TOOL USE IN A TECHNOLOGY-RICH LEARNING ARRANGEMENT FOR THE CONCEPT OF FUNCTION

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Abstract
This paper describes the design of a technology-rich learning arrangement for the function concept. In line with the instrumental approach of tool use, aspects of conceptual understanding of the notion of function are linked with the tasks and with techniques in the technological tool, i.e. an applet embedded in an electronic learning environment. Some data from the pilot teaching experiment are presented.

INTRODUCTION
Since the origin of mankind, people have been using tools as extensions of the body. Vygotsky already pointed out that an instrument constitutes "a new intermediary element situated between the object and the psychic operation directed at it" and thus mediates the activity (Vygotsky, 1930/1985, p. 42). More recently, the complex nature of tool use has acquired considerable attention within cognitive ergonomics (Rabardel, 2002) and cultural-historical activity theory (Engeström et al., 1999). Research suggests a close relationship between tool use, cognitive development and social practice. Identification of the nature of these relationships and exploiting them for educational goals, however, is a non-trivial issue. The central problem with which education is confronted, therefore, is to identify the relation between the use of technological tools and learning, and to use these insights as guidelines for technology-rich teaching practices. How can the use of technological tools be embedded in innovative learning/teaching arrangements so that it improves learning?

A recent, promising technological development concerns small interactive environments called applets (or ‘thinklets’), which are accessible through the Internet. For algebra, many applets are available, which aim at developing mental models and relating skills and concepts (e.g. see www.wisweb.nl). So far, however, neither the influence of the use of such applets in the teaching and learning of algebra nor the role of the teacher in such an arrangement have been systematically investigated.

This paper reports on the start of a research study on the use of applets for the acquisition of a rich function concept. The study focuses on students of grade 8 (13-14 years old), and is currently in its first phase. In line with an earlier pilot project (Boon & Drijvers, 2006) the aim is to investigate the power of combining the affordances of technology with by-hand work, group work and other forms of teaching.
RESEARCH QUESTIONS AND THEORETICAL FRAMEWORK

The three-year research project, entitled 'Tool Use in Innovative Learning Arrangements for Mathematics', started in January 2006 and focuses on the following research questions:

1. How can applets be integrated in an instructional sequence on the function concept, so that their use fosters learning?

2. How can teachers orchestrate tool use in the classroom community?

An important part of the theoretical framework of this study is based on the Vygotskian notions on tool use (Vygotsky, 1930/1985). Two related recent theoretical developments are relevant. First, the further development of cultural-historical activity theory by Engeström and his colleagues considers the activity system of which the tool, the user, the task and the social context are part (Engeström et al., 1999). The importance of the community of practice in which the learning takes place is stressed (Wenger, 1998). The notion of the activity system provides an interpretative framework to describe and analyse the social context of the classroom, and in particular interactions between students and guidance by the teacher. As such, it also addresses the second research question.

As a second recent elaboration of the Vygotskian notions on tool use, the instrumental approach to tool use stresses the distinction between the artefact – the object in use, for example a calculator – and the instrument (Lagrange, 2005; Rabardel, 2002; Trouche, 2004). The instrument consists of the artefact and the cognitive instrumentation schemes, which are needed to be able to use the artefact. In short, ‘instrument = artefact + instrumentation scheme’. While constructing the instrument, students develop these mental schemes in which techniques for its use and insights into the concepts behind it co-evolve in a close relationship. By means of instrumental orchestration, the teacher guides this process of instrumental genesis (Drijvers & Trouche, in press). Because of the conceptual aspects within the cognitive instrumentation schemes, the instrumental approach is a valuable framework to investigate the relation between tool use and learning.

These two theoretical perspectives both stress the way tools mediate students’ actions, as well as the relation between techniques for tool use, cognitive development and social context. The third component of the theoretical framework consists of domain-specific theories on the teaching and learning of mathematics. With the philosophy of realistic mathematics education as an overall background (Freudenthal, 1983), more specific theories on the learning of the function concept and the reification of function are considered (Vinner, 1983; Sfard, 1991).

RESEARCH SETUP AND METHODOLOGY

The research questions aim at ‘understanding how’ instead of ‘knowing whether’. The research setup consists of three cycles of design research. Each of the cycles includes a design phase, a teaching experiment and a data-analysis phase. Throughout
the cycles, the length of the teaching experiment and the number of students involved increase. Also, the data that are gathered range from initially purely qualitative to more quantitative in the third cycle.

The design phase includes the design of student activities and tasks and the design of the technological tools that will be used. This is the main topic of this paper. However, the design of the teaching arrangements – the organization of the lessons and the way the teacher deals with theory, task and tool – is crucial as well. According to the instrumental approach, the orchestration of tool use is an important task for the teacher. By means of instrumental orchestration, the teacher fine-tunes the tool use in the classroom and thus, to continue the metaphor, conducts the different instruments formed by the students into one harmonic orchestra (Drijvers & Trouche, in press). In the design of the learning sequence, the issue of orchestration is dealt with by developing a teacher guide, in which didactical scenarios are described that suggest working arrangements as well as possible ways to use them.

