Mathematical power of special-needs pupils: An ICT-based dynamic assessment format to reveal weak pupils’ learning potential

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Abstract
This paper reports a study aimed at revealing special-educational-needs pupils’ learning potential by means of an ICT-based assessment including a dynamic visual tool that might help pupils when solving mathematics problems. The study focused on subtraction problems up to 100, which require ‘borrowing’. These problems, in which the value of the ones-digit of the subtrahend is larger than the ones-digit of the minuend, are known as a serious difficulty for weak pupils in mathematics. Seven of such problems from a standardised test were placed in the ICT environment. Data were collected from two test conditions: the standardised written test format and the ICT version of the test items including the tool that provided pupils with a set of virtual manipulatives. The 37 pupils involved in the study were 8–12 years old and from two special-education schools in the Netherlands. Comparison of the performance scores in the two formats showed that an ICT-based assessment format, including a dynamic visual tool, can reveal weak pupils’ learning potential and strategy use. The study also pointed out that ‘partial-tool use’, ie, not carrying out the complete subtraction operation with the tool, can provide sufficient support to find the correct answer.

1This study has been presented as a poster at the Fourth Biennial EARLI/Northumbria Assessment Conference 2008 (Peltenburg & Van den Heuvel-Panhuizen, 2008)
Introduction

As in regular primary education, it is crucial that teachers in special education who deal with pupils with learning difficulties (LD) in mathematics have a good understanding of their pupils’ capabilities, i.e., their calculation skills. In the Netherlands, the CITO Monitoring Test for Mathematics (Janssen, Scheltens & Kraemer, 2005) is a frequently used assessment instrument for collecting information about the pupils’ development. This instrument, which has the characteristics of a standardised test (e.g., no auxiliary resources are allowed), has been designed for pupils in regular primary education. Therefore, the question arises, whether this instrument gives full opportunities to pupils in special education with LD in mathematics to show what they are capable of.

The study reported here looks beyond the achievement scores that are to be found with a standardised test and intends to expose the pupils’ competence that in standardised tests might remain below the surface. In order to do this, seven test items on subtraction in the number domain up to 100 which have been taken from the CITO Monitoring Test for Mathematics End Grade 2 are placed in an ICT environment and extended with a dynamic visual tool which the pupils can use when solving the problems. The research question investigated in this study is whether these ICT-based test items can reveal more about the LD pupils’ potential with respect to subtraction than the standardised test items do.

Theoretical background

Before we describe how the study was carried out and what results we gained from it, we summarise some findings from research literature that guided the setup of our study. When investigating LD pupils’ potential with respect to subtraction, we first needed to have a better image of the pupils’ difficulties with subtraction problems and the role of manipulatives in overcoming obstacles when solving these problems. Special attention goes to what ICT-based virtual manipulatives have to offer in connection with dynamic assessment. Finally, we address the functions ICT had in our study: (1) offering the pupils an environment for doing mathematics; and (2) providing us the opportunity to trace the pupils’ solution processes.

Subtraction with numbers up to 100

When pupils learn to calculate in the number domain up to 100, subtraction is mostly more difficult for them than addition (Kraemer, Van der Schoot & Engelen, 2000, p. 59), or, as Riccomini (2005, pp. 233–234) points out, subtraction is particularly problematic for many pupils and for pupils with LD the difficulties are ‘even more troubling’.

Basically, there are three different strategies to solve subtraction problems of the type ‘a – b = ?’: (1) taking away b from a, (2) adding on from b until a is reached, or subtracting from a until b is reached, and (3) by tinkering, e.g., taking away from a a number that is different from b, but is easier to handle and then correct it afterward, or changing the problem into an easier problem by keeping the difference the same. Torbeys, De Smedt,
Stassen, Ghesquière, and Verschaffel (in press) call these three strategies respectively (1) direct subtraction, (2) indirect addition or indirect subtraction, and (3) subtraction by compensating.

