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| **Worksheet 3: Time dilation**  **Answers** |
| **Names:** |

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| **Thought experiment** |

A muon is a cosmic particle. A muon has an average lifetime (half life) of *t1/2* = 1.5 · 10-6 s and a velocity of *v* = 0.98*c*.

1000 muons travel from an altitude of 2.2 km to the Earth's surface. This journey takes *t* = 6.8 - 10-6 s. Because of their low half life, only 31 out of 1000 muons would reach the Earth's surface.

In practice, however, we measure that 496 out of 1000 muons reach the Earth's surface.

Explain why more muons are measured on Earth than expected.

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| Answer as you see fit. |

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| **Simulation activity** |



**Task 1**

In this task, we’ll create a simulation of a light clock. Scan the QR code to see an example of the simulation.

*Figure : a screenshot of a light clock*

Perform the following steps:

1. Select a laser by clicking on the icon in the left column and place it in the centre of the screen. The scale of the coordinate system automatically changes from meters to light seconds (1 light second is the distance light travels in 1 second, so 1 ls = 3.00 - 108 m). Chart

   Description automatically generated
2. In the laser's pop-up menu, you can specify in which direction(s) the laser will emit light. Press the arrows on your keyboard to make sure the laser only emits light upwards.
3. Place a mirror three squares above the laser and adjust the angle of the mirror to 90 degrees so that it is horizontal. Place a second horizontal mirror 3 squares below the laser. An example is given in figure 1.

We define two events:

Event A: the light beam is emitted by the laser

Event B: the light beam travels back and forth once

The time interval between event A and B is one **tick** of the light clock (similar to the ticking of a mechanical clock).

1. Watch the simulation. What is the duration of one tick of the light clock?

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| *∆t* = 12 s |

**Task 2**

Read the following thought experiment and answer the questions below. *Answer questions 1 and 2 before observing the simulation.*

*Albert and Marie are two scientists. Marie moves at a horizontal velocity of 0.4c (40% of the speed of light) relative to Albert. Albert and Marie both have a light clock available in their own reference frame. Albert measures the duration of one tick of Marie's light clock.*

1. Complete this sentence: in Albert's reference frame, one tick of Marie's light clock takes
   * **Longer than** one tick of his own light clock
   * Less long than one tick of his own light clock
   * As long as one tick of his own light clock
2. Comment on your prediction below.

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| We express one tick of Marie's light clock in Albert's reference frame.  The distance between event A and B is longer (because the laser and mirrors move to the right so the light has to travel a zigzag trajectory). However, the speed of light is the same in both reference frames. So in Albert's reference frame, one tick of Marie's light clock will take longer than one tick of his own light clock. |



1. Create a simulation of the thought experiment in Relativity Lab. Do this by inserting a second light clock with a horizontal velocity of "**0.4c**". Scan the QR code to see an example of the simulation.
2. What is the duration of one tick of Marie's light clock in Albert's reference frame?

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| *∆t'* = ± 13 s |

1. Revisit back the explanation of your prediction. What would you like to change about this now that you have observed the simulation?

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| Answer as you see fit. |

**Task 3**

In this assignment, we are going to repeat the thought experiment from the previous task in Marie's reference frame. *Answer questions 1 and 2 before looking at the simulation.*

1. Complete this sentence: in Marie's reference frame, one tick of Albert's light clock takes
   * **Longer than** one tick of her own light clock
   * Less long than one tick of her own light clock
   * As long as one tick of her own light clock
2. Comment on your prediction below.

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| This thought experiment is the same as in task 2, only the roles of Marie and Albert are reversed.  The distance between event A and B is longer (because the laser and mirrors move to the left so the light has to travel a zigzag trajectory). However, the speed of light is the same in both reference frames. So in Marie's reference frame, one tick of Albert's light clock will take longer than one tick of her own light clock. |

1. Verify your answer through a simulation in Relativity Lab.
2. Revisit the explanation of your prediction. What would you like to change about this now that you have observed the simulation?

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| Answer as you see fit. |

1. What general conclusion can you draw about the measurement of time intervals in different reference frames?

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| A light clock that is moving in the chosen reference frame ticks slower than a light clock in its own reference frame. A light clock can be used to measure the time interval between two events. So the time interval between two events depends on the chosen reference frame. |

**Task 4**

In this task, we will explore the role of relative velocity[[1]](#footnote-1) in the thought experiment of the previous tasks.

1. Complete this sentence: at a relative velocity of 0.8*c*, the difference in the duration of one tick of Albert’s and Marie’s light clocks is
   * **Greater than** at a relative velocity of 0.4*c*
   * Smaller than at a relative velocity of 0.4*c*
   * The same as for a relative velocity of 0.4.
2. Comment on your prediction below.

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| With increasing relative velocity, the distance light travels in one tick also increases. This means that the difference in the duration of one tick of Albert’s and Marie’s light clock is increased at increasing relative velocity. |

1. Give the two light clocks a relative velocity of 0.8*c*. Observe the simulation in both reference frames.
2. What is the duration of one tick of the moving light clock?

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| *∆t'* = ± 20 s |

1. Revisit the explanation of your prediction. What would you like to change about this now that you have observed the simulation?

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| Answer as you see fit. |

1. Muons move at a velocity of 0.98*c* relative to the Earth. How long does one tick of a light clock last at this relative velocity?

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| *∆t'* = ± 60 s |

**End of the simulation activity**

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| **Thought experiment (continued)** |

*Revisit the question below, but this time using the knowledge you have gained during this lesson.*

Explain why more muons are measured on Earth than expected.

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| We define two events:  A: the muons start their journey from an altitude of 2.2 km  B: the muons reach the Earth's surface  We express these events in the Earth's reference frame.  We measure the duration between event A and B using a light clock travelling along with the muons. In Earth's reference frame, this light clock ticks slower than a light clock in Earth's own reference frame (see Task 3). So the duration between A and B is smaller in the Earth's reference frame than in the muons' own reference frame. If the muons' travel time is smaller, fewer muons will decay during the journey. As a result, more muons reach the Earth's surface than expected. |

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| **Check questions** |

1. Rachael is on a train passing a platform at 0.*8c.* She claps her hands at a frequency of 1 Hz (i.e. once per second) according to her watch. Charlie is standing on the platform. He is also clapping his hands with a frequency of 1 Hz according to his watch. According to Rachael, Charlie claps at a lower rate than she does.

Read the three statements below:

1. Charlie agrees with Rachael about clapping at a slower pace
2. **According to Charlie, Rachael is actually clapping at a slower pace**
3. According to Charlie, they are clapping at the same pace

Explain which statement is correct.

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| We measure the frequency of Rachael's clapping using a light clock travelling along with her. In Charlie's reference frame, this light clock ticks slower than a light clock in Charlie's own reference frame. So, according to Charlie, Rachael claps at a slower pace. |

1. A passenger sits in a train carriage passing a platform at 0.*8c.* He drops his watch on the floor. In the train's reference frame, it takes 0.20 s for the watch to reach the floor of the carriage.

In which reference frame is the proper time of this movement measured? Please explain your answer.

1. The reference frame of the train
2. The reference frame of the platform
3. Neither of the above

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| The proper time between two events is measured in the reference frame in which the two events take place at the same position. This is the case only in the reference frame of the watch. So neither statement is true. |

**End of check questions**

1. "Relative velocity" refers to the difference in velocity between Albert and Marie. [↑](#footnote-ref-1)