The use of applets to improve Indonesian student performance in algebra

This research was funded by a grant from the Indonesian Ministry of Education, project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794.

This research was carried out in the context of the Dutch Interuniversity Center for Educational Sciences

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Jupri, Al
The use of applets to improve Indonesian student performance in algebra/ A. Jupri – Utrecht: Freudenthal Institute for Science and Mathematics Education, Faculty of Science, Utrecht University / FIsme Scientific Library (formerly published as CD-β Scientific Library), no. 89, 2015.

Dissertation Utrecht University. With references. Met een samenvatting in het Nederlands.

ISBN:	978-90-70786-27-4
Keywords:	algebra education / initial algebra / digital technology / applets / linear equations in one variable / Indonesia
Design:	Vormgeving Faculteit Bètawetenschappen
Printed by:	Drukkerij Wilco, Amersfoort

THE USE OF APPLETS TO IMPROVE INDONESIAN STUDENT PERFORMANCE IN ALGEBRA

HET GEBRUIK VAN APPLETS OM DE PRESTATIES VAN INDONESISCHE LEERLINGEN IN ALGEBRA TE VERBETEREN

(met een samenvatting in het Nederlands)

Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit Utrecht op gezag van
de rector magnificus, prof. dr. G.J. van der Zwaan, ingevolge het besluit van
het college voor promoties in het openbaar te verdedigen op woensdag 28 januari 2015
des middags te 2.30 uur

door
Al Jupri
geboren op 10 mei 1982
te Serang, Indonesia

Promotoren: Prof. dr. J.A. van Maanen
Prof. dr. M.H.A.M. van den Heuvel-Panhuizen
Prof. dr. P.H.M. Drijvers

To the memory of Safira Annaja
To Azkiya Alghautsina and Rohma Mauhibah
To my parents, my parents in law, my sisters and my brothers

There is a stage in the curriculum when the introduction of algebra may make simple things hard, but not teaching algebra will soon render it impossible to make hard things simple.
(Tall & Thomas, 1991)

Table of contents

Preface

If there is a group of people who fall in love with mathematics late, then I would probably be part of this group. Although I do not know the reason, I still remember that I fell in love with mathematics at the beginning of grade IX of junior secondary school in 1996; this is very late compared to friends who have loved mathematics since primary school. Among other things, one mathematical formula that I have always remembered since that time – as my teacher suggested to memorize it – is $(a + b)^2 = a^2 + 2ab + b^2$. I tried to make it easier to remember and to understand by replacing the variables by numbers. Later, I realized that this approach can also be useful to check whether a mathematical formula is correct or not. For example, by replacing a and b with 1 and 2, respectively, I can easily calculate $(1 + 2)^2 = 3^2 = 9$; and by substituting the two values into $a^2 + 2ab + b^2$, I can also find a numerical value. If the results of the calculations for the left and the right side of the formula are not the same, then I am sure that something is wrong. I did the same thing to check other formulas in mathematics. Since that time, I experience mathematics, particularly algebra, as fascinating and meaningful.

In senior high school I was lucky to have a teacher who taught mathematics not as a collection of rules and formulas, but as meaningful knowledge. He often proved the mathematical formulas at stake, to show that they do not come down to us from heaven, but can be derived from definitions and rules which were already known. The first formula that I remember to be derived is the general "*abc*" formula for solving any equation of the form $ax^2 + bx + c = 0$. Even though it was not discussed during mathematics lessons, this motivated me and I realized that proving a mathematical formula is often not easy. Whenever I am able to prove, or to understand a proof and then do the proof by myself, I perceive mathematics as more and more interesting.

My fascination for proving formulas also made me write a paper that addressed ten different proofs of the Pythagorean theorem, including the Pythagorean triples and historical elements. In this paper, I combined three classical mathematical domains: geometry, number theory and algebra. Of these three, algebra became my favorite; my curiosity was raised by my school teachers who always mentioned that algebra is the masterpiece of Al-Khwarizmi, one of the great scholars in the Islamic golden age. It is an important contribution of the Islamic world to the development of science today.

To pursue a masters degree in mathematics education I went to Utrecht University, the Netherlands (2006-2008). In the course on the history of mathematics, taught by Prof. Jan van Maanen, I had an opportunity to read the English translation of the *Al-Kitab al-mukhtasar fi hisab al-jabr wa-l-muqabla* (The compendious book on calculation by completion and balancing), Al-Khwarizmi's masterpiece which my school teachers always mentioned. I was happy to write about this book and to present a term paper about it and the related work of the 'father of algebra' on linear and quadratic equations. This history class further increased my interest in equations. This inspired me to choose the topic of equations in my mathematics thesis project. In this project, I addressed cubic and quartic equations. My supervisor, Dr. Roelof Bruggeman, suggested that I should address equations not only in the real and complex fields, but also in other rings, including matrices. This was not the end of my equations adventure.

During a PMRI (Realistic Mathematics Education Project in Indonesia) workshop in Yogyakarta in 2010, Prof. R. K. Sembiring and Dr. Maarten Dolk encouraged me to do a PhD study in the Netherlands. As a first step I presented my initial ideas for this to them and to the PMRI members. Soon after that, I entered a selection procedure to get a PhD grant from the Directorate General of Higher Education of Indonesia, under the project DIKTI BERMUTU. I wrote a PhD research proposal on initial algebra and on the transition from arithmetical to algebraic thinking in particular. This proposal was inspired by the beautiful book entitled *Positive Algebra* by Martin Kindt – thanks to Henk van der Kooij who gave me a copy of this book – and by Barbara van Amerom's (2002) dissertation about the reinvention of early algebra. After being awarded the grant, I started a PhD in February 2011 at the Freudenthal Institute for Science and Mathematics Education (FIsme), Utrecht University, under supervision of Prof. Jan van Maanen, Prof. Marja van den Heuvel-Panhuizen, and Prof. Paul Drijvers. My new adventure on equations began.

Inspired by my interest in equations and also in line with the Indonesian mathematics curriculum, I decided to address linear equations as a core topic in the transition from arithmetic to algebra when finalizing my PhD research plan. At that time I hypothesized that students often fail in learning mathematics because they cannot relate the mathematical knowledge they learned in primary school (arithmetic, focusing on numbers) and the algebra knowledge, which focuses on using variables, in secondary schools. My method, in junior secondary school, using numbers to understand mathematical formulas, is a way to connect arithmetic and algebra. Another important decision was choosing Realistic Mathematics Education (RME) theory as a didactical background and design research as the research methodology. Having made these two decisions, I planned to investigate the effect of meaningful algebra teaching on student conceptual understanding and skills.

In a later phase, we decided to slightly change the research plan. My daily supervisor, Paul Drijvers, proposed to include digital technology, and in particular applets embedded in the Digital Mathematics Environment (DME), as an essential element in my research. To be honest, I was skeptical about the effectiveness of technology for mathematics teaching. How could students understand mathematics or algebra using digital tools? How would students acquire mathematical insight and understanding using digital technology? I was afraid that students would be trapped in learning how to use the digital tool rather than to learn mathematics, and would not acquire conceptual understanding. In my view, a tool would only be useful if students had already understood the mathematical concepts; a tool is just an assistant to carry out cumbersome and complex mathematical calculations and manipulations. I did not think of a tool, the other way around, helping to develop conceptual understanding and skills. As the data collection would be carried out in Indonesia, I also worried about the schools' technological infrastructure for integrating technology in mathematics teaching. To integrate applets and the DME in learning, both computers and an internet connection would be needed. These two reasons made it difficult for me to directly accept Paul's suggestion.

Before making a decision, I had the opportunity to learn how the applets for algebra work. I found that the applets, small online programs, are interesting in the sense that they are meaningful, not necessarily as an assistant for doing mathematics, and they seemed promising for the development of conceptual understanding and skills. Peter Boon, the designer of the applets at FIsme, informed me that the applets' design was inspired by the pedagogical view of RME theory. In the meantime, I learned that computer laboratories and internet access are available in most secondary schools in Indonesia, thanks to the Indonesian government policy of introducing ICT as a new subject for secondary school students. With these two considerations, I finally accepted the suggestion of my supervisors. The applets for algebra, embedded in the DME, then became a crucial element in my PhD study. Did it mean that I was not skeptical anymore about the use of technology in algebra teaching?

Eradicating skepticism was not as easy as all that. On the one hand, I tried to integrate digital technology as well as possible in the designed teaching sequence. On the other hand, I tried to ensure that students would also be skilled doing mathematics with paper and pencil. Therefore, in the learning arrangement activities, I looked for a balance between the use of digital technology and of working with paper and pencil. This, for instance, led to taking individual paper-and-pencil tests after the digital activities. My personal goal was to see a direct effect of the use of technology on student conceptual understanding and paper-and-pencil skills.

By just looking at the final result in the form of this dissertation, and knowing that the topic of equations is my favorite, people may get the impression that my PhD journey was a smooth and easy one. In reality, it was not. Four years of working on a PhD is not a short time, and much has happened during this period. From the academic perspective, the first year was difficult: even though I had a research plan, details about its what, why, and how were not yet clear. Searching and reading literature were my main activities. As an inexperienced researcher, comprehending research papers in English was often not easy, and sometimes frustrating! Also writing in an academic style was difficult and receiving many suggestions for improvement did not improve my self-confidence. In this situation, self-motivation, perseverance, tenacity, and patience were needed. Non-academically, it was one of the most difficult episodes in my life. About seven months after getting married, I was away from my wife while she was pregnant with our first daughter. Even if technology helped us to communicate with each other virtually every time, it did not replace us being close to each other. Things became more difficult when our first daughter died, ten days after her birth. Even if it was very hard, I decided to continue. I remembered a great motivation from my teachers in Indonesia who often recited two consecutive verses in the Qur'an which mean: "So, verily, with every difficulty, there is relief; Verily, with every difficulty there is relief."

Even if in the second year my study was getting clearer, the challenges of academic and personal difficulties remained. For instance, more than thirty revisions were needed – back and forth between my supervisors and me – before submitting the first paper to a research journal. Of course, less time was needed for each of next papers in the third and the fourth year of the study. In short, I can only say that sweat and tears were involved in each step of the journey.

This PhD research would certainly not have been accomplished without the help of many people. First of all, I would like to express my sincere gratitude to Jan van Maanen, my first promotor, who always encouraged and motivated me to work diligently throughout this four-year study. He is the first person, that I know and I feel, who believed that I would be a proper candidate to do a PhD at Utrecht University. His critical comments and suggestions on my work, and in particular on mathematical aspects, have always made me be careful each time when I met him in our regular monthly meeting. The way he expressed his comments was relaxed, but very serious, and often made me laugh. This made me enjoy the meetings. What I always loved about meetings with him is that he would often tease me with mathematical insights and inspirations. For example, around the beginning of my first year, when I was struggling with academic life, he asked me how many hours a day I spent to do my work. I replied in desperation that I needed 24 1/2 hours a day. He praised me that it is good to work hard as a PhD researcher! As a professor, he jokingly admitted that he even needed 25 hours a day to do his work. He liked 25 because it is a square number. I just smiled, no further response. But then he immediately exclaimed that 24 1/2 is also a very good number! It is exactly half of a square number, that is, $\frac{49}{2}=\frac{7^2}{2}$. To me this was mathematically inspiring and memorable!

Second, I would like to express many thanks to Marja van den Heuvel-Panhuizen, my second promotor. I recognize her as a fantastic researcher. I needed a lot of effort, time, perseverance, and patience to meet her quality standards. When I was already confident about my work, she questioned it easily with her brilliant comments, insightful remarks and consecutive suggestions. Experience, talent, and hard work may explain this wonderful behavior. One moment of mathematical inspiration that I will always remember happened when celebrating my second daughter's birth, who was born on 30 May 2013. On 2 July 2013, I informed my FIsme colleagues via email that my daughter was one month and a few days old, and so I invited them to celebrate. At the celebration event, Marja criticized me: I should express my daughter's age as precisely as possible, and not using 'a few days'!

Third, I would like to express my gratitude to Paul Drijvers, my third promotor and daily supervisor, who patiently guided me during this PhD journey. He is the person who knows every step of my work and my development as a researcher. Also, he is the one who motivated me not only in academic, but also in non-academic matters. I do not know how to express my gratefulness to him. In his supervision, he indirectly taught me how to give comments and suggestions carefully without hurting hearts, how to motivate without underestimating confidence, and how to acknowledge mistakes without being embarrassing. I guess these qualities will be useful for my future career. I recognize him as a direct, to the point, and systematic researcher. This, in my view, is reflected in his written work, which in my perception is structured like mathematics. To me, his papers are nice to read, easy to understand, look perfect, and are not boring to read and invite to be read again. The written work in this manuscript is certainly much influenced by his taste in writing. Similar to my other promotors, Paul has also enriched me with a lot of mathematical knowledge. For example, in his limited leisure time, he responded to an algebra problem which I posted on Twitter late on a Friday night in 2013, i.e., to find the value of $x^{2013}+\frac{1}{x^{2013}}$ if $x+\frac{1}{x}=-1$. The answer came immediately.

Fourth, I would like to express my thanks to FIsme colleagues and staff members. Peter Boon allowed me to use the applets in my research and also helped me with technical problems. Sietske Tacoma helped me while designing tasks in the applet environments. Wim van Velthoven solved some issues in the technical storage of student work. Martin Kindt often enriched me with mathematical knowledge by showing his fantastic new work. Aad Goddijn often helped me to solve challenging mathematical problems. Betty Heijman helped me on practical administrative matters at the beginning of my study. Nathalie Kuijpers corrected my written English and helped me to do the lay-out of this manuscript. Heleen Verhage, Wil Hofman and Mark Uwland helped me with administrative financial matters. Ank van der Heiden-Bergsteijn, Liesbeth Walther, Ellen Komproe, and Mariozee Wintermans helped me while working with the copy machine and reserving a meeting room.

Fifth, I would like to thank many people who supported me when I was staying in the Netherlands. Oom Richard and tante Nanda helped me in many daily life matters. Coby provided a memorable room for the about three years and four months that I stayed in her big house. Ari acquainted me with these three kind people and helped me in many practical things. Bang Andi, Mas Untung and Kang Arie helped and cured me while I was sick in this foreign country. Mas Yusuf provided his home where I stayed for the remaining year of finishing this PhD journey. Thanks to all other Indonesian friends here: Bang Pardi, Mas Bambang, Mas Agus, and others.

Sixth, I would like to express my gratitude to the following people in Indonesia. Prof. Dadang Sunendar and his staff were responsible for my scholarship and financial matters in my home university, *Universitas Pendidikan Indonesia* (Indonesia University of Education), Bandung. Turmudi, PhD, the head of the department of mathematics education, Indonesia University of Education, motivated me during this four-year study period. Dr. Rizky Rosjanuardi often kept in contact and encouraged me to be a good researcher. Also, thanks to my other colleagues at the department of mathematics education, Indonesia University of Education, for their support and attention.

Seventh, I must express my gratitude to many people, school principals, teachers and students, who were involved during data collections in Indonesia. Without their help, this study would not have been accomplished. Among others, they are Bu Mahmudah, Bu Farihah, Bu Fanny, Pak Zainudin, Pak Nanang, Pak Tri, Pak Nur Ikhwan, Pak Ali, Pak Maman, Pak Nidhom, Pak Ashari, and Pak Fahrudin.

Eighth, and final, I dedicate this work to my parents, my sisters and brothers, and in particular to my wife and my daughters. I am sure my parents are always praying for me every time for my success in the past, in the present, and in the future. I have no word to express my gratitude to both of them. I also thank my three sisters and two brothers who I believe always pray for me. My great thanks should surely be expressed to my parents in law who are, like my own parents, praying for my success. Finally, if there is a word to express the greatest gratitude, I will still not use that word to express to my wife, the most beautiful woman in the world, Rohma Mauhibah. Her love and loyalty to me make me fall in love with her again and again. She really helped me during this PhD journey, and her competent preparation of the data collection made my life easier. Even if it is very difficult, she sacrifices her life for me, readily waits for me, takes care of our daughters without my close presence, and prays for me every time. We believe that our first daughter, Safira, is now in heaven, and that we will meet her in the hereafter. Azkiya, our second daughter, makes our life complete and lively. Often while I was cycling to Utrecht University campus, I had imaginary dialogues with her. In one dialogue, she asked me why to study algebra education. I could not answer her question satisfactorily. She then told me that my name is similar to the word of algebra, particularly to the word *aljabar* or *al-jabr*. I just told her that my seniors even called me Aljabar Jupri. She smiled and laughed at me!

Chapter 1 Introduction

1.1. Research context

Education is widely recognized as a key factor in a nation's development. Indonesia, as a developing country, and the fourth most populated country in the world, faces many challenges to enhance the quality of its education to prepare its citizens for the future (Kemendikbud, 2013; Depdiknas, 2006). The characteristics of a qualified citizen as an educated human include, inter alia, having good morality and personality; and being an independent, creative, and knowledgeable individual (Kemendikbud, 2013; Depdiknas, 2006). These characteristics are expected to be developed through educational processes via various subjects, including mathematics.

Mathematics, as a compulsory subject for Indonesian secondary school students, plays a significant role in developing a qualified citizen who possesses the mentioned characteristics. Through learning mathematics, students are expected to not only acquire mathematical knowledge, but also become independent, responsible, and creative problem solvers. Moreover, mathematics also contributes to the development of the student's morality and personality (Swadener & Soedjadi, 1988). This also holds for algebra, which is recognized as a core topic within school mathematics (Harvey, Waits, & Demana, 1995; Katz, 2007; Morgatto, 2008).

In addition to this motivation for algebra, the mastery of algebraic skills may act as a gateway for pursuing advanced studies at university level (e.g., Harvey, Waits, & Demana, 1995; Katz, 2007; Morgatto, 2008). However, it is well known that students all over the world experience difficulties in learning algebra (e.g., Drijvers, 2003; Kolovou, 2011; Van Amerom, 2002; Warren, 2003). Moreover, algebra is recognized as a subject that is not only hard to learn, but also difficult to teach (Stacey, Chick, & Kendal, 2004; Watson, 2009). Although these issues with learning and teaching algebra are a worldwide phenomenon, they hold for Indonesia in particular: Indonesian students showed low scores in recent international comparative studies compared to students in other countries. For example, in the 2007 Trends in International Mathematics and Science Study (TIMSS), Indonesian students' average score in the domain of algebra was below the international average, in 36th position out of 48 countries (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008; Mullis, Martin, & Foy, 2008). Moreover, compared to Southeast Asian countries, Indonesian students' average algebra score was significantly below that of students from Thailand, Malaysia, and Singapore respectively (Gonzales et al., 2008). In TIMSS 2011, Indonesian students were ranked 38th out of 42 participating countries in the domain of algebra (Mullis, Martin, Foy, & Arora, 2012).

These results give rise to a why-question with respect to the low algebra scores of Indonesian students: Why do Indonesian students seem to experience more difficulties in learning algebra than students in most other countries? A first possible answer might be the educational factor. How is algebra taught in Indonesia? Although we cannot answer this question in detail yet, it is known that in spite of curriculum revisions over the last decades (e.g., Depdiknas, 2006), most mathematics lessons in Indonesia are still delivered in a traditional way (see, for instance, Johar, 2010; Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002). Preserving the traditional way of teaching algebra includes, for example, the use of drill-and-practice as a teaching method and the memorization of formulae as a central point in teaching. Another aspect of algebra education in Indonesia is that students start to learn algebra in the first semester of grade VII directly in a formal way (Kemendikbud, 2013; Depdiknas, 2006). This means that the students are not prepared for learning algebra through having experiences with informal algebra in elementary school. As a consequence, algebra might be difficult for students because there is no smooth transition from primary school arithmetic to secondary school algebra, i.e., the two topics are treated as different, not as connected to each other. In our view, these two possible answers to the why-question are still general and hypothetical; therefore, a more specific and scientific explanation for the question is needed. Investigating the difficulties students encounter while learning algebra, which is an initial step to answer the why-question, constitutes the first main focus of this study.

A natural next question, then, is how to improve Indonesian students' performance in algebra. One promising approach concerns the use of Information and Communication Technology (ICT). Recently, several studies have shown that the use of ICT contributes to the improvement of students' algebra performance in secondary school (Bokhove & Drijvers, 2010b; Kieran & Drijvers, 2006; Rakes, Valentine, McGatha, & Ronau, 2010; Trouche & Drijvers, 2010). Moreover, in other studies, ICT was found to be a powerful learning environment for early algebra in primary school (Kolovou, Van den Heuvel-Panhuizen, Bakker, & Elia, 2008; Kolovou, 2011). In these latter studies it was also shown that ICT worked as an environment for homework. In view of the earlier collected positive experiences with using ICT for mathematics teaching in Indonesia (Widjaja & Heck, 2003; Zulkardi, 2002), making use of ICT to create a powerful environment for teaching algebra to

Indonesian students might be a promising avenue. Thus, investigating the use of ICT in algebra education is the second focus of this present study.

In short, the main aim of this PhD study is to investigate the abovementioned why and how-questions for improving Indonesian algebra education.

1.2. The Indonesian educational system: a brief overview

To understand the educational context of the present study one can consult Figure 1.1, which outlines the Indonesian educational system. General education in Indonesia includes two streams: an academic and a professional (vocational) stream. Either academic or professional, each stream includes two types: Islamic (religious) education and public education. Both Islamic and public education involve private and governmental schools. The Ministry of Religious Affairs (MORA) is responsible for Islamic education and the Ministry of Education and Culture (MoEC) for public education. Whereas Islamic education follows the two curricula released by MORA and MoEC in parallel, public education only addresses the MoEC curriculum. As a result, Islamic Junior Secondary School students receive less time than their peers in public schools for, for instance, the subject of mathematics.

Even though a child can go to kindergarten from age 4 or 5 on, education in Indonesia is compulsory for students from age 7 to 15, including six years of primary school and three years of junior secondary school level. After completing compulsory education, a student may go to secondary education, either senior secondary school or vocational school depending on passing the admission criteria. After completing secondary education, a student can enter higher education and may choose either the academic or the professional stream, again depending on admission criteria. For the academic stream, higher education consists of three levels: bachelor (S1), master (S2), and doctorate (S3). The higher education professional stream includes the following programs: Diplomas (D1, D2, D3, or D4), First Professional Program (SP1), and Second Professional Program (SP2). In this study, we focus on investigating the algebraic performance of junior secondary school students, the gray area in Figure 1.1.

Age	Grade	Education level	Academic		Professional			
			MORA	MoEC	MORA/MoEC			
27		Higher education	Islamic Doctorate Program (S3)	Doctorate Program (S3)	Second Professional Program (SP2)			
26								
25								
24			Islamic Master Program (S2)	Master Program (S2)	First Professional Program (SP1)			
23								
22			Islamic Bachelor Program (S1)	Bachelor Program (S1)	Dipl4 (D4)			
21						Dipl3 (D3)		
20							Dipl2 (D2)	
19								Dipl1 (D1)
18	12	Secondary education	Islamic Senior Secondary School	Public Senior Secondary School	Islamic Vocational School		Public Vocational School	
17	11							
16	10							
15	9	Compulsory education	Islamic Junior Secondary School	Public Junior Secondary School				
14	8							
13	7							
12	6		Islamic Primary School	Public Primary School				
11	5							
10	4							
9	3							
8	2							
7	1							
6	K2	Early childhood education	Islamic Kindergarten	Public Kindergarten				
5	K1							
4								

Figure 1.1. The Indonesian educational system (Adapted from Hendayana, Supriatna, & Imansyah, 2011, p. 44)

1.3. Research questions

To investigate the issues described in section 1.1, we now define the mathematical topic, the research aims and the corresponding research questions. As secondary school algebra is too broad to investigate within the frame of this study, we need to further restrict ourselves. As the low performances of Indonesian students in the TIMSS studies became manifest in grade VIII (13-14 year-old), it seems appropriate to address this grade. However, these low performances may result from student difficulties in the beginning of algebra learning, which in Indonesia starts in grade VII (12-13 year-old). Therefore, we choose to investigate algebra learning for students in grade VII and partly grade VIII, who are engaged in the transition from arithmetic (taught in primary school) to algebra learning.

A central topic at the beginning of algebra concerns linear equations in one variable and the related linear inequalities, which already include core algebra concepts, such as the notions of variable and algebraic equivalence. Therefore, the two general questions elaborated in section 1.1 – the why-question of low student performances in algebra and the how-question of improving algebraic student performance – will be addressed for this topic within the algebra curriculum.

Table 1.1. Research aims and research questions in this PhD study

No.	Research aims	Research questions
1.	To investigate students' difficulties in initial algebra in the domain of linear equations and inequalities in one variable.	What are Indonesian students' difficulties in initial algebra learning, particularly in solving linear equations in one variable and the related linear inequalities?
2.	To identify and explain student difficulties in initial algebra and in solving equations, in particular which emerge in an ICT-rich learning setting using the operational and a structural perspective.	What are student difficulties in solving equations in one variable which emerge in an ICT-rich approach and how can operational and structural views on equations explain these difficulties?
3.	To identify and explain student difficulties in initial algebra and in solving word problems, in particular which emerge in an ICT-rich learning setting using the mathematization perspective.	What are student difficulties in mathematizing word problems in the domain of linear equations in one variable?
4.	To investigate the effect of digital technology on student performance in initial algebra and in the domain of equations in one variable in particular.	Does an intervention with digital technology enhance students' performance in initial algebra?
5.	To describe the relationship between the use of a digital tool for algebra and students' understanding from the perspective of instrumentation theory.	Which schemes do students develop for solving equations using algebraic substitution with the Cover-up applet and which relationships between techniques and understanding are developed?

Concerning the use of ICT to improve the algebra performance, we choose to use applets for algebra embedded in the Digital Mathematics Environment (DME) – a high-performing online environment which is developed at the Freudenthal Institute, Utrecht University, the Netherlands – for the following reason. According to four groups of criteria (algebra pedagogy, tool use, assessment, and general features), 27 external experts in the field involved in the Delphi study considered the DME to be the most suitable environment for research in algebra education (Bokhove & Drijvers, 2010a). From these criteria, the main DME's characteristics that are suitable for algebra learning include stability and performance, correct display of algebraic notations, ease of use, mathematical soundness, and storage of student work (Bokhove & Drijvers, 2010a).

Taking the above into account, we formulate the research aims and the corresponding research questions of this study in Table 1.1. Research questions 1-3 address mainly the why-question, while research questions 4-5 mainly deal with the how-question.

1.4. Theoretical perspectives

In relation to the research questions phrased in the previous section, we need to select and elaborate theoretical perspectives that can be used appropriately as foundation for the investigation. Clearly, the research questions require theoretical foundations from algebra education and the use of ICT in mathematics education. However, as these two fields are so wide and are still general, more specific choices are needed. From the field of algebra education research, we focus on the following three theories:

1. The theory on student difficulties in algebra, which encompass the difficulties in applying arithmetical operations in numerical and algebraic expressions (e.g., Booth, 1988; Warren, 2003), in understanding the concept of variable (e.g., Herscovics & Linchevski, 1994; Wagner, 1983); in understanding algebraic expressions (e.g., Tall & Thomas, 1991); in understanding the use of the equal sign (e.g. Kieran, 1981); and in mathematization (e.g., Treffers, 1987). This lens is used for classifying types of difficulties or mistakes made by students while solving problems in the beginning of learning algebra.
2. The theory on operational and structural views on algebraic objects and activities, that is, the view of conceiving an abstract concept that can be seen operationally as a process and structurally as an object (Sfard, 1991). This lens is used for comprehending the causes of the difficulties, particularly when students are solving symbolic algebra problems.

3. The theory on mathematization, which refers to the activity of transforming a problem situation into the symbolic mathematical world and vice versa, as well as reorganizing and (re)constructing the world of mathematics (Freudenthal, 1991; Gravemeijer, 1994; Treffers, 1987; Van den Heuvel-Panhuizen, 2003). This mathematization perspective is used for comprehending student difficulties when dealing with word problems.

Concerning the use of ICT in mathematics education, and in algebra education in particular, we focus on ICT as an environment to develop students' conceptual understanding, and to make them practice algebraic techniques (Drijvers, Boon, & Van Reeuwijk, 2010). To describe the subtle interplay between the use of ICT and student conceptual understanding as well as procedural skills, we make use of instrumentation theory, addressing students' schemes and techniques while solving algebra problems in either digital or paper-and-pencil environments (Artigue, 2002; Trouche, 2004; Trouche & Drijvers, 2010).

1.5. Research methods

This study was carried out in three stages. Stage 1 concerned an interview study to identify and understand student difficulties in initial algebra. In Stage 2, an ICT-rich teaching intervention was designed and field tested in a small-scale pilot experiment. Stage 3 concerned a larger teaching experiment involving eight classes in four different schools. Table 1.2 outlines the study. We will now briefly address each stage of the study.

Table 1.2. Outline of the study

Stages	Brief description of the study	Research questions
1	Interview study: A small-scale test and interviews	1
2	Pilot study: A small-scale classroom experiment	2 & 3
3	Larger-scale experiment: pre-posttest control-group experiment	4 & 5

1.5.1. Stage 1: Interview study

The aim of Stage 1 was to investigate Indonesian students' difficulties when starting to learn algebra, and to find out why they experience these difficulties. To do so, we carried out an exploratory interview study, and triangulated the results with findings from literature.

In the literature study we surveyed: studies that refer to the learning of algebra in general and to the transition from arithmetic to algebra in particular; results of international comparative studies such as TIMSS and PISA (Programme for International Student Assessment); and Realistic Mathematics Education

(RME) theory and its view on the learning and teaching of algebra. Next, we administered an individual written test on algebra involving 51 grade seventh Indonesian students and follow up interviews with 37 of these students. The students came from two Islamic schools and one Public school.

1.5.2. Stage 2: Pilot study

In Stage 2, we designed an ICT-rich algebra teaching sequence, and field tested it in a small-scale pilot experiment. The teaching sequence consisted of four 80-minutes mathematics lessons on the topic of equations in one variable and related word problems. The designed lessons were to replace the corresponding regular text book chapter. The materials for the teaching sequence (paper-and-pencil tasks, digital tasks, intermediate assessment tasks, and a teacher guide) were designed and piloted in two grade VII classes, which involved 51 students in two Islamic schools in Indonesia. The digital tools integrated in the teaching experiment included two applets called Algebra Arrows and Cover-up which were embedded in the Digital Mathematics Environment.

1.5.3. Stage 3: Larger-scale experiment

In Stage 3, a larger teaching experiment was carried out. Again, the algebra topic was equations in one variable and related word problems. The teaching sequence included a re-designed version of the sequence in Stage 2, which focused on the equations of the form $f(x) = c$, and additional sequence focusing on equations of the form $f(x) = g(x)$. To evaluate this teaching sequence, an experiment with a pre-and-posttest control group design was carried out. The teaching experiment consisted of four-80 minutes mathematics lesson; eight classes in four different schools enrolled. Four applets – called Algebra Arrows, Cover-up, the Balance Model, and the Balance Strategy – were integrated in the teaching experiment.

Taking these three stages into account, the study as a whole has the characteristics of design research consisting of two cycles, the first one including the interview study and the small-scale pilot experiment; and the second one including the larger experimental study. Each cycle consisted of three phases: a preparatory phase, in which the results of the interview study functioned mainly as the foundation for the development of a learning sequence; an experimental phase, in which the designed learning sequence was implemented in the classroom setting; and a retrospective analysis phase, in which the results were analyzed and reflected upon (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Gravemeijer, 2004).

1.6. The dissertation's structure

This dissertation consists of seven chapters. This first chapter provides an overview of the study. Chapters 2-6 contain articles that have been submitted to, or published in different research journals in the field of mathematics education. Chapter 7 presents the study's overall conclusions, reflections and recommendations. Table 1.3 summarizes the dissertation's structure and shows the relation between chapters, publications, and research questions (RQ).

Table 1.3. Dissertation outline

Chapters and titles	Publication	RQ
Chapter 1: Introduction		
Chapter 2: Difficulties in initial algebra learning in Indonesia	Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal, 26*(4), 683–710. DOI: 10.1007/s13394-013-0097-0.	1
Chapter 3: Student difficulties in solving equations from an operational and a structural perspective.	Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014). Student difficulties in solving equations from an operational and a structural perspective. *Mathematics Education, 9*(1), 39–55.	2
Chapter 4: Student difficulties in mathematizing word problems.	Jupri, A., & Drijvers, P. (Accepted). Student difficulties in mathematizing word problems. *EURASIA Journal of Mathematics, Science & Technology Education*.	3
Chapter 5: Improving grade 7 students' achievement in initial algebra through a technology-based intervention.	Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (submitted). Improving grade 7 students' achievement in initial algebra through a technology-based intervention.	4
Chapter 6: An instrumentation theory view on students' use of an applet for algebraic substitution	Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (submitted). An instrumentation theory view on students' use of an applet for algebraic substitution.	5
Chapter 7: Conclusion		

Chapter 2 describes an interview study investigating student difficulties in initial algebra through an individual written test and follow up interviews. The results of this interview study serve as a point of departure for designing the small-scale experimental study and the larger experimental study reported in chapters 3, 4, 5 and 6, respectively.

Chapter 3 reports a part of the small-scale experimental study results involving 51 Indonesian grade VII students who use two applets – called Algebra Arrows and Cover-up Strategy applets – embedded in the DME, and equations in one variable in particular. The operational and structural perspective reveals to be a fruitful framework to explain student difficulties in solving equations.

Chapter 4 complements the report of the small-scale experimental study by describing student difficulties in solving word problems in algebra using the mathematization perspective.

Chapter 5 addresses the results of the larger experimental study involving grade VII (12-13 year-old) Indonesian students who use four applets – called Algebra Arrows, Cover-up Strategy, Balance Model, and Balance Strategy applets – embedded in the DME for algebra.

Chapter 6 describes a case study analyzing a part of the larger experiment data. It addresses the relationship between the use of a digital tool for algebra and students' algebraic understanding on algebraic substitution using the Cover-up Strategy applet in the light of the instrumentation theory.

Finally, Chapter 7 concludes this study with a summary of the main results, corresponding reflections, and practical and theoretical recommendations for teaching as well as for future design and research.

As each of the chapters 2-6 in this dissertation is intended to be published as a scientific paper, and therefore should be understood independently from other chapters, some overlap could not be avoided.

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Chapter 2 Difficulties in initial algebra learning in Indonesia

Publication:

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal, 26*(4), 683–710. DOI: 10.1007/s13394-013-0097-0.

Abstract Within mathematics curricula, algebra has been widely recognized as one of the most difficult topics, which leads to learning difficulties worldwide. In Indonesia, algebra performance is an important issue. In the Trends in International Mathematics and Science Study (TIMSS) 2007, Indonesian students' achievement in the algebra domain was significantly below the average student performance in other Southeast Asian countries such as Thailand, Malaysia and Singapore. This fact gave rise to this study which aims to investigate Indonesian students' difficulties in algebra. In order to do so, a literature study was carried out on students' difficulties in initial algebra. Next, an individual written test on algebra tasks was administered, followed by interviews. A sample of 51 grade seven Indonesian students worked the written test, and 37 of them were interviewed afterwards. Data analysis revealed that mathematization, i.e., the ability to translate back and forth between the world of the problem situation and the world of mathematics and to reorganize the mathematical system itself, constituted the most frequently observed difficulty in both the written test and the interview data. Other observed difficulties concerned understanding algebraic expressions, applying arithmetic operations in numerical and algebraic expressions, understanding the different meanings of the equal sign, and understanding variables. The consequences of these findings on both task design and further research in algebra education are discussed.

Keywords algebra, difficulties, Indonesian students, linear equations, linear inequalities

2.1. Introduction

Algebra is a core topic within mathematics and in secondary school mathematics in particular. It is instrumental for achievements in other mathematical domains such as, analytical geometry, calculus and statistics. Algebra serves not only as a language for science, but also as a gateway to advanced mathematics and higher education. Furthermore, algebraic knowledge and skills are relevant in daily and professional life either directly or as a prerequisite (Katz, 2007; Kendal & Stacey, 2004). Therefore, successful algebra education is a precondition for achievements in mathematics education in general. Initial algebra education, which encompasses the students' first steps in this domain, is of course a crucial phase in algebra education (Van Amerom, 2002).

Throughout the world, however, students experience difficulties in learning algebra (e.g., see Booth, 1988; Drijvers, 2003; Herscovics & Linchevski, 1994; Kolovou, 2011; Warren, 2003). Moreover, algebra has been increasingly recognized as a subject that is not only hard to learn but also hard to teach well (Stacey, Chick, & Kendal, 2004; Watson, 2009).

Although these difficulties in the learning and teaching of algebra are a worldwide phenomenon, the case of Indonesian algebra education deserves special attention. Indonesian students showed low scores in the recent TIMSS 2007 study: the Indonesian average score in the domain of algebra was 405, which is far below the international average of 500 (Gonzales et al., 2008). Moreover, Indonesian students' algebra performance was significantly lower than the scores of students from other Southeast Asian countries, such as Thailand, Malaysia and Singapore, where students' average scores for algebra were 433, 454 and 579, and rankings were 29th, 20th and 3rd, respectively.

This fact gives rise to the questions of why Indonesian students have such low algebra scores and why they seem to experience more difficulties in learning algebra than students in other countries. As an initial step to address these questions, this present study aims to investigate Indonesian students' difficulties in initial algebra.

2.2. Theoretical background

2.2.1. A closer look at Indonesian students' performance in initial algebra

Figure 2.1 shows an algebra task retrieved from the TIMSS 1999 study. It concerns connecting the corresponding linear equation to a given verbal statement. Only 37% of the Indonesian participants were able to solve it, which was significantly below the international average of 65% (Mullis et al., 2000). The TIMSS 2007 study shows similar result for Indonesian students solving the algebra task shown in Figure 2.2 on solving a linear equation in one variable presented in a context. This task was solved correctly by only 26% of Indonesian students which was significantly below the international average of 34% (Mullis et al., 2008).

n is a number. When n is multiplied by 7, and 6 is then added, the result is 41. Which of these equations represents this relation?
A. $7n + 6 = 41$
B. $7n - 6 = 41$
C. $7n \times 6 = 41$
D. $7(n + 6) = 41$

Figure 2.1. TIMSS 1999 algebra task (Mullis et al., 2000)

In Zedland, total shipping charges to ship an item are given by the equation $y = 4x + 30$, where x is the weight in grams and y is the cost in zeds. If you have 150 zeds, how many grams can you ship?
A. 630
B. 150
C. 120
D. 30

Figure 2.2. TIMSS 2007 algebra task (Mullis et al., 2008)

As in many algebra curricula, linear equations in one variable is a central topic in the Indonesian initial algebra program (Depdiknas, 2006). The above two examples indicate that Indonesian students have serious difficulties with this topic. Therefore, in this study we focus on linear equations in one variable, and the related linear inequalities. As many researchers (e.g., Herscovics & Linchevski, 1994; Linchevski & Herscovics, 1996; Pillay, Wilss, & Boulton-Lewis, 1998) have addressed linear equations in one variable to comprehend students' learning and thinking in the transition from arithmetic to algebra and students' capability to understand and use variables in particular, this seems an appropriate topic to further elaborate on.

2.2.2. Difficulties in initial algebra

What does existing research in initial algebra education tell us about students' learning difficulties? Some literature uses the term "difficulties" (e.g., Herscovics & Linchevski, 1994; Warren, 2003) or "conceptual difficulties" (e.g., Tall & Thomas, 1991), while others speak of "errors" (e.g., Booth, 1988). We decided to use the term "difficulties" as we consider errors as manifestations of the difficulties. The literature on initial algebra for 10-14 year-old students led us to identify five types of difficulties in initial algebra which we will now describe.

2.2.2.1. Applying arithmetic operations

Many studies show that 12-13 year-old students often fail to add or subtract like algebraic terms and sometimes detach symbolic expressions from the operations (e.g., Herscovics & Linchevski, 1994; Linchevski, 1995; Linchevski & Herscovics, 1996). Also, students (11-14 year-olds) misapply commutative as well as associative properties when carrying out subtractions or divisions (Booth, 1988; Pillay, Wilss, & Boulton-Lewis, 1998; Warren, 2003), and fail to use the distributive property of a multiplication over an addition (Booth, 1988; Pillay, Wilss, & Boulton-Lewis, 1998). In our view, these difficulties reveal students' limited mastery of addition, subtraction, multiplication and division; of applying the priority rules of arithmetic operations in calculations; and of using properties of numerical operations. We understand properties of numerical operations as commutative, associative, inverse and distributive properties within both additive and multiplicative situations. All together we summarize these issues as difficulties in applying *arithmetic operations* in both numerical and algebraic expressions and we abbreviate this type of difficulties as ARITH.

2.2.2.2. Understanding the notion of variable

Concerning the literal symbols that are so crucial in algebra, research—carried out with 10-14 year-old students—reveals that students have difficulties to distinguish different roles of literal symbols such as placeholder, generalized number, unknown, or varying quantity (Booth, 1988; Drijvers, 2003; Linchevski & Herscovics, 1996; Rosnick, 1981; Usiskin, 1988; Van Amerom, 2002). As a placeholder, a literal symbol is seen as an empty 'container' in which a numerical value can be stored or from which it can be retrieved. As an unknown, a literal symbol is used in a problem solving process in which the goal is to find a solution of an equation. As a generalized number, a literal symbol acts as a pattern generalizer symbolizing equivalence: all values substituted for the literal symbols will result in true statements, for instance, $2x + 5x = 7x$. As a varying quantity, a literal symbol is used in a functional relationship either as an input argument or as the output function value. We summarize issues with these roles as a category of difficulty in *understanding the notion of variable*, and we abbreviate this type of difficulties as VAR.

2.2.2.3. Understanding algebraic expressions

In addition to the different views on literal symbols, students also have to recognize that an algebraic expression, such as $x + 10$, has a dual nature: it represents a calculation process as well as being an algebraic object in its own right (Drijvers, 2003; Van Amerom, 2002, 2003). In the literature, this is called the process-object duality (Sfard, 1991); the inability to switch between the process and the object view is called the process-product obstacle (Tall & Thomas, 1991). Other obstacles are identified, such as the inability to disentangle the order in which the algebraic expressions must be understood and processed, sometimes conflicting with the order of natural language. This is called the parsing obstacle. For example, in dealing with $12 - 5x$, students may read from left to right as $12 - 5$ giving 7, and consider the full expression to be equivalent to $7x$; in dealing with $x + 3$, students may read it as x and 3, and interpret this as $3x$ (Tall & Thomas, 1991). The expected answer obstacle is the incorrect expectation of having a numerical answer for an algebraic expression. This causes a related difficulty which is called the lack of closure obstacle, which is the discomfort from attempting to handle an algebraic expression which represents a process that cannot be carried out (Tall & Thomas, 1991). One characteristic of an ability to manipulate algebraic expressions technically as well as with insight that causes difficulties in algebra is the gestalt view on algebraic expressions (Arcavi, 1994, 2005). This concerns the ability to consider an algebraic expression as a whole, to recognize its global characteristics, and to foresee the effect of a manipulation strategy. According to Bokhove and Drijvers (2010), the gestalt view includes both pattern salience, i.e., the recognition of visual pattern in expressions and equations; and local salience, i.e., the attraction by local algebraic symbols, such as inequality signs and minus signs in inequalities or equations. The latter may lead to looking at symbols in isolation without taking the whole expressions into consideration. Furthermore, Bokhove and Drijvers perceive the gestalt view as enabling the learner to take strategic decisions about what to do next and to resist or succumb to the visual salience. We summarize these difficulties as a category of difficulty in *understanding algebraic expressions*, and we abbreviate these as AE.

2.2.2.4. Understanding the different meanings of the equal sign

Another difficulty in initial algebra learning concerns the equal sign. In arithmetic, the equal sign often invites carrying out a calculation and writing down a numerical answer, whereas in algebra, it usually means 'is algebraically equivalent to' (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; Ketterlin-Geller, Jungjohann, & Chard, 2007; Kieran, 1981; Linchevski, 1995; Pillay, Wilss, & Boulton-Lewis, 1998). With the former insight, students may interpret 2 + 3 = ... as adding 2 and 3 to get the specific answer 5 and may not view 2 + 3 = 3 + 2; 2 + 3 = 1 + 4; or 5 = 2 + 3 as possible solutions to the same task. The latter insight, however, is needed to understand equivalence, for example while rewriting $x + 2 = 3x + 4$ as $x = 3x + 2$. In this study, the difficulty in understanding the different *meanings of the equal sign* in arithmetic and algebra is abbreviated as EQS.

2.2.2.5. Mathematization

The final difficulty in initial algebra concerns mathematization, a core concept of the approach to teaching mathematics that is called Realistic Mathematics Education (RME) (Freudenthal, 1991; Treffers, 1987). This mathematization has both horizontal and vertical mathematization aspects. The difficulty in horizontal mathematization concerns going from the world of real phenomena to the world of symbols and vice versa: in other words, to translate back and forth between the world of the problem situation and the world of mathematics (Treffers, 1987; Van den Heuvel-Panhuizen, 2000, 2003). These transitions are demanding for students and in particular 10-14 year-old students (Ketterlin-Geller et al., 2007; MacGregor & Stacey, 1998; Van Amerom, 2002, 2003; Warren, 2003; Watson, 2009). Activities of horizontal mathematization include, for instance, formulating a problem in a different way, discovering relations and regularities, and transferring a real world problem to a mathematical problem or to a known mathematical model (De Lange, 1987). These horizontal mathematization activities are in line with the first two problem solving heuristics proposed by Polya (1973), namely understanding the problem and devising a plan. The difficulty in vertical mathematization concerns dealing with the process of reorganization within the mathematical system itself, i.e., the process of moving within the symbolic world (Treffers, 1987; Van den Heuvel-Panhuizen, 2000, 2003). Instances of vertical mathematization activities involve combining, integrating, formulating, and manipulating algebraic models while solving equations and inequalities; formulating a new mathematical concept; and proving regularities and generalizing (De Lange, 1987; Treffers, 1987; Van den Heuvel-Panhuizen, 2000). In this study, we abbreviate this category of *mathematization difficulties* as MATH.

2.2.3. Research question

Table 2.1 provides an overview of the five categories as they emerged from the literature study. Of course, these five categories are not independent, but form a set of intertwined and related difficulties. The first three categories in particular might co-emerge, as they all concern algebraic meaning: the meaning of numerical operations, variables and algebraic expressions, which are so significant in initial algebra. Whereas the ARITH, VAR, AE and EQS categories come from the algebra domain, the mathematization category (MATH) is rooted in the theory of RME—which concerns mathematics education in general. As such, the MATH category complements the other categories of difficulties.

Table 2.1. Overview of the five categories of difficulties in initial algebra learning

	ARITH
1.	Related operations: Carrying out operations such as addition, subtraction, multiplication and division of numbers or algebraic expressions
2.	Related rules: Following the rules of order of operations when dealing with numbers or algebraic expressions
3.	Related properties: Applying properties of numerical operations, i.e., commutative, associative, inverse, or distributive, when dealing with numbers or algebraic expressions
	VAR
4.	Roles of literal symbols: Understanding literal symbols as placeholders, generalized numbers, unknowns and varying quantities
	AE
5.	Parsing obstacle: Understand the order in which the algebraic expressions must be understood and processed, which may conflict with the order of natural language
6.	Expected answer obstacle: An incorrect expectation to get a numerical answer for an algebraic expression
7.	Lack of closure obstacle: The discomfort in attempting to handle an algebraic expression which represents a process that cannot be carried out
8.	Lack of gestalt view: The inability to deal with algebraic expressions' visual salience, including both pattern salience and local salience
	EQS
9.	Different meanings: The understanding of the different meanings of the equal sign in arithmetic (carrying out a calculation and writing down an answer) and in algebra ("is algebraically equivalent to")
	MATH
10.	Horizontal mathematization: The process of translating back and forth between the world of the problem situation and the world of mathematics
11.	Vertical mathematization: The process of reorganization within the mathematical system or the process of moving within the symbolic world

Taking the above types of difficulties which emerged from the literature study as a point of departure, the research question of this study is:

What are Indonesian students' difficulties in initial algebra learning, particularly in solving linear equations in one variable and the related linear inequalities?

2.3. Methods

To address the research question, we conducted an explorative study in which an individual written test on algebra was administered, followed by student interviews on the written work.

2.3.1. Sample

The subjects of the study were 51 Indonesian students taken from two samples: 33 students from the 2011 grade VII cohort took part in the summer of 2011, and 18 students from the 2012 grade VII cohort were involved in the summer of 2012. The two samples had the same characteristics: they finished grade VII (13/14 year-old), in which they had studied linear equations and inequalities in one variable in its first semester (Depdiknas, 2006). The students came from three different schools, one a public school, and the two others religious schools. As such, this school selection is representative for the Indonesian educational system, which consists of these two types of schools, and includes both urban and rural schools. In these three schools, the teaching of mathematics seems to be traditional—i.e., a teacher explains mathematical concepts with corresponding examples and gives exercises (mainly bare mathematics tasks), while students pay attention, take notes and do the exercises. This teaching approach is quite common in Indonesia (see, for instance, Johar, 2010; Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002). Based on formative assessments as well as on a summative test at the end of the previous semester, the mathematics teachers in each of the schools selected the students to be included in the study, including high, medium and low achievers in a balanced manner.

2.3.2. Data collection

In each school, data were collected by means of a written task and follow-up interviews. First, students were asked to solve a set of algebra tasks with paper and pencil individually for thirty to forty minutes, and they were informed that their solutions would not be graded so that they would feel free to use their own solution methods. We planned to give 3-5 additional minutes in case students did not finish their work within the given time. However, all students seemed to have ample time to solve all the tasks. During the written test, students were not allowed to use calculators as they had not been allowed to use them during the learning process, in formative and in summative tests. The goal of this written test was to identify student difficulties with the algebra tasks.

Students had a break after the written test. This time was used to select students' written work based on a preliminary selection made through observation during the written test. In this way the interviewer selected students for the additional individual interviews, which had as a goal to gather more detailed data on the occurring types of difficulties. Out of the 33 participating students who did the written test from the 2011 sample (ten, thirteen and ten students from the first, second and third school, respectively), nineteen were interviewed afterwards. The number of interviewed students from the first, second, and third schools was six, eight and five, respectively. In the 2012 sample, all eighteen students who did the written test (eight, five and five students from the first, second and third schools, respectively) were interviewed afterwards. The interviews were conducted and videotaped on the same day as the written test and took about 15-20 minutes each. The written test and interviews in the three schools were carried out on different days, i.e., during one day in each of the schools.

During the semi-structured interviews, the students' written solutions of tasks were presented and they were encouraged to explain their reasoning. The interviewer did not intervene to get correct or incorrect solutions. As a guideline for carrying out the interviews, general starting questions and follow-up questions had been prepared to both focus on investigating students' difficulties and to allow flexibility during the interviews. The general interview questions included: Do you understand this problem? How did you solve this problem? Could you explain your solution? And how do you check whether your solution is correct or not? The follow-up questions included, for instance: Why did you make this step? What did you mean by this step? What is the next step? What does it mean? The former type of questions was used at the beginning of an interview, and the latter was used while the interview was taking place and depended on students' responses.

If a student did not solve one of the tasks, the interviewer asked whether he or she understood the task and then asked further questions depending on his or her reactions. For example, if a student did not solve a task, such as solve $4x + 7 < 15$, the interviewer would ask him or her to read the task aloud to identify whether the student understood if the task was about inequality or not. Next, the interviewer asked questions to the student based on his or her mistakes or misunderstandings. Although in the 2011 sample there were two students who had left one item blank, the interviews revealed that this was caused by their inability to solve the task and not by a lack of time.

2.3.3. Tasks

The tasks used in this study, all algebra problems on linear equations and inequalities in one variable topic, were taken from three sources. The first source consisted of Indonesian mathematics textbooks for students in grade VII, both governmental and private publishers' textbooks. The second source consisted of the TIMSS 2003 released items (IEA, 2003) on algebra, particularly the linear equations and inequalities in one variable topic. The third source consisted of the set of PISA 2006 released items on algebra which in PISA is labeled as Change and Relationship (OECD, 2006). An overview of the sixteen tasks can be found in the Appendix 2.1.

For the 2011 sample, the sixteen tasks were divided evenly into four sets, consisting of tasks 1-4, 5-8, 9-12 and 13-16, respectively. The sets contained two bare and two contextual algebra tasks and were considered to be equivalent in terms of the degree of difficulty. Bare algebra tasks are tasks that are not related to contexts either within mathematics or other subjects (tasks 3, 4, 7, 8, 10, 11, 15 and 16), whereas contextual algebra tasks are (tasks 1, 2, 5, 6, 9, 12, 13 and 14). Criteria for the tasks were that they should concern linear equations and inequalities; and because of the mathematization issue, the set should cover bare and contextual algebra tasks in a balanced manner. In the written test and interviews, the four sets were spread randomly so that each student was randomly assigned one of the four task sets.

For the 2012 sample, the same set of five tasks—namely tasks 5, 7, 11, 13 and 16—was used for all students. We decided to use the tasks on the topic of linear equations in one variable only as, based on the experience with the 2011 sample, this topic is representative as a means to reveal the five categories of difficulties in initial algebra. Because of the mathematization issue, the set covered bare algebra tasks (tasks 7, 11 and 16) and contextual algebra tasks (tasks 5 and 13) in a balanced manner. In order to serve students' differences, the set of tasks also included relatively easy and more difficult tasks (tasks 5, 7 and 11 being relatively easy, and the rest being more difficult).

2.3.4. Possible difficulties within the tasks

Before administering the written test and the interviews, we listed possible difficulties that students might encounter while solving the tasks. The bare algebra tasks might challenge students to deal with difficulties in the ARITH, VAR, AE, or EQS categories, whereas the contextual algebra tasks might challenge students to handle difficulties in all five categories. Table 2.2

summarizes the tasks and the categories of difficulties that students might encounter in the written test and in the interviews.