During the teaching experiment, the following data are gathered. In the light of the first research question, video recordings are made of classroom teaching and group work; screen videos of pairs of students working with the computer are captured. Students’ work with the applets is saved on a central server and photocopies of their written work are made. In the light of the second research question, the teacher’s behaviour is videotaped, both in classroom teaching and in individual interaction with students. In this way, we hope to be able to trace exploitation modes of the didactic scenarios that are either more or less successful in the process of instrumentation and concept development. Data analysis is carried out by qualitative analysis and coding of the data; in the following teaching experiments, more quantitative techniques will be included.

This paper focuses on the design ideas concerning the notion of function, the role of the tools and the learning arrangement; it ends with some indicative data from the pilot teaching experiment.

**FUNCTION CONCEPT**

The study aims at developing an innovative learning arrangement for a rich and versatile function concept. This is not a trivial goal. Vinner and Dreyfus point out that there often is a considerable difference between the formal function definition and the students’ concept image (Vinner, 1983; Vinner & Dreyfus, 1989; see also Meel, 1999). Malle (2000) describes two main faces of the function concept image: the input – output assignment and the co-variation, in which a change of the independent variable effects the dependent variable. The ideas of co-variation and dependency are stressed by Freudenthal as well:

The function is a special kind of dependence, that is, between variables which are distinguished as dependent and independent. (...) This - oldfashioned - definition stresses the phenomenologically important element: the directedness from something that varies freely to something that varies under constraint. (Freudenthal, 1983, p. 496).
Based on student interviews before the teaching experiment, we expect the starting function concept image for most of the students to be the local calculation procedure, which can be applied to one single input number at the time, and then results in an output number. The goals of the learning arrangement include the following four aspects of the function concept, which are of course not independent from each other.

1. An input – output assignment
   The function is an input – output assignment that helps to organize and to carry out a calculation process. This somewhat vague notion gradually gets more nuances: how does the output depend on the input, how does the input determine the output? Its originally local character becomes more global: the function is not only a relationship between input and output numbers, but also one between domain set and co-domain. Functions can be compared with respect to global properties, such as increasing / decreasing or asymptotic behaviour. This opens the horizon for issues of co-variation.

2. A dynamic process of co-variation
   This concerns the notion that the independent variable running through the domain set causes the dependent variable to run through the co-domain. The dependent variable co-varies with the independent. At first, the linked change is observed in a somewhat phenomenological way. Then, the question of how the process of joint dynamics is made to happen. What happens to the output if the input increases by 1 unit? How can one observe this in the table or the graph, or explain it with the formula?

3. A mathematical object
   A function is a mathematical object which can be represented in different ways, such as arrows, tables, graphs, formulas, each of which shows a different ‘face’ of the same object. This concept image is an integrated function notion, which allows for reasoning with functions on a global level: how is a certain property of the graph reflected in the table or the formula, how can one decide if two functions belong to the same ‘family’?

**TOOL USE**

In the learning arrangement, both material and digital tools are used. The material tools include a quadrilateral that can be changed (Fig. 1 left picture), posters to use in presentations, cards with operation symbols to make ‘living operation chains’ (Fig. 1 right picture), and cards with function representations to be matched. Typical in the design of the learning sequence is the aim to establish a close link between the material and the digital tools, by means of resemblance of representations. Here, we focus on the digital tools, i.e. an applet embedded in a simple electronic learning environment.
Figure 1 Sliding a quadrilateral and making a ‘living operation chain’

The main digital tool that plays a central role in the learning arrangement is an applet called ‘AlgebraArrows’, developed by Peter Boon. AlgebraArrows (AA) offers means to construct input-output chains of operations. These chains can be applied to single numerical values as well as to variables. In the latter case, tables, formulas and (dot) graphs can be shown. Chains can be extended, linked, compared and compressed. Fig. 2 shows some of the main features of the applet.

The applet is embedded in a simple electronic learning environment, which is called the Digital Mathematics Environment (DME). The DME is an answer to the fleeting character of the work with applets, that both students and teachers often experience. In the DME, student work is saved on a central server. This allows the student to review the work, correct it, and continue it in any location with internet access, particularly at home. For the teacher, the DME offers a means of checking the students’ progress, of monitoring the learning process by means of all-class result overviews and eventually of grading the students’ work.

The DME also offers ways to set tasks and questions and for answering them outside the framework of the embedded applet. Fig. 2 shows how students can read the task in the upper left of the screen, work in the applet window on the right, and formulate their answer in a textbox on the left of the screen.

**TOOL USE AND CONCEPT DEVELOPMENT**

The instrumental approach to tool use stresses the relationship between developing a concept, using a technique with the tool, and carrying out a specific task (Kieran & Drijvers, 2006). How can the three aspects of the targeted rich function concept be connected to tasks in the learning arrangements and affordances of the tools, i.e. the applet en the electronic environment? Let us briefly go through the three aspects.

