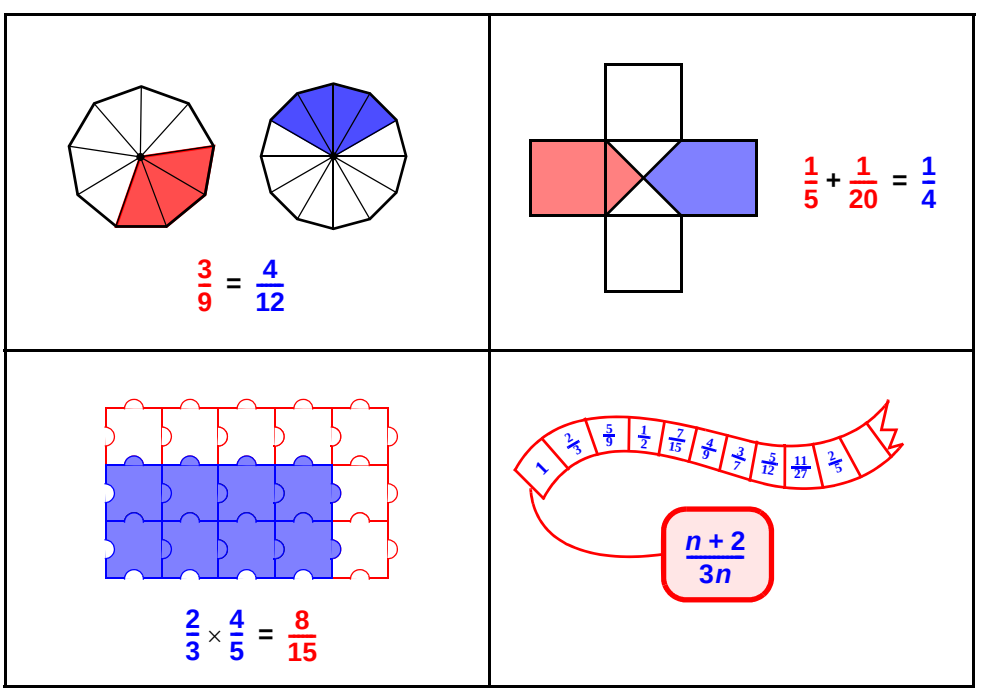
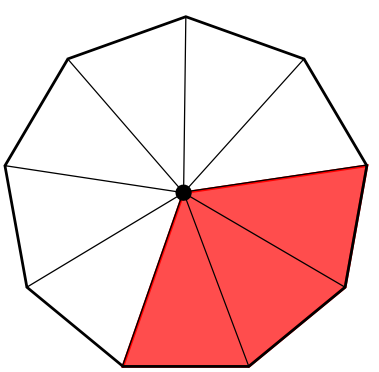
Fraction Tools

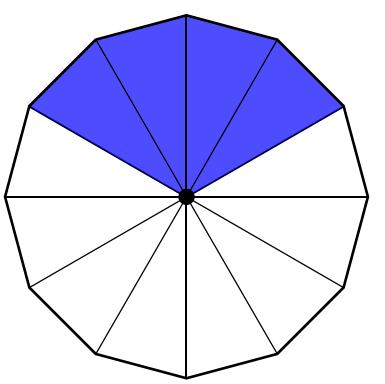


Martin Kindt & Truus Dekker

Section A

Comparing Fractions

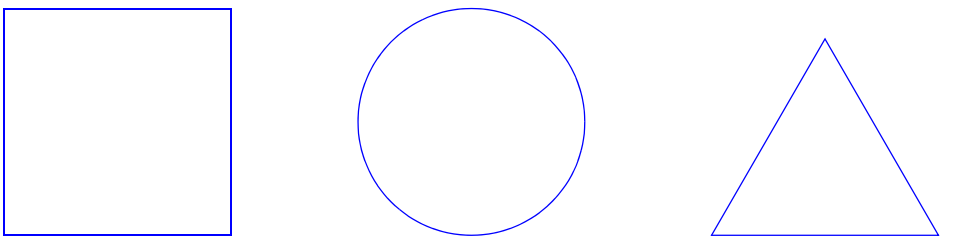




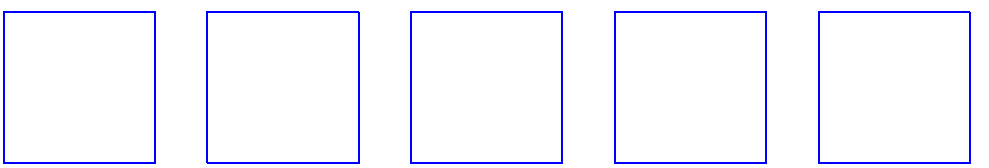
$$\frac{3}{9} = \frac{4}{12}$$

Parts and pieces (1)

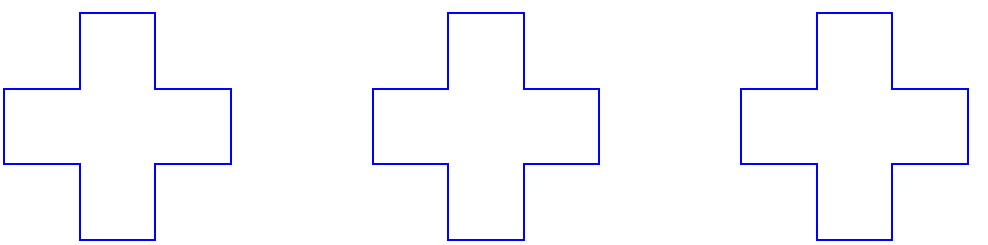
1. Of each figure, color $\frac{1}{6}$ part. Be as precise as possible.



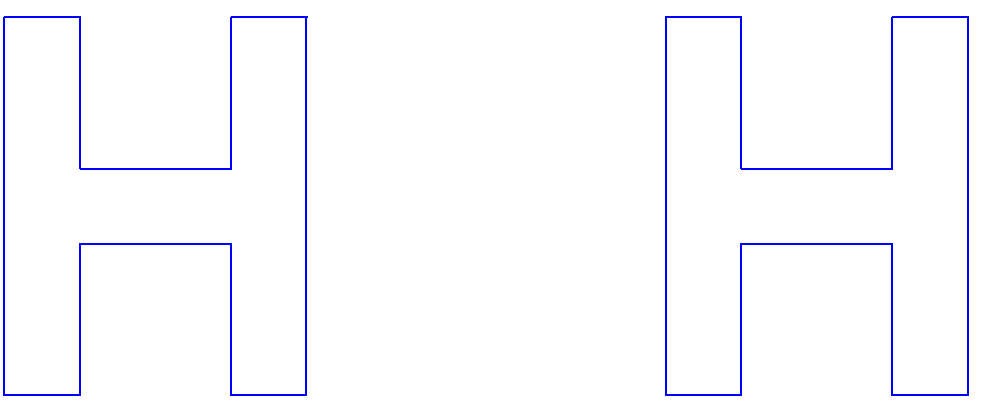
2. Of each figure, color $\frac{1}{4}$ part. Each time, use a different way.



