Non-Routine Problem Solving Tasks in Primary School Mathematics Textbooks – A Needle in a Haystack

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ABSTRACT: In this paper, we report on a study in which we investigated the nature of numerical problem solving tasks as presented in primary school mathematics textbooks in the Netherlands. Although several factors influence what mathematics teachers teach children, there is much evidence that the curriculum and the textbooks are important determinants of what children are taught and what they learn. Contradicting results from TIMSS and poor performances of Dutch fourth graders on a test on mathematical problem solving were the immediate reasons for this textbook analysis study. We analyzed the fourth-grade textbook materials of six mathematics textbook series. The analysis tool used was designed through an iterative process of classifying tasks involving number problems according to their cognitive demand. The final version of the framework contains three main categories of tasks. The first category involves straightforward tasks in which the operation is given. The third category involves non-routine, puzzle-like tasks that require higher-order understanding. The second category lies between these extremes. The tasks that belong to this category are called “gray-area tasks”. They do not have a puzzle-like nature in themselves, but may prompt children to do investigations and develop a mathematical attitude that is needed for solving non-routine mathematical problems. The analysis of the textbook series revealed that only a very small proportion of the tasks included in the textbooks is of the third category, which is the category with the highest cognitive demand. In some textbooks series these non-routine puzzle-like tasks are completely absent. This result raises questions and concerns about the mathematical nature of arithmetic education in Dutch primary schools.

Keywords: mathematical problem solving; non-routine tasks; primary school; opportunity to learn; textbook analysis.
1. INTRODUCTION

1.1. Background of the study and its research question

Problem solving is the heart of mathematics (Halmos, 1980) and is supposed to play a crucial role in mathematics education. The significance of problem solving is evident in many curricula and educational policy documents (Stacey, 2005; and see the Special Issue of ZDM – The International Journal on Mathematics Education edited by Törner, Schoenfeld, and Reiss, 2007). Moreover, much attention is paid to the topic of problem solving in research literature (Lesh & Zawojewski, 2007; Lester & Kehle, 2003; Schoenfeld, 1985; Törner et al., 2007). As a consequence, problem solving is one of the key competences assessed in international assessments studies, such as TIMSS and PISA, which compare students’ achievements in mathematics (Dossey, McCrone, & O’Sullivan, 2006).

For the Netherlands, the PISA findings on problem solving were rather disappointing. Dutch 15-year-old students scored relatively low on real-life problem solving, which in the PISA study covers a wide range of disciplines including mathematics, science, literature, and social studies. The students in the Netherlands were placed twelfth of 40 OECD-countries for problem solving, while for mathematics in general, they obtained the fourth position. It is also noteworthy that of all OECD-countries, the difference in scores between mathematics in general and problem solving is the largest in the Netherlands (HKPISA Centre, 2006; PISA-NL-team, 2006).

These PISA findings are more or less in agreement with the results from the Dutch POPO (Problem Solving in Primary School) project that aims at getting a better understanding of the mathematical problem solving performance of Dutch primary school students. The first study carried out in this project investigated the problem solving competences and strategies of the 25% best achievers in mathematics in grade 4 (Van den Heuvel-Panhuizen, Bakker, Kolovou, & Elia, in preparation). Unlike in PISA, problem solving was interpreted in this study as solving non-routine puzzle-like numerical problems. Preliminary results of the POPO study already revealed that the students have considerable difficulties with non-routine problem-solving tasks that require higher-order mathematical thinking (Van den Heuvel-Panhuizen & Bodin, 2004; see also Doorman et al., 2007).

In the present study, we attempted to uncover a possible reason for Dutch students’ poor performance in problem solving by investigating to what degree the students get the chance to learn problem solving. We explored this so-called “opportunity to learn” (Husen, 1967) by analyzing how much mathematical problem solving Dutch textbooks series provide to teachers and, consequently, to students. The guiding research question for our study was: What proportion of the number-related tasks in textbooks documents can be qualified as problem-solving tasks that require higher-order thinking? In the aforementioned POPO study, student data were collected.
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from the middle of grade 4, therefore we carried out the textbook analysis on the textbook series documents that are used in the first half year of grade 4.

1.2. Determining role of textbooks in the Netherlands

In the Netherlands, textbook series are published by commercial publishers and a school team can freely decide which textbook series to purchase and use. The textbook authors are free to determine the content and the layout of the textbooks, and are even free in choosing a particular underlying teaching principle. The only requirement for textbooks is that they are in agreement with the core goals published by the Dutch Ministry of Education. Should a textbook series not meet this criterion, the school inspectorate would give schools the advice not to use it. However, the core goals are rather limited in their description and leave much space for different interpretations. Consequently, there is much free space in interpreting these goals. Therefore, textbook series can differ greatly in how they translate these goals into teaching-learning activities and in how they are structured in different kind of documents. As a consequence, by including particular mathematical content and activities and excluding others, textbooks can influence students’ opportunity to learn.

Another reason why textbook series affect to a great extent what is taught in Dutch classrooms is that Dutch teachers use their textbook series as a daily guide for organizing their teaching, both with respect to the teaching content and the teaching methods (Van den Heuvel-Panhuizen & Wijers, 2005). According to the last National Assessment of the Educational Achievement (PPON) (Janssen, Van der Schoot, & Hemker, 2005), almost all teachers reported that they follow the textbook and only rarely deviate from it.

The determining role of Dutch textbooks is also clear if we think of the innovation in mathematics education that has taken place in the Netherlands since the beginning of the 1970s and that had its breakthrough in the mid 1980s. This reform is largely attributed to the implementation of the new reform-based textbook series (De Jong, 1986; Van den Heuvel-Panhuizen, 2000).

