Designing Rich Numeracy Tasks

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In Australia, numeracy is regarded as a general capability to be developed across the whole school curriculum, not just mathematics. This paper draws on a research study that aimed to help teachers in ten schools design numeracy tasks and implement investigative numeracy pedagogies across the middle school (Grades 6-9) curriculum. Teachers were introduced to a rich model of numeracy that gives attention to real-life contexts, application of mathematical knowledge, use of representational, physical, and digital tools, and positive dispositions towards mathematics. These elements are grounded in a critical orientation to the use of mathematics. The paper identifies ways in which collaboration between the researchers and teachers influenced the design of numeracy tasks.

Keywords: Numeracy; Teacher development; Influences on task design

Numeracy is a term used in many English-speaking countries to denote the capacity to deal with quantitative aspects of life. It is often considered to have a similar meaning to terms such as *quantitative literacy* (Steen, 2001) or *mathematical literacy* (OECD, 2004). For example, Steen proposed that the elements of quantitative literacy include: confidence with mathematics; appreciation of the nature and history of mathematics and its significance for understanding issues in the public realm; logical thinking and decision-making; use of mathematics to solve practical everyday problems in different contexts; number sense and symbol sense; reasoning with data; and the ability to draw on a range of prerequisite mathematical knowledge and tools. Some of these elements are visible in the PISA definition of mathematical literacy as:

> an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, p. 15)

Steen (2001) argues that, for numeracy to be useful to students, it must be learned in multiple contexts and in all school subjects, not just mathematics. In Australia, support for this challenging notion has come from several sources. A recent national review of numeracy education undertaken by the Australian government recommended that numeracy be recognised as "an across the curriculum commitment" (Council of Australian Governments, 2008, p. 7). In addition, the newly developed Australian Curriculum – the first ever nationally mandated curriculum in this country – identifies numeracy as one of seven general capabilities that apply across all discipline content (Australian Curriculum, Assessment and Reporting Authority, 2012). However, the new curriculum documents do not provide teachers with detailed guidance in recognising the numeracy demands of the subjects they teach, in designing tasks and learning sequences that embed numeracy across the curriculum, or in making decisions about pedagogies that support numeracy learning. This paper draws on data from a one year project that helped teachers to design numeracy tasks and implement numeracy pedagogies across the school curriculum in Grades 6-9. It was conducted before the release of the Australian Curriculum, with the intention of developing and testing a theoretically informed model of numeracy that could guide teachers' curriculum planning, task design, and pedagogical decision making in the context of an existing state-based curriculum framework (South Australian Department of Education and Children's Services, 2005). For the purposes of this project, a "numeracy task" is considered to be an activity that engages students in learning and/or applying some mathematics within a given curriculum context.

This paper aligns with Theme E: Features of task design informing teachers' decisions about goals and pedagogies. It addresses the following question:

- *How does collaboration between researchers and teachers influence the design of rich numeracy tasks?*

Numeracy Model

We developed the model shown in Figure 1 to affirm the value of definitions of numeracy widely accepted in Australia (e.g., Department of Education, Training and Youth Affairs, 2000), while introducing a greater emphasis on tools as mediators of mathematical thinking and action (Sfard & McClain, 2002) and a critical orientation to the ways mathematics is used to support arguments and influence opinions (Jablonka, 2003). The original purpose of the model was to describe the characteristics of a *numerate person*. However, when we worked with teachers we realised the model also provides principles for the design of rich *numeracy tasks*. Thus the elements of the model, elaborated below, were the task design principles that we encouraged teachers to use.

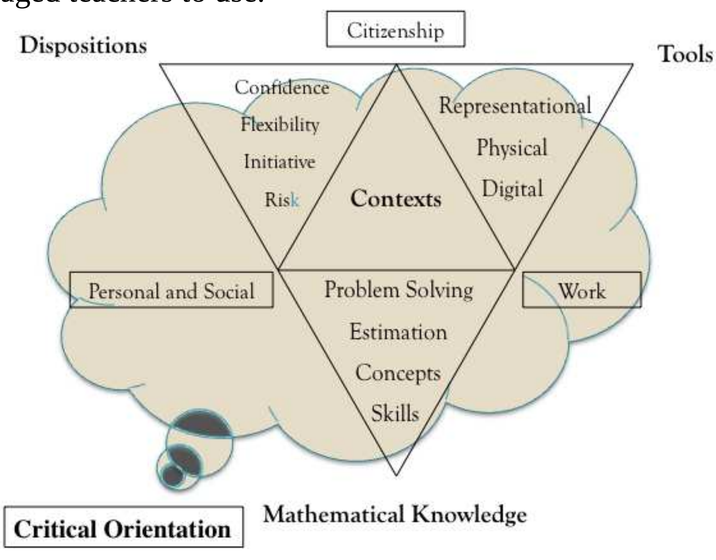

Figure 1. Numeracy model

A numeracy task should require application of *mathematical knowledge*. In a numeracy context, mathematical knowledge includes not only fluency with accessing concepts and skills, but also problem solving strategies and the ability to make sensible estimations (Zevenbergen, 2004).

