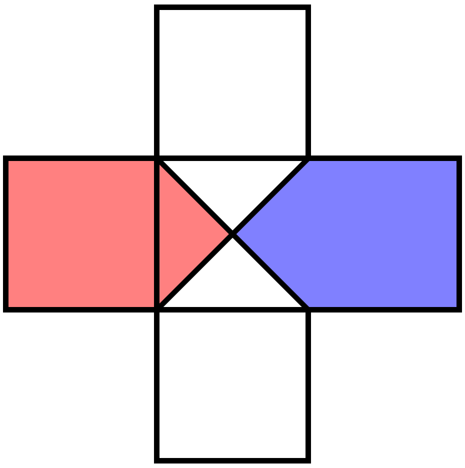
Section B

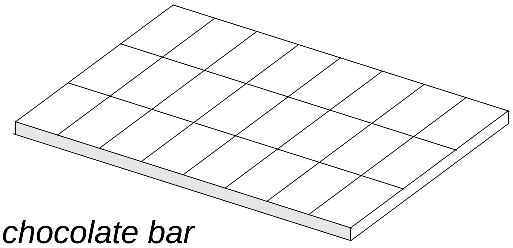
Fractions

addition and subtraction



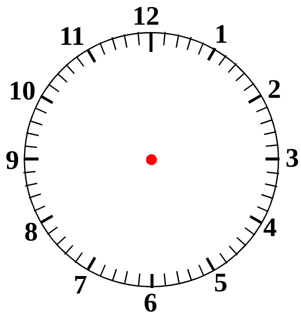
$$\frac{1}{5} + \frac{1}{20} = \frac{1}{4}$$

Adding parts



chocolate bar

clock



1. Fill in

$\frac{1}{3}$ bar + $\frac{1}{4}$ bar = ... pieces

= $\frac{\dots}{\dots}$ bar

$\frac{1}{3}$ hour + $\frac{1}{4}$ hour = ... minutes

= $\frac{\dots}{\dots}$ hour

Find fitting fractions. Write fractions in the simplest form.

$\frac{1}{3}$ bar + $\frac{1}{6}$ bar = ... bar

$\frac{1}{6}$ bar + $\frac{1}{8}$ bar = ... bar

$\frac{5}{8}$ bar + $\frac{5}{12}$ bar = ... tablet

$\frac{2}{3}$ hour + $\frac{1}{6}$ hour = ... hour

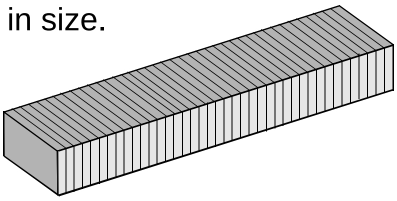
$\frac{2}{3}$ hour + $\frac{1}{5}$ hour = ... hour

$\frac{3}{10}$ hour + $\frac{5}{12}$ hour = ... hour

2. There are 40 slices of gingerbread, all equal in size.

Find fitting fractions.

Write fractions in the simplest form.



$\frac{1}{10}$ bread + $\frac{1}{5}$ bread = ... bread

$\frac{1}{10}$ bread + $\frac{1}{4}$ bread = ... bread

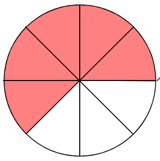
$\frac{1}{8}$ bread + $\frac{1}{5}$ bread = ... bread

$\frac{3}{8}$ bread + $\frac{3}{5}$ bread = ... bread

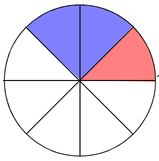
3. Now think of four problems that would fit on this page yourself.
Do not forget to provide answers!

Complete and reduce

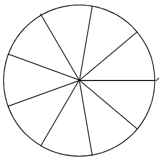
4. Complete to 1



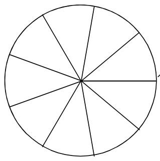
$\frac{5}{8}$ + ... = 1



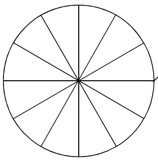
$\frac{1}{8} + \frac{1}{4}$ + ... = 1



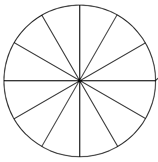
$\frac{5}{9}$ + ... = 1



$\frac{1}{9} + \frac{1}{3}$ + ... = 1



$\frac{7}{12}$ + ... = 1



$\frac{7}{12} + \frac{1}{3}$ + ... = 1

5. Complete to the nearest whole number

$2\frac{3}{4}$ + ... = 3

$3\frac{5}{7}$ + ... = ...

$7\frac{3}{8}$ + ... = ...

$8\frac{3}{7}$ + ... = ...

$\frac{26}{9}$ + ... = 3

$\frac{29}{8}$ + ... = ...