3. In each figure, color the indicated part.



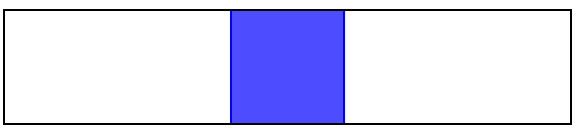
$\frac{1}{5}$ $\frac{1}{4}$ $\frac{1}{3}$



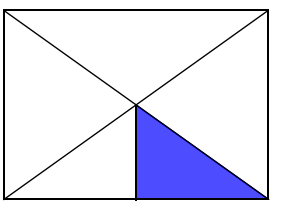
$\frac{1}{6}$ $\frac{1}{12}$

Parts and pieces (2)

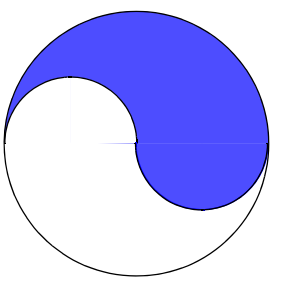
4. Which part of each figure is colored? Fill in the table.



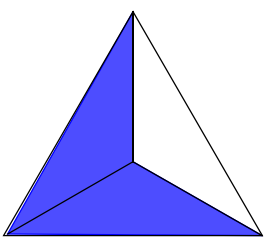
A



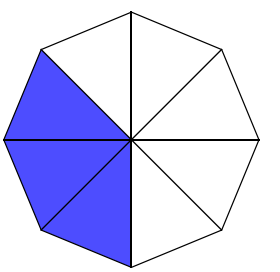
B



C



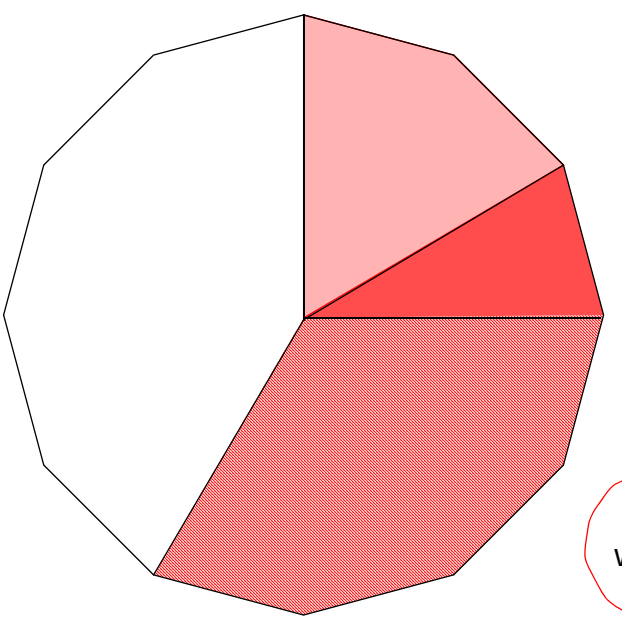
D

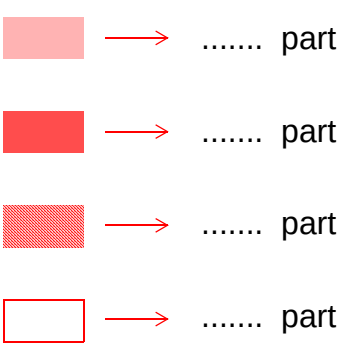


E

Figure	A	B	C	D	E
Fraction					

5. This twelve-sided polygon is divided in four pieces. Which parts?



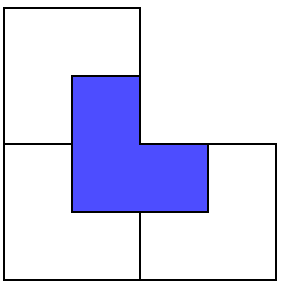


If you add up all fractions, what is the answer?

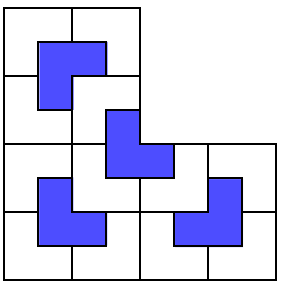
.....

Fractions and figures

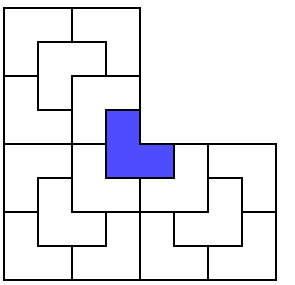
6. Which part of each figure is **not** colored?



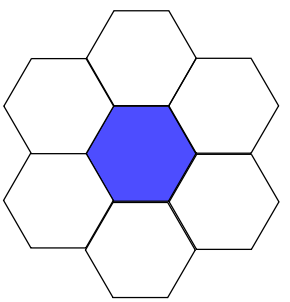
..... part



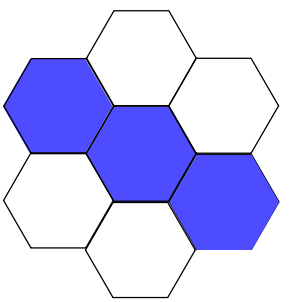
..... part



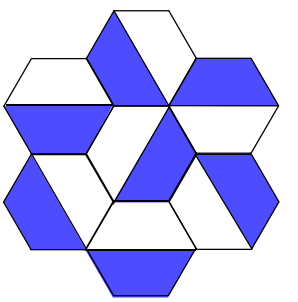
..... part



..... part

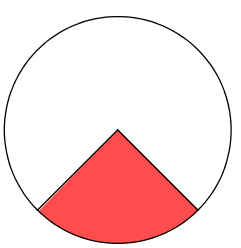


..... part

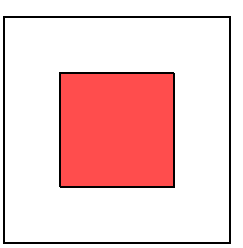


..... part

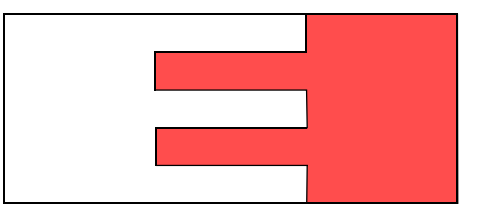
7. Look **carefully** because it is **wrong**!
   Fix the drawing or change the fraction.



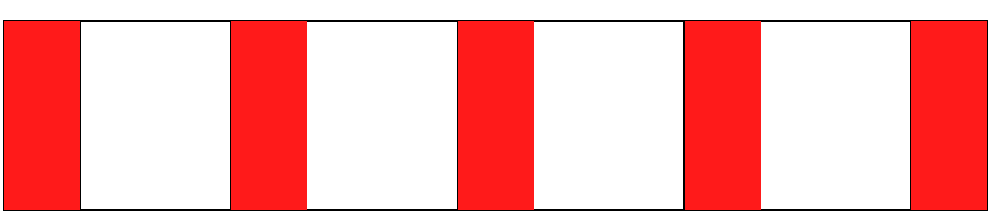
$\frac{1}{3}$



$\frac{1}{2}$



$\frac{1}{2}$

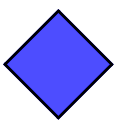


$\frac{1}{3}$

Part and whole

8.

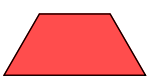
$\frac{1}{4}$ part is shown.
Draw the whole figure.



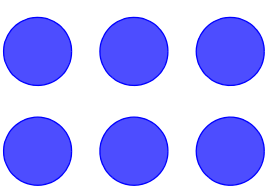
$\frac{1}{6}$ part is shown.
Draw the whole figure.

$\frac{2}{3}$ part is shown.
Draw the whole figure.

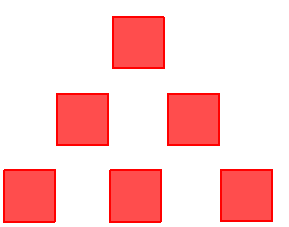
$\frac{3}{4}$ part is shown.
Draw the whole figure.



$\frac{3}{5}$ part is shown.
Draw the whole figure.