In sum, we may say that there are enough reasons to look at the Dutch textbooks series when we want to understand why Dutch students have poor results in problem solving. Yet, the determining role of Dutch textbooks on what is taught is not the only reason for this low performance.

1.3. Growing international interest in textbooks

Worldwide, over the last decades, school mathematics textbooks and curriculum materials have received a growing interest. More and more, they have been found to be important factors in influencing the teaching of mathematics and the output of that teaching (Braslavsky & Halil, 2006; Cueto, Ramírez, & León, 2006; Doyle, 1988; Nicol
& Crespo, 2006; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Stein, Schwan, Henningsen, & Silver, 2000). For that reason, the Third International Mathematics and Science Study (TIMSS) (Schmidt et al., 1997) carried out a thorough analysis of the curriculum guides and textbooks of the 50 participating countries.

For the Dutch curriculum guides and textbooks, these findings of TIMSS (Schmidt et al., 1997) were rather daunting. The analysis revealed that for Population 1 (third and fourth-grade students) the main focus was on procedural skills. Problem solving was almost absent. Later, these results were confirmed by the national analyses of the TIMSS data. These showed that, by the end of grade 4, more than half of the time has been invested on numbers and only 13% on patterns, comparisons and connections (Meelissen & Doornekamp, 2004). However, at the same time the Dutch students in Population 1 showed high mathematics achievement scores in TIMSS (Meelissen & Doornekamp, 2004; Schmidt et al., 1997). The aforementioned facts – high general mathematics scores, low scores on problem solving, and mathematics textbook series with limited attention paid to problem solving – combined with the determining role that Dutch textbooks have in primary school mathematics education brought us to analyze more deeply what the textbooks offer students in terms of tasks that ask for mathematical problem solving.

Before we describe how the textbook analysis was carried out and what results we got from it, we continue with two literature reviews that guided the setup of our analysis. In order to obtain knowledge about how textbooks can contribute to students’ ability of mathematical problem solving, we first needed to have a better image of what we mean by problem solving. Based on our experiences in the POPO project our focus is on non-routine, puzzle-like tasks that imply higher-order understanding and application of higher-order skills. At the primary school level, this distinction touches on the difference between plain arithmetic and mathematics. In other words, the tasks we had in mind were tasks where mathematics comes into numerical problem solving. The first literature review deals with this issue. The second review elaborates the concept of opportunity to learn and the ways in which this can be assessed.

2. LITERATURE REVIEW

2.1. Problem solving

In the previous section, we briefly explained how we interpreted problem solving in this study. Here we embed this interpretation in the existing research literature about problem solving. It will become clear that problem solving is not an unequivocal concept (Törner et al., 2007).

Some authors call every task a problem and use the definition of problem solving as the process from the givens to the goal in which the goal is finding the right and often the one and only solution (Moursund, 1996). In this interpretation, problem solving is
seen as doing calculations with numbers that are presented either as bare numbers or in a context. The second, in particular, is often called problem solving, although it might be just solving rather straightforward word problems.

2.1.1. Non-routine character Other interpretations of problem solving are more plausible. Many researchers have emphasized that in problem solving the path from the givens to the solution is not a straightforward one. Such an interpretation is reflected in TIMSS and PISA. For example, PISA 2003 focuses mainly on real-life problem solving covering a wide range of disciplines. The PISA researchers used the following definition of problem solving: “Problem-Solving is an individual’s capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science or reading” (OECD, 2003, p. 156). In TIMSS 2003, problem solving is included in the cognitive domain of reasoning. Here, much attention is given to the non-routine character of the problems. “Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and the skills required for their solution have been learned” (Mullis et al., 2003, p. 32). In this interpretation, genuine problem solving is the counterpart of solving routine problems aimed at getting practice in particular methods or techniques and in problem settings that are more familiar to students.

Although at first glance the non-routine characteristic of problem-solving problems may appear obvious, it is not clear-cut. The difficulty is that a non-routine task may itself become routine. According to Zhu and Fan (2006, p. 612) the characterization of a problem as a problem largely “depends on the person who is dealing with the situation”. Stein et al. (2000) also mentioned prior knowledge and experiences as important factors when deciding what tasks can count as tasks for problem solving. Furthermore, we should be aware of the fact that tasks can have both routine and non-routine aspects (Mamona-Downs & Downs, 2005).

2.1.2. Genuine problems for students What is essential in the case of real problem solving is that the problems are genuine problems for the students. In the words of Kantowski (1977, p. 163) this means that “[a]n individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him. […] A problem differs from an exercise in that the problem solver does not have an algorithm that, when applied, will certainly lead to a solution.”

2.1.3. Interpreting the problem situation When the problem on which the students have to work is really a problem for them – in the sense that it is not clear in advance which calculation has to be carried out – the solution process often requires many steps back and forth until the student is able to unravel the complexity of the problem
situation. Furthermore, students have to be aware of how the given numbers or quantities relate to one another in order to find a way to the solution (O’Brien & Moss, 2007). This ability to find an underlying pattern in a problem was also recognized by Lesh and Zawojewski (2007) as a crucial aspect of problem solving. They emphasized that problem solving is a goal-directed activity that “requires a more productive way of thinking about the given situation […] The problem solver needs to engage in a process of interpreting the situation, which in mathematics means modeling” (ibid, p. 782). Similar thoughts were expressed by Kilpatrick, Swafford, and Findell (2001) who argued that the problem solving competence involves the construction of mental models.

2.1.4. Higher-order thinking

The aforementioned interpretation of problem solving, in which the problems are true problems and the solution strategy is not immediately clear at the moment that the problem is presented, is in line with our interpretation. We want to focus on problem solving as a cognitive activity that requires both an insightful approach to the problem situation and strategic thinking. In other words, there is something more involved in non-routine mathematical problem solving than carrying out a calculation in an appropriate way. Our point of view implies that problem solving is a complex activity that requires higher-order thinking and goes beyond procedural skills.

