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Primary school students' strategies in early algebra problem solving supported by an online game

Marja van den Heuvel-Panhuizen · Angeliki Kolovou · Alexander Robitzsch

Abstract In this study we investigated the role of a dynamic online game on students' early algebra problem solving. In total 253 students from grades 4, 5, and 6 (10–12 years old) used the game at home to solve a sequence of early algebra problems consisting of contextual problems addressing covarying quantities. Special software monitored the students' online working when solving the problems. Before and after the intervention a paper-and-pencil test on early algebra was administered. The data analysis revealed that the online working contributed to the students' early algebra performance. There was a significant gain in performance across all grades. The highest effect was found in grade 6. Out of the three strategy profile clusters that could be distinguished in the whole sample, the cluster dominated by using extreme values and the cluster characterized by the trial-and-error strategy were most influential on the gain in early algebra performance. The students' level of online working, which was defined as a combination of online involvement and strategy use, appeared to have a marginally significant effect on the gain score for the total sample. Per grade there was no significant effect, yet the levels of online working were significantly related to grade. Free playing was mostly performed in grade 4, looking for answers in grade 5, and exploring relations slightly more in grades 5 and 6. About 17 % of the effect of grade on the gain score was mediated by the level of online working.

Keywords Primary school mathematics · Online learning environment · Dynamic game · Early algebra · Strategy profiles

1 Introduction

In primary school mathematics the focus is mainly on developing numeracy and calculation skills. However, this is too narrow an interpretation of the mathematics curriculum. Several

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researchers (Goldenberg, Shteingold, & Feurzeig, 2003; Harel, 2008) have pointed out that mathematics also involves seeking and exploring patterns, making conjectures and performing experiments, and applying heuristics to solve nonroutine problems. These algebra-related activities should also be a vital part of primary school mathematics to offer students opportunities for developing more sophisticated thinking skills (Kaput, Carraher, & Blanton, 2008). Moreover, algebra in primary school is essential for adding coherence, depth, and power to the mathematics curriculum and can be a step towards making this mathematical domain accessible to all students and prepare them for the learning of algebra in later grades (Kaput, 2008).

However, preparing students for algebra does not mean adding a scaled-down version of secondary school algebra topics into the primary school grades (Smith & Thompson, 2008). Rather, it means changing the primary school arithmetic curriculum in such a way that it promotes algebraic thinking, for example by infusing opportunities for investigating and generalizing patterns (Kaput, 1999). Along these lines, the National Council of Teachers of Mathematics (NCTM) suggested that algebra is a strand that permeates all levels of schooling from prekindergarten through grade 12. For example, the concept of function can build on experiences with numerical patterns in primary school (National Council of Teachers of Mathematics (NCTM), 2000). Rich problem contexts play an indispensable role in eliciting functional reasoning (Carraher & Schliemann, 2007). In particular, contextual problems in which students have to deal with covarying quantities might induce reasoning about relations between quantities, which is an essential aspect of algebraic reasoning (Humberstone & Reeve, 2008). Similarly, Smith and Thompson (2008, p. 96–97) argue that “problem situations involving related quantities serve as the true source and ground for the development of algebraic methods.”

Therefore, in our study aimed at seeking ways to enrich the primary school arithmetic curriculum with algebra-related activities we focused on contextual problems with covarying quantities. These problems can be solved informally, i.e., without using symbolic algebra, but can also lay a foundation for developing formal algebraic skills (Kieran, 2004; Smith & Thompson, 2008). For the sake of conciseness we call these problems *early algebra problems*.

Another focus in our study is the use of Information and Communication Technology (ICT). There is evidence that ICT has an overall positive effect on mathematics achievement (Li & Ma, 2010; Slavin & Lake, 2008), and it is considered particularly significant in the teaching and learning of algebra (see, e.g., Kieran, 2007; Bokhove & Drijvers, 2012). Moreover, according to Kieran (2007, p. 727), “the results regarding the effects of technology use on the initial learning of algebra have been quite remarkable.”

Additionally, with respect to the use of ICT, recently, a significant body of research has focused on the effectiveness of computer games for educational purposes in general (see, e.g. Hays, 2005) and for the learning of mathematics in particular (Randel, Morris, Wetzel, & Whitehill, 1992). Moreover, some positive results of using games were also reported for the learning of algebra (Kebritchi, Hirumi, & Bai, 2010).

Based on the abovementioned findings, we started a research project investigating the use of an ICT-based game for developing primary school students’ ability to solve early algebra problems. Our main goal was to create opportunities for students to experience covariation of quantities by solving a sequence of early algebra problems with a particular online game designed for this project. In an earlier study in this project we found that students in grade 6 who were involved in the experimental group, which was offered the ICT environment with the online game, outperformed students in the control group on a paper-and-pencil test on early algebra problems (Kolovou, Van den Heuvel-Panhuizen, & Köller, 2013). In the present study we wanted to shed light on how students on different levels of development, i.e., students in grades 4, 5, and 6, worked with the game and on how their online working is related to their performance in the paper-and-pencil test.

2 Theoretical background and research questions

2.1 Algebra in primary school

According to Kieran (1996) school algebra involves three types of activities: generational (i.e., forming expressions and equations), transformational (i.e., factoring, expanding, substituting, solving equations and so on), and global meta-level activities (i.e., problem solving, modeling, noticing structure, studying change, generalizing, analyzing relations, justifying, and proving). These latter, global meta-level activities, which do not exclusively belong to algebra, are "ideal vehicles for conceptualizing a non-symbolic or pre-symbolic approach to algebraic thinking in the primary grades" (Kieran, 2004, p. 148). In other words, even when students are not using symbolic expressions they can develop algebraic thinking (see, e.g., Radford, 2010, 2012). In fact, algebra builds on students' proficiency in arithmetic and develops it further (Kilpatrick, Swafford, & Findell, 2001). Caspi and Sfard (2012, p. 46) suggest that "algebraic thinking occurs whenever one scrutinizes numerical relations and processes in search of generalization." Similarly, Carraher, Martinez and Schliemann (2008) focus on algebra as generalized arithmetic of numbers and quantities, thus moving from computations on particular numbers and measures toward thinking about relations among sets of numbers and measures, especially functional relations.

In line with Carraher et al. (2008) other researchers have also emphasized functional thinking as a core strand of (early) algebraic reasoning (Kaput, 2008; NCTM, 2000). According to Smith (2008, p. 143), "functional thinking is representational thinking that focuses on the relation between two (or more) varying quantities, specifically the kinds of thinking that lead from specific relations (individual incidences) to generalizations of that relation across instances." Function tables (Schliemann, Carraher, & Brizuela, 2001), function machines (Warren, Cooper, & Lamb, 2006), and patterning activities (Moss & Beatty, 2006) have been successfully implemented to support this reasoning in primary school students.

Another means to support functional thinking in primary school is to provide students with contextual problems with covarying quantities. Not having been taught algebra yet, primary school students can only solve these problems by applying context-connected, informal reasoning. This means that students have to understand how the quantities are related and that they should be able to use these relations to draw inferences and infer numerical values from those that are given. Smith and Thompson (2008) call these processes 'quantitative reasoning'. Although grounded in arithmetic, these solution methods can be the stepping stones to developing algebraic reasoning.

Several studies have indeed indicated that students can successfully engage, even from pre-K grade on, in tasks that require functional thinking before the study of formal algebra (Blanton & Kaput, 2004; Beatty, 2010; Warren et al., 2006). However, according to our experiences, primary school students with no previous instruction in early algebra, have difficulties when solving problems with interrelated quantities (Elia, Van den Heuvel-Panhuizen, & Kolovou, 2009); in particular, students' difficulties with such tasks lie in not being able to take into account more than one quantity simultaneously. Yet, the development of the ability of dealing with covarying quantities, which is crucial for functional thinking, entails a major conceptual shift from working with one varying quantity to taking into account two or more varying quantities simultaneously (Blanton & Kaput, 2004). Therefore, it is crucial for students to work in an environment in which they can experience the interdependency of quantities.

