Chapter 10
Realistic Mathematics Education in the Chinese Context—Some Personal Reflections

Xiaotian Sun and Wei He

Abstract In this chapter, we start with a historical review of how Professor Hans Freudenthal and Realistic Mathematics Education (RME) became known in China, and how the academic exchange between Chinese scholars in the field of mathematics education and researchers at the Freudenthal Institute initiated and continued later. Then we discuss the positive impact of RME. Specifically, we cite some living examples for how the theoretical and empirical research substances related to RME-influenced mathematics curriculum development in China. These examples include the fields of curricular policy making, textbook design and classroom teaching.

Keywords Hans Freudenthal · Freudenthal Institute · Realistic Mathematics Education · Curricular policy · Textbook design · China

10.1 Historical Review

10.1.1 Hans Freudenthal's Visit to China

Before 1985, there were no connections between mathematics education in China and the Netherlands. Chinese scholars and educators in mathematics education had little knowledge about Professor Hans Freudenthal, the Freudenthal Institute, and Realistic Mathematics Education (RME). However, all things began to change after 1985. Professor Zehan Jiang, a famous Chinese mathematician and a member of the Chinese Academy of Sciences worked at Peking University, read Freudenthal's (1973) book *Mathematics as an Educational Task* during his visit abroad, which gave him a new perspective on understanding mathematics education. Instead of just

X. Sun (✉) · W. He
Minzu University of China, Beijing, China
e-mail: sunxt0761@sina.com

W. He
e-mail: cunhw@sina.com

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appreciating this book, Jiang introduced it to one of his former student Changpei Wang, whose main research interest was in mathematics education. At that time, Professor Wang was the Dean of the Faculty of Mathematics of the Beijing Institute of Education and the main goal of this faculty was doing research in mathematics education to improve teachers' practice. In this way, Freudenthal and his book were introduced in the field of mathematics education in China. It can be considered as the first time that mathematics education in China and RME in the Netherlands met.

Wang had taught mathematics in primary and secondary education for about 17 years. Compared to his colleagues, he had been making a greater effort to learn English. His strong interests and perseverance in learning English made it possible for him to read the English version of the book *Mathematics as an Educational Task*. Wang was deeply attracted by the ideas in that book and he was extremely eager to meet the author. As the reform and openness policy in China increased, scholars were encouraged more to go abroad to learn from the world. Therefore, the supportive political environment became an important factor in making Wang's wish come true. Finally, at the CIEAEM (International Commission for the Study and Improvement of Mathematics Teaching) conference in London in 1986, Wang got the opportunity to meet and talk with Freudenthal face-to-face. It was this meeting that started a new era of exchange in mathematics education between China and the Netherlands.

A direct result of the meeting was that Wang arranged a short visit to the Netherlands. In that week, by having more in-depth discussions and conversions with Freudenthal, Wang gained a preliminary understanding of how to put the ideas of RME into practice. During Wang's first visit to Freudenthal and their continuous discussion by exchange of letters afterwards, another plan was proposed: Wang sincerely invited Professor Hans Freudenthal to give lectures in China. After careful preparation, Freudenthal visited China in 1988, when he was 82 years old. He went to Shanghai and Beijing, and gave three separate but interrelated lectures on the themes of research in mathematics education, research in mathematics curriculum, and the future and development of mathematics education. The lectures were given at East Normal University, Beijing Normal University and the People's Education Press. By giving these lively presentations, Freudenthal elaborated the fundamental ideas and important principles described in his books to his Chinese audiences in a beautiful prose tone.

The Chinese educational authorities paid great attention to Freudenthal's visit. Mr. Bin Liu, the Vice-Minister of Education at that time, had a meeting with Freudenthal and hosted a dinner party to welcome this knowledgeable and insightful researcher in mathematics education. All this did not only show the Chinese authorities' respect for Professor Hans Freudenthal, but also demonstrated their great interest in his work and RME. After Freudenthal's visit, Professor Ruifen Tang from East China Normal University further continued the introduction of Freudenthal and his work based on Freudenthal's lectures and by publishing a series of three papers titled "Professor Hans Freudenthal's Answers to the Questions in Mathematics Education" in the journal named *Mathematics Teaching*. As these three papers were widely circulated and read, scholars, researchers, educators in mathematics education, teachers and even some students began to become familiar with the name of Hans Freudenthal and RME, as a new theory in mathematics education which is almost totally different from how mathematics is taught in China, but equally effective in guiding teaching and learning. Another important result of this visit is that the book *Mathematics as an Educational Task* was translated into Chinese and published. Since then, references have been made to this book very often. In the China National Knowledge Infrastructure (CNKI) database it has been cited more than 2000 times in journal papers, master's and doctoral theses, and conference papers and other publications. Also, our experience is that it has been widely spread among mathematics teachers in primary and secondary schools.

In addition, it is worth noting that a number of young researchers, who had just graduated from university, listened to the lectures given by Freudenthal in Shanghai and Beijing; some of them even took their courage to enter into Freudental's room in the hotel to have a face-to-face conversation. One of those young researchers was Jian Liu. He was only in his twenties at that time. Now he is the Director of the China Educational Research Centre for Creativity and one of the chief designers of the mathematics curriculum standards for compulsory education of China. According to Professor Liu, he could remember vividly the details of the conversation in Freudenthal's room: Professor Hans Freudenthal, with his grey hairs, took out his handwritten poster used in the lecture, put it on the floor, and explained it carefully. Due to the language barrier, Liu could only understand a small part of what Freudenthal tried to explain. However, how this world-renowned mathematician and mathematics educator discussed with them friendly and equally was stamped in his heart. Even until now, Liu feels full of respect when he is talking about this scene.

The book *Revisiting Mathematics Education*, with a subtitle *China Lectures* (Freudenthal, 1991) was published in 1991, one year after Freudenthal passed away. In the lectures given in China, Freudenthal summarised his lifelong thoughts and experiences. To some extent, visiting China became a perfect curtain-call performance in his career. For China, his visit was of paramount importance. It provided an irreplaceable impetus to the future development of mathematics education in China.

10.1.2 Chinese Scholars' Visits to the Freudenthal Institute

Freudenthal's visit brought new idea of teaching and learning mathematics. However, in the early 1990s, the discussion about RME in China remained at a theoretical level; no connection between RME theory and Chinese classroom practice occurred. In other words, Chinese researchers' interests in RME had not turned into the practice of using RME to guide or change teachers' daily practice in mathematics education. The reason is that there was a long tradition of mechanistic teaching in China. To be more specific, teaching procedures were the main part of the curriculum; teaching-to-the-test was the goal; in class, students learned what the teachers taught, listened to what the teachers said and did what the teacher asked them to do; and training how to solve particular problems correctly and quickly was the basic approach for enhancing students' achievement. During that period, the development of labour intensive industry was the primary goal of society in China. Therefore, to some extent, it was reasonable that mechanistic teaching was taking the leading role in mathematics education. In such societal and educational situations, it was hard to use RME in China. Another reason why Freudenthal did not become 'hot' in China at that time was that a short visit was not enough for people in China to have a thorough understanding of RME. Therefore, a bridge which could lead to more and deeper exchange in mathematics education between China and the Netherlands needed to be built.

It was under this background that a young researcher from Minzu University of China, Xiaotian Sun, got a scholarship from the Chinese government to visit the Mathematics Institute at Utrecht University in 1992. Xiaotian Sun had keen interests in doing research in mathematics education and a dream of conducting a reform in this field. In 1990, Sun published a paper titled "The Change of the Mathematics Textbooks", in which he called for changing mechanistic teaching and carrying out reform in the mathematics curriculum.