Several authors have elaborated the distinction between higher and lower types of cognitive engagement of students. More than thirty years ago, Skemp (1976, p. 2) discerned relational and instrumental understanding in which relational understanding is “knowing both what to do and why”, whereas instrumental understanding is “rules without reasons”. While instrumental understanding suggests memorizing an increasing number of procedures, relational understanding involves building conceptual structures. In a similar way, Stein et al. (2000) based their Task Analysis Guide on the difference between tasks of low-level and higher-level demands. The first category includes memorization tasks and algorithmic tasks unrelated to the underlying meaning, whereas the second requires engagement with conceptual ideas and complex, non-algorithmic thinking. Doing mathematics as a high-level cognitive demand includes tasks where there is no pathway suggested by the task and where the focus is on looking for the underlying mathematical structure. Although problem solving is located more on the side of the high-level demands, Silver (1986) reminds us that problem solving involves elements of both sides. Moreover, as stated by Stein et al. (2000), the cognitive demands of a task can change during a lesson. A task that starts out as challenging might not induce the high-level thinking and reasoning that was intended as the students actually go about working on it. However, according to them, in any case it is clear that challenging tasks appear to be a prerequisite to elicit high-level thinking.
2.1.5. To conclude this review on problem solving

In sum, we can say that although the interpretations differ, there is consensus that genuine problem solving refers to a higher cognitive ability in which a straightforward solution is not available and that mostly requires analyzing and modeling the problem situation. In order to be a true problem for students, it should not be a routine problem. On the other hand, the review makes it clear that the distinction between tasks with a low-level demand and a high-level demand is not fixed; the developmental level and experience of the students also determine whether a task is a true task for problem solving.

2.2. Opportunity to learn

In this section, we review relevant research literature related to procedures and methods that have been used for assessing what mathematical content is taught. The findings of this review are used for developing our textbook analysis instrument.

Many studies have shown that there is a strong correlation between the content that is taught and the achievements of the students (Leimu, 1992; Floden, 2002; Haggarty & Pepin, 2002; Törnroos, 2005; Cueto et al., 2006). Whether primary school students are able to solve non-routine mathematical problems will therefore largely depend on whether they have been taught to solve these kinds of problems. The generative concept behind the correlation between what is taught and what is learned is the so-called “opportunity to learn” (OTL). According to Floden (2002) the most quoted definition of OTL comes from Husen’s report of the First International Mathematics Study (FIMS). This report describes OTL as “whether or not ... students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test” (Husen, 1967, pp. 162-163).

Although OTL seems to be a clear-cut concept, there are several reasons it is not. According to Schmidt et al. (1997) there is an intricate system of factors that affect the so-called the potential educational experiences. Moreover, cross-national comparisons of textbooks, teachers’ mediation and students’ access to the textbooks have shown that students get significantly different opportunities to learn (Haggarty & Pepin, 2002). However, “having an opportunity to learn is a necessary prerequisite for learning, but a learning opportunity is no guarantee of students really learning” (Törnroos, 2005, p. 325).

Despite the complexity of the concept, several ways of measuring OTL have been developed. Roughly speaking they include using teacher reports, document analysis, and classroom observations.

2.2.1. Questionnaires

The first measurements of OTL were based on questionnaires in which teachers had to indicate whether particular mathematical topics or kinds of problems were taught to students. Such questionnaires were used in the international comparative studies FIMS, SIMS and TIMSS, which were carried out by
the International Association for the Evaluation of Educational Achievement (IEA) (Floden, 2002). An example of this approach is the study by Leimu (1992). He gathered OTL ratings from teachers who made an item-by-item judgment concerning exposure to a topic, or teaching of the knowledge and principles necessary for solving a problem, as it applied to the group of students in question. These OTL ratings included emphasis placed by teachers on particular contents and expected student success in those contents. A similar approach was applied in the Dutch version of TIMSS 2003 (Meelissen & Doornekamp, 2004), where OTL was used to evaluate whether the TIMSS test items fit the implemented curriculum. They selected 31 TIMSS test items and asked 129 teachers whether they would include these items in a test that would contain everything their students had been taught up to that moment. However, the interesting thing in Leimu’s (1992) approach of OTL was that he also asked students whether they had had an opportunity to learn the contents required for a correct solution of the test items.

2.2.2. Curriculum and textbook analysis

Another approach applied in TIMSS was looking at what content is offered in curricula and – in connection with this – in textbook series. Schmidt et al. (1997, p.4) see the curriculum as “a kind of underlying ‘skeleton’ that gives characteristic shape and direction to mathematics instruction in educational systems around the world” and that provides “a basic outline of planned and sequenced educational opportunities.”

A textbook analysis to measure the OTL was also applied in a study by Törnroos (2005) that examined whether the test items of TIMSS fit the curriculum. Each of the 162 test items for mathematics was judged on the question of whether the textbook contained adequate material to enable the student to answer the item correctly. A scale from 0 to 2 was used, where the codes ranked from 0 (inadequate material) to 2 (fully adequate material). The values 0, 1, and 2 were used to describe the opportunities to learn offered by the textbooks.

The method of curriculum and textbook analysis developed in TIMSS (Schmidt et al., 1997) is a natural extension of the informal analyses of curriculum guides and textbooks in earlier IEA studies. The basis for this analysis was a mathematics framework containing content areas, performance expectations, and discipline perspectives. This last term refers to what kind of ideas textbooks reflect about mathematics.