$\frac{22}{7}$ + ... = ...

$\frac{98}{11}$ + ... = ...

6. Reduce to the nearest whole number

$\frac{26}{9}$ – ...... = 2

$\frac{29}{8}$ – ...... = ...

$\frac{22}{7}$ – ...... = ...

$\frac{98}{11}$ – ...... = ...

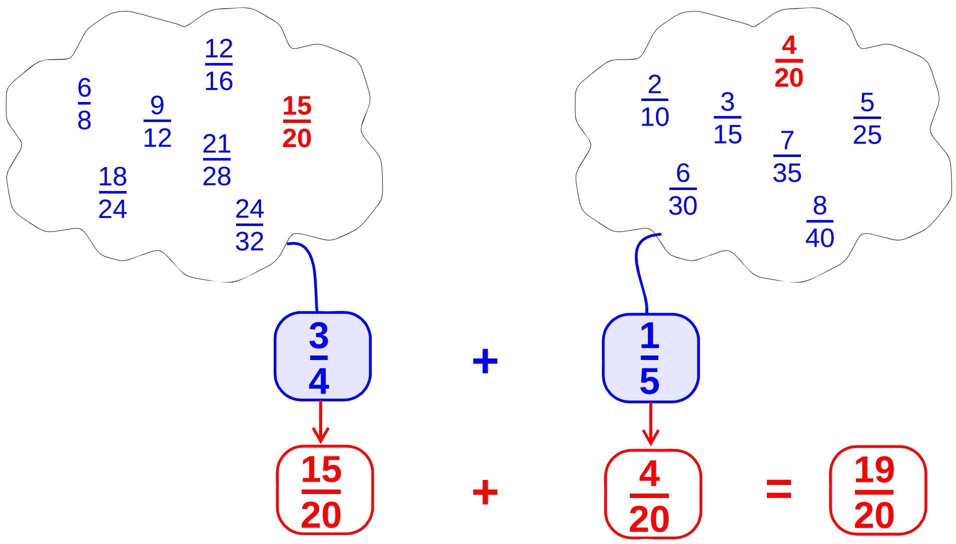
$\frac{17}{8}$ – ...... = ...

$\frac{17}{9}$ – ...... = ...

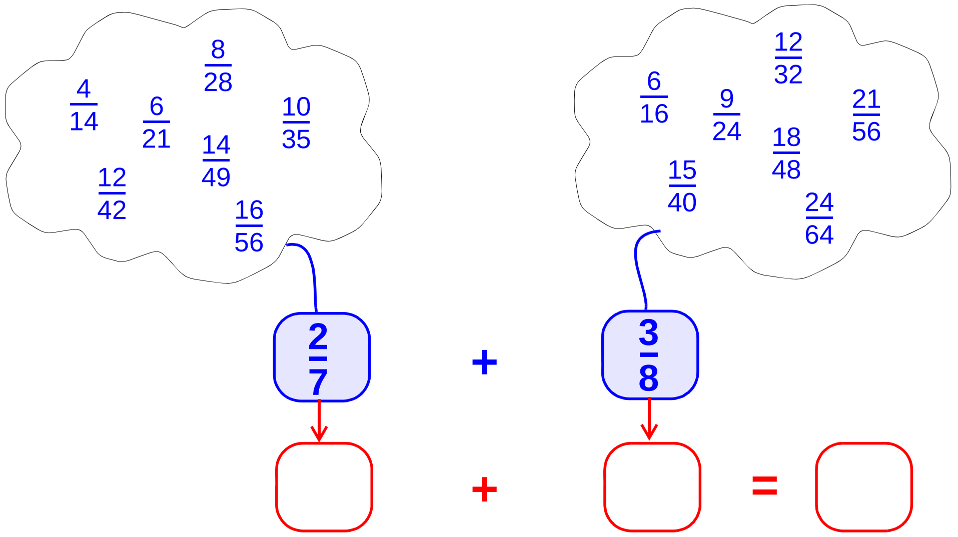
$\frac{77}{9}$ – ...... = ...

$\frac{99}{7}$ – ...... = ...

