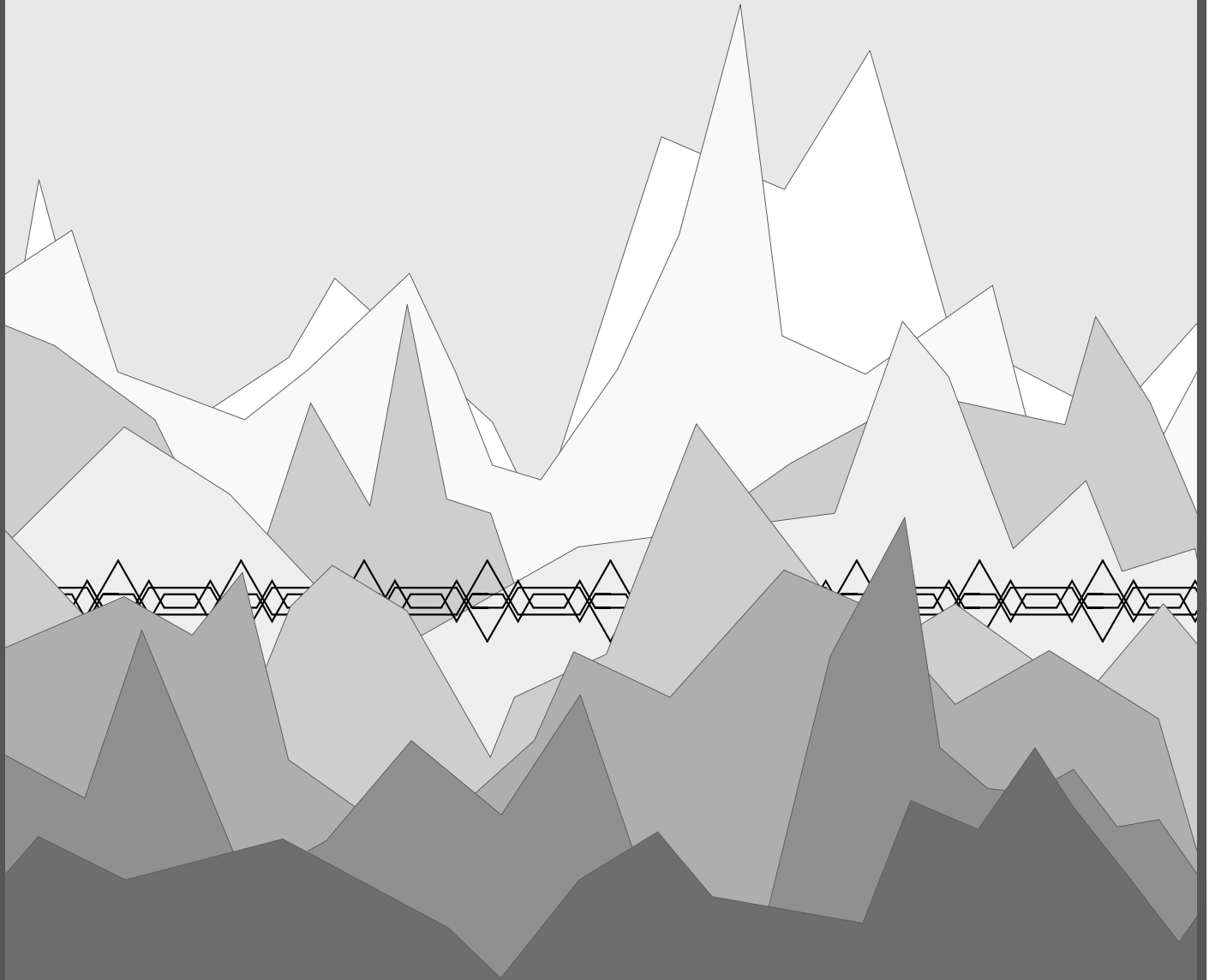


Zigzag
and the
anti-periodic



Mathematics B-day
Friday 19 november 2010

Read this before starting to work

The Mathematics B-day assignment 2010 is about piecewise linear functions and their graphs that will be studied mainly with help of the computer. Here is a short description of the 5 parts of the assignment.

Part A: Introduction

In part A you will get a clear understanding of the basics of the piecewise linear functions, in particular the functions *zig* and *zag*. You are also introduced to the use of the software package *Geogebra*. A *Geogebra* manual is added as an appendix. All tasks in part A are called 'exercise', because they are meant to become familiar with drawing of and reasoning about the functions *zig* and *zag* with use of the computer. These exercises are not worth to be included in your final report.

Part B: Kink design: symmetric and anti-periodic patterns

Part B is about infinite continuous patterns with specific characteristics. It is not only about drawing with the computer, but you also have to *explain* what you discover with the computer. Questions like that are called *Reasoning- and Design questions (R&D)*. These R&D-questions will be part of your final report. Your own creativity should be used in *Own Production-questions*.

Part C: One-Peak functions

In part B you designed anti-periodic patterns. In part C you are asked to prove that every anti-periodic pattern can be designed only using the basic function *zig*. The R&D-questions will be described in your final report.

Part D: 2-dimensional zigzagging with speed and animation

After the theoretical work in part C, you will here learn three new elements that will increase your zigzag creativity. These elements will be very helpful when working in part E on the final tasks.

Part E: Final task

Design a static or dynamic (which means: moving) pattern with use of all techniques you found in parts A through D.

Like with the Own Production-questions, pictures with explanations are part of the final report and you hand in digital products as .ggb files.

Rules to follow for the Mathematics B-day 2010

What do you do, and when?

Make sure that most of Parts A-D are worked through before 1pm. In that case you should have enough time to work on the Own Production-questions of parts B and C and the final task of part E, and for the final report.