Most difficult are the subtraction problems that require ‘borrowing’—which means that the value of the ones-digit of the subtrahend is larger than the ones-digit of the minuend (eg, 62 – 58). Here many errors occur (Brown & Burton, 1978, p. 163; Brown & VanLehn, 1982, p. 119; Beishuizen, Van Putten & Van Mulken, 1997, p. 89; Kraemer et al, 2000; Fiori & Zuccheri, 2005, pp. 327–329). A frequently made mistake in these problems is processing the digits in the reverse way (in the case of 62 – 58, subtracting 2 from 8 instead of 8 from 2). Some authors call this mistake the ‘smaller-from-larger’ bug (Ashlock, 2005, p. 113; Beishuizen, 1993, p. 89; Riccomini, 2005, p. 234) describes the ‘borrowing across zero’ error, where a pupil attempts to borrow from a zero (eg, in the case of 102 – 37), and does not continue to borrow from the column to the left of the zero. This would appear to be an extension of the borrowing error previously described. Other errors that can occur, are ‘not carrying the tens’ (eg, 62 – 58 = 14), ‘mistakenly subtracting all’ (eg, 75 – 32 = 70 – 30 = 40, 40 – 5 = 35, 35 – 2 = 33) and ‘wrongly combining steps’ (eg, 58 – 34 = 20, 8 + 4 = 12, together 20 – 12 = 8) (Beishuizen, pp. 309–310; Beishuizen et al, p. 89; Woodward & Gersten, 1992).

### Subtraction with manipulative materials

A common approach to assist pupils who have difficulties in operating with numbers is to provide them with manipulatives, such as blocks and counters (Driscoll, 1983; Sowell, 1989; Suydam, 1986). These manipulatives can make abstract ideas and symbols more meaningful and understandable for pupils.

Instead of using physically concrete materials, virtual ICT-based manipulatives can be used (Lee & Chen, 2008). Virtual manipulatives might also provide further advantages over physical manipulatives since they allow pupils to perform specific mathematical transformations on numbers easily. For example, whereas physical base-ten blocks must be exchanged (eg, when subtracting, pupils may need to exchange one 10 for ten 1s), pupils can break a computer base-ten rod into ten 1s. Clements (2002, p. 168) claims that such actions are more in line with the mental actions that we want pupils to learn. Thus, computer manipulatives can provide unique advantages such as offering a flexible and manageable manipulative, one that, for example, might ‘snap’ into position (Clements & McMillen, 1996).

### Dynamic assessment

The standardised way of assessing requires an objective approach in which pupils are not allowed to receive any help, nor are they allowed to use manipulatives. In contrast to this static way of testing, where the test administrator must avoid giving any information that might be helpful, in ‘dynamic assessment’ (Campione, 1989; Feuerstein, 1979, 89–125) a learning environment is created within the assessment situation, in which the pupil’s reaction to a given learning task is observed and the assessment task
suitably adjusted. Thus, a characteristic of this method of assessment is that it has a certain flexibility and sensibility, which makes it possible to reveal hidden competences of pupils. Comparative research conducted into static and dynamic assessment has shown that information about pupils that remains invisible in static assessment will come to light through dynamic assessment. Furthermore, weaker pupils in particular might benefit the most from this assessment method (Allsop et al., 2008; Van den Heuvel-Panhuizen, 1996).

An ICT environment to give pupils support to do mathematics
Several researchers have stressed the possibilities of computer-based assessment (CBA) for low achievers (Babbitt & Miller, 1996; Kumar & Wilson, 1997; Ryba, Selby & Nolan, 1995; Singleton, 2004; Woodward & Rieth, 1997). Bottge and Rueda (2006) developed and evaluated the effectiveness of a multimedia CBA compared to a paper-and-pencil assessment (PPA). The CBA measured the same concepts as the PPA, with the additional benefit of providing pupils access to information for solving the mathematical problem rather than having to recall the relevant information from the instructional period. The study showed that the CBA eliminated some of the cognitive demands for the low-achieving pupils and thus enabled them to more fully demonstrate their understanding of the mathematical concepts they had learned. With respect to this reduction in cognitive demand, ICT can offer LD pupils a structured, stylised assessment environment in which they can easily keep track of their actions. Further, in comparison with ‘real life’ actions, such as moving beads on a string, the same actions in the ICT environment can require less concentration and cognitive demand (Kumar & Wilson).

