

# Mathematics B-day 2014

Friday November 14, 9:00 – 16:00 hr



## Lights Out



The mathematics B-day is sponsored by





# Warming up: play the game!

The assignment is about the game 'Lights Out' that has been played worldwide since 1995. The game is played on a grid of five rows by five columns with a total of 25 lights. Some of the lights are on at the start of the game. The point of Lights Out, as can be seen from the name, is to switch off all the lights. You can click on lights (they are in fact buttons with a light in them). If you click on a light, you toggle the state it's in (so if the light was on, it goes off; if it was off, you switch it on). There is an important addition: if you toggle a light by clicking on it, you will also toggle its direct neighbours (left, right, top, bottom).

Today, you will find out that there is a lot of mathematical puzzling to do in Lights Out! You will also look into variants of the game that are played on different-sized grids.

First, to warm up for the day, here are a few problems for the original game. Open the app via: <http://www.fisme.science.uu.nl/wisbdag/opdrachten/LightsoutEN/Lightsout.html> and take some time to explore the options in the app. Your teacher will have an offline version (Windows only) available.

## Puzzle 1 (easy)

Create a 5x5 grid in the app een grid, and use the option 'toggle lights individually' to make the starting configuration on the right. Now deselect the option 'toggle lights individually' and select the option 'Execute clicks immediately' under 'Play the game'. Solve the puzzle (i.e. turn off all the lights). A 0 means 'off', a 1 means 'on'. This puzzle can be solved in five clicks.

1	1	0	1	1
1	0	1	0	1
0	1	1	1	0
1	0	1	0	1
1	1	0	1	1

## Puzzle 2 (medium)

Solve the next puzzle (it can be done in six clicks).

1	0	1	0	1
1	0	1	0	1
0	0	0	0	0
1	0	1	0	1
1	0	1	0	1

## Puzzle 3 (difficult)

Is this puzzle too difficult right now? No problem!

You may even think after a lot of fruitless clicking: *it's impossible*.

In this case you are wrong, but we can already tell you that only 25% of the 33 554 432 possible starting configurations is solvable. Solvable means that there is a solution; it is possible to turn off all the lights. Unsolvable means that no matter what you try it is impossible to turn off all the lights.

Later in the day you will learn techniques that help you decide for a given puzzle *whether* it is solvable and *how* you can find the solution in that case.

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

# Practical points

## The structure of the assignment and schedule for the day

In part ONE you will explore Lights Out, and try out various things yourself. Part ONE is primarily intended for the morning. There are quite a few assignments where you have to draw or write something down. The assignments in this part are numbered. Later (for instance also in the second part) you can refer to something you did before. For example: "In nr. 13 we saw that ... . We are using that again now."

The exploratory questions, which are called **[Exploration]**, are meant to introduce something new, and these questions are not included in your final report. For the other questions, which are called **[Assignment]**, you have to include an answer with correct reasoning in your final report.

**Warning!** *The final part (E) of part ONE is rather difficult and theoretical. Only work on that if you still have enough time in the morning. This part is specifically important for answering Main question D in part TWO.*

In part TWO (own investigation) you will work on more complex and open assignments. Part TWO is primarily intended for the afternoon. In this part you can apply what you learned in part ONE. You will work on one or more of the Main assignments A, B, and C. Additionally, if you want you can also work on Main assignment D, which means you will have extra work! Keep in mind that answering two Main assignments in depth will be appreciated more than superficially treating three or four Main assignments in part TWO. The answers to these Main assignments are to be included in your final report.

## What do we expect from you?

In your *final report* you will describe your results from the *assignments* in part ONE and the Main assignments in part TWO.

Present your report so that it is clear and convincing. Of course you will include relevant explanatory figures as illustrations. Make sure to be clear for an audience that does not take part in the Mathematics B-day, but which does have a sufficient mathematical background. This means that you must introduce the problems clearly and that, where relevant, you will fall back on your explorations and arguments from part ONE.

In short: you will write a clear report, supported by mathematical arguments. The way the report is presented will play a part in assessment!

For the final report it may be useful to keep track in a digital document of what you do in the morning. Keep in mind that you must be able to hand in the completed final report by four o'clock in the afternoon!