What must be handed in on paper?

Elaborations, together with explanations and pictures of all R&D-questions, all Own Production-questions and the Final task are in the final paper report that you have to hand in. It would be nice if these elaborations are included in a coherent text that can be read by a person who does not know the assignment. He/she should be able to enjoy reading your text and look at your illustrations and at the same time understand mathematically what you discovered this day.

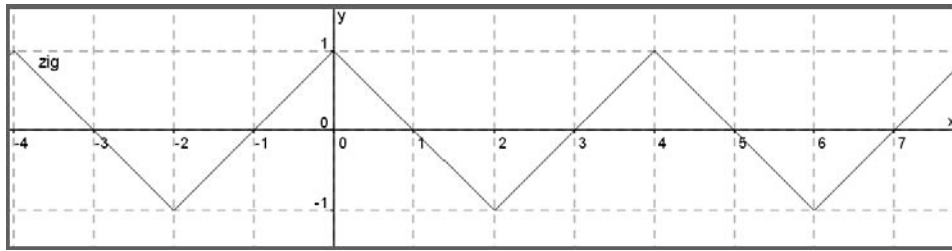
How to deliver the digital files?

Don't hand in too many files!! Select the-best-of and limit it up to six files. To avoid national chaos, there is a strict rule for coding your files. Your school has a school number for the math B-day and your team has a number. These numbers appear in the coding of the files you hand in, followed by a 'real' title. So, if you are a student of school 3071 and are a member of team 23 and you designed a *snake* in *Geogebra*, that file will be coded as: **3071_23_snake.ggb**. Don't use other ways of coding and please don't make any mistakes with that. In the paper report you use these codes for the pictures as a reference to the file and explain how they work.

Part A: Introduction

The zig-function is a periodic kink function

This is the graph of the function $\text{zig}(x)$.



From the picture it is clear what the characteristics of this function are:

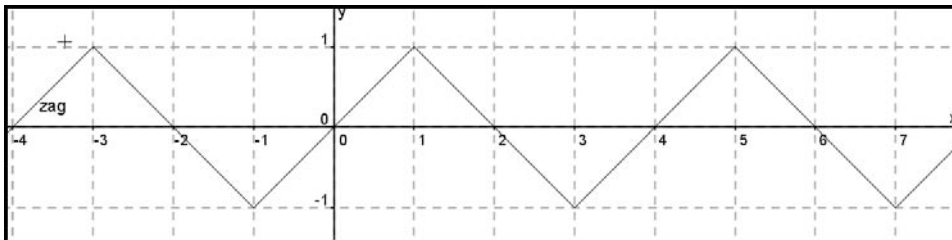
- $\text{zig}(0) = 1$
- the graph of zig moves up and down (with fixed slope 1 or -1) between the heights 1 and -1
- the graph repeats itself: zig has period 4. The points of the graph in which the slope changes from 1 to -1 or from -1 to 1 are called the *kink points*. In the case of zig , the kink points are $(0, 1)$ and $(2, -1)$. The periodic behaviour of the graph assures that all points $(4k, 1)$ and $(4k + 2, -1)$, with k any integer number, are kink points too.

Exercise 1

- Find the value of $\text{zig}(x)$ for $x = 1$, for $x = 2006$, for $x = 0.7$ and for $x = -511.3$.
- For what values of x in the interval $[0, 8[$ has $\text{zig}(x)$ the value $3/4$? (See foot note¹)
- Find the kink points of $2 \cdot \text{zig}(x)$ and of $0.5 \cdot \text{zig}(x - 1)$.

The family of zig and zag

This is the graph of zag , the brother of zig .



Exercise 2

- Find the value of $\text{zag}(x)$ for $x = 2$, for $x = 2007$, for $x = 1.7$ and for $x = -510.3$
- Find the kink points of zag .

The zig -function is an example of a *periodic kink function*.

The general characteristics of a periodic kink function are:

- the function is periodic
- the graph is a sequence of straight line segments with different slopes that are connected head to tail. The points in which they are connected are called the *kink points*.

The slopes of the line segments of such a kink function don't necessarily need to be 1 and -1. And positive and negative slopes don't need to alternate, like with zig and zag .

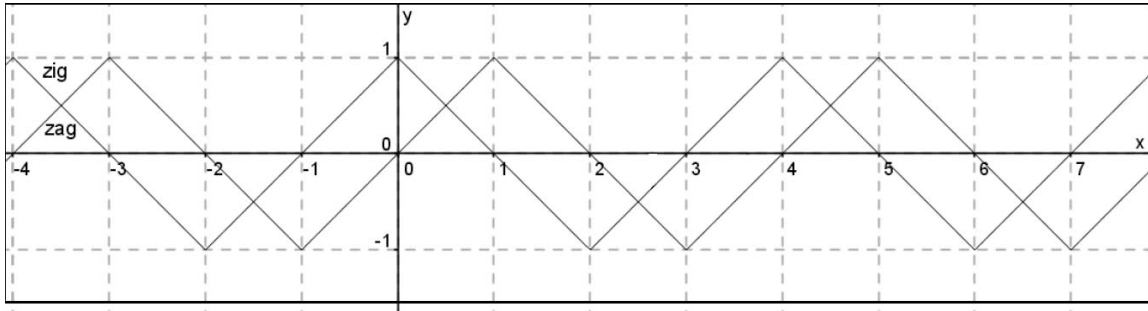
¹ The notation $[0, 8[$ stands for: the number 0, all numbers between 0 and 8, but NOT 8 itself.

