Mathematical Creativity and School Mathematics: Indicators of Mathematical Creativity in Middle School Students

Eric Louis Mann, Ph.D.

University of Connecticut, 2005

As students progress through the educational system their interest in mathematics diminishes. Yet there is an ever increasing need within the workforce for individuals who possess talent in mathematics. The literature suggests that mathematical talent is most often measured by speed and accuracy of a student's computation with little emphasis on problem solving and pattern finding and no opportunities for students to work on rich mathematical tasks that require divergent thinking. Such an approach limits the use of creativity in the classroom and reduces mathematics to a set of skills to master and rules to memorize. Doing so causes many children's natural curiosity and enthusiasm for mathematics to disappear as they get older. Keeping students interested and engaged in mathematics by recognizing and valuing their mathematical creativity may reverse this tendency.

The identification of creative potential is challenging. Prior research into the identification of mathematical creativity has focused on the development of measurement instruments. Scoring of these instruments is time consuming and subject to scorer interpretation due to the variety of possible responses. Thus, their use in schools has been very limited, if used at all, since their creation. This study seeks a simpler means to obtain indicators of creative potential in mathematics. Existing instruments, the *Creative Ability in Mathematics Test*, the *Connecticut Mastery Tests*, the *Fennema-Sherman*

Mathematics Attitude Scales, What Kind of Person are You? from *the Khatena-Torrance Creative Perception Index* and the *Scales for Rating the Behavioral Characteristics of Superior Students* were used to conduct a standard multiple regression analysis. This analysis explored the relationship between mathematical creativity and mathematical achievement, attitude towards mathematics, self-perception of creative ability, gender and teacher perception of mathematical talent and creative ability. Data were gathered from 89 seventh graders in a suburban Connecticut school. The regression model predicted 35% of the variance in mathematical creativity scores. Mathematical achievement was the strongest predictor accounting for 23% of the variance. Student attitudes towards mathematics, self-perception of their own creative ability and gender contributed the remaining 12% of variance. Interpretation of the relative importance of the independent variables was complicated by correlations between them.

Mathematical Creativity and School Mathematics:
Indicators of Mathematical Creativity in Middle School Students

Eric Louis Mann

B.A., Albion College, 1974
M.S., University of Southern California, 1980
M.A.T., The Colorado College, 1997

A Dissertation
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy
at the
University of Connecticut
2005

APPROVAL PAGE

Doctor of Philosophy Dissertation

Mathematical Creativity and School Mathematics:

Indicators of Mathematical Creativity in Middle School Students

Presented by

Eric Louis Mann, B.A., M.S., M.A.T.

Major Advisor ______________________
M. Katherine Gavin

Associate Advisor ______________________
Sally M. Reis

Associate Advisor ______________________
Joseph S. Renzulli

Associate Advisor ______________________
Del Siegle

Associate Advisor ______________________
E. Jean Gubbins

University of Connecticut

2005

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter I

Introduction

In *Rising Above The Gathering Storm: Energizing and Employing America for a Brighter Economic Future* (Committee on Science, Engineering, and Public Policy, 2005) members of the National Academy of Science developed a list of recommended actions needed to ensure that the United States can continue to compete globally. The top recommendation was to increase America's talent pool by vastly improving K-12 mathematics and science education (pp. 91-110).

One of the strengths of the United States economic growth has been the creativity of its citizens. Inherent in the recommendations above is the need for growth and innovation, both of which are fueled by creativity. This study investigates several means of identifying mathematical creativity as a first step in identifying and nurturing this talent.

Statement of the Problem

In answering the question, *why measure creativity*? Treffinger (2003) offered eight general roles for creativity measurement. Of those eight, two are relevant to this study:

- Help to recognize and affirm the strengths and talents of individuals and enable people to know and understand themselves, and
- Help instructors, counselors, or individuals discover unrecognized or untapped talents. (p. 60)

Hong and Aqui (2004) studied academically gifted mathematics students and students with creative talent in mathematics and found significant differences in cognitive strategies with the creatively talented group being more cognitively resourceful. Resourcefulness, persistence, and the desire to explore alternative methods of solution are all characteristics of the potentially creative mathematical thinker identified by Carlton (1959). Traditional tests to identify mathematically gifted students do not identify creativity (Kim, Cho, & Ahn, 2003), but rather value accuracy and speed. This implies that mathematical talent is measured by computation with little emphasis on problem solving and pattern finding and no opportunities for students to work on rich mathematical tasks that require divergent thinking. Limiting the identification of mathematical talent to the current methods ignores the very group of students who offer the greatest potential for the advancement of mathematics.

As students progress through the educational system their interest in mathematics diminishes. The U.S. Department of Education (2003) reports that 81% of fourth graders have a positive or strongly positive attitude towards mathematics but four years later only 35% of eighth graders share that attitude. At the post-secondary level less than 1% of degree-seeking baccalaureate students choose mathematics as their major field of study (National Center for Educational Statistics, 2005). Current emphases on convergent thinking and rapid response have failed to reverse the trend. Limiting the use of creativity in the classroom reduces mathematics to a set of skills to master and rules to memorize. Doing so causes many children's natural curiosity and enthusiasm for mathematics to disappear as they get older, creating a tremendous problem for mathematics educators who are trying to instill these very qualities (Meissner, 2000). Keeping students interested and engaged in mathematics by recognizing and valuing their mathematical creativity may reverse this tendency.

The first step in focusing on mathematical creativity is the identification of this characteristic in students. However, current identification tools are inadequate. The identification of creative potential is challenging. Although a few tests of mathematical creativity have been developed (Balka, 1974a; Evans, 1964; Getzels & Jackson, 1962: Haylock, 1984; Jensen, 1973; Prouse, 1964), scoring of these instruments is time consuming and subject to scorer interpretation due to the variety of possible responses. Thus, their use in schools has been very limited, if used at all, since their creation. This study seeks a simpler means to obtain indicators of creative potential in mathematics to assist classroom teachers in the identification of this potential in middle school students. Using existing instruments, several factors within the educational setting were examined that may be indicative, individually or collectively, of a student's mathematical creativity potential. The factors considered included achievement in mathematics, attitude towards mathematics, self-perception of creative ability, and teacher perception of mathematical talent and creative ability. Data on performance by gender were collected to explore potential differences as well. Mathematical creativity was measured using the *Creative Ability in Mathematics Test* (*CAMT*) (Appendix A) developed by Balka. His instrument was developed as a measure of mathematical creativity based on input from 244 mathematicians, professors of mathematics education and classroom teachers of mathematics. His definition of mathematical creativity was the score obtained on his instrument, which will be how mathematical creativity is defined within this study as well. Validity and reliability data for this instrument are presented in Chapter III. Data on student attitudes towards mathematics, mathematics achievement, and student and teacher perceptions were analyzed for their value in predicting mathematical creativity scores, as measured by *CAMT*.

It is hoped that by finding simpler ways to identify creative potential an increase in the recognized talent pool of future mathematics can be achieved at a younger age. It is also hoped that identifying mathematical creativity in students will encourage teachers to nurture this aspect of mathematical talent; an aspect that is perhaps the most important one for mathematicians who will make significant contributions to the field.

<u>Research Questions</u>

This study examined several factors in the educational setting and their relationships to mathematical creativity. The following research question with its subcomponents formulated the basis of the research:

> *Is there a measure, or combination of measures, that accurately predicts student performance on the Creative Ability in Mathematics Test (CAMT)?*

(a) Does a measure of student achievement in mathematics contribute to the prediction of student performance on the CAMT, after controlling for teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics, student perception of his/her creative ability and gender?

(b) Does teacher perception of student general creativity contribute to the prediction of student performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of mathematical

talent, student attitude towards mathematics, student perception of his/her creative ability and gender?

(c) *Does teacher perception of mathematical talent contribute to the prediction of student performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of general creativity, student attitude towards mathematics, student perception of his/her creative ability and gender?*

(d) *Does student attitude towards mathematics contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student perception of his/her creative ability and gender?*

(e) *Does student perception of his/her creative ability contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics and gender?*

(f) *Does gender contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics and student perception of his/her creative ability?*

Summary

This chapter provided a rationale for this study and identified the research questions that guided the investigation. The negative trend in individual interest in mathematics was noted, as was the failure of traditional classroom emphasis on convergent thought and computational speed in reversing this trend. An understanding of mathematics is needed in almost every occupation and the need to find and develop talent is in the best interest of both the individual and society as a whole. The rationale for expanding the effort to find mathematical talent beyond those who are academically gifted was discussed.

In Chapter II, a review of literature on mathematical creativity and factors related to the identification of creativity are described. The methodology used in this study is explained in Chapter III with the results of the research described in Chapter IV. Chapter V includes a discussion of the implications of this research, the limitations of the study and suggestions for further research.

Chapter II

Review of Literature

Definition of Mathematical Creativity

An examination of the literature that has attempted to define mathematical creativity found that the lack of an accepted definition for mathematical creativity has hindered research efforts (Ford and Harris 1992; Treffinger, Renzulli and Feldhusen, 1971). Treffinger, Young, Selby and Shepardson, 2002 acknowledged that there are numerous ways to express creativity and identified over 100 contemporary definitions. Runco (1993) describes creativity as a multifaceted construct involving both "divergent and convergent thinking, problem finding and problem solving, self-expression, intrinsic motivation, a questioning attitude, and self-confidence" (p. ix). Haylock (1987) summarized many of the attempts to define mathematical creativity. One view "includes the ability to see new relationships between techniques and areas of application and to make associations between possibly unrelated ideas" (Tammadge, as cited in Haylock). The Russian psychologist Krutetskii characterized mathematical creativity in the context of problem formation (problem finding), invention, independence, and originality (Haylock; Krutetskii, 1976). Others have applied the concepts of fluency, flexibility, and originality to the concept of creativity in mathematics (Haylock, 1997; Jensen, 1973; Kim et al., 2003, Tuli, 1980;). In addition to these concepts, Holland (as cited in Imai, 2000) added elaboration (extending or improving methods) and sensitivity (constructive criticism of standard methods). Singh (1988) defined mathematical creativity as the "process of formulating hypotheses concerning cause and effect in a mathematical situation, testing and retesting these hypotheses and making modifications and finally communicating the results" (p. 15).

Studies of mathematical creativity (Balka, 1974a; Evans, 1964; Getzels & Jackson, 1962; Haylock, 1984; Jensen, 1973; Meyer, 1969; Prouse, 1964; Singh, 1988) have sought to measure mathematical creativity either in terms of flexibility, fluency and originality of a student's response to problems presented or in terms of the development of mathematical problems from situational data. In his article on creative ability in mathematics, Balka (1974b) introduced a comprehensive set of criteria for measuring mathematical creative ability based on the works of Guilford; Harris and Simberg; Torrance; and Meeker. He addressed both convergent thinking, characterized by determining patterns and breaking from established mindsets, and divergent thought defined as formulating mathematical hypotheses, evaluating unusual mathematical ideas, sensing what is missing from a problem, and splitting general problems into specific sub-problems. In reviewing Balka's (1974a) criteria, breaking from established mindsets was a defining feature in the efforts of others to understand the creative mathematician.

Haylock (1997) and Krutetskii (1976) both believed that overcoming fixations was necessary for creativity to emerge. Both, like Balka, focused on the breaking of a mental set that places limits on the problem-solver's creativity. Trying a variety of approaches to solving problems, each in a systematic way, can be confused with exhibiting mathematical creativity. In an earlier work, Haylock (1984) discussed the difference between creativity and being systematic in mathematical problem solving. By applying learned strategies, a student can systematically apply multiple methods to solve a problem but never diverge into a creative one; never exploring areas outside the individual's known content-universe.

Carlton (1959) analyzed the educational concepts of 14 eminent mathematicians. From her analysis emerged a list of 21 characteristics of the potentially creative thinker in mathematics (Appendix B). She found no single defining characteristic of creative talent in mathematics but inferred that the mathematically creative gifted child would demonstrate a subset of these characteristics. Carlton's analysis also distinguished between two types of creative mathematical minds. Among the mathematicians in her study she found that Klein, Hadamard, Poincaré, Böcher, and Hilbert drew distinctions between logical and intuitive minds. Intuitionalists are described as those who use geometrical intuition, are capable of "seeing in space," and "have the faculty of seeing the end from afar" whereas the logicians work from strict definitions, reason by analogy and work step-by–step through "a very great number of elementary operations" (Carlton, pp. 234- 236). Another difference identified by Carlton was made by Cajori where he separated creative minds into two categories, "the alert, quick minds and the slow, although frequently more profound, minds" (Carlton, p. 243).

Sternberg (personal communication, February 8, 2005) has found that the culture within the United States predominantly equates intelligence with speed of response. However, his research supports a different view, one in which a high intelligence is associated with up-front global planning and reflection. Success in school mathematics, where talent is often measured by speed and accuracy of computation, is much easier for the logical, formal, and fast mind. Cajori had similar reservations with timed mental tests. He wrote, "these intelligence tests measure only fleeting performances of the mind.

They do not take cognizance of the power of the sustained effort to which serious study habituates the individual and the frequent subconscious mental action in such experiences" (Cajori, 1928, p. 15). Hadamard (1945) believed that creativity in mathematics requires the intuitive mind with ample time for reflection and incubation of ideas. Because of this disconnect between time for reflection and measures of computational speed, many students who have the potential to make significant contributions become intimidated and conform to simply follow the crowd, and deny their creative nature (Csikszentmihalyi & Wolfe, 2000).

In summary, there is no single accepted definition of mathematical creativity. However, the literature supports Runco's (1993) multi-dimensional view. Carlton's (1959) work identified 21 characteristics of mathematical creativity. Carlton also reported differences in the types of mathematically creative minds; one that is logical and the other intuitive, one that is quick and alert and the other slow and reflective. The wide variety in definitions and characteristics has created challenges in the identification and development of mathematical creativity. Instruments developed to identify potential mathematical creativity have used the concepts of flexibility, fluency and originality in student responses as a way to quantify student responses (Balka, 1974a; Evans, 1964; Getzels & Jackson, 1962; Haylock, 1984; Jensen, 1973; Meyer, 1969; Prouse, 1964; Singh, 1988). However, no information was found in the literature that addressed the application of the instruments in an education setting.

Development of Mathematical Creativity

Mathematical creativity is difficult to develop if one is limited to rule-based applications without recognizing the essence of the problem to be solved. The visionary classrooms described by leaders in the National Council of Teachers of Mathematics (NCTM) (2000) enable students to

> confidently engage in complex mathematical tasks…draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. (NCTM, 2000, p. 3)

For many adults, this vision is unlike the mathematics classrooms they remember from their youth where time was spent *learning from the master.* In this setting, the teacher demonstrated a method with examples and then the students practiced with similar problems (Pehkonen, 1997). For these adults, the concept of mathematics is of "a digestive process rather than a creative one" (Dreyfus & Eisenberg, 1996, p. 258). However, mathematics is not a fixed body of knowledge to be mastered; the essence of mathematics is what mathematicians do (Poincaré, 1913; Whitcombe, 1988).

Köhler (1997) discussed an experiment by Hollenstein in which one group of children worked on a mathematics exercise presented in the traditional method. This method is described by Romberg and Kaput (1999) as a three-segment lesson: correction of the previous day's homework, teacher presentation of new material and student practice. The problems in the experiment were complete or closed in that they were constructed so that a single correct answer existed (Shimada, 1997). A second group was given the conditions on which the first group's exercise was based and asked to develop and answer problems that could be solved using calculations. The open-ended nature of the task given to the second group did not limit them to a set number of problems. This group created and answered more questions than were posed to the first group, calculated more accurately and arrived at more correct results. Researchers at Japan's National

Institute for Educational Research conducted a six-year research study that evaluated higher-order mathematical thinking using open-ended problems (problems with multiple correct answers). In a round-table review of the study, Sugiyama from Tokyo Gakugei University affirmed this approach as a means to allow students to experience the first stages of mathematical creativity (Becker & Shimada, 1997).

Doing what mathematicians do as a means of developing mathematical creativity (as opposed to replication and practice) is consistent with the work at The National Research Center on the Gifted and Talented (Reis, Gentry & Maxfield, 1998; Renzulli 1997; Renzulli, Gentry & Reis, 2004, 2003). Emphasis is placed on creating authentic learning situations where students are thinking, feeling, and doing what practicing professionals do (Renzulli, Leppien & Hays, 2000; Tomlinson et al., 2001). The fundamental nature of such authentic high-end learning creates an environment in which students apply relevant knowledge and skills to the solving of real problems (Renzulli, Gentry & Reis, 2004).

The solving of real problems also entails problem finding as well as problem solving. Kilpatrick (1987) described problem formulation as a neglected but essential means of mathematical instruction. Real world problems are ill-formed and require one to employ a variety of methods and skills to solve the problem. In addition to equations to solve and problems designed to converge on one right answer, students need the opportunity to design and solve their own problems. In his *Creative Mathematical Ability Test,* Balka (1974a) provided participants with mathematical situations from which they were to develop problems. Mathematical creativity was measured by the flexibility, fluency and originality of the problems the participants constructed. By working with these types of mathematical situations, students are encouraged to use their knowledge flexibly in new applications.

Student Achievement and Mathematical Creativity

The essence of mathematics is thinking creatively, not simply arriving at the right answer (Dreyfus & Eisenberg, 1966; Ginsburg, 1996). Yet typical school mathematics programs often focus on what the student does rather than what the student thinks (R. B. Davis, 1986). Hong and Aqui (2004) studied the differences between academically gifted students who achieved high grades in school math, and the creatively talented in mathematics, those students with a high interest, active and accomplished in math but not necessarily high achieving in school math. Hong and Aqui found significant differences in cognitive strategies used by the two groups with the creatively talented being more cognitively resourceful. This is not to say that students cannot be both academically gifted and creatively talented in mathematics. However, as they were examining differences, their study did not include students with strengths in both areas.

Neither group of students should be neglected, yet Ching (1997) found hidden talent to be rarely identified by typical classroom practices. Traditional tests to identify the mathematically gifted do not identify or measure creativity (Kim et al., 2003) but often reward accuracy and speed. These tests identify students who do well in school mathematics (Hong and Aqui's academically talented) and are computationally fluent, but neglect the creatively talented in mathematics.

The definition of mathematical giftedness varies depending on the identification tools used and the program offered. Regardless of the definition used, finding students with mathematical giftedness is a challenge for both educators and society. Often giftedness in mathematics is identified through classroom performance, test scores and teacher recommendations. Yet, the literature suggests that a high level of achievement in school mathematics is not a necessary ingredient for high levels of accomplishment in mathematics. Sternberg (1996) summarized conversations with a number of mathematicians when he wrote:

> …performance in mathematics courses, up to the college and even early graduate levels, often does not effectively predict who will succeed as a mathematician. The prediction failure occurs due to the fact that in math, as in most other fields, one can get away with good analytical but weak creative thinking until one reaches the highest levels of mathematics. (p. 313)

Mayer and Hegarty's (1996) research focused on problem understanding. They found that student difficulties in mathematics lie with understanding and representation of the problem, not in the execution of computational tasks. In an environment where computation is the basis of assessment, high achievement is possible without mathematical understanding. Pehkonen (1997) discussed the balance between knowledge/logic and creativity. In schools where education is one-sided emphasizing knowledge and logic, students develop the left hemisphere of the brain but neglect the right. For achievement beyond traditional school mathematics, a balance between the right and left hemispheres is needed. Yet many students leave school with the right side, the creative side, of the brain undeveloped. The research finding of Pehkonen and Hong and Aqui (2004) suggests an apparent detachment between school mathematics and mathematical accomplishments. Not only are the identified mathematically gifted being neglected, there is a significant probability that some talented students are overlooked by current practices in school.

