A Gateway to Numeracy Mieke van Groenestijn

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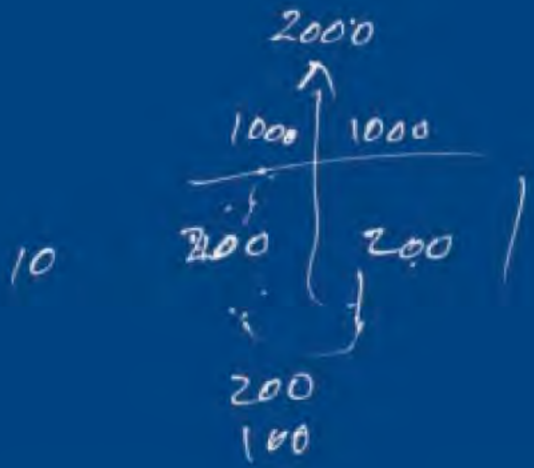

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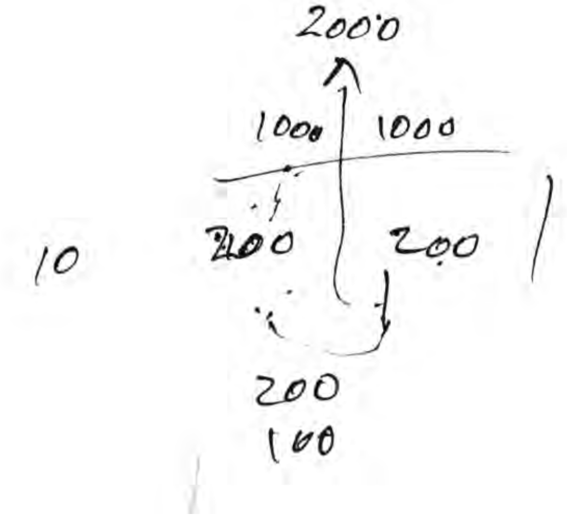

Tevens verschenen als dissertatie ter verkrijging van de graad van doctor in de sociale wetenschappen aan de Universiteit Utrecht.

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The cover illustration shows a computation of Azeb, one of the learners in this study.

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Preface

Working in the field of adult numeracy is still pioneer work. Despite about twenty years of development the concept of *numeracy* is hardly known in and out of adult education. On the contrary, the word *mathematics* is very well known and often recalls negative school memories to learners in adult education. This study explores the concept of numeracy and shows how adults in adult basic education deal with their mathematical knowledge and skills, as learned in former school years. It offers a design for a numeracy program in adult basic education.

Writing a dissertation cannot be done without support of many people. In general, many people contributed to this study without being aware of it. First there were the learners in adult basic education of whom I learned that mathematics is very important and that being numerate is more than only being familiar with numbers. They made clear to me that adults want to learn the math they really need to manage their everyday lives. Second, there were the teacher-students in my courses who discussed their problems concerning learning and teaching mathematics in adult basic education and asked critical questions. They helped to uncover the actual problems they encountered in their classes. Third, there were my enthusiast colleagues, in particular in the beginning of adult basic education, who also worked hard on developing practical instructional materials for learning mathematics by adults. We started from scratch and learned a lot from each other.

More specifically concerning this study I want to thank a few persons. I am very grateful to the learners of the Adult Learning Center in Nieuwegein, ROC Utrecht. Their contributions to this study helped to uncover and explore an almost unknown field in adult education. This could lay a basis for further research. Also many thanks to their teachers, in particular Piet van Rheenen and Peter Pot. Without their enthusiastic participation this dissertation could never have been written.

The many fruitful discussions with the authors of “Supermarket Strategy” and “In Balance” have contributed to the development of my theoretical thoughts concerning learning mathematics by adults. In parallel, I learned a lot from my friends of our international numeracy team: Iddo Gal, Myrna Manly, Mary Jane Schmitt, Dave Tout and Yvan Clermont.

I am very grateful to Koeno Gravemeijer en Iddo Gal for their great support at the writing of this dissertation. They helped shaping my thoughts and making my thoughts visible on paper. I also want to thank Myrna Manly and my son Wouter for their corrections to the English text.

Finally, I want to thank Willem for his endless patience. I am now ready to welcome our first grandchild.

Overview of Sections and Chapters

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Introduction

Background

This study has its roots in my work in adult basic education (ABE) and in my interest in adult numeracy. I started as a volunteer teacher in an adult literacy course in 1981. Many years of working with adults who had little or no formal schooling have laid the basis for my ideas about learning and teaching in ABE. Since 1986 I have been involved in the development of mathematics education to adults in ABE. I observed and interviewed many adult learners and trained ABE teachers to teach mathematics. In parallel I got involved in international research into numeracy of adults. All these experiences together are the breeding ground for this study.
The actual start of this study was in December 1997 when a group of thirty-seven newcomers started a mathematics course in an ABE learning center in Nieuwegein, near Utrecht in the Netherlands. That time adult education in the Netherlands was changing into a new system of adult education. ABE would be merged with other parts of adult education and with vocational education. The new system became operational in August 1998. This change would also have consequences for mathematics education in ABE.

The ABE population is only a small part of the total population in adult education (about 10%). Most of the learners in ABE are second language learners (about 80%). Hence ABE focuses mainly on learning Dutch and becoming familiar with the Dutch society. The direct purpose of ABE is that learners stream on to vocational education and to the labor market as soon as possible. The fact that mathematics, or numeracy, is also a precondition for success in vocational education and work is of less interest. Mathematics education to learners in ABE has always taken place in the shadow of learning Dutch as a second language. However, internationally, interest in mathematics and numeracy for adults has been growing in recent years. Results of studies in the U.S. and international comparative studies show that numeracy is of real international concern, i.e. the Young Adult Literacy Survey in 1986 (YALS) and the National Adult Literacy Survey in 1992 (NALS) in the U.S., and the following International Adult Literacy Survey (IALS) in 1996 (Van der Kamp and Scheeren, 1996, Dossey, 1997, OECD, 1997, NCES, 1998, Houtkoop, 1999)[i]. The IALS comparative study was held in twelve countries, the Netherlands among them. In these surveys information was acquired on three scales: prose literacy, document literacy and quantitative literacy. The results were shocking. According to the YALS, for example, about 40% of the young adults (age 21-25) could correctly compute the change they should get back from an amount tendered after having received the bill for a meal in restaurant and having computed 10% for the tip they should leave. Only one in five could read a bus schedule. The results on YALS showed that *"young adults lack many of the 'walking around' quantitative skills we would expect of almost any citizen."* (Dossey, 1997, p.46-47) Between one-third and half of the population achieved low on the scale of quantitative literacy in all three surveys. (Van der Kamp and Scheeren, 1996, Dossey, 1997, Houtkoop 1999). This indicates that these adults may have big problems with the interpretation of and giving meaning to quantitative information. Hence they may be hampered in their functioning in personal and societal life and work. The results of these studies showed that literacy and numeracy are still of great concern in western countries.

These studies made clear that adults also need to have mastered some numeracy skills in addition to literacy skills, even though one might suspect the opposite with all technological developments in the past decades. After all, technological means have been developed to liberate us from many mathematical tasks. On the contrary, due to the developments of technological tools, new mathematical skills are required for being able to deal with such new tools. Hence the question emerges what numeracy skills adults should have acquired to be numerate in personal life, society and work. In addition, in order to keep up with these technological developments, the need for lifelong learning has become clear. Consequently the need for the development of numeracy programs has been growing.

Setting

Central in this study are a group of low-schooled second language learners in ABE at the Regional Education Center Utrecht, department Nieuwegein, in the Netherlands. After about four months intensive language training these learners were strongly advised to do a numeracy course to be better prepared for vocational education and work. The learners would first take a math test in December 1997. In discussion with the learning center the learners took two tests: the new national *Cito* math test for adults and the experimental *In Balance* placement test, both published in 1996[ii]. This was done to be able to compare both tests on feasibility, effectiveness and results. With those two tests qualitative information could be acquired about the mathematical knowledge and skills of these adults at the start of a numeracy course. After that the learners were offered a six-week program on percentages for one hour per week, based on the *In Balance* instruction materials. The program was finished with a post test, composed of the same *Cito* and *In Balance* percent tasks as at the placement test, and with some interviews with individual learners. The results on both tests and the learning experiences during the course are the empirical ingredients for this study. They show the characteristics of these learners and serve as the seeds in a breeding ground for development of a theory on learning and teaching mathematics in ABE.

Main Theme

Internationally numeracy education to adults in ABE has a short history and is mainly built on experiences in mathematics education to children. Very often numeracy courses consist of just teaching mathematics. The difference between numeracy and mathematics in adult education is often not clear. Sometimes the two words are just used as synonyms. In the Netherlands the word "numeracy" [in Dutch "gecijferdheid"] is hardly used in adult education. Internationally teachers in adult education struggle how to organize numeracy courses that satisfy the wishes and needs of adults and that make sense in the frame of adult numeracy. The quality of programs often depends on the capability and experiences of individual (voluntary) teachers. There is little or no theoretical foundation for adult numeracy courses and no common base how to organize such courses. The existence of standards, curricula and national reporting systems is no guarantee for the quality of local numeracy courses. Little is known about the effectiveness of numeracy courses in ABE. Hence the main question in this study is:
What content should be offered in a numeracy program for learners in adult basic education and how should it be organized?

To find an answer to this question four subjects will be explored and elaborated thoroughly in four sub-studies. The first three studies provide the building blocks for the fourth study.

The first study concerns the questions "What is numeracy?" and "Who are the learners in ABE?". Here we focus on the development of adult basic education in general, on the concept of numeracy and on the problematique concerning numeracy in ABE.

In the second study the emphasis has been laid on the question "What do learners in ABE know about mathematics when they enter ABE?" To answer this question we may first wonder how to assess adults and what kind of assessment tools are appropriate to acquire information about the learners' mathematical knowledge and skills when they enter ABE. After that we will go into depth about the actual mathematical knowledge and skills of a group learners in ABE.

In the third study we will focus on the development of a theoretical basis for learning mathematics by adults in ABE in the frame of functional numeracy. This study will be supported by examples of actual learning situations acquired in a learning-teaching experiment in ABE.

In the fourth sub-study a framework has been developed for a program on functional numeracy education for learners in ABE, based on the information acquired in the first three sub-studies.

Set-up of this study

The four studies mentioned above have been elaborated in the following sections and chapters:

Section 1: Numeracy in Adult Basic Education

Chapter 1 briefly describes the development of adult basic education in the Netherlands. It outlines the new system for adult and vocational education. The features of the population are described. Within these developments, mathematics education has its own history, which will be addressed as well. In this chapter the research questions for this study have been set.

In chapter 2 the concept of numeracy is analyzed. After a brief historical overview of the definitions of quantitative literacy, mathematical literacy and numeracy, as it developed internationally, and research done into these areas, a working definition is set for numeracy in this study. This definition also focuses on numeracy needed for the future in the frame of lifelong learning. The four components identified in this definition serve as keystones for the development of a theory in chapter 5 and for program building in chapter 7.

Section 2: Numeracy Skills of Adults in ABE

To be able to develop a numeracy program for adults we first need to know what adults actually know about mathematics and can do when they enter ABE. Chapter 3 starts with a brief study into the developments in assessment of mathematics in elementary school. After that goals are set for assessment of adults and criteria are described for appropriate assessment tools. Three assessment tools for adults are discussed: the *Supermarket Strategy*, the *In Balance* placement test and the *Cito* test for adults. The latter two tests have been used in this small scale study to acquire qualitative information about mathematical skills of learners in ABE. The results of this field study have been analyzed in chapter 4. The conclusions provide building blocks for the learning of mathematics by adults and for the content of a numeracy program in ABE.

Section 3: Numeracy Learning and Teaching in ABE

In chapter 5 theories about the learning of adults in general and about the learning of mathematics in particular are discussed. The developments in elementary school concerning action theory, constructivism and realistic mathematics education (RME) are a point of departure for a theory about learning mathematics by adults. The four components of numeracy, identified in chapter 2, are the keystones in this theory development. The theoretical ideas developed here are matched in chapter 6 with learning experiences observed in the learners group during the six-week program on percents.

Section 4: Development of a Numeracy Program for ABE

In chapter 7 a framework for a program for Functional Numeracy Education in ABE is designed based on the ingredients of the six previous chapters.

Section 5: Conclusions and Discussion

Finally, in chapter 8 we will look back at the research questions. Suggestions for follow-up studies and topics for discussion are posed.

Characteristics of this study are:

Exploratory and descriptive
This study is exploratory and descriptive in a threefold way. First, it explores the concept of numeracy and its meaning for adults in personal life, society and work. It describes research done into adult numeracy. Second, it pictures the numeracy knowledge and skills of a relatively unknown population in foundational education. Third, this study explores, analyzes and sketches the field of numeracy education in foundational education.

Qualitative analytic
Results of adult learners in foundational education on mathematics placement tests and observations concerning their learning of mathematics in classroom sessions have been analyzed. This analysis yields much qualitative information that serves as ingredients for theory building and program development for functional numeracy.

Theory oriented
A study of theories about learning by adults in general and of theories about learning of mathematics in particular, provides points of departure for theory building and an instructional model for numeracy learning and teaching in adult basic education.

Developmental
The information acquired in the first three sections in this study has been used for the development of a numeracy program that should lead to functional numeracy.

This study aims to contribute qualitatively to the development of knowledge in the field of functional numeracy and the learning of mathematics by under-schooled adults in Adult Basic Education.

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[i] More about these studies and other research into numeracy of adults is mentioned in chapter 2.

[ii] The Cito mathematics test for adults was developed as part of a series of independent tests for the new system of adult and vocational education by the national center for development of test materials *Cito.* The mathematics test was published in 1996. (Cito, 1996) The *In Balance* placement test was published as part of the *In Balance* series, a set of instruction materials and tests on mathematics for adults in ABE. This series was published in the years 1996-2000. (van Groenestijn et al, 1996, 1999, 2000)

1 Brief History of Adult Basic Education in the Netherlands

1.1 Introduction

To set the stage for this study this chapter provides a brief historical sketch of the development of Adult Basic Education (ABE) in the Netherlands from the early sixties. Though this overview examines a local, national situation, it can be seen as part of the international developments in adult education in the light of lifelong learning. Dutch adult education changed in 1996 to a coherent system of general and professional education courses, completed with certifications. This system enables citizens to keep up with (technological) developments in society. ABE became part of this system and was renamed “Foundational Education”.

The changes in adult education not only involve the structure of the organization but also the content of the programs. They also affect mathematics education in ABE. The development of mathematics education to adults in ABE in the Netherlands started from scratch in 1986 and has been developing since. The domain-specific theory for Realistic Mathematics Education (RME) was the starting point for this development. This theory - originally developed for primary and secondary school - was generally accepted in the Dutch ABE without any comment. However, there has never been systematic research in the Netherlands into how under-schooled and illiterate adults in ABE learn mathematics and into its implications for teaching mathematics to these adults. In fact, there are still no empirical grounds to accept that RME was the right choice at that time. Now may be the time for a critical reflection. Learning more about national and international developments in this field may help to lay a theoretical foundation for learning and teaching mathematics in ABE. As a start, this chapter outlines the developments in ABE in the Netherlands and states the situation as it is now.

Section 1.2 of this chapter describes the development of Adult Basic Education. In section 1.3 the main features of the population in ABE are described. After that the focus in section 1.4 moves to the development of mathematics education in ABE in the Netherlands. The chapter closes with the wording of the research questions for this study in section 1.5.

1.2 Development of ABE in the Netherlands

1.2.1 Early adult education projects

After several years of informal youth and young adults projects, women and workmen projects in the Netherlands, mainly organized by Churches, companies and social organizations in the first half of the twentieth century, the need for a better organization of adult education increased. This was in particular due to the rebuilding of the country after the second world war and in relation to the economic expansion in western countries.

Ideas about adult education became more substantial after the UNESCO conferences on "Adult Education" in Montreal in 1960 and on "New trends in Adult Education" in Marly-le-Roy in 1967, where initial thoughts about the need for lifelong learning were elaborated. In a continuously developing society, where circumstances in personal life and work change, the need for lifelong learning became evident. The conferences stressed the importance of the development of individual adults to be active and critical citizens. Adult education should be encouraged and systematically organized by governments in a coherent system of education for children, young adults and adults. Policy makers were to be responsible for the elaboration of this. (Snijders & Van der Zwaart-de Gast, 1987). In reaction to this concept of lifelong learning for personal wellbeing, the Organization for Economic Cooperation and Development (OECD) proposed recurrent education for the interest of economic developments. In our technological, rapidly and continuously developing and changing society, people should have a chance to brush up on and expand their school knowledge and skills on a regular basis, because of the need for qualified employees. Recurrent education aimed short courses and learning units that are completed with certifications. Such courses should be part of a coherent system of adult education. In this way all kinds of courses and activities, often organized by companies for their own employees, could be acknowledged and certified. Such a system should enable adults to acquire qualifications in their course of life. In addition it could give access to further education and training. (Snijders & Van der Zwaart-de Gast, 1987).

Many years later, in 1992, the OECD published the results of a study on "Adult Literacy and Economic Performance" in which adults basic skills, in particularly literacy skills, were directly placed in the context of economic consequences for industrialized western countries. It appeared that illiteracy and semi-literacy was becoming of greater concern in western countries, because the many under-schooled employees could hamper the technological and economical developments. However, *"It is not that schools are turning out demonstrably less literate graduates than in the past, but that the ways in which adults need to apply literacy skills are becoming far more demanding"* (Stern and Tuijnman, 1997, p. 2).

In this international climate the Dutch adult education system was developed further. Between 1960 and 1980 many projects started, e.g. literacy projects, second language projects for immigrants, home projects for Islamic women, youth projects, women projects, open school projects, parent courses. Local adult education centers organized all kinds of courses. Companies started organizing vocational training courses. Some national initial television courses and courses for distance learning started during those years as well.
Around 1980 all those different projects were reorganized to six projects: literacy projects, educational activities for cultural minorities, Open School projects, educational networks, preparation to retirement and qualifying educational activities for vocation. This reorganization was part of preparations for a more coherent system of adult education. In the summer of 1983 three initial acts concerning arrangements for Adult Education were accepted by the Dutch Government: the act for arrangement of Adult Basic Education (ABE), the act for arrangement of harmonization of Adult Education and the act for arrangement of paid educational leave. (see Snijders & Van der Zwaart-de Gast, 1987)

Following these arrangements ABE started in August 1986 and aimed to provide "educational activities that enable adults to acquire knowledge, skills and attitudes that are at least necessary to function in personal and societal life". (Ministerie van O&W, 1986) Those activities should consist of language, mathematics and societal knowledge and skills. Therefore the program provided Dutch for native citizens, Dutch as a second language for immigrants and elementary knowledge of the English language. Adults should be able to communicate effectively in written and spoken communication in personal and societal situations. The mathematics component provided elementary mathematical knowledge and skills to enable adults to manage their own personal and societal life. Societal knowledge and skills aimed to empower adults to communicate effectively in personal and societal interaction. It was strongly recommended to teach these three components in an integrated way. ABE also provided activities to recruit new learners and to bridge the gap between ABE and further education and vocational training in order to create access for advanced ABE learners.

ABE was meant for adults with no more than eight years of schooling: primary school with a maximum of two years secondary school not completed. It aimed to empower and enable adults to manage their family life, work situations, to function in society as critical citizens and to prepare people for further education and vocational training. ABE should create a bridge to further education and vocational training for adults who have had no schooling in a long time or needed some brushing up.
At that time ABE had to change from a variety of volunteer projects into a professional system with qualified teachers. From August 1987 until January

1991 was the grace period during which volunteers could become qualified by completing additional courses.

1.2.2 Current situation: Regional Education Centers

The Act of Adult and Vocational Education was introduced on January 1, 1996, and became operational in January 1997. (Ministerie van Onderwijs, Cultuur en Wetenschappen, 1994, 1998). Between August 1995 and August 1998 all institutions for adult education and vocational education were merged to a new education system, spread over regional institutes called “Regional Education Centers” [in Dutch: Regionaal Opleidingscentrum, (ROC)]. ROCs provide education to adults and to young students coming from secondary school. At the same time a new qualification structure for all general education and vocational courses was introduced. (table 1.1) Each ROC provides an education stream and a vocation stream, based on qualification levels for general education [in Dutch: KSE] and for vocational education [in Dutch: KSB].

The qualification levels 1 and 2 in general adult education concern elementary and foundational education and replace Adult Basic Education. Level 2 provides access to courses in further general education at level 3 and to courses in vocational education at level 1. In both streams an individual can progress vertically and horizontally. That means, for instance, that a person who passed basic level 3 can start courses at level 4 in general education but also at level 2 in vocational education. National standards for mathematics in the general adult education stream were published in 1997 (Kemme, Sormani & Weijers, 1997).

Level 2 in vocational education is a basic qualification. It is the minimum qualification for entering the labor market. Holders of a basic qualification are able to carry out relatively complex routines and standard procedures, with responsibility for their own work only. The assistant level is for those who are not able to attain a basic qualification, giving them the opportunity to obtain some sort of qualification nonetheless. Holders of qualifications on level 3 in vocational education are supposed to be able to carry out complex tasks, to take responsibility for their own actions and those of others, and to monitor and supervise implementation of standard procedures by others. At level 4 in vocational education one can choose from specialist or management courses. Management courses require non-job-specific skills like tactical and strategic thinking for managing professional situations.

Table 1.1 Overview Dutch Qualification System for General Adult Education and for Vocational Education.

<table>
<tr><td colspan="4">University + Higher Professional Education</td></tr>
<tr><td colspan="2">General Adult Education (KSE)</td><td colspan="2">Vocational Education (KSB)</td></tr>
<tr><td>level 6</td><td>Advanced level</td><td>level 4</td><td>a. Middle Management level
b. Specialist</td></tr>
<tr><td>level 5</td><td>Start-2 level</td><td>level 3</td><td>Professional level</td></tr>
<tr><td>level 4</td><td>Start-1 level</td><td>level 2</td><td>Basic professional level</td></tr>
<tr><td>level 3</td><td>Basic level</td><td>level 1</td><td>Assistant level</td></tr>
<tr><td>level 2</td><td colspan="3">Foundational level (Threshold level)</td></tr>
<tr><td>level 1</td><td colspan="3">Elementary level (Self-help level)</td></tr>
</table>

In the general education stream, level 3 is the minimum qualification one should have to participate in societal and vocational situations. Courses on that level encompass content from the first two years of secondary school. Level 4 in general education is equivalent to a Dutch VMBO diploma, internationally comparable to high school and the General Educational Development Diploma (GED) in the U.S. and to the General Certificate of Secondary Education (GCSE) in the U.K. Certifications at levels 5 and 6 provide access to higher professional education and university.

The total number of participants in general education in the year 2000 was 165,100 of which 48,800 participants were at the levels 1-3. The total number of participants in vocational education was 416,300 in the year 2000. (Ministerie OcenW, 2001)

This system is comparable to the British national qualification framework that also started in the mid nineties and provided national standards for literacy and numeracy in 2000. The core curriculum for numeracy was published recently in 2001. (Basic Skills Agency, 2001). There the original foundation level (ABE) was divided into three entry levels, which covers the Dutch levels 1 and 2 in general adult education. The Dutch system can also be compared with the levels in the Australian National Reporting System which is based on the National Framework of Adult English Language, Literacy and Numeracy Competence (Coates et al, 1995)

Keystones for the Dutch system are “education to measure” and “quality”. The system is based on learning outcomes, outcome-financing and quality evaluation. Teacher-free learning, cooperative learning, distance learning, apprenticeship and individual support are key activities in this system that enable adults to learn at their own pace and time in order to prevent drop-out. A portfolio system was introduced to collect assessment results, qualifications and certifications, etc. In this way the Dutch adult education provides a system for lifelong learning.

In this study the name Adult Basic Education (ABE) will be used to indicate the education at levels 1 and 2 in the general adult education stream. This concerns education to illiterate, semi-literate and under-schooled adults, native citizens as well as immigrants. This is done to make this study recognizable across countries. There were it specifically concerns the Dutch situation it will be referred to by levels KSE-1 and KSE-2.

1.3 The Population in Adult Basic Education

The first two levels of general education provide foundational education and is in fact the same as in the former ABE. Courses at these levels are accessible for adults with no more than eight years of schooling, primary education with a maximum of two years of secondary not completed education. In this population we distinguish two main groups: Dutch native speakers and second language learners.

The native adults can be divided into two sub-groups. The first group exists of adults who have completed elementary school more or less successfully and may have some specific questions like learning more about budgeting, learning about fractions, decimals and percent, learning more about the metric system. The second group are adults who failed in elementary school in earlier days and may be frustrated by severe problems with all kinds of elementary skills as well as math anxiety.

The second language learners can be divided similarly into two sub-groups: adults who completed elementary school with some follow-up courses in their home countries, and adults with little or even no schooling. Main sub-populations in the Netherlands are Turkish and Moroccan people, but there are also many learners from other countries e.g. Bosnia, China, Eritrea, Ethiopia, Hungary, Iran, Poland, Somalia, Suriname, Tsjech Republic. The Dutch society is also confronted with about 60,000 refugees a year with an educational background on all levels. These people stay in our country for an undefined length of time, depending on their refugee status. Special programs for refugees and other newcomers have been organized. Low level courses have been organized in Foundational Education. In the four biggest cities in the Netherlands (Amsterdam, Rotterdam, The Hague and Utrecht) around eighty

percent of the learners in ABE are second language learners. The ratio of native to non-native adults who participate in courses at levels 1 and 2 is about 30% - 70% for the entire country.
This mixture of target groups means that learning in ABE mainly concerns learning Dutch as a second language and learning through Dutch as a second language. A variety of courses for learning a second language have been developed during past twenty years. For every other activity in ABE it means that learning through a second language plays a big role. The language component is of great importance in all spoken and written instruction. It starts already at the intake procedure when applicants have to take assessment tests. This is of particular interest for this study when we talk about mathematics tests. What kind of tasks can be required of second language learners when they apply for a mathematics course? What minimum of language mastery do they need to be able to do a math course through a second language? What kind of tests do we need to test adults in general and second language learners in particular? This issue will be discussed in chapter 3.

Another problem is that the current adult education system is based partly on teacher-free learning. This policy is to enable learners to learn more and better in their own time and pace, and to take responsibility for their own learning, also in the frame of lifelong learning. This requires advanced learning skills. Adults in ABE often have little or even no school experience, so have acquired few learning skills for adequate learning in teacher-free learning settings. In combination with learning a second language this may cause problematic learning situations. For this reason, many teachers in ABE have doubts about teacher-free learning and wonder how to overcome these problems. This also concerns learning mathematics. This topic will be addressed in chapters 5 and 7.

1.4 Development of mathematics education in ABE

In light of the developments in ABE, a small project-group of enthusiastic ABE teachers started developing a mathematics program in 1986. Mathematics would become part of the ABE program and, apart from some informal and individual ideas, nothing had been developed on mathematics for adults in previous years. The project was based on Realistic Mathematics Education (RME), a new mathematics program for primary and secondary school, developed by the Research Group on Mathematics Education (OW&OC) at the University of Utrecht, currently the Freudenthal Institute. The purpose of the project-group was to develop a teacher-training course in mathematics for teachers in ABE, as part of the qualification courses, as described in section 1.2.1. The content of the program was also meant to provide basic ingredients for mathematics programs in ABE.

Yet, there had never been research into numeracy skills in adult education in the Netherlands. The word “numeracy” [in Dutch: “gecijferdheid”] was hardly known. However, everybody in the project group realized that adult math programs should encompass more than just mathematics and RME was a very good base for that. The group focused on mathematics based on real-life contexts that could be of general interest of adults. The project-group started from scratch and their efforts finally resulted in many interesting math topics that could be discussed with adults in ABE. These topics were also the ingredients for the first ABE teacher training courses that started in September 1987 in Utrecht.

These first courses were very experimental. Though there was a program, in fact little was known about the learning of math by under-schooled adults and about what and how to teach math to these adults. Besides, most of these teachers were second language teachers and were not at all used to teaching mathematics. Teachers came with many questions, such as: *“How do we know what they know? How do we know what they need to know and what they want to know? How can we assess adults’ math skills? How do we know where to start? How can we organize groups with so many differences in levels?”*

There were many more questions than answers. In fact, these questions set the base for this study. Little was known about what adults know when they enter ABE, what kind of math they need and should be taught, how to compose groups and how to teach. In particular, teaching to second language learners presented a problem because they had learned different types of computations, for instance different algorithms, different ways of working with fractions and proportions, different ways of reading time. Having a mix of different cultures in one learners’ group made teaching mathematics very complex. Also, semi-literate and illiterate adults showed a wide variety of mathematical skills. As a start, the teachers in the Utrecht courses were trained in interviewing techniques to learn to talk with adults about their mathematical knowledge and about the ways they do mathematics in real life situations. These interview techniques came from the “Kwantiwijzer”, a diagnostic assessment tool for elementary school (Van den Berg and Van Eerde, 1983, see also chapter 3). As the main stimulus a supermarket leaflet was used that offered possibilities to talk about different topics such as quantities, kilograms, grams, liters, decimals and doing money computations. That was the start of “Supermarktstrategie” [Supermarket Strategy]. This packet was published in 1992 after a two-year project. (van Groenestijn, van Amersfoort and Matthijsse, 1992). It provided goals for a curriculum and a placement test. (see extensive description in chapter 3). It offered a simple coherent structure for mathematical subjects on different levels. All tasks were set in everyday life contexts. The packet could help with testing adults and with designing a program for ABE. It set a new way for assessment of adults and helped teachers to learn more about the actual mathematical knowledge and skills of adults in ABE. However, this packet was just experimental and there were no instructional materials at that time. Teachers still had to compose their own instructional materials, mainly by selecting materials from elementary school instruction booklets and experimental products they created themselves.

Based on this “Supermarket Strategy”, an initial study was done in 1992-1993 into numeracy skills and problem solving strategies of adults in ABE in the Netherlands (van Groenestijn, 1993). The study focused on a group of twenty Moroccan adults, ten illiterate adults and ten adults with little schooling. The study provided qualitative information about the actual mathematical competencies, needs and wishes of adults in ABE. It resulted in a set of recommendations for numeracy learning and teaching in ABE.

The follow-up of that study and the Supermarket Strategy was the development of the “In Balance” materials. This new project started in 1993 and lasted until 1999 (van Groenestijn et al, 1994-1999, see chapter 3). The products include a new placement test, instructional materials, formative and summative tests and teachers guides, all especially designed for ABE. The series focuses on real-life numeracy contexts and tries to combine adults’ informal knowledge and problem solving strategies with formal procedures based on RME, Constructivism and Action Theory. The series was completed in 1999 with three units composed of tasks that aim to bridge the gap between the second level of the current adult education system and the first level of vocational training.
The development of these materials and the background studies underlying them, were the source of this study.

In the same period other products were published for ABE in the Netherlands. In December 1991, SVE, the institute for the development of adult education that time, currently Cinop, published goals for mathematics in ABE. (Luyten and Sormani, 1991). This was followed by an experimental program for bridging ABE courses with vocational programs (Matthijsse, 1994).
In 1998 the new National Institute for the Development of Adult Education (Cinop) published a module system and goals for the new levels 1 to 4 in general adult education. (Kemme and Sormani, 1998). At the same time the Dutch National Institute for Test Development (Cito) started developing an item bank with test items, also for placement (see also chapter 3) (Cito, 1996). Recently Cito began with the development of a computer adaptive test that aims to offer a broad ranch of possibilities for the entire adult education.
Many more materials were published over the years, often on single mathematical topics like fractions, percent, measurement, money computations, the use of a calculator, etc. These cannot all be mentioned here. A few publications that contributed to the development of mathematics in ABE came from Goffree and Stroomberg (1989), Ter Heege, Van Zon and Goffree, (1989), Ter Heege (1992, 1997) and Verschaffel and De Corte, eds. (1995).
At that time many enthusiastic teachers and researchers worked on the development of mathematics education in ABE in the Netherlands. However, it is strange to note that so little real research was undertaken to try to find out

what adults in ABE actually need and to develop a theory for the learning of mathematics by under-schooled and illiterate adults in ABE. RME was accepted in ABE without any critical reflection.

1.5 Research questions

Step-by-step mathematics education for learners in ABE had been created in the Netherlands. Developments happened in parallel, but since RME was the starting point of all materials most products are quite well synchronized. Hence the materials for the levels 1 and 2 of Cinop, Cito and the In Balance materials can be compared quite well.

Though all these developments have improved adult education in general and mathematics education in particular, we may wonder if mathematics education in ABE has been developing in the right direction. There are several points that beg for a critical reflection.

Mathematics education to learners in ABE in the Netherlands started from scratch in 1986 and reaped the fruits of RME that was originally developed for primary and secondary school. However it may be argued whether RME is indeed the right and only way for learning and teaching mathematics in ABE. This concerns two main points: the *content* that should be offered and *the way in which* mathematics should be taught. Hence the main question in this study is: *What content should be offered in a numeracy program for learners in Adult Basic Education and how should such a program be organized?*

In this discussion four elements can be distinguished.

1) In the light of lifelong learning we must think about what kind of mathematical knowledge and skills are really needed to be equipped for the future. It seems that RME can provide an appropriate set of mathematical tools, but the Dutch developments should be compared with international developments to get more background information and to set a wider perspective. For this the concept of “numeracy” needs a careful analysis. The next step is to determine what mathematical content adults need to function optimally in personal, societal and work life. Here in the Netherlands RME programs are offered. In other countries, where mathematics in school developed in a different way, the word “mathematics” is hardly used in adult basic education. In the U.S., Australia and the U.K. “numeracy” programs have been developed. What is the difference? This asks for an in-depth study about the international developments on the concept of “numeracy” in order to place the content in the Dutch ABE programs in the frame of worldwide lifelong learning. This sets the first research questions:

- *What is numeracy?*
- *What mathematical content should be offered to adults in ABE to be equipped for the future?*

2) In order to fine-tune the content of ABE programs to the competencies, needs and wishes of adults in ABE, it is essential to learn what adults actually know when they enter ABE courses. Here two questions emerge:

- *How do we assess the learners' mathematical knowledge and skills when they enter ABE?*
- *What mathematical knowledge and skills have adults acquired when they enter ABE?*

More specifically, the question about how to assess the learner's mathematical knowledge and skills when they enter ABE, can be divided into the following sub-questions:

a - What do we want to know?
b - What are criteria for good assessment tools for under-schooled and not-schooled adults?

Answers to these questions should provide qualitative information about what mathematical knowledge and skills adults in ABE have acquired and what content should be offered.

3) Learning and instruction in ABE is the next point for discussion. As noted before, RME was not developed for ABE, but was generally accepted as the way of learning and teaching mathematics in ABE in the Netherlands, but in fact, little is known about the actual ways of learning mathematics by under-schooled and not-schooled adults. There are specific theories about adult learning. There are also more general theories on learning, like action theory and constructivism. How can mathematics education in ABE profit from these theories? With the developments towards independent, teacher-free and cooperative learning in the frame of lifelong learning, we may wonder *what way of learning and teaching math/numeracy could be appropriate for adults in ABE.*
Findings here could be a base for building a theory on learning and teaching mathematics/numeracy to under-schooled and illiterate adults.

4) Findings to the questions above may provide building blocks for answering the final question in this study: *How to develop a program for functional numeracy education in Adult Basic Education that prepares adults to be equipped for the future?*

These research questions are elaborated in the next chapters.

2 Numeracy, a dynamic Concept

2.1 Introduction

> **Example 2.1: The birthday cake**
> Jannie, a Dutch woman in our literacy project and mother of three teenage kids, brought a cake with her to class. She told us that the cake remained after her daughter's birthday and we all enjoyed the cake. After a while it happened again and she told us that the cake remained after her husband's birthday. Again we enjoyed her cake, but I asked her why a whole cake remained. She told us she had a recipe for cake that fits nicely in her two baking molds. As she found it difficult to take half of all ingredients, she always baked two cakes, but most of the time one cake remained. Since she attended our literacy class for a few years and she loved baking cakes, we often enjoyed her birthday cakes.

Jannie was one of the learners who attended our first official local literacy classes in the early eighties. Learners like Jannie made it very clear that illiteracy was not the only problem we had to tackle. Many of our learners also asked for help with shopping, money, completing forms, doing measurement computations, reading time tables, reading and understanding weight, volume on packages, etc.. With Paulo Freire's theory about "learning from experiences" in mind (Freire, 1972), many literacy courses turned out to be a place where adults could share their everyday life problems and learned to manage real life situations. Learning reading, writing and arithmetic was only a small part of it. Voluntary teachers in these projects were often shocked by what they encountered in their classes. They were not well prepared for this; they were just volunteers. "Illiteracy" was an unknown phenomenon in the Netherlands and the words "numeracy" and "innumeracy" did not even exist in the Dutch language.

Now, two decades later, a lot of research has been done in several countries into mathematical knowledge and skills of adults. One of the first studies is Cockcroft's report *Mathematics Counts* (Cockcroft, 1982), that identified adults' mathematical skills mainly as feeling "at-home" with numbers. After that study, more research was begun into this topic and different words and definitions were introduced to describe adults' mathematical skills in real life situations, like "quantitative literacy", "quantitative reasoning", "mathematical literacy" and also "numeracy" and "innumeracy", as in YALS, 1986, NALS, 1992, IALS, 1992, ALL, 1999. (Paulos, 1988, Steen, 1990, 1997, Dossey, 1997) Though these definitions may differ in words, and differences between these terms are not always clear, the basic idea is unanimous: *real-life mathematics differs strongly from school mathematics.* This became very obvious in out-of-school studies from, among others, Lave, (1984, 1988, 1991), Saxe (1988), Carraher, Carraher and Schliemann (1985), Nunes, Schliemann and Carraher (1993).

At the beginning of the new millenium the world is flooded with numbers. Newspaper headlines, news bulletins, advertisements, reports about societal, economical, medical and technical research are full of quantitative information. Increasing use of computers at work and in personal life requires more complex quantitative thinking. In this world we may wonder what it means to be "numerate" or "innumerate" in the context of managing everyday life, work and societal situations, and where being numerate begins and being innumerate ends. Is there a minimum level of numeracy? What is innumeracy? We may assume that there is quite a difference between baking cakes and taking part in a highly developed technological society, but when is a person "innumerate"? Is baking two cakes instead of one a good "numerate" solution for not knowing how to take half of the ingredients of the baking recipe? Steen writes about the importance of being numerate in our society:

> *"As information becomes ever more quantitative and as society relies increasingly on computers and the data they produce, an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time"*
> *(Steen, 1997, in "Why Numbers Count")*

Developments in western countries in the area of technology are still going on in a continuously increasing pace. It is like traveling in a high speed train to a far-away station, but we don't know where and when the train stops. The gap between being numerate and being innumerate is only becoming wider. Who are the people who are doomed to be innumerate? What don't they have that other people do have? What are characteristics of the lucky people who can keep up with technological developments and may call themselves "numerate"? When do we decide that others missed the train and stay behind? Finding answers to these questions asks for extensive international research.

This chapter describes the developments in the field of literacy and numeracy in past three decades. It focuses on different concepts of numeracy as they developed world wide and sets the frame for *functional numeracy*.

2.2 Developments in adult literacy and numeracy

2.2.1 UNESCO Reports on Adult Education

Reading, writing and arithmetic, the three "R"-s, as a means for communication and for transmitting information are as old as mankind and indispensable to human life. People have always tried to communicate through symbols indicating words or quantities. It is therefore incomprehensible that, at the beginning of the twenty-first century, nearly 900 million people have not acquired literacy skills. Furthermore there are millions who have been unable to sustain them, even in the most prosperous countries. It is also alarming that around 140 million children all over the world are still out of school. These facts give a dim prospect of adult literacy in the near future (UNESCO, 1997, 2000).