Exercise 3

- Sketch the graph of a periodic kink function that has period 5 and that has the points (1, 0), (2, 3) and (4, -1) as the only kink points on the interval $[0, 5>$ (the slopes of the graph don't need to be 1 and -1).
- Is (1001, -1) a kink point of this function?

Translation

In the figure you see the graphs of *zig* and *zag* together.

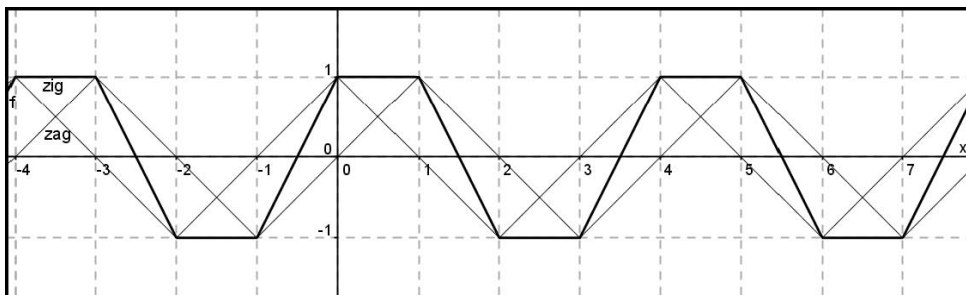


Exercise 4

- In the same figure, add the graph of $\text{zig}(x - 2)$. You get that graph by translating the graph of $\text{zig}(x)$ over the distance 2 to the right.
- Also add the graph of $\text{zig}(x - 3)$. Why do you get the same result when you add $\text{zig}(x + 1)$ instead of $\text{zig}(x - 3)$?
- Explain, with good argumentation, why $\text{zag}(x) = \text{zig}(x - 1)$

Adding, subtracting, multiplying functions.

There are more options for operations on graphs and functions than translation (exercise 4). You can also add functions. Drawn in one figure you see $\text{zig}(x)$, $\text{zag}(x)$ and the sum-function $\text{zig}(x) + \text{zag}(x)$. The graph of the sum function is drawn a little more thick than the other two.



Exercise 5

- Find out whether the graph of the sum function is drawn the right way by checking the addition $\text{zig}(x) + \text{zag}(x)$ on the interval $[0, 4>$ in the kink points of $\text{zig}(x)$ and $\text{zag}(x)$. Why is it decisive to check the values in the kink points of $\text{zig}(x)$ and $\text{zag}(x)$ and why you don't you have to check all inbetween points of the graph?
- Sketch the graph of $\text{zig}(x) + \text{zig}(x - 1/2)$. If needed you can use a table with the information you need: the kink points of $\text{zig}(x)$ and the kink points of $\text{zig}(x - 1/2)$.
- Why is this true for all values of x : $\text{zig}(x) + \text{zig}(x - 1) + \text{zig}(x - 2) + \text{zig}(x - 3) = 0$.

Further investigations with help of the computer: Geogebra

By drawing, calculating and reasoning you have developed some basic insight in the function $zig(x)$, the translated family members $zig(x - a)$ and the connections between them. From now on we will start drawing these graphs with the software package Geogebra.

But before we start with that, the exercises on this page are meant to get familiar with how to use Geogebra for that goal.

Exercise 6

Geogebra allows an easy way to draw graphs.

- a. Start Geogebra; see the manual if needed.
- b. Put the cursor in the *Input field* at the bottom of the screen.
Type $f(x) = 2 - x/2$ and finish with Enter.
Check:
 - that the graph is drawn correctly,
 - that the formula for f also appears on the left side of the screen: the Algebra View,
 - that you can *hide* the graph by clicking the small dot left to the formula and make the graph *appear* by clicking again.
- c. Type in the input field: $g(x) = f(x) + f(x + 2)$. As you can see a new graph is drawn that uses the already defined function $f(x)$. The function f is called a 'free object' and g a 'dependent object'
- d. When you right-click the mouse on f or g (in the Algebra View or on the graph in the Graphing View) you see a menu. When you choose *Object properties* it is possible to give the graph a different color and to change the weight of the line thickness. There are also other features that can be activated. Find out how that works.
Also find out how you can use the page *Basic* to change the formula for f from $f(x) = 2 - x/2$ to $f(x) = 3 - x/3$. Does g also change?
- e. A final introductory activity:
 - Move the Graphics View with SHIFT + dragging the mouse.
 - Zoom In and Zoom Out with the scroll wheel of the mouse.Or use the most right button at the top of the screen. If needed: see the manual.
- f. Delete f and g . That is different from *hide*! When you delete f first, g will be deleted automatically. Of course, because g depends on f .

Important in Geogebra: Names of formulas

When you type the formulas the way it was shown, so like

$$f(x) = 2 - x/2 \quad \text{and not like} \quad y = 2 - x/2$$

it is okay. In that case you can invoke the formula with its name f and do things like

$$g(x) = f(x) + f(x + 2).$$

You are free to choose your own names, but these are the rules to obey:

- always start the name with a letter; after that you can use numerals
- no spaces in the name. Geogebra gives 'error' and may start strange behaviour!

This was a short introduction on the use of Geogebra.

We now start to use Geogebra for zig and his family.

Zig and Zag in Geogebra

Exercise 7

There are two options to create the functions $zig(x)$ and $zag(x)$ with Geogebra.

ONE: Open in Geogebra the file *zigzag.ggb*. [You will be told by your teacher where the file can be found]. You see zig and zag on the screen and the formulas (if the formulas are not visible, select Algebra View under View in the menubar).

TWO: Very carefully type the following functions in the Input field:

$$zig(x) = abs(x - 4 * floor(x/4) - 2) - 1$$

$$zag(x) = zig(x - 1)$$

Exercise 8

Exercising with zig and zag .

