten op dit soort vaardigheden. Als volgende stap stellen we voor de toetsitems niet alleen binnen de toetsen, maar ook tussen de twee toetsen te herverdelen door discriminerende items op te sporen. Hiermee kunnen de toetsen genormaliseerd worden in moeilijkheid en kan een volgende stap worden gezet in het validatieproces.

Ten slotte

Aan het einde van het ontwikkel- en onderzoekstraject is er een innovatieve en duurzame cursus Toegepaste Wiskunde gerealiseerd met een stabiel team van docenten die een vaste plek in het curriculum heeft gekregen en inmiddels door meer dan 1400 studenten in de eerste drie jaar is gevolgd. Door de keuze van een expliciete focus op TmL in contextrijk materiaal en te combineren met IBL onderwijsstrategieën, hopen we bij te dragen aan studenten die beter voorbereid zijn hun wiskundige kennis en vaardigheden toe te passen om de problemen van morgen op te lossen. Niet alleen de aanpassingen in de leerstof, maar ook de didactische veranderingen zijn vernieuwend geweest in onze academie en hebben ook andere academies, alsmede diverse andere hogescholen geïnspireerd en navolging gekregen.

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Curriculum Vitae

Nathalie J. van der Wal was born in Hilversum (the Netherlands) on October 26th, 1971. After her secondary school education, she studied molecular sciences at Wageningen University, specialising in applied mathematics. In 1999, she graduated on two research projects, regarding extinction and fixation times of two alleles under the influence of selection and genetic drift, and the splash dispersal of fungal plant pathogens during rainfall.

In 2002, she passed state exams in music theory at the Schumann Akademie in Zwolle. After running her own company in vocal coaching, in 2009 she returned to her roots, starting as a lecturer in mathematics, statistics and chemistry, and as an intern supervisor and student coach at Avans University of Applied Sciences in 's-Hertogenbosch and Breda.

In 2014, Nathalie received a grant from Avans University; additionally, in 2017, she was awarded a Doctoral Grant for Teachers from the Dutch Organisation for Scientific Research (NWO), which enabled her to conduct a PhD project at the Freudenthal Institute of Utrecht University. Her research addressed both theoretical knowledge about the nature of so-called *Techno-mathematical Literacies* (TmL) and their application in education as well as the design and implementation of an applied mathematics course for higher technical professional education in the School of Life Sciences and Environmental Technology at Avans University.

Since 2017, in addition to her work as a PhD researcher and a lecturer, Nathalie has been working as a Digital Faculty Consultant for McGraw Hill Education. As such, she provides advice on how to use digital products in education.

In September 2020, Nathalie started as a postdoc at the Statistics group of the interfaculty Delft Institute of Applied Mathematics (DIAM) at Delft University of Technology. Participating in the Programme of Innovation in Mathematics Education (PRIME), she continues doing research on the innovation of mathematics education for engineers.

Publications related to this dissertation

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Van der Wal, N.J., Bakker, A., & Drijvers, P. (Submitted). *Assessing the development of Techno-mathematical Literacies in an innovative mathematics course for future engineers.*

Other scientific publications

Van Herwaarden, O. A., & Van der Wal, N. J. (2002). Extinction Time and Age of an Allele in a Large Finite Population. *Theoretical Population Biology*, *61*(3), 311–318. DOI: 10.1006/tpbi.2002.1576.

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Pielaat, A., Van den Bosch, F., De Gee, M., & Van der Wal, N. J. (2000). A theory on the vertical dispersal of splash-borne pathogen units influenced by arable crop characteristics. *Splash: The dispersal of fungal plant pathogens in rain events* (pp. 87–102). (Doctoral dissertation). Wageningen, the Netherlands: Wageningen Universiteit.

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105 Tacoma, S. (2020). *Automated intelligent feedback in university statistics education*

104 Zanten, M. van (2020). *Opportunities to learn offered by primary school mathematics textbooks in the Netherlands*

103 Walma, L. (2020). *Between Morpheus and Mary: The Public Debate on Morphine in Dutch Newspapers, 1880-1939*

102 Van der Gronde, A.G.M.P. (2019). *Systematic Review Methodology in Biomedical Evidence Generation.*

101 Klein, W. (2018). *New Drugs for the Dutch Republic. The Commodification of Fever Remedies in the Netherlands (c. 1650-1800).*

100 Flis, I. (2018). *Discipline Through Method - Recent history and philosophy of scientific psychology (1950-2018).*

99 Hoeneveld, F. (2018). *Een vinger in de Amerikaanse pap. Fundamenteel fysisch en defensie onderzoek in Nederland tijdens de vroege Koude Oorlog.*