A numeracy task should promote *positive dispositions* – such as confidence, initiative, and a willingness to apply mathematical knowledge flexibly and adaptively. Affective issues have long been held to play a central role in mathematics learning and teaching (Leder & Forgasz, 2006), and the importance of developing positive attitudes towards mathematics is emphasised in national and international curriculum documents (e.g., National Council of Teachers of Mathematics, 2000; National Curriculum Board, 2009).

A numeracy task should involve using *tools*. Sfard and McClain (2002) discuss ways in which symbolic tools and other specially designed artefacts “enable, mediate, and shape mathematical thinking” (p. 154). In school and workplace contexts, tools may be representational (symbol systems, graphs, maps, diagrams, drawings, tables), physical (models, measuring instruments), and digital (computers, software, calculators, internet) (Noss, Hoyles, & Pozzi, 2000; Zevenbergen, 2004).

Because numeracy is about using mathematics to act in and on the world, numeracy tasks should be embedded in a range of *contexts* (Steen, 2001). These contexts may be drawn from real life or curriculum areas other than mathematics.

Numeracy tasks should develop a *critical orientation* in students since numerate people not only know and use efficient methods, they also evaluate the reasonableness of the results obtained and are aware of appropriate and inappropriate uses of mathematical thinking. Numeracy tasks could ask students to evaluate quantitative, spatial or probabilistic information used to support claims made in the media or other contexts. They could also encourage students to consider how mathematical information can be used to manipulate, disadvantage or shape opinions about social or political issues (Jablonka, 2003).

The design of rich numeracy tasks according to the principles outlined above is not sufficient to enable learning. We argue that teachers also need to adopt *investigative pedagogies* to fully realise the numeracy opportunities that such tasks afford. Diezmann, Watters, and English (2001) define mathematical investigations as “contextualized problem solving tasks through which students can speculate, test ideas and argue with others to defend their solutions” (p. 170). We consider this definition applies equally well to numeracy investigations.

The numeracy model was used in three ways: (1) to analyse the numeracy demands of the South Australian school curriculum (Goos, Geiger, & Dole, 2010); (2) to support teachers’ curriculum planning (Goos, Dole, & Geiger, 2011); and (3) to trace changes in teachers’ understanding of numeracy (Goos, Geiger, & Dole, 2011). This paper is primarily concerned with (2), and it extends our previously published analyses by focusing on the design of numeracy tasks and implications for pedagogy.

Project Overview

Teachers were recruited from ten schools with diverse demographic characteristics: four primary schools (Kindergarten-Grade 7), one secondary school (Grades 8-12), four small schools in rural areas (Grades 1-12), and one school that combined middle and secondary grades (Grades 6-12). Each school nominated two teachers, thus ensuring that participants could collaborate with a colleague in their own school as well as teachers from the other schools. They included generalist primary school teachers who taught across all curriculum areas as well as secondary teachers qualified to teach specific subjects (mathematics, English, science, social education, health and physical education).

There were three elements to the research plan: (1) an audit of the middle years curriculum to identify the numeracy demands inherent in all curriculum areas; (2) three whole-day professional development workshops at the beginning, middle, and end of the project; and (3) two daylong visits to each school for lesson observations, discussion of planning documents and teaching approaches, and audio-recorded interviews with teachers and students. The overall project design is summarised in Table 1. More details on data collection and analysis methods can be found in Goos, Dole and Geiger (2011).

Time	Researcher activity	Teacher activity
February: Curriculum audit	Identify numeracy demands in all curriculum areas	
March: Workshop #1	Introduce numeracy model; present findings from curriculum audit; provide sample numeracy tasks	Analyse numeracy task design via reference to model; plan for implementation
June: School visits	Observe lessons; provide feedback on planning, task design and pedagogies	Incorporate feedback into planning for further implementation
August: Workshop #2	Provide feedback on first round of school visits; present stimulus materials for task design	Share tasks and strategies tried so far; practise task design with emphasis on critical orientation
October: School visits	Observe lessons; provide feedback on planning, task design and pedagogies	Incorporate feedback into planning for further implementation
November: Workshop #3	Report on student perceptions of numeracy; present stimulus materials for task design	Practise task design; reflect on professional learning trajectories