Table 2.2. Tasks and possible categories of difficulties

Category of difficulties	Tasks															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
ARITH	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
VAR	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
AE	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
EQS			✓				✓		✓	✓	✓				✓	✓
MATH	✓	✓			✓	✓			✓			✓	✓	✓		

Table 2.2 shows that the difficulties in the ARITH category are challenged in all algebra tasks as they all require students to carry out calculational operations. Similarly, except for task 2 which only requires arithmetical operations, the VAR and the AE categories might emerge in all tasks as the variables and algebraic expressions play a crucial role. All together, the first three categories might frequently co-emerge. The EQS category of difficulty possibly emerges in tasks that explicitly use the equal sign, namely the tasks 3, 7, 9, 10, 11, 15 and 16. The MATH category of difficulty might show up in contextual algebra tasks which require students to formulate mathematical forms, or in bare tasks that require students to 'build' mathematics. Examples 1 and 2 illustrate the tick marks shown in Table 2.2.

Example 1

A bare algebra task used in the written test is to solve the following equation:

Solve for x : $3(x - 5) = 2x - 7$

This equation, Task 3 in the set, is categorized as a bare algebra task as it is not related to any context. This task might invite difficulties in ARITH, VAR, AE and EQS categories. Concerning the ARITH category, the task requires the student to apply the distributive property, namely when expanding $3(x - 5)$ into $3x - 15$; to apply an additive inverse property, e.g., when simplifying $3x - 15 = 2x - 7$ into $3x = 2x - 7 + 15$; and to subtract algebraic expressions, e.g., from $3x - 2x = -7 + 15$ to $x = -7 + 15$. If a student was unable to apply these operations, she or he would encounter the ARITH category of difficulty. Concerning the VAR category, the task requires the student to understand the variable x as an unknown to be found. Concerning the AE category, the task might invite the student to simplify, for instance, $3x - 15$ into $-12x$, which suggests a misunderstanding of the meaning of the algebraic expression $3x - 15$ and which is categorized as a lack of closure obstacle (Tall & Thomas, 1991); the task requires the student to visually recognize the brackets as an invitation to expand $3(x - 5)$ before further calculations. Finally, the task might provoke the student to simplify $3x - 15 = 2x - 7$ into $3x = 2x - 7 + 15 = 3x - 2x = 8$, i.e., misunderstanding the meaning of the equal sign as an algebraic equivalence which falls into the EQS category.

Example 2

A contextual algebra task used in the written test is to solve the following:

> The sum of three consecutive positive integers is not greater than 63. Find boundaries for each of the numbers.

This task, Task 14 in the set, is categorized as a contextual algebra task as it is related to a mathematical context, namely the challenge to find boundaries for each of the numbers that satisfies the statement in the task. Possible difficulties in this task include the MATH, VAR, ARITH, and AE categories. Concerning the MATH category, the task requires the student to translate the word problem into a mathematical form, such as $x + (x + 1) + (x + 2) \leq 63$, in which x, $(x + 1)$ and $(x + 2)$ represent three unknown consecutive integers. The student might encounter difficulties in translating the word problem into the mathematical form, namely the inequality. This difficulty includes the difficulty to use variables, as varying quantities, to represent three consecutive integers which fall into the VAR category. With regard to the ARITH category, if a student is able to get $x + (x + 1) + (x + 2) \leq 63$, then the inequality requires to be rewritten into $3x + 3 \leq 63$ which is the application of the associative property of addition. Finally, the task might invite the student to simplify $x + (x + 1) + (x + 2)$ into $6x$, i.e., experiencing a lack of closure obstacle (Tall & Thomas, 1991), which is included as the AE category.

2.3.5. Analysis of the student data

The data of the study include student worksheets of the written test, interview video registrations (audio-video data of interviews), and interview field notes. A unit of analysis or case in this study is a student's written work on one single test item or a video clip which covers the interview on one single task. A case may reveal more than one category of difficulty and a particular difficulty may include more than one sub-category. Therefore, the total number of difficulties may exceed the number of cases.

For the 2011 sample, the data were analyzed in three steps with the help of software for qualitative data analysis—in our case Atlas.ti. First, the data were organized and clipped into cases which serve as units of analysis. Next, the categorization described in sub section 2.2.2 (see Table 2.1) was used as an initial lens, and was elaborated through its use in the preliminary analysis. In this way, an analytical framework was developed in a bottom-up way. Finally, this framework was applied once more through coding the dataset.

For the data from the 2012 sample, the framework developed in the 2011 analysis was applied through coding the dataset. To establish inter-observer reliability, a second coder who was a mathematics educator not involved in this study analyzed 20% of the cases of both the 2011 and the 2012 sample after being given an explanation about the framework and the code book manual for data analysis. With a Cohen's Kappa of 0.77, the agreement between the first author and the second coder at the level of sub-categorization was found to be substantial (Landis & Koch, 1977).

2.3.6. Framework for analyzing student responses

In this section, we describe the analytical framework which elaborates each of the five categories identified in sub section 2.2.2. The corresponding examples within Tables 2.3-2.7 illustrate the categorizations.

2.3.6.1. Difficulties in the ARITH category

Table 2.3 shows the framework for the ARITH category. Students' capabilities in carrying out arithmetic operations (related operations) were grouped into two sub-categories: mistakes in carrying out operations on numbers, and mistakes in carrying out operations on algebraic expressions. Concerning the priority rules of arithmetic operations (related rules), students made mistakes in applying these either in numerical or in algebraic expressions. Concerning students' mastery in applying properties of numerical operations (related properties) we categorized students' mistakes into three sub-categories: (i) misapplication of the commutative property in calculating a division in numerical expressions; (ii) misuse of the distributive property of a multiplication over an addition; and (iii) an improper use of the additive inverse property in solving an equation.

Table 2.3. The ARITH category

Difficulties in the ARITH category	Sub-category	Examples
1. Related operations	A student makes mistakes when carrying out addition, subtraction, multiplication, or division of numbers	• $70/2 = 140$ • $n = 140/70 = 20$
	A student makes mistakes when carrying out addition, subtraction, multiplication, or division of algebraic expressions (difficulties in combining like terms)	• $2x + x = 2x^2$ • $2x + 3x = 5x^2$
2. Related rules	A student does not follow the rules of order of arithmetical operations in numerical or in algebraic expressions	• $17 - 3 + 5 = 17 - 8$ • $6x + 2x - 8 + 2 = 8x - 10$
3. Related properties	A student misapplies a commutative property in calculating a division in numerical expressions	• $p = 70 / 140 \Longrightarrow p = 2$ • $70/p = 140 \Longrightarrow p = 2$
	A student misuses a distributive property of a multiplication over an addition in algebraic expressions	• $2(2x + 5) = 4x + 5$
	A student does not use an additive or multiplicative inverse in solving an equation	• $5x + 2 = 10 - 3x \Longrightarrow 5x - 3x = 10 + 2$ • $x - 9 = 13 \Longrightarrow x = 13 - 9 = 4$

2.3.6.2. Difficulties in the VAR category

Table 2.4 shows the framework for the VAR category. Concerning understanding the meaning of variables, students made mistakes in interpreting a literal symbol as having one single value rather than more than one in an inequality; and in substituting a particular number in an equation.

Table 2.4. The VAR category

Difficulties in the VAR category	Sub-category	Examples
4. Roles of literal symbols	A student interprets that a literal symbol has only a single value rather than more than one value (variable as a varying quantity).	Solution for $4x + 7 < 15$ is $x = 7 + 4 = 11 + 4 = 15$
	A student substitutes a literal symbol in an equation with a particular value and the result is incorrect (variable as an unknown)	A substitution value in the task $3x + 5 = 17 - x$. In this task, each of the terms $3x$ and x is replaced by 12

2.3.6.3. Difficulties in the AE category

Table 2.5 shows the framework for the AE category. Concerning obstacles related to algebraic expressions, students encountered the parsing obstacle, the expected answer obstacle, and the lack of closure obstacle.

Concerning the gestalt view on algebraic expressions, students ignored local salience in an algebraic expression, such as neglecting the inequality sign while solving $5x + 2 \geq 10 - 3x$; and ignored pattern salience aspects, such as neglecting an algebraic expression of two terms within a bracket while expanding $2(2x + 5)$ into $4x + 5$.

Table 2.5. The AE category

Difficulties in the AE category	Sub-category	Examples
5. Parsing obstacle	A student experiences a conflict between the order in natural language and in algebraic language	• $17 - 8x = 9x$ • $x + 9 = 9x$
6. Expected answer obstacle	A student expects to have a numerical answer for an algebraic expression	• From $4x + 7 < 15$ to $x = 7 + 4 = 11 + 4 = 15$ • $15 + 2x = 17$
7. Lack of closure obstacle	A student adds or subtracts algebraic terms and numbers to get an algebraic term within an algebraic expression	• $2x + 3 = 5x$ • $2x - 1 = x$
8. Lack of gestalt view	A student ignores local salience in an algebraic expression, such as the inequality sign <, the variable x, a positive or negative sign of an algebraic term	• From $4x + 7 < 15$ to $x = 7 + 4 = 11 + 4 = 15$ • $4x + 7 = 11x$ • $5x + 2 = 10 - 3x \Longrightarrow 5x - 3x = 10 + 2$
	A student ignores pattern salience in an algebraic expression, such as an algebraic expression with two terms and within a bracket	• $x + 5 = 5x$ • $2(2x + 5) = 4x + 5$

2.3.6.4. Difficulties in the EQS category

Table 2.6 shows the framework for the EQS category. This category includes two different mistakes where students do not understand the meaning of the equal sign as an algebraic equivalence. First, students made a notational error as a result of a combination of operations—which is called a "running" statements error (Jones & Pratt, 2012; Saenz-Ludlow & Walgamuth, 1998). Second, students ignored the equal sign and applied an incorrect simplification on algebraic expressions. For example, they changed $3(x - 5) = 2x - 7$ into $3(3x + 12) = 3(15x)$, i.e., added x and $2x$ to be $3x$, and added 5 and 7 to be 12 (ignoring the negative signs of –5 and –7), and next changed $3x + 12$ into $15x$.

Table 2.6. The EQS category

Difficulties in the EQS category	Sub-category	Examples
9. The different meanings of the equal sign	A student does not understand the meaning of the equal sign as algebraic equivalence, such as the student makes a notational error as a result of a combination of operations	• $30{,}000 - 4{,}000 = \frac{26{,}000}{2} = 13{,}000$ • $x = 7 + 4 = 11 + 4 = 15$
	A student does not understand the meaning of the equal sign as algebraic equivalence, such as the student ignores the equal sign and applies an incorrect simplification on algebraic expressions	• $3(x - 5) = 2x - 7 \Longrightarrow 3(3x + 12) = 3(15x)$

2.3.6.5. Difficulties in the MATH category

Table 2.7, finally, shows the framework for the MATH category. Concerning horizontal mathematization difficulties, students encountered difficulties in translating phrases into mathematical notations, in reformulating the given (word) task into a mathematical form, in interpreting mathematical concepts and patterns, in substituting information into a mathematical formula and in using a given formula.

Concerning vertical mathematization difficulties, students encountered difficulties in combining as well as integrating information either given in the task or given as a result of calculations while solving symbolic algebra tasks; in using the algebraic method; in manipulating symbols when solving symbolic algebra tasks; and in applying equation solving methods.

Table 2.7. The MATH category

Difficulties in the MATH category	**Sub-category**	**Examples**
10. Horizontal mathematization	(i) A student mistranslates words or phrases into mathematical notations	• "is not greater than" is mistranslated into $\geq$
	(ii) A student fails to formulate an equation or an inequality from the given (word) problem	• reformulating an equation • reformulating an inequality
	(iii) A student encounters a difficulty in interpreting a mathematical concept and a pattern, in substituting information into a formula, and in using a formula.	• a cube is misinterpreted as a square • recognizing a pattern • misusing a formula
11. Vertical mathematization	(i) A student encounters difficulties in combining, in integrating, or in using information either given in the task or given as a result of calculation in solving symbolic algebra problems	• $x - y = 5 \Longrightarrow 5 = x + y$ and $\frac{x}{2} = 3 \Longrightarrow x = 2 \times 3 = 6$
	(ii) A student uses an arithmetical method rather than an algebraic method to solve symbolic algebra problems	• $4(x + 5) = 80 \Longrightarrow 80 : 4 - 5 = 20 - 5 = 15$
	(iii) A student encounters a difficulty in manipulating symbols when solving symbolic algebra problems	• $4(x + 5) = 80 \Longrightarrow 4x + 20 = 80$
	(iv) A student misapplies equation solving when simplifying algebraic expressions	• $(3x - 4) + (3x - 4) + (x + 1) + (x + 1)$ $= 3x - 4 + 3x - 4 + x + 1 + x + 1$ $= 8x - 8 + 2$ $= 8x - 8 + 2 - 2$ $= 8x - 8$ $= 8x - 8 + 8$ $= 8x$

2.4. Results

Table 2.8 summarizes the number of tasks done by students in the written test and the percentages of correct responses. Table 2.9 provides the same data for the interviews, and the two tables show little difference. All students solved tasks 1 and 15 correctly, maybe because these tasks can be solved by using arithmetical calculations only. In contrast, no student was able to solve tasks 4, 9, 12 and 14. This seems to be caused by two reasons. First, the tasks 4, 12 and 14 concern (relatively complex) linear inequalities and these last two tasks require students to reformulate inequalities from contexts. Second, students involved in this study were 13-14 years old, whereas task 9 (from the PISA 2006 study) was intended for 15-16 year-old students.

Table 2.8. Written test: Number of tasks done by students and percentage solved correctly

Tasks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
N	5	5	5	5	24	6	24	6	12	12	30	12	28	10	10	28
NC	5	3	1	0	4	4	11	2	0	9	23	0	7	0	10	12
%C	100	60	20	0	17	67	46	33	0	75	77	0	25	0	100	43

N: Number of students who have done a task; NC: Number of students who solved a task correctly; %C: percentage correct

Table 2.9. Interviews: Number of tasks addressed and solved correctly

Tasks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
N	5	5	5	5	23	5	23	5	6	6	24	6	21	3	3	21
NC	5	3	1	0	4	3	10	1	0	4	18	0	7	0	3	10
%	100	60	20	0	17	60	44	20	0	67	75	0	33	0	100	48

N: Number of students who have done a task; NC: Number of students who solved a task correctly; %C: percentage correct

To see difficulties encountered by students within each of the five categories, Table 2.10 summarizes the result of the data analysis from both the written test and the interviews in terms of the framework. In total, the two samples provide 222 cases (33 students × 4 tasks + 18 students × 5 tasks, see column 3) from the written test, in which 166 cases (19 students × 4 tasks + 18 students × 5 tasks) provide extra information from the interviews. As one case may have more than one category of difficulty, and a particular difficulty may include more than one sub-category, the number of codes may exceed the number of cases. An elaboration of Table 2.10 at the task level can be found in Appendix 2.2.

Table 2.10. Observed difficulties in algebra: Category frequencies and percentages

Category	Sub-category	Difficulties in written test by all students (total 33 × 4 + 18 × 5 = 222 cases)	Difficulties in written test by interviewed students (total 19 × 4 +18 × 5 = 166 cases)	Difficulties by interviewed students in written test and during interviews (total 19 × 4 + 18 × 5 = 166 cases)
ARITH				
1	(i)	6	4	4
	(ii)	4	4	7
2	(i)	4	1	1
3	(i)	3	1	1
	(ii)	6	5	10
	(iii)	13	13	15
	Total	**36 (16%)**	**28 (17%)**	**38 (23%)**
VAR				
4	(i)	2	2	3
	(ii)	3	2	17
	Total	**5 (2%)**	**4 (2%)**	**20 (12%)**
AE				
5	(i)	6	4	11
6	(i)	4	4	8
7	(i)	5	1	5
8	(i)	11	6	11
	(ii)	6	4	17
	Total	**32 (14%)**	**19 (11%)**	**52 (31%)**
EQS				
9	(i)	16	16	17
	(ii)	0	0	3
	Total	**16 (7%)**	**16 (10%)**	**20 (12%)**
MATH				
10	(i)	4	2	8
	(ii)	55	40	42
	(iii)	6	2	4
11	(i)	0	0	1
	(ii)	1	1	2
	(iii)	3	3	5
	(iv)	1	0	0
	Total	**70 (32%)**	**48 (29%)**	**62 (37%)**

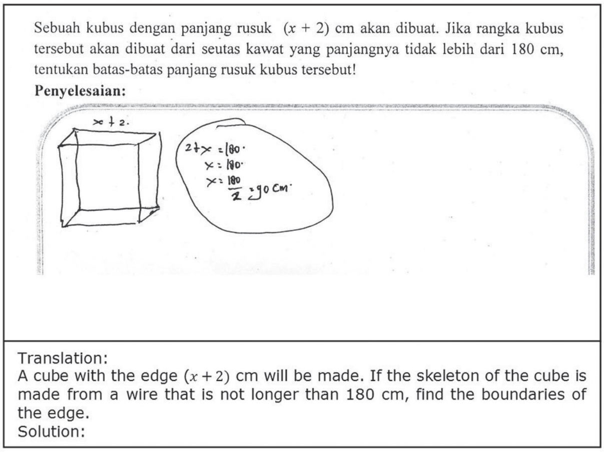
2.4.1. Findings from the written test

We now discuss the findings presented in Table 2.10 in more detail. The column 'Difficulties in written tests by all students' shows the observed difficulty in the cases from the written test data. It reveals that the mathematization category (MATH) caused the most frequent difficulties, namely in 70 (32%) out of 222 cases. Sub- category 10(ii) on reformulating equations or inequalities was the most frequent one, followed by the interpretation of mathematical concepts, patterns and formulas (10(iii)); mistranslations of phrases into mathematical notations (10(i)); difficulties in manipulating symbols (11(iii)), applying algebraic methods to solve symbolic algebra tasks (11(ii)); and a misapplication of equation solving (11(iv)). Similar results occurred in the written test data of the interviewed students (see the fourth column), namely 48 (29%) out of 166 cases. In short, written test data show that mathematization difficulties were frequent and that the problems concerned horizontal mathematization in particular.

The second category in frequency concerned difficulties in applying arithmetic operations in both numerical and algebraic expressions (the ARITH category), with 36 (16%) out of 222 cases. The observations were divided over sub-categories: using an additive inverse property in solving an equation (3(iii)); using a distributive property of a multiplication over an addition in algebraic expressions (3(ii)); carrying out arithmetical operations in numerical expressions (1(i)); carrying out arithmetical operations in algebraic expressions (1(ii)); following priority rules of arithmetical operations (2(i)); and misapplying the commutative property in numerical expressions (3(i)). This result was similar to the written test data of the interviewed students.

The third category in frequency concerned difficulties in algebraic expressions (the AE category), with 32 (14%) out of 222 cases. In this category there was a small difference with the written test data of the interviewed students, with nineteen (11%) out of 166 cases. The observations were divided over sub-categories as follows: the local salience of algebraic expressions (8(i)); the pattern salience of algebraic expressions (8(ii)); the parsing obstacle (5(i)); the lack of closure obstacle (7(i)); and the expected answer obstacle (6(i)). Apparently, algebraic expressions are difficult for the students: all sub-categories are involved and no sub-category was dominant. The categories of difficulties on the equal sign (EQS) and the notion of variable (VAR) were observed less frequently.

To illustrate these results, we present two examples of a student's written work and the way in which we applied the framework. Figure 2.3 shows the first example on task 12. The student exhibited two categories of difficulties, coded as MATH and ARITH categories. For the former, the student could not formulate the word problem into the inequality $12(x + 2) \leq 180$; instead he reformulated it as $(2 + x) = 180$. This is categorized as MATH, and as a problem of setting up an inequality from the given word problem in particular (sub-category 10(ii)). This difficulty seems to be caused by the inability to use the number of the cube's edges, to translate the word phrase "is not longer than" and to integrate this into the inequality $12(x + 2) \leq 180$. In addition, the student seemed to overlook the task of finding the boundaries of the edge. This indicates that the student lacked understanding of the problem as a whole as the first problem solving heuristic (Polya, 1973). For the ARITH category, if $(2 + x) = 180$ were a correct reformulation of the given task, the next step should have been $x = 180 – 2 = 178$ instead of $x = \frac{180}{2} = 90$. This mistake is categorized in the ARITH category, in the sub-category of an incorrect use of the additive inverse property in solving an equation (3(iii)). As indicated in Table 2.2, beforehand we noticed that task 12 might invite difficulties in the ARITH, VAR, AE and MATH categories. In this observation, however, only the ARITH and the MATH categories of difficulties were identified.


Sebuah kubus dengan panjang rusuk $(x + 2)$ cm akan dibuat. Jika rangka kubus tersebut akan dibuat dari seutas kawat yang panjangnya tidak lebih dari 180 cm, tentukan batas-batas panjang rusuk kubus tersebut!

Penyelesaian:

Translation:
A cube with the edge $(x + 2)$ cm will be made. If the skeleton of the cube is made from a wire that is not longer than 180 cm, find the boundaries of the edge.
Solution:

Figure 2.3. A student's written work on task 12 with MATH and ARITH difficulties

There are two cases in this student's work that seem to be included within the VAR and AE categories. Although rewriting $x + 2$ as $2 + x$ is correct, it may suggest that the student was uncomfortable with operating the variable x first and 2 afterwards, probably because he is more familiar with 2 than with x. This suggests a VAR issue. Second, the student's rewriting $2 + x = 180$ as $x = 180$ (which is strange, as the number 2 is missed in this step), and to $x = \frac{180}{2} = 90$ might suggest that he interpreted $(2 + x)$ as $2x$ (the parsing obstacle from the AE category). However, these two interpretations were not explicitly written in the student work, and therefore, the VAR and AE codes were not assigned. This shows that the analysis of written work is limited, which is why additional interview data were needed in the analysis.

The second example concerns task 11 shown in Figure 2.4. The ARITH category of difficulty that can be identified from the student's written work concerns the step from $x - 9 = 13$ to $13 = x + 9$. In this step, the student seems to improperly use an inverse property of addition (sub-category 3(iii)), i.e., by changing –9 into +9 as if this number moves to the other side of the equation. If this is the case, he should also change the signs of x and 13; yet he did not do so. This might be a notational error but we have no information on that. Therefore, we coded this as the ARITH category and in the sub-category 3(iii). The AE category of difficulty that can be identified from Figure 2.4 includes $x + 9 = 9x$ and $13 - 9x = 4x$. Concerning the former, the student might read $x + 9$ as x and 9, and might interpret them as $9x$ and vice versa. With regard to the latter, the student might read $13 - 9x$ as $13 - 9$ giving 4, and might interpret the full expression as $4x$. Both mistakes are considered parsing obstacles as the student encountered a conflict between the order in natural language and the order in algebraic language (sub-category 5(i)).

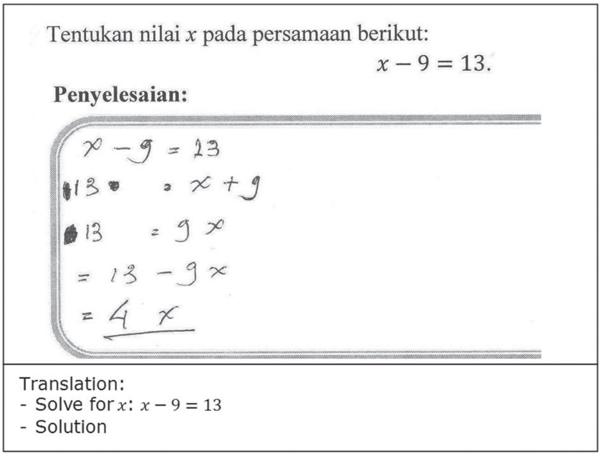


Figure 2.4. A student's written work on task 11 with ARITH and AE difficulties

Beforehand, we thought that this task might invite difficulties in the ARITH, VAR, AE and EQS categories (see Table 2.2). In the written work, however, only the ARITH and AE categories were observed. Even if one argued that the student encountered an EQS category of difficulty, namely from the step $13 = 9x$ to "$= 13 - 9x$", we perceive this as an application of an additive inverse property which falls into the ARITH category. In addition, the difficulty in the AE category might also be caused by the difficulty in the VAR category (which is inherent within the understanding of algebraic expressions). Overall, it may be due to these interpretations that the frequencies of the EQS and VAR categories are so low.

2.4.2. Findings from the interviews

Table 2.10's columns on 'Difficulties in written test' for all students and for the students who were interviewed show little differences. The final column on 'Difficulties in interviews', however, confirms the findings from the written test: mathematization, algebraic expression and the arithmetic operation categories are the most frequently observed, but their relative frequencies have increased. This suggests that the interviews do offer additional information.

Concerning the MATH category, again, the sub-category of reformulating equations or inequalities (10(ii)) was the most frequent one. As this sub-category is in line with the first two problem solving heuristics proposed by Polya (1973), namely understanding the problem and devising a plan, it seems that solving contextual (word) algebra tasks is a serious problem for Indonesian students. Although the Indonesian mathematics curriculum suggests emphasizing problem solving activities and solving contextual mathematics tasks in particular (Depdiknas, 2006), we observed in both the governmental and private publishers textbooks (e.g., Adinawan & Sugijono, 2007; Budhi, 2007; Nuharni & Wahyuni, 2008; Wagiyo et al., 2008) that contextual tasks appear only at the beginning of chapters as motivational tasks and at the end of chapters as applications. Thus, this fact might contribute to this result.

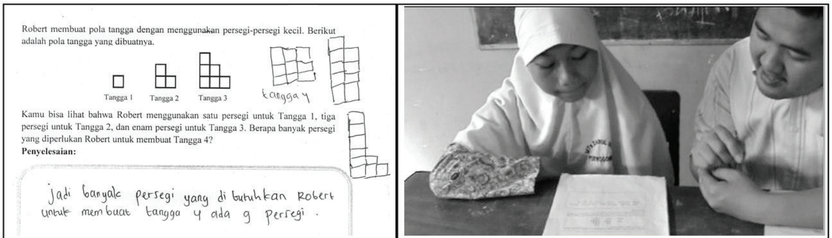
The difficulty in the AE category might be caused by when the interviews took place. Based on the curriculum, the linear equations and inequalities in one variable topic was taught in the first semester of grade VII (Depdiknas, 2006). This study, however, was conducted during the second semester which focused on geometry topics. We conjecture, therefore, that the students had forgotten the linear equations and inequalities in one variable concepts and as such made mistakes, particularly in this category. This suggests that the students' learning processes may lack the conceptual understanding that might have prevented them from forgetting. The sub-category of the gestalt view on pattern salience was the most frequent, followed subsequently by the gestalt view on local salience, the parsing obstacle, expected answer obstacle, and the lack of closure obstacle. From these results, it seems that students had only learned the algebra algorithmically without a conceptual understanding of algebraic expressions. For instance, we observed that students made mistakes simplifying $17 - 8x$ into $9x$ (parsing obstacle) and simplifying $2x + 3$ into $5x$ (lack of closure obstacle).

With regard to difficulties in the ARITH category, these might be caused by the fact that the arithmetic operations now should be carried out in the context of algebra which relates to the understanding of variables and algebraic expressions. For example, in simplifying $2x + x$ into $2x^2$, the mistake probably occurred because students might not understand the meaning of the variable in the term of $2x$ and might not understand the algebraic expression $2x + x$ which requires to add rather than to multiply variables.

Concerning the EQS category, the low frequency of its occurrence was surprising. Beforehand, we expected that this category would frequently occur for tasks that explicitly use the equal sign. However, we observed that this was not always the case. In fact, the EQS difficulty also appeared in the tasks that require students to reformulate equations as a consequence of combining operations. An explanation for the low frequencies could be that this difficulty is fairly subtle, relates to other difficulties, and does not occur at the concrete-observational level. Moreover, this could also be caused by the number of tasks that have the scope for the EQS category is less frequent as compared to other tasks (see Table 2.2).

Finally, the VAR category was expected to co-occur with the AE category. However, the results were different. The low frequency of this category might be because this type of difficulty could be inherent within the AE and the ARITH categories.

To illustrate these results, we present two examples from student interviews and the way in which we applied the framework. Figure 2.5 shows the task and the interview setting of the first example.



Translation:
Robert builds a step pattern using squares. Here are the stages he follows.
Stage 1, stage 2, stage 3.
As you can see, he uses one square for Stage 1, three squares for Stage 2 and six for Stage 3. How many squares should he use for the fourth stage?
Solution:
Thus, the number of squares needed by Robert for the fourth Stage is 9 squares.

Figure 2.5. A student interview on Task 2 revealing MATH difficulty

By observing this student‘s written work, it is difficult to determine the difficulties that the student encountered. The following interview transcript sheds new light on the student’s thinking.

I: What does this problem mean?

S: [Reads the problem and her written work solution, as well as pointing the Stages 1-3 and the corresponding number of squares].

I: How did you determine the number of squares for Stage 4?

S: [Again, she reads her written work solution that the number of squares for the fourth Stage is nine]

I: Why? [Why is the number of squares for the fourth Stage nine?]

S: Because $6 + 3 = 9$ [Six means the number of squares in Stage 3, and three means the number of squares in Stage 2]. Because the difference of the number of these [the number of squares in Stage 2 and 3] is 3 [She did not consider the difference of the number of squares between Stages 1 and 2]

I: Are you sure the difference [of the number of squares between Stages] is three?

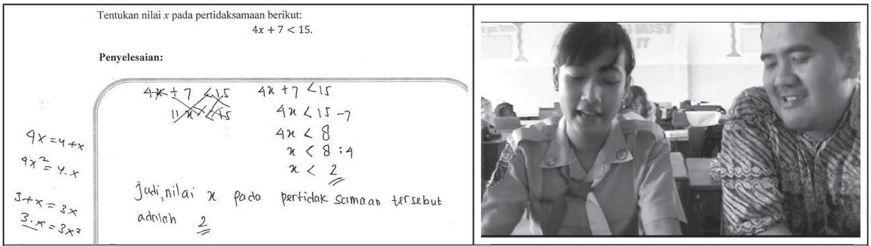
S: Yes!

I: Show me that the difference is three.

[After showing the difference between the number of squares of Stages 3 and 2, she draws the figure of Stage 4. She draws three different figures, and the number of squares for Stage 4 is nine].

The interview transcript reveals that the student encountered difficulties in the MATH category, as she could not identify the pattern of the differences of the number of squares between the stages (sub-category 10(iii)).

The second example concerns task 4. Figure 2.6 shows one student's written work. Although the student drew an inappropriate conclusion at the end of her solution, namely "So, the value of x that satisfies the inequality is 2", the written solution is true. Noticing the crossed out part, namely from $4x + 7 < 15$ to $11x < 15$, it seems that the student encountered the AE category of difficulty and the lack of closure obstacle in particular (sub-category 7(i)), i.e., she added the algebraic term $4x$ and the number 7 to get $11x$. However, as this part was crossed out and the student's solution is true, we did not assign this an AE category.



Translation:
Solve for x : $4x + 7 < 15$.
Solution:
So, the value of x that satisfies the inequality is 2.

Figure 2.6. A student interview on Task 4 with AE and ARITH category of difficulty.

The following interview transcript again provides additional information.

I: Please, let me know how you solved this problem.

S: This is about inequality!

I: It is alright, please let me know what you did!

S: [Reads her written solution $4x + 7 < 15$

$4x < 15 - 7$
$4x < 8$
$x < 8 : 4$
$x < 2$

I: What does $4x$ mean: does it mean $4 + x$ or 4 times x? [As in the crossed out part of the written solution she added $4x$ and 7 to get $11x$, the interviewer asks the meaning of $4x$]

S: $4 + x$.

I: So, $4x$ is equal to $4 + x$ [write $4x = 4 + x$]. Is it what you mean?

S: Yes!

I: What is the meaning of $4x^2$

S: 4 times x.

I: What is the result of $3 + x$?

S: $3x$

I: What is the result of 3 times x [write $3.x$]?

S: $3x^2$

The transcript reveals that the student encountered the AE category of difficulty and the parsing obstacle in particular (sub-category 5(i)), i.e., she understood that $4x$ means $4 + x$ (conflicting between the order of natural and algebraic languages). This difficulty might cause the lack of closure difficulty as indicated in the analysis of the written work only. Furthermore, she calculated, for instance, 4 times x to be equal to $4x^2$, which falls into an ARITH category of difficulty and the inability to carry out multiplication of algebraic expressions (sub-category 1(ii)).

2.5. Discussion and conclusions

2.5.1. Overview of student difficulties in initial algebra

The research question in this study concerns Indonesian students' difficulties in initial algebra learning, and in solving linear equations and inequalities in particular. From this study's data we conclude that the students' most important difficulties in initial algebra concern mathematization (MATH), the understanding of algebraic expressions (AE), applying arithmetic operations in numerical and algebraic expressions (ARITH), understanding the different meanings of the equal sign (EQS), and understanding the notion of variables (VAR), respectively.

Concerning the MATH category, the frequently observed sub-categories include (re)formulating equations or inequalities; interpreting mathematical concepts, patterns and formulas; and mistranslating words, phrases or sentences into mathematical notations, respectively. This difficulty seems to relate to horizontal mathematization and to problem solving skills which are required to solve contextual (word) algebra problems. In particular, these sub-categories are in line with the first two problem solving heuristics described by Polya (1973): understanding the problem and devising a plan. In other words, the Indonesian students in this study seem to lack problem solving skills needed for initial algebra learning. As a possible explanation, this lack might result from textbooks in initial algebra education and mathematics education in elementary school which focus more on computational skills than on problem solving as is the case in the Netherlands (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009). Therefore, we conjecture that the characteristics of mathematization as proposed by the theory of Realistic Mathematics Education and of problem solving are not so common in the learning and teaching processes in Indonesian education.

Concerning the AE category, the frequently observed sub-categories include the pattern salience of algebraic expressions; the local salience of algebraic expressions; the parsing obstacle; the expected answer obstacle; and the lack of closure obstacle. In our view, this relates to students' lack of conceptual understanding in algebra, and in the notions of variables and algebraic expressions in particular (Bokhove, 2011; Drijvers, 2010), which might be caused by an imperfect or incomplete transition from the world of numbers to the world of symbols. Another tentative explanation is that this difficulty is caused by a teaching emphasis on calculation as opposed to understanding the meaning of an expression. In other words, the algebra that is taught might be too algorithmic and not directed enough towards understanding both how and why.

Concerning the ARITH category, the frequently observed sub-categories comprise the improper use of the additive inverse property in solving an equation; improper use of the distributive property of multiplication over an addition; mistakes in carrying out addition, subtraction, multiplication or division of algebraic expressions and numbers; and not following the rules of the order of operations. The mistakes that students made might be due to the fact that the arithmetic operations had to be carried out in the context of algebra which relates to the understanding of variables and algebraic expressions. In addition, this difficulty might also go back to elementary school with students lacking of proficiency in arithmetic calculations, understanding in using priority rules and structure properties such as commutative and distributive laws.

Concerning the EQS category, the frequently observed sub-category concerns mistakes resulting from combining operations This seems to be caused by students' lack of understanding of the meaning of the equal sign as algebraic equivalence (Herscovics & Linchevski, 1994; Ketterlin-Geller, Jungjohann, & Chard, 2007; Kieran, 1981; Linchevski, 1995; Pillay, Wilss, & Boulton-Lewis, 1998).

Finally, the low frequency of the VAR category might be caused by the fact that this type of difficulty is subtle and cannot easily be identified from student work and interviews; also it is inherent to the AE and the ARITH categories.

2.5.2. Limitations

This study has several limitations to discuss. First, this study's data was collected during the second semester of grade seven which focused on geometry topics. This timing might have made retention of algebraic knowledge difficult for the students. Second, we recognize that all components of the five categories of difficulties are related each other. However, we have not analyzed these relations further in the frame of this study. The categorization itself is by no means exhaustive. It may need improvements or additional categorizations if applied to other studies. Third, regarding the analysis of student difficulties, this study addressed the rationale of why students encountered these difficulties only to a limited extent. For example, in analyzing a student mistake of $13 - 9x = 4x$, we have categorized it in the AE category and as a parsing obstacle in particular (Tall & Thomas, 1991)—so that the student might read $13 - 9x$ as $13 - 9$ giving 4, and might interpret the full expression as $4x$. We did not yet, however, address the rationale behind the difficulty. Fourth and final, for the purpose of this study, the framework which we set up has worked quite well. Still, it has only been applied to a small sample of students and to a specific case of initial algebra, and therefore needs further investigation.

2.5.3. Implications

The results of this study give rise to several implications for algebra education in general and for algebra education in Indonesia in particular. First, the finding that mathematization is the most frequently observed category of difficulty suggests a problem associated with contextual algebra. This is in line with the TIMSS 1999 and TIMSS 2007 results in which Indonesian students had very low performances compared to international averages in solving contextual algebra tasks (Mullis et al., 2000; 2008). Do these limited mathematization skills explain Indonesia's overall low performance on studies such as TIMSS 2007 (36st position out of 48, see Mullis et al., 2008) and PISA 2009 (61st position out of 65, see OECD, 2010)? We wonder if this result is general for Indonesian grade VII students.

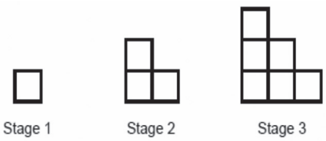
In line with the above, the finding that mathematization and understanding algebraic expressions were the two most frequently observed categories of difficulty may suggest that initial algebra education in Indonesia should pay attention to both contextual algebra and bare algebra in a carefully balanced way. In other words, there should be equilibrium between bare and contextual algebra tasks and approaches. Contextual algebra tasks constitute not only applications of algebraic concepts, but also starting points for concept development towards more advanced algebraic concepts. Both in the bare algebra and the contextual algebra problems, the emphasis should be not only on procedural fluency, but also on understanding why the procedure works. In this way, students are expected to develop an algebraic expertise which includes both procedural skills and conceptual understanding (Bokhove, 2011; Drijvers, 2010).

Third, regarding the questions proposed in the Introduction section, namely why Indonesian students have low algebra scores and why they seem to experience more difficulties in learning algebra than students in other countries, a first tentative conjecture that would need further investigation is the educational factor: how is algebra taught and when is algebra introduced to Indonesian students? In spite of the curriculum revisions in the last decades (Depdiknas, 2006), most mathematics teaching in Indonesia still seems to be traditional (see, for instance, Johar, 2010; Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002). In other words, there is a discrepancy between the intended and the implemented curriculum (Schmidt et al., 1997). Preserving the traditional way of teaching algebra includes, for example, a central role for the memorization of formulas. Another educational aspect might be that Indonesian students immediately start to learn algebra in a formal way, in the first semester of Grade VII (Depdiknas, 2006). Therefore, the students are not prepared to learn algebra through experiences with informal algebra in elementary school. Perhaps, the difficulties students encounter in initial algebra, as we reported in this paper, are a consequence of the directly formal and traditional algebra teaching which is still prevalent in Indonesia.

Acknowledgment
This study was funded by the Indonesia Ministry of Education project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794. We would like to thank Jan van Maanen for his valuable and constructive comments and suggestions. We also thank the teachers and students for their participation, as well as the external assistant and the second coder for their contributions.

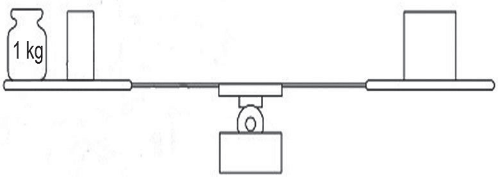
Appendix 2.1.
The following algebra tasks are used in the written test before the interviews.

1. There is a number and if 14 is added to it, then it is equal to 60. Find the number!
2. Robert builds a step pattern using squares. Here are the stages he follows.



As you can see, he uses one square for Stage 1, three squares for Stage 2 and six for Stage 3. How many squares should he use for the fourth stage?

3. Solve for x: $3(x - 5) = 2x - 7$.
4. Solve for x: $4x + 7 < 15$.
5. Amir and Tono together have Rp 30,000. If Amir's amount of money is Rp 4,000 more than Tono's, find each of their amounts.
6. The objects on the scale make it balance exactly. On the left pan there is a 1 kg weight (mass) and half a brick. On the right pan there is one brick. What is the weight (mass) of one brick?



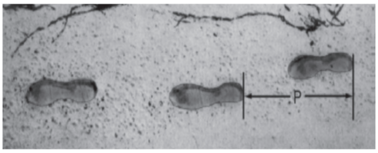
A. 0.5 kg

B. 1 kg

C. 2 kg

D. 3 kg

7. If $4(x + 5) = 80$, then $x = ...$
8. Solve for x: $5x + 2 \geq 10 - 3x$.
9. The picture shows the footprints of a man walking.



The pace length p is the distance between the rear of two consecutive footprints. For men, the formula $\frac{n}{P}$ = 140, gives an approximate relationship between n and p where,

n = number of steps per minute, and
p = pace length in meters.

(i) If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pace length? Show your work.

(ii) Bernard knows his pace length is 0.80 meters. The formula applies to Bernard's walking. Calculate Bernard's walking speed in meters per minute and in kilometers per hour. Show your working out.

10. If $x - y = 5$ and $\frac{x}{2} = 3$, what is the value of y?
    A. 6
    B. 1
    C. –1
    D. –7
11. Solve for x: $x - 9 = 13$.
12. A cube with the edge $(x + 2)$ cm will be made. If the skeleton of the cube is made from a wire that is not longer than 180 cm, find the boundaries of the edge.
13. A rectangle has length and width $(3x - 4)$ cm and $(x + 1)$ cm, respectively: (i) Write a formula for its perimeter; (ii) If the perimeter of the rectangle is 34 cm, find the area of the rectangle.
14. The sum of three consecutive positive integers is not greater than 63. Find boundaries for each of possible numbers.
15. If $L = 4$ when K = 6 and M = 24, which of the following is true?
    A. $L = \frac{M}{K}$
    B. $L = \frac{K}{M}$
    C. $L = KM$
    D. $L = K + M$
    E. $L = M - K$
16. Solve for x: $3x + 5 = 17 - x$.

Note: Tasks 2 and 9 are taken from PISA 2006

(http://www.oecd.org/dataoecd/14/10/38709418.pdf); Tasks 6, 7, 10 and 15 are taken from TIMSS 2003 (http://timss.bc.edu/PDF/T03_RELEASED_M8.pdf); Other tasks are from Indonesian mathematics textbook series.

Appendix 2.2.

Table A.1 Observed difficulties in written test: Frequencies and percentages (all students, total 33 × 4 + 18 × 5= 222 cases)

Cat. / Diff.	Tasks 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total	%
ARITH	0	0	0	0	0	1	2	1	5	0	6	2	12	0	0	7	36	16
VAR	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0	2	5	2
AE	0	0	0	4	0	1	2	1	0	0	4	4	5	0	0	11	32	14
EQS	0	0	0	1	9	0	4	0	0	0	0	0	1	0	0	1	16	7
MATH	0	0	0	0	23	7	2	0	4	0	0	9	14	10	0	1	70	32

Table A.2 Observed difficulties in written test of interviewed students: Frequencies and percentages (total 19 × 4 + 18 × 5 =166 cases)

Cat. / Diff.	Tasks 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total	%
ARITH	0	0	0	0	0	1	2	1	3	0	5	1	10	0	0	5	28	17
VAR	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0	1	4	2
AE	0	0	0	4	0	1	1	1	0	0	4	2	0	0	0	6	19	11
EQS	0	0	0	1	9	0	4	0	0	0	0	0	1	0	0	1	16	10
MATH	0	0	0	0	21	5	2	0	2	0	0	3	11	3	0	1	48	29

Table A.3 Observed difficulties in the interviews: Frequencies and percentages (total 19 × 4 + 18 × 5 = 166 cases)

Cat. / Diff.	Tasks 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total	%
ARITH	0	0	0	2	0	1	8	1	3	0	5	1	11	0	0	6	38	23
VAR	0	0	0	3	0	0	8	0	0	0	3	0	0	0	0	6	20	12
AE	0	0	0	10	3	1	11	1	0	0	4	2	9	0	0	11	52	31
EQS	0	0	3	1	9	0	5	0	0	0	0	0	1	0	0	1	20	12
MATH	0	2	1	0	25	5	5	0	2	1	0	4	13	3	0	1	62	37

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Chapter 3 Student difficulties in solving equations from an operational and a structural perspective

Publication:

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014). Student difficulties in solving equations from an operational and a structural perspective. *Mathematics Education, 9*(1), 39–55.

Abstract In Indonesia, as in many other countries, mathematics teachers, educators and researchers are confronted with student difficulties in initial algebra. To investigate and understand these difficulties, we carried out a pilot study involving 51 Indonesian grade seven students who use a digital mathematics environment for algebra. The notions of operational and structural conceptions offer a framework for explaining student difficulties in solving equations. These include difficulties with arithmetical skills, the use of the equal sign, understanding algebraic expressions, and understanding the concept of variable. The operational and structural perspectives provide guidelines for future task design and research.

Keywords algebra education, digital mathematics environment, equations in one variable, operational and structural views

3.1. Introduction

Over the last decade Indonesian students had very low performances in mathematics and particularly in algebra, as revealed in the Trends in International Mathematics and Science Study (TIMSS). In TIMSS 2007, on the topic of algebra, Indonesian students were in 36th position out of 48 participating countries (Mullis et al., 2008). In TIMSS 2011, similarly, Indonesian students were in 38th position out of 42 countries (Mullis, Martin, Foy, & Arora, 2012). As an initial step to explain these low performances, an explorative study was carried out to investigate student difficulties in initial algebra learning in Indonesia (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). This study revealed five types of difficulties in initial algebra that relate to a lack of both conceptual understanding and algebraic procedural skills, two competencies that are crucial aspects of the algebraic expertise needed by secondary school students (Bokhove & Drijvers, 2010; Drijvers, 2010).

In order to acquire a better understanding of these student difficulties, we have conducted a pilot study in which students work on equations in one variable using two applets, one of which invites an operational view on algebraic expressions and the other a more structural view. In this paper we present the findings of this pilot study, whose main results concern the identification of student difficulties and the understanding of these difficulties from both an operational and a structural view on equations and expressions.

This paper first describes the research aim and theoretical background, including difficulties in initial algebra from the literature; and both operational and structural views on mathematical conceptions. Next, the research question and research method are described. The results section elaborates student difficulties in the light of the operational and structural views. Finally, the conclusion and discussion section reflects on the results which inform future research and task design in particular.

3.2. Research aim and theoretical background

This paper aims to identify student difficulties in initial algebra and in solving equations in one variable in particular which emerge in an Information and Communication Technology (ICT)-rich approach, and to explain the difficulties from operational and structural views on the algebraic activity involved. We argue that this theoretical perspective offers insight into student conceptual difficulties and understanding in the field of initial algebra.

3.2.1. Difficulties in initial algebra learning

The existing research literature in initial algebra education serves as a frame of reference for this study and has led us to identify five types of difficulties in initial algebra: applying arithmetical operations in numerical and algebraic expressions, understanding the notion of variable, understanding algebraic expressions, understanding the different meanings of the equal sign, and mathematization (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). Let us briefly describe each of these types of difficulties.

First, concerning the category of applying arithmetical operations in numerical and algebraic expressions (abbreviated as the ARITH category), research literature shows that students often fail to add or subtract similar algebraic terms (e.g., Herscovics & Linchevski, 1994; Linchevski, 1995). Also, students misapply commutative as well as associative properties when carrying out subtractions or divisions (Booth, 1988; Warren, 2003), and fail to use the distributive property of a multiplication over an addition (Booth, 1988). In our view, these difficulties reveal students' limited mastery of addition, subtraction, multiplication and division; of applying the priority rules of arithmetical operations in calculations; and of using properties of numerical operations. These difficulties seem to originate in the way arithmetic is taught at in primary school, focusing on calculation at local level before the problem as a whole is overseen.

Second, concerning the category of understanding the notion of variable (the VAR category), research reveals that students have difficulties to distinguish a literal symbol as a variable that can play several roles, such as the role of a placeholder, a generalized number, an unknown, or a varying quantity (Booth, 1988; Herscovics & Linchevski, 1994).

Third, the category of understanding algebraic expressions (AE) includes the parsing obstacle (understanding the order in which the algebraic expressions must be processed, which may conflict with the order of natural language), the expected answer obstacle (an incorrect expectation to get a number for an algebraic expression), the lack of closure obstacle (the discomfort in handling algebraic expressions that cannot be simplified any further), and the lack of gestalt view of algebraic expressions (Arcavi, 1994; Tall & Thomas, 1991).

The fourth category concerns understanding the different meanings of the equal sign (EQS). In arithmetic, the equal sign often invites carrying out a calculation and writing down a numerical answer, whereas in algebra, it usually means 'is algebraically equivalent to' (Herscovics & Linchevski, 1994; Kieran, 1981).

The fifth, and final, category of mathematization (MATH) distinguishes horizontal and vertical mathematization. The difficulty in horizontal mathematization concerns going from the world of real phenomena to the world of symbols and vice versa: in other words, to translate back and forth between the world of the problem situation and the world of mathematics (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). The difficulty in vertical mathematization concerns dealing with the process of reorganization within the mathematical system itself, that is, the process of moving within the symbolic world (Treffers, 1987; Van den Heuvel-Panhuizen, 2003).

3.2.2. Operational and structural views

The present study addresses the topic of equations in one variable and linear equations in particular, in which the variable appears only on one side of the equation. To understand the difficulties that students encounter while solving these equations, we wonder whether operational and structural views on algebraic activity might offer an insight. These views originate from Sfard's theory of reification—i.e., the transformation of a process performed on an accepted object to become a new mathematical object. The following two quotations illustrate this duality.

> An analysis of different mathematical definitions and representations brings us to the conclusion that abstract notions, such as number or function, can be conceived in two fundamentally different ways: *structurally*—as objects, and *operationally*—as processes. These two approaches, although ostensibly incompatible, are in fact complementary. (...) the processes of learning and of problem-solving consist in an intricate interplay between operational and structural conceptions of the same notions. (Sfard, 1991, p.1)
>
> To sum up, the history of numbers has been presented here as a long chain of transitions from operational to structural conceptions: again and again, processes performed on already accepted abstract objects have been converted into compact wholes, or reified (from the Latin word *res*—a thing), to become a new kind of self-contained static constructs. Our conjecture is that this model can be generalized to fit many other mathematical ideas. (Sfard, 1991, p.14)

In the case of the present study, equations can be conceived from both operational and structural views. For example, in the light of the operational view, the equation $-3(x + 1) - 22 = 8$ can be seen as a series of calculational process: Add 1 to a certain given number (x), next multiply by -3, and finally subtract by 22 to get 8; in the light of the structural view, this equation can be conceived as equivalence between two objects (algebraic expressions), namely the algebraic expressions $-3(x + 1) - 22$ and 8. The flexibility in switching this process-object view on algebraic expressions signifies a mature understanding of mathematical thinking (Drijvers, 2003).

According to Sfard, "in the process of concept formation, operational conceptions would precede the structural" (Sfard, 1991, p.10). Furthermore, she distinguishes three hierarchical stages of concept formation: interiorization, condensation, and reification, respectively. In the interiorization stage, a student becomes acquainted with a process, for instance an equation as a calculational process. In the condensation stage, the student is more capable to view a process as a whole. These two stages are gradual processes. The third stage of reification, however, is a sudden process, i.e. "a sudden ability to see something familiar in a totally new light." (Sfard, 1991, p.19). A model of concept formation has a hierarchical character, for instance, a process X becomes an object X, next the object X turns into a process Y, and the process Y becomes an object Y, and so on.

Other theoretical lenses that seem to be more specific within the reification theory and the structural view in particular for algebra are symbol sense and structure sense. The term 'symbol sense', even if it is not precisely defined, refers to an ability to have a feeling for and to give meaning to mathematical symbols such as algebraic expressions, formulas, and equations. Two characteristics of symbol sense which fit with the structural view include an ability to read through and to manipulate algebraic expressions to gain a feel for and an understanding of the problem; and an ability to realize the need to check for the meaning of symbols during the implementation of an equation solving procedure or during the inspection of a result (Arcavi, 1994, 2005).

Structure sense, which is a more specific perspective on the structure of algebraic expressions, is a flexible and creative ability to identify all equivalent forms of algebraic expressions (Linchevski & Livneh, 1999). This structure sense idea is elaborated by Hoch and Dreyfus (2009) as well as Novotna and Hoch (2008) for the case of secondary school algebra. Students are said to show structure sense if they can (1) recognize a familiar structure in both its simplest form and in a more complex form, (2) deal with a compound term as a single entity, and (3) choose appropriate manipulations to make best use of a structure. A key feature of structure sense is the substitution principle, i.e., when an algebraic sub-expression is substituted by a dummy variable and vice versa, the structure of the expression as a whole remains the same.

3.3. Research question

The integration of ICT not only seems to be a promising avenue for improving algebra education (e.g., Bokhove, 2010; Bokhove & Drijvers, 2010), it may also offer a vehicle to further study student difficulties in initial algebra and in equations in one variable in particular. We argue that identifying student difficulties and understanding these difficulties from operational and structural perspectives can lead to a better insight on student conceptual understanding and skills. Taking the above into account, we formulate the following research question:

What are student difficulties in solving equations in one variable which emerge in an ICT-rich approach and how can operational and structural views on equations explain these difficulties?

3.4. Methods

This section addresses the design of instruments, the participants, the data collection, and the data analysis.

3.4.1. Design of instruments: Applets and tasks

This study is part of a larger project in which a learning arrangement was designed, consisting of student material, including paper-and-pencil tasks, digital tasks, intermediate formative paper-and-pencil assessment tasks, and a final written test. A teacher guide informs the learning arrangement activities.