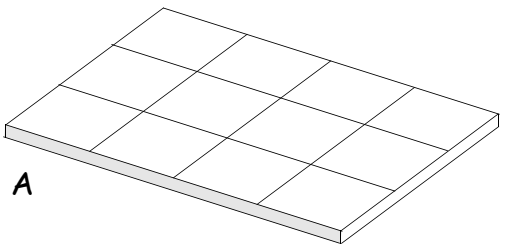
$\frac{3}{5}$ part is shown.
Draw the whole figure.

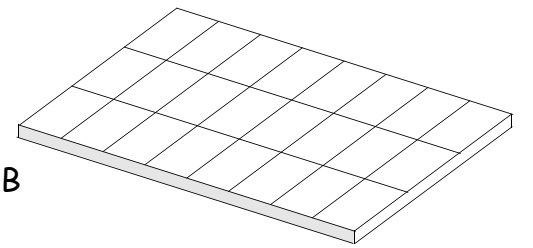


9. Now think of a similar problem yourself. Also provide the answer.

Numerator and denominator (1)

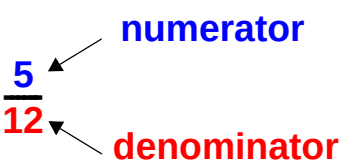
Two bars of white chocolate: one has 12 pieces, the other has 24 pieces.





Of bar A, 5 pieces are broken off. That is $\frac{5}{12}$ part of the whole bar.

The number 12 **names** the total number of pieces (in the bar)
The number 5 **counts** the number of pieces taken off.
Of the fraction $\frac{5}{12}$,
12 is named the **denominator**
5 is named the **numerator**



10. You also want $\frac{5}{12}$ part of chocolate bar B.

    How many pieces should be broken off bar B? .....

    Which fraction part of the whole bar? .....

11. You have broken off $\frac{5}{12}$ part of a bar and you want to share this fairly with somebody.

    Which part of the whole bar does each person get? .....

We will name the part of a chocolate bar a 'portion' for now.
Two (or more) fractions, with different numerators and denominators could represent equal portions of chocolate.

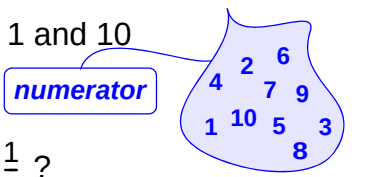
Example: a portion of $\frac{1}{3}$ of a bar is equal to a portion of $\frac{4}{12}$ bar.

12. Find some new examples of different fractions that represent equal portions.

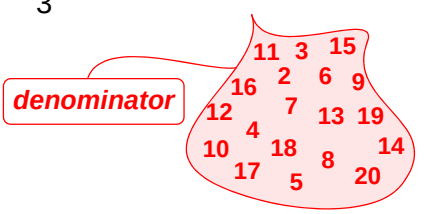
Numerator and denominator (2)

If two fractions have different numerators and denominators and represent equal portions, these fractions are named **equivalent**.

13. Think of all fractions with a *numerator* between 1 and 10 and a *denominator* between 2 and 20.



Which of all these fractions are equivalent with $\frac{1}{3}$?



And which fractions are equivalent with $\frac{3}{5}$?

14. Draw a connecting line between equivalent fractions:

$\frac{6}{24}$	$\frac{7}{24}$	$\frac{8}{24}$	$\frac{9}{24}$

$\frac{10}{40}$	$\frac{3}{9}$	$\frac{3}{8}$	$\frac{28}{96}$

15. Consider the bottom row of four fractions in question 14.
For which fraction(s) does an equivalent fraction exist with a smaller numerator and denominator?
For each of these fractions, find the one with the smallest numerator and denominator.

16. Find the fraction with the smallest numerator and denominator which is equivalent with

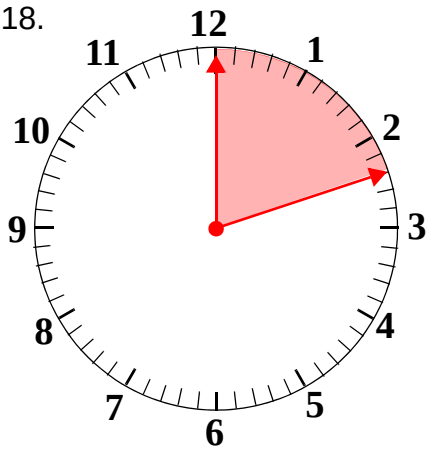
$\frac{9}{36}$ = $\frac{12}{36}$ = $\frac{10}{34}$ = $\frac{10}{90}$ = $\frac{20}{70}$ =

$\frac{11}{55}$ = $\frac{22}{55}$ = $\frac{25}{55}$ = $\frac{50}{110}$ = $\frac{11}{110}$ =

17. Of a fraction, the numerator equals 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15.
The denominator equals 2 + 6 + 10 + 14 + 18 + 22 + 26 + 30.
Find an equivalent fraction. Numerator and denominator should be as small as possible.

Fractions on the clock

18.

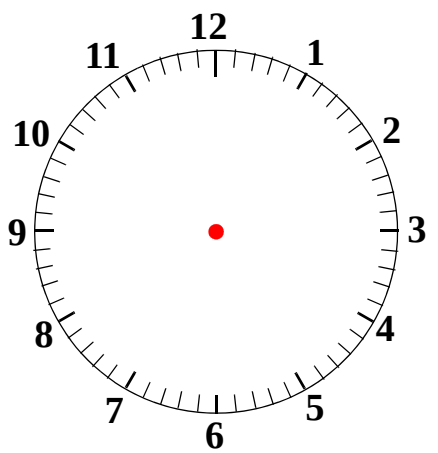


The seconds hand of the clock has run 12 seconds.

This is $\frac{\text{.....}}{\text{.....}}$ part of one minute

How many seconds is $\frac{3}{10}$ minute?

.....



The part of the clockface colored red in the first figure, is named a **sector**.

Color a sector that belongs to

$\frac{4}{15}$ minute.

19. Fill in the blanks

$\frac{7}{12}$ minute = ... seconds

$\frac{11}{20}$ minute = ... seconds

90 seconds = $\frac{\text{.....}}{\text{.....}}$ minute

54 seconds = $\frac{\text{.....}}{\text{.....}}$ minute

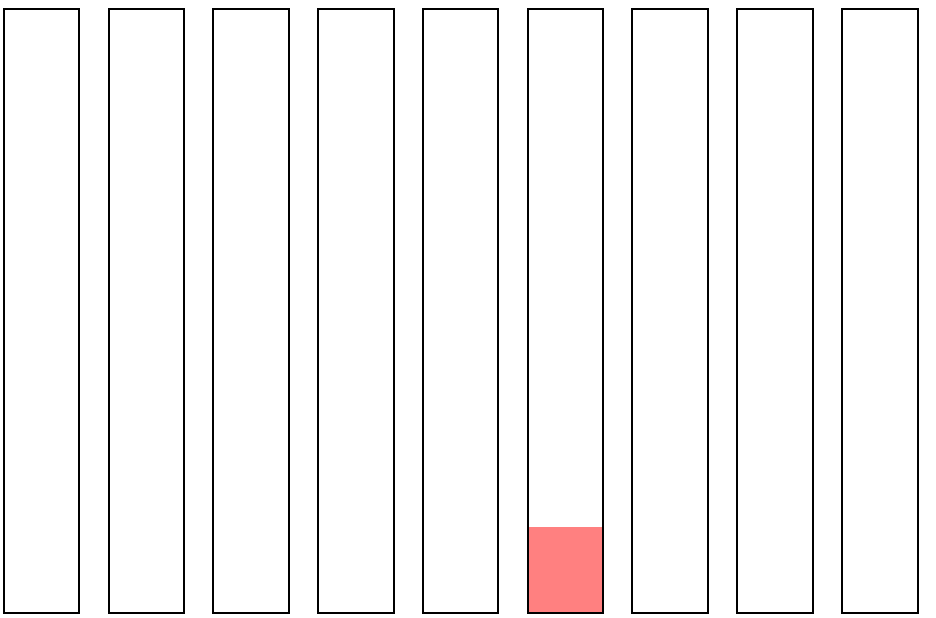
20. The seconds hand first runs $\frac{1}{5}$ minute and next $\frac{1}{6}$ minute.

