Crossing a desert in a jeep

Imagine:

You want to cross a big desert in a Jeep. There is an endless supply of water, food and gas at your starting point, but across the desert there is absolutely nothing. Your Jeep is not big enough to take enough supplies with you in order to cross the desert in one time, not enough by far!

What to do? Can you make it?

Introduction to the problem

On the front page you find a short summary of the problem you will be working on today: transporting supplies you need during your journey. We will only consider the problem of the needed quantity of gas; we disregard food and water.
You are not able to drive through the whole desert on just one full tank. So, you will have to construct gas storage rooms in one or more locations along the way. In the end you want to keep the costs as low as possible; the total quantity of gas used, must be as small as possible.

During this activity you will be working on this problem by investigating which way to cross the desert is the cheapest and whether you are able to cross any arbitrary distance.
A good way of working would be to solve a few simple cases first, next to think about whether you can cross every distance and finally to design a general strategy for crossing a big desert.
For that reason, the problem is split up in three parts:

- Part A: two smaller problems that you are asked to solve explicitly,
- Part B: a theoretical investigation to show that you can reach any distance,
- Part C: designing a strategy and apply it to a very big distance.

Restricting the problem

You are standing at the border of a desert with your Jeep. There are no gas stations in the desert. On your destination there will be enough gas present. So you need to transport everything you need on your trip from the starting point. We will make a few assumptions, which will be valid while investigating the problem.

- For each driven distance, the Jeep uses a fixed amount of gas, despite the load of the jeep or the quality of the road.
- The Jeep will not break down.
- The Jeep has only one tank. The gasolin in the tank is used for driving and part of the remaining gasolin will be used to build up the storage.
- You will be driving along one road only.
- You can construct storage rooms in every place along the road. How these rooms are built is not important for us.

Such assumptions are necessary to keep the problem manageable. There are also some options, for example: do you place the storage rooms on equal distances from each other or do you allow the distances to vary. It is very likely that during this day you will make some assumptions or choices. Make sure that you state those clearly and explain why you make them.

Study guide

Final assignment

The purpose is that you hand in a report in which the results of the different problems are presented as a coherent piece of work.
First describe clearly for which type of problem you made a model. Clarify this by using examples. Also explain why making a model in the first place is convenient.
Make sure that it is clear to the reader which assumptions you made.
Show that your strategy gives 'good' solutions. If the occasion arises, compare your model with earlier made models which were less good, to strengthen the good points of the definite model.
Do not be afraid to state possibilities to improve your model. Explain why your ideas would lead to improvement.
Include clear calculations and arguments for a solution. Do not just give the final answer.
In short:

Describe in an independent readable report your model in such a way,

- ***that it is clear to the reader which assumptions were made,***
- ***that the reader will be convinced that your model gives the cheapest solutions,***
- ***that it demonstrates what effect your model has on a example situation and that it states what quantity of gas comes with it,***
- ***that the introductionary problems A, B and C and their solutions are integrated into the report as a whole. That means that these problems are not just attached to the report as loose ends.***

An independent readable report is a report, which one should be able to read as a whole without any prerequisites. The report must be understandable for people who have not seen the problem. So include a clear description of the problem and argumentation for the given solution. You can assume that the reader has enough mathematical background to comprehend the mathematics included in the report.
The report must be **printed** or written in **black pen**, due to xeroxing.
If you assimilate figures, make sure you print them or make them in black pen. Clarify by usage of good indexing which part of the text the figures reflect on.

Grading Criteria

The report is assessed on three points:

1. Mathematical content: make sure this is complete and correct.
2. Presentation: think of construction, readability and appearance of the report.
3. The quantity of gas found in the last problem. Try to keep this amount as low as possible.

This assignment consists of three parts and a final assignment. Each of the three parts consists of two problems. These problems are to be assimilated in the final assignment. In the final assignment you need to use the results of your investigation in the problems and report them clearly.
Below you find a global time schedule, which can assist you to manage the given time.

In part A you are going to solve two smaller, precisely formulated problems explicitly. Spend ca. two hours on this part.
In part B you investigate whether you can (in theory) cross any arbitrary distance. Spend ca. one hour on this part.
In part C you design a model en apply it to a large, precisely formulated problem. Spend ca. two hours on this part.
In the final assignment you make a report of everything you did. This is a lot of work! Do not start working on it later than 14.00 hours.

Tips

- Read through the whole problem to get a provisional view of the tasks to be completed.
- Split up, if necessary, the tasks.
- But…. make sure you discuss the results you found, especially problem B1, together, it will help you to write down good argumentations.
- Retain enough time for the final assignment!

GOOD LUCK!!!