The designed learning arrangement includes activities with two applets called Algebra Arrows and Cover-up Strategy, the first one inviting an operational view on algebraic expressions and the second one a more structural view. The first one, Algebra Arrows, is an applet which offers the possibility to construct and use chains of operations on numbers and formulas. Initially, this applet was designed to support the construction of input-output chains of operations as a model of a dependency relationship in the function concept. (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012). In this study, the applet was used to solve equations. Figure 3.1 shows how the equation $\frac{2x-7}{3} + 11 = 40$ can be solved using the Algebra Arrows. Solving an equation through this applet is similar to the informal reverse strategy. As an equation can be interpreted as a calculational process, the reverse strategy is essentially a process of undoing this calculational process to find solutions of the equation. Therefore, in our view, this strategy relies on an operational view on equations. Through working with the Algebra Arrows, students are expected to get a better insight into the equation as a calculational process.

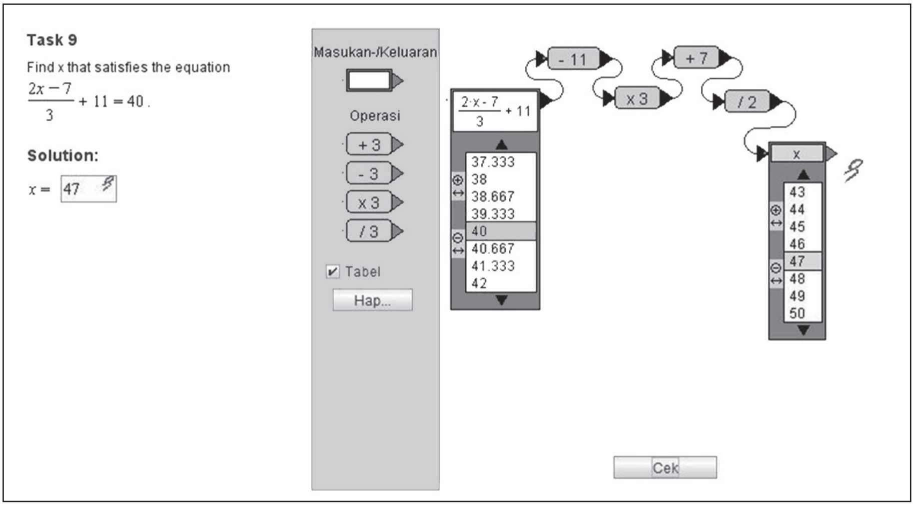

Figure 3.1. Equation solving with the reverse strategy using the Algebra Arrows applet

The second applet, the Cover-up Strategy applet, allows for solving a certain equation type (equations in one variable which appears in one side only) by subsequently selecting a part of the expression in an equation with the mouse and finding its value. For example, Figure 3.2 shows an equation solving scenario with the cover-up strategy to solve the equation $\frac{2012-x}{2} - 1006 = 0$ using this applet. In step 1, a student highlights the expression $\frac{2012-x}{2}$ and the applet provides $\frac{2012-x}{2}$ = ... in the next line. In step 2, the student fills in 1006, and the applet gives a tick mark which signifies that it is correct (otherwise a cross mark will appear). This scenario proceeds until step 6 and ends up at $x = 0$ as the solution of the equation (which is signified by the emergence of the final feedback from the applet, namely "The equation is solved correctly!"). To properly apply the cover-up strategy in solving an equation, students should first perceive the equation as an equivalence of two objects (algebraic expressions). Next, in each step they should be able to identify the part (structure) of the equation to be covered. In this way, this strategy relies on a structural view on equations and expressions. Through working with the Cover-up Strategy applet, students are expected to get a better object view on the equation and the algebraic sub-expressions that are part of it (Boon, 2006).

For both applets, online student activities were designed focusing on solving linear equations in one variable. Preliminary versions of these activities with the two applets were tested in a group of nine Indonesian master students in mathematics education. Based on this, some improvements were incorporated.

The applets and the online tasks can be accessed through the Digital Mathematics Environment (DME), i.e., a web-based electronic learning environment which offers interactive mathematical tools for algebra, graphing geometry, and other domains. The DME allows for the design of open online tasks and appropriate feedback (Boon, 2006; Drijvers, Boon, Doorman, Bokhove, & Tacoma, 2013). Through the DME students can learn mathematics and in particular algebra with conventional notations and techniques, learn any time and any place (as far as technological conditions are met), and save their work.

The activities with the Algebra Arrows applet took part during the first two lessons of the teaching sequence, whereas lessons 3 and 4 included Cover-up Strategy applet activities. Lesson 5 consisted of a final written test, covering the topics of the four lessons. In the first four lessons the digital activity consisted of a demonstration in which the teacher demonstrated how to work with the applets, group work and discussion. When students worked in groups, the teacher controlled the activity, gave help when necessary and discussed important issues, such as frequent mistakes made while solving the digital tasks.

3.4.2. Participants

The observations took place in two schools in Indonesia. One class with 41 grade seven students (12-13 year-old) was chosen from the first school, and ten grade seven students (12-13 year-old) participated from the second school. The students from the second school, including high, medium and low achievers, were selected by their mathematics teacher to participate in this study. The experiment as a whole in each school took five 80-minutes lessons.

3.4.3. Data collection

Data that were collected from each school consisted of video registrations of four teaching sessions, student written work from each assessment and from the final written test, and field notes.

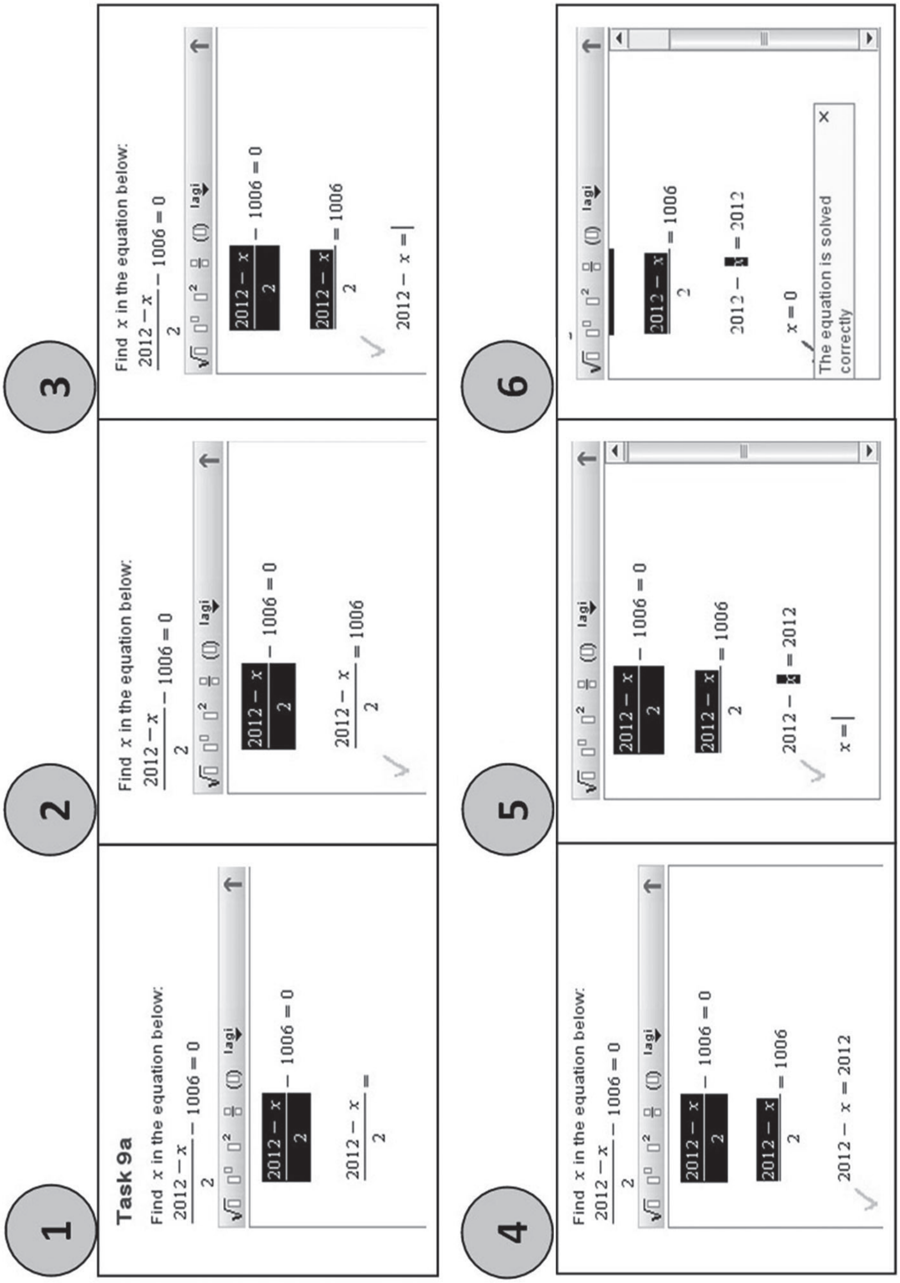

Figure 3.2. An equation solving scenario using the Cover-up Strategy applet

3.4.4. Data analysis

The data analysis was carried out in two steps. In the first step, a preliminary analysis on video registration—with software for qualitative analysis (Atlas.ti in this case)—and on student digital group work as well as on individual written work was carried out. With the difficulties in initial algebra as a framework, this preliminary analysis included: marking and transcribing crucial moments in paper-and-pencil activities and in classroom discussions as well as in student digital group work; examining and assigning difficulties on student written work (including the final written test) for each single task (which serves as a unit of analysis). A unit may reveal more than one category of difficulty. This analysis produced results on student difficulties in solving symbolic equation tasks using the two applets.

The second step of the analysis consisted of an in-depth analysis on student difficulties in solving symbolic equation tasks from the operational and structural perspectives. To confirm the analysis of the written work, transcriptions from observations during the learning activities using the digital technology were used. Thus, the results of the analysis integrate the quantitative data from the intermediate formative assessments and the qualitative analysis of the video data from students' activities in the Digital Mathematics Environment.

3.5. Results

The results include an analysis of the data of the student work with the two applets as well as the final written test. The main results involve individual written student work after the work with the applets, and are illustrated by student group work in the DME-sessions. The findings from the final written test are used to confirm the results of these analyses.

3.5.1. Student difficulties while applying the reverse strategy

The Algebra Arrows activities focused on equation solving with the reverse strategy (RS). A total of fifty students participated in this activity. The results of these students for the four tasks they worked on with paper and pencil at the end of the lesson are summarized in Table 3.1. Columns 1-5 subsequently present: tasks, number of students who solved the tasks correctly (#C), type of equation solving strategy used by students, type of student difficulties revealed in each task, and the operational and structural aspects which might explain student difficulties. Corresponding percentages (relative to the total number of participating students) are provided for columns 2-4.

Table 3.1. Results from data analysis of the Algebra Arrows lesson (N = 50)

Equations to solve	#C (%)	Strategy (%)	Difficulties (%)	Nature of the difficulties
1. $2x - 11 = 29$	40 (80)	RS (100)	ARITH: inverses (8)	Structural
			calculational errors (2)	Operational
			EQS: notational errors (10)	Structural
2. $\frac{x}{12} - 11 = 10$	40 (80)	RS (100)	ARITH: calculational errors (2)	Operational
			EQS: notational errors (22)	Structural
3. $-3(x + 1) - 22 = 8$	6 (12)	RS (100)	ARITH: priority rules (32)	Operational & structural
			inverses (12)	Structural
			calculational errors (18)	Operational
			EQS: notational errors (16)	Structural
4. $\frac{3x-7}{7} + 3 = 4$	17 (34)	RS (100)	ARITH: priority rules (28)	Operational & structural
			inverses (8)	Structural
			calculational errors (22)	Operational
			EQS: notational errors (44)	Structural

Tasks 3 and 4 seem to be difficult for most students. Six students (12%) solved task 3 correctly; and seventeen students (34%) solved task 4 correctly. Although there were students who solved tasks 1 and 2 incorrectly, the frequencies were not high (20%). As this lesson dealt with the reverse strategy, it is no wonder that all students used this strategy to solve the tasks. However, we noted that the strategy used by students had differences in terms of representations, namely the reverse strategy with and without arrow chains. Next, the types of difficulties that emerged in student work included the arithmetical (ARITH) and the equal sign (EQS) category. Mistakes in applying priority rules, in calculations (mainly) dealing with negative numbers and fractions, and in inverses were three sub-categories within the ARITH category, while the notational error of the use of the equal sign was a sub-category within the EQS category. Finally, concerning the use of the operational and structural views, the lack of an operational view may explain the occurrences of calculational errors; the lack of a structural view may explain the occurrences of mistakes in additive or multiplicative inverses and notational errors of the use of the equal sign; and the lack of both operational and structural views can explain mistakes in applying priority rules of arithmetical operations. In other words, lack of either the operational or structural view or of both views on equations might cause these types of difficulties. For example, mistakes in priority rules might happen because students lacked the operational and structural views on equations.

To illustrate these findings, we present two representative examples from written student work on task 3. Figure 3.3 (left part) shows an example of student work containing an ARITH category of difficulty and the use of the reverse strategy with arrow chains. The difficulty concerns a mistake in using the additive rather than multiplicative inverse: instead of dividing by –3 to get (x + 1), the student added +3. This mistake seems to occur because the student did not understand the meaning of the algebraic expression –3(x + 1) as a multiplication of –3 and (x + 1). In other words, the student lacked structure sense, which has to do with the structural view on the equation.

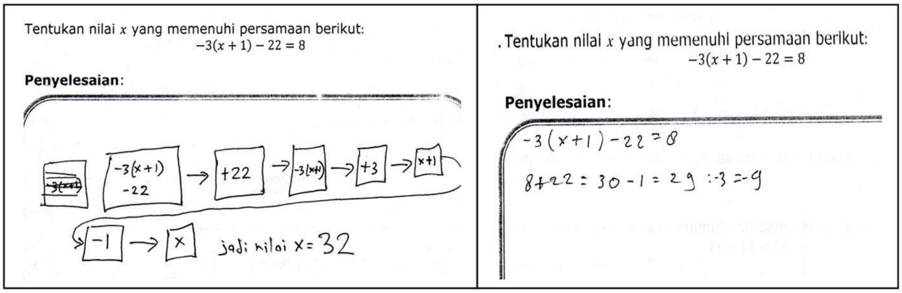

Figure 3.3. Representative examples of written student work on task 3

Figure 3.3 (right part) shows an example of student work containing the ARITH and EQS category of difficulties and the use of the reverse strategy without arrow chains. The ARITH category includes mistakes in applying priority rules of arithmetical operations and in calculation dealing with fractions; and the EQS category includes notational errors in the use of the equal sign. In the light of the operational view, the priority rules mistake seems to occur because the student did not understand the equation as a calculational process, and was not able to undo this process properly: after adding 22 to 8 to get 30, the student did a subtraction of 1 and a division by –3 afterwards, rather than to do a division by –3 and a subtraction of 1 respectively to get the solution. In the light of the structural view, this mistake seems to emerge because the student did not understand the expression –3(x + 1) as a multiplication of –3 and (x + 1), which means that the student lacked structure sense. A similar priority rules mistake working on the equation 3(2x – 1) + 7 = 28 which has a similar structure to task 3, taken from observation shown in the following transcript, corroborates this finding.

A pair of students is working on the equation 3(2x – 1) + 7 = 28 and the researcher is observing.

Student 1: [Using the Algebra Arrows, he will simplify. 3(2x – 1) + 7. First, he would like to subtract 7 from this expression].

Student 2: [It must first be] divided by [3]!

Student 1: No. [It should be subtracted by 7. Student 2 does not complain because the expression becomes simpler into 3(2*x* – 1). Student 1 will simplify 3(2*x* – 1) by adding +1].

Student 2: It should be subtracted [by 1].

Student 1: No. It is minus [within 2*x* – 1], is not it? So 1 must be added. [He comes up with Figure 3.4].

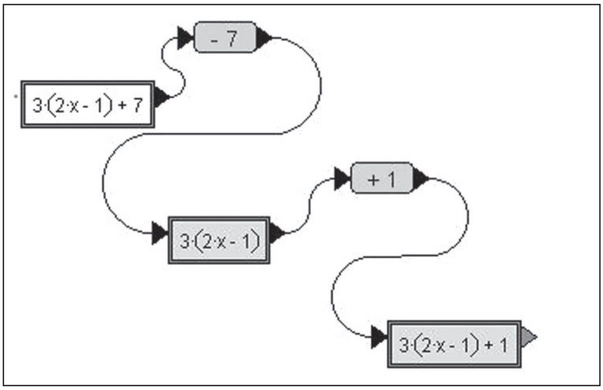

Figure 3.4. A priority rules mistake in Algebra Arrows applet environment

Student 1 & 2: [Laughing]. It is bigger [and more complicated]!

Student 2: So, it is incorrect! [He erases the incorrect part, but seems not to know what to do next.]

Even if the calculational error dealing with a division (fractions) has to do with arithmetical skills, we perceive this as the lack of an operational view on the equation, namely the inability to do or to undo a calculational process properly. Finally, we conceive that the notational errors in the use of the equal sign were a consequence of the use of the reverse strategy in the equation solving, which has to do with lacking a structural view of the meaning of the equal sign as an equivalent between two (algebraic or numerical) expressions.

3.5.2. Student difficulties while applying the cover-up strategy

The Cover-up activity focused on solving equations with the cover-up strategy (CS). In total, 51 students participated in this lesson. Table 3.2, which has the same headings as Table 3.1, summarizes the results of these students on the four tasks they worked on with paper and pencil at the end of the lesson.

First, concerning the number of students who solved the tasks correctly (#C), tasks 6, 7 and 8 seem to be difficult for most of students. Five students (10%) solved task 6 correctly; thirteen students (25%) solved task 7 correctly; and three students (6%) solved task 8 correctly. Although there were students who solved task 5 incorrectly, the frequency was not high (24%). Second, concerning strategy, even if this lesson focused on the cover-up strategy use, the data showed that not all students used this strategy to solve tasks 5-8: The reverse strategy (RS) was still used. Third, regarding difficulties, the ARITH and the EQS category appeared in student work with the reverse strategy, while the ARITH, EQS, and AE category emerged in student work with the cover-up strategy. If we zoomed in on student work where the reverse strategy was applied, the mistakes in applying priority rules, calculational errors, and inverses were sub-categories within the ARITH category; and notational errors in the use of the equal sign was a sub-category within the EQS category. These results were in line with the findings in the Algebra Arrows activity. Closely looking at student work with the cover-up strategy, sub-categories of difficulties within the ARITH category included calculational errors dealing with negative numbers and fractions, and inverses. Although the EQS category and in particular notational errors in the use of the equal sign appeared, the number was not as frequent as the number of the same mistakes in student work with the reverse strategy. The AE category (the parsing obstacle and lack of closure obstacle) of difficulty emerged in student work with the cover-up strategy, but not in student work with the reverse strategy. Fourth, and final, concerning operational and structural views, the lack of an operational conception may explain the occurrences of calculational errors; the lack of a structural conception may explain the occurrences of mistakes in additive or multiplicative inverses and notational errors of the use of the equal sign; and the lack of both operational and structural views may explain the occurrences of mistakes in applying priority rules of arithmetical operations, parsing obstacle and lack of closure obstacle. In other words, these mistakes occurred because of a lack of either the operational or structural view or of both views on equations. For instance, the inverse mistake occurred because students lacked the structural view on equations.

Table 3.2. Results from data analysis of the Cover-up lesson (N = 51)

Equations to solve	#C (%)	Strategy (%)	Difficulties (%)	Nature of the difficulties
5. $7(x + 1) = 49$	39 (76)	CS (65)	ARITH: inverses (6)	Structural
			calculational errors (4)	Operational
			EQS: notational errors (2)	Structural
		RS (35)	ARITH: priority rules (4)	Operational & structural
			EQS: notational errors (12)	Structural
6. $\frac{5}{2-x} = 1$	5 (10)	CS (68)	ARITH: calculational errors (26)	Operational
			EQS: notational errors (8)	Structural
			AE: lack of closure (2)	Operational & structural
		RS (32)	ARITH: calculational errors (21)	Operational
			EQS: notational errors (4)	Structural
7. $\frac{3x-5}{9} + 8 = 11$	13 (25)	CS (61)	ARITH: inverses (8)	Structural
			calculational errors (35)	Operational
			EQS: notational errors (4)	Structural
		RS (39)	ARITH: priority rules (12)	Operational & structural
			inverses (10)	Structural
			calculational errors (6)	Operational
			EQS: notational errors (6)	Structural
8. $6 + 7(4 – 5x) = 20$	3 (6)	CS (39)	ARITH: calculational errors (29)	Operational
			AE: parsing obstacle (4)	Operational & structural
		RS (61)	ARITH: priority rules (45)	Operational & structural
			calculational errors (16)	Operational
			EQS: notational errors (25)	Structural

To illustrate these findings, we elaborate two representative examples from written student work on task 8. Figure 3.5 (left part) shows an example of student work with the cover-up strategy containing the AE category of difficulty and the parsing obstacle in particular, that is, the student did not understand the order in which the algebraic expression $6 + 7(4 – 5x)$ must be processed. In the light of the operational view, it seems that the student did not understand the meaning of the equation as a calculational process properly: multiply a given number (x) by 5, next this is subtracted from 4, then multiplied by 7, and finally add 6 to get 20. Rather, the student understood the equation as 6 + 7 =13, and add $(4 – 5x)$ to get 20. In the light of the structural view, the student seems to fail at choosing the first part (structure) of the equation to be covered to get a next step: the student covered $(4 – 5x)$ directly rather than $7(4 – 5x)$. This means the student did not understand how to carry out the cover-up strategy. As a consequence, we perceive this as lack of a structural view on the equation and of structure sense in particular.

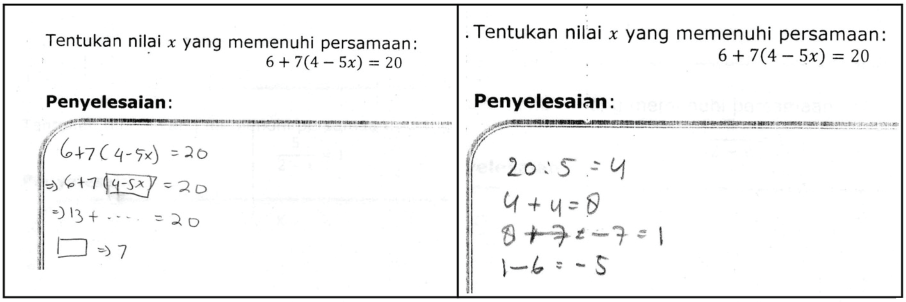

Figure 3.5. Representative examples of written student work on task 8

Failures to perform the cover-up strategy properly were also observed in the learning processes in both two participating schools, described in the transcript below.

The teacher, in school one, after observing students directly highlight x while doing $5(x + 1) = 40$, which causes a difficulty in determining the x value, suggests to all students that the expression that must be covered first is $(x + 1)$. They seem to follow this suggestion. However, we still observe this same mistake when students work on the equation $7(3x – 2) – 2 = 5$.

A similar difficulty occurs in school two. The teacher observes a pair of students working on $6x + 7 = 19$. The students seem to not know what to fill in after getting $x = ...$ (Figure 3.6, left screen).

Teacher: If you have this equation [$6x + 7 = 19$], what should be covered first?

Students 1 & 2: [They keep silent. The teacher reminds them of her example in the demonstration. Next, Student 1 covers $6x$, but does not know what to do next.]

Teacher: So, what is the value of $6x = ...$? [No reply. The teacher explains that means “blah-blah-blah added to 7 equals 19”.]

Student 2: So, it is 12.

Student 1: [He fills in 12 and gets $6x = 12$ as in Figure 3.6, right screen]

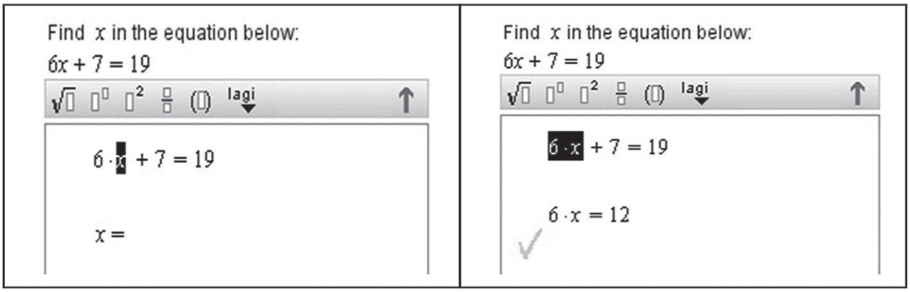

Figure 3.6. Student work in the Cover-up Strategy applet environment

Teacher: Good! Now you can cover-up *x*. [Student 1 covers-up *x* and fills in 2 which is the correct solution of the equation, with the applet providing a final feedback].

Figure 3.5 (right screen) contains student work with the reverse strategy showing the ARITH category of difficulty, and priority rules of arithmetical operations mistakes in particular. In the light of the operational view, this type of mistake seems to occur because the student did not understand the equation as a calculational process, and was not able to undo this process properly: rather than subtract 6, the student divided by 5 first, and so on. In the light of the structural view, this mistake seems to occur because the student did not understand the expression $7(4 - 5x)$ as a multiplication of 7 and $(4 - 5x)$, which means that the student lacked structure sense.

3.5.3. Confirmation of student difficulties from the final written test

Were the results of the Algebra Arrows and Cover-up activities confirmed by the final written test data? A total of 47 students from schools one and two participated in the final written test. Table 3.3, which has the same headings as Table 3.1, summarizes the results of these students on the two tasks they worked on with paper and pencil in this test. It shows that task 9 seems to be easy (94% of students solved it correctly), but task 10 seems to be difficult for most of the students (4% of students solved it correctly). Furthermore, as found in the Cover-up activity data, the type of strategies revealed in student work consisted of the cover-up (CS) and the reverse strategy (RS).

Table 3.3. Results from data analysis of the final written test ($N = 47$)

Equations to solve	#C (%)	Strategy (%)	Difficulties (%)		Nature of the difficulties
9. $12x + 1 = 49$	44 (94)	CS (38)	ARITH:	inverses (4)	Structural
		RS (62)	ARITH:	priority rules (2)	Operational & structural
			EQS:	notational errors (9)	Structural
10. $\frac{4(1-2x)}{5} - 3 = 1$	2 (4)	CS (55)	ARITH:	calculational errors (47)	Operational
			VAR:	unknown (6)	Structural
		RS (45)	ARITH:	priority rules (32)	Operational & structural
				calculational errors (6)	Operational
				inverses (4)	Structural
			EQS:	notational errors (13)	Structural

In relation to the use of the reverse strategy, types of difficulty emerged in the data included the ARITH category (mistakes in applying priority rules, calculational errors and inverses) and the EQS category (notational errors in the use of the equal sign). These findings confirm the applets activity data. In relation to the cover-up strategy, the types of difficulty consisted of the ARITH (calculational errors, and inverses mistakes) and the VAR category (understanding the variable as an unknown). This means that not all types of difficulties that appeared in the final test also appeared in the Cover-up observational data and vice versa. For example, the VAR category and understanding of the variable as an unknown in particular is a type of difficulty that did not appear in the Cover-up lesson, but did show up in the final test data.

Concerning the role of the operational and structural perspective, similar to the data in Table 3.2, the lack of the operational view may explain the occurrences of calculational errors; the lack of the structural view can explain the occurrences of mistakes in additive or multiplicative inverses, notational errors of the use of the equal sign, and in understanding the variable as an unknown; and the lack of both operational and structural views may explain the occurrences of mistakes in applying priority rules of arithmetical operations.

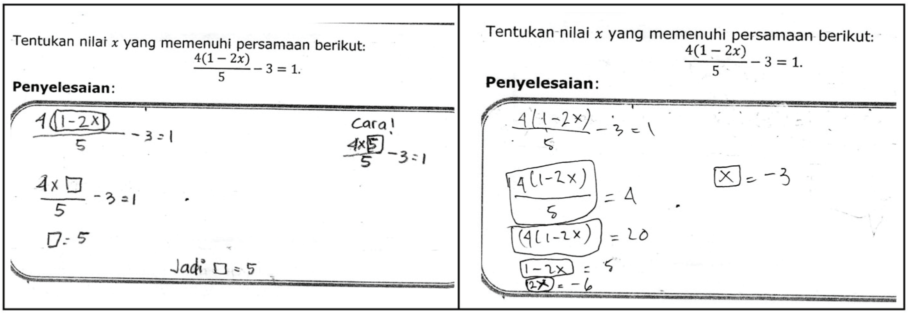

Figure 3.7. Representative examples of written student work on task 10

To illustrate these findings, we present representative examples of student work on task 10. Figure 3.7 (left part) shows an example of student work with the cover-up strategy containing the VAR category and understanding the variable as an unknown in particular. Although the student seems to be able to identify sub-expressions within the equation that must be covered, she seems to forget that the final goal of equation solving is to find the value of x rather than to end up at a box—which represents $1 - 2x$. The decision to end with this box may indicate that she lacked symbol sense (checking the solution in particular), which means lacking a structural view on the equation.

Figure 3.7 (right part) illustrates student work with the cover-up strategy containing an ARITH category and a calculational error dealing with negative numbers in particular: rather than to conclude $2x = -4$ from $1 - 2x = 5$, the student deduced $2x = -6$. Our interpretation is that this calculational error signifies a lacking operational view on the equation and on performing arithmetical calculations in particular.

3.6. Conclusions and discussion

The research question addressed in this paper concerns the identification of student difficulties in solving equations which emerge in the ICT-rich approach, and the understanding of these difficulties from operational and structural views. The results lead to the following conclusions. The difficulties that appeared in student work can be classified in two classes related to the equation solving strategies. First, while applying the reverse strategy (frequent in using the Algebra Arrows applet), the main difficulties include arithmetical skills and the equal sign category. The arithmetical skills category concerns mistakes in applying priority rules of arithmetical operations, in determining additive or multiplicative inverses, and calculational errors dealing with negative numbers and fractions. The equal sign category encompasses notational errors in the use of the equal sign only.

Second, while using the cover-up strategy (which relates to the Cover-up Strategy applet), the main difficulties are in the arithmetical skills category, including calculational errors dealing with negative numbers and fractions, and mistakes in determining additive or multiplicative inverses; understanding the concept of variable category and understanding the variable as an unknown in particular; understanding algebraic expressions category, including the parsing obstacle and the lack of closure obstacle; and the equal sign category, and notational errors of the use of the equal sign in particular.

Our analysis of the data suggests that limited operational and structural understanding of equations may explain these difficulties. A limited operational view may account for calculational errors, e.g., dealing with negative numbers and fractions, in the sense that these errors reflect an inability to do or undo a proper calculational process and, as such, limited operational view on equations. A limited structural view on equations may explain mistakes in additive or multiplicative inverses, notational errors of the use of the equal sign, and understanding the variable as an unknown. Mistakes in additive or multiplicative inverses may be caused by a lack of insight in the structure of algebraic expressions involved, and, for instance, mixing up multiplication and addition of sub-expressions. Notational errors concerning the equal sign may result from a lacking insight in the structural meaning of the equal sign as expressing an equivalent relation between two expressions. The mistake on understanding a variable as an unknown reflects a lack of symbol sense. Difficulties that may result from both limited operational and structural views on equations include misapplying priority rules of arithmetical operations, the parsing obstacle and the lack of closure obstacle. The priority rules mistakes may be explained by a lack of understanding of expressions as representing ordered calculational processes—which concerns an operational view; and of a misunderstanding of the structure of an algebraic expression. The parsing obstacle and the lack of closure obstacle occur because of a limited understanding of the operational meaning of algebraic expressions as representations of calculational processes. This may be caused by following the order of natural language rather than algebraic rules, and by the inability to identify relevant sub-expressions in the equation solving process.

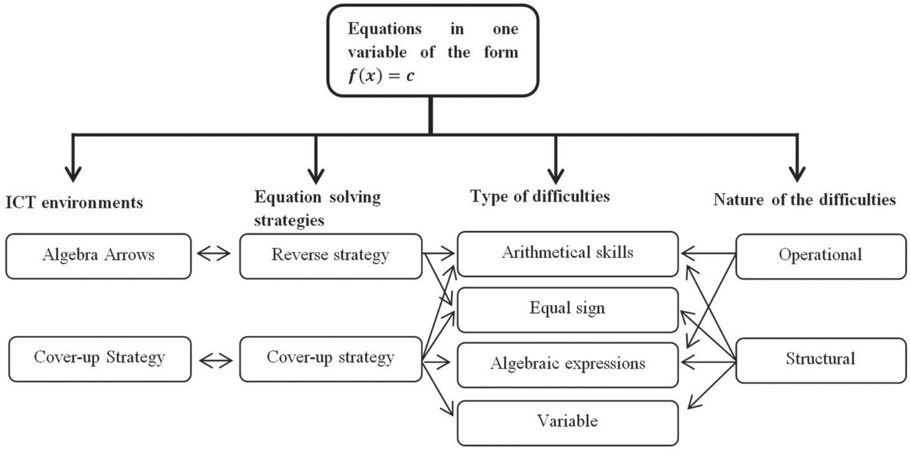

Figure 3.8. The relations between the study's components

To visualize the study, the diagram in Figure 3.8 shows the main components and their relationships. It subsequently depicts the ICT environments (applets) involved in this study, the different equation solving strategies included in the teaching sequence, the type of difficulties manifest in student work, and the nature of difficulties from the operational and structural views. The double-sided arrows represent relationships between equation solving strategies and applets; the single-headed arrows connect the strategies to the observed difficulties, and type of difficulties to their operational and structural nature.

Let us briefly reflect on these relationships. Even if we expected students to develop an operational view through the Algebra Arrows activity and a structural view through the Cover-up activity, students' written work revealed considerable difficulties that may be caused by several factors. The first factor concerns student readiness for integrated operational and structural thinking on mathematical conceptions. The process-object views signify a mature understanding of mathematical thinking which may only be reached in higher grades of secondary school (Drijvers, 2003), which participants in this study, who are in the transition phase from primary to secondary level, do not yet have. These students are not ready to develop integrated and flexible operational and structural views on equations and algebraic expressions. In terms of Skemp's (1976) vocabulary, students are not yet prepared to reach relational understanding—that is knowing what to do and why and-even if Skemp did not mention this explicitly-includes relating operational and structural conceptions—of equations and algebraic expressions. As a consequence, students understand equations and algebraic expressions instrumentally and primarily have a limited operational view on these concepts. The second factor concerns the limited amount of time spent on the intervention. According to Sfard (1991), the reification of a mathematical notion is a long and time consuming process, whereas the ICT intervention in this study took a relatively short period. The third factor concerns the role of ICT in the learning processes. Although the use of ICT may motivate students to engage in the learning of mathematics (e.g., Barkatsas, Kasimatis, & Gialamas, 2009), we consider that the ICT intervention in this study is not yet as effective as intended for integrating the flexible operational and structural conceptions. Even if the Algebra Arrows applet fits to develop an operational conception and the Cover-up Strategy applet is more appropriate to promote a structural conception, neither of these is proper for developing both conceptions in a flexible, integrated manner. The fourth factor, related to the third, concerns the teacher's ability to use ICT for promoting operational and structural views. Although this is not the focus of this study, we might pay attention to it in future research by better preparing and training teachers. The fifth and final factor concerns the appropriateness of the designed tasks and their presentation for promoting operational and structural conceptions. The following points may inform task design and presentation for future research:

- To reduce student difficulties in calculational errors dealing with negative numbers and fractions—which is one of the most frequent difficulties revealed in student work—we suggest to design tasks that consist of equations that avoid these issues. In this way, algebraic difficulties will be isolated and can be addressed in a separate way before returning to arithmetically more complex tasks.

- As a means to foster the development of integrated and flexible operational and structural views on equations and algebraic expressions, an important didactical approach to the reverse and cover-up strategies presented here might provide students with the opportunity to use both strategies for the same equation and to compare the two strategies. Also, the reasons for students to prefer one of the two might be investigated.

- As a didactical idea to promote student development of a structural view, structure sense, and symbol sense, we suggest to confront students at an earlier stage with non-linear equations that can be solved using the cover-up strategy, such as $\frac{32}{3x-4} + 3 = 19$, $\frac{64}{7(x+1)+1} = 8$, and $(x + 2)^2 = 36$ rather than allowing for this at a later stage. We conjecture that students who are able to solve this type of equations correctly improve on these three notions. This approach also promotes the cover-up rather than the reverse strategy in the equation solving.

- To develop a more general equation solving strategy, and as an addition to the two equation solving strategies which only work for equations of the form $f(x) = c$, we suggest to add the balance strategy. This strategy can be applied to solve equations of the more general form $f(x) = g(x)$, and highlights the notion of algebraic equivalence. Doing so will also provide a more comprehensive insight into student conceptual understanding of and difficulties with the concept of equations in one variable.

Acknowledgments

This study was funded by the Indonesia Ministry of Education project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794. We would like to thank Jan van Maanen for his valuable and constructive comments and suggestions. We also thank the teachers and students for their participation as well as an external assistant for her contributions.

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Chapter 4 Student difficulties in mathematizing word problems in algebra

Publication:

Jupri, A., & Drijvers, P. (Accepted). Student difficulties in mathematizing word problems. *EURASIA Journal of Mathematics, Science & Technology Education*.

Abstract To investigate student difficulties in solving word problems in algebra, we carried out a teaching experiment involving 51 Indonesian students (12/13 year-old) who used a digital mathematics environment. The findings were backed up by an interview study, in which eighteen students (13/14 year-old) were involved. The perspective of mathematization, i.e., the activity to transform a problem into a symbolic mathematical problem, and to reorganize the mathematical system, was used to identify student difficulties on the topic of linear equations in one variable. The results show that formulating a mathematical model—evidenced by errors in formulating equations, schemas or diagrams—is the main difficulty. This highlights the importance of mathematization as a crucial process in the learning and teaching of algebra.

Keywords algebra education, digital mathematics environment, linear equations in one variable, mathematization, word problems

4.1. Introduction

Solving word problems is among the main difficulties in algebra for many secondary school students all over the world (see, for instance, Bush & Karp, 2013; Carpraro & Joffrion, 2006; MacGregor & Stacey, 1998; Van Amerom, 2003). In Indonesia, student difficulties with solving word problems were revealed in the Trends in International Mathematics and Science Study (TIMSS) in 2007; for instance, only eight percent of the Indonesian participants were able to solve the word problem shown in Figure 4.1. This result was significantly below the international average of 18 percent (Mullis et al., 2008). Similar results can be found for other word problems.

> Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?
>
> Show your work.

Figure 4.1. TIMSS 2007 algebra word problem (Mullis et al., 2008)

To help Indonesian students to overcome these low performances in solving word problems in algebra, we wonder whether digital tools might be of value. Over the last decade, ICT-use has become widespread in mathematics education (e.g., Barkatsas, Kasimatis, & Gialamas, 2009; Bokhove & Drijvers, 2010; Kabaca, 2013), and research on the integration of ICT in algebra education suggests a positive influence on student achievement in general (Li & Ma, 2010), and in solving word problems in particular (Ghosh, 2012).

In an earlier interview study, mathematization, that is, the activity of organizing any kind of reality with mathematical means (Freudenthal, 1991; Treffers, 1987), was identified as one of the obstacles that students experience in initial algebra (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). To better understand the nature of the difficulties with solving word problems while using digital tools, we use the lens of mathematization.

To further investigate student difficulties in solving word problems from the perspective of mathematization, we set up a teaching experiment that included technology-rich lessons on solving word problems on the topic of linear equations in one variable. Here we report on this teaching experiment. Below, we first describe a theoretical background, including a brief description of difficulties in initial algebra, and the theory of mathematization. Next, the research question and methods are addressed. The results section elaborates student difficulties observed in the teaching experiment in the light of the mathematization perspective. These findings are triangulated with earlier interview data (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). Finally, we reflect upon the results in the conclusions and discussion section.

4.2. Theoretical background

4.2.1. Difficulties in initial algebra learning

The term "difficulties" in this section's title refers to obstacles that cause errors or mistakes made by students when dealing with algebra problems. By "initial algebra" we mean formal algebra topics—such as arithmetical operations on algebraic expressions, and linear equations and inequalities in one variable—which are in the curriculum for 12-14 year-old students in Indonesia as in many other countries.

From the existing research literature and from an interview study, we earlier identified the following five categories of difficulties in initial algebra (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014):

- The category of applying arithmetical operations in numerical and algebraic expressions (abbreviated as ARITH) includes difficulties in adding or subtracting similar algebraic terms (e.g., Herscovics & Linchevski, 1994; Linchevski, 1995); also difficulties in using associative, commutative, distributive, and inverses properties; and in applying priority rules of arithmetical operations (e.g., Booth, 1988; Bush & Karp, 2013; Warren, 2003).

- The category of understanding the notion of variable (VAR) concerns difficulties to distinguish a literal symbol as a variable that can play the role of a placeholder, a generalized number, an unknown, or a varying quantity (Booth, 1988; Bush & Karp, 2013; Herscovics & Linchevski, 1994).

- The category of understanding algebraic expressions (AE) encompasses the parsing obstacle, the expected answer obstacle, the lack of closure obstacle, and the gestalt view of algebraic expressions (Arcavi, 1994; Tall & Thomas, 1991). The parsing obstacle refers to understanding the order in which the algebraic expressions must be processed, the expected answer obstacle concerns the expectation to get a numeric result rather than an algebraic expression, and the lack of closure obstacle refers to discomfort in handling algebraic expressions that cannot be simplified any further.

- The category of understanding the different meanings of the equal sign (EQS) concerns difficulties in dealing with the equal sign, as an equal sign in arithmetic usually invites a calculation, while it is a sign of equivalence in algebra (Bush & Karp, 2013; Herscovics & Linchevski, 1994; Kieran, 1981).

- Finally, the category of mathematization (MATH) concerns the difficulty to translate back and forth between the world of the problem situation and the world of mathematics, and in the process of moving within the symbolic world (Treffers, 1987; Van den Heuvel-Panhuizen, 2003).

The first four categories are elaborated in Jupri, Drijvers, and Van den Heuvel-Panhuizen (2014). To shed new light on student difficulties in dealing with word problems, the present paper focuses on the fifth category of mathematization.

4.2.2. Mathematization

The notion of mathematization originates from the theory of Realistic Mathematics Education. It refers to the activity of organizing and studying any kind of reality with mathematical means, that is, translating a realistic problem into the symbolic mathematical world, and vice versa, as well as reorganizing and (re)constructing within the world of mathematics. 'Reality' can either refer to real life, to fantasy world, or to mathematical situations as far as they are meaningful and imaginable to the student, for example because their essential elements have been previously experienced and understood by the student (Freudenthal, 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2000; Van den Heuvel-Panhuizen & Drijvers, 2013).

Within mathematization, horizontal and vertical mathematization are distinguished (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). Horizontal mathematization refers to the activity of transferring a realistic problem to a symbolic mathematical problem through observation, experimentation, and inductive reasoning (Treffers, 1987). Activities that characterize horizontal mathematization include, for instance, identifying the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, and discovering relations (De Lange, 1987). Solving word problems—including the problems that combine both symbolic expressions and natural language—appeals to horizontal mathematization.

Vertical mathematization refers to the activity of reorganizing and (re)constructing within the world of symbols which includes solving the problem, generalization of the solution and further formalization (Treffers, 1987). Activities that characterize vertical mathematization include, for instance, manipulating and refining mathematical models, using different models, combining and integrating models, and generalizing (De Lange, 1987). Freudenthal (1991) points out that vertical mathematization includes both mechanical—in the sense of automatized procedures—and comprehensive aspects of reorganizing and (re)constructing within the world of symbols: "... symbols are shaped, reshaped, and manipulated mechanically, comprehendingly, reflectingly; this is vertical mathematization." (Freudenthal, 1991, p. 41-42).

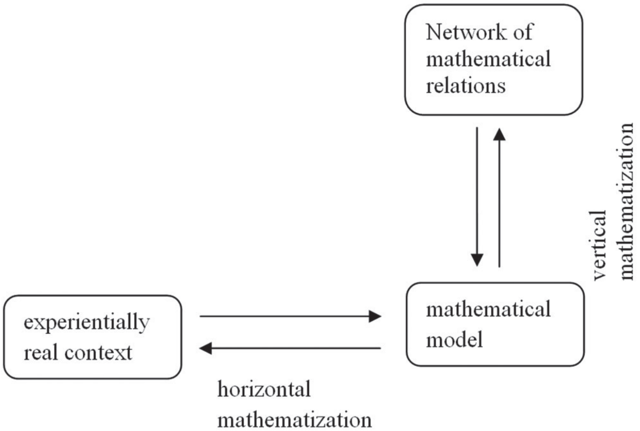


Figure 4.2. Horizontal and vertical mathematization (based on Drijvers, 2003, p. 54)

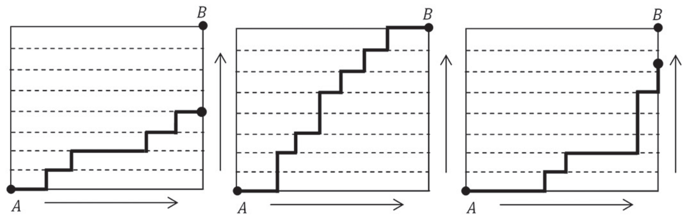


Figure 4.3. Different routes of mathematization (De Lange, 1987, p. 45)

In all phases of mathematical activity, the two types of mathematization complement each other (De Lange, 1987). Figure 4.2 depicts the global idea of horizontal and vertical mathematization activity. De Lange (1987) elaborates on the interplay between horizontal and vertical mathematization activity. He states that the process of mathematization undertaken by students in the learning processes is personal and may take different routes depending on the students' perception of the realistic situation, their skills, and their problem solving abilities. Figure 4.3 depicts the different routes of possible mathematization processes. Rather than expecting all students to travel the same route from *A* to *B*, the routes may be different and may not end up in the same point. These may include many horizontal steps and few vertical ones, or vice versa.

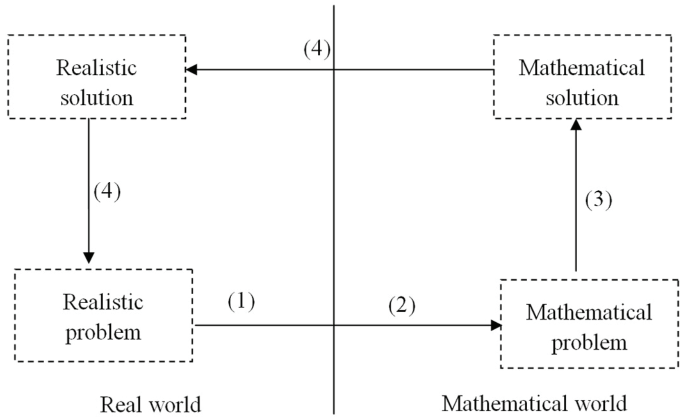


Figure 4.4. The mathematization cycle (based on De Lange, 2006, p.17)

According to De Lange (2006) the process of mathematization as it is carried out by the student has a cyclic character (see Figure 4.4). First, given a meaningful problem situated in reality, the student who acts as a problem solver starts the process by understanding the problem and identifying the relevant mathematical concepts within it (1). Next, based on the identified mathematical concepts, the problem solver phrases the problem in terms of a mathematical model (2). Third, the mathematical problem included in the model is solved and the student reflects on the solution process (3). Finally, the student is able to interpret the mathematical solution in terms of the original, realistic situation (4). The first two steps transform a realistic problem into a symbolic mathematical problem, and as such concern horizontal mathematization. The third step takes place within the symbolic mathematical world, and therefore characterizes vertical mathematization. Step four, the interpretation of the mathematical solution in terms of the realistic solution again concerns horizontal mathematization. If the interpretation of the realistic solution in terms of the original realistic problem includes verifying all conditions in the problem, generalizing the solution procedure and recognizing a possible application of this procedure in other similar problems, then vertical mathematization is involved.

> A rectangle has length and width $(3x - 4)$ and $(x + 1)$ cm, respectively. If the perimeter of the rectangle is 34 cm, find the area of the rectangle.

Figure 4.5. A problem to illustrate a mathematization cycle

To illustrate the cyclic character of the mathematization process, we consider the problem shown in Figure 4.5, which was taken from the interview study

(Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). Even if the length and the width are already expressed in symbolic form in this task, this problem involves the horizontal mathematization of setting up mathematical expressions for the perimeter and the area:

- *Given a problem situated in a reality, the mathematization process is started by understanding the problem to identify the relevant mathematical concepts within the problem (1).* As a problem solver, a student should realize that the reality involved in the task is a mathematical reality in the domain of geometry. We consider that the object involved, namely the rectangle, is imaginable in the student's mind as it has been present since primary school. The relevant mathematical information in the task includes the length, the width, and the perimeter of a rectangle.

- *Formulating the problem into a mathematical model (2).* Based on the identified mathematical concepts, the student should transform the given information by formulating, for instance, the following mathematical model: $2[(3x - 4) + (x + 1)] = 34$ and $A = (3x - 4)(x + 1)$, where A is the area of the rectangle. This action has transformed the problem into a mathematical problem.

- *Mathematical problem solving* and *a reflection* on the solution process *(3).* For the student who is a novice in algebra, a mathematical model in the form of an equation is still new, and as such the solution process is still not a routine procedure. Therefore, to solve the equation $2[(3x - 4) + (x + 1)] = 34$, the student should be able to plan an efficient strategy. For instance, the student should decide whether to first divide 34 by 2, next simplify the equation into $4x - 3 = 17$, and eventually get $x = 5$; or to first apply a distributive property to get $2(3x - 4) + 2(x + 1) = 34$, to multiply and then simplify, and to finally obtain $x = 5$. Substituting this value for x into $A = (3x - 4)(x + 1)$, the student will get $A = 66$. As a reflection on the solution process, the student can check whether it is correct by for instance substituting the value of $x = 5$ into the equation $2[(3x - 4) + (x + 1)] = 34$, and see if the equivalence is maintained; or by scrutinizing each step of the solution process.

- *Interpretation (4).* The student is able to interpret the solution $A = 66$ in terms of the realistic solution, i.e., as the area of the rectangle. Next, to understand this in terms of the realistic problem, the student can verify all conditions given in the problem using the obtained mathematical results. By substituting $x = 5$ into $(3x - 4)$ cm and $(x + 1)$ cm, the student will find 11 cm and 6 cm as the length and the width of the rectangle, respectively.

In this way, the student can confirm that the perimeter of the rectangle is indeed 2(11 + 6) = 34 cm and that the given conditions are met.

4.3. Research question

We consider that identifying and understanding student difficulties in solving word problems from a mathematization perspective can lead to a better insight into students' learning of algebra. Therefore, we focus on the following research question:

What are student difficulties in mathematizing word problems in the domain of linear equations in one variable?

In line with the literature (e.g., Carpraro & Joffrion, 2006; Clement, 1982; Stacey & MacGregor, 2000), by "word problems" in algebra we mean algebra tasks that are at least partially represented in natural language. Solving these tasks—which may include graphs, images, tables, geometric figures, or mathematical symbols—involves transformation into mathematical models, such as equations or inequalities, if algebraic methods will be used in the problems solving. As such, these tasks appeal for horizontal mathematization.

4.4. Methods

To answer the research question, a teaching experiment was carried out because we would like to study student learning rather than just capture student thinking at one specific moment, as was done in the interview study. In addition, results from a teaching experiment include exemplary teaching materials and teaching practices that can inform teachers. We included digital tools in the teaching experiment, as we expect this can support students' mathematization processes, while offering an explorative and expressive environments for doing mathematics (e.g., Drijvers & Doorman, 1996; Drijvers, 2000; Drijvers, Boon, & Van Reeuwijk, 2010). Finally, we re-analyzed part of the older interview data for two reasons: (i) Mathematization, which is the main lens in this study, was one of categories of difficulties that emerged in the interviews. Therefore, after analyzing the teaching experiment data, we looked back at the interview data to investigate in retrospective whether the teaching experiment findings match with the interview results; and (ii) even if these two studies have different settings, i.e., different students and teaching approaches, we claim that mathematization difficulties are so general that they should be recognizable in both data sets.

Below, we describe the methods of the teaching experiment study and provide some information on the interview study, the data of which were used for a triangulation.

4.4.1. The design of the teaching experiment

The learning arrangement that we designed consisted of student activities including digital tasks within applets embedded in a digital environment; intermediate formative paper-and-pencil assessment tasks; a final written test; and a teacher guide.

As digital environment, the Digital Mathematics Environment (DME) was used. The DME is a web-based electronic learning environment which offers: (i) interactive mathematical tools for algebra, geometry, and other domains; (ii) a design of open online tasks and appropriate immediate feedback; (iii) conventional mathematical notations and techniques; (iv) access to the environment at any time and place, as long as technological conditions, especially the availability of internet connection and web-browser, are met; and (v) a storage of student work (Boon, 2006; Drijvers et al., 2013). The DME applets Algebra Arrows and Cover-up Strategy were included in the designed arrangement.

Algebra Arrows is an applet which offers the possibility to construct and use chains of operations on numbers and formulas, and provides automatic calculations. Initially, it is designed to support the construction of input-output chains of operations as a model of a dependency relationship in the function concept (Doorman et al., 2012). In this study, the applet was used as a support for solving word problems.