## Information on the Internet

You can find a lot of information about playing Lights Out on the internet, but the information will hinder rather than help you for the problems you will work on today.

***Good luck, but above all: have fun with this mathematical challenge!!***

## Part ONE

You indicate the **position** of a light on the grid with (*row, column*). A click on light (*i, j*) is written down as  $K_{i,j}$ . If there is only one row, you click on light *j* with  $K_j$ . A **click series** is performing multiple clicks, for example  $K_{1,3} K_{4,2} K_{3,3}$ . The **null configuration** is the configuration in which all lights are off (i.e. 0).

### A: Click series

#### 1. [Exploration]

You can make this configuration on a 4x7 grid by using the app (use 'Configure the grid' and 'Make a configuration'):

1	1	0	0	1	1	0
0	0	0	0	1	1	0
0	1	0	1	1	0	0
0	0	0	0	0	1	0

The underlined numbers indicate which lights will be clicked (you can do this yourself by selecting the option 'make a plan' in the app):

1	1	0	<u>0</u>	<u>1</u>	1	0
0	<u>0</u>	<u>0</u>	0	<u>1</u>	1	0
0	<u>1</u>	0	1	1	0	0
0	0	0	0	0	1	0

Before you click on 'execute plan' you can already work out that afterwards light (2,3) will not be toggled and light (1, 5) will be.

- Explain how that is connected to the number of clicked neighbours.
- On a 1x99 grid, all the lights are on. You perform the click series  $K_1 K_2 K_{50} K_{92} K_{94}$ . Which lights are now off?

#### 2. [Exploration]

Make a random configuration on a 1x6 grid, and write down this starting configuration.

- Perform the click series  $K_1 K_3 K_4 K_5 K_3 K_5$  and write down the resulting configuration.
- Return the grid to the starting configuration and perform the click series  $K_1 K_4$ . Write down the resulting configuration.
- The final configurations in **questions a) and b)** turn out to be identical. Explain why that is the case.

#### 3. [Exploration]

Make a configuration on a grid of your choice, and think of a three click long click series; call the clicks *A*, *B* and *C*. Verify that  $A B C$  has the same effect as  $B A C$  and as  $C A B$ . Also explain why this is the case.

### Summary

- You can remove double clicks from a click series. A click only matters if it occurs an odd number of times.
- The order of clicks in a click series is unimportant.

The above has important consequences that will be of constant relevance today.

*Consequence 1: You can always go back.*

#### 4. [Exploration]

If you perform the same series of clicks twice in succession, you will be doing every click double. So, in one word, what happens?

#### 5. [Exploration]

If from a specific start configuration you have found a click series that takes you to the null configuration and you repeat that same click series, what will be the final configuration?

*Consequence 2: The solvable configurations are precisely those that you can make by clicking, starting from the null configuration.*

#### 6. [Exploration]

Explain why consequence 2 has to be valid.

#### 7. [Exploration]

- Use the app to make a  $3 \times 4$  grid. Click the button 'all lights on'. Click the button 'make a plan' and click to switch on all the lights along the edge of the grid, including the corners. Do *not yet* click the button 'execute the plan'. Predict what will happen when you perform this click series.
- Click the button 'execute the plan'. Does the result match what you predicted?
- What will happen if you click on all the lights along the edge of the grid again?
- In the same way, investigate what happens if, on an  $n \times n$  grid, you use all the lights along the diagonal  $((1,1), (2,2), (3,3)$  up to and including  $(n,n)$  as a click series.
- First toggle all the lights off using the button 'all lights off'. Now click on every light once. Predict the result for  $m \times n$  grids for some values of  $m$  and  $n$ .

#### 8. [Assignment A]

We will now look at a  $1 \times n$  grid (voor  $n = 1, 2, 3, 4, \dots$ ) with all lights turned on in the start configuration.

*Claim: There is a click series for every value of  $n$  to reach the null configuration.*

- Show that this statement is true for all values of  $n$ .
- For every value of  $n$  there is also a shortest click series to reach the null configuration. Investigate the relationship between the length of that click series and  $n$ .