Find equal denominators



7. Find fitting fractions



8. Find fitting fractions

$\frac{1}{2} + \frac{2}{7} = \ldots + \ldots = \ldots$

$\frac{1}{2} - \frac{2}{7} = \ldots - \ldots = \ldots$

$\frac{3}{4} + \frac{1}{7} = \ldots + \ldots = \ldots$

$\frac{3}{4} - \frac{3}{7} = \ldots - \ldots = \ldots$

$\frac{3}{8} + \frac{1}{5} = \ldots + \ldots = \ldots$

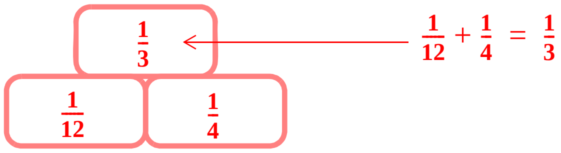
$\frac{3}{8} - \frac{1}{5} = \ldots - \ldots = \ldots$

$\frac{5}{8} + \frac{2}{5} = \ldots + \ldots = \ldots$

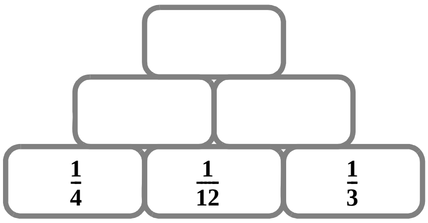
$\frac{5}{8} - \frac{3}{5} = \ldots - \ldots = \ldots$

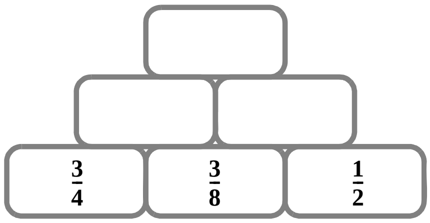
Fraction Walls

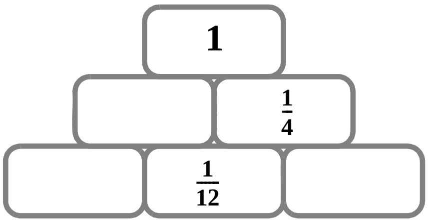
Rules of the game. Add the numbers on two neighbourly "bricks" and write the result in the brick that rests on the two bricks.

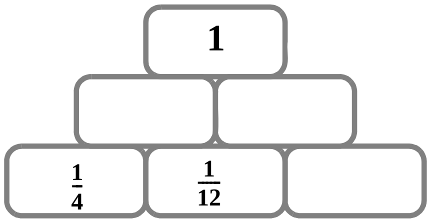


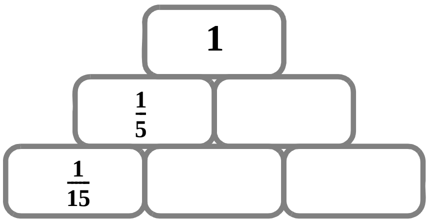
9. Fill in missing fractions or whole numbers

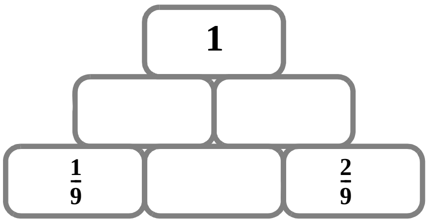




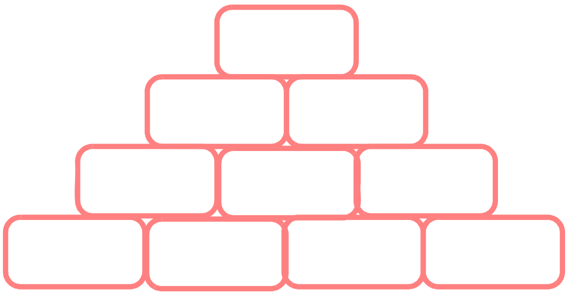






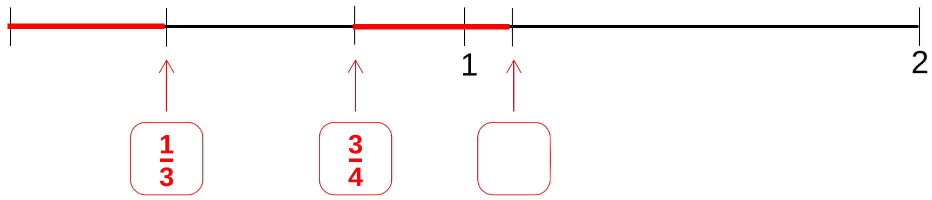


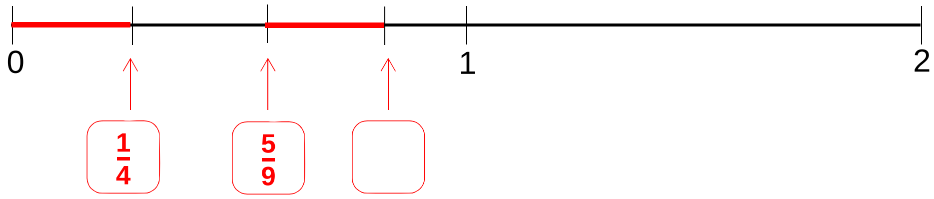
10. Now make your own fraction wall.
The top brick must show a *whole number*.