Before Sun travelled to the Netherlands, he visited Xiaoda Zhang, the Director of the Mathematics Research Group on Secondary Education of the People's Education Press. Zhang was responsible for Freudenthal's visit in Beijing in 1988 and the person who accomplished the meeting between Freudenthal and Mr. Bin Liu. When Zhang knew that Sun was going to study in the Netherlands, he was very happy. Then Zhang found an edition of the newspaper *Reference News* from his piles of documents that contained an introduction about the best ten secondary schools in the world and one of them was in Zeist in the Netherlands. By showing this piece of newspaper, Zhang explained Sun the worldwide influence of mathematics education in the Netherlands. He suggested Sun to create the opportunity to study at the Freudenthal Institute, because this could be more than helpful for the coming mathematics curriculum reform in China. When Sun arrived in the Netherlands, he started with learning graph theory at the Mathematics Institute according to his visit plan. Later on, he tried to get into contact with the Freudenthal Institute, and could study at both institutes. Shortly after that, he decided to focus all his effort on learning the theory and application of RME at the Freudenthal Institute.

In 1994, Sun invited Professor Jan de Lange, at that time the director of the Freudenthal Institute, to visit China. De Lange is one of Freudenthal's students; in his thesis *Mathematics, Insight and Meaning*, De Lange (1987) illustrated how RME was used in mathematics textbooks in senior high school. Together with Freudenthal's work that mainly focuses on the application of RME in primary education and junior high school, the research done at the Freudenthal Institute had covered all basic education phases in the Netherlands. In addition, researchers at the Freudenthal Institute were also involved in the development of the textbook series *Mathematics in Context* (Wisconsin Center for Education Research & Freudenthal Institute, 1997–1998) that was in line with the mathematics curriculum standards of the NCTM. This textbook series was chosen to be used in many places in the United States of America.

When Sun's first stay at the Freudenthal Institute was finished, he felt that it was not long enough, and with support from the Chinese government he had a second chance to visit Freudenthal Institute in 1999. Then, he worked very intensively with Marja van den Heuvel-Panhuizen, who at that time was involved in a large project funded by the Dutch Ministry of Education to develop teaching-learning trajectories for primary school mathematics.

With his two stays at the Freudenthal Institute and the accompanying learning experiences, Professor Xiaotian Sun systematically introduced what he learnt about RME to his Chinese audience. Through 15 papers and some book chapters, Sun explained in detail what main message Freudenthal tried to deliver and what RME is. Based on his observations in Dutch schools, Sun also gave a detailed explanation of some concepts which were difficult to be understood in the Chinese education context, such as context problems and mathematisation. In this way, Sun re-introduced RME in China from more different perspectives. In addition, Sun introduced the Dutch standards and curricula in primary and basic secondary education and the syllabus in senior high school. He analysed the Dutch mathematics textbooks which were designed under the guidance of RME. To make clear how statements in books are related to teachers' practice, he observed many classes in primary and secondary schools. Without audio or video equipment, Sun made substantial observation notes about what happens in real Dutch classrooms. All his work and reports aroused the attention of Chinese researchers and mathematics teachers for RME once again.

More importantly, as a result of the advent of the information era, modern information technology has become an indispensable part of social and personal life. How to prepare students for the society, especially how to foster creativity, became urgent questions to be answered. The government and educational society in China realised that reform was necessary. However, having clear goals for a needed reform is not enough. Most difficult is to have the theoretical power on which one can rely to guide concrete practice towards these goals. Since such educational theory was not generated in China, we needed to find the answer in the rest of the world. Therefore, Sun's knowledge about RME and the experiences accumulated by him during his stay at the Freudenthal Institute became a very important resource for the curriculum design group that was responsible for the trial version of the curriculum reform outline of basic education. Sun was one of three people who coordinated the work of the national design group and who were responsible for the design. In this way, the Chinese mathematics curriculum reform was inspired by RME principles such as letting students become the owner of their own learning, using mathematics to understand and reflect on reality, and making the learning of today the foundation of the creativity tomorrow.

Instead of visiting the Freudenthal Institute individually, Professor Wei He and two young researchers from Minzu University of China formed a research group to take part in a summer school organised by the Freudenthal Institute in 2014. It was the first time that researchers from a university in China visited the Freudenthal Institute in a group. During the summer school, they got a better understanding of how RME is put into practice in different educational stages. Especially they got more insights in how RME is concretised in textbook design and classroom instruction. Under the guidance of lecturers from the Freudenthal Institute, they experienced the process of how mathematisation happens in real instruction. This brand-new way of learning made the Chinese research group feel that mathematisation, as a theoretical word in their mind, became more practical. In this way, they deepened their understanding of RME.

Besides the researchers from China mentioned above, Xiaoyan Zhao, a Chinese student who obtained her master's at Nanjing Normal University got a grant from the China Scholarship Council, supported by the Chinese government, and did a PhD study[1] at the Freudenthal Institute. Under supervision of Professor Marja van den Heuvel-Panhuizen and Doctor Michiel Veldhuis she worked on a project on Chinese teachers' classroom assessment which was an extension of a project on Dutch teachers carried out in the Netherlands. The Chinese part of the project started with a literature review about how Chinese mathematics teachers in primary school used classroom assessment as reported in papers written by them. Also, a questionnaire survey was carried out about teachers' classroom assessment practice. Later, an explorative pilot study was done around the use of particular classroom assessment techniques. Finally, in a larger quasi-experiment it was investigated whether the teachers, through using these techniques, gained new insights in their students' learning.

10.1.3 Two Forums on the Theory and Practice of RME Held in China

The first forum took place in 2000. Then, two important researchers from the Freudenthal Institute: Doctor Marja van den Heuvel-Panhuizen and Doctorandus Martin Kindt, were invited to give a series of lectures at Forum on the Theory and Practice of Realistic Mathematics Education. This first forum was held in two places: in Beijing and in Changchun. Different from the lectures given by Freudenthal, who focused more on the theoretical parts of RME, Van den Heuvel-Panhuizen's and Kindt's presentations mainly emphasised the application of RME in mathematics curriculum and classroom teaching, for example, they addressed how to teach and learn arithmetic in primary education and geometry in secondary education. Another difference with the lectures given by Freudenthal was the audience. In 1988 a small number of scholars and researchers from universities and research institutes attended the lectures, while in the forums held in 2000, hundreds of educators and in-service mathematics teachers were able to participate. In addition, instead of mainly focusing on lecturing such as Freudenthal did, Van den Heuvel-Panhuizen and Kindt adopted another way of communicating with their audience. They saved sufficient time and provided many opportunities for the audience to ask questions freely. The people in the audience asked what they wanted to know most and the speakers answered the questions by illustrating what happens in Dutch classrooms. Both the content in the presentations and the ways in which it was presented highly inspired the audience. The mathematics teachers, particularly, showed great interests in the the-

[1]On March 7, 2018, Xiaoyan Zhao successfully defended her PhD thesis at Utrecht University.

ory and application of RME. This forum also reflected that the curriculum reform in mathematics education in China began to step forwards to another stage: from involving researchers and scholars and focusing on the theoretical part of RME to paying more attention to mathematics teachers' understanding and the application of RME. Although, generally speaking, it is widely accepted that there is a gap between research results and teachers' practice and that it can be very difficult for researchers and teachers to be on the same wavelength, in this forum, both groups agreed with the ideas of RME and showed common interests and attention.

