Uncertainty, a Metaphor for Mathematics Education?[1]

Leen Streefland & Marja van den Heuvel-Panhuizen
Utrecht University, Utrecht, The Netherlands

The present contribution urges an uncertainty principle for mathematics education, with special emphasis upon assessment. We need to open space for creativity and improvement, once we recognize the impossibility of precise testing in the traditional sense. The traditional optimism about (mathematics) education based on standard testing, with its aura of infallibility, may do more harm through doubtful claims for certainty than through the unavoidable uncertainties, with all their dangers, which we need to recognize and face.

To measure is to know

Research in the natural sciences at one time served as a model for the development of research in education. This origin is still reflected by the use of measurement in educational research today. Not only does measurement pervade research design, but at the level of testing and assessment of teaching-learning processes, it is also about determining the performance of students, and not only with great precision, but with abandonment of all subjectivity as well.

The adage of the natural sciences 'to measure is to know' that once graced the entrance to the laboratory of the famous Dutch physicist Kamerlingh Onnes would not be unbefitting on many a classroom door either. That would certainly have been true some two decades ago, when education was placed under a ban of goal-oriented testing and hence teaching threatened to become test-directed. Even today, however, when there is a revival of interest in testing, it seems feasible that this danger of test-oriented teaching could return.

However, there is a big difference between then and now. As far as the issue is concerned in The Netherlands, former interest in testing was particularly prompted by negative opinion. There was opposition to the test movement from the viewpoint of realistic mathematics education[2] resulting in alternative approaches. The current upswing of interest in testing, on the other hand, has evolved from general developments in mathematics education itself - and in the particular case of The Netherlands, from the theory of realistic mathematics education (cf. van den Heuvel-Panhuizen, 1996).

New ways by which to test in mathematics education call for new methods of assessment (cf. see de Lange, 1987). This relationship between teaching and assessment also exists in the reverse as far as realistic theory is concerned. lt would not be what it is, if the consideration of the implications for assessment would not contribute to its own enrichment. Streefland (1990) gives an example in defining internal and external assessment in developmental research. He indicated that what was initially used as internal assessment of the teaching-learning process, later became the main source of external assessment of the (prototype) of courses, which are modeled after the outcomes of research.

Again, this brief presentation aims to contribute to the enrichment of the theory of realistic mathematics education. Its core idea evolved from the professional activities of both writers with respect to the role of reflection in realistic mathematics education as well as the assessment of realistic mathematics education.

Optimism continues to persist

Traditionally, testing and evaluation of teaching - learning processes are viewed from an optimistic perspective. One takes into regard what is being taught and brings in fitting instruments to measure its effectiveness. Such measurement is given a stamp of objectivity by leaving both the correction of data and their processing to a computer. As a consequence, the results are beyond suspicion in terms of the consequential decisions, even if these are far-reaching. This is what is claimed by tradition.

As indicated earlier, both the format and the content of assessment are undergoing considerable change due to a different view on mathematics education. However, this does not necessarily imply the abolishment of the certainty concerning what one is capable of and the optimism concerning its reliability with respect to assessment. Lesh and Lamon (1992) posit in the preface to their survey:

> ... and assessment is conceived as an ongoing documentation process, seamless with instruction, whose quality hinges upon its ability to provide complete and appropriate information as needed to inform priorities in instructional decision making. (1992, p. VI) (Italics placed by LS/MvdHP)

We can endorse the first part of the quotation, indeed, but what we have difficulty with is the 'complete and appropriate' information the authors claim, and also the autonomous ability of assessment, as if it does not involve human beings. Even if it is completely clear how the collection of information must take place, our ability to realize it might be insufficient, without even speaking about being complete. What a student does not write down or does not say, we simply do not know. The greatest caution should be exercised in this regard.

A first intermezzo from physics

Actually, we could well have known what was just revealed. lt is a known fact that inaccuracy is inherent to measurement in scientific research. In measuring the temperature of a substance, for instance, there is loss of heat. In other words, the temperature of the substance is influenced by the act of measurement. Classical physics provides a solution for this problem. The inaccuracy can be made negligibly small by employing a different, more accurate instrument. In consequence, while endorsing this metaphor of measurement in the tradition of classical physics, one need not lose one's optimism about the possibilities of assessment. On the contrary, it promotes optimism with respect to the measureableness.

A second intermezzo from physics

The German physicist Max Planck (1 8 5 8 -1947) conceived light as transmitted by way of small parcels, so-called quanta. Albert Einstein (1879-1955) accepted the idea that light itself consists of such separate, tiny parcels. It helped him to explain the photoelectric effect. It was for this that he received the Nobel Prize in 1922, and not for his relativity-theory, as one might have expected. In 1925, however, Werner Heisenberg (1901-1976) stated that by proving the existence of photons (quanta of light) uncertainty had crept into physics. Hawkins (1993, p. 177) formulated this as follows:

> To see where a particle is, you have to shine light on it. But Einstein had shown that you couldn't show a very small amount of light: you had to use at least one packet or quantum. This packet of light would disturb the particle and cause it to move at a speed in some direction. The more accurately you wanted to measure the position of a particle, the greater the energy of the packet you would have to use and thus the more it would disturb the particle.

This phenomenon became known as 'the uncertainty principle of Heisenberg'. In summary, it means that measuring both the position and the velocity of a particle with unrestricted accuracy cannot be done at the same time. Accurate measurement of the magnitude inevitably implies the measurement of the other with greater inaccuracy, and vice versa. This precisely reflects the difference with measurement according to classical physics.

Suitable as a mathematical-didactical metaphor?