## B: Working with zeroes and ones and toggle diagrams

A click on a light will toggle the state of that light and its direct neighbours. So this changes the configuration on the grid.

An example on a 4x5 grid. You can represent the result at click  $K_{3,3}$  like this:

Start configuration		click $K_{3,3}$		Final configuration
1 1 0 1 0		0 0 0 0 0		1 1 0 1 0
1 0 0 0 1		0 0 1 0 0		1 0 1 0 1
0 0 1 1 0	+	0 1 1 1 0	=	0 1 0 0 0
0 1 1 0 1		0 0 1 0 0		0 1 0 0 1

The ones on the grid in the start and final configuration are the burning lights. The ones at click  $K_{3,3}$  represent something else: namely that those five lights will be toggled by the click (from 'on' to 'off' or the other way around). Here the zeroes represent that these lights are not being toggled.

We call the 4x5 rectangle under  $K_{3,3}$  the **toggle diagram** of  $K_{3,3}$

The '+' and '=' in the figure indicate that you can 'calculate' the final configuration. You need to apply the following 'calculation rules' in each cell:

$0 + 0 = 0$  a light that is off and is not toggled stays off,  
 $1 + 0 = 1$  a light that is on and is not toggled stays on,  
 $0 + 1 = 1$  a light that is off and is toggled is turned on,  
 $1 + 1 = 0$  a light that is on and is toggled is turned off.

The effect of two subsequent clicks (first  $K_{3,3}$  and then  $K_{2,4}$ ) you can represent as follows:

Start configuration		click $K_{3,3}$		click $K_{2,4}$		Final configuration
1 1 0 1 0		0 0 0 0 0		0 0 0 1 0		1 1 0 0 0
1 0 0 0 1		0 0 1 0 0		0 0 1 1 1		1 0 0 1 0
0 0 1 1 0	+	0 1 1 1 0	+	0 0 0 1 0	=	0 1 0 1 0
0 1 1 0 1		0 0 1 0 0		0 0 0 0 0		0 1 0 0 1

You can also first calculate the result of the two clicks together and then determine the effect of the click series ( $K_{3,3} K_{2,4}$ ):

Start configuration		$K_{3,3} + K_{2,4}$		Final configuration
1 1 0 1 0		0 0 0 1 0		1 1 0 0 0
1 0 0 0 1		0 0 0 1 1		1 0 0 1 0
0 0 1 1 0	+	0 1 1 0 0	=	0 1 0 1 0
0 1 1 0 1		0 0 1 0 0		0 1 0 0 1

The rectangle under  $K_{3,3} + K_{2,4}$  is of course called the toggle diagram of click series  $K_{3,3} K_{2,4}$

Pay attention to the difference between click series and toggle diagram!

### Summary

- a click series shows which lights are clicked on;
- the toggle diagram of the click series shows which lights are toggled.

### 9. [Exploration]

Take a grid, for example  $7 \times 4$ , and think of a click series of 3 clicks. You can find the toggle diagram for this click series like this: Turn off all the lights and perform your click series. Done! Now explain why you SEE the toggle diagram of the click series on the screen.

### 10. [Exploration]

On a  $7 \times 7$  grid, what does the toggle diagram look like if in the click series the lights of the British flag are switched on: all lights on the two diagonals and all lights in row 4 and in column 4.

### 11. [Exploration]

Quite a number of really different click series can be performed on a given grid (*really different* meaning: every light is clicked on no more than once and the order is not important).

- How many really different click series are there on a  $1 \times 4$  grid?
- How many really different click series are there on a  $2 \times 2$  grid?
- Find click series for the following four toggle diagrams: (on  $1 \times 4$  and on  $2 \times 2$ ):

1	0	0	1
---	---	---	---

0	1	0	0
---	---	---	---

0	0
0	1

1	0
0	1

### 12. [Assignment B]

Think about the  $5 \times 5$  grid. Thinking and reasoning is enough for this assignment; trying things out in the app will not bring you much further.

In **question 8** you found that from the null configuration of the  $1 \times 5$  grid you can turn everything on with two clicks, and of course turn everything off too.  $K_1 K_4$  was one of those click series.