Apart from having a framework to classify what is in the curriculum and the textbook, a very crucial thing is that one first defines the unit of analysis. According to Schmidt et al. (1997), the first step in the document analysis process was to subdivide each document into smaller units of analysis on which more detailed analyses could take place. In textbooks, the most fundamental unit type was a lesson. Subsequently these units were subdivided into smaller blocks, containing narrative blocks, graphic blocks, exercise and question sets, suggested activities, and worked mathematical examples. After a document had been divided into units and blocks, each block was
described by assigning codes based on relevant aspects of the mathematics framework.

A more fine-grained approach in defining the unit of analysis was used by Stein et al. (2000) and Cueto et al. (2006). They both considered a task the smallest unit of an activity in a workbook or a student’s notebook. By a task, they meant every question that requires an answer from a student.

2.2.3. Classroom observations More recently, direct observations of classrooms have been implemented to overcome some of the limitations of the approaches used in the aforementioned studies. For example, questionnaires are rather economical and simple for the purpose of large-scale administration and statistical analysis; however, it is difficult for teachers to describe classroom events and interactions using questionnaires (Hiebert et al., 2003). Furthermore, textbook analysis captures the influence of the written curriculum on learning, but “the influence of curriculum materials on student learning […] cannot be understood without examining the curriculum as designed by teachers and as enacted in the classroom” (Stein, Remillard, & Smith, 2007, p. 321).

Therefore, the TIMSS 1999 video study as a supplement to the TIMSS 1999 student assessment has sampled eighth-grade mathematics lessons from six countries where students performed better than their peers in the United States on the TIMSS 1995 mathematics assessments (Hiebert et al., 2003). The TIMSS 1999 video study expanded on the earlier TIMSS 1995 video study which included only one country, Japan. In total, 638 mathematics lessons from seven countries (including the 1995 data from Japan) were analyzed in order to describe and compare teaching practices among countries. In particular, the TIMSS 1999 video study examined the structure and the mathematical content of the lessons, and specific instructional practices, all shaping students’ learning opportunities. Furthermore, questionnaire items for teachers and students were designed to help understand and interpret the videotaped lessons (Hiebert et al., 2003).

Several codes were developed and applied to the video data regarding different aspects of teaching. “[The] coding of [the] classroom lessons was based on segmenting the lesson into meaningful chunks. This requires identifying a unit of classroom practice that can be identified reliably so that its beginning and end points can be marked” (Stigler, Gallimore, & Hiebert, 2000, p. 92). In the TIMSS 1999 video study, mathematical problems were the primary unit of analysis. Each mathematical problem was coded as addressing a specific topic and a scheme for coding procedural complexity was developed; problems were sorted into low, moderate, and high complexity (Hiebert et al., 2003). In addition, they examined whether problems required reasoning and the mathematical relationships between the problems were coded.

Classroom observations were also used in a study about OTL in Chicago’s public schools (Smith, 1998). More specifically, this study addressed the issue of how teachers
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make use of school time to create learning opportunities for their students, because “the amount and quality of time available for instruction directly shapes school outcomes and student achievement” (ibid, p.3). Data from three years of school and classroom observations from fifteen schools were coded as a series of activities segments. The observation framework included instructional and non-instructional activities, such as activities linked to academic and non-academic learning, classroom management, transition time, and so on. Furthermore, interviews with teachers and administrators were used as an additional source of information. Subsequently, these data were used to calculate the hours of instruction typically delivered to the students by Chicago’s public schools.

2.2.4. To conclude this review on opportunity to learn Generally spoken, three different methods have been used to measure OTL. These methods differ not only in their focus of analysis but also in costs, time, and the reliability of the collected data. Each method has its advantages and limitations. However, the review made clear that all three approaches require a framework for analyzing the content that is taught, and a unit of analysis. Despite of the complexity of the learning processes which makes it not easy to say when learning takes place and — as a consequence — when students are offered an opportunity to learn, according to Hiebert and Grouws (2007, p. 379), “opportunity to learn can be a powerful concept that, if traced carefully through to its implications, provides a useful guide to both explain the effects of particular kinds of teaching on particular kinds of learning”. In agreement with Hiebert and Grouws we think that opportunity to learn is “more nuanced and complex than simply exposure to subject matter” (ibid.). However, taking into consideration the determining role that textbooks play in the Netherlands, we decided to do a textbook analysis to investigate whether Dutch students encounter problem solving tasks that require higher-order thinking.

3. METHOD

3.1. Analyzed textbooks

In the textbook analysis that we carried out, we included the textbook documents of the first half year of grade 4 for the six main textbook series that are currently used in Dutch primary schools: Pluspunt, De Wereld in Getallen, Rekenrijk, Talrijk, Wis en Reken, en Alles Telt. The last National Assessment of the Educational Achievement (PPON) (Janssen et al., 2005) showed that approximately 40% of the Dutch primary schools were using the textbooks series Pluspunt. Nearly 20% were using De Wereld in Getallen and 15% were using Rekenrijk. Other textbook series were used by less than 5% of the schools. The textbook series analyzed are the same as those that were used in the schools participating in the earlier mentioned POPO study (Van den Heuvel-Panhuizen, Bakker, Kolovou, & Elia, in preparation) in which we investigated the problem solving of high-achieving students.

The six textbook series cover the grades 1 to 6 and also include documents for
kindergarten classes, which are part of primary school in the Netherlands. Most of the
textbook series consist of a two-volume lesson book (e.g. Student book 6A and 6B),
additional documents such as workbooks and books with master pages meant for
repetition or enrichment, and a teacher guide that explains how to use the textbooks
series.

Table 1 gives a detailed overview of the documents for grade 4 (the Dutch “groep
6”) that were included in the analysis (for every textbook series, the first document in
the list is the main book). The teacher guides and assessment materials were excluded
from the textbook analysis.