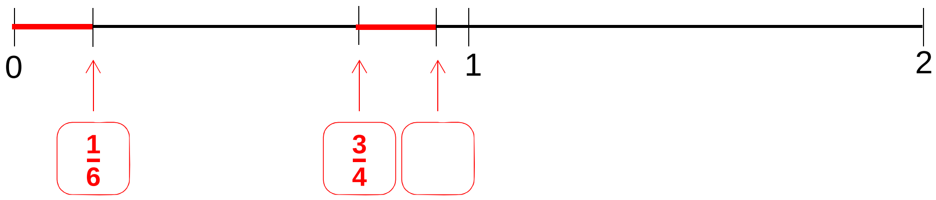


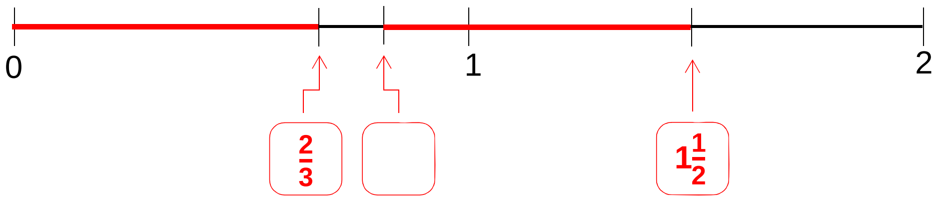
On a number line

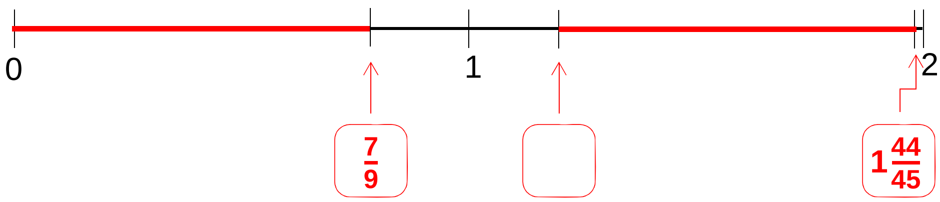
11. The two line segments are equal in length. Which fraction should be written in the empty box? Which problem is shown?











Addition tables

12. Write appropriate fractions in each box.

+	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{2}$		$\frac{5}{6}$		
$\frac{2}{3}$				
$\frac{3}{4}$				
$\frac{4}{5}$				

+	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{6}$				
$\frac{1}{7}$				
$\frac{1}{8}$				
$\frac{1}{9}$				

+				
$\frac{3}{10}$	$\frac{4}{5}$	1	$\frac{7}{10}$	$\frac{11}{20}$
			$\frac{9}{10}$	
		$\frac{7}{20}$		
	$\frac{1}{8}$			

+			$\frac{5}{6}$	$\frac{3}{8}$
$1\frac{1}{8}$				
		2	1	
$2\frac{1}{8}$				
	2			1

13. Create two addition tables using fractions.

+				

+				

String of sums

In this string of sums, the answer to each problem can be found by using the result of the previous problem.

14. Fill in the answers

$$\frac{1}{2}+\frac{1}{3} = \frac{5}{6}$$

$$\downarrow$$

$$\frac{1}{4}+\frac{1}{6} = \ldots$$

$$\downarrow$$

$$\frac{1}{4}+\frac{1}{6}-\frac{1}{12} = \ldots$$

$$\downarrow$$

$$\frac{1}{4}+\frac{1}{6}+\frac{1}{12} = \ldots$$

$$\downarrow$$

$$1\frac{1}{4}+2\frac{1}{6}+3\frac{1}{12} = \ldots$$

$$\downarrow$$

$$\frac{5}{4}+\frac{13}{6}+\frac{37}{12} = \ldots$$

$$\downarrow$$

$$\frac{5}{8}+\frac{13}{12}+\frac{37}{24} = \ldots$$

$$\downarrow$$

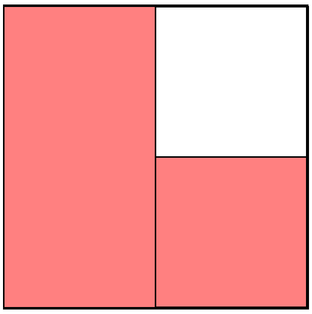
$$\frac{15}{8}+\frac{39}{12}+\frac{111}{24} = \ldots$$

$$\downarrow$$

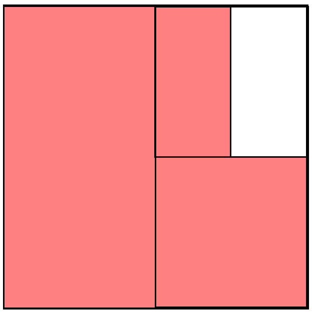
$$\frac{1}{4}+\frac{15}{8}+\frac{39}{12}+\frac{111}{24} = \ldots$$

Half of half of half of ...