- Type $sum(x) = zig(x) + zag(x)$ and check the graphs of exercise 5.
- Predict the kink points of the graph of $h(x) = zig(x) - 1/2 zig(2x)$ and check your prediction with Geogebra. Keep this function h for future use!

Exercise 9

(You may skip this exercise because the long formula for zig is not needed for the remaining questions of this assignment. Nevertheless, if you study the structure of that formula, you learn a technique that may be helpful in other situations ...)

Use the following stepwise instruction to discover what happens in the long formula of zig .

$$zig(x) = abs(x - 4 * floor(x / 4) - 2) - 1$$

If you go through this formula step by step, building it up from the inside, the stepwise change will show why the graph of zig has this typical shape.

- Start with $floor(x)$ as input
 $floor(x)$ is the biggest whole number that does not exceed x . That is why you get all these horizontal line segments. Because: $floor(6.5) = floor(6) = 6$
- Stretch the graph horizontally with factor 4 in the x - direction:
 $floor(x/4)$
- Stretch the graph vertically with factor 4 in the y - direction:
 $4 * floor(x/4)$
- The difference with the graph of $y = x$ continuously grows from 0 to 4:
 $x - 4 * floor(x/4)$

Explain yourself what happens to the graph at steps 5, 6 and 7:

- $x - 4 * floor(x/4) - 2$
- $abs(x - 4 * floor(x/4) - 2)$
- $abs(x - 4 * floor(x/4) - 2) - 1$

Part B Kink design: symmetric and anti-periodic patterns

***A thing of beauty is a joy for ever*²**

For sure! And certainly when it is about a nice pattern that continues endlessly, like with the periodic formulas of this day. You are allowed to object that it's up to the viewer to decide about the beauty: *Beauty is in the eye of the beholder*³. But there is no doubt about the two kink graphs of the first R&D question that you are asked to complete to a regular pattern of four graphs with symmetries. These questions - elaborated and explained- must be included in your final report!

R&D-question 1. A consistent pattern of 4 functions, part 1

- a. In an empty screen, show the graph again of
$$h(x) = \text{zig}(x) - 1/2 \text{zig}(2x)$$
and also draw the graph of the related formula
$$j(x) = \text{zig}(x) - 2 \text{zig}(x/2)$$
- b. Prove *algebraically*, that the two formulas are nicely related in the following way:
$$j(x) = -2 h(x/2)$$

A hint for question b, about 'Prove algebraically'

In fact you are asked to show that the two formulas $j(x)$ and $-2 h(x/2)$ are actually the same. Start the proof with the more complex formula and replace x , in all positions where you find an x , by $x/2$:

$$2 h(x/2) = -2 (\text{zig}(x/2) - 1/2 \text{zig}(2x/2)) =$$

Elaborate, simplify, etc. Do you end up with the formula of $j(x)$?

- c. On aesthetic⁴ grounds, decide which one of the two following formulas fits best the pattern of h and j :
- $$k(x) = 3/2 - h(x) \quad \text{or} \quad k(x) = 5/2 - h(x)$$
- d. Add a fourth function to the three you already have, to complete a nice pattern of four graphs that has a horizontal axis of symmetry.

Note: parts of the Graphics View screen can be copied to your report this way:

- Use ESC or click the button 'Move'
- By dragging a grey rectangle, select an area in the Graphic View that you want to copy
- In the menu **Edit**, choose the option **Graphics View to Clipboard**
- Go back to your report (MsWord or any other program): **Paste!**

Optimising Beauty with a Slider

Is $k(x) = 3/2 - h(x)$ the better choice, or $k(x) = 5/2 - h(x)$? Who knows...

Maybe there is a better one of the form $k(x) = a - h(x)$ for a still unknown value of a .

Geogebra can be a very efficient help when you want to find the aesthetically best value of a . This is how you do it:

Step 1: Create a slider with which you can make a move easily through a range of values.

Step 2: Use the adapted formula $k(x) = a - h(x)$ instead of the one you used before. You also need to adapt the fourth formula (in R&D question 1d) in order to keep the symmetry.

Step 2 is obvious, but Geogebra needs Step 1 first, because otherwise a is not known to Geogebra.

² John Keats, in the narrative poem Endymion, 1818.

³ Margaret Wolfe Hungerford, in the novel Molly Bawn, 1878.

⁴ Aesthetics is the science of taste, style and beauty

Design the slider this way:

- Type in the Input Field: $a = 1$.
- a is now in the Algebra View. Make a **visible** by clicking the small dot left to a . The slider appears in the Graphics View.
- Later on (or now), you can change the properties of this slider by right-clicking the mouse and choose for Object Properties.

R&D-question 2. A consistent pattern of 4 functions, part 2

- Design the slider and adapt the formulas for h and the fourth function.
Be aware: if you want to change formulas, you first need to use ESC, or click the button Move. See manual part 4 and 5.
- Use the ruler to find some aesthetically attractive pictures.
- The graph of $k(x) = a - h(x)$ is, as you must have noticed, the mirror image of $h(x)$ in a horizontal line. What is the equation (formula) of that horizontal line?

Own Production-question 1. Design patterns with several graphs

It is allowed to use more sliders or to use a slider in different positions within one formula. With these sliders you can compose nice and sometimes unexpected patterns, using the function zig , translations of zig and variations like $\text{zig}(2x)$, $\text{zig}(x/3)$.

Design two different attractive patterns with or without horizontal symmetry.

Symmetrical patterns, symmetric functions

We looked at horizontal symmetry. Mirror symmetry of one function in the vertical line $x = 0$ was all the time present. The graph of $\text{zig}(x)$ is symmetric in that vertical mirror line. The next R&D-question is about reflecting in vertical lines, like in $x = 0$.