2.2 Problem variation

Recognizing covariation of quantities and discovering their general relations implies that students have to distinguish what remains invariant in the problem situation as the values of the quantities change. In order to achieve this, students should be offered various problem situations in which they encounter different instances of the same relation. Such an approach fits to the variation theory as it is developed by Marton and his colleagues (e.g., Marton & Booth, 1997). From the variation theory perspective a necessary condition for learning is the possibility to experience variation and distinguish critical aspects in the phenomenon under study and simultaneously become aware of the possible values that these aspects can take (Marton & Tsui, 2004).

Because noticing structure and identifying patterns is the essence of mathematics, it is no wonder that variation theory found its way in designing instructional material for mathematics education. For example, Sun (2011) argued that problem variation is an indigenous Chinese practice aimed at having students discern the invariant feature of relations among concepts and solutions that may lead to mathematical abstraction.

Although experiencing patterns of variation is significant for learning mathematics in general, it is especially relevant to the teaching and learning of algebra (Al-Murani, 2006), since the ability to generalize from particular instances implies that one can distinguish between what changes and what remains invariant. Watson and Mason (2006, p. 94) "[…] see *generalization* as sensing the possible variation in a relation, and *abstraction* as shifting from seeing relations as specific to the situation, to seeing them as potential properties of similar situations." Both processes are essential for algebraic thinking. Therefore, according to Li, Peng and Song (2011), teaching with problem variation can help students with the transition from arithmetic to algebra.

2.3 ICT and games in mathematics education

According to the National Council of Teachers of Mathematics (NCTM, 2000), technology influences not only how mathematics is taught and learned but also what is taught and when it is taught. Technological tools enable the quick and accurate execution of routine procedures, which allows more time for conceptualizing and modeling (NCTM, 2000). Digital tools provide access to external representations and feedback (Zbiek, Heid, Blume, & Dick, 2007) and support active engagement and interactive learning (Roschelle, Pea, Hoadley, Gordin, & Means, 2000). Li and Ma (2010) in their meta-analysis of the effects of computer technology on mathematics achievement ascertained an overall positive impact. Similarly, a meta-analysis by Cheung and Slavin (2011) revealed a positive but small effect of educational technology applications on mathematics achievement. The positive effects of technology on mathematics learning can be attributed to its unique characteristics, such as interactivity and immediate feedback (Roschelle et al., 2000). Especially feedback that prompts students to reflect on their own thinking is crucial for learning (Hewitt, 2012).

Besides research on the impact of technology on mathematics learning in general, several studies have focused on their role to support algebra learning in different grade levels (Hoyles & Sutherland, 1989; Lannin, 2005; Suh & Moyer, 2007) with significant positive results (Rakes, Valentine, McGatha, & Ronau, 2010). Nathan (1998) showed that a computer environment in which undergraduate students were able to construct and test models of relations between quantities in problem situations supported students in algebra problem solving. A study by Suh and Moyer (2007) also demonstrated that the unique characteristics of a virtual balance encouraged relational thinking and promoted algebraic reasoning in a group of third-grade students. Furthermore, with appropriately designed software young students can even overcome difficulties with formal algebraic notation (Hewitt, 2012).

A recent development in the application of technology in mathematics education is the use of computer games. In fact, mathematics education has already a long tradition of game playing activities to support the learning of mathematics (e.g., Oldfield, 1991). Currently, Holton, Ahmed, Williams and Hill (2010, p. 403) consider mathematical play as “that part of the process used to solve mathematical problems, which involves both experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion.” Moreover, Holton et al. (2010, p. 404) emphasize that “mathematical play provides a non-threatening environment where incorrect solutions are not read as mistakes and may lead to a better understanding of the problem and/or the confrontation of misconceptions.”

The computer has not only brought us new digital versions of the traditional games, but the rapid technological developments also generated new possibilities for interaction, feedback and dynamic representation of mathematical concepts. In an extensive literature review, Mitchell and Savill-Smith (2004) concluded first of all that computer games are engaging for students. In addition, they found that mathematical concepts are sometimes easier to explain with a computer game than with concrete materials. In a review on the effectiveness of games for educational purposes, Randel et al. (1992) pointed out that computer games are reported to be very effective in improving mathematics achievement scores in a range of topics and different grade levels in primary and middle school. For example, Sedighian and Sedighian (1996) found that computer-based mathematical game environments, such as the ‘Super Tangrams’, offered sixth graders successful and challenging experiences with respect to two-dimensional transformation geometry. Furthermore, Klawe (1998) concluded that computer games can be highly effective in enhancing the learning of mathematics for students in grades 4 to 8, especially in facilitating learning complex concepts and skills.

More recently, Kebritchi et al. (2010) showed that three-dimensional computer games used for practicing algebraic concepts had a positive impact on the mathematics achievement of students in grades 9 and 10. These positive results are also corroborated by our finding that the sixth graders who received an intervention including an online game outperformed those in the control group in solving early algebra problems (Kolovou et al., 2013).

The aforementioned initial research findings about using ICT-based games for teaching students early algebra indicate that it is a promising teaching approach. Yet, there are still many questions to be answered about this way of teaching algebra; in particular, whether games are helpful for particular domains of algebra and for which grade level. Moreover, new advancements in technology, which offer opportunities to monitor students while working online, enable researchers to gain more knowledge about the learning processes in early algebraic thinking.

2.4 Research questions

In this study we focused on how students in primary grades worked in an online environment including a dynamic game by means of which they could experience situations with covarying quantities. In particular, we sought to answer the following research questions:

1. How do students in the upper level of primary school (grades 4 to 6) work in an online environment including a dynamic game to solve a sequence of early algebra problems based on the idea of problem variation?
2. Does the online working have an effect on the students' performance in a paper-and-pencil test on early algebra problem solving?

3 Method

3.1 Overview of the study

To answer the research questions an experiment was set up with an intervention including an online computer activity with a dynamic game (see Section 3.5.1). The game playing did not take place at school but at home. Before and after the intervention a paper-and-pencil pretest and posttest (see Section 3.4) were administered. The experiment covered in total a time span of 6 weeks. The intervention lasted 3 weeks in which the students were asked to work online with three sets of problems (see Section 3.5.2). Monitoring software (see Section 3.5.3) was used to collect data on the students' online working.

3.2 Procedure

The intervention included three whole-class sessions, one per week, which were led by one of the authors. In the first session the students were given instruction on how to get access to the online environment. Each student received a unique account with which the students could log in in this environment at any computer at any time. They were told that they could stop and resume their online working without limitations. Furthermore, they got a short demonstration of the features of the game and the first set of problems.

These problems were presented on a worksheet on which the students could write their answers. The online activity was not part of the students' compulsory homework. At the end of the first and the second week in which they played the game, the students presented their answers in a whole class discussion of 15 min. Emphasis was placed on articulating relations between the quantities in the problem situations, which the students had discovered during playing the game. The discussion was led by one of the authors.

The classroom teachers had a minimum role in the intervention; they only asked their students to go online and play the game. Before starting the experiment both the students and their parents were informed that data were collected about the students' online working with the game.