How many seconds in total? .....

Which fraction has the same value as $\frac{1}{5} + \frac{1}{6}$? .....

Large and small

21. Of each bar, color the part that is given at the bottom.
Start coloring at the bottom of the bar.



Below each fraction, write the fraction that represents the part which is *not* colored. What do you notice?

22. The top row of fractions with a numerator of 1, can be extended to the right.
Each time, the denominator gets larger.
The part that should be colored then gets.....
Is there a smallest possible fraction, with numerator 1 ? .....
Explain why or why not.

Now consider the bottom row of fractions below the bars.

Which fraction represents the larger part, $\frac{7}{8}$ or $\frac{8}{9}$?

23. In order to write '**is smaller than**' we use the symbol **<** .

Instead of $\frac{1}{6}$ bar is smaller than $\frac{1}{5}$ bar, we write in short $\frac{1}{6} < \frac{1}{5}$.

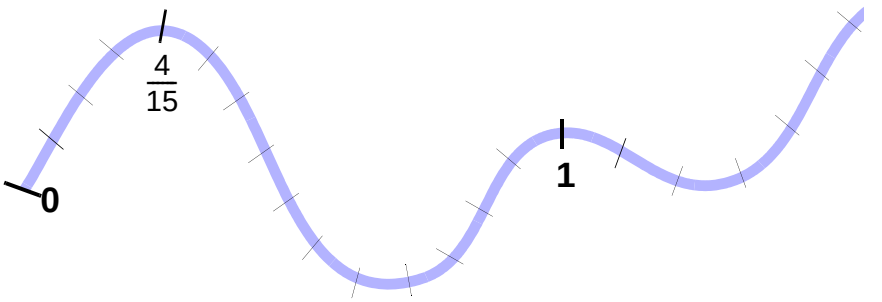
You may also write $\frac{1}{5} > \frac{1}{6}$, which means one fifth **is greater than** one sixth.

Use the appropriate symbol, < or > and fill in the blank.

$\frac{1}{4}$ .... $\frac{1}{5}$ $\quad$ $\frac{3}{4}$ .... $\frac{4}{5}$ $\quad$ $\frac{1}{30}$ .... $\frac{1}{20}$ $\quad$ $\frac{8}{9}$ .... $\frac{9}{10}$ $\quad$ $\frac{19}{20}$ .... $\frac{9}{10}$

Fractions on a number line (1)

24. This curved number line is divided in equal parts.



At each stroke between 0 and 1, write an appropriate fraction.
Make the numerator and denominator as small as possible!

How could you continue past 1 ? .....

Draw a stroke that is exactly in the middle between $\frac{7}{15}$ and $\frac{8}{15}$

Which fraction belongs at this stroke? .....

25. A number line can be used as a help to solve fraction problems.

Find appropriate fractions (numerator and denominator as small as possible!);

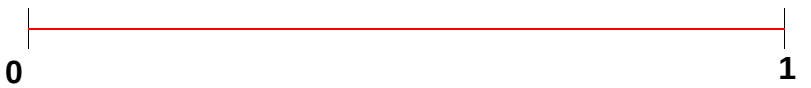
$\frac{1}{15} + \frac{2}{15} = \ldots$ $\quad$ $\frac{4}{15} + \frac{7}{15} = \ldots$ $\quad$ $\frac{8}{15} + \frac{7}{15} = \ldots$

$\frac{7}{15} + \frac{1}{3} = \ldots$ $\quad$ $\frac{7}{15} - \frac{4}{15} = \ldots$ $\quad$ $\frac{11}{15} + \frac{1}{3} = \ldots$

Now make some fraction problems yourself for which a number line can be used as a help. Also provide answers.

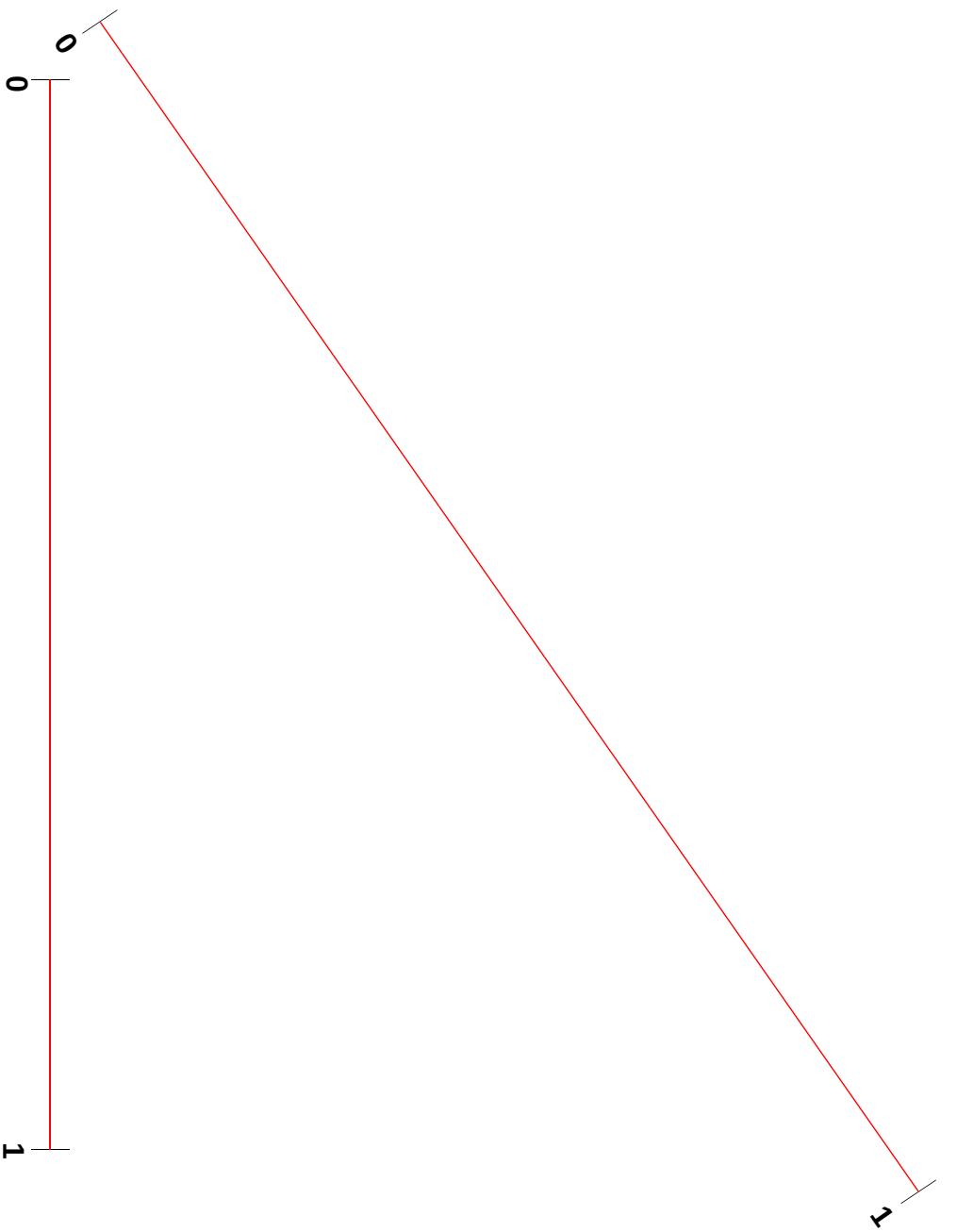
Fractions on a number line (2)

26.



