Mathematics education as cultural induction

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Samenvatting

In deze bijdrage aan het OW & OC-symposium op 2 mei 1986 poneert Alan Bishop de stelling dat wiskunde-onderwijs alleen bestaat omdat wiskunde een deel van onze cultuur is en een niet onbelangrijk deel.
Om die reden moet een discipline 'wiskunde-onderwijs' zijn basis hebben in de idee van een inductie in dat belangrijke aspect van onze cultuur. Vandaar de titel: 'Wiskunde-onderwijs als culturele inductie'.

This article will be in the spirit of a report on 'work in progress'. I am writing a book, to try to pull together various ideas which I have been developing over the last fifteen years, and my report today will focus on the most fundamental aspect of my work – the cultural perspective on mathematics education.

In these fifteen years or so there have been two different, but yet related, areas of concern which have challenged me to think very hard about our priorities in mathematics education, and in research in mathematics education.
The first concern is with what I call 'cultural interfaces' – the problems of math education in developing countries, the difficulties faced by immigrant children in 'Western' schools, and most recently the cultural handicap of deafness.
The second concern is with the over-emphasis on the teaching and learning of skills in mathematics to the exclusion of any consideration of 'values' in maths education.
Recently I have been trying to use these concerns to help me understand more about mathematics as a part of our culture, and what this implies for us in the field of mathematics education, hence the title of my talk today "Mathematics education as cultural induction".

In relation to the title of the Symposium. "*The Foundation of a Discipline of Mathematics Education*", I would argue that Mathematics *Education* only exists because Mathematics is part of our culture, and also because Mathematics is known by all to be such a *significant* part of our culture. Therefore a *Discipline* of Mathematical Education must have at its foundation the idea of an *induction* into that significant aspect of our culture.
That seems such an obvious statement to me and I presume that it is one which everybody here would agree with. However the idea of Mathematics Education as Cultural Induction is also a profound idea, because it implies so much about what we should be doing in Mathematics Education, and about how we should *think* about what we are doing in Mathematics Education. I hope today that I will be able to persuade you about the importance of this idea.
Let us begin by considering Mathematics as a part of our culture. How are we to understand it, from the perspective of cultural *induction*?

I have often begun stories and talks with the words "When I was in Papua New Guinea . . .", but that kind of experience, and the general influence of evidence from studies of other cultures has made me very aware of contrasts, of differences, of conflicts – of 'our' culture and 'their' culture.

When I was writing last year about this, I focussed on these differences too much. I was so busy trying to describe the differences that I failed to give sufficient attention to the similarities and to what *it* is which is different between these cultures.
This year I have concentrated more on the similarities and that has produced some important results for me, and I would like to share some of them with you.

E.B. Tyler gives us a good start, in his book "*Primitive Culture*".
"Culture, or civilisation, taken in its wide ethnographic sense, is that complex whole which includes knowledge, belief, art, morals, law, customs, and any other capabilities and habits acquired by man as a member of society" (p. 1).

That definition is fine, and we can easily agree with maths being part of cultural knowledge. But it doesn't help us a great deal – particularly if we are to understand the similarities and differences between cultures.
What we need to introduce is the idea of 'environment' – of physical and social environment. Culture is the product of human interactions, and cultural differences exist because of differences in the physical and *social* environment with which individuals engage. Of course it is not just a simply matter of a one-way process, because culture also affects environment. The way individuals engage with their physical and social environment is culturally shaped. But it is true, and it is surely also obvious that the physical and social environment helps to shape culture. So, how should we link 'environment' to 'culture'?
When you look at cross-cultural studies and at studies of other cultures you do begin to realise that there are activities which all cultures engage in. There are the obvious ones like eating, speaking and killing but there are less obvious ones also. Anthropologists will talk of these as 'cultural universals'. Our mathematically trained minds will make us question the *precise* notion of 'universal' of course but I'm sure you can pretend to be a non-mathematical anthropologist for a moment!

I have analysed in detail as many as I could of the 'cultural' studies which relate in any way to mathematics, and I have concluded that there are six activities which can be called 'universal', in the anthropological sense, which have been, and which continue to be significant for the development of mathematics in culture. Note please, not just in our culture, but in any culture.
These six activities are not mathematical activities (like multiplying or solving equations) they are *environmental* activities through which culture has developed, and particularly through which mathematical culture has developed.
Maths as a cultural phenomenon exists in some form, and to some extent in any culture, and I will argue that it is these six universal activities which help to develop mathematical culture anywhere.