$K_1$  toggles two lights,  $K_4$  the other three. The toggle diagram has five ones. But there is a different way ...

- The toggle diagram of click series  $K_1 K_4$  is the same as that for  $K_2 K_5$ . Verify this.

We say it like this:  $K_1 K_4$  and  $K_2 K_5$  are **parallel click series**.

Of course this is only special if the click series themselves are different.

You might think that this is an exceptional situation. But it is not! In a few steps, we will now find out that there are many pairs of parallel click series on a  $1 \times 5$  grid!

- If you perform  $K_1 K_4 K_2 K_5$ , in the end nothing happens. Why is this?



From now on we will call a click series that does nothing a ***silent click series***.

- c) Explain why the click series  $K_1 K_2$  and  $K_4 K_5$  are parallel as well.
- d) Find a click series that is parallel with the click series that only consists of  $K_1$ .

There can be clicks that occur in both series in two parallel click series. This is something that you will need now.

- e) Find a click series that is parallel with  $K_1 K_2 K_3$

Using the following recipe, you can find parallel click series:

*After a click series you selected yourself, place the series  $K_1 K_2 K_4 K_5$  and remove the double clicks.*

The click series you end up with is parallel with the one you started with.

- f) Check that this recipe works for the examples already given, and give an explanation.
- g) On the  $1 \times 5$  grid there are 16 pairs of parallel click series. Why is this?

### 13. [Exploration]

- a) You can also make parallel click series on a  $1 \times 8$  grid. Make a partner for  $K_1 K_4 K_7$
- b) What is the smallest grid that has parallel click series?

Keep in mind what we found in **question 6**:

The solvable configurations are precisely those that you can make by clicking, starting from the null configuration.

### 14. [Exploration]

- a) Every grid has as many possible configurations as possible click series. Why is this?
- b) Sometimes there are not as many toggle diagrams as click series. Why is this?
- c) So, on some grids you cannot reach every configuration from the null configuration. Why is this?

So, on some grids there are start configurations for which you can find no click series that results in the null configuration.

A pity.... But at least we know.

Now let us go and look for the ones that can be solved and how.

## C: $1 \times n$ grids and 'mutually toggling' lights

In this part you will learn a technique that gets you pretty far on  $1 \times n$  grids. It's an introduction. In Main assignment A in part TWO you can finish the work yourself.

### 15. [Exploration]

In this Exploration you will use a  $1 \times n$  grid, with  $n > 3$ .

- Which toggle diagram goes with click series  $K_2 K_3$ ?  
What does that mean for lights 1 and 4? And for lights 2 and 3?
- Which toggle diagram goes with click series  $K_i K_{i+1}$ ?  
What does that mean for lights  $i - 1$  and  $i + 2$ ?
- Which toggle diagram goes with click series  $K_1 K_2$  and with click series  $K_{n-1} K_n$ ?  
Which lights will now be toggled?

This idea of clicking two 'neighbouring lights' can be applied to all  $1 \times n$  grids.

### 16. [Exploration]

On this  $1 \times 15$  grid the crosses mark which lights are to be clicked on:

			x	x		x	x		x	x				
--	--	--	---	---	--	---	---	--	---	---	--	--	--	--

What toggle diagram goes with this click series?

Using this technique you can toggle two lights, where the distance between the two lights is three places, without affecting the other lights. We call this 'mutual toggling'. By repeating the technique you can also bridge distances that are a multiple of three. In the example the distance was 9.

### 17. [Assignment C]

As an example we will look at a  $1 \times 10$  grid.

Using this technique, the lights in the group  $\{1, 4, 7, 10\}$  can mutually toggle each other.

- Which group of mutually toggling lights do you get with light 2?  
And with light 3?

Lights 1 and 2 mutually toggle, as do 9 and 10.

- On a  $1 \times 10$  grid the only lights that are on are 1, 3, 7 and 9.  
Find (use 'make a plan') a click series that lets you reach the null configuration.

- Now, the only lights that are on are 1, 3 and 8.  
What click series will now let you reach the null configuration?