At the Yomtien World conference on Education for all, in 1990, it was declared that

> *"every person shall be able to benefit from educational opportunities designed to meet their basic learning needs. These needs comprise both essential learning tools (as literacy, oral expression, numeracy, and problem solving) and the basic learning content (such as knowledge, skills, values, and attitudes)...........". It was also stated that "Basic Education is more than an end in itself. It is the foundation for lifelong learning and human development on which countries may build, systematically, further levels and types of education and training" (UNESCO, 2000, p. 15)*

During the UNESCO-World Conference on Adult Learning in Hamburg (CONFINTEO, July 1997) in the international year of Lifelong Learning, the right to literacy and basic education was emphatically re-emphasized and was formulated in article 11 of the "Hamburg Declaration on Adult Learning":

> *"Adult Literacy: Literacy, conceived broadly as the basic knowledge and skills needed by all in a rapidly changing world, is a fundamental human right. In every society, literacy is a necessary skill in itself and one of the foundations of other life skills. There are millions, the majority of whom are women, who lack opportunities to learn or who have insufficient skills to be able to assert in this right. The challenge is to enable them to do so. This will often imply the creation of preconditions for learning through awareness-raising and empowerment. Literacy is also a catalyst for participation in social, cultural, political and economic activities, and for learning throughout life. We therefore commit ourselves to ensuring opportunities for all to acquire and maintain literacy skills, and to create, in all Member States, a literate environment to support the oral culture. The provision of learning opportunities for all, including the unreached and the excluded, is the most urgent concern."*

Article 12 follows with:

> *"The recognition of the Right to Education and the Right to Learn throughout life is more than ever a necessity; it is the right to read and write, the right to question and analyse, the right to have access to resources, and to develop and practise individual and collective skills and competences".*

In "The Agenda for the Future of Adult Learning" the UNESCO commits itself to:

- *Linking literacy to the social, cultural and economic development aspirations of learners (art. 25)*
- *Improving the quality of literacy programmes through building links with traditional and minority knowledge and culture (art. 26)*
- *Enriching the literacy environment (art. 27)"*

In our developed prosperous western countries we can hardly imagine that such a declaration on literacy and so many points to work at, are necessary in the year 1997. Yet, there are still too many people who lack sufficient literacy and other basic skills for being able to manage their own life in the information society. It is expected that by the year 2005 the total number of illiterate people in the world will be about 850 million. This number includes about 10 million citizens from the developed countries. In addition these countries are confronted with many immigrants coming from developing countries and countries in transition.

The UNESCO 2000 report on World Education, the follow up of the Hamburg Conference, states that, though the number of illiterate adults is now decreasing and the majority of all young people now go to school and participate in formal education beyond elementary and fundamental stages, there are still too many adults who have acquired insufficient basic skills (UNESCO, 2000). In this report it is also stressed again that learning in school only when one is young, is not sufficient anymore. In this rapidly changing society "learning throughout life" is essential for "*adapting to the evolving requirements of the labor market and for better mastery of the changing timeframes and rhythms of individual existence*." (UNESCO, 2000, p.14). In this frame the UNESCO states that:

> *"In the broad conceptual framework of lifelong educational system, - a system which should ultimately provide every individual with a flexible and diversified range of useful learning options throughout his or her lifetime - formal, non-formal and informal education are clearly complementary and mutually reinforcing elements." (page 41)*[1]

This system of lifelong learning should enable all citizens to acquire the knowledge, skills and attitudes needed to be critical citizens and to function optimally in personal, work and societal life. However, as the information technology in western societies develops further, the knowledge gap between literate and illiterate citizens grows due to increasing literacy demands. This increasing gap may cause illiterate and semi-literate people to try to hide their lack of literacy skills. They typically find all kinds of strategies to survive, often so that literate people do not recognize them as being illiterate or semi-literate. When the problem of "hidden illiteracy" is recognized, society often has great difficulties accepting this problem as a societal problem, in particular if it concerns native citizens. This is a vicious circle and may be the very heart of this time-consuming fight against illiteracy.

In fact the fight for the human right for literacy was started by Paulo Freire (1968) in the sixties, only thirty years ago. His theory about *"learning from experiences"* has been put into practice all over the world. Many pages on literacy have been written since, but in the year of Freire's death (May 1997), only a few weeks before his planned visit to the UNESCO Conference in Hamburg, the fights for world-wide literacy were still ongoing.

2.2.2 The Cockcroft Report

That illiteracy and innumeracy were not restricted only to developing countries was already obvious soon after the start of Freire's literacy campaigns. The British survey "Make it Count" from David Stringer in 1979 (Cockcroft, 1982) mentioned that

> *"There are indeed many adults in Britain who have the greatest difficulty with even such apparently simple matters as adding up money, checking their change in shops or working out the cost of five gallons of petrol. Yet these adults are not just the unintelligent or the uneducated. They come from many walks of life and some are very highly educated indeed, but they are hopeless at arithmetic and they want to do something about it."*

The conclusion was that

> *"During this investigation the firm impression has built up - in the investigator's mind, at least - that functional innumeracy is far more widespread than anyone has cared to believe."*
> (Cockcroft 1982, chapter 2, page 5)

Cockcroft also mentioned that

> *"failure and consequent dislike of mathematics was often ascribed to a specific cause when young. Such causes included change of teacher or of school, absence through illness, being promoted to a higher class and becoming left behind, having an irascible or unsympathetic teacher who failed to resolve difficulties, or even over-expectation on the part of the parents, usually fathers. Criticism by husbands or wives or by other members in the family, especially comment about slowness or the need to use pencil and paper instead of performing a calculation mentally, also eroded confidence and contributed to decreasing use of mathematics."*
> (Cockcroft, 1982, chapter 2, page 8)

He wonders (page 9-10, art. 31) what the mathematical needs of adult life are by describing some examples:

> *"....... it may be clear that those who don't travel by bus or train probably have no need to consult timetables; those who don't drive a car have no need to buy petrol; those who do not have meals in hotels or restaurants have no need to be able to calculate a service charge..... There are, however, very few people who do not at some time need to be able to read numbers, to count, to tell the time or to undertake a minimal amount of shopping.*
> *(art. 32) Therefore, whilst realising that there are some who will not achieve all of them, we would include among the mathematical needs if adult life the ability to read numbers and to count, to tell the time, to pay for purchases and to give change, to weigh and to measure, to understand straightforward timetables and simple graphs and charts, and to carry out any necessary calculations associated with these."*

He stresses (art. 33) that

> *"it is important to have the feeling for number which permits sensible estimation and approximation.......and which enables straightforward mental calculation to be accomplished." He ends with the words: (art. 34) "Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much." (page 10)*

Cockcroft describes adult numeracy as

> *"an ability to cope confidently with the mathematical needs of adult life" as described above. (page 10, art. 35).*

About being numerate he writes:

> *"We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an* 'at-homeness' *with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demand of his everyday life. The second is an 'ability to have some appreciation and understanding of information which is presented in mathematical terms', for instance in graphs, charts or tables or by reference to percentage increase or decrease. Taken together, these implies that a numerate person should be expected to be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication.............Our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation." (page 11, art. 39)*

The Cockcroft Report was an important source of information and was the start of further research into numeracy in many western countries, especially in Britain, the U.S.A. and in Australia.

2.2.3 Developments in Australia

In 1977 the Education Department of Western Australia (Willis, 1990, Reeves, 1994) published a policy paper on literacy and numeracy which stated:

> *"The term* 'numerate' *is understood to mean* mathematical literacy....... *A person is considered to be literate and numerate when he has acquired the skills and concepts which enable him to function effectively in his group and community, and when his attainment in reading, writing and mathematics make it possible for him to continue to use these skills to further his own and his community's development."*

Reeves' report provides detailed information about skills needed to be considered numerate. Topics mentioned in this report are:

- Mastery of basic number facts (tables)
- Competence in operations (+, -, x, /) with whole numbers, fractions, decimals, percentages, money and measurements
- Skills in estimation in relation to these operations, and the habit of making estimates
- Skill in interpreting graphs
- Sound proportion concept
- Statistical literacy based on experiences with chance processes.

This document maintains a more traditional view by emphasizing the mathematical side of numeracy. After its publication numeracy in Australia focused more on the practical and functional application and use of mathematics. Also, awareness was growing that numeracy may have a different content for persons in different situations. Numeracy skills of a car service man in a garage could be quite different from a nurse in a hospital. The Australian Beazley Committee definition is typical:

> *"Numeracy is the mathematics for effective functioning in one's group and community, and the capacity to use these skills to further one's own development and of one's community."*
> *(Beazley, 1984)*

About one decade later Reeves (1994) described being numerate as possessing some mix of mathematical skills, knowledge, understanding, intuition and experience which could be expressed as senses, namely number sense, spatial sense, data sense, measurement sense, formula /relationship sense. In addition, there are kinds of action that are evident in numerate behavior. These are:

> *"being fluent, that is being comfortable and confident in their use of Mathematics in different situations and applications of a mathematical nature; using their mathematical sense as a base on which to build further learning both in and through Mathematics; reflecting on their personal use of Mathematics, and the uses made by others."*

With this broadening, "numeracy" has attained a deeper meaning than "mathematics".

In 1994 The Queensland Department of Education in Australia defined numeracy as: "Numeracy involves abilities that include interpreting, applying and communicating mathematical information in commonly encountered situations to enable full, critical and effective participation in a wide range of life roles."

From this date definitions and descriptions about numeracy show an increasing broadening from exclusively mathematical components to mathematics in combination with several other skills. People became increasingly aware that numeracy not only involves "being familiar with math", but also "how to use math" in different situations and that "being numerate" can encompass different knowledge and skills for individuals in different situations.

In 1996 Kindler et al stated that numeracy is organized into four broad categories or domains, according to different purposes and functions of using mathematics. *Numeracy for Practical Purposes* addresses aspects of the physical world that involve designing, making, and measuring. *Numeracy for Interpreting Society* relates to interpreting and reflecting on numerical and graphical information in public documents and texts. *Numeracy for Personal Organization* focuses on the numeracy requirements for personal organizational matters involving money, time and travel. *Numeracy for Knowledge* is another domain of numeracy that describes the mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings and/or assumptions (Gal et al, 1999)

Several courses and instructional materials have been developed for adult numeracy programs since the late eighties (Marr and Helme, 1987, 1991, 1995; Goddard, Marr and Martin, 1991; Marr, Anderson and Tout, 1994). In conjunction a teacher training program was developed and also put into effect. (Willis, 1990, Tout, 1991, Tout and Johnston, 1995, Johnston, 1992, 1995,).

2.2.4 Developments in the U.S.A.

Everybody Counts

In the United States of America, soon after the Cockcroft Report, the report "Everybody Counts" (1989) was published. In this report the authors refer to two kinds of "literacy": "verbal" and "mathematical". Numeracy is described as "*mathematical literacy"*. The report holds that the two kinds of literacy

> *"....although different, are not unrelated. Without the ability to read and understand, no one can become mathematically literate. Increasingly, the reverse is also true: without the ability to understand basic mathematical ideas, one cannot fully comprehend modern writing such as that which appears in the daily newspapers."*

The report goes on to say that

> *"Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one must be able to grasp the implications of many mathematical concepts - for example, chance, logic and graphs - that permeate daily news and routine decisions." The report suggests that "citizens must become mathematically literate enough to distinguish evidence from anecdote, recognise nonsense, understand chance, and value the notion of proof. In short, bombarded daily as they are "with conflicting quantitative information", they "need to be aware of both the power and the limitations of mathematics". (Reeves, 1994)*

The report laments that most learners spend most of their school mathematics time learning only "practical arithmetic".

> *"Secondary education is particular devoid of exposure to modes of mathematical thought required for intelligent citizenship." (Reeves, 1994)*

Meanwhile John Allen Paulos (1988) published his book "Innumeracy" that describes numeracy by discussing the opposite. He describes many examples of how knowledgeable citizens are being plagued by innumeracy, an inability to deal comfortably with the fundamental notions of number and chance.

YALS

In 1986 an important start was made with national research into quantitative literacy skills of adults as part of the National Assessment of Educational Progress project (NAEP). In a first study, the Young Adult Literacy Skills (YALS), young adults (21-25) were assessed on tasks related to three literacy scales: prose literacy, document literacy and quantitative literacy. These scales measure

> *"the ability to apply the knowledge and skills to understand and use the information from texts similar to newspapers and popular magazines; to locate and use information contained in job applications, schedules, maps, tables, and indexes; and to apply arithmetic operations embedded in printed materials, such as recording checks in a check register, computing the amount of a tip, completing an order form, or determining the amount of a loan." (Dossey, 1997, p. 46)*

In this description quantitative literacy has been defined as: *"the knowledge and skills required to apply arithmetic operations, either alone or sequentially, using numbers embedded in printed material" (e.g. balancing a checkbook, completing an order form).* (Gal et al, 1999)

The results of this first attempt led to the conclusion that *"performances on the scale were less than what the nation required in human talent for long-term international competitiveness"* (Dossey, 1997). In the area of quantitative literacy, for example, 72 percent could handle tasks such as totaling a deposit slip and correctly subtracting on the check register, but only about 40% percent could correctly figure out the change they should get back from a meal and the amount of tip they should leave, given what they had ordered, the amount of money they had tendered, and a copy of the menu.

Based on these early attempts to measure aspects of quantitative literacy, one began to consider how to define literacy broadly related to school mathematics programs, the demands of employment and the needs of society. In the American Heritage Dictionary of the English Language "literate" is defined as "able to read and write" and as "knowledgeable, educated". These definitions provide some directions for understanding the meaning of quantitative literacy. (Dossey, 1997, p. 48). Dossey concludes that

> *"everyone who is quantitatively literate would be capable of using written, spoken, or graphic sources dealing with number, spatial, or data information in achieving goals and functioning in everyday life. In short, quantitative literate people are capable of manipulating aspects of mathematical knowledge to understand, predict, and control situations important to their lives. Such people have the ability to reason in numerical, data, spatial, and chance settings; to integrate and apply mathematical concepts and procedural skills; and to develop and interpret models related to the problems they encounter". (Dossey, 1997, p.48)*

NALS

The follow-up of YALS was the National Adult Literacy Survey (NALS), done in 1992. In this study the three components of literacy as used in YALS, i.e. prose, document and quantitative literacy, were assessed on a five-level scale. (Dossey, 1997). Briefly summarized, the conclusions were that

> *"....roughly 50% of American adults would either have major difficulty with or be fully unable to handle real world tasks such as (a) using a bus schedule to determine departure time for a bus going to certain destination, or how long it takes to travel from one spot to another; (b) identifying a trend on a simple graph showing yearly changes in sales figures; (c) using a calculator to find the difference between the regular and the sale price printed in an advertisement; (d) estimating the cost-per-ounce of a grocery item based on the unit price label that is found in many supermarkets; (e) reading and understanding a table summarizing results from a survey of parents and teachers about school-related issues; or (f) calculating interest charges associated with a home loan. An additional one quarter of adults interviewed for the NALS had difficulty with even simpler tasks such as determining the shipping charges for office supplies and completing an order form, finding the overall difference in cost of buying theater and bus tickets for attending two different shows, or identifying a certain figure on a pay stub." (Gal, 1993, p.2)*
>
> *"Performance on these and other tasks used in the NALS suggests that many everyday or work-related literacy tasks involving either manipulation of numbers or comprehension of quantitative information that are embedded in*

> *various types of text prove difficult for a large proportion of adults in the United States" (Gal, 1993, p.2)*

NCAL Report

In 1993 the technical report "Issues and Challenges in Adult Numeracy", an extensive inventory study on research in adult numeracy in the US, was published by the National Center on Adult Literacy (NCAL), University of Pennsylvania. (Gal, 1993)

The NCAL report mentions that there is no consensus on the meaning of numeracy. Two extreme views on numeracy are described. One view equates numeracy with basic math skills, much in the same way that literacy may be viewed as reading and writing skills. In this view, numeracy encompasses the lower end of mathematics, or whatever math educators attempt to achieve in the early grades. Another view of numeracy covers a much larger set of issues than basic computational skills, and focuses on adults' capacities to interact with the quantitative aspects of the adult world. In the NCAL report numeracy is viewed as *"encompassing a broad set of skills, knowledge, strategies, beliefs and dispositions that people need to autonomously engage in and effectively manage situations involving numbers, quantitative or quantifiable data, or information based on quantitative data."* (Gal 1993, 1997)

Implications for adult education in the U.S.A.

In response to the results of the previous studies, a group of adult educators in the U.S.A. formed an initiative for change in 1996. They started a practitioners network to develop a framework for adult education and to improve the learning and teaching of mathematics in adult education. This network was called the "Adult Numeracy Practitioners Network" (ANPN). Currently known as the Adult Numeracy Network (ANN), it sponsors an Internet discussion group and arranges meetings across the U.S.A. Network members created a preliminary framework of numeracy skills that need to be emphasized in adult literacy classes, the Massachusetts Adult Basic Education Math Standards (Leonelli and Schwendeman, 1994). The ANPN accepts that mathematics for adults is more than computation: *"It is a set of concepts, principles and relationships which serves as a powerful symbol system and tool for describing and analyzing our world."*

The ANPN Massachusetts framework focuses on twelve standards of which four broad beliefs, seven content themes and one evaluation standard are adapted from the NCTM-standards (NCTM, 1989). The first four standards and the evaluation standard are the core of the Massachusetts ABE Math Standards. They form the basis for all recommended methodologies which follow. The four broad standards are: Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning; and Mathematical Connections. The seven remaining standards deal with individual content areas: Estimation; Numbers, Operations and Computation; Patterns, Relationships and Functions;

Algebra; Statistics and Probability; Geometry and Spatial Sense. The matching of the seven content themes with the four broader standards has the potential to further enhance the understanding of needed skills and dispositions.
The standards are based on a field research done by twenty ABE teachers to implement the 1989 NCTM K-12 standards in ABE. (Leonelli, Merson and Schmitt, 1994).
A follow-up study based on the Massachusetts ABE Math Standards resulted in *"A Framework for Adult Numeracy Standards: the Mathematical Skills and Abilities Adults Need to be Equipped for the Future"* (Curry, Schmitt, Waldron, 1996). It explored the Massachusetts standards in the view of about two hundred adult learners, teachers and stakeholders across the country in light of future numeracy demands. The main conclusion was that numeracy must be an integrated component of adult education programs and of workplace learning environments in a basic and systemic way, not as an afterthought.

Preliminary programs based on these standards are published for ABE by Goodridge, Leonelli, et al *(Number Sense: ABE Math Curriculum Frameworks Unit, 1998)* and by Huntington, Leonelli, et al *(ABE Priority Math Curriculum, 1998.)*

2.2.5 International Surveys

IALS

In the years 1990-1996 an international follow-up of the NALS was conducted by the Organization for Economic Cooperation and Development (OECD): the International Adults Literacy Skills survey (IALS). This was the first international comparative survey on adult literacy. This survey was done in several OECD countries (OECD, 1997), among others Canada, Germany, Ireland, the Netherlands, Poland, Sweden, Switzerland, and the United States[2]. In this survey the same three literacy domains were used as in YALS and NALS: prose literacy, document literacy and quantitative literacy. Quantitative literacy was described here as: "*The knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a check-book, figuring out a tip, completing an order form, or determining the amount of interest on a loan from an advertisement."*

In brief, results from this survey show that a relatively small percentage of adults achieved the upper two levels 4 and 5 of prose, document and quantitative literacy. Sweden was on top with 36%, followed by Germany (24%), Canada and United States (23%), the Netherlands (20%), and Poland (7%). In comparison, a larger percentage of adults in most countries function on the lower levels 1 and 2, e.g. Poland with 69%, the United States with 46%, the

Netherlands with 36% and Sweden with 25%. (Dossey, 1997, Statistics Canada, 1996, OECD, 1997, Houtkoop, 1999). (see appendix 1)

That these results may have consequences for economic situations in each country, and therefore for policy makers and employers, may be clear. Conclusions from the IALS reports have led to insight into the need to further development of systematic international comparative large scale assessments on adults' basic skills.

ALL

In the follow up survey of IALS, the Adult Literacy and Life Skills Survey (ALL) that will be fielded in 2003 in several countries, quantitative literacy has been replaced by a thoroughly elaborated numeracy domain. Whereas quantitative literacy includes simple arithmetical operations embedded in printed materials, numeracy covers a broad scale of mathematical operations in diverse situations. The ALL numeracy development team[3] described numeracy as: *"The knowledge and skills required to effectively manage the mathematical demands of diverse situations."* (Gal, van Groenestijn, Manly, Schmitt and Tout, 1999)

It is the opinion of the numeracy development team that "numeracy" itself cannot be tested, only "numerate behavior". With this purpose in mind and to be able to create items for the numeracy assessment, the team operationalized the definition as:

> *"Numerate behavior involves managing a situation or solving a problem in a real context by responding to mathematical information that is represented in a range of ways and requires the activation of a range of enabling processes and behaviors."* (Gal et al, 1999)

Three studies provided information for the fundamental ideas on numeracy in the future world. Steen (1990) defined six broad categories pertaining to: *Quantity, Dimension, Pattern, Shape, Uncertainty*, and *Change*. Rutherford & Ahlgren (1990) described networks of related ideas: *Numbers, Shapes, Uncertainty, Summarizing data, Sampling,* and *Reasoning*. Dossey (1997) categorized the mathematical behaviors of quantitative literacy as: *Data representation and interpretation, Number and operation sense, Measurement, Variables and relations, Geometric shapes and spatial visualization,* and *Chance*. From these three closely related categorizations the ALL numeracy team drew a set of five fundamental ideas that characterize the mathematical demands met by adults in diverse situations at the beginning of the 21st century: quantity & number, dimension & shape, pattern & relationships, data & chance, and change. More specifically, the operational definition says:

Table 2.1 Numerate Behavior

Numerate behavior involves:
managing a situation or solving a problem in a real context
everyday life
work
societal
further learning

by responding
identifying or locating
acting upon
- order/sort
- count
- estimate
- compute
- measure
- model

interpreting
communicating

to information about mathematical ideas
quantity & number
dimension & shape
pattern & relationships
data & chance
change

that is represented in a range of ways
objects & pictures
numbers & symbols
formulae
diagrams & maps
graphs
tables
texts

and requires activation of a range of
enabling knowledge, behaviors, and processes
mathematical knowledge and understanding
mathematical problem-solving skills
literacy skills
beliefs and attitudes

Numerate behavior involves ***managing a situation or solving a problem in a real context*** (everyday life, work, societal, further learning) ***by responding*** (identifying, interpreting, acting upon, communicating about) ***to mathematical information*** (quantity & number, dimension & shape, pattern & relationships, data & chance, change) ***that is represented in a range of ways*** (objects & pictures, numbers & symbols, diagrams & maps, graphs, tables, texts, formulae) ***and requires the activation of a range of enabling processes and behaviors*** (mathematical knowledge and understanding, mathematical problem solving skills, literacy skills, beliefs and attitudes). (Gal et al, 1999; table 2.1).

By choosing one element from each of these five subcategories, one can compose a definition for each particular situation, for example: Numerate behavior involves managing a situation or solving a problem *in everyday life* by *acting upon* (*estimation with money)* to information concerning *quantity and number* that is represented by *pictures (in advertisements in leaflets)* and requires the activation of *computational and estimation skills*.
This makes the definition applicable in almost all situations in which people have to manage a mathematical situation. Based on this definition an item bank of about 120 items was developed for the numeracy domain of the ALL survey.

International Numeracy Survey

In 1996 a small scale international numeracy survey was fielded in seven countries, initiated by the Basic Skills Agency in London. Participating countries were: UK, France, the Netherlands, Sweden, Denmark, Japan and Australia. Respondents were asked to do twelve fairly easy computational tasks. (see appendix 2). The numeracy tasks included addition and subtraction of decimals, simple multiplication, calculation of area, percentages and fractions. Comparing the percentage of respondents who managed to give the correct answer for all tasks, Japan emerged at the top with 43 percent, followed by France (40 percent), and the Netherlands (38 percent). Respondents in the United Kingdom performed worst with only 20 percent accurately completing all 12 tasks successfully. (The Basic Skills Agency, 1997) The study revealed three significant outcomes:

- Regarding socio-economic class: the results show that those with the poorest numeracy skills are from working class households. Only 35% managed to answer 10 or more of the 12 tasks correctly.
- Women performed worse than men in the tasks; while 54% of men correctly answered 10-12 questions, only 39% of women were able to achieve this score. The average number of correct answers achieved by men was 8.6 compared to 7.3 for women.
- Across the different age groups, 35-54 year olds out-performed 16-34 year olds. While the former group scored an average of 8.2 correct answers out of 12, this dropped to 7.8 for the latter group.

Though in this study "numeracy" was seen as doing mathematical operations on school-based tasks, it does indicate that the performance on such school-related tasks is disappointing and presumably undermines numerate behavior.

PISA

In the OECD Programme for International Student Assessment (PISA, 2000), an international assessment of 15 year olds that looks at how well they are prepared for life beyond school and that was fielded in 32 countries in the year 2000, four types of skills were assessed: skills and knowledge that prepare students for life and lifelong learning, reading literacy, mathematical literacy and scientific literacy.

For this survey "mathematical literacy" was described as: *"An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen".*

Mathematical literacy entails the use of mathematical competencies at several levels, ranging from performance of mathematical standard operations to mathematical thinking and insight. Results of this study have not yet been published.

2.2.6 Developments in the Netherlands

Attention to adult literacy started in the sixties when many people from Mediterranean countries came to and settled in the Netherlands. At the same time the Dutch population became aware of the phenomenon of "illiteracy" and "semi-literacy" among Dutch inhabitants and the necessity to start literacy campaigns for Dutch citizens.

Though there had not been national surveys and reports about adult literacy and numeracy like in Britain, the United States and Australia, it was assumed that roughly about 4% of the Dutch citizens could be semi-literate (nearly 600,000 people). This percentage was estimated by workers in the field based on numbers of school dropouts in those years and based on UNESCO information and the UNESCO conference in Marly-le-Roy in 1967. Adding to these the many not-schooled and low-schooled immigrants, approximately 10% of the Dutch population (about 1.5 million people) was assumed to have great difficulties with reading, writing and arithmetic. Many literacy projects started in the seventies, developed and directed by volunteers and mostly based on the ideas of Paulo Freire. The programs focused primarily on reading, writing and speaking Dutch, but in occasional situations volunteers also tried to help adults with mathematical problems.

The first time the word “gecijferdheid” [numeracy] came into the picture in the Dutch language was in a keynote address of Van der Blij (1987) who discussed the opposite of it during the annual Panama Conference of OW&OC (the current Freudenthal Institute) in 1986. He wondered if there could be something like “mathematical illiteracy” compared to “illiteracy”. The central topic in his address was the meaning, pronunciation, the writing, use, misuse and misunderstanding of numbers in newspaper articles and advertisements.
In his inaugural lecture as professor at the Freudenthal Institute, Treffers (1989) addressed Van der Blij’s lecture and discussed the prevention of innumeracy in primary school [“Het voorkomen van ongecijferdheid op de basisschool”], in relation to Realistic Mathematics Education. In 1991, Goffree published an article about numeracy in Adult Basic Education (Goffree, 1991). Both elaborated an aggregate of knowledge and skills that could be part of numeracy or could show innumeracy but they did not come to a definition of numeracy for the Dutch situation. A few years later Goffree (1995) described numeracy as “making use of the computation system in a functional way” [in Dutch: “het functioneel gebruik maken van het rekensysteem”].

Though the word and the concept of numeracy were hardly known in the Netherlands, it was obvious that mathematics education for adults needed to include more than just mathematics. These ideas were in alignment with the RME approach to mathematics in primary and secondary school in the Netherlands. The concept of numeracy was, in fact, embedded in the RME approach, though not clearly distinguished from mathematics. RME incorporates real life situations and offers a mathematics program in which “applicability” of mathematics in real life situations is the first priority. (Streefland, 1991)

Internationally the results of the IALS Study (Houtkoop, 1999) made clear that numeracy should also be a serious point of attention in adult education. The Netherlands participated in the IALS survey in the year 1996 (see section 2.2.5). The results were rather shocking: 7.2% of the younger people in the age of 16-49 achieved only level 1 and 21.6% only level 2 on the quantitative scale, out of five levels. This is nearly 30% of the younger people of the sample. Older people in the age of 50-72 achieved similar results: 19.3% scored on level 1 and 34.1% on level 2. In total nearly 36% scored on the lower levels 1 and 2. (Van der Kamp and Scheeren, 1996, Houtkoop, 1999. See appendix 1)

2.3 Numeracy, a dynamic Concept

A closer look at the definitions which were stated in the previous section, teaches us that there are no two definitions of numeracy that are the same. In the early sixties numeracy was mainly seen as a small part of mathematics and was expressed as "quantitative literacy", including only arithmetical operations with whole numbers. (see Cockcroft (1982) and Reeves (1994)). Later the concept of numeracy became broader. Nowadays, when looking at the definitions from Gal (1993) and the ALL-numeracy team (Gal et al, 1999), numeracy encompasses a broad set of knowledge and skills that not only cover mathematical knowledge and skills, but also include the capability to *manage* situations that ask for interpretation, communication and making decisions concerning mathematical information.

A further look into these definitions leads us to two questions: *how can we build on these definitions* and *what constitutes these broad sets of knowledge and skills* mentioned in almost all definitions from past recent years?
The essence of all definitions is that numeracy is about "*mathematics embedded in a situation*" in a very broad sense. Second, math is a "*functional"* part of real life situations that adults "*have to manage*". In addition to these main topics, issues are mentioned as "*feeling confident with numbers*", being able to "*communicate*" about mathematical topics, math as a tool for "*describing and analyzing our world*", math for "*effective functioning in one's group and community*", math for "*critical and effective participation in a wide range of life roles*" and the capacity to "*further one's own development*".

These descriptions tell us that numeracy:
- includes functional mathematics
- is more than the traditional mathematics learned in school
- is always embedded in a real life situation
- includes managing a mathematical situation
- includes interpretation of and critical reflection on mathematical information
- includes communication and reasoning about mathematical information
- may differ per person, depending on one's situation.
- is the base for further learning.

The last aspect, coming from Beazly (1984), is an important factor for adults to keep up with the developments in society and technology and for doing additional courses in the frame of lifelong learning. Society and work environments demand that people will be able to adjust flexibly to continuous changes in an environment that is highly dominated by quantitative information that often is embedded in texts and visualizations in high tech applications. Therefore numeracy cannot only exist of a fixed set of mathematics knowledge and skills, based on (traditional) mathematics from the past. It also requires

competencies that enable adults to engage in independent or autonomous lifelong learning, to acquire new mathematical knowledge and skills to minimize the gap between their own knowledge and skills and the knowledge and skills that are required in order to keep up with new, technological developments. Numeracy is a *dynamic concept* that encompasses mathematical content that may change over the years or may still be invented in the future, due to technological developments.
Taking the above into account we come to the following definition:

Numeracy encompasses the knowledge and skills required to effectively manage mathematical demands in personal, societal and work situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology.

The first part of this definition is derived from the ALL definition, the second part describes the necessity to keep up with societal and technological developments and to work on one’s own development, as indicated by Beazly (1984).

The essence of numeracy is that it has its own content for every individual person - it depends on his or her personal and societal life and real life experiences - and that it is only a part of the person’s total knowledge and skills. Everybody carries a backpack filled with a mix of real-life experiences and school knowledge and skills, built upon a variety of language, mathematical, cultural, social and emotional aspects. We would argue that these aspects should not be seen as loose elements but as an *entity*. They can be distinguished but not separated from each other. It is only possible to stress a specific part of this entity in a specific situation. Numeracy must always be seen as part of this broader set of knowledge, skills and feelings.

2.4 Components of numeracy

Starting from the definition in previous section and looking back to the analysis of the preceding definitions, we may wonder what kind of knowledge and skills are actually needed for being numerate in the present and the future and how to operationalize the concept numeracy for adult education programs. The numeracy elements mentioned in the previous section can be captured in four main components of numeracy: mathematical knowledge and skills, management skills, skills for processing new information and learning skills.
These four components are elaborated in the following subsections.

2.4.1 Mathematical knowledge and skills

The mathematics in real life, as elaborated in the ALL numeracy framework (Gal et al, 1999), encompasses information based on five “big ideas”: quantity and number, dimension and shape, pattern and relationships, data and chance, and change, from Steen (1990), Rutherford & Ahlgren (1990) and Dossey (1997). Together these fundamental ideas cover all mathematical issues in societal, work and everyday life. They include underlying mathematical skills that are necessary to deal with these “big ideas”. This section describes the five big ideas in short, based on the ALL Numeracy framework (Gal et al, 1999) and may clarify the content of each and the needed skills to deal with these five “big ideas”, in order to create a base for adult numeracy education. (table 2.2)

Quantity and number are the essence and ingredients of people’s need to measure and structure their environment in order to find attributes and terms that enable them to communicate. People identify, locate, sort, model, structure, count, estimate, compute and talk about quantity and number in their personal, societal and professional situations.

Dimension and shape are the ingredients that landscape the world in which we live, and that we use to re-landscape our environment, using “things” in one, two or three dimensions. Geometric objects, shapes and forms are basic ingredients for this landscaping. To be able to work with this people also need to have developed skills including identify, locate, sort, model, shape, structure, count, compute and estimate, to give and follow directions.

Patterns and relationships are the fundamental ingredients of mathematics. Steen (1990) argues that, in contradiction to traditional math, one should not only think of patterns and relations between numbers, algorithms, ratios, shapes, functions and data, but also of attributes (like linear, symmetric), actions (e.g. model, classify, control, prove) abstractions (e.g. symbols, logic, similarity), behaviors (e.g. motion, chaos, iteration), attitudes (e.g. wonder, meaning, beauty, reality) or dichotomies (e.g. discrete vs. continuous, algorithmic vs. existential, exact vs. approximate).
Patterns and relationships are the key to dealing with the world around us and the basis of logical mathematical thinking and reasoning. It should be built up in multiple parallel vertical strands from informal experiences in kindergarten to scientific education at the university. (Steen, 1990)

Data covers information about variability, sampling, error, or prediction, and related statistical topics such as data collection, data displays, and graphs. Modern society demands that adults interpret and produce organizers of data such as frequency tables, pie charts, graphs and sort out relevant from irrelevant

Table 2.2 Big Ideas

Mathematics		
Big Ideas	Content	Required skills
Quantity and Number	whole numbers, proportions, percent, fractions, decimals	- identify, locate - order, sort - count - compute and estimate - model - measure
Dimension and Shape	measurement, geometry, space, shape, direction, time	
Pattern and Relationships	patterns, formulas	- mathematical thinking and reasoning - recognize and create patterns - compute, evaluate and derive formulas - describe relations between quantities, e.g. volume and weight
Data and Chance	statistics, graphs, probability	- reading and understanding data, graphs, pie charts, tables, etc - describe randomness - describe patterns of variation in measurements and data that may influence outcomes
Change	rates, percent change	- represent, understand, recognize, apply and control changes - recognize trends - recognize exponential growth patterns

data. ***Chance*** covers “big ideas” related to probability, subjective probability, and relevant statistical methods.

Change describes the mathematics of how the world changes around us. Individual organisms grow, populations vary, prices fluctuate, objects traveling speed up and slow down. Change and rates of change help provide a narration of the world as time marches on. Additive, multiplicative, exponential patterns of change can characterize steady trends; periodic changes suggest cycles and irregular change patterns connect with chaos theory. Describing weight loss in a simple task compared to calculating compound interest.

These five fundamental ideas set a broad range of mathematical concepts and activities as part of numeracy for the present and a future world and may lead to optimal numerate behavior. To be able to develop numeracy programs these big ideas will have to be explicitly represented in content and skills recognizable for teachers and learners. However, Steen's basic idea is not to separate them in subtopics, but to integrate subtopics into these overarching concepts. This may have consequences for school programs. Table 2.2 shows an attempt to make these big ideas recognizable and operational in numeracy programs.

The table is based on the ingredients of an internal working scheme used by the ALL numeracy team for the development of items for the ALL project. The table should be read from left to right, from right to left and from top to bottom in mutual coherence. It reflects an iterative process from big ideas to detail (content and skills) and back to big ideas. The five big ideas should go hand in hand as parallel strands, not separated in ladder strands. Based on table 2.2 mathematical content appropriate for Adult Basic Education can be derived. (see chapter 7)

2.4.2 Management skills

Mathematical problems in real life are always embedded in functional situations. In his most recent publication Gal (2000, p.13) identifies three types of situations in which numerate behavior becomes apparent:

- ***Generative situations*** *that require actors to count, quantify, compute or otherwise manipulate numbers, quantities, items, or visual elements, and eventually create (generate) new numbers.*
- ***Interpretative situations*** *demand that people make sense of, and grasp the implications of, verbal or text-based messages that may be based on quantitative data but that do not involve direct manipulation of numbers.*
- ***Decision situations*** *demand that people find and consider multiple pieces of information in order to determine a course of action, typically in the presence of conflicting goals, constraints, or uncertainty.*

Mathematical problems in real-life situations may vary from very simple to very complex but they are always concrete and functional, for example budgeting, arranging a loan, a mortgage, dealing with best-buy problems, carpeting a room, painting walls, calculating the costs of a vacation with the family, calculating the average costs of a car per month, work-related problems like keeping storage, delivery of goods, building houses, and society-related problems, like elections or environment issues. To manage such problems adults need to have acquired a broad aggregate of skills, amongst other things:

- *generative mathematical understanding and insight* to give meaning to and to interpret numbers and to plan appropriate mathematical actions
- *literacy skills* to read and understand the problem and to reason about it;
- *communication skills* to be able to acquire information, to share the problem with others, if necessary, to learn from others how they would solve the problem and to work cooperatively;
- *problem solving skills* to locate, identify, analyze and structure the problem, to plan steps, to select appropriate actions, to actually handle the problem and to make decisions.
- *reflection skills* to be able to control the situation, to check computations, to evaluate decisions, to come to contextual judgements and to learn from experiences.

Such management skills are often assumed to develop spontaneously in the course of life. We may argue that it is necessary to explicitly pay attention to the training of such skills. Such may enable adults to develop appropriate skills for different types of mathematical situations.

2.4.3 Skills for processing new information

A critical element of the technological society is information. People are overwhelmed with information that they have to analyze and to interpret as a consumer and an informed citizen. Interpretation of information demands understanding and giving meaning to verbal, text-based or visualized messages in commercials, newspaper reports, graphs, charts, tables, etc. It involves, among other things, that adults are able to distinguish important information from less or not important information by interpreting it and reflecting on what it means for their own situation. Based on this they can make further interpretations, draw conclusions and make decisions in communication with family, friends, etc., and with colleagues in work situations. A lot of information is based on numbers, quantities, measures, ratios or percentages. Other information may sometimes involve no numbers at all but refer to notions that are part of statistics (Gal, 1997, Gal and Garfield, 1997). For example, people need to be able to make sense of published results of studies and surveys reported in the media or in a workplace context. To do this, appropriate skills are required to read information, to locate and identify what is new information, to analyze and structure it, to reflect on what is new, to communicate with others about it and to reflect on what the new information means for one’s own life. Though this process often goes without any computation, people must have acquired adequate reading skills and must understand the computations that underlie the information.