Figure 4.6 shows how a word problem is solved with the Algebra Arrows applet. 1) The applet provides a word problem to solve, a window for the solution process, input-output boxes (*kotak masukan-keluaran*) which can be dragged and connected with operation boxes in the solution window, and a white box with an unknown (x) in the solution window. 2) A student translates the word problem word-by-word – i.e., translating words or phrases into mathematical operations (through dragging and connecting operation boxes with the input box) – into a mathematical expression. 3) By clicking the Table button, the table appears below the expression $\frac{3x-5}{5}$ and automatically provides some of its values. Considering the problem, the appropriate value of the expression is 5, which means the equation representing the problem is $\frac{3x-5}{5} = 5$. Finally, 4) through applying a reverse-strategy, the student solves the equation to find the value of $x = 10$.

The Cover-up Strategy applet provides an environment to set up equations based on the given word problems, and allows for solving equations in one variable of the form $f(x) = c$. The equation solving process is carried out by subsequently selecting a part of the expression in an equation with the mouse and finding its value. For example, Figure 4.7 shows a scenario for solving a word problem with the Cover-up Strategy applet. In step 1, a student is expected to formulate an equation based on the given word problem. As he made a mistake, the applet gives feedback, namely a crossed mark in red signifying an incorrect action. If the student formulated a correct equation, namely $\frac{y+2}{3} = 1$ as shown in step 2, the applet provides a tick mark in yellow signifying a correct action. In step 3, the student highlights the expression $y + 2$ and the applet provides $y + 2 = ...$ in the next line. In step 4, the student fills in, and the applet gives a yellow tick mark signifying a correct response. This scenario proceeds until step 6 and ends up at $y = 1$ as the solution of the equation (signified by the green tick mark and the final feedback: "The equation is solved correctly!").

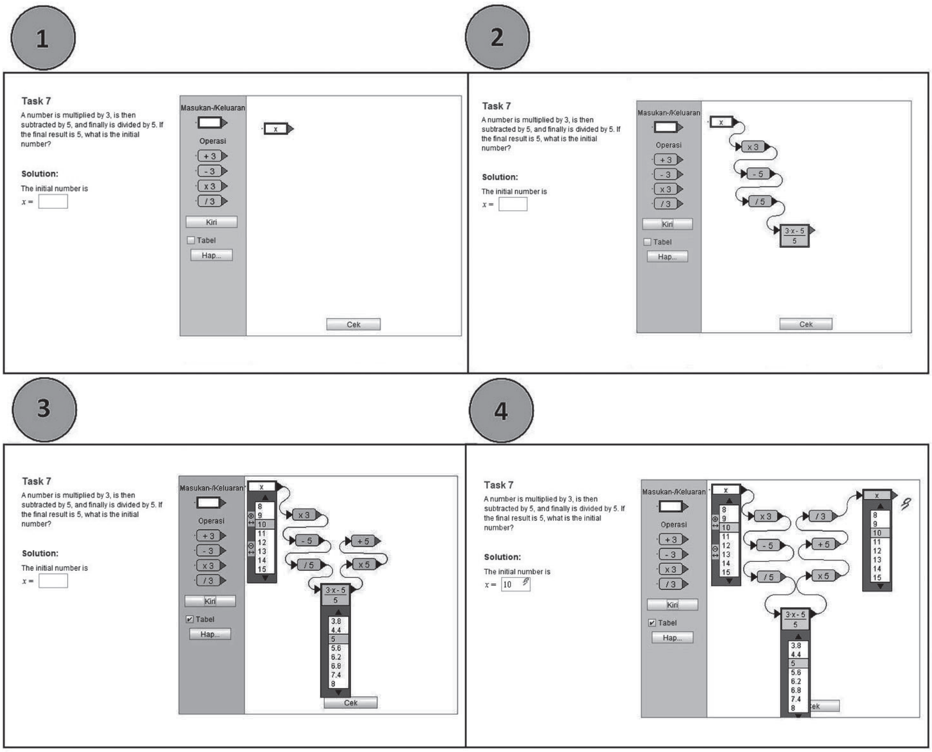


Figure 4.6. A scenario for solving a word problem using the Algebra Arrows applet

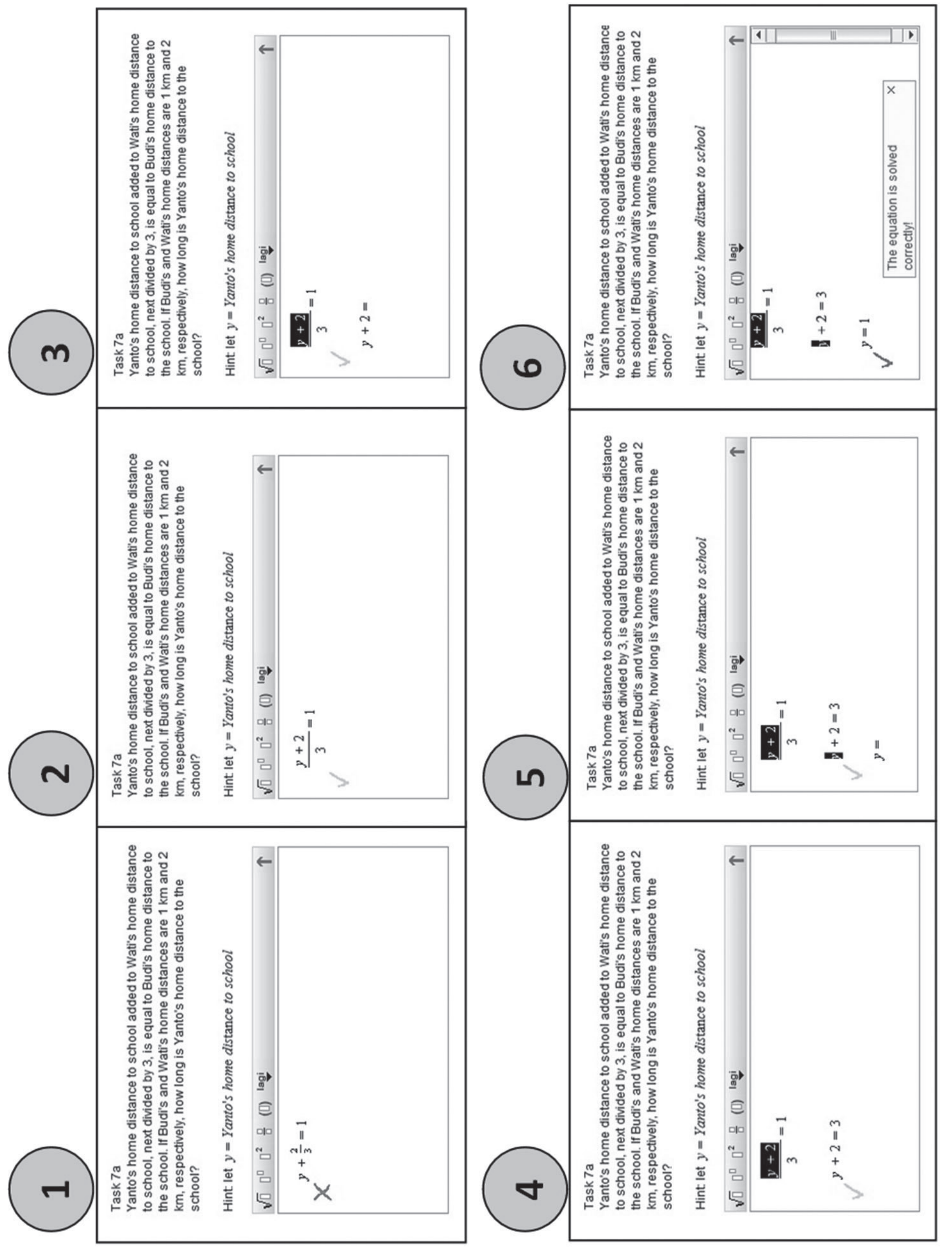


Figure 4.7. A scenario for solving a word problem using the Cover-up Strategy applet

Nine Indonesian master students in mathematics education tested preliminary versions of the activities with the above two applets. Their inputs were incorporated in order to improve the activities presented to the students involved.

A teacher guide was designed for five lessons. Lessons 1 and 2 were enriched with the Algebra Arrows applet and respectively focused on word problems and symbolic equations. Lessons 3 and 4 included the use of the Cover-up Strategy applet and subsequently focused on symbolic equations and word problems. Lesson 5 consisted of a final written test covering the topics of the four previous lessons. The experiment took 80 minutes for each lesson. The learning sequence in each of the first-four lessons consisted of three parts: paper-and-pencil activity, digital activity, and paper-and-pencil assessment as well as reflection. The paper-and-pencil activity included posing problems and classroom discussion. The digital activity consisted of a demonstration of an applet, student group digital work and discussion. During the digital activity, the teacher or the observer gave help to groups of students when necessary, including guiding students during the learning process. In the end-of-the-lesson formative assessment, students were requested to do paper-and-pencil tasks individually. The tasks were designed based on the tasks used in the DME session and initially referred to Indonesian mathematics textbooks. Finally, the teacher guided students to reflect upon the lesson.

4.4.2. Data collection

The teaching experiments were conducted in two schools in Indonesia. One complete class with 41 grade seven students (12-13 year-old) was chosen from the first school, and ten grade seven students (12-13 year-old) participated from the second school. The ten students who were selected by their mathematics teacher to participate in this study included high, medium and low achievers in a balanced manner. Data that were collected from each school consisted of video registrations of four teaching sessions (including paper-and-pencil-board activity, group digital work, and classroom discussion), student written work from each assessment and from the final written test, and field notes. During the periods of group work with the applets, the video registration focused on two groups of students. As the teacher usually did not take care of these video groups too much, the researcher-observer to some extent guided these two groups if needed, as to give all students the same treatment.

4.4.3. Data analysis

The analysis of the data from the teaching experiment was carried out in three phases. In the first phase, a preliminary analysis of video registration of student digital group work as well as on individual written work was carried out with software for a qualitative analysis (Atlas.ti). This analysis included marking and transcribing crucial moments in paper-and-pencil activity and in classroom discussions as well as in student digital group work; examining and assigning difficulties on written student work (including a written final test) for each single task—which serves as a case of analysis. In total there are 394 cases of data. To confirm the analysis of the written student work, transcripts from observations during the learning activities with the digital technology were used. Thus, the results of analysis integrate the quantitative data from the intermediate formative assessments and the qualitative analysis of the video data from students' activities in the digital mathematics environment.

The second phase of analysis consisted of an in-depth analysis of student difficulties from the perspective of mathematization (see section 4.2.2). We classified student difficulties identified in the first phase into four subcategories. First, difficulties in understanding words, phrases, or sentences, and ignoring parts of the problem were classified in the subcategory of understanding the problem. Second, difficulties in formulating equations, schemas, or diagrams were classified under the subcategory of formulating mathematical models. Third, mistakes made in the solution process were grouped into the subcategory of symbolic mathematical problem solving. We argued that types of difficulties in applying arithmetical operations (ARITH), in understanding the notion of variable (VAR), in understanding algebraic expressions (AE), and in understanding the different meanings of the equal sign (EQS) can be included in the third subcategory because they normally occur during the solution processes. Fourth, difficulties in checking the solution process were included in the subcategory of reflection. To check the inter-rater reliability, a second coder—an external research assistant not included in this study—analyzed 20% of the cases after being given an explanation and the coding manual for data analysis. With a Cohen's Kappa of 0.91, the agreement between the first author and the second coder was found to be almost perfect (Landis & Koch, 1977).

To check the findings of the teaching experiment, the third phase of analysis concerns triangulation with the interview data from an earlier study and of word problems on linear equations in one variable in that study in particular (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). The interview study involved eighteen Indonesian students who finished grade seven (13/14 year-old). These students were asked to solve a set of five algebra tasks (two of which are on word problems) with paper and pencil individually for thirty minutes. Next, interviews were conducted, during which the students were encouraged to explain their reasoning in their written work. The interview data had been analyzed before, and an additional analysis of the two algebra word problems involved in the interviews was carried out using a similar mathematization framework and coding schemes focusing on mathematization.

4.5. Results

This section presents the results of the teaching experiment which were backed up with the findings from the interview study. The main results of the teaching experiment include individual written student work after the use of the applets and if necessary are confirmed by observations of student group work in the DME sessions. The written final test findings are used to corroborate the results of this analysis. To confirm these findings, we revisit findings from earlier student interviews.

4.5.1. Student mathematization difficulties revealed in the Algebra Arrows lesson

A total of 49 students participated in the lesson which focused on solving word problems with the Algebra Arrows applet. Table 4.1 summarizes the results of these students for the three tasks they worked on with paper and pencil at the end of the lesson. Columns 1-4 subsequently present: tasks, number of students who solved the tasks correctly (#C), types of difficulties revealed, and mathematization subcategories which might explain the difficulties. Corresponding percentages, relative to the total number of participating students, are provided for columns 2 and 3.

Task 3 seems to be difficult for most of the students. Of the 49, fourteen students (29%) solved this task correctly. Even if task 3 has the same structure as task 2—namely, the mathematical models of these two tasks are similar—it seems that task 3 is more difficult. This could be caused by the fact that task 3 requires students to work with negative numbers which is often demanding for them.

Table 4.1. Results from data analysis of the Algebra Arrows lesson (N = 49)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
1. You have a number. The number is subtracted by 7, next the result is divided by 5. If the final result is 11, what was the starting number?	43(88)	- misunderstand or ignore words, phrases, sentences (10) - errors in formulating equations, schemas, diagrams (10) mistakes in solution processes: • ARITH: calculation errors (2) • EQS: notational errors (25) - checking the solution process (2)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection
2. A number is multiplied by 2, the result is then subtracted by 4, and finally is divided by 5. If the final result is 3, what was the starting number?	31(63)	- misunderstand or ignore words, phrases, sentences (4) - errors in formulating equations, schemas, diagrams (4) mistakes in solution processes: • ARITH: calculation errors (12), inverse errors (10). • EQS: notational errors (35) - checking the solution process (25)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection
3. A number is multiplied by 2, next the result is added to 50, and finally is divided by 5. If the final result is 5, what was the starting number?	14(29)	- misunderstand or ignore words, phrases, sentences (25) - errors in formulating equations, schemas, diagrams (12) - mistakes in solution processes: • ARITH: priority rules (10), calculation errors (25), inverse errors (4). • EQS: notational errors (35). Checking the solution process (45)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection

The data for each task revealed that difficulties in the solution processes were the most frequent. From a mathematization perspective, this means that students encountered difficulty in vertical mathematization and in the subcategory of mathematical problem solving in particular. To illustrate these findings, we present two representative examples of written student work on task 3. Figure 4.8 (left part) shows an example of written student work containing difficulties in understanding a phrase, and in the solution process.

First, rather than to translate the phrase "*sebuah bilangan* (a number)" into an unknown, for instance, the student translated it as an integer number "one". As a consequence, she translated the problem into an incorrect mathematical model: $1 \times 2 + 50 : 5 = 5$. In this case, if the student had understood the phrase "*sebuah bilangan*" correctly, she would probably have got a correct model. Second, if we assume that the model is correct, then the student did an incorrect calculation, namely $52 : 5 = 25$ instead of $52 : 5 = 10\frac{2}{5}$. From the perspective of mathematization, the first difficulty concerns horizontal mathematization and understanding the problem in particular. The second difficulty concerns vertical mathematization and the subcategory of mathematical problem solving—namely, lack of proficiency in arithmetical calculation—in particular.

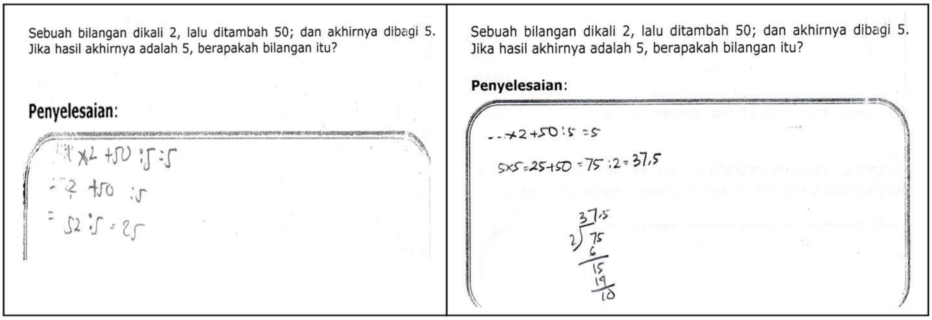


Figure 4.8. Representative examples of written student work on task 3

Figure 4.8 (right part) shows an example of written student work which contains mistakes in the solution process (subcategory of mathematical problem solving) and an indication of not checking the solution (subcategory of reflection). It seems that the student understood the problem and was able to translate it into a correct mathematical model. However, she made two mistakes in the solution process. First, she made an additive inverse error: instead of subtracting 50 from 25 and next dividing by 2, she added 50 to 25. The second mistake concerns an improper use of the equal sign: the student wrote down $5 \times 5 = 25 + 50 = 75 : 2 = 37.5$, which is incorrect since, for instance, 5×5 is not equal to $25 + 50$. Furthermore, after getting 37.5, she seemed not to check this by substituting it into the model. This indicates that she forgot to check the solution process. In the light of mathematization, the student encountered difficulty in vertical mathematization and the mathematical problem solving and the reflection subcategories in particular. The frequent difficulties in the solution processes, which were also observed in digital group work, seem to be a direct consequence of automatic calculation provided by the Algebra Arrows applet during the learning process. As a consequence, when students were working on word problems on paper, they were not used to doing the solution processes, in particular the calculation, by themselves.

Aside from the above findings, our data on student group digital work shows that five out of eight groups (25 students) failed to deal with word problems, in which the context concerns real life and is not merely on number. The following observation excerpt on the group digital work corroborates this finding.

A group of five students was doing the following task:

Tom is 7 years older than Safira. If Safira is 4 years old, how old is Tom?

After reading the task, the group was puzzled.

Student 1: [Reads the task out loud].

Student 2: It seems 7 – 4, does not it? [She suggests Student 1 to do her idea].

Student 1: [She represents 7 – 4 using the applet].

Student 2: Wait! It must be 7 + 4 – 4.

Student 1: [She represents 7 + 4 – 4 on the computer].

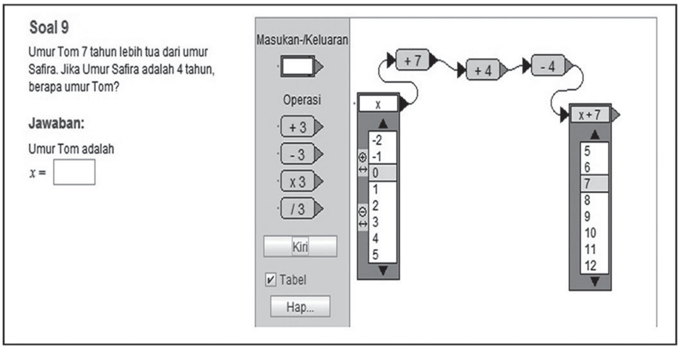


Figure 4.9. A student's work on the Algebra Arrows applet

Student 3: What number should be clicked below the expression $x + 7$ [to get a direct value of x]?

Student 1: Seven.

Student 3: Why is x [Tom's age] zero? [See Figure 4.9].

[Even if they succeeded eventually after getting a guidance, this group took time to ponder the task].

During the observation, one of the students could solve the task mentally, i.e., without using the applet. The obstacle for this group of students was that they could not represent the word problem into a mathematical expression using the Algebra Arrows properly. This could be because the structure of the task that could not easily be translated into a mathematical expression. In short, the above excerpt suggests that students seemed to encounter difficulty in understanding the problem and as such implies difficulty in formulating a mathematical model from the problem. This means that the main difficulties encountered by students when dealing with word problems—in which the context relates to real life—concern horizontal mathematization.

4.5.2. Student mathematization difficulties revealed in the Cover-up Strategy lesson

A total of 51 students participated in the lesson which focused on solving word problems with the Cover-up Strategy applet. Table 4.2 which has the same headings as Table 4.1 summarizes the results of these students for the three tasks they worked on with paper and pencil at the end of the lesson.

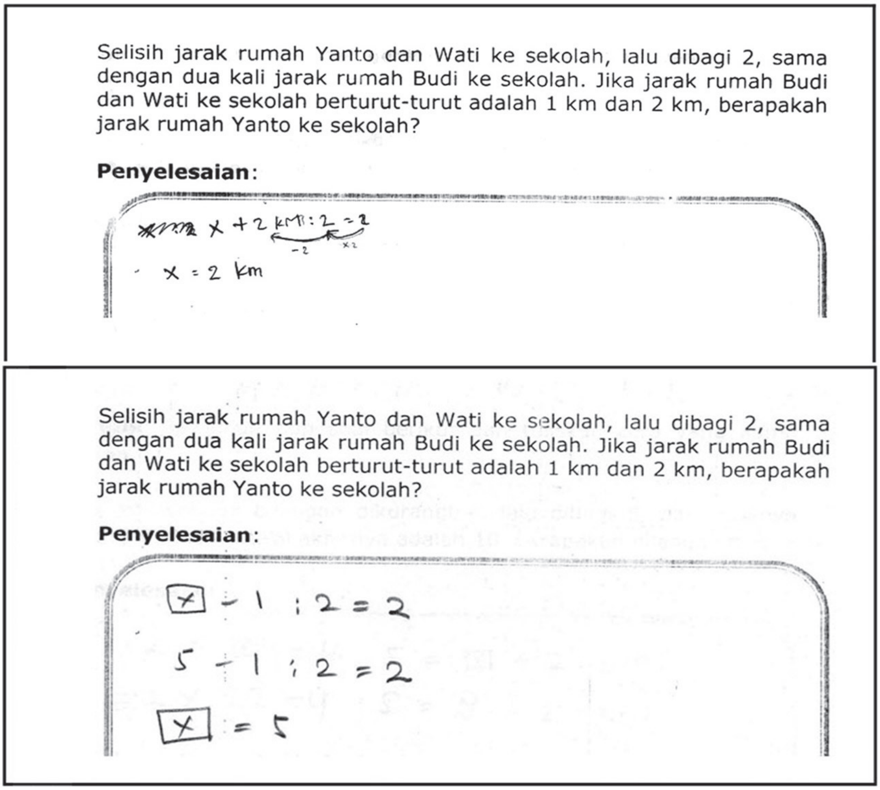
Task 6 seems to be difficult for most of the students. Of the 51, six students (12%) solved this task correctly. Even if task 4 is a typical problem of the Algebra Arrows lesson, still this task is more difficult than task 5. This could be because the structure of task 4 is more complex than task 5, and as such it is difficult to translate and to solve.

All categories and the corresponding subcategories of difficulties that emerged in this lesson were the same as the findings of the Algebra Arrows lesson. Although difficulties in the solution processes were still frequent, other difficulties occurred more often. The two most frequent subcategories of difficulties that emerged in each task were: (i) understanding words, phrases, sentences; and (ii) formulating equations, schemas, or diagrams. From the perspective of mathematization, the first subcategory concerns understanding the problems, and the second subcategory concerns formulating mathematical models. In other words, the emergence of these two subcategories of difficulties signifies difficulties in horizontal mathematization.

Table 4.2. Results from data analysis of the Cover-up Strategy lesson (N = 51)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
4. Two times a number is subtracted by 4, next divided by 5, and finally added by 2. If the final result is 10, find the number.	21(41)	- misunderstand or ignore words, phrases, sentences (33) - errors in formulating equations, schemas, diagrams (26) - mistakes in solution processes: • ARITH: inverses errors (4), priority rules (2) calculation errors (2) • EQS: notational errors (24) - checking the solution process (35)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection
5. Adin's height is divided by 3, next the result is added to Budin's height. If the final result is equal to 180 cm and Budin's height is 130 cm; find Adin's height.	32(63)	- misunderstand or ignore words, phrases, sentences (16) - errors in formulating equations, schemas, diagrams (16) - mistakes in solution processes: • ARITH: inverses (10), calculation errors (6) • EQS: notational errors (12) - checking the solution process (16)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection
6. The difference of the distances from Yanto's and Wati's homes to their school divided by 2 is equal to twice the distance from Budi's home to the school. If Budi's and Wati's home distances are 1 km and 2 km, respectively, find the distance between Yanto's home and the school.	6(12)	- misunderstand or ignore words, phrases, sentences (84) - errors in formulating equations, schemas, diagrams (82) - mistakes in solution processes: • ARITH: inverses (2), calculation errors (12) • EQS: notational errors (14) - checking the solution process (8)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection

We clarify these findings by two representative examples of written student work on task 6. Figure 4.10 (upper part) shows student work containing the difficulty in understanding a phrase which causes a mistake in formulating an equation. Rather than translating the phrase, "The difference of the distances from Yanto's and Wati's homes to their school," into, for example, $x - 2$ (in which x and 2 represent respectively Yanto's and Wati's homes' distances to their school), the student translated it into $x + 2$. This led to an incorrect equation. Figure 4.10 (lower part) shows a similar mistake. Even if the student seems to understand the aforementioned phrase, he assigned 1 rather than 2 as the distance of Wati's home to the school. This then also led to an incorrect equation. From a mathematization perspective, both examples illustrate difficulties in horizontal mathematization and in understanding the problem and formulating mathematical models in particular.


Selisih jarak rumah Yanto dan Wati ke sekolah, lalu dibagi 2, sama dengan dua kali jarak rumah Budi ke sekolah. Jika jarak rumah Budi dan Wati ke sekolah berturut-turut adalah 1 km dan 2 km, berapakah jarak rumah Yanto ke sekolah?

Penyelesaian:

$x + 2$ km : 2 = 2

$x = 2$ km

Selisih jarak rumah Yanto dan Wati ke sekolah, lalu dibagi 2, sama dengan dua kali jarak rumah Budi ke sekolah. Jika jarak rumah Budi dan Wati ke sekolah berturut-turut adalah 1 km dan 2 km, berapakah jarak rumah Yanto ke sekolah?

Penyelesaian:

$x - 1 : 2 = 2$

$5 - 1 : 2 = 2$

$x = 5$

Figure 4.10. Representative examples of written student work on task 6

The difficulties in understanding problems and in formulating mathematical models were also observed in digital group works. The following observation illustrates this.

A group of students was working on the following task:

"Wenny's and Yudi's ages together are 27. If Yudi is 9 years younger than Wenny, how old is Wenny? Hint: Let w = Wenny's age."

After reading the problem, one student typed an equation on the computer, namely $w + 9 + w = 27$. The applet provided direct feedback that the equation was incorrect. Next, the observer suggested the group to reread the problem.

Student A: So, it must be subtracted! [He erases the incorrect equation and types $w - 9 - w = 27$. Student B presses the enter button to check, but it is still incorrect. See Figure 4.11.]

Student B: Why is it still wrong?

[After thinking and getting the observer's guidance, the group succeeds eventually.]

This observation suggests that the difficulty in formulating a mathematical model (equation) is caused by students' limited understanding of the problem.

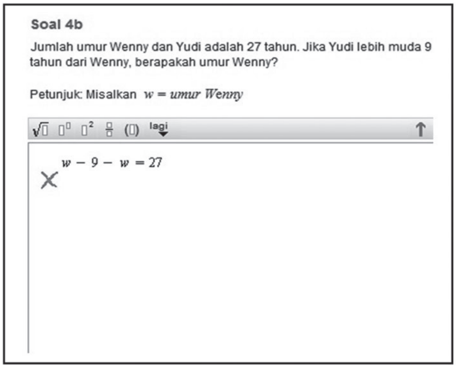


Figure 4.11. A student mistake in the Cover-up Strategy applet

4.5.3. Student mathematization difficulties revealed in the final written test

The results of the final written test were used to confirm the findings of the Algebra Arrows and Cover-up activities. A total of 47 students participated in the final written test. Table 4.3 which has the same headings as Tables 1 and 2 summarizes the results of these students on the two word problems they worked on with paper and pencil in this test. It shows that students performed well on task 7 (87% correct results), but encountered difficulties in dealing with task 8 (28% correct results).

Table 4.3. Results from data analysis of final written test (N =47)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
7. You have a number. The number is subtracted by 2, the result is then multiplied by 7, and finally 4 is added. If the final result is 25, what was the starting number?	41(87)	- misunderstand or ignore words, phrases, sentences (2) - errors in formulating equations, schemas, diagrams (2) - mistakes in solution processes: • ARITH: inverses errors (2), priority rules (4) • EQS: notational errors (23) - checking the solution process (11)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection
8. The sum of distances of Tom's and Jerry's homes to the city center divided by 9 is equal to the distance of three times Udin's home to the city center. If Udin's and Jerry's home distances are 1 km and 7 km, respectively, find the distance between Tom's home and the city center.	13(28)	- misunderstand or ignore words, phrases, sentences (68) - errors in formulating equations, schemas, diagrams (68) - mistakes in solution processes: • ARITH: priority rules (4), calculation errors (15) • EQS: notational errors (9) - checking the solution process (0)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection

In general, the difficulties in the final written test were the same as found in the Algebra Arrows and Cover-up Strategy activities. Mistakes in the solution processes (mathematical problem solving subcategory) and in checking solutions (reflection subcategory) were two most frequent difficulties on task 7—which is a typical task addressed in the Algebra Arrows activity. These results confirmed the findings of the Algebra Arrows activity, namely most of the students encountered difficulties in vertical mathematization and in mathematical problem solving and reflection in particular.

Difficulties in understanding words, phrases, or sentences (*understanding the problem*); and in formulating equations, schemas, or diagrams (*formulating mathematical models*) were the two most frequent difficulties on task 8. In other words, most of the students encountered difficulties in horizontal mathematization. These results also confirmed the findings of the Cover-up Strategy lesson because task 8 is a typical problem addressed in this lesson.

4.5.4. Backing up the findings with data from student interviews

Table 4.4. which has the same headings as Tables 4.1., 4.2. and 4.3. summarizes the results of interviews for the two word problems worked by students. Both tasks seem to be difficult for most of the students. Of the eighteen, one student (6%) solved task A and seven students (39%) solved task B correctly.

In general, the difficulties revealed in the interviews results match the teaching experiment findings. Even if mistakes in the solution processes occurred quite often, we observed that mistakes in formulating equations, schemas or diagrams were the most frequent. From a mathematization perspective, these findings show that difficulties in the horizontal mathematization and formulating mathematical models subcategory in particular were the most important obstacles revealed in the interviews.

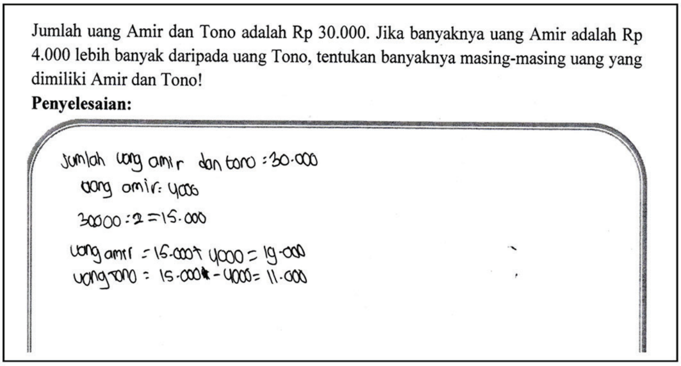


Figure 4.12. A student's work with an incorrect arithmetical method on task A

Concerning task A and in relation to the difficulties in formulating mathematical models, we observed that fourteen students (78%) used incorrect arithmetical methods—which include incorrect arithmetical models—to solve the task. Such arithmetical methods include, for instance, dividing 30,000 by 2, next adding and subtracting 4,000 to 15,000 to find the amounts of Amir's and Tono's money, respectively. Figure 4.12 shows an example of such incorrect arithmetical methods.

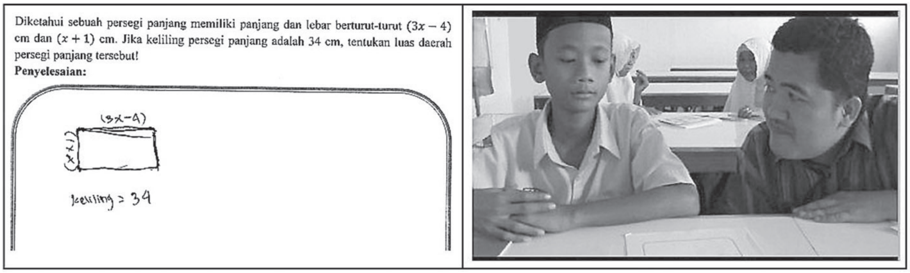


Figure 4.13. A student interview showing an inability to formulate a math model

We observe that the difficulty in formulating mathematical models for task B, which was also frequent, seems to be caused by students' lacking understanding and abilities to connect mathematical concepts from different mathematical strands, such as connecting algebra and geometry. The following interview excerpt provides evidence for this.

I (interviewer): Please can you read the problem? [As the solution space of the student is blank, the interviewer asks the student to understand the problem.]

S (student): [Reads the problem aloud.]

I: Do you know what a rectangle is? [To check whether the student understood what he reads].

S: [Draws a rectangle on his paper.]

I: Which are the length and the width of the rectangle?

S: [Points the length and the width of the rectangle correctly.]

I: In the problem you read, what is the length [of the rectangle]?

S: $3x - 4$.

I: What is the width of the rectangle?

S: $x + 1$. [He writes down $3x - 4$ and $x + 1$ beside the length and the width of the rectangle, respectively. See Figure 4.13.]

I: What is the value of the perimeter given in the problem?

S: 34 cm.

I: Do you know the perimeter [of this rectangle]?

S: [Keeps silent. But then he points to the length and the width of the rectangle and seems to round the rectangle.]

I: Okay, can you write [the formula for] the perimeter of this rectangle?

S: [He writes "*Keliling* = 34" on his paper. *Keliling* means the perimeter.]

I: So, based on your explanation before, can you write an equation representing the perimeter?

S: I do not know [to write it].

Table 4.4. Results from data analysis of the interviews (N = 18)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
A. Amir and Tono together have Rp 30,000. If Amir's amount of money is Rp 4,000 more than Tono's, find each of their amounts	1 (6)	- misunderstand or ignore words, phrases, sentences (22)	Horizontal math: Understand the problem
		- errors in formulating equations, schemas, diagrams (83)	Formulate math model
		- mistakes in solution processes: ● ARITH: associative errors (6) ● EQS: notational errors (28) ● VAR: Unknown (11)	Vertical math: Math problem solving
		- checking the solution process (44)	Reflection
B. A rectangle has length and width $(3x - 4)$ cm and $(x + 1)$ cm, respectively. If the perimeter of the rectangle is 34 cm, find the area of the rectangle.	7 (39)	- misunderstand or ignore words, phrases, sentences (6)	Horizontal math: Understand the problem
		- errors in formulating equations, schemas, diagrams (50)	Formulate math model
		- mistakes in solution processes: ● ARITH: distributive (17), calculation errors (22), inverse errors (6). ● EQS: notational errors (6). ● AE: lack of closure (22) Expected answer (17) Parsing obstacle (6) Gestalt view (22)	Vertical math: Math problem solving
		- Checking the solution process (39)	Reflection

This excerpt shows that even if the student understood the problem, he could not formulate an equation because he was unable to represent the concept of the perimeter of the rectangle with an algebraic expression. The lack of closure obstacle might explain this inability to deal with algebraic expressions.

4.6. Conclusions and discussion

The research question addressed in this paper concerns identifying student difficulties in solving word problems in the topic of linear equations in one variable using a mathematization perspective. The results described in the previous section lead to the following conclusions. First, the main difficulties in students' written work after the Algebra Arrows lesson concern the solution processes and to a lesser extent, checking solutions. These findings suggest that the main obstacle concerns vertical mathematization and the mathematical problem solving and reflection subcategories in particular. Second, the main difficulties shown in the students' written work at the end of the Cover-up Strategy lesson concern understanding words, phrases or sentences; and formulating equations, schemas or diagrams. These findings suggest a lack of ability in horizontal mathematization, and understanding problems and formulating mathematical models in particular. Third, the findings from both lessons are confirmed by the results of the final written test: the difficulties in vertical mathematization emerge in student work on typical tasks of the Algebra Arrows activity, whereas the difficulties in horizontal mathematization appear in student work on typical tasks of the Cover-up activity.

How do we explain these differences? Factors that may explain the Algebra Arrows activity findings include: (i) the context for most of the tasks involved in this activity, namely number, is familiar to the students; (ii) the structure of the tasks is relatively operational in the sense that it gives an opportunity to translate them word-by-word into mathematical models; and (iii) the automatic calculation provided by the Algebra Arrows applet avoids calculation errors. As a consequence, students did not encounter serious difficulty in understanding problems and in formulating mathematical models, but found more obstacles in the paper-and-pencil solution processes and reflection—as they are not used to do calculations by themselves. These two mathematization subcategories characterize difficulties in vertical mathematization. However, our observation on student digital group work in the Algebra Arrows lesson suggests that the main obstacle encountered by students when dealing with word problems—in which the context concerns real life and not merely number—concerns understanding the problems and formulating mathematical models which characterize difficulties in horizontal mathematization. Factors that may explain the Cover-up activity findings include: (i) the contexts of tasks are various and are closer to real life than in the Algebra Arrows lesson; (ii) the structure of the tasks is complex and as such is difficult to translate directly into a mathematical model. As a consequence, students encounter obstacles in understanding problems and in formulating mathematical models. These two mathematization subcategories characterize difficulties in horizontal mathematization. All together, we conclude that the main difficulties encountered by students who deal with word problems concern transforming problems into mathematical models, i.e., in horizontal mathematization.

The data from the interviews confirm the above findings. Even if difficulties in the solution processes—which can be included in the subcategory of mathematical problem solving—appeared quite often, the most frequent difficulties revealed in the interviews concern formulating equations, schemas or diagrams. In the light of mathematization, these findings indicate that the main obstacle concerns horizontal mathematization and formulating mathematical models in particular.

As a discussion of these results, we might explain student difficulties in formulating mathematical models as an effect of the prevailing conventional teaching approach in Indonesia, in which students tend to do more routine bare algebra tasks than algebra word problems (e.g. Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002). As a result of this tradition, students may not acquire adequate mathematization skills. Furthermore, this teaching tradition often relies heavily on textbooks (Sembiring, Hadi, & Dolk, 2008). Future research on analyzing Indonesian textbooks on the topic of algebra might be fruitful to investigate if adequate resources are available for developing mathematization skills.

Concerning the effect of the ICT-rich approach, we conjecture that student difficulties in understanding problems, in formulating mathematical models, and to a lesser extent in symbolic mathematical problem solving are at least partially caused by a lack of a transfer between the digital and paper-and-pencil environments. On the one hand, students learn to deal with word problems with applets, in which immediate feedback and to some extent automatic calculations are available during the learning process; on the other hand, students are tested to do word problems with paper and pencil without feedback and automatic calculations. These are apparently two different conditions. For future research on developing better mathematization skills, it seems to be useful that feedback and automatic calculations in the applets gradually fade out in the applets and are varied in a systematic, didactical manner (Bokhove & Drijvers, 2012).

Finally, to improve the design of the learning arrangement, we retain the following ideas:

- To reduce student difficulties in transforming words, phrases, or sentences into mathematical expressions—such as translating, "the difference of distances between *A* and *B*; the sum of distances of *P* and *Q*; etc."—we suggest to give students more translation practices on this. In this way, they will become familiar with translating such phrases into appropriate mathematical expressions.
- As a didactical idea, to develop a better problem solving skills dealing with word problems, we suggest to use four subcategories of mathematization as a problem solving strategy in the learning and teaching processes (see section 4.2.2). In this way, student difficulties can be observed more easily and teachers can give appropriate help to students during the learning processes.
- To extend skills in solving word problems on the topic of equations in one variable, without being exhaustive we suggest to widen the scope of mathematical models of the problems: not only of the form $f(x) = c$ as addressed in the present research, but also of the forms $f(x) = g(x)$ and $f(x) = \frac{c}{g(x)}$. Doing so will provide a more comprehensive insight into student conceptual understanding of and difficulties with word problems.

Acknowledgment

This study was funded by the Indonesian Ministry of Education project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794. We would like to thank Jan van Maanen for his constructive comments and suggestions, as well as Marja van den Heuvel-Panhuizen for her valuable comments on an earlier version of this paper. We also thank the teachers and students for their participation as well as an external assistant for her contributions.

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Chapter 5 Improving grade 7 students' achievement in initial algebra through a technology-based intervention

Publication:

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (submitted). *Improving grade 7 students' achievement in initial algebra through a technology-based intervention*.

Abstract Digital technology plays an increasingly important role in daily life, mathematics education and algebra education in particular. To investigate the effect of a technology-rich intervention about initial algebra on the achievement of 12-13 year-old Indonesian students, we set up an experiment. The experimental group's intervention focused on equations in one variable and is characterized by an alternated use of paper-and-pencil and digital work, and by the intertwinement of word problems and bare algebra problems. The control group was taught in a regular way without digital tools. In total 266 students from eight classes in four schools took part in a pretest and a posttest. The results showed that the experimental group's (*N* = 139) gain score was significantly higher than the control group's (*N* = 127) score with a medium effect size. Also, a school factor was found to affect student achievement. The qualitative analysis of student written and digital work during the teaching experiment corroborated the quantitative results. Both results confirm the effectiveness of this type of technology-rich intervention for enhancing student achievement in algebra.

Keywords algebra education, digital technology, equations in one variable, secondary education

5.1. Introduction

Algebra is a core topic in secondary school mathematics curricula, and is recognized as a gateway to either advanced study or professional work in today's society (Katz, 2007; Kendal & Stacey, 2004). Mastering algebra is crucial for students' futures all over the world.

Indonesia is no exception to this. The 2007 Trends in International Mathematics and Science Study (TIMSS), however, shows that Indonesian students score low on algebra and were in 36th position out of 48 participating countries (Mullis, Martin, & Foy, 2008). In TIMSS 2011, similarly, Indonesian students were ranked 38th out of 42 countries in the domain of algebra (Mullis, Martin, Foy, & Arora, 2012). This raises the question of how to improve Indonesian student performance in the algebra domain. What are possible approaches to enhancing students' conceptual understanding and skills in algebra, and how effective would such an approach be?

Information and Communication Technology (ICT) plays an increasingly important role in daily life, in education, and mathematics education in particular. The National Council of Teachers of Mathematics (NCTM), for instance, claimed that "technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology" (NCTM, 2008, p.1). Also, there is research evidence that the use of ICT can have positive effects on students' mathematics achievement (Li & Ma, 2010) and perception of mathematics (Bakker, Van den Heuvel-Panhuizen, & Robitsch, in press; Barkatsas, Kasimatis, & Gialamas, 2009), and can support students in both exploring and expressing mathematical ideas (Ghosh, 2012). In algebra education, ICT use contributes significantly to its learning and teaching (Rakes, Valentine, McGatha, & Ronau, 2010). For example, the use of digital tools in algebra education can promote students' development of both symbol sense and procedural skills (Bokhove & Drijvers, 2010b), can be effective for improving conceptual understanding and procedural skills of secondary school students (Bokhove & Drijvers, 2012), and may foster the development of the notion of the function concept (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012). Furthermore, the use of a digital environment can support students' mathematical problem solving skills and can contribute to their ability in solving informal algebra problems (Kolovou, Van den Heuvel-Panhuizen, & Köller, 2013; Van den Heuvel-Panhuizen, Kolovou, & Robitzsch, 2013).

In response to the worldwide use of technology in education, the Indonesian Ministry of National Education set up a policy that introduces ICT as a new subject for secondary schools (Depdiknas, 2007). Furthermore, the curriculum documentation suggests to integrate the use of ICT in all school subjects, including in mathematics. Even if ICT is not a panacea for all problems, and its impact is subtle and depends on the learning environment (such as technological infrastructure, task design and didactical approach), this integration is expected to enhance the quality of the learning and teaching of mathematics, and in particular to improve student achievement. However, the integration of digital tools in mathematics teaching is relatively new in Indonesia (PPPPTK Matematika, 2013). As a consequence, the potential of ICT for enhancing the quality of mathematics and algebra education in Indonesia is still unexploited.

The above considerations led us to set up an ICT-rich teaching experiment aiming to improve Indonesian student achievement in algebra and in the domain of equations in one variable in particular. In this paper, we report on the results of this enterprise. We first describe the context of the study. Next, we elaborate on a theoretical background, addressing difficulties in initial algebra, the role of ICT in the teaching and learning of algebra, and the research question. Then, we elaborate on the research method. The results section presents the quantitative and qualitative results of the study. Finally, the conclusions are summarized and discussed.

5.2. The context of the study

To understand the context of the study, we first provide some background information about mathematics education, teacher education, teaching practices, and the use of technology in Indonesia. As a developing country, Indonesia is struggling in enhancing the quality of its education, as is reflected in the low TIMSS scores mentioned above. In international comparative studies Indonesian student performances were in the lower positions. For example, in the recent 2011 TIMSS, the Indonesian students' performance in mathematics was ranked in 38th place out of 42 countries, and in the domain of science its position was ranked as 40th out of 42 (Martin, Mullis, Foy, & Stanco, 2012). To explain the low Indonesian results for mathematics and algebra on international comparative tests, we have proposed two hypothetical reasons (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014a). First, in spite of mathematics curriculum revisions in the last decades, most mathematics teaching still seems to be traditional (e.g., Sembiring, Hadi, & Dolk, 2008), in the sense that 'drill-and-practice' approaches are prevalent, including memorization of algebraic formulas. As such, there is a discrepancy between the intended and the implemented curriculum. Second, Indonesian students immediately start to learn algebra in a formal way, in the first semester of grade

VII (Depdiknas, 2006), whereas the students did not learn any algebraic topic at primary school level. As a consequence, they are not prepared to study algebra. According to Mohandes (2000), the low Indonesian performances in the TIMSS studies in both science and mathematics are caused by student absenteeism and the large number of subjects that is taught in school.

Teacher education in Indonesia consists of a four-year bachelor program. For mathematics teacher education, the content of this program includes mathematics courses (70%), didactics courses (20%), and general courses, including a six months internship in a school and a bachelor thesis. Prospective secondary school teachers in the mathematics education program obtain limited training for the use of digital technology in courses with titles like *Multimedia in Mathematics Teaching course*. Practicing mathematics teachers may receive such trainings from the center of in-service teacher training (PPPPTK[1]). With a bachelor degree in mathematics, one can also become a licensed mathematics teacher after a one-year teacher training program.

Concerning Indonesian teaching practice, there are 35-45 students in an average class. Due to this large class size, class management is an important issue for teachers. The teaching approaches which are prevalent in Indonesian classrooms are teacher-centered (Sembiring, Hadi, & Dolk, 2008). In mathematics lessons, for instance, the common lesson structure is as follows: the teacher explains the mathematical concept under consideration, next he or she gives some worked examples relating to the concept, provides an exercise, and finally closes the lesson and sets homework; students listen, take notes, and do exercises.

As ICT was introduced as a new compulsory subject for secondary schools in 2007, computer laboratories are now available in most secondary schools. However, the integration of ICT in other school subjects, such as mathematics, is not yet mandatory (Depdiknas, 2007). This means that computers are mainly used for the subject of ICT. Other subjects can only use the computer laboratory to a limited extent. In mathematics education, calculators are not permitted in courses and in formative and summative tests; nevertheless, digital tools are occasionally used in the lessons. This limited integration of ICT in education is in contrast with what is happening in Indonesian society. As the fourth most populated country in the world, Indonesia is a big market for various technological products from, for instance, Japan, South Korea, China, European countries, and United States. Furthermore, more than 80

[1] PPPPTK is the center for in-service training, and it is under Indonesian Ministry of Education. It is responsible to give training to teachers all over Indonesia. With this responsibility, the teachers that are trained are selected, and usually representative of other mathematics teachers.

million Indonesians are now accessing the internet (Kemkominfo, 2014) for various purposes, such as for mobile communication with messenger services and for social media.

5.3. Theoretical framework and research question

The theoretical framework of this study concerns both difficulties in initial algebra, and the role of ICT in algebra education.

5.3.1. Difficulties in initial algebra

As a first step before being able to improve algebra education, we should have a clear view on what is hard in initial algebra. Therefore, based on an earlier literature review study (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014a), we identified five categories of difficulties in initial algebra. First, difficulties in applying arithmetical operations in numerical and algebraic expressions (abbreviated as ARITH) concern difficulties in adding or subtracting similar algebraic terms (e.g., Herscovics & Linchevski, 1994; Linchevski, 1995); in applying associative, commutative, distributive, and inverses properties; and in applying priority rules of arithmetical operations (e.g., Booth, 1988; Bush & Karp, 2013; Warren, 2003).

Second, difficulties in understanding the notion of variable (VAR) include different views on the different roles it can play: the role of a placeholder, a generalized number, an unknown, or a varying quantity (Booth, 1988; Bush & Karp, 2013; Herscovics & Linchevski, 1994).

Third, the difficulties in understanding algebraic expressions (AE) include the parsing obstacle, the expected answer obstacle, the lack of closure obstacle, and the gestalt view of algebraic expressions (Arcavi, 1994; Tall & Thomas, 1991).

Fourth, the difficulties in understanding the different meanings of the equal sign (EQS) concern difficulties in dealing with the equal sign, which usually invites a calculation in arithmetic, while it is a sign of equivalence in algebra (Bush & Karp, 2013; Herscovics & Linchevski, 1994; Kieran, 1981).

Fifth and final, the category of mathematization (MATH) concerns the difficulty to transform any kind of reality in the world of the problem situation to the world of mathematics and vice versa, and to reorganize and to (re)construct the symbolic world of mathematics (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). By 'reality' we mean real life, but also mathematical situations that are meaningful and imaginable in mind (Freudenthal, 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2000; 2005; Van den Heuvel-

Panhuizen & Drijvers, 2013). The activity of transforming a realistic problem into a symbolic mathematical problem through observation, experimentation, and inductive reasoning is called horizontal mathematization, while the activity of reorganizing and (re)constructing within the world of symbols which includes solving the problem, generalization of the solution and further formalization is called vertical mathematization (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). These two activities are complementary during the process of mathematical activity (De Lange, 1987).

The aforementioned five categories serve as a point of departure for analyzing observable student difficulties in learning of algebra. To better understand and explain the background of these difficulties, in a further study we use the lens of operational and structural views on algebraic activity (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014b). This lens originates from Sfard's theory of reification, that is, a transformation of a process performed on an accepted mathematical object to become a new object. According to Sfard (1991), an abstract notion, such as an algebraic expression, can be perceived in two different complementary ways: operationally as a process and structurally as an object. For example, the equation $\frac{x-3}{5}+4=11$, can operationally be conceived as a series of processes of arithmetical calculation, i.e., subtract 3 from a certain given number (x), next divide by 5, and finally add 4 to get 11; and it can structurally be seen as an equivalence between two objects, namely the algebraic expressions $\frac{x-3}{5}+4$ and 11. According to Drijvers (2003), the flexibility in switching this process and the object view on algebraic expressions simultaneously signifies a mature understanding of mathematical thinking. In this study, this structural-operational duality is used to understand student activity while solving equations, as well as the difficulties encountered while doing so.

5.3.2. The role of ICT in the learning and teaching of algebra

In which way might the use of ICT contribute to the learning and teaching of algebra? In answering this question, Drijvers, Boon and Van Reeuwijk (2010) distinguish three didactical functions of technology in algebra education: a tool for doing mathematics, an environment for practicing skills, and an environment for developing algebraic concepts. In the first function, technology acts as an assistant to carry out algebraic routine procedures, such as expanding algebraic expressions and drawing graphs, and the user does not necessarily know and understand how the technology produces the outcomes, but may be triggered to find this out. The reconciliation of ICT-tool techniques and paper-and-pencil methods may be a particular educational challenge (e.g., Kieran & Drijvers, 2006). In practice, while doing algebra, a student has the initiative on whether or not to use the technology to carry out the routine procedural work: s/he is probably able to carry out the routine procedures by hand, but chooses not to spend energy on that and to outsource the work to the technology (Drijvers et al., 2010).

As an environment for practicing algebraic skills, the second function of technology in algebra education, technology may offer immediate feedback to students' responses, solutions and strategies (e.g., Bokhove, 2010). In this function of technology, randomization of tasks may allow for variation of tasks and to avoid repetition (e.g., Bokhove & Drijvers, 2010b). Moreover, the compatibility of problem solving strategies in the technological environment and in the paper-and-pencil environment is crucial for transfer of notation and skills to the latter environment (e.g., Bokhove & Drijvers, 2010a). In practice, a student can determine by her- or himself, depending on her or his ability in mastering the skills, when to use the technology or not for practicing (Drijvers et al., 2010).

In the third function, as an environment for developing concepts and mental models, technology aims to evoke a specific thinking process and to guide the development of algebraic thinking. In doing so, the technology, for instance, helps to visualize a concept of an equation and to present it in a dynamic way (e.g., Drijvers & Barzel, 2012); and helps to generate various examples for provoking exploration and generalization (e.g., Kieran & Drijvers, 2006). According to Beeson's glass box principle (1998), the transparency of the representations and techniques of the ICT environment is crucial for fostering conceptual understanding, because it provides an opportunity for students to perceive how the technology produces mathematical outcomes. In practice, this didactical function is guided by the teacher, as this function of technology requires a careful didactical analysis of the relationship between the use of the tool with its representations and techniques on the one hand, and the mathematical thinking and skills that the students are supposed to acquire on the other (Drijvers et al., 2010).

For the purpose of this study, i.e., improving student achievement in initial algebra, the use of technology is devoted to the second and the third function: practicing algebraic skills and developing algebraic concepts. Ideally speaking, these two functions go hand in hand and are supported by ICT in an integrated way: conceptual understanding underpins the acquisition of skills, and the mastery of procedural skills, in turn, may strengthen the conceptual understanding. Therefore, we look for activities with ICT tools in which the representations and actions are closely related to the targeted conceptual development; next, problem solving activity should be routinized as to foster procedural skills. As we describe in section 5.4.2.1, these considerations guided the choice for specific applets and for specific tasks.