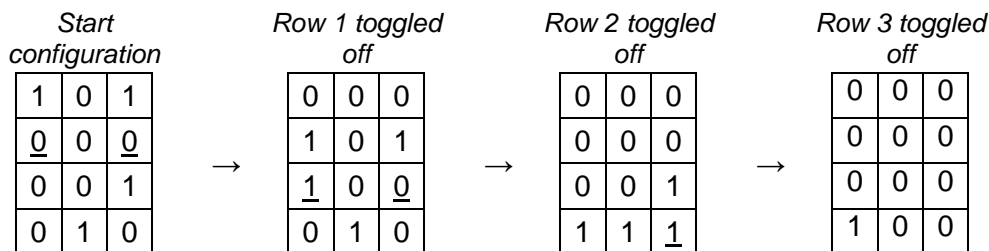
## D: Hunting

A technique that is often mentioned online as a way to approach Lights Out puzzles is called *hunting*.

With a given start configuration you can, working vertically, toggle all lights that are on in all rows except the bottom row to off by clicking, in the row below the current row, on all the lights in the columns where the current row has ones.

### An example as Exploration.

With the given start configuration on a 4×3 grid below, click series  $K_{2,1}$   $K_{2,3}$  (the underlined lights in row 2) will toggle the lights (1, 1) and (1, 3) to off. After that,  $K_{3,1}$   $K_{3,3}$  in row 3 will toggle the two burning lights in row 2 to off. Finally,  $K_{4,3}$  in row 4 will toggle off the burning light in row 3.



## 18. [Exploration]

a) Make the start configuration given above, and empty the first three rows by hunting.

If you want to solve the start configuration, you are almost there after hunting; except for the bottom row. For this row there is a trick, for which we will first show that it works, so that you do indeed reach the null configuration.

b) Now click on light (1, 3) and again hunt the lights down.

Surprise! The null configuration!

The obvious question now is: how do you reach the mysterious idea to click (1,3) rather than any other light or combination of lights in the first row? Because you do not like hocus pocus, do you?

The honest answer is:

In row 1 all eight possible combinations of clicks have been tried one by one. Clicking on only (1,3) turned out to be correct. It was a bit of a search, but it is limited to performing only the eight possible click series in the first row.

We will however check what happened exactly in the hunting after clicking (1,3).

## 19. [Exploration]

Start with an empty 4×3 grid.

a) Click on light (1,3). **Very important:** Do not use 'make a configuration', but click on (1,3) from 'execute clicks immediately'.

b) Hunt down.

Your final result is exactly the final result after hunting from the given start configuration!

In the real plan, in **question 18**, we did the following consecutively:

- hunting from the start configuration
- clicks on (1,3)
- hunting again
- ... *and done*.

c) Explain why the 'and done' was to be expected.

Other lights and combinations of lights work exactly the same way.  
We do first have to try out the eight options in the first row, though!

For the 4×3 grid there are only 8 possible click series in row 1. You have already seen the result of starting with light (1,3) in **question 18**.

## 20. [Exploration]

Start with a null configuration on the grid every time, and immediately begin with 'execute clicks immediately' (do NOT use 'toggle lights individually').

For the eight possible click series hunt to the fourth row from an empty grid, and write down the final configurations in the table (the result for light (1,3) is already entered):

<i>click series in row 1</i>	<i>hunting result in row 4</i>			
no clicks	....			
only (1, 1)	....			
only (1, 2)	....			
only (1, 3)	<table border="1"><tr><td>1</td><td>0</td><td>0</td></tr></table>	1	0	0
1	0	0		
(1, 1) and (1, 2)	....			
(1, 1) and (1, 3)	....			
(1, 2) and (1, 3)	....			
all three lights	....			

Now take a random start configuration on a 4×3 grid for which you want to find the solution (if there is one...). Call that start configuration *A*.

Through hunting you reach a situation with ones only possibly in row 4. With some luck you end up in the null configuration immediately, but that is the exception rather than the rule!

If there are still ones in row 4, you look for a click series, starting from the null configuration, that will also give the same result in row four *A* after hunting as the result from hunting in *A*.

So: In the table, look for the right click series in row 1 on the empty grid that will, after hunting, yield the result of hunting *A*.