3.2. Textbook analysis instrument

The development of the textbook analysis instrument required in the first place
that we identified what we consider the unit of analysis. Secondly, we had to define
more precisely what we mean by problem-solving tasks.

3.2.1. Unit of analysis. Since each textbook series differs with regard to the format
and the number of the pages, we needed to determine a unit of analysis that fits all the
six textbook series. This means that we had to decide how small or how large a unit
should be. To avoid extremely large counts, we decided not to count every operation in
a column (see Figure 1) but take a larger unit size.

In our study, a unit consists of a page section, mostly consisting of a row of bare
number tasks or a picture with a sequence of questions. Such a section can be
considered a didactical unit in the sense that the tasks in it belong together and mostly
address a particular learning content. Every page has about three to six of such sections.
Sometimes the textbooks have pages that count as one section. The subdivision in
sections is characteristic for every textbook series; all the six textbook series we
analyzed have this structure. The sections can be used for whole class discussion, group
work, or individual work. Some of the sections have been designed for the more able
students; a symbol then indicates that the section includes challenging tasks. Figures
1-3 show what a regular page and the subdivision in sections look like in respectively De
Wereld in Getallen, Rekenrijk, and Talrijk. On these sample pages we indicate by
accolades what we took as a unit of analysis.

3.2.2. Categories of problem-solving tasks. The next step in the design of the
textbook analysis instrument was the development of a framework of categories of
problem-solving tasks to classify the units. To develop a clear-cut definition of
problem-solving tasks, we needed several rounds. In the first round, we just marked the
units that contain tasks that can be considered as non-routine mathematical problems.
That means that we were looking for tasks that place a greater cognitive demand on
students than tasks that merely require basic computational skills. Since we found
extremely few of such genuine non-routine puzzle-like tasks in the textbooks, we
decided to make an extra category for what we called “gray-area tasks”. Next, we explain our categories more precisely.

The puzzle-like tasks include problems that do not have a straightforward solution, but that require creative thinking, for example, splitting a number into three or four successive numbers. Since fourth-graders do not have any algebraic tools at their disposal, they cannot apply a routine algebraic procedure, but have to tackle such tasks by a problem-solving strategy, such as trial-and-error, or systematic listing of possible solutions.

The tasks that fall into the gray-area category are not really puzzles and are not really straightforward either, but can trigger strategic thinking and stimulate non-routine approaches. In other words, such gray-area tasks can provoke and prepare the development of problem-solving strategies. Examples of gray-area tasks are problems in which the students have to investigate all possible combinations in which one can throw two dice, problems in which they have to search for a pattern in a series of numbers, and problems like the second task of the three following measurement tasks. These three tasks differ noticeably in cognitive demand and illustrate the difference between straightforward tasks, gray-area tasks, and puzzle-like tasks.

1. You have a soup cup (300 ml). How can you use it to measure 2100 ml of water?

2. You have a soup cup (300 ml), a mug (200 ml) and a glass (250 ml). Show different ways in which you can use these containers to measure 1500 ml of water.

3. You have a 5-liter and a 3-liter jug. How can you take 4 liters of water out of the big bowl using two jugs? You may pour water back into the bowl.

The first task requires the plain application of an algorithm to find that 7 cups make up 2100 ml. The second task can be solved by combining containers that make up the required quantity. Actually, this problem is an example of an “own-construction problem.” This means that the students can explore the different ways to reach 1500 ml. However, in case all possible solutions are required, one has to construct a model and tackle the problem systematically. The third task, that is taken from Alles Telt, Student book 6A (p. 37), is a real puzzle because the solution is not a straightforward one. It requires building a model of the situation in which one has to find a sequence of steps to set apart an amount of 4 liters of water.

As the next step in developing the textbook analysis tool, we subdivided the two problem-solving categories (puzzle-like and gray-area tasks) into more specific types. The puzzle-like tasks were partitioned in context problems and bare number problems. In both sub-categories, the “equations” form a main group. These problems – such as
“Fill in the numbers: … – … = 3200. The first number must be the double of the second number” (Rekenrijk, Master copies book 6A, p. 3) – describe relationships between two or more variables or between quantities. When using algebra, these problems can be solved by means of equations with unknowns. Other sub-categories that have been distinguished are the “switch problems” (within the category of context problems) and the “magic frames” (within the category of bare number problems). The last sub-category, for example, includes tasks in which grids have to be filled with numbers in such a way that horizontally and vertically the totals are the same. The earlier discussed tasks in which a 5-liter and a 3-liter jug have to be used to get 4 liters of water, is an example of a “switch problem” because a quantity of water has to be transferred back and forth between the containers. Another example that can be considered a switch problem is the famous Towers of Hanoi problem.

The category of gray-area tasks is subdivided in tasks about numbers and operations, patterns, and combinatorics. The first sub-category includes non-algorithmic tasks with numbers, for example, making number sentences out of a given number of numbers and reasoning about calculation chains. To avoid getting too many sub-categories the gray-area tasks are not subdivided into context problems and bare number problems. The sub-categories include both kinds of problems.

Table 2 shows the complete framework of problem-solving tasks that we developed. It contains examples of tasks for each of the two categories (puzzle-like and gray-area tasks) and their sub-categories taken from the six analyzed textbook series.

3.2.3. Coding procedure. The framework of problem-solving tasks served as the guideline for coding the units in the six textbook series. First, for each textbook series the total number of units was determined, then each of the units was classified according to the categories and subcategories included in the framework. If a unit neither fit the puzzle-like tasks nor the gray-area tasks no classification was given. In case a unit consisted of a sequence of tasks that included both of the two main categories (puzzle-like and gray-area problems), the highest category was coded. Moreover, if the tasks of a unit belonged to more than one sub-category (for example, within the category of puzzle-like problems the unit can contain context problems and bare number problems), then the most prevalent sub-category was coded.