3.3 Participants

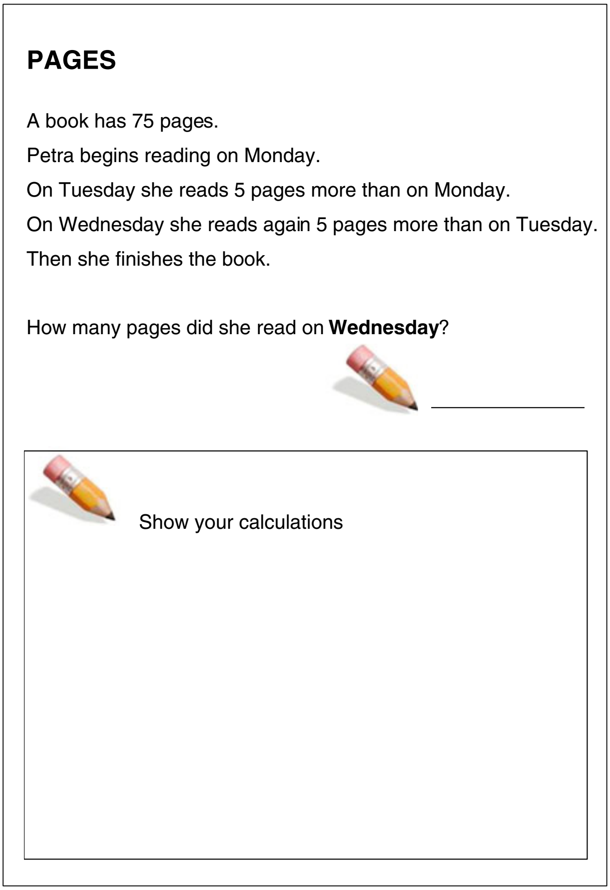
For reasons of convenience we contacted schools in the city of Utrecht. We did this until we had five schools that consented to participate in the study. Every school took part with one fourth-grade, one fifth-grade and one sixth-grade class. The schools are located in various city districts so that the sample included students with a diversity of socio-economic backgrounds.

The participating classes in the five schools contained 318 students (10–12 years old). Since playing the online game was not compulsory the students could decide whether they logged in or not and how often they worked in the online environment. In total 253 students (80 % of the total number of students) logged in at least once. The proportions of logged-in students in grades 4, 5, and 6 were 74 %, 88 %, and 78 %, respectively. We did not find significant differences in the pretest performance between the students who logged in and those who did not log in; $t(309) = -1.72$, $p > .05$, $d = .25$. Because the present study focused on the online working of students, our analysis included only the data of the students who logged in.

3.4 Test on early algebraic problem solving

To measure the students' performance on early algebra we used a paper-and-pencil test that included six contextual problems with covarying quantities (see Kolovou et al., 2013). For example, one of the items was about reading a book (see Fig. 1).

This *Pages* item, like the other items included in the test, can be solved by a formal algebraic approach; in this case setting up and solving a system of three linear equations with three unknowns (i.e., $a+b+c=75$, $a+5=b$, and $b+5=c$), but it can also be solved by reasoning informally about the relations between the quantities. The latter implies that students solve the *Pages* item, for example, by reasoning as follows: "Petra has read a particular number of pages on each of the 3 days; and on Tuesday 5 extra pages; and on



PAGES

A book has 75 pages.

Petra begins reading on Monday.

On Tuesday she reads 5 pages more than on Monday.

On Wednesday she reads again 5 pages more than on Tuesday.

Then she finishes the book.

How many pages did she read on **Wednesday**?

Show your calculations

Fig. 1 *Pages* item

Wednesday two times 5 extra pages. That gives a total of 15 extra pages. When I put aside the extra pages I have 60 pages left for the 3 days. This means for each day 20 pages. On Wednesday Petra reads these 20 pages and the 10 extra pages, which makes a total of 30 pages."

Since in the Netherlands formal algebra is not introduced until the first year of secondary school and the Dutch primary school mathematics curriculum hardly includes any early algebra tasks (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009), the students in our sample had neither formal algebraic methods at their disposal, nor experience in informal solution methods for this type of problems.

The test on early algebra was used as a pretest and a posttest. For each student the proportion of correct items was computed as an individual raw test score ranging from 0 to 1. To measure the reliability of the test, in terms of the internal consistency, that is, to what extent the six items assess a homogeneous competence, we calculated the Cronbach's alpha. Based on the students who did the pretest and the posttest (n=232) we found for the pretest a Cronbach's alpha of .82 for the total sample, which indicated sufficient reliability. For the grades 4, 5, and 6 the Cronbach's alpha's were .69, .75 and .81, respectively. For the posttest the values were slightly larger. Here the Cronbach's alpha for the total sample was .86 and for the different grades .74, .84, and .82 respectively.

3.5 The online environment

3.5.1 The game

Our goal was to provide students with an environment that allows them to discover relations between quantities. Therefore, we designed a dynamic java applet called *Hit the target*[1] (Fig. 2), which is an interactive simulation of an archery game. In this game students can manipulate quantities and discern the relations between them. The screen of the game displays five features: a target, a pile of arrows and a bow, a board that displays the number of hits and misses, a board that displays the game rule (indicating the points for a hit and the penalty points for a miss), and a scoreboard.

The game can be played in several ways. The students can shoot arrows one by one by dragging them to the bow or let the computer shoot them at once by entering a number of hits, misses and random arrows and clicking on the shoot button (that appears in the computer shooting mode). Furthermore, the students can set the game rule by filling in the points added or subtracted per hit or per miss or the computer sets the rule randomly. Because of practical reasons, i.e., the restricted space to display arrows on the target, the maximum number of arrows to be shot consecutively was restricted to 150. However, in problems where the total number of arrows exceed this number, this limitation might also be conducive to triggering students to look for a general solution and, in this way, can support relational thinking.

Because the features of the game are dynamically linked, the students can experience that quantities covary when playing the game. During the shooting the values on the scoreboard update rapidly to inform students about their score. The same happens when students after the shooting remove arrows from the target by dragging them back to the supply pile of arrows. A modification in the game rule also results in a different score. The game offers instant feedback by showing the consequences of the students' actions. In this way, students

[1] The game *Hit the target* was developed by the first author of this article and programmed by our colleague Huub Nilwik at the Freudenthal Institute of Utrecht University.

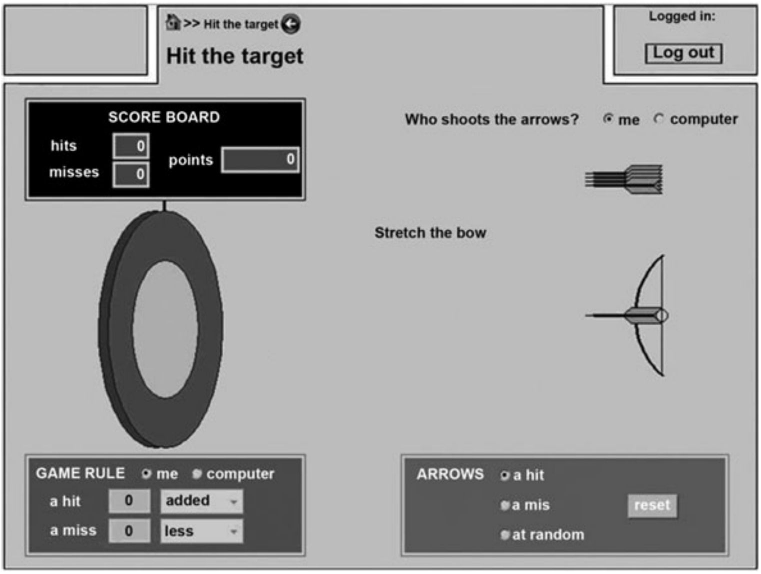


Fig. 2 Screen view of *Hit the target*

are prompted to realize that the number of hits and misses, the score and the game rule are related to each other.

3.5.2 The intervention problems

During the three periods of a week in which the students worked online with the game, they had to solve eight problems which were split into three sets (see Fig. 3). Some of the problems contained only one question, while others consisted of two questions (a and b), which made a total of 14 questions. The problems varied from finding a single solution to generating a general solution.

The problems are based on the principle of variation (see Section 2.2). In order to help students grasp the relations between the quantities (i.e., the number of arrows, the values of the game rule and the number of points in the score), they need to be able to recognize throughout the sequence of problems what changes in the values of the quantities and what remains invariant. This means, for example, that after it is asked in *Problem 3a* to find the game rule for 15 hits, 15 misses, and 15 points, the question in *Problem 3b* is whether there are other game rules that lead to the same result. Subsequently, in *Problem 4a* the students are given a variation of *Problem 3a/b*, i.e., they have to find the game rule for getting 16 hits, 16 misses, and 16 points. In this latter case, the game rule is the same as in 15 hits, 15 misses, and 15 points, although the values of the hits, misses and points are different. By working on such sequences of problems based on the idea of problem variation, students have opportunities to distinguish what remains invariant in the problem situation as the values of the quantities change, which might bring them to the general rule behind the covarying quantities.