An ICT environment as a window to the pupil’s mind
In addition to advantages of reducing cognitive demand while working on a task, ICT can, as Singleton (2004) points out, help teachers gain a deeper understanding of key difficulties. Or, as Clements (1998) argues, the ICT environment offers ‘windows to the mind’ for many teachers and researchers. The potential of computer environments to provide insight into pupils’ cognitive processes makes them a fruitful setting for research on how this learning takes place (Kolovou, Van den Heuvel-Panhuizen, Bakker & Elia, 2008). This is possible because ICT can register detailed information on pupils’ strategies, and so provide a vehicle for orchestrating higher quality assessment (Woodward & Rieth, 1997, pp. 517–521). For example, capturing software enables us to record every command and statement pupils make while working in an ICT environment and thus provides insight into pupils’ cognitive processes. This allows us to assess their strategies in more precise ways than can paper-and-pencil tasks (Kolovou et al.; Van den Heuvel-Panhuizen, 2007).

Method
Participants
In total, 37 pupils from two schools for special education in Utrecht, the Netherlands, participated in the study. Most of these pupils are of Dutch (43%) or Moroccan nationality (35%). The pupils’ ages were between 8;9 and 12;11 (average age = 10; 5 years). The mathematical level of these special-needs pupils was about end Grade 2. This
means that they have a developmental delay in mathematics that ranged from 1 to 4 years, since 8–9 year olds are normally in Grade 3 and the 11–12 year olds are normally in Grade 6. The two schools used the same mathematics textbook series.

Data collection
Two types of performance scores were collected. First, in April–May 2008, performance data were collected by a Flash ICT assessment environment, especially developed for this study to function as a dynamic assessment environment. This ICT environment consists of the seven subtraction problems in the number domain up to 100. The problems all require borrowing. The items were taken from the CITO Monitoring Test for Mathematics End Grade 2, but were redesigned for the ICT environment. The pupils worked for about 15 to 20 minutes individually in the ICT version of the test with one of the authors of this paper sitting next to them. In the background, Camtasia Studio software was running to record a screen video and an audio file of the pupil’s working.

Second, performance data were collected from the CITO Monitoring Test for Mathematics End Grade 2. This data collection took place in June 2008. The pupils did the complete test, including the seven subtraction items. The reason for administering first the ICT version, followed by the standardised test version, is that we wanted to avoid a retest effect in the scores of the ICT version. Along with the two types of performances data, other data from the pupils were collected, such as age, sex, and nationality. Furthermore, we received allowance to extract information about the pupils’ specific difficulties from their dossiers on developmental background.

ICT version of the test
The ICT version of the test contains scans from seven items on subtraction with borrowing. Every item (consisting of text and a picture illustrating the context) is displayed on the screen and the accompanying text is spoken by the computer. The pupil can hear the problem again by clicking on a button, which shows a small ear. After the problem is read aloud, a sentence follows to stimulate the pupil’s thinking: ‘How are you going to solve the problem?’ An important characteristic of the ICT environment is the flexible and natural way pupils can work on it. For example, for filling in an answer to a problem, the pupil can drag one or more digits to the answer field. In case of a two-digit number, the pupil can easily change the positions of the digits if necessary. In this way, pupils can already fill in what they know. They are not obliged to first fill in the tens and then the ones, which can be a burden on their working memory.

Each test session started with filling in the pupil’s name, grade, date of birth and the name of the school. The pupil was informed about the test procedure and that it was allowed to use the tool with virtual manipulatives. Before the pupil started with the test, he or she was given the opportunity to explore the tool.

A dynamic visual tool
The tool that is included in the ICT version of the test items offers the pupils a set of virtual manipulatives, consisting of a 100 board with a 10 by 10 grid and divided in
four parts with a 5–5 structure. Next to the 100 board, there is a stock of counters, also structured in 5s. The pupils can select a number of counters, drag them to any place on the board, and rearrange or remove them. The tool can be used in any way that is helpful for the pupils. In particular, we expected that this tool would help pupils to overcome obstacles in solving subtraction problems that require borrowing. Our hypothesis was that, through an onscreen visual representation of the problem, the pupils would be less inclined to reverse the processing of the ones-digits. For example, in the case of 62 – 58, the tool can prompt them to find a solution for subtracting 8 from 2 by opening up the next ten.