2.4.4 Learning Skills

Based on the previous section we may state that in a rapidly changing and developing society, people need to develop skills and strategies for lifelong learning to be able to accommodate and adjust flexibly to new information and developments in the course of life. This is also strongly advised by UNESCO (1967, 1990, 1997, 2000). This certainly concerns the maintenance and development of numerate behavior beyond school. Most school systems and ways of teaching are not yet appropriate for developing skills for lifelong learning. Learning in school mainly focuses on achieving standards and passing exams. The content of education mostly consists of knowledge and skills developed and acquired in the past. However, learning only from the past will not be sufficient anymore for being equipped for the future. People need to be prepared for learning in the future beyond school (see PISA, 2000). School programs will also have to focus on knowledge and skills that have not yet been invented but is being developed or will be invented in the near future. For that people need to develop learning skills that may help them to learn autonomously and independently from teachers. This demands a change in thinking about learning and teaching. In the contemporary developments in adult and vocational education (in the Netherlands) four important movements can be identified:

- *There is a growth in general awareness of the need of lifelong learning*. This is what UNESCO has been working on in past decades. Only when people are aware of the need for lifelong learning they will start thinking how to arrange that. Learning is often associated with learning in school. However, learning not only happens in school situations and during school time, but increasingly more in out-of-school situations. Work situations change due to the development of more sophisticate machines. Computer applications offer increasingly more possibilities. Hence there has been a break-through in adult education that adults will have to take their learning processes in their own hands to be able to flexibly adjust to such changing situations. Awareness of this situation is the first step for adults to develop the necessary skills for self-directed learning.

- *More emphasis on facilitation of self-directed, autonomous learning*. Brookfield (1986) already described the role of the teacher in adult education as the “facilitator of learning” rather than “teacher”. His thoughts are based on Rogers” (1969) theory about “Freedom to Learn”. Knowles (1990) stresses people’s own perspectives and responsibility for their own learning processes by which they become less independent from teachers. Institutions will have to develop programs in which learners have more freedom to choose their own learning routes and can learn in their own way, pace and time.
These developments lead to a change of the teacher’s as well as the learner’s role. The learner will take part more actively in his own learning program and

will become an active learner and his own teacher. The teacher's role will be changed into a more guiding, supporting and co-learning one. Classes will end and will make place for small self-directing learning groups.

- *More emphasis on problem-based learning environments in which adults can actively participate.* Solving problems asks from learners to think creatively and collaboratively. They may have to look for information, to structure and analyze complex situations, to think critically, to reason about possible solving strategies, to discuss working procedures, to make decisions, to use additional tools such as calculators. Such learning situations may help adults to become curious and to develop knowledge and skills for dealing with new complex problem situations. This is the basis for independent learning. Curry, Schmitt and Waldron (1996, p.22) describe statements from learners in which they show the importance of problem solving: One employer summed up the problem-solving process this way: "Our philosophy in the workplace is 'whatever it takes'. We will use whatever it takes to make it work, we will try it. And to be open to try."

- *Facilities for lifelong learning have been created.* Adults need learning centers where they can get the information they want and can meet other adults for learning together. Careful attention must also be paid to facilitating and encouraging distance and at home learning and to the quality of instruction materials required for self study.

The above are essential preconditions and means for the development of effective learning skills that will enable adults to learn autonomously and independently beyond school and in out-of-school situations. They may help and encourage adults to adjust and accommodate their knowledge and skills to new developments in society in the present and in the future.

In the current Dutch adult and vocational education system, described in chapter 1, new programs have been developed to meet the conditions for lifelong learning by problem-based, cooperative and interactive learning. Learning-to-learn, teacher-free and independent learning have been an official part of the Dutch curriculum since 1998.

2.5 Levels of Numeracy

The four components described in the previous section (mathematical knowledge and skills, management skills, skills for processing new information, learning skills) shape a rich dynamic concept of numeracy. All four components are necessary in an integrated and coordinated way for being numerate. However, we may assume that the mathematical component is a major factor. The other three components are also valid for prose and document literacy, scientific literacy and statistical literacy. Two questions now arise:
- What is innumeracy?
- Are there possible levels of numeracy?

Gal (1993) wonders if there is a certain level of mathematical knowledge that qualifies a person as being numerate. And this is exactly the question we ask ourselves as well. When can a person be called "numerate"? What is the minimum level of being numerate? Where does "innumeracy" begin? In *"Issues and Challenges",* an elaboration of the NCAL report, Gal (1993) reflects:

> *"It might be useful to reflect on what being numerate entails in the context of a K-12 school system that, in the US as well in many other countries, seems to have adopted a more or less agreed upon ladder of mathematical skills. From learning the four basic operations, students routinely move to the proportional topics (fractions, decimals and percentages), then to algebra, geometry, trigonometry and calculus. Measurement and some graphing skills are visited at various points. However, several questions arise: Is there a certain point on this ladder above which one is considered numerate? If so, what might it be? How high on this ladder does one need to climb in order to be an effective citizen, worker, shopper, or parent? Do some of these roles entail a separate ladder with different steps? Finally, to what extent is the ladder metaphor relevant or helpful to an adult educator who has to plan instruction for adult students seeking to improve their functional skills?" (Gal, 1993, p. 28).*

In general, we may argue that becoming numerate does not develop in alignment with the ladder metaphor, as described above, because, for example, learning about proportions (including fractions, decimals and percentages) can also start in parallel with learning the four basic operations. Concepts as double, doubling, half and halving are already present in a developmental stage at a very young age. Dividing by four, for instance, can be seen as "taking a quarter" of something. Four times 25 cents can be pronounced as "four times a quarter (of a dollar)". Such concepts and simple operations involving proportional thinking, can be explored in parallel to the four basic operations and to measurement. Adults in adult basic education often show (some) knowledge about the traditional "higher order operations" in combination with lack of knowledge when doing basic operations, like multiplication and division. Hence, the question still is what exactly defines “being numerate”.

Based on the four components we can now roughly indicate what factors may cause numerate or innumerate behavior. In general it can be said that insufficient mastery of one or more components and/or lack of coordination between the four components may cause innumerate behavior, e.g. by lack of effective management of mathematical situations. Since the mathematical component is only one out of these four components, it does not always need to be lack of mathematical knowledge and skills that causes innumerate behavior. It is quite likely that people do not know how to use the mathematics they learned in school in real life situations, or that the mathematics they use in practice does not match the mathematics they learned in school. This is illustrated, for example, by studies about differences in school mathematics and e.g. mathematics in the workplace (see for example Hoyles and Noss, 1998, Forman and Steen, 1999, Evans, 2000, Straesser, 2000). However, it may be assumed that lack of mathematical knowledge can be a main factor at innumerate behavior since the mathematical component is the core of numerate behavior.
That said, we may wonder whether there are also levels in numeracy and in numerate behavior. Given that the four components have been developed in parallel and/or in an integrated way, there may be a gradual advancing on a scale of

- increase of mathematical knowledge and skills, informal as well as formal
- managing problems varying from very simple to very complex
- acquiring and processing information varying from guided to independent
- development of learning skills varying from guided learning in school to independent, autonomous learning in out-of-school situations.

Defining levels in such a scale might be arbitrary, especially in real life because every adult uses the knowledge and skills he needs and which he is able to apply effectively (Johnston, 1995, 1998). However, the results of the surveys described in section 2.2 (YALS, NALS, IALS, and the International Numeracy Survey) may show that such a level indication is legitimate and necessary to be able to arrange effective education settings. Roughly we could distinguish three levels of numeracy with no clear borders but gradually moving in each of the four fields from elementary numeracy, functional numeracy into optimal numeracy, as shown in table 2.3:

Table 2.3: Numeracy Levels

Numeracy components and levels:	elementary----▶ functional----▶ optimal numeracy
Mathematical knowledge and skills	-------------▶
Management skills	-------------▶
Skills for processing new information	-------------▶
Learning skills	-------------▶

Elementary numeracy encompasses minimal knowledge and skills necessary for managing mathematical situations in personal everyday life and for functioning in simple work situations with perhaps a little help from others. Such an elementary set is the basis for further learning in all kinds of further education and vocational programs.

Functional numeracy encompasses a broad set of functional knowledge and skills, including elementary numeracy skills, that enable people to manage mathematical situations in their personal and their families' societal life and work situations effectively and autonomously and to be a critical citizen.

Optimal numeracy encompasses all extra knowledge and skills, in addition to people's functional numeracy skills, that enable them to act in broader societal and/or political communities, to perform higher-level professional tasks and to take part in leisure activities that include a lot of complex calculation and computing. Among them are professions based on mathematical knowledge like mathematicians, statisticians, accountants, engineers, art like music, painting, sculpture, drawing, e.g. Escher, and also web design and web art, and sports like bridge and chess.

2.6 Conclusions

Numeracy is a dynamic concept. Starting in the early eighties as “being familiar with numbers” as part of mathematics and as “quantitative literacy” as part of literacy, it has changed in past decades to be a broad concept that includes mathematical topics and literacy as well as management, problem solving, data processing and learning skills.
The development of numeracy and numerate behavior is a never ending process. New technological developments may require that the content of numeracy must be adjusted from time to time, and consequently also adults’ numerate behavior.

Looking at the numeracy scale of elementary, functional and optimal numeracy in section 2.5, we may assume that new developments in society will mainly take place at the right side of the scale, in the area of high-functional and optimal numeracy. This means that the scale may widen. The distance between elementary numeracy and the area of the high-functional and optimal numeracy becomes larger. As a consequence, people at the left side of the scale, who are still at the elementary level and the lower end of functional numeracy, will have to work harder to keep up with the developments at the right side. Good learning skills here are very important. People who function at the lower end are very vulnerable. These might be the people Steen meant in his quote in section 2.1 about the peasant in Gutenberg’s time.

Knowing this, we may be concerned for all those people, in particular young people, who scored at the lower levels in the IALS survey, and will score in the near future on level 1 and 2 in the ALL survey. The results of these surveys may provide good insight into serious nature of the actual numeracy skills in diverse countries. Also, the analysis of mathematical knowledge and skills of adults described in chapter 4 may show more about the actual numeracy levels of adults in ABE in the Netherlands.

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[1] By *informal education* is meant the truly lifelong learning process whereby individual acquires attitudes, values, skills and knowledge from daily experiences and the educative influences and recourses in his or her environment.
Formal education indicates the hierarchically structured and chronologically graded educational system, running from primary school through university, including technical and professional education.
Non-formal education refers to any educational activity organized outside the established formal system. (UNESCO, 2000, page 41)

[2] In the first phase of IALS (1994, 1996), adults from 14 countries were tested based on methodology that combined household survey research and methods of educational testing. A second cohort of 10 countries conducted surveys in 1998 and 1999 (SIALS). In 1994/6, participating countries were: Canada, France, Germany, Ireland, the Netherlands, Sweden, Switzerland, and the United States; in 1996, Australia, the Flemish community in Belgium, Great Britain, New Zealand, and Northern Ireland participated. The second full round of data collection in 1998/9 (SIALS) includes: Chile, the Czech Republic, Denmark, Finland, Hungary, Italy, Malaysia, Norway, Slovenia, and Switzerland.

[3] The ALL Numeracy Working Group is comprised of:
Iddo Gal, University of Haifa, Israel; Mieke van Groenestijn, Utrecht University of Professional Education, Netherlands; Myrna Manly, El Camino College, California; Mary Jane Schmitt, National Center for the Study of Adult Learning and Literacy, Harvard University, USA; Dave Tout, Language Australia

3 Numeracy Assessment in Adult Basic Education

3.1 Introduction

Assessing adults in the new adult education system is of increasing importance since the features of lifelong learning asks for growing accountability of educational effectiveness. National reporting systems, as in Australia and in the Netherlands and those currently being developed in the U.S. and other countries, demand reliable assessment and portfolio systems. In this framework the development of numeracy assessment tools is a challenge for adult education developers because there is little tradition in this field and, as discussed in chapter 2, numeracy includes a wide range of skills. The questions of what, why and how to assess adult learners challenges organizations, curriculum, program and test developers to think about effective assessment tools that provide quantitative and qualitative information. Such information should be available for learners and teachers in order to improve learning and instruction, for program and curriculum developers to develop effective programs and for funders because of accountability. Good assessment includes a coherent system of placement, formative and summative tests and, based on that, program evaluation.

To learn more about the population in ABE, a central part of this study is qualitative and quantitative analysis of the numeracy skills adults have acquired when they enter an ABE numeracy course (chapter 4). The results provide empirical information to set a foundation for numeracy learning and teaching in ABE (chapters 5 and 6). Practice and theory together lay the basis for a numeracy program that meets their needs and wishes (chapter 7). The placement test itself is crucial because, in practice, teachers experience that about a quarter of potential ABE learners never start a course after the intake interview and placement test, or stop after a few weeks when they feel frustrated or are placed in the wrong course. This indicates strongly that placement needs careful attention and should be done in a way that motivates and encourages adults to actually start. For this purpose three main issues are discussed in this chapter: the characteristics of the ABE population (section 3.2), goals of numeracy assessment (section 3.4), criteria for a placement test (section 3.5).These are related to developments on assessment in compulsory education (section 3.3) Three tests are described in section 3.6, especially developed for ABE in the Netherlands: the *Supermarket Strategy*, the *In Balance* test and the *Cito* placement test for adults. The latter two are used in the field study described in chapter 4 for analyses of numeracy skills of a group of ABE learners.

3.2 Characteristics of the ABE population

As described in chapter 1, the population of ABE in general but certainly in the Netherlands is very diverse. We can distinguish two main sub-populations: native speakers and second language learners. Native speakers, almost always born and raised in the Netherlands, can be spread over two sub-groups: learners who have finished primary school with varying success and may have had some secondary school experience with a maximum of ten years schooling in total including kindergarten, and learners who failed at school in earlier days and may have serious problems with all kinds of basic skills.
Second Language Learners come from abroad at a later age and can also be divided into two main sub-groups: learners with school education in their home countries similar to Dutch adults described above, and learners with very little or even no school education. The latter are often illiterate or semi-literate.
Each of these four subgroups has it's own characteristics, which will be discussed in following sub-sections.

3.2.1 Native speakers

Dutch native speakers who attended school with some level of success are often able to do many kinds of mathematical operations they learned in school, but their procedures are sometimes based on partial insight. They may want teaching or re-teaching on specific issues as fractions, percents, the metric system, budgeting, or other courses to be better prepared for further education and vocational courses. These learners are used to processing paper-based information and to taking all kinds of tests, but they may have negative school memories and frustrations like test anxiety. They often expect “school-math” in ABE to help them to acquire certificates in follow-up courses.

The other sub-group of native speakers are often the drop-outs who may have serious problems with reading, writing and arithmetic. They come to ABE with many negative school memories, feelings of uncertainty, math anxiety and more problems. Concerning mathematics they may remember problems with the four basic operations, long pages with difficult sums, times tables they could never learn by heart, learning fractions and percents they never understood, difficult sums with decimals, the metric system, reading time, doing operations with time, etc. These learners may have developed fragmentary or partial knowledge and skills, sometimes on very commonplace things, as shown in the following two examples.

Example 3.1: Reading Time
Ellen, a Dutch women, comes to class. She shows a note her daughter (13) wrote: “Look at this. She writes: Am home at 15.30 (Ellen pronounces it as “fifteen dot thirty”). What does she mean?”

> One of the other learners reacts: “She’ll be home at half past three”
> Ellen, totally confused: “Oh….. Why can’t she just say that?”

The confusion Ellen shows may indicate that she is not familiar with the digital clock, which is very commonly used in the Netherlands. It appeared she had no problems with reading time on clock with hands.

> **Example 3.2: Hans, the Truck Driver**
> A teacher explains that 1 kilometer is the same as 1000 meters. Some examples and a few tasks on this topic have been discussed by teacher and learners.
> Suddenly Hans, a truck driver, reacts spontaneously: “I don’t believe this. It’s NOT possible! Meters are used around the house, kilometers are done on the road.”

For Hans kilometers and meters are two different systems, used in different situations. These situations cannot be compared and do not match each other. They serve different purposes. Though, he knows a lot about kilometers as a truck driver. He knows all distances between towns and cities in the Netherlands including the time he needs to get there with his truck.

Adults with such specific problems may carry many school frustrations, partial knowledge and misconceptions but also a lot of practical knowledge, like Hans shows. They are not or less used to processing information from paper, but used to acting three-dimensionally in actual real life situations. Because of their problems with reading, writing and doing computations on paper, it may be difficult for them to take paper-based tests. These are often the people who don’t actually start a course after they take a placement test.

3.2.2 Second Language Learners

Second language learners who received school education in their home countries often expect a similar way of education in Dutch schools. However, learning-teaching situations here, and in other western countries, may be quite different from those in their home countries. Also, mathematics programs in primary school here have changed drastically in past years. Mathematics programs in other countries, in particular in Mediterranean, Middle Eastern and African countries are often based on more traditional education and hence quite different from the Dutch system. In addition, due to cultural differences and sometimes a different alphabet, they may have different notation systems for algorithms, different ways of expressing ratios and fractions, different ways of reading time, etc.. Smaller differences, such as in pronunciation of numbers, in notation systems and doing mathematical operations are even present among neighboring countries like Netherlands, UK, Germany, Belgium and France. In the early years of ABE, Dutch teachers were not used to these cultural differences. It caused many problems in learning-teaching situations, and served as an eye-opener for many teachers to learn about these differences. Acceptance of these cultural differences played a big role in the development of mathematics programs and instructional materials for second-language learners and also affected the development of assessment tools.

It may be obvious that the performance of non-native adults on paper-based math tests strongly depends on their mastery of the Dutch language. A precondition for doing math courses is that second-language learners have achieved a minimum level of communication language, reading and writing (Cito, NIVOR-1, 1996) . For newcomers this first part of intensive language training takes about three months. After that they can start doing mathematics and other courses. After about six months they can apply for vocational courses to become certified, either in their own professions or in new professions. Depending on their immigration status, they are then allowed to enter the labor market.

Despite their intensive language training these adults often continue to perform less well on paper-based math tests than might be expected given their school education in their home countries, because there are questions and words in the tasks they do not understand. There will always be language problems in tasks and tests that cannot be foreseen and that may differ from person to person. There may also be tasks in the test they are not used to, like working on unfamiliar contexts, doing estimations, rounding money amounts in a task, creating a strategy for solving a math problem. Sonia (S), a Moroccan woman, 23 years old, has been in the Netherlands for about seven months. In an individual interview the teacher (T) asks her about the following coffee problem.

Example 3..3: The coffee problem

The illustration shows two common Dutch coffee machines with an equal amount of water in both, but a different amount of coffee. The question is: “Which coffee will be stronger?” Comparing these two coffee machines in the illustration should lead to answer B.

T: Look at the next task. Can you read the text?
S: (reads aloud) Which coffee will be stronger? (no problem to read the text)
S: (repeats) stronger…… (takes her dictionary)
T. what are you looking for?
S: mmm…... (studying her dictionary)
T: what word are you looking for?
S: stronger …. (cannot find it)
T: (helps) strong….stronger
S: stronger……. (cannot find it)
T: what does it mean: “strong coffee”?
S: good coffee
T: then, what means “stronger”?

S: (still looking in her dictionary)… cannot find it
T: good coffee, what does that mean?
S: tasty….tasty coffee. I think it's B
T: (tries a different way) Which coffee is very strong?
S: Tasty or not tasty? It's B. Don't drink coffee, only tea.

Here we encounter two problems at the same time: a language problem and a cultural problem. Many Moroccan women drink only tea, not coffee. The item itself was meant to be a simple ratio and reasoning item with only a few words in it and a visual explanation in the context. Sonia had a language problem with the comparative of "strong". In addition she does not drink coffee and may not be familiar with Dutch coffee machines and so with the illustration. In this interview it did not become clear whether she came to the right answer based on reasoning or if it was just a guess. The answer was just "B". She could not explain why. Here we see that context items, even if they are interesting for discussion and reasoning in group sessions, are not always usable for testing.

It may be assumed that learners in literacy courses are not used to school-like activities and testing at all. They have often mastered all kinds of informal literacy and numeracy skills and procedures to be able to manage their own lives but are not able to show their capabilities in paper-and-pencil tests. Preliminary research in this area shows that oral testing by means of interviews and using hands-on materials are strongly preferable for these adults. (van Groenestijn, 1993). Hands-on materials like measuring cups, milk containers, a measuring tape, money, real cookies, etc. may help people to actually do mathematics so that they can show their skills as they work with them. The woman in the following example could not read and write, but could do computations in her head.

Example 3.4: Caramel Cookies
Zaara, a Moroccan woman in a literacy course, was shown a packet with in it 8 caramel cookies.
Teacher: How many packets would you buy if you wanted 40 cookies?
Zaara responded:
"Eight ….. is difficult…. ten is easy…, ten, ten,.... ten, ten….is forty.
Not ten…. one more….. five packets."

In the final sentence she refers to the number of cookies in the packet which is less than ten, i.e. eight. Not clear here is whether she did an estimation in the final step or that she computed it. But that is not important. The answer was clear. She knows that 40 would be 4 packets of 10, but there are fewer than 10 cookies in a packet, hence the answer should be more. She shows a good understanding of multiplication in this context, (or is it counting?), and could do it mentally, but she could not write down her operation. Compare this to Jannie, the woman from chapter 2 who loved baking cakes, who was asked the same question. She started writing down the 8-times table as learned in school and made errors in the answers, so she did not get 40. After that she started drawing cookies and counted them all until she had 40 cookies and then she had problems with the addition 8+8+8+8+8. After about 20 minutes and a lot of help she figured out that it should be 5 packets. Jannie tried to recall all her school knowledge and skills but was in fact handicapped because she had not learned how to solve such problems in an alternative way. Zaara, not hampered by any school education, had developed good mathematical procedures in principle. (van Groenestijn, 1993).

Learners in literacy courses often appear to have developed good mathematical strategies to survive in personal lives but their skills are limited to a few basic operations like counting and doubling strategies and most are not able to do computations on paper (van Groenestijn, 1993). Hence they cannot be tested merely through paper-based assessment materials. Oral interviews are the only way to get more information about what they know, how they think and what kind of procedures they apply.

3.2.3 Preliminary impressions and questions

It may be obvious that these four subgroups in ABE are quite different with respect to competencies and needs. The examples show also that learning and teaching mathematics in ABE will differ from primary and secondary school education. Though actual mathematical skills of adults in ABE may be on the same level, or even lower than in general primary and secondary education, learning and teaching are quite different because adults carry many real life experiences which affect their mathematical actions. Thus mathematical assessment in ABE, in particular in placement procedures, needs to include more than only school mathematics. Assessment must also enable adults to show how they can use mathematics in real life situations. Three initial questions arise here regarding placement procedures for ABE in general.

1) *What way of testing is most appropriate for incoming learners at placement procedures?*
Discussions about oral interviews, paper-and-pencil tests and computer adaptive tests are continuing in the Netherlands. To reduce time and cost the latter two tests are often preferred. Current computer adaptive tests (CAT) offer the possibility of keeping the placement test as short as possible depending on individual performance. They result in a proficiency score that indicates level of placement, often without qualitative information. This raises the question as to what exactly we want to learn from a placement test. Is it sufficient to know only a placement level score or do we also want to learn more about the learners' way of doing certain tasks in order to get information for learning and

teaching. Moreover, computer-based tests require skills that learners in ABE may not have developed. The primary selection criterion is which type of test is the most appropriate for each subgroup and which would provide the best qualitative placement information. Then different types of tests may be required for different subgroups in ABE. For higher-level learners trained in contemporary educational systems with experience with computer-based learning and calculators, CAT can be a very efficient way of testing. Also, while their test-anxiety should not be underestimated, most adults in ABE can handle a paper-based test. It is important that such tests be prepared by and administered by experienced teachers so that unusual difficulties can be recognized and taken into account. For example, in addition to the expected language problems, second-language learners may encounter types of tasks and notations that are unfamiliar to them. Finally, as described earlier, adults in early literacy courses cannot do either of these types of tests. The only possibility to learn more about their competencies is through oral interviews.

2) *What content should be tested at placement?*
Computer-adaptive tests can assess a person's proficiency level very precisely. The computer selects items from different levels in the item bank and determines the learner's exact proficiency level by going up and down through the item pool on the scale until it comes closest to the learner's actual proficiency level. The choice of items made by the computer depends on the item difficulty level and not on typical content. This means that a learner may do e.g. 4 items on basic operations, 6 on measurement and none on money computation or percent. The test ends when the computer knows the exact proficiency level. The computer assigns a proficiency score.

A traditional paper-based mathematics test often contains many tasks from all fields of mathematics on different levels from easy to difficult based on mathematical complexities. Here the learners' strengths and weaknesses in different fields of mathematics can be determined. Due to the developments in CAT testing two questions arise here:
- Is it necessary to test all fields of mathematics at a placement test?
- What is easy and what is difficult?

To start with the latter: in particular in adult education it appears that some mathematical operations which are thought to be mathematically difficult can be easy for adults who are used to doing it, for example computing VAT with the help of a calculator. However, that does not always mean that their operations when computing VAT are based on insight in percents and the same adults may have problems doing similar computations on paper.

The answer to the first question depends on what information we want to acquire from placement tests. How diagnostic should a placement test be? If we want to learn more about what adults can actually do in order to fine-tune the program to the needs of individual learners, we need more qualitative information. Also, in many ABE programs it is becoming more common to offer only parts of a program, like only units on percents or fractions. Then it is necessary to know what content areas learners have already mastered to analyze what they still should learn. Hence a proficiency level alone will not provide sufficient information. This discussion is going on in the Netherlands. Based on this, new CAT programs have been developed in which, to certain limitations, choices can be made to test particular fields. However, the result is still only a proficiency score. No qualitative information about computation and problem solving procedures can be acquired. CAT may be a perfect means to learn a placement level in a very short time, but we may argue what we learn from it to help ABE learners efficiently. Hence the question still stands: *What do we want to know about learners' competencies, skills and problem solving procedures at the start of a numeracy course? What is necessary to be tested?*

3) *What types of tasks should be offered in a placement test?*
At this point the question evolves what type of tasks should be offered to attain qualitative information about the learners' mathematical knowledge and skills. Here we enter a discussion about context-based and context-free tasks, multiple-choice and open-ended questions, tasks that ask for informal and/or formal mathematical operations and "do"-tasks. Placement tests often ask only for formal school knowledge and skills worded as "word-problems" or just as "sums", sometimes even as a multiple choice test. For example, see the discussion about the TABE test in the US (Cumming and Gal, 2000). Context-based tests provide more information, though it is often assumed that adults who are used to traditional math programs may have problems with context items. However, context items may offer possibilities to also apply more informal creative procedures. Moreover, we often see that adults can do mathematics in "real" situations. They have developed all kinds of skills and procedures that cannot be tested through paper-based tests that ask for formal procedures. Context-based assessment may help to learn more about such informal procedures. Here the question arises how to create "real-life" situations in which adults can *do* such tasks and what could be the value of it for placement in a numeracy course? In the Supermarket Strategy (van Groenestijn et al, 1992, see section 3.6.1) a few tasks were included concerning payment and getting change using real money. The caramel cookies described in the previous section were part of a multiplication context, and it was also possible to perform some tasks on weighing and measuring with hands-on materials when necessary. In this way learners like Zaara and Jannie could show their informal real-life knowledge and skills. In vocational education it is becoming more common for learners to do paper-based assignments that test theoretical knowledge, in combination with doing physical tasks in actual vocational settings to test job-related knowledge and skills. In ABE mathematics or numeracy tests are still only "theoretical". To date it is not clear what type of tasks should be offered at the placement in ABE to get insight in adults knowledge and skills to be able to offer an effective program for these adults. It is also not yet clear what type of knowledge and skills should be learned in ABE and what type of tasks should be included. ABE classes are often still arranged the same way as common school-

based settings in which adults learn through paper-and-pencil tasks sitting on a chair, whilst real life situations call for action.

These three questions may make obvious that developing test materials for adults in ABE is a complex matter, particularly for placement. A placement test should offer appropriate information about adults in order to place them in the right courses. However, is an indication for placement by proficiency level sufficient or do we also need some qualitative and diagnostic information to help these adults learn in the right way? If so, how much and what kind of information is needed for that? The main question still remains: *What are the goals of assessment in general and for placement assessment in particular?* A second question follows subsequently: *What are the criteria for assessment tools to meet the purposes?* And here can be added: *What can we learn from assessment in school mathematics education?*

3.3 Developments in Assessment of School Mathematics

3.3.1 A few studies of school mathematics

In order to learn more about mathematics assessment we will first focus on a few studies concerning school mathematics.

De Lange (1987, 1995) formulated five principles for assessment of school mathematics:

- the main purpose of testing is to improve learning and teaching
- assessment should enable students to demonstrate what they know rather than what they do not know (positive testing)
- assessment tasks should operationalize the learning goals as much as possible
- the quality of mathematics assessment is not determined by its accessibility to objective scoring
- assessment tools should be practical.

All five principles are important for ABE but the main points here are in particular De Lange’s first two principles: improve learning and positive testing. The first principle implies that teachers should learn from test results how to improve their teaching in order to have their learners learn better. The second principle includes that we should start from the learners’ competencies and not from their failures. These are important issues for learning and teaching in ABE where many learners were already confronted with their incompetencies in former school education.

In 1989 the NCTM published the first *Curriculum and Evaluation Standards for School Mathematics* in the United States in 1989 (NCTM, 1989). These present a consensual vision of the mathematical content that all learners should have an opportunity to learn - the *content standards*. These content standards were revised in 2000. In 1991 the *Professional Standards for Teaching Mathematics* were published by NCTM, followed in 1995 by the *Assessment Standards for School Mathematics* (NCTM, 1995). Key principles of the assessment standards are the three clear principles worded in a report of the Mathematical Sciences Education Board (see Cumming and Gal, 2000, p.316)

- the Content principle: assessment should reflect the mathematics that is most important for learners to learn
- the Learning Principle: assessment should enhance mathematics learning and support good instructional practices
- the Equity Principle: assessment should support every learner's opportunity to learn important mathematics.

  (NCTM, 1995)

The NCTM standards (NCTM, 1989) reflect a change in thinking about learning and teaching mathematics in general in the US since the nineties and by that regarding the content of mathematics education. They indicate in fact that 1) the content may differ per person depending on individual goals and 2) the learning and teaching focuses *also* on the process of learning and teaching and not only on the result. A third point here is that assessment should reflect the *content* to be learned. The NCTM assessment serves four purposes:

- monitoring students' progress and promote learning and growth
- making instructional decisions to improve instruction
- evaluating students' achievements and recognizing accomplishment
- evaluating and modifying programs

These purposes should be achieved by a balanced coherent assessment system in which all pieces fit together. The NCTM assessment standards indicate the importance of assessment in the development of mathematics education and in the guidance of individual learners' progress. Though the NCTM advises ways of assessment that are appropriate for the program and the learners, including different ways of testing, many standardized tests are still very traditional, see for instance the TABE and the GED for adults. (Duncan and De Avila, 1988, GED Testing Service, 1991)

Romberg (1995) and Lajoie (1995) argue that the current traditional norm-referenced standardized achievement tests do not justify the student' actual mathematical knowledge. In traditional tests one can never know what a student really understands. Counting the number of correct answers to a series of brief questions contradicts current views of mathematics as an intellectual discipline. Vast differences exist between the tasks learned in school and the tasks mathematicians or users of mathematics actually carry out. Learning inside the classroom differs from learning outside of the classroom. Inside the classroom

students are taught to manipulate symbols and abstract principles, but outside the classroom learning is often concrete and situated in the context in which it will be used. Classroom activities should be in accordance with the way mathematics is learned and used in real life situations. Requests for more authentic classroom activities also call for authentic assessment.
The NCTM Standards (NCTM, 1989) express that “knowing” mathematics is “doing” mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. This activity purpose is different from mastering concepts and procedures. Instruction should persistently emphasize “doing” rather than “knowing that”. Assessment, then, should be based on a view of the learning of mathematics as a socially constructed process, not a fixed hierarchy of skills and concepts be mastered. Mathematics cannot be broken down in pieces that can be mastered by the student in a linear, sequential fashion. (Romberg, 1995, p.5-6). Romberg compares learning mathematics to a mosaic, “with specific bits of knowledge situated in some larger design that is continually being reorganized or redesigned in an organic manner.” (Romberg, 1995, p.5)

Lajoie (1995), p. 10-11, and p.19-37) defines seven principles for authentic assessment:

1 It must provide us with multiple indicators of the learning of the individual in the cognitive and conative dimensions that affect learning.
2 It must be relevant, meaningful and realistic.
3 It must be accompanied by scoring and scaling procedures that are constructed in ways appropriate to the assessment tasks.
4 It must be evaluated in terms of whether it improves instruction, is aligned with the NCTM Standards, and provides information on what the student knows.
5 It must consider racial or ethnic and cultural biases, gender issues, and aptitude biases.
6 It must be an integral part of the classroom.
7 It must consider ways to differentiate between individual and group measures of growth and to provide for ways of assessing individual growth within a group activity.

This set of criteria was developed as a guideline for authentic assessment of school mathematics. In particular the first five criteria could help to set a frame for assessment in ABE, but these criteria are not yet sufficient. In numeracy programs for adults we want to focus on real-life related mathematics which includes problems that may have different ways of solving and sometimes also different solutions. This also includes mathematics knowledge and skills acquired in out of school situations, not only in school situations. Besides we should learn more diagnostic information in placement assessment. The developments in the Netherlands within RME and also in Action Theory provide more content information. Together with the assessment criteria from Van den

Heuvel (1996) and Van Eerde (1996), described below, we get closer to goals and principles for assessment in ABE.

A specific study about mathematics assessment in RME was done by Van den Heuvel and Gravemeijer in primary school (Van den Heuvel and Gravemeijer, 1991, Van den Heuvel, 1996). They showed that assessment can be done in an active, productive and constructive way and can indeed improve learning and teaching. Van den Heuvel (1996) elaborated this study thoroughly in her thesis, starting from De Lange's principles, and states that assessment should do justice to RME principles and should include:

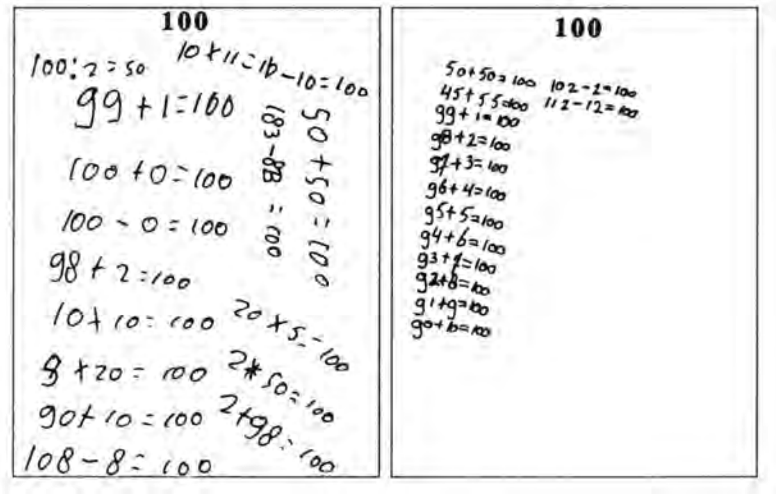
- the subject of mathematics (as a meaningful human activity) and the educational goals linked to the subject (in the breadth, the depth, and in terms of applicability)
- the manner of instruction (through contexts, models, students' own contributions, reflection, interaction and integration of learning strands)
- the learning process (the students' own activities, their levels of understanding and of mathematization, and the discontinuous nature of learning processes). (Van den Heuvel, 1996, p.165).

These three starting points lead to the following five principles: (Van den Heuvel, 1996, p. 166-175).

- *From passive to active assessment*

Mathematics is a human activity and fosters meaningful, creative solutions. Hence students' own productions take an important place in assessment in RME. The most important criterion for good assessment is that it elicits the students' own thinking process. Active involvement in assessment may also mean that students, in learning situations, can participate directly in their own assessment in that sense that they can provide information about their own knowledge and understanding. For instance, by asking them to design both an easy and a difficult problem for themselves, they contribute to the development of assessment problems. Such tasks also provide the opportunity for teachers to do real time evaluation. This can even be done with bare sums. The task in figure 3.2, for instance, requires the student to write down "*as many sums as possible with the answer 100*". Both students show their understanding in number and basic operations. The first one produces many isolated sums but shows his insight in number and basic operations. The second students proceeds systematically by changing the first item or by applying commutativity. (see also De Lange, 1995, Van den Heuvel and Gravemeijer, 1991, Van den Heuvel, 1996)

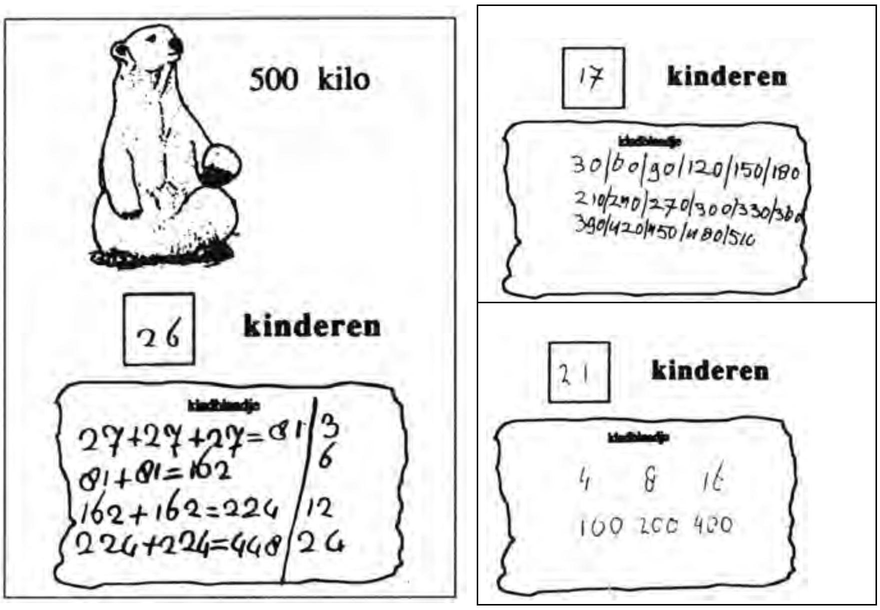
Figure 3.2 “As many sums as possible to 100”



- *From static to dynamic assessment*

Learning and teaching is a dynamic process in RME. Interaction and participation seem essential for cognitive development. Moreover, students may achieve the same goals by following different learning paths (Freudenthal, 1973, 1991). This standpoint also characterizes the socio-constructivist approach in which assessment should provide information for dynamic instructional decision-making. (Van den Heuvel, 1996, p. 167, Cobb, Yackel, and Wood, 1992c). An example of such dynamic assessment is the “polar bear”-item, one of a series of experimental assessment items in RME in Dutch primary school. (see figure 3.3) (Van den Heuvel and Gravemeijer, 1991, De Lange, 1995). The item poses the question: *How many children weigh the same as this bear?* Only the weight of the polar bear is given. It is left to the students to estimate how much a child generally weighs. The child in the left example starts probably from its own weight, the other children prefer to use a round number, 30 and 25 kg. All show a different way of getting at 500. The first child applies a doubling strategy, the second one repeated addition, the third child applies a table of ratio. All three show their insight in weight, relationship and estimation.

Figure 3.3 The Polar Bear



Another problem, the "best buy"-item (figure 3.4), requires the student's understanding of percent and reasoning skills. The first question requires the student to reason about percent: *"In which of the two shoe shops do you think the sales price of the tennis shoes is the lowest? Explain why you think so"*. To keep both items the same it was decided to present a picture of the same shoes in both advertisements, but without showing a price. To overcome the uncertainty how to interpret answers because of the openness of this question, the developers decided to add an additional "safety-net question":

"Is it also possible that the sales price of the shoes in the other shop is the lowest? Explain your answer."