The final coding was done by the first two authors. This was followed by a reliability check by the third author who was not involved in the development of the framework of problem-solving tasks. The reliability check was based on a second coding of a part of the main books of the three textbook series. (De Wereld in Getallen, Arithmetic book 6A; Pluspunt, Lesson book 6; and Alles Telt, Student book 6A). In this selection we included all types of problems. This second coding was 96% in agreement with the coding of the first two raters.

After the coding procedure was completed, for each textbook series in total and
for each of the documents that belong to a textbook series, the absolute frequencies of all the categories were determined. Then, the relative frequencies were calculated reflecting what percentage of the total number of units belonged to a particular category.

4. RESULTS

The most important results from the textbook analysis are that the textbooks differ in many aspects and that the majority of the number tasks included in the textbook series are straightforward problems and do not really require problem solving. Before we deal with this main result, we discuss some other differences between the textbook series that were revealed by our analysis.

Table 1 shows that the textbook series differ greatly in size. All the figures in this table belong to the textbook materials that are meant for half a year of teaching in grade 4. The first striking thing to note is that the textbook series do not have the same number of documents. *De Wereld in Getallen* consists of two books, while *Pluspunt* and *Wis en Reken* involve four books. The other three textbook series contain three books. The number of pages is also quite different. *De Wereld in Getallen, Alles Telt,* and *Rekenrijk* have approximately 200 pages, while *Pluspunt, Talrijk,* and *Wis en Reken* have more than 300 pages. We also found differences in the number of units. *Wis en Reken, De Wereld in Getallen,* and *Pluspunt* have between 400 and 500 units while *Alles Telt, Talrijk,* and *Rekenrijk* have almost twice that number. Several factors can explain this difference. Apart from differences in the number of books and the number and format of pages, the units in the textbook series do not look alike. *De Wereld in Getallen* has fewer units than *Rekenrijk,* but as can be seen in Figures 1 and 2, the units in the first textbook series contain more tasks than the second. Because of the difference in the number of units we only compared the presence of problem-solving tasks in each of the six textbooks in relation to the total number of units in the textbook series.

The results from the coding show that the percentages of puzzle-like tasks – the tasks that require genuine problem solving – vary between the textbook series, but are extremely low (see Table 3 and Figure 4). They range from 2.43% to 0%. *De Wereld in Getallen* contains the highest percentage – 2.43% of the total units include puzzle-like problems tasks. *Rekenrijk* follows with 1.40%, whereas *Talrijk* and *Alles Telt* include less than 1% puzzle-like tasks (0.66% and 0.77% respectively). Two textbook series, *Pluspunt* and *Wis en Reken* do not contain any puzzle-like tasks at all. In these two textbook series all problem-solving tasks belong to the gray-area category.

If we take the whole category of problem-solving tasks (that means puzzle-like problems and gray-area tasks) the frequency is still remarkably low. The textbook series *De Wereld in Getallen* and *Talrijk* hold the highest percentage of problem solving tasks – both 13%, whereas the other textbook series contain less than 10% problem solving tasks. *Alles Telt* includes the smallest percentage – just 5%.
The results revealed that problem solving and – in particular – puzzle-like tasks have a marginal place in the six textbook series. Moreover, we found that the problem-solving tasks (gray-area tasks and puzzle-like tasks together) are distributed differently over the different documents of the textbook series (see Figure 5). In some textbook series, the problem-solving tasks are primarily included in the main book, while in other textbook series they are spread out over all documents. For example, in *De Wereld in Getallen* 96% of the problem-solving tasks is in the main book, the *Arithmetic book*. This is different, for example, for *Pluspunt* where the *Book with assignments* and the *Extra book* contain more problem-solving tasks (29% and 37% respectively) than the *Lesson book* (22%) which is the main book. In two of the textbook series, a relatively large part of the problem-solving tasks are in the *Master copies books*: in *Talrijk* 43% and in *Rekenrijk* 46%. Having the problem-solving tasks in these documents (mostly containing enrichment material) does not really guarantee that all students will get the opportunity to work on these tasks. The same is true for the problem-solving tasks that are in the main book of *Talrijk*; mostly these tasks are denoted by a special symbol which means that the tasks are meant for the better students.

When we zoomed in on the special group of puzzle-like problems (see Figure 6) we found again that the textbook series differ in how these problems are distributed over the documents. Here, again *De Wereld in Getallen* stands out. This textbook series does not only have the largest proportion of puzzle-like problems, but also has these problems all in the main book, which probably gives the largest chance to students to work on these problems. *Rekenrijk*, on the contrary, has 64% of the puzzle-like problems in the *Master copies book*. This means that more than half of the puzzle-like tasks of *Rekenrijk* are to be found outside the main book. In *Talrijk* 40% of the puzzle-like tasks is also included in the *Master copies book*. To sum up, not only is the number of puzzle-like tasks in the Dutch textbook series very small, but these tasks are often not included in the main book either. In fact, one has to wonder whether the majority of Dutch students encounter any puzzle-like tasks at all.

### 5. DISCUSSION

The disappointing performance of Dutch students in problem solving led us to scrutinize the main mathematics textbook series for grade 4. The percentages of problem-solving tasks that we found in the six textbook series involved in our study – by which we covered the textbook series used by about 85% of the schools – were correspondingly disappointing. In the textbook series with the highest proportion of puzzle-like tasks, the percentage of these tasks was slightly over 2%. Even when we expanded the strict category of puzzle-like tasks with the gray-area tasks, the highest percentage found was 13%.

