Low attainers and opportunities to do mathematics

P. Scherer
University of Dortmund

1 mathematics education in schools for learning disabled

German schools for children with learning disabilities show extremely heterogeneous learning groups: On the one hand, there are the learning disabled with general learning difficulties, on the other hand students who show non-uniform performances, and also the maladjusted children. The learning processes are comparable with those of non-handicapped children, however, the processes are characterized by a temporal extension and a higher error-rate. The teacher is confronted with individual handicaps, for example defects in the language or visual perception, failure of concentration or reduced memory and transfer so that a high degree of differentiation and training is needed (Kanter, 1978).
The curriculum for mathematics in special education points out that the children should be able to develop their own methods of problem solving. Concerning the training of the basic skills, the danger of getting stuck in purely schematic thinking through mechanistic drill and practice is clearly stated (KM, 1977, 7). In contrast to these standards, textbooks and classroom practice do not come up to these high expectations. The generally shared opinion is that low-attaining pupils cannot cope with more demanding and complex problems. Usually the reasons for failure are seen in the student him- or herself, and the teaching methods are rarely called into question. As a consequence, the demands are lowered, and the student's activities are confined to bare reproduction. Accordingly, the isolation of difficulties and learning step-by-step are guiding principles in special education. Mechanistic drill and practice replace insightful learning (Baier, 1977; Grölz, 1983).

2 opportunities through investigative learning and productive exercises

In Germany the situation in special education is quite different from the regular school system: In contrast to regular schools, particularly primary schools, in special education investigative learning is mostly disregarded. Actual didactic research in special education focuses on diagnosis with regard to deficiencies (Moog, 1993; Niegemann, 1988). You will find only a few papers that criticize textbooks (Kornmann et al., 1993; Wagner et al., 1991) and might effect a change of classroom culture. Nevertheless positive experiences for other subjects are reported in special education (Wittoch, 1991). Theoretical papers on the teaching of mathematics that argue in favour of investigative learning have already been published (Böhm, 1988; Höck, 1986; König, 1976). In England and the Netherlands there are encouraging classroom experiences with low-attaining pupils (Ahmed, 1985; Trickett & Sulke, 1988; Van den Heuvel-Panhuizen, 1991).

This leads to the question to what extent the investigative learning approach is also appropriate for children with special needs or especially for them. Investigative learning does not only include challenging problem solving activities but more general active acquisition of knowledge in contrast to passive reception. Productive work is to be understood in contrast to bare reproduction of knowledge. It should enable pupils to think, to construct and to extend their knowledge (Winter, 1984; Wittmann, 1990). In the following, four crucial questions will be discussed:

getting information about abilities versus diagnosing deficiencies?

There are several possibilities for diagnosing learning difficulties. Some exercises show what the pupils do not know. But they do not show why; nor do they give information about what the children are able to. What can productive exercises accomplish in this context? One possible assignment is open problems that give freedom to the student in choosing tasks and methods of calculation.

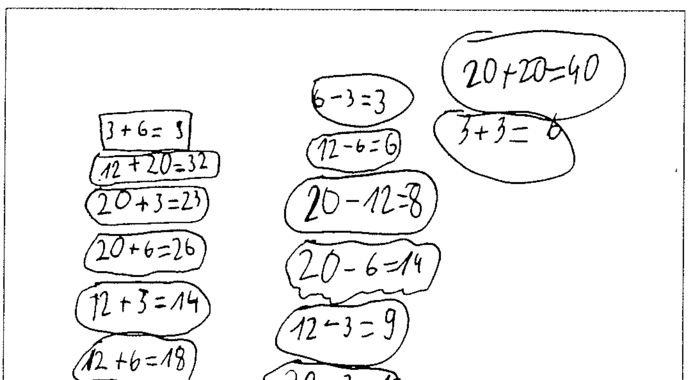


figure 1

An example is the following problem:[1] Find addition and subtraction tasks with the numbers 3, 6, 12, 20.[2] This problem may help to clarify existing misconceptions, but at the same time it enables the child to show his or her abilities. For example, making use of arithmetic structures and properties, the individual extent of systematic work, etcetera (fig.1). Moreover, this exercise covers a natural differentiation. It includes tasks on different levels, that are neither fixed nor determined by the teacher.

holistic approach versus learning step-by-step?