98 Stubbé-Albers, H. (2018). *Designing learning opportunities for the hardest to reach: Game-based mathematics learning for out-of-school children in Sudan.*

97 Dijk, G. van (2018). *Het opleiden van taalbewuste docenten natuurkunde, scheikunde en techniek: Een ontwerpgericht onderzoek.*

96 Zhao, Xiaoyan (2018). *Classroom assessment in Chinese primary school mathematics education.*

95 Laan, S. van der (2017). *Een varken voor iedereen. De modernisering van de Nederlandse varkensfokkerij in de twintigste eeuw.*

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81 Sun, L. (2014). *From rhetoric to practice: enhancing environmental literacy of pupils in China.*

80 Mazereeuw, M. (2013). *The functionality of biological knowledge in the workplace. Integrating school and workplace learning about reproduction.*

79 Dierdorp, A. (2013). *Learning correlation and regression within authentic contexts*.

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74 Stolk, M. J. (2013). *Empowering chemistry teachers for context-based education. Towards a framework for design and evaluation of a teacher professional development programme in curriculum innovations.*

73 Agung, S. (2013). *Facilitating professional development of Madrasah chemistry teachers. Analysis of its establishment in the decentralized educational system of Indonesia.*

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63 Prins, G. T. (2010). *Teaching and learning of modelling in chemistry education. Authentic practices as contexts for learning.*

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61 Ormel, B. J. B. (2010). *Het natuurwetenschappelijk modelleren van dynamische systemen. Naar een didactiek voor het voortgezet onderwijs.*

60 Hammann, M., Waarlo, A. J., & Boersma, K. Th. (Eds.) (2010). *The nature of research in biological education: Old and new perspectives on theoretical and methodological issues – A selection of papers presented at the VIIth Conference of European Researchers in Didactics of Biology.*

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35 Margadant-van Arcken, M., & Van den Berg, C. (2000). *Natuur in pluralistisch perspectief – Theoretisch kader en voorbeeldlesmateriaal voor het omgaan met een veelheid aan natuurbeelden.*

34 Janssen, F. J. J. M. (1999). *Ontwerpend leren in het biologieonderwijs. Uitgewerkt en beproefd voor immunologie in het voortgezet onderwijs.*

33 De Moor, E. W. A. (1999). *Van vormleer naar realistische meetkunde – Een historisch-didactisch onderzoek van het meetkundeonderwijs aan kinderen van vier tot veertien jaar in Nederland gedurende de negentiende en twintigste eeuw.*

32 Van den Heuvel-Panhuizen, M., & Vermeer, H. J. (1999). *Verschillen tussen meisjes en jongens bij het vak rekenen-wiskunde op de basisschool – Eindrapport MOOJ-onderzoek.*

31 Beeftink, C. (2000). *Met het oog op integratie – Een studie over integratie van leerstof uit de natuurwetenschappelijke vakken in de tweede fase van het voortgezet onderwijs.*

30 Vollebregt, M. J. (1998). *A problem posing approach to teaching an initial particle model*.

29 Klein, A. S. (1998). *Flexibilization of mental arithmeticsstrategies on a different knowledge base – The empty number line in a realistic versus gradual program design.*

28 Genseberger, R. (1997). *Interessegeoriënteerd natuur- en scheikundeonderwijs – Een studie naar onderwijsontwikkeling op de Open Schoolgemeenschap Bijlmer.*

27 Kaper, W. H. (1997). *Thermodynamica leren onderwijzen.*

26 Gravemeijer, K. (1997). *The role of context and models in the development of mathematical strategies and procedures.*

25 *Acampo, J. J. C. (1997). Teaching electrochemical cells – A study on teachers' conceptions and teaching problems in secondary education.*

24 *Reygel, P. C. F. (1997). Het thema 'reproductie' in het schoolvak biologie.*

23 Roebertsen, H. (1996). *Integratie en toepassing van biologische kennis – Ontwikkeling en onderzoek van een curriculum rond het thema 'Lichaamsprocessen en Vergift'.*

22 Lijnse, P. L., & Wubbels, T. (1996). *Over natuurkundedidactiek, curriculumontwikkeling en lerarenopleiding.*

21 Buddingh', J. (1997). *Regulatie en homeostase als onderwijsthema: een biologie-didactisch onderzoek.*

20 Van Hoeve-Brouwer G. M. (1996). *Teaching structures in chemistry – An educational structure for chemical bonding*.

19 Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education.*

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17 De Jong, O., Van Roon, P. H., & De Vos, W. (1995). *Perspectives on research in chemical education.*

16 Van Keulen, H. (1995). *Making sense – Simulation-of-research in organic chemistry education.*

15 Doorman, L. M., Drijvers, P. & Kindt, M. (1994). *De grafische rekenmachine in het wiskundeonderwijs.*

14 Gravemeijer, K. (1994). *Realistic mathematics education.*

13 Lijnse, P. L. (Ed.) (1993). *European research in science education.*

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9 Streefland, L. (Ed.) (1991). *Realistic mathematics education in primary schools*.