Table 1. Project Design

Influencing the Design of Numeracy Tasks

We claim that there are several aspects of the project that influenced the design of rich numeracy tasks and enactment of associated investigative pedagogies. The first was the numeracy model itself, the elements of which provided a set of principles for task design. The second was our numeracy audit of the South Australian Curriculum Framework, which identified distinctive numeracy demands for each school subject taught in Grades 6-9. For example, the subject called Society and Environment is organised into four strands: time, continuity and change; place, space and environment; societies and cultures; and social systems. The numeracy audit found that data analysis and spatial sense are the most relevant elements of *mathematical knowledge* for this subject. *Contexts* for numeracy development included the study of social, economic, political and ecological systems. Students were expected to develop *dispositions* enabling them to “to be active citizens who can make informed and reasoned decisions and act on these” (DECS, 2005, p. 291). The use of *tools* such as maps, measuring instruments, online data sources, and spreadsheets for collecting and analysing information was vital to learning in this subject. The goal of enabling students to participate as ethical, active and informed citizens, requires development of a *critical orientation* to viewing information and interpreting data. Presenting the audit findings to teachers was intended to raise awareness of their subject’s numeracy demands.

The third influence on task design was the way in which the research team used the three workshops to (1) immerse teachers in numeracy tasks we had created, (2) model investigative numeracy pedagogies, (3) guide the analysis of tasks using the design principles provided by the numeracy model, and (4) invite the teachers to practise designing their own numeracy tasks. For example, in the first workshop we engaged teachers in cross-curricular numeracy investigations suitable for use with middle years students. These included investigations of Barbie dolls' physical proportion (with links to the health and physical education curriculum), the occurrence of the Golden Rectangle in art, design and nature (linked to the arts and design studies curricula), and planning for participation in the Tour Down Under, a bicycle race similar to the Tour de France (linked to the society and environment curriculum). In the first round of school visits we found little evidence of a critical orientation in the lessons we observed. In interviews with teachers it emerged that they were unsure about how to embed this element of the numeracy model into their planning and practice. Therefore, at the second workshop we presented a range of stimulus materials drawn from print and digital media sources and asked teachers to work together to develop these into tasks that would promote a critical orientation in their students, without losing sight of the other elements of the numeracy model. In both workshops we provided a task design/analysis template that listed each element of the numeracy model, asked how the task developed numeracy with respect to each element, and invited teachers to identify school subjects that could provide a context for using the task.

The fifth influence on task design was the researchers' provision of in situ feedback to teachers during school visits. We were able to suggest ways of modifying tasks used in the lessons we observed to give greater prominence to elements of the numeracy model that appeared to be under-represented. The final influence on task design was the structured sharing of practice by teachers at the second and third workshops. At the second workshop all teachers were asked to bring evidence of one task or lesson sequence they had tried with their class, to describe to the whole group how the task had been implemented and how well (or not) it had worked, explain what they learned from this experience and how they would use this evaluation in their subsequent planning. This workshop provided an opportunity for teachers to see how colleagues in other schools went about designing rich numeracy tasks. At the third and final workshop, and as a result of our analysis of school visit data, we invited four teachers who exemplified different types of professional learning trajectories to report on their experiences. One of these is the teacher whose abbreviated case study is presented below to illustrate how she made decisions about the design of numeracy tasks with an investigative flavour. Although this project emphasised numeracy across the whole school curriculum, the example below shows how numeracy tasks can be designed within mathematics.

Teacher Decision Making about Task Design

Maggie taught mathematics and science at a large secondary school in a rural town. She was in only her second year of teaching. The class with which she worked for this project was a Grade 8 mathematics class.

First school visit

Initially Maggie struggled to come to grips with how to highlight the numeracy within mathematics, but she decided to focus on teaching mathematics in real life contexts that would be of interest to her students. She planned a task based on the television program *The Amazing Race*. Students worked for 3 weeks in the computer laboratory to complete the task, which involved organising an adventure holiday around the world, given an itinerary and a budget of $10,000. They also had to complete a number of challenges for which they earned an additional $2000 each. The challenges, which included *Diving with Sharks* in Cairns, *Skiing* in Switzerland, and visiting *The Roman Colosseum*, had a focus on using directed number in context. In *The Roman Colosseum* challenge students were also required to use formulas in the context of comparing areas of the Colosseum and the Melbourne Cricket Ground, as well as looking at exchange rates and converting between currencies.

Members of the research team observed the second lesson of this unit. Students appeared motivated and well prepared, and they were able to explain the task to us when we questioned them. Maggie noted that some previously disengaged students were interested in the task, while a few others remained aloof. Some students seemed so engaged that they acted as though the task was real; for example, when Maggie asked one boy “Where are you up to?”, he replied “I’m on my way to Paris!”.

