Digital design:
RME principles for designing online tasks

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The theory of Realistic Mathematics Education provides principles that can be applied to task design. In this paper, we investigate how these principles apply to the design of online tasks. To do so, we present examples of tasks on algebra, calculus and geometry designed in the Digital Mathematics Environment. As a result, we conclude that the principles of guided reinvention, didactical phenomenology, and emergent modeling can inform and guide digital design, but that some aspects work out differently compared to the design of paper-and-pencil tasks.

Keywords: Didactical Phenomenology, Digital Design, Digital Mathematics Environment, Emergent Modeling, Guided Reinvention, Realistic Mathematics Education

Introduction

Task design is widely recognized as an important, but complex and subtle activity. Based on the experience of skilled designers, design guidelines and heuristics have been identified (e.g., see Watson & Mason, 2006). Whereas the design and use of digital content nowadays plays an increasingly important role in mathematics education, most of these design principles are based on and applied to the design of paper-and-pencil tasks. The question, therefore, is how such design principles apply to digital design, and how the rich experience and knowledge in the field of designing paper-and-pencil tasks can be transferred to the case of digital design.

To address this question, we limit ourselves to three principles that emerge from the theory of Realistic Mathematics Education and may inform design: guided reinvention, didactical phenomenology, and emergent modeling. We revisit some tasks designed in the Freudenthal Institute's Digital Mathematics Environment in the field of algebra, calculus and geometry education.

RME Design Principles

Realistic Mathematics Education (RME) is a domain-specific instruction theory for the teaching and learning of mathematics. According to RME, mathematics should be seen as an activity (Freudenthal, 1973), and students, rather than being receivers of ready-made mathematics, should be active participants in the educational process, in which they develop mathematical tools and insights by themselves. This point of departure led to the following RME principles that inform task design: guided reinvention, didactical phenomenology, and emergent modeling (Freudenthal, 1973; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 1996).

According to the principle of *guided reinvention*, students should be given the opportunity to experience a process similar to that by which a given mathematical topic was invented. Even if this primarily is a teaching principle, it has consequences for task design: tasks – or sets of tasks – should invite students to develop 'their own' mathematics. This process, however, needs guidance from the teacher, to help to further develop sensible directions, to leave 'dead-end streets' and to ascertain convergence towards shared knowledge according to the standards within the mathematical community.

Didactical phenomenology concerns the relation between the thought object −the 'nooumenon'− and the phenomenon –the 'phainomenon'− from the perspective of teaching and learning. In particular, it addresses the question how mathematical 'thought objects' can help in organizing and structuring phenomena in reality. The challenge for the task designer, then, is to find such meaningful phenomena that beg to be organized and structured by the targeted mathematical knowledge.

According to the *emergent modeling* perspective, a model may play different roles during different phases of activity. Initially, a model is context-specific: it refers to a meaningful problem situation that is experientially real for the student, and is a model of that situation. Then, through working with the model, it gradually acquires a more generic character and develops into a model for mathematical reasoning that is possible because of the development of new mathematical objects in a more abstract framework of mathematical relations that the model starts to refer to. This notion is elaborated into a four-level structure that represents levels of mathematical activity (Gravemeijer, 1994). For the task designer, the challenge is to find suitable situations that ask for the development of such models, and allow for a process of progressive abstraction.

While applying these RME principles to the design of paper-and-pencil tasks is not straightforward, their use for the design of digital content is even more challenging. Although early research on the use of graphing calculators identified opportunities for an RME-based teaching approach (Drijvers & Doorman, 1996), later studies describe the tension between RME principles and the integration of computer algebra software (Drijvers, 2000). Apparently, the match between RME and ICT is not self-evident.

The Digital Mathematics Environment

As technology for teaching mathematics the Freudenthal Institute's *Digital Mathematics Environment* (DME) is used. The DME integrates a content management system, a learning management system and an authoring environment. The content consists of online modules in the form of Java applets. The learning management system offers means to distribute content among students and to monitor student progress.

The authoring tool is the DME's design environment. Authors, such as teachers, text book authors, or educators, can use the tool for adapting existing online modules or for designing new ones, based on existing materials and basic tools such as graphing and equation editing facilities. While designing, the author can split up the screens in different windows, add applications and tasks, and design feedback. Knowledge of the underlying Java programming language is not required; rather, an intuitive and mathematical interface makes the digital design accessible to a wide audience (Figure 1).