Hong and Aqui's (2004) division of mathematical talent into the academically gifted and creatively talented is important in the consideration of talent development. The academically gifted student may excel in the classroom by demonstrating high achievement, or schoolhouse giftedness, which is valued in traditional educational settings. These students' abilities remain relatively stable over time (Renzulli, 1998). Those academically gifted in mathematics are able to acquire the skills and methodologies taught often at a much more rapid pace than less able students and perform well on standardized testing. The academically gifted usually demonstrate their mastery of the utilitarian aspects of mathematics, but neither speed nor accuracy in computation or the analytical ability to apply known strategies to identified problems are measures of creative mathematical talent. Hadamard (1945) described individuals he labels "numerical calculators" as "prodigious calculators – frequently quite uneducated men – who can very rapidly make very complicated numerical calculations…such talent is, in reality, distinct from mathematical ability" (p. 58). Thus in an environment that values skill and speed, it is possible to be academically gifted but lack mathematical creativity.

While speed of information processing is important in testing situations in which students' mathematical thinking is assessed using standardized tests, it is less important when a mathematician spends months or even years exploring a variety of mathematical strategies to solve ill-defined problems (Sternberg, 1996). Current tests of number or numerical facility emphasize speed with stress imposed by severe time limits and accountability on the accuracy of the solutions (Carroll, 1996). However, the next generation of mathematicians must be shown the "wellsprings of mathematics; creativity, imagination, and an appreciation of the beauty of the subject" (Whitcombe, 1988, p.14). In an analysis of cognitive ability theory and the supporting psychological tests and factor analysis, Carroll noted that despite six to seven decades of work, the relationships between the discrete abilities measured by psychometric tests and performance in mathematics remains unclear.

<u>Testing, Accountability and Mathematical Creativity</u>

The new open-ended assessments used by many state department of education officials often place little value on creative solutions. Problems with test scoring in Connecticut's 2003-2004 mastery tests illustrate some of the issues where strict guidelines focusing on accuracy are the norm. "There is an art to scoring…there is subjectivity…our work is to remove as much of that variable as possible" according to Hall, CTB/McGraw-Hill's director of hand-scoring (Frahm, 2004). While accuracy is important, strict emphasis on accuracy when assessing a child's conceptual understanding of mathematics discourages the risk taker who applies her/his knowledge and creativity to develop original applications in solving a problem (Haylock, 1984). Such an individual would be in the company of Poincaré, Hadamard and Einstein, all eminent scientists and mathematicians who confessed to having problems with calculations (Hadamard, 1945).

Mayer and Hegarty (1996) report converging evidence that students leave high school with adequate skills to accurately carry out arithmetic and algebraic procedures but inadequate problem solving skills to understand the meaning of word problems. A good mathematical mind is capable of flexible thought and can manipulate and investigate a problem from many different aspects (Drefyus & Eisenberg, 1996). Procedural skills without the necessary higher-order mathematical thinking skills, however, are of limited use in our society. There is little use for individuals trained to solve problems mechanically as technology is rapidly replacing tedious computational tasks (Kohler, 1999; Sternberg, 1996). Often the difference between the errors made by eminent mathematicians and students of mathematics is a function of their insight into and appreciation of mathematics not their computational skills (Hadamard, 1945).

With the increased emphasis on accountability from the *No Child Left Behind Act of 2001* (U.S. Department of Education, 2005), teachers are under even more pressure to teach to the test rather than to work toward developing in their students a conceptual understanding of mathematics. Encouraging students to take risks and look for creative applications reintroduces variability in scoring that assessment teams are working to eliminate. Discouraging risk taking limits student exposure to genuine mathematical activity and dampens the development of mathematical creativity (Silver, 1997). For substantial and permanent progress in a child's understanding of mathematics, an appreciation of "the difficult-beautiful-rewarding-creative view of mathematics" (Whitcombe, 1988, p. 14) must be developed. However, rather than developing an appreciation for mathematics by focusing on qualities of mathematical talent, teachers who only emphasize algorithms, speed and accuracy provide the creative student negative reinforcement, often through skills-based remediation tasks. Thus many talented students do not envision themselves as future mathematicians or in other professions that require a strong foundation in mathematics (Usiskin, 1999).

Mathematical School Experiences and Mathematical Creativity

In 1980 the National Council of Teachers of Mathematics identified gifted students of mathematics as the most neglected segment of students challenged to reach their full potential (NCTM, 1980). In 1995, the NCTM Task force on the Mathematically Promising found little had changed in the subsequent 15 years (Sheffield, Bennett, Beriozábal, DeArmond, & Wertheimer, 1995).

All students, especially those with potential talent in mathematics, need academic rigor and challenge as well as creative opportunities to explore the nature of mathematics and to employ the skills they have developed. Young children explore mathematics naturally and yet the skills-based mathematics encountered in many classrooms fails to connect their natural curiosity with the established curriculum of mathematics. Instead, they are immersed in a classroom environment where mastery and understanding are assessed based on the ability to rapidly solve problems presented in a straightforward manner (Carpenter, 1986; Ginsberg, 1986; Schoenfeld, 1987). Haylock's (1997) research suggests that students' mathematical experience and techniques may limit their creative development. Hashimoto (1997) found that, in general, most classroom teachers think there is a single correct answer and only one correct method to solve a mathematics problem. If taught that there is only one right answer or only one correct method, a student's concept of mathematics as an application of mathematical techniques is reinforced. Köhler (1997) illustrates this point in a discussion with an elementary classroom teacher about a student who had arrived at the correct answer in an unexpected way.

> "While going through the classroom, that pupil asked me [the teacher] whether or not his solution was correct. I **was forced** to admit that it was. That is what you get **when you don't tell the pupils exactly what to do**…." The teacher now reproaches himself for **not having prevented this solution.** He is obviously influenced by an insufficient understanding of what is mathematics, by the image of school as an institution for stuffing of brains…. (p. 88) (emphasis added)

Devlin (2000) identifies four faces of mathematics as (1) computational, formal reasoning and problem solving, (2) a way of knowing, (3) a creative medium, and (4) applications. Of these four, he states that current educational practices in elementary and secondary education focus on the first and touch on the fourth, ignoring the other two. In her foreword to *Making Sense: Teaching and Learning Mathematics with Understanding* (Hiebert et al., 1997), Mary Lindquist, a past president of the National Council of Teachers of Mathematics, shares comments from mathematics students who achieved high grades in school. A sixth grader's comment that, "It doesn't make much sense. But, we are in math class, so I guess it does here," and a calculus student's comment that, "In math, I do things just the opposite way from what I think it should be and it almost always works" (p. vii), are illustrative of the impact such instruction can have. Pehkonen (1997) suggested that the constant emphasis on sequential rules and algorithms may prevent the development of creativity, problem solving skills and spatial ability. If the instruction focuses on memorization rather than meaning, then the student will correctly learn how to follow a procedure, and will view the procedure as a symbol-pushing operation that obeys arbitrary constraints.

Creativity needs time to develop and thrives on experience. Drawing from contemporary research, Silver (1997) suggested, "creativity is closely related to deep, flexible knowledge in content domains; is often associated with long periods of work and reflection rather than rapid, exceptional insight; and is susceptible to instructional and experiential influences" (p. 750). Ponicaré's (1913) essay on mathematical creation also discussed the need for reflection. He described his discovery of the solution to a problem on which he had worked for a considerable amount of time arriving as a sudden illumination as he stepped onto a bus on a geologic excursion. This illumination was "a manifest sign of long, unconscious prior work…which is only fruitful, if it is on the one hand preceded and on the other hand followed by a period of conscious work" (Poincaré, p. 389). This period of incubation appears to be an essential aspect of creativity requiring inquiry-oriented, creativity enriched mathematics curriculum and instruction (Silver). Whitcombe (1988) described an impoverished mathematics experience as one in which instruction only focuses on utilitarian aspects of mathematics and is without appropriate interest-stimulating material and time to reflect. Such experiences deny creativity the time and opportunities needed to develop.

Indicators of Mathematical Creativity

The National Council of Teachers of Mathematics Task Force on the Mathematically Promising (Sheffield et al., 1995) characterized our promising young mathematics students in light of their ability, motivation, belief [self-efficacy], and opportunity/experience, all considered variables that must be maximized to fully develop a student's mathematical talent. G. A. Davis (1969) considered developing creativity in students of mathematics in terms of three major parameters: attitudes, abilities, and techniques (methods of preparing and manipulating information). While 26 years separate these efforts, they offer similar recommendations. In searching for potentially creative student mathematicians, using existing creativity instruments is difficult to do for entire grade levels due to the time involved in scoring such instruments. Yet, relying solely on teacher recommendations provides an incomplete picture of the students (Hashimoto, 1997; Köhler, 1997).

Teacher Perceptions of Students' Mathematical Ability and Mathematical Creativity

Prouse (1964) reported a significant correlation (r =.30, p =.01) between teacher ratings of student creativity and student performance on a test of mathematical creativity. Both of the instruments were developed by Prouse with reported reliabilities of .42 for the teacher ratings and .88 for the test of mathematical creativity. No other studies of the relationships between teacher perceptions and measures of mathematical creativity were found. However, in analyzing problems in the assessment of creative thinking Treffinger, Renzulli and Feldhusen (1971) cited studies by Holland; Wallend and Stevenson; Rivlin; Reid, Kind and Wickwire; Torrance; and Yamamoto that found teacher judgments favor high IQ and high achieving students. Gear (as cited in Mayfield, 1979) found many examples of inaccuracy of teacher judgments when rating gifted students. Mayfield's (1979) study of 573 third graders found that teacher ratings of intelligence corresponded to student achievement on standardized tests but that teachers were unable to judge student creativity. In summarizing the result of workshops on the assessment of creativity at the University of Hertfordshire, University of Portsmouth and at Indiana University-Purdue University Indianapolis, Jackson (2005) wrote "A teacher's perceptions of creativity are too limited and biased (to their own values) to be the only catcher" (p. 2). If this finding holds, then talent identification programs that rely solely on achievement and teacher recommendations may be overlooking students who would benefit from inclusion in such a program.

If teacher perceptions of creativity are inaccurate then the recognition and development of creative potential within the classroom is difficult. To improve teacher ratings the *Scales for Rating the Behavioral Characteristics of Superior Students*

(SRBCSS) (Renzulli et al. 2004) were developed to obtain teacher judgments on characteristics of high ability students. These scales are often used, in conjunction with other instruments, as a means of identification for gifted education services as well as a means to assess student strengths. The *SRBCSS* sub-scales for mathematics and creativity were used for all participants in the present study to assess their predictive ability of student performance on a measure of mathematical creativity.

<u>*Student Attitudes Towards Mathematics*</u>

Goldin (2002) stated that a student's affective system is central to her cognition and that its influence can enhance or inhibit cognitive activities. Yates (2002) drew a distinction between students who are task involved and those who are ego orientated. Students with a task focus seek challenges and persist when difficulties are encountered, traits that Carlton (1959) identified as characteristics of mathematical creativity. Ego orientated students focused on their performance relative to others and put forth effort only as needed to avoid failure. Evans (1964) and Tuli (1980) reported a significant relationship between attitudes towards mathematics and mathematical creativity. Using Amabile's (1989) ingredients of creativity, Starko (2001) also discussed the role of interest in intrinsic motivation for the development of creativity. The greater a child's intrinsic motivation, the greater the likelihood of creative applications and discoveries. McLeod's (1992) review of research on affect in mathematics education found a positive correlation between attitude and achievement across grade levels. Plucker and Renzulli (1999) suggest a positive attitude may be an indicator of creative potential. In the development of the *CAMT*, Balka (1974a) did not collect attitudinal data.

The *Fennema-Sherman Mathematics Attitude Scales* (1976) were developed to study domain specific attitudes that were thought to be related to the learning of mathematics. Fennema-Sherman developed nine different scales summarized in Table 1.

Table 1

Fennema-Sherman Mathematics Attitude Scales (1976)

Scale	Definition
1. *Attitude Toward Success in Mathematics*	The degree to which students anticipate positive or negative consequences as a result of success in mathematics.
2. *Confidence in Learning Mathematics*	The level of confidence in one's ability to learn and to perform well on mathematical tasks. Not intended to measure anxiety, confusion, interest or enjoyment.
3. *Mathematics Anxiety*	Measures feelings of anxiety, dread, nervousness and associated bodily symptoms related to doing mathematics. Not intended to measure confidence or enjoyment of mathematics.
4. *Effectance Motivation in Mathematics*	Measures effectance as applied to measure and ranges from lack of involvement to active enjoyment and seeking of challenges. Not intended to measure interest or enjoyment of mathematics.
5. *Teacher Scale*	The student's perception of his/her teacher's attitudes toward them as learners of mathematics.
6. *The Mother* 7. *The Father Scale*	The student's perception of his/her mother's/father's interest, encouragement and confidence in the student's ability.
8. *Mathematics Usefulness*	Student beliefs about the usefulness of mathematics currently and in relationship to their future.
9. *Mathematics as a Male Domain*	The degree to which students see mathematics as a male, female or neutral domain.

Scales that address student intrinsic motivation and attitudes (*Attitude Toward Success in Mathematics, Confidence in Learning Mathematics*, *Mathematics Anxiety, Effectance Motivation in Mathematics*) were selected for use in the present study.

Student Self-Perceptions of Creative Ability

In assessing personality characteristics as a means of measuring creative potential, Treffinger (2003) wrote, “questions that ask the individual if he or she is creative, inventive, ingenious, or original may have a high degree of accuracy for prediction of future creative interests” (p. 72). The *Khatenna-Torrance Creative Perception Inventory* (Khatenna & Torrance, 1976) was developed to provide information on student attitudes and perceptions of their creativity. Khatenna and Torrance report that the inventory has been widely used for the identification of creative individuals in schools settings and in research. This inventory has two independently developed measures. Both are suitable for group administration for children in grade 4 – 12 (ages 10 to 19) and may be individually administrated to children in grades 1 – 3 (Khatenna & Torrance). Treffinger found the instruments useful as a means to provide some information on a student’s personal creative characteristics but insufficient as a comprehensive measure of creativity. Feldhusen and Goh (1995) concluded that multiple means of measurement are necessary for the assessment as creativity is a multidimensional construct. The use of the *Khatenna-Torrance Creative Perception Inventory* adds another means to assess student creativity and compare general creativity score with domain specific mathematical creativity.

Gender Differences Regarding Mathematical Creativity

Evans (1964), Jensen (1973) and Prouse (1967) reported significantly higher mathematical creativity scores for females than males. In his study, Evans (1964) analyzed data collected from 42 students in eighth grade, 42 students in seventh grade, 21 students in sixth grade and 18 students in fifth grade at the University School, University of Michigan. He reported eighth grade girls outscored boys with significant differences in 11 of 15 measures. With seventh grade girls, significant differences were noted in 7 of the 15 measures. No significant gender differences were found for the remaining grades. In his summary, he stated that differences noted may be due to sample bias in favor of girls as well as attitude and motivation factors.

Jensen's (1973) study involved sixth graders at three schools in Texas. While the difference in mathematical creativity between the schools was not significant ($\chi^2 = 1.44$, $p > .05$), the gender differences varied across schools with a significant difference favoring females noted in one of the three schools ($\chi^2 = 14.59$, $p = .001$, $n = 89$) and no difference at the other two ($\chi^2 = .65$, $p > .05$, $n = 40$; $\chi^2 = 2.52$, $p > .05$, $n = 103$). Prouse (1967) investigated creativity in seventh graders in 14 classrooms in 5 schools in Iowa. He reported a significant mean difference in composite creativity scores favoring females ($t = 3.24$, $p < .05$, $n = 312$).

However, research by Schmader, Johns, and Barquissau (2004) found that many college women still endorse the stereotypical views that men are superior to women in mathematics. Such a belief may have a negative effect on women's involvement in mathematics related fields. While the research is inconclusive, gender differences in mathematical creativity may emerge as a means of finding unrecognized talent. This study presents an opportunity to add additional data to the field in the area of possible gender differences.

Measurement of Mathematical Creativity

The works cited above discuss creativity's importance in a global manner but never really define it as a construct in measurable terms. To assess efforts to develop creative potential, a means of identification and measurement is needed. There have been several instruments developed to measure mathematical creativity (Balka, 1974a; Evans, 1964; Getzels & Jackson, 1962; Haylock, 1984; Jensen, 1973; Singh, 1988). Of this group, Balka's *CAMT* was the only available instrument with a sufficient discussion of validity and reliability and on that basis was selected for use in this study.

Getzels and Jackson's (1962), *Make-Up Problems* test provided an internal consistency reliability coefficient of .81 based on a data obtained from 45 randomly selected participants. The instrument design and validation process in Getzels and Jackson's work involved a much smaller participant pool and lacked the in-depth discussion available with Balka's work. Jensen's (1973*) How Many Questions Game* was a modification of Getzels and Jackson's instrument. No reliability or validity measurements are provided in her dissertation. Likewise, Evans (1964) and Haylock (1984) provided their instruments but no statistical data on reliability or validity. Prouse (1964) estimated his test reliability at .42 using split-half technique with the Spearman-Brown prophecy formula. For his instrument, Singh (1988) reported high item and factor validity and a test-retest reliability of .84. Unfortunately, Singh's text does not include a copy of the instrument and was designed to measure changes in mathematical creativity as a result of treatments involving teaching-learning strategies.

Balka (1974a) defined mathematical creativity as the score obtained on his instrument. His instrument was developed based on responses to his *Creative Ability in Mathematics Survey* distributed to a randomly selected group of 100 mathematicians, 100 university mathematics educators, and 100 secondary school mathematics teachers. The overall response rate to the survey was 81.3%. Of the 25 criterions on the survey, only those that received at least 80% agreement from at least one of the groups were retained. The resulting criteria to measure creative mathematical potential are the following:

1. Ability to formulate mathematical hypotheses concerning cause and effect in mathematical situations;
2. Ability to determine patterns in mathematical situations;
3. Ability to break from established mind sets to obtain solutions in a mathematical situation;
4. Ability to consider and evaluate unusual mathematical ideas, to think through the possible consequences for a mathematical situation;
5. Ability to sense what is missing from a given mathematical situation and to ask questions that will enable one to fill in the missing mathematical information;
6. Ability to split general mathematical problems into specific sub problems (Balka, pp. 52-62).

A comparison of the Balka's remaining criteria with Carlton's (1959) 21 characteristics of the potentially creative thinker in mathematics is provided in Table 2.

