Implementing feedback in a digital tool for symbol sense

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This article reports on research at the crossroads of on the one hand the procedural-conceptual cut in algebra in the transition from secondary education to higher education, and on the other hand the potential of ICT. Three key topics come together: algebra didactics (symbol sense), theories on tool use (instrumental genesis) and assessment (feedback and formative assessment). Our focus in this article is on formative assessment and feedback. A prototypical digital algebra environment was designed, consisting of thirty tasks, which focus on symbol sense. The activities were piloted in five one-to-one think-aloud sessions with students from pre-university grade 12. The results of the one-to-one sessions were analyzed and used to make a feedback design for the tasks. The feedback design was implemented in a second revision of the tool. The revision will be used for experiments on a larger scale with a web-based tool for acquiring, practicing and assessing algebraic skills.

1 INTRODUCTION

During the last twenty years the relationship between procedural skills and conceptual understanding has been widely debated. This relationship played and still plays a central role in the ‘Math war’ (Becker, 1998; Kieran & Drijvers, 2006) discussion. An important issue in this debate is how students should acquire algebraic expertise: by practicing algorithms, or by focusing on reasoning and strategic problem solving activities. One approach sees computational skills as an essential ingredient for understanding mathematical concepts (US Department of Education, 2007). The second approach has more focus on conceptual understanding (ibid.). Although there often is agreement on the fact that both procedural skills and conceptual understanding are important, there is disagreement on their relationship and the priorities among the two. Apart from this discussion, the last decades also saw an advent in the use of technology in mathematics education. National Council of Teachers of Mathematics acknowledges the potential of ICT for learning in their position statement (2008). In our research we combine these two issues: we want to use the potential of ICT to address algebraic skills, on both a procedural and a conceptual level. A pre-study concerned the choice and design of a digital tool for algebra tools (Bokhove & Drijvers, in press). Figure 1 sums up a selection of these criteria. The next cycle involved one-to-one sessions with students. These sessions had two goals. One was to see whether digital activities could invite symbol sense behaviour; we report on this in a different article (Bokhove & Drijvers, 2009). A second goal was to elicit the design of feedback for a second revision of the tool. This article reports on this second goal.

2 CONCEPTUAL FRAMEWORK

To place our focus on feedback in a more general framework, we briefly describe the three topics that are involved in our study: algebra didactics (symbol sense), theories on tool use (instrumental genesis) and assessment (feedback and formative assessment).

Algebra

According to Arcavi (1994) algebraic skills have a basic skill component and a symbol sense component. Both basic skills and symbol sense should be addressed in education, as they are closely related: understanding of concepts makes basic skills understandable, and basic skills can reinforce conceptual understanding. Here, symbol sense lies in the realm of conceptual understanding: an intuitive feel for when to call on symbols in the process of solving a problem, and conversely, when to abandon a symbolic treatment for better tools. Arcavi (1994) describes several ‘behaviours’ of symbol sense. Figure 2 shows the dimension Drijvers (2006) sees: basic skills involve procedural work with a local focus and emphasis on algebraic calculation, while symbol sense involves strategic work with a global focus and emphasis on algebraic reasoning.

Algebraic Expertise

<table>
<thead>
<tr>
<th>Basic skills</th>
<th>Symbol sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural work</td>
<td>Strategic work</td>
</tr>
<tr>
<td>Local focus</td>
<td>Global focus</td>
</tr>
<tr>
<td>Algebraic calculation</td>
<td>Algebraic reasoning</td>
</tr>
</tbody>
</table>

Figure 2. Algebraic expertise as a dimension: from basic skills to symbol sense

Bokhove and Drijvers (2009) studied four categories of items with symbol sense opportunities: solving equations with...
common factors, covering up sub-expressions, resist visual salience in powers of sub-expressions and recognizing common factors. The categories are sometimes subtly interlinked. For example the category on recognizing common factors involves common factors that are not clearly apparent in an equation. The category on equations with common factors, on the other hand, have clearly visible common factors on both hand sides of the equation. One could even say that after recognizing common factors (category four) one is left with a category one equation. However, the type of symbol sense involved, is different. One thing all these categories have in common is a combination of gestalt view or resistance to visual salience. A gestalt view on algebraic expressions involves the ability to consider an algebraic expression as a whole, to recognize its global characteristics, to ‘read through’ algebraic expressions and equations, and to foresee the effects of a manipulation strategy. Resistance to visual salience refers to the ability to resist visually salient properties of expressions, and their implicit invitation to carry out specific operations. The following task provides an example of the first category:

**Category 1: Solving equations with common factors**

This category can be characterized by common factors on the left and right hand side of the equation. Example:

Solve the equation:

\[(x^2 - 7x + 12) \cdot (8x - 11) = (x^2 - 7x + 12) \cdot (3x + 14)\]

Later on we will return to this example in our case study.