Investigative learning and productive exercises are usually connected with holistic approaches, and especially for low attainers it is important to get an overview and see relationships (Trickett & Sulke, 1988, 112).

In special education, holistic approaches are usually avoided in favour of splitting up the subject matter into fragments. In this context Donaldson (1978) distinguishes between the mastering of all the individual patterns or relationships of a system on the one hand, and understanding the nature of a system on the other hand. It will inevitably take a child some time to learn all the sets of correspondences. The question is simply whether he will do it better if he is correctly informed about the kind of thing to expect. (Donaldson, 1978, 105).

An example from addition may illustrate this point. The main strategy for addition-table, the counting-on, becomes ineffective for bigger domains of numbers. For the first steps according to the step-by-step approach stipulated by the curriculum (KM, 1977, 30) it can be successful. That means that it is not necessary for the children to make use of new strategies, for example the decomposition of sums. However, splitting a subject into little fragments does not contribute to solving certain difficulties like understanding the structure of our number system.

allowing individual strategies versus stipulating fixed ways of solutions?

Usually step-by-step learning is connected with a special way of solving the tasks and with fixed notations. In many cases, the stipulated way is not the students way so that the student may not be able to comprehend the teachers approach for lack of transfer capabilities. Here the strategy of avoiding mistakes is found. The opinion that all situations that could lead to mistakes and failures are to be avoided, is also found. But this would exclude the acquisition of new contents and strategies for acting, and there would be hardly a chance for productive thinking and learning (EDK 1991, 12; Wittoch, 1976, 60).

My experiences have shown that without special instruction pupils use their own informal strategies, for example for adding two-digit numbers (fig. 2). And these strategies can be quite different from those prescribed by the textbook. But usually the children are not allowed to use their own strategies.

Rechne aus!

35 + 23 = 30+5+3+20=8+50=58

figure 2

making use of the pupil's ideas and knowledge versus only supplying of subjects?

Mathematics education has to consider what ideas pupils have in mind and how to make use of these ideas (Höck, 1986). Curriculum and textbooks in special education do not use the pupils knowledge, and generally prefer a step-by-step instruction for a new topic (KM, 1977, 32f, 44). That should now be explained for multiplication.

For example, rectangular arrays appear in our environment so that you can talk about them, and make use of children's knowledge (Wittmann & Müller, 1990, 108). Rather often the textbooks for special education choose rectangular arrangements that make insight more difficul.

For instance the tasks 5×8 and 8×5 are represented by two different arrays (fig.3).[3] It might be difficult to see the commutative property in these arrangements as the left example might lead the pupils to the task 4×10. Nevertheless the intended task by the textbook is 5×8.

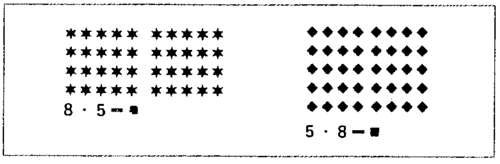
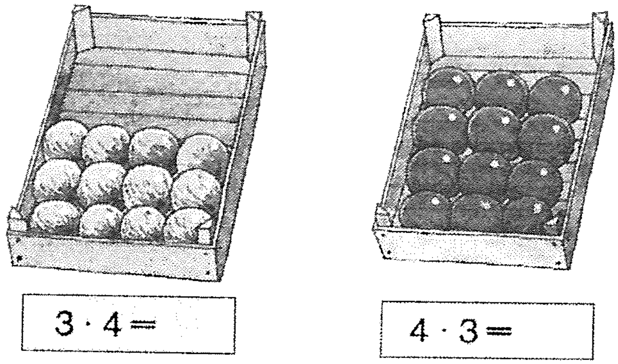


figure 3: example from a textbook

Even when the commutative property is obvious (fig.4). In general, one rectangular arrangement belongs to one multiplication only.