The problems also varied with respect to the given and unknown quantities. For example, in *Problem 1* the students had to determine the number of hits and misses based on the score and the game rule, while in *Problem 3a/b* the score and the number of hits and misses were given and the students had to find the game rule. This variation in the given and unknown quantities might also support the development of relational thinking.

In week 1
Problem 1: For every hit you gain 3 points and for every miss 1 point is taken away from your score. How many hits and misses do you have to shoot to get 15 points in total?
Problem 2: For every hit you gain 3 points and for every miss you gain 1 point. How many hits and misses do you have to shoot to get 15 points in total?
Problem 3a: What is the game rule to get 15 points in total with 15 hits and 15 misses?
Problem 3b: Are there other game rules to get 15 hits, 15 misses, and 15 points?

In week 2
Problem 4a: What is the game rule to get 16 points in total with 16 hits and 16 misses?
Problem 4b: Are there other game rules to get 16 hits, 16 misses, and 16 points?
Problem 5a: What is the game rule to get 100 points in total with 100 hits and 100 misses?
Problem 5b: Are there other game rules to get 100 hits, 100 misses, and 100 points? Can you explain your answer?

In week 3
Problem 6a: What is the game rule to get 30 points in total with 15 hits and 15 misses?
Problem 6b: Are there other game rules to get 15 hits, 15 misses, and 30 points?
Problem 7a: For every hit you gain 2 points and for every miss 1 point is taken away from your score. How many hits and misses do you have to shoot to get 0 points in total?
Problem 7b: Are there any other solutions possible??
Problem 8a: For every hit you gain 2 points and for every miss 1 point is taken away from your score. You have 10 arrows in total. How many hits and misses do you have to shoot to get 5 points in total?
Problem 8b: Are there any other solutions possible?

Fig. 3 Problems given to the students during the intervention

Similarly to the items in the pre- and posttest, all the intervention problems refer to equations which can be solved by an algebraic procedure. However, in case students have not yet been taught such a procedure the problems can also be solved in an informal context-based way. In fact, the game was developed to offer students opportunities to apply informal solution strategies, try out solutions and receive instant feedback. The scoreboard of the game displays the result of shooting a particular number of arrows and choosing a particular game rule. This feedback does not explicitly provide information about the correctness of a solution (i.e., the game does not evaluate a solution as 'correct' or 'wrong'). Nor does it give hints about the actions to be taken to reach a correct answer. In this sense the software offers support that is "neutral, non-judgmental and non-explanatory" (Hewitt, 2012, p. 144). Earlier, Nathan (1998) called this type of feedback, which is generated from an unintelligent tutoring system, situation-based feedback (see also Kolovou & Van den Heuvel-Panhuizen, 2010). Yet, situation-based feedback enables students to ascertain the correctness of an answer and might be of help for correcting the answer in case it is wrong. By modifying the quantities to reach a correct answer students might discover relations between them, detect patterns and make generalizations by grasping the invariant structure of a problem. In other words, students might be able to see the general relation between the quantities, despite the fact that the particular values vary. Since the computer can perform the calculations very quickly–there is little time between one result and the next–students are able to experience the effects of varying the values.

The game and part of the intervention problems were piloted in a study with 24 fourth-grade students (Kolovou, Van den Heuvel-Panhuizen, Bakker, & Elia, 2008). The results of the pilot showed that the game indeed supported students in solving the problems. The students used various strategies ranging from trial-and-error to applying a general rule. In the present study we did not only extend the number of students to get more robust results, but we also increased the number of problems and the duration of the experiment to get a better understanding of how the students use the online environment to solve early algebra problems and what is the effect of the online working on their performance in a paper-and-pencil test on early algebra.

3.5.3 The monitoring software

The game was included in the so-called Digital Mathematics Environment (DME),[2] which is the software that was keeping track of the students' online working. The DME created for every student a log file including all the actions performed in the online environment by that student. Moreover, the DME structured these actions into sessions and events and generated for each student a list of events as shown in Fig. 4.

A *session* contains the shooting actions carried out each time a student logged in in the online environment. Also, information is provided on the date, time and duration of a session. Every shooting action is registered as an *event*, which shows the number of hits, the number of misses, the game rule, as well as the shooting mode and the game rule mode in which the student operated.

3.6 Data preparation and analyses

3.6.1 Preparation of the pretest and posttest data

First, of each answer given to the items in the pretest and posttest it was determined whether the answer was correct or not. Then, for each student the proportion of correctly answered items was computed as the individual raw test score.

3.6.2 Preparation of the log files data

Students' log files provided us with two types of data: the online problem solving strategies and the online involvement. In both cases, a single event was taken as the unit for coding the data.

Online problem solving strategies First we identified for each event whether it was a "focused event" or an "unfocused event". An event was characterized as focused in case the student in that event or in following connected events (i) came to a correct answer, (ii) gave a partially correct answer, or (iii) only tried to find an answer. We considered an event as unfocused when the students were just shooting arrows randomly without any relation to the problem to be solved.

For every focused event we determined which question (or problem) was answered. Then we established the problem solving strategy or strategies. For this we used a coding scheme (see Table 1) that was established after several rounds of examining the online actions of the students. Our goal was to find a list of strategies that covers all possible approaches the students applied.

In total we identified 12 different strategies which could be classified into 'answer-focused strategies' and 'relation-focused strategies'. Altering or ignoring information, using

[2] The DME is developed by our colleague Peter Boon at the Freudenthal Institute of Utrecht University.

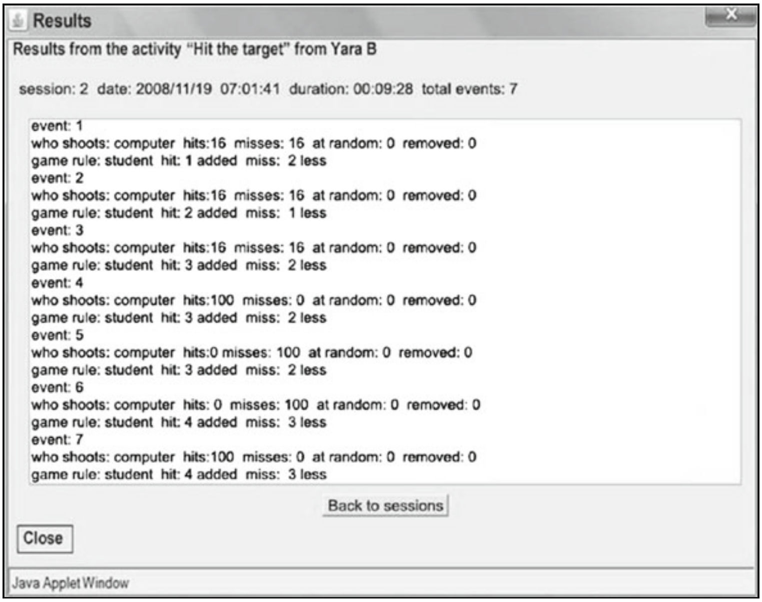


Fig. 4 Screen view of log data generated by the DME

extreme values, repeating an answer, and trial-and-error signify that the student was focused on providing a (correct) answer, while solving an analogous problem, cancelling out values, applying a general rule, applying an erroneous rule, reversing a solution, splitting the problem, systematic trialing, and transposing values indicate that the student explored relations between quantities.

Online involvement As a first indicator of a student's involvement in the online environment we determined the logged-in time (in minutes) and the number of focused events. Moreover, we also took into account the proportion of focused events of the total events (focused and unfocused) of a student. In addition, we determined the number of problems and questions the students worked on.

Although the abovementioned focused actions can be considered as most important for the learning process, the unfocused events can also give students experience in how the game works and how the quantities are related. In a way these activities contribute to "get a sense of what is going on" (Mason, 1980). Therefore, we took these events as an additional indication of the involvement in the online environment. Two examples of unfocused events can be seen in the log file displayed in Fig. 5. The student involved had to solve the problems presented in week 1 (see Fig. 3). The two events are characterized as unfocused, because the number of hits and the number of misses that the student shot did not fit to any of these problems. Moreover, the game rule was determined by the computer and not by the student. This means that the student was not trying specific cases to find a solution to one of these problems.

