The Motivational Power of Mini-games for the Learning of Mathematics

Vincent Jonker, Monica Wijers, Frans van Galen Freudenthal Institute, Utrecht University, Utrecht, The Netherlands <u>v.jonker@fi.uu.nl</u> <u>m.wijers@fi.uu.nl</u> <u>f.vangalen@fi.uu.nl</u>

Abstract

Computer games are part of many children's daily life. Playing computer games is done in leisure time. Nevertheless, there is also a growing interest in serious games, games that are made to learn or to train a skill that is useful outside the game.

In this paper we describe the results from a six-week pilot-study on the use of the mini-game 'Crack the Number Safe' that is designed for exploring divisibility. We address the following questions:

- 1. What are characteristics of mini-games like 'Crack the Number Safe' that motivate students to play? How gamelike are these games?
- 2. What do individual students do and learn when they play 'Crack the Number Safe'?

Outcomes show that children experience 'Crack the Number Safe' as a game. Most children like to play this game more than once. They play the game in leisure time. From the analysis of the 'think aloud' protocols recorded during game-play it is clear that children are able to use their prior knowledge about division and that in some cases they discover new division rules.

Keywords: game-based learning, mini-games, mathematics education, division

1. Introduction

Numerous studies show that games can engage students in learning activities. These studies mostly focus on 'big' games, like MMOG (massively multiplayer online games, e.g. see Copier 2007) and less on the use of online casual games or mini-games (Juul 2007). Online computer games have gained in popularity along with the increase in public access to broadband networks. It is known that especially younger students (aged 8-12) often play online casual games on game portals like addictinggames.com and funnygames.com (ISFE 2008; Rohrl 2009; McFarlane 2002). For all types of educational games 'big' or 'mini' we can state that for these to be successful in realizing learning effects it is important that the player is and stays motivated to play. The motivation is influenced by what is called the 'flow' of the game (Csikszentmihalyi 1990). This is the balance between skills and challenge. If this balance exists the player will get better and better and will be able to meet bigger challenges and in this way get more and more involved in the game and reach a state of 'flow'.

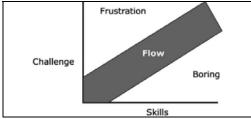


Figure 1: The Flow according to Csikszentmihalyi (1990)

From research (Zagal and Bruckman 2008; Salen and Zimmerman 2004) we know that characteristics like goals, rules, narrative, game level(s) and the possibility of user-generated content have a strong influence on the gameplay and the motivation of players. These characteristics will therefore determine whether or not players keep playing and – for educational games – whether or not players will learn something.

We discuss a small scale qualitative study on motivation and learning effects of playing an educational mini-game 'Crack the Number Safe' that is designed for exploring division.

In the game research we conduct we address the following three questions:

- 1. What are characteristics of mini-games like 'Crack the Number Safe' that motivate students to play? How gamelike are these games?
- 2. What do individual students do and learn when they play 'Crack the Number Safe'?

3. How can a teacher profit from the use of mini-games and integrate these in their regular math lessons?

In this paper we focus on the first and second research question.

2. Mini-games

Large games like MMORPG (role playing), games like Simcity or the Tycoon series and games like Civilization provide a rich environment in which players learn and use skills to solve complex tasks. Players analyse, plan, collaborate and emerge themselves in the rules and community of the virtual worlds of the game. Numerous research shows that this contributes to learning in general (Squire 2002; Copier 2007; Kafai 1996).

Some authors like Prensky (Prensky 2000; Prensky 2005) have argued that only this type of large games is valuable for learning. We don't agree with this point of view. Mini-games can contribute to learning as well, but – and in this respect we do agree with Prensky – they contribute to other types of learning, they address other skills. Some studies for example show opportunities for learning with mini-games in the area of mathematics education (Alexopoulou 2006; Barmby 2009)

With respect to their use in education mini-games have the advantage that they are 'everywhere', they are online and often for free, their use only requires a computer or any handheld device with an internet connection. Furthermore there is an advantage in that they are 'mini'. Mini' in this context means three things:

- Dimensions. The mini-games fit within the window of a browser like Internet Explorer, Firefox, etc. The dimensions are less than 1500 pixels wide and less than 1000 pixels high. Sometimes the whole screen is used for the mini-game. It is questionable if a fullscreen game must still be called a mini-game;
- One episode. The mini-game is restricted to one episode, scene or image, with a restricted 'rule-set'. This does not mean that there is no animation or that the game does not contain a certain amount of sub scenes or even levels. Having one episode within the game means that the user easily grasps how to play the game. There is no steep learning curve, neither does learning to play require an investment of many hours. This is an important motivation for playing mini-games and it makes them easier to use in education;
- Short period of playing. The playing time is restricted. Often a mini-game can be played within a maximum of about 10 to 15 minutes. This means that the player can replay the same game often (or play a number of levels). It is also possible and easy to switch from mini-game to mini-game within one session. In educational settings the restricted playing time is an advantage.