Figure 3.4 The Best Buy

a In which of the two shops do you think the sale price of the tennis shoes is the lowest? Explain why you think so. Ever Sports is the best deal because, it has the most discounts. 40% is more than 25%.

b Is it also possible that the sale price of the shoes in the other shop is the lowest? Explain your answer Yes, because the shoes might have been cheaper before the discount.

Van den Heuvel's analyses show that such items provide information about the students' actual understanding and reasoning which can be the start for further learning in interactive situations. By this, assessment is also part of the learning process itself. She formulates three main points of departure for assessment in realistic mathematics education:

- *From objective to fair*

Van den Heuvel concludes, based on previous examples, that fair grading is preferable to objective scoring. (p. 168). Fairness of assessment prevails over objectivity and standardization.

- *From limited certainty to rich uncertainty*

The three principles mentioned above, question the idea that students' progress can be determined precisely by assessing what students know and can do. She points out that "nevertheless, people have always been optimistic about the potential of written tests to precisely map out learning achievements." (p. 169)
In education it is assumed that any progress can and should be documented by testing but in fact there are always many doubts in assessment, so testing uncertainty is a positive option that may provide rich information. Open-ended questions may provide rich qualitative information that is appropriate to inform instructional decision making.

- From *problems* on different levels to *problems and answers* on different levels

According to Van den Heuvel, the final point of departure encompasses the problem that level classifications are often unsuccessful because designed tests produce profiles that fail to correspond with the conjectured hierarchies. Designed test items do not necessarily include the intended level of cognitive process. This may become visible in the mathematical operation the student applies, as shown at the bear item (figure 3.3). According to Freudenthal the *way in which* a learner solves the problem determines the actual attainment of a given level and not simply the ability to answer the question. (See Freudenthal, 1991). It means that in assessment the way in which the learner solves a problem should count more than the correct answer.

Van den Heuvel's points of view can be summarized with the following statements:

- Accepting the RME starting points includes that assessment should be done in a way that mirrors the way of learning and teaching in RME
- Assessment in RME is not only to assess the cognitive development of the learner but is also the source of information for further development of learning and instruction in RME.
- Assessment in RME is a good source for further development of theories about assessment.

Van den Heuvel argues that assessment should be done in a way that is appropriate for the purpose it supposes to assess. Traditional ways of testing do not fit the RME way of learning and teaching. Though her study mainly concerns formative assessment in school mathematics, it is also very helpful when developing criteria for assessment in adult education.

Another interesting study for mathematics assessment in adult education is the "Kwantiwijzer" project from Van Eerde (1996). Van Eerde describes (p.100) a move from developing an assessment tool for diagnostics of mathematical problems to developing a procedure for assessing the student's way of thinking. Her study is mainly based on Action Theory and RME (see chapter 5). She analyzes six characteristics of process diagnostics:

- Diagnostic assessment starts in the learning setting and hence it is directly related to the way of teaching in the class.
- Assessment items should reflect notions about the process of learning mathematics rather than item difficulty merely based on statistical procedures
- Process diagnostics describes very precisely what students know and can do and what they do not yet know. Hence process diagnostics provide direct information for instruction, better than product assessment can.
- Process diagnostics focuses on the student's actions. Hence possible problems in student learning can be determined earlier and more accurately.
- The result of process diagnostics provides information for instruction that fits in the learning and teaching process.

- If the teacher is capable of doing process diagnostics, it may help to prevent learning problems.

An important contribution to the diagnosis of mathematics problems in Van Eerde's research is her elaboration of techniques to analyze the student's mathematical actions in oral interviews. This can be done by:

- observation (looking at what children do during the interview session)
- interviewing techniques:
  a- introspection (ask the student to think aloud)
  b- retrospection (asking the student after the action was done what he did)
  c- continued questioning (repeating the question in a different way and more in depth)
- mirroring (reflecting the student's action by repeating and demonstrating the student's own action)
- problem variation (offering a different problem of the same level difficulty, a more difficult problem or an easier problem)
- offering assistance (by providing hands-on materials, pre-structuring a solving procedure, solving a problem together and then having the student do a similar problem, referring to a solving procedure learned in the class room situation, drawing attention to emerging errors)
  (Van Eerde, p. 133)

Based on the results of such a diagnostic interview the interviewer is able to describe the student's actual action level and his zone of proximate development, as described in Action Theory (see chapter 5). This provides direct information for instruction.

3.3.2 Preliminary conclusions

The developments in assessment of school mathematics are helpful at defining goals and principles for ABE. According to De Lange (1987) and the NCTM Standards (1989), assessment should be organized in a coherent system of placement, formative and summative assessment. Three main components can be distinguished: monitoring the students' progress, improving learning and instruction and evaluation of the program. For ABE a fourth component should be added: accountability to funders and policy makers, because ABE, as for most courses in adult education, at least in the Netherlands, is funded based on outcomes.

An important issue introduced by De Lange is "positive testing". Education should start from what learners already know and not from what they do not know. This is the basic principle for competence learning. Furthermore, assessment should show alignment with the program and should be practical. Finally, qualitative information is more important than objectivity.

Romberg (1995) and Lajoie (1995) add to this that assessment of mathematics should be done in a way that justifies mathematics as a real-life science. Assessment of mathematics cannot be done in a straightforward linear, sequential fashion by breaking down mathematics in pieces, but should cover knowledge and skills required in real life situations. Lajoie's main point is that assessment must be relevant, meaningful and realistic. This means in particular that out-of-school knowledge is an important factor for placement in ABE courses.
Van den Heuvel and Van Eerde offer valuable information for assessment in adult education. Van den Heuvel emphasizes the development of formative assessment in a way appropriate for RME in primary school. She shows that assessment can be done in many different, constructive ways and can provide appropriate, qualitative information. She offers many suggestions that can be of help in the development of assessment in ABE. Van Eerde moves the mind from developing diagnostic assessment tools to developing a procedure to find out the learners' way of mathematical thinking. She offers valuable diagnostic tools, in particular the observation and interviewing techniques. Process diagnostics focuses on the learners' mathematical actions. Hence it can better determine what learners know and how they do mathematics. This provides direct information for learning and teaching. Based on the discussion above we can now move to goals and criteria for numeracy assessment in ABE. (sections 3.4 and 3.5)

3.4 Goals for numeracy assessment in ABE

Assessment in contemporary adult education in the Netherlands is moving toward a more "authentic" portfolio system. Broad information has been assembled about each adult who starts learning in a regional learning center [in Dutch: ROC, see chapter 1]. The system includes a broad placement procedure, formative and summative assessment, and support with application procedures for work. Adult and vocational education are part of the same system. Though the overall system appears to be good, in practice we may wonder whether it works well for each individual learner. In this system computer adaptive testing (CAT) has been promoted for more subjects, including mathematics. This is, of course, understandable because in such a gigantic system it is efficient in most situations to use quick scans for a rough indication for placement and for monitoring results in general. However, for placement in numeracy courses in ABE we may wonder if CAT provides sufficient detailed information. It depends on what information is necessary for effective learning and teaching.

Focusing on numeracy assessment in ABE as part of this broad system, goals and criteria for assessment need to be set. Yet, little is known about assessment of adults. Cumming and Gal (2000) mention three general goals for assessing adults' numeracy skills:

- For the learner: initial diagnosis for placement, formative and summative assessment
- For the curriculum: goal setting, planning and modification
- For the program: evaluation, reporting and accountability.

In these goals they indicate that assessment is not only important to learn more about the individual learner, but also and perhaps more about the curriculum and the program. Assessment can help to adjust the learner's and the program's purposes in order to help the learner in the best way possible. This does not mean that the program purposes should be adjusted but the way in which to achieve these purposes should be fine-tuned to the learner's capacities and needs. Concerning placement assessment Cumming and Gal state that

> *"Diagnostic assessment in adult numeracy should encompass a broad definition of numeracy and seek to:*
> - *Establish the learner's goals*
> - *Determine what knowledge, strategies, and reasoning processes the adult learner already possesses, whether formally or informally*
> - *Determine what needs to be learned in the context of the learner's goals and the learning setting (e.g. a workplace)*
> - *Be able to indicate which strategies should be used in instruction."*
>
> (Cumming and Gal, 2000, p. 311)

This fourfold goal-setting indicates that for good instruction it is necessary that placement assessment includes initial diagnostics related to the context in which the learner will be placed. This means, for instance, that placement in a workplace setting requires different assessment procedures than in an ABE setting. Placement assessment in an ABE learning setting depends on the way learning in ABE is arranged. Such is almost always comparable to school-like situations. However, we may wonder whether a school-like situation is the ideal learning setting for ABE learners (see chapter 7).

Based on the above introduction and the elaboration in a previous section the following goals for numeracy assessment in ABE can be set. These goals concern monitoring the learner's progress, curriculum and program evaluation.

1. Learn more about the mathematical knowledge and skills adults have acquired when they enter ABE in order to determine what needs to be learned and to place them in the right courses.

Insight into adults' formal and informal knowledge and skills is necessary in order to learn more about their competencies and to help them determining their goals and wishes. Therefore the placement test should provide qualitative and quantitative information.

To prevent incorrect placement and early drop outs, it is important to have a good placement test that covers a well-elaborated program with multiple possibilities. The placement must be part of a broad intake procedure. The organization of groups of learners should be in accordance to levels of the placement test and the learners' learning wishes.

2. Learn more about adults' problem solving procedures in order to improve instruction.
Insight into adults' problem solving procedures is an important source of qualitative information for learning and instruction. In particular, information about informal and formal procedures, doing estimations and computations, the use of the calculator, individual idiosyncratic procedures, etc., may help teachers to learn more about mathematical problem-solving strategies of adults, cultural differences, etc., but may also help them to improve their teaching and develop appropriate ways of instruction. The main purpose of teaching should be to enable adults to improve not only their mathematical knowledge and skills, but also their own ways of learning. (De Lange, 1987, p. 179-180)

3. Learn more about adults' numeracy skills to be able to fine-tune educational programs to their needs and wishes.
In traditional mathematics programs it is very common that learners try to overcome gaps between their own knowledge and skills and that which is required to pass tests or exams. However, competence learning starts from the learner. ABE programs must be adaptive and must be fine-tuned as closely as possible to the learners wishes and learning strategies in order to achieve their goals. It means that learning routes and instruction may differ from person to person. Numeracy programs should provide possibilities for both flexible learning and flexible teaching. Hence it is important to know at the start of a course what the learner knows and wants to achieve.

4. Monitor and document the individual learner's progress in the course of the numeracy program in order to guide the learners through their own learning routes and to prevent learning problems and drop-out.
In general, when adults start taking (numeracy) courses in adult education, their intention is to finish it, and it is often their second and last chance. In practice however, many adults drop out for different, sometimes unclear reasons. (40-60% in the Netherlands). It may be for reasons involving the pace in which they have to learn is too fast, or they cannot keep up with others in their group, or dates when exams are planned don't fit their personal agendas. They stay home to care for their kids, or have a change in work shifts. To be able to help them to get a grip on their own learning process and to guide adult learners through their learning routes, it is necessary to develop a system of assessment, tutoring and individual support to monitor and encourage learners to go on with their learning activities in combination with more flexible programs.

5. Be able to evaluate numeracy curricula in order to improve numeracy education in general.
Internationally, numeracy learning and teaching, in particular in ABE, is still in the developmental stage. Assessment reports may help improve learning and teaching in individual programs. If provided on a regular basis, numeracy assessment reports could also be very helpful for the evaluation of numeracy programs in general in order to further develop the knowledge about adults learning mathematics. National and international research (see chapter 2) could play a central role in this development.

6. Enable policymakers and program developers to adjust policy and numeracy programs to new demands and developments in the labor market and in personal and societal life.
Developments in technology increase the need for adults to continuously adapt to changing citizenship and workplace demands. Therefore it is vital that policymakers and employers have up-to-date information about their citizens' and employees' numeracy skills in order to plan lifelong learning opportunities. In this frame, assessment results from both local learning centers and large scale surveys, like the ALL project, are important input. (European Commission, 1996; Gal et al, 1999)

3.5 Criteria for numeracy assessment tools in ABE

Based on the above goals criteria for assessment and assessment tools in ABE can be formulated.

1. Numeracy assessment in ABE should be done in an appropriate way for learners in ABE.
Influenced by the current developments in adult education, i.e. outcomes based learning, continuous intake, more individual learning routes, development of brief courses, less teaching time and more time for self-study activities, teachers and institutes prefer a system of placement and formative procedures that does not take much valuable teaching time. Teachers still prefer paper-and-pencil tests with minimal grading to oral interviews. However, this can be a pitfall, because the results of written tests do not always cover the learners' actual skills. Given the sub-populations in ABE three types of tests are advised in combination:
- oral interviews for non-native and un-schooled adults and for native speakers who are assumed to have reading problems or severe school or test anxiety. Such should become clear in a first intake interview or during literacy classes. Non-native learners in literacy classes are often placed in low level math courses without any testing, though many of them show good, informal math skills and strategies. In oral interviews, done by well-trained interviewers, these learners can show better what they know about mathematics. (van Groenestijn, 1993)

- paper-and-pencil tests in combination with individual evaluation talks.

This can be done with learners who are better used to doing tests and have no test anxiety. The test should be adaptive by offering possibilities to take a long or short version of the test in multiple forms. It should also offer possibilities for learners to show their computations.

- quick scans by computer adaptive tests or a short version of the paper-and-pencil test for advanced learners who only come for specific courses on a higher level and are used to working with computers.

2. Assessment should enable adults to show what knowledge, skills and procedures they have mastered when they enter ABE, rather than what they do not have.

Adults have often a lot of informal knowledge and many skills acquired in real life situations in addition to and sometimes as a replacement of formal school knowledge and skills. Such knowledge is based on individual experiences and can be work-related and specific for some individual situations. Therefore tasks in placement tests should not only test tacit formal school knowledge and skills, but should enable adults to also show what they learned in informal real-life situations. This requires recognizable everyday and work-based tasks that can be solved in formal and informal ways. This sets the basis for positive testing. (De Lange, 1987, 1995)

3. Numeracy assessment in ABE should provide insight into mathematical procedures and problem solving.

Tasks should offer mathematical problems in contexts, derived from real life situations, that can be solved in multiple ways and offer possibilities for creative solution procedures. Tasks may also have many correct answers. In addition to positive testing, such tasks enable adults to show the mathematical thinking and knowledge they have mastered and how they apply that in a constructive way. They may use formal procedures they learned in school, but they can also show their own invented, creative, informal procedures. Accompanying computations in the test sheets may provide insight into procedures on different levels of mathematical action, e.g. smart ways of counting, repeated addition or doubling procedures instead of multiplication. Such tasks set the basis for constructive testing and can be compared to Van den Heuvel's active and dynamic assessment (Van den Heuvel, 1996)

Analyses of this information may tell more about functional math skills that adults have developed in real life situations. This may change the focus of the teachers' teaching from school math to real life math activities. For example: we may wonder why we still try to teach times tables or traditional multiplication and long division algorithms in ABE if it appears that adults don't need that in real life situations but can better apply their own alternative ways of doing multiplication and division.

4. Placement tests in ABE should reflect the goals, content and levels of the math curriculum so that adults know what they can expect during the course and can be placed in right course.
A placement test for adults should be based on the level classification that covers the program. It should offer tasks that reflect the content in different areas on all levels and the type of tasks learners can expect during the course. Results on the test must be presented to the learners in such a way that they can clearly see their places on the levels in the underlying level classification. It must also show their strengths and weaknesses in the different areas of mathematics.

5. Numeracy assessment in ABE should allow second-language learners to apply the mathematical procedures and algorithms that they learned in their home countries.
This criterion is important in intercultural learning settings, though it is not always clear what kind of differences can be expected. Cultural differences become apparent in, for example, different notation systems and pronunciation, but also in mathematical contexts. (see for instance the coffee problem in example 3.3. It is very difficult to create items for placement tests that offer cross-cultural possibilities. The use of different notation systems in placement tests should be allowed. Also, notations can be pronounced or written in different ways, e.g. 1/5 can be pronounced as “one-fifth” or “one-over-five” or “one-out-of-five” or “one-in-five”. Reading such text notations in a second language may confuse adults with the pronunciation in their own language. In a context that says “One-fifth of our learners in this learning center come from abroad, how much is that in percent?”, it may be better to use the notation 1/5 and make it visible in a graph instead of just words. Visual support may help to prevent misunderstanding.

6. Text used in a paper-and-pencil math test should not hamper second-language learners to take a mathematics test.
It may be difficult for second language learners to take a paper-and-pencil test. It requires at least a minimum level of mastery of the instructional language. The question is that if the test is supposed to test mathematics skills, to what level of difficulty can text be used in the mathematics tasks? In particular for placement tests we may argue that the language barrier should be as low as possible.
Text in contexts may cause problems for second language learners to understand the mathematics in the problem. Some text is unavoidable, inherent to the context, other text is communication text that can be changed and adjusted without changing the context problem. Such communication text in contexts can be limited to elementary common everyday language. In addition contexts can be supported by visual aids, preferably photos, which may make context situations recognizable.
Instructional text and text in questions must be structured in a clear way. Repeating text and questions can be offered in the same way and style in order

to make similar types of text and tasks recognizable. If this is done, then learners may still stumble over single words, like Sonia did at the coffee problem, but the language barrier can be reduced as much as possible in every task.

Learners who have not yet mastered the second language well, should be allowed to take the mathematics test in their own language, if possible, or to do it orally with the help of an interviewer.

3.6 Development of numeracy assessment tools in the Netherlands

In the past decade three numeracy tests were developed for adult education. The first test, "Supermarkt Strategie" [in English: "Supermarket Strategy"] (van Groenestijn et al, 1992), is an independent adaptive and diagnostic test for adults in ABE. The test was done through individual oral interviews. As a result of this "Supermarket Strategy" the learner mathematics series "In Balance" was published, including a placement test plus formative tests (van Groenestijn et al, 1996). The placement test is a semi-adaptive paper-and-pencil test that can also be done in an oral interview. The test uses the same structure as the design of the series and refers for placement to the level classification of the series. Also in 1996 the Dutch National Center for Test Development (Cito, 1996) published an independent paper-and-pencil mathematics test for adult education in general. These three tests are discussed in the following sections. (See also chapter 1 for development of numeracy education in the Netherlands)

3.6.1 Supermarket Strategy

Resulting from a project that lasted from 1989 until 1992, the first example of assessment of mathematics published for the former ABE, was "Supermarktstrategie", (van Groenestijn et al, 1992, van Groenestijn, 2000). The name refers to the use of mathematics contexts in the package derived from real life situations and to the diversity of strategies adults may have to solve such real life problems.

The direct drives to develop this package were questions from ABE teachers, mostly volunteers, in teacher training courses for ABE teaching qualification during the years 1987-1990 (see chapter 1). At that time mathematics in ABE was totally new and nobody knew exactly what to do. Teachers came with questions, such as: *How do we know what they know? How do we know what they need to know and what they want to know? How can we assess learners' math skills? How do we know where to start? How can we organize groups with so many differences in levels? We need a curriculum!* There were many more questions than answers. We started from scratch.

The very first context, used in these courses, was an advertisement of a local newspaper. A set of possible questions was developed at that advertisement to test mathematical knowledge and skills. The teachers were trained in interviewing techniques as used in "Kwantiwijzer", described in section 3.3. (Van den Berg and Van Eerde, 1983, 1986, Van Eerde, 1996). Teachers started interviewing their learners and became enthusiastic about this way of assessment. A second result of these interviews was that these teachers changed their teaching style more into "asking questions" rather than "teaching". They also started encouraging their learners to try alternative problem-solving strategies instead of traditional algorithms and other "school" procedures for learners who had problems with such school skills. This was a break-through in ABE and the start of the project "Supermarket Strategy".

After extensive study and discussion with the experienced teachers in the teacher-training courses, a grid emerged for a curriculum based on three main fields: basic skills, proportions, and measurement and geometry. These fields were subdivided into seven related fields of knowledge and six levels of mastery, as shown in table 3.1. Goals were formulated in each cell of the table. These goals are not shown here as they cover multiple pages.

There is a coherence within the horizontal and vertical structure in this level classification. Horizontally the subject matter per field is built up from six levels of complexity or difficulty, from elementary to advanced. Vertically, there is a coherence within and between the three main areas and their seven sub-fields. This coherence is a main issue in RME. A specific topic such as "doubling and halving", for instance, appears at level 2 and serves as the starting point for multiplication and division, based on addition and subtraction. It is also the starting point for working with proportions and the metric system at the same level. At level 3 "doubling and halving" procedures come back in a broader perspective at multiplication, division and at working with fractions, decimals and percentages. At this level it is aimed that learners can make optimal use of multiplication and division. Additionally, they must have insight into the use of benchmark fractions, decimals and percents. This goes in line with benchmark measures in the metric system up to a half, a quarter and a tenth of something like weight, volume or length. The level indication was designed on this vertical coherence.

Table 3.1: Supermarket Strategy grid: Goals and Level Classification.

		Elementary Levels		**Intermediate Levels**		**Advanced Levels**	
Mathematics Areas	**Content**	**1**	**2**	**3**	**4**	**5**	**6**
Basic Skills	1. number, counting, addition, subtraction						
	2. multiplication and division						
Proportions	3. proportions and percent						
	4. fractions and decimals						
Measurement and Geometry	5. metric system and geometry						
	6. money computations						
	7. time and calendar						

Horizontally the classification of difficulty is characterized by:

- An advance in computational skills from simple tasks at the elementary level to more advanced, complex mathematical procedures at the higher levels.
- A gradual transition from concrete, informal real-life contexts at the lower levels to more abstract and formal contexts at the higher levels. For learners at the elementary level, real objects such as money and a measuring cup may be used to solve the tasks. At the intermediate level the contexts are mostly represented by graphics and by symbols and numbers. At the advanced level the contexts are represented more abstractly through the use of symbols and are supported by pictures, schemes and/or other graphics. Note that representations are important at all levels to support insight.
- A progression from simple one-step problems at the lower levels into more complex multi-step ones at the higher levels.

In each cell the goals are formulated for that particular cell, based on the structure described above. Initially, these goals helped teachers to develop mathematical activities in a planned structure. The next step was to develop assessment contexts in which this coherence could be applied, in order to enable adults to solve these mathematical problems in multiple ways. These contexts were based on real-life situations, gradually increasing in complexity and abstraction from level 1 to 6. A context is comprised of a photo of something referring to a real life situation. Text used in contexts is restricted to a minimum and exists mostly of common everyday language. There is only one question, sometimes two, to each context. Learners are free to solve the context problem in their own way, using mental math, computation procedures on paper or using a calculator.

A special advertising leaflet (flyer) was developed, in collaboration with a large supermarket chain in the Netherlands, to provide a realistic visual and informational context for the supermarket-related problems. The leaflet presents, just like the real thing, many pictures of different kinds of products in full color, with accompanying measures and prices. The products are grouped and displayed in such a way that a discussion can focus on many issues, starting from simple issues such as the shapes and relative sizes or volumes of different containers (e.g., milk cartons, bottles) or the number of elements in different pictures (e.g., bottles in boxes, carrots in a bag). The discussion can culminate with advanced issues, such as those involved in decisions about the mix of prod ucts that can be purchased if the buyer has a given amount of money, or the price of a certain piece of cheese if the price per kilo is shown in the leaflet. The use of an "authentic" leaflet enables the interviewer and the interviewee to talk together in an informal way about several topics, in a way that resembles an adult conversation and does not look like a traditional test. This conversational approach, based on observations and interviewing, can provide the examiner with information about the adult's problem-solving strategies, whilst allowing the adult to reflect on them as well.

By individual interviewing, applying Van Eerde's (1996) interviewing techniques, teachers could trace the individual mathematical knowledge and procedures, ways of reasoning and interpreting skills of their adult learners, gaining insight into their capacities, needs and wishes. At the end of the interview a profile was acquired providing qualitative information about the adult's mathematical proficiency. This scheme, with the described goals per area and level, was and can still be of great help for gaining insight into adults' knowledge and skills and for developing programs on different levels, placing adults in learning-groups (classes) and for testing progress. An additional benefit was that the program prompted teachers to stop teaching in a traditional linear fashion of first teaching basic skills, then fractions, decimals and percentages and some measurement activities in between. Now they could build up a balanced program based on the necessary activities across different areas and levels.

The Assessment Procedure

The main purpose of Supermarket Strategy is placement. In addition it can be used to assess progress in the class setting and in particular also as a diagnostic instrument to analyze mathematics problems. In an intake situation the examiner/teacher determines the learner's skill profile using an adaptive process. An individual test consist of only a subset of the 60 context cards. In other words, not all problems will be presented to each learner. The examiner will start at a difficulty level (usually 2 or 3) that seems most suitable for the examinee, based on a "locator" problem. Assessment will stop when it appears the examinee is unable to solve more problems in a reasonable manner.

An example for a locator task is the "bread" problem, which helps select which of skill levels 1,2,3 or 4 to start with. The leaflet shows pictures of different types of breads and rolls. The adult is asked to choose two different types of bread. What would he have to pay for it? If the adult answers well, the next question is more complex, e.g., "Now you want to buy 6 rolls; which ones do you choose and what would you have to pay for it?" As part of this process, the teacher analyses the learner's ways of adding and multiplying.

The placement decision called for in an intake process does not require that all skills are assessed in an equal manner. Often, only the first three or four fields will be assessed for an initial placement. Despite the order of fields in table 3.1, field 6 (money computation) is always the first one to be actually used after the locator problem, because most people are familiar with money and it is very important in daily life. In doing computations with money the learner also shows his or her familiarity with the four basic operations.

Table 3.2 shows a possible order of administration of problems to a hypothetical learner. The path followed by the examiner through the grid is determined by the quality of the learner's responses. For each question the learner can get 2, 1 or 0 points, which indicate that:

2 The learner gives a good solution in a smart, efficient way, is confident in his/her solution and is able to reflect on it.
1 The learner is able to solve the problem, but slowly and perhaps with many mistakes and restarts. He/she is not certain about his/her solution.
0 The learner does not understand the context and is not able to solve the problem or may come to a solution in a very slow and laborious way.

If the learner is assigned a 2, the next question will be in the same field but at the next higher level. If the learner gets a 1, the next question will be at the same level but in the field below. If the learner gets a 0, the next question will be in the same field but at the next *lower* level. Solution procedures, like doing mental math or using a calculator, can be administered in an accompanying item list. Computations on paper can be used for analysis of the learner's computation procedures. Consequently, at the end of this adaptive assessment process, a qualitative profile of the learner becomes evident.

Table 3.2: Skill grid used in actual assessment for a virtual learner

Sequence	level 1	level 2	level 3	level 4	level 5	level 6
locator	2	2	1			
field 6			2	1		
field 1			1	0		
field 2			1			
field 3		1	0			
field 4		1				
field 5						
field 7						

The pattern in this table shows the final "profile" of the learner. Based on this pattern, plus the accompanying administration form, the teacher knows the strengths and weaknesses of the learner and the placement level. This procedure appears to be very effective. An experienced interviewer needs about 45 minutes to determine the profile of the learner. The result provides the teacher with qualitative information about the learner's competence and possible problems. This helps to determine learning goals and to tune instruction to the learner's needs.

Reflections on Supermarket Strategy

Reflecting on this experimental Supermarket Strategy assessment tool we may conclude that this assessment procedure comes close to the principles of De Lange, Romberg, Lajoie, Van den Heuvel and Van Eerde, as described in section 3.3. The developments went in parallel that time. The use of authentic materials was strongly advised in the Dutch ABE system and was based on Paulo Freire's pedagogy "learning from experiences" (see chapter 5). The word "test" was more or less "forbidden" in ABE and the word "assessment" in the meaning as it is used now, did not yet exist in the Dutch education system.

The intention of the developers was to design a "test" that did not look like a test and offered the possibility to "talk" with adults in an informal way about mathematical real life issues and the problems they might encounter but at the same time could indicate their level of mathematical actions. For that purpose the leaflet was designed, which created an "authentic" mathematical situation. It was assumed that adults would seldom score on only one "level", due to various experiences in their societal and work related situations. The "profile" would provide more accurate information. This format exemplifies De Lange's and the NCTM's principle that assessment should provide qualitative information for learning and instruction and Romberg's and Lajoie's principles (1995) that assessment should justify "authentic" mathematics and must be relevant, meaningful and realistic. It is also in accordance with the RME principles. (see chapter 5)

The development of the Supermarket Strategy contexts went in parallel with Van den Heuvel's experiments with realistic assessment in primary school. In the "bread" locator test, for instance, adults can indicate their own proficiency by choosing types of bread and rolls for their own families, accompanying prices and number of rolls and compute the total they would have to pay. In that way they created their own "sums" with different answers and their individual computations were on quite different levels. All the other contexts also offered possibilities for the learners to show their best work. Thus testing was "positive" and focused on competence, according to De Lange. The effect on the adults was also positive, as one adult reported after a one-hour assessment talk: *"I never learned as much as today"*.

The observation and interview techniques coming from Van Eerde's Kwantiwijzer project, are a perfect means to direct the interview, to have the adult reflect on his own actions and to get qualitative information. These are indispensable when diagnosing mathematical skills and possible problems.

After this first experimental assessment tool in ABE the reorganization of adult education in the Netherlands started, which had consequences for the Supermarket Strategy. The new system required placement tests that could be taken in a short time and would take little time from teachers to grade. Paper-and-pencil tests and computer adaptive tests seemed to be more effective than time-consuming individual oral interviews. Such tests would also be less dependent of the knowledge of individual teachers.

3.6.2 In Balance

As a follow-up of Supermarket Strategy, a small workgroup started in 1993 developing the first series of instruction booklets for adult learners in ABE: "In Balans" [in English: "In Balance"]. The project initially aimed to provide instructional materials in addition to the test materials of the "Supermarket Strategy", but soon it became clear that the project should produce more complete materials for ABE. Due to the developments in adult education the curriculum had to be revised and refined. Good instructional materials were necessary in combination with a coherent system of placement and formative tests. Teacher guides and teacher-training programs were also badly needed. It was time for a complete adult numeracy program in ABE, though the word "numeracy" was still hardly known.

The "In Balance" series provides such a program. The series focuses on real-life numeracy problems in contexts and tries to combine adults' informal and formal knowledge and problem solving strategies, based on RME-insights and the principles of socio-constructivism. (see chapter 5). The series was completed in 1999 with three units on level 5*,* composed of tasks that were intended to bridge the gap between ABE and the first level of vocational education. (see chapter 1)

3.6.2.1 Design of the series and placement test

The units are designed on a revised level classification of the Supermarket Strategy and are synchronized with the national curriculum that was developed in parallel by Kemme, Sormani and Weijers (1997). The series was based on the new level classification in adult and vocational education (see table 1.1, p.11), so that the new materials would cover the new system on the lower levels.

The "In Balance"-series is composed of 11 units spread over 5 levels. Eight units are spread over an A and a B series over four levels. Level 5 provides 3 units as bridging links to General Education at level 3 and to three main sections in Vocational Education at level 1. See table 3.3.

The units in the A series focus on number and the four basic operations in an integrated way and related to proportions. The B series focuses on the metric system with decimals integrated and based on proportions. Proportions as the linking issue between the A and B series also include tasks on fractions and percents. Money computations and tasks involving shape, dimension, time and calendar, reading and understanding simple statistical graphs are also included in the B-series.

Table 3.3 Overview In Balance series

Elementary Level (level 1 in general education)		Foundational Level (level 2 in general education)		Bridging Level (to level 3 in general education and to level 1 in vocational education)
Level 1	Level 2	Level 3	Level 4	Level 5
A1	A2	A3	A4	Technology Economics Health and Care
B1	B2	B3	B4	
Assessment materials Teacher guides Copy sheets		Assessment materials Teacher guides Copy sheets		Assessment materials Teacher guides Copy sheets

A lot of attention is paid to doing estimations, smart computations, development of good notation systems and use of the calculator. The A and B series on level 1 to 4 work in parallel and must be used in combination with each other, not after one another. The series ends on level 5 with three units in which all skills are practiced in an integrated way in work contexts. This is the bridging level to vocational education.

The series is built up in a concentric way. This means that each level creates a starting situation for new learners and offers a complete program on that level. In this way learners can start on each level, apart from the previous levels.
All A-units and all B-units have the same structure, but on different levels and with a wider perspective at the higher levels. There is a gradual progression and transition from simple, one-step real-life tasks on level 1 to complex, multi-step and more abstract tasks on level 4.

The placement test is designed on the structure of the instruction units with 10 assessment items at each unit from level 1 to 4 (20 items per level) which cover the content of each unit. (see table 3.3) At level 5 the 20 items in the placement test are spread over 5 general real-life items and 5 for each work section. The total of 100 items covers the entire program. At the time of the field research of this study (1997-1998) only the first four levels were published.

The “In Balance” placement test is a semi-adaptive test and can be applied in several ways in individual situations for diagnostic purposes. The standard procedure prescribes that the test is taken in two phases. In the first phase the learners do the sub-tests A1 to A4. Depending on the results on the A-subtests, they do only two sub-tests of the B-series in phase 2: combination B1-B2, or

B2-B3, or B3-B4. The total procedure takes about 90 minutes in total. However, the full test is seldom taken. Adults who are expected to start at a higher level can do the placement test from level 3. They do not need to do the levels 1 and 2. Learners starting in vocational training are advised to take the test on levels 4 and 5. Even there a selection of items can be made depending on the work setting. Hence the placement test time always takes about one hour, with a maximum of 90 minutes. Learners who start at a higher level and learners who do the higher B-sub-tests receive bonus points for the sub-tests they are assumed to have mastered. The level placement depends on the total score in the A+B sub-tests. This may vary from 0-100.

Table 3.4

Level 1	Level 2	Level 3	Level 4	Level 5
0 - 20	21 - 40	41 - 60	61 - 80	81 -100

3.6.2.2 Characteristics of the In Balance placement test

The test is designed as a paper-and-pencil test that can be done with groups of learners. For learners at a higher ABE level and with school education, this should not be a problem. To make the test more accessible for native adults with reading and writing problems and for non-native speakers, the use of instructional language in the test is limited to very common everyday language in short single sentences. Instruction reiterates often with the same words for similar tasks, also on different levels. For native learners who have reading problems and also for non-native speakers, an interviewer can read the text aloud where learners show problems.

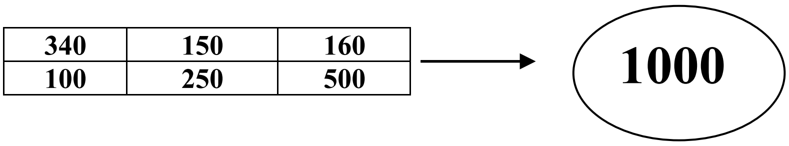
The test can also be done through individual oral interviews. For this the same interview techniques can be used as in the Supermarket Strategy. Interviewers are allowed to use real materials to support pictures and drawings on paper, where necessary. In this way illiterate and semi-literate adults, who may not be used to processing written information, can also be tested.

Though most tasks rely on real life contexts with supporting photos or drawings, there are also a few “school-like” tasks at each level. The decision to include them was a dilemma, but was made in particular for learners who are also used to traditional school math and may expect this when they come back to school. These tasks offer these learners the opportunity to show what they still know, but should not hamper other learners who may have forgotten a lot and those who have created their own (informal) way of doing computations. These items are composed in such a way that they can be solved in different ways. Items in the In Balance test are characterized by:

1) Computation tasks:
- Context items with one question to each item. These items can be solved in multiple ways. It is assumed that adults will always show their best way of solving such problems.

- "get the number" - items. Such items were developed to enable adults to show their familiarity with numbers by creating their own sums, e.g. the number 1000, by selecting numbers from a range of numbers, for instance:

Figure 3.4: Get the number. Check the numbers you need to get at a total of 1000.



340	**150**	**160**
100	**250**	**500**

These tasks are used instead of traditional sums where only one possible answer can be given. Such problems come close to Van den Heuvel's free productions, but are restricted to a few possible correct answers to make a scoring system possible. (see also figure 3.1)

- A few traditional tasks at each level, like 19x35 and 445÷5, which can be solved in various ways. Here adults can show former school knowledge but also own inventive idiosyncratic computations. Test leaders should be aware of the different algorithms and other notation systems learners may use. Where necessary they may show alternative notations for learners, like 445/5.

2) *Estimation tasks*
These tasks are always embedded in contexts and indicated by the word "about", e.g. showing a few shopping items with price labels on it, accompanied by the question: "About how much would you have to pay?"

3) *Non-computational task*
These items ask for insight or knowledge without doing computations. They require insight in numbers (place value), reading and interpretation of data (graphs), everyday life knowledge, insight in dimension or shape, reading clocks and calendars.

Adults are asked to do their computations next to the tasks in the booklet. Thus a numerical score is possible to indicate the level, but a qualitative analysis can also be done. The numerical score can be divided into sub-scores for the A and B tasks. Hence an indication can be given concerning basic operations and applied operations. Qualitative analyses can be done on the kind of operations the adult applies by analyzing his computations on paper. (See chapter 4)

3.6.3 Cito

In 1996 the National Center for Test Development in the Netherlands (Cito), published the first Cito national placement test on mathematics for adults on levels 1, 2 and 3 in the new adult education system. (Cito, 1996). The test is an independent paper-and-pencil test. Second language learners can take this test if they have mastered reading level 1 of the national Cito test (Dutch for second language learners). The Cito mathematics test is based on IRT scaling (Item Response Theory). Each item indicates a certain proficiency and hence the test result provides a proficiency score within the levels 1 through 3. The Cito test was validated based on a sample of about 1200 learners in adult education (Straetmans, 1994). The test is used in this study to compare the results of the experimental *"In Balance"* placement test with an independent standardized test.

The *Cito* test, like the In Balance test, is also a 2-phase test (2x 45 minutes). The test is compiled of three sub-tests. In the first phase adults take the C1 subtest that consists of 12 items on level 2, the middle level of the *Cito* test. Depending on the results in this first phase (<7≥ correct), adults take the C2-Low or C2-High sub-test in the second phase of the test, with items respectively on levels 1 and 2 or levels 2 and 3.The total number of items to do in the Cito test is 25. Each sub-test is composed of items from all main areas in mathematics. Scores can only be correct or incorrect (1-0). The final proficiency score is based on the total score on C1+ C2-Low (total score in the range of 0-96) or C1+C2-High (total score in the range of 79-184). (see figure 3.5)

Figure 3.5 Cito procedure and proficiency levels

Adults are supposed to do their computations next to the items. There are a few items involving mental math and estimation. The items are characterized by:

1) computation tasks
- short information, almost always textual, in combination with one question.
- contexts in which the information is visualized in a picture
- context-less tasks, e.g. 12.85 + 39 + 37.86 =

2) mental math tasks
These tasks are indicated by a grey bar accompanied by the words “in the head” and shaded space next to the item to alert the learner that no computations are expected there. One example is: 9.9 + 9.9 =

3) estimation tasks
There are two estimation tasks, only in the first phase test (C1). These are indicated by the word “estimate” . Both are multiple-choice questions about percents. The first item is based on visual information, the second one on textual information.

Each task has only one correct answer. Where appropriate, explanation of difficult words in each item is provided.

This *Cito* test was, in fact, the precursor of the *Cito* computer adaptive tests. The test itself appears to be effective for indication of placement level, though little qualitative information can be acquired because there is no clear design of sub-fields. Experienced teachers can take time to analyze the computations in the paper version of the test, which may tell a bit more about the learners' computation skills, but no systematic information can be acquired. This means that, particularly at the lower levels 1 and 2, the teacher will need to do a supplementary diagnostic test in order to have qualitative information about the learners' way of doing computations.

3.7 Summary

To learn more about numeracy skills of adults and to be able to organize learning and teaching in ABE in the best possible way, it is necessary to have a coherent assessment system for placement, formative and summative assessment of numeracy. Placement assessment plays a particularly important role in this because people have a variety of educational and cultural backgrounds. Roughly, four sub-populations with their own characteristics can be distinguished for each. Placement assessment should be done in an appropriate way for each sub-population.