Table 2

A Comparison of Mathematical Creativity Criteria

Balka's (1974a) Criteria	Calton's (1959) Characteristics of Potentially Creative Thinker in Mathematics*
1. Ability to formulate mathematical hypotheses concerning cause and effect in mathematical situations;	1. The speculating or guessing about what would happen if one or more hypotheses of a problem are changed (7).
2. Ability to determine patterns in mathematical situations;	2. The tendency to generalize particular results, either by finding a common thread of induction or by seeing similar patterns by analogy (12). A desire to improve a proof or the structure of a solution (3).
3. Ability to break from established mind sets to obtain solutions in a mathematical situation;	3. Pleasure derived from adding to the knowledge of the class by producing another solution or another proof beyond those that the class considered (9).
4. Ability to consider and evaluate unusual mathematical ideas, to think through the possible consequences for a mathematical situation;	4. A seeking for consequences of connections between a problem, proposition, or concept and what would follow from it (4).
5. Ability to sense what is missing from a given mathematical situation and to ask questions that will enable one to fill in the missing mathematical information;	5. Intuition as to how things should result (14).
6. Ability to split general mathematical problems into specific sub problems	6. The making up or seeing of problems in data or in situations which arouse no particular curiosity in other children (2).

* Numbers in parentheses correspond to the number of the characteristic in Appendix B.

Summary

A child's growth in mathematics involves more than just mastering computational skills. Identification of mathematical talent using only speed and accuracy of computation neglects those who are creative and reflective. Mathematical talent requires creative applications of mathematics in the exploration of problems, not replication of the work of others. The challenge is to provide an environment of practice and problem solving that stimulates creativity, while avoiding the imposition of problem-solving heuristic strategies (Pehkonen, 1997). Such an environment will enable the development of mathematically talented students who can think creatively and introspectively (Ginsburg, 1996).

This review of literature provides evidence for the importance and the development of mathematical creativity. Research has shown that mathematical creativity is an essential aspect in the development of mathematical talent and yet it is difficult to measure or identify. While the literature supports the development of mathematical creativity, it also reports that little is being done to identify or develop mathematical creativity in schools today. Further research is necessary to develop identification tools so that effectiveness of interventions to encourage talent development can be measured.

Chapter III
Methods and Procedures

In Chapter III the sample population and the research design are described. The instruments used, the rationale for their selection and methods of scoring are explained.

Sample Population

Eighty-nine seventh graders in a suburban Connecticut middle school were used as a convenience sample for this study. Seventh graders were selected as the closest match in age to the groups involved in the development of the instruments used in this study. The school administration and teachers expressed interest and support for the study and offered access to all students, not just those with high math achievement. Table 3 provides an overview of the school population. Grade level data were not collected.

Table 3

Student Population Profile of the Participating School (n=674) 2003-2004

% of students eligible for free/reduced-price meals	9.9%
% of students with non-English home language	7.4%
% of students who attended the school the previous year	91.0%
% of students receiving bilingual and ESL services	1.0%
% of students receiving special education services	14.0%
% of students involved in gifted and talented programs	16.2%
% of minority students	15.7%

Source: State of Connecticut Department of Education
http://www.csde.state.ct.us/public/cedar/profiles/index.htm

All participants in the study scored at or above the proficient level in mathematics as measured by the *Connecticut Mastery Tests (CMT)* (Connecticut State Board of Education, 2001). There was no difference in teacher perceptions of mathematical ability by gender with both male and female mean scores of 40.7 on the *Scales for Rating the Behavioral Characteristics of Superior Students - Mathematics* (*SRBCSS-M*) (Renzulli et al., 2004). Tables 4 and 5 provide a summary of the participants' achievement and teacher perceptions.

Table 4

Mathematical Achievement of Study Participants*

Gender	*n*	Minimum	Maximum	Mean	Std. Deviation
Female	47	220	400	283.9	33.4
Male	42	221	344	285.5	29.2

* 2004 Connecticut Mastery Test Mathematics Scores.
Advanced: 293-400, Goal: 245-292, Proficient: 215-244, Basic: 191-214.

Table 5

Teacher Perceptions of Study Participants' Mathematical Ability*

Gender	*n*	Minimum	Maximum	Mean	Std. Deviation
Female	47	20	59	40.7	9.9
Male	42	19	56	40.7	9.9

Scales for Rating the Behavioral Characteristics of Superior Students – Mathematics (Renzulli et al., 2004).

The desired sample size was $n \geq 110$ as determined by the rules of thumb developed by Green and provided in Tabachnick and Findell (2001) of $n \geq 50 + 8m$ for multiple correlations and $n \geq 104 + m$ for testing individual predictors, where m equals the number of independent variables. In this study there are six independent variables. All of the seventh grade students were offered the opportunity to participate and 83% (139) completed some of the instruments. Incomplete data were obtained on 50 students which resulted in their removal from the study population. The reasons for the removal of students from the study included the lack of achievement data on students new to the school, participants choosing to opt out of the study as the data were collected or students choosing not to complete one of more of the instruments. The sample size used in this study was 89. While the sample size was less than recommended by Green, it was considered acceptable.

Research Design

To answer the general research question, a standard multiple linear regression was conducted. This method was chosen as it is designed to predict the dependent variable (mathematical creativity) from a linear combination of the independent variables (achievement in mathematics, attitude towards mathematics, student perceptions of creative ability, teacher perceptions of mathematical talent and creative ability, and gender) with maximum accuracy (Glass & Hopkins, 1996). The linear regression model used was:

$$y_{CMAT} = \beta_0 + \beta_1(CMT) + \beta_2(FS) + \beta_3(WKOPAY) + \beta_4(SRCBSS_M) + \beta_5(SRCBSS_C) + \beta_6(gender) + e_{CMAT}$$

- Y_{CMAT} is the combined scores flexibility, fluency and originality scores from the *CAMT* (Balka, 1974a) (Appendix A)
- CMT is the student's scaled score on the *Connecticut Mastery Test of Mathematics* (Connecticut State Board of Education, 2001) (Appendix C)
- FS is the student's score on the *Fennema-Sherman Mathematics Attitude Scales* (Fennema & Sherman, 1976) used in this study (Appendix D)

- WKOPAY is the composite score on the *What Kind of Person Are You Inventory* of the Khatena Torrance Creative Perception Inventory (Khatena & Torrance, 1976) The instrument is available from the publisher: Scholastic Testing Service.
- $SRBCSS_M$ is the teacher's rating of the student on the *Scales for Rating the Behavioral Characteristics of Superior Students in Mathematics* (Renzulli et al., 2004) (Appendix E)
- $SRBCSS_C$ is the teacher's rating of the student on the *Scales for Rating the Behavioral Characteristics of Superior Students in Creativity* (Renzulli et al., 2004) (Appendix E)

Instrumentation

Creative Ability in Mathematics Test (CAMT) (Balka, 1974a)

The *CAMT* was developed and tested with a sample of 500 middle school students (grades 6, 7 and 8) (Balka, 1974a). Balka's use of content experts and the high response to his survey (81.3% of 300 content experts surveyed) provides a high level of confidence in the content-validity of the instrument. Balka reported the reliability of the *CAMT* as $r_{xx} = .72$ (Cronbach's alpha) and a standard error of measurement of 7.24. Reliability analyses for the present study data yielded comparable results with a Cronbach's alpha of $\alpha = .86$ and a standard error of measurement of 5.16.

Balka (1974a) conducted a factor analysis using a principal components analysis with an orthogonal rotation of the survey results from 490 sixth, seventh and eighth grade students. Two factors described as relatively independent were identified, one divergent and one convergent. His factor matrix is reproduced in Table 6. This analysis matches the instrument's designed factors with factor A containing the divergent items and factor B the convergent ones.

Table 6

Rotated Factor Matrix for the CAMT (n = 490) (Balka, 1974a, p. 106)

Variable	Factor A Divergent Tasks	Factor B Convergent Tasks
Item II	**0.66**	0.03
Item III	**0.64**	0.26
Item V	**0.72**	-0.10
Item VI	**0.58**	0.27
Item I	0.18	**0.51**
Item IV	-0.06	**0.86**

Item analysis for Item I indicated that it was not suitable for measuring the designed criterion as a high percentage of students answered it correctly (Balka, 1974a, p. 112). Removing Item I from the data analysis left a single item factor for the convergent tasks. As the scoring for convergent items on the *CAMT* is binary (1 for a right answer, 0 for a wrong one) (Balka, p. 84), there is no assessment of creativity of the student's approach to arriving at the solution and thus a single item to measure a students' ability to arrive at a right answer is of questionable value. Therefore, only the items in factor A, items II, III, V and VI, the divergent tasks, were used in the regression analysis within the present study.

Connecticut Mastery Test, Third Generation – Mathematics (Connecticut State Board of Education, 2001)

The study participants' composite scaled scores on the 2003-2004 school-year (sixth grade) *Connecticut Mastery Test of Mathematics* were obtained for use in this study. Connecticut reports student achievement data for fourth, sixth and eighth grades. Seventh grade tests are given but not subject to the same level of review. Reliability for sixth grade mathematics portion of the mastery test is reported as .96 (Cronbach's alpha) (Connecticut State Board of Education, 2005). Sample test items can be found in Appendix C.

Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976).

The population used for the development of the scales was comprised of 180 males and 187 females in grades 9 through 12 (Fennema & Sherman, 1976, p. 13). Within the present study, data on student attitude were collected using four of the *Fennema-Sherman Mathematics Attitude Scales*. The items of the four scales selected for this study were combined and randomly listed on a single survey*, How I Feel About Math* (Appendix D) that was distributed to the participants of this study. This approach is consistent with the recommendation of the instrument developers when scales are used in sets of two or more (Fennema & Sherman).

The *Attitude Towards Success in Mathematics Scale* measures the degree to which students anticipate positive or negative consequences as a result of success in mathematics. The *Confidence in Learning Mathematics Scale* measures a student's confidence in learning and performing mathematical tasks. The *Mathematics Anxiety Scale* is intended to measure feelings of anxiety, dread, and nervousness associated with mathematics. The *Effectance Motivation Scale* is designed to measure a student's motivation for involvement in mathematics. Fennema and Sherman (1976) reported split-half reliabilities for each scale. A comparison of the reported reliabilities with those calculated from data collected in this study is provided in Table 7.

Table 7

Comparison of Reported and Calculated Reliabilities for the Fennema-Sherman Scales

Scale	Fennema-Sherman Reported 1976 (split-half)	Present Study Data Calculated (Cronbach's alpha)	Number of items
Attitude Towards Success in Mathematics Scale	.87	.78	11
Confidence in Learning Mathematics Scale	.93	.94	12
Mathematics Anxiety Scale	.89	.91	12
Effectance Motivation Scale	.87	.86	12

Three of the four reliabilities from this study compared favorably with those found by Fennema and Sherman. A review of the data was done to seek an explanation of the disparity in reliabilities for the attitude scale. This review discovered an omission of an item from this subscale on the instrument distributed to the study participants that may have contributed to the reduced reliability value.

Fennema-Sherman (1976) provides conversions from raw to *t*-scores using their sample of 588 females and 642 males. However, gender differences are considered in the conversion tables, with different *t*-scores by gender. As gender is an independent variable in the present study's regression, raw scores were used.

Fennema and Sherman (1976) conducted a principal components factor analysis of the combined scales. They excluded the *Mathematics Anxiety* scale because of a .89

correlation between the anxiety and confidence scales. The results of their analysis are provided in Table 8.

Table 8

Factor Loading From Principal Components Analysis for the Fennema & Sherman Scales (1976, p 19).

Scale	Factor	A		B		C		D	
	Sex	F	M	F	M	F	M	F	M
Confidence in Learning Mathematics		**.88**	**.87**	.22	.19	.01	.07	.12	.12
Mother		.34	.21	**.80**	**.79**	.09	.10	.12	.23
Father		.16	.14	**.88**	**.87**	.10	.04	.10	.08
Attitude Towards Success in Learning Mathematics		.12	.23	.19	.25	**.90**	.15	.22	**.92**
Teacher		**.73**	**.68**	.35	.32	.05	.37	.21	.07
Mathematics as a Male Domain		.11	.13	.12	.10	.18	**.96**	**.95**	.13
Usefulness of Mathematics		.36	.44	**.66**	**.64**	.33	.14	.01	.12
Effectance Motivation in Mathematics		**.75**	**.84**	.23	.19	.38	.02	.11	.20

To assess the viability of combining scores to create a composite score for use in the present study, a principal components factor analysis was done using the total scores from each of the individual scales. As the study data had a correlation between anxiety and confidence of .87, similar to that reported by Fennema and Sherman, anxiety was deleted (Table 9). The factor analysis using the remaining three scales extracted a single factor with an eigenvalue of 1.84 and 61.2% of the variance explained. The students' combined score on the three scales was used as the measure of attitude towards mathematics in the regression analysis.

Table 9

Intercorrelations Between the Raw Scores on Fennema-Sherman Mathematics Attitude Scales for Effectance, Confidence, Anxiety and Attitude (n = 89)

Measure	1	2	3	4
1. Effectance	--			
2. Confidence	.650**	--		
3. Anxiety	.596**	.871**	--	
4. Attitude	.325**	.237*	.091	--

** Correlation is significant at $p \leq .01$(2-tailed)

Scales for Rating the Behavioral Characteristics of Superior Students (Renzulli et. al., 2004)

Creativity (*SRBCSS-C*) This scale was designed to measure teacher estimates of student creativity characteristics in grades 3-12. The reported reliability (Cronbach's alpha) is .84. The reliability calculated from the present study data was .96.

Mathematics (*SRBCSS-M*) This scale was designed to measure teacher estimates of student mathematical talent in grades 3 – 12. The reported reliability (Cronbach's alpha) is .97. Using the present study data, a reliability value of .97 was also found.

What Kind of Person Are You? (Khatena & Torrance, 1998)

The developers reported a split-half reliability of .59 with a Spearman-Brown correction. Present study data yielded a Cronbach's alpha .63 for the 50 item instrument.

Data Collection

The *CAMT*, the *Fennema-Sherman Mathematics Attitude Scales* and the Khatena-Torrance *What Kind of Person Am I?* were administered during a regular 90 minute mathematics class period during the Spring of 2005. Students were given the *CAMT* first and as they finished they were given the attitude and creativity surveys. A small percentage of students did not complete all the surveys during the class period and were offered additional time. Data from the *Connecticut Mastery Tests* were retrieved from student records. Teacher perceptions of student creativity and mathematical ability were obtained via the *SRBCSS* scales completed by each participant's mathematics teachers.

Scoring of Instruments

With the exception of the *CAMT* and the *Connecticut Mastery Test,* scoring the instruments involved simple tabulations of responses. Each student's total scaled mathematics score from the *Connecticut Mastery Test* was obtained from his or her academic records. A random selection of 15 *CAMT*s representing approximately 17% of the total sample were selected and scored by two individuals using guidelines developed by Balka (1974a) (Appendix G). Differences in scores were discussed and agreement among the scores was achieved. The remainder of the tests were scored by a single individual. The flexibility score on the *CAMT* reflected the number of problems a student generated. Fluency was measured by the different categories of answers. Originality scores were based on category weights that reflected the percentage of Balka's sample population that provided an answer within a particular category. A weight of 0 was assigned to those categories that 5% or more of the sample population included in the set of problems they created. Categories which 1% to 4.99% of the population included in their problems received a weight of 1. If less then 1% of the population included a problem in a category, then a 2 was assigned. The originality score was calculated by multiplying each answer by its respective weight and then totaling the resulting products (Balka, p. 69).

Data Analysis – Assumptions of Statistical Tests

Normality, linearity and homoscedasticity

Prior to analysis, each of the independent variable data sets was assessed to determine the degree to which a normal distribution was represented. Skewness and kurtosis ratios (Table 10) were examined and issues of normality were highlighted for the *CMT-Mathematics* scores. All of the measures were considered within acceptable limits.

Table 10

Skewness and Kurtosis Ratios for the Independent Variables (n = 89)

Variable	*Skewness*[a]	*Skewness Ratio*	*Kurtosis*[b]	*Kurtosis Ratio*
CMT - Mathematics	.59	2.32	1.64	3.23
SRBCSS – Mathematics	-.55	-2.14	-.51	-1.00
SRBCSS – Creativity	-.11	-.48	-.44	-.87
WKOPAY?	-.55	2.14	.23	.46
Fennema-Sherman scales	-.21	.84	-.44	-.87

a. Std. Error = .26.
b. Std. Error = .51.

Next the data sets were examined for univariate outliers. Three cases were identified. The first case (Case 53, participant 374) scored 3.4 standard deviations below the mean on the *What Kind of Person are You.* For this participant all other data were within one standard deviation from the mean. The second and third cases (case 24 and 31, participants 338 and 345) scored 2.6 and 3.7 standard deviations respectively above the mean on *CMT – Mathematics*. These two cases contributed to the non-normality of the distribution of scores. The recalculated kurtosis ratio without these data points for *CMT-Mathematics* was .32. As the study goal was to examine alternative options for the identification of creative potential in mathematics, all three univariate outliers remained in the data set.

An examination for multivariate outliers was done using SPSS regression and the RESIDUALS=OUTLIERS (MAHAL) syntax, as described in Tabachnick and Findell (2001, p. 93). The identification number was used as the dummy dependent variable. Evaluating the Mahalanobis distance as a χ^2 with six degrees of freedom and $p < .001$, no multivariate outliers were identified. The data satisfied the assumption of multivariate normality and the relationships between the variables was homoscedastic (Tabachnick & Findell, 2001, p. 79).

<u>*Mutlicollinearity*</u>

Examination of the bivarate correlations and colinearity tolerances (Table 11) of the independent variables raised issues of multicollinearity between *SRBCSS – Mathematics* and the other variables. However, as the regression model ran successfully in SPSS all variables were retained and evaluated.

Table 11

Correlations and Collinearity Tolerances for the Independent Variables (n=89)

Measure	1	2	3	4	5	6	Tolerance
1. Gender	--						.902
2. *SRBCSS – Mathematics*	-.01	--					.386
3. *SRBCSS – Creativity*	.18	.64**	--				.509
4. *WKOPAY*	.24*	.06	.28**	--			.864
5. *Fennema-Sherman*	.06	.46**	.26*	-.03	--		.745
6. *CMT Mathematics*	.03	.63**	.42**	-.00	.44**	--	.570

** Correlation is significant at $p \leq .01$ (2-tailed).
* Correlation is significant at $p \leq .05$ (2-tailed).

Chapter IV
Results

This chapter presents the results of the statistical data collected from the study participants' academic records (mathematics achievement), classroom teacher perceptions (general creativity and mathematical ability), self-reported attitudes (attitudes towards mathematics and perceptions of creativity) and performance on a measure of mathematical creativity.

Research Question

Is there a measure, or combination of measures, that accurately predicts student performance on the Creative Ability in Mathematics Test (CAMT)?

(a) Does a measure of student achievement in mathematics contribute to the prediction of student performance on the CAMT, after controlling for teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics, student perception of his/her creative ability and gender?