R&D-question 3. Finding Mirror symmetry

The mirror symmetry of function zig in the vertical line $x = 0$ can be described this way:

$$\text{zig}(-x) = \text{zig}(x)$$

This is an algebraic notation that simply states that zig has the same value for two values x and $-x$ on the horizontal axis that are located symmetrically compared to $x = 0$.

- Why is the function $\text{zig}(x - 1/2)$ **not** mirror-symmetrical in the vertical line $x = 0$?
- Prove that the function $\text{zig}(x + 1/2) + \text{zig}(x - 1/2)$ is mirror-symmetrical in line $x = 0$.
- The value $1/2$ is just an example.

For all values of a in $[0, 4>$, investigate symmetry in vertical lines of the functions $\text{zig}(x - a)$ and $\text{zig}(x + a) + \text{zig}(x - a)$. Where are the axes of symmetry located?

R&D-question 4. Mirror symmetry, different axes

The function $zag(x)$ is not symmetric in the line $x = 0$, but it is in the vertical line $x = 1$. This can be formulated algebraically by describing that the value of zag is the same for two values on the horizontal axis that are located symmetrically compared to $x = 1$:

$$zag(1 - x) = zag(1 + x)$$

- Prove algebraically: $zag(1 - x) = zag(1 + x)$, using the formula of zag .
- What are the axes of symmetry of the functions h en j in R&D-question 1?
- Suppose: the functions $n(x)$ and $m(x)$ both have axis of symmetry $x = b$. Prove algebraically that in that case $3 \cdot n(x) - 7 \cdot m(x)$ also has an axis of symmetry in $x = b$.
- A formula such as $3 \cdot n(x) - 7 \cdot m(x)$ is called a linear combination of the functions n and m . Formulate a general proposition (question c can be part of it) about the symmetry of linear combinations.
Hint: your proposition might start like this:

If the functions ..., ... are symmetric in $x = c$,

Anti-symmetry

The function zag is not symmetrical in the vertical line $x = 0$, because $zag(-x)$ and $zag(x)$ don't have the same value. But this is true:

$$zag(-x) = -zag(x)$$

Functions with this characteristic are called **anti-symmetric** in the line $x = 0$.

Function zig is anti-symmetric in $x = 1$, because:

$$zig(1 - x) = -zig(1 + x)$$

Exercise 10

- Are there other vertical lines, different from $x = 1$, in which zig is anti-symmetric?
- Is there maybe a proposition, like the one about symmetry of R&D-question 4, about anti-symmetry and linear combinations?

Periodicity

Most of this day we look at functions with period 4.

A function f has period 4 if the following is true for all values of x :

$$f(x + 4) = f(x)$$

Such a function automatically has period 8, or 12 or ...

The proof goes like:

$$f(x + 8) = f(x + 4 + 4) = f(x + 4) = f(x)$$

R&D-question 5. Periodic and (anti)symmetric

- Prove algebraically: If f has period 4 and an axis of symmetry is $x = a$, then f also has an axis of symmetry $x = a + 4$.
- Prove, rectify or give a counterexample:
 - linear combinations of functions with period 4 also have period 4 .
 - If f has period 4 with axis of symmetry $x = a$, then f also has an axis of symmetry $x = 4 - a$.

Finally: Anti-periodic functions

This holds for both *zig* and *zag*: in the second half of the interval $[0, 4>$ the values of the function are opposite to the values they have in the first half of the interval. Formulated algebraically it is about comparing the values belonging to $x + 2$ en x :

$$\text{zig}(x + 2) = -\text{zig}(x)$$

$$\text{zag}(x + 2) = -\text{zag}(x)$$

Functions having this characteristic are called **anti-periodic with period 2**.

And again: linear combinations of functions that ... Etc.

R&D-question 6. Inventing examples

Draw graphs of functions with period 4. Design three different ones:

- one that IS anti-periodic with period 2 and IS also anti-symmetric in $x = 2$
- one that IS NOT anti-periodic with period 2 and that IS anti-symmetric in $x = 2$
- one that IS anti-periodic with period 2 and IS NOT anti-symmetric in $x = 2$.

R&D-question 7. Anti-periodic, and thus also periodic!

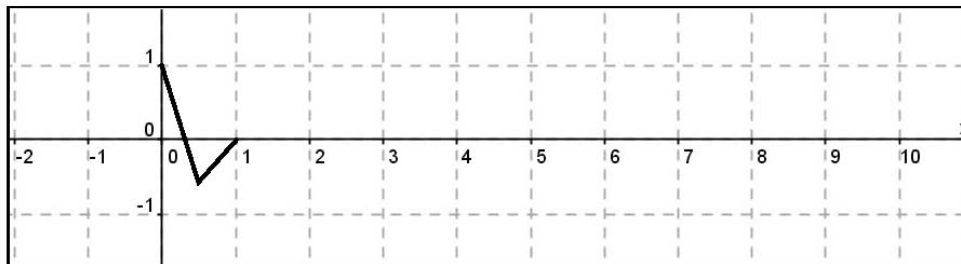
Prove algebraically:

a function that is anti-periodic with period 2, is automatically periodic with period 4.

R&D-question 8. Completing the graph

The two connected line segments you see in the figure below is part of the graph of a function that is anti-periodic with period 2 and also symmetric in $x = 0$.

Complete the graph.

**Summary**

You have been introduced to the four characteristics (anti)symmetry and (anti)periodicity of functions and graphs. You have also seen that if the functions $m(x)$ and $n(x)$ has one of these characteristics with a certain axis or period, linear combinations of $m(x)$ and $n(x)$ also have that characteristic.