From then on, the academic exchanges in RME between China and the Netherlands were not only limited to the researchers' level, but extended their impact to teachers by evoking them to reflect on mathematics education and change their classroom practice. More and more mathematics teachers in primary and secondary education began to read the book *Mathematics as an Educational Task*. During the same period, the book *Revisiting Mathematics Education: China Lectures* was translated into Chinese and published. The two books, together with the first Forum on the Theory and Practice of Realistic Mathematics Education, helped people in China get a better understanding of RME and pushed the cooperation and exchange in mathematics education between China and the Netherlands to a peak in that year.

The second Forum on the Theory and Practice of Realistic Mathematics Education was held in 2013 in Beijing. Professor Marja van den Heuvel-Panhuizen and Doctor Michiel Doorman were invited to give lectures in Beijing, Chengdu and Jinan. Van den Heuvel-Panhuizen started working at the Freudenthal Institute in 1987 and made a great contribution to the development of RME in the post-Hans Freudenthal time. In the second forum, she gave three speeches in which she addressed the principles of RME, recent research in mathematics education from an RME perspective, and research on RME textbooks used in the Netherlands. The Chinese audience, after more than 10 years, got a second chance to gain more insight in RME and receive an update of the research done at the Freudenthal Institute. While Van den Heuvel-Panhuizen was explaining the principles of RME, the Chinese audience noticed that 'free productions', which was one of the key features of RME about 20 years ago, was not mentioned this time. Instead, the 'mathematisation under the guidance of the teacher' was introduced. In fact, the idea of 'free productions' was hard to be understood or put into practice in the Chinese educational context. Whereas 'mathematisation under the guidance of the teacher' was closer to the situation in China, because it not only affirms students' primary role of learning mathematics—mathematisation is considered to be the starting point of mathematics learning and teaching—but also emphasises the importance of teacher' guidance during the process of mathematisation. As a result, this idea was quickly accepted and supported by the Chinese audience.

While looking back at the process of evolvement of RME—one of the most important theory systems in mathematics education—it is clear that RME is not a finished or closed system, but is continuously open to new developments and innovations according to the ever-changing society and accumulated experiences of people. Just as a famous Chinese saying goes, it keeps pace with the times. Only when a theory can move along with the latest changes in the environment that it is generated from and serves to, it can have lasting vitality and the power to extend without limit in both theoretical and applicable aspects. Van den Heuvel-Panhuizen's interpretation of such characteristic of RME deepened our understanding of RME, which made us feel more respectful to this theory.

Doorman had been doing research in RME and its application in secondary mathematics education for nearly 20 years. In this second forum, he introduced the curriculum design and teaching practice of calculus under the guidance of RME, which aroused great interest in the audience. It is almost 30 years since calculus was included in the mathematics curriculum in secondary education in China. But there was not really a clear teaching trajectory for calculus. The main problem was that the distinction between the educational goals of learning calculus in secondary school and in tertiary education was not clear. It seemed that what students have learned in secondary school was repeated in tertiary education. For example, the teaching trajectory of calculus in secondary school starts with introducing the slope of the tangent of a graph, the average velocity and the instantaneous velocity. The same path is also taken for teaching calculus in tertiary education. Doorman pointed out the educational value of learning calculus in secondary school from the perspective of mathematisation. This means helping students to understand and analyse the relationship between finity and infinity by using various and rich contexts. It also means facilitating them to quantify the relationship rather than simply remembering the definition of the derivative, calculating it and using it to solve problems without context. In his lecture Doorman first showed in a trace graph how a hurricane approached the mainland. Then, the next step of teaching was asking students to explain how the trace graphs changed by studying the points of tangency and the slopes of the tangent. In this way, together with other contexts, the definition of limit is introduced step by step. Along these lines, it was clearly illustrated how to use RME to guide the teaching in secondary education.

Organising forums on the theory and practice of RME in China means building a platform for more researchers and mathematics teachers in China to meet and discuss with the successors of Freudenthal and gain insight in RME as a theory system in mathematics education, its application in the curriculum and in teaching, and the relation to how teachers and students perform in practice. Such forums are held every few years in China with the purpose of updating the knowledge of RME. This continuous attention to and regular updating of knowledge regarding RME reveal that there is a special interest in RME in China, especially in the context of the mathematics curriculum reform. These forums in which Chinese participants are informed about RME in the Netherlands, are not common for the cooperation in the field of mathematics education between China and other countries. Therefore, these forums reveal that there is a special interest in RME in China; in particular this is the case in the context of the mathematics curriculum reform in China.

10.2 The Influence of RME in the Chinese Context

It has been 30 years since the academic communication in mathematics education between China and the Netherlands started in 1985. In the years since then, RME research done by Freudenthal and other researchers at the Freudenthal Institute have provided rich and precious experience, which is considered an important resource by Chinese scholars and educators for developing mathematics education in China. Particularly, RME exerted an effect on the latest curriculum reform in mathematics education in China in terms of policy making, new textbook design and change in classroom teaching.

10.2.1 The Influence of RME on Curricular Policy Making

In China, the curricular policy is embodied in the publication of curriculum standards by the Ministry of Education of the People's Republic of China (MOE). There is no doubt that these standards, being the only applicable document formulated according to the intention of the educational policy, is of great importance. Scholars and researchers are largely responsible for establishing them. After being proved by an evaluation team organised by governmental authorities, the curriculum standards released by MOE had a statutory status.

In the publication *Mathematics Curriculum Standards for Full-time Obligatory Education*[2] (MOE, 2001), one can find detailed information about (1) fundamental ideas about mathematics and mathematics education, (2) the objectives of mathematics education, (3) mathematical content, and (4) suggestions for instruction, assessment, and the design of mathematics textbooks and other materials. From all these descriptions in the Curriculum Standards, it is evident that its design was obviously influenced by RME, because many keywords and expressions which echo the basic characteristics of RME had never appeared in similar official documents before 2001. Several examples are provided in the following.

> Mathematics is the process that people understand and describe the external world qualitatively and quantitatively, progressively conceptualise and generalise rules or theory, and put them into practice. (MOE, 2001, p. 1)

Here, mathematics is defined as a process, as a process of human's activities. It is consistent to how mathematics and mathematisation are interpreted in RME. In Curriculum Standards it is also stated:

> Students are the owner of their mathematics learning [and]
>
> mathematics teaching has to be built on students' cognition development and the experience they have already got. (MOE, 2001, p. 2)

[2]Hereafter referred to as 'Curriculum Standards'.

Such expressions put students in the central place in all teaching activities and emphasise students' existing knowledge and experience as the starting point of teaching activities. These ideas are different from the traditional view of mathematics as a discipline and the traditional teacher-centred educational style, but more related to the connotation of 'realistic' in RME.

> The content learned in mathematics is realistic, […], should be potential to let students be actively involved in observation, doing experiment, conjecture, proving, reasoning, communication and other mathematics activities. (MOE, 2001, p. 2)

This sentence outlines the relationship between mathematics learned in school and the realistic external world around students. In this way, the sentence answers to the fundamental question that where the mathematics learned in school is from and what it serves for. This point is highly relevant to one of the basic characteristics of RME that mathematics is a human activity.