Levels of online working Having data about the applied strategies and the online involvement offered us the opportunity to combine these data and identify for every student qualitatively different ways or 'levels' of online working. For example, some students performed many focused events but applied mainly answer-focused strategies, while others carried out less focused events but all with relation-focused strategies.

Table 1 Scheme for coding the online problem solving strategies

Strategy type	Strategy	Description	Example (*Problem #*)
Answer-focused	Altering/Ignoring information (Alter)	The student arrived at the intended result, but only part of the information was used.	The student shot 15 hits and 0 misses and assigned 1 point to each hit, ignoring the given 15 misses (*Problem 3a*).
	Using extreme values (Extreme)	The student assigned the value 0 to one of the unknown quantities.	The student used the game rule +1 (1 point added for hit) −0 (0 points less for a miss) (*Problem 3a*).
	Repeating answer (Repeat)	The student repeated a (correct) answer to a problem to provide more answers to this problem, instead of finding a new solution.	The student repeated a correct answer to *Problem 3a* (e.g., +3 −2) in order to solve *Problem 3b*.
	Trial-and-error (TE)	After one or more trials the student came up with a (correct) answer.	The student applied several game rules until she came up with the intended result.
Relation-focused	Solving analogous problem (Analogous)	The student substituted the numbers of a problem with smaller numbers.	Instead of shooting 100 hits and 100 misses the student shot 10 hits and 10 misses (*Problem 5a*).
	Cancelling out values (Cancelout)	The partial (negative) score of the misses cancels out the partial (positive) score of the hits. The total score becomes 0.	The student shot 15 hits and 15 misses and applied the game rule +1 −1 with total score 0 instead of 15 (*Problem 3a*).
	Applying a general rule (General)	The student applied a general solution.	The student applied the general rule where the sum of points per hit and points per miss is 1, e.g.: +100 −99 (*Problem 3b*).
	Erroneously derived rule (Erroneous)	Based on a correct answer to a problem the student applied an erroneously derived rule to provide more answers to this problem.	Based on the correct game rule +2 −1 the student applied the game rule +4 −2 (i.e., ratio of *points per hit* to *points per miss* is 2:1) (*Problem 3b*).
	Reversing solution (Reverse)	The student reversed a correct answer to a problem to provide more answers to this problem.	The student reversed a correct answer to *Problem 3a* (e.g., +2 −1) in order to solve *Problem 3b*, i.e., used the game rule −1+2.
	Splitting the problem (Split)	The student answered a problem in two steps and added the partial scores to calculate (mentally) the total score.	The student shot first 100 hits and 0 misses, then 0 hits and 100 misses, and added the two partial results (*Problem 5a*).
	Systematic trialing (Sys)	The student adjusted systematically the values until a (correct) answer was found.	The student applied consecutively the game rules: +6 −3, +6 −4, +6 −5 (*Problem 3a*).

Table 1 (continued)

Strategy type	Strategy	Description	Example (*Problem* #)
	Transposing values (Transpose)	The student exchanged the values of the arrows and the points.	The student shot 2 hits and 1 miss and used the game rule +100 −100, instead of 100 hits and 100 misses (*Problem 5a*).

Based on the online involvement (in particular, the number of focused events and worked questions) and the type of strategies used (i.e., answer-focused or relation-focused strategies) we distinguished three levels of online working. Level 1, free playing, included students that put little or no effort in answering the intervention problems; in particular, the students who performed below the threshold of three focused events and three worked questions. Level 2, mainly looking for answers, included students who predominantly tried to answer the intervention problems. This means that they exhibited an activity equal or beyond the threshold of three focused events and three worked questions and that their main concern was to comply with our instructions and provide correct solutions. This type of activity is indicated by the use of answer-focused strategies. Level 3, exploring relations, includes students who also exceeded the minimum amount of activity (three focused events and three worked questions) and used relation-focused strategies.

3.6.3 Statistical procedures

Interrater reliability of the coding To test the reliability of the codes, which the second author assigned to the students' online activity per focused event, an additional rater coded a random subsection of the log files data (including 8 % of the total of 4013 events). The calculated Cohen's Kappa's indicated that the coding was quite reliable. For identifying which intervention problem a student worked on we found a Cohen's Kappa of .83 and an 85 % agreement. For coding the various strategies (for which multiple coding was possible) we found Cohen's Kappas that ranged from .61 to 1.00 with a mean of .84 and 90 % to 100 % agreement. This is a reasonable result given the fact that some strategies were only rarely employed.

Testing the effect of the online working In total we investigated three types of influences of the online working on the students' performance in the paper-and-pencil test on early

session: 1 date: 2008/11/08 07:21:56 duration: 00:03:54 total events: 1

event: 1
who shoots: student hits: 5 misses: 0 at-random: 0
game rule: computer hits: 2 added misses: 8 less

session: 2 date: 2008/11/10 08:14:53 duration: 00:26:06 total events: 10

event: 1
who shoots: computer hits: 150 misses: 0 at-random: 0
game rule: computer hits: 4 added misses: 3 less

Fig. 5 Log file of a student's unfocused events algebra. First, we tested whether there was a *general effect* of the online working. For this analysis we computed the gain score (posttest score minus pretest score) and conducted a one-sample *t*-test to assess whether there was a significant change in performance.

Next, we examined the effect of strategy use. For this we first determined for each student who applied at least one strategy ($n=166$) an individual strategy profile. This means that for every strategy used by a student we calculated the proportion of the number of events related to this strategy of the student's total number of focused events. The proportions for all the strategies that were used by a student formed together the student's individual strategy profile. Then we carried out a K-means cluster analysis to identify strategy profile clusters. Hereafter, we used a one-sample *t*-test to examine in the whole sample for each cluster separately whether the mean of the gain score differed significantly from zero.

The third type of influence that was tested was the *effect of the level of the online working*. For this we carried out for each grade an analysis of variance with the gain score as the dependent variable. Furthermore, we did a regression analysis in which we compared the effects of the different levels of online working on the gain scores. In this analysis we took Level 1 as the baseline level. To test if the effect of grade on gain score was mediated by the level of online working, we performed a mediation analysis (MacKinnon, Fairchild, & Fritz, 2007). The corresponding path analysis was conducted in the R package lavaan (Rosseel, 2012).

For all analyses we ignored the clustered sampling of students within schools. A multilevel analysis of the gain score as the dependent variable—which was the main dependent variable in our analyses—revealed that the intraclass correlation was .037. Due to this small intraclass correlation we did not expect substantially biased statistical inference as a result of ignoring the clustered data structure.

4 Results

4.1 Students' working in the online environment

To answer the first research question with respect to the ways students used the online environment to solve a sequence of early algebra problems, we first provide two examples of students' online working. Next, we discuss students' online working (logged-in time, number of events, number of focused events, percentage focused events, number of worked problems, and number of worked questions) and their strategy use.

4.1.1 Student's online working: two examples

To illustrate how the students used the online environment to solve the intervention problems we describe here the online working of two sixth-grade students.