(b) Does teacher perception of student general creativity contribute to the prediction of student performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of mathematical talent, student attitude towards mathematics, student perception of his/her creative ability and gender?

(c) *Does teacher perception of mathematical talent contribute to the prediction of student performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of general creativity, student attitude towards mathematics, student perception of his/her creative ability and gender?*

(d) *Does student attitude towards mathematics contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student perception of his/her creative ability and gender?*

(e) *Does student perception of his/her creative ability contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics and gender?*

(f) *Does gender contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics and student perception of his/her creative ability?*

Research Findings

To investigate the research questions, a standard multiple regression was performed. The multiple regression analysis evaluated how well the independent variables predicted student performance on a measure of mathematical creativity in the educational setting. The independent variables were mathematical achievement, teacher perceptions of student mathematical and creative talent, student perceptions of their own creativity, student attitudes towards mathematics and gender. The criterion variable was the student score on the divergent tasks from the *Creative Ability in Mathematics Test* (Balka, 1974a). The linear combination of variables was significant, F (6, 82) = 7.47, p <.0001. Four of the independent variables, mathematical achievement, student perceptions of their own creativity, student attitudes towards mathematics and gender, contributed significantly to the prediction of the participant's score on the *CAMT*. The 95% confidence intervals for these four variables are provided in Table 12 and confirm the significance of the variables as zero does not fall within the interval.

Table 12

95% Confidence Intervals for the Coefficients of the Independent Variables

Variable	*95% Confidence Interval for B*	
	Lower Bound	*Upper Bound*
Significant Predictors		
CMT - Mathematics	.063	.282
Gender	-11.840	-1.196
Fennema-Sherman	.021	.270
WKOPAY	.085	1.089
Non-Significant Predictors		
SRBCSS – Creativity	-.271	.609
SRBCSS – Mathematics	-.480	.356

The sample multiple correlation coefficient was .595, indicating that approximately 35% of the variance in mathematical creativity scores was accounted for by the linear combination of independent variables. Under Cohen's (1988) guidelines, the correlation coefficient indicated a moderately strong relationship. Table 13 provides a summary of data used in the regression analysis, Table 14 a summary of the regression analysis and Table 15 a summary of the part and partial R^2 for the independent variables.

Table 13

Means, Standard Deviations, and Intercorrelations for the Composite Score on the CAMT Divergent Items and the Independent Variables (n=89)

Variable	*M*	*SD*	1	2	3	4	5	6
CAMT Score	23.5	14.4	.48**	.38*	.31**	.18	.39**	-.13
Independent Variables								
1. *CMT - Mathematics*	284.7	30.6	- -	.63**	.42**	-.00	.44**	.03
2. *SRBCSS – Mathematics*	40.6	9.8		--	.64**	.06	.46**	-.01
3. *SRBCSS – Creativity*	34.7	8.1			--	.28**	.27*	.18
4. *WKOPAY*	25.67	5.4				--	-.03	.24*
5. *Fennema-Sherman*	129.1	23.7					.--	.06
6. Gender								--

* $p < .05$. ** $p < .01$.

Table 14

Regression Analysis Summary for the Composite Score on CAMT Divergent Items

Variable	*B*	*SEB*	*β*
(Constant)	-59.6	14.17	
CMT – Mathematics	.17	.06	.37**
SRBCSS – Mathematics	-.06	.21	-.04
SRBCSS – Creativity	.17	.22	.09
WKOPAY	.59	.25	.22*
Fennema-Sherman	.15	.06	.24*
Gender	-6.52	2.68	-.23*

Note. $R^2 = .35$ ($n = 89$, $p < .01$).
* $p < .05$. ** $p < .01$.

Table 15

Summary of Partial and Part R^2

Dependent Variable	Partial R^2	Part R^2
Gender	.068	.047
SRBCSS-Mathematics	.001	.001
SRBCSS-Creativity	.007	.005
WKOPAY	.062	.042
Fennema-Sherman	.062	.042
CMT-Mathematics	.107	.078

As the correlation between the *SRBCSS-Creativity* and *SRBCSS-Mathematics* and the *CAMT* scores were significant but neither independent variable contributed significantly to the regression, a post hoc evaluation of the correlations was done using the method recommended by Larzelere and Mulaik (as cited in Tabachnick & Findell, 2001). This evaluation revealed that the correlation between *CAMT* and *SRBCSS-Creativity* was not significantly different from zero, F (6, 82) = 1.41, p = .35. Examination of the squared part correlations yielded expected R^2 change of .01 if *SRBCSS-Creativity* was dropped from the regression model. The correlation between *CAMT* and *SRBCSS-Mathematics* was significantly different from zero, F (6, 82) = 2.28, p = .04 yet the squared part correlation was .0007. Thus *SRBCSS-Mathematics* scores are redundant to or mediated by the linear combination of the other independent variables.

Research Question – Part (a)

> *Does a measure of student achievement in mathematics contribute to the prediction of student performance on the CAMT, after controlling for teacher perception of student general creativity, student attitude towards mathematics, student perception of his/her creative ability and gender?*

The *CMT-Mathematics* score was used as the measure of student achievement in mathematics. The regression coefficient for *CMT-Mathematics* was significant, t = 3.14, p =.002. For every one-point increase in *CMT-Mathematics* scores, *CAMT s*cores increased .17 points. Squared partial correlations represent the amount of variance in *CAMT* scores explained by the *CMT-Mathematics* scores after the effects of the other independent variables have been removed. With this data set and linear combination of independent variables, the *CMT-Mathematics* score represents 10.7% of the variance in the *CAMT,* scores after controlling for teacher perception of student general creativity and mathematical ability, student attitude towards mathematics, student perception of his/her creative ability, and gender. Squared part correlations provide the proportion of variance in *CAMT* explained by the *CMT-Mathematics* beyond what is explained by the other independent variables, or the unique relationship between the *CMT-Mathematics* and *CAMT,* after the variance shared with other variables is removed. The importance of an independent variable is best measured by squared part correlation that equals the decrease in R^2 if the independent variable were removed. Within the context of this analysis, the contribution to R^2 from *CMT-Mathematics* was .078. Therefore, this analysis supported the inclusion of a measure of mathematical achievement in a linear combination of independent variables for creative ability in mathematics.

<u>Research Question – Part (b)</u>

Does teacher perception of student general creativity contribute to the prediction of student performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of mathematical talent, student attitude towards mathematics, student perception of his/her creative ability and gender?

The *SRBCSS-Creativity* was used to assess teacher perceptions of student general creativity. In the models, the regression coefficient for this variable was non-significant, $t = .764$, $p = .447$. An examination of the part and partial R^2 show that this variable's contribution to the prediction of *CAMT* scores was minimal contributing 0.5% to the variance explained over and above what was explained by other variables in this model.

Thus, teacher perceptions of general creativity were not a significant contributor to the prediction of student scores on the *CAMT* in this study.

<u>Research Question – Part (c)</u>

> *Does teacher perception of mathematical talent contribute to the prediction of student performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of general creativity, student attitude towards mathematics, student perception of his/her creative ability and gender?*

The *SRBCSS-Mathematics* was used to assess teacher perceptions of student mathematical talent. In the models, the regression coefficient for this variable was non-significant, $t = -.297$, $p = .767$. An examination of the part and partial R^2 show that this variable's contribution to the prediction of *CAMT* scores was minimal contributing 0.1% to the variance explained over and above what was explained by other variables in this model. Thus, teacher perceptions of mathematical ability were not a significant contributor to the prediction of student scores on the *CAMT* in this study.

<u>Research Question – Part (d)</u>

> *Does student attitude towards mathematics contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student perception of his/her creative ability and gender?*

The combined scores on the Fennema-Sherman subscales, *Attitude towards Mathematics*, *Confidence in Learning Mathematics* and *Effectance Motivation* were used to assess student attitude for this question. Regression analysis revealed that student attitude was a significant predictor of scores on the *CAMT,* $t = 2.23$, $p = .02$. For every one point increase in the score on the *Fennema-Sherman* subscales on average, a .15 increase in *CAMT* scores can be expected. Scores on the *Fennema-Sherman* subscales represent 6.2% of the variance in the *CAMT* scores, after controlling for mathematical achievement, teacher perception of student general creativity and mathematical ability, student perception of his creative ability, and gender. Within the context of this analysis, the contribution to R^2 from *Fennema-Sherman* subscales was .042. Therefore, this analysis supported the inclusion of a measure of student attitudes in a linear combination of independent variables for creative ability in mathematics.

Research Question – Part (e)

> *Does student perception of his/her creative ability contribute to the prediction of performance on the CAMT after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics and gender?*

The *What Kind of Person Are You?* instrument from the *Khatenna-Torrance Creative Perception Inventory* was used to assess student self-perceptions of creativity. The regression coefficient for student perceptions of general creativity was significant, $t = 2.23$, $p = .02$ with an average increase in *CAMT* scores of .15 for each one point increase in the student's score on this instrument. The contribution of student perception is the same as found for student attitudes with scores on the *What Kind of Person Are You?* representing 6.2% of the variance in the *CAMT* scores, after controlling for mathematical achievement, teacher perception of student general creativity and mathematical ability, student attitude towards mathematics and gender. Within the context of this analysis, the contribution to R^2 from students' perception of their own creativity was .042. Therefore, this analysis supported the inclusion of this measure in a linear combination of independent variables for creative ability in mathematics.

Research Question part (f)

> *Does gender contribute to the prediction of performance on the CAMT, after controlling for student achievement in mathematics, teacher perception of student general creativity, teacher perception of mathematical talent, student attitude towards mathematics and student perception of his/her creative ability?*

Gender data was entered into SPSS with a code of 0 for female and 1 for male. In the regression model, gender had a statistically significant regression coefficient, $t = 3.14$ $p = .002$. On average, females scored 6.5 points higher on the *CAMT* than did males after controlling for the other variables in the regression model. Gender differences represented 6.8% of the variance in the *CAMT* scores, after controlling for mathematical achievement, teacher perception of student general creativity and mathematical ability, student perception of her creative ability and student attitude towards mathematics. Within the context of this analysis, the contribution to R^2 from gender was .047. Therefore, this analysis supported the inclusion of gender in a linear combination of independent variables for creative ability in mathematics.

Chapter V

Discussion, Implications and Recommendations

This chapter provides a summary of the findings from the regression model used in this study followed by a discussion of the implications drawn from the results. An acknowledgment of the study's limitations and suggestions for future research are also included.

Discussion

This study was undertaken with the hope that simpler ways to identify creative potential in mathematics could be found allowing for earlier identification and a deepening of the recognized talent pool of future mathematics. Within the regression model used in the present study, mathematical achievement was the strongest predictor of student performance on the *CAMT*. It accounted for 23% ($.48^2 = .23$) of the variance in creativity scores while the other variables contributed only 12% (35% - 23% = 12%). However the interpretation of the relative importance of the independent variables is complicated by correlations among them. For example, the *Fennema-Sherman* attitude scales were a significant predictor in this regression model with a bivariate correlation with mathematical creativity of .39. In contrast, both of the *SRBCSS* scales also showed moderately strong correlations with the mathematical creativity scores obtained from the instrument used in the present study (.38 and .31) yet were not significant. Table 16 provides a summary of the bivariate and partial correlations with *CAMT* for the six independent variables used in this study.

From the review of literature it was not anticipated that mathematical achievement as measured by standardized test scores would prove to be the strongest predictor. Prouse (1967) had reported a correlation $r = .53$, $p = .01$ between performance on his test of mathematical creativity and the *Iowa Tests of Basic Skills* composite scores. While he had sub-scores for problem solving and computation, no analysis of the relationship of these sub-test scores to mathematical creativity was reported. Prouse's analysis focused on the relationship of overall academic achievement and mathematical creativity. Jensen (1973) reported a weak, positive relationship between measures of mathematical achievement and mathematical creativity. However, she cautioned against the use of traditional mathematical achievement tests as a predictor of mathematical creativity as there are high achievers with low creativity scores and highly creative individuals who do not perform well on achievement tests.

Table 16

The Bivariate and Partial Correlations of the Independent Variables and Mathematical Creativity Scores on the Divergent Items of the CAMT

Independent Variables	Correlations between each independent variable and the CAMT score	Correlation between each independent variable and the CAMT score controlling for all other independent variables
Mathematical Achievement (*CMT- Mathematics*)	.48**	.33**
Student Attitudes Towards Mathematics (*Fennema-Sherman Scales*)	.39**	.33*
Self-perception of individual's creativity (Khatena-Torrence *WKOPAY?)*	.18	.25*
Teacher's perception of the student's mathematical ability *(SRBCSS-Mathematics)*	.38*	-.03
Teacher's perception of the student's creativity *(SRBCSS-Creativity)*	.31*	.08
Gender	-.13	-.26*

* $p < .05$. ** $p < .01$.

Jensen (1973) included data on participants' mathematical achievement and scores on a test of mathematical creativity she created, based on the *How Many Questions* test developed by Getzels and Jackson (1962), in her work. Similar to the *CAMT*, participants in her study were presented with situational information from which they were to develop problem statements. Student achievement data for mathematical computation and problem solving were obtained from the participants' scores on the *Metropolitan Achievement Test*. Using Jensen's data, a multiple regression was run using computation and problem solving scores as a predictor of mathematical creativity. The model was significant, F (2, 229) = 13.0, p >.001 and explained 10% of the variance in mathematical creativity scores on the test created by Jensen. A summary of the model results is provided in Table 18. As both measures of mathematical creativity and mathematical achievement differed between the present study and Jensen's work, direct comparisons can not be made, but it is significant that in both studies similar relationships were found.

Table 17

Regression Analysis Summary for the Jensen (1973) Data on Mathematical Computation and Problem Solving as Predictors of Mathematical Creativity

				Correlations		
Variable	*B*	*SEB*	*β*	*Zero-Order*	*Partial*	*Part*
(Constant)	-7.1	5.5				
Computation	2.7	1.2	.17*	.409	.225	.206
Problem Solving	2.0	2.28	.18*	.397	.199	.182

Note. R^2 = .10 (n = 232, p < .01).
* p < .05.

Student perceptions of their own creativity and attitudes towards mathematics were significant predictors of performance on the *CAMT* however teacher perceptions of were not. Teachers were asked to complete the SRCBSS scales without any orientation or training on the use of the instrument. While the teachers were aware of the study's purpose it is likely that additional and more detailed discussion was needed. Within the context of traditional mathematics classrooms, student opportunities to demonstrate the types of creativity measured by *SRCBSS-C* are restricted by the classroom environment thus teacher observations are made on limited information. A regression analysis using only *SRCBSS-C* and *SRCBSS-M* as predictors of student performance on the *CAMT* was done. In this model, SRCBSS-C remained a non-significant predictor but *SRCBSS-M* was a significant predictor explaining 6.3% of the variance in *CAMT* scores.

Teachers need opportunities to observe students' creative abilities in a variety of mathematical settings. Information on classroom sociomathematical norms such as acceptance of risk taking, discourse, collaboration and others would have been helpful in assessing the effectiveness of the *SCRBSS Scales* as predictors of performance on the *CAMT*. Teachers completed the scales rapidly and with no training. The results suggest that teachers' ratings in the present study were based more on mathematical achievement than mathematical creativity.

<u>Implications</u>

Haylock (1997) found that students who are equal in mathematical achievement may have significant differences in performance on measures of mathematical creativity. Feldhusen and Westby (2003) asserted that an individual's knowledge base was the fundamental source of their creative thought. Students who have not yet attained sufficient mathematical knowledge and skills may be unable to demonstrate creative mathematical thinking. A scatter plot of the participants' *CAMT* scores versus their *CMT-Mathematics* scores (figure 1) offers support for this relationship between the levels of mathematical achievement and the student responses to divergent-production tasks with a positive slope to the regression line; however, there are exceptions.

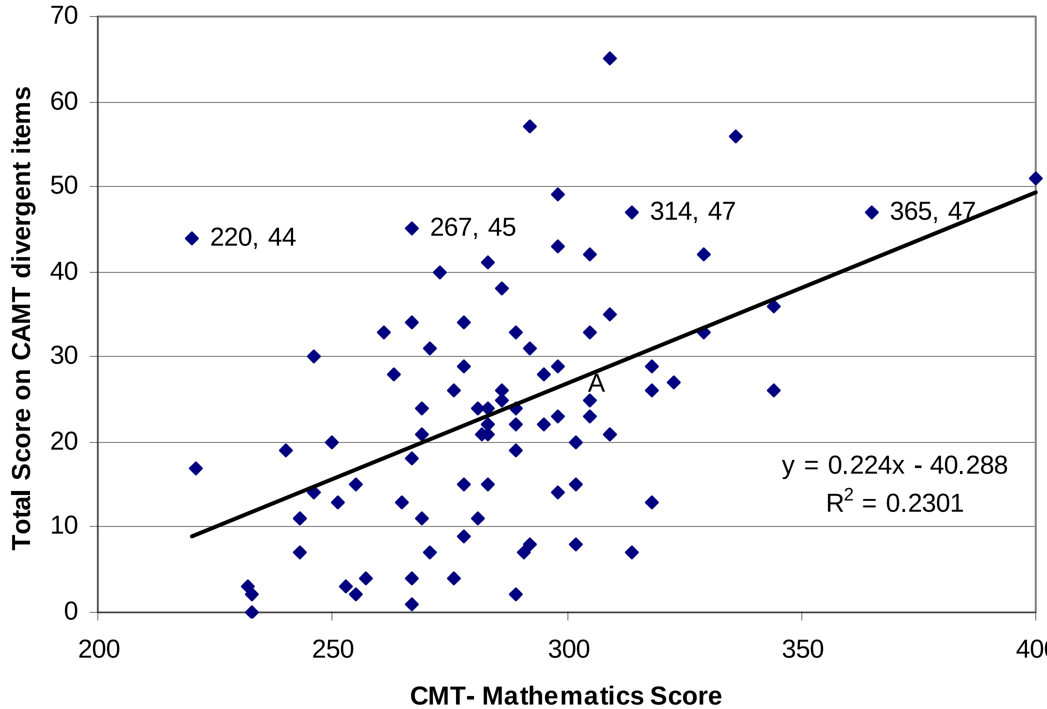


Figure 1 Total Scores on the CAMT Divergent Items versus CMT-Mathematics Scores

The four labeled points on the scatter plot were selected to demonstrate a range of achievement scores with similar creativity scores. Feldhusen and Westby (2003) also theorized that those with high levels of achievement may be constrained by rigidity in their thought patterns.

Plucker and Beghetto (2004) proposed a conceptual model of development of creative domain specificity (Figure 2) that flows from a superficial level of creativity at low levels of experience to a fixed perspective within the domain as high levels of experience are gained. Within this model, a level of interest as well as experience is needed for creativity to emerge. Plucker and Beghetto believe that the optimal condition for creative production falls within a flexible region between generality and specificity. Cunningham (as cited in Haylock, 1987) asserted that drill and the learning of fixed procedures, common for many in school mathematics, may contribute to a child's predisposition for rigidity of thought. This may place limits on a child's ability to formulate problems in situations like those presented on the *CAMT*. Rather than develop problems for which the method of solution is unknown, student responses would be limited to types for which they have had experience solving algorithmically.