Another significant feature of RME, which does not belong to the classical teaching philosophy in China with its emphasis on teachers and teaching, but which is reflected in Curriculum Standards, is that

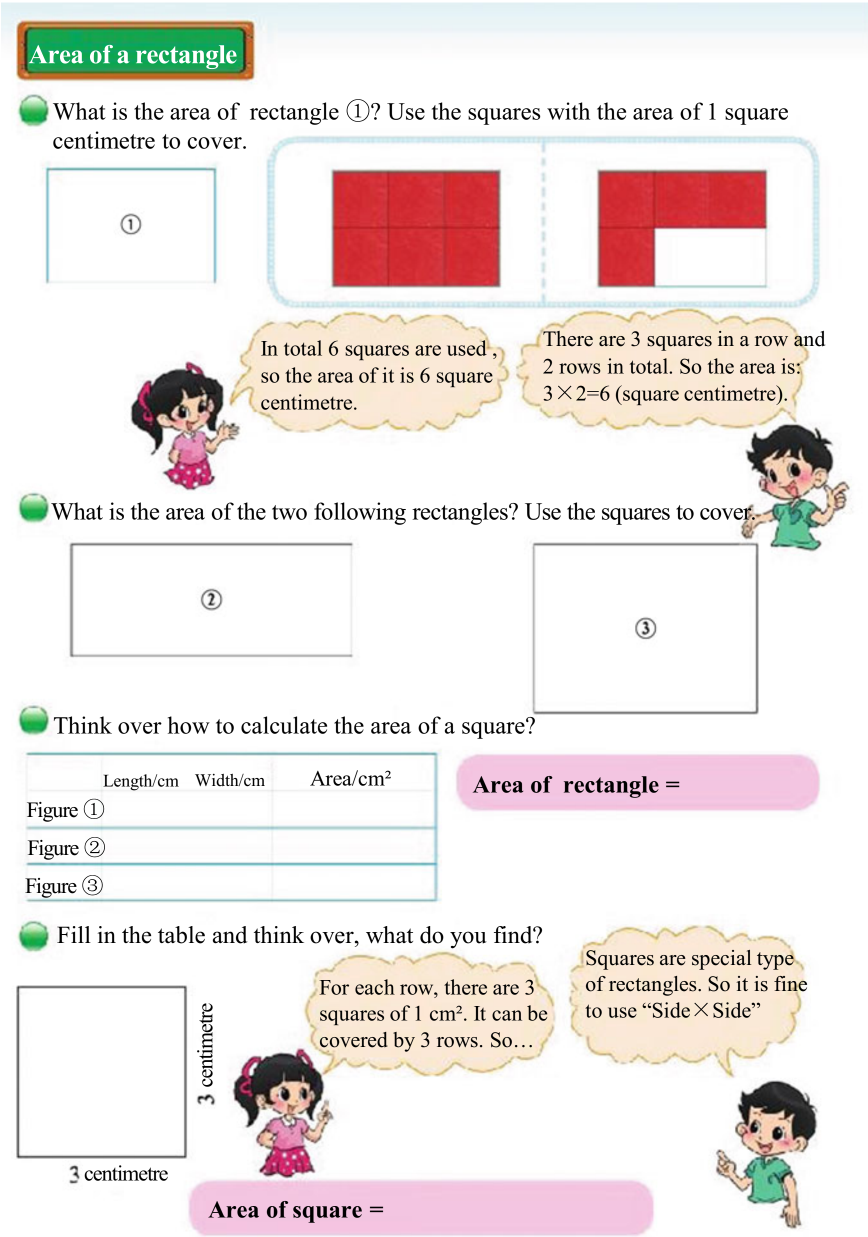
> practical activity, initiative and independent exploration, cooperation and communication are all important approaches of learning mathematics […] (MOE, 2001, p. 2)

Although, Bloom's taxonomy was the basic requirement of mathematics teaching in China for a long time, in the new Curriculum Standards, in addition to 'knowing', 'understanding', 'remembering', and 'applying', three new requirements are added, namely 'undergoing', 'experiencing', and 'exploring'. Clearly, these three words are all associated with mathematisation. Therefore, we can conclude that many elements in the Curriculum Standards are interrelated with RME.

10.2.2 The Influence of RME on Textbook Design

Figure 10.1 shows a page from the Chinese textbook *The Primary Mathematics for the New Century* (Liu, 2014). The pictures at this page refer to the theme that is addressed here. Such a picture is called a 'context picture'. Based on the information provided in this picture, especially information related to numbers and shapes, a series of questions are posed one by one, and the last question of the series is intended to be answered by an explicit conclusion about the concept that is at issue.

In contrast to what is shown in Fig. 10.1, where one context is used, generally two or three contexts around one mathematics theme and their corresponding questions series are used. In this way, the core of what has to be learned is approached from different perspectives. This example makes clear that Chinese textbook designers acknowledge the importance of context and mathematisation. Although there is still much room for improving the choice and design of rich context problems, and providing opportunity to students to explore problem situations by themselves, the



Area of a rectangle

What is the area of rectangle ①? Use the squares with the area of 1 square centimetre to cover.

①

In total 6 squares are used, so the area of it is 6 square centimetre.

There are 3 squares in a row and 2 rows in total. So the area is: $3 \times 2 = 6$ (square centimetre).

What is the area of the two following rectangles? Use the squares to cover.

②

③

Think over how to calculate the area of a square?

	Length/cm	Width/cm	Area/cm²
Figure ①			
Figure ②			
Figure ③			

Area of rectangle =

Fill in the table and think over, what do you find?

3 centimetre

3 centimetre

For each row, there are 3 squares of 1 cm². It can be covered by 3 rows. So…

Squares are special type of rectangles. So it is fine to use "Side × Side"

Area of square =

Fig. 10.1 Page from the textbook *The Primary Mathematics for the New Century* (Liu, 2014); the page is meant for Grade 3 and illustrates the structure used in such a textbook: starting with a context problem followed by a series of questions recently designed textbook series pay much attention to students' mental and cognitive development, which is totally different from the traditional textbooks that start from definition and examples.

Generally speaking, starting with a context problem, followed by a series of questions to lead students to what they are supposed to learn, is the basic style in most textbooks series used in primary and secondary mathematics education after the curriculum reform. Until the 21st century, this way of structuring textbooks did not exist in the history of mathematics textbook design in China. To a great extent, this change was inspired by the RME idea of using context problems and mathematisation. In addition, mathematics textbooks designed within the RME approach were other important resources for the textbook design in China. For example, the textbook *Mathematics in Context* (Wisconsin Center for Education Research & Freudenthal Institute, 1997–1998) served as a model for Chinese reformed textbooks.

The style of a context followed by series of question series is not only adopted in textbooks designed for primary education, but also in textbooks for secondary education, see Fig. 10.2.

As well as in the style of structuring textbooks, the influence of RME on textbook design can be found in the aspect of content. Like in RME, in Chinese textbooks attention is paid to connecting mathematics to reality. For example, besides knowing and understanding the number system, students should also develop knowledge about daily life numbers, see Fig. 10.3.

Another clear example of adapting the content can be found in the domain of geometry. The content of geometry in traditional primary school textbooks in China involves mainly measurement, including the definition of area and volume with the main focus on calculation. So, for a long time, the concept of space which is so important for students was not included in the mathematics textbooks. However, after the curriculum reform, there was a big change in this approach. In the newly designed textbooks the concept of space was added, which is illustrated by pages from primary school textbooks shown in Figs. 10.4, 10.5, 10.6 and 10.7. From these textbook pages, one can see the impact of RME. In Fig. 10.8 this influence can also be recognised in a textbook used in junior high schools.

In China, the focus in mathematics teaching and learning has been always on problem-solving. However, what were considered as problems here were mainly bare number problems and simple word problems. Moreover, such problems were grouped into certain types of problems. Therefore, a textbook starts with a sample problem which represents a type of problems. By reflecting and generalising the way of solving this sample problem, students are expected to know how to solve that type of problems. Then, exercises aimed at enhancing students' ability to solve particular types of problems are provided in textbooks. Since exercises in textbook are always considered to be not enough, more exercises and learning material follow. The result is that Chinese students generally become the most hard-working students. The fixed procedure of grouping problems into types and solving these types of problems is effective in getting a good result in examination. However, in this approach to mathematics education not enough attention is given to prepare students for daily life and their future professional career. The question of what students can

1 Exploration of the Pythagorean theorem

In Fig 1-1, the upper part of a telegraph pole needs to be connected to the ground by a steel rope.