Results

Comparison of scores in the two formats

Our observations included a total of 518 instances (37 pupils each did seven problems in two formats). A case is considered to be the combination of two responses, one for the standardised test format, and one for the ICT test format. Thus, Table 1 shows the results for the 259 cases. It shows that the percentage of correct answers was higher for the ICT version of the seven items (53%) than for the items in the standardised test format (34%). In 30% of the cases the pupils did not find a correct answer in the standardised test format, but their answer in the ICT version was correct. There were 36% of the cases which were incorrect in both formats.

Tool use and pupil’s performance

The choice of the pupils to make use of the tool in the ICT format matched their performance on the standardised test quite well. That is, tool use was higher when the pupils did not find a correct answer in the standardised test (49%), (see Table 2), than when the pupils found a correct answer (21%) in the standardised test format.

As Table 2 shows, using the tool does not, of course, guarantee that pupils find the correct answer. It also appeared that, incidentally, the tool worked counterproductively in some cases. In these eight cases the pupils answered the standardised test items correctly, but answered the ICT version of the items incorrectly, despite using the tool. In order to identify the pupils’ strategies when using the ICT tool, we analysed all screen

Table 1: Cross tabulation of correct and incorrect answers in both test formats

<table>
<thead>
<tr>
<th>ICT version of test items</th>
<th>Correct answer</th>
<th>Incorrect answer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test items standardised test</td>
<td>Correct answer</td>
<td>24% (61)</td>
<td>11% (28)</td>
</tr>
<tr>
<td></td>
<td>Incorrect answer</td>
<td>30% (77)</td>
<td>36% (93)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>53% (138)</td>
<td>47% (121)</td>
</tr>
</tbody>
</table>

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Pupils’ strategies and tool use in the ICT environment

Analysis of the screen videos of the cases (n = 65) in which the pupils found a correct answer and used the tool, showed that in 82% of the cases the pupils applied a taking-away strategy (the subtrahend is taken away from the minuend), in 9% an adding-on strategy (bridging the difference between the subtrahend and the minuend) and in 5% a comparing strategy (comparing the minuend and the subtrahend). In some cases, it was not clear how the pupil’s work with the tool related to the answer that he or she had filled in. These cases were coded ‘Unclear’: later on, we will illustrate this.

A surprising finding in our study was that applying one of the previously mentioned strategies did not necessarily mean that the pupils performed this strategy completely on the 100 board. In 38% of the cases in which the correct answer was found and the tool was used (n = 65), the tool use was incomplete; see Table 3.

Table 4 shows the number of cases using each of the strategies and which each led to the correct answer for the seven subtraction problems. Taking away was by far the most frequently used strategy for solving the problems (53 cases). When this strategy was applied, the pupil first represented the minuend by dragging counters from the stock to the board. Then, the subtrahend was taken away, either mentally or by dragging counters back to the stock, either on the left part or on the right. Finally, the pupil determined the remaining number of counters.

The adding-on strategy was applied when either the subtrahend was represented or nothing was represented on the board. There were six cases in which the adding-on
strategy was used. It is remarkable that even in the case of $62 - 58$, this strategy was not applied and in the case of $48 - 39$ only once. Similar results were found in Torbeyns et al (in press).

The strategy that is described as the comparing strategy is used three times by two of the pupils. For example, when Farah (for privacy reasons all names of the pupils have been changed) was solving the problem $48 - 39$, she represented the minuend and the subtrahend by dragging counters to the board. Figure 1 shows that she is just dragging the nine 1s of 39. Then she waited a couple of seconds and gave the correct answer.

Table 4 also shows that in three instances it was unclear which strategy the pupils used. For example, when solving the problem $71 - 3$, Saniya represented 70 counters on the board, which we considered as an incomplete representation of the minuend. Then Saniya thought for a couple of seconds and filled in the correct answer.