All six of these activities are, then, motivated by some perceived environmental need, all of them stimulate and are stimulated by various cognitive processes, and all of them involve special kinds of language and representations. They all help to develop the symbolic technology which we call mathematics. So, here are my activities, with some of their symbolic possibilities.

Counting

We begin with perhaps the most obvious activity suggesting mathemtical development, and one which is probably the best researched mathematical activity in the cultural literature. Counting and the association of objects with numbers clearly has a long history.
Even with so-called 'primitive' peoples there is plenty of evidence of counting. Harris' (1980) survey of Aboriginal mathematics shows that the "one-two-many" caricature of a primitive counting system is, like all caricatures, both an exaggeration and also only part of the story. While "almost all Australian languages contain only two or three cardinal numbers" (p. 13) there is clearly much use made of body-counting – an extension of finger counting. Also she points out that "where Western mathematics emphasises many calculations with large numbers the Aborigines have always been intimately concerned with individuals and small numbers" (p. 14). That concern is reflected in a "richness" of language for small number.

The various taboos associated with the dangers of counting people or animals may be a joke to some people, but they may have contributed to the development of abstract number by requiring the use of *indirect* methods of counting (Zaslavsky).

If we move to a country with many separate peoples we can find Lancy (1983) classifying 225 different counting systems into the following four types:

Type I	–	a body parts tally system with the number of body parts varying from 12 to 68;
Type II	–	a tally system using counters, like sticks. The base number is usually between 2 and 5;
Type III	–	mixed bases of 5 and 20 using compound number names like "two hands and a foot" to mean 15;
Type IV	–	base 10 system with several discrete, rather than compound number names.

Studies like these surely convince us, if we needed convincing of it, that there are not just two systems of numbers – "civilised" and "primitive" – as used to be the conventional public wisdom, but a rich variety of systems, varying in line with the environmental need, and existing in all societies.

Locating

I choose this word to characterise the activities relating to finding your way around, knowing your home area, working the land, travelling without getting lost and relating objects to each other. As might be expected all societies have developed different ways to code and symbolise their spatial environment, and different societies find different aspects to be of significance. As with counting, where we found the Aborigines concerned with small detail in contrast to our worries with the very large, we might expect rural dwellers to be concerned with other spatial aspects than are urban groups.
Surprisingly this activity has not been regarded as significant, mathematically, as number aspects, at least judging by the limited attention given to it by those researchers ostensibly interested in mathematics. But in terms of both mathematical behaviours

and mathematical concepts, particularly in geometry, activity in large-scale forms a very significant foundation.

The Aborigines' ability to find their way in what to anyone else would be a featureless landscape has been part of Australian folk-lore for many years. When asked by an anthropologist what they do if they get lost, some Aborigines answered "We go home". They had no concept of getting lost! Lewis' (1976) fascinating study of their route-finding and spatial orientation shows us that the people he studied carried an internalised compass system in their heads – as one indigenous informant pointed out "Aborigines knew north, south, east and west before the white man's compass" (p. 265). They could talk about this system and their use of it – its relationship to the sun, and to the temperature of the wind – and their languages reflect this ability. Far more significant however, was their intricate knowledge of the landscape in relation to their myths, and to their history within that landscape.

Another study which looks in detail at another culture's way of conceptualising large-scale space is Pinxten's (1983) work about Navajos in North America. This comprehensive study attempts to set out the Navajo philosophy and phenomenology of space, and provides us with good evidence that 'locating' is a clearly universal activity and is one which provides a rich set of geometrical concepts and language.

Moreover, it is no accident that *on paper*, North is up, 'horizontal' means across the page and 'vertical' means straight down it. It is no accident that we use two and three dimensional axis systems, and it is no accident either that much of the informal language and imagery of geometry is drawn from movements and locations in large-scale space, e.g. 'turn through 90°', 'a straight line between two points', 'height of a triangle', 'rotate about a point'. It is no accident that geometry has developed from measuring the earth.

Measuring

Measuring is another universally significant activity for the development of mathematical ideas. Measuring is concerned with comparing, with ordering and with valuing, and all societies value certain things. We must be careful not to search for precision, and systems of units though – these only develop inm relation to particular environmental needs.
For example in Papua New Guinea, Jones (1974) collected data from several informants about quantities and measures which included statements like these:
"The local unit of distance is a day's travel which is not very precise."