3. Mathematics education

In Realistic Mathematics Education (RME) the focus is on learning with understanding, rather than on learning to follow procedures (Gravemeijer 2004; Drijvers 2004). In a process of 'guided reinvention' (Freudenthal 1983) students construct a meaningful and coherent network of mathematical concepts, procedures strategies and their relations. Construction of knowledge is not merely an individual process, it is part of what happens in the classroom as a community of practice (Wenger 1998). Interaction is therefore an important aspect of RME. Besides social interaction, interaction with tools – models, representations, simulations etc. – is crucial for the learning of mathematics (Van den Heuvel Panhuizen 2003; Treffers 1987; Gravemeijer 1994). Tools that allow students to generate their own productions and constructions contribute to students deeper understanding (Gravemeijer and Drijvers 2004). This type of tools can often be designed as games. A large collection of mini-games dedicated to the learning of mathematics and fitting the RME principles has been designed over the past 10 years (Jonker and Wijers 2008). These mini-games (also called Th!nklets) are presented online on a dedicated free game-portal (http:// www.thinklets.nl). Web statistics show that Th!nklets are played outside of classroom settings as well as during school hours. We see the most frequent use during after school hours between 4 and 8 pm (see Figure 2).

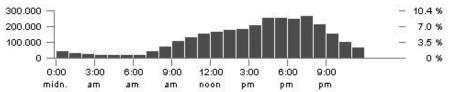


Figure 2: Average distribution of visitors of the dutch Th!nklets.nl over the day, recorded from 3 July 2008 to 7 May 2009.

For quite a large number of Th!nklets we also have data from online surveys (Jonker and Van Galen 2004). These data give insight in the facts and figures of the players: their average age is 11 years; most players play the mini-games on their own; an online session has a mean duration of about 10 minutes. These data combined lead us to believe that Th!nklets are seen as enough fun by students to play after school and in this respect can be compared to casual mini-games on other web-portals.

3.1 Division

In mathematics education in primary school multiplication and division are important topics. Multiplication and division are part of the mathematics curricula all over the world. The way these topics are addressed as well as the subtopics that are included will differ. In the Netherlands multiplication and division tables as well as written calculation procedures for multiplication and long division are being taught. Factorisation and rules for divisibility are not part of the regular core curriculum for primary schools.

It is evident that automated and flexible use of knowledge of basic facts and skills in the domain of multiplication and division contribute to a better understanding of other, related, mathematical topics. Flexible knowledge can only be formed when the learning process focuses on understanding instead of on memorizing facts and procedures.

We are interested in finding out whether students are able to use their knowledge of multiplication and division in a flexible way in order to factorize numbers and whether they are able to discover or become aware of certain divisibility rules while playing a mini-game specifically designed to address divisibility.

4. The mini-game 'Crack the Number Safe'

To address division a set of mini-games was created, as part of the bigger collection of Th!nklets. The mini-game 'Crack the Number Safe' is part of this collection. We chose 'Crack the Number Safe' to be studied more thoroughly for two main reason. The first reason is that this mini-game is an example of a typical division task that is accessible for students in a range of grade levels (see description in Figure 3).

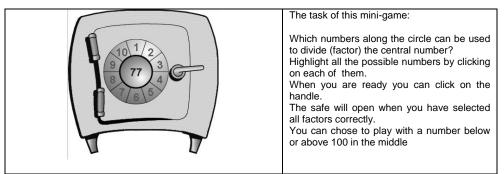


Figure 3: Th!nklet 'Crack the Number Safe

A second reason for choosing this specific mini-game is its popularity as a game. Web statistics show that it ranks second in the Top 5 of the most popular Th!nklets that deal with divisibility.

5. The mathematics in 'Crack the Number Safe'

As described in Figure 3 the task in the mini-game is identifying all factors between 1 and 10 of a given number. The given number (The Safe Number) is either between 1 and 100 or between 101 and 999. This can be selected at any moment in the game. Numbers are chosen at random from the selected range. Students identify the factors by clicking on them so they get highlighted. This action can always be undone.