Figure 6 shows the log file of student A involving three sessions of her online activity. Table 2 presents the same three sessions in a compact format including the total score per event. The log file reveals that student A first found a solution to *Problem 3* by trial-and-error. In *Problem 4* she arrived at the general rule for obtaining the result "16 hits, 16 misses, 16 points". The application of the game rule "+100 −99" suggests that the student understood that the size of the numbers does not matter; any game rule is correct, as long as the sum of points per hit and points per miss is 1. Later on student A applied this general rule in solving *Problem 5* by shooting 10 hits and 10 misses (i.e., using the strategy of solving an analogous problem). Her solution "+1000 −999" is again not dependent on the particular values of the points per hit and the points per miss, but on the general rule "sum is 1".

session: 1 date: 2008/11/17 04:13:58 duration: 00:05:59 total events: 5

Problem 3

event: 3
who shoots: computer hits: 15 misses: 15 at-random: 0
game rule: student hits: 4 added misses: 2 added

event: 4
who shoots: computer hits: 15 misses: 15 at-random: 0
game rule: student hits: 1 added misses: 1 added

session: 2 date: 2008/11/17 04:20:43 duration: 00:02:52 total events: 2

Problem 3

event: 2
who schoots: computer hits: 15 misses: 15 at-random: 0
game rule: student hits: 5 added misses: 4 less

session: 3 date: 2008/12/01 04:02:24 duration: 00:36:03 total events: 7

Problem 4

event: 1
who shoots: computer hits: 16 misses: 16 at-random: 0
game rule: student hits: 2 added misses: 1 less

event: 2
who shoots: computer hits: 16 misses: 16 at-random: 0
game rule: student hits: 3 added misses: 2 less

event: 3
who shoots: computer hits: 16 misses: 16 at-random: 0
game rule: student hits: 100 added misses: 99 less

Problem 5

event: 4
who shoots: computer hits: 10 misses: 10 at-random: 0
game rule: student hits: 2 added misses: 1 less

event: 5
who shoots: computer hits: 10 misses: 10 at-random: 0
game rule: student hits: 1000 added misses: 999 less

event: 7
who shoots: computer hits: 10 misses: 10 at-random: 0
game rule: student hits: 50 added misses: 49 less

Fig. 6 Log file of the focused events of student A

Table 2 Focused events of student A

Problem	Session	Event	Hits	Misses	Game rule		Score
					Points per hit	Points per miss	
	Week 1						
15 hits, 15 misses, 15 points	1	3	15	15	+4	+2	90
		4	15	15	+1	+1	30
	2	2	15	15	+5	−4	15
	Week 2						
16 hits, 16 misses,16 points	3	1	16	16	+2	−1	16
		2	16	16	+3	−2	16
		3	16	16	+100	−99	16
100 hits, 100 misses, 100 points		4	10	10	+2	−1	10
		5	10	10	+1000	−999	10
		7	10	10	+50	−49	10

The second example refers to two sessions of the online activity of student B. Table 3 presents these two sessions in a compact format including the total score per event. Student B also applied trial-and-error for solving *Problem 3*. From her log file we can derive that she used the results from previous attempts in subsequent trials. Based on her initial answer "+2 −1", in the following event she applied the erroneous rule "+4 −2", but subsequently she adjusted the values of the game rule, which resulted in a correct answer, namely "+4 −3". To solve *Problem 4* and *Problem 5* she applied the strategy of transposing values and she provided several pairs of values for the hits and misses that led to the intended score. It seems that the student was able to provide a series of correct solutions due to her discovery of the underlying relation between the quantities. It is also noteworthy that the student applied game rules with a similar underlying structure, i.e., "+1 −0", "+2 −1" and "+3 −2", to solve the three problems, *Problem 3*, *Problem 4*, and *Problem 5*, which share the common structure of an equal number of hits, misses and points.

It is the sequence of problems that allowed students to grasp that there may be different game rules that result in the same situation, e.g., the situation "15 hits, 15 misses, 15 points", but that they all satisfy a general relation (i.e., "the sum of 1"). Actually, the students realized that this general relation applies to different situations (not only to the situation of "15 hits, 15 misses, 15 points", but also to "16 hits, 16 misses, 16 points" and "100 hits, 100 misses, 100 points"), which are actually all instances of the same situation at a more general level (i.e., "x hits, x misses, x points").

4.1.2 Students' online working in general

Online involvement The average total time that the students were online was about half an hour (see Table 4). During this logged-in time they performed 16 events on average. About half of these events were focused meaning that they were aimed at solving one of the intervention problems. On average the students worked on three of the eight problems, covering four of the 14 questions.

Table 4 shows large standard deviations which means that there was considerable diversity in the way the students utilized the online environment. The positive large values for skewness indicate that the distributions for the students' online involvement are strongly asymmetric (e.g., many students performed only a few events and a few students performed

Table 3 Focused events of student B

Problem	Session	Event	Hits	Misses	Game rule		Score
					Points per hit	Points per miss	
	Week 1						
15 hits, 15 misses, 15 points	1	5	15	15	+2	−1	15
		6	15	15	+4	−2	30
		7	15	15	+4	−3	15
	Week 2						
16 hits, 16 misses, 16 points	2	1	1	0	+16	−16	16
		2	4	3	+16	−16	16
		3	2	1	+16	−16	16
100 hits, 100 misses, 100 points		4	1	0	+100	−100	100
		5	4	3	+100	−100	100

Table 4 Students' online involvement

	Grade 4 (*n*=79)		Grade 5 (*n*=78)		Grade 6 (*n*=96)		Total (*n*=253)		
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*Skewness*
Logged-in time (in minutes)	35.70	41.63	25.75	18.69	32.22	54.46	31.31	42.15	4.76
Number of events	17.04	14.77	17.81	13.12	13.34	9.90	15.87	12.61	2.12
Number of focused events	5.68	7.61	9.60	8.15	7.48	8.07	7.57	8.03	1.23
% focused events	26.7	26.9	50.6	31.1	46.2	33.7	41.4	32.5	9.60
Number of worked problems (max 8)	2.11	2.32	3.42	2.36	2.94	2.41	2.83	2.41	.53
Number of worked questions (max 14)	3.10	3.76	4.92	3.70	4.46	3.91	4.18	3.86	.71

many events). The students of grade 5 exhibited the highest number of events, focused events, worked problems, worked questions, and the highest percentage of focused events, despite the fact they had the lowest logged-in time.

Online problem solving strategies In order to solve the problems, the students applied spontaneously various strategies. Figure 7 shows for each grade the percentage of students who used a particular strategy.

The mostly applied strategy was trial-and-error (49 % to 64 % of the students), while the strategy of systematic trialing was performed only by a few students, with the highest frequency (8 %) in grade 6. A high percentage of students (43 % to 56 %) used an extreme value to find a solution. Among the less frequently applied strategies were solving an analogous problem, splitting a problem into two sub-problems, and using the strategy of

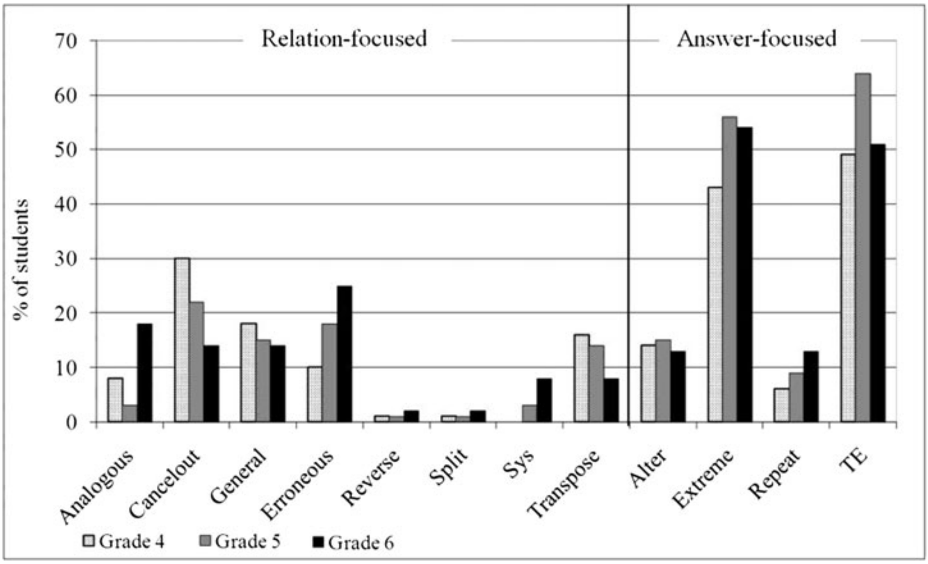


Fig. 7 Strategy use in grades 4, 5, and 6 (*n*=253)

transposing values. The reason for the low percentages of these strategies is that they were only evoked when students tried to solve *Problem 5.* Since the maximum number of arrows to be shot at once in the game is restricted to 150, the students cannot shoot 100 hits and 100 misses at once; which means that they should come up with another way to solve this problem.