**Tool use**

Tool use is an integrated part of human behaviour. Vygotsky (1978) sees a tool as a mediator, a "new intermediary element between the object and the psychic operation, directed at it". Verillon and Rabardel (1995) distinguish artefact and instrument. The *artefact* is just the tool. The *instrument* denotes the relationship between a person and the artefact. Only when this relationship is established one can call it a "user agent". The mental processes that come with this are called *schemes*. In short: instrument = artefact + instrumentation scheme. Trouche (2003) distinguishes *instrumentation* (how the tool shapes the tool-use) and *instrumentalisation* (the way the user shapes a tool). *Instrumental genesis* is the process of an artefact becoming an instrument. In this process both conceptual and technical knowledge play a role ("use to learn" and "learn to use"). To overcome the contrast between pen-and-paper and ICT based learning, an ICT environment has to correspond with traditional techniques (Kieran & Drijvers, 2006).

**Assessment**

Black and Wiliam (2004) distinguish three functions for assessment:

- supporting learning (formative)
- certifying the achievements or potential of individuals (summative)
- evaluating the quality of educational programs or institutions (evaluative)

Summative assessment is also characterised as assessment of learning and is contrasted with formative assessment, which is assessment for learning. Black and Wiliam (1998) define assessment as being ‘formative’ only when the feedback from learning activities is actually used to modify teaching to meet the learner's needs. From this it is clear that feedback plays a pivotal role in the process of formative assessment.

**Feedback**

In the learning process adapting instruction to meet students learning needs showed substantial benefits, for example in studies by Carpenter et al. (1989) and Black et al. (2003). As the role of feedback had to be taken into account, drill-and-practice use of the computer made formative assessment difficult. An interesting question is whether the use of "more intelligent" new technology makes a difference in this respect. Bangert-Drowns et al. (1991) found that not being able to see the answer before trying a question is better. Also giving details of the right answer, instead of just wrong or right, seemed more effective, as other research has also confirmed (Dempster, 1991, 1992; Elshout-Mohr, 1994).

Several studies –including Nyquist (2003) seem to show more effectiveness in assessment when using feedback. The stronger the feedback the larger the effect seems to be (Elawar & Corno, 1985). Reviews conducted by Natriello (1987), Crooks (1988), Bangert-Drowns et al. (1991) and Black and Wiliam (1998) showed that that not all kinds of feedback to students about their work are equally effective. It is therefore sensible to distinguish feedback types. There are several models that distinguish feedback types. Nyquist (2003) reviewed 185 studies in higher education, developing a typology of different kinds of formative assessment:

- *Weak formative assessment*: students are given only the knowledge of their own score or grade, often described as "knowledge of results";
- *Feedback only*: students are given their own score or grade, together with either clear goals to work towards or feedback on the correct answers to the questions they attempt, often described as "knowledge of correct results";
- *Moderate formative assessment*: students are given information about the correct results, some explanation, and some specific suggestions for improvement.
- *Strong formative assessment*: students are given information about correct results, some explanation, and specific activities to undertake in order to improve.

This distinction emphasizes the important role feedback plays in formative assessment.

Hattie and Timperley (2007) did a meta-review of the effectiveness of different types of feedback. The feedback effects of cues and corrective feedback are deemed best. Seeking feedback is governed by cost/benefit ratio. In general, feedback is psychologically reassuring, and people like to obtain feedback about their performance, even if it has no impact on their performance. The model provided by Hattie and Timperley (ibid.) distinguishes three questions that effective feedback answers:

*Where am I going? (the goals)*

FeedUp
Each feedback question works at four levels (focus of the feedback):

- Task level: how well tasks are understood/perform (FT)
- Process level: the main process needed to understand/perform tasks (FP)
- Self-regulation level: self-monitoring, directing and regulating of actions (FR)
- Self level: personal evaluations and affect (usually positive) about the learner (FS)

Hattie and Timperley (ibid.) also provide some statements on the effectiveness of (combinations of) feedback types, including that FS feedback is least effective, simple FT feedback is more effective than complex FT feedback, FT and FS do not mix well (“Well done, that is correct” is worse than “Correct” only), FT is more powerful when it’s about faulty interpretations, not lack of information. Furthermore they state that we should be attentive to the varying importance of the feedback information during study of the task.

