Mathematics Day for lower secondary 2015:
Fish and peas

During a schoolday, you will be working in groups of 3 or 4 students on an open ended problem. At the end of the day, there will be a written report as a result of your work. Below you will find a checklist we strongly advise yo use:

The method of working	*check*

- Read the complete assignment. Think about:
   o approach
   o dividing tasks
   o use of computer
   o time table	

- Cooperate: confer with each other on a regular basis about your planning, and if you're still on the right track; don't forget at the end of the day you do need time to produce a report that is a coherent whole;	
- Don't be satisfied too easily. Try different appoaches in the assignments, and mention these in your report.	
- Justify your choices, describe your method of working.	
- Produce a coherent report, not just a 'list of answers'.	
- You report must be written in such a way that it can be read independently from the assignment. The report will consist of three parts: the INTRODUCTION with the answers to questions 1-4; FINAL ASSIGNMENT PART 2, with FINAL ASSIGNMENT PART 1 as an appendix to FINAL ASSIGNMENT PART 2.	
- Don't forget to paginate the pages.	
- Put the names of the teammembers and the name of the school on the frontpage.	

The judgement

"The only good answer" is not the issue in this assignment: there is not one good answer, there are several possible right answers at the different assignments. In assessing the work, the jury will mainly pay attention to:

- whether your way of working is clear;
- whether your choices and results are justified clearly;
- what kind of approach you chose, whether you worked in a systematic way, whether the use of mathematics is correct, meaningful and clear;
- how you dealt with the FINAL ASSIGNMENT. The FINAL ASSIGNMENT will be more important in judging than the INTRODUCTION ASSIGNMENTS.
- whether the report is a coherent whole which can be understood without the assignments next to it.

Lots of fun, and good luck!

Fish and peas

Introduction
Sometimes it is not possible to count the exact number of something, e.g. fish in a pond. You then have to think of other ways to estimate the total amount. We are going to work on that in this assignment.

Introduction assignments
Pelle has a large pond behind his house with a special breed of fish, the Prussian Carp. He would like to know how many carps live in the pond and thinks of the following method:
- He catches a number of fish;
- he marks those fish with a special colouring agent and then throws them back in the pond alive;
- the following day (after the fish are mixed well again) he catches another number of fish;
- he counts the number of marked fish and on the basis of that he estimates the total amount of fish in the pond.

The first day Pelle catches 15 fish, he marks these fish and puts them back in the pond. The following day he catches 20 fish of which 4 are marked. He also puts these fish back.

Assignment 1
On the basis of Pelle's results, how many fish do you estimate that there are in total in this pond? Give a clear description of how you have determined this and why you believe this is a suitable method.

Assignment 2
Another day later, Pelle again catches an amount of fish and he counts the marked fish. Now he has caught 6 fish of which 2 are marked. Do you want to adjust your estimation of assignment 1? Clarify your answer.

Formulas
Several formulas can be used to make an estimation on the basis of the number of caught and marked fish. We provide two.

Formula 1

$$N = \frac{K \cdot n}{k}$$

Formula 2

$$N = \frac{(K+1)(n+1)}{(k+1)} - 1$$

N = the estimation of the total amount of fish in the pond
K = the amount of fish that has been caught and marked on day 1
n = the amount of fish caught on day 2
k = amount of fish caught on day 2 that had already been marked

Assignment 3
Explain these formulas and show how they work. Compare them to your own method or algorithm from assignment 1 and 2.
Which formula provides better estimations according to you? Why?

Catch more often
Until now you have estimated the total amount of fish on the basis of two catches:
- On day 1 you catch and mark the fish and put them back;
- on day 2 you catch more fish and count the marked ones.

Naturally, you can catch fish for a number of days in a row: So on day 3, 4, etc. where you continue to put the fish back.
You now have to think of a new formula or algorithm to estimate the total amount of fish.

Assignment 4
Pelle (see assignment 1) continues to catch and count for a number of days.
Here are the results:
On day 1 Pelle caught 15 fish, he has marked all of them and put them back.
The 6 days after that he catches the following:

Day 2:	caught 20	of which 4 marked
Day 3:	caught 6	of which 2 marked
Day 4:	caught 12	of which 5 marked
Day 5:	caught 8	of which 0 marked
Day 6:	caught 2	of which 2 marked
Day 7:	caught 6	of which 1 marked

Estimate the total amount of fish in the pond on the basis of these data. Clearly illustrate in which way you have done this. Use what you have discovered from assignment 1 to 3 and explain why your method produced a good estimation.

Final assignment

The final assignment consists of two parts:
1) executing and describing an experiment and
2) advising fish biologists

Experiment

In aforementioned assignments it is difficult to determine whether the estimation of the total number of fish has been executed well. After all, you do not have any information on the actual amount. Therefore, you will now execute an experiment in which you simulate the catching and marking of the fish. This will provide insight in how the estimation methods work and which method provides the best estimations in what situations.

Your teacher will provide you with a large number of smaller 'objects' (e.g. split peas) and a marker. You will also need a small box or a sack.

- Count off a large amount of objects (about 100) and put them in a box or sack;
- draw a number (write down this amount), mark them (e.g. with a water-resistant marker) and put them back;
- mix everything well and draw another amount: write down the number that you draw and the number that have been marked;
- using these data, estimate the total number of objects – use the aforementioned formulas and/or your own algorithms from assignments 1 to 4;
- compare the result of the estimations with the actual amount (which you have counted at the start).

Final assignment part 1

1. Repeat all the steps of the experiment a large number of times varying anything you like. Vary e.g.: The total number of objects to start with, the amount that you catch (*K, n*) etc…

   Keep a close record of everything (on paper).

   Report on your experiments and the accompanying results. This report is an appendix to the advice you will give in final assignment part 2.

Advice

Fish biologists are in search of a method to estimate how many fish (of a certain species) are in a lake. They have planned to do this by catching and marking fish. Costs are involved with such a method. E.g. an amount is figured for every caught fish (even if they throw back the fish).

Final assignment part 2

Write a piece of advice for the fish biologists in which you discuss the method which produces the most reliable estimations according to you. Explain why you believe this is the case. Explain how the fish biologists have to operate and exemplify how your method works in different circumstances e.g. if small or large amounts of fish swim around. In your advice you have to keep the costs into consideration.