Big ideas and understandings as the foundation for elementary and middle school mathematics

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1. **NUMBERS** - the set of real numbers is infinite, and each real number can be associated with a unique point on the number line.

2. **THE BASE TEN NUMERATION SYSTEM** - the base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

3. **EQUIVALENCE** - any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

4. **COMPARISON** - numbers, expressions, and measures can be compared by their relative values.

5. **OPERATION MEANINGS and RELATIONSHIPS** - the same number sentence (e.g. 12-4 = 8) can be associated with different concrete or real-world situations, AND different number sentences can be associated with the same concrete or real-world situation.

6. **PROPERTIES -** for a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra.

7. **BASIC FACTS and ALGORITHMS -** basic facts and algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones.

8. **ESTIMATION -** numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process.

9. **PATTERNS -** relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.

10. **VARIABLE -** mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.

11. **PROPORTIONALITY -** if two quantities vary proportionally, that relationship can be represented as a linear function.

12. **RELATIONS and FUNCTIONS -** mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.

13. **EQUATIONS and INEQUALITIES -** rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

14. **SHAPES and SOLIDS** - two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes.

15. **ORIENTATION and LOCATION -** objects in space can be oriented in an infinite number of ways, and an object's location in space can be described quantitatively.

16. **TRANSFORMATIONS** - objects in space can be transformed in an infinite number of ways, and those transformations can be described and analyzed mathematically.

17. **MEASUREMENT -** some attributes of objects are measurable and can be quantified using unit amounts.

18. **DATA COLLECTION -** some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it.

19. **DATA REPRESENTATION -** data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.

20. **DATA DISTRIBUTION** - there are special numerical measures that describe the center and spread of numerical data sets.

21. **CHANCE -** the chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.

After initial conversations Cambridge Mathematics would add the following to this list (in no particular order)

22. **PROOF** – mathematical statements can be proved or disproved using previously established statements, self-evident truths or assumed statements. This may be through the use of physical objects, diagrams, manipulatives, or algebra.

23. **CLASSIFICATION** – abstract and concrete mathematical items can be grouped according to their characteristics.