If the children are free and encouraged to show their own ideas, different images of multiplication are found. For example, the pupils[4] were given an egg carton and asked what tasks they could see in this object. The children wrote down additions and multiplications like 2 + 2 + 2 + 2 + 2 = 10; 5 + 5 = 10; 2 × 5 = 10; 4 + 4 + 2 = 10; 5 × 2 = 10.

So the connection between addition (the well-known content) and multiplication (the new content) could be pointed out. :



figuur 4: example from a textbook

Some misconceptions became obvious (fig.5). One student said 'Two times three', but wrote down 3 × 3, where the three was written down two times. Another student calculated 5 + 1 instead of 5 × 1.

Analogously to addition several students calculated 6 × 0 = 6. It has to be taken into consideration whether a step-by-step approach in which the tasks mentioned above do not appear at the beginning, might lead to the fixing of misconceptions.

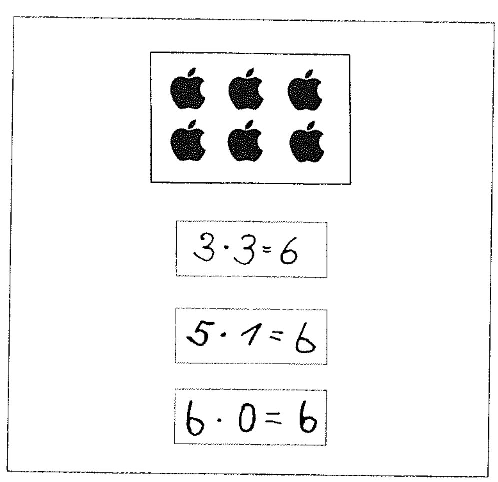


figure 5

When children are asked to illustrate multiplication tasks with circular counters, usually linear structures or rectangular arrangements, structured by lines or rows are found (fig.6a).

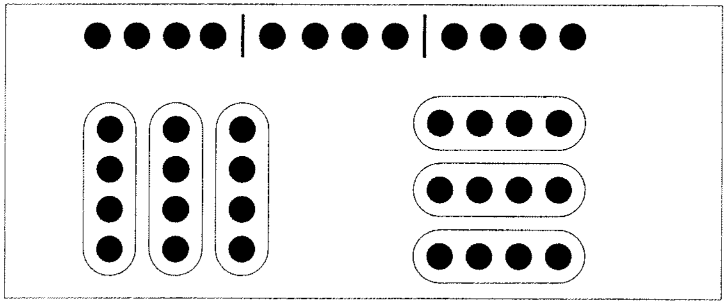


figure 6a

You can also find an illustration of the task but not of the result (fig.6b). Here you would have to ask if the children are not used to illustrate problems, and if there is a rather quick transition to a symbolic level.

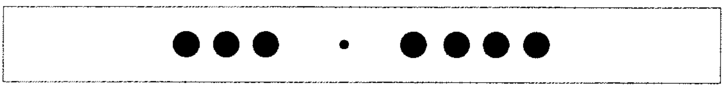


figure 6b

In the rectangular arrangements the pupils have the possibility to integrate different images and to discover how effective different strategies are.