Part A: Introductory problems

A1

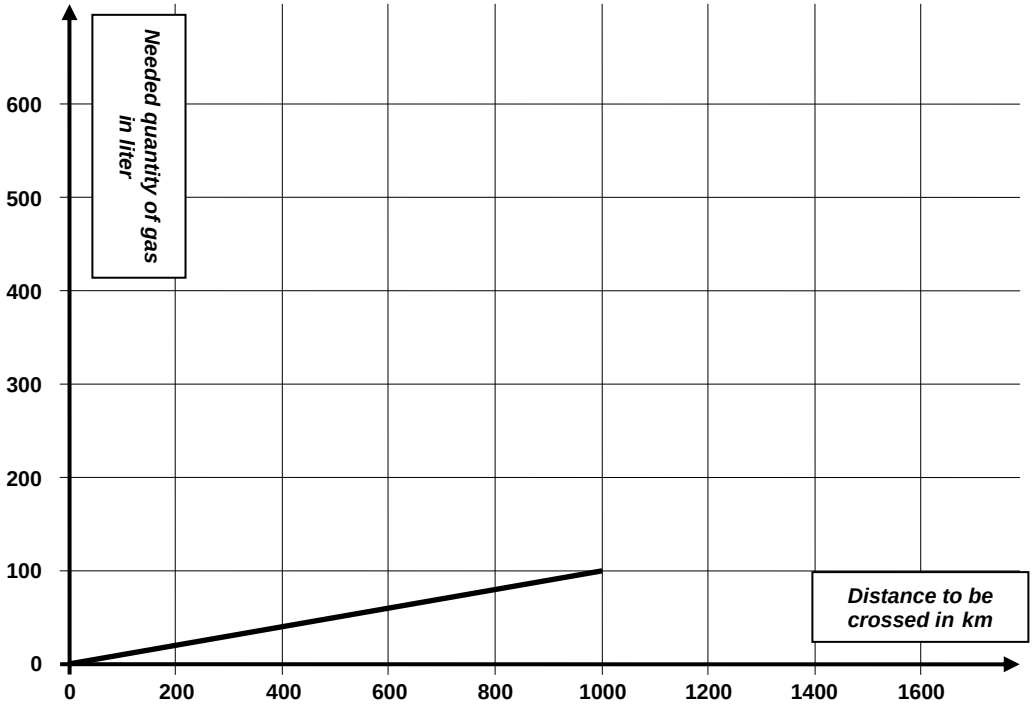
You need to cross a distance of 1100 kilometers with a jeep, which can transport a total of 100 liters.
The jeep drives 1 on 10, that is: it consumes 1 liter for each 10 kilometers it drives. You will need to construct (at least) one storage room. Assume for this problem you construct just one storage room. Construct the storage room at such a location that the quantity of gas needed to cross the 1100 kilometers is as small as possible.
Illustrate your calculations and arguments with any visual tools.

A2

You need to cross a distance of 1600 kilometers with a jeep, which can transport a total of 100 liter.
The jeep drives 1 on 10. Maybe you will need to construct more than one storage room. Construct the storage room(s) at such location(s) that the quantity of gas needed to cross the 1600 kilometers is as small as possible.

Part B: Is it possible?

Below you see a part of a graph. It contains the minimum quantity of gas needed to cross the distance to the distance. The first piece of the graph, from 0 to 1000 km, has been drawn.



B1
Finish the graph up to 1600 km by drawing points of the minimum quantities needed (according to you) to cross the distance. Explain how you created this graph and also explain the found shape of the graph.
Remark: The graph above has been added enlarged as an appendix.

You probably noticed that the graph increases very fast. Maybe the growth is so strong that at some point it will hit eternity, like for example occurs with the graph of

$$f(x) = \frac{1}{2000 - x}$$

Thus an important question is:
Is it possible to cross every distance or does it stop at some point?
Problem B2 is meant to show that it is possible in theory. Keep in mind the fact that in problem 2 the emphasis will not be on the optimal solution, but on the existence of the quantity needed to cross any arbitrary distance.

B2

1. Show that it is possible, without any storage rooms, to get any arbitrary quantity of gas and your jeep, 100 km from the starting point.
2. Try to find, with each other, a closed argument that you can cross any arbitrary distance (by constructing storage rooms).