Own Production-question 2. Patterns with anti-characteristics

Design two different interesting patterns with period 4

- composed of one or more graphs of kink functions
- containing graphs with diverse (anti)symmetries in $x = 0$, $x = 1$, $x = 2$ and $x = 3$
- and the shape of the pattern can be influenced through the use of sliders.

In the paper report you show the results of different variations (different slider values) as pictures. Also choose one Geogebra-file to add to the final report.

Part C: One-Peak functions

In part B you constructed anti-periodic patterns by using all kinds of combinations of *zig*-functions and their transformed family members. In this part you will show that you can – in a certain sense – construct any anti-periodic pattern from these basic functions. The corresponding *Reasoning- and Design questions* will of course be included in your final report.

First we will therefore construct anti-periodic functions that have the value 1 in one kink point, in another kink point (necessarily) the value -1, but that will have value 0 everywhere else: *one-peak functions*. Later we will combine this type of functions.

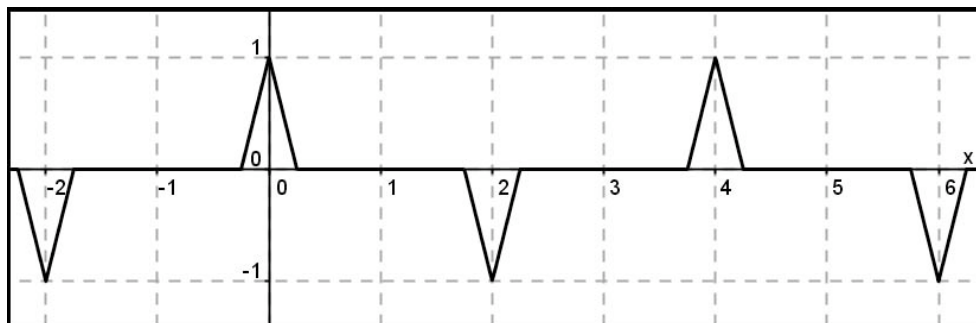
The construction starts from the symmetrical function $zig(x - a) + zig(x + a)$.

R&D-question 9. Making the slopes equal to 0

- Enter this function in Geogebra in the Input field: $azi(x) = zig(x - a) + zig(x + a)$
- We will use this function for fairly small values of a . Investigate what happens to the graph when a goes from 0 to 1.
- The sloping sides of this function are parts of the sides of $2\ zig(x)$. Or: when we take the difference of the function $2\ zig(x)$ and the function from part **a**, that difference should be equal to zero for most values of x , but not in the neighbourhood of $x = 0$, $x = 2$ and $x = 4$. Give it a try in Geogebra.
- Now give a reasoning to explain that the formula

$$top(x) = 1/(2a) (2\ zig(x) - zig(x+a) - zig(x-a))$$

for the value $a = 1/4$ has this graph. Test it with Geogebra!



- Also draw the graphs for the values $a = 0.1$ and 1 .
- Explain:
 - the function $top(x)$ is anti-periodic (with period 2)
 - the width of the foot of the mountain, the distance between the beginning and the end of the mountain, is $2a$.
 - the top value is 1 (for every value $a > 0$).
 - Add the functions $top(x - a)$ and $top(x - 2a)$. For these two functions the top of the graph is exactly above one foot of the mountain of $top(x)$.

Constructing functions with given kink points

When you have an anti-periodic kink function with period 4 and you know the kink points in the interval $[0, 2>$, then you know everything about the function. Because the anti-periodicity determines how the function behaves at $[2, 4>$ and the periodicity tells you the rest as well. Now for the first example think of five kink points in $[0, 2>$, equally distributed as follows:

$$0 \qquad 1 \cdot 4/10 \qquad 2 \cdot 4/10 \qquad 3 \cdot 4/10 \qquad 4 \cdot 4/10$$

We will create a function that, for these values of x , has the following values

$$4 \qquad 3.5 \qquad 2 \qquad -1 \qquad -2$$

The plan: choose the right *one-peak function* for each of the kink points and add. Select for the a -parameter in $\text{top}(x)$: $a = 4/10$. With that you divide $[0, 4]$ into exactly 10 equal intervals. The foot of the one-peak function with top in 0 will be exactly on the second kink.

So $4 \cdot \text{top}(x)$ has the value 4 in the first kink point and in the other kink points value 0:

$$4 \cdot \text{top}(x): \qquad 4 \qquad 0 \qquad 0 \qquad 0 \qquad 0$$

To get the value 3.5 correct on the second kink point, we use $3.5 \cdot \text{top}(x - 4/10)$:

$$3.5 \cdot \text{top}(x - 4/10): \qquad 0 \qquad 3.5 \qquad 0 \qquad 0 \qquad 0$$

The function $4 \cdot \text{top}(x) + 3.5 \cdot \text{top}(x - 4/10)$ already hits the first two kink points, and has zero as value for the other kink points!

R&D-question 10. Finish the example

- Write down the function that has the five given kink points.
- Test your result with Geogebra.

We call the function you just made a *five-point function* because you have defined 5 kink points on the interval $[0, 2>$. So for a whole period of 4 there are 10 kink points. It should be evident that the method will also work if there are 10, or 17, or – in general – n kink points in the interval $[0, 2>$

- What value of a should you choose in those situations? Give a formula in n that yields the correct value for a when you have n kink points in the interval $[0, 2>$.

Anti-periodic functions with n kink points in $[0, 2>$ are called **n -point functions**. We will make a number of n -point functions with Geogebra. Take care: Geogebra will of course use one value for a for everything that it knows. So if you say that $a = 4/12 = 1/3$ to work with 6-point functions, the 5-point functions you made earlier will go wrong. There is a simple solution: make a new file for your 6-point functions.