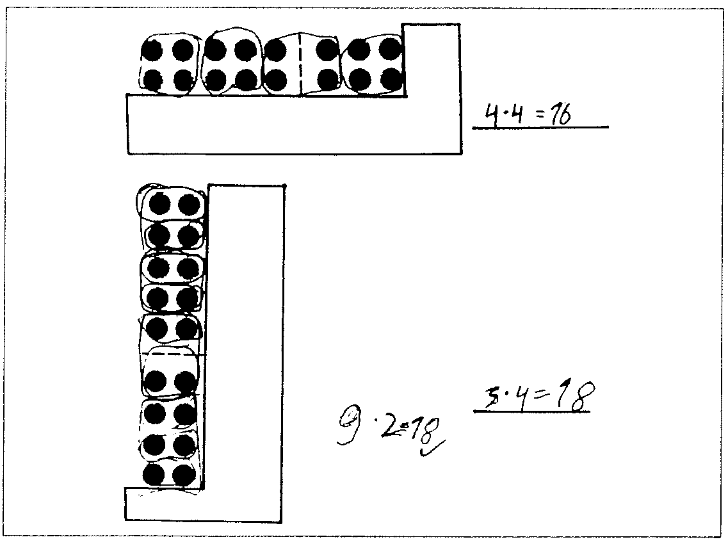


figure 7

Given a 2 × 8-field, one student figured out the task 4 × 4 by the image of a die (fig.7). The same strategy did not work for a 9 × 2-field. So this student was challenged to find a better method, and he finally succeeded by using lines.

3 conclusions

Investigative learning and productive work cannot completely solve the diverse problems of low attainers in mathematics. Probably the new learning arrangement will cause a lot of difficulties in the beginning, as the children have to become familiar with working in an active and self-responsible manner.
However, there are good reasons for investigative learning especially for low attainers:

- The pupils can show what they are capable of, for instance as shown in the first example (see fig.1). The level is not fixed at the beginning, and that can boost the pupils' confidence (Wittoch, 1976, 40).
- Misconceptions and difficulties can be identified more easily and at an early time. If the pupils only have to work on given right tasks, some special mistakes will not become obvious (see the subtractions in fig.1 or the multiplications in fig.6). The usual strategy of avoiding all difficulties and complexities will have short-term success, but in the long term the failure is predetermined.
- Children approach open problems with enthusiasm because the results are assessed not only as right or wrong, and therefore the fear of failure is reduced (Grossmann, 1975; Wittoch, 1991).
- The restriction to one way of solving a task bears the risk that this special way cannot be understood by the students. It was shown that the children are able to find their own methods and strategies for example for adding two-digit-numbers (see fig.2).
- The holistic approach offers a lot of possibilities of a natural differentiation, as the students can work on several levels of difficulties and be successful at their own one (Wittoch, 1976, 11).
- Experiences have shown that the learning disabled are often underestimated or misjudged. That we often bar their ways on which they - even surprisingly - would succeed by a more open approach. Supporting the children means making certain demands.

notes

1 The worksheets for addition and subtraction are for children who attended grade three of a school for learning disabled at the age of ten.
2 The children were free to choose three or more numbers, nevertheless all pupils built tasks with only two numbers.
3 In Germany the mathematical symbol of multiplication is .
4 The following examples are from eleven-year-old children who attended grade four of a school for learning disabled.

references

Ahmed, A. et al. (1985). *Mathematics for low attainers.* Chichester: Her Majesty's Stationary Office.

Baier, H. (1977). Allgemeine Prinzipien der Erziehung und des Unterrichts in der Schule für Lernbehinderte. In: G.O. Kanter & O. Speck (hrsg.) (1977). *Handbuch der Sonderpädagogik, Band 4. Pädagogik der Lernbehinderten.* Berlin: Marhold, 252-277.

Böhm, O. (1988). Die Übung im Unterricht der Schule für Lernbehinderte. *Zeitschrift für Heilpdagogik*, 39, 73-85.

Donaldson, M. (1978). *Childrens minds.* London: Fontana/Croom Helm.

EDK - Schweizerische Konferenz der kantonalen Erziehungsdirektoren (1991). *Fehler! - Fehler?* Bern: Sekretariat EDK.

Grölz, H. (1983). Problemorientierte Unterrichtsmethoden in der Schule für Lernbehinderte. *Zeitschrift für Heilpädagogik*, 34, 645-653.