Where are $\frac{1}{2}$, $\frac{1}{5}$, $\frac{4}{5}$ and $\frac{1}{10}$?

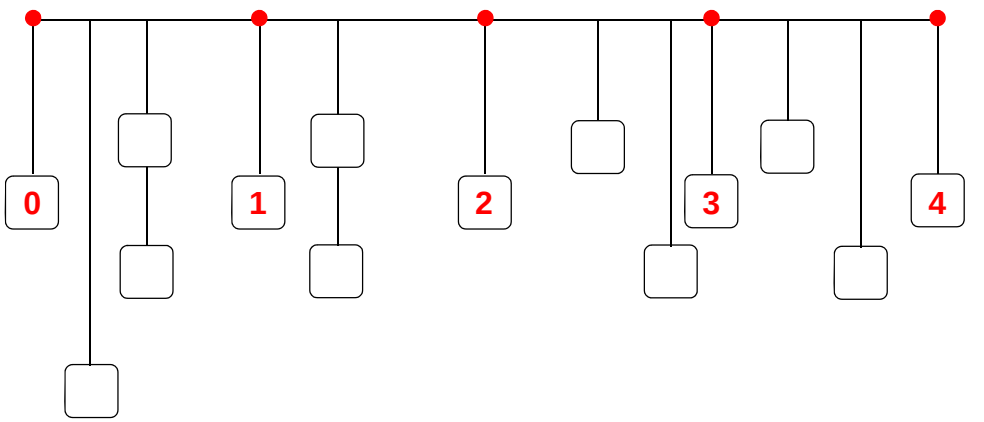
Indicate the position of each fraction on the number line and write the fraction underneath. Do the same for each number line.



Fractions on the clothesline

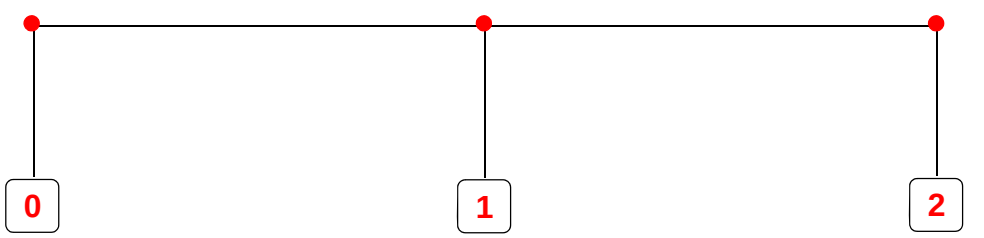
27. Write the fractions on the right labels:

$2\frac{1}{2}$ $1\frac{1}{3}$ $\frac{1}{2}$ $\frac{10}{3}$ $\frac{1}{4}$ $\frac{3}{6}$ $3\frac{2}{3}$ $\frac{4}{3}$ $2\frac{5}{6}$



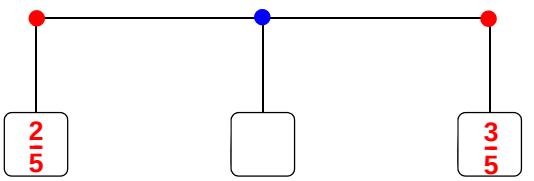
28. Hang the labels in the right place. Be as precise as possible.

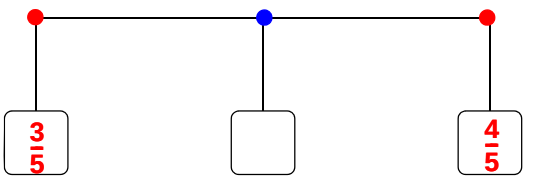
$\frac{5}{10}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{6}{5}$ $\frac{5}{5}$ $1\frac{4}{5}$ $1\frac{5}{6}$ $\frac{13}{12}$

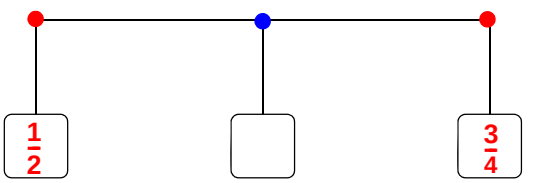


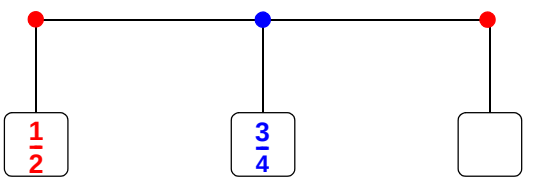
Fractions between fractions

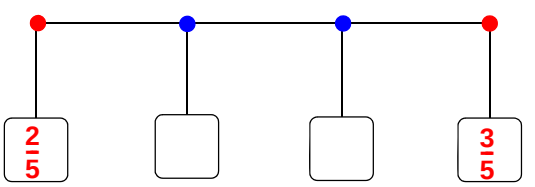
29. Write appropriate fractions on the labels.
Make the denominator and the numerator as small as possible.

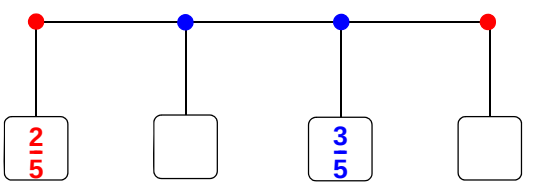


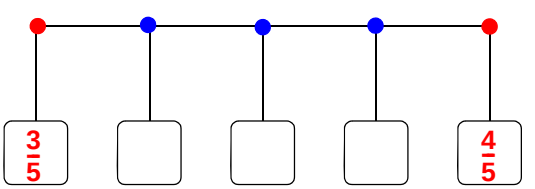


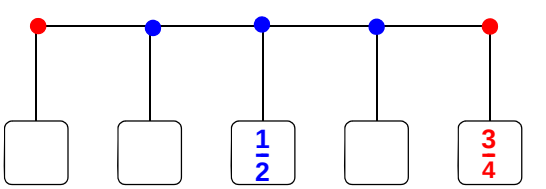






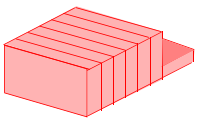






Fraction and whole

30. 7 slices of gingerbread, shared fairly by 3 children.
How many slices for each? .....



11 pancakes, shared fairly by 4 children.
How much for each child? .....

How could you explain that $\frac{16}{5}$ is equal to $3\frac{1}{5}$?

And why is $4\frac{2}{3}$ equal to $\frac{14}{3}$?

31. Split off as many 'wholes' as possible:

$\frac{25}{4}$ = ... $\frac{25}{6}$ = ... $\frac{35}{8}$ = ... $\frac{40}{9}$ = ... $\frac{42}{9}$ = ...