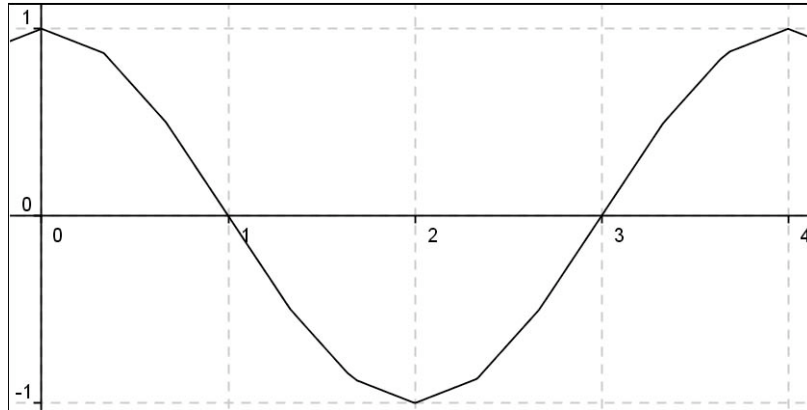
SAFELY do this as follows:

- Save the work you made under a name you choose yourself.
- Save a copy of the work under a different name through **Save as** (under File) and use a name with a 6 in it, for example
`schoolnr_teamnr_Top6functions.ggb`
- Hide what you no longer want to see but may still need. You can also – carefully – use 'Delete'.

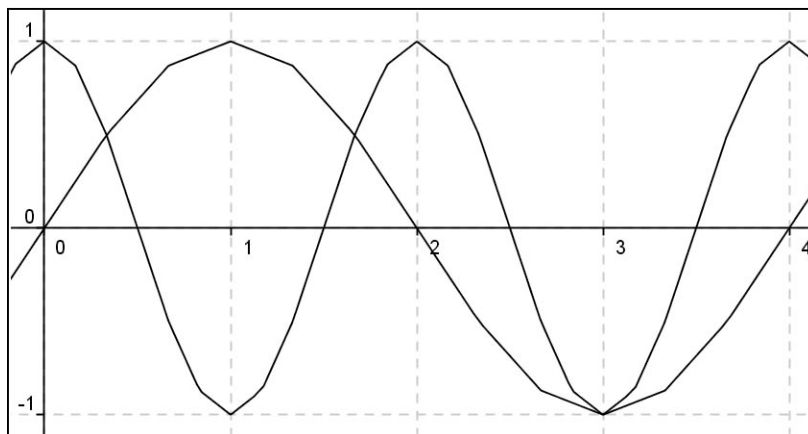
You are going now to construct a 6-point function for which the graph looks a bit like that of the function $\cos(x)$. That function is called **wave6A(x)**. The function has period 4 and is symmetrical in $x = 0$ and in $x = 2$ and anti-symmetrical in $x = 1$ and in $x = 3$. In addition, $(0, 1)$, $(1/3, 0.87)$ and $(2/3, 0.5)$ are the three kink points in $[0, 1]$

R&D-question 11. Wave functions

- Enter $a = 1/3$.
- Design $\text{wave6A}(x)$, as shown below, as economically as possible.



- Also design $\text{wave6B}(x)$ and $\text{doublewave6}(x)$, using transformations of wave6A .



Own production-question 3. Patterns with n-functions

Design two different attractive patterns with period 4 using the resources in part **B** and **C**.

Part D: 2-dimensional zigzagging with speed and animation

You will learn three new elements here that will significantly increase your creativity (in the field of zigzagging). A single example will include many possibilities. These will be useful in Part E, the final part of the assignment.

New element I: Speed

$\text{zig}(x)$ goes up and down exactly once in the interval $[0, 4>$. If we now look at $\text{zig}(3x)$, also with x from 0 to 4, $3x$ will run through three periods: from 0 to 4, from 4 to 8, from 8 to 12. So that is up and down three times in the interval $[0, 4>$. Instead of 3 you can use other speed factors; and you can combine speeds. You have used this speed variable before. An exciting example will follow.

New element II: Animation

You made slider buttons before. If you right click on a slider button, you will see the option 'Animation'. Geogebra itself will move the number you selected. A cool option to use!

R&D-question 12. Speed differences, decrease and increase in animation

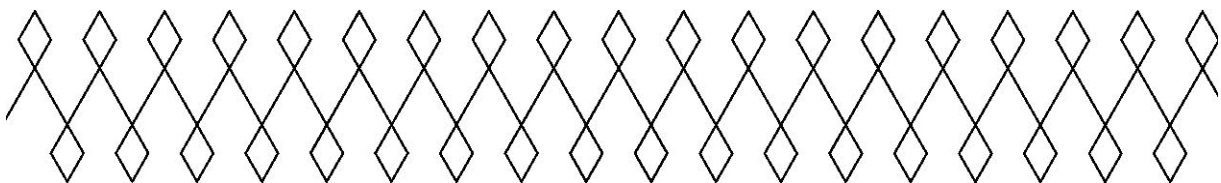
- Enter the function $\text{zw}(x) = \text{zig}(x) + \text{zig}(1.1 x)$.
If necessary, zoom out until you can see a range of at least 0 -100. Because that flattens the graph too much, vertically multiply the whole by a factor 5 or 6.
- Explain the periodic decrement and increment of the graph. What is the total period?
- Now the function family $\text{zig}(x) + \text{zig}(c x)$ is the obvious choice, with c running in small steps from 1 to for example 3. Make this and *animate* it. It looks really good. For what value of c does the function have a period of 400?