We are in excellent company. That much is beyond dispute. But what is very concise and incomplete contemplation of certain facts from physics worth in a journal on mathematics education? The answer is both simple and surprising; namely that it is timely to formulate a mathematical-didactical uncertainty principle, particularly with respect to assessment of teaching-learning processes.

Let us attempt a small thought experiment in which all complexity has been set aside. Imagine a particular item for assessment, which provokes certain results from which we draw conclusions regarding the state of the learning process of the students involved. How confident can we be about the certainty of our conclusions? Will one single measure suffice? Our first reaction will be to confront the students with more items of the same type. Paradoxically, uncertainty would then not only decrease, but at the same time would also increase. This is due to the fact that we do not know the influence of the items on the learning process. Or do we? Bear in mind that while assessing the learning process, it continues. In other words, assessment is doomed to uncertainty. This uncertainty cannot be restored easily, or so it seems. lt concerns uncertainty that demands a different metaphor than the one of measuring temperature where the measure of inaccuracy can be made arbitrarily small. In our view, we have found such a metaphor in the principle of uncertainty of Heisenberg. lt not only indicates that the observation influences, or even changes, the observed phenomenon. Moreover, there are two sides to the question, both in physics and in mathematics education. In physics, it is about position and velocity of a particle; in mathematics education, we must deal with both teaching activities and individual learning processes. The more intensively we want to investigate the occurrence of a certain learning effect, the stronger the phenomenon that we wish to observe and to determine will be influenced. lt is this analogy that leads us to propose to enrich the theory of realistic mathematics education by a 'mathematical-didactical uncertainty principle'.

The embodiment in realistic theory

As indicated earlier, the second metaphor is richer than the one of measuring temperature. Correction will now be impossible. Uncertainty persists. However, this can have a very positive effect. One needs only to consider the fact that the coin of teaching inevitably has a reverse side of learning. In our view, this complex process can never be fully grasped. But there is more.

The metaphor of uncertainty also fits the theory of realistic mathematics education. In developmental research connected to realistic mathematics education aimed at bringing forward potential teaching (elaborated in prototypes of courses), it is essential that the teaching strand be intertwined with the individual learning strands (Streefland, 1994). Together with other tenets of realistic theory (such as doing constructions and productions, interactive teaching in service of both individual learning processes, and the progression in mathematization of the group involved), this offers a certain guarantee that the teaching and learning of mathematics, based on courses that are derived from the prototype, will be in balance. What is more, the learning process that takes place in a teaching experiment that is part of developmental research is tested internally. From this, a source for later external assessment of courses derived from the prototype will result. lt then turns out that the measure of uncertainty will be both acceptable and manageable, even though it persists. Once again, we consider this an advantage.

Traditional optimism about the assessibility of (mathematics) education characterized by an aura of infallibility has been doomed even more to the certainty of being in the right than has uncertainty. Anyone who aims at achieving certainty in testing inevitably rejects all doubts and criticism in advance. However, if one recognizes uncertainty and endorses the mathematical didactical principle of uncertainty, one thereby creates space and freedom for oneself, room for development and improvement, room for assessment, which in turn means room to measure. Those who support this principle do not want to measure precisely at all. If a criterion of precision does exist it will certainly have a completely different meaning, because it is a precision that is more flexible, more dynamic than the former criterion. This is illustrated, for instance, by developmental research on assessment that exploits student-generated problems, which means that the material reflecting the individual learning processes is considered, and used as a source for assessment (van den Heuvel-Panhuizen, Middleton & Streefland, 1995). For this reason, it may be assumed that the efforts made in developmental research will reveal as much as possible of the process of teaching and learning, so that genuine intertwinement of the two can be realized. This increase in both scope and level of awareness may be the greatest strength of the exposed mathematical-didactical uncertainty principle.

This article ha beens published in the Journal of Mathematical Behaviour, 17 (4), 393-397, 1999.

Notes

1. The physical correctness of this communication was checked by Prof. Piet Lijnse of the Center of Didactics of Mathematics and Sciences of the University of Utrecht, The Netherlands. Our thanks to him.
2. For further information on the theory of realistic mathematics education, the reader is referred to The Legacy of Hans Freudenthal, edited by the first author (cf. Streefland, 1993) .

References

de Lange, Jan (1987). *Mathematics, insight and meanin*g. Utrecht: OW&OC.

Hawkins, Stephen (1993). *Black holes and baby universes and other essays*. New York: Bantam Books.

Lesh, Richard, & Lamon, Susan J. (Eds.). (1992). *Assessment of authentic performance in school mathematics*. Washington: AAAS Press.

Streefland, Leen (1990). Developmental research and tests - sine functions as a paradigm. *In Proceedings of the 2nd Bratislava International Symposium on Mathematics Education* (pp. 78-98). Bratislava: Comenius University.

Streefland, Leen (Ed.). (1993). *The legacy of Hans Freudenthal*. Dordrecht: Kluwer Academic Publishers (reprint of) Educational Studies in Mathematics, Vol. 25, (pp. 1-2).

Streefland, Leen (1994). Theoretiseren in ontwikkelingsonderzoek. (Theorizing in developmental research). *Tijdschrift voor didactiek der b-wetenschappen, 12* (1), 21-35.

van den Heuvel-Panhuizen, Marja (1996). *Assessment and realistic mathematics education.* Utrecht: CD-β press, Center for Science and Mathematics Education.

van den Heuvel-Panhuizen, Marja, Middleton, June, & Streefland, Leen (1995). Student-generated problems: Easy and difficult problems on percentage. *For the Learning of Mathematics*, 15, 21-27.