In this chapter a few important developments were described on assessment in school mathematics in the beginning of the nineties. More emphasis was placed on positive, competence focusing, active, constructive and authentic testing (De Lange, 1987, Romberg, 1995, Lajoie, 1995, Van den Heuvel and Gravemeijer, 1991, Van den Heuvel, 1996). The NCTM assessment standards (1995), De Lange (1987), Van den Heuvel (1996) and Van den Heuvel and Gravemeijer (1991) stated also that assessment was meant not only to keep track of the learners' progress but also to improve learning and teaching. Interviewing techniques for diagnostic assessment moved the attention from product assessment to process assessment (Van Eerde, 1996). New types of context tasks, and a focus on the learners' reasoning and problem solving strategies rather than on the correct answer, offered new possibilities in this development.

In parallel to these developments assessment in adult education in the Netherlands became gradually more important due to the new adult education system (see chapter 1). The first package, "Supermarket Strategy", developed for ABE, based on the RME principles and constructivism, was published in 1992 as an experimental assessment tool. (van Groenestijn et al, 1992). The main ideas are in accordance with the developments in RME assessment described above. After discussion with Van Eerde and Van den Berg the interviewing techniques of the "Kwantiwijzer"-project were adapted in the Supermarket Strategy.
The successor to this package, the "In Balance" placement test, was published in 1996 as part of a mathematics series for adults. (van Groenestijn et al, 1996). This assessment package was based on the same principles as Supermarket Strategy but was mainly designed for written assessments. Nonetheless it still offers possibilities for oral interviews.
At the same time an independent mathematics placement test for adults was published by Cito (Cito, 1996). This package is based on IRT scaling and opens the way to a new assessment system. These three assessment packages are described in this chapter. The last two tests, *In Balance* and *Cito*, are used in the field study of this research with a group of adult learners in ABE. The results of the *In Balance* test are compared to the results of the *Cito* test and are described in the next chapter. The qualitative analysis of the learners' computations and procedures on both tests provides valuable information on adults' numeracy knowledge and skills when they enter ABE. (see chapter 4)

4 Numeracy Skills in Adult Basic Education

4.1 Introduction

To learn more about the numeracy skills adults have acquired at the start of an ABE mathematics course, a group of 37 newcomers at an ABE learning center near Utrecht, the Netherlands, was assessed in December 1997 and January 1998. The learners came from Bosnia, Bulgaria, China, Czech Republic, Eritrea, Ethiopia, Iran, Morocco, Poland, Suriname and Turkey. They were partly refugees, partly immigrants. Their ages varied from 19 to 43. All learners received school education in their home countries with a minimum of four and a maximum of ten years. In the Netherlands they received three months of intensive language training. They all had achieved level 1 of the Cito language test. This was a precondition to start the mathematics course and made it reasonable to assume they all could do a paper-and-pencil test.

For this field study the In Balance placement test (IB) and the Cito placement test (Cito) were used. (See descriptions in chapter 3.). The Cito test was used to be able to compare the IB quantitative results with an independent test. Since the Cito test is based on IRT scaling (Item Response Theory) and was tested on a sample of about 1200 respondents in ABE in 1993 (Straetmans, 1994), it can be assumed that the results from that test are reliable. The results from both tests are analyzed in this chapter. Five learners in this research were removed from these analyses because they took only one of both tests. The remaining group of N=32 was comprised of 11 men and 21 women.

Their numeracy skills are examined alongside the goals for assessment of adults, described in chapter 3. More specifically, the analyses of both tests aim to provide:

1 quantitative information for level placement;
2 qualitative information about competencies and needs of adults in ABE concerning their mathematical knowledge and skills;
3 building blocks for mathematical content for numeracy programs in ABE, fine-tuned to the needs and competencies of ABE learners;
4 information about the learning of adults for improvement of learning and teaching in ABE;
5 In addition, where appropriate specific problems encountered concerning second language learners will be mentioned. Information acquired here could lead to further research into testing second language learners in general and to improvement of the IB test in particular. Problematic items detected here will be evaluated in a separate study.

The quantitative results on both tests, IB and Cito, are presented in section 4.2. (objective 1). In section 4.3 the content of the learners' work is analyzed qualitatively (objectives 2-5). This is done alongside the five main categories of the IB test: number and basic operations, proportions, measurement, money, and data. Preliminary conclusions are drawn at each subsection, related to the purposes of this chapter mentioned above. In section 4.4 we look back at the results and characteristics of the learners, the assessment goals as described in chapter 3 and the criteria for assessment tools, also described in chapter 3, related to the instruments used in this study.

4.2 Quantitative Results

All learners were tested with both tests, IB and Cito. This was done in two phases, according to the standard procedure, as described in chapter 3. There it was explained that the IB placement test is a semi-adaptive test. This means that all learners take all A-subtests in the first phase, starting from level 1. In the second phase they do only two B-subtests, depending on the results at the A-subtests. Learners who achieve level A1 or A2 in the first phase take B1 and B2 in the second phase. Learners who achieve level A3 do only subtests B2 and B3 in the second phase. For subtest B1 they receive 10 bonus points because it is assumed they have mastered level 1. Learners who achieve level A4 in the first phase take only subtests B3 and B4 in the second phase and receive 20 bonus points for subtests B1 and B2. Due to this system all learners (N=32) did the A-subtests. However, the number of learners at the B-subtests is different per level. The actual N at each B-subtest is shown in table 4.1.

Table 4.1 N at the B-subtests of IB placement test

B1	B2	B3	B4
N=7	N=19	N=25	N=13

A similar procedure takes place for the Cito test. This test is also done in two phases. All learners take the first phase test, which is on level 2 of the KSE levels (see chapter 1). In the second phase, learners who achieve less than 7 points in the first phase take the C2-Low test, with items on levels KSE 1 and 2. Learners who achieve 7 points or more in the first phase take the C2-High test, with items on levels KSE 2 and 3. In this study 21 learners did the C2-Low test and 11 learners the C2-High test. An overview of results per learner can be found in appendix 3.

Based on the descriptions of both tests in chapter 3 and how they fit into the Dutch adult education system (see chapter 1), the quantitative results of the IB and Cito placement tests can be compared as in table 4.2. Since level 5 of IB was only published in October 1999, this level was not part of this study. Table 4.2 shows the frequencies per level for these 32 persons.

Table 4.2 Frequencies per level on IB and Cito test

IB test			
IB level	proficiency score	# correct	% correct
1	0-20	2	6%
2	21-40	5	16%
3	41-60	12	38%
4	61-80	13	40%
5	81-100	----	----
		N=32	100%

Cito test			
KSE level	proficiency score	# correct	% correct
1	0-59	8	25%
2	63-105	22	69%
3	108-184	2	6%
		N=32	100%

About a quarter of the learners are still at the elementary level of numeracy (KSE-1). About three-quarters of the group function on the lower end of the foundational level (KSE-2). Two learners achieved the basic level at Cito (KSE-3). To indicate the coherence between the results on IB and Cito, table 4.3 was composed.

Table 4.3: Cito-levels versus IB-levels

C-Level	IB-Level				N
	1	2	3	4	
1	2	5	1		8
2			11	11	22
3				2	2
N	2	5	12	13	N=32

Overall, the IB and Cito level classifications are quite comparable, though, there are a few differences within the levels. For example: two learners achieved level 3 at the Cito test. Their proficiency scores were respectively 108 and 124. Score 108 at Cito is exactly on the low border of Cito level 3. The same two learners did not have very high proficiency scores at the IB level 4 test, only 64 and 66.25 points, which is low in level 4 of IB. A few other persons achieved higher scores at the IB-test, but lower at Cito. The highest score at the IB-test was 71.5 points (out of 80) and this person achieved 83 points at Cito, which is in the middle of level 2 at Cito. (see overview of results in appendix 2.) One person scored only 10 points on Cito by which she achieved proficiency level 59. This is exactly on the high border of Cito level 1. At IB she scored 50.25, which is in the middle of IB level 3.

From tables 4.2 and 4.3 we may learn that:
1) About a quarter of the learners in this study need instruction at the elementary level of numeracy. The other learners have acquired more skills and function at the beginning of the foundational level. Considering that the Dutch adult education system consists of six KSE levels, these learners are very vulnerable. Mastery of level KSE-3 has been seen as a minimum level to participate in work, vocational education and further education.

2) The results of individual learners on both tests indicate that there are qualitative differences between both tests, because some learners who achieve higher at IB perform less high at Cito, and vice versa. This suggests that an evaluation of both tests would be valuable. (This will not be done in this study)

3) Given the similarities between IB and Cito, but taking into account conclusion 2, we may assume that the IB level classification provides a more detailed insight in sub-levels for placement. These levels are in accordance to the level classification of the IB instruction materials. In the Cito placement test 22 out of 32 learners achieved level KSE-2. We may wonder if such a rough classification provides sufficient information for instruction, since it includes nearly three-quarters of the learners in this study. This ratio was also determined in the Cito test. (Straetmans, 1994)

Differences in results between men and women on the IB test are shown in table 4.4. It shows a fairly equal spread of men and women over the four levels for the learners in this study. Women are not disadvantaged in number and in level. Further research with a larger sample is desirable to examine actual differences in math skills between men and women.

Table 4.4 Frequencies per level for men and women in IB-test

IB levels	#Men	#Women	%Men	%Women
level 1	0	2	-	5
level 2	2	3	18	19
level 3	3	9	27	43
level 4	6	7	55	33
level 5				
Total N=32	N=11	N=21		

Ahead of the content analysis it should be said that the IB test includes some yes/no items. These items require insight. The learners' computations to these items and scores over 80% correct on the lower levels show that they did not influence the actual results or perhaps only marginally. The results indicate that there is little chance that these items influence the learners' actual knowledge and skills. However, these items will be evaluated in a separate study.

4.3 Content analyses

In this section the learners' work on the IB placement test will be analyzed in five subsections, quantitatively and qualitatively: number and basic operations (4.3.1), proportions, fractions, decimals and percents (4.3.2), measurement and dimension (4.3.3), money (4.3.4), and reading and understanding data (4.3.5). The content analysis will be done along the following steps:

1 Each subsection starts with a brief introduction of the subject regarding the content of that subject and the type of items to test that subject.
2 After that a brief overview of the quantitative results for that subsection is presented, continuing a comparison to the results of the Cito test.
3 Subsequently, learners' work on a few items per subcategory is analyzed. The main criterion for selection of items and learners' work is that it must contribute to qualitative information about the population. Items that offered no specific information are not discussed. Repetitive information is avoided. We focus mainly on the results of the IB test. If learners' work on the Cito test provides specific information, this is added.
4 Possible language problems and evaluation of items are discussed, when appropriate.
5 Preliminary conclusions about items are *italicized.*
6 Finally, conclusions are drawn per subsection concerning:
  - possible building blocks for improvement of mathematics programs
  - information about learning and teaching

This analysis procedure will be applied in all five subsections.

A first impression of the results in the five categories is presented in table 4.5.

Table 4.5 Mean scores and percent correct on subcategories (rounded)

presented in sub-section	sub-categories	Total # items in IB	IB % correct	Total # items in Cito	Cito % correct
4.3.1	number and basic operations	26	70	10	71
4.3.2	proportions, percents, fractions	11	45	15	27
4.3.3	measurement, including decimals	24	54	5	47
4.3.4	money	16	58	7	44
4.3.5	reading and understanding data	3	54	1	9
	Total # items	80		38	
	Mean score on IB and Cito		56		50

The percentages shown represent the mean percentage correct of all items in each sub-category, for example: there are 26 items in the category "number and basic operations". The percent correct (70%) is the mean of all percentage correct score on each of these 26 items. The percentages are rounded. An overview of all scores per item is added in appendix 4.

A preliminary impression, based on table 4.5, is that the results are poor. The highest percentage correct in the IB test was attained in number and basic operations, 70% correct. This is about the same in the Cito-test. The second highest score is in the money items, 58% correct. In Cito it is 44%. In the subcategories of proportions, measurement and data the mean scores are 45%, 54% and 54% respectively. The results on the Cito test were even lower: 27%, 47% and only 9%. In category five, reading and understanding data, there is only one item in the second phase Cito-High test (C2H-10), so not all learners reached that item. In IB there is one item concerning data at each level, so there is more response (IB3-0). However, the data item at IB level 3 was counted as proportion in this table, because it asks for insight and computation with the benchmark "quarter". Hence table 4.5 shows only 3 items on data in the IB test. More details about the results will be elaborated in the subsections.

The required procedures in the tests are mental math, computations on paper, estimations or combinations of these procedures. The use of a calculator is not allowed in both tests, IB and Cito. The intention is that learners show what they can do on their own, without the help of a calculator. All tasks on both tests require simple or no computations. Only simple computations are required in the test and many items are meant to test insight and don't even ask for computations. Of course, during the numeracy course the learners are allowed to use a calculator and they are also trained in how to use the calculator in a sensible way. Since the learners in ABE are all adults, it would be interesting to see if they incorporate alternative procedures in their computations that they have developed in real-life situations after their school years. In the analyses of these test materials we also focus on these aspects. However, assessment situations are often not ideal situations for doing such observations. Adults may revert to the "school behavior" they were used to years ago. Though for this study and in this test situation the learners were encouraged to do their computations in their own best way, many of them told us afterwards that they had "forgotten" the rules they learned in school and did not "recall" how to do some specific tasks. This could mean that they did not feel free to try to find alternative solutions but tried to recall expected "school math" answers. It could also mean that they could not see a relation between the test items and real life situations. In oral settings, a good interviewer can switch to more informal procedures in such situations. This is more difficult in paper-and-pencil test situations. At the IB test the leader could remind a learner to try to find an alternative way to solve the problem, when a learner told him he had forgotten how to do it, but was not allowed to ask other questions that could lead to a different way of solving the problem.

4.3.1 Number and Basic Operations

Counting, place value, addition, subtraction, multiplication and division are in fact one rich unity. By doing operations with numbers people show number sense and their familiarity or unfamiliarity with numbers. In real-life situations we see many variations and combinations of these skills: algorithms are often combined with mental math, people create their own algorithms and ways of jotting down computations, estimations replace precise calculations, addition and subtraction can replace each other in many situations, and that is also the case for multiplication and division. In previous research it was reported that adults have mastered addition and multiplication procedures better than subtraction and division procedures. (van Groenestijn, 1993). To acquire broad information about the adults' ways of doing basic operations the IB assessment materials offer a rich variety of items:

1. A few specific items for counting which offer possibilities for various counting strategies;
2. Context items in which counting, addition and multiplication can be applied;
3. Two context items for multiplication/division strategies;
4. Three “Get at” items for addition where people have multiple choices out of a few numbers to create a number, i.e. 25, 100 and 1000. There are more correct answers to these items;
5. A few rounding/estimation items asking “Which sum is about the same?”;
6. A few traditional sums that can be solved by doing algorithms but that can also be solved in alternative ways;
7. Two place value items.

This set of items, spread over four levels, offers sufficient possibilities for adults to show their skills and insight into basic operations. Adults who are used to doing computations in a traditional way can apply the procedures they learned in school, including their own algorithms. The items also offer possibilities for alternative strategies. The assumption is that adults always show their best way of problem solving. Specific items will be discussed at the appropriate places. To compare with the results on Cito, table 4.6 shows a more traditional clustering on basic operations in the IB and Cito tests.

Table 4.6 Mean scores on Number and Basic Operations

	Number and Basic Operations	IB # items	IB % correct	Cito # items	Cito % correct
1	Counting	4	81	-	-
2	Addition/Subtraction	13	72	5	90
3	Multiplication/Division	7	66	4	69
4	Place value	2	50	1	13
	Total # items	26		10	
	Mean score		70		71

There are no specific counting items in the Cito test. Yet, on the IB counting items, 19% of the learners failed. Table 4.6 also shows that multiplication and division are more difficult than addition and subtraction. Place value also appears to be a difficult subject. Only half of the learners passed these items on the IB test and on Cito only 13% correct.

4.3.1.1 Specific Counting Items

Counting is an everyday life activity necessary to determine a quantity or a measure. It is almost always related to real objects, e.g. counting tables and chairs in a restaurant for a number of guests, or boxes in a storehouse to determine what is in stock, or measuring the length of a garden to build a fence. Counting is therefore the basis of number sense, insight into quantity, and for doing computations with numbers. (Treffers and de Moor, 1990).

To make counting of quantities easier and faster, objects are often structured in some way, e.g. in piles of boxes. Then counting can be done by addition, multiplication and various counting strategies. This is also the case when counting measures, e.g. if one needs to measure the length of a fence that is being held by posts, equally divided along the fence, one only needs to measure the length of one part between two posts and can estimate the total length by the fence by multiplying with the number of parts between all posts.

For most people counting real objects in an actual situation is not difficult. However, counting objects in a two dimensional representation appears to be difficult for about 20% of the learners. In the IB test, two counting items present photos of real objects, counting eggs in boxes (item A1-1) and counting chairs (A2-1), one item shows piles of sketched bricks (A2-2), and one counting item shows a number line (A3-2). The number line is used here to see if people can also count on a more abstract level, using a mental model, without a visual presentation of real objects. The results of the counting items are shown in table 4.7

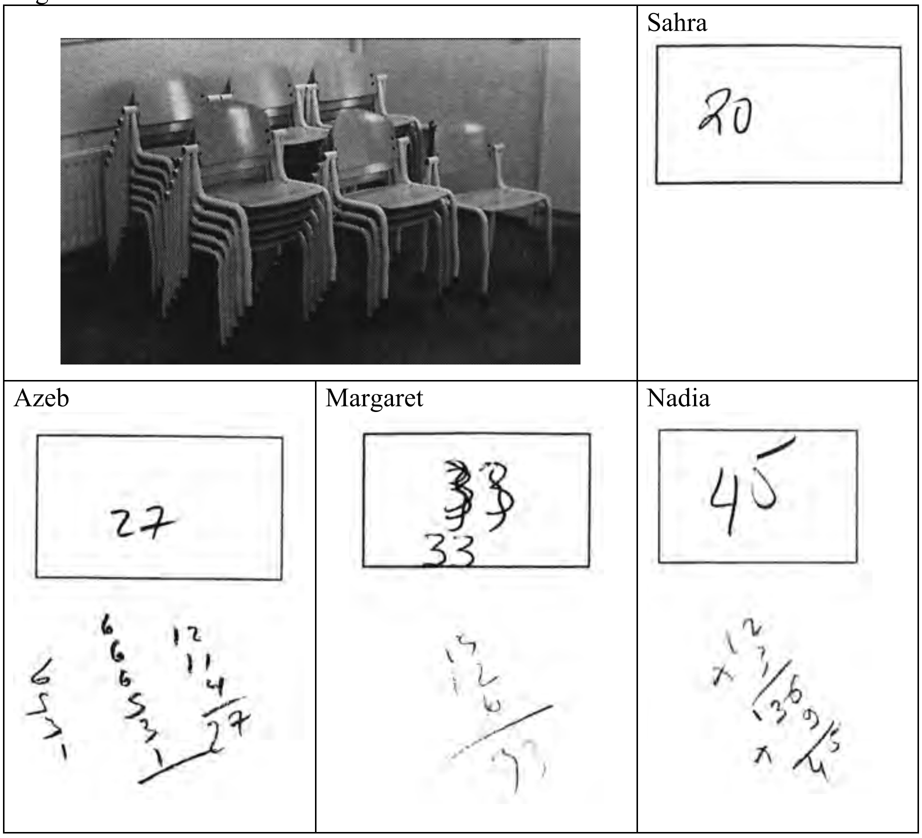
Table 4.7 Results of counting items

Item ID	Name	# correct	% correct
A1-1	Eggs	30	94
A2-1	Chairs	21	66
A2-2	Bricks	25	78
A3-2	Number line	28	88

Item A1-1, counting eggs in a tray, was easy. The eggs are structured in boxes of 10 eggs and in a 6x5 tray. Only Sahra and Sausan counted the total possible number of eggs in the boxes (30 eggs) and in the tray (also 30 eggs) instead of the actual number of eggs shown.

The chairs item was more complex. The chairs are piled and presented in a photo (figure 4.1). The piles at the back are equal in height (3x6), the piles in front differ in height (5+3+1). The correct answer is 27. Not all chairs are visible. The learners need to know how to work with dimension in the photo to count correctly. Eleven learners failed on this item.

Figure 4.1 Item A2-1 - Chairs



Sahra probably did an estimation or just a guess. She wrote only “20”. There is no systematic way to arrive at the number 20, but maybe she could have thought "It's more than 10, probably about 20." The fact that she failed on both, the eggs and the chairs items, and using only round numbers, may indicate that she really has problems with counting objects presented in a photo, and perhaps also with working with instructions on paper. This is at any rate an alert.

Azeb shows a quick way of counting by adding sub-totals.

Margaret’s computation was not quite clear. She did an addition of 15+12+6 = 33. She probably counted the 15 visible chairs in front, added the two piles of 6 in the back and added again the first pile of 6 chairs at the side.

Nadia did 12x3 +9 = 45. The other learners wrote only wrong numbers without indicating counting procedures.

Three learners (Fadumo, Mehmed and Silva) counted a total of 17 chairs. They probably only counted the visible chairs in front and from the side plus the two chairs on top of the two piles in the back. Farangis and Mohamed counted only the visible chairs in front and at the side (15) and forgot to count the chairs at the back.

The bricks in figure 4.2 show a structure of objects on a bit more abstract level than the photo of the chairs in the previous item. The correct answer is 43 bricks. Here we see difficulties similar to the chairs item, though this item appeared to be slightly easier. The structure of the piled bricks presented in the photo appeared to be clearer than the structure of the piled chairs.

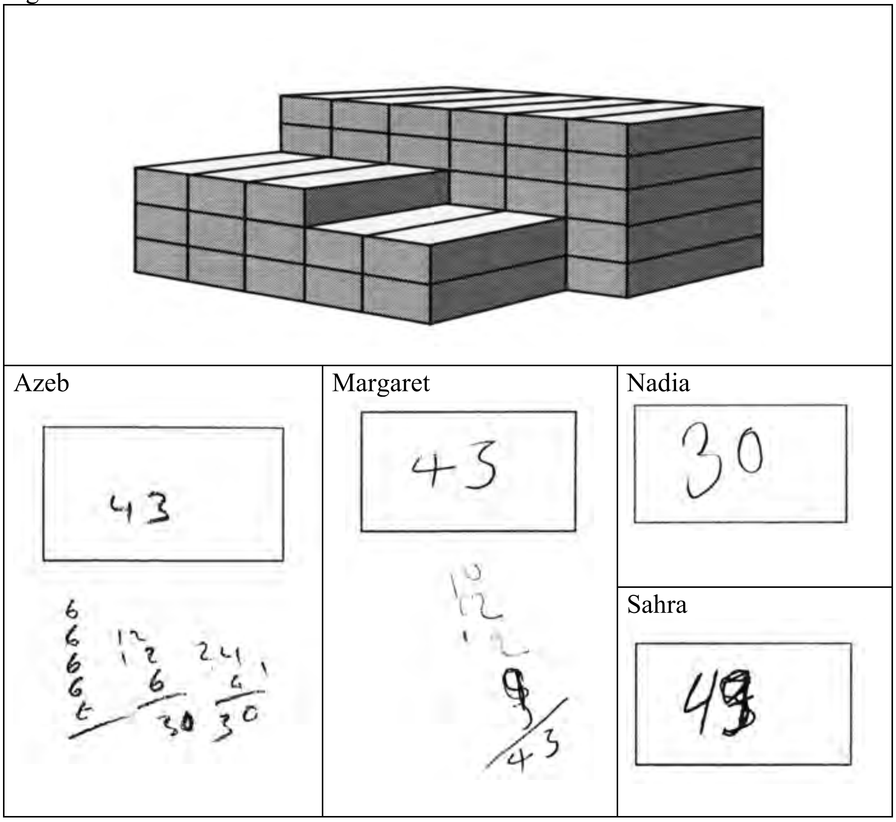
Azeb applies the same way of counting in the bricks item as with the chairs.

Margaret’s computation shows that she counted subtotals and added those at the end. Within these subtotals she combined 6 and 3 to 9. She shows a structured way of counting and adding. Nadia possibly counted only the visible bricks. Sahra must have counted the bricks several times. In her answer it is not clear whether she means 43, 47 or 49.

Fadumo counted 35 bricks and Farangis came to 37 bricks. (not in the figure). Interesting here is to interpret how they may have miscounted. Farangis could probably have counted the visible bricks (total 30) and could have added also the 5 and 2 front sides of the bricks at the top of both piles (total 37). Fadumo could have done the same, but “forgot” to count the two bricks on top of the front pile or miscounted somewhere (total 35).

Fadumo and Farangis may have problems with dimension, identification and location of the chairs and bricks represented in the photo and figure, but the question is how they would have counted the piled chairs and bricks in reality. These counting results may indicate that these persons need instruction using real life things in combination with representations on paper to train them in using paper instruction materials.

Figure 4.2 Item A2-2 - Bricks

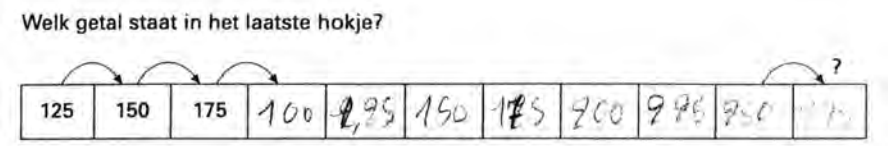


On levels 1 and 2 the learners are asked to count with visual representations of real objects, represented in photos and graphs. (see previous section). On level 3 the counting tasks continue with counting with benchmarks on a number line.

The first purpose of this item is to see if the learner is able to count with numbers in an abstract mental model. Many adults may not be familiar with a number line because of their past traditional school mathematics or their experiences in different school systems in other countries. Also, in everyday life it is rare that adults would use an empty number line. They may be more familiar with a tape measure, for instance. However, because this is a level 3 item, it expects the learner to be able to skip-count with numbers on a more abstract level. The second purpose is to see what kind of counting strategies the learner use. Because of these factors, it was decided to keep the number to count by simple, i.e. 25.

The task at this item is “Complete the line. What number should be in the last box?” The learners are not told to count with the number 25. The learners have to derive the number 25 from the jumps shown in the first three cells. The arrows indicate to count further to the next cell. (see figure 4.3)

Figure 4.3: A3-2- Zaara number line



Welk getal staat in het laatste hokje?

Zaara misunderstood the question and started counting from 100. (figure 4.3) However, she showed that she is able to count with jumps of 25. She just did not understand the number line.
In general this task appeared to be easy: 28 correct answers out of 32. (88% correct). Two persons skipped the whole item; one person miscounted and forgot to count 350.

On the one hand, the fact that most learners completed this task correctly (28 out of 32), indicates that they understand counting on a number line. This is hopeful since the number line is a common instruction tool in math education nowadays. It also indicates that most learners understand this visual instruction and can count with a benchmark number like 25, which may be of help when learning and teaching estimation and doing computations with round numbers. On the other hand, the task itself appears to be very easy and did not give much insight into the learners' counting strategies, because most learners completed all cells correctly. Hence this item needs to be revised.

4.3.1.2 Counting, Addition and Multiplication

In many situations structured objects can be 'counted' by counting, addition or multiplication strategies, or by applying combinations of these three. Table 4.8 shows context items that offer possibilities for all three types of computations. The table shows that even on these simple items some adults have problems. The items ask for simple computations related to three dimensions, real life experience (knowing that crates and boxes are piled on the floor and how to count such piles), and/or simple multiplication.

Table 4.8 Results on simple contexts for counting, addition and multiplication

Item ID	Name	# correct	% correct
A1-9	Crates (3-dimensional counting, addition, multiplication)	26	81
A2-5	Chocolate bars and Pudding cups (counting, addition, multiplication)	25	78
B2-8	Boxes (3-dimensional counting, addition, multiplication (concept volume))	22	69
A3-6	Glasses (multiplication 6x6)	27	84

The crates in item A1-9 offer the possibility to apply counting, addition and multiplication procedures in different ways. Most people only entered the (correct) number as the answer and showed no visible ways of counting or multiplication procedures. This context requires three-dimensional counting that could make it more difficult for low achievers.

Mohamed and Philomene counted 15 crates. They possibly only counted the visible sides. Nadia counted 9 crates. She apparently counted the visible crates on top. Zafiroula shows an addition algorithm.

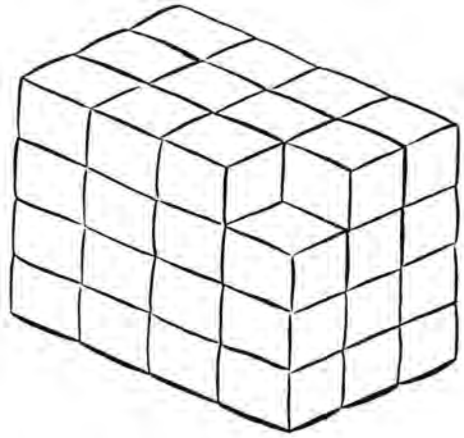
Figure 4.4 - Crates

	Zafiroula 9 6 15 3 18

The boxes item in figure 4.5 shows a variety of counting, addition and multiplication strategies.

Jing obviously counted only the visible front sides of the boxes and counted the row of three twice (total 26). Sonia (not shown in the figure) probably did the same as Jing but counted the row of three boxes only once (total 23). Silva counted possibly only the front side (4x4=16) and subtracted one box (total 15). Enver shows a good addition (16+16+15=47). Mehmed shows a multiplication plus an addition. Azeb started from the total. She shows a multiplication and a subtraction. Nadia got lost in this item. She might have thought “10x10x10 = 1000” and based her computation on that.

Figure 4.5 B2-8 Boxes

<table>
<tr><td rowspan="2"></td><td>Enver
16
16
15
47</td></tr>
<tr><td>Mehmed
12x3
36
+11
47</td></tr>
<tr><td>Jing
4+4+4+3+3+4+4=26</td><td>Silva
4x4 = 16 -1 = 15</td></tr>
<tr><td>Nadia
4x4 =8
4000x4=8000-1-79000
x</td><td>Azeb
12x4 48
48-1 =47</td></tr>
</table>

A preliminary conclusion here may be that this item is a good item to discover people's counting, addition, subtraction and multiplication strategies. This item shows again, in addition to the previous counting and multiplication items, that many learners in this study have problems with working with three-dimensional illustrations on paper. We may wonder how these learners would count a similar pile of boxes in reality. It suggests that these learners may need real materials in learning settings in combination with assignments on paper.

The results on the simple multiplication items, A2-5 (chocolate bars and pudding cups), indicate that many learners have mastered these simple multiplications, though a few learners had problems here. These items will not be discussed here since they do not add new information to the previous items.

Item A3-6 at level 3 is a simple multiplication context that shows 6 boxes of which 4 are closed. On each box is written "6 glasses". The content of the boxes can only be viewed in the two boxes on top. The respondent should recognize that the total number of glasses in the boxes is 6x6 = 36 glasses. The context offers possibilities for people who don't know the times tables, to do the computation in multiple ways. This context presented no difficulties for most of the learners. (27 out of 32 correct)

4.3.1.3 More complex multiplication and division contexts.

The following four items are still simple computation items, only one-step items, but can also be solved using alternative strategies.

Numbers are chosen in such a way that mental procedures can be applied, e.g. the multiplication task at item A4-7 is 19x35. It can be done by first doing 20x35 and then subtract 35. The results for these items are shown in table 4.9.

Table 4.9 Results on multiplication and division items

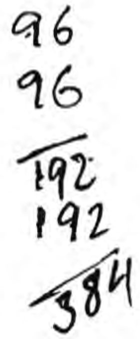
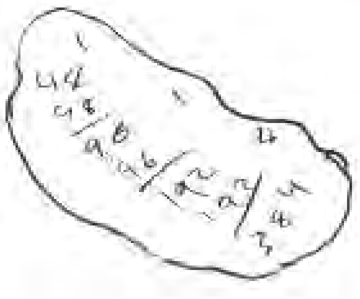
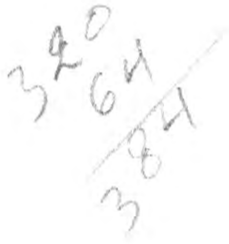
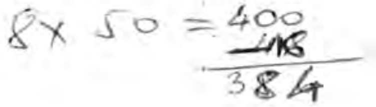
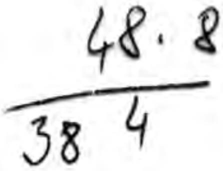
Item ID	Name	# correct	% correct
A3-7	multiplication (times tables)	23	72
A4-5	multiplication (boxes, smart calc., algorithms)	17	53
A4-6	multiplication - division (glasses)	14	44
A4-7	multiplication - division (algorithms)	16	50

Item A4-5 offers many possibilities for doing counting, addition and multiplication strategies in various ways. It shows a pile of boxes with the following question: "*How many rolls of adhesive tape in total?*" (Figure 4.6, p.102)

The text in the context shows boxes with on it the word "plakband", means "adhesive tape" and "48 rollen", means "48 rolls" In fact, the text in the context is not relevant, only the number 48 is important. It could also have been 48 rolls of "cookies" or something else. The word adhesive tape appeared not to be a problem. There was only one person who did not know this word. Showing her an actual roll of adhesive tape, that was present in the class room, was enough to clear this question.

The words “how many rolls” and “in total” are critical in the question. The word “plakband” (“adhesive tape”) can be identified in the picture. If the question would have been “How many rolls of cookies in total?”, with the word “cookies” on the boxes, this would not have changed the difficulty level of language and text and also not the mathematical operation and difficulty level of the item.

Figure 4.6 A4-5 Multiplication

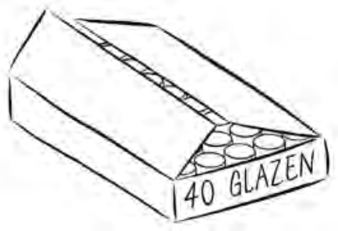
How many tape rolls in total?	Zafiroula
 	
Azeb	Balbir
	
Himzo	Zeki
 	

The computational part in this context did not appear to be a problem for about half of the group: 17 learners out of 32 could solve the problem in the right way.

(see section 4.2.2). This was nice in the range of correct-incorrect answers at level 4. The item offers many possibilities to get at the right answer.
There are a few interesting computations:
Zafiroula and Azeb show a doubling strategy.
Balbir shows his two-step computation based on mental math: 320+64 = 384.
Himzo computed 8x50 mentally and after that he subtracted 16.
Zeki shows a Turkish algorithm.
These learners apply various strategies which can be used in instruction situations.

Many situations that ask for division can be solved by multiplication instead. In everyday life situations multiplication is more frequently used than division. In adult education it often occurs that adults master multiplication better than division. Multiplication can often be learned as a way of smart counting without the necessity to learn times tables or algorithms. Many adults who don't master algorithms can often apply alternative multiplication or counting and addition strategies. As found in previous research, (van Groenestijn, 1993), such alternative strategies are often context-based and context-bound. It means that people may tune their counting or multiplication strategies to the composition of the context, or in other words, their way of counting may be dependent on the structure of items in a context. The following context (A4-06) offers a problem that can be solved by applying multiplication or division procedures. The text says: *A restaurant owner orders 3000 glasses. The glasses are packed in boxes. How many boxes should he buy?*

Figure 4.7 A4-6: How many boxes to buy?

<table>
<tr><td colspan="2">
</td><td>Zeki
3000
40 x
0000
12000
120.00</td></tr>
<tr><td>Mehmed
3000:40=75
2800
=200
200-
000</td><td>Nevzeta
3000: 40 =75
280
200
200</td><td>Abdinasir
3000 |40
280 75
200</td></tr>
</table>

Mehmed, Nevzeta, Abdinasir, and a few other learners show good, though different division algorithms. It is interesting to see the differences between

Mehmed and Nevzeta. Mehmed apparently started with 2800. This could indicate that Mehmed started with the quantity in mind, whereas Nevzeta applied the more traditional procedure and started with 280 - just ciphering. From Zeki it was not clear whether he had misunderstood the question or had not read the question at all. He does not show insight into the context. At this item he applies an algorithm different from the one in the previous task.

About half of the learners had no difficulties with this task (17 correct answers out of 32). One learner, who was deleted from the final group of N=32 because she did only the IB test and not the Cito test, had a language problem. The information on the box shows there are 40 glasses in one box and the picture shows the arrangement in the box (5x8). However, the Dutch plural of “doos” (means “box”) is “dozen” (means “boxes”). Many Moroccan people have problems with the Dutch vowels, so when the word “dozen” is pronounced in a bit wrong way, it may sound like “duizend”, means “thousand.” The learner got very confused by this and thought the glasses were packed per thousand. She thought the answer should be 3.

4.3.1.4 “Get at” items for addition.

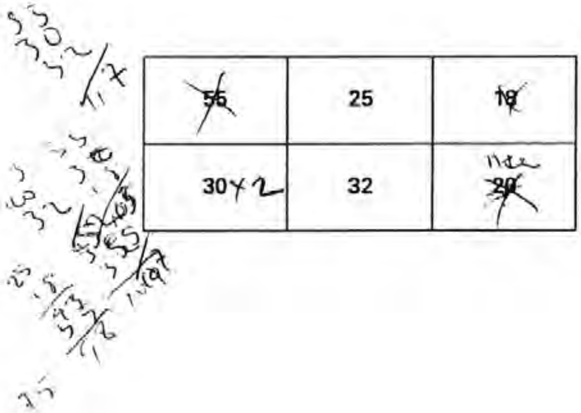
For addition three “get at” or “get the number” items were created at three levels (A1-3, A2-3, A3-3). This was done because in real life situations people have to create their own sums and computations rather than “doing sums” as they are presented in school. These items create the opportunity for adults to show their familiarity with doing operations with round numbers. The numbers are 25, 100 and 1000 respectively. These tasks can be done mentally. There are many possible correct answers for each task. People should be able to do this mentally by recognizing the numbers and their properties, e.g. know that 12+8=20, or that 3+7 sums up to 10, and 60+40 is 100. Table 4.10 shows that, though the results are good, even at the very low levels 1 and 2, a few learners were not able to do these tasks. On the other hand, many learners could do it.

Table 4.10 “Get at” items

Item ID	Name	# correct	% correct
A1-3	create your own sum (get at 25)	30	94
A2-3	create your own sum (get at 100)	29	91
A3-3	create your own sum (get at 1000)	25	78

Most learners had no problems with the first two items, though a few people still missed them. In the second item, for example, Margaret apparently understood the task but could not find the right numbers. She shows in her additions that she is not familiar with characteristics of numbers. She tries to find the answer by trial and error.

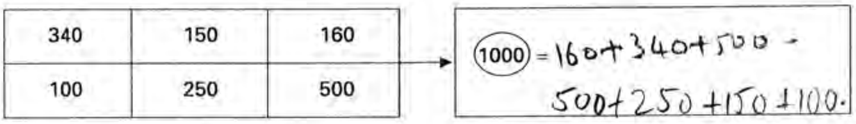
Figure 4.8 A2-3 - "Get at 100" - Margaret



The question at level 3 is: *Which numbers can be used to get at a total of 1000? Check those numbers. Write down the sum.*

Nadia shows her full understanding of this task by showing two solutions, though she did not cross the right numbers in the left table.