Harris' survey among the Aboriginal groups showed other features though which were equally revealing of both skills or needs: "People 'measure' via a mental picture or by 'eye'. There's hardly anyone here who can't buy a dress for a relative by simply looking at the dress – they nearly always buy the correct size" (p. 52).

Clearly estimating 'by eye' is a word-wide non-verbal technique for measuring objects, but as the quality grows in significance and the number of objects increases, so the language develops both the number words of order (first, second, third, etc.) and the 'objectification' of the quality (e.g. from 'heavy' to 'weight').

Regarding the development of units, and systems of units, there is again a clear progression, with the main idea being that of the stronger the environmental and social need the more accurate the measure. Gay and Cole (1967) refer to the unit called 'Kopi', a cup, much used for measuring rice, and it was clear in their study how much more skilled the Kpelle were in comparison with the American researchers at estimating how many cupfuls of rice were in a certain container. They also give examples to show how units can combine – one informant said that a bucket contains 24 cups of rice and the tin (another 'standard') contains 44 cups of rice – very close to the actual arithmetical measures. Rice for the Kpelle is a very important commodity, hence the internal coherence and the complexity of measures associates with it.

Zaslavsky (1973) also refers us to body-measures used for length (the Ganda of Uganda refer to the *mukono*, the same as cubit, the distance from the elbow to the tip of the outstretched middle finger), a basket holding about ten pounds, a package of coffee beans, and a bundle of sweet potatoes, all 'standard' measures to the local people, but with that element of inaccuracy which allows for social and commercial negotiation! She quotes the old Ethiopian proverb "Measure ten times, tear the cloth once".

Gay and Cole documented a nice example of this 'social' dimension of measuring – whereby "the cup the trader uses to buy rice has the bottom rounded out by long and careful pounding, but the cup he uses to *sell* rice does not have the rounded bottom. This is the source of his profit" (p. 64).

Leach (1973) catches this societal aspect of measurement well when he says: "It is a peculiarity of scientific society that an ideal scale should be one which is unambiguous and exact; under other conditions people have preferred scales which were easy to use. Where the criterion of a good scale is its convenience, too much precision may even be a nuisance (p. 139).

So accuracy is not necessarily to be valued highly, it depends on the purpose and importance of measuring. But all societies engage in plenty of measuring activities.

Designing

Another universal and important source of mathematical ideas are the many designing activities pursued by all cultures. Where the 'locating' activities refer to locating oneself mainly within the natural environment, the activities of designing concern all the man-made objects and artifacts which cultures create for their home life, for trade, for adornment, for warfare, for games and for religious purposes. In addition designing can concern larger-scale space, as with houses, villages, gardens, fields, roads and towns.

The essence of designing is the process of transforming a part of nature, of taking some natural phenomenon be it wood, clay or ground, and making it into something else – perhaps a carved ornament, a cooking pot or a garden. Designing involves imposing some structure on nature. When one is walking and finds a branch of a tree on the path one removes the unnecessary parts, cuts it to a convenient length and makes a walking stick. One has a 'design' in mind which one is able to impose on that branch.

Designing involves imagining nature without the unnecessary bits and perhaps even emphasising some aspects more than others. To a large extent, then, designing concerns abstracting a shape from the natural environment. That is why I have chosen to focus (for mathematical purposes) on 'designing' rather than on 'making'. The actual product is not important mathematically, whereas it *may* be for developing scientific ideas, where one *is* concerned with the actual properties of objects. What is important for us is the plan the structure, the imagined shape, the perceived spatial relationship between object and purpose, the abstracted form and the abstracting process. For example, whether we are discussing cave paintings of animals, wood carvings of humans, or Eskimo rock carvings of sea mammals, the designer has chosen to emphasise some features and to ignore others. The *idea* of form or shape is developed by designing.
Moreover the designed object serves as the representation of the design by which other objects can be constructed. Man has of course developed other ways to represent designs, notably by drawing in the sand, or by constructing models, or later by drawings on paper and on electronic screens. All of these developments have been created by the need to consider aspects of the designed form without having to actually make the object. This need is of course greatest when designing very large-scale 'objects' such as agricultural land, gardens, obelisks and monuments, but it will also be important if the material from which the object is to be made is expensive or rare, or both. It is easy to see how these environmental and social needs have created a demand for *designing*, which in turn has developed important mathematical ideas concerning shape, size, scale, measure, and many other geometric concepts.