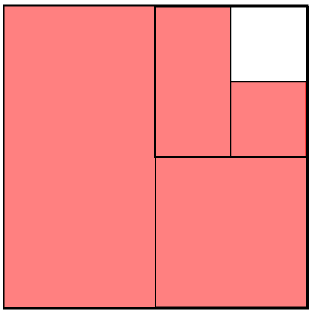
15. Fill in appropriate fractions



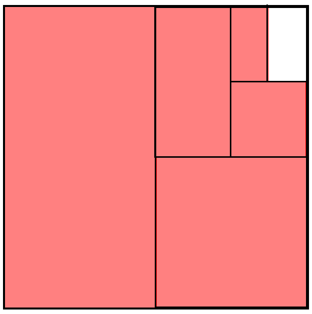
$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \dots$$



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \dots$$



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \dots$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \frac{1}{512} + \frac{1}{1024} = \dots$$

What does each letter stand for?

16. The letters ***a***, ***b***, ***c***, ***d***, ***e***, ***f***, ***g***, ***h*** each stand for a whole number.
Find these eight numbers.

$\frac{1}{5} + \frac{1}{13} = \frac{a}{65}$ → ***a* = ...**

$\frac{2}{5} - \frac{3}{8} = \frac{b}{40}$ → ***b* = ...**

$\frac{5}{14} + \frac{2}{21} = \frac{19}{c}$ → ***c* = ...**

$\frac{4}{7} + \frac{d}{8} = \frac{53}{56}$ → ***d* = ...**

$\frac{11}{20} - \frac{e}{30} = \frac{1}{60}$ → ***e* = ...**

$\frac{f}{100} - \frac{1}{25} = \frac{1}{50}$ → ***f* = ...**

$\frac{3}{g} + \frac{4}{g} = \frac{7}{11}$ → ***g* = ...**

$\frac{h}{5} + \frac{h}{10} = \frac{9}{10}$ → ***h* = ...**

Check that $a + b + c + d + e + f + g + h = 100$

Fractions in Egypt (1)

In Egypt, 4000 years ago, some people already were good in doing calculations with fractions. This is shown by problems on papyrus from that period. A sample problem is;
How should 7 loafs of bread be shared equally amongst 12 workers?
The solution went like this
Break all loafs in two equal parts. Each worker gets one half. Now divide the two halfs that are left over in six equal parts.

17. Show that this problem is represented by the problem $\frac{7}{12} = \frac{1}{2} + \frac{1}{12}$

18. Use the Egyptian method; 15 workers share 8 loafs of bread.
Write the fractions problem that represents this problem.

The Egyptions only used fractions with **numerator** 1, like

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, and so on

These fractions are named **unit fractions**.
The only exception was the fraction $\frac{2}{3}$ named '*the two parts*' by them.
All other fractions were split into two or more unit fractions by the Egyptians.
The denominators of these unit fractions were different.
Example. Right $\mathbf{\frac{2}{7} = \frac{1}{4} + \frac{1}{28}}$, but wrong $\frac{2}{7} = \frac{1}{7} + \frac{1}{7}$

19. a. Check that this division is right $\mathbf{\frac{2}{7} = \frac{1}{4} + \frac{1}{28}}$
b. Split into two different unit fractions

$\frac{3}{8}$ = … + …	$\frac{5}{12}$ = … + …	$\frac{3}{5}$ = … + …
$\frac{4}{9}$ = … + …	$\frac{4}{7}$ = … + …	$\frac{11}{18}$ = … + …
$\frac{2}{9}$ = … + …	$\frac{3}{10}$ = … + …	$\frac{13}{22}$ = … + …

Fractions in Egypt (2)

20. Sometimes one fractions can be split into unit fractions in different ways.

Example $\frac{7}{12} = \frac{1}{2} + \frac{1}{12}$, but also $\frac{7}{12} = \frac{1}{3} + \frac{1}{4}$

Check the two splittings.

Now split $\frac{7}{24}$ into two ways

Also split $\frac{11}{30}$ in two ways

21. Find another fraction that can be split into unit fractions in two different ways.

22. Often more than two unit fractions are necessary when splitting a fraction.

Example $\frac{4}{5} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$

Split into three different unit fractions

$\frac{7}{8}$ = … + … + …

$\frac{11}{12}$ = … + … + …

$\frac{8}{9}$ = … + … + …

$\frac{23}{24}$ = … + … + …

23. When doing calculations, the Egyptians often used the method of *doubling* numbers. For that purpose, they used long lists of fractions split into a fraction with *numerator* 2. The denominators of the fractions to be split into unit fractions were limited to *odd* numbers.