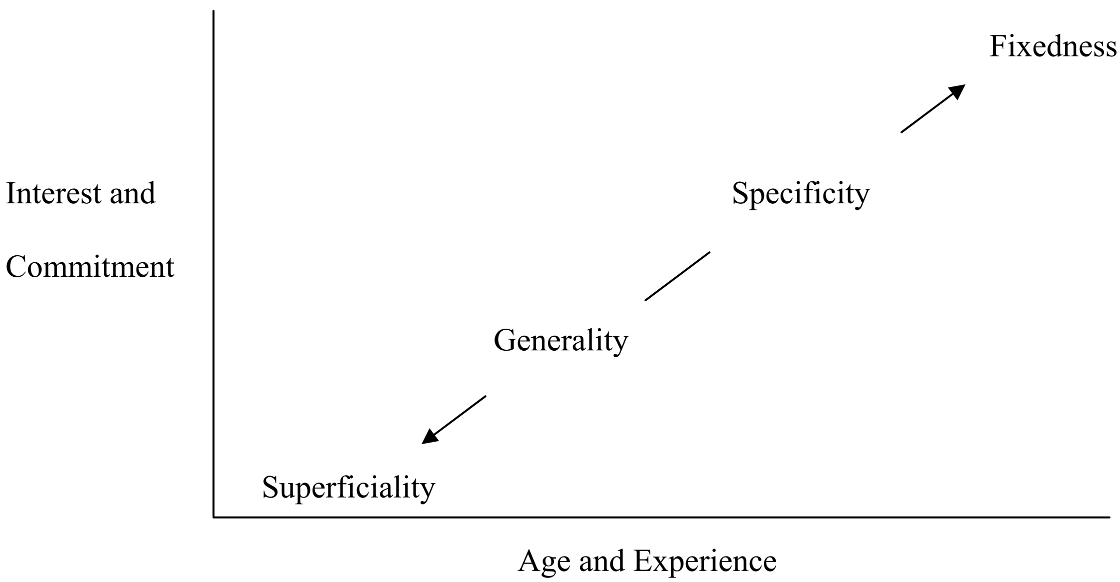


Figure 2 Conceptualization of Domain Specificity and Generality of Creativity (Plucker and Beghetto, 2004, p. 161)

An examination of students with high scores on the *CAMT* was done. Rather than a single cut-off value for the identification of talent, Reis and Renzulli (1982) advocated the use of a talent pool composed of the top 15% to 25% of the student population. Z-Scores were computed for the statistically significant continuous independent variables, *CMT-Mathematics*, *Fennema-Sherman Scales* and *WKOPAY*. Figure 3 illustrates the relationship between these scores and performance on the *CAMT* divergent tasks for the 14 students with a z-score ≥ 1.036

($_{85}Z$ =1.036). Figure 4 provides data on the relationships between these independent variables and *CAMT* performance for the 16 students with z-scores ≤ -1.036. With such a small sample caution in stating implications is warranted. However, the data illustrate that the greatest variation in independent variable scores was in mathematical achievement for students who achieved the highest scores on the *CAMT.* This suggests that a threshold level of knowledge and skills are required for high levels of performance on the *CAMT;* yet, after the threshold level is reached mathematical achievement scores are less important in the prediction of potential creativity. This should not be confused with Getzels and Jackson's (1962) threshold theory of creativity in which a minimum level of intelligence is required for creativity to develop but beyond that level there is no correlation between creativity and intelligence. The distinction here involves levels of intelligence versus levels of achievement. While intelligence may be a factor in achievement, there are many other factors such as motivation, personality, environment, encouragement, self-efficacy, that have an impact on developing creativity.

A further exploration of the relationship between mathematical achievement and mathematical creativity as measured by the *CAMT* was done. Two additional simple linear regressions were conducted. For both, the model $y_{CMAT} = \beta_0 + \beta_1(CMT) + e_{CMAT}$ was used. The first regression was done using students who scored above the sample mean score for the *CAMT* (23.49) and the second those who scored below the mean. The results (Tables 18 and 19) offer further support to the theory that there is experience threshold necessary for creativity to emerge. Below the mean score on the *CAMT* mathematics achievement was a significant predictor, above the mean it was not.

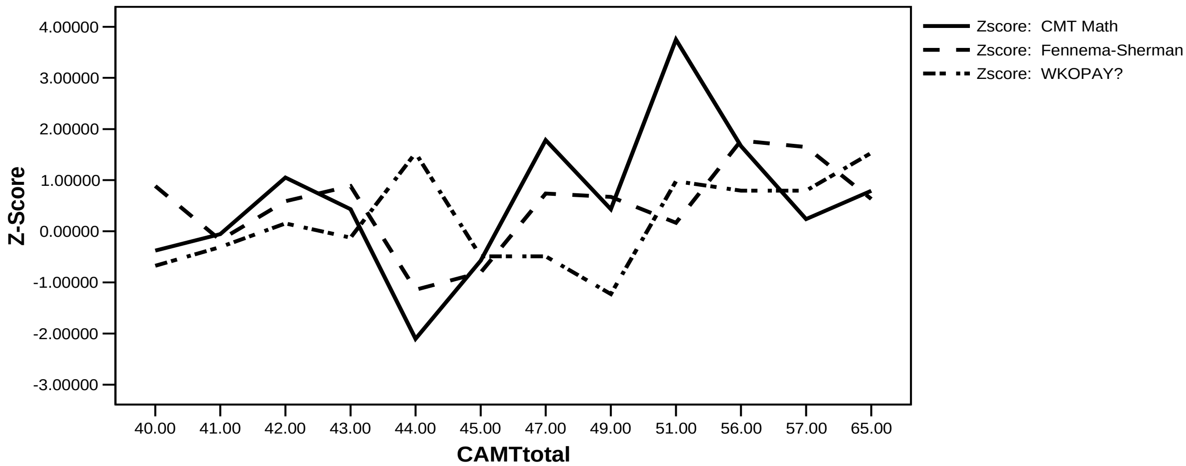


Figure 3 The Relationships Between Scores on the Significant Independent Variables and the Total Score on the CAMT Divergent Items for the Top 15% of the Study Sample

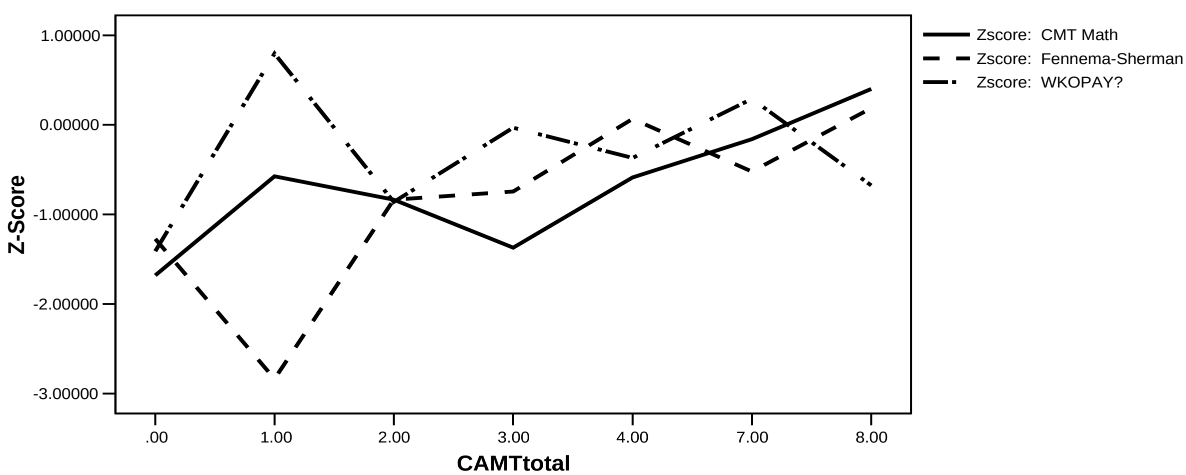


Figure 4 The Relationships Between Scores on the Significant Independent Variables and the Total Score on the CAMT Divergent Items for the Bottom 15% of the Study Sample

Table 18

Means, Standard Deviations, and Intercorrelations for the composite score above and below the sample mean on the CAMT divergent items and CMT-Mathematics

	CAMT < mean score				*CAMT* > mean score			
Variable	*M*	*SD*	*R*	*n*	*M*	*SD*	*R*	*n*
CAMT Score	12.79	7.25	.37**	47	35.48	10.27	.28	42
Independent Variable								
CMT - Mathematics	273.60	24.78			297.10	32.28		

** $p = .01$.

Table 19

Regression Analysis Summary for the composite score above and below the sample mean on CAMT divergent items

	CAMT < mean score			*CAMT* > mean score		
Variable	*B*	*SEB*	*β*	*B*	*SEB*	*β*
(Constant)	-16.96	11.12		9.41	14.45	
CMT - Mathematics	.11	.04	.37**	.09	.05	.28

Note:
$R^2 = .14$ ($n = 47, p = .01$).
$F(1, 45) = 7.22$, $p = .01$.
** $p = .01$.

Note:
$R^2 = .08$ ($n = 42, p = .08$).
$F(1, 40) = 3.29$, $p = .08$.

Creativity is most often described in terms of the person, product, process and press (Runco, 2004). Within the context of the present study the focus was on person (attitudes and perceptions) and product (outcomes in terms of the flexibility, fluency and originality of responses). Press was defined by Rhodes as “the relationship between human beings and their environment” (as cited in Runco, p. 662). Runco suggests that classroom expectations to conform, characteristic of many educational settings, may contribute to a drop in student originality. Yet breaking from established mindsets or fixations on process (Balka, 1974a; Haylock, 1997; Krutetskii, 1976) was a defining feature in the efforts of others to understand the creative mathematician. Urban (as cited in Seo, Lee & Kim, 2005) defined creativity in terms of cognitive aspects, personality and environment. Thus, environment and experience may play a significant role in the emergence of creative behaviors.

Creative personality theory also draws on the role of experience in the development of creativity. Selby, Shaw and Houtz (2005) wrote about the role of experience in individual choice. Experience leads to choices made when forming and solving problems. These choices lead to additional experiences that either positively or negatively contribute to future choices. These experiences in turn become the basis for an individual's creative style. From their review of divergent creativity personality theories, Selby, Shaw and Houtz found that environmental interaction was important in the development of creativity. If problem solving is taught through "bottom line teaching" where the student is held accountable only for the method of problem solving presented by the teacher (Crosswhite, 1987), then an environment which discourages creativity is formed. Such an environment perpetuates the common misconception that creativity and mathematics are unrelated (Pehkonen, 1997).

The regression model in this study left 65% of the variance in mathematical creativity scores unexplained. Data on the participants' experiences with divergent problem solving activities and multi-year experiences with school mathematics may account for some of the unexplained variance. The emerging construct of an individual's creative problem solving style (Selby, Shaw & Houtz, 2005) provides a different lens through which to study developing creativity in mathematics. Selby, Treffinger, Isaksen and Lauer (2002) have developed an instrument entitled *VIEW: An Assessment of Problem Solving Style* for use with individual ages 12 through adult. This instrument provides information on three dimensions of an individual's problem solving style; orientation to change, manner of processing, and ways of deciding. As with other human attributes, levels of creativity should be considered as a continuum. In the traditional mathematics classrooms ability, is recognized as a continuum, however; student preferences and style are often not. Creating an environment that acknowledges problem solving style differences, similar to instruction in thinking or learning styles, in which students can develop an understanding of their creative problem solving style, offers promise in the development of mathematical creativity.

Limitations

The application of the results of this study to other populations is restricted due to several limitations inherent in the design. The sample was drawn from a population attending a local middle school. The community was comprised predominately of middle and upper-middle class families with limited cultural diversity. Thus, the results of this study are not generalizable to other populations. In addition, *CAMT* has had limited testing. The *CAMT* measures only a student's ability to formulate problems and does not fully address the set of criteria Balka (1974a) developed as measures of mathematical creativity. Another limitation involved the use of written language. Some students had difficulty expressing their ideas. For example, in response to Item II, one participant wrote "The number of diagonals decrease the number of triangles not formed." The child's intention is unclear. Is he saying that additional triangles are formed as the number of diagonals increase by using the double negative or is his understanding of the vocabulary incorrect? The group administration of the test did not offer the opportunity to follow up with this student. The need to interpret the students' written responses increased the subjective effects on scoring of the *CAMT*.

Group administration also influenced student performance by imposing artificial constraints. On several occasions pairs of students would finish within minutes of each other and then work together on other activities. Social considerations (i.e., working with a friend) may have influenced student performance. Time constraints imposed by school schedules may have been a factor when students declined the opportunity for extra time to complete the instruments. Contextual issues were also noted when scoring *CAMT* responses. Item III asked the students to consider the things that would happen if the only item they could use to draw geometric figures was a globe used for geography or a large ball. Many of the students responded with answers related to latitude or longitude or other geography specific comments. Other students wrote about using the ball to play games in the classroom rather than as a surface on which to draw. Without the opportunity to do individual student follow-ups, it was not possible to determine the student's intent; were these serious attempts to respond or simply an effort to put something on paper?

In the 31 years that have transpired since Balka (1974a) developed the CAMT there have been significant advances in the mathematics education research. National and state standards now exist. The National Science Foundation has funded the development of several curriculums that have a greater emphasis on inquiry and problem solving. While Balka's work provided a thoughtful and rigorously developed instrument it needs to be revalidated within the context of what is now known about how children develop their mathematical abilities.

Suggestions for Future Research

Is there a relationship between experience in mathematics and an individual's level of mathematical creativity?

It appears that a level of experience is necessary for creativity to emerge within a domain. However experience alone does not determine an individual's level of creativity in mathematics. Balka (1974 a, b), Haylock (1997), and Krutetskii (1976) all discuss the need to overcome rigidity of thought for creativity to emerge. Selby, Shaw and Houtz (2005) suggest that different levels of creative problem skills be identified and that appropriate learning experiences be created that match learning experiences to a child's developing level of creativity. This approach to the development of creativity is equitable in that everyone can become a better problem solver, and it is as an important consideration in the development of curricular materials.

Is mathematical creativity within individuals consistent across the various fields of study within mathematics?

Students with spatial strengths may do better with geometric concepts than with abstract mathematics. Students with linguistics strengths may be better at analyzing and formulating problems. Differences in learning styles, creative-problem solving styles, or creative personality may contribute to creative talent in a particular field of mathematics. These hypothesis need to be investigated.

Does the introduction of more complex mathematics at an earlier age have a positive impact on mathematical creativity?

Project M^3: Mentoring Mathematical Minds (http://www.projectm3.org) is an on-going research project at the Neag Center for Gifted Education located at the University of Connecticut, which provides younger students (third, fourth and fifth grade) the opportunity to work with mathematical concepts traditionally introduced in later grades. Initial results show statistically significant growth in student conceptual understanding and problem solving. At the end of this study, do students involved in Project M^3 score higher on measures of mathematical creativity than students in the study's control group?

Are there creative differences by gender in the solving of mathematical problems?

A considerable amount of research has focused on gender differences in mathematics. The present study along with Jensen (1973) and Prouse (1964) have found differences favoring females in the identification of mathematical creativity. Could such differences be caused by social/environmental factors, parental/teacher expectations, physiological differences in growth and development or through some other identifiable factors? Further exploration of this difference is needed.

Are there observable indicators of creative thought and "playfulness" with mathematics that are useful for identifying mathematical talent?

Mathematical talent exists in various levels. A few of these students will emerge as the eminent mathematicians of their generation and others will become skilled practitioners in the field. Runco (2004) notes that "everyday" creativity, which is used as a means of coping and solving day-to-day problems is an emerging area of research. Carlton (1959) found that non-deductive methods such as intuition, experimentation, induction and analogy, speculation, and analysis of errors are also methods used in creative mathematical thought. Indicators of everyday creativity and non-deductive problem solving in mathematics would provide additional tools for identification of mathematical talent.

Are there changes to pedagogical practices that will provide greater opportunities for mathematical creativity to emerge and develop?

Haylock (1997) saw the challenge, not in the identification of creative ability, but in the development of pedagogical practices that are effective in moving students towards creative thought within the context of school mathematics. This study and the literature suggest the need to focus future research on the characteristics of developing creativity in mathematics. Such an understanding is needed to identify effective teaching practices and appropriate curricular materials.

Conclusion

This study examined several factors in the educational setting and their relationships to mathematical creativity in search of a simpler means to identify potentially creative students of mathematics. Statistically significant predictors of student performance on a measure of mathematical creativity were identified. Mathematical achievement was the most significant predictor of performance, explaining 23% of the variance in scores on the *Creative Ability in Mathematics Test* (Balka, 1974a) however 65% variance in scores remained unexplained. Other significant predictors were gender, attitude towards mathematics and self-perceptions of creativity. Teacher perceptions of student mathematical ability and creativity, though highly correlated with the dependent variable, were not significant predictors. High correlations between the independent variables complicated the analysis of the regression model.

Data from this study suggest that there is a relationship between mathematical experiences (knowledge and skills) and creativity in mathematics as measured by the *Creative Ability in Mathematics Test* (Balka, 1974a). However, questions on limitations of this instrument in the measurement of mathematical creativity were also raised within the present study. Future research focusing on providing rich mathematical problem solving experiences tailored to an individual's developing style of creativity may prove a significant means of developing creative mathematical talent in all students.

References

Amabile, T. M. (1989). *Growing up creative: Nurturing a lifetime of creativity*. Buffalo, NY: C.E.F. Press.

Balka, D. S. (1974a). The development of an instrument to measure creative ability in mathematics. *Dissertation Abstracts International, 36*(01), 98. (UMI No. AAT 7515965)

Balka, D. S. (1974b). Creative ability in mathematics. *Arithmetic Teacher, 21,* 633-636.

Becker, J. P., & Shimada, S. (Eds.). (1997). *The open-ended approach: A new proposal for teaching mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Cajori, F. (1928). *Mathematics in liberal education*. Boston: The Christopher Publishing House.

Carlton, V. L. (1959). An analysis of the educational concepts of fourteen outstanding mathematicians, 1790-1940, in the areas of mental growth and development, creative thinking, and symbolism and meaning, *Dissertation Abstracts, 20*(06), 2131. (UMI No. AAT 5904782)

Carpenter, T. P. (1986). Conceptual knowledge as a foundation of procedural knowledge. In James Hiebert, (Ed.), *Conceptual and procedural knowledge: The case of mathematics.* (pp. 113–132). Hillsdale, NJ: Lawrence Erlbaum Associates.

Carroll, J. B. (1996). Mathematical abilities: Some results from factor analysis. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 3-25). Mahwah, NJ: Lawrence Erlbaum Associates.

Ching, T. P. (1997). An experiment to discover mathematical talent in a primary school in Kampong Air. *International Reviews on Mathematical Education, 29*(3), 94-96. Retrieved March 10, 2003, from http://www.fizkarlsruhe.de/fix/publications/zdm/adm97

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). New York: Academic Press.