If the upper part of the pole is 8 m away from the ground and the touching point of the steel rope on the ground is 6 m away from the pole, how many metres the steel rope needs to be.

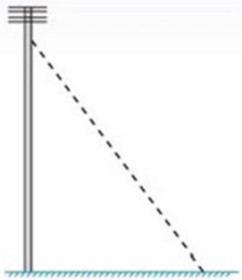


Fig 1–1

In a rectangular triangle, if two arbitrary sides are determined, the third side is also determined. Between the three sides, there is a certain relationship. In fact, people in ancient time have already found there is a particular relationship between the square of each side. Let us go to explore together!

(1) Draw a rectangular triangle on the grid paper and measure its three sides. See what the relationship between the square of the three sides is, and discuss what you find with your classmates.

(2) In Fig 1-2, what are the square of the three sides? Is it fit to the rule you just discussed? How do you calculate? Discuss with your classmates. How about the rectangular triangles in Fig 1-3?

(3) If the legs of a rectangular triangle are 1.6 and 2.4 unit respectively, do you think the rule still apply? Give your reason.

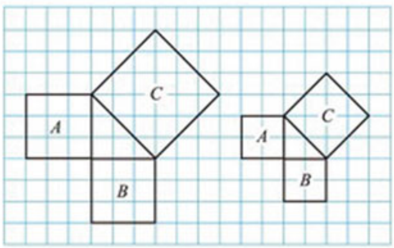


Fig 1–2

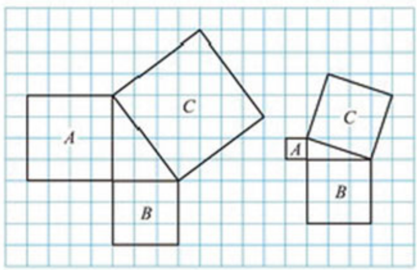


Fig 1–3

Fig. 10.2 Page from the textbook *Mathematics* (Ma, 2013) designed for junior high school; the page is meant for Grade 8 and illustrates the structure of starting with a context which is followed by a series of questions

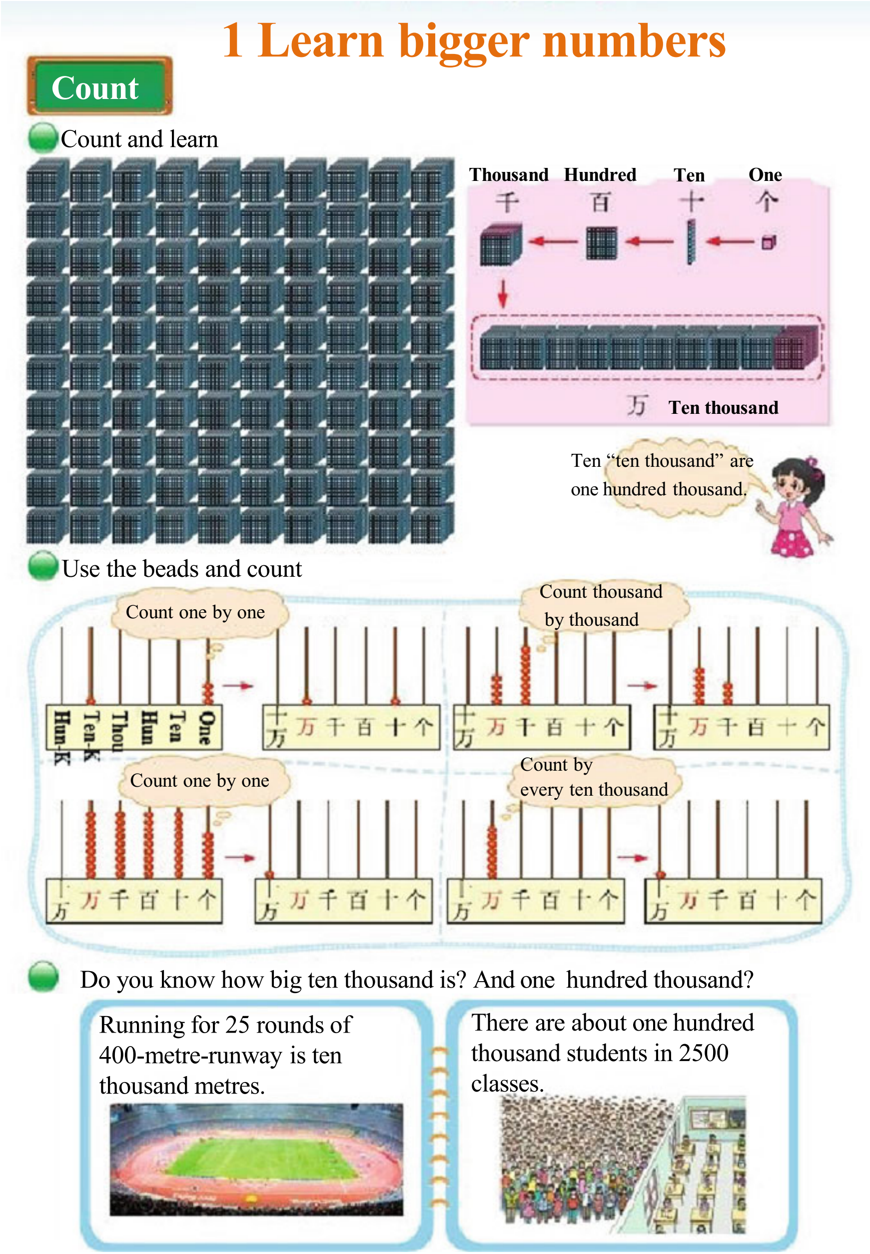


Fig. 10.3 Page from the textbook *The Primary Mathematics for the New Century* (Liu, 2014) meant for primary school Grade 4; in this page, the students have to learn bigger numbers