Why do you think they did not use *even* denominators?

In one of these old lists, the denominators are odd and divisible by 3.
Here you see part of such a list.
Find the next three entries on the list.

$\frac{2}{9} = \frac{1}{6} + \frac{1}{18}$

$\frac{2}{15} = \frac{1}{10} + \frac{1}{30}$

$\frac{2}{21} = \frac{1}{14} + \frac{1}{42}$

$\frac{2}{27} = \frac{1}{18} + \frac{1}{54}$

……………………

The inheritance

24. An old Indian in the Andes writes in his last will how, after his death, his herd of llama's should be divided amongst his four sons. According to this will, the eldest son will inherite one third of the herd, the second son will get one quarter and the youngest two sons, twins, will get one fifth of the herd each.

Half a year later, the Indian dies. At that moment, his herd consists of 59 llama's. The four sons are desperate, how to share these equally according to the will?
They go to an Indian, known as a very wise man, for advice.
He says; I have got 1 llama which you may borrow and use to find a fair division.
Now there are 60 llama's and so the eldest son gets 20 llama's, the second son gets 15 and the youngest sons get 12 llama's each. This adds up to 59 and the wise man is given back his own llama.

A strange story, but by using fractions you can explain how this worked.
Just try!

25. Now think of a similar story about an old woman and her three daughters.

Statements about fractions

26. Next to each statement, write TRUE or NOT TRUE.

$2 + 3 = 5$
$5 + 7 = 12$ so $\frac{2}{5} + \frac{3}{7} = \frac{5}{12}$

$1 - \frac{28}{57} = \frac{29}{57}$

$1\frac{1}{4} + \frac{1}{3} = \frac{3}{4} + \frac{5}{6}$

The result of $\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7}$ is a whole number

The result of $\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} + \frac{7}{8}$ is a whole number

$\frac{5}{6} + \frac{1}{7}$ is smaller than 1

Each non-unit fraction between 0 and 1 equals the sum of two unit fractions

$\frac{5}{8}$ en $\frac{8}{13}$ are on the number line less than $\frac{1}{100}$ apart

Find two true statements and two untrue statements yourself.

Together one

27. In this magic square, consisting of fractions, the sum of all fractions in a horizontal row, in each vertical column and on each diagonal equals 1.

$\frac{2}{5}$		
	$\frac{1}{3}$	
$\frac{8}{15}$		

Three fractions are already filled in. Finish the magic square.

28. $\frac{1}{2} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} + \frac{1}{40} = 1$

You should not take that for granted, of course, but check!

Almost without any calculations, you immediately see the outcome of

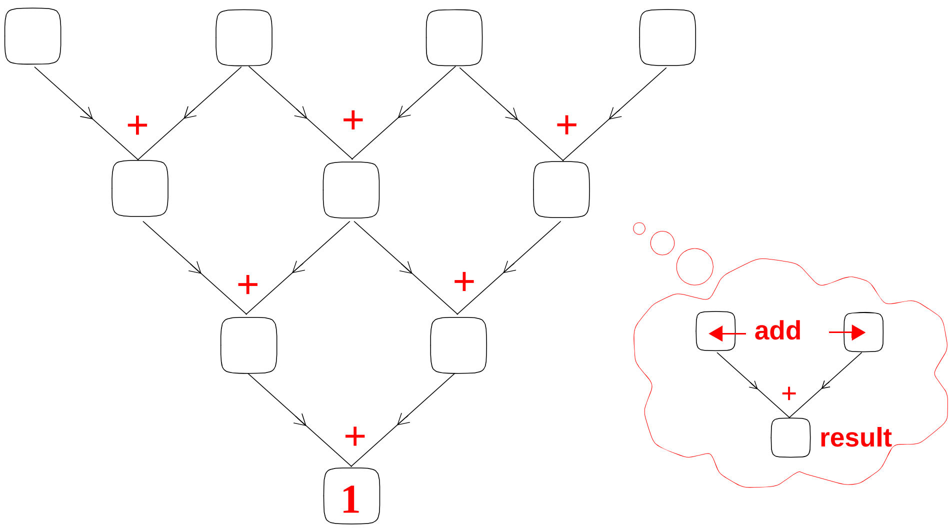
$$\frac{1}{4} + \frac{1}{12} + \frac{1}{16} + \frac{1}{20} + \frac{1}{24} + \frac{1}{80}$$

Explain how that is possible.

Now find the sum of *seven* different unit fractions with the outcome 1.