Two 'support' mechanisms are provided in the game: students can click the handle of the safe to get feedback on the numbers they have selected so far. If all factors are correctly identified the safe opens. If not, the number of factors that is correctly identified is given. Students have to interpret this feedback. For example "7 out of 10 factors correct' means that 3 mistakes have been made: either a number is incorrectly identified as factor or a proper factor has been overlooked. A calculator is

available at all times. It is a special calculator that does not have a button for division. It can be used to multiply or to subtract.

Crack the safe is a suitable task for division since the following strategies may be used by the students to identify the factors of a given number are:

- using theirknowledge of the Multiplication Tables between 1 x 1 and 10 x 10;
- using divisibility rules;
- using other calculation strategies.

We expect students to use their knowledge of multiplications between 1×1 and 10×10 to identify pairs of factors of numbers below 100. . If the number is the result of 6×8 then 6 and 8 are both factors.

We expect students to know and use divisibility rules for 5 and 10 (the products will end in a 5 or a zero) and maybe the rule for 2 (products ending in 0, 2, 4, 6, 8) including the names 'odd' and 'even'. Since divisibility rules are not part of the curriculum we do not expect students to know rules for the factors 3, 4 and 9. Of course 1 as a factor is a special case.

Furthermore we expect them to use other calculation strategies especially for the number > 100. They may use trial and error or a more systematic way of 'adding on' from a known produc. Splitting the number is also an effective strategy if it is done correctly (see examples in Table 1). We expect that not all students will know how to do this.

Table 1: Three examples of Safe Numbers with their possible solution: 42 and 81

42	We expect students to easily find 6 and 7 as a pair. They may know that 2 is a factor since 42 is even because it ends in a 2; they may not easily find 3 because 42 is 3 times 14 and 14 is bigger than 10. Adding on from 10 x $3 = 30$ using 33, 36, 39, or splitting 42 in $30 + 12$ and concluding that both 30 and 12 can divided by 3 are other foreseeable strategies.
81	 Students may reason like this: 20 x 3 = 60, the remainder is 21, is this divisible by 3? Yes 20 x 4 = 80, the remainder is 1. So 81 is not divisible by 4. 10 x 7 = 70, the remainder is 11. So 81 is not divisible by 7.

We expect some students will discover and learn some new rules while playing the game. This could be rules like:

- If 9 is a factor of a number, then also is 3.
- If 8 is a factor of a number, then also are 2 and 4
- If 6 is a factor of a number, then also are 2 and 3 and the other way around.
- Even factors 2, 4, 6, 8 have only even numbers as multiples.

The observations will reveal what students actually do, which of these strategies they use, whether they use other strategies as well and how successful they are.

6. Research Method

The research is a small-scale qualitative study. During a six-week period 18 students of grades 4 and 6 of a primary school in Utrecht (the Netherlands) were observed while playing 'Crack the Number Safe'. These students worked individually or in pairs outside their classroom with one researcher/observer present. The screen capture software 'iShowU' was used to tape all sessions. All mouse movements and clicks as well as all keyboard actions were captured, the webcam above the screen recorded audio and video of the player. All recordings combined can be reviewed in the same screen (see Figure 4).



Figure 4: Review screen of the tool iShowU (screen capture software)

The observer/researcher prompted the students to 'think aloud' and guided the process only if necessary Sessions had an average duration of about 15 minutes. During one session 2 to 6 different Safe Numbers were played. Background data on the players age, gender, grade level, average math grade, was collected as well.

6.1 Method of analysis

We follow the methodological ideas of developing a 'humble theory' (diSessa and Cobb 2004; Parnafes 2008), conducting a moment-by-moment analysis, zooming in on the fine details of the studied processes, rather than proving or applying a theory, the objective is to make theoretical innovations (see also Glaser and Strauss 1956).

Data were organised and analysed in two ways:

- 1. By student. In the analysis per student we focused on what students do and if they are motivated to play and on the math students used and learned (strategy use). The 'what students do' is related to the interaction with the game; The focus on the math shows the use and development of strategies and of understanding division. In this analysis we take into account their average math grade and also compare the data of the students from grade 4 with students from grade 6.
- 2. By Safe Number. Three groups can be distinguished:
 - Numbers below 100 with only factors below 10
 - Numbers below 100 with also factors above 10 (for example 84, 99 but also 72)
 - Numbers between 100 and 1000

In the analysis we focused on which characteristics of each number students used, the results for that number (which factors were found and in what order, solution correct/incorrect) etc.