1. **An input – output assignment**

   The chain of operations in the AA applet, applied to a single numerical value, reflects the function as an input – output assignment. In Fig. 3, two chains concerning different mobile phone offers invite a global investigation of properties. The technique of putting numerical values in the input window allows for comparison of function values. A variable input allows for a more global comparison, in which graphs, tables and formulas can be involved.
Figure 2 Main features of the applet AlgebraArrows embedded in the DME

Figure 3 An input – output assignment

2. A dynamic process of co-variation
The applet allows not only a change of input value, but also the study of the co-variation by means of tables and graphs. The chain in Fig. 4 represents the cost of one particular mobile phone offer. The dynamics of the co-variation can be investigated by techniques of substituting different values, scrolling through the table, tracing the graph and studying the formula. The task for the students is to find the change of cost per minute change of the input variable – the number of minutes of phone calls.
3. A mathematical object

The different representations available in the applet, combined with appropriate tasks focusing on the way they are interrelated, invite a more holistic view on a function as a mathematical object. A means to stress the object character of the function is to consider families of functions. Fig. 5 shows the chains and the graphs of the stopping distance as a function of speed for the case of a motorbike, a car and a truck. The technique of changing the last operation in the chain allows discovering the general features in the graphs and to find properties of the ‘family’ of functions, of which three members are represented.
SOME FINDINGS FROM THE PILOT EXPERIMENT

Let us first briefly present some quotations from the student output windows.

- In the task presented in Fig. 3, one pair of students wrote down:
  “He can better use Offer1 if he calls 25 minutes per month, but if he calls less
  than 20 minutes, he’d better choose Offer2”
  Many students focus on the break-even point as the intersection point of the two
  graphs.

- In the task presented in Fig. 4, one pair of students wrote down:
  “The numbers increase with 15 cents each time.”
  An other pair was more vague:
  “The minutes and the costs both increase.”
  Many students noticed the co-variation, but had difficulties with clearly
  expressing their perception in a detailed way.

- In the task presented in Fig. 5, most students focus on the graph:
  “They all increase in a kind of curve.”
  In some exceptional cases, the chain of operations was considered:
  “It gets squared in all three cases and then divided. But just divided by a
  different number.”
  Once more, many students noticed similarities, but had difficulties to clearly
  express them.

Overall, it was noticed that the combination in the DME of applet facilities and ‘book
facilities’ for posing questions and for answering them was powerful. Still, the
quotations above indicate that students found it difficult to reflect on their work with
the applet and write down precise mathematical conclusions. Also, the arrangement
with strong links between the screen work and paper-and-pencil work had some
practical complications. For example, once the students were working with the
computer, they hardly looked at the hardcopy teaching materials. As a feed forward
of the pilot, we decided to make the digital materials more self-containing through
extra help facilities and digital copies of the written materials, to avoid the
complexity of dealing with screen and paper at the same time.

The global learning trajectory turned out to work well, though it was noticed that it
was too much driven by the mathematical concept and not enough by the problems
that students could deal with by means of the applet. Also, we underestimated the
students’ difficulties of choosing and naming the independent variable in applied
situations.

The relation between applet techniques and conceptual thinking was clearly observed
in the activities with the applet. However, it was not always clear whether
instrumental genesis during the successive activities included student reasoning or
was merely based on trial-and-improve. The options to enhance a dynamic view of
function could be better exploited, which is one of the aims of the next research
cycle.
In the post experiment interview, the teacher reported that the applet embedded in the DME was a powerful means to experience “that formulas, tables, graphs and all those things have to do with each other”. Also, she reports that at the end of the teaching sequence, students “did much better in reasoning than at the start”. However, she stated that technical classroom management was more complicated than in regular teaching, because of the mixed and integrated use of media and the somewhat complex teaching scenarios.

CONCLUSION

The pilot teaching experiment served as a first field test for the designed activities. Therefore, the conclusions mainly have a feed forward character for the next research cycle.

Concerning the first research question on the relation between the use of the applet and learning, we conclude that the applet offers interesting activities that foster conceptual development. The close relationship between applet techniques and paper-and-pencil activities guaranteed an integrated conceptual understanding and a transfer between applet notations and paper-and-pencil notations. In that sense, the ‘mixed media’ approach was fruitful. Still, a more detailed observation of the instrumental genesis process is needed. A more detailed description of the intended learning process will be developed, which delineates how successive techniques support each other and finally foster conceptual development. Such a description would allow to investigate whether the instrumental genesis is the result of learning history and imagery of the students or of trial-and-improve strategies (Gravemeijer et al., 2003; Doorman, 2005).

Concerning the second research question on the orchestration by the teacher, we conclude that the teaching arrangement was demanding for the teacher, who spent too much of her time on organizational matters. Furthermore, the DPME offered interesting possibilities for the teacher to monitor the students’ progress and to adapt her lessons to that. As far as teacher – student interactions during applet work is concerned, this turned out to be particularly helpful if the teacher’s explanation related to both technical and conceptual elements. Finally, applet work in pairs was not enough for collective instrumental genesis; classroom demonstrations and discussions played an essential role in capturing the students’ results and in provoking a collective convergence of mathematical thinking.

REFERENCES