Playing

This may seem initially to be a rather strange activity to include in a collection of cultural activities relevant to the development of mathematical ideas, until one realises just how many games have mathematical connections. It is even more important to include it, when approaching education from a cultural perspective, because of the vast amount of documentation of games and playing around the world. One is forced then to realise just how significant 'play' has been in the development of culture. All cultures play and what is more important, they take their play seriously! By that I mean that it is essential not to treat play as a relatively unimportant aspect of cultural life.

Johan Huizinga (1949) in his classic book "*Homo Ludens*" says: "The spirit of playful competition is, as a social impulse, older than culture itself and pervades all life like a cultural ferment..." (p. 173).

Certainly games, their description, analyses, and roles, feature widely in the anthropological literature. Interestingly, although the characteristics of 'play' as described by Huizinga can be seen in these descriptions of games, the notion of 'game' is more restricted than that of 'play'. It seems as if 'playing is the general activity '(which is why I chose it as the section heading) and the idea of 'game' is a formalisation of playing. We can certainly think of the game as the 'representation' of playing.

As with 'designing', the quality of the form developed in play becomes valued for its own sake. Because, as Huizinga pointed out, play is essentially unserious in its goals, its performance becomes its own reward. So string figures and other play-forms become of interest in themselves, and the roots of artistic and aesthetic appreciation can be seen.

Once the play-form itself becomes the focus, and a 'game' develops, then the rules, procedures, tasks and criteria become formalised and ritualised. Games are often valued by mathematicians because of their rule-governed behaviour which it is said, is like mathematics itself. I think that it is not too difficult to imagine how the rule-governed criteria of mathematics have developed from the pleasures and satisfactions of rule-governed behaviour in games. Within what Huizinga delightfully calls the "Magic Circle" of the game, rule-bound behaviour is the prime concern. And there is no doubt that people everywhere, adults and children, enjoy participating in the rule-bound behaviour of games, perhaps because they are, unlike reality, social settings where the players all know the rules and all agree to play by those rules. Perhaps we all secretly wish for more consistency in our social reality?

Explaining

The sixth and final 'universal' activity I call 'explaining',and it is this activity which lifts human cognition above the level of that associated with merely experiencing the environment.

Explaining is the activity of exposing connections between phenomena and the "quest for explanatory theory" as Horton (1967) describes it "is basically the quest for unity underlying apparent diversity; for simplicity underlying apparent complexity; for order underlying apparent disorder; for regularity underlying apparent anomaly" (p. 209). It is the security of things familiar which probably makes us seek 'sameness' or similarity, and language is of course a fundamental 'similarity' representation. 'Bird', 'stone', 'happy', 'run', are words which represent classes of similar phenomena and explaining is thus as universal as language.

It may seem that merely attaching a label to something is hardly worth calling explaining but a few examples from our own culture may help – "a professional footballer is an entertainer", "a teacher is a policeman", "religion is the opium of the masses". All of these sentences establish connections between different phenomena thus explaining aspects of those phenomena.

So, at an elementary level of explaining, the nouns, adjectives, verbs and adverbs of our languages, and sentences which link these, help us in our quest for "unity underlying apparent diversity". But although classifying is a universal activity, the classifications obtained are not. The diversity of languages brings a diversity of classifications.

But what about explaining more complex phenomena, in particular dynamic phenomena, the processes of life, and the ebb-and-flo of events?
Here, the fundamental and universal representation is the 'story'. Every culture has its stories, its folk-tales, and its story-tellers, and the "Once upon a time..." phrase is known everywhere, even if the actual wording is different.
The time-scale can of course be much longer and the story becomes the history. "Once upon a time..." grows into "In the beginning was the word...". The events of long ago become shrouded in mystery, and the growth of mysticism, of myth, of legend, and of religious belief can be found everywhere.

The 'story' is then a universal phenomenon, and from our point of view, in thinking about mathematical culture, the most interesting aspect is the ability of the language to connect discourse in rich and varied ways. In research terms, attention has focussed on the 'logical connectives' in a language which allow prepositions to be combined, or opposed, extended, restricted, exemplified, elaborated, etc. From these the ideas of proof have developed along with criteria of consistency, elegance and conviction.

These then are my six 'universal' activities, and I suggest to you, moreover, that since these are universal activities, mathematics also exists within all cultures in some form, to some extent, and with more-or-less significance for the people.