7. Results

We analysed the data of 18 playing sessions with individual students (see Table 2).

_				
	Number of Safe Numbers per session	4.5 numbers		
	Mean playing time	16 minutes		
	Mean time per Safe Number	3.50 minutes		
	Mean time Low Number (0-100)	2.50 minutes		
	Mean time High Number (101-999)	4 minutes		
	Mean number of Handle actions per Safe Number	1.6 times		

Table 2: Summarized data of eighteen sessions of playing 'Crack the Number Safe'.

7.1 Results of students: game perpection and strategy use

A remarkable observation is that children showed a lot of mathematical activity that in many standard mathematics lessons would lead to remarks like 'stupid' and 'boring', but with 'Crack the Number Safe' we observed that children enjoyed the game. They all liked to keep on playing more numbers. The observers had to end the session in each case.. When asked afterwards if they liked the game all students said they did, of course this can be partly based on politeness.

In this mini-game it is important to work systematically. Otherwise students loose track of which factors they already checked. When we look at the data per student we notice that several ways of working systematically have been used:

- Checking factors in the order from 1 to 10. (A)
- First checking 'easy' factors: (1), 2, 5 and 10 (B) or pairs of factors below 10 (C). Then check the other factors either in order (A) or at random.
- No system (D)

From the data we conclude that which strategy is used, not only depends on the student, but also on the SafeNumber (see section 7.2). Very few students (3 out of 18) systematically check all factors in order. This are all boys from grade 6. The large majority of students use a mix of strategies B and C. Some of the students develop a more systematic way only after they have played 2 or 3 SafeNumbers. Only a small number of students (3 out of 18) seem to use no systematic strategy at all.

For checking whether a number is a factor of the Safe Number several strategies were used. Some of which are incorrectly expanded to numbers it doesn't hold for.

- *'Table jumping'*. Some children easily 'jump' through the products of Multiplication Tables to check for a factor. For example to check if 4 is a factor they jump along 4, 8, 12, 16. Sometimes they use bigger jumps like in 16, 32, 48, 64.
- Check the last digit. The strategy to make a decision depending on the last digit of the safe number works for 0, 5 and even numbers, but it does not work in all cases. For example '78 not divisible by 3 because 8 cannot be divided by 3' is not correct.
- *Remove the last 0.* This works with some numbers: '80 can be divided by 4 because 8 can', but it gives a problem with other numbers, for example some children reason that 100 cannot be divided by 4 because 4 is not a factor of 10..
- Splitting the number. Splitting a number and check if a factor holds for both parts is an effective strategy. Although a number of mistakes can be made if the number is not split in an approriate way.
- Using base10. Especially for numbers below 100 (but sometimes also for the larger numbers) the base10 facts are used to split the numbers. 78 divided by 6 -> 10 x 6 = 60, and then check the remainder (28 divided by 6)
- *Middle 0.* Numbers with a middle 0 (e.g. 603) sometimes provoke the often incorrect strategy of splitting the number in the first two digits and the last one. For example in 603 finding factors for 60 and then reason they can be taken 'times 10' and trying to divide the remainder.
- Taking apart the Hundred digit. Numbers with 3 digits are split up in hundreds and the rest (e.g. 687 is split up in 600 and 87). A problem is that some children only check the divisibility of the remainder (87 in this case) and exclude the hunderds. This strategy only holds for factors of 100 which are 1, 2, 4, 5 and 10.

A final observation about what students did is that they all worked through the game by reasoning and calculating without using any support. They did not use the calculator provided in the game, neither did they use paper and pencil. Even for larger numbers and difficult factors they made all calculations mentally. We conclude this section with an example of a protocol (Figure 5).

John, grade 4. Age 10. Safe Number: 96

- He starts with 1: 'that's one that always fits'
- 2, because 96 is even
- 3, is possible (3 x 30 = 90 and 2 x 3 = 6)
- 4: 40 and another 40 makes 80 and then 4 x 4 makes 16
- 6: 60 and 36, both in the Multiplication Table of 6
- He clicks the handle. One number is not correct.
- John is silent for a minute
- Then he says: 10 does not fit
- 9 does not fit (10 x 9 and then it does not fit)
- So it is 7 or 8. After some hesitation he chooses 8 (he split up 96 in 80 and 16)

Figure 5: Example of a protocol

7.2 Results per Safe Number

In the 18 sessions 81 Safe Numbers have been played, including some numbers that occur more than once. These are distributed as shown in Table 3.