This task placed mathematics in the real life *context* of an adventure holiday. It targeted *mathematical knowledge* of directed numbers and operations with integers (money calculations), using digital (internet) and representational (charts, tables) *tools*. We did not observe teacher actions that promoted positive *dispositions* towards numeracy, but students were clearly motivated and confident in tackling the task and trying out different combinations of flights and accommodation bookings that would fit within their budget. A *critical orientation* does not seem to have been built into this task. However, after the lesson we suggested to Maggie that this orientation could be promoted via questioning, such as that we observed when Maggie helped a student compare advantages and disadvantages of booking cheap backpackers’ accommodation. In this way we attempted to show how the numeracy model informed not only task design but also teachers’ pedagogies.

Second school visit

When interviewed before the lesson observation, Maggie said she had given a lot of consideration to the types of tasks she wanted to design for the second research cycle. She was dissatisfied with the length of the *Amazing Race* investigation, as this tended to discourage some students and to make it difficult to complete for any who missed some lessons. As a result of our emphasis in the second workshop on developing a critical orientation, she also decided to give more attention to this element of the numeracy model in designing the next investigation, *Approaches to a Healthy Lifestyle,* which comprised a number of smaller tasks.

In one task, students investigated the relationship between the heights and walking speeds of everyone in the class. The mathematics embedded in the investigation included elements of collecting, representing, reducing and analysing data. In the lesson we observed, students were to make scatter plots using Excel in order to determine whether there was a pattern in the data they had collected. In earlier lessons they had collected height data and calculated the mean, median and mode. In another lesson students had marked out a 40 metre section of a 100 metre running track and then found the time it took to walk this distance. With this information students had calculated their walking speeds in metres per second, metres per minute and kilometers per hour.

Students all appeared engaged with the task and each group or individual produced a scatter plot, although the appearance of the graphs varied depending on the scales chosen or on the choice of variable for the *x* and *y* axes. Most students were able to describe a general trend in the data and use this to make a prediction about what Maggie's walking speed might be, based on her height. Interestingly, many students gave most attention to their own data point within the scatter plot with comments such as "This is me (pointing at the appropriate data point) and "This is how tall I am and how fast I walk". Using personal data seemed to be effective for engaging students with the task. From a student's perspective, the activity was about them and how they compared to the rest of the class.

Students expressed surprise that the scatter plot was not linear, so that taller people did not necessarily walk faster. Maggie challenged them to explain why this should be the case. Some groups suggested that alternative variables – with associated alternative hypotheses – should be explored, including, for example, the relationship between walking speed and leg length or between walking speed and stride rate. One group suggested there might be a stronger relationship between a person's height and their maximum walking pace rather than their natural walking pace.

Maggie chose an engaging *context* that made use of students' personal details to introduce the *mathematical knowledge* that was used in this lesson. The use of personal data encouraged positive *dispositions* towards involvement in and completion of the task. This task required knowledge of how to produce a scatter plot from a data set using Excel and the capacity to make predictions from trends in the data. Maggie asked students to use *representational tools* such as scatter plots and *digital tools* in the form of computers and Excel. By challenging students to explain the variance in their data from the anticipated linear relationship, Maggie introduced a *critical orientation* to the task. A critical orientation was evident in most tasks in the *Approaches to a Healthy Lifestyle* investigation. For example, in the culminating task students were to compare data on the number of overweight and obese Australians in various age groups, and make an argument for whether or not the government needed to introduce a healthy eating policy for South Australian schools.

Implications and Concluding Comments

The work we reported here has implications for teachers, researchers, and curriculum developers. First, it provides some evidence that a focus on task design, when supported by a theoretical model of numeracy that is readily accessible to teachers, can influence teacher learning and development. For example, when Maggie reflected on what she had learned during the course of the project, she identified her readiness to make use of more extended tasks when teaching mathematics. However, she tempered this view by arguing that tasks needed to be made up of self-contained sub-tasks that allowed students to move towards smaller achievable goals. For her, the level of engagement she observed while students were working on numeracy investigations was a compelling case for their inclusion within mathematics classes. Nevertheless, other teachers in the project found it more difficult to decide "how long" a numeracy investigation should be to allow students enough time to explore all aspects of a task without losing interest, and how much guidance to give students in structuring the investigation.

A challenge for researchers is to design larger scale studies that do not simply rely on recruiting more schools or teachers. This is the case for most educational research and is not unique to our project. To tackle these challenges we are currently conducting a follow up study that aims for both scale and sustainability by developing numeracy curriculum leadership within schools, so that the numeracy model and associated task design principles become integrated into schools' planning processes for implementation of the new Australian Curriculum.

The current version of the Australian Curriculum offers some support for recognising the numeracy demands of different school subjects, for example, by providing a numeracy learning continuum together with icons and filters that link numeracy capabilities to relevant curriculum content. However, additional opportunities for developing students' numeracy capabilities are invisible unless one knows how to "see" them and how to design and implement tasks for classroom use. The numeracy model and ways of working with teachers outlined in this paper may prove useful in supporting teachers to fulfil the numeracy intentions of the Australian Curriculum.

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