**Strategies, tool use and incorrect answers**

Table 5 shows the number of cases using each of the strategies and which each led to an incorrect answer for the seven subtraction problems. Taking away was the most frequent used strategy for solving the problems (15 cases). When this strategy was applied, the pupil first represented the minuend, then, the subtrahend was taken away.
Although it is not always clear how the pupils came up with their incorrect answer, as Table 3 shows, some cases provided us clear-cut information on their strategies and the errors they made. For example, when Mimoun solved the problem 62 – 58, he represented the minuend by dragging counters to the board. Then, he moved the cursor over the counters starting at the most right bar. He stopped when he arrived halfway to the leftmost column. He counted the five upper counters, and filled in five in the answer field. This strategy can be described as a taking-away strategy, because Mimoun took away 58 counters from 62 counters. Unfortunately a small counting error had crept in.

Another example is Ayman’s strategy for solving the problem 48 – 39. Ayman used an adding-on strategy. He first tried to solve the problem mentally. He said: ‘39, 38, 39, 40, 41, 42, 43, 44, 45, wait. I want, wait, the board’. Then he opened the 100 board and selected two or three counters a time, which he dragged to the board. He counted out loud and probably to be sure he counted the counters on the board again, starting at 40.

<table>
<thead>
<tr>
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<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Taking-away</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>*5</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Adding-on</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>*2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Comparing</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Unclear</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>4</td>
<td>10</td>
<td>9</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>38</td>
</tr>
</tbody>
</table>

*Examples of this strategy are given in the text.
until he arrived at 48. Finally, he filled in his answer: 48. It is important to notice that, although Ayman did not find the correct answer, his strategy was in essence a good strategy for finding the answer.

**Conclusions and discussion**

The results of this study showed that the use of an ICT-based assessment including a dynamic tool had a positive effect on pupil scores. This effect was found, even though the pupils had never used the tool before. The fact that pupils with LD found ways in which they could benefit from using the tool, demonstrated convincingly their mathematical power.

In this study the pupils were stimulated to think about their learning process and their understanding of the subtraction problems involved in the test, because for each problem, they had to decide whether or not to use the aid tool. The availability of this tool can elicit pupils’ reflection on their calculation skills and, consequently, effect that they become more active in their learning. Such a tool makes the assessment learning oriented (Wiliam, 2008). It is important to note that most pupils appeared to be quite capable of judging their mathematical proficiency and therefore could decide when they could benefit from tool use. In other words, choice of tool use was based upon well-considered understanding of one’s own competences. This conclusion sharply contrasts with the lack of metacognitive skills that is generally attributed to LD pupils, and can be considered as yet another clue to the hidden learning potential of these pupils.

Furthermore, the study gave important information on the way the pupils used the tool. It was brought to the fore that finding the correct answer by using the tool does not necessarily mean that the pupils have to perform their strategy completely on the 100 board. Our results showed that ‘partial-tool use’ can also provide sufficient support to find the correct answer. This conclusion may be in contrast with the idea that in order to realise good understanding of number operations the materialised steps should be equivalent to the mental steps. Moreover, partial tool use could also be an indication of being in a transition phase between working on a concrete level and more abstract level.

The study showed two opportunities of ICT in assessing pupils with LD. First, ICT can provide unique advantages for offering pupils flexible and manageable manipulatives in a dynamic format by which they can overcome obstacles in solving subtraction problems with borrowing. Second, the ICT format makes it possible to examine pupils’ actions and thinking processes in detail, which allows to assess their strategies in more precise ways than can paper-and-pencil formats.

Although this study showed that ICT tool use can help LD pupils overcome obstacles in solving subtraction problems, more research is necessary. So far, data have only been collected from 37 pupils. To have a more solid understanding of weak pupils’ learning potential in mathematics, more pupils should be involved in future research as well as more mathematical domains. Another point is that it is not clear as to how the tool

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affected the pupils’ strategies. Particularly, we do not know whether the 100 board encouraged or discouraged specific strategies. For this reason, our next step will be collecting data on pupils’ strategies with and without tool use in the ICT environment. Other tools will be involved as well, as these could be more efficient and perhaps be more favourable for solving subtraction problems that require borrowing. Giving pupils opportunities to choose between tools, may provide even a better insight in the discrepancy in performance scores between the ICT version and the standardised test version.

References

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