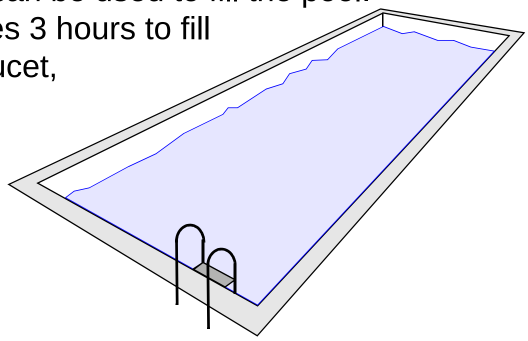
Do the same for *eight* different unit fractions.

29. Fill in fractions in the 'addition tree'. All fractions must be different.



Filling a swimming pool

30. Water in a swimming pool should be refreshed regularly.
There are two faucets that can be used to fill the pool.
Using the first faucet, it takes 3 hours to fill the pool, using the other faucet, it takes 5 hours.



If both faucets are opened at the same time and run for exactly one hour, the swimming pool is filled for a little over one half.

Explain this by using fractions.

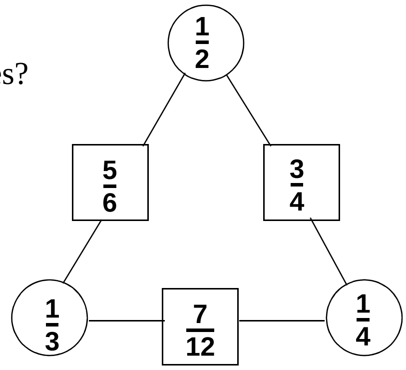
If after this first hour, the faucets are kept open for another half hour, the pool is filled four fifth.

Explain this as well.

How many minutes are still necessary to fill the pool completely?

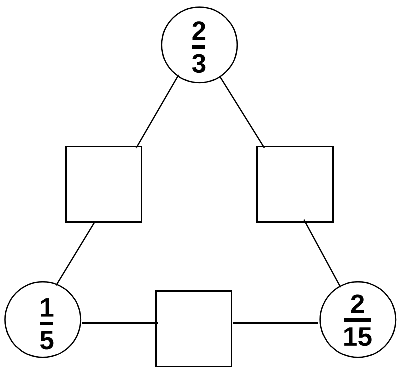
Circuit sums

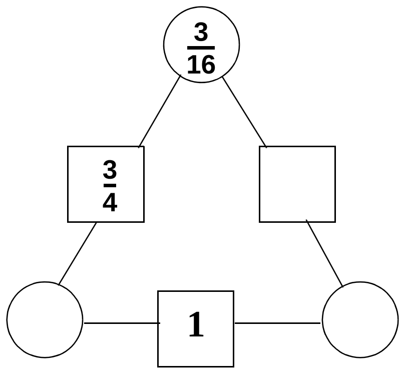
31. What is the relation between the fractions in the circles of this circuit? And the squares?

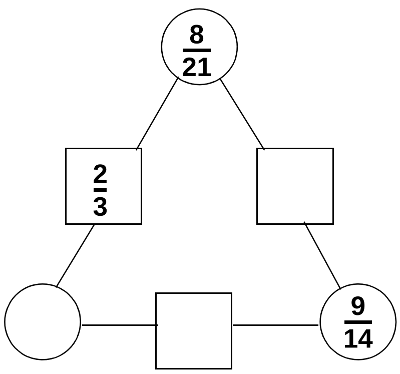


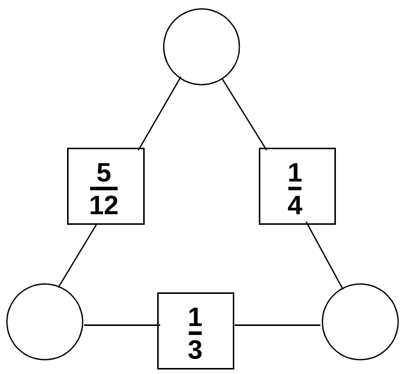
Add the three fractions in the circles.
Do the same with the fractions in the squares.
Compare the two results. What do you notice?
Could you have predicted this without doing a calculation? How?

Fill in fitting fractions with the smallest possible numerator and denominator









The sum of two unit fractions (1)

32. ***a*** and ***b*** represent whole numbers that are 17 if added together.
The number ***a*** is smaller than ***b***, but greater than 1.
In short, $a + b = 17$, $a < b$ and $a > 1$.

Fill in the table

a	b	$a + b$	$a \times b$	$\frac{1}{a} + \frac{1}{b}$
2	15	17	30	$\frac{1}{2} + \frac{1}{15} = \frac{17}{30}$
3		17		
4		17		
5		17		
6		17		
7		17		
8		17		

33. ***a*** and ***b*** represent whole numbers with 210 as the outcome if multiplied.
Again the number ***a*** is smaller than ***b***, but greater than 1.
In short $a \times b = 210$, $a < b$ and $a > 1$.