5.3.3. Research question

This study addressed the following research question:

Does an intervention with digital technology enhance students' performance in initial algebra?

This general research question was specified as follows:

- The intervention in this study concerns a teaching arrangement in which digital work and paper-and-pencil work are used alternately (a blended approach). Also, bare algebra tasks and word problems are used in an intertwined way, rather than addressing application at the end of the learning process.

- The digital technology in this study is the Digital Mathematics Environment (DME) which is developed at the Freudenthal Institute, Utrecht University, the Netherlands, and four applets in particular, called Algebra Arrows, Cover-up Strategy, Balance Model, and Balance Strategy. These four applets provide opportunities for both concept development and procedural work, and are described in more detail in section 5.4.2.1.

- The domain of initial algebra addressed in this study includes a central topic in this domain, i.e., equations in one variable and the related word problems, which is in line with the content of the Indonesian mathematics curriculum. In this curriculum, algebra is introduced to grade VII students (12-13 year-old). The algebra topics in this grade include linear equations and inequalities in one variable, simplifying algebraic expressions, proportions, and sets (Depdiknas, 2006). At the primary school level, students were taught arithmetic and geometry, but no algebra. Therefore, the topic of linear equations in one variable is new for the participants in this study.

As a conjecture, we hypothesize that students who experience an intervention with the digital technology will outperform their peers who are engaged in regular learning settings. In addition to this, we wonder whether a school factor may play a role. Even if the role of the teachers is important in the intervention, the focus of this study concerns the alternated and integrated use of digital technology and paper-and-pencil, and the task design, i.e., the balanced use between bare problems and word problems.

5.4. Methods

To investigate the influence of the ICT-rich intervention on students' algebra performance in the topic of equations in one variable, a pretest-posttest-control-group experiment was set up. In this section we address the sample, the instruments and the learning environment, the intervention procedure, and the data collection and analysis.

5.4.1. Sample

In total, 266 grade seven students (12-13 year-old) were involved in this experiment: 139 students were in the experimental group and 127 students in the control group. The experimental group included four classes from two schools (two classes from each school), and the control group also included four classes, from two other schools (also two classes from each school). The schools were all located in one of the regencies – including sub-urban and rural areas in a balanced manner – in the Central Java province, in Indonesia; all schools came from similar socioeconomic backgrounds, and followed the same Indonesian mathematics curriculum.

Indonesian education authorities have a school accreditation system, according to which schools are graded. The highest grade is *A* (excellent). In the case of the religious schools in this study, schools with grade *A* usually have a good reputation because they have various academic and non-academic achievements and are chaired by respected clerics. As a consequence, parents tend to send their children to *A*-certified schools. Of the schools in this study, one school in the experimental group and one school in the control group are *A*-certified schools and the other two schools are not. We assume that the *A*-certified schools and the non-*A*-certified schools are representative of other schools as the certification is officially conducted using the same criteria. So the classes at *A*-certified schools are expected to be representative for other *A*-certified schools, not for all Indonesian classes. Taking this school categorization into consideration, which from here onward is called School type, we wondered whether it would influence student achievement.

The background of classifying School type into *A*-certified and non-*A*-certified school is as follows. Initially, we classified the schools as respectively favorite and non-favorite schools. A favorite school, according to people's perception, is a school that has a good reputation because, inter alia, it is chaired by a respected cleric (an influential figure in the community), has various academic and non-academic achievements, and has good infrastructure and facilities. However, this categorization is difficult to measure. Therefore, we used the accreditation grade which is officially used as a measure of school quality in Indonesia.

5.4.2. Instruments and learning environment

The learning arrangement consisted of four applets with digital tasks for the learning of equations in one variable embedded in the DME, the daily intermediate paper-and-pencil assessment tasks, and the paper-and-pencil tasks for the pretest and posttest. A teacher guide informs the teacher about this learning environment and the activities to be done.

5.4.2.1. Applets

The designed learning sequence included activities with four applets: Algebra Arrows, Cover-up Strategy, Balance Model, and Balance Strategy. These four applets are embedded in the Digital Mathematics Environment (DME), which is a web-based electronic learning environment. The DME and the four applets have been developed by Peter Boon at the Freudenthal Institute (Boon, 2006). The DME provides: (i) interactive digital tools for algebra, geometry, and other domains; (ii) a design of open online tasks and appropriate immediate feedback; (iii) access to the environment at any time and place, as long as technological infrastructure and conditions are met, and (iv) a storage for student work (Drijvers, Boon, Doorman, Bokhove, & Tacoma, 2013). According to four groups of criteria (algebra didactics, theories on tool use, assessments, and general characteristics of digital tools), DME is considered suitable for research in algebra education addressing the co-emergence of procedural skills and conceptual understanding (Bokhove & Drijvers, 2010a). The DME is found to be suitable for algebra education because of the mathematical soundness, i.e. the correct display of algebraic notations, the ease of use, and the storage of student work.

The Algebra Arrows applet

The Algebra Arrows applet is designed to offer students the possibility to construct and use chains of operations on numbers and algebraic formulas, and as such to foster students' view of function as an input-output chain of operations representing a dependency relationship (Doorman et al., 2012). If $f(x)$ is a given function, $f(x) = c$ represents an equation in one variable and it can be interpreted as: which input value in the chain of operations defined by f provides c as an output value? Figure 5.1 shows how the equation $\frac{a-9}{7} = 10$ is solved using this interpretation and the Algebra Arrows applet. Solving an equation through this applet, then, comes down to applying a reverse strategy, that is, a process of undoing a series of operations. As such, in line with the operational view on algebraic expressions, this applet invites students to see an equation as a series of operational process (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014b).

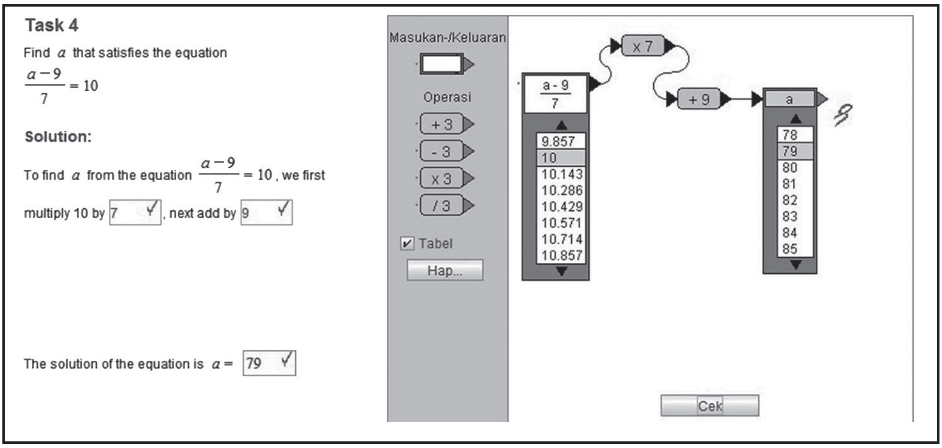


Figure 5.1. Equation solving with the Algebra Arrows applet applying the reverse strategy

The Cover-up Strategy applet

The Cover-up Strategy applet was designed to solve equations in one variable of the form $f(x) = c$ through global substitution (Wenger, 1987) by subsequently selecting a part of the expression in an equation with the mouse and finding its value. Figure 5.2 shows a scenario of using the Cover-up applet to solve the equation $\frac{18}{5a-2} = 6$. In step 1, a student follows the first hint provided, namely choosing a part of the equation that should be highlighted first. In step 2, the student highlights the expression $5a - 2$ and the expression $5a - 2 = ...$ appears automatically in the next line. In step 3, the student fills in and the applet gives a yellow tick mark signifying a correct action (otherwise a cross mark emerges). This scenario continues until step 6 and ends with $a = 1$ as the solution of the equation (signified by a green tick mark and the final feedback: “The equation is solved correctly!”). In practice, a student does not necessarily follow all these six steps, but may also take shortcuts, such as going directly to step 5 after step 3.

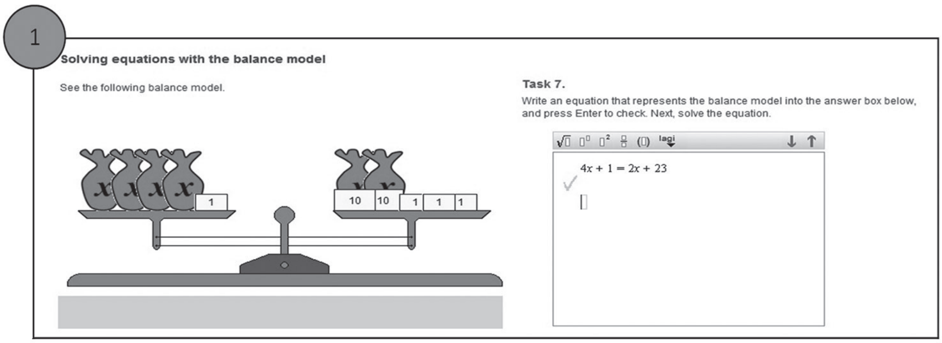
In applying the cover-up strategy, a student should first see the equation as an equivalence of two objects (algebraic expressions). Next, s/he should be able to identify a sub-expression within the equation that is to be covered and will be assigned a numerical value. In this way, in line with the structural view on algebraic expression, this strategy seems to invite students to have a structural

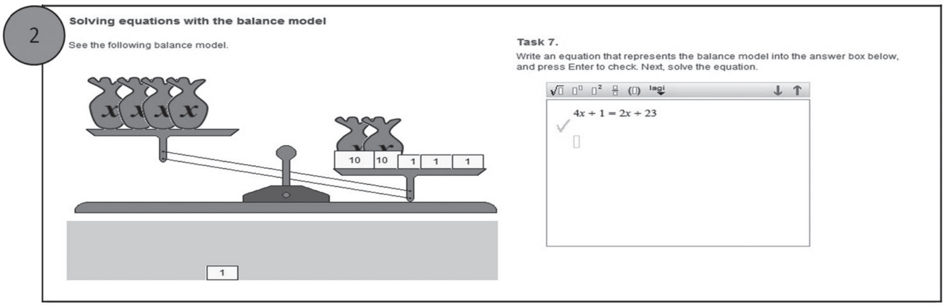
Figure 5.2. An equation solving scenario using the Cover-up Strategy applet

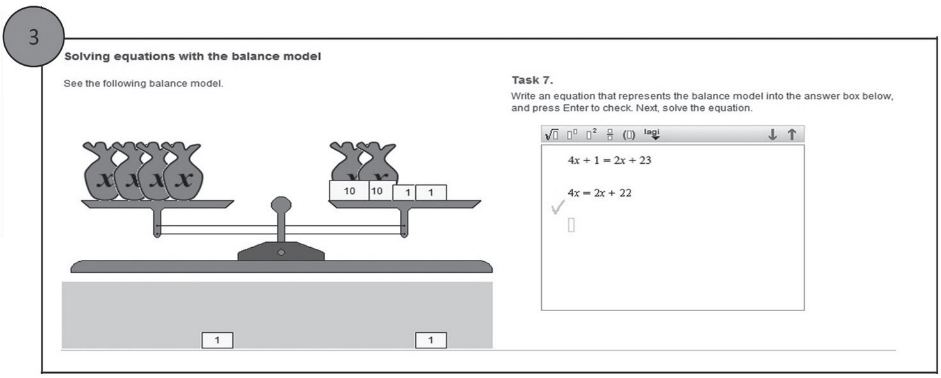
view on equations and expressions (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014b). This is the main reason to use the Cover-up Strategy applet after the Algebra Arrows applet. Another reason is that the Cover-up applet can be used to solve a wider repertoire of equations than just linear ones, which was not the case for Algebra Arrows applet.

The Balance Model applet

The Balance Model is an applet that provides equations and the corresponding virtual dynamic models which can be used for solving the equations. In this case, it applies to linear equations in one variable of the form $f(x) = g(x)$, i.e., equations with the variable appearing in both sides of the equations.







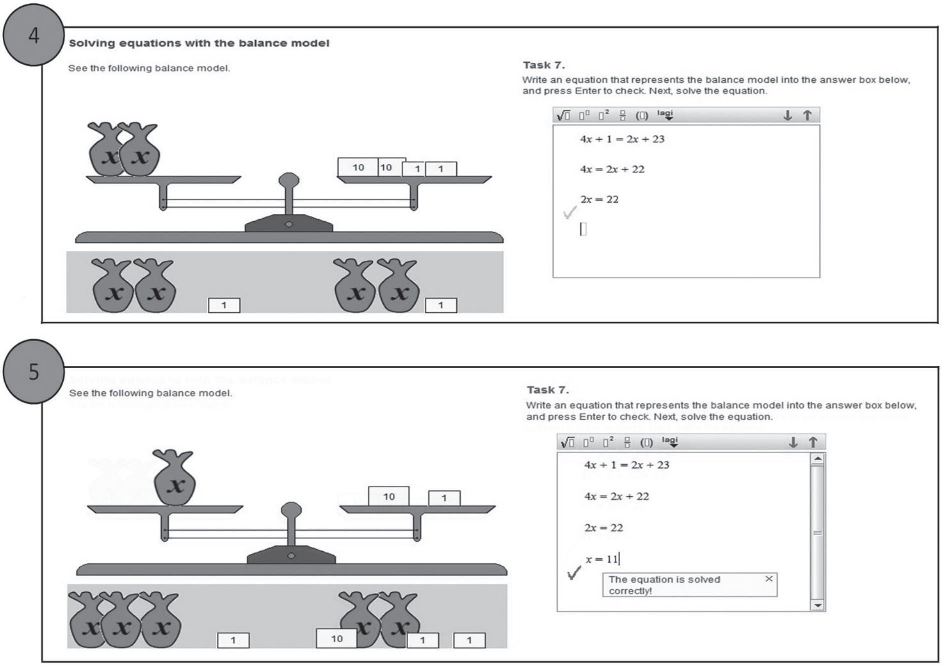


Figure 5.3. An equation solving scenario with the Balance Model applet

Mathematically, this type of equations is an extension of linear equations of the form $f(x) = c$ addressed in the activities in the Algebra Arrows and Cover-up applets. Therefore, this Balance Model applet activity was placed after the previous two activities.

Figure 5.3 shows an equation solving scenario with the Balance Model applet. In step 1, given a balance model, a student is required to write an equation that represents the model. In step 2, the student moves one block (representing the value 1) as an initial step to solving the equation $4x + 1 = 2x + 23$. Step 3 represents a condition in which the student has moved a block of value 1 from each side of the equation. This scenario proceeds until step 5 which ends with $x = 11$ (signified by the emergence of feedback: "The equation is solved correctly!"). Again, a student does not necessarily follow all these steps consecutively and s/he may find shortcuts. The students are expected to perceive solving the equation both as a process of applying the same operation to both sides, and as an equivalence of two algebraic equations; operational and structural perspectives are intertwined.

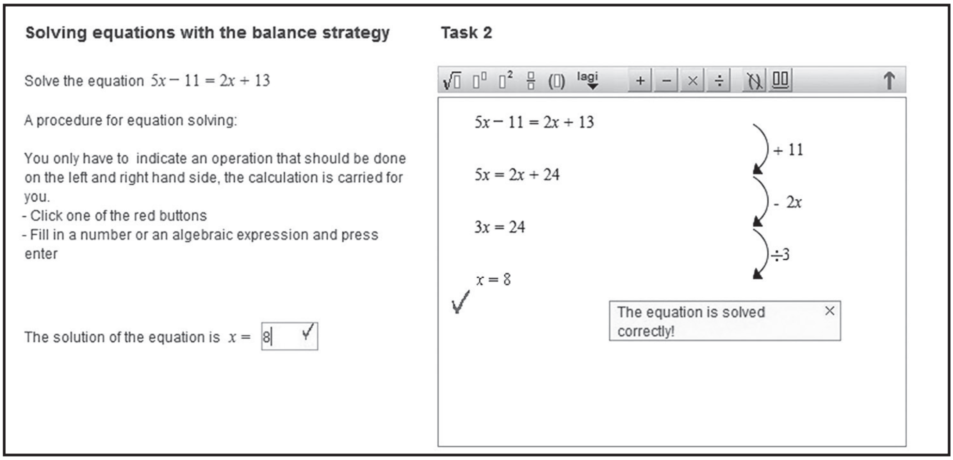


Figure 5.4. Equation solving with the Balance Strategy applet

The Balance Strategy applet

The Balance Strategy applet can be used to solve linear equations in one variable of the form $f(x) = g(x)$ using strategies that have been developed with the Balance Model – namely do the same operations on both sides of equations – without providing models in the solution processes. As a design decision, this applet provides an abstraction of the Balance Model applet, and is therefore used after it. Figure 5.4 shows how the equation $5x - 11 = 2x + 13$ is solved through the Balance Strategy applet. Equations can be provided in the task, or can be set up by the student from word problems. Similar to the case of the Balance Model applet, both operational and structural views on equations play an important role in this applet.

5.4.2.2. Type of tasks

The tasks addressed in the intervention consisted of two types: bare problems and word problems. Bare problems are tasks that are not related to contexts either within mathematics or other subjects, such as $5x - 11 = 2x + 13$, whereas word problems are. To develop an integrated and balanced view on the topic, and to not only consider word problems at the end of the teaching sequence, students work on these two types of problems alternately. Of course, throughout the intervention, these problems are ordered from relatively easy to more difficult.

5.4.2.3. Daily intermediate assessment, pretest and posttest

The daily intermediate assessment was carried out for 15-20 minutes in each of four lessons, in which each student was individually required to write down and to show his or her solution on paper. The reason to use paper-and-pencil work is that students in Indonesia are not allowed to use technology in formative and summative tests. Table A.1 (Appendix 5.1) presents tasks used in daily intermediate assessments at the end of each of the applet activities.

The tasks for the pretest and the posttest were the same. However, we used some different words or phrases in the questions, such as: rather than using the word "equation", we used the word "an expression" in the pretest, and rather than using the phrase "find a solution" we used the phrase "find a value of ..." in the pretest. The pretest and posttest were each administered for 60 minutes, in which each student was required to write down and show his or her solution on paper individually. Table A.2 (Appendix 5.2) presents the tasks used in the pretest and posttest. The Cronbach's alpha for the pretest was $\alpha = 0.65$ (acceptable); for the posttest it was $\alpha = 0.81$ (good).

5.4.3. Intervention procedure

The intervention for the experimental group included: (i) an individual 60-minutes paper-and-pencil pretest; (ii) four 80-minutes lessons on equations in one variable, partly in whole-class teaching and partly in groups (of 3-5 students with mixed abilities) in which the students work on the designed tasks making use of the four applets described above; and (iii) an individual 60-minutes paper-and-pencil posttest, similar to the pretest. Each of the four lessons ended with a daily intermediate paper-and-pencil assessment after each activity with the applet. Even if the results of these daily intermediate assessments in this study did not contribute to students' final grades as they were not reported to the teachers, students worked on these tasks seriously. Probably, they expected the results would contribute to their grade, as is usually the case in similar situations.

The control group worked on a 60-minutes paper-and-pencil pretest, attended the regular teaching on equations in one variable without digital technology but including exercises and daily tests, and a 60-minutes paper-and-pencil posttest, similar to the pretest. In the control condition lessons, the teacher explained the concept of equations in one variable with the corresponding examples and provided exercises, while the students took notes and did the exercises. The control condition and the experimental condition share the topic, the whole-class instruction by the teacher, and the daily written assessment; the differences concern the intertwinement of bare algebra tasks and word problems—in the control condition word problems are used as applications of the concept at the end of the learning process, and the alternation of working with the applets and working with paper and pencil.

In total, the duration of the experiment was six meetings: two meeting in which the students were tested and four lessons (see Table 5.1). The teacher organized these three parts according to the teacher guide. In the paper-and-pencil activity, the teacher posed problems and guided whole-class discussion. In the digital activity, the teacher demonstrated how to work with the applets, guided students into the group digital activity, and led discussion. While the teacher demonstrated techniques for use of an applet, students (in groups) followed the demonstration. For accessing the DME during group work, each group was given a unique account so that the digital work was stored and could be retrieved either by the teacher or by the researcher for the analysis. Next, students were requested to individually do the paper-and-pencil daily intermediate assessment tasks. Finally, the teacher guided students to reflect upon the lesson. In carrying out the teaching intervention, the teachers were supported by a teacher guide provided by the researcher. The teacher guide contains teaching strategy steps (introduction, demonstration, digital activity, and daily intermediate assessment), problems to pose in the whole class discussion with corresponding solutions and predictions of student responses, problems in the applets with the corresponding answers, problems for daily intermediate assessments and the corresponding solutions, and a guide for accessing the DME and the applets. This teacher guide was explained to the teachers by the researcher prior to intervention. The teachers in the intervention classes had already been involved in the small-scale pilot experiment with the Algebra Arrows and Cover-up applets in the previous year with other students (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014b; Jupri & Drijvers, accepted). In case that the teachers encountered technical obstacles during the demonstration of the applets or during the digital group activity, an external research assistant was available to give help.

Table 5.1. Intervention set-up

Meeting	Intervention	Experimental group	Control group
1	Pretest: Individual written test	✓	✓
2	Lesson 1: Algebra Arrows activity	✓	
3	Lesson 2: Cover-up Strategy activity	✓	
4	Lesson 3: Balance Model activity	✓	
5	Lesson 4: Balance Strategy activity	✓	
6	Posttest: Individual written test	✓	✓

During the intervention lessons, the researcher, while video-taping one group of students in each class for the purpose of data collection, helped these students by acting as a substitute teacher, while the teacher took care of the other groups of students. In this way, each group received appropriate guidance during the learning process. The group that was video-taped in each class was based on the teacher's recommendation, i.e., consisted of mixed ability students who were communicative in front of a camera during the intervention. By mixed ability students we mean students with heterogeneous mathematical abilities. One observed group consisted of female students only, and the other three observed groups are all male students. Even if female and male students are mixed in experimental classes, there are no mixed gender groups – this is common in religious schools in which the teachers decide on group composition.

5.4.4. Data collection

The data that were collected from each of the control groups consisted of individual written student work from pretest and posttest. In addition, the data that were collected from each experimental group consisted of video registrations of one group, student digital work, student written work from four daily intermediate assessments, and field notes. Table 5.2 provides an overview of data collected from the experiment.

Table 5.2. Overview of data

Class	Types of data				
	Pretest	Posttest	Digital DME data lessons 1-4	Video lessons 1-4	Written daily intermediate assessment lessons 1-4
Experimental 1	✓	✓	✓	✓	✓
Experimental 2	✓	✓	✓	✓	✓
Experimental 3	✓	✓	✓	✓	✓
Experimental 4	✓	✓	✓	✓	✓
Control 1	✓	✓			
Control 2	✓	✓			
Control 3	✓	✓			
Control 4	✓	✓			
Total (N)	263	266			

5.4.5. Data analysis

To analyze the quantitative data, we used statistical methods with the help of SPSS software. First, as the tasks in the pretest and posttest were the same, we used gain scores, the difference between posttest and pretest scores, as the dependent variable. The Kolmogorov-Smirnov and Shapiro-Wilk tests showed that Gain was not normally distributed ($p < .01$). However, as the sample size is large and the *P-P* and *Q-Q* plots do not show crucial variations, we applied parametric techniques to analyze the data, including *t*-test and

ANCOVA (Field, 2009). A further exploration of the collected data showed that there was a significant difference between the pretest scores of the experimental and the control group ($t(199) = 5.93$, $p < .001$). Also, there was a significance difference in the pretest scores for *A*-certified and non-*A*-certified groups ($t(248) = 2.26$, $p < .05$).

Second, using *t*-tests for two independent samples, we compared the gain scores for the experimental and control groups. Third, an ANCOVA was carried out with School type (i.e. *A*-certified and non-*A*-certified schools) as a covariate. Fourth, to investigate relationships between the achievements in the daily intermediate assessments and the output variables, a correlation analysis was carried out. Finally, a descriptive qualitative analysis on written and digital student work as well as observations during the lessons in experimental conditions was carried out with the help of Atlas.ti software, to complement the aforementioned quantitative analysis.

5.5. Results

This section addresses the results of both quantitative and qualitative analyses. The quantitative results, provided in sections 5.5.1–5.5.3, include a comparative analysis of gain scores between the control and experimental groups; the effect of the School type on student achievement; and the correlation between the results of daily intermediate assessment tests, pretest and posttest scores. The qualitative analysis, presented in section 5.5.4, includes an analysis of student written work which is backed up with the observation data during the intervention.

5.5.1. Overall comparison between experimental and control groups

Figure 5.5 shows a bar graph of the mean gain score for each class of the experimental and control group. The graph shows that the classes in the *A*-certified schools had better gain scores than those in non-*A*-certified schools and that within the *A*-certified schools and the non-*A*-certified schools, the classes in the experimental group benefited more from the intervention than the classes in the control group.

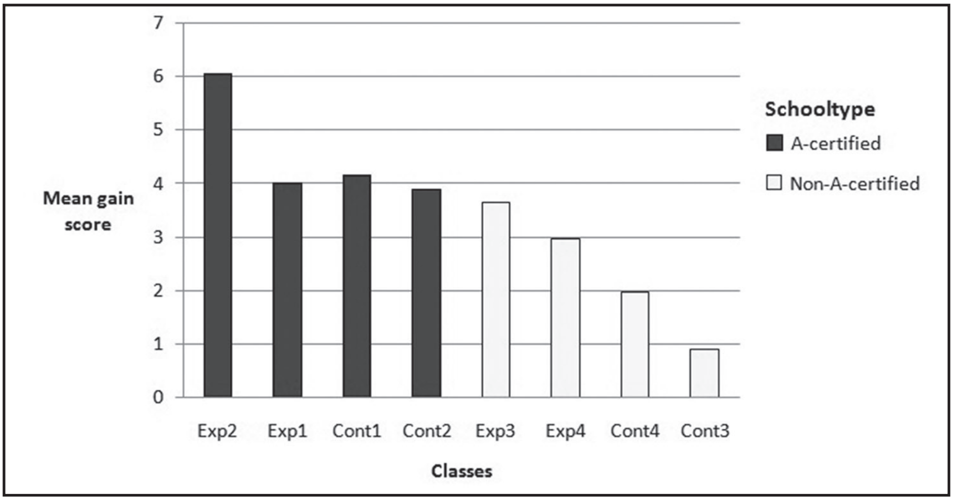


Figure 5.5. Mean gain for the experimental and control classes

Table 5.3 provides the descriptive statistics of the mean gain scores and the result of the *t*-test for independent samples grouped by the condition. The results show that the mean gain score of the experimental group was significantly higher than that the control group (Cohen's $d = 0.32$). As such, the hypothesis that the experimental group would outperform the control group was confirmed by the data.

Table 5.3. Mean gain score in experimental and control group, t-test result and effect size

Condition	Number of students	*M* gain score	*SD* gain score	*t*	*p*	*d*
Experimental group	131	4.6293	2.4057	t=-5.23	< .001	.32
Control group	119	3.0233	2.4459	*df* = 248		

5.5.2. The effect of school type

A-certified schools have a selective admission procedure, and, as a consequence, have better students than non-*A*-certified schools. Even if better students are not necessarily good in mathematics, they are often highly motivated in the learning process. As such, one might conjecture that students from *A*-certified schools will benefit more from the intervention than students from non-*A*-certified schools. To investigate this, the data were split up with respect to the School type variable: two are *A*-certified schools and the other two are not.

Table 5.4 shows the mean gain score in the experimental and control group for each school type. In the experimental group, the students from the *A*-certified schools benefited more from the intervention than the students from the non-*A*-certified schools. However, also students from the non-*A*-certified schools gained from the intervention. Within these schools the students in the experimental group did better than the students in the control group. Nevertheless, the experimental students from the non-*A*-certified schools had a lower mean gain score than students from the *A*-certified schools in the control group. These results suggest that even if the intervention affects student achievement, the school type is also an important factor influencing the results.

Table 5.4. Mean gain score in experimental and control group per school type

Condition	School type			
	A-certified		Non-*A*-certified	
	Mean gain score	Number of students	*Mean* gain score	Number of students
Experimental group	5.3263	79	3.5705	52
Control group	4.2413	64	1.6061	55

Table 5.5 shows the results of the ANCOVA test on the dependent variable Gain (mean gain score), with Condition (experimental or control group) as the independent variable and School type (*A*-certified or non-*A*-certified) as covariate. The results show that both Condition and School type had a significant effect on the improvement of the students' achievement with small to medium effect sizes. However, there was no significant interaction between Condition and School type. In other words, the hypothesis that students in *A*-certified schools would benefit more from the intervention than students in non-*A*-certified schools was not confirmed.

Table 5.5. Results from the ANCOVA test on the dependent variable Gain, with Condition as the independent variable and School type as covariate

Condition	Number of students	Gain	Main effect Condition	Main effect School type	Interaction effect
Experimental group	131	4.629	$F(1,247) = 28.13$ $p < .001$ $d = 0.32$	$F(1,247) = 61.39$ $p < .001$ $d = 0.45$	$F(1,247) = 2.51$ $p = 0.12$
Control group	119	3.023			

5.5.3. Correlations between daily intermediate assessment, pretest and posttest

Table 5.6 presents correlations between daily intermediate assessment scores (ScoreL1, ScoreL2, ScoreL3, and ScoreL4), pretest and posttest scores. All correlations are significant and positive.

Table 5.6. Pearson correlations between intermediate assessment scores and output variables

	Pretest	ScoreL1	ScoreL2	ScoreL3	ScoreL4	Posttest
Pretest	1	.388**	.379**	.375**	.229**	.423**
ScoreL1		1	.581**	.523**	.375**	.475**
ScoreL2			1	.618**	.357**	.436**
ScoreL3				1	.391**	.455**
ScoreL4					1	.260**
Posttest						1

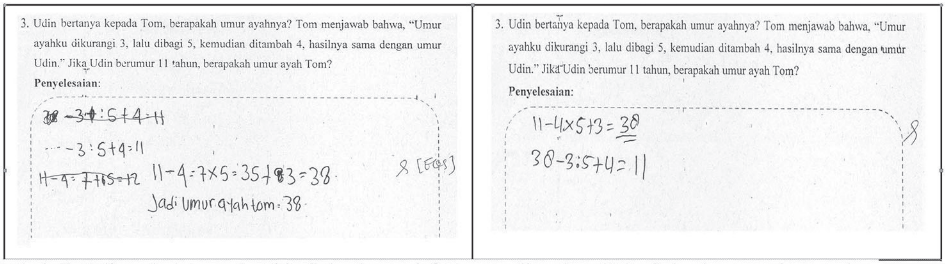
** Significant at the 0.01 level (1-tailed).

5.5.4. Illustrative student work during the intervention

To illustrate the improvement of student performance, as shown in the quantitative results, we describe the work of one group of students during the four lessons of the experimental intervention. This group consists of five 12-13 year-old male students with mixed ability from an *A*-certified school and is considered to be representative of other groups that were video-taped during the intervention. In this paper, these five students are named Saiful, Danang, Rafi, Syafii and Taufiq. For each lesson, we start the description with a typical task from the lesson, taken from the daily intermediate assessment, and the student results. We interpret these results in terms of the theoretical framework described in section 5.3., and if necessary back this up with appropriate evidence from the lesson observation. Next, we look at the results of these students' written work on a task from the pretest and posttest that is quite similar to the one from the daily intermediate assessment.

5.5.4.1. Lesson 1: Algebra Arrows activity

Figure 5.6 shows two examples of written student work on Task 3 of the daily intermediate assessment after the Algebra Arrows activity. All five students in the observed group solved this task correctly, all of them by using the reverse strategy.

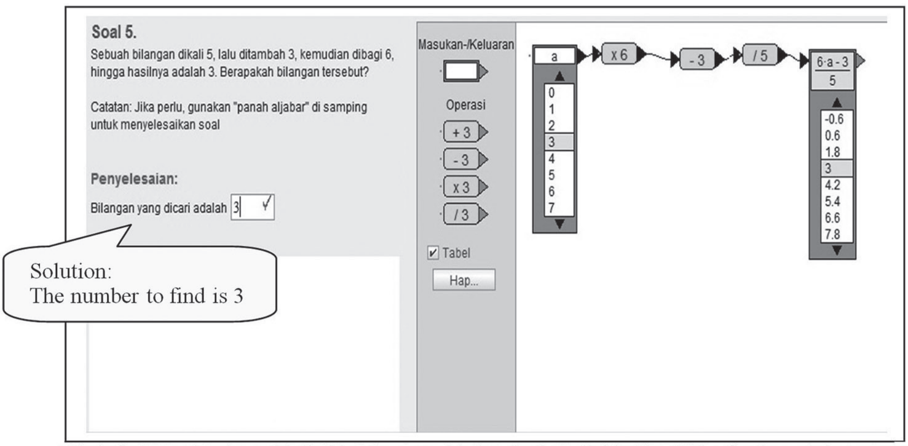


Task 3. Udin asks Tom what his father's age is? Tom replies that, "My father's age subtracted by 3, divided by 5, next added to 4, the result is equal to your age." If Udin is eleven years-old, how old is Tom's father?

Figure 5.6. Taufiq's (left) and Saiful's (right) written work on Task 3 of Lesson 1

In Figure 5.6 (left part), Taufiq first transformed the word problem into an informal equation: ... – 3 : 5 + 4 = 11. Next he solved the equation using the reverse strategy as shown in the line below. Though the final answer is correct, the student made a notational mistake in the use of the equal sign, i.e., rather than to write 11 – 4 = 7; 7 × 5 = 35 and 35 + 3 = 38, Taufiq wrote 11 – 4 = 7 × 5 = 35 + 3 = 38.

In Figure 5.6 (right part), rather than first transforming the word problem into an equation, Saiful directly used the reverse strategy to solve it, i.e., 11 – 4 × 5 + 3 = 38. Next, he checked the answer by substituting it into the equation: ... – 3 : 5 + 4 = 11, that is, by replacing the dots with the answer. Even if the answer is correct, the written notation of the reverse strategy is not appropriate as this violates the priority rules of arithmetical operations. The proper notation for the solution would be (11 – 4) × 5 + 3 = 38. The immediate use of the reverse strategy for solving the word problem was probably a direct consequence of the learning process in the digital activity in which this group used the same strategy directly, as for instance shown in Figure 5.7 and described in the corresponding excerpt below.



Task 5. A number is multiplied by 5, next added to 3, then divided by 6, the final result is 3. What was the number?

Figure 5.7. An example of student digital work in Lesson 1

Observer: Here, the unknown number to find is not given yet. So, you should determine for yourself, with for instance *a*, *b*, *c* or *n*. This task is similar to Task 1, is not it?

[The students choose *a* as the variable. Interesting to note here is that rather than creating an equation to represent the word problem, students directly apply the reverse strategy to solve the problem.]

Danang: [Puts *a* into the input box, clicks 3 from the table.] This [3] should be multiplied by 6, next subtracted by 3.

Saiful: [Then] divide by 5.

Danang: [He does the solution process in the computer to find the unknown number, that is, 3. He puts this into the answer box and presses enter. The answer is correct as shown in Figure 5.7.]

A pretest and posttest task similar to this daily intermediate task is Task 4 (see Appendix 5.2). In the posttest, four students solved it correctly: two students directly used the reverse strategy, and three students used the reverse strategy after formulating equations. In the pretest, there was no correct answer for Task 4.

These results reveal two points. First, even if the Algebra Arrows' notation did not emerge in written student work while solving problems, the two types of reverse strategies used by students in the daily intermediate assessment and in the posttest seem to follow from the use of the applet. Second, the pretest and posttest results of the students in this group show positive gain scores and as such illustrate the improvement of student achievement.

5.5.4.2. Lesson 2: Cover-up Strategy activity

We consider Task 3 of the daily intermediate assessment Lesson 2 – i.e., solve for positive *a*: $\frac{24}{(a+2)^2-1} = 3$ – as a typical task for recognizing student understanding of the cover-up strategy. Out of the five students who used the cover-up strategy, three students solved Task 3 correctly. Figure 5.8 presents two examples of written student work on this task. The left part shows a correct solution, and the right part shows an incorrect one.

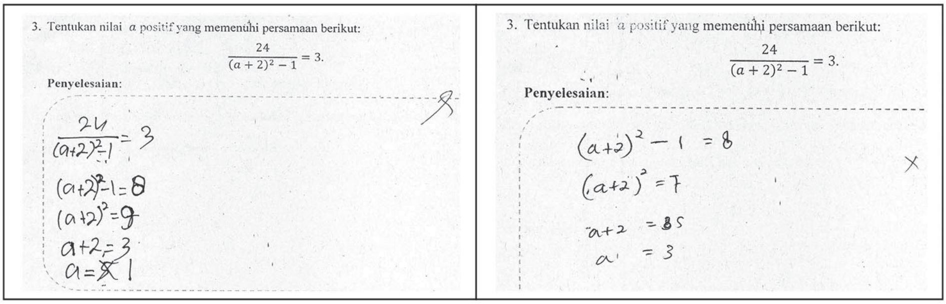


Figure 5.8. Saiful's (left) and Rafi's (right) written work on Task 3 of Lesson 2

In Figure 5.8 (right part), Rafi was successful in applying the cover-up strategy for the first step, i.e., determining the part of the equation to cover and filling in a numerical value for it, namely $(a + 2)^2 - 1 = 8$. However, in the next two steps, Rafi made mistakes. In step 2, rather than to fill in 9 for the value of $(a + 2)^2$, Rafi assigned 7, which is an additive inverse mistake. This suggests that rather than using the cover-up strategy, Rafi used the reverse strategy in an incorrect way for this step. In step 3, Rafi seems not to understand how to find the inverse of a square: he subtracted 2 from 7 to get 5 rather than to find a square root.

A similar difficulty in applying the cover-up strategy was observed during the digital group work, i.e., students used an improper reverse strategy to solve an equation that can be solved easier with the cover-up strategy. This was probably the origin of student difficulties that were observed in the daily intermediate assessment, as described in the following excerpt.

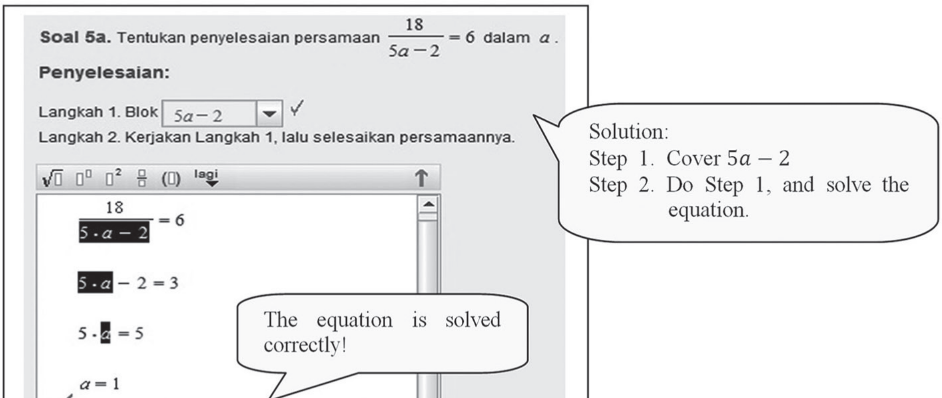


Figure 5.9. An example of digital student work in Lesson 2

The group is working on Task 5a in the digital activity, i.e., solve for *a*: $\frac{18}{5a-2}=6$. The observer (i.e., the researcher who acts as a substitute teacher) finds that students have difficulties in identifying the expression to cover in the first step.

Observer: Okay, what is the first step you should do?

[The students follow step 1 as given in the task, but they are not sure which part of the equation should be covered first.]

Observer: Which part of the equation should you cover first? [The students are still hesitating.]

Saiful: $6 \times 18 = 108$, and $108 + 2 = 110$, next $\frac{110}{5}$.

[He used an incorrect reverse strategy rather than the cover-up strategy to deal with the equation. So, the observer suggests students follow step 1 properly.]

Observer: Just choose and follow step 1. [After some guidance, the students are finally able to apply the cover-up strategy to solve the equation, and their solution is shown in Figure 5.9.]

Task 6 of the pretest and posttest, i.e., solve for *m*: $\frac{64}{3(m+1)-1}=8$ is similar to the daily intermediate Task 3 of this lesson. Four students solved this posttest task correctly using the cover-up strategy. In the pretest, two students ended up with correct answers and seemed to use an informal guess-and-check strategy.

The increase in the number of students who got correct answers in the posttest as compared to the pretest is in line with the improved achievement reported earlier. Two additional remarks are noteworthy. First, the written work in both the intermediate assessment and the posttest reveals that students have consistently used the cover-up strategy in ways that are quite similar to the cover-up strategy in the applet environment. This suggests a transfer of the applet strategy to paper-and-pencil environment. The transparent and visual character of the Cover-up Strategy applet may explain this. Second, mistakes in written student work concern the arithmetical category of difficulties, including calculation errors and errors in applying inverse properties (see section 5.3.1), but they had nothing to do with difficulties in the algebraic expressions or the variable category. This suggests that the applet invites students to develop on a structural view rather than on an operational view on algebraic expressions.

5.5.4.3. Lesson 3: Balance Model activity

Task 3 of the daily intermediate assessment Lesson 3, i.e., solve for x: $3x + 22 = 6x + 1$, is a typical task for recognizing student understanding in the Balance Model activity. The result shows that all students solved this task correctly. Four students presented a solution process similar to the one they had learned—indicating that it had influenced their thinking and actions—and one student provided the final answer only. While solving this task, students apparently had a visual image of an equation as a balance in mind. So, solving an equation comes down to maintaining the equilibrium of the balance, and to finding an answer as the weight of an object.

A digital activity task similar to this daily intermediate task is task 7, in which students are required to write an equation from the given model and then to solve it: the equation to solve is $4x + 1 = 2x + 23$. While solving this equation, the students performed an action on the model in each step (by moving a bag or a block representing a weight) and represented the action in the form of an equivalent equation. After this group arrived at the equation $3x = 2x + 11$ and one of the students moved a bag (representing x), one of other students concluded that $x = 11$. Their solution to this equation is shown in Figure 5.10.

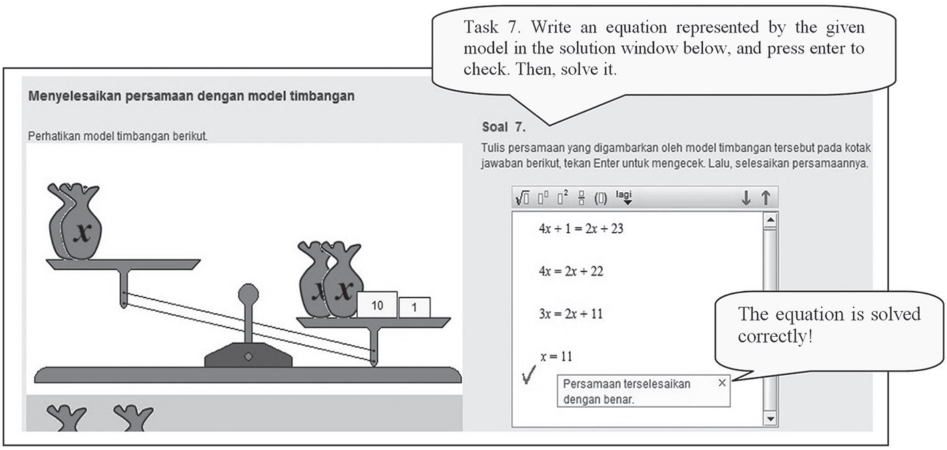


Figure 5.10. An example of digital student work in Lesson 3

Task 8 in the pretest and posttest, i.e., solve for x: $12x – 11 = 4x + 13$, is quite similar to Task 3 of the daily intermediate test of this lesson. The result showed that all five students of this group solved this posttest task correctly using the balance model strategy, whereas only two students solved it correctly in the pretest using an informal guess-and-check strategy.

In addition to the improvement of student achievement, a point to note from these results concerns the balance strategy that students used in their written work. Even if the students did not have the balance models at hand during their paper-and-pencil work, their solution strategies seem to follow the Balance Model applet approach directly. That is in line with the study by Vlassis (2002) on the balance model for solving linear equations in one variable; solving an equation boils down to maintaining an equilibrium of the left and the right side of the equation; finding a solution comes down to finding a numerical value of the variable, representing the weight of an object in a balance.

5.5.4.4. Lesson 4: Balance Strategy activity

A typical task to determine student understanding in the Balance Strategy activity is Task 3 of the daily intermediate assessment, i.e., solve for x: $9(x-1) = 2(x-1) + 21$. There are at least two different methods to implement the balance strategy for solving this equation. First, students can subtract $2(x-1)$ from both sides of the equation to obtain $7(x-1) = 21$ in the first step, next divide both sides by 7 and finally add 1 to find $x = 4$ as the solution. This first method is actually a combination between the balance strategy and the cover-up strategy. To do this, a structural view on the algebraic expressions in the equation plays an important role. Second, students can initially apply the distributive property to remove the brackets in the equation to get $9x - 9 = 2x - 2 + 21$, and next carry out the balance strategy (i.e., for instance, add 9, subtract $2x$, and divide by 7 to both sides, respectively) to get the solution $x = 4$. The results show that all five students solved this task correctly using the second method, but that no student used the first method. This suggests that the integration of the balance strategy and other equation solving strategies in this activity is subtle; it was not observed in these students' written work.

Another way to see student understanding in this Balance Strategy activity is by analyzing student work in solving word problems in algebra. Our observation showed that it was often difficult for students to transform word problems into appropriate equations. This difficulty is partly caused by, for instance, an inability to translate phrases into correct algebraic expressions. From the perspective of mathematization, such difficulty concerns understanding problems and formulating corresponding mathematical models. In the excerpt below, we provide an example of observation for a word problem task showing student difficulty in formulating a mathematical model.

Students are working on the following task:

Father is 39 years old now. If two times Tom's age is added to his father's age, the result is equal to 5 times Tom's age three years later. How old is Tom now?

After reading the task, students try to represent the word problem in an equation. The observer reads the task phrase-by-phrase to guide students in representing the problem in an equation.

Observer: Two times Tom's age...

Saiful: $2t$.

Obsever: Okay, good! Now, it is added to the father's age.

Saiful: $2t + 39 = ...$

Observer: Good! Now, it is equal to five times Tom's age three years later.

Danang & Rafi: [So, it is $2t + 39 = ...$] $5t + 3$

Observer: Which one should be multiplied by 5?

Danang: t.

Observer: Is it only t or $(t + 3)$? Please enter what you wrote.

Saiful: [He types $2t + 39 = 5t + 3$, and presses enter.] Incorrect!

Observer: It says 'five times Tom's age three years later'. So, what should be multiplied by 5?

Saiful: $t + 3$

Observer: Okay, so it means 5 times $(t + 3)$. [The students represent it correctly as: $2t + 39 = 5(t + 3)$]

Observer: Good! [Next the students remove the bracket in the equation.]

Saiful: So, now it is $3t + 39 = 5t + 15$

Observer: Good! Now, what is next?

[Next students solve the equation as shown in Figure 5.11.]

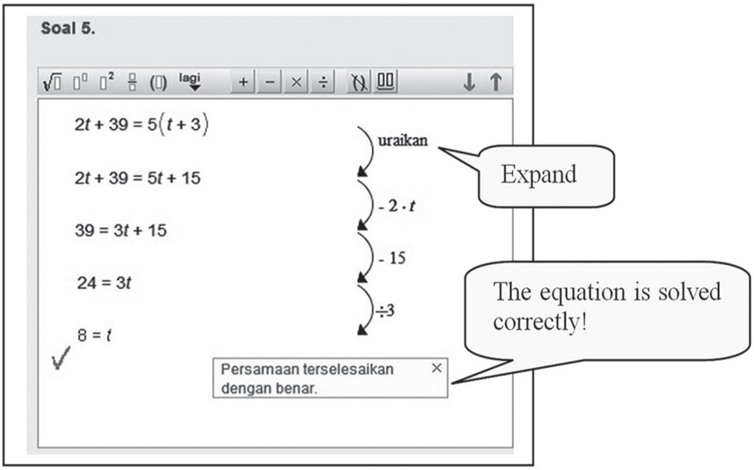


Figure 5.11. An example of digital student work in the Lesson 4

Task 7 of the pretest and posttest concerns a word problem that requires students to formulate an appropriate equation and then solve it (see Appendix 5.2). The posttest result shows that four students solved this task correctly using the balance model and in combination with the balance strategy, while only two students solved it correctly in the pretest using an informal guess-and-check strategy.

From these observational findings, we retain two points. First, the scarce use of a combination of equation solving strategies in student work seems to be a consequence of the absence of tasks that require students to do so. Only inserting tasks that can be solved with more than one strategy in the digital activity apparently is not enough to influence student thinking and strategies. Apparently, the integration of different equation solving strategies requires specific attention. Second, concerning student difficulties in transforming word problems into appropriate equations, intensive attention from the teacher during the learning process of transforming word problems into equations seems necessary.

5.6. Conclusions and discussion

In this paper we set out to answer the following research question for the case of equations in one variable and the related word problems by using four applets embedded in the Digital Mathematics Environment:

Does an intervention with digital technology enhance students' performance in initial algebra?

The first hypothesis was that students who were engaged in the intervention with digital technology would outperform their peers in the regular learning setting in solving equations in one variable and related word problems. The results of this study confirm this hypothesis, that is, students in the experimental group had a significantly higher mean gain scores than the students in the control group, with a small to medium effect size.

The second hypothesis concerned the impact of school factor on student achievement, that is, students from *A*-certified schools were expected to benefit more from the intervention than students from non-*A*-certified schools. The results show that both intervention condition and the school type had a significant effect on students' gain score with small to medium effect sizes. However, there was no interaction effect between the condition and the school type. Thus, the second hypothesis was not confirmed by the data.

Concerning the results of this study, we note six important points to discuss. First, the medium effect size (Cohen's $d = 0.32$) found in this study – which aims to improve student achievement – is in line with the results of a recent review study on the methods of instructional improvement in algebra (Rakes et al., 2010). In that review study, studies that focus on conceptual understanding show an observed weighted effect size of more than twice the magnitude of the effects of studies focusing on procedural work (Cohen's $d = 0.47$ and Cohen's $d = 0.21$, respectively). In sum, the effect size of the present study, which lies in between the two effect sizes of the review study, is in agreement with the results of other studies into improvement of algebra education and the use of technology in algebra education in particular.

Second, concerning the School type as an important component that affects student achievement, the result of the present study is in line with the study on the effect of online tasks for algebra in the domain of linear and quadratic equations (Drijvers, Doorman, Kischner, Hoogveld, & Boon, 2014). In that study, students from schools with good ICT facilities – which also applies to the *A*-certified schools in the present study (see Figure 5.5) – performed better than students from schools with less prepared ICT facilities. However, a point that should be taken into account is that categorizing school type by *A*-certified and non-*A*-certified reflects the perceived quality only to a limited extent. Therefore, the influence of school type on student achievement and the effect of a particular intervention need further investigation using measurable characteristics that include both perceived qualities and objective evaluation data.

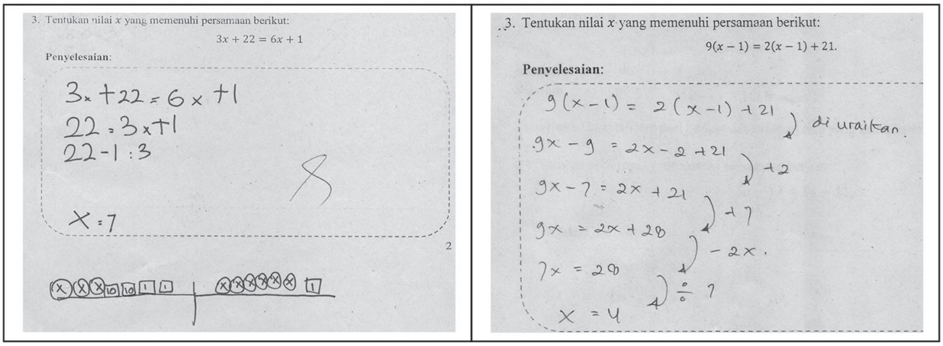


Figure 5.12. Transfer from applet visualization to paper-and-pencil notation

Third, in spite of the success in improving student achievement, the duration of the intervention was not long in terms of the use of applets. Each of the four applets was used in only one 80 minute lesson. Actually, we think this is quite short, even if Rakes et al. (2010) found that the duration of an intervention does not account for differences in effectiveness on student achievement. Furthermore, because of the limited duration of the intervention, it was quite difficult to determine a specific applet's role in the development of student conceptual understanding and procedural skills. Therefore, we are left with a number of questions. Does each of the four applets used in this study influence student development in an equal manner? Does the combination of the four applets for developing student conceptual understanding in the topic of equations in one variable produce an optimum effect? What would be the effects of the applets use on student algebraic skills, for instance, if one specific applet is used over a longer period, such as two or three lessons? What is the effect of the use of an applet on student algebraic skills when used over a longer period? In line with Artigue (2002), who claims that instrumental genesis – i.e., a process of a tool becoming an instrument – is a time-consuming process, using an applet more extensively might result in even more solid conceptual understanding and procedural skills. This might be detected, for instance, through the transfer of the applet's notations or visualizations in paper-and-pencil work (Bokhove & Drijvers, 2010a; Kieran & Drijvers, 2006). Figure 5.12 shows two examples of such transfer, which was not that frequently observed in our study: the left part shows the transfer of visualization of the Balance Model applet, and the right part depicts the transfer of notation of the Balance Strategy applet.

Fourth, we would like to stress that the use of technology in the experimental classes was only one aspect of the teaching intervention as a whole. Each lesson included an introduction of the topic delivered interactively by the teacher, i.e., the teacher interacted with students in a whole-class discussion, a demonstration of the applet's use, a digital group work session, and daily intermediate assessment. Thus, the results of the present study cannot be attributed to the digital technology only, but should take into account the effect of the intervention as a whole. We have, for example, the impression that group work efficiency and motivation during the lessons in class Experimental 2 (Exp2) contributed to the high gain in this class (see Figure 5.5).