Committee on Science, Engineering, and Public Policy (2005). *Rising above the gathering storm: Energizing and employing America for a brighter economic future*. Washington, DC: National Academies Press.

Connecticut State Board of Education. (2005). *The 2004 Connecticut Mastery Tests: Generation 3 Technical Report*. Hartford, CT: Author.

Connecticut State Board of Education, (2001). *Connecticut Mastery Tests: Third Generation Mathematics Handbook.* Retrieved, February 14, 2004 from http://www.state.ct.us/sde/dtl/curriculum/mathcmt3/currmath_publ_cmt3.htm

Crosswhite, F. J., (1987). Cognitive science and mathematics education: A mathematician's presepective. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 253 - 277). Mahwah, NJ: Lawrence Erlbaum.

Csikszentmihalyi, M., & Wolfe, R. (2000). New conceptions and research approaches to creativity: Implications of a systems perspective for creativity in education. In K.A. Heller (Ed.), *International handbook of giftedness and talent* (pp. 81-93). Oxford, UK: Elsevier Science.

Davis, G. A. (1969). Thinking creatively in adolescence: A discussion of strategy. In R.E. Grinder (Ed.), *Studies in adolescence* (pp. 538-545). New York: Macmillan.

Davis, R. B. (1986). Conceptual and procedural knowledge in mathematics: A summary analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 265-298). Hillsdale, NJ: Lawrence Erlbaum Associates.

Devlin, K., (2002). The four faces of mathematics. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century* (pp. 16-27). Reston, VA: National Council of Teachers of Mathematics.

Dreyfus, T., & Eisenberg, T. (1996). On different facets of mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 253 - 284). Mahwah, NJ: Lawrence Erlbaum Associates.

Evans, E.W. (1964). Measuring the ability of students to respond in creative mathematical situations at the late elementary and early junior high school level. *Dissertation Abstracts, 25* (12), 7107. (UMI No. AAT 6505302)

Feldhusen, J. F., & Westby, E. L. (2003). Creative and affective behavior: Cognition, personality and motivation. In J. Houtz (Ed.), *The educational psychology of creativity* (pp. 95 – 105). Cresskill, NJ: Hampton Press.

Feldhusen, J. F., & Goh, B. E. (1995). Assessing and accessing creativity: An integrative review of theory, research, and development. *Creativity Research Journal*, *8*, 231 – 247.

Fennema, E., & Sherman, J. A. (1976). *Instruments designed to measure attitudes towards the learning of mathematics by females and males*. Madison, WI: University of Wisconsin.

Ford, D. Y., & Harris, J. J, III. (1992). The elusive definition of creativity. *Journal of Creative Behavior, 26,* 186-198.

Frahm, R.A., (2004, March 21). Feverishly fixing an error; company races to correct inaccurate scoring of Connecticut's mastery test*, The Hartford Courant,* pp. A1.

Getzels, J. W., & Jackson, P. W. (1962). *Creativity and intelligence: Explorations with gifted students*. New York: Wiley.

Ginsburg, H. P. (1996). Toby's math. In R. J. Sternberg & T. Ben-Zee (Eds.), *The nature of mathematical thinking* (pp. 175-282). Mahwah, NJ: Lawrence Erlbaum Associates.

Glass, G. V., & Hopkins, E. D. (1996). *Statistical methods in educational psychology* (3rd ed.). Needham Heights, MA: Allyn & Bacon.

Goldin, G. A. (2002). Affect, meta-affect and mathematical belief structures. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 59-72). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Hadamard, J. (1945). *The psychology of invention in the mathematical field.* Mineola, NY: Dover Publications.

Hashimoto, Y. (1997). The methods of fostering creativity through mathematical problem solving. *International Reviews on Mathematical Education, 29*(3), 86-87. Retrieved March 10, 2003, from http://www.fiz-karlsruhe.de/fix/publications/zdm/adm97

Haylock, D. W. (1984). Aspects of mathematical creativity in children aged 11-12 (Doctoral dissertation, Chelsea College, University of London, 1984). *British Thesis Service*. (The British Library System No. 0009803014).

Haylock, D. W. (1987). A framework for assessing mathematical creativity in school children. *Education Studies in Mathematics, 18(1),* 59-74.

Haylock, D. (1997). Recognizing mathematical creativity in school children. *International Reviews on Mathematical Education, 29*(3), 68-74. Retrieved March 10, 2003, from http://www.fiz-karlsruhe.de/fix/publications/zdm/adm97

Heibert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., et al. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hong, E., & Aqui, Y. (2004). Cognitive and motivational characteristics of adolescents gifted in mathematics: Comparisons among students with different types of giftedness. *Gifted Child Quarterly, 48,* 191-201.

Imai, T. (2000). The influence of overcoming fixation in mathematics towards divergent thinking in open-ended mathematics problems on Japanese junior high school students. *International Journal of Mathematical Education in Science and Technology, 31,* 187-193.

Jackson, N. (2005). *Assessing students' creativity: Synthesis of higher education teacher views.* Unpublished manuscript, The Higher Education Academy. Retrieved October 24, 2005 from www.heacademy.ac.uk/embedded_object.asp?id=21882&prompt=yes&filename=Assessing%20creativity

Jensen, L. R. (1973). The relationships among mathematical creativity, numerical aptitude and mathematical achievement. *Dissertation Abstracts International, 34* (05), 2168. (UMI No AAT 7326021)

Khatena, J., & Torrance, E.P. (1976). *Khatena-Torrance Creative Perception Inventory.* Bensenville, IL: Scholastic Testing Service.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-148). Mahwah, NJ: Lawrence Erlbaum Associates.

Kim, H., Cho, S., & Ahn, D. (2003). Development of mathematical creative problem solving ability test for identification of gifted in math. *Gifted Education International, 18*, 184-174.

Köhler, H. (1997). Acting artist-like in the classroom, *International Reviews on Mathematical Education, 29*(3), 88-93. Retrieved March 10, 2003, from http://www.fizkarlsruhe.de/fix/publications/zdm/adm97

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children.* Chicago: University of Chicago Press.

Mayer, R, E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 29 - 54). Mahwah, NJ: Lawrence Erlbaum Associates.

Mayfield, B., (1979). Teacher perception of creativity, intelligence and achievement. *The Gifted Child Quarterly, 23*, 812-817.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization, In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575 – 596). New York: Macmillan.

Messiner, H. (2000, July-August). *Creativity in mathematics education.* Paper presented at the meeting of the International Congress on Mathematical Education, Tokyo.

Meyer, R. W. (1969). *The identification and encouragement of mathematical creativity in first grade students.* Unpublished doctoral dissertation, University of Wisconsin, Madison.

National Center for Education Statistics. (2005). *Education Statistics Quarterly*, 6, 1-2. Retrieved February 9, 2005 from http://nces.ed.gov/programs/quarterly/vol_6/1_2/5_1.asp

National Council of Teachers of Mathematics. (1980). *An agenda for action*. Reston, VA: National Council of Teacher of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teacher of Mathematics.

Pehkonen, E. (1997). The state-of-art in mathematical creativity. *International Reviews on Mathematical Education*, *29,* 63-66. Retrieved March 10, 2003, from http://www.fiz-karlsruhe.de/fix/publications/zdm/adm97

Plucker, J. A., & Beghetto, R. A. (2004). Why creativity is domain general, why it looks domain specific and why the distinction does not matter. In R. J. Sternberg, E.L. Grigorenko, & J. L. Singer, (Eds.). *Creativity from potential to realization* (pp. 153 – 167), Washington, DC: American Psychological Association.

Plucker, J. A., & Renzuli, J. S. (1999). Psychometric approaches to the study of human creativity. In R. J. Sternberg (Ed.). *Handbook of creativity* (pp. 35-61). New York: Cambridge Press.

Poincaré, H. (1913). *The foundations of science*. New York: The Science Press.

Prouse, H.L. (1964). The construction and use of a test for the measure of certain aspects of creativity in seventh-grade mathematics. *Dissertation Abstracts, 26* (01), 394. (UMI No AAT 6500500)

Prouse, H. L. (1967). Creativity in school mathematics. *The Mathematics Teacher, 60*, 876-879.

Reis, S. M., & Renzulli, J. S. (1982). A case for a broaden conception of giftedness. *Phi Delta Kappan, 63*, 619-620.

Reis, S. M., Gentry, M., & Maxfield, L. R. (1998). The application of enrichment clusters to teachers' classroom practices. *Journal for the Education of the Gifted*, *21*, 310-324.

Renzulli, J. S. (1978). What makes giftedness? Reexamining a definition. *Phi Delta Kappan, 60,* 180-184, 261.

Renzulli, J. S. (1997). *How to develop an authentic enrichment cluster*. Storrs, CT: The National Research Center on the Gifted and Talented. Retrieved Nov 12, 2002, from http://www.gifted.uconn.edu/sem/semart01.html

Renzulli, J. S. (1998). The three-ring conception of giftedness. In S. M. Baum, S. M. Reis, & L. R. Maxfield (Eds.), *Nurturing the gifts and talents of primary grade students*. Mansfield Center, CT: Creative Learning Press. Retrieved Nov 12, 2002, from http://www.gifted.uconn.edu/sem/semart13.html

Renzulli, J. S., Gentry, M., & Reis, S. M. (2003). *Enrichment clusters: A practical plan for real-world, student-driven learning.* Mansfield Center, CT: Creative Learning Press.

Renzulli, J. S., Gentry, M., & Reis, S. M. (2004). A time and palace for authentic learning, *Educational Leadership, 26,* 73-77.

Renzulli, J. S., Leppien, J. H., & Hays, T. S. (2000). *The Multiple Menu Model: A practical guide for developing differentiated curriculum.* Mansfield Center, CT: Creative Learning Press.

Renzulli, J. S., Smith, L. H., White, A. J., Callahan., C. M., Hartman, R. K., Westberg, K.L., et al. (2004). *Scales for rating the behavioral characteristics of superior students.* Mansfield Center, CT: Creative Learning Press.

Romberg, T. A., & Kaput, J. J. (1999). Mathematics worth teaching, mathematics worth understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 3-18). Mahwah, NJ: Lawrence Erlbaum.

Runco, M. A. (1993). *Creativity as an educational objective for disadvantaged students* (RBDM 9306). Storrs, CT: The National Research Center on the Gifted and Talented, University of Connecticut.

Runco, M. A. (2004). Creativity. In S. T. Fiske, D. L. Schacter, & C. Zahn-Waxler (Eds.), *Annual Review of Psychology* (pp. 657 – 687). Palo Alto: CA, Annual Reviews. Retrieved, Nov 20, 2005, from http://arjournals.annualreviews.org/doi/full/10.1146/ annurev.psych.55.090902.141502

Schmader, T., Johns. M., & Barquissu, M. (2004). The cost of accepting gender differences: The role of stereotype endorsements in women's experience in the math domain*. Sex Roles, 50,* 835-850.

Schoenfeld, A. H. (1987). Cognitive science and mathematics education. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-216). Mahwah, NJ: Lawrence Erlbaum.

Selby, E. C., Shaw, E. J, & Houtz, J. C. (2005). The creative personality. *Gifted Child Quarterly, 49*, 300 – 314.

Selby E.C., Treffinger, D. J., Isaksen, S. G., & Lauer, K. J. (2004). Defining and assessing problem solving style: Design and development of a new tool*. Journal of Creative Behavior, 38*, 221-243.

Seo, H, Lee, E A., & Kim E. H. (2005). Korean science teachers' understanding of creativity in gifted education. *The Journal of Secondary Gifted Education, 14*, 98-105.

Sheffield, L. J., Bennett, J., Beriozábal, M., DeArmond, M., & Wertheimer, R. (1995). Report of the task force on the mathematically promising. *NCTM News Bulletin,* 32.

Shimada, S. (1997). The significance of an open-ended approach, In J. P. Becker, & S. Shimada (Eds.), *The open-ended approach: A new proposal for teaching mathematics* (pp. 1-9). Reston, VA: National Council of Teachers of Mathematics.

Silver, E. A. (1997). Fostering creativity though instruction rich in mathematical problem solving and problem posing. *International Reviews on Mathematical Education, 29,* 75-80. Retrieved March 10, 2003 from http://www.fiz-karlsruhe.de/fix/publications/ zdm/adm97

Singh, B. (1988). *Teaching-learning strategies and mathematical creativity.* Delhi, India: Mittal Publications.

Starko, A. J. (2001). *Creativity in the classroom: Schools of curious delight*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Sternberg, R. J. (1996).What is mathematical thinking? In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 303-318). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Tabachnick, B. G., & Fidell, L. S. (2001). *Using multivariate statistics* (4th ed.). Boston, MA: Allyn and Bacon.

Tomlinson, C. A., Kaplan, S. N., Renzulli, J. S., Purcell, J. H, Leppien, J. H., & Burns, D. E. (2001). *The parallel curriculum.* Thousand Oaks, CA: Corwin Press.

Treffinger, D. J. (2003). Assessment and measurement in creativity and creative problem solving. In J. C. Houtz, *The educational psychology of creativity* (pp. 59-93), Cresskill, NJ: Hampton Press.

Treffinger, D. J., Renzulli, J. S., & Feldhusen, J. F. (1971). Problems in the assessment of creative thinking. *The Journal of Creative Behavior,* 5, 104-112.

Treffinger, D. J., Young, G. C., Selby, E.C., & Shepardson, C. (2002). *Assessing creativity: A guide for educators* (RM02170). Storrs, CT: The National Research Center on the Gifted and Talented, University of Connecticut.

Tuli, M. R., (1980). Mathematical creativity as related to aptitude for achievement in and attitude towards mathematics (Doctoral dissertation, Panjab University, 1980). *Dissertation Abstracts International, 42* (01), 122.

U.S. Department of Education, National Center for Education Statistics. (2003). *Comparative indicators of education in the United States and other G-8 countries: 2002* (NCES Publication No. 2003-26). Washington DC: Author.

U.S. Department of Education, Office of Elementary and Secondary Education (2005). *Public law print of PL 107-110, the No Child Left Behind Act of 2001.* Retrieved, September 20, 2005, from http://www.ed.gov/policy/elsec/leg/esea02/index.html

Usiskin, Z. (1999). The mathematically promising and the mathematically gifted. In L. J. Sheffield (Ed.), *Developing mathematically promising students* (pp. 57-69). Reston, VA: National Council of Teacher of Mathematics.

Whitcombe, A. (1988). Mathematics: Creativity, imagination, beauty, *Mathematics in School, 17,* 13-15.

Yates, S. M. (2002). Longitudinal measurement of student motivation and achievement in mathematics. In S. Goodchild & L. English (Eds.), *Research mathematics classrooms: A critical examination of methodology* (pp. 155-197). Greenwich, CT: Information Age Publishing.

Creative Ability in Mathematics

Name:____

Grade:__________ Age:__________ Boy or Girl? __________

Directions

The items in the booklet give you a chance to use your imagination to think up ideas and problems about mathematical situations. We want to find out how creative you are in mathematics. Try to think of unusual, interesting, and exciting ideas – things no one else in your class will think of. Let your mind go wild in thinking up ideas.

You will have the entire class time to complete this booklet. Make good use of your time and work as fast as you can without rushing. If you run out of ideas for a certain item go on to the next item. You may have difficulty with some of the items; however, do not worry. You will not be graded on the answers that you write. Do your best!

Do you have any questions?

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Appendix A

ITEM I

Directions

Patterns, chains, or sequences of numbers appear frequently in mathematics. It is fun to find out how the numbers are related. For example look at the following chain:

2 5 8 11 ___ ___

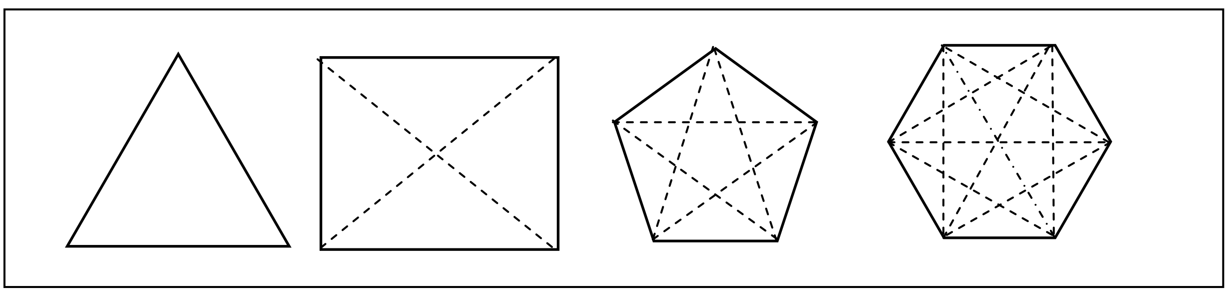
The difference between each term is 3; therefore, the next two terms are 14 and 17. Now look at the chain shown below and supply the next three numbers.

1 1 2 3 5 8 13 21 ___ ___ ___

ITEM II

Directions

Below are figures of various polygons with all the possible diagonals drawn (dotted lines) from each vertex of the polygon. List as many things as you can of what happens when you increase the number of sides of the polygon. For example: The number of diagonals increases. The number of triangles formed by the number of diagonals increases.



1.____

2.____

3.____

4.____

5.____

6.____

7.____

8.____

9.____

10. ____

11.____

12.____

Item III

Directions

Suppose the chalkboard in your classroom was broken and everyone's paper was thrown away; consequently, you and your teacher could not draw any plane geometry figures such as lines, triangles, squares, polygons, or any others. The only object remaining in the room that you could draw on was a large ball or globe used for geography. List all the things which could happen as a result of doing your geometry on this ball. Let your mind go wild thinking up ideas. For example: If we start drawing a straight line on the ball, we will eventually end up where we started. (Don't worry about the maps of the countries on the globe.)

1.____________________
2.____________________
3.____________________
4.____________________
5.____________________
6.____________________
7.____________________
8.____________________
9.____________________
10. ____________________
11.____________________
12.____________________
13.____________________
14.____________________
15.____________________
16.____________________
17.____________________
18.____________________
19.____________________
20. ____________________
21.____________________
22.____________________

Appendix A

ITEM IV

Directions

Write down every step necessary to solve the following mathematical situation. Lines are provided for you to write on; however there may be more lines than you actually need.

> Suppose you have a barrel of water, a seven cup can, and an eight cup can. The cans have no markings on them to indicate a smaller number of cups such as 3 cups. How can you measure nine cups of water using only the seven cup can and the eight cup can?

____

____

____

____

____

____

ITEM V

Directions

Suppose you were given the general problem of determining the names or identities of two hidden geometric figures, and you were told that the two figures were related in some manner. List as many other problems as you can which must be solved in order to determine the names of the figures. For example: Are they solid figures such as a ball, a box, or a pyramid? Are they plane figures such as a square, a triangle, or a parallelogram? If you need more space, write on the back of this page.

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____

Item VI

Directions

The situation listed below contains much information involving numbers. Your task is to make up as many problems as you can concerning the mathematical situation. You do not need to solve the problems you write. For example, from the situation which follows: If the company buys one airplane of each kind, how much will it cost? If you need more space to write problems, use the back of this page.