## 21. [Exploration]

Show that this strategy of hunting and if necessary hunting again, does indeed work from a null configuration.

The method works for all configurations on a 4×3 grid, and you only have to make the table once to use it for every start configuration on a 4×3 grid.

Which leaves the question...

## 22. [Assignment D]

Can all random start configurations on a 4×3 grid be solved?

Use the table from **question 20** to investigate.

## E: Configurations and powers of two

This Explorations section is fairly general. You will learn nothing about solving specific situations. But you will gain a good insight over the whole of possible configurations on any grid. And about how many start configurations are solvable. Key words for this part are: *Smart counting* and *clever divisions*.

**Warning!** *This part is pretty difficult and fairly theoretical. Only work on this if you still have enough time in the morning. This part is especially relevant if you want to tackle Main assignment D in part TWO.*

### COUNTING the TOTAL number of Configurations

#### 23. [Exploration]

- a) On the 4×3 grid you have a total of 12 lights. Each of these lights can be either on or off. Convince yourself that this means that there are  $2^{12}$  possible configurations.

The 2×6 and 1×12 grids also have  $2^{12}$  configurations. For the number of configurations only the total number of lights is important. Not the shape of the grid! We refer to this number with  $N$ .

- b) What is the number of possible click series on a grid with  $N$  lights?  
We will also count 'click nothing' as a click series.

#### The class of connected configurations

Using a toggle diagram (created through a click series) you can go from one configuration to another configuration. You already know:

- If you can get from configuration  $A$  to configuration  $B$  via toggle diagram  $S$ , then you can also go from  $B$  to  $A$  via  $S$ . In that case we call  $A$  and  $B$  **connected configurations**. Pay attention:  $A$  is also connected to itself, through the toggle diagram with all zeroes (i.e.: do nothing)!

- If you can get from configuration  $A$  to configuration  $B$  via toggle diagram  $S$ , and from  $A$  via another diagram  $T$  to  $C$  as well, you can also go from  $B$  to  $C$  with a click series. Or, put another way: Configurations that are connected to  $A$  are also connected to each other.

#### 24. [Exploration]

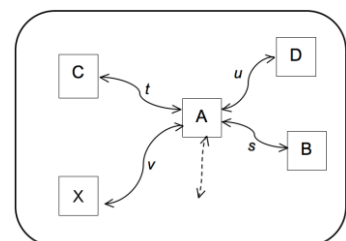
Explain: the number of configurations that is connected to configuration  $A$  is equal to the number of possible toggle diagrams.

Keep in mind: quite often there are fewer toggle diagrams than the number of all possible click series and thus fewer toggle diagrams than possible start configurations.

(If you do not believe this, count the number of possible configurations on a 1×2 grid and the number of possible click series and the number of toggle diagrams.)

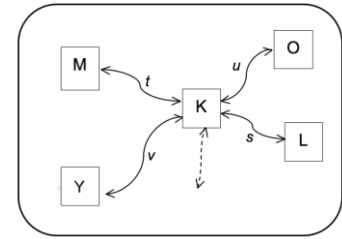
The whole class of configurations that are connected to  $A$  we call **configuration class A**.

In the figure on the right some connections between  $A$  and other configurations and toggle diagrams have been drawn. Of course the figure is not complete, there are probably many more. From  $A$ , for every existing toggle diagram an arrow runs to a configuration that is connected to  $A$ . The configurations within the class are all also connected by toggle diagrams, but that has not been drawn here. The rounded rectangle depicts the boundary of configuration class  $A$ .



### ***The class division of the whole***

If configuration  $K$  is not in the class of configuration  $A$ , then there is an arrow running from  $K$  to every existing toggle diagram for a configuration that is connected to  $K$ . All these configurations together form configuration class  $K$ .



### **25. [Exploration]**

Imagine:  $A$  and  $K$  are not connected, and  $X$  is a configuration in configuration class  $A$  and  $Y$  is a configuration in configuration class  $K$ . Then  $X$  and  $Y$  are not connected to each other.

a) Can you reason that out?

Your explanation might for instance start with the observation:  
'if  $X$  and  $Y$  were connected, then ... and ... would also be ...'