Fifth, as the integration of ICT in mathematics teaching is a complex enterprise (Lagrange, Artigue, Laborde, & Trouche, 2003), we expect the teacher to be an important factor in the success of the intervention. In our experiment, this role included the ability to demonstrate how to use the applets to the whole class as effectively as possible; to help students when they encountered technical obstacles; and, in particular, to guide students to acquire conceptual understanding and procedural skills in algebra through working with the digital tools. As this teacher factor was not systematically investigated in this study, we acknowledge that here we only have a limited view on the factors that explain the interventions' success.

Sixth and final, the qualitative results described in section 5.5.4 to illustrate and corroborate the quantitative results are still limited and focused only on the students' written and digital work, which we analyzed with respect to difficulties in algebra and solution strategies. We did not yet, however, consider to a larger extent and in a more concrete manner the impact of the technology-based intervention on students' conceptual understanding and procedural skills from an instrumental approach perspective (e.g., Artigue, 2002; Trouche, 2004; Trouche & Drijvers, 2010). This type of impact of technology needs to be further elaborated to conceive the interrelationship between student thinking and the use of the digital tools.

Acknowledgment

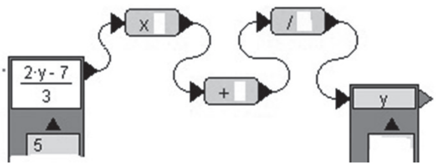
This study was funded by the Indonesia Ministry of Education project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794. We would like to thank Jan van Maanen for his critical and constructive comments and suggestions, and Peter Boon for designing the four applets used in the present study. Finally, we want to thank the teachers and students for their participation and an external assistant for her contributions.

Appendix 5.1

Table A.1 Tasks used in daily intermediate paper-and-pencil assessments

Tasks for daily intermediate assessment in Lesson 1: Algebra Arrows activity

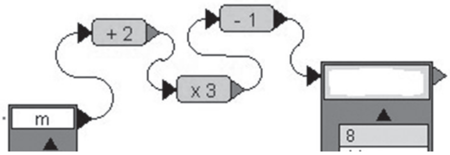
1. A number is multiplied by 3, next added by 2, and divided by 5, the result is 4. Find the number.
2. Consider the figure below



   a. Complete the missing numbers in the chain of operations above.

   b. Solve the equation $\frac{2y-7}{3} = 5$ for y.

3. Udin asks Tom what his father's age is? Tom replies that, "My father's age subtracted by 3, divided by 5, next added to 4, the result is equal to your age." If Udin is 11 years-old, how old is Tom's father?
4. Consider the figure below



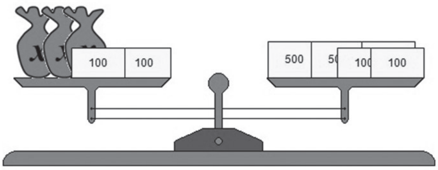
   a. Write an equation described by the chain of operations above.
   b. Solve the equation you got in part a) for *m*.

Tasks for daily intermediate assessment in Lesson 2: Cover-up Strategy activity

1. Solve for x: $5(x - 1) + 6 = 21$.
2. Yanto's money is Rp 5000 less than Zainudin. If Zainudin and Yanto's money together is Rp 23000, how much is Zainudin's money?
3. Solve for positive a: $\frac{24}{(a+2)^2 - 1} = 3$.
4. Tom, Jerry and Udin are friends of each other at the same school. One day, after a mathematics test, it is known that Jerry got a 5 and Udin got a 7. Tom says that, "Two times my grade added to Jerry's grade, next divided by 3, and the result is equal to Udin's grade." Find Tom's grade.

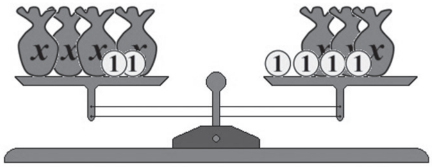
Tasks for daily intermediate assessment in Lesson 3: Balance Model activity

1. See the figure below.



The above condition describes a daily life situation, with three bags of sugar – with the weight of each bag unknown (x), added to 200 grams of weights, the total weights are equal to 1700 grams. Find the weight of a bag of sugar.

2. See the balance model below.



   a. Represent the balance condition above into an equation.
   b. Solve the equation you got in part a).

3. Solve for x: $3x + 22 = 6x + 1$.

Tasks for daily intermediate assessment in Lesson 4: Balance Strategy activity

1. Solve for a: $6a - 13 = 3a + 11$.
2. I have a number. If the number is added to 7, next divided by 2, then they are equal to two times the number subtracted by 1. Find the number.
3. Solve for x: $9(x - 1) = 2(x - 1) + 21$.
4. It is known that the weight of a sack of sugar is 38 kg. If this sack of sugar is added to three sacks of flour, then they are equal to a sack of corn added to a 100 kg sack of rice. If a sack of flour has the same weight as a sack of corn, find the weight of a sack of corn.

Appendix 5.2

Table A.2 Tasks used in pre-and posttest

1. Calculate:
   a. $7 + 3 \times 2 - 4 = \ldots$
   b. $6 + 3 \times (9 - 5) = \ldots$
2. Consider the equation $\frac{2n+1}{5} = 6$. Find the value of $2n + 1$.
3. Solve the equation $\frac{8p+3}{7} = 5$ for p.
4. When Azkiya is 12 years-old, mother says that the result of her father's age added to 5, next divided by 6, then added by 4, is the same as Azkiya's age. Find Azkiya's father age.
5. Suppose $(2y - 1)^2 + 3 = 12$. Find the value of $2y - 1$.
6. Solve for m: $\frac{64}{3(m+1)-1} = 8$
7. Tom, Jerry, Udin, Adin and Budin are friends. Udin, Adin and Budin are triplets. One day, when playing on a teeter-totter, an interesting phenomenon occurred. Tom and Udin in the left hand side of the teeter-totter have the same weight as Jerry, Adin and Budin on the other side. This has put the teeter-totter in an equilibrium condition. If Tom's weight is 60 kg, Jerry's weight is 35 kg, and the triplets are all the same weight, find Udin's weight.
8. Solve for x: $12x - 11 = 4x + 13$.

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Chapter 6 An instrumentation theory view on students' use of an applet for algebraic substitution

Publication:

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (submitted). An instrumentation theory view on students' use of an applet for algebraic substitution.

Abstract In this paper we investigated the relationship between the use of a digital tool for algebra and students' algebraic understanding from an instrumentation theory perspective. In particular, we considered the schemes that students developed for algebraic substitution using an applet called Cover-up. The data included video registrations of three seventh grade Indonesian students (12-13 year-old) using the applet. The results showed that while solving equations and related word problems, the students developed schemes for algebraic substitution in which technical skills and conceptual understanding are intertwined. The schemes gradually were adapted to solve larger classes of equations. We found that crucial factors in this development – called instrumental genesis – are the characteristics of the applet and the task design, the role of a teacher, and the interaction among students.

Keywords algebra education, digital technology, equations in one variable, instrumental genesis, instrumentation, scheme.

6.1. Introduction

Proficiency in algebra is a gateway for secondary school students to pursue advanced studies at university level (Harvey, Waits, & Demana, 1995; Katz, 2007; Kendal & Stacey, 2004; Morgatto, 2008). Therefore, the acquisition of algebraic expertise, including conceptual understanding and procedural skills, is an issue at international level (e.g., Bokhove, 2011; Kendal & Stacey, 2004; Van Stiphout, 2011). Also in Indonesia much importance is attributed to students' algebra competence (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014a). However, the significance ascribed to proficiency in algebra is in sharp contrast with Indonesia's 38th position out of 42 participating countries in the domain of algebra in the 2011 *Trends in International Mathematics and Science Studies* (Mullis, Martin, Foy, & Arora, 2012). These low results raise the question of how to improve student achievement in algebra.

Over the last decades, educational stakeholders over the world have highlighted the potential of digital technologies for mathematics education. The National Council of Teachers of Mathematics (NCTM), for instance, in its position statement, claims that "technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology" (NCTM, 2008, p.1). In Indonesia, the Ministry of National Education releases a policy to introduce ICT as a new subject for secondary schools, and suggests integrating the use of ICT in all school subjects, including mathematics (Depdiknas, 2007).

Whether ICT really helps and in what circumstances is not that obvious yet. On the one hand, there is research evidence that underpins the plea for technology-rich mathematics education. Review studies in mathematics education show that the use of ICT impacts positively student mathematics achievement (Li & Ma, 2010) as well as students' attitude towards mathematics (Barkatsas, Kasimatis, & Gialamas, 2009). Specifically for algebra education, Rakes, Valentine, McGatha and Ronau (2010) show that the use of ICT significantly affects student achievement and conceptual understanding as well as procedural skills. In line with this, we found that students who enrolled in a digital technology-rich intervention significantly outperformed their peers in the control condition without digital tools (Jupri, Drijvers, & Van den Heuvel-Panhuizen, submitted). These results suggest that digital technology may enhance student learning of algebra.

On the other hand, however, digital technology is not a panacea for all issues in mathematics education and its integration turns out to be a non-trivial matter (Trouche & Drijvers, 2010), as is shown by the modest effect sizes found in the above studies, and even the absence of significant positive effects in others (e.g., see Drijvers, Doorman, Kirschner, Hoogveld, & Boon, 2014). Because the transfer between work in a digital environment and the traditional paper-and-pencil work is not self-evident, teachers find themselves faced with the challenge of integrating new media in an appropriate way (Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013). Moreover, fundamental questions about how and why digital technology works are waiting to be answered. Therefore, in the study reported in this paper, we aimed to contribute to the investigation of how the use of digital technology in algebra does foster students' algebraic thinking. In particular, we addressed the relationship between using a digital tool for algebra and the targeted algebraic understanding as well as mastery of procedural skills.

6.2. Algebraic substitution

To investigate students' algebraic thinking, we focused on algebraic substitution, which is an important and sometimes indispensable method in algebra for, e.g., simplifying algebraic expressions, solving equations, and solving integration problems.

From a mathematical point of view, algebraic substitution includes (1) replacing a more complex expression by one variable, and (2) replacing one variable by a more complex expression.

6.2.1. The first type of substitution

A well-known example of the first type of algebraic substitution, replacing a more complex expression by one variable, is provided by Wenger (1987):

Solve the equation $v\cdot\sqrt{u}=1+2v\cdot\sqrt{1+u}$ for v.

Many students do not see this equation as being linear in v and therefore are unable to solve it (Wenger, 1987; Gravemeijer, 1990). The reason is that the sub-expressions $\sqrt{u}$ and $\sqrt{1+u}$ are not considered as objects, as entities that can be covered or replaced with arbitrary variables without caring for their content. Rather, students see the square root signs as strong cues calling for algebraic manipulations. In other words, students do not have a 'global substitution principle' at their disposal that triggers them to consider sub-expressions such as $\sqrt{u}$ and $\sqrt{1+u}$ as objects (Wenger, 1987). Such a global look at sub-expressions can be stimulated by putting square or oval tiles on the sub-expressions by which the object's interpretation is elicited (Freudenthal, 1962). For instance, Wenger's equation can be represented as follows:

$$v\cdot\square=1+2v\cdot\bigcirc$$

This method of putting tiles on sub-expressions, which is called the 'cover-up method', is used by several authors (Kindt, 2010; Vlassis, 2002), especially in the initial stage of solving equations. According to Kindt (2010), this substitution method increases students' manipulation skills by stimulating them to simplify algebraic expressions to a familiar, standard form.

Furthermore, Kindt (2010) stated that if the cover-up method is kept up for a time, and if sufficient variation of tasks is provided, it will encourage students to develop more formal strategies for solving problems. As an example, a relatively complex non-linear equation, such as $\frac{2015}{\sqrt{4053-2x}} = 403$, can be given to students in initial algebra to trigger them to use the cover-up method. This means that they can cover $\sqrt{4053-2x}$ with a tile and notice that its value is 5, which means that $4053 - 2x = 25$. Next, by covering $2x$ they will find that $2x = 4028$. Finally, by covering x, they will conclude that $x = 2014$ is the equation's solution.

6.2.2. The second type of substitution

The second type of algebraic substitution, that is, replacing one variable by a more complex expression, is important to understand composite functions and to combine different equations. For example, to find a formula for the composite function $f(2m - 1)$ if $f(x) = x^2 + x$, the variable x must be replaced by the expression $2m - 1$, which after some intermediate steps leads to $f(2m - 1) = 4m^2 - 2m$. As another example, Figure 6.1 shows the screen of a symbolic calculator (TI-89), on which $x = \frac{-b}{2}$ is substituted into the expression $x^2 + bx + 1$ which leads to $1 - \frac{b^2}{4}$ as its result.

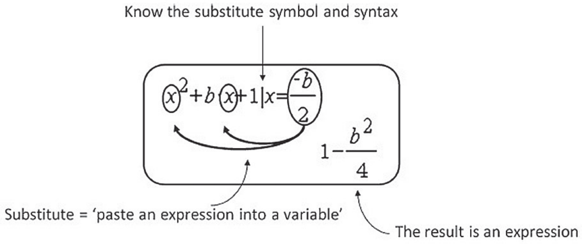


Figure 6.1. Algebraic substitution on a TI-89 (Drijvers, Godino, Font, & Trouche, 2013, p. 35)

6.2.3. The difficulty of substitution

Whether carried out with pencil and paper or in a digital environment, a main underlying difficulty of algebraic substitution concerns the process-object duality of an expression. In the above examples, an algebraic expression should be perceived not only operationally as a calculation process on variables, but also, and more important here, structurally as an algebraic object that can be replaced by another one. The in this case less appropriate process view on an algebraic expression often precedes an object view (Sfard & Linchevski, 1994) and may result in the so-called lack of closure obstacle (Tall & Thomas, 1991), the discomfort in dealing with algebraic expressions that cannot be simplified any further and that do not have a numerical value. Integrating a process view and an object view requires the reification of an algebraic expression as a mathematical object (Sfard, 1991). It is this reification process that is difficult to achieve for students, but is needed for algebraic substitution.

6.3. Substitution in a digital environment: the Cover-up applet

The Cover-up[1] applet is an online digital environment developed to foster students' understanding of algebraic substitution and the reification of expressions.

This applet allows the student to solve equations of the form $f(x) = c$, by subsequently highlighting with the mouse an expression within an equation and assigning a value to it. Figure 6.2 shows an example of how the equation $\frac{48}{8(z+1)} = 3$ can be solved. In step 1, the equation to solve is displayed in the solution window and needs to be studied by the student. In step 2, the expression $8(z + 1)$ can be highlighted; then, the applet automatically shows the expression $8(z + 1) = \ldots$ in the next line. In step 3, the value 16 is filled in for the selected expression. The applet produces feedback in the form of a yellow tick mark signifying a correct action (otherwise a red cross mark appears). This solution process proceeds, for instance, until step 6 and ends with $z = 1$ as the solution of the equation, leading to a green tick mark and the final feedback: "The equation is solved correctly!" In practice, a student does not necessarily follow all these six steps, but may also take shortcuts, such as jump to step 6 immediately after step 3, or make a detour and may need more than six steps to solve the equation.

1 The Cover-up applet is developed by Peter Boon, Freudenthal Institute, Utrecht University, the Netherlands.

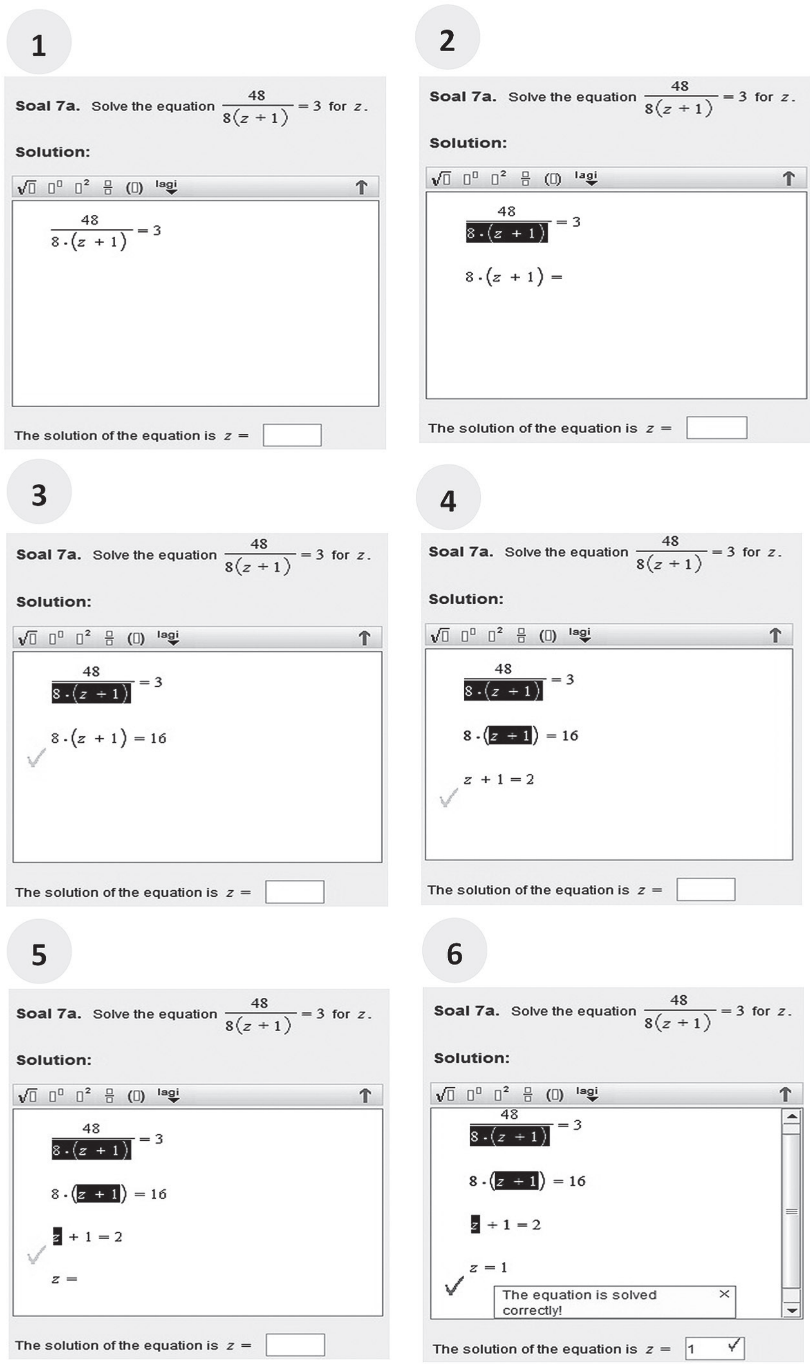


Figure 6.2. An equation solving scenario using the Cover-up applet

There are three main reasons for using the Cover-up applet for fostering students' understanding of algebraic substitution and the reification of expressions. First, the activities with the applet invite students to simultaneously develop an operational and a structural view on algebraic expressions: selecting expressions by highlighting them stresses their object character, whereas assigning numerical values relates to the outcome of a calculation process (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014b). Second, the Cover-up applet can be used to solve various kinds of equations and not just linear equations in one variable of the form $f(x) = c$. Third, the way to use the applet is close to the intended way of thinking about equations and expressions, whereas student in the meantime have the freedom to explore different pathways within this approach.

6.4. Instrumentation theory as a lens

As mentioned in the introduction, the relationship between using a digital tool for algebra and the targeted algebraic understanding and mastery of procedural skills was the focal issue in this paper. We investigated this relationship for the case of carrying out algebraic substitution with the Cover-up applet. To get an in-depth understanding of this relationship, we chose the perspective of instrumentation theory. We now briefly review the core elements of this theory, also called the instrumental approach, for using digital tools in mathematics education.

As a point of departure, we notice that tools are not neutral (Hoyles & Noss; 2003); rather, they are active agents. On the one hand, their use is shaped by the knowledge and skills of the user, but on the other hand the user's knowledge and skills are shaped by the opportunities and constraints of the tools. This reciprocal interplay between, in our case mathematical insight and ways of using digital tools, and its development over time in particular, is what instrumental genesis is about (e.g., Artigue, 2002; Trouche, 2004; Trouche & Drijvers, 2010). Within this instrumentation theory, the following terms play a key role: artefact, tool, technique, scheme, and instrument (e.g., Drijvers, Godino, Font, & Trouche, 2013; Trouche & Drijvers, 2010).

An artefact is an object, either material or not. A graphing calculator is an artefact, and mathematical language can be considered an artefact as well. In many cases, a further specification of an artefact may be needed. For example, while studying the use of a graphing calculator for the notion of equation, one might want to distinguish its graphing module from its equation solver, and as such distinguish two artefacts within the object as a whole. This is a matter of choosing an appropriate granularity for the purpose of study. In this study, the main artefact is the Cover-up applet, which can be used for solving equations, but paper and pencil form an important pair of artefacts as well. If an artefact is used for carrying out a specific task, such as solving an equation, we call it a *tool* (Trouche, 2004).

An artefact is useless as a tool as long as the user has no idea for which task or how to use it. This is where the notion of *technique* comes in. In line with Artigue (2002) we define a technique as a manner of solving a task using an artefact. As such, a technique has a pragmatic aspect: a task needs to be solved, there is a goal to achieve. In the meanwhile, a technique also has an epistemic value in that the way the artefact is used reflects the rules and methods the user has in mind, and the underlying mathematical insights (Lagrange, 1999). Techniques can be observed in the user's behavior while using the artefact. The development of a comprehensive repertoire of increasingly complex techniques is an important aspect of instrumental genesis (Trouche, 2000; 2004). The main techniques for using the Cover-up applet are described in the previous section (see Figure 6.2); solving equations on paper also entails the application of techniques.

Techniques, however, do not stand on their own, but are based on cognitive foundations. It is these foundations that form the schemes. Based on the work of Piaget, Vergnaud (1996) defines a *scheme* as an invariant organization of behavior for a given class of situations. A scheme has an intentional aspect (it involves a goal), a generative aspect (it involves rules to generate activity, such as carrying out a technique), an epistemic aspect, and a computational aspect (Vergnaud, 2009). The epistemic aspect involves two related types of operational invariants, concepts-in-action and theorems-in-action, that reflect the – often implicit – notions and rules a user has in mind while developing and using a technique. The schemes at stake in the study presented here concern solving equations and the related word problems through algebraic substitution using the cover-up strategy and will be addressed in more detail in the next section.

Schemes and techniques both share conceptual and technical elements, both share epistemic and pragmatic values, and both involve using an artefact for solving a specific type of tasks. Nevertheless, an important difference between the two is that schemes are invisible, whereas techniques are observable. In fact, we consider techniques as the observable manifestations of the invisible schemes (Drijvers, Godino, Font and Trouche, 2013). An *instrument*, now, is a mixed entity of scheme, technique, artefact and task. As such, it is the amalgam of all the 'players' involved when a student solves a mathematical task using a digital tool (Trouche & Drijvers, 2010; Trouche, 2004).

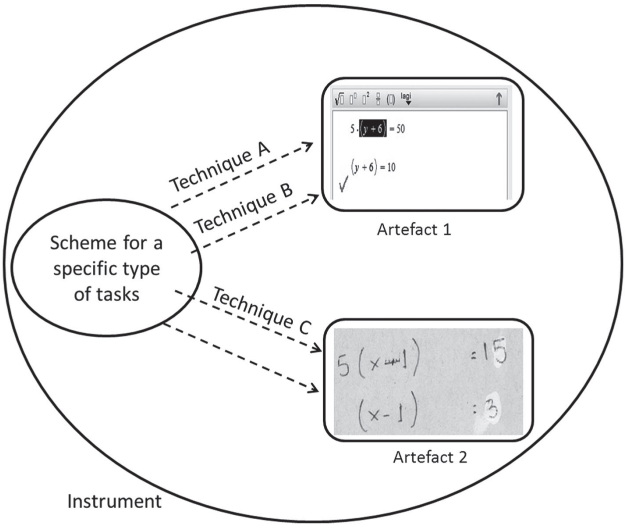


Figure 6.3. An instrument including different artefacts and techniques

If a type of task can be solved by using different artefacts, but with different, related techniques, the corresponding scheme, the different artefacts and techniques can be regarded as one single instrument (see Figure 6.3). In this study, the artefacts in play while solving equations through algebraic substitution were the Cover-up applet and paper and pencil, but we considered the corresponding techniques to be closely related.

Based on the above *instrumental genesis* can be defined as the process of the user developing instruments, consisting of cognitive schemes and observable techniques for using a specific artefact for a specific class of tasks. Instrumental genesis, in principle is an individual process and usually takes place in a social context, which in this study consist of students who work in groups.

Many studies have used instrumentation theory to address the relation between user and tool in the problem solving process (e.g., Artigue, 2002; Drijvers et al., 2013; Guin &Trouche, 1999; Lagrange, 1999; Trouche, 2004; Trouche & Drijvers, 2010). However, elaborated examples of schemes are still scarce. To contribute to this, we will now provide a description of a conjectured scheme for solving equations using algebraic substitution with the Cover-up applet.

6.5. Instrumentation schemes for solving equations using algebraic substitution with the Cover-up applet

To further investigate algebraic substitution, we first set up a conjectured instrumentation scheme for solving symbolic equations of the form $f(x) = c$ and then for related word problems. Table 6.1 summarizes the conjectured scheme for solving equations using algebraic substitution with the Cover-up applet. This scheme includes conceptual and technical elements which are related to each other. Even if the scheme is described for the paradigmatic task of solving the equation $\frac{48}{8(z+1)}=3$ for z, this description has a generic character and applies to every equation of the form $f(x) = c$.

Table 6.1. Conjectured scheme for solving equations using algebraic substitution with the Cover-up applet

Step	Conceptual aspect	Related technical aspect
1.	By scanning the equation, recognizing the equation as being of the form $f(z) = c$, so of the form < expression> = <numerical value>, with the unknown appearing only once on the left hand side. Realizing that the task is to rewrite this equation in the form < unknown> = <value>, which provides its solution. In this case, the expression is $\frac{48}{8(z+1)}$ and the numerical value is 3. As a consequence, the cover-up strategy can be applied.	No specific techniques involved in this step.
2.	By further inspection, recognizing the structure of the expression in left hand side of the equation. In this case, for example, the division of 48 by $8(z + 1)$ should be recognized as the central operator.	No specific techniques involved in this step. The equation has already been given in the solution window, i.e., $\frac{48}{8(z+1)} = 3$.
3.	Identifying a sub-expression to be covered as to start the cover-up strategy. In this case, this could be $8(z + 1)$.	Highlighting the identified sub-expression using the mouse. The applet puts the sub-expression in a new line and adds the equal sign. In this case, the result would be $8(z + 1) = \ldots$.
4.	Assigning a numerical value to the covered sub-expression to make a new equation becomes true. In this case, this value would be 16.	Typing the value after the equal sign, and pressing enter. In this case, the result would be $8(z + 1) = 16$ with a yellow tick mark signifying a correct action.
5.	(If necessary) repeating steps 3 and 4 to the new equation obtained in step 4 until the equation is simplified to $z = \ldots$ In the example, this would lead to $z = 1$.	Highlighting a sub-expression from the new equation, typing a numerical value, and pressing enter. In this case, the sub-expressions and corresponding numerical values would be: $z + 1 = 2$, and $z = 1$, respectively. Finally, the solution is indicated by the feedback provided by the applet: a green tick mark and the text "The equation is solved correctly!"

In terms of the operational-structural duality, steps 1, 2, and 3 in Table 6.1 mainly appeal for a structural view on equation as the equivalence of an expression and a number, and on expression as an algebraic object. However, to assign a numerical value to the selected sub-expression (step 4) asks for an operational view on the equation: if the output of the central operation is known, the value of the operand can be found. It is this integration of operational and structural views that makes this scheme, and the corresponding use of this applet, relevant for reification. Note that the screen captures provided in Figure 6.2 match with the above scheme description.

For solving word problems, which can be translated into equations of the form $f(x) = c$, the previous scheme has to be extended. The conjectured scheme for solving word problems with the Cover-up applet is summarized in Table 6.2.

Table 6.2. Conjectured scheme extension for solving word problems with the Cover-up applet

Step	Conceptual aspect	Related technical aspect
a.	Recognizing the possibility of solving the word problem through re-phrasing it in terms of a mathematical equation.	Reading the word problem aloud (if necessary).
b.	Setting up the equation, i.e., transforming each phrase into an algebraic expression, and altogether the word problem into an equation.	Typing the equation using the Cover-up applet's equation editor (if necessary), and pressing enter to check whether the equation is correct or not.
	After setting up an equation from the word problem, the next steps are the same as described in Table 6.1.	

6.6. Research question

Based on the lens of instrumentation theory we can specify our initial questions phrased in the Introduction and decided to examine the relationship between using a digital tool for algebra and the targeted algebraic understanding as well as mastery of procedural skills through the following theory-guided research question:

Which schemes do students develop for solving equations using algebraic substitution with the Cover-up applet and which relationships between techniques and understanding are developed?

In this question, the 'schemes' should be understood in the perspective of instrumentation theory. In particular, these include a scheme for solving equations using algebraic substitution with the Cover-up applet, and an extended one for solving related word problems. The problems on which the focus is in this research question are (mainly linear) equations of the form $f(x) = c$ with the unknown, in this case x, appearing only once on the left hand side, as well as related word problems.

6.7. Method

To answer the research question we carried out a case study. This case study was part of a larger experimental study (Jupri, Drijvers, & Van den Heuvel-Panhuizen, submitted) in which a learning arrangement was designed for learning to solve (mainly) linear equations in one variable which in Indonesia is part of the grade VII curriculum. The designed learning arrangement included activities with the Cover-up applet (see section 6.3), which is embedded within the Digital Mathematics Environment (DME).

The DME is a web-based learning environment providing (i) interactive digital tools for algebra, geometry, and other mathematical domains; (ii) a design of open online tasks and immediate feedback; (iii) access to the environment at any time and place, as long as technological infrastructure and conditions are met, and (iv) a storage for student work (Boon, 2006; Drijvers, Boon, Doorman, Bokhove, & Tacoma, 2013). In a Delphi study (Bokhove & Drijvers, 2010) where four groups of criteria (algebra didactics, theories on tool use, assessments, and general characteristics of digital tools) were used to evaluate digital environments for mathematics education it was shown that the DME compared to other digital tools was recognized as a suitable environment for research in algebra education addressing the co-emergence of procedural skills and conceptual understanding.

The case study was based on one lesson carried out in one seventh-grade classroom. The lesson was given by the classroom teacher who was informed on how to implement the learning arrangement activities through the teacher guide. The lesson lasted for 80 minutes and consisted of three respective parts. First, a paper-and-pencil activity was done and included posing problems and whole-class discussion. This was followed by a whole-class demonstration of how to work with the Cover-up applet and a group-based digital activity done by students. Finally, the students were requested to do individually paper-and-pencil tasks and the teacher was guiding the students to reflect upon the lesson.

To analyze the relationship between the use of the Cover-up applet and students' conceptual understanding and skills, we focused on the data of one group of three male Indonesian students (12-13 year-old). In the paper these students are named Ali, Quni and Widan. The group we chose was based on the teacher's recommendation with the criteria: consisted of students with heterogeneous mathematical abilities and the students would feel free to do mathematical activities in front of the camera. Based on the information provided by the teacher and the students' marks, these three students can be considered to be representative of the class as a whole. During their work the three students were observed and video-recorded by the first author. Moreover, during the lesson, the first author also helped these students. In this way, he acted as a substitute teacher, while the teacher took care of the other groups of students in the class.

The analyzed data included the video recordings of the group of three students during the digital activity, the corresponding student digital work stored in the DME, written student work on individual tasks, and observation notes. An integrative qualitative analysis on these data, with the help of Atlas.ti software, was carried out to investigate the students' scheme development and the relationship between the use of the applet and the targeted algebraic understanding.

6.8. Results

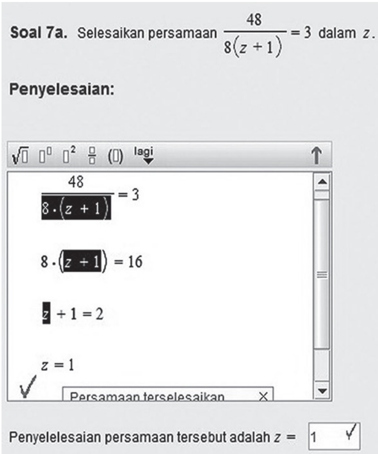
In this section we present the results of the observations of the group work during the Cover-up activity. For both bare problems and word problems, we first provide an analysis of one paradigmatic task with the conjectured scheme described in section 6.5 as a frame of reference. Next, we describe one scheme that the students used for solving the task. Finally, to follow students' development throughout the activity, we present our observation on schemes and related techniques for all tasks treated in this one lesson, discuss the observation and relate to student written work.

6.8.1. Students' scheme for solving equations using algebraic substitution with the Cover- up applet

An observation of a group working on one of the tasks

Table 6.3 presents a two-minute observation of the group's work on Task 7a. In the right column we provide corresponding commentaries, which are based on the conjectured scheme presented in Table 6.1.

Table 6.3. A commented observation of the group's work on Task 7a

Observation	Commentary
Task 7a. Solve the equation $\frac{48}{8(z+1)} = 3$ for z.  	Task 7a is a bare problem in the Cover-up activity. The figure that is below the task shows the corresponding student digital work stored in the DME.
Widan reads out the task aloud.	Reading the equation aloud might help the students to recognize the equation as being of the form $f(z) = c$ and to realize that z = < value > would provide the solution of the equation (step 1); and to see the structure of the algebraic expression in the left-hand side of the equation. In particular, it is important to perceive the division of 48 and $8(z + 1)$ the central operator that produces 3, the numerical value in the right side (step 2). This enables the student to determine the sub-expression to cover and to assign a numerical value.
Quni: Please cover that part [$8(z + 1)$]. [Ali highlights $8(z + 1)$ with the mouse, the applet yields $8(z + 1) = \ldots$ in the next line]. Good! Quni: The value of [$8(z + 1)$] is... Ali: This is 48 [divided by [$8(z + 1)$] equals 3]. So, [the value of $8(z + 1)$ is] 16. Quni: Yes, yes, you are right! It is 16. [He types 16 and presses enter. It is correct.].	Quni and Ali correctly apply the cover-up strategy to solve the equation. That is, they recognize the first sub-expression to cover, i.e., $8(z + 1)$, and they are able to assign a correct numerical value for that, i.e., 16 (steps 3 and 4).
Widan: How did you get 16? Quni: What is the value of 3 x 16? Widan: Yes, it is 48. [3 x 16= 48].	The question posed by Widan indicates that he initially does not understand why his friends assign 16 to $8(z + 1)$, which means he probably does not recognize the division as the central operation of the expression in the left-hand side. Therefore, Quni explains to Widan by asking the value of 3 x 16.

Quni: $8(z + 1) = 16$. So, $z + 1 = 2$. Ali: Yes it is 2 [he highlights $z + 1$, types 2, and presses enter. It is correct.] Quni: Now, z, z, z [to be covered]. And its value is 1. [Ali highlights z, types 1 and presses enter. It is correct. Also, he inputs the solution in the answer box, and presses enter.]	Quni and Ali carry out step 5. Quni is able to identify a sub-expression to cover from the new equation $8(z + 1) = 16$, i.e., $z + 1$, and assigns 2 to it. Ali agrees, and carries out the technique in the applet. Finally, Quni identifies z to cover from $z + 1 = 2$ and assigns 1 to it. Ali carries out the technique. Both of them finally get $z = 1$ as the solution of the equation.
The students immediately proceed to a next task without checking their solution mentally or orally.	Students do not check the solution because the applet has already provided feedback in each step, thus confirming a correct action and solution.

In the light of the conjectured scheme, this observation shows that the group's scheme is in line with the conjectured scheme described in Table 6.1, even if one of the students did not fully understand the solution process. To summarize this observation, Figure 6.4 visualizes the main conceptual elements of the students' scheme: recognizing the equation as suitable for the cover-up strategy, and wanting to rewrite in the form < unknown > = < value >, identifying a sub-expression to cover, assigning a numerical value to the covered sub-expression, and repeating these steps as long as needed.

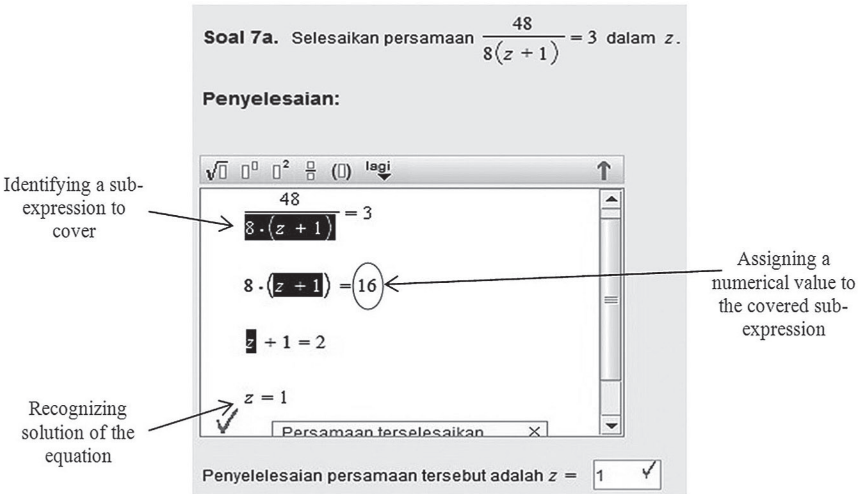


Figure 6.4. The main elements of the substitution scheme observed in the group of three students

An analysis of the group's work over the different tasks within one lesson

Table 6.4 summarizes students' schemes and related techniques of the observed group for bare algebra problems treated during the one-lesson

Cover-up activity. From this observation we noted that even if the three students were finally able to solve the given equations by applying the cover-up strategy using the applet, they still encountered difficulties while doing so. The main difficulties encountered by students concerned arithmetical calculations errors, as shown for tasks 3a, 8, and 9a. For instance, when solving task 3a, the students assign 10 as a numerical value to $2p + 5$ instead of 15. Also, the observer in some cases gave too much guidance as shown in the observations of tasks 2a, 3a, 4a, 8, and 9a.

Three points in this observation deserve further attention. First, Widan seemed to experience difficulties – in the sense that he often could not follow his peers' thinking – while solving equations during the activity. In our view, these difficulties were caused by Widan's limited understanding of an equation as a structural equivalence between two objects (an algebraic expression and a number). This lack of understanding was manifest when working on the tasks 6a, 7a, 9a and 9b. For example, Widan did not understand why his peers assigned 16 to $8(z + 1)$ while solving the equation $\frac{48}{8(z+1)} = 3$.

Table 6.4. Students' schemes for the different tasks within one lesson

Task	Observation: scheme and techniques
Task 2a. Solve the equation $5(y + 6) = 50$ for y. Hint: Cover $y + 6$ at the first step and assign a value to it.	As suggested by the observer, students follow the hint. They assign the values to $y + 6$ and y correctly. An observed technical obstacle concerns covering parts of the equation, i.e., they initially highlight $5(y + 6)$ rather than $y + 6$.
Task 3a. Solve the equation $4(2p + 5) = 60$ for p. Hint: Cover $2p + 5$ at the first step and assign a value to it.	Students follow the hint, but they assign 10 (i.e., students conclude that 60 divided by 4 is 10) rather than 15 to $2p + 5$. After the observer explains that the equation means "4 times something equals 60", students assign 15 to $2p + 5$. Once the equation is reduced to $2p = 10$, the technical obstacle of covering $2p$ rather than p appears again. They are finally able to find $p = 5$ as the solution.
Task 4a. Solve the equation for w. $\frac{8(3w+2)}{5} - 2 = 6$ Hint: Cover $\frac{8(3w+2)}{5}$ at the first step and assign a value to it.	After getting the observer's explanation, students follow the hint and assign 8 to $\frac{8(3w+2)}{5}$. Next, they identify correct values for $8(3w + 2)$, $3w + 2$, $3w$ and w, respectively, by themselves.
Task 5a. Solve for a. $\frac{18}{5a-2} = 6$ Hint: Choose one of three sub-expressions from the answer box to cover at the first step.	Even if Ali assigns 3 to $5a – 2$ correctly, Quni misunderstands it as $5 – 2 = 3$. Next, Quni suggests to cover a directly and assigns 3 to it, rather than assigning $5a$ as suggested by Ali. Overall, the students finally solve the equation correctly. The technical obstacle of covering $5a – 2$ appears at the initial step of solving the equation.

Task 6a. Solve for x. $$\frac{30}{2x+3}+4=6$$ Hint: Choose one of three sub-expressions from the answer box to cover at the first step.	After reading the task, students are able to identify parts of the equation to cover and to assign proper numerical values to those parts by themselves. However, Widan seems to not fully understand when Ali says that he will cover $\frac{30}{2x+3}$ at first.
Task 7a. Solve for z. $$\frac{48}{8(z+1)} = 3$$	Overall, students are able to identify parts of the equation to cover and to assign proper numerical values to those parts. However, Widan seems to not fully understand why his colleagues assign 16 to $8(z + 1)$ (see Table 6.3 for a detailed description).
Task 8. Solve for positive q: $$(q + 3)^2 = 49.$$ Hint: Choose one of three sub-expressions from the answer box to cover at the first step.	After reading a worked example and getting an explanation from the observer, the students work on Task 8. Even if Quni identifies $(q + 3)$ to cover in the first step, he assigns 9 rather than 7 to it. Overall, the students are able to solve the equation.
Task 9a. Solve for positive x: $$(x - 1)^2 + 3 = 12.$$	With the observer guidance, students solve the equation without a serious difficulty. A calculation mistake appears when Widan assigns 4 to $x - 1$ rather than 3 as suggested by Quni.
Task 9b. Solve for positive r: $$(2r + 1)^2 + 1 = 26.$$	After reading the equation aloud, Quni identifies 5 for the value of $2r + 1$. When this group concludes 2 for r from $2r = 4$ Widan does not know why his colleagues assigned it – indicating that he does not understand the solution process.

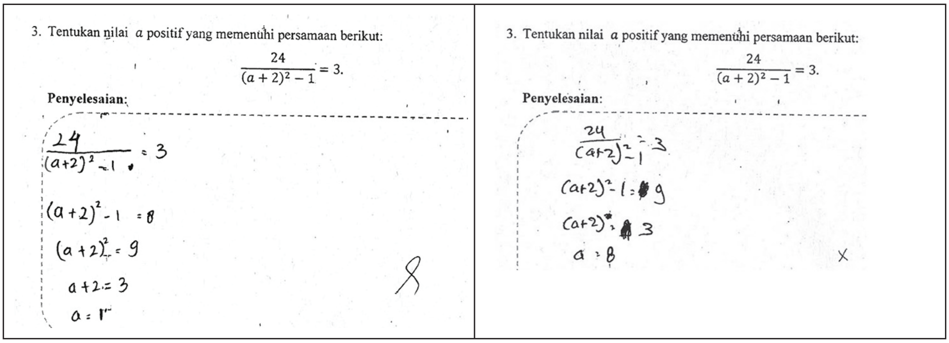


Figure 6.5. Exemplary written work by Ali (left) and Widan (right) after engaging in the Cover-up activity

This result shows that the three students in the group acquired different conceptual understanding and skills. This also is manifest in their individual written work on a paper-and-pencil task as shown in Figure 6.5. The left part shows Ali's work. It is a correct solution and Ali is able to select appropriate expressions as well as able to assign correct numerical values to them, and to successfully apply the cover-up strategy. The right part shows Widan's work, which is similar to Quni's work. Even if Widan is able to select appropriate expressions, he in some cases assigns incorrect values to them. For example, he initially assigned 8 to $(a + 2)^2 - 1$, but then changed it to 9. Also, Widan initially assigned 9 to $a + 2$, but he changed it to 3 (which is correct, surprisingly enough!). We conjecture that Widan assigned 8 to a because he calculated 24 divided by 3. This final step suggests that he mixes up the variable a and the expression $(a + 2)^2 - 1$.

The second point concerns the applet's technical limitations. Whereas covering an expression within an equation with the mouse was expected to foster reification, the applet proved to be too sensitive to mouse movements. As a consequence, students often highlighted an expression that they did not intend to cover, such as covering $\frac{18}{5a-2}$ instead of $5a - 2$ in $\frac{18}{5a-2} = 6$. Once students got used to working with the applet, this phenomenon became less frequent. Speaking in general, technical limitations, which in themselves have nothing to do with conceptual understanding, may influence and even hinder the artefact's use, and as a consequence may impede the instrumental genesis.

The third point concerns the order of tasks. While designing the instructional sequence, we ordered the tasks according to their conjectured difficulty, as reflected in many curricula: linear equations first (Tasks 2a-4a), next rational equations (Tasks 5a-7a) of the form $\frac{k}{g(x)} = c$, where k and c are constants, and $g(x)$ is a linear expression, and finally quadratic equations (Tasks 8-9b). Indeed, the data show that linear equations were easier than rational equations, but the rational equations seemed to be more difficult than the quadratic ones. However, the students were able to apply the scheme and techniques to new types of equations, which can be seen as a modest form of instrumental genesis.

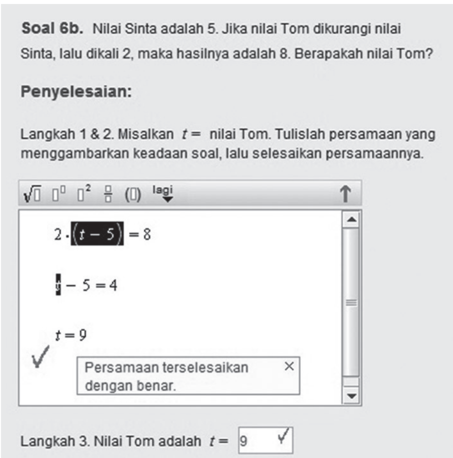
To summarize, the observed scheme was in line with the conjectured scheme described in Table 6.1 and Figure 6.4. As presented in Table 6.4, the main obstacles encountered by students included arithmetical calculation errors. This scheme and the related techniques were applied to increasingly complex equations, which suggest the development of a structural view on equations and expressions, and instrumental genesis.

6.8.2. Students' scheme for solving word problems using algebraic substitution with the Cover-up applet

An observation of a group working on one of the tasks

The word problems addressed in the activity after the proper transformation of problems into mathematical models all concern linear equations of the form $f(x) = c$. Table 6.5 presents a three-minute observation of the group's work on Task 6b. In the right column we provide commentaries based on the conjectured scheme presented in Table 6.2 (steps a-b) and Table 6.1 (steps 1-5).

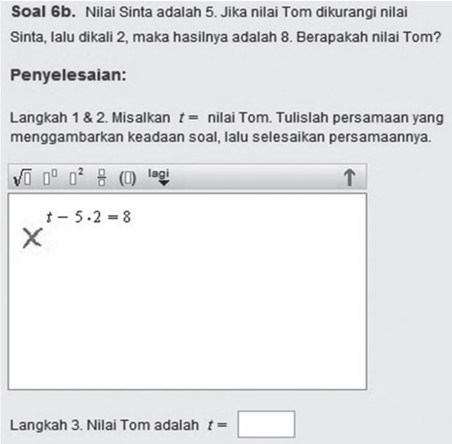
Table 6.5. A commented observation of a group's work on Task 6b

Observation	Commentary
Task 6b. Sinta's grade is 5. If Tom's grade is subtracted by Sinta's grade, next multiplied by 2, the result is 8. Find Tom's grade. Hint: Let *t* be Tom's grade.*  	Task 6b is a word problem in the Cover-up activity. The figure below is the student digital work stored in the DME.
Students read out the task aloud together.	Reading the task aloud seems to help the students to recognize that the word problem can be transformed into a mathematical equation, to identify the given information and an unknown, and to prepare possible strategies for finding the unknown (step a).
Quni and Widan: *t* is subtracted by 5, next divided by 2. Ali: No, it is multiplied by 2. Quni and Widan: Yes, it is incorrect. It must be multiplied by 2. Quni: *t* is subtracted by 5, and then multiplied by 2.	Students are trying to set up an equation (step b). However, Quni and Widan mistranslated the second phrase of the problem. Ali corrects them.

<table>
<tr>
<td>Quni and Widan: t minus 5, multiplied by 2, equals 8. [Quni types $t - 5 \times 2 = 8$.]

Ali: Enter!

Quni, Ali and Widan: [After Quni presses enter] Incorrect! Why is it still wrong? [A red cross appears in the solution window signifying that the formulated equation is incorrect.]



Observer: Maybe you typed the equation incorrectly! Please you type it again! Please you type 2 [first]. [Quni erases the previous incorrect equation, and types 2 firstly.]

Observer: So, 2 times...</td>
<td>Even if the three students understand the word problem, they are not able to express it in a proper equation. In other words, they have difficulties in setting up an equation (step b).

Observing this situation, the observer gives guidance.</td>
</tr>
<tr>
<td>Quni, Ali and Widan: [2 times] t subtracted by 5, equals 8. [One of the students types $2(t - 5) = 8$.]

Observer: Enter!

Quni, Ali and Widan: [One of students presses enter! It is correct!] Ooo...

Observer: Do you understand why I suggested you to type the 2 [in front of] $(t - 5)$? [No reply, maybe they are thinking].

Observer: Because, as you said, first you do t minus 5, and then multiplied by 2. The bracket means something that should be carried out firstly.</td>
<td>The observer suggests to type 2 in front of $t - 5$, and, as an indirect consequence, to use a bracket for $t - 5$, so that they get a correct equation. With the observer's guidance, students are finally able to set up a correct equation (step b). However, the observer seems to give too much guidance as if he is part of the group wanting to solve the task correctly.</td>
</tr>
</table>

Ali: Cover *t* – 5. [Quni highlights *t* – 5, the applet provides *t* – 5 = … in the next line.] Quni, Ali and Widan: [*t* – 5 =] 4. Quni: types 4 and presses enter. It is correct.] Quni: [The value of *t* is] 6. Quni: Eh, no, [it is] 4. Widan: [No! It is] 5. Ali: [Hi, the value of *t* must be] 9. Quni and Widan: Yes, 9, you are right! [Quni types 9, so it becomes *t* = 9.] Ali: Enter! Quni: [Presses enter] Correct! [So, the solution is *t* = 9.]	Quni and Ali recognize the equation of the form *f*(*t*) = *c* and its structure (steps 1 and 2). Even if they make mistakes during the solution process, they are finally able to solve the equation using the cover-up strategy (steps 3-5), i.e., they are able to identify the sub-expression to cover (step 3), assign a numerical value to the covered sub-expression (step 4), repeat steps 3 and 4 while applying the cover-up strategy (step 5) and recognize *t* = 9 as the solution.
The students proceed directly to the next task without checking the solution.	They do not check the solution because the applet has already given feedback in each step that their actions are correct.

* This task is adapted from a task in the Indonesian textbooks. Even if the task is considered to be less appropriate from a didactical point of view – one can argue whether grades can be multiplied – we decided to use it to connect to the Indonesian textbook.

Concerning students' mistakes when assigning a numerical value to *t* in the equation *t* – 5 = 4, we conjecture that the students referred to the equation 2(*t* – 5) = 8 when determining the value of *t* for the equation *t* – 5 = 4. Quni seemed to see the addition as the central operation in the expression 2(*t* – 5) rather than the multiplication of 2 and *t* – 5. As a result, Quni assigned 6 to the value of *t*. Next, when he saw 2(*t* – 5) as a multiplication of 2 and (*t* – 5), he assigned *t* = 4. Widan, who assigned *t* = 5, might have guessed an arbitrary value. The mistake of recognizing the central operation for the expression 2(*t* – 5) as an addition of 2 and (*t* – 5) suggests that Quni lacks a structural view on the algebraic expression. As an aside, we notice that he also interchanged *t* – 5 for *t*, which concerns the difficulty in understanding a variable.

From the observations such as the one described in Table 6.5, we conclude that the students' scheme is in line with the conjectured scheme outlined in Tables 6.2 and 6.1. To summarize, the main conceptual elements in this scheme – which is an extension of the scheme visualized in Figure 6.4, include: setting up an equation, and working towards the form < unknown > = < value >, which provides the solution of the equation. The step of setting up an equation is the difference between this scheme and the scheme in Figure 6.4. For this step, the role of the applet concerns providing feedback stating whether a formulated equation typed in the solution window is correct or not: when it is correct, a yellow tick mark appears, otherwise a red cross mark emerges. In this way, students can improve their ideas while formulating an equation.

An analysis of the group's work over the different word problems within one lesson
Table 6.6 summarizes the students' scheme and related techniques of the observed group for the case of word problems treated during the one-lesson Cover-up activity. Similar to the observation for the bare problems as shown in Table 6.4, the students' main observed obstacles included arithmetical calculation mistakes (such as the ones in tasks 3b and 4b) and mistakes in transforming word problems into equations. Technical obstacles included the use of the equation editor, such as typing fractional expressions in the solution window, which was not needed in the bare equation solving tasks. The already noted difficulties of highlighting expressions with the mouse reappeared. Also, the observer gave too much guidance while the students were solving the tasks, such as for task 6b.

From the above observations, we conclude that the students' scheme development for solving word problems is in line with the conjectured scheme described in Tables 6.2 and 6.1. Also, similar to the case of bare equation problems, this scheme and techniques were applied to increasingly complex equations, which again suggest the development of a structural view on equations and expressions and instrumental genesis.