> An airline company is considering the purchase of 3 types of jet passenger airplanes, the 747, the 707 and the DC-10. The cost of each 747 is $15 million; $10 million for each DC 10; and $6 million for each 707. The company can spend a total of $250 million. After expenses, the profits for the company are expected to be $800,000 for each 747,$500,000 for each DC-10, and $350,000 for each 707. It is predicted that there will be enough trained pilots to man 25 new airplanes. The overhaul base for the airplanes can handle 45 of the 707 jets. In terms of their use of the maintenance facility, each DC 10 is equivalent to $1\frac{1}{3}$ of the 707's and each 747 is equivalent to $1\frac{2}{3}$ of the 707's.

.

1.____

2.____

3.____

4.____

5.____

6.____

7.____

8.____

9.____

10. _______________________________________

11._______________________________________

12._______________________________________

13._______________________________________

14._______________________________________

15._______________________________________

Appendix B

Characteristics of the Potentially Creative Thinker in Mathematics

1. An esthetic sensibility, expressed in an appreciation of the harmony, unity and analogy present in mathematical solutions and proofs and in an appreciation of the structure of the field.
2. The making up or seeing of problems in data or in situations which arouse no particular curiosity in the other children.
3. A desire to improve a proof or the structure of a solution.
4. A seeking for consequences or connections between a problem, proposition, or concept and what would follow from it.
5. Desire for working independently of both teacher and other pupils.
6. Pleasure out of communicating concerning mathematics with others of equal ability and interest.
7. The speculating or guessing about what would happen if one or more hypotheses of a problem are changed.
8. Pleasure derived from adding to the knowledge of the class by producing another solution or another proof beyond those which the class has considered.
9. Pleasure out of working with the symbols of mathematics.
10. The producing or conjecturing concerning other meanings for symbols than those the teacher has revealed.
11. The making up of mathematical symbols of his/her own.
12. The tendency to generalize particular results either by finding a common thread of induction or by seeing similar patterns by analogy.
13. The ability to see a whole solution at one time or to visualize a proof as a whole.
14. Intuition as to how things should result.
15. A vivid imagination concerning the way things appear in space, the relation of things to each other.
16. A vivid imagination concerning the resulting paths or relationships of objects which have motion.
17. A tendency to speculate concerning unusual applications for the results obtained by the class.
18. The belief that every problem has a solution.
19. Persistence in working on particularly difficult problems or proofs.
20. Boredom with repetition or working of a large number of problems which she/he has well in hand.
21. Ability to perform many operations without thinking.

Carlton, L. V. (1959*), An analysis of the educational concepts of fourteen outstanding mathematicians, 1790-1940, in the areas of mental growth and development, creative thinking and symbolism and meaning* (pp. 415 – 417). Unpublished doctoral dissertation, Northwestern University, IL.

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

Grade 6

1	**Place Value**
A	Solve problems involving 100 and 1,000 more or less

On Monday 285 concert tickets were sold. By Tuesday 100 more tickets had been sold. How many tickets had been sold all together?

275
287
385*
395

1	**Place Value**
B	Identify alternative forms of expressing whole numbers less than 10,000 using expanded notation

Which means the same as 3,815?

3000 + 800 + 10 + 5*
3000 + 800 + 10 + 50
3000 + 80 + 100 + 5
300 + 80 + 100 + 5

1	**Place Value**
C	Identify alternative forms of expressing whole numbers less than 10,000 using regrouping

Which means the same as 7,500?

75 hundreds*
75 tens
75 thousands
75 ones

1	**Place Value**
D	Use place value concepts to interpret the meaning of numbers

In which number does the 3 have the greatest value?

4351
3451*
5431
391

60

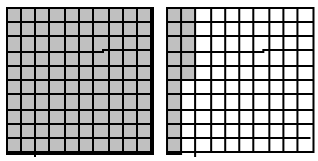
Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

2 Pictorial Representations of Numbers

A Relate decimals (0.01-2.99) to pictorial representations



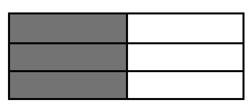
The shaded part of this picture shows which decimal number?

1.35
1.25
1.15*
1.05

2 Pictorial Representations of Numbers

B Relate fractions and mixed numbers to pictures and vice versa

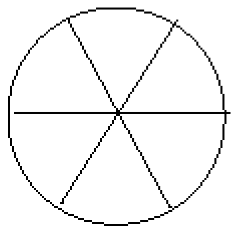
The shaded part of this picture shows which fraction?



3/8
3/4
3/5
3/6*

2	**Pictorial Representations of Numbers**
C	Relate fractions and mixed numbers to pictures and vice versa

Shade in 1/6 of this shape.



3	**Equivalent Fractions, Decimals and Percents**
A	Rename equivalent fractions

Wally eats breakfast 1/3 of the mornings he goes to school. Which is another way to describe this?

Wally eats breakfast 12 out of 20 school mornings.
Wally eats breakfast 8 out of 16 school mornings.
Wally eats breakfast 6 out of 8 school mornings.
Wally eats breakfast 5 out of 15 school mornings. *

3	**Equivalent Fractions, Decimals and Percents**
B	Relate equivalent mixed numbers and improper fractions

Which fraction is equivalent to 2 3/5?

10/5
12/5
13/5*
14/5

Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

4 Order, Magnitude and Rounding of Numbers
A Order whole numbers less than 100,000

Renee is studying the Great Lakes in geography class. She made this chart to show the maximum depth of each of the lakes.

Lake	Depth feet
Erie	116
Huron	850
Michigan	923
Ontario	393
Superior	1,330

Which shows the depth of each lake arranged from GREATEST to LEAST?

Erie, Huron, Michigan, Ontario, Superior
Superior, Ontario, Michigan, Huron, Erie
Erie, Ontario, Huron, Michigan, Superior
Superior, Michigan, Huron, Ontario, Erie *

4 Order, Magnitude and Rounding of Numbers
B Order mixed numbers [including fractions] and decimals

The chart shows the heights of some of Bill's friends.

Which of Bill's friends was the TALLEST?

Mac
Sam *
Ted
Johnny

Friend	Height Feet
Mac	$4\frac{1}{4}$
Sam	$4\frac{3}{4}$
Ted	$3\frac{7}{8}$
Johnny	$4\frac{1}{2}$

63

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

4	**Order, Magnitude and Rounding of Numbers**
C	Describe magnitude of whole numbers less than 100,000

Amanda collected between 4,600 and 5,800 pounds of newspaper for recycling. Which could be the amount she collected?

5120*
3960
4410
5970

4	**Order, Magnitude and Rounding of Numbers**
D	Describe magnitude of mixed numbers [including fractions] and decimals

Karen bought between 4 1/2 and 7 1/2 pounds of meat for the barbecue. Which could be the amount she bought?

4 1/4 pounds
5 1/4 pounds*
7 3/4 pounds
3 3/4 pounds

4	**Order, Magnitude and Rounding of Numbers**
E	Round whole numbers in a context

There were 6,852 people at the game. This number is ABOUT

5000
6000
7000*
8000

4	**Order, Magnitude and Rounding of Numbers**
F	Round decimals in a context

Susan bought 2.1 pounds of pet food. This amount rounded to the NEAREST whole number is

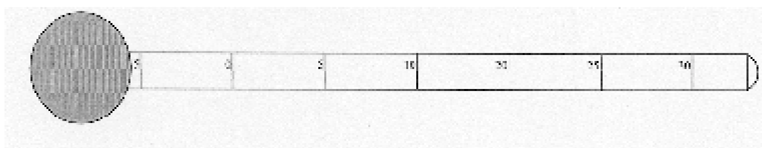
1 pound
2 pounds*
3 pounds
4 pounds

64

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation (Connecticut Board of Education, 2001)

4	**Order, Magnitude and Rounding of Numbers**
G	Locate **numbers** on number lines, scales and grids

The average July temperature in Connecticut is 21 degrees Celsius. Mark this temperature on the thermometer with a heavy line.



5	**Models for Operations**
A	Identify the appropriate operation or number sentence to solve a story problem

During inventory of a hardware store Jason counted 16 boxes of bolts. Each box contained 48 bolts. Which number sentence could be used to find out how many bolts were in all of the boxes?

Add 48 and 16
Subtract 16 from 48
Multiply 48 by 16*
Divide 48 by 16

5	**Models for Operations**
B	Write story problems from number sentences

Write a story problem that can be solved using the number sentence
$8 \times 4 = \square$.

6	**Basic Facts**
A	Multiply and divide facts

Solve this problem

$8 \times 7 =$ $63 \div 7 =$

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3^{rd} Generation
(Connecticut Board of Education, 2001)

7	**Computation with Whole Numbers and Decimals**
A	Add and subtract 2-, 3- and 4-digit whole numbers and money amounts less than \$100 [expressed with decimal notation]

Solve this problem

\$69.99 – \$2.34 =

7	**Computation with Whole Numbers and Decimals**
B	Multiply and divide multiples of 10 and 100 by 10 and 100

\$10 x 40 =

\$4000
\$400*
\$40
\$4

7	**Computation with Whole Numbers and Decimals**
C	Multiply and divide 2- and 3-digit whole numbers and money amounts less than \$10 by 1-digit numbers

Solve this problem

345 x 4 =

\$8.56 ÷ 4 =

8	**Computation with Fractions**
	Add and subtract fractions and mixed numbers with like denominators

1/4 + 2/4 =

3/8
3/6
3/5
3/4*

Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

9	**Solve Simple Word Problems**
A	Solve 1-step problems involving whole numbers and money amounts

Solve this problem

A package of 15 computer disks cost $47.25. Each disk costs the same amount. How much did each disk cost?

9	**Solve Simple Word Problems**
B	Solve 2-step problems involving whole numbers and money amounts

Solve this problem

Rob's baseball card collection was organized in 3 boxes with 175 cards in each box. He hopes to have 4 times as many cards as he has now after his visit to the Baseball Card Collectors' Convention. How many cards does Joe hope to have after the convention?

9	**Solve Simple Word Problems**
C	Solve and explain solutions to 2-step problems involving whole numbers and money amounts

Jenn bought 3 tops that each cost $12.95. She gave the clerk a $50 bill. If there is no tax on the tops, how much change should Jenn receive?

Explain how you got your answer.

10	**Numerical Estimation Strategies**
A	Identify the best expression to find an estimate

Ellen needs to subtract 31,919 from 79,899. Which of the following would be BEST for Ellen to use to ESTIMATE the difference?

80,000 – 30,000 *
80,000 – 40,000
70,000 – 30,000
70,000 – 40,000

67

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

10	**Numerical Estimation Strategies**
B	Identify whether and why a particular strategy will result in an overestimate or an underestimate

To ESTIMATE the product of 521 and 613, Ernesto multiplied 500 x 600. Would Ernesto's estimate be MORE or LESS than the actual product?

More, because he rounded both numbers up.
More, because he rounded both numbers down.
Less, because he rounded both numbers up.
Less, because he rounded both numbers down.*

11	**Estimating Solutions to Problems**
	Estimate a reasonable answer to a problem

Charlie rode on the bus 37 ½ miles the first week of school and 28 ½ miles the second week. ABOUT how many miles did he ride on the bus during the two weeks?

Fewer than 60
A little more than 60
A little less than 70*
More than 70

14	**Time**
A	Solve problems involving elapsed time

Rocco saw a movie that began at 5:10 p.m. and ended at 6:40 p.m. How long was the movie?

1 hour 45 minutes
1 hour 40 minutes
1 hour 20 minutes
1 hour 30 minutes*

68

Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation (Connecticut Board of Education, 2001)

14	**Time**
B	Solve problems involving the conversion of measures of time

A jogger runs for 20 minutes each day. How many hours does he spend running in 4 days?

2 hours
1 hour and 5 minutes
1 hour and 30 minutes
1 hour and 20 minutes*

15	**Approximate Measures**
A	Estimate **lengths and areas**

If the shorter tree is about 5 feet tall, the height of the taller tree is ABOUT

15 ft
12 ft
8 ft*
6 ft

16	**Customary and Metric Measures**
A	Solve problems involving the conversion of measures of length

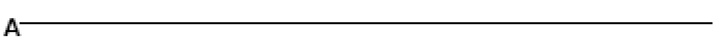
A basketball player is 209 centimeters tall. How many meters is that?

2 meters and 9 centimeters*
5 meters and 8 centimeters
20 meters and 9 centimeters
29 meters

69

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

16	**Customary and Metric Measures**
B	Measure length to the metric or customary unit specified

A  B

Use your centimeter ruler to measure the length of the line segment between points **A** and **B** to the NEAREST half centimeter.

Length: _________

16	**Customary and Metric Measures**
C	Measure and determine perimeters or areas

Use your centimeter ruler to measure the lengths of each side of this rectangle. Label the lengths of the sides and determine the PERIMETER of the rectangle.

PERIMETER = ____

16	**Customary and Metric Measures**
D	Identify appropriate metric or customary units of measure (length, capacity, mass) for a given situation

The length of the floor in a gym is BEST measured in

meters.*
centimeters.
kilometers.
liters.

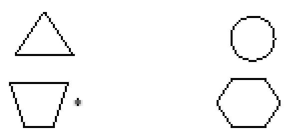
Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

17	**Geometric Shapes and Properties**
A	Identify and draw geometric shapes and figures

Which shape is a quadrilateral?



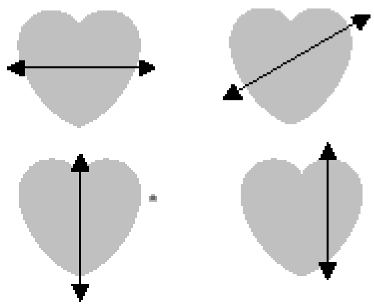
17	**Geometric Shapes and Properties**
B	Draw, describe and classify geometric shapes and figures

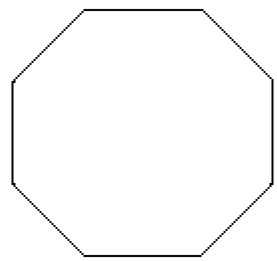
Draw a trapezoid. Then explain why the figure you drew is a trapezoid.

Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

18	**Spatial Relationships**
A	Identify and draw lines of symmetry

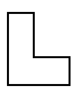
Which picture shows a line of symmetry?



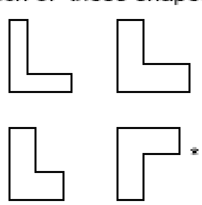


Draw exactly 2 lines of symmetry on this figure.

18	**Spatial Relationships**
B	Identify congruent figures



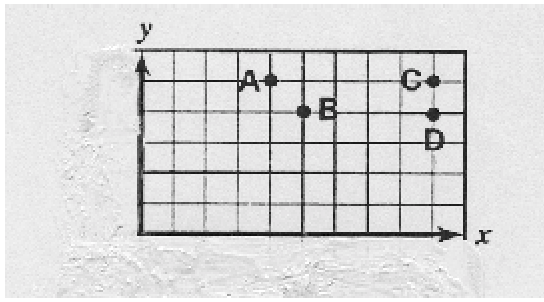
Which of these shapes appears to be congruent to the figure above?



Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

18	**Spatial Relationships**
C	Locate points on grids

What letter is located at (5,4)?



A
B *
C
D

19	**Tables, Graphs and Charts**
A	Identify correct information from graphs, tables and charts

Class	**Number of Cans collected**
Mr. Green	652
Mr. Gomez	507
Ms. Castro	553
Ms. Powell	605

How many classes collected more than 500 cans?

1
2
3
4 *

73

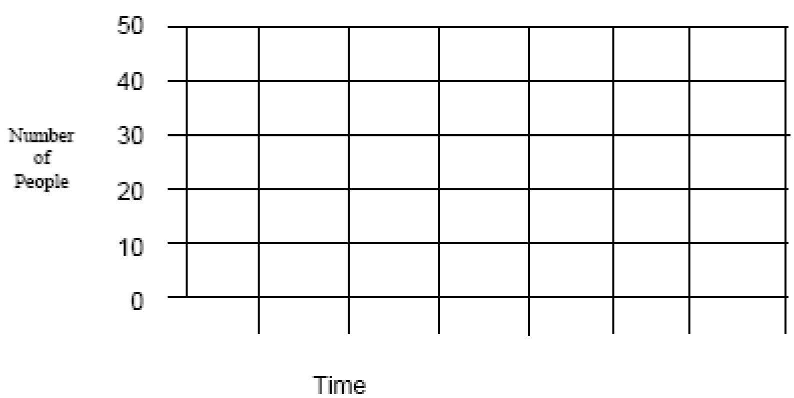
Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

19	**Tables, Graphs and Charts**
B	Create bar graphs and pictographs from data in tables and charts

The table shows the number of people on each hourly tour of the museum.

Time	Temperature Fahrenheit
9:00 am	24
10:00 am	45
11:00 am	38
12:00 noon	18
1:00 pm	41
2:00 pm	28
3:00 pm	35

Complete the BAR graph to show the same information.



Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

20 Statistics and Data Analysis

Draw and justify reasonable conclusions from data in tables, graphs and charts

This table shows the number of votes each candidate received in an election.

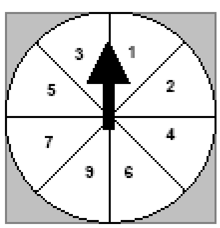
Candidate	Votes
Downey	3,821
Jones	2,746
Smith	4,110
McCoy	2,241
Carter	5,984
Roberts	1,970

Jill claims that Smith received ABOUT twice as many votes as McCoy. Based on the data in the chart above, is Jill's statement accurate? Use the data in the table to explain why or why not.

21 Probability

Solve problems involving elementary notions of probability and fairness, including justifying answers

Joe and Jill take turns spinning this spinner. Joe gets a point if the arrow lands on an even number and Jill gets a point if it lands on an odd number. Is this game fair?



Yes, because there are 8 choices.
Yes, because the outcomes are equally likely.
No, because there are more odd than even numbers.*
No, because there are more even than odd numbers.

Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

22	**Patterns** Extend or complete patterns involving numbers and attributes, and identify or state rules for given patterns.

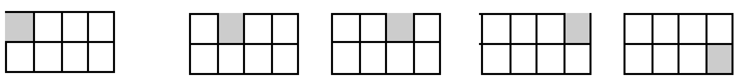
The numbers follow a pattern.

3, 9, 27, 81, ___

Which number should be next in the pattern?

135
243*
245
397

These shapes follow a pattern.



Which shape should be next in the pattern? Draw the next shape and explain why you think that it is the next shape in the pattern.

23	**Algebraic Concepts** Solve simple 1-step equations

What is the value of in this equation?

43 + = 65

12
99
22*
108

76

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

24 Classification and Logical Reasoning

Solve problems involving the organization of data

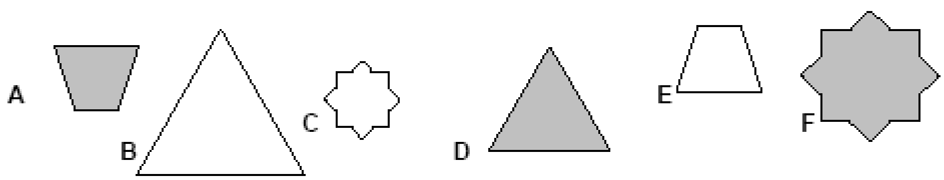
Ann, Joe, Harry and Don collected shells at the beach yesterday.