These principles are the basis of the choice of our tool and will also be the basis of our student-inquired feedback design for an algebra tool.

3 RESEARCH QUESTIONS

Our research methodology will aim at elaborating possible and desired feedback for our digital tool.

The questions we set out to answer in this article are:
1. Can the feedback design of a digital tool be improved through student inquiry?
2. What methodology can be used to elaborate feedback desired by students?

4 METHODOLOGY

The methods include one-to-one sessions with students, post-analysis of feedback occurrences and subsequent revision of the prototype. Apart from validating whether the proposed activities in the prototypical digital environment indeed invite symbol sense (Bokhove & Drijvers, 2009), five one-to-one sessions with students were also held to explicate what feedback was desired on what moment. As the tool was also intended for home-use we chose to focus on individual usage of the tool, and thus one-to-one sessions. Four pre-university grade 12 students were in the so-called nature/health stream. A fifth student took the advanced math for science course. The students all had C+ grades for mathematics. An additional sixth one-to-one session was held with a mathematics student, to compare the data with expert behaviour. The sessions were recorded with audio and camera, and the content of the computer screens by screen recording software (Camtasia).

During the sessions, the students worked through the digital activities. They were asked to think aloud while working. If a student was not able to do (part of) an item, the observer asked what information would help in proceeding. On occasions where student used wrong strategies or made specific procedural choices, the observer asked the student what he or she was thinking. This informed possible feedback for a future revision of the prototype. After the first part of each session, the observer and the student went through the student’s work and reflected on the solutions, discussing the student’s arguments and alternative solution paths, as well as possible (feedback) improvements of the prototype.

Analysis of screen casts took place with software for qualitative data analysis (ATLAS ti), and involved aggregating data on desired feedback. This meant that for every task we constructed an overview of all the feedback that was desired by the students. Next, this feedback was implemented in the revised prototype. In the next paragraph we give an example of a feedback-design-in-progress for the example from figure 2.

<table>
<thead>
<tr>
<th>Input</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 3$ or $x = 4$ or $x = 5$</td>
<td>Correct</td>
</tr>
<tr>
<td>$(8x - 11) = (3x + 14)$</td>
<td>You probably divided by $(x^2 - 7x - 12)$. You lost some solutions for $x^2 - 7x - 12 = 0$</td>
</tr>
<tr>
<td>$(8x - 11) = (3x + 14)$</td>
<td>You have plotted the graphs of the functions on both sides of the equation and determined how many points of intersection there are.</td>
</tr>
<tr>
<td>$5x^3 - 60x^2 + 235x - 300 = 0$</td>
<td>You probably don’t know how to solve this third order equation. If not, then look at the initial equation. Do you see corresponding factors on the left and right hand side?</td>
</tr>
<tr>
<td>$x^2 - 7x + 12 = 0$ or $(8x - 11) = (3x + 14)$</td>
<td>You recognized similar factors on both sides.</td>
</tr>
</tbody>
</table>

Figure 4. Step one. The table shows a part of the feedback.
5 RESULTS

The results consist of:

- The described student-driven methodology for elaborating desired feedback for digital tools;
- A feedback design for our initial prototypical tool;
- A revised tool;

These results are exemplified in a case study example.

6 CASE STUDY EXAMPLE

In this paragraph we provide an example of the feedback elaboration process through a feedback design for one of the tasks in our prototypical tool. The task we will present is the task from figure 2.

Solve the equation:

\[(x^2 - 7x + 12) \cdot (8x - 11) = (x^2 - 7x + 12) \cdot (3x + 14)\]

Subsequently the steps that are involved are:

Step 1: we distilled desired feedback occurrences from the screencasts and made a feedback design.
Step 2: we implemented the design in our second revision. See figure 5.
Step 3: we tested our implementation. See figure 6.