6.9. Conclusions and reflection

To investigate the relationship between the use of a digital tool and student conceptual understanding we examined which schemes students develop for solving equations using algebraic substitution with the Cover-up applet. In particular, we focused on the relationship between the technique of highlighting expressions with the mouse, and the ability to identify and select appropriate expressions, which requires both an object and a process view.

A first conclusion is that the scheme which students developed for solving an equation using algebraic substitution with the Cover-up applet is in line with the conjectured scheme formulated in Table 6.1. It includes recognizing the equation as suitable for the cover-up strategy and the task to rewrite it in the form < unknown > = < value >, identifying a sub-expression within the equation to cover as well as assigning a numerical value to it (in each cover-up strategy step), and repeating this until the desired form is found. Within this scheme we noticed the interplay between on the one hand the techniques of highlighting an expression, typing a numerical value for the highlighted part, and pressing enter to check, and on the other hand the ability to see

Table 6.6. Students' schemes over the different tasks within one lesson

Tasks	Observation: scheme and techniques
Task 2b. Budin's height is 130 cm. If Adin's height is divided by 3, next added to Budin's height, the final result is 175 cm. Find Adin's height. Hints: Given the equation $\frac{a}{3} + 130 = 175$ representing the word problem, students are required to choose one out of three options from the answer box for the meaning of *a*.	Through questions, the observer guides students. In this way, students are able to apply the cover-up strategy step-by-step: identify the respective sub-expressions $\frac{a}{3}$ and *a*, and assign numerical values. An observed obstacle includes typing the fractional expression $\frac{a}{3}$ with the equation editor.
Task 3b. Two times a number is added to 3, then divided by 5, and finally added by 1. If the final result is 4, find the number. Hint: Let *m* be the number to find, students are required to select one out of three equations representing the problem from the answer box.	Students choose $\frac{2m+3}{5} + 1 = 4$ as the equation expressing the word problem correctly. Next, they type it correctly through the equation editor. Even if the observer guides them, the students improperly identify the first sub-expression to cover: $2m$, $2m + 3$ and *m* respectively. As a result, they cannot assign numerical values to these expressions. Next, after correctly choosing $\frac{2m+3}{5}$ as the sub-expression to cover, students make calculation mistakes. This may indicate that students do not understand yet how to apply the cover-up strategy. By the observer's guidance, students are finally able to solve the equation.
Task 4b. Udin is 4 years older than Tom. If Tom's and Udin's ages are 30, find Tom's age. Hint: Let *t* be Tom's age, students are required to choose one out of three equations representing the problem.	After typing $t + t + 4 = 30$, students do not simplify the equation to $2t + 4 = 30$. Rather, they directly cover $t + t$, and finally assign $t = 13$. Calculation mistakes appear during the process of assigning from the equation $t + t = 26$. The technical obstacle of covering a sub-expression also appears when covering $t + t$.
Task 5b. The price of a glass of ice is Rp 1000. Doni has Rp 2000. If the price of a bowl of meatballs is subtracted by the price of a glass of ice, next divided by 3, then the results is equal to Doni's money. How much is a bowl of meatballs? Hint: Let *b* be the price of a bowl of meatballs.	Even if students are able to translate the word problem in a proper equation, they type it with a minor incorrect notation: rather than typing for example 1000 for the price of a glass of ice, students typed it as Rp 1000. Next, with the help from the observer, students are able to type the equation correctly. Then, they properly apply the cover-up strategy (despite some calculation mistakes).
Task 6b. Sinta's grade is 5. If Tom's grade is subtracted by Sinta's grade, next multiplied by 2, then the result is 8. Find Tom's grade. Hint: Let *t* be Tom's grade.	Students find it difficult to set up a correct equation from the word problem. With the observer's guidance, students are finally able to do this. However, calculation errors emerge while applying the cover-up strategy in solving the equation (see Table 6.5 for a detailed description).

expressions as objects and to identify an appropriate expression to advance towards the desired form.

Second, the scheme that students develop for solving corresponding word problems is in line with the conjectured scheme described in Tables 6.2 and 6.1 as well. This scheme includes setting up an equation from the word problem, entering it, and putting into action the scheme described above. As such it is an extension of the previous scheme, which includes some additional interplay between technique and understanding. For instance, entering an algebraic expression corresponding to a phrase in the word problem using the equation editor is a technique that reflects the mental activity of recognizing the algebraic structure within that phrase.

Third, as the equations in the digital activity gradually get more complex, we observed that the students' schemes develop in the sense that their application is extended to a wider category of problems. The fact that the schemes and techniques 'survived' when facing an increasing complexity is considered a form of instrumental genesis. Further instrumental genesis would be expected over a more extended period of use.

Reflecting on these conclusions, we feel that three factors play an important role in fostering the co-emergence of techniques and understanding, and as such the instrumental genesis: the characteristics of the applet and the corresponding task design, the role of the observer who acted as a teacher for the students, and the interaction among students. Concerning the first factor, central in the characteristics of the applet is that the techniques that the applet makes available – in this case a quite limited set of possible techniques – correspond to mathematical notions and operations. We might call this the applet's mathematical fidelity. The applet's feedback helped students to overcome algebraic errors and to improve their method. For word problems, the applet also provided feedback on the syntactical correctness of the equations the student enters.

The Cover-up applet can be criticized because of the limited construction room it provides to the students. Indeed, the repertoire of possible techniques that the applet makes available is small. From a didactical point of view, this is a drawback; for a case study on instrumental genesis, however, this limitation allows us to focus on instrumental genesis.

Concerning the task design, ordering the tasks from a relatively simple to more complex problems helped the students to gradually develop their thinking and as such contributed to the development of schemes and techniques. As an aside, the relatively simple tasks in this study are more complex than the tasks in the regular Indonesian mathematics curriculum. In fact, one of the interesting features of the cover-up strategy is that it can easily be applied to any equation of the form $f(x) = c$ with the variable appearing only once on the left hand side, and is not restricted to linear equations. The hints provided indirect guidance to students on initial actions in the solution process. In the design of word problems, we observed that student difficulties in setting up equations are not caused by their inability to understand each word or phrase in the problem, but by their inability to represent them in an appropriate expression or equation (Jupri & Drijvers, accepted). This concerns the process of transforming the problem situation into the world of mathematics, also called horizontal mathematization (Treffers, 1987; Van den Heuvel-Panhuizen, 2003).

The second crucial factor concerns the role of the observer, who acted as a teacher for the observed group. On the one hand, this is part of an orchestration that guides students' instrumental genesis, as the regular teacher did for the other groups of students. On the other hand, we acknowledge that the observer gave strong guidance in some cases, which may have affected the instrumental genesis. In line with Swan (2008), for future teaching design, we recommend the observer to take a more distant stance to avoid this effect.

The third and final factor concerns the interaction among students during the group activity. Even if the three students helped each other during the problem solving, they seem to have gained different conceptual understanding and skills. In particular the higher ability student, Ali, acquired a better understanding than his peers and he played an important role in the group's success in solving the problems. Apparently, the group work interaction did not lead to similar individual instrumental geneses, even if it did contribute to these individual processes.

As a final reflection, we wonder how the specific conclusions on scheme development for algebraic substitution can be extrapolated to the general issue of the relationship between using a digital tool and the targeted mathematical understanding, of the "why and how" of using digital technology in mathematics education. Even if this paper reports on a small case study, we feel that its main conclusions go beyond its case. The correspondence between techniques that the digital environment invites and the targeted mathematical understanding, that is so well phrased in the vocabulary of the instrumentation theory, is an indispensable condition for a fruitful use of digital tools for mathematical learning. Using digital technology in mathematics education will really work, is our strong conviction, because of this intertwinement of technique and mathematical concept, according to which the techniques encapsulate mathematical thinking. This is a start to answering the "why"-question: digital tools work because they allow us to represent mathematical ideas in an efficient and challenging way. As a consequence, answering the "how"-question might start with designing tasks and orchestrating the learning process in a way that exploits the affordances of the so crucial connection between technique and mathematical understanding. This being said, we are of course aware that these reflections are but a start in the engaging enterprise of fruitfully integrating digital technology in mathematics education.

Acknowledgment

This study was funded by the Indonesia Ministry of Education project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794. The first author thanks Jan van Maanen for his supervision. We thank Peter Boon for designing the Cover-up applet, the teachers and students for their participation, and research assistant Rohma Mauhibah for her contributions.

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Chapter 7 Conclusion

7.1. Summarizing the findings

In this final chapter we complete this study with a conclusion and a reflection on the overall findings. To do so, we first summarize the answers to the research questions. Next, we reflect upon the overall findings from different angles. This is followed by theoretical and practical recommendations for algebra teaching, for algebra pedagogy and digital tools design, and for future research. Finally, we close this chapter with an epilogue about future algebra education in Indonesia.

To summarize the findings, we provide answers to each of the five research questions investigated in this study.

> **Research question 1:** What are Indonesian students' difficulties in initial algebra learning, particularly in solving linear equations in one variable and the related linear inequalities?

From the interview study we concluded that mathematization, i.e., a transforming process of problem situations into mathematical models and vice versa, and the reorganization of the mathematics itself, constituted the most frequent observed category of difficulty. Other observed difficulties included the categories of understanding algebraic expressions, applying arithmetical operations in algebra, understanding the different meanings of the equal sign, and understanding the notion of variable. Each of these five categories included sub-categories of specific difficulties encountered by students. For example, the difficulty in formulating equations or inequalities concerned the most frequent difficulty within the category of mathematization; and the difficulty in using the additive inverse property in solving an equation concerned the most frequent difficulty within the category of applying arithmetical operations. These five categories were then used as a point of departure for the rest of the study.

> **Research question 2**: What are student difficulties in solving equations in one variable which emerge in an ICT-rich approach and how can operational and structural views on equations explain these difficulties?

From the analysis of the small-scale pilot teaching experiment data we concluded that the difficulties which appeared in written student work after solving equations as the effect of the ICT-rich approach, include two types related to the equation solving strategies. First, while applying the reverse strategy with paper-and-pencil after the Algebra Arrows applet activity, the main difficulties consisted of applying arithmetical operations and the equal sign categories. Whereas calculation errors, mistakes in using the properties of arithmetical operations, and mistakes in using the priority rules of arithmetical operations were the difficulties in the category of applying arithmetical operations, notational mistakes were the main problems within the equal sign category. Second, while using the cover-up strategy by hand after the Cover-up applet activity, the main observed difficulties fell in the categories of applying arithmetical operations (such as calculation errors), of understanding the concept of variable (such as understanding the variable as an unknown), of understanding algebraic expressions (such as the parsing obstacle, the expected answer obstacle, and the lack of closure obstacle), and of the equal sign (such as notational mistakes in the use of the equal sign). The parsing obstacle concerns the (mis)understanding of the order in which algebraic expressions must be processed, which may conflict with the order in natural language; the expected answer obstacle means expecting a numerical value for an algebraic expression; and the lack of closure obstacle refers to the discomfort in dealing with algebraic expressions that cannot be simplified further (Tall & Thomas, 1991). The operational and structural perspectives were fruitful to understand the above student difficulties in solving equations in the following way:

- A limited operational view may cause calculation errors, which reflect an inability to carry out or to reverse a calculation process;
- A lack of insight into the structure of algebraic expressions may cause the additive or multiplicative inverses mistakes, because these mistakes may stem from students' incorrect view on the relationship between expressions within an equation;
- Notational errors in using the equal sign may result from a lacking insight into the structural meaning of the equal sign as expressing the equivalence of two expressions;
- The mistake of understanding a variable as an unknown reflects a lack of meaning to an equation, which reflects a lack of symbol sense;
- The priority rules mistakes may be explained by a lack of understanding of an expression as representing an ordered calculation process, which concerns an operational view, and of a misunderstanding of the structure of an algebraic expression; and

- The parsing obstacle and the lack of closure obstacle occur because of a limited view of the operational meaning of an algebraic expression which represents a calculation process.

> **Research question 3:** What are student difficulties in mathematizing word problems in the domain of linear equations in one variable?

To answer this question, we analyzed a part of the results of the pilot experiment, particularly for the case of solving word problems, using a mathematization perspective. This led to three conclusions. First, the main difficulties in students' written work after the Algebra Arrows activity concern the solution processes and, to a lesser extent, checking solutions, i.e., the third and fourth steps of the mathematization cycle (De Lange, 2006), which can be categorized as difficulties in vertical mathematization. However, our observations of student digital group work suggests that the main difficulties encountered by students concern understanding the problems and formulating mathematical models, i.e., the first and second steps of the mathematization cycle, which characterizes the difficulties in horizontal mathematization. Factors that explain the results of student written work after the Algebra Arrows activity include: (i) the context of the tasks, namely about numbers, is familiar for the students; and (ii) the structure of the tasks is relatively easy to translate into mathematical models. As a consequence, students did not encounter serious difficulty in understanding problems and in formulating mathematical models, but found more obstacles in the solution processes and checking solutions—because in this paper-and-pencil environment there is no feedback on whether their solutions are correct or not. Second, the main difficulties in the written student work after the Cover-up activity concern understanding problems and formulating mathematical models, i.e., the first and second steps of the mathematization cycle, which concern the difficulties in horizontal mathematization. Factors that explain the results of student written work after the Cover-up activity include: (i) the contexts of tasks are various and closer to real life than in the Algebra Arrows activity; (ii) the structure of the tasks is difficult to translate directly into a mathematical model. As a consequence, students encounter obstacles in understanding problems and in formulating mathematical models. Third, the findings from both lessons are confirmed by the results of the final written test: the difficulties in vertical mathematization emerge in student work on typical tasks related to the Algebra Arrows activity, whereas the difficulties in horizontal mathematization appear in student work on typical tasks of the Cover-up activity. Altogether, we found that the main difficulties encountered by students who deal with word problems concern transforming problems into mathematical models, i.e., in horizontal mathematization. The data from the interviews confirmed these findings. Even if these difficulties in the solution processes were frequently observed, the most frequent difficulties revealed in the interviews concern translating word problems into mathematical models.

> **Research question 4:** Does an intervention with digital technology enhance students' performance in initial algebra?

In the larger-scale experiment we found that the experimental group's mean gain score (4.63) was significantly higher ($p < .01$) than the control group's mean gain score (3.02) with a small to medium effect size (0.32). The quantitative results were confirmed by the findings of the qualitative analysis of the digital and written work of one group of students during the four lessons of the experimental intervention. In other words, the intervention proved to be effective for enhancing student achievement in algebra. This effectiveness can partly be observed, for instance, from problem solving strategies that were used by the students and from observable difficulties that they encountered in both digital group work and written work. We consider that the similarities between the students' problem solving strategies in written work and in digital work reflect a direct effect of the applets use. Also, the difficulties which were observed in observations of both digital activity and written work to a certain extent depict student conceptual understanding and procedural skills.

> **Research question 5:** Which schemes do students develop for solving equations using algebraic substitution with the Cover-up applet and which relationships between techniques and understanding are developed?

A case study analysis led to three conclusions. First, the scheme that students develop for solving an equation with the Cover-up applet includes: (1) recognizing the equation as suitable for the cover-up strategy and the task to rewrite it in the form < unknown > = < value >; (2) identifying a sub-expression from the equation to cover as well as assigning a numerical value to it (in each cover-up strategy step), and (if necessary) repeating this until the desired form is found. Second, the scheme that students develop for solving a word problem using the Cover-up applet includes: setting up an equation from the word problem; entering it in the solution window; and putting into action the scheme used for solving an equation as described previously. Within these schemes we notice the relationships between techniques and understanding, such as highlighting a sub-expression, typing a numerical value for the highlighted part, and pressing enter to check are techniques that reflect the mental identification of an appropriate sub-expression from the equation and the assignment of a corresponding numerical value to it. Third, as the problems used in the cover-up activity are ordered from relatively easy to more difficult, we observed that the students' scheme for solving an equation seems to develop in the sense that it is applied to new, more complex types of problems. Even if it is hard to observe instrumental genesis from one Cover-up activity, we interpret the latter observation as a modest support for the students' scheme development.

7.2. Reflecting upon the results

In this section we reflect upon the overall results of this study and its limitations. First, we reflect on the theoretical perspectives, on the methodology and design of the study, and on the generalization of the findings. Then, we elaborate on the limitations of the study.

7.2.1. Reflection on the theoretical perspectives

In this study, we used several theoretical perspectives. These include theories on student difficulties in initial algebra, operational and structural views on algebraic activity, mathematization, the role of ICT in algebra education, and the instrumentation theory on tool use. We now provide our reflections on each of these theoretical perspectives.

Student difficulties in initial algebra

The first reflection concerns the inventory of five categories of student difficulties in initial algebra which were described in Chapter 2. This categorization resulted from both a top-down and a bottom-up approach. The top-down approach consisted of a literature study in the field of mathematics and algebra education, and the bottom-up approach made use of a synthesis of the analysis of written student work and interviews. As a consequence, the inventory plays a double role: as a frame of reference for analyzing data and as an important result of the study. We conjecture that the categorization is by no means exhaustive, as it may need extensions when applied to other algebraic topics. For example, if the categorization is applied to identify student difficulties in the topic of functions, a new (sub)-categorization may be needed to include, for instance, difficulties in making a graph of a function. From this reflection, we deduce two contributions concerning the role of this perspective in the field of algebra education: (1) a theoretical contribution, the categorization itself being a systematic summary of student difficulties as reported in the literature; (2) a practical contribution in that the study provides an example of how to put this categorization into practice in teaching and research.

Operational and structural views on algebraic activity
The operational and structural perspective on algebraic activity – which originates from reification theory (Sfard, 1991) – was used to explain the causes of student difficulties in solving algebraic equation problems, and was mainly addressed in Chapter 3. Concerning this, we noted the following two experiences while applying this perspective. First, the operational and structural perspective did indeed seem to be a fruitful lens to understand student difficulties. In line with Sfard (1991), most of students seemed to prioritize an operational view, and in general they lacked an integrated operational and structural view on algebraic expressions and equations. This may relate to the fact that the students in this study were still novices in algebra, and as such they need more experiences, effort and time to acquire this integrated view. We found that a limited operational view mainly explains student difficulties in the arithmetical category; and a limited structural view mainly explains student difficulties in the categories of understanding algebraic expressions, variables, and the equal sign. As such this study illustrates the practical application of reification theory. Second, the operational and structural perspective in algebra relates to the notions of symbol sense (Arcavi, 1994; 2005) and structure sense (Novotna & Hoch, 2008), that both include a structural view on algebraic expressions and equations.

Another reflection concerns the use of this perspective as the background for algebraic substitution, which is addressed in Chapter 6. Algebraic substitution is the foundation of the cover-up strategy for solving equations, as it concerns replacing a more complex expression by one variable. To properly apply this strategy, an integrated structural and operational view on algebraic expressions is necessary.

Mathematization
In this study, the notion of mathematization is mainly used for analyzing and explaining student difficulties dealing with word problems in algebra (see Chapter 4). As a first remark on this notion, we should realize that the term "mathematization" that is used here is a more elaborated version of the fifth category of difficulties in initial algebra described in Chapter 2. In particular, this concerns the mathematization cycle (De Lange, 2006). Second, even if the use of this lens in this study seems to be restricted to only dealing with word problems, the mathematization perspective actually concerns all types of mathematical problems. We used this lens for addressing word problems

because the first two steps of the mathematization cycle, i.e. understanding problems and formulating mathematical models, are manifest when dealing with this type of problem. Third, in our view, the contributions to the field of algebra education that result from this study using this perspective are twofold: (1) this study provides an insight into student difficulties in solving word problems in the topic of linear equations in one variable; and (2) the study provides a concrete example of how to put this lens into practice for identifying and explaining student difficulties dealing with word problems in algebra.

The role of ICT in algebra education

Drijvers, Boon and Van Reeuwijk (2010) distinguish three didactical functions of ICT in algebra education: as a tool for doing mathematics, as an environment for practicing skills, and as an environment for concept development. The first didactical function hardly plays a role in this study, because the applets involved are not developed for outsourcing mathematics. We consider that the practicing skills function and the concept development function both play an indirect but crucial role, that is, as a background in the design of the learning arrangements. For instance, the design of feedback in the applets allows for students to gradually develop both conceptual understanding and procedural skills through working with the applets. Even if in this study the teacher introduced the equation concept before the students did the digital group work activities, the development of procedural skills and conceptual understanding in our opinion are not independent: both develop simultaneously and in alternation, in iterative ways.

Instrumentation theory

The instrumentation perspective played an important role for interpreting student behavior during digital group work with the Cover-up applet, as elaborated in Chapter 6. The notions of techniques and schemes were fruitful for interpreting student conceptual understanding and procedural skills while solving equations: they provide means to address the subtle interplay between mathematical thinking – in this case about algebraic substitution – and techniques for using the digital artefact, in this case the mouse movements to select the expressions to be substituted. This interpretation was concretized in the form of a formulation of student schemes for solving different type of problems.

Our other reflections concerning the use of this perspective include the following. First, of the three factors that play a role in the instrumental genesis (addressed in Chapter 6) – i.e., the characteristics of the applet and task design, the role of the teacher, and the interaction among students within the group – we consider that the interaction among students in the group plays a direct and crucial role in individual instrumental genesis. The reason is that during the interaction, each student is engaged in simultaneous hands-on and minds-on activity. The hands-on activity takes place while the student is using the tool, in this case the Cover-up applet, and the minds-on activity occurs when the student is thinking and helping others in the problem solving process. Second, we contend that a solid effect of the applet use can be traced by the transfer of the student strategies in written student work. In our view this reveals the observable behavior of the student schemes which include both applet and paper-and-pencil techniques.

7.2.2. Reflection on the study's design and methodology

In this study, we applied qualitative and quantitative methods for addressing partial studies: qualitative methods in the interview and the small-scale pilot experimental studies and quantitative methods in the experimental study. Overall, as stated in Chapter 1, the study can be characterized as a design research study. Concerning this claim, we acknowledge that we did not make an explicit elaboration of the three design research phases – preparatory phase, teaching experiment phase, and retrospective analysis phase – for each cycle. Rather, we presented partial studies as described in Chapters 2-6. In this sense, the design research methodology in this study was not followed in a strict manner.

Concerning the integration of ICT in the teaching experiment, we note two points to reflect on. First, we emphasize that the use of technology in the teaching experiment is only one aspect of the teaching intervention as a whole, including the introduction of the topic, the demonstration of the use of applets, the group work digital activity, a whole-class discussion and the daily intermediate paper-and-pencil assessments. As such, the results of the experiment cannot be attributed to the digital technology's effect only.

Second, the Algebra Arrows and the Cover-up activities in this study address equations of the form $f(x) = c$, and the Balance Model and the Balance Strategy activities address equations of the form $f(x) = g(x)$. Together, the learning sequence implemented in this study offers a relatively complete set of strategies for the topic of (linear) equations in one variable.

7.2.3. Reflection on generalization of the findings

This study focused on the domain of (linear) equations in one variable, related inequalities, and related word problems, which are central in initial algebra.Within this topic, the main difficulties students encountered concern the notion of an algebraic expression which needs to be understood as both a process and an object, and the difficulties in translating word problems into mathematical and algebraic models. These two issues also play a central role in other topics, such as quadratic equations and corresponding word problems: in quadratic equations, too, expressions need to be flexibly considered as processes and objects, and in word problems leading to algebraic equations, students need to carry out similar transformation and translation activity as in the case of linear equations. Therefore, we expect these findings to be generalizable to other topics within beginning algebra, and we conjecture that the same theoretical lenses, in this case the process-object duality and the notion of mathematization, would apply in these topics, too.

With respect to the use of ICT in the learning setting, an important finding is that a seamless match is needed between the targeted mathematical thinking (in this study, for example, the perception of relevant algebraic expressions as 'units' to be covered up to carry out algebraic substitution in the Cover-up applet), the technique in the digital environment, and the transfer to similar paper-and-pencil techniques. Again, we believe that the same holds for the use of digital tools for more sophisticated algebra tasks, or for other technological environments. For example, when students use an advanced computer algebra system to carry out algebraic manipulations, they definitelly will need similar skills to make the software work for them. In that sense, we expect our findings to be generalizable to such situations, and we conjecture that the instrumentation lens we used in this study will be fruitful in such situation as well.

7.2.4. The study's limitations

Reflecting on the overall study, we note some limitations. For stage 1, in which the interview study was carried out, we identify two main limitations. First, the interview study included 51 students who provided written work, 37 of whom were interviewed. One may question if this sample size is large enough to represent all Indonesian students. Indeed, a larger sampe size would of course have been better, but we have no indication that the current sample is biased.

Second, the framework that resulted from the interview study consists of five categories of student difficulties. In some cases, a student mistake could be categorized in more than one category. This means that the intertwinement and interdependency of the five needs further investigation, which we were unable to carry out within the frame of this study.

In stage 2, the pilot teaching experiment stage, we identify two main limitations. First, as the pilot experiment was carried out within the frame of design research, we made use the main results of the stage 1 and incorporated them in the design of the learning sequence. This incorporation included, for instance, making predictions of student problem solving strategies and possible difficulties that might emerge. Such predictions can take shape in a so-called hypothetical learning trajectory (HLT, Simon, 1995). However, we did not explicitly elaborate the HLT. Rather, we incorporated the main elements of the HLT in the analysis which focused on investigating student difficulties as emerged in written and digital student work. We consider this point as a limitation in the sense that we did not follow the design research method strictly; if the study were to be replicated, the present results could easily be phrased in terms of a HLT.

Second, we designed the learning sequence on the topic of equations in one variable of the form $f(x) = c$ and of the form $f(x) = g(x)$. Due to time constraints in the Islamic schools involved in this study, however, the pilot experiment only addressed the form of $f(x) = c$. The results of this experiment, therefore, informed the larger experiment in stage 3, in which we also addressed the equations of the form $f(x) = g(x)$, to a limited extent.

The limitations of the larger experiment (stage 3) also encompass two main points. First, as the four schools involved in this experiment are all Islamic junior secondary schools, this means that not all school types in the Indonesian educational system are involved – in this case Public junior secondary schools are not involved. As a consequence, we could not compare the results of students in these two different types of schools. As students in Public schools have more time for mathematics lessons than their peers in Islamic schools, however, it seems plausible to expect that the results there would be even better than in Islamic schools.

Second, in the case study we focused only on the observation of one group of students in each of the four experimental classes. On the one hand, this decision allowed us to get a deeper insight into students' development of conceptual understanding and procedural skills; on the other hand, while analyzing written and digital student work, we sometimes traced interesting data that might have contributed to further evidence and stronger conclusions, but the limited observational data made it impossible to further investigate this.

7.3. Proposing recommendations

In this section we propose recommendations concerning algebra education in general, the integration of ICT in algebra education, and algebra teaching on the topic of (linear) equations in one variable in particular. The recommendations are inspired by this study's findings, and are synthesized into more general suggestions. They concern algebra teaching, algebra didactic and digital tools design, and future research.

7.3.1. Recommendations for algebra teaching

The first recommendation for algebra teaching concerns the use of word problems and bare algebra problems in the learning process. Throughout this study, we found that mathematization and understanding algebraic expressions are the two most frequently observed categories of difficulty. The mathematization category often appeared in student work with word problems, and the understanding algebraic expressions category appeared frequently in student work on bare algebra problems. Considering these findings, we recommend that for initial algebra education both word problems and bare algebra problems should be addressed, and in an intertwined way. Word problems should be addressed not only as applications at the end of a lesson, but also as core elements and starting points in the learning process. Also, the emphasis should be not only on procedural skills, but also on conceptual understanding of algebraic expressions (Bokhove, 2011; Drijvers, Goddijn, & Kindt, 2010).

> *We recommend using both word problems and bare algebra problems in an intertwined way from the beginning.*

The second recommendation relates to student difficulties dealing with word problems. To develop better problem solving strategies when dealing with word problems in algebra, we recommend that teachers use the four steps of the mathematization cycle (De Lange, 2006) as a teaching strategy in the learning and teaching processes. This cycle includes: (1) understanding the problem; (2) phrasing the problem in terms of a mathematical model, such as an equation; (3) solving the mathematical problem included in the model and reflecting on the solution process; and (4) interpreting the mathematical solution(s) in terms of the original problem situation. This cycle provides guidelines to teachers on how to react to student difficulties and how to appropriately help students.

> *We recommend using the four steps of the mathematization cycle as a teaching strategy when dealing with word problems in algebra.*

To foster the development of integrated operational and structural views on equations and algebraic expressions, we recommend teachers to encourage students to use different strategies or a combination of strategies for, for instance, solving the same algebraic equation problem. This can be done, for instance, respectively by asking students to use more than one strategy for the same equation and to compare the different strategies; and by using a combination of strategies for solving an equation to obtain an efficient and an elegant strategy. A final, crucial, but often forgotten step while solving an algebra problem is checking the solution and the solution process. This final step is carried out as a check, but also refers to the meaning of the equation, i.e., the solution set are the values that turn the equation into a true statement when substituted.

Students should be encouraged to use different strategies or combination of strategies to solve the same algebra problem, and to check ther solution(s).

To develop student conceptual understanding and flexible procedural skills, we recommend teachers to confront students with algebra problems beyond the scope of the topic under consideration. For example, when addressing linear equations in one variable, students are confronted at an earlier stage with particular non-linear equations that can be solved using the cover-up strategy – such as $\frac{2015}{3x-7}=403$, and $(x-2011)^2 = 16$ – rather than allowing this at a later stage. Our experience in this study reveals that even if student mistakes may emerge when dealing with relatively complex problems, students are able to solve these equations using the cover-up strategy. According to Kindt (2010), students will be able to flexibly apply their procedural skills in equation solving, and will develop more formal strategies meaningfully, if the cover-up strategy is kept up for a time and sufficient variation of tasks is provided.

Confront students with related problems beyond the scope of the topic under consideration in the initial stage of problem solving to develop both conceptual understanding and procedural skills.

The fifth recommendation concerns the use of ICT in algebra teaching. In line with the recommendation proposed by Drijvers (2003), when starting the use of an ICT-rich approach in mathematics teaching, the teacher and students are suggested to establish a didactical contract (Brousseau, 1997) concerning the use of paper-and-pencil and the use of digital technology in the learning setting. In this way, students will be aware of what the teacher expects from them.

We recommend to establish a didactical contract between the teacher and students for the ICT-rich learning setting, that indicates when to use paper-and-pencil and when to use digital technology.

The sixth recommendation for algebra teaching concerns guiding student instrumental genesis. One factor that plays a role in the instrumental genesis concerns the role of the teacher in guiding students during the learning process. In our observation, we acknowledge that in some cases the teacher gave strong guidance – which affected student instrumental genesis – in the sense that students are often able to use procedural skills by means of the digital tool by only following the teacher's instructions in the problem solving process, though they do not really have conceptual understanding. Therefore, to avoid this effect, we recommend that teachers take a distant stance in orchestrating students' instrumental genesis. In addition, we recommend that teachers discuss student obstacles (technical or conceptual obstacles) during digital activity processes using projection facilities. In this way, all students can be aware of, anticipate, and avoid similar obstacles in their own work.

We recommend taking a distant stance while guiding students during the learning process as to foster students' instrumental genesis.

7.3.2. Recommendations for the design of algebra tasks

For designing algebra tasks, we propose the following recommendations. To reduce student difficulties in calculation errors, for instance when dealing with negative numbers and fractions, we recommend designing two types of tasks. First, tasks that consist of algebra problems that avoid these issues (calculation complexities). In this way, the algebraic difficulties can be isolated and can be addressed separately. Second, tasks that are similar to the first type, but contain calculation complexities. In this way, we can compare the results of student work for these two types of tasks, and in turn the tasks can be used to improve future task design.

We recommend developing two types of algebra tasks: (1) focus only on algebra; (2) focus on both arithmetic and algebra.

Another recommendation concerns task design heuristics, based on theoretical and practical references. Concerning this, we propose the following recommendations:

- If the design aims to foster the development of integrated and flexible operational and structural views on equations and algebraic expressions, for example for the case of the use of reverse and cover-up strategies for solving equations, we recommend to design tasks that explicitly call for students to use both strategies for the same equation and to compare the two strategies.

- If the design aims to promote the development of a structural view, structure sense, and symbol sense, for instance for the case of equations in one variable, we recommend: (1) to adopt or to adapt well known problems in the literature which address these issues (see for instance, Arcavi, 1994; 2005; Bokhove & Drijvers, 2010; Wenger, 1987; for problems addressing symbol sense); and (2) to confront students at an earlier stage with non-linear equations that can be solved using the cover-up strategy, such as $\frac{48}{3x-4}=16$ and $\frac{5}{6x+7}+8=9$.

- If the design aims to extend students' skills in solving word problems on the topic of equations in one variable, without being exhaustive, we recommend to widen the scope of mathematical models of the problems: not only of the form $f(x) = c$ and $f(x) = g(x)$ as addressed in the present study, but also of the forms $f(x)=\frac{c}{g(x)}$ and $\frac{f(x)}{g(x)}=c$.

We recommend making explicit use of task design heuristics based on specific theoretical or practical frames of reference that relate to the aims of the study.

7.3.3. Recommendations for the design of digital tools for algebra

In this study we used applets, embedded in the DME, as the main digital tools. The experiences with these applets lead to some recommendations for the design of digital tools. The first three points apply specifically to the applets used in this study, and are followed by some more general recommendations.

The first applet used, called Algebra Arrows applet, was initially designed to foster students' view of function as an input-output chain of operations (Doorman et al., 2012). In this study we used this applet for solving equations of the form $f(x) = c$ and related word problems. Although this worked out quite well, we observed that it sometimes confused students during the learning process, because, for instance, the applet does not provide an equal sign between $f(x)$ and c when representing equations of the form $f(x) = c$. Therefore, we recommend that digital tools designers add an option within this applet for dealing with solving equations. For this equation solving option, we would expect that the applet can represent $f(x) = c$ as an equation. With this first recommendation, we recommend that once we click 42 in the table below the expression $8x + 2$, the applet automatically provides $8x + 2 = 42$, i.e., the equal sign and 42 appear on the right side of the expression $8x + 2$. Similarly, we suggest this would also hold for the solution. Figure 7.1 depicts this recommendation, i.e., showing the equation solving process for the equation $8x + 2 = 42$.

We recommend adding specific equation option to the Algebra Arrows applet.

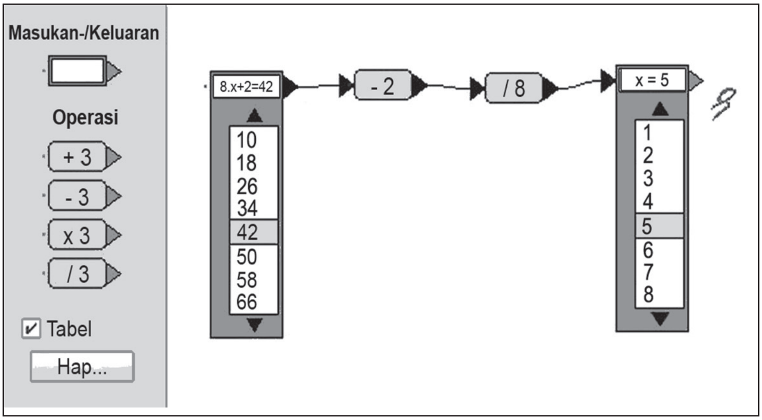

Figure 7.1. Additional equation solving option in the Algebra Arrows applet

The second applet, the Cover-up applet, can only be used to solve equations of the form $f(x) = c$. However, some equations of the form $f(x) = g(x)$ can also be solved easily, or at least partly, through the use of the cover-up strategy. For example, in the equation $8(x - 1) = 5(x - 1) + 21$ covering up $(x - 1)$ twice might be a step ahead. As an alternative approach, the equation can be solved through the cover-up strategy after applying the balance strategy in the initial step—i.e., by subtracting from both sides $5(x - 1)$ to get $3(x - 1) = 21$. It would be interesting to extend the Cover-up applet's functionality to include this type of procedures.

> *We recommend the extension of the Cover-up applet so that it allows for covering up expressions that appear twice in equations of the form f(x) = g(x), and for a combination of cover-up and balance strategies.*

The third applet, the Balance Model applet, provides one representation for a variable, i.e., using a bag containing x. However, for different equations, the size of the bags is the same, whereas at the end the bags have different values for x. This sometimes causes student misunderstanding because they thought that with the same sizes of the bags in different equations, the values of x would be the same. Therefore, we recommend digital tool designers to develop different bag sizes in different situations to avoid this issue.

> *We recommend designing different bag sizes in different situations in the Balance Model applet.*

Concerning students' difficulties in setting up mathematical models from word problems, in line with the findings in other studies and particularly in an Indonesian context (e.g., Wijaya, Van den Heuvel-Panhuizen, Doorman, &

Robitzsch, 2014), we observed that the causes may stem from students' inability to represent, for example, a phrase into a corresponding algebraic expression, and the whole problem in a proper equation. To date, the applets do only help students with feedback on whether their formulated equations are correct. However, the applets do not necessarily help with, or offer suggestions for, formulating mathematical expressions or equations. Taking this into account, we recommend designing applets that can help with, or provide suggestions for formulating a verbal phrase into a mathematical expression and finally formulating the whole problem into a proper mathematical model. This can be carried out, for instance, by providing pop-ups containing hints on how to transform a phrase into a mathematical expression, and finally how to formulate the whole problem into a mathematical model. In this way, students are expected to develop better transforming skills from a problem situation to a mathematical model, which is called horizontal mathematization.

> *We recommend designing applets that provide suggestions for, and feedback on, translating word problems into algebraic forms.*

A next recommendation concerns technical obstacles of using the equation editor in the applets' environments. For a novice user, writing mathematical expressions using the equation editor is not a trivial matter. We often observed that, for instance, students made mistakes when writing fractional expressions using the equation editor provided by the applets. We therefore suggest developing a more user-friendly equation editor. This can be carried out, for instance, by providing pop-ups that contain an explanation or an example on how to enter a mathematical expression.

> *We recommend additional online assistance for using a mathematical equation editor in the applet environment.*

A good internet connection is necessary to use the applets during the learning process. In our experience, this often caused stressful technical obstacles in this study when limited and unstable internet access occurred. To overcome this issue, once the applets are installed, we would expect that the applets could then be used offline. Next, when the computer is connected to the internet, the results of offline digital work could then be imported and stored online.

> *We recommend that applets can be used offline, with the opportunity to upload and store student work as soon as an internet connection is available.*

7.3.4. Recommendations for future research

Based on the findings of this study, we note some recommendations for future research. The first recommendation concerns the use of theories on student difficulties in initial algebra. For the purpose of this study, the five categories of difficulties have been applied throughout this study and have worked quite well. Still, it has only been applied to a specific case of algebra, that is, for the topic of equations in one variable, related linear inequalities and related word problems. Therefore, for the future, we recommend to apply this categorization in other algebra topics, such as in the topics of simplifying algebraic expressions, algebraic proportions, and function. In this way, as the categorization itself is by no means exhaustive, we expect that the categorization can be improved or expanded with new (sub) categories.

We recommend investigating the application of the five categories of student difficulties to other topics in initial algebra.

The second recommendation concerns the category of mathematization as the most frequent difficulty observed in student work (see Chapters 2 and 4). For the case of Indonesia, we might hypothetically explain student difficulties in this category as the effect of the prevailing conventional teaching approach, in which students are more used to doing routine bare algebra tasks than algebra word problems (e.g. Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002). In the Indonesian situation, word problems are usually used as applications of mathematical concepts, which are given to students at the end of certain mathematical topics on a limited scale, and depending on the available time. As a consequence, students may not acquire adequate mathematization skills. Since this conventional teaching approach often relies heavily on textbooks (Sembiring, Hadi, & Dolk, 2008), therefore, for future research, we recommend analyzing Indonesian textbooks on initial algebra topics, for instance, the topic of linear equations in one variable. In this way, we can investigate whether adequate resources are available within the textbooks for developing student mathematization skills.

We recommend an analysis of Indonesian textbooks on algebra topics to investigate whether opportunities for mathematization are provided.

The third recommendation concerns further research on the category of understanding algebraic expressions as the second most observed difficulty in student work (see Chapter 2). In this study, for instance, we often found that students encountered the parsing obstacle, that is, the difficulty to understand the order in which algebraic expressions must be understood and processed which may conflict with the order of natural language (Tall & Thomas, 1991). For example, students simplified $3 + 7x$ into $10x$ because they processed the expression from the left to the right, following natural language, that is $3 + 7 = 10$ and then concluded that the result is $10x$. This may be caused, for instance, by the way in which arithmetic is taught, with too much emphasis on local aspects and too little on global aspects of the tasks. To reduce and even avoid this obstacle, we therefore propose research about the too direct focus on immediate execution of binary arithmetical operations.

We recommend research on the too immediate executions of binary arithmetical operations.

The fourth recommendation concerns the investigation of rationales for student difficulties. In this study, we have used, for instance, the operational and structural perspective to explain student difficulties dealing with equations (see Chapter 3), and the mathematization perspective to understand student difficulties dealing with word problems (Chapter 4). We found that these perspectives are fruitful to explain the difficulties. For future research, therefore, we recommend: (1) to apply these theoretical notions to other topics in algebra, such as the concept of function; (2) to get a complementary and probably better insight into rationales underpinning student difficulties in initial algebra. While doing so, we suggest to use theoretical notions, such as symbol sense (Arcavi, 1994; 2005), structure sense (Linchevski & Livneh, 1999; Novotna & Hoch, 2008), and the emergent modeling perspective (Gravemeijer, 1994).

We recommend research addressing the rationales of student difficulties in initial algebra through applying the operational and structural perspective, symbol sense, structure sense, and emergent modeling perspectives.

The fifth recommendation concerns the use of feedback to foster student mathematization skills in the digital environment. For the effect of the ICT-rich approach to student mathematization skills, we consider that student difficulties in understanding problems, in formulating mathematical models, and to a lesser extent in symbolic mathematical problem solving are at least partially caused by the lack of transfer between digital and paper-and-pencil environments. On the one hand, when learning to deal with word problems in the digital environments, students are provided with immediate feedback and to some extent automatic calculations; on the other hand, students are tested to do word problems with paper and pencil without feedback. Therefore, for future research, we recommend to apply one of the feedback principles as proposed by Bokhove and Drijvers (2012), namely timing and fading. In this way, we expect to reduce the gap between digital and paper-and-pencil environments to improve the transfer of mathematization skills.

For the design of a digital environment for mathematics, we recommend the conscious use of feedback design principles to foster the development of the students' mathematization skills.

The sixth recommendation concerns the duration of the use of digital technology. Regardless of its success in improving student achievement, the larger teaching experiment in this study is quite short in terms of the use of a specific digital technology and an applet in particular. Concerning this, we wonder what would be the effects of the use of the applets on student algebraic skills, for instance, if one specific applet is used for a longer period? In line with Artigue (2002), who claims that instrumental genesis is a time-consuming process, we consider that: (1) if an applet in the teaching experiment is used more extensively, the effect on student conceptual understanding and procedural skills would probably be more manifest; (2) the instrumentation theory – to study the interaction between the use of a digital tool and student thinking – would probably have been more powerful if the digital tool had been used over a longer period. We therefore recommend conducting longitudinal studies on the use of digital technology to investigate these two issues.

We recommend longitudinal studies on the use of digital technology.

The seventh recommendation concerns the observations of groups of mixed ability students. We wonder whether this is the optimal way to foster student development of algebraic expertise. In our study, we observed that even when students help each other during the problem solving, they seem to have gained different conceptual understanding and skills. For future research, therefore, we recommend to also observe groups with homogeneous ability students to see whether this orchestration yields different and better results.

We recommend research on the impact of mixed and homogeneous ability groups on student achievement.

The eighth, and final, recommendation concerns the effect of the school factor on student achievement. This recommendation is not necessarily for algebra topics, but can also be applied to other mathematical topics, or even other subjects. In the larger experimental study, we found that the variable school type affects student achievement in algebra. However, we consider that school classification according to a certification only to a limited extent reflects the quality of the school. Therefore, for the future, the impact of school type on student achievement needs to be further investigated, in a way that involves measurable criteria that include both people's perception and official evaluation.

We recommend to further investigate the effect of school type on student algebraic achievement.

7.4. Epilogue

This study is an initial endeavor to improve algebra education in Indonesia. Even if many initiatives are still waiting to be undertaken in the future, we expect that the findings of this study can contribute to attain this aim in different ways. First, the effort to investigate student difficulties and to understand these using appropriate theoretical perspectives is a start to answer the why-question of the low performance of Indonesian students in algebra, even if it only provides a partial answer. Also, if we extend this effort to other algebra topics, we believe that more concrete answers to the why-question will emerge. Of course, other factors – such as the role of teachers, students' motivation, the school environment and infrastructure, parental participation, and government policies – should be considered seriously to provide a complete answer to the question, and as such research addressing them is worthwhile to do.

Second, the integration of ICT – and the online applets in particular – in the learning setting as a promising approach provides a partial answer to the how-question of improving Indonesian students' performance in algebra. The findings of this study provide scientific evidence of the effectiveness of the use of ICT in an appropriate learning setting, in Indonesia; and they also provide a firm foundation for conducting further research. As the implementation of ICT in education is advancing fast all over the world, we predict that ICT will be really integrated worldwide, including in Indonesia, in about ten or fifteen years. ICT will become a main element in the learning and teaching process: not only in algebra or mathematics, but also in other subjects. To make this prediction come true, other decisive factors supporting the use of ICT in education – including the teachers' interest and motivation, school infrastructures, and government policies – should be further investigated in research. In this way, we expect that the use of ICT will contribute to the improvement of algebra education in Indonesia.

Finally, as a consequence of the above, we consider that proper algebra education in Indonesia will really contribute to the development of characteristics of qualified citizens as stated in the curriculum, such as being independent, creative, and knowledgeable individuals. If this goal is achieved, Indonesian people are expected to be better equipped to face future challenges.

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Summary

This dissertation documents the development and the results of a mathematics education research in the domain of algebra that was conducted from 2011 to 2015 at the Freudenthal Institute for Science and Mathematics Education, Utrecht University, the Netherlands, and was funded by the Indonesia Ministry of National Education[1]. The title of this PhD research is "The use of applets to improve Indonesian student performance in algebra". Two main issues, which are relevant for contemporary and future mathematics education, form the main focus of this study: the learning of algebra and the integration of ICT in this learning process.

1. Background and research questions

Algebra is recognized as the core topic within secondary school mathematics and is important for pursuing advanced studies at university level as well as for professional work (Harvey, Waits, & Demana, 1995; Katz, 2007; Morgatto, 2008). However, students all over the world experience difficulties in learning algebra (e.g., Drijvers, 2003; Kolovou, 2011; Van Amerom, 2002; Warren, 2003). Also, algebra is recognized as a subject that is hard to teach (Stacey, Chick, & Kendal, 2004; Watson, 2009). Even if this concerns a worldwide phenomenon, Indonesian students showed remarkably low scores in recent international comparative studies: in the 2007 *Trends in International Mathematics and Science Study* (TIMSS), Indonesian students' average score in algebra was below the international average, in 36th position out of 48 countries (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008; Mullis, Martin, & Foy, 2008); in TIMSS 2011, Indonesian students were ranked 38th out of 42 participating countries in the domain of algebra (Mullis, Martin, Foy, & Arora, 2012).

These results give rise to a why-question about the low algebra scores of Indonesian students: Why do Indonesian students seem to experience more difficulties in learning algebra than students in most other countries? How is algebra taught in Indonesia? Even if we can provide hypothetical explanations for this issue, such as that the use of the drill-and-practice method in algebra teaching is still prevalent in Indonesia, a more specific and scientific explanation for the question is needed. Investigating this issue, in the form of identifying student difficulties in algebra and finding out reasons for the difficulties, constitutes the first focus of this study.

[1] Project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794

A natural next question is how to improve Indonesian students' performance in algebra. One promising approach concerns the integration of Information and Communication Technology (ICT) in algebra teaching. Education stakeholders worldwide have highlighted the potential of digital technologies for mathematics education. The National Council of Teachers of Mathematics (NCTM), for instance, claims that "technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology" (NCTM, 2008, p.1). In Indonesia, through a policy released by the Ministry of National Education, ICT is introduced as a new subject for secondary schools, and its integration in all school subjects, including mathematics, is suggested (Depdiknas, 2007). Also, research evidence underpins the potential of integrating ICT in mathematics education. For instance, review studies in mathematics education show that the use of ICT affects student mathematics achievement (Li & Ma, 2010) as well as student attitude towards mathematics (Barkatsas, Kasimatis, & Gialamas, 2009). Specifically for algebra education, Rakes, Valentine, McGatha and Ronau (2010) show that the use of ICT significantly affects student achievement, in particular in conceptual understanding and procedural skills. The use of ICT in algebra education is therefore the second focus of this study.

In short, the main aim of this PhD study is to investigate the abovementioned why-question and how-question to improve Indonesian algebra education. To do so, we decide to investigate algebra learning for grade VII students (12-13 year-old) who are in the transition phase from arithmetic to algebra learning, in the domain of linear equations in one variable and the related linear inequalities. In relation to the use of ICT in algebra teaching, we decide to use applets for algebra embedded in the Digital Mathematics Environment (DME), an online environment developed at the Freudenthal Institute, Utrecht University, the Netherlands. The reason for this choice is that the DME was rated the best by external experts on criteria of algebra, tool criteria, assessment feature, and general features (Bokhove & Drijvers, 2010). The main characteristics of the DME that make it suitable for algebra learning include stability and performance, correct display of algebraic notations, ease of use, mathematical soundness, and options to store and monitor student work (Bokhove & Drijvers, 2010a).

In this study we specifically investigate the following research questions:

1. What are Indonesian students' difficulties in initial algebra learning, particularly in solving linear equations in one variable and the related linear inequalities?

2. What are student difficulties in solving equations in one variable which emerge in an ICT-rich approach and how can operational and structural views on equations explain these difficulties?

3. What are student difficulties in mathematizing word problems in the domain of linear equations in one variable?

4. Does an intervention with digital technology enhance students' performance in initial algebra?

5. Which schemes do students develop for solving equations using algebraic substitution with the Cover-up applet and which relationships between techniques and understanding are developed?

Research questions 1-3 mainly address the why-question, while research questions 4-5 are formulated to deal primarily with the how-question.

2. Theoretical perspectives

The theoretical perspectives used in this study include two main fields: Algebra education and ICT in mathematics education. From algebra education, we focus on the theories on student difficulties in algebra, operational and structural views on algebraic activity, and mathematization.

Based on the literature study, we identified five categories of student difficulties in algebra. These include difficulties in applying arithmetical operations (e.g., Booth, 1988; Herscovics & Linchevski, 1994), understanding various faces of variable (e.g., Bush & Karp, 2013; Herscovics & Linchevski 1994; Wagner, 1983), understanding algebraic expressions (e.g., Tall & Thomas, 1991), understanding the different meanings of the equal sign (e.g., Kieran, 1981), and mathematization (e.g., Treffers, 1987) – that is the process of transforming problem situations into the world of mathematics, and vice versa, as well as reorganizing and (re)constructing within the symbolic world of mathematics. This categorization is used to identify difficulties or mistakes made by students when dealing with algebra problems, and in this study, in the topic of linear equations and inequalities in one variable as well as related word problems.

The perspective of the operational and structural views in algebraic activity which stems from the theory of reification (Sfard, 1991) is used to explain the rationale of student difficulties while solving equations. Central in this lens is the understanding of a mathematical notion, such as an algebraic expression, that can be conceived in two different complementary ways, namely as a process and an object. Other theoretical lenses involved while applying the operational-structural duality include symbol sense (Arcavi, 1994; 2005) and structure sense (Linchevski & Livneh, 1999; Novotna & Hoch, 2008). Symbol sense refers to the ability to give meaning to mathematical expressions, such as equations; structure sense concerns a flexible and creative ability to identify the structure of algebraic expressions.

The perspective of mathematization, which is grounded in the theory of Realistic Mathematics Education (RME), is used to describe and explain student difficulties when dealing with word problems in algebra. In the RME theory, mathematization is distinguished into horizontal and vertical mathematization (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). Horizontal mathematization concerns the activity of transforming a realistic problem to a symbolic mathematical problem, whereas vertical mathematization refers to the activity of reorganizing and (re)constructing the problem within the world of symbols (Treffers, 1987). In practice, while applying this perspective for analyzing student difficulties, we use the cyclic character of mathematization (De Lange, 2006). This mathematization cycle includes four steps: understanding the problem, formulating a mathematical model for the problem, solving the problem expressed in the model, and interpreting the solution in terms of the original problem. The first two steps and the fourth step are horizontal mathematization activity, while the third in particular concern vertical mathematization.

Concerning ICT in algebra education, we make use of instrumentation theory. We address three main didactical functions of digital technology in this field as identified by Drijvers, Boon and Van Reeuwijk (2010): function of digital technology as a tool for doing mathematics, as an environment for developing mathematical concepts, and as an environment for practicing algebraic skills. In this study, the last two functions are considered to be the most important as the background for designing learning arrangements (such as tasks within applets) and as a way to describe student algebraic performance as the effect of the use of the applets in the teaching process.

The instrumentation theory is used to understand the relationship between using a digital tool for algebra and the targeted algebraic understanding and procedural skills. To do so, we elaborate and exemplify the notions of artefact (a “thing”), tool, instrument (a psychological construct), scheme, and technique. Based on these elaborations instrumental genesis is defined as a process of an artefact becoming an integrated part of an instrument. If a type of task can be solved through different artefacts and related techniques, but with a similar scheme, we consider this as one single instrument at a meta level. The main application of this lens in this study concerns the analysis of students' techniques and the corresponding schemes while solving equations and related word problems (Trouche, 2004; Trouche & Drijvers, 2010).