Make one fraction:

$4\frac{1}{2}$ = $\frac{\cdots}{\cdots}$ $8\frac{1}{4}$ = $\frac{\cdots}{\cdots}$ $5\frac{3}{4}$ = $\frac{\cdots}{\cdots}$ $4\frac{3}{5}$ = $\frac{\cdots}{\cdots}$ $7\frac{1}{7}$ = $\frac{\cdots}{\cdots}$

32. $\frac{22}{7} = 3\frac{1}{7}$
$\frac{22}{3} = 7\frac{1}{3}$

An eye-catching pair! Find three similar pairs.

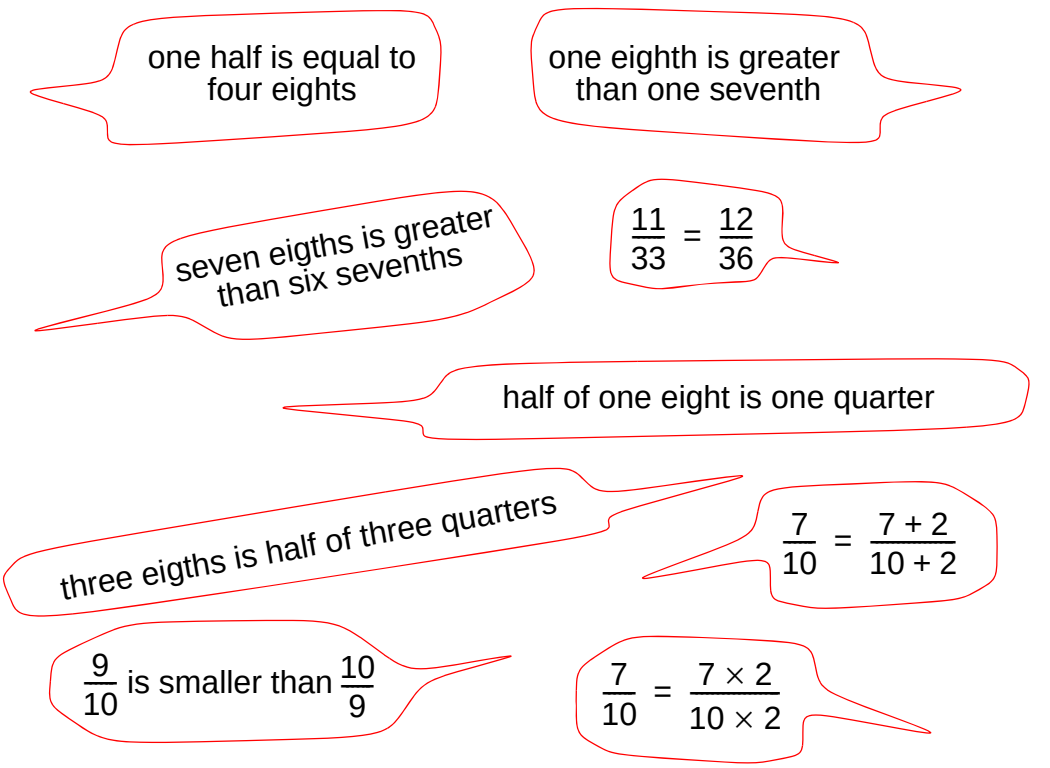
33. A soccer game takes $1\frac{3}{4}$ hours, including the break.
That is ..... minutes.

Sometimes, the referee allows for some extra minutes.

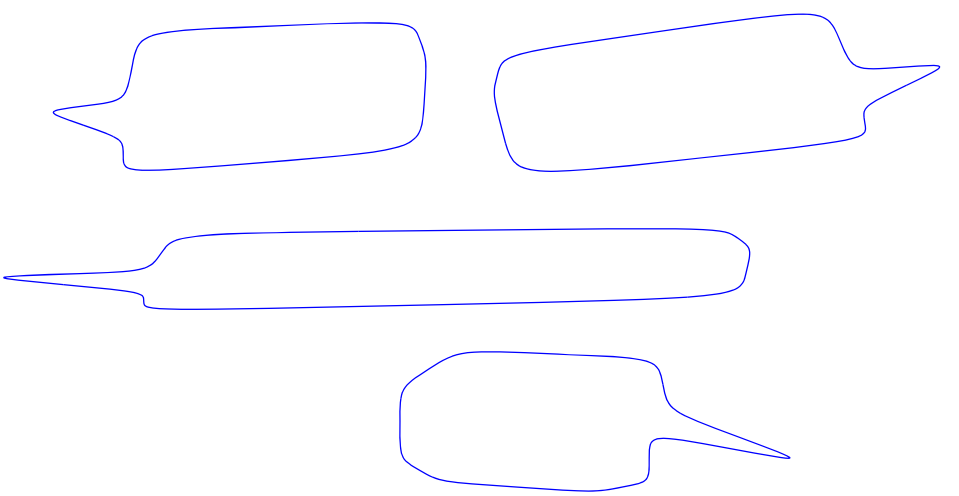
How much extra time did the game take if it finished after $1\frac{5}{6}$ hours?

Statements about fractions

34. Indicate whether the statement is TRUE or NOT TRUE



Find two true statements and two untrue statements yourself.



Equivalent fractions

35. On the dotted lines, write appropriate numbers.

$\frac{12}{\ldots} = \frac{2}{3}$	$\frac{5}{8} = \frac{625}{\ldots}$	$1\frac{1}{2} = \frac{\ldots}{12}$
$\frac{3}{4} = \frac{\ldots}{100}$	$\frac{8}{5} = \frac{1000}{\ldots}$	$2\frac{1}{2} = \frac{\ldots}{18}$
$\frac{\ldots}{49} = \frac{3}{7}$	$\frac{30}{48} = \frac{\ldots}{56}$	$2\frac{1}{2} = \frac{35}{\ldots}$
$\frac{15}{24} = \frac{10}{\ldots}$	$\frac{42}{48} = \frac{\ldots}{88}$	$3\frac{2}{3} = \frac{66}{\ldots}$

36. On the two dotted lines, fill in **equal** numbers. The statement must be true!

$$\frac{4}{\ldots} = \frac{\ldots}{9}$$

If different numbers are allowed on the dotted lines, there are eight possibilities to find a true statement.

Find as many of these possibilities as you can.

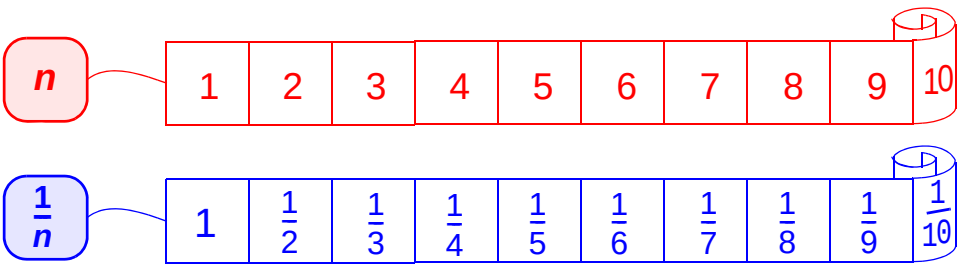
37. Write this fraction with the smallest possible numerator and denominator.

$$\frac{37 + 37}{37 + 37 + 37}$$

Now write another number to replace 37. Of course, the five numbers must be equal.
Does the value of the fraction change?