Figure 4.9 - "Get at 1000" - Nadia

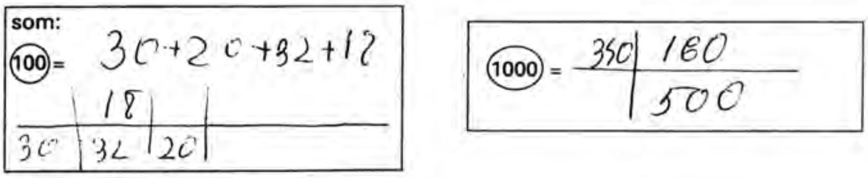


340	150	160
100	250	500

1000 = 160+340+500 -
500+250+150+100.

Enver found his own notation system and applied that at level 3 and level 4. He apparently looked at the characteristics of the numbers.

Figure 4.10 - Get at - Enver

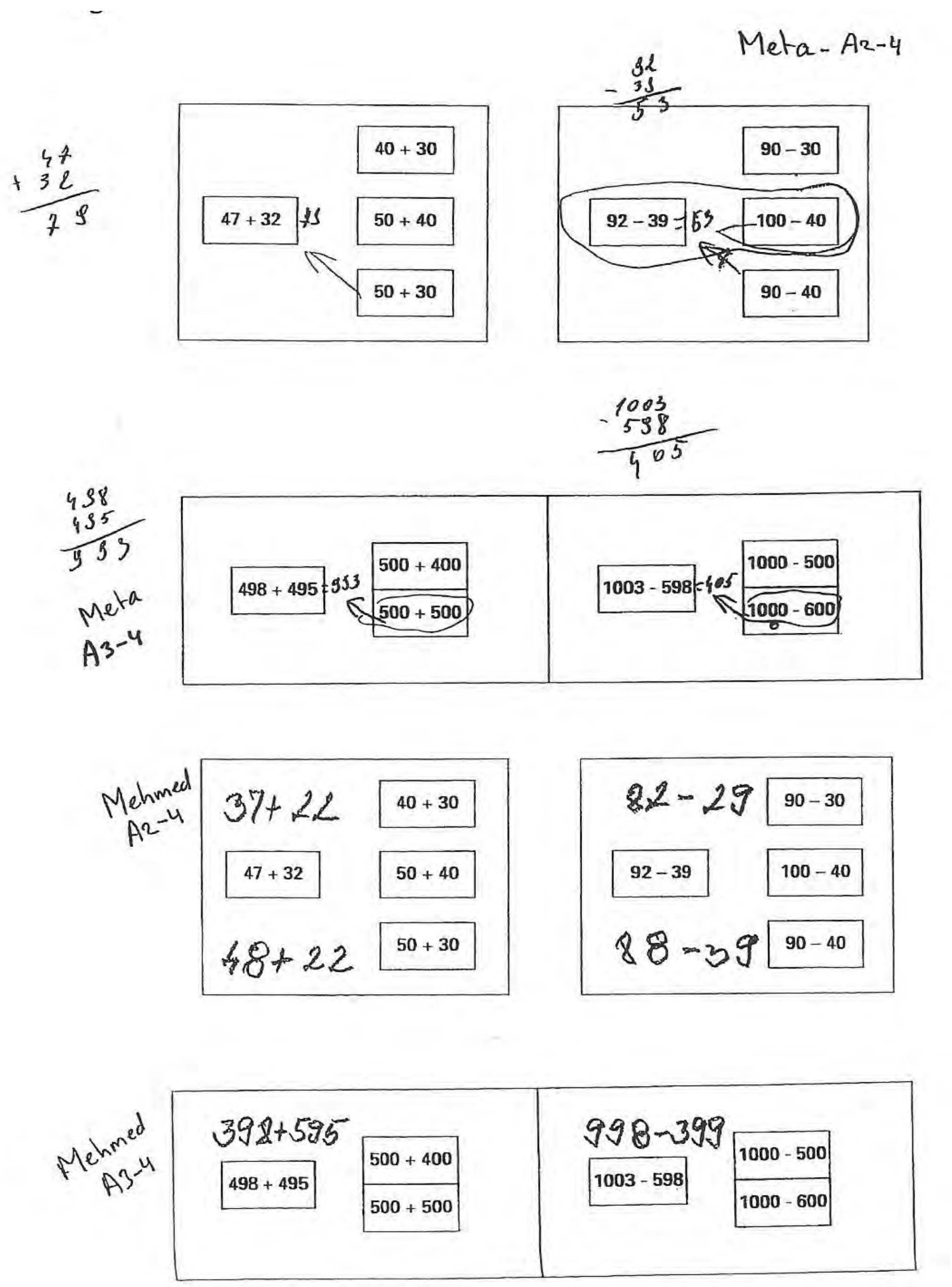


som:
100 = 30+20+32+18
18
30 32 20

1000 = 350 160
500

The results from these items are hopeful, especially because they show that most people are able to work with "nice" round numbers and they can make combinations of those numbers to get at a certain total. Doing computations with round numbers is an important means for learning estimation and mental math. On the other hand it is disconcerting that some people continue to use algorithms, even with very simple numbers. Perhaps "get at" problems can be part of activities that help to develop more creativity at working with numbers.

Figure 4.11 - Estimation - Which sum is about the same? - level 2 and 3



4.3.1.5 Rounding and Estimation

To learn more about rounding and estimation three items are inserted asking "*Which sum is about the same?*" (A2-4, A3-4 and A4-2)

Results on these tasks show the following:

Table 4.11 Rounding and estimation

Item ID	Name	# correct	% correct
A2-4	Which sum is about the same?	18	56
A3-4	Which sum is about the same?	22	69
A4-2	Which sum is about the same?	18	56

These items are not developed to do computations. It would be sufficient to compare the numbers in the sums, round to the nearest ten, and then make a choice which sums are about the same in the second column. However, many learners keep doing computations, maybe because of uncertainty, maybe because they are not used to doing estimations. A few examples are shown in the figures 4.11, 4.12 and 4.13. The descriptions are below.

Meta shows in A2-4 a good understanding but probably doubted in the second task and made the wrong choice. He did not choose the nearest round number but the higher round number.
At level 3 Meta shows a good understanding but he first applies an algorithm (addition and subtraction), while in fact, this was not necessary. After that he chooses the right answers.

Mehmed shows in A2-4 he understands the task, but not correctly. He creates his own new sums in the empty space next to the two sums on the right. This was not the intention of the task, but it shows his understanding of the meaning of the task. He applies the same procedure at A3-4

Figure 4.12 - Estimation - Which sum is about the same? - level 3

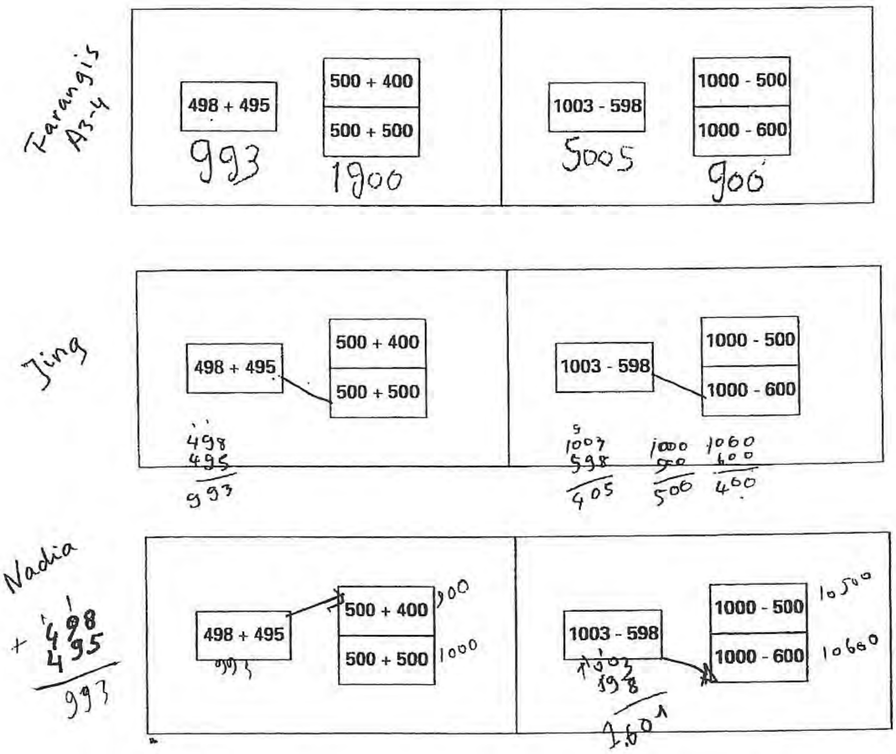
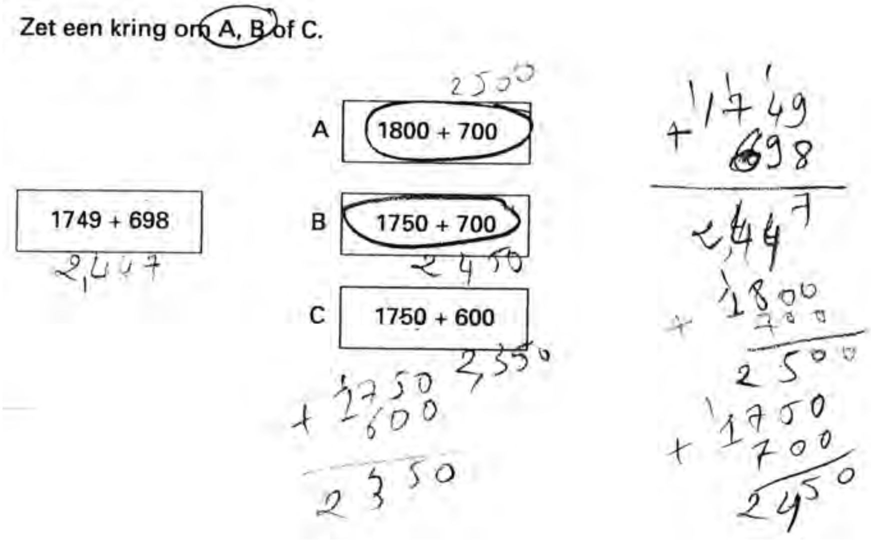


Figure 4.13 - Estimation - Which sum is about the same? - Nadia, level 4



At A3-4 (see figure 4.12) Farangis shows she does not understand the question. She just adds the numbers of the left and the right sums as separate sums. At the subtraction task she shows she has difficulties with zeros in a number. At the right side she apparently applied a subtraction at the two sums and after that she added the both answers (500+ 400=900).
Jing shows she understands the tasks but applied addition and subtraction algorithms and chose the correct answer.

Nadia can do the addition algorithm correctly, but then she chooses the wrong one from the right side sums. She apparently meant to choose the 500+400 sum, but we may wonder why she chooses 900 and not 1000. Perhaps because 900 sounds a bit the same as 993 and 1000 sounds much more? At the second task she shows she has problems with subtraction. She added 1003 and 598, instead of doing a subtraction. At the right side she applied an addition instead of subtraction and could have had problems with writing the large numbers. After that she connected the left sum with the one at the right below. This is the correct link, but in fact she probably meant to compare 1601 and 10600. At level 4 we see Nadia doing her addition computations and after that she makes her choice. (figure 4.13)

These items were meant to see whether the learners have a feeling for doing estimations. In fact it was not necessary to do any computations.
However, in particular in the examples of items at levels A3 and A4, we see that learners feel unsure and do computations on paper. This could be because they are not familiar with this type of tasks, but it may also be caused by their own feelings of uncertainty when estimating the answers, or the fact that they are used to always doing any computations on paper. Many learners, in particular those coming from non-western countries, seem not to be used to estimating. Mathematics asks for precise computations and that is what these learners want to show in this test.

4.3.1.6 Algorithms

Algorithms represent another type of tasks included in the assessment. Items A3-5, A4-3 and A4-4 offer the possibility to use the addition and subtraction algorithms, but they can also be solved in alternative ways. Most learners, however, did apply the traditional algorithms and didn't show alternatives. The third item, A4-4, intended to see if people would apply smart, alternative ways of computing. If one knows the structure of the numbers and can apply "smart computation", both answers in the tasks in that item can be found mentally. However, most of the learners applied an addition and subtraction algorithm.
For multiplication two times-tables problems with only numbers were added and two algorithms, one for multiplication and one for division. Compared to the context items, the results on algorithms were poor, as shown in table 4.12.

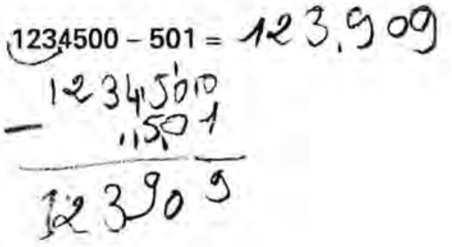
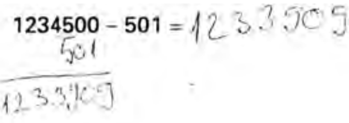
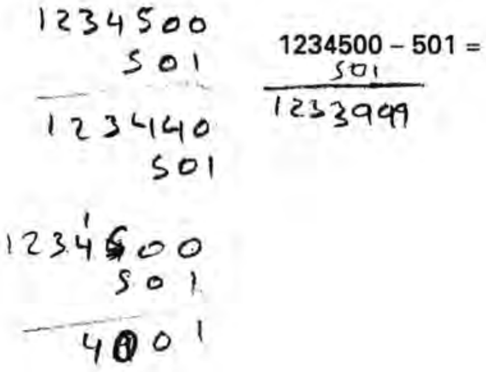
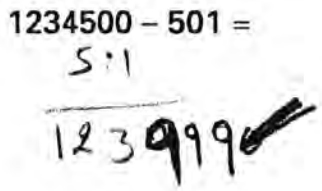
Table 4.12 Algorithms

Item D	Name	# correct	% correct
A3-5	Algorithms addition /subtraction to 1000	16	50
A4-3	Algorithms addition /subtraction over 1000	17	53
A4-4	(Algorithms) addition /subtraction over 1000 (smart computation)	12	38
A3-7	multiplication (times tables)	23	72
A4-7	multiplication - division (algorithms)	16	50

Most of the learners had no problems with the addition algorithm. However, in subtraction we see that many struggle with the zero in the number. A few examples are given of the subtraction algorithm in item A4-4 in figure 4.14.
Nadia applies a Turkish algorithm. Nadia, Nevzeta and Balbir have problems with the zeros in the subtraction algorithm. Azeb shows her tries but comes to a correct answer.

A preliminary conclusion here is that when these learners see an algorithm they automatically apply an algorithm. It is possible that they have not learned to look for alternative ways to compute. It is even remarkable that they created there own algorithms for item A4-4 and didn't try to find other ways for doing these sums. The fact that they can use algorithms is hopeful, but it also shows that they are limited in their computation skills. They may need to learn more creative ways of doing computations to become more flexible. Perhaps contexts in combination with estimation and the use of a calculator would help to develop such skills.

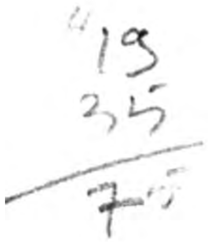
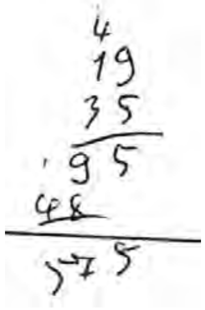
Figure 4.14 A4-4 - Subtraction

Nadia  	Nevzeta 
Azeb  	Balbir 

Multiplication and division algorithms

At level 4 two tasks are given which can be computed in several ways (item A4-7). They can be done by applying an algorithm but can also be done in various smart ways. However, almost all learners who achieved this level applied algorithms. They showed few problems. Only Sonia and Jing showed that they haven't mastered the multiplication algorithm. The multiplication task is 19x35.

Figure 4.15 item A4-7 Multiplication

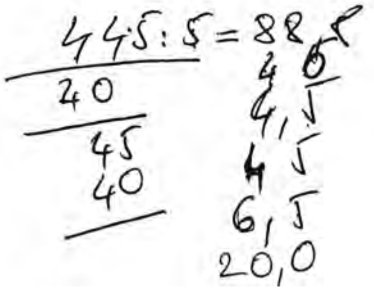
Sonia	Jing
	

Sonia applied an incorrect multiplication algorithm. She first did 5x9 = 45; write down 5, carry 4; after that she apparently did 3x1 plus 4 = 7. That may clarify her answer “75”

Jing also applied an incorrect multiplication algorithm. The first line went fine. In the second line she possibly first added 3, 1 and 4 (makes 8) and after that she possibly added again 3 and 1 (makes 4). Another clarification could be that she did 3x9 = 18, instead of 27, and carried the 1 to 3x1.

Nevzeta shows she had problems at the long division. The task is 445÷5.

Figure 4.16 A4-7 - Long division - Nevzeta



Though these two items offer possibilities for alternative strategies, most learners applied traditional algorithms. However, it is interesting to see all these various types of algorithms. It shows that algorithms are indeed culture-based and may vary from country to country. It indicates that teaching only one standard algorithm in a multinational and multicultural setting is not prudent. Teachers should be aware of these cultural varieties and accept all different algorithms. When learners do not master their own countries' algorithms we may wonder what type of algorithm or alternative procedures would be sensible to teach.

4.3.1.7 Place value

In ABE we often meet learners who learned the basic algorithms but can sometimes make incomprehensible mistakes, as we saw with Sonia and Jing. They get at answers that don't fit the numbers they work with and they don't know what happened during their computations. To be able to check the outcomes of their computations for correctness, they should keep the quantity of numbers in mind and should be able to reason whether an answer fits in the context or not. Keeping the quantity in mind is also the basis for doing mental operations and estimations. (Treffers & de Moor, 1990). Therefore, insight into the structure of numbers and in the place value of digits in a number is a necessity. Previous research shows that adults may even have problems with writing numbers, e.g. a semi-literate woman wrote 10050 but meant the number 150 (van Groenestijn, 1993). However, to find a good item to test the concept of "place value" in a placement test is not easy, in particular for testing second language learners. In IB a codification of the concept in combination with the question "What is it worth?" was chosen. An example was included as visual instruction for the second language learner. This experiment is described below. Here the Cito place value item has been added because of some interesting findings with that item. The results on the subject "place value" show a mean score of 50% correct on IB and 12.5% on Cito, which is low. Table 4.13 shows the results on IB and Cito specified per item:

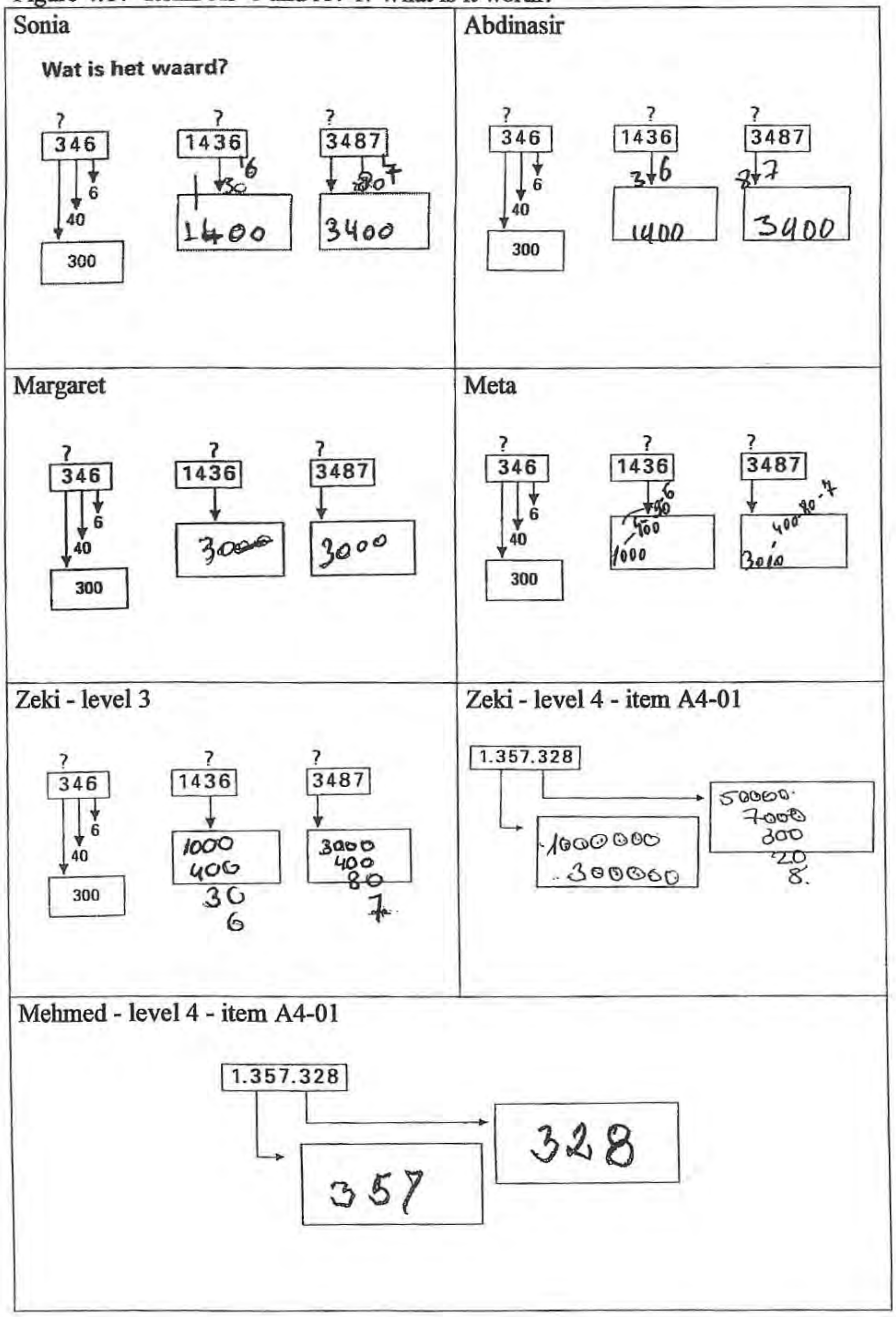
Table 4.13 Place value

Item ID	Name	# correct	% correct
A3-1	place value	22	69
A4-1	place value	10	31
C1-2	place value	4	12.5

Only Zebiba had a correct answer on all three items.

There were three other persons who scored correct on Cito, but on IB they scored correct or half correct only on level 3 and failed on level 4. Eight persons passed the IB level 3 and level 4 items but failed on the Cito item.

Figure 4.17 Items A3-1 and A4-1: What is it worth?



Sonia

Abdinasir

Margaret

Meta

Zeki - level 3

Zeki - level 4 - item A4-01

Mehmed - level 4 - item A4-01

The first codification is an instructional example. It shows a question mark above the digit 3 of 300, an explanation about the place value of the digits 3, 4 and 6 in the number 346 and the right answer in the box below the number as an example. The learner is supposed to fill in the other two empty boxes for the same digit 3. In these tasks the question mark above and the arrow below the 3's indicate that. This extra visual instruction should support the item question. The assumption is that learners who know about place value should understand this instruction. The results show that this item was indeed no problem for many learners: 22 correct answers out of 32. A few examples are discussed here. (See figure 4.17)

Sonia shows she has understood the explanation and applied the same elaboration to the second and third graph, using fourteen hundred and thirty-four hundred, which is common language use in the Netherlands but not the formal system. However, she had not quite understood the function of the question mark in combination with the arrow. She created her own new task in the second and third graph. In both tasks she created three lines as is in the first graph. In the second graph she drew a line from the first number(s) to the box and filled in 1400. The second and third line/arrow were used for the tens and units. In the third graph she applied the same structure. In her elaboration she shows her understanding of place value and partly understanding of the visual instruction, except that the question marks and the arrows did not lead her to the correct answer. Alas, she did not achieve level 4, so we could not compare what she would have done there at item A4-01. Abdinasir show the same idea but it is not clear if he understood the structure of the tens and the units.

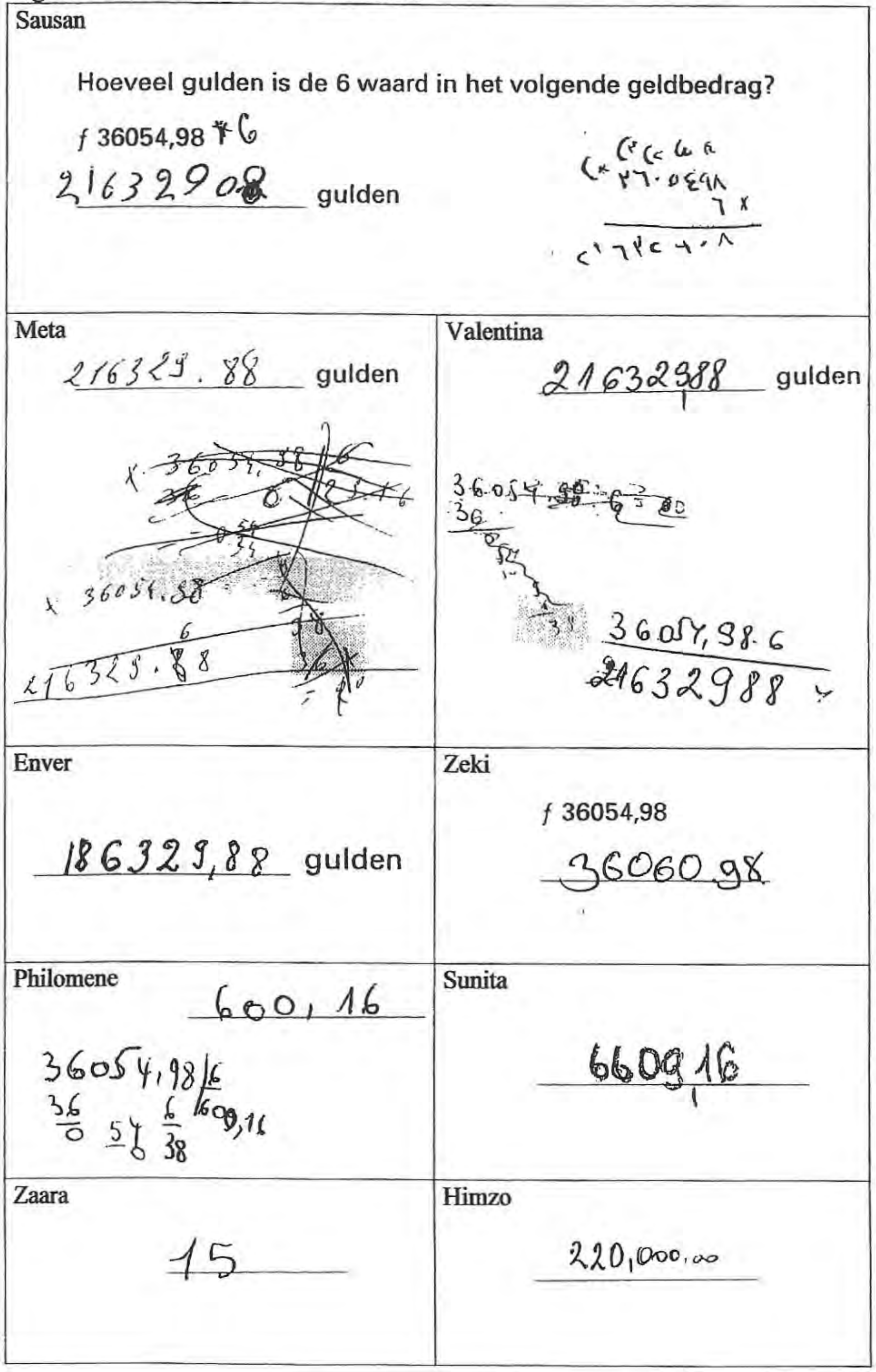
Margaret shows she fully understands the visual instruction. Meta and Zeki also understand the visual instruction but apparently not the question itself. They do not select the right numbers.

This item shows a variety of responses. Some learners thought in each box the first number should be mentioned, others made a combination of the first two numbers. They probably did not understand the question mark. However these elaborations show that these people have some understanding of "place value".

At level 4 a similar item is presented, A4-01, but the number is much larger 1, 357,328. The question is the same: *What is the 3 worth?*

Here is no example anymore, but the same digit 3 is used. The graph should remind the learner of the previous task at the lower level 3, though the format was a bit changed by the publisher due to space on the page. The number is more complex and, to be able to write the larger numbers, the boxes had to be bigger. Vertically this required more space than horizontally. The format of this item could be a bit distracting. This task asks for transfer of information. An extra problem here is that in this large number periods are inserted to structure the number. In other countries this could be commas. Surprisingly only two persons had questions about this, but it apparently caused problems for Mehmed.

Figure 4.18 Cito item C1-2 - Place value



Sausan

Hoeveel gulden is de 6 waard in het volgende geldbedrag?

ƒ 36054,98 × 6

2163290 gulden

٣٦٠٥٤٩٨

٦ x

٢١٦٣٢٩٠٨

Meta

216329.88 gulden

x 36054.98

6

216329.88

Valentina

21632988 gulden

36054,98.6

21632988

Enver

186329,88 gulden

Zeki

ƒ 36054,98

36060 98

Philomene

600,16

36054,98 | 6

6009,16

Sunita

6609 16

Zaara

15

Himzo

220,000.00

Analysis of this task in comparison with the previous one showed that learners who really knew about place value and finally ended up on level 4 of the test, had no problems with this item. In total, 10 learners answered correctly, out of 13 who achieved level 4.

The Cito place value item caused many more troubles. (C1-2; see figure 4.18)
The task says:

How many guilders is the 6 worth in the following amount of money?
f 36054,98
[in US notation it would be: *f* 3,6054.98]

Only 4 persons passed this item. All other learners did some computation with the numbers in the item like **multiplication or division**.
Sausan was one of the persons who did not understand that question. She applied a multiplication. She did the full amount times 6 and came to the answer 216,329.88. She first did her computation in Iranian writing and translated her final answer to our numbers.
Meta did both and chose the **multiplication**. He also got 216329.88
Valentina did the same. She did computations with multiplication and division algorithms and finally chose the answer of the multiplication.
Enver applied an incorrect multiplication (186,329.88). He forgot to add 3 to 18 at the beginning of the number, the end of his computation.
Zeki **added** the 6 to the amount of 36054,98 and came to the total of 36060,98
Philomene applied an incorrect **division.** Sunita applied a correct division: 6009,16
Other incorrect and unclear answers:
Zaara answered just 15. Himzo's answer is 220,000.00

Preliminary conclusion at this item: this task caused confusion for the learners. It could be that the learners interpreted this item as a word problem: there is some text, there are two numbers in it and something should be done with those numbers. Since it pertains a very large and a small number a multiplication or a division is a logic step. Another explanation can be that they simply did not understand the question (a language problem).

A few more general conclusions concerning place value are:
- *When an item is posed as a word problem we don't know whether the learners don't understand the concept of "'place value" or that they have difficulties to understand text based instruction.*
- *Visually based instruction may help to understand the question, but only to a certain level.*
- *It may be difficult for learners to analyze large numbers. (Cito and IB level 4)*

- *Learners who are familiar with numbers don't have problems with place value but may still have problems with the way the question is posed in the test.*

Unclear in this test situation is whether these non-native adults had problems with the way the questions are posed or with the concept of "place value", or with analyzing complex numbers. On the easy numbers at IB level 3, many learners did understand the question and came to a correct answer. At level 4 and at the Cito item, the instruction could be confusing, but also the complexity of the numbers could cause problems. This subject requires further investigation.

4.3.1.8 Preliminary conclusions at number and basic operations.

In this subsection we will summarize the preliminary conclusions concerning number and basic operations. Subsequently, these will result in building blocks and suggestions for a mathematics curriculum. The building blocks are marked with a number, e.g. *B1, B2,* etc. This system will be continued in all the remaining conclusion sections in this chapter. Results and characteristics of the learners will be summarized in section 4.4.1 (p. 170-173). Often some advice or a note for discussion has been added to the building blocks.
In general it can be said at this subsection that, though the results on "basic operations" are poor, people show skills that can be starting points for learning and teaching in adult education.

Conclusion 1
It is clear from the test results that most learners can count, but mis-perception of dimension in counting contexts in test and instructional materials on paper may cause problems for some people who are not used to working with paper-based instructional materials.

B1
It is preferable to use real objects in combination with representations of such counting materials on paper in learning settings so that adults can get used to presentations of three-dimensional objects in photos, drawings and schematic drawings.

Conclusion 2
Counting with benchmark numbers, such as 25, appears to be easy.

B2
The fact that these learners can count with benchmarks is hopeful because this can be a start for learning to estimate and to do mental calculations with round numbers.

Conclusion 3
The use of a number line appeared not to be a problem in this placement test, even for low level adults who possibly are not used to working with number line, due to their school history.

B3
The number line can be used to improve counting and doing simple addition and subtraction computations on a more abstract level. This can be built up from a very low level.

Conclusion 4
Most learners can do simple additions mentally. On the other hand, subtraction skills appear to lag behind, in particular with mental math.

B4
Addition is a starting point for learning the relation between addition and subtraction.

For discussion:
In practice we may wonder whether it is necessary to teach "subtraction" as a separate skill in adult education, or that it can be compensated by alternate addition strategies. This could be examined by offering contexts in which learners can apply both, e.g. show a person's weight on a pair of scales and mention his previous weight. Ask how many kilos this person gained or lost. Discussing different ways of solving such problems may help to develop insights into the connection between addition and subtraction and may improve skills on both. Eventually adults decide themselves what they prefer in such situations.

Conclusion 5
On items that ask for constructive reasoning and computations, like "get at ..." problems, learners show that they understand these kinds of problems and can do it quite well.

B5
Tasks that offer possibilities to apply the learner's own knowledge in a constructive way are for further development of working flexibly with round numbers and combinations of numbers.

For discussion:
Having learners construct their own sums, like "get at ..." problems, may help them to become more familiar with and flexible with numbers and may improve mental math and estimation procedures and skills. In particular, working with benchmark numbers may support this.

Conclusion 6

The results show that these adults have difficulties with estimation. When estimation is required, or for tasks involving rounding, they tend to first do exact computations on paper and after that they round up or down to the nearest ten or five.

B6

The fact that these adults can round up and down is a key link in learning to estimate.

"Which sum is about the same" is the type of task that may lead to creating key points for estimation without doing computations on paper.

For discussion:

Such sums could be preceded by rounding easy numbers to the nearest ten or five, to learn to create round numbers.

There is a difference between rounding an amount for payment in shopping situations and rounding for estimation.

Conclusion 7

In this study many learners show they have mastered algorithms. The problem is that they have not mastered alternative strategies that could make their computation skills more flexible. On the one hand the fact these learners can do algorithms is hopeful, but on the other hand it limits their computation skills when they don't actually understand the algorithms and don't know alternative strategies. (see conclusion 9)

B7

It is good to see that many of these learners can apply algorithms. An important side aspect is that these algorithms may vary because learners come from different countries. This creates the possibility to discuss their procedures and to develop more insights through which they may be able to develop more alternative and creative procedures.

Conclusion 8

On context items, like the boxes in A4-5 and A4-6, some learners show they have mastered traditional algorithms but others show that they can apply alternative procedures.

B8

Contexts compiled of boxes with all kinds of content offer good possibilities for learners to show the basic operation procedures and skills they have mastered and to develop and discuss more alternative procedures.

Conclusion 9
Place value is a difficult issue. Learners show their understanding of easy numbers less than 1000, but have problems when numbers become larger. The structure of larger numbers may confuse them. Visual instruction is helpful but only to a certain extent.

B9
Easy numbers may be building blocks for learning to analyze larger numbers, related to the positional system and the pronunciation of numbers. Knowing the place value of digits in numbers is essential for mental math and estimation.

4.3.2 Proportions (proportions, fractions, percent)

Proportions are a basic concept that occur in many everyday life, work and societal activities. People are familiar with all kinds of proportional thinking or reasoning like, for instance, figuring discounts, changing recipes for more or less people than the original recipe prescribes, preparing solutions for drinks or for cleaning jobs, preparing a mix of cement, sand, gravel and water for concrete in a certain ratio, preparing drugs, understanding statements in newspaper articles and news bulletins, e.g. three quarters of the Dutch population stopped smoking, 75% of the Dutch population go on vacation at least once a year, three out of four adults have a driving license, all prices are ¼ lower, etc. In general, it is assumed that adults understand such headlines in papers, news bulletins, commercials and in advertisement flyers. It is also assumed that adults know about the relation between proportions, fractions, percent and decimals and can handle various presentations of them. However, when we look at the results of the IB and Cito test, we may come to a different conclusion, but we should question whether people really have little or no knowledge and understanding of these concepts or that they cannot bridge the gap between school tasks and real life problems. For this it is interesting to know what people in this research actually know about proportions, fractions and percent and what kind of computations they can do.

Streefland (1988) emphasized proportions to be a starting point for learning and teaching fractions, based on realistic contexts. Based on his study the method of teaching fractions in RME in the Netherlands was radically changed in elementary school. Proportions are seen as the start and the core of teaching fractions, decimals and percent, based on real-life division situations. Accepting his theory, proportions are a central theme in the IB series and his pizza-problem is used as one of the proportion items in the IB placement test. By testing proportions one can also acquire insights into the learners' understanding of the basic concepts of fractions, decimals and percent. Items in the IB placement test are limited to insights into the meaning of these concepts, insights into the relationship between these four topics, operations with decimals and to main operations with fractions and percent. The idea behind this is that for everyday life situations it is sufficient for adults to have insight into these concepts and the relations between them, and to be able to do simple computations based on their understandings. For adult basic education it is not necessary to teach complex operations, in particular not with fractions, unless it is in courses that give access to further and professional education where this could be necessary. For work purposes and personal life simple operations based on benchmarks should be sufficient. For complex computations they could use a calculator or even, in appropriate situations, a computer.

Though decimals are also part of this field, these are tested mainly in the fields of money and measurement, since they form the core of the metric system. There are a few items that test the relationship between fractions and decimals in relation to the learners' understanding of measurement units. These items will be discussed in section 4.3.2.4 . Table 4.14 shows the results in this field.

Table 4.14 Mean scores on Proportions in IB and Cito placement test

	Proportions	IB # items	IB % correct	Cito # items	Cito % correct
1	Proportions	5	75	4	24
2	Fractions	2	31	4	25
3	Percent	4	15	4	19
4	Decimals	-		4	35
	Total # items	11		16	
	mean % correct		43		26

For simple graphs and tasks in the IB test that show proportions, the mean score is 75% correct, however, at the Cito test this is only 24%. In general, the items at the Cito test were more difficult.

In the fields of fractions and percent the mean scores are alarming. Only a quarter to 30% of the respondents were able to answer correctly on the fractions items and on the percent items it was even less. Tasks about decimals in the Cito test appear to be slightly easier, but still only less than half of the respondents were able to answer correctly on these items. This section is divided in four subsections: proportions, fractions, percent and a section about the link with decimals.

4.3.2.1 Proportions

In this section we will first discuss four IB items that are closely related to one another. These items were structured so that learners could show insight into proportions without using fractions, decimals or percent. In practice, however, the lower level items provide little additional information, because they don't require computations on paper and ask only for a correct- incorrect answer. The first three items appear to be real level 1 items (score of 88% correct and up). Table 4.15 shows the results on these items.