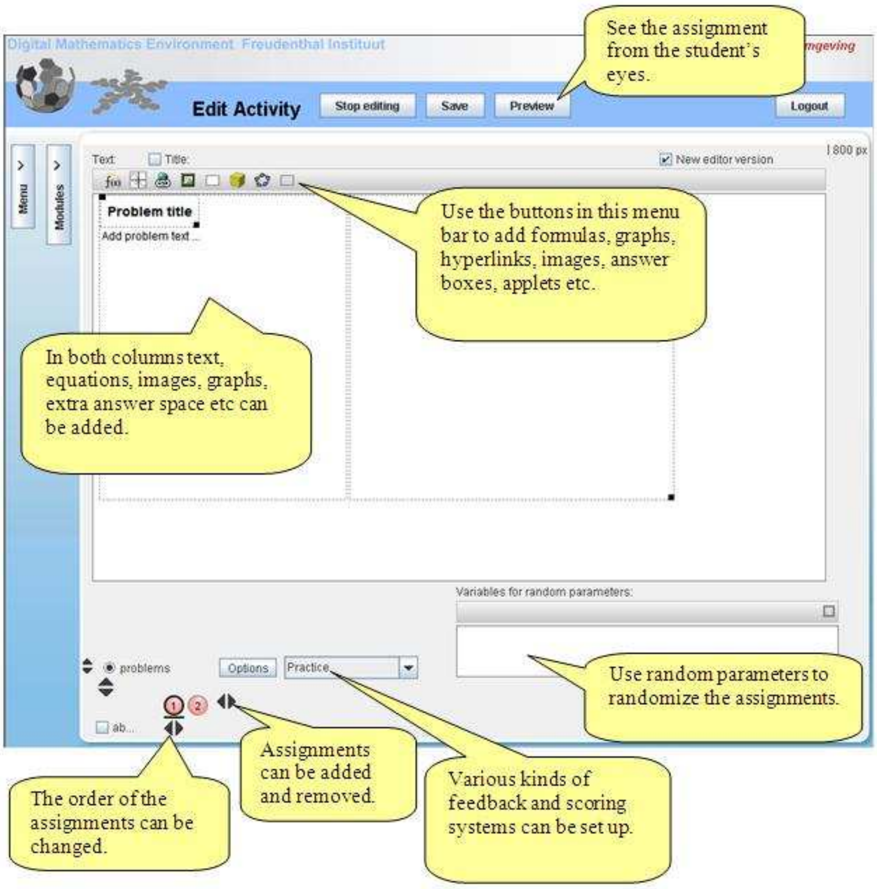


Figure 1 The Digital Mathematics Environment Authoring Tool

Examples of the results of digital design using the DME's authoring tool can be found at www.fi.uu.nl/dwo/demo/en. In this paper, we will briefly discuss some examples from algebra, calculus and geometry from the perspective of the above RME principles.

Digital Tasks for Algebra

As an example of the design of digital tasks for algebra, we consider the work by Bokhove, available at www.fi.uu.nl/dwo/voho/. The digital tasks focus on solving polynomial equations. However, the online units go beyond procedural practice and also focus on the development of symbol sense and strategic skills (Bokhove & Drijvers, 2010, 2012). Crucial factors in the design are the sequencing of the tasks (with sometimes thought-provoking equations) and the design of feedback, and its timing and fading in particular.

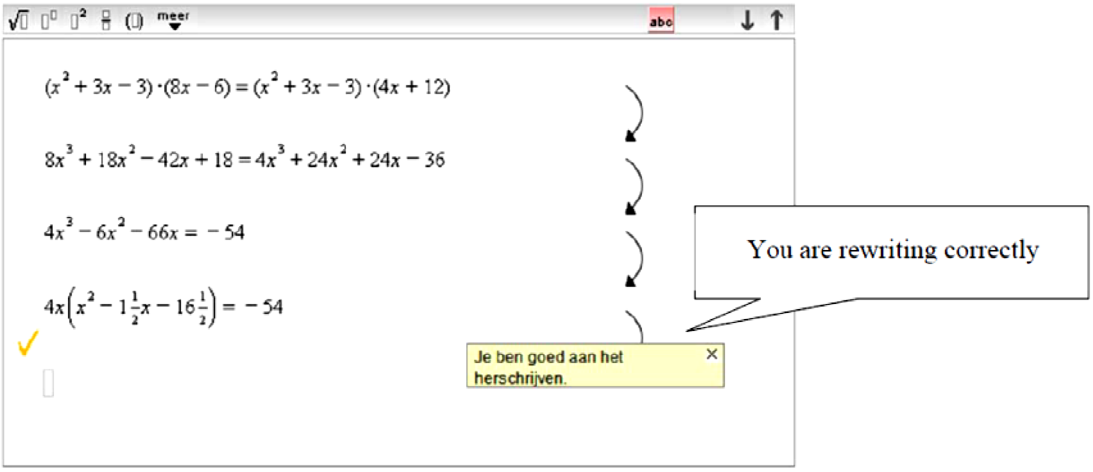


Figure 2 Solving an equation with unexpected difficulties (Bokhove & Drijvers, 2010, 2012)