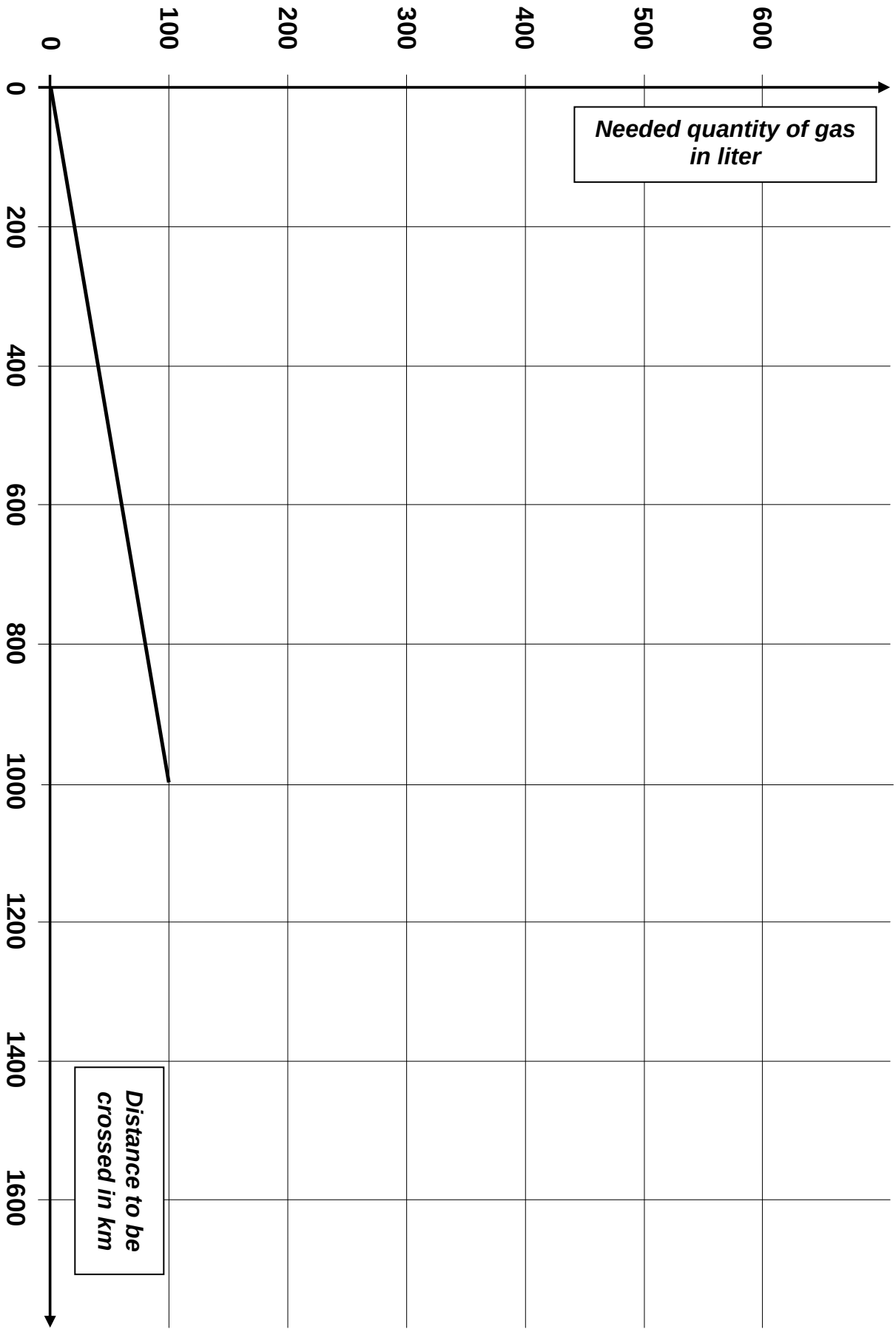
Part C: The model

From parts A and B you might have developed a feeling how to handle the jeep problem. Maybe you have found a clever method you calculate the total quantity of gas needed. De total amount of gas needed to cross a distance depends on where you place your storage rooms. You are now going to describe how, for an arbitrary given distance, you decide *how many* and *where* you place your storage rooms. The point is that you keep the total quantity of gas needed to cross the distance as low as possible.

C1
Make a model, which explains where you place your storage rooms if you want to cross a certain distance. Explain how much gas you store in each of the storage rooms. Also clarify in your model how you determine the total quantity of gas, which is needed to cross the distance. Recall the fact that this is what you want to minimize.

C2
You need to cross a distance of 3000 kilometers with a jeep, which can transport a total of 100 liter.
The jeep drives 1 on 10. Use your model to show where you place the storage rooms and how much gas you need to cross the distance.
It is of the utmost importance that it becomes clear that it is possible to cross the distance with the amount you say. Furthermore you are competing with the other participants of the Mathematics B-Day.
Whoever finds the smallest amount has an advantage over the other teams!

Good luck on writing the report!!!


0
100
200
300
400
500
600
Needed quantity of gas in liter
0
200
400
600
800
1000
1200
1400
1600
Distance to be crossed in km