Besides differences in the frequency of the strategies we also found that the online environment triggered the application of qualitatively different strategies. In addition to *answer-focused strategies* including altering or ignoring information, using extreme values, repeating an answer and trial-and-error, we also detected *relation-focused strategies*, including solving an analogous problem, cancelling out values, applying a general rule, applying an erroneously derived rule, reversing a solution, splitting the problem, systematic trialing, and transposing values.

Table 5 displays how often a relation-focused strategy was used per grade. The frequencies in the three grades were quite similar. The high positive value of skewness indicates that there were a few students in the sample who frequently used relation-focused strategies. Also the large standard deviation shows a substantial variability in the application of these strategies.

4.2 Effect of students' online working on early algebra performance

To answer the second research question with respect to the influence of the online working on students' performance in the paper-and-pencil test on early algebra, we first focus on their pre- and posttest differences. Next, we examine the effect of strategy use and of the levels of online working on students' gain in performance.

General effect of online working To examine the general effect of the online working we compared the mean scores in the six early algebra items in the pretest measurement and posttest measurement. These two mean scores were based on the proportions of correct items of each student who did both tests (in total 232 students). In the whole sample we found a high correlation between the pretest and posttest scores (r=.80, p<.01), which means that the order of the students' performances appeared to be similar in the pretest and the posttest. By applying a correction for attenuation which takes into account the unreliability of pretest and posttest scores, the correlation became nearly perfect (r=.95).

However, the average gain scores were not high (M=.09 in the total sample) and the items were quite difficult for the students, especially for the fourth graders. Nevertheless, as it is also shown in Table 6, we found for all grades a significant difference in performance.[3] When calculating the d-values, for which we used in each grade the standard deviation of the pretest score, we found moderate effect sizes with the largest gain in grade 6 (d=.37). Furthermore, the gain differed significantly between grades, $F(2,229) = 3.299$, $p<.05$, $\eta^2=.028$.

Identified strategy profile clusters When performing a K-means cluster analysis, we found that a three-cluster solution gave the best interpretable mean profiles of the students' strategy use (see Fig. 8).

One reason for considering this as a good solution was that it resulted in a clear peak for a dominant strategy in each cluster. A second reason was that having more than three clusters

[3] In this study, which focused on the effect of the online working we did not include data from a control group who did the test twice without working online. However, from another analysis (see Kolovou et al., 2013) based on data from sixth-grade students, including an experimental and a control group, we found that the gain score for the control group was nearly zero.

Table 5 Frequency of relation-focused strategies per grade

	Frequency of relation-focused strategies					
	n	*M*	*SD*	*Min.*	*Max.*	*Skewness*
Grade 4	79	1.84	3.08	0	16	2.49
Grade 5	78	1.82	2.88	0	14	2.23
Grade 6	96	1.89	3.02	0	18	2.76

resulted in too few students per cluster. When we assessed the stability of the cluster solution by a random split of the sample we found that in the two subsamples approximately the same clusters appeared with the same dominant strategy. This was confirmed by calculating the Pearson correlation of the mean profiles in both samples. For Cluster A we found $r=.72$, for Cluster B it is $r=.99$ and for Cluster C it is $r=.99$. For the Spearman rank correlation (r_S) the values were respectively .82, .78, and .76, which also supports our cluster solution.

Within the three-cluster solution, in Cluster A the most dominant strategy was cancelling out values followed by solving an analogous problem and applying a general rule. Cluster B was the cluster to which most of the students belonged. This cluster is characterized by the use of the trial-and-error strategy, while Cluster C is dominated by the strategy of using extreme values. Because the cluster analysis based on all grades already resulted in small sample sizes for the Clusters A and C, we did not repeat the cluster analysis per grade.

Effect of clusters of strategy profiles When we examined for each cluster whether the mean of the gain score differed significantly from zero we found a significant difference for Cluster C and Cluster B as well as for the students who did not apply a strategy at all, but we did not find a significant result for Cluster A (see Table 7).

However, for the students who did not apply a strategy the effect size of the change in performance (No strategy; $d=.16$), was similar to that of the students who belonged to the strategy profile cluster in which cancelling out values was the most dominant strategy (Cluster A; $d=.19$). The strategy profile cluster dominated by using extreme values (Cluster C; $d=.52$) and the cluster characterized by the trial-and-error strategy (Cluster B; $d=.32$) were most influential on the gain in early algebra performance. Especially the effect size of the gain for students in Cluster C ($d=.52$) was about twice as large as the gain in the total sample ($d=.29$).

Effect of levels of online working Determining for each student the level of online working revealed (see Table 8) that the percentage of students who performed free playing (Level 1) was the highest in grade 4, while the majority of students of grade 5 were mainly looking for

Table 6 Descriptives of pretest scores, posttest scores, and gain scores

	n	Pretest		Posttest		Gain score				
		M	SD	*M*	SD	*M*	SD	*t*	*p*	*d*
Grade 4	75	.07	.21	.12	.21	.05	.18	2.49	.015	.24
Grade 5	73	.26	.29	.33	.35	.07	.22	2.61	.011	.24
Grade 6	84	.39	.35	.52	.36	.13	.24	5.03	.000	.37
Total	232	.25	.31	.33	.36	.09	.22	6.00	.000	.29

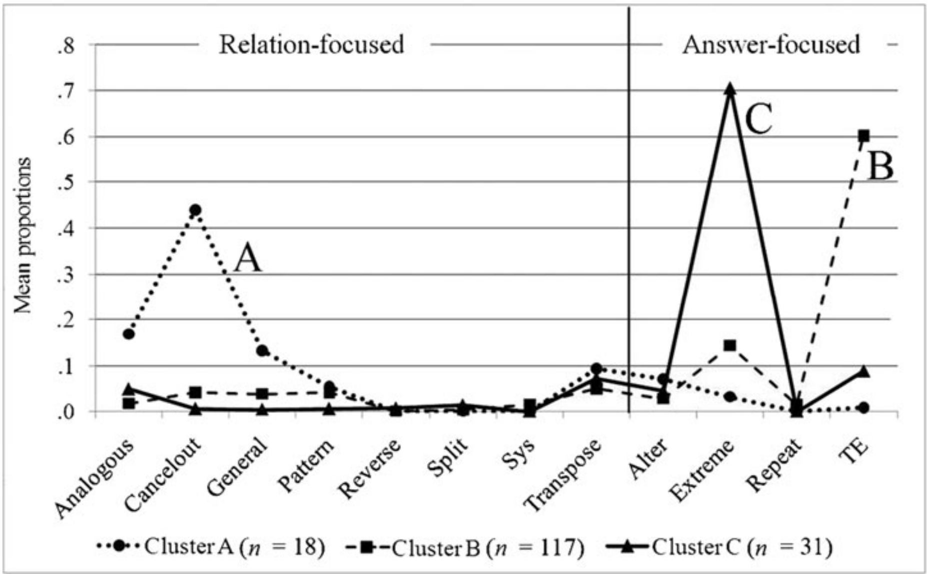


Fig. 8 Strategy profile clusters

answers (Level 2). The percentage of students who explored relations (Level 3) was slightly higher in grades 5 and 6 than in grade 4. A chi square test of independence of level and grade confirmed a significant association, $\chi^2(4, n=232) = 18.627$, $p<.01$, $Phi=.283$.

The variance analysis of the effect of the levels of online working on the gain scores revealed that there were no significant effects within each grade, but we found a marginally significant effect in the total sample (see Table 9). The η^2 effect sizes show that the level of online working was more influential in grades 4 and 6 than in grade 5.

The regression analysis in which we compared the effects of the different levels of online working on the gain scores revealed (see Table 10) that students at Level 3 had significantly higher gain scores than at Level 1 ($p<.05$) whereas students at Level 2 had marginally significant higher gains than the students at Level 1 ($p=.050$).