Because our study was aimed at investigating what textbooks have to offer to the students, we excluded additional materials that are not part of a textbook series. This might be a limitation of our study in cases where teachers do not stick to their textbook...
series documents, but also use additional instructional materials. According to the PPON report (Janssen et al., 2005) this is true for a large number of teachers. About two thirds of the teachers reported that they are using additional material for students who need extra support in mathematics. However, the question asked in the PPON study was clearly about additional material for practicing number operations. Therefore, it is not likely that these additional materials contain many puzzle-like problems. Consequently, we will not be far beside the truth when we say that, from the perspective of what is offered to students, non-routine problem solving tasks are rather scarce in Dutch primary school mathematics education.

Another limitation of our study is that we left out of our analysis how teachers interpret what is in the textbooks. According to Gilbert (as cited in Haggarty & Pepin, 2002) one can never conclude with confidence that what results from an analysis of a text is similarly realized in classrooms. Therefore, Gilbert emphasized that textbooks should be analyzed both in terms of their content and structure, and in terms of their use in classrooms. Earlier, Sosniak and Stodolsky (1993, p. 252), argued that “to understand textbook use, it is necessary to consider teachers’ thought and action and their relationships, teachers’ work within and across subjects, and the full context of teachers’ conditions of work”.

In our study, we restricted ourselves to analyzing the six textbook series with respect to the presence of problem-solving tasks. Taking into account that genuine problem solving – that prepares for algebraic thinking – is not included in the Dutch core goals and is not assessed in the CITO End of Primary School test and the tests of the CITO Monitoring System, we think that the results of our textbook analysis reflect to a large degree what is happening in classrooms. In other words, we can assume that non-routine problem solving gets almost no attention in Dutch primary schools. This, however, contrasts sharply with theoretical and societal claims of the importance of problem solving.

Although our study addressed the situation in the Netherlands, discrepancies between the intended curriculum and the curriculum that is reflected in the textbook series can also be present in other countries, as was, for example, recently revealed by an Australian study on proportional reasoning (Dole & Shield, 2008). In this Australian study, it was explored to what degree proportional reasoning was promoted by mathematics textbooks, and – similar to our study – the researchers found a predominance of calculation procedures with relatively few tasks to support conceptual understanding.

Disclosing possible inconsistencies between what we value as important to teach our students and the instructional materials we use to reach these educational goals, is of crucial importance to improve our teaching. Like Dole and Shield (ibid., p. 33) we see textbook analysis as “a potential means to raise awareness of instruction in key topics within the school mathematics curriculum” and consequently as a vital tool for
Problem Solving in Textbooks

educational progress. To realize this potential, further research is needed in this research domain of textbook analysis, which unfortunately and erroneously has a somewhat outmoded and moldy image, but from which we can learn so much.

REFERENCES


A. Kolovou et al.


A. Kolovou et al


Problem Solving in Textbooks


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### Table 2.

*Puzzle-like tasks and gray-area tasks*

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<th>Context problems</th>
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<td>Equations</td>
<td>How many three- and four-wheeled buggies are there in the shop if the total number of wheels is 125? Can you find different possibilities?</td>
</tr>
<tr>
<td></td>
<td><em>(Alles Telt, Student book 6A, p. 36)</em></td>
</tr>
<tr>
<td></td>
<td>There is €23400 in a suitcase. How many €200 and €500 banknotes are there if the €200 banknotes are twice as many as the €500 banknotes?</td>
</tr>
<tr>
<td></td>
<td><em>(Alles Telt, Student book 6A, p. 38)</em></td>
</tr>
</tbody>
</table>

Joris cycles from Utrecht to Leeuwarden. On his way he sees this sign. How many kilometers is it from Utrecht to Leeuwarden?

Some hours later Joris sees this sign. How much more must he cycle?

*(Alles Telt, Student book 6A, p. 51)*
Peter leaves at half past 3.
He rides 15 km per hour with his bike.

Anton leaves at 3 o’clock.
He walks 5 km per hour.

Cross the correct sentence:
☐ At 4 o’clock they haven’t met each other yet.
☐ At 4 o’clock they meet each other.
☐ At 4 o’clock they pass one another.

(Akenrijk, Master copies book 6A, p. 1)

A CD recorder costs €300 and an empty CD costs €1.50. A pre-recorded CD costs in the record shop €10. After how many CD’s is copying with the CD recorder cheaper than actually buying them?

(Akenrijk, Master copies book 6A, p. 14)

Switch problems
How can you take exactly 4 liters water out of the bowl using a 5 liter- and a 3 liter-jug? You can pour water back to the bowl.

(Alles Telt, Student book 6A, p. 37)
Bare number problems

Equations

Three times the same number.

(De Wereld in Getallen, Arithmetic book 6A, p. 36)

Fill in the numbers. The first number must be the double of the second number: ……... - ………... = 3200

(Rekenrijk, Master copies book 6A, p. 3)

<table>
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<tr>
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<th>difference(-)</th>
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<th>number b</th>
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<td>12</td>
<td>2</td>
<td>7</td>
<td>5</td>
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<tr>
<td>13</td>
<td>3</td>
<td></td>
<td></td>
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</table>

(Rekenrijk, Master copies book 6A, p. 15)

Find 3 successive numbers that make up the result.

……... + ….... + ….... = 270

(Talrijk, Arithmetic book D1, p. 12)
A. Kolovou et al

Magic frames

Use each of the numbers 10, 20, 30, 40, 50, 60 two times.

In every small square there is only one number. In the middle of the squares there is the sum of the rows and the columns.