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7 Elzenga, H. E. (1991). *Kwaliteit van kwantiteit*.

6 Lijnse, P. L., Licht, P., De Vos, W., & Waarlo, A. J. (Eds.) (1990). *Relating macroscopic phenomena to microscopic particles: a central problem in secondary science education.*

5 Van Driel, J. H. (1990). *Betrokken bij evenwicht*.

4 Vogelezang, M. J. (1990). *Een onverdeelbare eenheid*.

3 Wierstra, R. F. A. (1990). *Natuurkunde-onderwijs tussen leefwereld en vakstructuur*.

2 Eijkelhof, H. M. C. (1990). *Radiation and risk in physics education*.

1 Lijnse, P. L., & De Vos, W. (Eds.) (1990). *Didactiek in perspectief.*

ICO Publication List

388. Day, I.N.Z. (28-06-2018), *Intermediate assessment in higher education* (Leiden: Leiden University)
389. Huisman, B.A. (12-09-2018) *Peer feedback on academic writing.* Leiden: Leiden University.
390. Van Berg, M. (17-09-2018) *Classroom Formative Assessment. A quest for a practice that enhances students' mathematics performance*. Groningen: University of Groningen.
391. Tran, T.T.Q. (19-09-2018) *Cultural differences in Vietnam : differences in work-related values between Western and Vietnamese culture and cultural awareness at higher education*. Leiden: Leiden University
392. Boelens, R. (27-09-2018) *Studying blended learning designs for hands-on adult learners.* Ghent: Ghent University.
393. Van Laer, S. (4-10-2018) *Supporting learners in control: investigating self-regulated learning in blended learning environments*. Leuven: KU Leuven.
394. Van der Wilt, F.M. (08-10-18) *Being rejected*. Amsterdam: Vrije Universiteit Amsterdam.
395. Van Riesen, S.A.N. (26-10-2018) *Inquiring the effect of the experiment design tool: whose boat does it float?* Enschede: University of Twente.
396. Walhout, J.H. (26-10-2018) *Learning to organize digital information* Heerlen: Open University of the Netherlands.
397. Gresnigt, R. (08-11-2018) *Integrated curricula: An approach to strengthen Science & Technology in primary education.* Eindhoven: Eindhoven University of Technology.
398. De Vetten, A.J. (21-11-2018) *From sample to population*. Amsterdam: Vrije Universiteit Amsterdam.
399. Nederhand M.L. (22-11-2018) *Improving Calibration Accuracy Through Performance Feedback*. Rotterdam: Erasmus University Rotterdam.
400. Kippers, W.B. (28-11-2018) *Formative data use in schools. Unraveling the process*. Enschede: University of Twente.
401. Fix, G.M. (20-12-2018) *The football stadium as classroom. Exploring a program for at-risk students in secondary vocational education*. Enschede: University of Twente.
402. Gast, I. (13-12-2018) *Team-Based Professional Development – Possibilities and challenges of collaborative curriculum design in higher education*. Enschede: University of Twente.
403. Wijnen, M. (01-02-2019) *Introduction of problem-based learning at the Erasmus School of Law: Influences on study processes and outcomes*. Rotterdam: Erasmus University Rotterdam
404. Dobbelaer, M.J. (22-02-2019) *The quality and qualities of classroom observation systems*. Enschede: University of Twente
405. Van der Meulen, A.N. (28-02-2019) *Social cognition of children and young adults in context*. Amsterdam: Vrije Universiteit Amsterdam