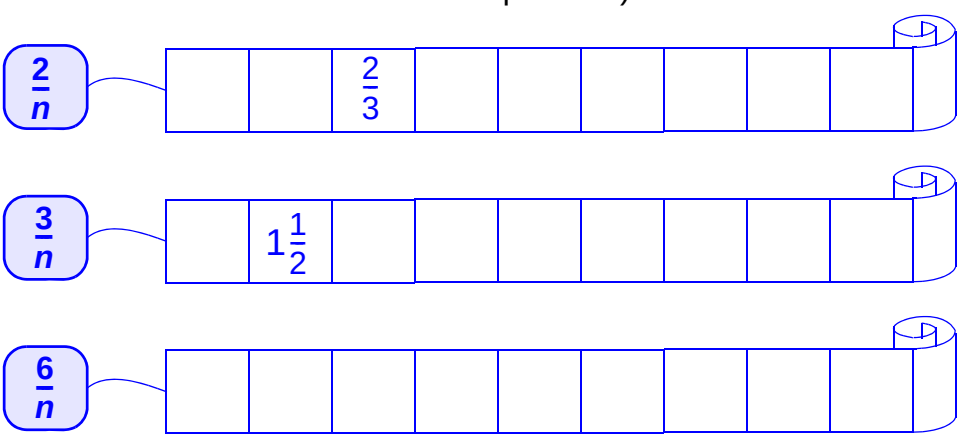
Strips and labels (1)

38.



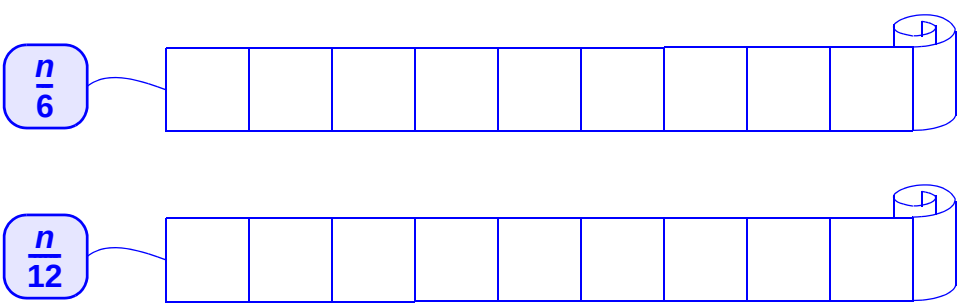
Explain why there is a number 1 on the strip which shows $\frac{1}{n}$ on its label.

Fill in fractions or whole numbers. Write each fraction in the simplest form (numerator and denominator as small as possible).



Which relationship exists between the numbers on the last two strips?

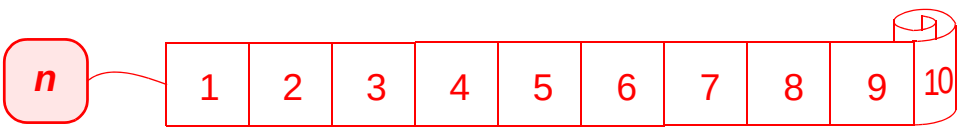
Again, fill in fractions or whole numbers. Each fraction in the simplest form.



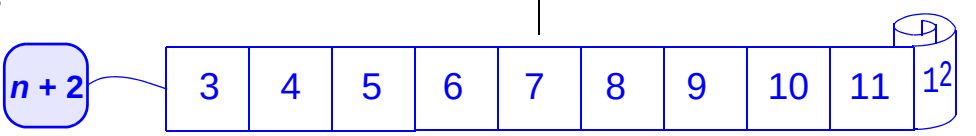
Which relationship exists between the numbers on the last two strips?

Strips and labels (2)

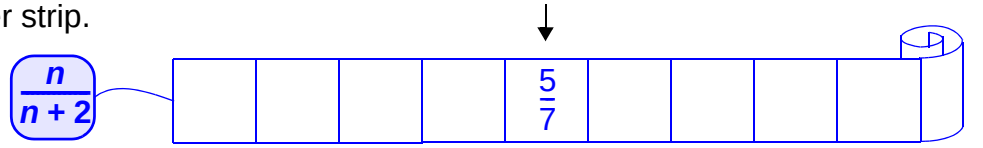
Again, here is a strip where ***n*** represents the numbers 1, 2, 3, ...



If 2 is added to each number on the strip you get the strip with ***n* + 2** on the label.

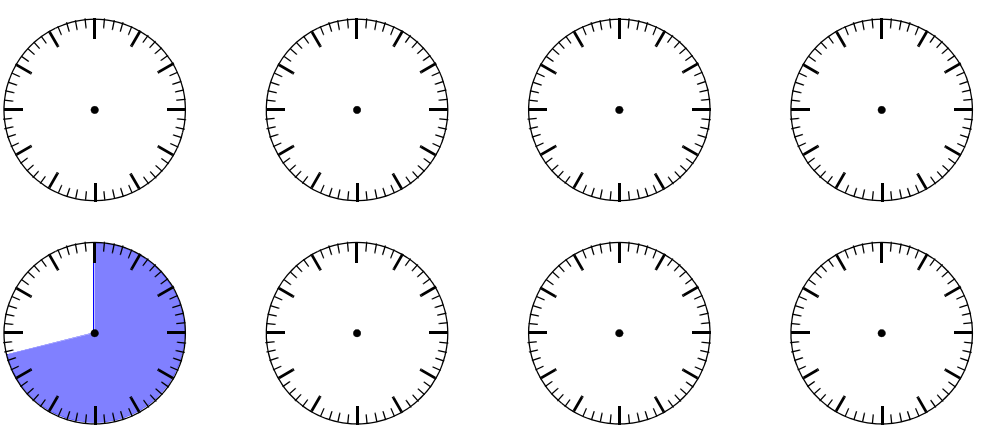


You can use the two strips to make fractions. The *numerator* is taken from the upper strip and the *denominator* is the corresponding number on the lower strip.

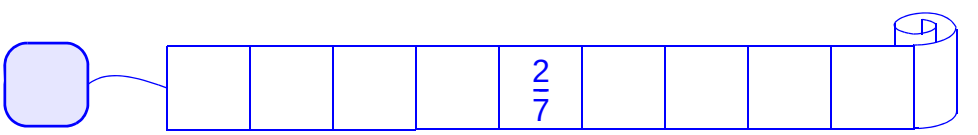


39. Fill in the strip (write fractions in the simplest form).

For the first eight fractions on the strip, color a corresponding sector of each clock. One sector is already done for you.