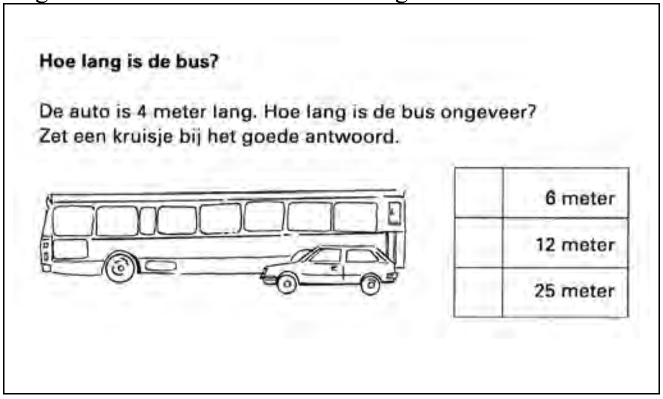
Table 4.15: Results on proportion items

Level	Item	Topic proportions	# correct	% correct
1	B1-3	coffee	30	94
	B1-4 (*)	What is the best buy (to 10 gld)	29	91
	B1-5 (**)	bus-car - (estimation-measurement)	28	88
2	B2-4 (*)	What is the best buy (100 gld)	22	69
4	B4-10 (*)	exchange rate, multiplication	10	31

The coffee item includes the basic concept of proportions. The illustration shows two Dutch coffee machines of the type frequently used in Dutch households, schools and offices. The machines show the same amount of water in both but each has a different amount of coffee. The question here is: which coffee will be stronger (tastier): A or B? Though this is a 50% chance item (correct-incorrect), the results were clear. Most learners had no problems with this item. They understand that more coffee with the same amount of water will result in stronger coffee. The two persons who did not score on this item just skipped the item. In an oral interview afterwards, one of these persons said she never drinks coffee, only tea (see chapter 3). Though this concerned only one person in this study, it could mean that incorrect scores on this item could also be due to unfamiliarity with such coffee machines or cultural differences, e.g. people who don't drink coffee don't know how such machines work.

In the Bus-Car item (B1-5, figure 4.19) people are asked to estimate the length of the bus in relation to the size of the car. It is not necessary to do a computation. The given sizes in the multiple choice answers were chosen in such a way that people could reason why two of them are not appropriate. The results show that most of the adults did understand the problem (88% correct). *This item could be recognizable from their own real life situations.*

Figure 4.19 - B1-5 - What is the length of the bus? Cross the right answer.



Hoe lang is de bus?

De auto is 4 meter lang. Hoe lang is de bus ongeveer?
Zet een kruisje bij het goede antwoord.

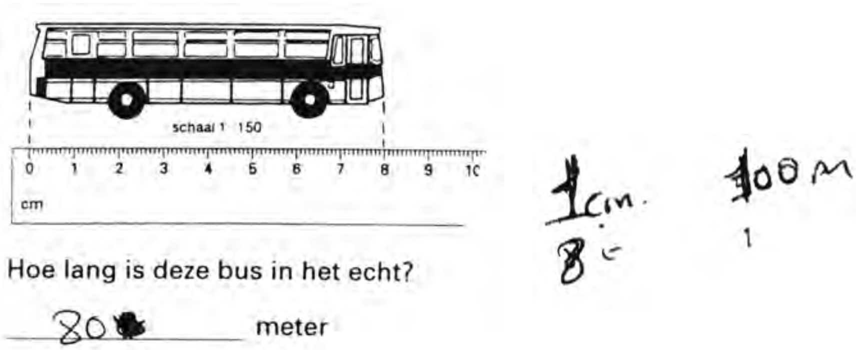
	6 meter
	12 meter
	25 meter

From the four learners who failed two said that the bus was 6 meters long and two mentioned 25 meters. There were no visible computations at this item. *Perhaps it would have been better to ask an open question instead of a multiple choice question, like in the following Cito item.*

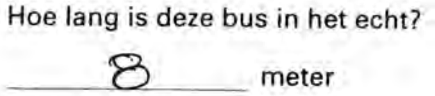
The Cito test also has a bus problem, but this item appeared only in the second phase High-test. It presents a scale (1:150) and a ruler to indicate the length of the bus (8 cm). (figure 4.20)

Figure 4.20 Cito 2H-6 Bus

Zebiba



Mehmed

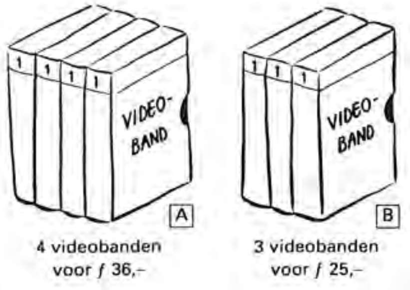


Zebiba shows that she knows about ‘scale’. She assumes that 1 cm = 100 m and computes that 8 cm is 800 meters. However, she understands that this would be too long for a bus and crossed out one zero. Mehmed assumed that 1 cm equals 1 meter and answered 8 meters. Only 3 learners out of 12 who took the Cito High test did this item correctly.

The IB test includes two best-buy items (B1-4 and B2-4). Because they offer only two answer choices both items have a 50% chance of being correct. Hence these items do not provide good information about the learners’ choices, in particular because the learners were not asked to show their computations. However, the results may show that the learners understood the first simple problem (91% correct), but had problems when the task was more complex, as is the case in the second item (69% correct). (see figure 4.21). Fadumo, for instance, answered 61. This may indicate that she probably just added both amounts and did not understand the question. Dina, who scored correctly, shows a nice computation in Iranian.

Since these “best-buy” problems are 50% chance items, it would be better to also ask them to show their computations, to get more information about the learners’ insight into proportions.

Figure 4.21 Best-Buy Problems

B2-4 Video-tapes  	Fadumo 61
Dina 	

Another interesting item on proportions is C1-10 in the Cito test. (figure 4.22) In Dutch the expression “1 on 12” means the car consumes 1 liter gas for 12 km driving. This way of expressing proportions, ratios, may vary in different countries. Mehmed from Bosnia, asked the test leader what these words mean. The test leader said: “What do you think?” He answered: “I think I can drive 12 kilometers for one liter”. The test leader confirmed. “I thought so already”, Mehmed said, “but we say it a bit differently. I only wanted to be sure”. After that he did a correct computation. Dina also asked, just to be sure, whether it meant “12 kilometers for one liter”.

Eleven learners passed this item, three skipped and eighteen failed. The incorrect answers were quite different. They varied from 3, 12, 24, 25, 28, 32, to 240, 260 and even 2000. From these examples we may assume that it is not clear whether people failed on this item because of the language in it or because of the computation.

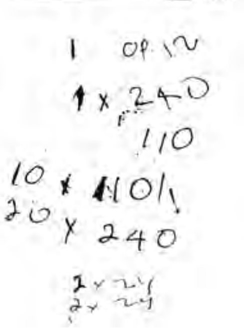
The results on this item show:

- *It may not be assumed that people understand specific expressions about proportions in e.g. advertisements or news bulletins.*
- *Proportions seem to be a good topic to create own notation systems based on informal mathematical procedures.*
- *Informal notations may create bridging links between the learner’s own idiosyncratic informal ways of thinking and formal mathematical procedures.*

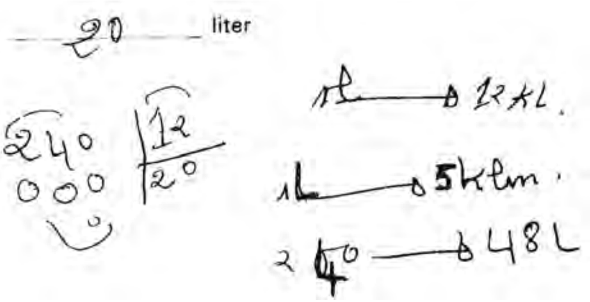
Figure 4.22 C1-10 Proportions

Harrie's car consumes 1 on 12. He drives 240 kilometers with his car.
How many liters fuel does the car need?

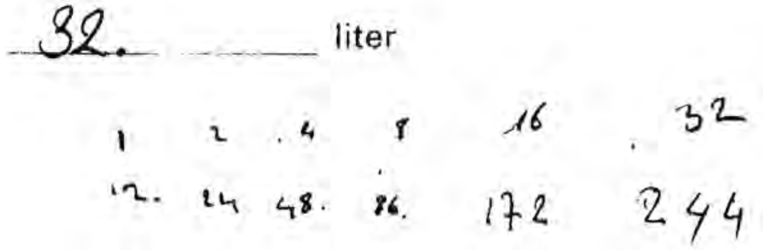
Margaret



Nadia



Zeki



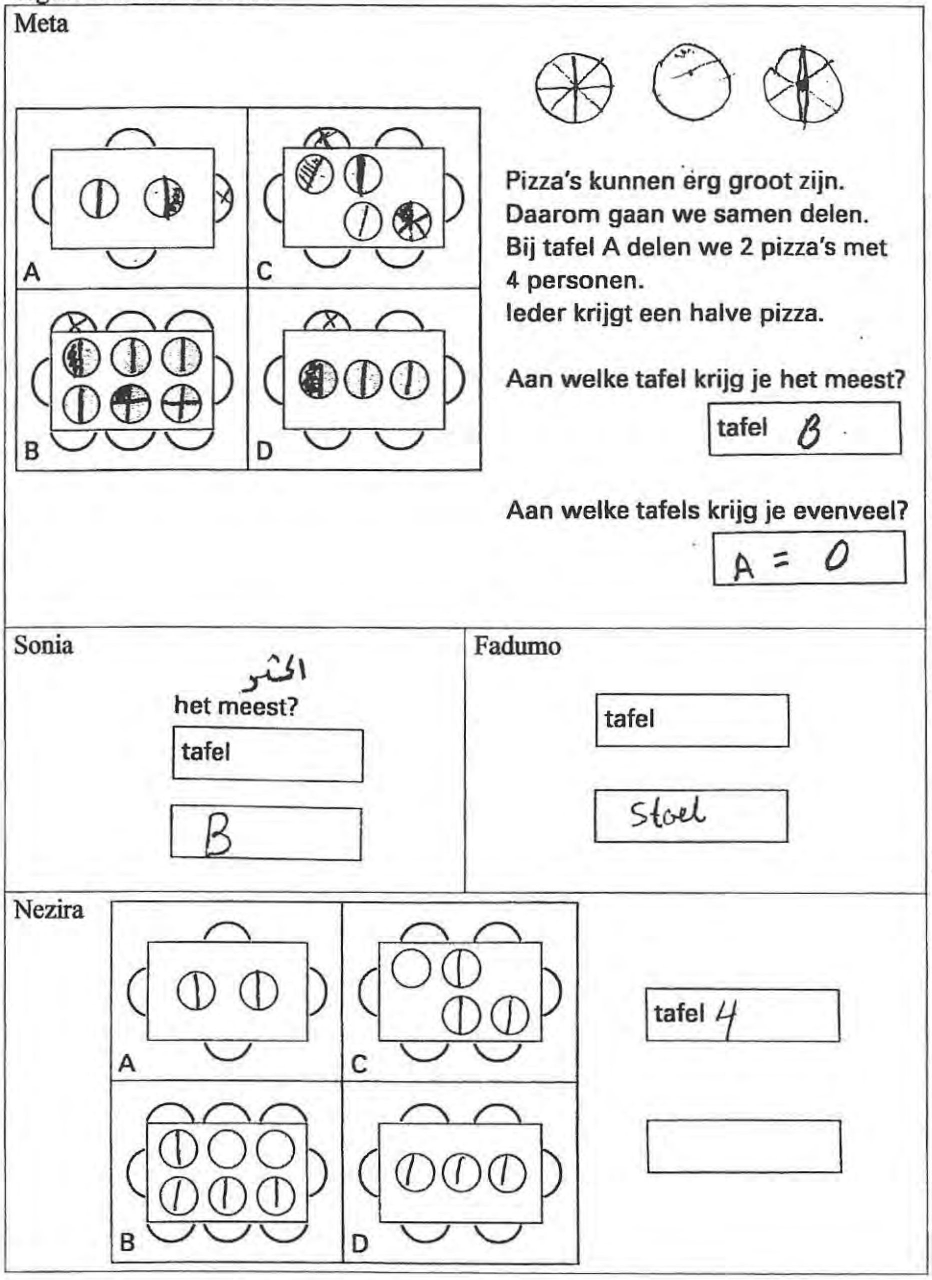
Though the item above is a bit problematic because of the ratio language in it, there were a few nice examples of informal computations.

Nadia, from Turkey, tried to first rewrite the "1 op 12" in a familiar way for her and started doing computations in an informal way (see example). Then she had second thoughts. She asked the test leader "It does mean 1 liter for 12 kilometers, doesn't it?" The test leader nodded. After that she completed a correct long division. Possibly she needed the informal notes to get her thoughts organized and felt unsure about the notation system.

Margaret also shows informal notes. She possibly did 10 times 12 but wrote 110 instead of 120 and after that she did 20 times is 240. She also could have thought 1 on 12, 2 on 24 because of her notes 2 x 24. She could have done this to check her answer.

Zeki shows a nice way of doubling. Alas he made a few errors in his computations. It would have been interesting to see what would have happened when this final number had been correct.

Figure 4.23 B3-4 Pizzas



Meta

Pizza's kunnen erg groot zijn.
Daarom gaan we samen delen.
Bij tafel A delen we 2 pizza's met
4 personen.
Ieder krijgt een halve pizza.

Aan welke tafel krijg je het meest?

tafel B

Aan welke tafels krijg je evenveel?

A = 0

Sonia

اكثر
het meest?

tafel

B

Fadumo

tafel

Stoel

Nezira

tafel 4

4.3.2.2 Proportions and Fractions

Two other problems on proportions, related to fractions, are the pizza item and the smoking item. The results on these items are:

Table 4.16: Results on Proportion items related to fractions

	Proportions - fractions	# correct	% correct
B3-4	pizza	2	6
B3-9	interview “smoking”	18	56
	mean scores fractions	10	31

The first item, the pizza problem, is used in the IB test to find out if learners really have insight into the basic concept of fractions. (figure 4.23). In fact, this item was a try-out in the IB test, because it was expected that many people would not be used to such a context. Most of them are not used to contexts at all in math tests. But it was possible that this item would appeal to their real life knowledge. The scores on this item were not surprising, but some were interesting.

Since this is a level 3 item, the majority of the group took this item (25 persons). Only 2 learners passed this item and 5 learners did it partially correct. The majority of the learners made some attempts and showed some understanding but most of them were not able to describe their findings.

To introduce the pizza problem a brief explanation was given to the pizza division at table A. Meta created a nice sketch of the division and showed he understands the context.

Sonia did not know the word “meest” [means “most”] and asked the test-leader. She said: veel-meer-meest [means: many- more- most]. Sonia understood this help and wrote “most” in Arabic above the word. After that she probably thought that her answer should be in the empty box below, so wrote “B”' in the lower box and just stopped. She possibly did not expect a second question.

Fadumo did not understand the question. She gave an associative answer in the second box, related to the word in the first box. The word in the first box shows “tafel” [means “table”]. She wrote “stoel” [means “chair”] and stopped.

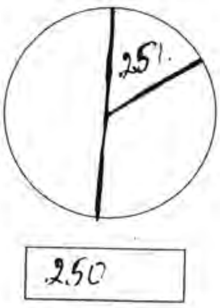
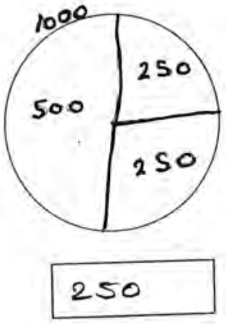
Nezira drew for each person half a pizza at each table, but the final answer is just “4”. Not clear is what she meant with this “4”. Perhaps she meant table D.

All the others just skipped the item or showed only unclear incorrect answers in the boxes.

Though the results on this item are poor, the various attempts by the learners to figure out the divisions, may indicate that they understand what to do with this context and that they have some understanding of fractions, but are not able to *figure out the right division and/or the correct wording or notation related to the result of the division. Given that the context on its own is unfamiliar to most of the adults who enter ABE, it shows that this context could be used as a basis for learning more about proportions and fractions. It offers many possibilities for discussion about proportions and fractions. On the other hand it can be said that the results on this item are alarming and shows the need for careful attention to developing insight into fractions.*

A second item about proportions and fractions was the interview item. The question at the task is: *1000 people are asked whether they smoke or not. A quarter of these people smoke. How many people are that? Indicate a quarter of the circle.* (figure 4.24)

Figure 4.24 B3-9 Interview

Sunita	Valentina	Hatice
		

Here the learners can show whether they understand simple statistical information when it is shown as proportions in a circle, as often presented in newspapers and news bulletins on television. The learners were asked to indicate a quarter of a circle. There were 17 correct answers and 2 partially correct answers out of 25. This was quite a difference from the previous item. A few persons gave the right answer but could not indicate a quarter of the circle correctly. Five learners wrote 750 as the answer. They subtracted 250 from 1000, like Sunita did. One person answered 7500. Most learners could draw a quarter of the circle, though their presentations are often unclear, see e.g. Valentina. Four persons drew a quarter of the circle but their answers were 750. One person drew a quarter and wrote 89. Two persons wrote only the number “250” in the circle. One person wrote the word “triangle” in the circle. Only Hatice drew a correct half and two quarters in the circle and wrote the correct numbers in it.

Two preliminary conclusions can be drawn here:

- *about half of these learners show that they don't actually have insights into a "quarter" and hence neither into other benchmark fractions. We may wonder how they process such information from public media.*
- *using a circle for visualization of benchmark fractions could help and could be a good mental model, because most of the learners could draw a quarter of the circle, though many people are not really familiar with it. This mental model could also link fractions with percent and decimals.*

In a similar way the Cito item C2-H-7 asked for "3/4 of 3000".

Figure 4.25 C2-H-7 - Dresses

<table>
<tr><td colspan="2">There are 3000 dresses in a store. 3/4 of them are damaged by a fire.
How many dresses are that?</td></tr>
<tr><td>Zebiba
 </td><td>Mehmed
 </td></tr>
</table>

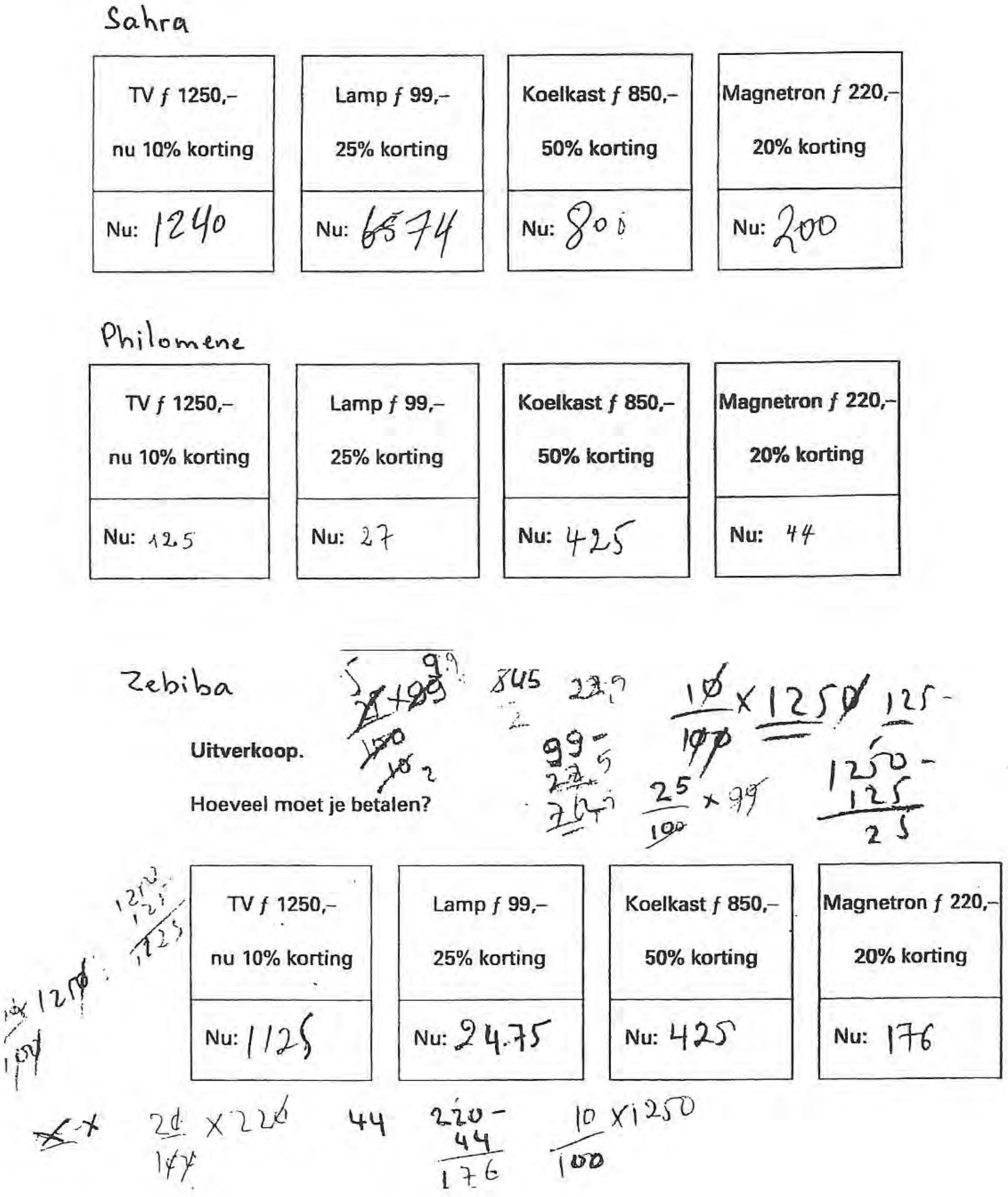
This item was posed as a word problem without any visual support. Only 3 out of 12 who took this item came up with the right answer. Most learners of this group only computed 1/4 of 3000 and forgot to subtract, like Zebiba did. Mehmed shows his computation based on halving plus addition. They were the only two who showed their computations.

Item C2-H-9 in the Cito High test shows an advertisement offering a part-time job which is 3/5 of a week's job. A full-time job is 40 hours per week. The question posed is how many hours a week a person will have to work for this job. Only two persons passed this item.

Two preliminary conclusions from these items:

- *In general, we may wonder how these learners deal with "fraction language" in everyday life.*
- *working with fractions on an abstract level, using symbols or quantities in word problems, appears to be more difficult than a visual representation of a situation with only a simple question.*

Figure 4.27: B3-3 Discount. What is the new price? (TV, Lamp, Refrigerator, Micro-wave)



Sahra

TV *f* 1250,– nu 10% korting	Lamp *f* 99,– 25% korting	Koelkast *f* 850,– 50% korting	Magnetron *f* 220,– 20% korting
Nu: 1240	Nu: ~~65~~74	Nu: 800	Nu: 200

Philomene

TV *f* 1250,– nu 10% korting	Lamp *f* 99,– 25% korting	Koelkast *f* 850,– 50% korting	Magnetron *f* 220,– 20% korting
Nu: 12,5	Nu: 27	Nu: 425	Nu: 44

Zebiba

Uitverkoop.

Hoeveel moet je betalen?

845

$\frac{10}{100} \times 1250$ 125 –

99 –

$\frac{25}{100} \times 99$

1250 –
125

TV *f* 1250,– nu 10% korting	Lamp *f* 99,– 25% korting	Koelkast *f* 850,– 50% korting	Magnetron *f* 220,– 20% korting
Nu: 1125	Nu: 24.75	Nu: 425	Nu: 176

$\frac{20}{100} \times 220$ 44 220 – 44 = 176 $\frac{10}{100} \times 1250$

4.3.2.3 Proportions and Percent

The next important field in proportions is percent. The IB test has 4 items on percent at levels 3 and 4, whereas the Cito test has five items of which two in the first phase of the test and three in the phase two High test. Given the results in table 4.17 we may conclude up front, that this field is very troublesome and needs special attention.

Table 4.17 Percent items in IB and Cito

Level	Item	Subject: proportions and percent	# correct	% correct
3	B3-3	find product (benchmark percentages)	2	6
4	A4-9	find product (15% discount of *f* 200,-)	9	28
	A4-10	find product (2% increase of rent)	1	3
	B4-3	proportion, find percent (15%)	7	22
		Mean scores percent	5	15
Level	Item	Subject	# correct	% correct
C1	C1-11	proportion, find percent (30%)	11	34
	C1-12	find product (96% of 5018)	16	50
C2-H	C2H-8	4/5 = ? % (compare fractions, percent)	2	6
	C2H-11	percent, proportions (60% of 40 hours)	3	9
	C2H-12	find percent gain (10%)	3	9
		Mean scores	7	22

The first item, B3-3, shows four advertisements with a percentage discount. The question is: What would you have to pay now? See figure 4.27.
The refrigerator item was the easiest. Twenty-three learners knew how to compute 50%. That is about two-thirds of the group. However, only 11 learners passed the TV and the Lamp item and only 10 learners passed the Micro-wave item. That is about one-third of the group. Only two learners passed all four items.

Many learners show misconceptions on these tasks.
Sahra thinks that the percentage equals the same amount in money, e.g. 10% equals 10 guilders, 25% is as much as 25 guilders, 50% is 50 guilders and 20% is 20 guilders. Most learners know that 50% is half of the amount. Three learners computed only the discount of all items, as Philomene did. Here they got a correct answer on the 50% item, but that could be by accident.

The 25% item was a bit problematic because of the amount of *ƒ* 99.00. Many learners felt uncertain about the answer being 74 or 75 guilders. Zebiba computed the correct discount (*ƒ* 24.75) but forgot to subtract. Only Meta achieved at the real correct answer (*ƒ* 74.25). He is also the one who passed the difficult "increase of rent" item (A4-10).

The 20% discount caused more problems. Though 10 learners did it correctly, there were still a few miscomputations. Six learners subtracted 20 guilders and got the answer *ƒ* 200.00. They possibly applied the strategy of 20% equals 20 guilders, though their answers to the other items were correct. One person subtracted only 22 guilders (= 10%) and got the answer *ƒ* 198.00

In these items the question was to find the new price, the result. In other types of items the question could be to find the percentage or to find the base (the original number or amount). The items on level 3 all require computing the new price with a benchmark percent. This type should be the easiest computation with percent. At level 4 there are three percent items. Two of them pertain to computation with 15%. In the first item the learner is required to find the new price for a discount of 15% (A4-9). In the second item the learner is required to find the percentage (B4-3). This second item was:

At a speed check on the A2 freeway, past weekend,
300 out of 2000 drivers drove too fast.
What percent of the drivers drove too fast? Check the right box.

10%	15%	20%	25%

The information is described in proportions, 300 out of 2000, but the question asks for an answer in percent. For this item there were 7 correct answers and 6 incorrect answers.

From the learners who failed this item two persons checked 10%, one checked 20% and two checked 25%. One learner skipped this item. Only one learner showed a computation, probably as kind of a check. This could mean that most learners did this task mentally but it is not clear how many learners guessed the answer. However, six out of the seven people who got the correct answer also scored correctly at B4-3. This could possibly mean that these 6 learners really understood these 15% items.

The final item on percent in IB is also at level 4: A4-10. This item asked to compute the new rent one will have to pay per month when the monthly rent of *ƒ* 875,00 is raised by 2%. Only one learner, Meta, passed this item. He writes the correct answer, *ƒ*892.50, but he does not show a computation in his work. Of the previous two items he only passed the telephone item. This could mean that he is able to compute results of percent problems, but that it is more difficult for him to interpret statistical information or to compute another type of items, finding the percentage.

A preliminary conclusion here concerning percents may be that about only one quarter of this group can do some computations with benchmark percent. Only one person is able to do computations with other percentages.
The percent items here mainly pertain to money computations. Processing percents based on statements worded in proportion language appears to be more difficult.

The Cito test offers two estimation items on percent in the first phase: C1-11 and C1-12. The first item shows a map of a kitchen garden with in it vegetables, potatoes and fruit, sketched in ratio: 50-30-20. The question here is: *30 percent of the garden has been used for ....*
This question was correctly answered by 11 out of 32 persons. There were no computations for this item.

The second item was a multiple choice question (C1-12). It asks the learner to estimate 96 percent of 5018 runners who finished a marathon. The test takers can choose from 20 - 48 - 200 and 4800. There were16 correct answers.

This may indicate that at least 50% of these learners have a sense of the concept 100%, though working with percent, or "seeing" percentages in figures, is more difficult, even with benchmark percents.

In the Cito second phase High test there are two items on doing computations with percent. The first one is about work (C2-H-11):

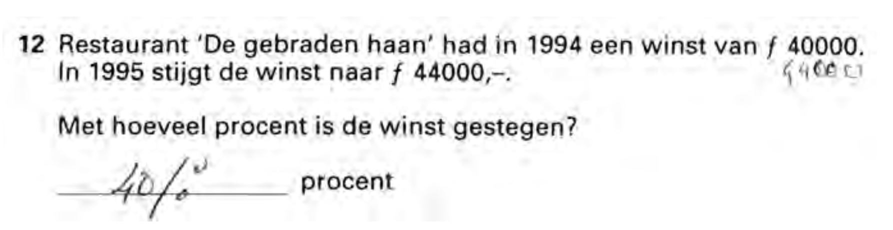
Figure 4.28 Zeki C2-H-11 - Work

Lisa has a job for 40 hours per week. She was ill for a long time and is now allowed to go back to work for 60% . How many hours per week is she allowed to work?	100/40 50/ 20 60/ 25 70/ 30 80/ 35 90 40

Only 3 learners passed this item. Zeki showed a nice computation, though his answer was wrong. (Figure 4.28, left side). He started his own ratio table, with 100/40, 50/20 but after that he went on with 60/25, 70/30, 80/35 and 90/40. His answer is 25%. (based on 60/25). He did not see that 90/40 in relation to 100/40, the numbers he started with, could never be correct. *However, he shows that a ratio table can be a good mental model to find percentages.*

In another item, however, he shows his misconception of percent. It asked to compute the percentage increase of profit made by a restaurant owner in one year. Here Zeki shows his miscomputation. He compared the profit of 1995 with that of 1994. The difference is *f* 4000.00. He did not relate this amount to 10% of 40,000.00. Possibly, he divided that amount by 100 to find the percentage and hence arrived at 40%.

Figure 4.29 Zeki - C2-H-12 Percentage increase of profit



12 Restaurant 'De gebraden haan' had in 1994 een winst van *f* 40000.
In 1995 stijgt de winst naar *f* 44000,–. 4400

Met hoeveel procent is de winst gestegen?

40% procent

The results from percent items may show that this subject is a worrisome area. It calls for careful attention in numeracy courses.

4.3.2.4 Relations between proportions, fractions, percent and decimals

Eventually adults should know about the relationship between fractions, percent and decimals as part of proportions. In IB decimals are tested in measurement. More on this is reported in the next section (4.4), but three specific items test the relation between fractions, the language of fractions and decimals. The results on these items are:

Table 4.18 Relationship between fractions, percent and decimals in IB and Cito

Item		# correct	% correct
B3-5	length (dec., m, cm)	11	34
B4-4	volume, length, weight (fractions, dec)	10	31
B4-6	weight (compare fractions, dec)	6	19
	Mean scores	9	28
C2-H-8	4/5 = ... % (compare fractions, percent)	2	6

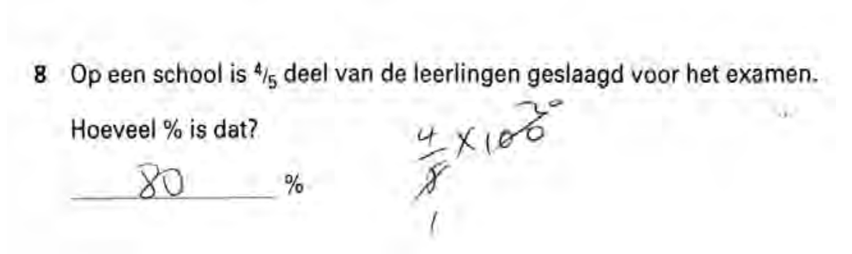
The items B3-5 and B4-4 are quite similar. The learner is asked to compare fraction language and decimals in measurement units in statements on different levels, e.g. at level 3: three quarters of a meter is 0.75 cm; 40 cm is more than half a meter. And at level 4: four cans of paint of 0.75 liters are as much as 2½ liters; 1½ kg potatoes is as much as 1.75 kg.

Given the results, the learners appeared to have many problems with these types of items. Similar results are noted on level 4 at item B4-06 where people are asked to read a price tag. The price tag shows a weight of 0.228 kg. The question asked is: Is this more or less than 1/4 kg?
Only 6 learners answered correctly.

The Cito test also has one item that tests the relationship between fractions and percent. (second phase High test: 2-H8). Only two learners passed this item. Zebiba, one of them, showed she knows the rule for this.

Figure 4.30: Exam - Zebiba

4/5 of the learners from one school passed their exams.
What percent is that?



8 Op een school is $^4/_5$ deel van de leerlingen geslaagd voor het examen.

Hoeveel % is dat? $\frac{4}{\cancel{5}_{1}} \times \overset{20}{1\cancel{00}}$

80 %

These results may show that insight into relationship between proportions, fractions, percent and decimals is a problematic field.

4.3.2.5 Preliminary conclusions on proportions.

The results on this part, IB, 43% correct and Cito, 26% correct, may indicate that there is still a long way to go in this field. In general, the learners have insight into proportions and can also do some simple computations with proportions, but when it comes to insight into and computations with percent and fractions, most of them fail. Only two learners are able to do more difficult computations with percent. About 15% of this group can do tasks with benchmark percents 50%, 25% and 10%. Insight into fractions and doing simple computations and comparisons with fractions show slightly better results (34% correct). A hopeful sign is that these adults show insights into proportions. (75% correct). The following conclusions can be drawn:

Conclusion 10
It appears that most learners in this study can do simple proportional reasoning like in "best buy"-problems, computing the consumption of gas over a certain distance related to consumption per kilometer, and other proportional comparisons.

B10
Proportional reasoning can be used as a basis for further development in doing computations with proportions and for developing insights into fractions and percent.

Conclusion 11
A few learners show informal mental models based on proportional thinking and reasoning.

B11
Individual informal mental models can be used to create proportion tables and to develop insights into proportional reasoning related to fractions and percent.
These learners are very sensitive to visual support from graphs, including circles, blocks, bars, tables, for instance, to overcome language problems.

Conclusion 12
The results on fraction tasks are alarming, though the respondents show in their attempts at the pizza problem that they do have a little notion of fractions.

B12
The pizza problem and other division contexts could be a good start for developing insight in fractions and doing basic operations with fractions.

Conclusion 13
Only one quarter of these learners are able to do computations with a benchmark percent. Only two learners could do all percent tasks. Learners' knowledge about percent often involves only percent as it relates to money computations.

B13
A hopeful sign is that a quarter of these learners can do computations with benchmark percentages. Benchmark percentages supported by visual mental proportion models could be building blocks for developing better insights into percent.

Conclusion 14
Knowledge about fractions and percent, also related to decimals, often consists of partial knowledge.

B14
Mental models could create links between knowledge about proportions, fractions, percent and decimals.

4.3.3 Measurement and Dimension

Measurement is an important field for the application of mathematical knowledge and skills. Especially in countries where the metric system is used, this field forms an integrated area of units of measure and the decimal system, related to proportions and the language of fractions. An additional advantage in many countries is that the money system also is based on the decimal system, though this is not always apparent in bills and coins. This means that, when people understand the decimal system and can do computations with decimals, they should also be able to work with measurement units, including money, and vice versa. Decimals are the link between the field of measurement and the field of proportions. In this section we will discuss the results from items on the metric system. Money computations will be discussed in the next section. The focus in ABE is mainly on practical knowledge that is useful for solving everyday life and simple work problems. In many situations, informal knowledge and skills are sufficient to manage real life situations. In more complex situations, formal school knowledge can also help, although transfer of school knowledge to work and other situations is not a straightforward issue (Evans, 2000). In fact, in real life situations the question is where informal practical knowledge stops and where formal school knowledge starts to be useful/necessary. Moreover, research into mathematics in real life situations increasingly shows that knowledge and skills needed for work and everyday life situations do not always match school knowledge (see e.g. Lave, 1988, Forman and Steen, 2000, Hoyles and Noss, 1998, 2000). In many work situations, job-specific skills are required. In this light it is interesting to see what adults actually know about measurement and dimension issues when they enter ABE. Table 4.19 presents a first impression of the results on measurement and dimension.

Table 4.19 Mean scores on measurement and dimension

	Measurement and dimension	IB # items	IB % correct	Cito # items	Cito % correct
1	Length	2	63	1	72
2	Area	1	13	-	
3	Volume	5	56	1	56
4	Weight	4	65	-	
5	Relation between L/V/W	1	31	-	
6	Shape/Space	4	54	-	
7	Time	6	50	3	34
8	Calendar (knowing the actual date)	1	84	-	
	Total # items	24		5	
	Mean score % correct		54		47

Table 4.19 shows that there is a big difference in the total number of items in the IB and Cito test. The Cito test has only 5 items in this field whereas IB has a total of 24 items. This difference reflects the fact that the IB test is meant to be a placement test for courses where the IB series is used. Items from all subjects appear on each of the different levels. The Cito test, on the other hand, measures proficiency in general so it is not necessary to test all subfields within measurement on all levels. Cito has only one item in the first phase test (volume), two items in the second phase low test (length and time) and two items in the second phase high test (both on time).

4.3.3.1 The metric system

The measurement items on level 1 focus mainly on informal, practical knowledge based on comparison of measures, related to estimation and proportions. For example: estimate the length of a bus related to the length of a car (4 meter). The bus and car are visualized in their proportion. The learner can choose here from three possible answers: 6 meter, 12 meter or 25 meter. He is not required to measure the objects in the picture. This task asks for practical and experiential knowledge and for common sense.

At level 2 the same type of questions are asked but now involve choosing the right units of measure in a certain situation: meter or centimeter, kilo or grams, e.g. make your choice, kilogram or gram: a person standing on a pair of scales weighs 65.......... (grams or kilogram)

At level 3 the learner is asked to read and compare measures within the metric system, e.g. “0.45 m is the same as 45 cm. Cross right or wrong.”

At level 4 the learner is asked to do simple computations. The learner is shown, for instance, a photo of 6 bottles of coffee creamer of 200 ml each. The question here is: “Is the total more or less than 1 liter? How much is it exactly?” Also, at level 4 the learner is asked to determine a total number of carpet tiles for a room. There are several possible ways to find the correct answer, e.g. he can just estimate the number of tiles in the length and width of the room and then choose the correct answer from the four possible answers given (as a multiple choice question). The learner is also asked to indicate the precise area of the room. This can be done by estimating and then choosing the correct answer out of four possible answers. Of course he can also compute it and then check the correct answer.

In short: the items in measurement are set up in a line from informal, practical knowledge and estimation at the levels 1 and 2, to more formal, but still practical knowledge and doing simple computations at levels 3 and 4. The results for all individual items are shown in table 4.20.

Table 4.20: Results measurement, metric system.

Level	Item	Topic	# correct	% correct
1	B1-5	length (bus-car)	28	88
	B1-6	weight (potatoes - sugar)	30	94
	B1-7	volume (compare milk containers)	28	88
2	B2-3	volume (compare containers)	21	66
	B2-5	length (m-cm) (choose unit)	29	91
	B2-6	weight (choose unit)	27	85
	B2-7	volume (compare milk containers)	16	50
3	B3-5	length (dec., m, cm) (compare units)	11	34
	B3-6	weight, (read price tag)	20	64
	B3-7	volume (l, cl) (add content of containers)	19	59
4	B4-4	volume, length, weight (fractions, dec)	10	31
	B4-5	area	4	13
	B4-6	weight (compare fractions, dec)	6	19
	B4-7	volume (add ml)	6	19
		Mean score	17	55
Level	Item	Topic	# correct	% correct
C1	C1-6	volume (liter, ml)	18	56
C2-L	C2L-6	Length (m-cm)	23	72
		Mean score	21	64

The table shows that the higher the level, the lower the results. The mean score at level 1 is about 90% correct, at level 2, about 73% correct, at level 3, about 52% correct, and at level 4, only 20% correct.

A first impression is that most adults have acquired informal knowledge and skills on the metric system. About three quarters of them have some knowledge about standard units of measure and can choose the appropriate unit in a measurement situation. About half of the group have some more knowledge about the measuring units within the metric system and can compare such units. But only 20% of these adults are able to do simple computations within the metric system.