- Joe collected more than Harry and Ann.
- Don collected the FEWEST shells.

Who collected the MOST shells?

Don
Harry
Joe*
Ann

Sort all 6 of these figures into 2 groups so that the figures in each group have something in common. Write the letter of each figure under one of the groups below. Then write a sentence that explains why you grouped them this way.



Group 1	Group 2

Describe your rule for sorting here:________________________________

____

____

Appendix C

Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation

(Connecticut Board of Education, 2001)

25	**Mathematical Applications**
A	Numerical

Joe and his family have $50 to spend at a restaurant. The members of his family are listed below:

Joe
Betty
Gil
Heather

The menu at the restaurant is as follows:

ENTREES	
Chicken Dinner	9.95
Hamburger	5.95
Cheeseburger	6.95
Fish Dinner	6.95
DRINKS	
Large Soda	1.50
Tea	.95
Coffee	1.25
Milk	1.00
DESSERTS	
Cake	2.50
Pie	1.95
Ice Cream	2.25

Each member of Joe's family orders 1 entrée, 1 drink and 1 dessert. Determine an order for the family that is under $50 and explain your mathematical thinking below.

78

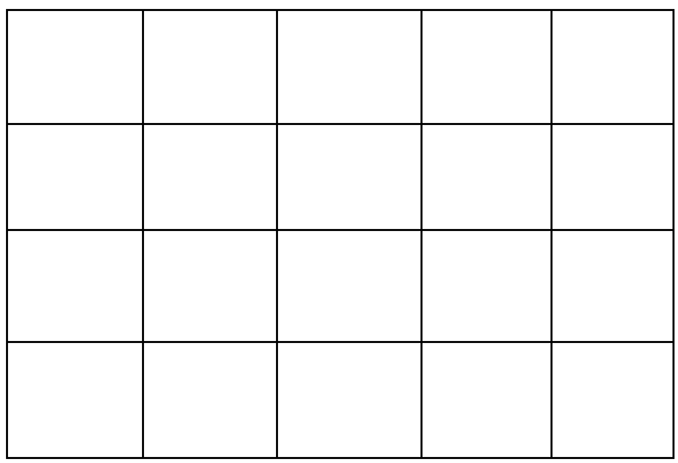
Appendix C
Sample Mathematics Items from the Sixth Grade, Connecticut Mastery Test - 3rd Generation
(Connecticut Board of Education, 2001)

25	**Mathematical Applications**
B	Spatial

Rob has these three kinds of stars:

He is making a design on his ceiling. His design will hold only 20 stars and he wants each row of his finished 4-row x 5-column design to match exactly every other row.

Use the grid below to draw a design that Rob could use. Be sure to use all 3 kinds of tiles, and explain how you know that each row matches every other row.



How I Feel About Math

(Based on the Fennema-Sherman Mathematics Attitude Scales)

Name:____

Directions

On the following pages are a series of statements. There are no correct answers for these statements. They have been set up in a way which permits you to indicate how strongly you agree or disagree with the ideas expressed. Suppose the statement is, "I like mathematics."

When you read the statement you will know if you agree or disagree with it.

	Disagree	Sort of Disagree	Not Sure	Sort of Agree	Agree
If you **agree** with the statement mark the box in the column for agree.					
I like mathematics.					X
If you **sort of disagree** mark that column.	Disagree	Sort of Disagree	Not Sure	Sort of Agree	Agree
I like mathematics.		X			
If you really are **not sure** mark the middle column.	Disagree	Sort of Disagree	Not Sure	Sort of Agree	Agree
I like mathematics.			X		

There are no "right" or "wrong answers. The only correct responses are those that are true for you.

This inventory is being used for research purposes only and your responses will be confidential.

Appendix D

	Disagree	Sort of Disagree	Not Sure	Sort of Agree	Agree
Generally I have felt secure about attempting mathematics.					
Figuring out mathematical problems does not appeal to me.					
If I got the highest grade in math I'd prefer no one knew.					
I get a sinking feeling when I think of trying hard math problems.					
I like math puzzles.					
Math has been my worst subject.					
Mathematics makes me feel uncomfortable, restless, irritable, and impatient.					
Mathematics is enjoyable and stimulating to me.					
I do as little work in math as possible.					
I almost never have gotten nervous during a math test.					
I can get good grades in mathematics.					
I don't think I could do advanced mathematics.					
The challenge of math problems does not appeal to me.					
I usually have been relaxed during math tests.					
Math doesn't scare me at all.					
I'd be proud to be the outstanding student in math.					
Winning a prize in mathematics would make me feel unpleasantly conspicuous.					
It would make people like me less if I were a really good math student.					
My mind goes blank and I am unable to think clearly when working with mathematics.					
A math test would scare me.					
When a math problem arises that I can't immediately solve, I stick with it until I have the solution.					
Math puzzles are boring.					
It wouldn't bother me at all to take more math courses.					

Appendix D

	Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
I am challenged by math problems I can't understand immediately.					
For some reason, even though I study, math seems unusually hard for me.					
Mathematics usually makes me feel uncomfortable and nervous.					
I'm not the type to do well in math.					
When I start trying to work on a math puzzle, I find it hard to stop.					
It would be great to win a prize in mathematics.					
I am sure I could do advanced work in mathematics.					
People would think I was some kind of a nerd if I got A's in math.					
I haven't usually worried about being able to solve math problems.					
I'm no good in math.					
Most subjects I can handle O.K., but I have a knack for messing up math.					
I think I could handle more difficult mathematics.					
I usually have been relaxed during math classes.					
When a question is left unanswered in math class, I continue to think about it afterward.					
I don't like people to think I'm smart in math.					
I would rather have someone give me the solution to a difficult math problem than to have to work it out for myself.					
I don't understand how some people can spend so much time on math and seem to enjoy it.					
I have a lot of self-confidence when it comes to math.					
Mathematics makes me feel uneasy and confused.					
It would make me happy to be recognized as an excellent student in mathematics.					
If I had good grades in math, I would try to hide it.					
Being first in a mathematics competition would make me pleased.					

Appendix E

Student:

MATHEMATICAL CHARACTERISTICS[*]
The student . . .

		Never	Very Rarely	Rarely	Occasionally	Frequently	Always
1	is eager to solve challenging math problems (A problem is defined as a task for which the solution is not known in advance).						
2	organizes data and information to discover mathematical patterns.						
3	enjoys challenging math puzzles, games and logic problems.						
4	understands new math concepts and processes more easily than other students.						
5	has creative (unusual and divergent) ways of solving math problems.						
6	displays a strong number sense (e.g., makes sense of large and small numbers, estimates easily and appropriately.)						
7	frequently solves math problems abstractly, without the need for manipulates or concrete materials.						
8	has an interest in analyzing the mathematical structure of a problem.						
9	when solving a math problem, can switch strategies easily if appropriate or necessary.						
10	regularly uses a variety of representation to explain math concepts (written explanation, pictorial, graphic, equations, etc.).						

CREATIVITY CHARACTERISTICS[*]
The student demonstrates . . .

		Never	Very Rarely	Rarely	Occasionally	Frequently	Always
1	imaginative thinking ability.						
2	a sense of humor.						
3	the ability to come up with unusual, unique, or clever responses.						
4	an adventurous spirit or a willingness to take risks.						
5	the ability to generate a large number of ideas of solution to problems or questions.						
6	a tendency to see humor in situations that may not appear to be humorous to others.						
7	the ability to adapt, improve, or modify objects or ideas.						
8	intellectual playfulness, willingness to fantasize and manipulate ideas.						
9	a non-conforming attitude, does not fear being different.						

[*]Renzulli, J. S., Smith, L. H., White, A. J., Callahan., C. M., Hartman, R. K., Westberg, K.L., Gavin, M.K., Reis, S.M., Siegle, D. & Sytsma, R. E. (2004). *Scales for rating the behavioral characteristics of superior students.* Mansfield Center, CT: Creative Learning Press.

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

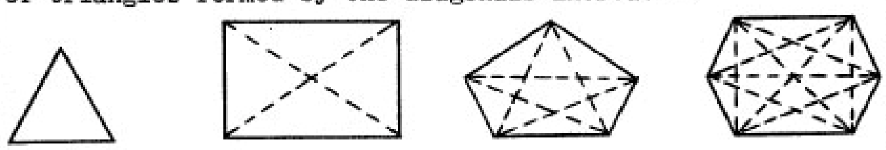
SCORING PROCEDURES AND WEIGHTS FOR CATEGORIES

EXPRESSED ON CAMT DIVERGENT ITEMS

ITEM II

DIRECTIONS

Below are figures of various polygons with all the possible diagonals drawn (dotted lines) from each vertex of the polygon. List as many things as you can of what happens when you increase the number of sides on the polygon. For example: The number of diagonals increases. The number of triangles formed by the diagonals increases.



Scoring

Fluency: One point for each relevant response
Flexibility: One point for each category expressed
Originality: Zero, one, or two points for each category expressed, weighted according to the following schedule of categories

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Number of shapes, kinds of shapes, designs increases	0	281	56.2%
Number of lines, line segments, folds	0	164	32.8
Number of vertices or corners	0	154	30.8
Number of points of intersection or crosses	0	135	27.0

Appendix F
CMAT Divergent Item Scoring Procedures (Balka, 1974)

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Size, area of shapes formed in interior change	0	121	24.2
Polygon becomes more dense with diagonal lines, becomes black	0	87	17.4
Number of angles formed by diagonals increases	0	37	7.4
Number of angles formed by sides of polygons increases	0	32	6.4
Lengths of sides, line segments, lines changes	0	30	6.0
Distance (diameter) across polygon changes	1	24	4.8
Name of polygon changes	1	17	3.4
Area of, size of figure might, probably changes, increases	1	17	3.4
Types, kinds of triangles change	1	17	3.4
Number of planes, half-planes increases	1	17	3.4
Number of diagonals from each vertex increases	1	16	3.2
Polygon acquires shape of circle, rounded	1	15	3.0
Parallel diagonals, lines appear	1	12	2.4
Perimeter of figure probably increases	2	9	1.8

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

203

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Size of interior angles of polygons	2	9	1.8
Symmetry	2	5	1.0
Kinds, types of angles formed	2	5	1.0
Drawing altitude to triangle or figure increases, doubles number of shapes, triangles	2	4	0.8
Center point appears	2	4	0.8
Total degree measure increases	2	2	0.4
Types of lines, horizontal, vertical	2	2	0.4
Size of angles formed by diagonals	2	1	0.2
Number of 3-dimensional figures increases	2	1	0.2
Number of intersecting planes	2	1	0.2
Equations of lines	2	1	0.2
Radius changes	2	1	0.2

Appendix F
CMAT Divergent Item Scoring Procedures (Balka, 1974)

204

ITEM III

DIRECTIONS

Suppose the chalkboard in your classroom was broken and everyone's paper was thrown away; consequently you and your teacher could not draw any plane geometry figures such as lines, triangles, squares, polygons, or any others. The only object remaining in the room that you could draw on was a large ball or globe used for geography. List all the things which could happen as a result of doing your geometry on the ball. Let your mind go wild in thinking up possible ideas. For example: If we start drawing a "straight" line on the ball, we will eventually end up where we started. Do not worry about the maps of the countries.

Scoring

Fluency:	One point for each relevant response
Flexibility:	One point for each category expressed
Originality:	Zero, one, or two points for each category expressed, weighted according to the following schedule of categories

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Figures, polygons would be distorted, round, stretched, curved	0	185	37.0
Straight lines would be curved	0	103	20.6
Entire figure could not be seen if very large	0	49	9.8
Figures would overlap, connect, touch if drawn large	0	48	9.6

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Measurement of distance, length is different	0	39	7.8
No planes would be present; no plane figures; plane would not be flat	0	34	6.8
Change in line direction would cause spiralling, intersections, unending line	0	29	5.8
Angle measurement would be different	1	19	3.8
Perfect circles could be drawn	1	16	3.2
Area of figures would be different	1	12	2.4
Radius, diameter, circumference could be found	1	11	2.2
Rays of angle would intersect	2	7	1.4
Figures would look 3-D	2	7	1.4
Two straight lines intersect in two points	2	4	0.8
If ball was large enough, geometry would not change much	2	3	0.6
Figures could cover ball	2	3	0.6
Volume is correct	2	2	0.4
Pythagorean Theorem would change	2	2	0.4

Appendix F
CMAT Divergent Item Scoring Procedures (Balka, 1974)

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Need to establish a new mathematical system	2	1	0.2
Axis of symmetry	2	1	0.2
Largest circle is "equator"	2	1	0.2
Straight angle would become closed curve	2	1	0.2
Imaginary line passing thru ball; three points determine triangle	2	1	0.2
Surface area of ball does not change	2	1	0.2

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

207

ITEM V

DIRECTIONS

Suppose you were given the general problem of determining the names or identities of two hidden geometric figures, and you were told that the two figures were related in some manner. List as many other problems as you can which must be solved in order to determine the names of the figures. For example: Are they solid figures such as a ball, a box, or a pyramid? Are they plane figures such as a square, a triangle, or a parallelogram? If you need more space, write on the back of this page.

Scoring

Fluency: One point for each relevant response
Flexibility: One point for each category expressed
Originality: Zero, one, or two points for each category expressed, weighted according to the following schedule of categories

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Does it have sides? How many sides?	0	268	53.6%
Are they round, curved, circular, radial?	0	262	52.4
Type of polygon	0	181	36.2
Does it have vertices, points? How many vertices, points?	0	135	27.0
Do they have congruent sides, same length?	0	86	17.2
Are they plane figures, flat, drawn on paper?	0	82	16.4

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

208

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Does it have depth? Is it 3-D, found in space?	0	81	16.2
Do they have diagonals? How many diagonals?	0	46	9.2
Are they congruent, equal, similar, same size?	0	36	7.2
Kinds of angles, degrees	0	32	6.4
What is volume, area, circumference, perimeter?	1	21	4.2
Open or closed figures, curves	1	20	4.0
Are opposite sides parallel?	1	20	4.0
Number of angles	1	18	3.6
Does it have faces, bases? What type of faces?	1	17	3.4
Does it have straight sides?	1	14	2.8
Number of planes, surfaces	1	11	2.2
How many edges?	2	7	1.4
Can one plane (solid) figure fit inside another?	2	7	1.4
Are they symmetrical?	2	6	1.2
Combination of curved and plane areas	2	4	0.8
Shape of surfaces	2	4	0.8
Does it have a radius?	2	3	0.6
Does it have any arcs?	2	2	0.4

Appendix F
CMAT Divergent Item Scoring Procedures (Balka, 1974)

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Formula for finding area, volume, perimeter	2	2	0.4
Is it on a line?	2	2	0.4
Mathematical equations	2	1	0.2
Is it concave, convex?	2	1	0.2

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

ITEM VI

DIRECTIONS

The situation listed below contains much information involving numbers. Your task is to make up as many problems as you can concerning the mathematical situation. You do not need to solve the problems you write. For example, from the situation which follows: If the company buys one airplane of each kind, how much will it cost? If you need more space to write problems, use the back of this page.

An airline company is considering the purchase of 3 types of jet passenger airplanes. The cost of each 747 is $15 million; $10 million for each DC 10; and $6 million for each 707. The company can spend a total of $250 million. After expenses, the profits of the company are expected to be $800,000 for each 747, $500,000 for each DC 10, and $350,000 for each 707. It is predicted that there will be enough trained pilots to man 25 new airplanes. The overhaul base for the airplanes can handle 45 of the 707 jets. In terms of their use of the maintenance facility, each DC 10 is equivalent to 1 1/3 of the 707's, and each 747 is equivalent to 1 2/3 of the 707's.

Scoring

Fluency: One point for each relevant response
Flexibility: One point for each category expressed
Originality: Zero, one, or two points for each category expressed, weighted according to the following schedule of categories

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Cost for buying certain number of one type plane	0	138	27.6%
Cost for buying certain number of two or three types of planes	0	127	25.4

Appendix F
CMAT Divergent Item Scoring Procedures (Balka, 1974)

211

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Number of planes which can be purchased for $250 million or part of	0	103	20.6
Number of DC 10's or 747's which overhaul base can handle	0	99	19.8
Profits for certain numbers of two or three types of planes	0	76	15.2
Profits for certain number of one type plane	0	44	8.8
Money remaining after purchasing certain number of planes	0	31	6.2
Difference in plane costs	0	30	6.0
Number of planes which overhaul base can handle of two or three types	0	29	5.8
Size, percent, comparison of DC 10 and 747	1	22	4.4
What would be best choice, best buy, most economical purchase of planes	1	17	3.4
Difference in profits	2	9	1.8
Number of years a plane needs to be operated to pay for itself	2	7	1.4
Number of planes which could be purchased from profit of others	2	7	1.4

Appendix F
CMAT Divergent Item Scoring Procedures (Balka, 1974)

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Purchase of different numbers of two or more types. Which is better deal, investment, more profits?	2	6	1.2
Maximum profit	2	6	1.2
Will there be enough pilots if company purchases certain number of planes?	2	5	1.0
How many of one type plane can be purchased for cost of certain number of different type?	2	4	0.8
Percent of garage used by planes	2	4	0.8
Ratio of costs to profits	2	4	0.8
Profit in certain period of time	2	3	0.6
Expense for certain number of planes	2	2	0.4
Difference, comparison of use of maintenance facility	2	2	0.4
If company purchases certain number of one type, can they purchase another type?	2	2	0.4
Purchase of a certain plane, kept for certain number of years, is there a profit?	2	2	0.4

Appendix F

CMAT Divergent Item Scoring Procedures (Balka, 1974)

213

Category Expressed	Weight	Number of Subjects Expressing Category	Percent of Sample
Number of one type plane which could use maintenance facility if certain number of other types were already using it	2	1	0.2
Number of pilots needed, trained in certain number of years	2	1	0.2
If company wants certain number of planes, what is the best choice?	2	1	0.2
Maximum use of maintenance facility	2	1	0.2
Ratio of profit to size	2	1	0.2
Cost of planes for maximum use of maintenance facility	2	1	0.2
Cost per month for plane	2	1	0.2

Eric Mahn

From: Don Balka [dbalka@saintmarys.edu]
Sent: Monday, January 24, 2005 8:27 AM
To: Eric.Mann@uconn.edu
Subject: Re: CAMT

Dear Eric,

I am delighted that you are interested in using CAMT. You do, indeed, have my permission to use the test.

Recent email messages have listed the EARCOME 3 conference in Shanghai this summer, where the theme will focus on creative ability in mathematics. I have pulled the dissertation from the shelf and have contacted a study group leader about submitting a proposal for the conference.

If I can be of any help, please contact me.

Don S. Balka
Mathematics Department
Saint Mary's College
547-284-4496