Table 3: Distribution of Safe Numbers played

	Below 100: Number from 1x 1 till 10 x 10	31 numbers		
	Below 100: Other numbers	29 numbers		
	Between 100 and 1000	21 numbers		

In general the factors most easily identified correctly are 5 and 10. All students know the division rules of numbers ending in 5 or 0. Identifying 1 as a factor is initially problematic for 5 of the 18 students, in the beginning they don't select the button with the 1. They need the feedback of the handle to 'reinvent' the fact that 1 is always a factor of the Safe Number.

For Safe Numbers between 1x1 and 10x10 almost all students use their knowledge of the multiplication tables to find some of the factors. The prevailing strategy for 18 of the 31 (57%) of this type of numbers is first identifying pairs of factors both below 10. For the squares, 4 of the 31 Safe Numbers (14%) students started with the factor 1.

For the remaining 9 numbers the order in which factors are identified is not based on how they occur in the multiplication tables, but for example on their numerical order. For 19 of the 31 numbers the answer is found without using the feedback option.

For Safe Numbers below 100 that are not in the multiplication tables between 1x1 and 10x10, the strategies and the order in which factors are identified differ from student to student. For 10 of the 29 numbers 1 is the first factor identified. In case of an even number the first factor identified is often 2. Identifying 2 as a factor is done in several ways. Some students know that they only have to check if the last digit is 0, 2, 4, 6 or 8. Other students know the term 'even' and implicitly use the last digit rule. Still a remarkable number of four students don't click the 2 while dealing with their first (even) Safe Number.

For Safe Numbers between 100 and 1000 we see that the solution process takes more time. More often students struggle with finding the factors, they go back and forth selecting and deselecting factors. For 17 of the 21 numbers the first factor identified is 1. This can be explained by the fact that the larger Safe Numbers are played after the lower ones: by then all students know that 1 is always a factor.

8. Conclusion and discussion

Children experience 'Crack the Number Safe' as being fun. This can be concluded in general from the data of online use where we notice that Crack the Number Safe is mainly played online in leisure time, more specific we conclude that children enjoy playing the mini-game from the data in the 18 protocols. Despite the fact that playing the game mainly asks for doing mental calculations, children are concentrated and like to play several Safe Numbers in one session, they seem to experience some kind of flow. They use the feedback option in a thoughtful way to check how far they have come in identifying factors and like to 'get the money from the safe' when all factors are correctly identified. The fact that the Safe is filled with coins when it is opened, contributes to the children's experience that they are playing a game.

From a designers point of view, however, we conclude that 'Crack the Number Safe' is missing some essential game aspects and that the goal of the game and the math goals are not fully aligned. There are no levels (Safe Numbers come at random), the game has limited feedback and there is no scoring mechanism. Of course the option to choose between easy (below 100) or difficult (between 100 and 1000) Safe Numbers is an alternative way to choose your own level, but this is primitive and not game-like. The feedback that follows the click on the handle is primitive and children have difficulty to interpret the message. The game is not supportive for specific strategy use, there are long periods in which the children are thinking and reasoning mentally without being able to interact with the game. Although a calculator is avalable on screen the children do not use this tool.

It is remarkable that despite these flaws children experience Crack the Number Safe as a gama and seem to enjoy playing it.

In answer to the second question: What do children learn when they play 'Crack the Number Safe'? we conclude that while playing the game students do indeed reason about divisibility. The game gives children the opportunity to use their knowledge on Multiplication Tables in a new environment. In this new and non-standard situation that has not been 'trained' in the classroom all students use this knowledge. Some students developed strategies on how to deal adequately with finding factors of 'large' numbers (>100). The two most common strategies that students developed were: adding on in jumps from a known multiple of the factor they were checking for, or splitting the Safe Number in two parts and check each of these. Sometimes for larger numbers, one of the remaining parts was split again. All students learned that the factor 1 is always a factor of each number and must be included in the final set of factors that divide the Safe Number properly. Some students made discoveries and learned new things about division rules: like for example that it is enough to check if the last digit is even to be sure that the number itself can be divided by 2, or that if a number is divisible by 10 also 2 and 5 are factors. As was shown in the results some of the students also came up with incorrect rules. In this research we focused on game sessions with individual students outside their classroom. A question that remains is whether and how this mini-game can be used in classroom sessions. We see opportunities for using this mini-game for discussions between students and for classroom discussions orchestrated by the teacher. The embedding of the game in the classroom will be the focus of the next step in this research.