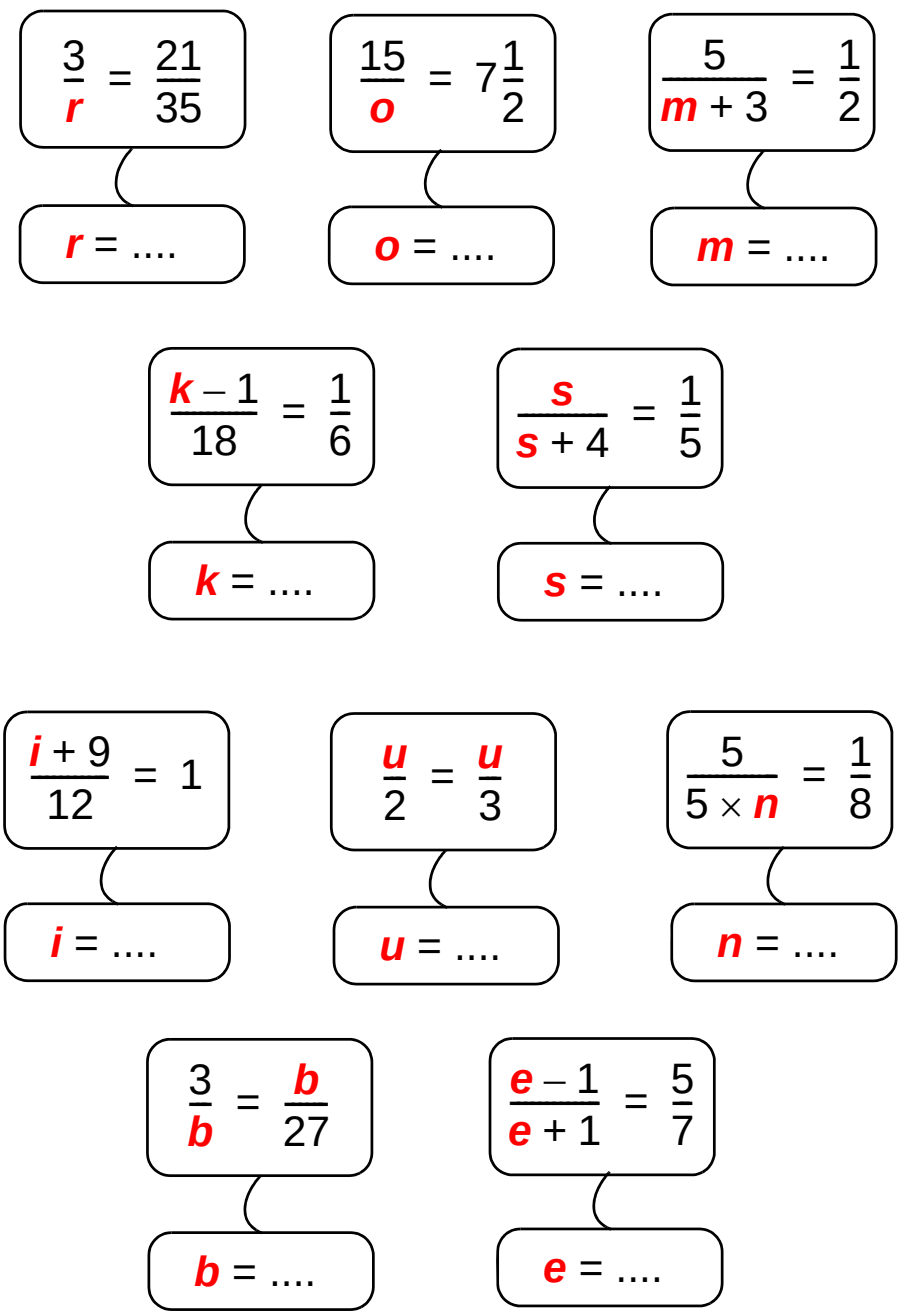


The sectors on each clock that are not colored have their own fraction.
Write these fractions on the strip below, in the right order.
What should be written on the label of the strip?



Breaking the code

40. The letters ***b, e, i, k, m, n, o, r, s, u*** represent one of the numbers **0, 1, 2, 3, 4, 5, 6, 7, 8** and **9**. Find the number that belongs to each letter.



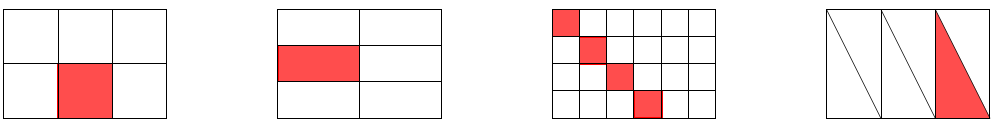
A statement with a code

7256 8079651 38 1071

What does it say?

Altogether now

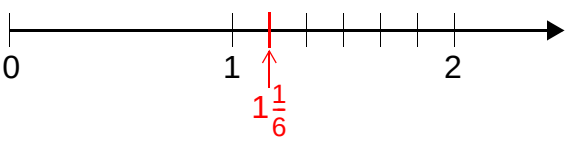
* Coloring $\frac{1}{6}$ part of a figure can be done in different ways.



* $\frac{5}{6}$ of some amount can be calculated by dividing the amount by 6 and multiplying the outcome by 5.

$\frac{5}{6}$ of 90 is $5 \times 15 = 75$. This is also written as $\frac{5}{6} \times 90 = 75$

* On the number line, $1\frac{1}{6}$ or $\frac{7}{6}$ is between 1 and 2.
The exact spot can be determined.



* The ***denominator*** of a fraction names the number of parts in which something is divided.
The ***numerator*** of a fraction shows the number of parts of the whole.

Of the fraction $\frac{8}{15}$, 8 is the numerator and 15 is the denominator.

* Two or more fractions indicating equal portions are named *equivalent*. Multiplying the numerator as well as the denominator of a fraction by the same number (or dividing by the same number) results in an equivalent fraction.

$\frac{3}{5} = \frac{24}{40}$ because $24 = 8 \times 3$ and $40 = 8 \times 5$

* Two or more fractions with equal denominators can be compared easily

$\frac{8}{15}$ is less than $\frac{3}{5}$ because $\frac{3}{5} = \frac{9}{15}$ and $\frac{8}{15} < \frac{9}{15}$

* Between each set of two fractions, new fractions can be found, even if the two are close together.

For example, between $\frac{31}{60}$ and $\frac{8}{15}$ is the fraction $\frac{21}{40}$,
because $\frac{31}{60} = \frac{62}{120}$ and $\frac{8}{15} = \frac{64}{120}$ and $\frac{21}{40} = \frac{63}{120}$

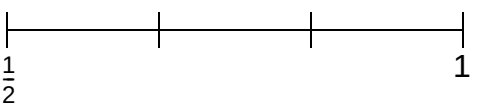
Mixed up!

1. What could the fraction $\frac{5}{12}$ mean? Write three examples.
   You may also use drawings.

2. Fill in the right words:
   In the sequence $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ the ..... each time gets two times as large and the ..... gets two times as small.

3. Find an appropriate fraction.
   For forty people, five equal pies have been bought.
   If shared fairly, each person gets ..... part of a pie.

4. The part of the number line between $\frac{1}{2}$ and 1 is divided in three equal parts.
   Which fractions belong to the indicated points?

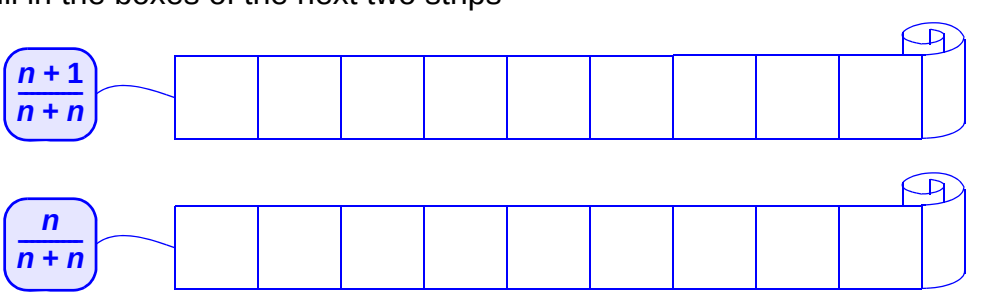


5. Tommy thinks that $\frac{1}{17}$ is more than $\frac{1}{16}$. Is he right?
   Explain why or why not.

6. On the number line, which fraction is exactly inbetween $\frac{1}{3}$ and $\frac{1}{5}$?

7. If you *add* 1 to the numerator and the denominator of a fraction, the result is an equivalent fraction. Is this statement true? Explain.

8. The label of a strip shows *n*.
   In the boxes on the strip are 1, 2, 3, 4, 5, 6, and so on.
   Fill in the boxes of the next two strips



9. *a* and *b* stand for two whole numbers. Which numbers do they represent?

$$\frac{8}{a+10} = \frac{1}{3} \quad \text{and} \quad \frac{b-3}{12} = \frac{1}{4}$$

10. Think of a problem that might fit in this section (A) of the book. Also provide a solution to your problem.