Fill in the table

a	b	$a + b$	$a \times b$	$\frac{1}{a} + \frac{1}{b}$
2	105	107	210	$\frac{1}{2} + \frac{1}{105} = \frac{107}{210}$
3			210	
5			210	
6			210	
7			210	
10			210	
14			210	

The sum of two unit fractions (2)

34. Consider the tables of problems 32 and 33.

In all cases, for the sum of the fractions $\frac{1}{a}$ and $\frac{1}{b}$ we find

* the *numerator* equals $a + b$ and

* the *denominator* equals $a \times b$

Using a *formula*, this can be noted in short

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{a \times b}$$

If for example $a = 3$ en $b = 5$, we find $\frac{1}{3} + \frac{1}{5} = \frac{3+5}{3 \times 5} = \frac{8}{15}$

Now find three other examples yourself and each time check whether the formula shows the right result.

$\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

Does this match with the result if the formula is used? Why?

a and b may also have the same value.

For example, $a = 7$ en $b = 7$

Using the formula, you find $\frac{1}{7} + \frac{1}{7} = \frac{7+7}{7 \times 7} = \frac{14}{49}$

Give your comment on this result.

What do you think of this formula $\frac{1}{a} + \frac{1}{a} = \frac{2}{a}$? Is it right or wrong?

Altogether now

Add fractions with the same denominator by adding the numerators.

Examples

$$\frac{2}{17} + \frac{3}{17} = \frac{5}{17}$$

$$\frac{17}{30} + \frac{23}{30} = \frac{40}{30} = \frac{4}{3} = 1\frac{1}{3}$$

If fractions have different denominators, before adding them you need to make *equal denominators* first.

Examples

$$\frac{2}{17} + \frac{5}{34} = \frac{4}{34} + \frac{5}{34} = \frac{9}{34}$$

$$\frac{17}{30} + \frac{17}{40} = \frac{68}{120} + \frac{51}{120} = \frac{119}{120}$$

Subtract fractions by making *equal denominators,* if necessary, and subtract the numerators.

Examples

$$\frac{13}{48} - \frac{11}{48} = \frac{2}{48} = \frac{1}{24}$$

$$\frac{17}{30} - \frac{17}{40} = \frac{68}{120} - \frac{51}{120} = \frac{17}{120}$$

Unit fractions always have a numerator which is equal to 1.

Each non-unit fraction smaller than 1 can be *split* into a sereis of different unit fractions.

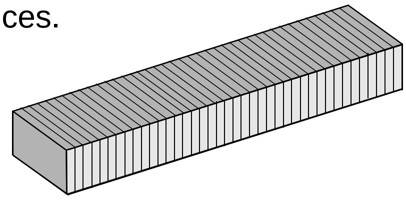
Examples

$$\frac{13}{48} = \frac{12}{48} + \frac{1}{48} = \frac{1}{4} + \frac{1}{48}$$

$$\frac{23}{30} = \frac{15}{30} + \frac{8}{30} = \frac{1}{2} + \frac{4}{15} = \frac{1}{2} + \frac{3}{15} + \frac{1}{15} = \frac{1}{2} + \frac{1}{5} + \frac{1}{15}$$

Mixed up!

1. The gingerbread is divided into 40 equal slices.
   Is the bread used up if $\frac{3}{8}$ part and $\frac{3}{5}$ part of the whole gingerbread is handed out?
   Explain how you found your answer.



2. What must be added?
   Complete to the nearest whole number $\frac{81}{12}$ + ... = ...

3. Fill in a fraction in the simplest form $\frac{5}{6} + \frac{3}{20}$ + ... = 1

4. A problem written on an Egyptian papyrus.
   How should 5 loafs of bread be shared fairly between 8 workers?

   Write the fractions problem that represents this problem.
   Remember that only unit fractions may be used.

5. For each statement, decide whether it is *true* or *not true*.
   Explain why you think so.

   a. $1\frac{7}{8} - \frac{27}{32}$ is less than 1.

   b. $\frac{2}{9}$ and $\frac{3}{10}$ added together are more than one half.

   c. If $\frac{2}{n} + \frac{3}{n} = \frac{1}{2}$ than *n* equals 10.

6. Fill in the addition table (fractions in the simplest form)

+			$\frac{1}{10}$	$\frac{5}{6}$	
$\frac{2}{5}$	1	$\frac{8}{15}$			
				$1\frac{1}{6}$	

7. Think of a question that would fit in this section (B) of the book.
   Also provide a solution.