You can say it like this: the configuration classes of  $A$  and  $K$  are completely separate.

b) All solvable start configurations are together in one configuration class! Explain!

The following is true as well of course: If  $B$  is in configuration class  $A$ , it is the case that configuration class  $B$  is exactly the same as configuration class  $A$ .

*Summarised:*

- The set of all configurations is now divided in configuration classes;
- All configurations in one configuration class are mutually connected;
- Configurations from different configuration classes are not connected;
- All configuration classes contain exactly the same number of configurations, namely exactly the same as the number of possible toggle diagrams.

### **26. [Exploration]**

- If you add the numbers of configurations in all classes to each other, the result is  $2^N$ . Why is that?
- Why are all configuration classes the same size? And what is that size (compared to the toggle diagrams)?
- Now explain this equation:

$$(\text{Number of configuration classes}) \times (\text{number of toggle diagrams}) = 2^N.$$

- Why can the numbers for 'number of configuration classes' and for 'number of toggle diagrams' only be of the form  $2^p$  with  $p = 0, 1, 2, 3, \dots$ ?
- Derive and explain (use a previous observation from the Explorations):

$$(\text{Number of configuration classes}) \times (\text{number of solvable configurations}) = 2^N.$$

## Part TWO: Own investigation

In part ONE, we explored various techniques such as *mutual toggling* (by forming groups of lights that can be mutually reached), and *hunting*. Also, a number of terms were used: *toggle diagram*, *parallel click series* and *silent click series*. These techniques and terms will prove their value once you start on the Main assignments below.

Choose wisely: It is better to do some main assignment thoroughly than to do all of them half and globally.

### Main assignment A: $1 \times n$ grids

Given is a  $1 \times n$  grid with a random start configuration. Use the Explorations from part A to find out which start configurations can or cannot be solved on a  $1 \times n$  grid. Try to analyse this for all values of  $n$ .

### Main assignment B: Hunting

'Hunting' and 'silent click series' can be used cleverly in analysing a random start configuration on a  $m \times n$  grid to see whether a given start configuration can be solved.

- Analyse the start configurations on a  $5 \times 5$  grid (the original game) that turn out to be solvable. Make use of all possible final configurations resulting from hunting on the first row from the null configuration.
- Now, can you solve puzzle 3 from the 'warming up' via a targeted click series?

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

- What can be discovered about a  $m \times n$  grid with a random start configuration? Analyse the  $m \times 3$  grids for example.

### Main assignment C: Everything on, everything off

It is said that on every  $m \times n$  grid that is completely filled with ones, a click series can be found that leads to the null configuration. We already investigated this for  $1 \times n$  grids in **question 8**.

- Investigate the solvability on a  $2 \times n$  grid.
- Investigate a few more cases, such as an  $n \times n$  grid for  $n = 2, 3$  and  $4$ .

## Main assignment D: configuration classes and click series

This main assignment refers back to the explorations in part ONE, section E. If you skipped that part of ONE due to lack of time, there is no point in trying this main assignment!

A nice proposition:

**Number of configuration classes = number of silent click series**

- a) Investigate this proposition.  
*Hint:* Divide the click series in groups of parallel click series.
- b) Some more fun research questions:

### **1. The number of solvable start configurations on a $m \times n$ grid.**

In that case we have  $N = m \times n$ . Base yourself on  $m \geq n$ .

If you are hunting with a given start configuration, you end up with a final configuration with only zeroes in row 1, 2, up to  $m-1$  and possible also ones in row  $m$ .

Two givens that you can make use of (do of course investigate their correctness):

- A start configuration and its final configuration are in the same configuration class.
  - There is a maximum of  $2^n$  different final configurations after hunting, so there is also a maximum of  $2^n$  configuration classes.
- Show that, in the case of a  $m \times n$  grid with  $m \geq n$ , there are always at least  $2^{(m-1) \cdot n}$  solvable configurations.

### **2. On a $5 \times 5$ grid only 25% of all start configurations is solvable**

→ Investigate this proposition.

*Hint:* Use what you found in Main assignment B and in the Explorations of part ONE, section E.