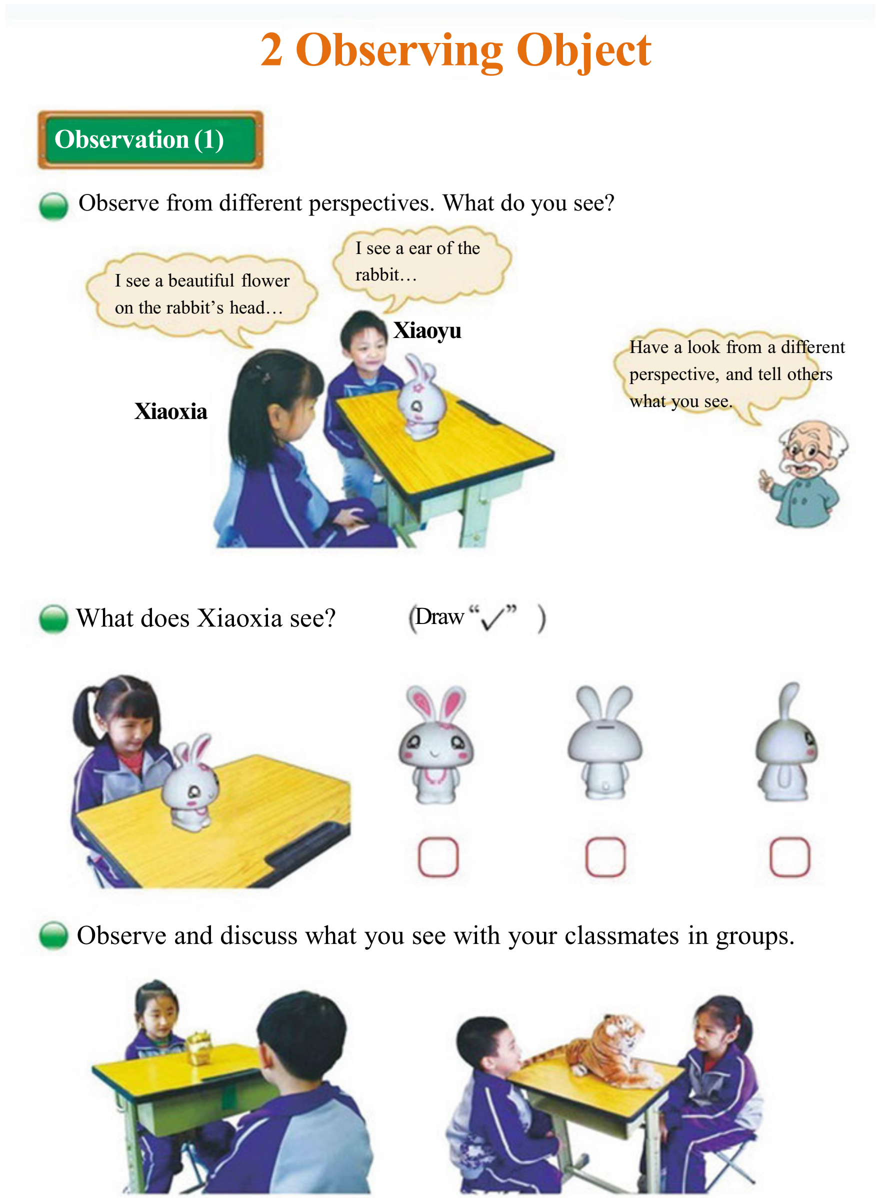
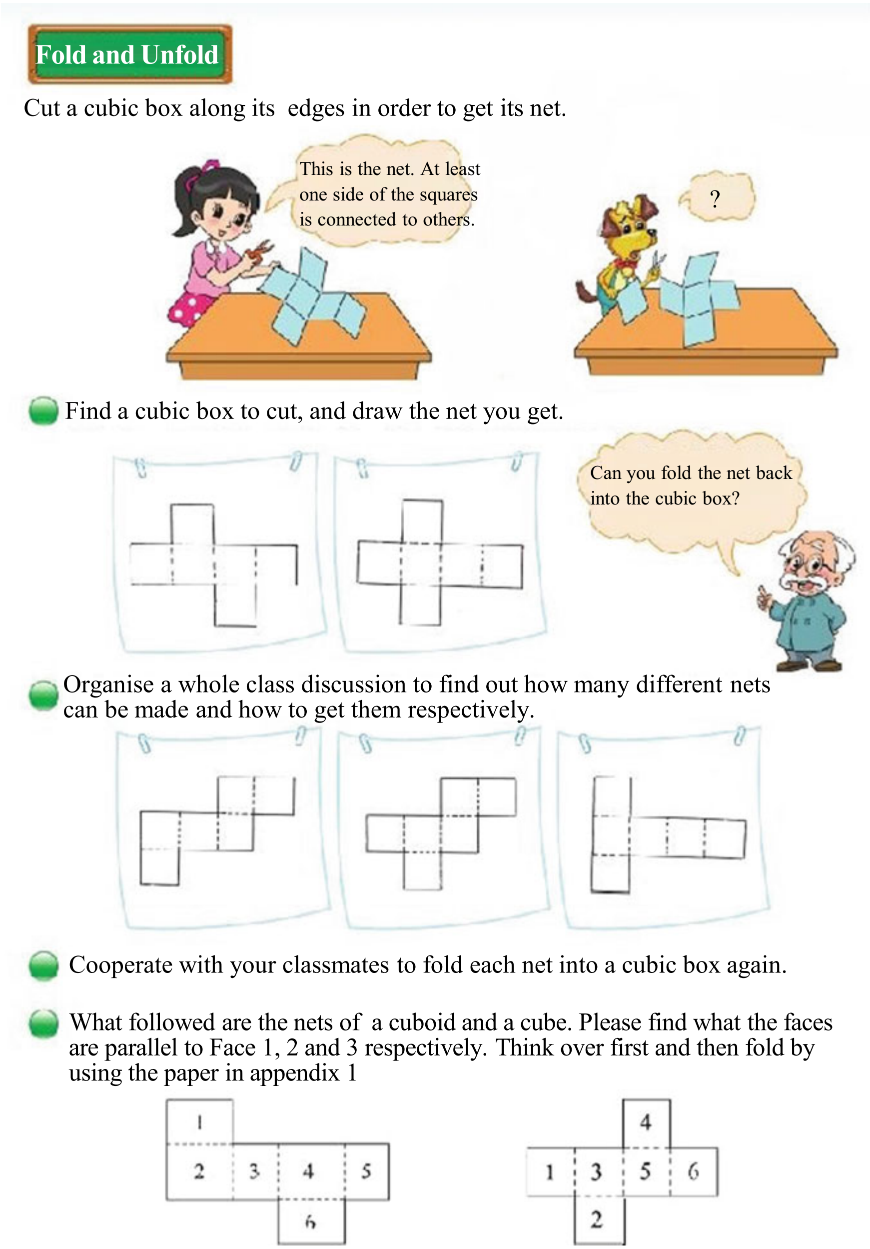


Fig. 10.4 Page from the textbook *The Primary Mathematics for the New Century* (Liu, 2014); the page is meant for Grade 1; the students have to observe objects from different perspectives