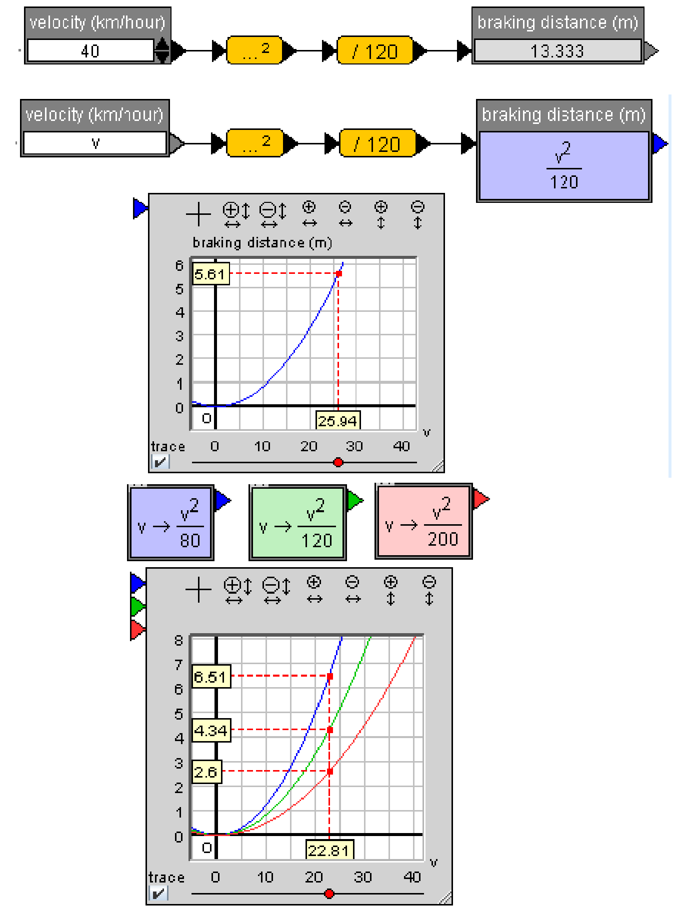
Figure 2 shows an exemplary equation in line 1, and a part of a student's work in lines 2-4. The feedback just refers to the algebraic equivalence of the subsequent equations the student enters; no feedback is provided on the problem solving strategy. This can be seen as a manifestation of the guided reinvention heuristic: this combination of task and tool provides the opportunity for students to reinvent efficient ways of solving equations. Eventually, students can continue entering equivalent equations without coming closer to the solution, but this exploration space is supposed to elicit a wish for efficiency.

From the didactical phenomenology perspective, this task may seem quite poor: what is the phenomenon at stake that would motivate students to engage in mathematics? In defense of this, one can argue that the target group of this online module consists of students in grade 12, who were to do the national examination soon, and who were familiar with the 'world of polynomial equations'. This familiarity makes that these tasks can be appropriate for the mathematization of the field and the development of new problem solving strategies.

Data shows that students work easily with this type of online tasks (Tacoma, Drijvers, & Boon, 2011). Instrumental genesis, the process of developing schemes to use tools to solve the tasks (Artigue, 2002), was an issue to a much lesser extent than in the case of computer algebra (Drijvers, 2002; Drijvers et al., 2012). Apparently, using CAS puts higher demands on instrumental genesis, and this is something to take into account as a designer.

Digital Tasks on Functions and Calculus

As an example of the design of digital tasks for (pre-)calculus we consider the online module Function and Arrow Chain, available at www.fi.uu.nl/dwo/prootool/en. The digital tasks focus on the development of conceptual understanding of the notion of functions, where a function is seen as an input-output assignment, as a dynamic process of co-variation, and as a mathematical object with different representations (Doorman et al., 2012). A crucial factor in the design is sequencing the tasks so these function models emerge in a natural way of increasing complexity and abstraction.