So how does this help us to understand Mathematics (with a capital 'M') as we know it, or as everyone else in our society knows it? First of all, it is clearly a specific kind of *m*athematics developed through a specific cultural and social history.
Moreover it is not just a collection of certain kinds of symbols and representations. Remember when I began this article I said that there were *two* concerns which motivated this work of mine. The first was the area of *cultural conflict*, and the second was the lack of reference to *values* in our maths education. This is precisely where these come in.
Mathematics as the cultural phenomenon we know cannot be separated from its cultural and social history. It is a product of interactions between man and his physical and social environment over many years and in many societies. I cannot hope to document these today, but in any case much of that history will be familiar to you.

One particular aspect which Anthropologists agree on is how significant technological growth is for cultural development. We have already seen this in the activities of designing and measuring, and it is also at the root of *causal* explanation. (Simple technological devices enable causation to be experienced in ways which natural events cannot do.) Moreover, part of this technological development has been with Mathematics itself. Maths is a *symbolic* technology – extending man's ability to control his environment symbolically in the same way that physical technology enables him to do this physically. Technological progress is based on man's wish to create more of a 'man-made' environment which it is assumed, will be more controllable than the so-called 'natural environment'. In fact, of course, that environment itself controls man, and thereby shapes the ideas, attitudes and values of cultural knowledge.
Mathematical development has thus been strongly related to the values of *control* and *progress* in the different societies through technological and scientific advance. The growth of *social* science is a particularly good example of the desire for control and progress in future societies through mathematically-based explanations of social phenomena, leading to ideas such as 'social engineering', 'social technology' and the whole 'Technological System'.

Two particular ideologies have enabled mathematics to become the most powerful 'explainer of phenomena' – *rationalism,* i.e. the spirit and canons of *logic*, and *objectism*, by which I mean the objectification of phenomena by our language. Through our cultural history the 'object' view of the world has grown in influence and 'rationalism' has significantly influenced the course of mathematical growth. These twin ideologies have helped to shape Mathematics in our culture and they too are part of the values of our Mathematics.

As well as these four components there are two more which I feel deserve our attention. First is what I call *openness*, which includes the idea that Mathematical truths can be tried and tested anywhere, and that they will hold true anywhere. The idea that anyone can learn the rules of the game of Mathematics is also in the spirit of openness. Moreover consider the words used to start Mathematical activity: "What if", "Suppose", "Let", – this is the language of opening up possibilities, the language of the hypothetical future.
There is also the value which I call *Mystery*. Even though Mathematical knowledge is open, there is still a great mystery about what Mathematics actually is. Whether you are a school-child, a man-in-the-street or even a professional Mathematician you will still wonder what on earth Mathematics is. As Bertrand Russell remarked "Mathematics is the subject in

which we never know what we are talking about, nor whether what we are saying is true". Moreover, just as Mathematics is mysterious, so are Mathematicians and what they do. They seem sometimes to want to keep it that way!

These value components associated with control, with progress, with rationalism, with objectism, with openness, and with mystery, are all part of *our* Mathematics. They are indeed so much a part of it that we take them for granted. Just as with the symbolic side of Maths, it is important to try to step outside our own culture by looking at other cultures, in order to understand Mathematics as part of culture, and to understand the socio-cultural *values* associated with *our* Mathematics.

All I have time to do in the rest of this article is to point out to you some implications of this analysis in ourline:

1. The anthropologists distinguish two different kinds of cultural induction:
   - enculturation: home culture;
   - acculturation: aliend culture.

   These are two different induction situations with very different moral and educational issues surrounding them.
2. The maths curriculum is an objectification of the mathematical culture for the purposes of induction. So, the curriculum should provide for both the symbolic technology of Maths and the values components. It needs a stronger social, historical and cultural base than it has at present.
3. Cultural induction focusses on the interpersonal and social nature of Mathematics education. It draws our attention to the intentional shaping of culture, and to the maths teachers as the person who does that shaping. Books and other materials are important aids for the teacher but it is the person called the teacher who enculturates the person called the learner.
4. Mathematics teacher education needs to give a much greater priority than it does at present to these cultural and historical ideas. Mathematics teachers, teacher educators, curriculum developers, and indeed the whole Mathematics education community is the group responsible, not just for the transmission of culture, and for the induction of the young, but ultimately for the preservation of mathematical culture.

Finally I hope I have been able to convince you of the claim with which I began – that cultural induction is the most important foundation stone of the discipline of Mathematics Education.
Cultural induction is the reason why ME exists at all and it therefore should influence profoundly our thoughts and ideas concerning ME. I hope I have shown you some ways in which that influence can be brought to bear on ME today.

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