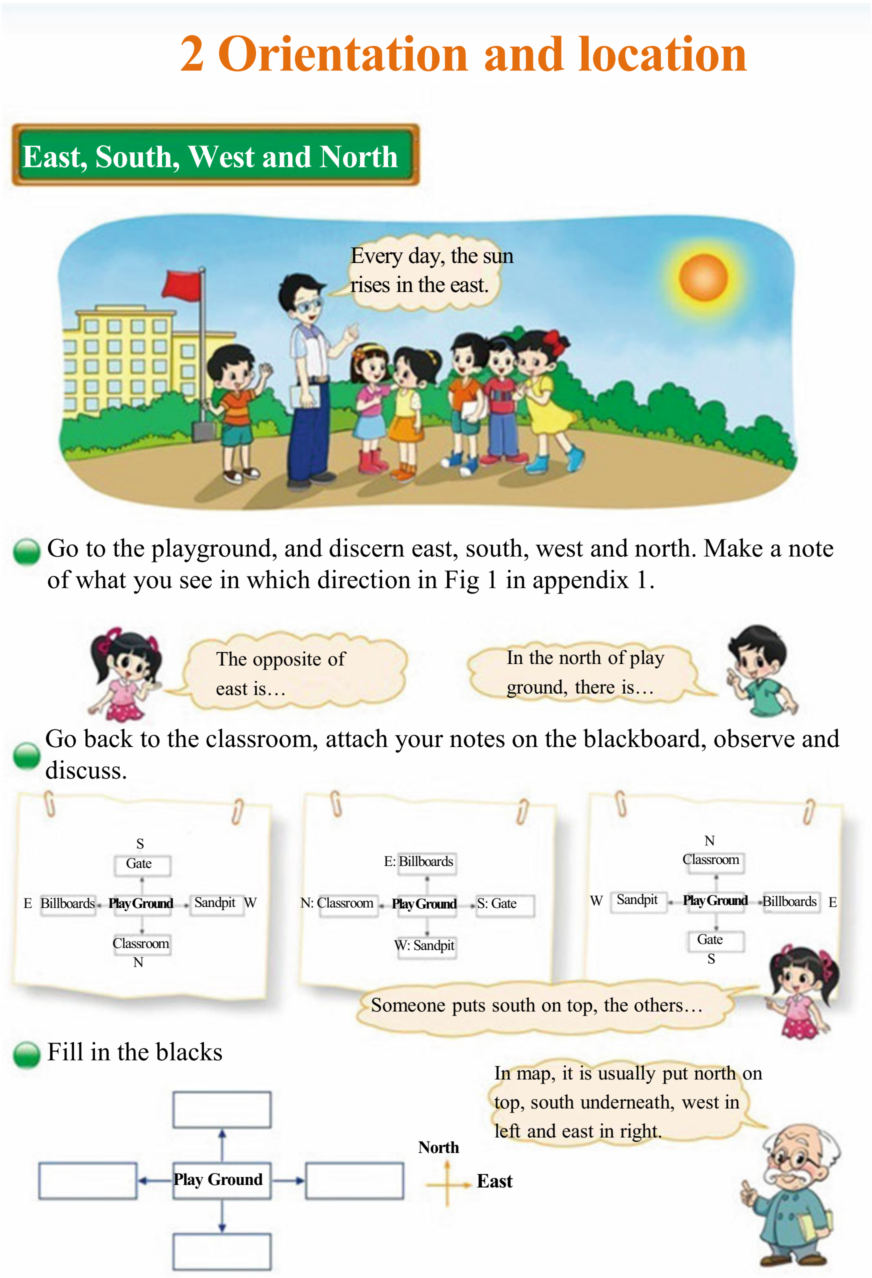


Fold and Unfold

Cut a cubic box along its edges in order to get its net.

- Find a cubic box to cut, and draw the net you get.
- Organise a whole class discussion to find out how many different nets can be made and how to get them respectively.
- Cooperate with your classmates to fold each net into a cubic box again.
- What followed are the nets of a cuboid and a cube. Please find what the faces are parallel to Face 1, 2 and 3 respectively. Think over first and then fold by using the paper in appendix 1

Fig. 10.5 Page from the textbook *The Primary Mathematics for the New Century* (Liu, 2014); the page is meant for Grade 5 and addresses the content of cubic figures and their nets



2 Orientation and location

East, South, West and North

Go to the playground, and discern east, south, west and north. Make a note of what you see in which direction in Fig 1 in appendix 1.

Go back to the classroom, attach your notes on the blackboard, observe and discuss.

Fill in the blacks

Fig. 10.6 Page from the textbook *The Primary Mathematics for the New Century* (Liu, 2014) meant for Grade 2; the page addresses the content of orientation and location

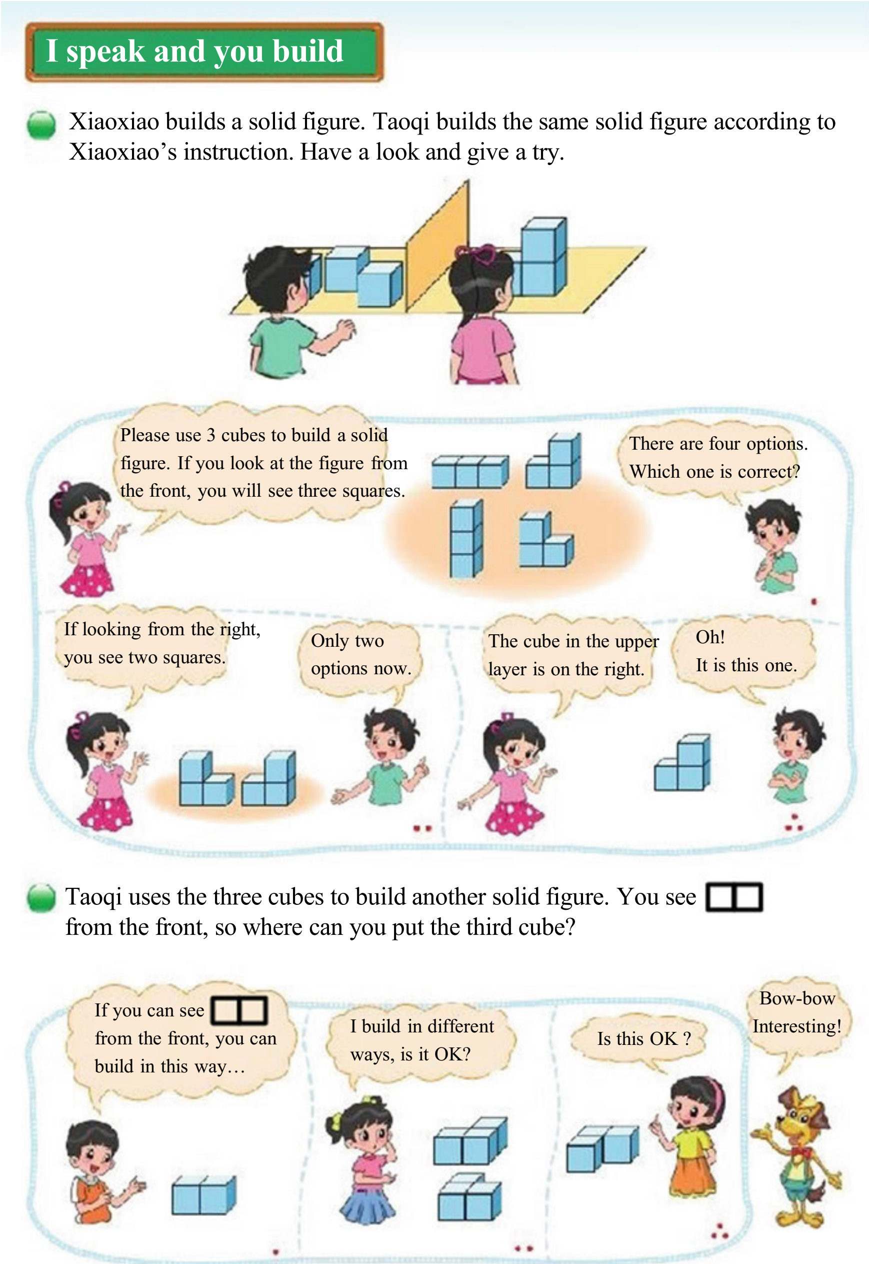
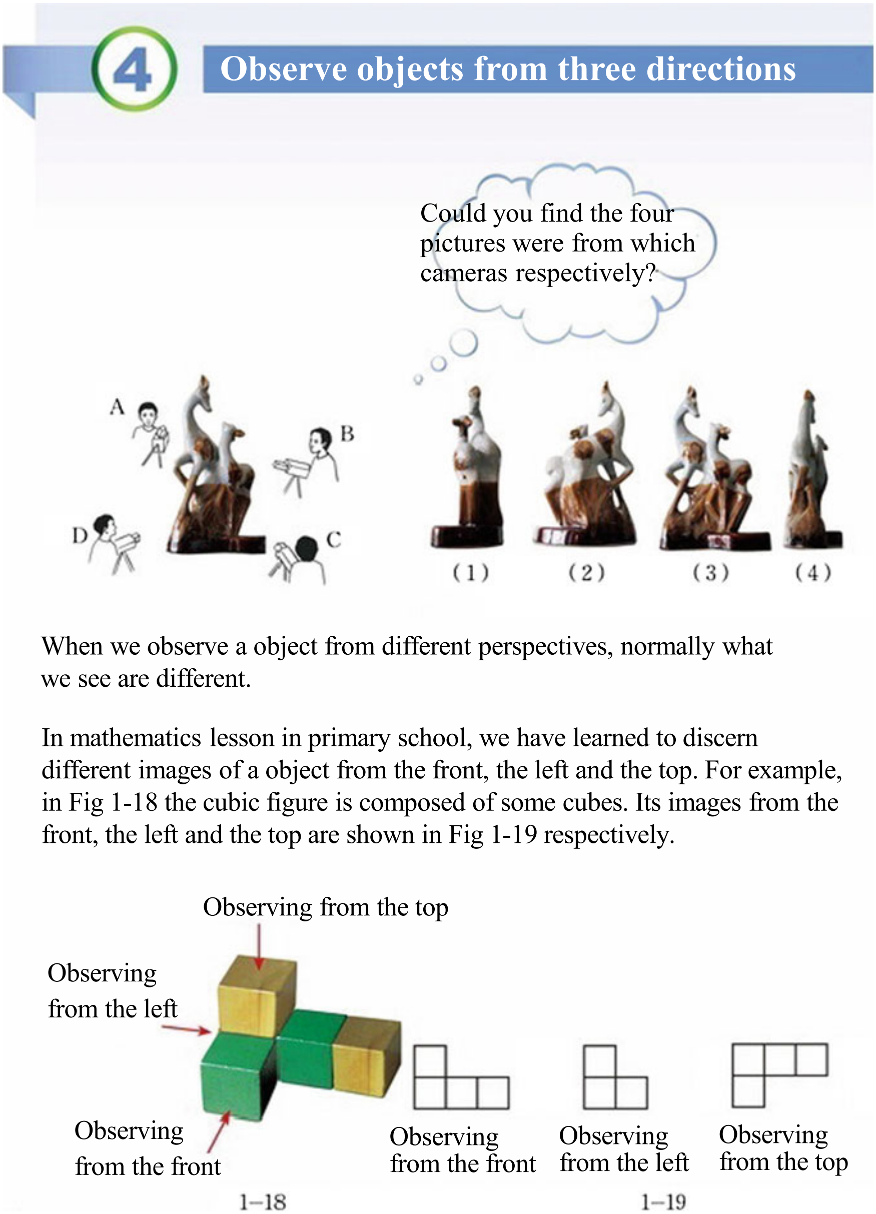