Step 1: feedback from one-to-one sessions

In this step we analyzed the screencast recordings of several student sessions. We distinguished remarks on desired feedback in all of the sessions, other welcome additions for the second revision of the tool, and other technical remarks. Grouping similar feedback resulted in a feedback design.

Figure 4 shows a selection of the design for the case study example. The first column is the input from the student, the second column denotes the feedback the students want and thus we desire in our tool, the third column provides some explanation on expected student behaviour. The last column shows the feedback type (Hattie & Timperley, 2007).

Step 2: implementation of the feedback design

Figure 5 shows the feedback design being implemented in our second revision. Figure 5 shows the feedback design being implemented in our second revision. The figure shows part of the authoring environment that is used ‘behind the scenes’. We can recognize several elements from figure 4.

At the top of the figure tabs denote the different feedback instances one exercise holds. In this case instance four of in total eight feedback instances is active and highlighted in white. Every feedback instance consists of an answer model. Every step in the solution process of the student is evaluated according to the answer models that are in the feedback implementation. Note that the answer model is randomized. This means that a through f are random integers (within boundaries that are defined elsewhere). In this case one can recognize the Quadratic Formula in the answer model, as applied to the quadratic term in the initial equation. Of course, our case study example with the quadratic part yields two solutions, the linear term of the equation yields one solution.

Under this box the authoring environment provides the opportunity to apply rules and limitations: “Equivalent” is checked. This denotes that the answer given by the student should be equivalent to the answer model. “Vorm” is checked. This option is linked to the box on the bottom right and demands that the student answer is in a certain algebraic form. As the digital tool has an emphasis on symbol sense we desire that a student recognizes similar terms. The “Vorm” box therefore contains the form after recognizing these similar terms. The four radio buttons signify what symbol is appropriate: a green “goed” (correct) symbol, an orange “half” correct symbol or a red “fout” (incorrect) symbol. “Door” can be used for answers that the tool deems not entirely correct but require the possibility to ‘follow-through’ on the answer. This opposed to the situation where the solution process suddenly stops. If all these conditions apply, the feedback in the box on the bottom left is shown and the score provided is given.

Step 3: testing the revision

Figure 6 shows part of what the feedback instance from figure 5 looks like when giving in the corresponding step. In this case the student has recognized common factors on both sides of the equation. The task is embedded in a full-fledged equation editor, and allows the student to make algebraic steps.

Figure 5. Step two: authoring custom feedback for one of the items.
7 CONCLUSION AND DISCUSSION

Our first question was whether the feedback design of a digital tool can be improved through student inquiry. We conclude that asking students when to use what feedback can improve a digital tool.

The second question was what methodology can be used to elaborate feedback desired by students. We have found our methodology fruitful:
1. Deploy the tool and ask students to ‘think aloud’;
2. Tabulate all the responses;
3. Make a feedback design;
4. Implement the feedback design in the tool;
5. Test the tool with help of the feedback design, and make another iteration. This provides a revision of the tool. It is possible to greatly improve a tool this way.

In our one-to-one sessions and explorations on the role of feedback we would like to bring up three points of discussion.

Firstly, the role of scores. Scoring seems to have a negative impact (Butler & Nisan, 1986). Butler (1988) even concluded that the effects of diagnostic remarks completely disappeared when grades were added. So here we have a dilemma: do we use grades/marks and feedback together, hoping motivation will overcome the disadvantages or not.

Secondly, it is very difficult to anticipate all the mistakes students can make. We in no way would claim that this methodology and this tool caters for all common misconceptions. The methodology described does, however, add student input to the domain of feedback design.

Thirdly, technology is always “on the move”. Along with feedback, technological developments influence the improvement of our tool. Whether these are actually implemented in our tool will depend on the added value. In the author’s opinion tools for education should not focus on new features or technologies but on its relevance in a classroom setting. Both education and design can profit from each other.

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REFERENCES


**BIOGRAPHICAL NOTES**

Christian Bokhove is a math teacher at St. Michael College in Zaandam, the Netherlands. He is involved in a Dutch PhD programme for secondary school teachers, called Dudoc. His current research is focused on the question how ICT tools can support acquiring, practicing and assessing relevant mathematical skills.