(De Wereld in Getallen, Arithmetic book 6A, p. 67)

GRAY-AREA TASKS
Numbers and operations

\[
\begin{array}{ccc}
\text{a} & 25 & 24 \\
 & 41 & 35 \\
 & 27 & 50 \\
\text{b} & 40 & 35 & 25 \\
 & 45 & 60 & 27 \\
\text{c} & 16 & 19 & 27 & 48 \\
 & 44 & 30 & 20 \\
\text{d} & 65 & 30 & 35 & 55 \\
 & 40 & 45 & 60 & 70 \\
\text{e} & 24 & 35 & 25 & 41 & 18 & 27 & 50 & 30 \\
 & 47 & 51 & 66 & 35 & 45 \\
\text{f} & 55 & 32 & 64 & 43 & 35 \\
\end{array}
\]

Make two groups of numbers that have an equal value.

(De Wereld in Getallen, Arithmetic book 6A, p. 37)
Try the calculation chain with three different numbers. What strikes you? Explain. Think of a calculation chain yourself.

start → think a number → add 6 → multiply by 2 → subtract 12 → half the number → done

(Rekenrijk, Student book 6A, p. 140)

When Jelmer had spent the half of the half of his money, he had the half of €150 left. How much money did he have at the beginning?

(Rekenrijk, Master copies book 6A, p. 38)

Find the numbers that are equally distant from 7500:

9750, 6950, 7950, 7050, 6925, 8075, 4050, 10950, 8050, 5250

less than 7500 more than 7500 difference with 7500

………….. …………… …………………

(Rekenrijk, Master copies book 6A, p. 15)

Lodewijk has to pay €56,55. He pays with three notes of 20 euros and some extra coins and gets back €5. How much money has he paid in total?

(Rekenrijk, Master copies book 6A, p. 11)
Find the path with the correct product.

*(Talrijk, Master copies book D, p. 7)*

Cross out 3 digits so that you get the biggest 4-digit number: 9150362

*(Talrijk, Master copies book D, p. 110)*

Do sums. Use 150, 20, 5, +, −, = and make 120, 280, 165, 275.

*(Wis en Reken, Math book 6.1, p. 30)*

**Patterns**

How many blocks from each color do you need for a tower with three floors? Fill in the table.

* (Alles Telt, Student book 6A, p. 45)
Problem Solving in Textbooks

Fill in the number line:

112 – 119 – 152 – ... – ... – ... - 232 – ... – ... – ...

(De Wereld in Getallen, Arithmetic book 6A, p. 102)

Combinatorics

Francis and Leo throw darts. In each turn they throw 3 darts. In the first turn Francis got 86 points. How did he throw the darts? Are there different ways?

(Alles Telt, Student book 6A, p. 62)

Pay the exact amount. Try it in at least five ways. Draw the money.

(De Wereld in Getallen, Arithmetic book 6A, p. 59)

3 1 6 8

Use all digits. How many different numbers can you make?

(Pluspunt, Lesson book 6, p. 43)

How do you pack 68 eggs in boxes of 4, 6, and 10 eggs?
Search for different ways.
(Pluspunt, Lesson book 6, p. 47)
In every maze find 6 different routes and results.

(Pluspunt, Extra book 6, p. 21)

Joop sold sausages (€1.50 each), pea soup (€2.75 per cup) and coffee (€1.25 per cup) for €880. How many sausages, cups of soup and coffee did he sell? Are different answers possible?

(Rekenrijk, Student book 6A, p. 14)

Take a pack of cards and remove all the jokers and the cards between 2 and 6. You want two cards of the same color. How many cards do you have to pull at the most?

(Rekenrijk, Master copies book 6A, p. 14)

Use two dice. In what ways can you throw 4, 7, and 10?

(Rekenrijk, Master copies book 6A, p. 16)

Draw all the possible sketches of a building that consists of 4 stones.

(Talrijk, Arithmetic book D1, p. 26)
### Table 3.

*Units in Dutch mathematics textbook series that contain problem-solving tasks*

<table>
<thead>
<tr>
<th></th>
<th>De Wereld in Getallen</th>
<th>Talrijk</th>
<th>Pluspunt</th>
<th>Rekenrijk</th>
<th>Wis en Reken</th>
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<td>469</td>
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<td>439</td>
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<td>(0)</td>
<td>(6)</td>
<td>(0)</td>
<td>(6)</td>
<td>(12)</td>
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<td>(5)</td>
<td>(0)</td>
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<td>(0)</td>
<td>(0)</td>
<td>(20)</td>
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<td>13^</td>
<td>13^</td>
<td>41</td>
<td>9^</td>
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<td>6^</td>
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^ percentage of the total number of units
**Figure Captions**

Figure 1. De Wereld in Getallen, Arithmetic book 6A, p.87

Figure 2. Rekenrijk, Student book 6A, p. 48

Figure 3. Talrijk, Arithmetic book 6A, p.1

Figure 4. Percentages of problem-solving tasks (puzzle-like tasks and gray-area tasks) per textbook series

Figure 5. Percentages of problem-solving tasks (puzzle-like tasks and gray-area tasks) per textbook series per document

Figure 6. Percentages of puzzle-like tasks per textbook series per document
Problem Solving in Textbooks

Figure 1.

Taak 36

1. Vlug en goed!

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2. Schrijf de antwoorden op.

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Maak: Kies uit: Kies uit:

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Zoek de hoogste en de laagste uitkomst

Zorg dat er aan alle kanten 34 uitkomt

Schat het aantal neushoorns

Hoeveel neushoorns waren er in 1982? En in 1978?
Hoeveel neushoorns waren er in 1973 meer dan in 1993?
Figure 3.
Problem-solving tasks (puzzle-like tasks and gray-area tasks) in:

- De Wereld in Getallen: Arithmetic book, Arithmetic workbook
- Rekenrijk: Student book, Workbook, Master copies book
- Alles Telt: Student book, Workbook, Master copies book
Figure 6.

Puzzle-like tasks in:

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