Finally, in a mediation analysis (MacKinnon et al., 2007) we investigated whether the influence of grade on gain score is mediated by the level of online working. This means that we tried to disentangle the effect of grade on gain score and the effect of the level of online working on gain score. In the mediation analysis we only considered students of grades 4 and 6 ($n=159$) and coded all students within grade 6 as 1 and students within grade 4 as 0. With respect to the level of online working, we took together the Level 2 and the Level 3 and coded these as 1 whereas Level 1 was coded as 0.

Table 7 Influence of profiles of strategy use on gain score

Profile of strategy use	Gain score					
	n	*M*	SD	*t*	*p*	*d*
No strategy	66	.05	.14	2.41	.018	.16
Cluster A	18	.06	.21	1.14	.269	.19
Cluster B	117	.10	.24	4.34	.000	.32
Cluster C	31	.16	.25	3.41	.002	.52

Table 8 Absolute frequencies (and percentages) of students who applied a particular level of online working per grade

	Grade 4	Grade 5	Grade 6	Total
Free playing (level 1)	44 (58.7)	21 (28.8)	34 (40.5)	99 (42.7)
Mainly looking for answers (level 2)	10 (13.3)	30 (41.1)	23 (27.4)	63 (27.2)
Exploring relations (level 3)	21 (28.0)	22 (30.1)	27 (32.1)	70 (30.2)
Total	75 (100)	73 (100)	84 (100)	232 (100)

The mediation analysis revealed (see Fig. 9) that the direct effect of grade on the gain score was significant (B=.068, p<.05), which means that sixth graders had larger gains than fourth graders. However, the indirect effect–which assesses the influence of grade on gain score mediated by changes in the level of online working–was not significant (B=.014, p=.105). Furthermore, this analysis showed that the total effects of grade on gain score amounts to .082 (= .068+.014) and that $17.1\%(= 100 \times (.082 - .068)/.082)$ of the total effect of grade on gain score was mediated by the level of online working.

5 Conclusions and discussion

This study investigated in what way an online dynamic game and a sequence of problems to be solved with this game at home, followed by a short discussion in class, can contribute to the early algebra performance of students in grades 4, 5, and 6. In an earlier study (Kolovou et al., 2013), involving students in grade 6 we found that the students who were in the experimental group and got an intervention with this game outperformed the students in a control group who did not get this intervention. In the present study, our focus was on the online working and on how the ways of online working influenced the students' early algebra performance.

5.1 Students' online working

The average time that the students were online was about half an hour, but there was considerable diversity in the students' online working. The learning environment with the dynamic game brought about the application of various types of strategies. Of the answer-focused strategies, trial-and-error and using extreme values were the most dominant. They were used by 43 % to 64 % of the students over the grades. The relation-focused strategies cancelling out, using an erroneously derived rule, and using a general rule were applied by 10 % to 30 % of the students.

Table 9 Analysis of variance of effect of levels of online working on gain scores per grade and across grades

	n	F	df_1	df_2	p	η^2
Grade 4	75	1.338	2	72	.269	.036
Grade 5	73	.504	2	70	.607	.014
Grade 6	84	1.875	2	81	.160	.044
Total	232	2.990	2	229	.052	.025

Table 10 Regression analysis of effect of levels of online working on gain score in the total sample with Level 1 as baseline level

	B	*SE*	*t*	*p*
Level 1	.05	.02	2.10	.037
Level 2 vs. level 1	.07	.03	1.97	.050
Level 3 vs. level 1	.07	.03	2.12	.035

5.2 Effect of students' online working

Comparing the pretest and the posttest scores of the students showed a significant gain in performance across all grades. The strategy profile cluster that was dominated by the answer-focused strategy of using extreme values and the cluster that was dominated by the answer-focused strategy of trial-and error had the strongest effect on the gain in performance. Using extreme values is interesting because it might not be a very common strategy in education and–although answer-focused–it can put students on the track of discovering the background structure of the quantities involved in the problem situation. By assigning the value zero to one of the input values this structure can become more overt.

The trial-and error strategy is interesting as well, because it was the most often applied strategy by the students and at the same time influential with respect to the gain in performance. This major role of trial-and-error is rather intriguing. One the one hand, it is often considered a poor guess-like method, related to finding a local solution to a problem. Moreover, as Lannin (2005) suggested, it might be the case that students do not reflect on the process when applying this strategy and therefore do not understand why a particular generalization is valid. In fact, the game might have allowed some students to find an answer without reflection. Yet, on the other hand, trial-and-error can be also regarded as a very powerful strategy. Through performing a sequence of purposeful trials students might grasp the relation between the input and output values so that they use this knowledge to determine solutions to algebra problems (Levin, 2008).

Another finding was related to the level of online working which combined the online involvement and the used strategies. Although we found only a marginally significant effect of the level of online working on the gain score and no significant effects per grades, a comparison of the different levels of online working per grade revealed that exploring relations or mainly looking for answers contributed more to a higher gain score than free playing. This is in line with research of, for example, De Jong (2005) who has shown that a simulation by itself is not sufficient to facilitate learning; rather, students should be steered by appropriate assignments that stimulate the generation and testing of hypotheses. In our study it means that in order to bring about learning effects, game playing should be accompanied by working on problems that can elicit the discovery of the relations between

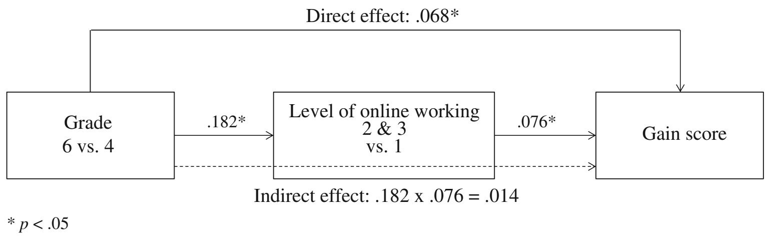


Fig. 9 Results from the mediation analysis

the quantities in the game. In addition to working on the problems, the sequence of the problems turned out to be important as well. The log files suggest that the variation in the problems was a crucial factor for discovering these relations. By working on the sequence of problems students could experience how the values covariate, which prompted the discovery of the general relation between these values.

5.3 Limitations and suggestions for further research

However, no matter how revealing our study was about how dynamic computer games can contribute to students' ability to solve early algebra problems, we have to be cautious with drawing too firm conclusions. Basically, our study focused only on one type of early algebra task and one type of online game. Further investigations are needed to develop a more general understanding of the use of computer games in teaching early algebra.

Other points that have to be taken into account concern the design of the study. We did not perform a random sampling procedure for selecting the schools. For that reason, our results might have restrictions with respect to generalizability. A further design issue is the size of our sample which may have led to the lack of significant effects of the levels of online working on the gain scores per grade. A larger sample size would have increased the statistical power of our study.

Another point that needs additional consideration has to do with the nature of the online environment. Some researchers doubt whether games and other student-driven animations can really contribute to the development of particular understanding. Because of their association with entertainment, dynamic visualizations might create an illusion of understanding (Bétrancourt, 2005) or result in a superficial engagement with the content (Lowe, 2004). Since students can check the correctness of an answer by shooting arrows, they might not need to engage in the more tedious work of exploring the relations between the quantities.

A point for further thought is the access to the students' thinking. Our conclusions are based on the log files of the students' online activity, which might not entirely capture the students' cognitive processes, such as the mental calculations that they performed. Nevertheless, the monitoring software allowed us to collect data which provided a step-by-step account of students' interaction with the computer environment. Although we are not sure whether the students worked online on their own, we assume that a voluntary activity that is not related to formal assessment is not likely to induce the need for assistance. In addition, if parents had been involved, it would have been more likely that the students would have worked on all the presented problems.

In sum, our results suggest that playing a dynamic computer game which was followed by short class discussions about the game playing can stimulate algebraic reasoning in the primary grades. Furthermore, home computing may create an effective learning environment supporting and extending school learning. Yet, it is also clear that further investigations are needed to get a more complete picture of and more robust findings about the use of online games for students' early algebra learning.

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