Fig. 10.7 Page from the textbook *The Primary Mathematics for the New Century* (Liu, 2014) meant for Grade 4; the page addresses the content of simple spatial reasoning



4 Observe objects from three directions

Could you find the four pictures were from which cameras respectively?

A B C D

(1) (2) (3) (4)

When we observe a object from different perspectives, normally what we see are different.

In mathematics lesson in primary school, we have learned to discern different images of a object from the front, the left and the top. For example, in Fig 1-18 the cubic figure is composed of some cubes. Its images from the front, the left and the top are shown in Fig 1-19 respectively.

Fig. 10.8 Page from the textbook *Mathematics* (Ma, 2013) meant for junior high school Grade 7; in this page students have to observe objects from different perspectives get from mathematics education except preparation for an examination has bothered us for a long time, even when facing the great performance of Shanghai students in PISA (Programme for International Student Assessment). At the beginning of the curriculum reform, we realised that it was needed to change both fundamental ideas of mathematics education at the level of educational policy making, and textbook design with respect to the structure of the textbooks, their content, and their connected materials. However, at a moment that the focus of teaching and learning was on solving types of problems for having high scores in tests, it was not easy to find the right direction to change mathematics education. At that moment, RME from the Netherlands provided a good example for us.

10.2.3 The Influence of RME on Classroom Teaching

Based on the principle of considering students as the owner of their mathematics learning, Chinese mathematics teachers are gradually shifting from a role as authority in class to a role as organiser, facilitator, and co-operator in students' mathematics learning. The proportion of instruction is reducing; while the proportion of students' exploration, communication, and cooperation is increasing. There is no doubt that it is a huge challenge for Chinese teachers, especially considering the deep-rooted Confucian culture in China that highly values the dignity and authority of the teacher.

From Hans Freudenthal, to Jan de Lange, to Marja van den Heuvel-Panhuizen, to Martin Kindt, and to Michiel Doorman, all the researchers at the Freudenthal Institute had put the emphasis in their lectures on students' active involvement in the learning process rather than on teachers' teaching in the sense of demonstrating what the students have to do. Especially, they underlined that teachers can reduce their unnecessary interventions by providing appropriate context problems which offer students opportunity for mathematisation. Many examples mentioned in these lectures have become classical cases used in China for mathematics teachers' professional development. By analysing and reflecting these cases, many Chinese mathematics teachers get a better understanding of RME, and try to change their former teaching practice of direct transmission. The two books written by Freudenthal are very popular among mathematics teachers in primary and secondary schools in China. Moreover, the book chapter written by Xiaotian Sun (2003) published in the book *The Development of Mathematics Curriculum from an International Perspective*, in which he vividly illustrated how RME by Dutch teachers is implemented in real classrooms, also became a resource for Chinese teachers to think over and learn from during the process of putting ideas of curriculum reform into practice.

Nowadays, students have become active learners in their mathematics class rather than learning passively. Instead of immersing themselves into exercise individually, students are given more change to voice their thought and to discuss with and learn from each other. Communication and exchange between teachers and students, between students and their peers are now mainstream in mathematics classes in China. It is delighting that Chinese students' ability of expressing themselves has been largely improved. The ability of articulating one's understanding is an important external indicator of the ability of thinking. Students' improvement in this aspect has been considered as one of the main achievements of the curriculum reform, which echoes the previously formulated expectation of the curriculum reform. Of course, this achievement is the result of multiple endeavours. One of them obviously is the contribution made by researchers from the Freudenthal Institute to bring RME from the Netherlands to China.

For a long time, China has put herself in the position of a student in the international class of mathematics education. Although Chinese students perform very well in PISA, there is still a long way to go for us as a student to learn from other countries. The improvement in curriculum reform in China is definitely related to the advanced international experience of mathematics reform worldwide. Scholars and researchers in China have done many studies on curriculum standards, textbooks and teaching practice in tens of countries, which provide important nutrition to improve curriculum reform in China. In this paper, we focus on reflecting the profound influence of RME, Hans Freudenthal and other researchers at the Freudenthal Institute in the Netherlands on the development of mathematics education in China. In our view, RME from the Netherlands has provided great power for the growth of the mathematics curriculum in China. At the same time, the application of RME in China also provides evidence of RME's power beyond the boundary of its cultural context.

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References

De Lange, J. (1987). *Mathematics, insight and meaning*. Utrecht, the Netherlands: OW&OC.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht, the Netherlands: Reidel.

Freudenthal, H. (1991). *Revisiting mathematics education—China lectures*. Dordrecht, the Netherlands: Kluwer Academic Publishers.

Liu, J. (Ed.). (2014). *The primary mathematics for the new century [a mathematics textbook for Grade 1-6]*. Beijing, China: Beijing Normal University Press (in Chinese).

Ma, F. (Ed.). (2013). *The mathematics [a mathematics textbook for Grade 7–9]*. Beijing, China: Beijing Normal University Press (in Chinese).

Ministry of Education of the People's Republic of China (MOE). (2001). *Mathematics curriculum standards for full-time obligatory education*. Beijing, China: Beijing Normal University Press.

Sun, X. (2003). The education and mathematics education of the Netherlands. In X. Sun (Ed.), *The development of mathematics curriculum from an international perspective* (pp. 149–159). Beijing, China: Higher Education Press (in Chinese).

Wisconsin Center for Education Research & Freudenthal Institute (Eds.). (1997–1998). *Mathematics in context: A connected curriculum for Grade 5–8*. Chicago, IL: Encyclopedia Britannica.