Acknowledgements: We would like to express our thanks to:

- the teachers of the pilot school;
- their students who enthusiastically played 'Crack the Number Safe'.

Literature

- Alexopoulou, E., Bennerstedt, U., Childs, M., Jonker, V., Kynigos, C., Pratt, D., et al. (2006). Literature review on the use of games in mathematical learning. Part II: Deployment. Report of the Learning Patterns for the Design and Deployment of Mathematical Games project. London: Knowledge Lab.
- Barmby, P., Harries, T., Higgins, S., and Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Education Studies in Mathematics*, 70, 217-241.
- Copier, M. (2007). Beyond the magic circle. A network perspective on role-play in online games. Utrecht, Utrecht.
- Csikszentmihalyi, M. (1990). *Flow: The Psychology of Optimal Experience*. New York: HarperCollins Publishers.
- diSessa, A., and Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *Journal of the Learning Sciences*, 13(1), 77-103.
- Drijvers, P. (Ed.). (2004). Classroom-based Research in Mathematics Education, Overview of doctoral research published by the Freudenthal Institute 2001-2004. Utrecht: Freudenthal Institute.
- Freudenthal, H. (1983). Didactical Phenomenology of Mathematical Structures. Dordrecht: Reidel.
- Glaser, B., and Strauss, A. (1956). *The Discovery of Grounded Theory. Strategies for Qualitative Research*. Chicago: Aldine publishing company.
- Gravemeijer, K., and Drijvers, P. (2004). *Tool Use in an Innovative Learning Arrangement for Mathematics*. Granted Research Proposal (NWO).
- Gravemeijer, K. P. E. (1994). Developing realistic mathematics education. CDbeta press, Utrecht.
- ISFE (2008). Video gamers in Europe. Prepared for the Interactive Software Federation of Europe (ISFE): Nielsen Games.
- Jonker, V., and Van Galen, F. (2004, 10-7-2004). KidsKount. *Mathematics games for realistic mathematics education in primary school.* Paper presented at the 10th International Conference on Mathematics Education (ICME), Kopenhagen, Denmark.
- Jonker, V., and Wijers, M. (2008). *Th!nklets for mathematics education. Re-using computer games characteristics in educational software*. Paper presented at the International Conference of the Learning Sciences (ICLS). from http://www.fi.uu.nl/en/icls2008/550/paper550.pdf
- Juul, J. (2007). Swap Adjacent Gems to Make Sets of Three: A History of Matching Tile Games. *Artifact Journal*, 2.
- Kafai, Y., and Resnick, M. (1996). *Constructionism in practice. Designing, thinking and learning in a digital world*. Mahwah, NJ: Lawrence Erlbaum Associates.
- McFarlane, A., Sparrowhawk, A., and Heald, Y. (2002). Report on the educational use of games. Cambridge: TEEM.
- Parnafes, O., DiSessa, A., Edelson, D. C., Hammer, D., Krakowski, M., and Sherin, B. (2008, June 24-28, 2008). How to study learning processes? Reflection on methods for fine-grain data analysis. Paper presented at the International Conference of the Learning Sciences, Utrecht, The Netherlands.
- Prensky, M. (2000). Digital Game-Based Learning.
- Prensky, M. (2005). In Educational Games, Size Matters. Mini-games are Trivial but "Complex" Games Are Not. An important Way for Teachers, Parents and Others to Look At Educational Computer and Video Games. Retrieved 07-05-09, 2009, from http://www.marcprensky.com/writing/Prensky-Size_Matters.pdf
- Rohrl, D. (2009). Casual Games White Paper 2008 2009, A Project of the Casual Games SIG of the IGDA IGDA.
- Salen, K., and Zimmerman, E. (2004). *Rules of Play. Game design fundamentals*. Cambridge, Massachusetts: MIT Press.
- Squire, K. (2002). Video games in education. *International Journal of Intelligent Games & Simulation*, 2(1).
- Treffers, A. (1987). Three dimensions. A model of goal and theory description in mathematics instruction The Wiskobas project. Dordrecht: Kluwer Academic Publishers.
- Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in Realistic Mathematics Education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics*, 54(1), 9-35.
- Wenger, E. (1998). Communities of Practice. Learning as a social system. Systems Thinker.
- Zagal, J., and Bruckman, A. (2008, June 24-28, 2008). *The Game Ontology Project: Supporting Learning While Contributing Authentically to Game Studies.* Paper presented at the International Conference of the Learning Sciences, Utrecht, The Netherlands.