New element III: Getting a grip on the horizontal movement

For all previous patterns we used graphs. You can imagine graphs on the computer as being made by a moving point that leaves a trail. The point makes two simultaneous movements:

- A regularly continuing movement in the x -direction from left to right.
- An (up and down) movement in the y -direction, determined by the formula.

You will never be able to make this pattern with that:



To be able to do that after all, Geogebra will need to determine the x -direction of the moving point through a formula that also allows the values of x to move in the other direction. Input of both the horizontal and the vertical movement, is possible with the Command *Curve* (or in Dutch *Kromme*)

Example

Enter in Geogebra (mind the square brackets!)

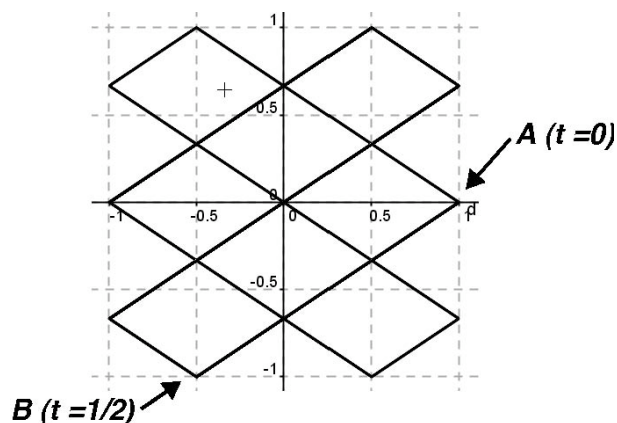
Curve [zig(3t), zig(2t + 1), t, 0, 4]

(English version)

Kromme[zig(3t), zig(2t + 1), t, 0, 4]

(Dutch version)

You will see the following figure:



Imagine that t is time running. Geogebra will now, for all values of t between 0 and 4, draw the points with x-coordinate $\text{zig}(3t)$ and y-coordinate $\text{zig}(2t + 1)$. If you want to do this by hand you would have to put all kink points in a table with x-coordinate $\text{zig}(3t)$ and y-coordinate $\text{zig}(2t + 1)$:

t	x-coordinate: $\text{zig}(3t)$	y-coordinate: $\text{zig}(2t + 1)$	letter in figure
0	1	0	A
1/2	-1	-1	B
2/3	-1	-2/3	
1			
.....			
.....			

Why the t in the curve command?

Geogebra is willing to use other names than t , but you have to tell who is running. You can do that in the spot in the command *after* the formulas. Later you will use extra parameters, and it would not be clear if you didn't indicate this in the command.

R&D-question 13.

- Complete the table; also mark the points C, D etc. in the figure.
- How many kink points are there in total?

Describing the trajectory of a point by using two functions, $x(t)$ and $y(t)$, is called the *parametric curves* method.

To make you aware of the many possibilities – before you start the final question – here you will find some suggestions.

R&D-question 14. Trying out possibilities

For this question you will fall back on functions you may have made before!

Make sliders p , q , r , s (select suitable ranges for the sliders yourself), enter, and look at the many possibilities you already have. Capture some images of the following curves.

- a. `Curve[zig(p t), zag(q t + s), t, 0, 20]`
- b. `Curve[t + r zig(p t), r zag(q t), t, 0, 20]` (when r is not too small, x will now sometimes run backward!)
- c. `Curve[t + r wave6A(p t), r wave6B(q t), t, 0, 20]`

R&D-question 15. Parametric curves and anti-periodic functions

The functions that were used for the x -coordinate in part 14b and c are not anti-periodic. If you limit yourself to anti-periodic functions for both the x - and the y -coordinate, you will get figures with a special symmetry.

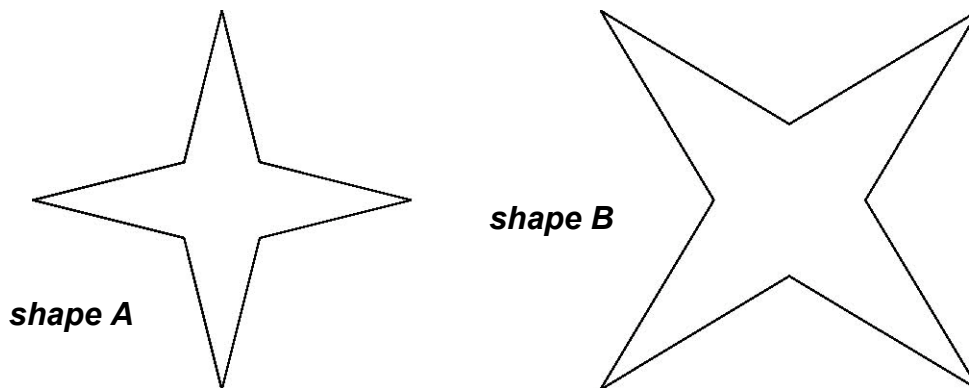
- a. What symmetry is that?
- b. Prove it!

Part E: Final task

For the **Final task** you will make some prescribed figures, and some you will make up yourself, both *using techniques from the previous parts*.

Final task A

Make a curve that continuously shifts from shape A to shape B and back again.



Final task B

It follows from the theory that, using only zig functions, you can easily draw a letter from the alphabet using n -point functions and the command Curve, but that this letter will be upside down in the part of the plane with negative coordinates.

Make the letter A, and optionally other letters of the alphabet as well, in that way.

Final task C

Make a static or dynamic (i.e. moving) pattern from your own design using all techniques from the previous parts.

Just like the creative assignments earlier, there will be *illustrations with explanations* in your work, and you will hand in your moving products digitally as well as .ggb-file.