

The circular pave-stones backyard- Packing problem-Teacher's notes

Content: An inquiry-based activity in geometry, linked to optimization problems and configuration of structures for maximum density.

Age: 12-17 years old

Intended use: Mathematics, geometry, links with work for professions that need the skill of configuration and optimization.

Time: 2 periods

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Concept:

To arrange objects in a way that leaves the minimum possible empty space between them is a skill needed in various professions, let alone in everyday life (for example the way workers put containers in a storehouse, the way workers arrange pave-stones in a pavement, how we arrange things in a luggage, etc).

Packing problems are a class of optimization problems in **mathematics** which involve attempting to pack objects together inside a given area, as densely as possible. The aim is to find the configuration with the maximal density.

Objectives of the activity:

By the proposed activity students are expected to be able to:

- Identify regularities in a structure of objects and to create patterns from the repetition of a shape.
- Approach the concept of infinite by identifying that the pattern is a micrography of the infinite surface, so measurements for the whole can be limited to the particular pattern
- Construct the concept of the percentage of a surface that is covered through an ordering with circles
- Provide the optimal packing for a structure of equal circles.

Suggested lesson plan with notes

Lesson 1

5 min briefly introduce the activity. You may want to talk about the general problems of packaging

10 min students work in pairs on activity 1 (pre-conceptions)

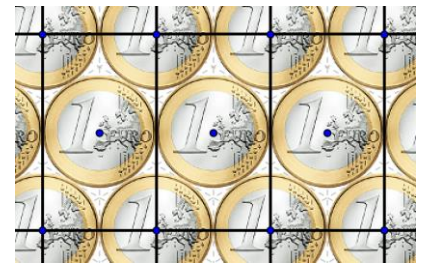
Students are given two different types of coin ordering (view students' worksheet). They are asked to speculate about which one of the two orderings leaves less empty space in the surface. Students enter in argumentative dialogue to support their views.

The activity here is not placed in a mathematical context; it rather tries to uncover students' pre-conceptions on the issue. The mathematical context is to be defined by the students through the activities that follow, mainly by the construction of the concept of the 'percentage of coverage'.

You may want to discuss the findings briefly in whole class.

15 min students work individually or in pairs on activity 2 (regularities). They are encouraged to identify regularities like:

- The centres of the coins are vertices of equilateral triangles (triangle ordering) in the first case and vertices of squares (square ordering) in the second case.
- For both of the orderings of the coins, students draw various patterns that are created by the repetition of a polygon. (For example, for the triangle ordering they could draw patterns with rhombus, hexagon, rectangle...)



15 min Students work on activity 3 (Percentage of coverage)

For each of the polygon that students identified in the previous phase, they calculate the percentage of its surface that is covered by the coins.

5 min Discuss the findings for activity 3 in class

It is interesting that this percentage depends only on the ordering of the coins (triangle ordering or square ordering) and NOT on the pattern that students chose.

Therefore, it is logical this percentage to be defined as “the percentage of the coverage of the surface” for each ordering.

Lesson 2

5 min reflect on findings from previous lesson

30 min students work in pairs or small groups on activity 4 (Further optional activities)

Based on the previous activities, students can enter on argumentative discussions on puzzles like the ones presented in activity 4 of the worksheet, which combines two patterns for triangle ordering:

You may also have students formulate their own research questions and investigate these.

15 min whole class reflection (activity 5)

Students are encouraged to reflect on the practical value of the activity. (For example they could be encouraged to reflect on how fruits are ordered in a grocery for space optimization, as well as to find similar examples).