Grossman, R. (1975). Open-ended lessons bring unexpected surprises. *Mathematics Teaching*, 71, 14-15.

Heuvel-Panhuizen, M. van den (1991). Ratio in Special Education. In: L. Streefland (ed.). *Realistic Mathematics Education in Primary School.* Utrecht: OW & OC.

Höck, M. (1986). Thesen zur mathematischen Begriffsbildung bei lernschwachen und lernbehinderten Kindern. *Zeitschrift für Heilpädagogik*, 37, 834-839.

Kanter, G. O. (1978). Lerngestörten- und Lernbehindertenpädagogik. In: H. Bach (ed.). *Sonderpädagogik im Grundriss*, 5, Berlin: Harnold, 105-112.

Kanter, G.O. & O. Speck (hrsg.) (1977). *Handbuch der Sonderpädagogik, Band 4. Pädagogik der Lernbehinderten.* Berlin: Marhold.

KM - Kultusminister des Landes NRW (Hrsg.) (1977). *Richtlinien und Lehrpläne für die Schule für Lernbehinderte (Sonderschule) - Mathematik.* Köln: VG Ritterbach.

König, H.W. (1976). Die Übung der Grundrechenarten im Mathematikunterricht der Schule für Lernbehinderte. *Zeitschrift für Heilpädagogik*, 27, 595-605.

Kornmann, R. et al. (1993). Lehrwerke als Lernbehinderung: Die Vernachlässigung des Kommutativgesetzes in den Mathematiklehrwerken für die Klassen 1-4 der Schulen für Lernbehinderte (Förderschulen). *Zeitschrift für Heilpädagogik*, 44, 600-605.

Moog, W. (1993). Schwachstellen beim Addieren Eine Erhebung bei lernbehinderten Sonderschülern. *Zeitschrift für Heilpädagogik*, 44, 534-554.

Niegemann, H.M. (1988). Neue Wege in der pädagogischen Diagnostik: Fehleranalyse und Fehlerdiagnostik im Mathematikunterricht. *Heilpädagogische For-*

schung, 14, 77-82.

Trickett, L. & F. Sulke (1988). Low attainers can do mathematics. In: D. Pimm (ed.). *Mathematics, Teachers and Children.* London: Hodder and Stoughton.

Wagner, H.J. et al. (1991). Die Null - eine vernachlässigte Grösse in elementaren Mathematiklehrwerken der Schule für Lernbehinderte? *Zeitschrift für Heilpädagogik*, 42, 442-479.

Westphal, E. (1977). Primarstufe. In: G.O. Kanter & O. Speck (hrsg.) (1977). *Handbuch der Sonderpädagogik, Band 4. Pädagogik der Lernbehinderten.* Berlin: Harnold, 252-277.

Winter, H. (1984). Begriff und Bedeutung des Übens. *Mathematik lehren*, 2, 4-16.

Wittmann, E. Ch. & G.N. Mller (1990). *Handbuch produktiver Rechenübungen. Bd. 1, Vom Einspluseins zum Einmaleins.* Stuttgart: Klett.

Wittmann, E. Ch. (1990). Wider die Flut der 'bunten Hunde' und der 'grauen Päckchen': Die Konzeption des aktiv-entdeckenden Lernens und des produktiven Übens. In: E. Ch. Wittmann & G.N. Mller (1990). *Handbuch produktiver Rechenübungen. Bd. 1, Vom Einspluseins zum Einmaleins.* Stuttgart: Klett, 152-166.

Wittoch, M. (1976). *Unterricht mit Schulversagern.* Köln: Kierpenhever & Uitsch.

Wittoch, M. (1991). Diagnose von Störungen - Erfahrungen mit Lernarrangements mit Kindern, die eine Schule für Lernbehinderte besuchen. In: J.H. Lorenz (hrsg.). *Störungen beim Mathematiklernen.* Köln: Aulis, 90-105.