A function as an input-output assignment: braking distance as a function of velocity

Investigation of the co-variation of velocity and braking distance through tracing the graph

Investigation of a family of functions, representing braking distances for three different vehicles

Figure 3 Different views on function in the Function and Arrow Chain module

Figure 3 shows some screens from this module, in which the function gradually develops from a numerical input-output engine, to a process of co-variation and, finally, a mathematical object that is part of a family of functions that can be compared. This sequence of screen shots reflects the emergent modeling heuristic: the context, in this case one of a vehicle's braking distance as a function of its velocity, leads to function models of increasing complexity and abstraction. The digital tool supports this development by means of offering techniques of increasing richness and an increasing repertoire of connected function representations. Of course, this approach requires a context that is suitable for this emergent modeling process. Finding such a context is a question of didactical phenomenology.

In an online calculus course for university freshmen, the co-variation idea is supported by a Geogebra applet for tracing graphs (www.fi.uu.nl/dwo/sk/en/). However, in the context of this intensive and short-period remedial course, the emergent modeling design heuristics was exploited to a lesser extent than was the case for the Function and Arrow Chain course.

Digital Tasks for Geometry

An example of the design of digital tasks for geometry is the module available at www.fi.uu.nl/dwo/dpict/en/. The digital tasks focus on exploring, discovering, and proving properties of bisectors, altitudes and medians in triangles. The latter aspect, the proving, provides a particular design challenge: how to design tasks that offer support and guidance for a proof, but leave room for reinvention?

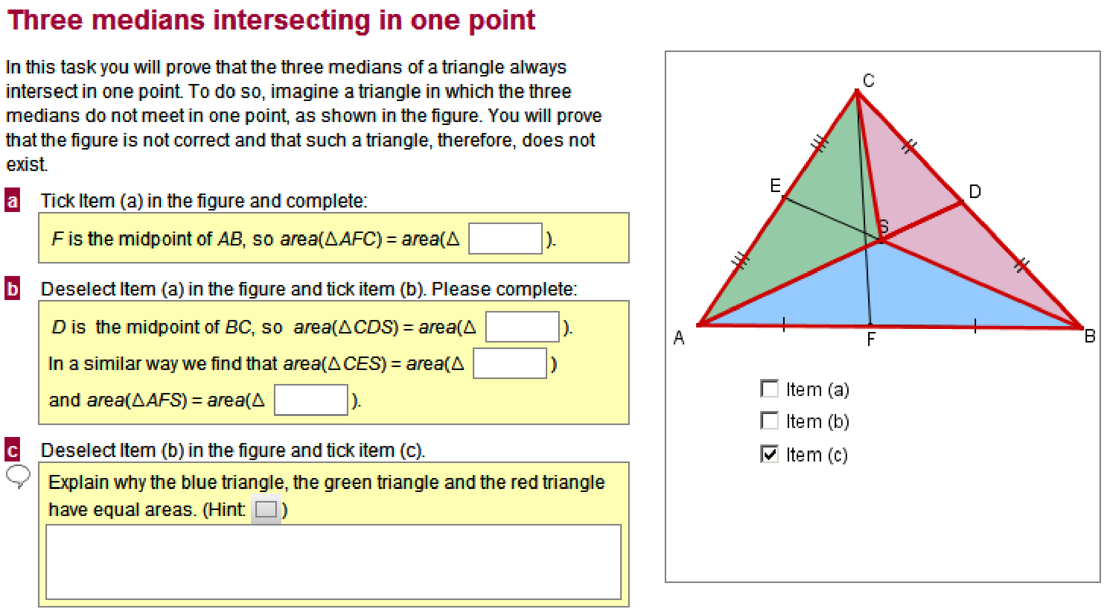


Figure 4 Proving that three medians of a triangle intersect in one point

While reading the proving task shown in Figure 4, it should be noted that Dutch grade 8 students, the target group for this module, have little experience in proving, and, therefore, need strong guidance: the structure of the proof and the corresponding sequence of images is suggested to the students. Still, writing down the final argument in the reasoning, as is requested in task c, is very difficult to them. The design principle of guided reinvention is easier to apply to tasks in which students explore the properties of bisectors, altitudes and medians in triangles: the dragging options of the dynamic geometry system, in this case Geogebra, in collaboration with DME's feedback, provide a strong learning environment in which students can really experience the geometrical situation and discover the targeted properties.