3. Methods and results

To investigate the research questions, we carried out this research in three stages. In Stage 1, an explorative interview study was carried out to identify the difficulties encountered by students in initial algebra and in the domain of linear equations and inequalities in one variable. The findings of this interview study formed the foundation for the next stages of the study. In Stage 2, we developed a teaching sequence on linear equations and field-tested it in a small-scale pilot experiment. The results of this pilot study were used as the basis for conducting a larger-scale experimental study in Stage 3. As such, the study has the characteristics of design research consisting of two cycles: the first cycle consisting of the interview and the small-scale pilot experimental study, and the second of the larger-scale experimental study. We now describe the partial studies in more detail and present the corresponding results.

The interview study

The interview study was carried out to address the first research question, that is, to investigate students' difficulties in initial algebra, and in the domain of linear equations and inequalities in one variable in particular. To do so, we first carried out a literature study, including a survey on the learning of algebra in general and the transition from arithmetic to algebra in particular; the results of international comparative studies, such as TIMSS and PISA; and the views on algebra education from the perspective of RME theory. Next, we designed and administered an individual written test involving 51 grade VII Indonesian students – who were about to enter grade VIII (13-14 year-old) – and carried out follow-up interviews with 37 of them. The students came from one public school and two Islamic (religious) schools. The tasks for this interviews study were retrieved from the TIMSS 2003 and PISA 2006 released items, and from Indonesian mathematics textbooks. The follow-up semi-structured interviews were meant to find out students' explanations on their written work.

From this interview study, we found that mathematization constituted the most frequent category of difficulty. Other difficulties, ordered from more frequent to less frequent, included the understanding of algebraic expressions, the application of arithmetical operations in algebra, the understanding of the different meanings of the equal sign, and the understanding of the notion of variable. We then used these five categories of difficulties as both a point of departure and the main frame of reference for the next partial studies. A complete report on this interview study can be found in Chapter 2.

A small-scale pilot study

This small-scale pilot study was carried out to address the second and third research questions, namely investigating student difficulties in solving equations and comprehending the difficulties from an operational and structural perspective, and investigating student difficulties when dealing with word problems in algebra from a mathematization perspective. To do so, we designed an ICT-rich teaching sequence in the domain of linear equations, and tested this in a teaching experiment. This teaching experiment involved 51 grade VII Indonesian students (12-13 year-old) from two classes of two Islamic schools. The teaching sequence included four lessons, and each lasted for 80 minutes. The designed teaching sequence was meant to replace the corresponding text book chapter. The ICT tools integrated in the teaching experiment were two applets – called Algebra Arrows and Cover-up – embedded in the DME. The structure of each lesson consisted of a paper-and-pencil activity, a whole class discussion, the demonstration of an applet, group digital work, a paper-and-pencil individual intermediate test, and reflection. Even if the Algebra Arrows was initially designed to support the development of the function concept (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012), in this study it was used for solving equations of the form $f(x) = c$ through applying the reverse strategy. In this way, we expected that students would get a better insight into the equation as a calculation process. The Cover-up applet was designed to solve equations of the form $f(x) = c$ with the mouse by subsequently selecting with the mouse a part of the expression in an equation and finding its value. We expected that this would foster an object view on the equation and its algebraic sub-expressions (Boon, 2006). The results of this small-scale pilot experiment are reported in Chapters 3 and 4.

Chapter 3 elaborates the results of the identification and the explanation of student difficulties in solving equations which emerge in an ICT-rich learning setting from the operational and structural perspective. The difficulties we observed in student digital and written work include two types related to the equation solving strategies. First, the main difficulties while applying the reverse strategy with paper-and-pencil after the Algebra Arrows activity included the categories of applying arithmetical operations, such as calculation mistakes, and using properties of arithmetical operations and priority rules; and of the equal sign, such as notational mistakes of the equal sign use. Second, the main observed difficulties while using the cover-up strategy by hand after the Cover-up applet activity consisted of categories of applying arithmetical operations, understanding the concept of variable, understanding the different meanings of the equal sign, and understanding algebraic expressions. The latter includes some well-known obstacles in literature, such as the parsing obstacle – the understanding of the order in which algebraic expressions must be processed –, the expected answer obstacle – expecting a numerical value for an algebraic expression –, and the lack of closure obstacle, referring to the discomfort in dealing with algebraic expressions that cannot be simplified further (Tall & Thomas, 1991). The operational and structural perspective was fruitful to explain student difficulties in solving equations. For example, a limited operational view, which reflects an inability to do or undo a calculation process, may cause calculation errors; and a lack of insight into the structure of algebraic expressions may cause the additive or multiplicative inverses mistakes, because of an incorrect view on the relationship between expressions within an equation.

Chapter 4 presents the results of the identification and the explanation of student difficulties in solving word problems using the mathematization perspective. To see whether the results suggest a general phenomenon, the findings of this study were triangulated with earlier interview data (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). We concluded three main findings. First, after the Algebra Arrows activity, we found that the main difficulties in students' written work concern vertical mathematization, including the difficulties in the solution processes (the third step of the mathematization cycle, De Lange, 2006) and, to a lesser extent, checking solutions. However, in student digital group work we observed that the main difficulties concerns horizontal mathematization, including the understanding of the problems and formulating mathematical models, which are the first and second steps of mathematization cycle. Second, the main difficulties shown in the students' written work after the Cover-up activity concerned horizontal mathematization, including the understanding of problems and formulating mathematical equations, which are the first and second steps of the mathematization cycle. Third, the results of written final test confirmed the findings of both Algebra Arrows and Cover-up Strategy lessons, i.e., the difficulties in vertical mathematization emerged in student work on typical tasks related to the Algebra Arrows activity, whereas the difficulties in horizontal mathematization appeared in student work on typical tasks of the Cover-up activity. As a conclusion, we found that the main difficulties encountered by students when dealing with word problems concern transforming problems into mathematical models (equations), i.e., in horizontal mathematization. Finally, we found that the results of the interviews confirmed these findings.

A larger-scale experiment

We carried out a larger-scale experiment to investigate the effect of digital technology use on students' performance in algebra. The algebra topic in the experiment concerned linear equations in one variable and related word problems. The experiment involved eight classes from four different schools: four classes from two different schools were in the experimental group, and the other four classes also from two different schools were in the control group. In the experiment, we implemented the ICT-rich approach intervention in the learning setting for grade VII Indonesian students (12-13 year-old): 139 students in the experimental group followed a digital technology-rich teaching sequence, while 127 students from the control group followed regular teaching without digital activities. The intervention in the experimental group consisted of individual written pretest, four lessons integrating the use of digital tools, and individual written posttest. Each lesson, which lasted 80 minutes, consisted of three parts: a paper-and-pencil activity with the whole class discussion; a demonstration of digital tool and group digital work activities, and an individual written test. The digital tools integrated in the teaching experiment included four applets, called Algebra Arrows, Cover-up Strategy, the Balance Model, and the Balance Strategy, which are embedded in the DME. Whereas the Algebra Arrows and Cover-up Strategy are the same applets as used in the small-scale pilot experiment and are designed for solving equations of the form $f(x) = c$, the Balance Model is designed for solving linear equations of the form $f(x) = g(x)$ with the help of visual models provided in the applet, and the Balance Strategy applet is designed for solving linear equations of the form $f(x) = g(x)$ using strategies developed in the Balance Model applet. By the regular teaching process we mean a learning setting in which a teacher explains the concept of equations in one variable with the corresponding examples and provides exercises, while students pay attention, take notes and do the paper-and-pencil exercises afterwards. The results of this larger experimental study were reported in Chapters 5 and 6.

Chapter 5 provides the results of the larger experimental study which addresses the fourth research question, i.e., investigating the effect of a digital technology-rich approach on student performance in algebra. Based on the quantitative analysis, we concluded that the experimental group's mean gain score (4.63) was significantly higher ($p < .01$) than the control group's mean gain score (3.02) with a medium effect size ($d = 0.32$). Furthermore, to illustrate the improved student performance as shown in the quantitative results, we presented the results of the qualitative analysis of the work of one group of students during the four lessons. We found that student written and digital work during the teaching experiment confirm the effectiveness of the digital technology-rich approach for enhancing student achievement in algebra. For example, we observed that written work strategies similar to digital work strategies reflect a direct effect of the applets' use; and the difficulties in both digital and written work to a certain extent depict student conceptual understanding and procedural skills.

Chapter 6 provides the findings of the case study, i.e., an in-depth analysis of a part of the larger-experimental study data, concerning the relationship between the use of digital tools in learning algebra and the corresponding student conceptual understanding from the perspective of instrumentation theory. In particular, we addressed the schemes students develop for solving equations using algebraic substitutions with the Cover-up applet. We analyzed video registrations of one group of three seventh grade Indonesian students (12-13 year-old) using the Cover-up applet for solving algebraic equations and word problems. We drew three conclusions. First, the scheme that students develop for solving an equation with the Cover-up applet includes: (1) recognizing the equation as suitable for the cover-up strategy and the task to rewrite it in the form < unknown > = < value >; (2) identifying a sub-expression from the equation to cover as well as assigning a numerical value to it (in each cover-up strategy step), and (if necessary) repeating this until the desired form is found. Second, the scheme that students develop for solving a word problem using the Cover-up applet includes: setting up an equation from the word problem; entering it in the solution window, and putting into action the scheme used for solving an equation as described previously. Third, as the scheme survives when it is applied to new and more complex types of problems, the students' scheme for solving an equation seems to develop within this one lesson. Taking this into account, although it is not easy to observe scheme development within one lesson, we interpret the survival of the scheme for solving problems of increasing complexity as a modest support for the students' instrumental genesis.

4. Conclusion

Chapter 7 presents the conclusion of this study which includes the summary of the main findings, reflections on the findings and the study's limitations, and practical as well as theoretical recommendations for algebra teaching, for algebra didactic and tools design, and for future research.

In the reflection part, we provide a reflection on the theoretical perspectives, on the design and the methodology of this study, and on generalization of the findings; and limitations of the study. Concerning the theoretical perspective of student difficulties, we would like to stress that this theory was developed in a combined top-down and bottom-up approach, i.e., by combining the results of the literature study and the findings of the interview study. Therefore, this lens plays a double role: as a frame of reference and as a main result of the study. With respect to the operational-structural duality, this lens was fruitful for explaining student difficulties dealing with algebraic problems, for linking to other lenses that are related to this perspective such as symbol sense and structure sense, and as a foundation for understanding algebraic substitution with either the Cover-up applet or paper and pencil. With respect to mathematization perspective, we made some remarks: (1) this lens relates to the category of difficulties in mathematization and can be used for dealing with not only word problems, but also symbolic mathematical problems in general; and (2) in practice, for analyzing student difficulties in word problems, this lens concerns the mathematization cycle (De Lange, 2006). With respect to the role of ICT in algebra education, we consider that the practicing skills and the concept development roles the digital environment may play are guiding the design in a somewhat implicit but crucial way. Finally, with respect to the instrumentation theory, the notions of techniques and schemes are fruitful for comprehending the relationship between conceptual understanding and the digital tool use. This is concretized in the form of an identification of student schemes for solving different type of problems.

The main reflection on the design and methodology of this study concerns the use of design research as the overall method. In the two research cycles, however, we did not elaborate explicitly the three design research phases – preparatory phase, teaching experiment phase, and retrospective analysis phase – in each cycle. Rather, we described partial studies in Chapters 2-6. In this way, the design research is used moderately, not in a strict manner.

The final reflection concerned the generalizability of the findings. The main difficulties students encountered in the domain of linear equations and inequalities in one variable concern the notion of an algebraic expression which needs to be understood as both a process and an object, and the difficulties in translating word problems into mathematical and algebraic models. These two issues may also play a central role in other topics, such as in quadratic equations and related word problems. Therefore, we expect the findings of this study to be generalizable to other topics in algebra, and we conjecture that the same theoretical lenses would apply in these topics, too. With respect to the use of ICT in the learning setting, an important finding is that a seamless match is needed between the targeted mathematical thinking, the technique in the digital environment, and the transfer to similar paper-and-pencil techniques. Again, we believe that the same holds for the use of digital tools for more sophisticated algebra tasks, or for other technological environments.

Concerning the overall study, we identified several limitations. In the interview study, we noted two main limitations: (1) even if the students who participated in this interview study came from Islamic and public schools and were selected by their teachers, included high, medium and low achievers, the sample is relatively small; as such it may raise a query whether it is representative for all Indonesian students in grade VII; and (2) whereas a mistake sometimes can be categorized in more than one category of difficulty, the intertwinement and dependencies among the five categories of difficulties – as the main frame of reference – are not addressed. In the small-scale pilot experiment, we also noticed two main limitations: (1) rather than to elaborate explicitly the hypothetical learning trajectory (HLT) and compare it with the results of the experiment, we incorporated only main elements of the HLT in the analysis; and (2) whereas in the design process we prepared the learning sequence for equations of the forms $f(x) = c$ and $f(x) = g(x)$, in the teaching experiment we implemented the sequence for equations of the first type only, due to time constraints. Finally, in the larger scale experiment, we again identified two main limitations: (1) as in the experimental group only Islamic schools and no public schools are involved, we could not compare the algebraic performance of students in these two different types of schools; (2) as in the case study, we focused only on one group of students in each of four experimental classes, some interesting data from other non-observed groups lacked observational data that might have contributed to further evidence and stronger conclusions.

In relation to the overall results of this study, we propose the following recommendations for algebra teaching, for algebra didactic and digital tools design, and for future research.

a. Recommendations for algebra teaching

- use both word problems and bare algebra problems in an intertwined way from the beginning;
- use the four steps of the mathematization cycle as a teaching strategy when dealing with word problems in algebra;
- encourage students to use different strategies or combination of strategies to solve the same algebra problem, and to check the solution(s);

- confront students with related problems beyond the scope of the topic under consideration in the initial stage of problem solving to develop both conceptual understanding and procedural skills;
- establish a didactical contract between the teacher and students in the ICT-rich approach learning setting, that indicates when to use paper-and-pencil and when to use digital technology;
- take a distant stance while guiding students during the learning process to foster their instrumental genesis.

b. Recommendations for algebra didactic and tools design

- develop two types of algebra tasks: (1) focus only on algebra; (2) focus on both arithmetic and algebra;
- when designing tasks, make explicit use of task design heuristics based on specific theoretical or practical frames of reference that relate to the aims of the study;
- add a specific equation option to the Algebra Arrows applet;
- extend the Cover-up applet so that it allows for covering up expressions that appear twice in equations of the form $f(x) = g(x)$ and for a combination of cover-up and balance strategies;
- design different bag sizes in different situations in the Balance Model applet;
- design applets that provide suggestions for, and feedback on, translating word problems into algebraic forms;
- add online assistance for using a mathematical equation editor in the applet environment;
- provide the possibility to use applets offline, with the opportunity to upload and store student work as soon as an internet connection is available.

c. Recommendations for future research

- investigate the application of the five categories of student difficulties to other topics in initial algebra;
- analyze Indonesian textbooks on the topic of algebra to investigate whether opportunities for mathematization are provided;

- research the too immediate executions of binary arithmetical operations, without overview over the task, which may cause mistakes such as $3 + 7x = 10x$;
- investigate the rationales of student difficulties in other topics within initial algebra through applying the theoretical perspectives used in this study, such as the operational and structural perspective, symbol sense, structure sense, and emergent modeling;
- conduct longitudinal studies on the use of digital technology;
- research on the effect of mixed and homogeneous ability groups on student achievement;
- investigate the effect of school type (Islamic or public in the Indonesian context) on student algebraic achievement.

As an epilogue, we acknowledge that this study is an initial endeavor to improve algebra education in Indonesia. In particular, we recognize that the effort to investigate student difficulties and to understand these using appropriate theoretical perspectives is a start to answer the why-question of the low performance of Indonesian students in algebra, even if we only provide a partial answer. Also, we are aware that the integration of ICT – and the online applets in particular – in the learning setting as a promising approach provides a partial answer to the how-question of improving Indonesian students' performance in algebra. However, if we can do these properly, algebra education in Indonesia will really contribute to the development of the nation to face future challenges.

Samenvatting

Deze dissertatie bevat de resultaten van een onderzoek naar algebraonderwijs, dat is uitgevoerd in de periode 2011-2015 bij het Freudenthal Instituut van de Universiteit Utrecht in Nederland met een beurs van het Indonesische ministerie van onderwijs[1]. De titel van deze dissertatie luidt "Het gebruik van applets om de prestaties van Indonesische leerlingen in algebra te verbeteren". De studie concentreert zich op twee thema's die van belang zijn voor het huidige en het toekomstige wiskundeonderwijs, namelijk het leren van algebra en de integratie van ICT in het leerproces.

1. Achtergrond en onderzoeksvragen

Algebra wordt gezien als een van de hoofdonderwerpen in het wiskundecurriculum van het voortgezet onderwijs en is van belang voor zowel universitaire vervolgopleidingen als voor beroepspraktijken (Harvey, Waits, & Demana, 1995; Katz, 2007; Morgatto, 2008). Toch ervaren leerlingen overal ter wereld moeilijkheden met het leren van algebra (zie bijvoorbeeld Van Amerom, 2002; Drijvers, 2003; Warren, 2003; Kolovou, 2011). Ook is algebra een erkend lastig onderwerp om te onderwijzen (Stacey, Chick, & Kendal, 2004; Watson, 2009). Hoewel het hier dus gaat om een wereldwijd probleem, laten leerlingen in Indonesië opvallend lage scores zien in internationale vergelijkende studies: in de *Trends in International Mathematics and Science Study* (TIMSS) in 2007 ligt de gemiddelde score van Indonesische leerlingen onder het gemiddelde, met een 36ste plaats onder 48 landen (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008; Mullis, Martin, & Foy, 2008); in TIMSS 2011 stonden Indonesische leerlingen voor algebra op de 38ste plaats onder 42 deelnemende landen (Mullis, Martin, Foy, & Arora, 2012).

Deze resultaten vormen aanleiding tot de waarom-vraag: waarom lijken Indonesische leerlingen meer moeilijkheden te hebben met het leren van algebra dan leerlingen in de meeste andere landen? Hoe ziet algebraonderwijs er uit in Indonesië? Ook al kunnen we hiervoor hypothetische verklaringen bedenken zoals de *drill-and-practice* aanpak van het Indonesische algebraonderwijs; een meer specifieke en beter onderbouwde verklaring voor dit verschijnsel ontbreekt nog. Het onderzoeken van dit probleem door de moeilijkheden van leerlingen met algebra te benoemen en de onderliggende oorzaken daarvan aan te wijzen vormt de eerste focus van deze studie.

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[1] Project BERMUTU IDA CREDIT NO.4349-IND, LOAN NO.7476-IND DAN HIBAH TF090794

Een natuurlijke vervolgvraag is hoe de algebraïsche prestaties van Indonesische leerlingen kunnen worden verbeterd. Naast andere mogelijkheden is de integratie van Informatie- en Communicatie Technologie (ICT) in het algebraonderwijs veelbelovend. Invloedrijke spelers in de onderwijswereld hebben de potentie van digitale hulpmiddelen voor wiskundeonderwijs onderstreept. De *National Council of Teachers of Mathematics* (NCTM) stelt bijvoorbeeld: "technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology" (NCTM, 2008, p.1). In Indonesië is ICT door het ministerie van onderwijs als verplicht onderwerp ingevoerd in het voortgezet onderwijs en is gesuggereerd om het ook te integreren in andere vakken, waaronder wiskunde (Depdiknas, 2007). Bovendien is er onderzoek dat de potentie van ICT voor het leren van wiskunde ondersteunt. Review studies tonen aan dat het gebruik van ICT een positieve invloed kan hebben op de wiskundeprestaties van leerlingen (Li & Ma, 2010) en tevens op hun houding ten aanzien van wiskunde (Barkatsas, Kasimatis, & Gialamas, 2009). Specifiek voor algebraonderwijs laten Rakes, Valentine, McGatha and Ronau (2010) zien dat het gebruik van ICT significant kan bijdragen aan de prestaties van leerlingen en aan inzicht en procedurele vaardigheid in het bijzonder. Het gebruik van ICT in het algebraonderwijs is daarom de tweede focus van deze studie.

Samengevat is het doel van dit promotieonderzoek om de bovenstaande vragen te onderzoeken met het doel het algebraonderwijs in Indonesië te verbeteren. Daartoe is ervoor gekozen om het onderzoek te richten op leerlingen van klas VII (12-13 jaar oud), die zich in de overgangsfase bevinden tussen rekenen en algebra, en op het onderwerp (lineaire) vergelijkingen in één onbekende met de bijbehorende ongelijkheden. Wat betreft het gebruik van ICT hebben we gekozen voor het gebruik van applets die deel uitmaken van de Digitale Wiskunde Omgeving (DWO) van het Freudenthal Instituut. De reden hiervoor is dat de DWO door externe deskundigen is beoordeeld als de beste omgeving voor algebraonderwijs als gekeken wordt naar algebraïsche inhoud, gebruikskenmerken, mogelijkheden voor toetsing, en algemene kenmerken (Bokhove & Drijvers, 2010). De belangrijkste karakteristieken die de DWO geschikt maken voor algebraonderwijs zijn stabiliteit en performance, de correcte weergave van algebraïsche notaties, gebruiksvriendelijkheid, wiskundige correctheid, en de mogelijkheid om het werk van leerlingen op te slaan (Bokhove & Drijvers, 2010a).

De volgende onderzoeksvragen staan in deze studie centraal:

1. Welke moeilijkheden ervaren Indonesische leerlingen in aanvankelijk algebraonderwijs, en met name bij het oplossen van lineaire vergelijkingen in één onbekende en de bijbehorende ongelijkheden?
2. Welke moeilijkheden van leerlingen komen naar voren in een ICT-rijke benadering van het oplossen van vergelijkingen in één onbekende en hoe kunnen een operationele en een structurele kijk op vergelijkingen deze verklaren?
3. Welke moeilijkheden hebben leerlingen met het mathematiseren van contextopgaven in het domein van lineaire vergelijkingen in één onbekende?
4. Kan een interventie met digitale technologie de prestaties van leerlingen in aanvankelijke algebra verbeteren?
5. Welke schema's ontwikkelen leerlingen voor het oplossen van vergelijkingen door algebraïsche substitutie met het *Cover-up* applet en welk verband ontstaat tussen de technieken en het begrip?

Onderzoeksvragen 1-3 hebben vooral betrekking op de waarom-vraag, terwijl vragen 4 en 5 in de eerste plaats de hoe-vraag betreffen.

2. Theoretische invalshoeken

De theoretische invalshoeken van deze studie zijn afkomstig uit twee gebieden: didactiek van de algebra en ICT in wiskundeonderwijs. Binnen de didactiek van de algebra ligt de nadruk op de moeilijkheden van leerlingen met algebra, de operationele en structurele kijk op algebraïsche activiteit, en het mathematiseren.

Op basis van een literatuurstudie zijn vijf categorieën van moeilijkheden van leerlingen met algebra geïdentificeerd: moeilijkheden met het toepassen van rekenkundige operaties (zie bijvoorbeeld Booth, 1988; Herscovics & Linchevski, 1994), inzicht in de verschillende kanten van het begrip variabele (Wagner, 1983; Herscovics & Linchevski 1994; Bush & Karp, 2013), inzicht in algebraïsche uitdrukkingen (Tall & Thomas, 1991), inzicht in de verschillende betekenissen van het gelijkheidsteken (Kieran, 1981), en mathematiseren (Treffers, 1987), dat wil zeggen het transformeren van probleemsituaties naar de wereld van de wiskunde en andersom en het reorganiseren en (re)construeren binnen de symbolische wereld van de wiskunde. Deze categorisering is gebruikt om moeilijkheden en fouten te duiden van leerling bij het werken aan algebraproblemen.

Het perspectief van de operationele en structurele kijk op algebraïsche activiteit, ontleend aan de reïficatietheorie (Sfard, 1991), is gebruikt om de moeilijkheden van leerlingen met het oplossen van vergelijkingen beter te begrijpen. Centraal in deze optiek staat het inzicht in een wiskundig begrip, zoals een algebraïsche uitdrukking, dat op twee elkaar aanvullende manieren kan worden beschouwd, namelijk als een proces en als een object. Andere theoretische invalshoeken bij het gebruiken van de operationele-structurele dualiteit zijn *symbol sense* (Arcavi, 1994; 2005) en *structure sense* (Linchevski & Livneh, 1999; Novotna & Hoch, 2008). *Symbol sense* verwijst naar het vermogen om betekenis te geven aan algebraïsche uitdrukkingen, zoals vergelijkingen; *structure sense* betreft een flexibel en creatief inzicht in de structuur van algebraïsche uitdrukkingen.

Het perspectief van mathematiseren, dat is geworteld in de theorie van Realistisch Wiskundeonderwijs (in het Engels afgekort tot RME), is gebruikt om de moeilijkheden van leerlingen te beschrijven en te verklaren bij het werken met contextopgaven in algebra. In RME wordt onderscheid gemaakt tussen horizontaal en verticaal mathematiseren (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). Horizontaal mathematiseren betreft het transformeren van een realistisch probleem in een symbolisch wiskundeprobleem, terwijl van verticaal mathematiseren sprake is bij het reorganiseren en (re)construeren van het probleem binnen de symbolische taal van de wiskunde (Treffers, 1987). Bij het toepassen van dit perspectief voor het analyseren van moeilijkheden van leerlingen hebben we gebruik gemaakt van het cyclische karakter van mathematiseren (De Lange, 2006). Diens mathematiseringscyclus bestaat uit vier stappen: het probleem begrijpen, een wiskundig model formuleren voor het probleem, het modelprobleem oplossen, en de oplossing interpreteren in termen van het oorspronkelijke probleem. De eerste twee stappen vallen onder horizontaal mathematiseren, terwijl met name de derde het verticaal mathematiseren betreft.

Wat betreft ICT in het algebraonderwijs maken we gebruik van instrumentatietheorie. We bekijken drie didactische hoofdfuncties van digitale technologie in algebraonderwijs, zoals onderscheiden door Drijvers, Boon en Van Reeuwijk (2010): ICT als gereedschap om wiskunde mee te doen, ICT als omgeving voor begripsontwikkeling en ICT als oefenomgeving voor algebraïsche vaardigheden. In deze studie zijn de laatste twee functies het belangrijkst voor het ontwerp van de leeromgeving (te denken valt aan de binnen de applets uit te voeren opdrachten) en als manier om de algebraprestaties van leerlingen te beschrijven als gevolg van het gebruik van de applets.

Instrumentatietheorie is gebruikt om de relatie te begrijpen tussen het gebruik van de digitale tools voor algebra enerzijds, en het beoogde algebraïsch inzicht en de bijbehorende procedurele vaardigheden anderzijds. Daartoe hebben we de begrippen artefact (een "ding"), *tool*, instrument (een psychologisch construct), schema en techniek uitgewerkt en toegelicht met voorbeelden. Gebaseerd op deze uitwerking is instrumentele genese gedefinieerd als een proces waarin een artefact een geïntegreerd onderdeel wordt van een instrument. Als een probleem kan worden opgelost met verschillende artefacten en technieken, maar met een vergelijkbaar schema, dan beschouwen we dit als één instrument op meta-niveau. De belangrijkste toepassing van deze theorie in deze studie betreft het analyseren van de technieken van leerlingen en de daarmee corresponderende schema's voor het oplossen van vergelijkingen en bijbehorende contextopgaven (Trouche, 2004; Trouche & Drijvers, 2010).

3. Methode en resultaten

Het onderzoek besloeg drie fasen. In fase 1 zijn door een exploratieve interviewstudie de moeilijkheden geïdentificeerd die leerlingen ervaren bij aanvankelijke algebra en bij het werken met vergelijkingen en ongelijkheden in één variabele. De bevindingen hiervan vormen de basis voor de volgende fasen van het onderzoek. In fase 2 hebben we een lessenserie ontwikkeld rond lineaire vergelijkingen en deze uitgetest in een kleinschalige pilot studie. De resultaten van deze pilot zijn gebruikt als basis voor een onderwijsexperiment op grotere schaal in fase 3. Zo heeft de studie dus het karakter van een ontwerponderzoek dat bestaat uit twee onderzoekscycli: de eerste gevormd door de interview studie en de kleinschalige pilot studie en de tweede door het grootschaliger onderwijsexperiment. We beschrijven deze deelstudies nu in meer detail en schetsen de resultaten.

De interview studie

De interview studie is uitgevoerd om de eerste onderzoeksvraag te beantwoorden, dus om de moeilijkheden van leerlingen te onderzoeken met aanvankelijke algebra en met lineaire vergelijkingen en ongelijkheden in één variabele in het bijzonder. Daartoe is eerst een literatuurstudie uitgevoerd, dat zich richtte op het leren van algebra in het algemeen en de overgang van rekenen naar algebra; de resultaten van internationale vergelijkende studies zoals TIMSS en PISA; en de visie op algebraonderwijs vanuit de theorie van RME. Vervolgens is een schriftelijke individuele toets ontwikkeld en afgenomen bij 51 Indonesische leerlingen die op het punt stonden om in klas VIII in te stromen (13-14 jaar oud), en zijn vervolginterviews gehouden met

37 van hen. De leerlingen kwamen van een openbare school en twee religieuze Islamitische scholen. De opgaven voor deze interviewstudie zijn geselecteerd uit de vrijgegeven items van TIMSS 2003 en PISA 2006 en uit opgaven uit Indonesische schoolboeken. De semi-gestructureerde vervolginterviews zijn bedoeld om de uitleg te achterhalen die leerlingen geven bij hun schriftelijk werk.

Uit deze interview studie is gebleken dat mathematiseren de meest voorkomende categorie van moeilijkheden is. Andere moeilijkheden, geordend van meer naar minder frequent, zijn het inzicht in algebraïsche expressies, het toepassen van rekenkundige operaties binnen de algebra, het inzicht in de verschillende betekenissen van het gelijkheidsteken, en inzicht in het begrip variabele. Deze vijf categorieën zijn gebruikt als vertrekpunt en referentiekader voor de volgende deelstudies. Een complete rapportage van de interviewstudie staat in hoofdstuk 2.

Een kleinschalige pilot studie

De kleinschalige pilotstudie had tot doel om de tweede en derde onderzoeksvraag te beantwoorden, dus om de moeilijkheden van leerlingen met het oplossen van vergelijkingen te onderzoeken en te begrijpen vanuit het operationele en structurele perspectief, en om de moeilijkheden van leerlingen met contextopgaven te onderzoeken vanuit de optiek van het mathematiseren. Om dit te doen hebben we een ICT-rijke lessenserie ontworpen rond lineaire vergelijkingen en deze getest in een onderwijsexperiment. Hierin waren 51 Indonesische leerlingen (12-13 jaar oud) uit klas VII betrokken, afkomstig uit twee groepen van twee Islamitische scholen. De lessenserie besloeg vier lessen van elk 80 minuten en was bedoeld om het overeenkomstige hoofdstuk uit het schoolboek te vervangen. De ICT-tools in het onderwijsexperiment waren twee applets, AlgebraPijlen en Cover-up geheten, die zijn ingebed in de DWO. De structuur van elke les bestond uit werken met pen en papier, een klassengesprek, het demonstreren van een applet, groepswerk met de computer, een schriftelijke individuele tussentoets en een terugblik.

Hoewel het applet AlgebraPijlen oorspronkelijk was ontworpen om de ontwikkeling van het functiebegrip te ondersteunen (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012), is het in deze studie gebruikt voor het oplossen van vergelijkingen van de vorm $f(x) = c$ met behulp van de omkeerstrategie. Op deze manier, was onze verwachting, zouden leerlingen meer inzicht krijgen in de vergelijking als een rekenproces. Het Cover-up applet was ontworpen om vergelijkingen van de vorm $f(x) = c$ op te lossen door achtereenvolgens met de muis een deelexpressie van de vergelijking te selecteren en de waarde daarvan te bepalen. We verwachtten dat dit zou bevorderen dat de vergelijking en zijn deelexpressies als objecten zouden worden beschouwd (Boon, 2006). De resultaten van deze kleinschalige pilot studie staan beschreven in de hoofdstukken 3 en 4.

Hoofdstuk 3 beschrijft de resultaten van de identificatie en de verklaring van de moeilijkheden van leerlingen met het oplossen van vergelijkingen die naar voren komen bij het werken met ICT vanuit het operationele en structurele perspectief. In het digitale en schriftelijke werk van leerlingen hebben we twee types moeilijkheden waargenomen, die samenhangen met de strategie voor het oplossen van vergelijkingen. Het eerste, belangrijkste type moeilijkheid trad op bij het toepassen van de omkeerstrategie met pen en papier na de activiteit met het applet AlgebraPijlen en betrof het toepassen van rekenkundige operaties (denk aan rekenfouten, eigenschappen van rekenkundige operaties en voorrangsregels) en het gelijkheidsteken (zoals notatiefouten hiermee). Het tweede type moeilijkheid trad op bij het met de hand toepassen van de *cover-up* strategie na afloop van het gebruik van het Cover-up applet en betrof het toepassen van rekenkundige operaties, het inzicht in het begrip variabele, het begrijpen van de verschillende betekenissen van het gelijkheidsteken, en het inzicht in algebraïsche uitdrukkingen. Dit laatste omvat de uit de literatuur bekende obstakels zoals het *parsing obstacle* over het inzicht in de volgorde waarin algebraïsche expressies moeten worden verwerkt, het *expected answer obstacle* over het verwachten van een numeriek antwoord voor een algebraïsche uitdrukking, en het *lack of closure obstacle*, dat verwijst naar het niet goed kunnen omgaan met algebraïsche uitdrukkingen die niet verder vereenvoudigd kunnen worden (Tall & Thomas, 1991). Het operationele en structurele perspectief was vruchtbaar om de moeilijkheden van leerlingen bij het oplossen van vergelijkingen te begrijpen. Een te beperkte operationele blik, bijvoorbeeld het onvermogen om een rekenstap te zetten of ongedaan te maken, kan rekenfouten veroorzaken; een beperkt inzicht in de structuur van algebraïsche uitdrukkingen kan fouten met de additieve of multiplicatieve inverses veroorzaken vanwege een incorrecte kijk op de relatie tussen uitdrukkingen binnen een vergelijking.

Hoofdstuk 4 kijkt met het begrip mathematisering als achtergrond naar de identificatie en verklaring van de moeilijkheden die leerlingen ervaren bij het oplossen van contextproblemen (vroeger meestal ingeklede vergelijkingen genoemd). Om na te gaan of de resultaten algemener geldig zijn, hebben we de bevindingen getrianguleerd met de gegevens uit de eerdere interviews met leerlingen (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). Dit heeft de volgende drie belangrijkste conclusies opgeleverd. Ten eerste vonden we dat de belangrijkste tekortkomingen in het geschreven leerlingenwerk na het werken met het AlgebraPijlen applet voortkomen uit het niet goed in staat zijn om verticaal te mathematiseren. Dit speelt in de fase van het oplossen van een probleem (de derde stap van de mathematiseringscyclus volgens De Lange, 2006) en in mindere mate bij het controleren van de oplossingen. Tijdens het digitale werk in groepen betroffen de belangrijkste obstakels echter de horizontale mathematisering: zowel het begrijpen van de vraagstelling als het opstellen van het bijbehorende wiskundige model, ofwel de eerste en tweede stap van de mathematiseringscyclus. Een tweede conclusie is dat de belangrijkste tekortkomingen in het geschreven leerlingenwerk na het werken met het Cover-up Strategy applet op het gebied van horizontale mathematisering liggen, in de eerste twee stappen van de mathematiseringscyclus. Ten derde bevestigen de resultaten van de schriftelijke eindtoets de eerdere bevindingen: bij taken die typisch voortbouwen op het werken met AlgebraPijlen ervaren leerlingen moeilijkheden met verticaal mathematiseren. Moeilijkheden met horizontaal mathematiseren troffen we vooral aan bij taken die typisch voortbouwen op het Cover-up applet.

We concluderen dat de leerlingen bij het oplossen van contextproblemen vooral moeilijkheden hebben met het omzetten van het probleem in een wiskundig model of vergelijking, dus met horizontaal mathematiseren. Deze conclusie wordt ondersteund door de resultaten van de eerder gehouden interviews.

Een experiment op grotere schaal

Vervolgens voerden we een experiment op grotere schaal uit om uit te zoeken welk effect digitale technologie had op de algebraïsche prestaties van leerlingen. Het algebraïsche onderwerp dat in het experiment centraal stond betrof (lineaire) vergelijkingen in één variabele en daarmee samenhangende contextproblemen. Vier klassen van twee verschillende scholen vormden de experimentele groep en vier klassen van twee andere scholen fungeerden als controlegroep. In de experimentele groep is de ICT-gebaseerde interventie uitgevoerd met 139 Indonesische leerlingen uit klas VII (12- tot 13-jarigen). De controlegroep bestond uit 127 leerlingen die het gebruikelijke onderwijs kregen zonder digitale activiteiten. De interventie in de experimentele groep bestond uit een individueel gemaakte schriftelijke pre-test, vier lessen waarin het gebruik van applets was geïntegreerd, en een individuele, schriftelijke post-test. Elke les duurde 80 minuten en had drie vaste, achtereenvolgende onderdelen: werk met pen en papier tijdens een bespreking met de hele klas, het demonstreren van het applet en groepswerk met het applet, en een individueel gemaakte schriftelijke toets. Tijdens deze lessen hebben we vier applets ingezet, te weten AlgebraPijlen, Cover-up, Weegschaalmodel en Weegschaalstrategie. Deze applets maken onderdeel uit van de eerder genoemde DWO. Terwijl AlgebraPijlen en Cover-up worden ingezet voor het oplossen van eerstegraads vergelijkingen van de vorm $f(x) = c$, waarvoor ze ook al gebruikt waren in de kleinschalige pilot, is het applet Weegschaalmodel ontworpen om het oplossen van eerstegraads vergelijkingen van de vorm $f(x) = g(x)$ te visualiseren. Het applet Weegschaalstrategie richt zich eveneens op vergelijkingen van de vorm $f(x) = g(x)$ en hierbij past de leerling de strategieën toe die hij via het applet Weegschaalmodel heeft leren kennen. Het gebruikelijke onderwijs, dat de over vier klassen verdeelde controlegroep kreeg, hield in dat de leraar het principe van eerstegraads vergelijkingen in één variabele uitlegde met toepasselijke voorbeelden en vervolgens oefeningen opgaf, terwijl de leerlingen opletten, aantekeningen maakten en aansluitend met pen en papier de oefeningen maakten. De resultaten van dit experiment op grotere schaal zijn beschreven in de hoofdstukken 5 en 6.

Hoofdstuk 5 bevat de resultaten van de experimentele studie op grotere schaal, die de vierde onderzoeksvraag betreft: kan een interventie met digitale technologie de prestaties van leerlingen in aanvankelijke algebra verbeteren? Op grond van een kwantitatieve analyse concludeerden we dat de gemiddelde leerwinst van de experimentele groep (4.63) significant hoger was ($p < .01$) dan de gemiddelde leerwinst van de controlegroep (3.02) met een kleine effectgrootte ($d = 0.32$). Daarnaast presenteerden we ter illustratie van de verbetering van de resultaten volgens de kwantitatieve analyse ook de resultaten van een kwalitatieve analyse van het werk van één groep leerlingen, die we vier lessen lang gevolgd hebben. We vonden dat het schriftelijke en digitale werk gedurende het experiment de effectiviteit bevestigen van de ICT-rijke benadering voor het verbeteren van de algebraïsche resultaten van de leerlingen. We namen bijvoorbeeld strategieën waar in het schriftelijke werk die sterk leken op strategieën in het digitale werk. Deze waarneming weerspiegelt een direct effect van het werken met de applets. De obstakels in zowel het digitale als het schriftelijke werk geven tot op zekere hoogte weer hoe ver de leerling gevorderd is met begripsvorming en praktische vaardigheden.

Hoofdstuk 6 bevat de bevindingen van een gedetailleerde analyse vanuit de instrumentatietheorie. Opnieuw hebben we daarvoor de data van het grootschalige experiment gebruikt, maar nu speciaal gericht op het verband tussen het leren van algebra met een digitaal hulpmiddel en de begripsontwikkeling bij de leerling. In het bijzonder hebben we gekeken naar de schema’s die leerlingen ontwikkelen voor het oplossen van vergelijkingen, als ze daarbij met het Cover-up applet algebraïsche substituties uitvoeren. Hiertoe analyseerden we videoregistraties van een groep van drie Indonesische leerlingen uit klas VII (12- tot 13-jarigen), die het Cover-up applet gebruikten bij het oplossen van vergelijkingen en contextproblemen. De analyse leidt tot de volgende drie conclusies.

Ten eerste bevat het schema dat leerlingen bij het oplossingsproces met het Cover-up applet hanteren de volgende elementen: (1) herkennen dat de vergelijking geschikt is om de cover-up strategie te gebruiken en het probleem in de vorm < iets onbekends > = < bepaald getal > te brengen; (2) een deel van de vergelijking aanwijzen dat in het applet bedekt kan worden, en tegelijkertijd daaraan een getalwaarde toekennen (en deze stap zo nodig enkele malen herhalen tot het probleem de gewenste vorm heeft).

Ten tweede zien we dat leerlingen het bovenstaande schema toepassen bij het oplossen van contextproblemen: ze vertalen het probleem in een vergelijking, voeren deze in het oplossingsvenster van het applet in en zetten bovenstaand schema in werking.

Ten derde lijkt het erop dat schema het dat de leerlingen binnen deze ene les ontwikkelen, intact blijft als ze het toepassen op nieuwe en complexere soorten problemen. Hoewel het niet eenvoudig is om schemaontwikkeling binnen één les waar te nemen, zien we het intact blijven van het schema als gevraagd wordt om problemen van toenemende moeilijkheidsgraad op te lossen als een voorzichtig teken van instrumentale genese.

4. Conclusie

Hoofdstuk 7 vat de belangrijkste bevindingen van deze studie samen. Daarnaast reflecteren we op de bevindingen en de beperkingen van de studie en doen we aanbevelingen voor het algebraonderwijs, de didactiek ervan, het ontwerp van digitale hulpmiddelen en toekomstig onderzoek.

In de paragrafen met reflecties beschouwen we de theoretische uitgangspunten, het ontwerp en de methodologie van deze studie, mogelijke generalisatie van de bevindingen en ook de beperkingen van de studie. Wat het theoretische uitganspunt van de leerproblemen betreft benadrukken we dat we deze is ontwikkeld in een gecombineerde *top-down* en *bottom-up* benadering, waarin we de resultaten van de literatuurstudie en de bevindingen uit de interviews hebben samengebracht. De aldus verkregen categorisering speelt dus een dubbele rol, die van referentiekader en die van onderzoeksresultaat. Wat betreft de dualiteit operationeel-structureel bleek deze invalshoek vruchtbaar voor het begrijpen van de moeilijkheden van leerlingen, voor het leggen van verbanden met gerelateerde theorieën zoals *symbol sense* en *structure sense*, en als basis voor het begrijpen van algebraïsche substitutie met het Cover-up applet of met pen en papier. Wat betreft het mathematiseren concluderen we het volgende: (1) deze invalshoek houdt verband met de overeenkomstige categorie van moeilijkheden van leerlingen en is niet alleen bruikbaar bij contextproblemen, maar ook bij problemen met symbolische wiskunde in het algemeen, en (2) in praktijk is het mathematiseren, en de moeilijkheden van leerlingen daarmee bij contextproblemen, goed te analyseren aan de hand van de mathematiseringscyclus van De Lange (2006). Met betrekking tot de rol van ICT in algebraonderwijs vinden we dat het oefenen van vaardigheden en het ontwikkelen van nieuwe begrippen, twee zaken waarbij ICT de leerling kan ondersteunen, het ontwerp op een impliciete maar belangrijke manier kunnen sturen. De begrippen "techniek" en "schema" uit de instrumentatietheorie zijn vruchtbaar om het verband te begrijpen tussen wiskundig inzicht en het gebruik van de digitale hulpmiddelen. Dit is geconcretiseerd in de identificatie van schema's die leerlingen ontwikkelen voor het oplossen van verschillende typen vergelijkingen.

De belangrijkste reflectie op het ontwerp en de methodologie van deze studie betreft de inzet van *design research* als de overkoepelende methode. In de twee cycli hebben we niet alle fasen van design research (voorbereiding, uitvoeren van het onderwijsexperiment en retrospectieve analyse) compleet uitgevoerd, maar deze fasen verwerkt in de deelstudies in de hoofdstukken 2 tot en met 6. Op deze wijze hebben we een lichte variant van design research uitgevoerd.

Onze afsluitende reflectie gaat over de generaliseerbaarheid van de bevindingen. Op het gebied van eerstegraads vergelijkingen en ongelijkheden in één variabele hadden leerlingen de meeste moeite met algebraïsche uitdrukkingen, die ze afwisselend als proces en als object zouden moeten zien, en met het vertalen van contextproblemen in wiskundige en algebraïsche modellen (in dit geval vergelijkingen). Deze twee typen obstakels spelen mogelijkerwijs ook een centrale rol bij andere onderwerpen, zoals vierkantsvergelijkingen en daarmee samenhangende contextproblemen. Daarom verwachten we dat onze bevindingen gegeneraliseerd kunnen worden naar andere algebraïsche onderwerpen en vermoeden we dat dezelfde theoretische invalshoeken ook toepasbaar zijn op deze onderwerpen. Met betrekking tot de rol van ICT als hulpmiddel bij het leren is een belangrijke conclusie dat er een naadloze samenhang moet zijn tussen het beoogde wiskundig denken, de technische uitwerking in de digitale omgeving en de doorwerking bij het werken met pen en papier. Ook deze conclusie is volgens ons geldig voor de inzet van digitale hulpmiddelen voor complexere algebraïsche taken, of voor het werken in andere technologische omgevingen.

De studie als geheel kent een aantal beperkingen. Voor de interview studie waren dat de volgende: (1) hoewel de leerlingen in de studie afkomstig waren van zowel Islamitische als openbare scholen en zij door de leraren uit alle niveaus geselecteerd waren (laag, midden en hoog), is de omvang van de onderzoeksgroep betrekkelijk klein; je kunt je dus afvragen in hoeverre deze groep representatief is voor alle Indonesische leerlingen van klas VII, en (2) de fouten die leerlingen maken zijn soms niet eenduidig te categoriseren; de verknooptheid en onderlinge samenhang van de vijf categorieën, die ons centrale referentiekader vormen, zijn geen apart onderzoeksthema geweest. Ook de kleinschalige pilot heeft volgens ons twee beperkingen: (1) in plaats van het hypothetische leertraject (HLT) compleet uit te werken en de uitkomsten van het experiment ermee te vergelijken, hebben we alleen de belangrijkste elementen van het HLT in de analyse betrokken; en (2) ook al hadden we in de ontwerpfase een serie activiteiten voorbereid over vergelijkingen van de typen $f(x) = c$ en $f(x) = g(x)$, zijn we door de beperkte beschikbare tijd alleen toegekomen aan de activiteiten voor het eerste type. Ook in het grootschalige experiment, ten slotte, zien we twee beperkingen: (1) in de experimentele groep waren alleen Islamitische en geen openbare scholen betrokken. We kunnen daarom geen vergelijking maken tussen de prestaties van leerlingen in deze twee verschillende schooltypen; (2) voor de gevalsstudie concentreerden we ons in elk van de vier experimentele lessen slechts op één groep leerlingen, waardoor data van andere, niet geobserveerde groepen buiten beeld zijn gebleven; deze zouden nader bewijsmateriaal geleverd kunnen hebben, waardoor we sterkere conclusies hadden kunnen trekken.

De resultaten van de studie overziend doen we nog een aantal aanbevelingen ten behoeve van het algebraonderwijs, de didactiek ervan, het ontwerp van digitale hulpmiddelen, en vervolgonderzoek.

a. Aanbevelingen voor algebraonderwijs

- bied vanaf het begin contextproblemen en 'kale' algebrasommen verweven met elkaar aan;
- gebruik bij algebraïsche contextproblemen de vier stappen van de mathematiseringscyclus als onderwijsstrategie;
- moedig leerlingen aan om verschillende strategieën te gebruiken of strategieën te combineren om één en hetzelfde algebravraagstuk op te lossen en om de oplossingen te controleren;

- confronteer leerlingen met verwante problemen buiten de reikwijdte van het geplande onderwerp, om zodoende in de beginfase van het probleemoplossen de leerlingen zowel inzicht als praktische vaardigheid te laten ontwikkelen.
- sluit in een ICT-rijke onderwijssituatie een didactisch contract af tussen leraar en leerlingen dat aangeeft wanneer de leerlingen met pen en papier werken en wanneer met digitale hulpmiddelen;
- bewaar in het leerproces enige afstand in de begeleiding van leerlingen om zo bij hen de instrumentele genese te laten plaatsvinden.

b. Aanbevelingen voor algebradidactiek en het ontwerp van hulpmiddelen

- ontwikkel twee soorten algebraïsche taken: (1) taken die geheel op de algebra gericht zijn; (2) taken die tegelijk op rekenen en algebra betrekking hebben;
- maak bij het ontwerpen van taken expliciet gebruik van ontwerpheuristieken die gebaseerd zijn op theoretische of praktische kaders die passen bij het doel van de studie;
- voeg een optie "Vergelijkingen oplossen" toe aan het AlgebraPijlen applet;
- breid het Cover-up applet zo uit dat het mogelijk wordt om uitdrukkingen die twee keer voorkomen in vergelijkingen van de vorm beide te bedekken, en maak het mogelijk om de strategieën Cover-up en Weegschaal te combineren;
- breid het ontwerp van het applet Weegschaalmodel zo uit dat bij verschillende situaties zakken van verschillende grootte gebruikt kunnen worden;
- ontwerp een applet dat aanwijzingen geeft voor en met feedback reageert op het vertalen van een contextprobleem in een algebraïsch model;
- voeg online hulp toe aan de vergelijkingseditor binnen de applet-omgeving;
- maak het mogelijk om applets offline te gebruiken en bouw de mogelijkheid in om het werk van leerlingen te uploaden en op te slaan zodra er een internetverbinding beschikbaar is.

c. Aanbevelingen voor toekomstig onderzoek

- onderzoek de toepasbaarheid van de vijf in deze studie gehanteerde categorieën van moeilijkheden van leerlingen op andere onderwerpen in het aanvankelijk algebraonderwijs;
- analyseer de algebrahoofdstukken in Indonesische schoolboeken en ga na in hoeverre ze mogelijkheden bieden voor mathematiseren;
- doe onderzoek naar de te directe uitvoering van binaire rekenkundige bewerkingen zonder eerst de gehele opgave te overzien, een aanpak die mogelijk het grote aantal fouten van het type $3 + 7x = 10x$ kan verklaren;
- onderzoek de achtergronden van moeilijkheden met andere onderwerpen binnen de aanvankelijke algebra vanuit de theoretische invalshoeken die in de studie behandeld zijn (het operationele en structurele perspectief, *symbol sense, structure sense en emergent modeling*);
- voer longitudinale studies uit naar het gebruik van digitale hulpmiddelen;
- onderzoek de invloed van de groepssamenstelling (groepen met leerlingen van verschillend niveau of homogeen samengestelde groepen) op de prestaties van leerlingen.
- onderzoek het effect van het type school (Islamitisch of openbaar in de Indonesische context) op de algebraprestaties van leerlingen.

Bij wijze van slotwoord erkennen we dat deze studie een eerste stap is naar de verbetering van het algebraonderwijs in Indonesië. In het bijzonder realiseren we ons dat onze inspanningen om de moeilijkheden van leerlingen te onderzoeken en deze vanuit geschikte theoretische perspectieven te begrijpen tenminste het begin is van het beantwoorden van de waarom-vraag naar de geringe algebraïsche prestaties van Indonesische leerlingen, ook al beantwoorden we deze vraag maar gedeeltelijk. Ook zijn we ons ervan bewust dat de integratie van ICT en van de online applets in het bijzonder als een beloftevolle aanpak binnen de onderwijssituatie een gedeeltelijk antwoord geeft op de hoe-vraag naar de verbetering van de algebraïsche prestaties van Indonesische leerlingen. Als we al deze stappen op een goede manier zetten, dan zal het algebraonderwijs in Indonesië werkelijk bijdragen aan de ontwikkeling van het land, zodat het toekomstige uitdagingen het hoofd kan bieden.

Curriculum vitae

Al Jupri was born on 10 May 1982 in Serang, Indonesia. After finishing senior secondary school in 2000 from *SMA Negeri 1* Anyer, Serang, Banten, Indonesia, he continued his study to *Universitas Pendidikan Indonesia* (Indonesia University of Education), Bandung, and then received a bachelor degree in mathematics education in 2004.

From January 2005 to October 2006 he worked as a lecturer at the Department of Mathematics Education, Faculty of Mathematics and Science Education, Indonesia University of Education. He taught calculus, elementary number theory, analytical geometry, and selected topics of school mathematics for prospective secondary school teachers.

After a two-year study, he was awarded a Masters degree (MSc) in mathematics education from Utrecht University in November 2008.

From December 2008 to January 2011, he worked again as a mathematics education lecturer at the Indonesia University of Education. Since then he has published several mathematics books for secondary school students and pre-service teachers, has published newspaper articles in the field of mathematics education, and has presented his papers at both national and international seminars.

In February 2011 he started a PhD research in the field of algebra education at the Freudenthal Institute for Science and Mathematics Education, Utrecht University, the Netherlands, supervised by Prof. Dr. Jan van Maanen, Prof. Dr. Marja van den Heuvel-Panhuizen, and Prof. Dr. Paul Drijvers. His PhD research was accepted by the Interuniversity Center for Educational Sciences (ICO) research school in the Netherlands, for which he fulfilled all requirements to be an ICO PhD member.

In February 2015 he will again work at the department of mathematics education of Indonesia University of Education, as a lecturer, and hopes to be a productive author and researcher in the field of mathematics education.

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