406. Schep, M. (06-03-2019) *Guidance for guiding. Professionalization of guides in museums of art and history*. Amsterdam: University of Amsterdam
407. Jonker, H.M. (09-04-2019) *Teachers' perceptions of the collaborative design and implementation of flexibility in a blended curriculum*. Amsterdam: University of Amsterdam
408. Wanders, F. H. K. (03-05-2019). *The contribution of schools to societal participation of young adults: The role of teachers, parents, and friends in stimulating societal interest and societal involvement during adolescence*. Amsterdam: University of Amsterdam
409. Schrijvers, M.S.T. (03-05-2019) *The story, the self, the other. Developing insight into human nature in the literature classroom*. Amsterdam: University of Amsterdam
410. Degrande, T. (08-05-2019) *To add or to multiply? An investigation of children's preference for additive or multiplicative relations*. Leuven: KU Leuven.
411. Filius, R.M. (23-05-2019) *Peer feedback to promote deep learning in online education. Unravelling the process.* Utrecht: Utrecht University
412. Woldman, N. (24-05-2019) *Competence development of temporary agency workers*. Wageningen: Wageningen University
413. Donszelman, S. (06-06-2019) *Doeltaal-leertaal didactiek, professionalisering en leereffecten*. Amsterdam: Vrije Universiteit Amsterdam
414. Van Oeveren, C.D.P. (12-06-2019) *ITHAKA gaf je de reis*. Amsterdam: Vrije Universiteit Amsterdam
415. Agricola, B.T. (21-06-2019) *Who's in control? Finding balance in student-teacher interactions*. Utrecht: Utrecht University
416. Cuyvers, K. (28-08-2019), *Unravelling medical specialists self -regulated learning in the clinical environment.* Antwerp: University of Antwerp
417. Vossen, T.E. (04-09-2019) *Research and design in STEM education*. Leiden: Leiden University
418. Van Kampen, E. (05-09-2019) *What's CLIL about bilingual education?* Leiden: Leiden University
419. Henderikx, M.A. (06-09-2019) *Mind the Gap: Unravelling learner success and behaviour in Massive Open Online Courses*. Heerlen: Open University of the Netherlands
420. Liu, M. (13-09-2019) *Exploring culture-related values in Chinese student teachers' professional self-understanding and teaching experiences*. Utrecht: Utrecht University
421. Sun, X. (13-09-2019) *Teacher-student interpersonal relationships in Chinese secondary education classrooms*. Utrecht: Utrecht University
422. Wu, Q. (02-10-2019) *Making Construct-Irrelevant Variance Relevant: Modelling item position effects and response behaviors on multiple-choice tests*. Leuven: KU Leuven
423. Jansen, R.S. (11-10-2019) *Dealing with autonomy: Self-regulated learning in open online education.* Utrecht: Utrecht University
424. Van Ginkel, S.O. (23-10-2019) *Fostering oral presentation competence in higher education*. Wageningen: Wageningen University

425. Van der Zanden, P. (05-11-2019) *First-year student success at university: Domains, predictors, and preperation in secondary education*. Nijmegen: Radboud University Nijmegen
426. De Bruijn, A.G.M. (14-11-2019) *The brain in motion: Effects of different types of physical activity on primary school children's academic achievement and brain activation*. Groningen: University of Groningen
427. Hopster-Den Otter, D. (28-11-2019) *Formative assessment design: A balancing act*. Enschede: University of Twente
428. Harmsen, R. (10-12-2019) *Let's talk about stress. Beginning secondary school teachers' stress in the context of induction programmes*. Groningen: University of Groningen
429. Post, T. (11-12-2019) *Fostering inquiry-based pedagogy in primary school: a longitudinal study into the effects of a two-year school improvement project.* Enschede: University of Twente
430. Ackermans, K. (20-12-2019) *Designing Video-Enhanced Rubrics to Master Complex Skills*. Heerlen: Open University of the Netherlands

Universiteit Utrecht

Due to an increased use of technology, engineers' workplace practices have changed. So-called Techno-mathematical Literacies (TmL) are necessary skills for current and future engineers. This dissertation describes four studies that investigate how to develop these skills in first-year higher technical professional education (HBO). First, seven TmL frequently used by engineers, such as data literacy and sense of number, were identified in an interview study with fourteen engineers in different technical domains. Second, with these TmL as central learning goals, an innovative course in applied mathematics was designed and four categories of teaching strategies to stimulate TmL learning were observed. The course was implemented in the School of Life Sciences and Environmental Technology at Avans University of Applied Sciences in the Netherlands. For three years, eleven lecturers taught the new course to over 1400 students. Third, in an interview study on teachers' experiences, participating lecturers reported challenges in teaching the new course, in particular related to their changing role. In the fourth study, an instrument was designed to measure TmL skills. The results showed a positive effect on students' development of some of the TmL, e.g., sense of error and technical software skills. Altogether, the PhD study yielded theoretical knowledge about the nature of TmL, practical knowledge on the implementation of TmL in education, and a course in applied mathematics that is adjustable for all technical domains.