In the case of this online module, classroom observations show that attention needs to be paid to students' and teachers' instrumental genesis: the interplay between Geogebra and the DME in this module is powerful but it may also be demanding and initially complex to novice users.

Conclusion

In this short paper, we set out to investigate the application of the RME principles of guided reinvention, didactical phenomenology, and emergent modeling for the case of the design of digital tasks in the online DME. To do so, we considered three exemplary tasks. The first example, the task on solving a relatively complex equation, shows that guided reinvention in this case concerns the development of new problem solving strategies, invited by tasks that cannot be solved with the strategies available so far. The didactical phenomenology heuristic here does not lead to the use of real life contexts, but rather takes the world of polynomial equations as a point of departure. In the second example, a real life context –in this case the stopping distance situation– does form the starting point. Emergent modeling heuristics are manifest in the gradual abstraction of the students' view on function. The third example on geometrical proof again takes a guided reinvention perspective, in which the designers chose strong guidance. Again, the didactical phenomenology here does not lead to a real life problem situation, but to a problem in the world of geometry, that is expected to be experientially real to the students.

What can we conclude about the three RME principles and their application to digital design? The principle of guided reinvention seems to apply well to digital design. ICT offers opportunities for exploration and investigation, and in this way for reinvention. The design choice to confront students with unexpected examples can be seen as a way to invite reinvention as well. In general, this guided reinvention approach might suffer from constraints of the technology that may limit the students' exploration space, such as requirements for input formats and styles, and pre-designed tools that may incorporate too much guidance.

The didactical phenomenology heuristics is also valuable for digital design, but it seems that the phenomena that play a central role in the task do not necessarily come from real life: ICT already forms a meaningful 'world' on its own for the student, in addition to the world of mathematics. Having a real life context as an entrance to these two worlds may lead to cognitive overload. This is a question to be considered carefully in the design process.

Emergent modeling can be a fruitful design heuristics for digital design. As in the case of paper-and-pencil design, the models need to lend themselves for further development towards increasing mathematical abstraction and complexity. Specific for the case of digital design is that these emerging models need to be supported by the digital tools available, for example by an increasing repertoire of representations and techniques in the digital environment, or by increasing options to dynamically use these representations, connect them, and switch between them (Duval, 2006).

As an overall conclusion, this brief exploration of the issue suggests that the three RME principles are valuable for digital design, even if some appropriation is needed compared to the design of paper-and-pencil tasks, such as taking into account the constraints of the digital tools and the fact that the technological environment forms an additional 'world' to the student. The transfer of skills, developed in the ICT environment, to paper-and-pencil, for example, may need specific attention, as well as the instrumental genesis involved in the learning process.

References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245-274.

Bokhove, C., & Drijvers, P. (2012). Effects of feedback in an online algebra intervention. *Technology, knowledge and learning, Online First*. http://www.springerlink.com/content/c5v8pg33730p2303/?MUD=MP.

Bokhove, C., & Drijvers, P. (2010). Symbol sense behavior in digital activities. *For the learning of mathematics, 30*(3), 43-49.

Boon, P. (2009). A designer speaks: Designing educational software for 3D geometry. *Educational Designer, 1*(2). Retrieved June, 19th, 2012, from http://www.educationaldesigner.org/ed/volume1/issue2/article7/

Doorman, M., Drijvers, P., Gravemeijer, K., Boon, P., & Reed, H. (2012). Tool use and the development of the function concept: from repeated calculations to functional thinking. *International Journal of Science and Mathematics Education, online first*.

Drijvers, P. (2000). Students encountering obstacles using a CAS. *International Journal of Computers for Mathematical Learning, 5*(3), 189-209.

Drijvers, P., & Doorman, M. (1996). The graphics calculator in mathematics education. *Journal of Mathematical Behavior, 14*(4), 425-440.

Drijvers, P., Godino, J.D., Font, V., & Trouche, L. (2012). One episode, two lenses; A reflective analysis of student learning with computer algebra from instrumental and onto-semiotic perspectives. *Educational Studies in Mathematics, online first*.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103-131.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Reidel Publishing Company.

Gravemeijer, K.P.E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-ß Press / Freudenthal Institute.

Tacoma, S., Drijvers, P. & Boon, P. (2011). The Use of a Digital Environment to Improve First Year Science Students' Symbol Sense. *Journal of Computers in Mathematics and Science Teaching*, *30*(4), 403-428.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht: CD-ß Press / Freudenthal Institute, Utrecht University.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning*, *8*(2), 91-111.