To indicate the type of tasks, a few items and some learners' work are described below. For many items no computations are required.
At level 1 the items are related to real life experience. The first item B1-5, asks for an estimation of the length of the bus, related to the given length of the car. This item has already been described in section 4.3.2. The weight item, B1-6, shows photos of a 2.5 kg bag of potatoes and a 1 kg bag of sugar. The learner is asked to indicate from the photo which bag is heavier. These bags are standard ones available in Dutch supermarkets. This means that experience can help the learner to find the correct answer. A similar task is required in the volume item in which the learner is asked how many small containers (1/2 liter) he should buy if he wanted 3 large containers (1 liter) that are sold out.
In the length and weight items at level 2 people are asked to choose an appropriate measure for length and weight. The learner is asked to complete the sentences. Most people could do these tasks.

Figure 4.31 Length and Weight

item B2-5	item B2-6
choose meters or centimeters	choose kilograms or grams
a room is about 8 ......long the book is 5 ...... thick a car is about 4 .... long my pen is about 12 .... long	5 tomatoes weigh about 400 .... the woman weighs 65 ....

The first volume item in level 2, item B2-3, shows a milk container and a pan. The question says: empty a container that holds one liter milk into a pan that can hold 4 liter. How full is the pan? Check the right answer.

Figure 4.32 - Milk containers

Here it becomes more difficult. How to imagine a certain volume in a new container with a different area and different volume? Only 21 adults answered correctly. That means that 11 persons cannot imagine what happens when they empty the container in the pan.

The results at level 1 and 2 show that most adults can relate the tasks with standard measures in a real life context. It is not clear from the persons who failed whether they had problems with reading the text or understanding the photos or a combination of both. It is also possible that they don't know about units of measure, cannot estimate or cannot link measurement information on paper with actual measures in real life, e.g. cannot imagine how long one meter is and how long a bus is related to a car, or that 5 tomatoes cannot weigh 400 kilogram.

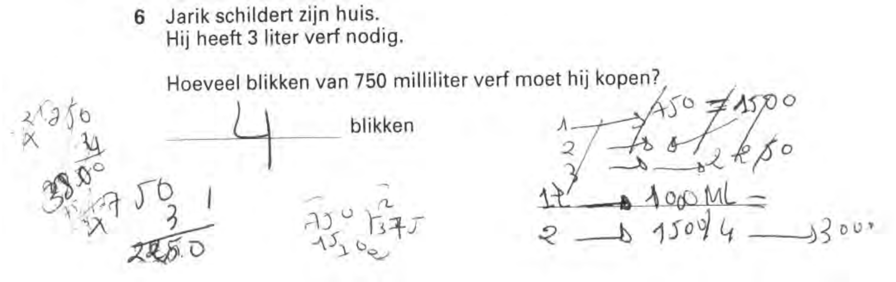
The point is that actual measurement always takes place in a real situation and never as a task on paper. One needs to "see" measures and to actually do measurement computations to develop a real sense of measures. This is difficult to test in a paper and pencil test. There may be a question whether these adults are familiar with doing such measurement tasks on paper and how they would do such tasks in informal real life situations.

At level 3, the tasks focus more on insight into the metric system related to decimals and on doing simple comparisons and computations within the metric system with different measurement units. For instance, people are asked to read a price tag regarding weight or to do a simple computation with decimals in the volume item. Here we see that the results go down. When different units of measure are used in one simple task, like conversion from meters to centimeters, the task becomes much more difficult. This is also the case in the volume item where people are asked to add one liter, half a liter and a quarter liter and to check the right total.

At level 4, people are asked to compare measures written in different ways, using decimals and fractions, and to do simple computations, e.g. 6 times 200 ml. (item B4-7) determining the total number of carpet tiles for a floor and computing the area of that room (item B4-5). Here the results go down drastically. Only four persons answered correctly on both questions at this last item, one person passed only the tiles item and four others only the area item.

The item questions in Cito are somewhat similar. The first one (C1-6) asks the respondent to compute how many cans of paint a person needs to buy if 3 liters of paint are needed and he wants to buy it in 750 ml cans. Nadia shows an interesting computation.

Figure 4.33 Paint - Nadia



6 Jarik schildert zijn huis.
Hij heeft 3 liter verf nodig.

Hoeveel blikken van 750 milliliter verf moet hij kopen?

4 blikken

The question is: “Jarik needs 3 liters of paint to paint his house. How many 750 ml cans of paint should he buy?”

Nadia shows a few formal computations on the left side and in the middle, and informal computations on the right side. It is not clear what came first.

On the left side we see a multiplication, 4 times 750, but she made errors and came up with 3800. We may wonder whether this was her first computation or a check at the end for the informal computation on the right side. In the second computation she did 3x750. Here she might have chosen to multiply the “3” from “3 liter” by the content of one container “750 ml”. She also tried a long division. These algorithms might be attempts to find out what to do.

On the right side we see two other attempts: first she tries to compute how many liters of paint she gets by three times adding 750 ml. (2250). Then she needs a step on the side: 1 liter ➔1000ML. Here she refers to her formal knowledge about liters and milliliters. Finally she computes in an informal way:

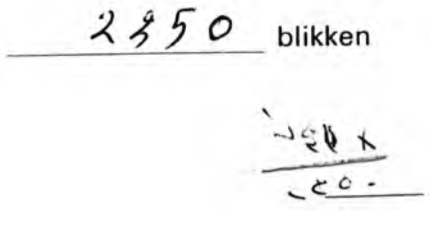
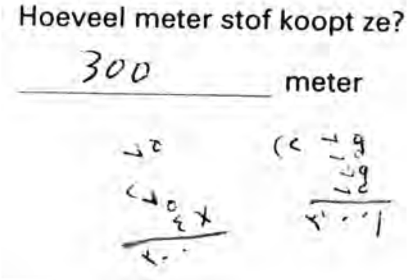
2 ➔ 1500 / 4 ➔ 3000.

Here we see that she finds the correct answer by doubling the volume of the cans: 2 cans hold 1500 ml, 4 cans hold 3000 ml. This informal computation is based fully on insight.

She applies similar computations to other items, the fuel item C2-L-10 among others, where she computed the number of liters of fuel needed for a distance of 240 kilometer.

Sausan shows a different computation for this paint item. She just applies a multiplication. Her answer is 2350. (See figure 4.34). She does the same at the curtains item (C2-L-6): “Anna wants to buy fabric for curtains for four windows. She needs 75 cm for one window. How many meters fabric does she need?” Again she applies a multiplication, though she also shows repeated addition in her Iranian computations. She added 75 four times. Her answer is 300, but she does not convert that to meters. Not clear here is if she really knew that she had to add (or multiply) 4x75 in this context or that she just did it, because she also applied multiplications in both the paint item and the place value item (see figure 4.19).

Figure 4.34 Sausan

Sausan - Paint	Sausan - Curtains
	

We also saw this happen for the place value item: learners seem to just apply a computation, either multiplication or division, without wondering what they should be doing and whether it fits in the context. In this particular situation we may wonder whether language problems play a part for Sausan or that she simply does not understand such tasks. She shows she can apply algorithms in her own language.

Preliminary conclusions here are that these learners can do simple measurement tasks that refer to real life situations, but if the tasks involve more abstract comparisons and computations in the metric system they do not know what to do. Language problems may be causing them to not understand the tasks. On the other hand, we may wonder whether these learners understand the mathematics embedded in the tasks. In fact, some learners show blind ciphering, just as they did int the place value item in section 4.3.2.7.

4.3.3.2 Dimension

Living in a three-dimensional world requires a well-developed sense for shape and space. Everyone develops such a sense in their own way and based on their own specific knowledge and skills, which is often a mix of informal practical knowledge and skills and some former school knowledge. An additional component is the measurement of time. Reading clocks and calendars and managing time are key skills in our current society.

Testing dimensionality in a paper-and-pencil test is difficult, particulary in a placement test. A main constraint is that such tests are always two-dimensional. This should not be a problem for adults who are used to paper-based information, but this is not the case for all adult learners in ABE. We may question if it is necessary to test dimensionality for placement, but it seems to provide specific information, as seen at the counting and multiplication items in section 4.3.1.1 and 4.3.1.2. The ways of counting and multiplication people applied at the "bricks", "crates" and "box" items are good insights into how people deal with dimension on paper. Based on the results for those items we infer that there are adults who cannot imagine a three dimensional structure in a two dimensional space. However, this does not indicate that these same persons would also have problems with counting structures, like counting bricks, crates or boxes in actual three dimensional situations.

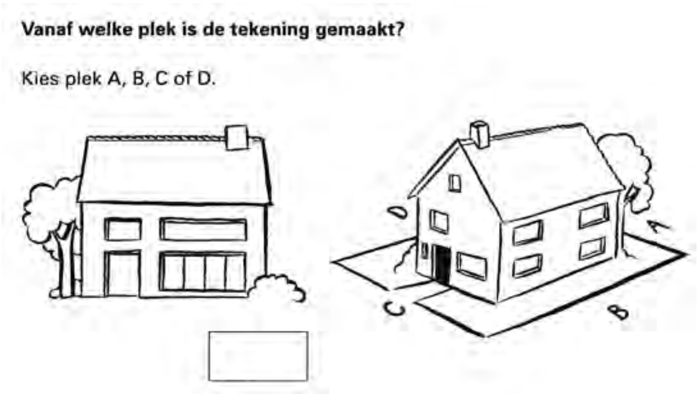
In the field of dimension and shape four items were developed for IB. The boxes item (B2-8) was also part of that, though it was discussed at counting and was also meant to assess volume.

Table 4.21 Dimension

Level	Item	Topic	# correct	% correct
1	B1-8	dimension (photo)	27	84
2	B2-8*	dimension (boxes)	22	69
3	B3-8	shape (traffic signs)	18	56
4	B4-8	shape (cube map)	1	3
		Mean score	17	53

A common item in many math programs in primary and secondary school is the photo item. A photo or drawing of an object is shown together with the same object from a perspective that shows two visible and two invisible sides of that object.

Figure 4.35 - B1-8- From which side was the drawing made?



Vanaf welke plek is de tekening gemaakt?

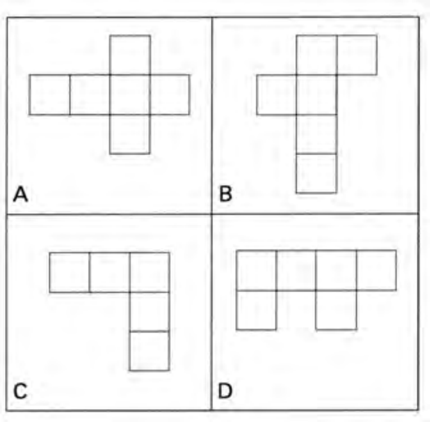
Kies plek A, B, C of D.

The question here is "From which side was the drawing made?" Amazingly, 27 persons passed this item. It is striking to note that of the 5 adults who failed this level 1 item (about 15 %), all five learners also miscounted at the “chairs”-item (A2-1) and three miscounted at the “boxes”-item (B2-8) as well. Here too, the chairs and boxes were shown in three-dimensional piles.

On level 3 an item is presented about traffic signs. The learners are asked to match traffic signs with the names of their shapes: a square, a rectangle, a triangle and a circle. Only 18 learners did this correctly.

A much more difficult item is on level 4: “Find the map(s) of a cube.” This is also a common item in many math programs. Only one person in this study was able to discern the right maps of the cube. Two persons found only one of the two possibilities.

Figure 4.35 Cubes



The results on these two items concern us as dimensionality is the essence of geometry, necessary in many vocations and an essential part of many vocational training courses. Also, in many simulation programs, one needs to be able to see three dimensions in a two-dimensional area.

In the field of time and calendar quite a few people have problems with reading the time, in analog and digital format, and with doing simple computations with time. On the calendar task most of the learners can fill in the correct date. See table 4.22

Table 4.22 Time and calendar

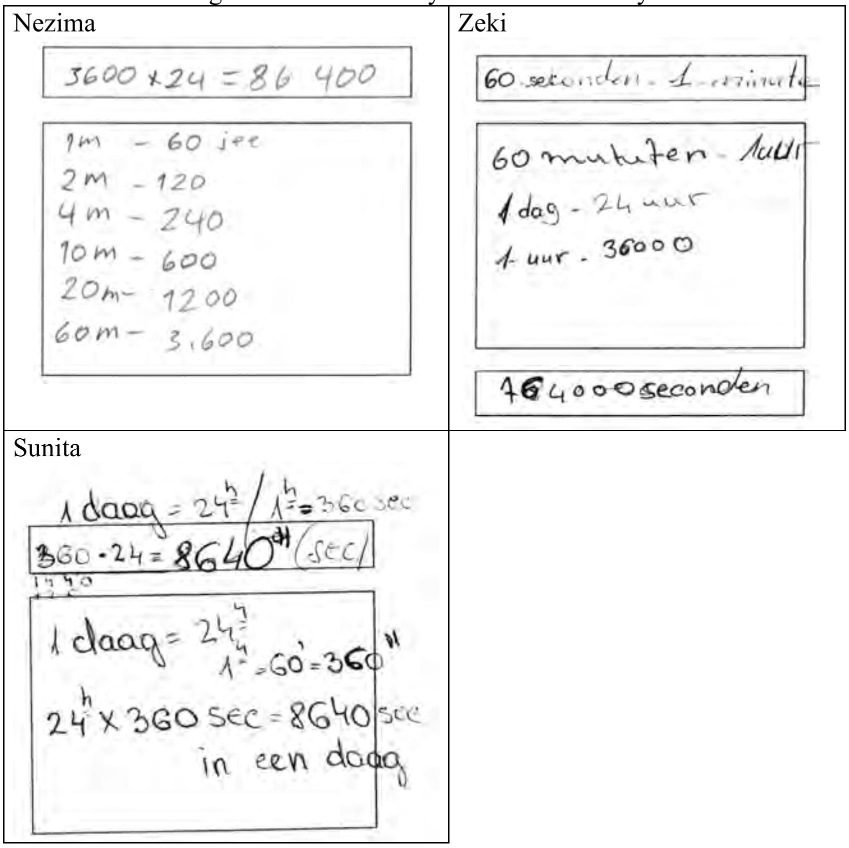
Level	Item	Topic	# correct	% correct
1	B1-2	Time, reading clock (hours, half hours)	21	66
	B1-10	Time, determine arrival time (travel 1½ hour)	26	81
2	B2-2	Time, reading clock (minutes)	17	53
3	B3-2	Time, compute hours + min (work schedule)	10	31
	B3-10	Time, compute seconds + min (passport photo)	17	53
4	B4-2	Time, compute # seconds in a day	4	13
		Mean score	16	50
2	B2-10	date (today's date-calendar)	27	84

On level 3, item B3-2, people are asked to read a work schedule for an employee and to compute the total number of working hours per week for that person. Only 10 persons passed that item. *This means that two-thirds of this learners' group are not able to read a work schedule.*

On the passport photo item, B3-10, only 17 persons know that 300 seconds pass by in 5 minutes. *This means that half of the group does not know how to compute the total number of seconds in five minutes and perhaps does not even know that there are 60 seconds in a minute.*

On level 4 only four learners are able to compute the total number of seconds in one day. (A4-10). Three learners show an interesting computation. (figure 4.36). Nezima shows a correct computation based on her own thinking model. She starts with 1, 2, 4 minutes but then she changes to jumps of 10 and 20 to find the number of seconds in one hour. Based on that she computes 3600x24 and finds the correct answer. Zeki knows exactly the number of seconds per minute and the number of minutes per hour, but ends up with an incorrect multiplication. This happens to Sunita in a similar way.

Figure 4.36 How many seconds in one day?



4.3.3.3 Preliminary conclusions for measurement and dimension

An initial conclusion here is that, though the results on measurement and dimension items in this study are poor, it is not clear whether these adults also cannot do similar computations in real life situations. The point is that measurement tasks in real-life situations rarely occur only as paper-based tasks. They normally occur in a three-dimensional environment, sometimes requiring some computations on paper. One needs to be able to “see” measures in their actual situations to be able to develop a real sense of measures. However, it may concern us to see that about a quarter of these learners show problems with three-dimensional tasks on paper and that about half to three-quarters of them had problems with easy measurement computations in the metric system.

Based on the analyses in previous sub-sections the following preliminary conclusions can be drawn:

Conclusion 15
Most adults in this learners' group can deal with standard units of measure in a realistic context with representations of real-life materials. (about 72% correct on levels 1,2,3)

B15
Real-life materials should be the basis for further development in the metric system, e.g. compare a milk container that can hold one liter with quarter liter cups and discuss the notations of the measure units on these containers.

Conclusion 16
When items involve more formal computations on paper and conversion of units of measure, the percentage correct goes down dramatically. (about 20% correct on level 4)

B16
Formal computations on measurement should always be related to real life situations and materials, supported by photos or other representations on paper.

Conclusion 17
Knowledge about the relationships between different units when measuring length, weight and volume, as part of the metric system, is fragmentary. (31% correct)

B17
Integrating mental models based on proportions and notations for the different units of measure are needed to learn to see the coherence between length, volume and weight as part of the metric measurement system.

Conclusion 18
The results show that the learners' capability is often limited to doing computations with single units of measure, like doing computations with length, but they have problems when they have to compute a composite measurement, like area. Such computations are often based on applying formulas.

B18
Based on computations with single units of measure, a careful build-up is recommended for doing computations based on more complex measurement concepts based on formulas.

Conclusion 19
Supporting measurement tasks visually by means of photos and illustrations of real life materials and by mental models may help to create links from formal school measurement tasks to real life measurement situations.

B19
Measurement tasks should be related to the learners' real life experiences.
It is recommended to apply measurement tasks in real life measurement situations or to use representations of real life measurement situations in school situations.

Conclusion 20
Working with three dimensions in paper-based tasks appears to be difficult for some learners. This appeared before in counting issues based on three dimensional structures. The question here may be whether these learners would also have problems with doing measurement computations based on three dimensions in reality.

B20
Tasks involving working with three dimensions in paper-based tasks should be built up with the help of real objects that can actually be measured or counted. (see also B1)

Conclusion 21
About one-third to a half of the learners show problems with reading the time. Doing simple computations with time is even more difficult for them. Only about half of the group is able to do computations with time and only one-third can read a work schedule and compute working time.

B21
Since reading time schedules and doing computations with time are essential for job-related purposes and employability, this topic requires careful attention.

4.3.4 Money

Of all mathematical applications computations with money play the most common and most important role in everyday life. Thus we might expect that everyone can deal with money operations, at least in some way. People need to be able to do their shopping, manage their household cost, pay their bills, pay the rent of their houses, care for kids who need food and clothes, arrange their own assurances, calculate the cost of a vacation, etc.
In education money is ideal for creating real-life contexts for practicing basic operations, estimating and computing mentally. Hence it is important to know what kind of skills adults in ABE have developed concerning money issues and how they can deal with it in their personal, work and societal life. In this study it still pertains to the Dutch guilder as the currency system. From January 2002 the system will change to the euro currency.

In the IB placement test, items are offered that mainly reflect everyday life situations. Three types of money tasks have been formulated: pay and get change, estimate an amount, and basic operations with money. The results on these three types in the IB and Cito test are shown in table 4.23

Table 4.23 Mean scores on Money at IB and Cito

	Money	IB # items	IB % correct	Cito # items	Cito % correct
1	Pay and get change	7	64	1	38
2	Estimate an amount	5	53	-	-
3	Basic operations with money	4	58	5	53
	Total # items	16		6	
	Mean scores		59		46

In an initial impression, again, the results are poor. In general, it can be said that about 60% of the learners in this group can deal with money sufficiently, at least on paper. The Cito test even shows this percentage to be around 45%, less than a half of the group. These results do not necessarily indicate how these learners would deal with money in an actual situation.

4.3.4.1 Pay and get change

The IB test offers three types of items for the topic "pay and get change".

In the first type of items people can show their knowledge of and familiarity with bills and coins of the Dutch currency system. The bills and coins are shown in a table and the person is asked to indicate with which bills and coins a certain amount of money can be paid.

In the second type people are asked to round an amount. This is a basic skill in the Netherlands since there are no single cents in the Dutch currency system anymore. Amounts are rounded to the nearest five cents. However, this system will be changed when the euro becomes operational in 2002: then the cent will come back. Also, since the introduction of electronic payment in the Netherlands the cent came back into the payment system and the exact amount could be paid again, but only electronically. Nowadays, it is possible to pay electronically in almost all supermarkets, gas stations, stores, shops, etc. These changes, particularly the new currency system of the euro, will influence people's proficiency to work with money.

In the third type of items people are asked to pay the exact amount for a few purchases in a supermarket and to compute how much change they will get when they pay with a bill of 10, 25 or 100 Dutch guilders. These are the common operations in everyday life situations. Paying amounts larger than 200 guilders will seldom occur because of the possibility for electronic payment. The results on these specific items are shown in table 4.24.

Table 4.24 Pay and get change

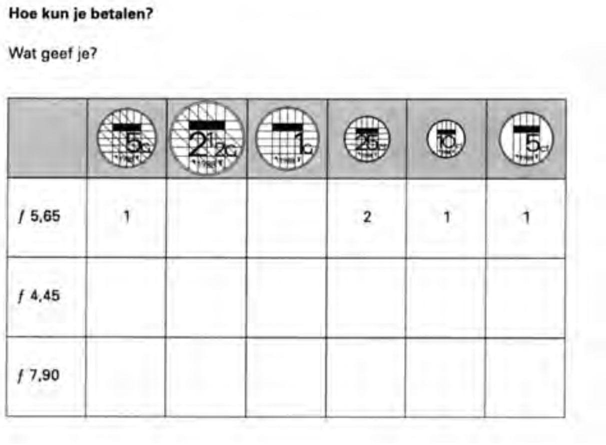
Item	Topic	# correct	% correct
A1-6	pay and get change up to *ƒ* 10,00	25	78
A1-7	pay an amount up to *ƒ* 10.00	17	53
B1-1	rounding an amount	28	88
A2-7	pay and get change up to *ƒ* 100.00	23	72
A2-8	get change from *ƒ* 100.00	23	72
A2-9	pay an amount up to *ƒ* 100.00	20	63
A3-9	pay and get change up to *ƒ* 200.00	14	44
	Mean score	21	67

Paying the exact amount with bills and coins.

To gain insight into the learners' knowledge about the Dutch money system, items ask respondents to show which bills and coins they would use to pay a certain amount. Two items were constructed to offer the possibility for learners to do this in their own best way (items A1-7 and A2-9). The first item (A1-7) shows only coins in a table along with the amounts to be paid, *ƒ* 4.45 and *f* 7.90. Learners are free to check the coins they want. They need to know the notation system. One example is given as a sample in the first line. The second item (A2-9) also shows a table but with all bills and coins up to 100 guilders. The amounts here are *f* 68.35 and *f* 87.95.

The text with the table is short. It only says: "How can you pay? What do you give?" This was done to keep the reading level as low as possible. In combination with the visual instruction in the first line, this should be sufficient information. The test leader was allowed to read the text and to explain the sample item in the first line. Respondents were allowed to put checks in the cells to indicate the number of coins wanted, or to put numbers in the cells.

Figure 4.37 A1-7 Pay an amount up to 10 guilders



Hoe kun je betalen?

Wat geef je?

	5	2½	1	25	10	5
f 5,65	1			2	1	1
f 4,45						
f 7,90						

This item shows a way of constructive testing. The item offers possibilities for learners to show their own insights into the money system. There are several possible correct answers to each task. The only problem here is that learners need to be able to understand the table. It was not clear whether this mathematical activity could be expected of learners on level 1. Seventeen learners answered correctly on both tasks in this item, 8 adults passed only one item, 7 adults failed on both items. Only two learners left the table blank. It is possible that these two learners did not understand the table and skipped it. Two learners completed only one of the lines, but did it correctly. All other tables show correct answers or attempts.

With these results we may assume that completing the table was not a problem for most of the learners, though we don't know whether the learners who made errors, would have succeeded better with real money.

Similar conclusions can be drawn for the second item, A2-9. The question is: How can you pay the exact amount?

Figure 4.38 A2-9 Pay an amount up to 100 guilders - Zaara

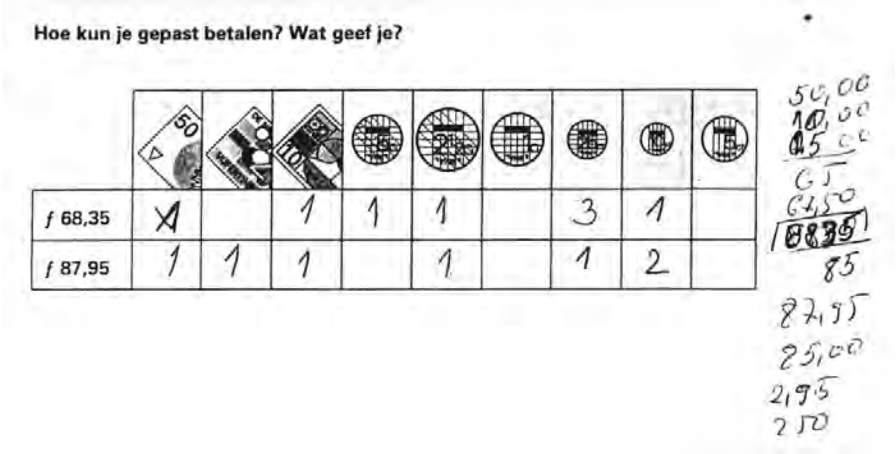
Hoe kun je gepast betalen? Wat geef je?

f 68,35	1		1		2	3	1	1	1
f 87,95	1	1		1	3		3	2	

The total amount is still less than 100 guilders. The table shows all bills and coins of the Dutch currency under 100 guilders, not including the 100-bill. In this item no example is given. The adult should know, from the first table, how to fill in this table. Twenty persons were able to do both tasks correctly. Eight persons passed only one out of the two items.
Zaara shows she understands this task, but has problems at the end with the cents. In the first task she added a five cent coin. She probably saw that it should end on "5" but forgot she had that already with the 25 cent coin. In the second task she compiled 95 cents, but forgot she had already 50 cents in the three 2.50 guilder coins. She did the level-1 item correctly.

Enver shows his understanding, however, he is not able to do it mentally. He needs his computations alongside the table.

Figure 4.39 Enver



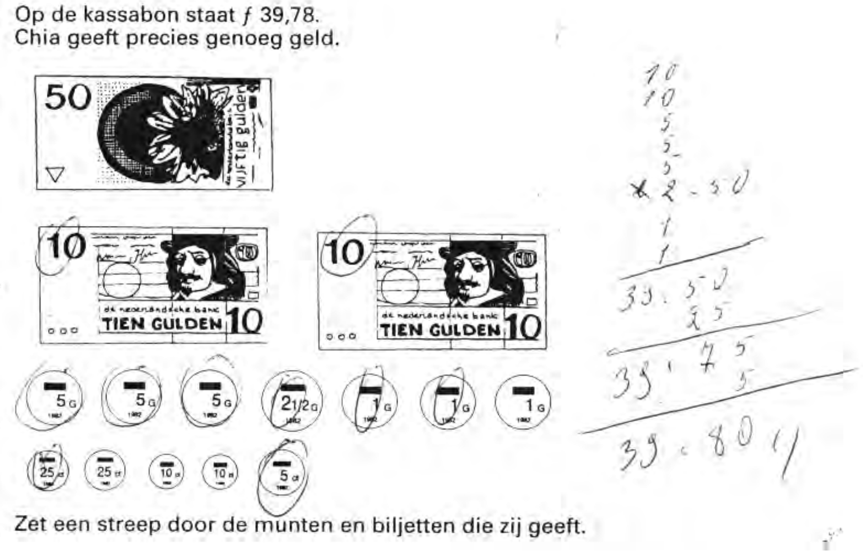
Hoe kun je gepast betalen? Wat geef je?

f 68,35	1		1	1	1		3	1	
f 87,95	1	1	1		1		1	2	

50,00
10,00
05,00
65
67,50
6835
85
87,95
25,00
2,95
2,50

Cito presents a similar item, though more difficult. Interesting here is that people have to compose the amount of ƒ 39.78 from a pre-selected series of bills and coins, like what one might have available in his or her wallet, simulating a real situation. Based on that people need to select those bills and coins that equal the required amount. Rounding the amount to the nearest five or ten cents is also included in this item. The item is good on its own, but it's weakness is that there is only one correct answer. Results: only 12 correct answers out of 32 (about 40%) Meta shows his understanding and a correct rounding. (figure 4.40).

Figure 4.40 C1-9 Pay an amount - Meta


Op de kassabon staat ƒ 39,78.
Chia geeft precies genoeg geld.

Zet een streep door de munten en biljetten die zij geeft.

A prerequisite to working with the Dutch currency is to be able to round an amount to the nearest five or ten cents. To assess this, one item is included on the lowest level in IB: B1-1.

It requires rounding of two amounts from everyday life, the prices of a container of milk and a piece of cheese. The prices for these items are ƒ 8.49 and ƒ 1.04. An example has been given to show what is meant and what people are supposed to do: a photo of a pack of sugar (1 kilo) with two amounts: the exact amount of ƒ 1.99 and the rounded amount of ƒ 2.00. About 88% of the learners did this correctly.

A preliminary conclusion here is that about 1/3 of these adults do not have good insight into the Dutch currency system. In addition, though in some situations it is not clear whether people failed because of the money system or because of not understanding the table, the impression is that most people did understand the instruction in the table and that people who failed made only a few mistakes in the choice of bills and coins.

The third type of items at pay-and-get-change pertains to shopping. In this test the shopping items build up from a total of 10 guilders to 25 guilders and to 100 guilders. See figure 4.41.
Twenty-four learners out of 32 (75%) were able to compute the total amount of three different items and/or to give change from *f* 10.00 .

Figure 4.41 - A1-6 - Shopping - Pay and get change

Boodschappen.

Questions are:
1. You want to buy these items. What do you have to pay?
2. How much change do you get from *f* 10.00 ?

Mohamed	Nadia	Meta
10 gulden	4, 33	5,33
5.10	6,33	4.67
		2,55 1.79 99 5,33

The rule for computing a total amount is to first add precisely and then round the amount to the nearest five or ten. However, an easy way of counting is to round first, then add and finally compensate for the rounding, for example: 1 guilder plus 1.80 makes 2.80. After that add 2.50. That makes 5.30. Finally compensate the 2 added cents with the 5 taken away, so add 3 cents. The precise amount to be paid is 5.33 or, rounded, 5.35. This kind of computation can obviously only be done with a few amounts, not with a long list of shopping-items.
It is expected that to the question: "What do you have to pay?", people will mention the rounded amount. However, the exact amount was also marked as

correct. The same was the case for the amount to get back from the cashier. Here both amounts, *f* 4.65 and *f* 4.67 were counted as being correct.

Though the item questions seem clear, Mohamed did not understand this task. To the first question he responded with “10 guilders”. He may have assumed that he should pay there with 10 guilders, because in the second question it is asked how much change he will receive from 10 guilders. This is a very logical error, but it may indicate that the question is not quite clear. His answer to the second question (*f* 5,10) cannot be derived from his computations. It is not clear whether this is the amount he is supposed to get as change, or that this is the total amount he has to pay for the three items. Perhaps the first question ought to be changed to “How much do you have to pay?”
Nadia shows an error in the addition and gets 4.33 instead of 5.33. In the second question she only looks at the guilders. She writes 6 guilders change. After that she adds the 33 cents to the amount. This type of error is very common in ABE.
Meta computed the total amount by applying the addition algorithm. He only shows exact computations and no rounding.
Nadia and Meta do not apply the rounding as should be done in the Netherlands. The amount to be paid should be rounded to *f* 5.35, if they would pay in cash.

4.3.4.2 Estimate an amount

Good estimation procedures used in combination with good mental math strategies and the calculator are important in today’s society. This opinion is part of the Dutch RME and is verified in everyday life and work situations.
The need to master traditional algorithms is less important. However, it is desirable to be able to do computations on paper based on mental math in addition to estimating and using a calculator.
The process of estimation usually includes: rounding up or down to nice or to the nearest round numbers, then doing the computation with round numbers, and at the end, knowing about how much more or less the estimation is than the exact answer would be, within certain boundaries. Estimating with money while shopping is a very common situation in real life and is an ideal means to test people’s understanding of estimation. The IB test offers a few items on this topic, A1-8, A1-10, A2-10, A3-10 and B3-1. The results on these items are shown in table 4.25

Table 4.25 Items at estimation

	Estimation	# correct	% correct
A1-8	Estimate a total amount up to *ƒ* 20.00	14	44
A1-10	Estimate a total amount up to *ƒ* 25.00	26	82
A2-10	Estimate an amount up to *ƒ* 25.00	11	34
A3-10	Estimate a total amount to *ƒ* 200.00	10	31
B3-1	Price per kg. , estimate price of 2,5 kg fruit	24	75
	Mean score	17	53

The illustration in item A1-10 shows Fatima with her purchases and a bill of 25 guilders. The total amount can be estimated and is about 26 guilders. The only question is "Does Fatima have enough money?" The learner is asked to circle the right word, yes or no. The learner is free to do the computation in his or her own way. Though there is a 50% chance of choosing the correct answer, the results were surprising. Many learners show computations for this item, meaning that most of them looked seriously at this item and did their computations before they circled yes or no. On the other hand it is worrisome that so many learners carried out computations, because the item was meant for estimation based on mental math.

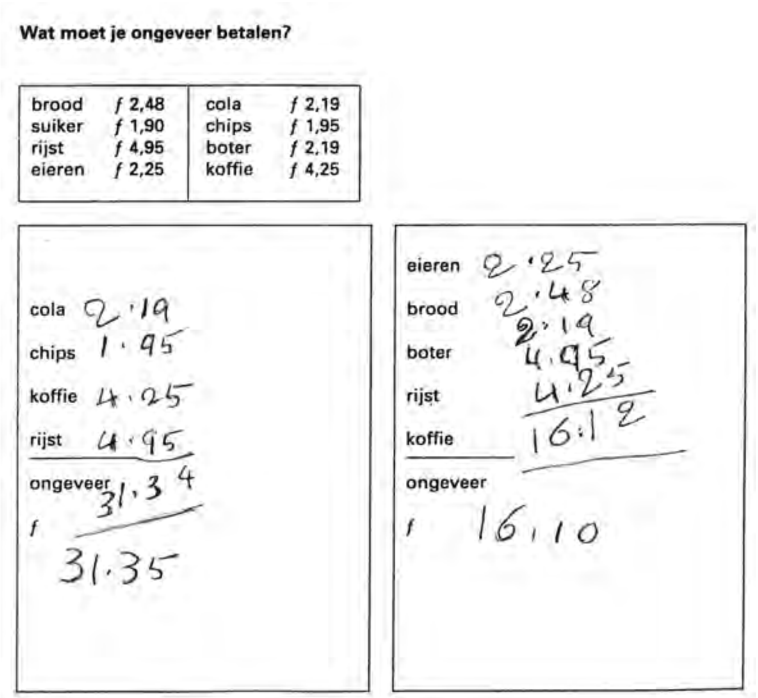
The expected process was: round the prices in the shopping basket up or down, add the rounded numbers and compare with the bill. Since this is a level 1 item, the numbers were chosen so that it is easy to round them up or down to the next guilder or 50 cents, to make the estimation easier. Though it was not necessary to do a precise computation, many learners show precise addition algorithms. In total 23 learners arrived at the correct answer (72%).

The same things happen with item A1-8. Even though this item pertains to very small amounts and only a few shopping items, and people can choose a suitable estimated amount from the choices below the shopping lists, most of them show addition algorithms, even when they first rounded the amounts.

Such is also the case for item A2-10 (see figure 4.42).

Here we see similar procedures. In the left box the learner is asked to estimate the total amount to pay for: coke, chips, coffee and rice (items). In the right box the same question but then for: eggs, bread, butter, rice and coffee (5 items). The learner is expected to do an estimation by rounding the prices to the nearest rounded amount. The prices are created in such way that the nearest five or ten leads to a nice round number, but prices can also be rounded to the nearest guilder or 2.50 coin or 5 guilders coin to be considered a good estimation. In practice we see that many adults first do a precise addition using the addition algorithm and then either round or not. There is no estimation, only a rounding of the amount at the end.

Figure 4.42 A2-10 - Estimation. About how much would you have to pay? - Mohamed



Wat moet je ongeveer betalen?

brood	*f* 2,48	cola	*f* 2,19
suiker	*f* 1,90	chips	*f* 1,95
rijst	*f* 4,95	boter	*f* 2,19
eieren	*f* 2,25	koffie	*f* 4,25

cola 2·19
chips 1·95
koffie 4·25
rijst 4·95
ongeveer 31·34
f 31·35

eieren 2·25
brood 2·48
2·19
boter 4·95
rijst 4·25
koffie 16·12
ongeveer
f 16,10

Mohamed (figure 4.42) adds using the algorithm and is not surprised when the answer results into more than 30 guilders for a few shopping items. It might be possible that he turned 13 guilders to 31 and did not notice the difference. Though the question asks for estimation, he added precisely and rounded only to the nearest five and ten.

The same type of problem appears at level 3 in item A3-10, where people are asked to estimate larger amounts. Though the numbers are easy to round to nice numbers, most people continue doing computations on paper, using the addition algorithm, and then they round the number to the final amount they would have to pay.

The question here is whether these adults are not used to doing estimations with money, or doing estimations in general. This may depend on what is common in their home countries. A preliminary impression is that they learned (in school) how to do algorithms, but did not learn how to estimate. Here it also becomes clear that some items are not suitable to assess estimation. Some items tend to suggest doing precise computation. Further investigation must be done into appropriate types of items to be used for testing estimation.

4.3.4.3 Basic operations with money.

In addition to estimation and "pay-and-get-change" items, a few items are included about simple precise addition, multiplication and division, shown in table 4.26.

Table 4.26

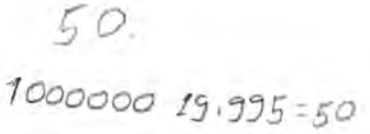
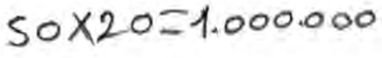
		# correct	% correct
B2-1	Addition to 10 gld	25	78
A3-8	Addition to 1000 gld	25	78
A4-8	Multiply an amount up to *f* 1000.00	16	50
B4-1	How many cars for one million?	8	25
	Mean score	19	58

On level 2 (B2-1) there are two additions up to 10 guilders of two shopping items each. People are asked to check the right answers, including rounding. One computation has been given as a sample. This item appeared not to be difficult since 25 learners scored correct on both items and 6 learners on one item. This also appears on level 3, where two amounts have to be added: the amount of a laundry machine (*f* 870.00) and that of a microwave (*f* 390.00). Such amounts are apparently easy to work with, whereas the estimation item on the same level but with lower amounts, caused more problems.

In the range of difficulty we see that addition is not a problem for most people, 78% correct, but when multiplication and division are required, the percentage correct goes down to 50% and 25%. Since these items are on level 4, this is not really surprising. However, the multiplication item (A4-8) on its own was not so difficult: "What would you have to pay for 4 new car tires of *f* 235.00 each?" Hence it is alarming that only 50% could do this multiplication correctly.

The final item in this series is a problem where people can apply either addition, multiplication or division. The item shows an advertisement of a car, priced *f* 19,995.--. The question is: "How many of these cars could you buy for one million guilders?" (figure 4.43)

Figure 4.43 B4-1 - How many cars could you buy for one million guilders?

Zeki  Hoe reken je dat uit? 19995 50 999750 Antwoord: 999750	Valentina  Hoe reken je dat uit? 1000000:19.995=50 19.995.50 00000 4 99975 999750 Antwoord: 50
Nezima  50. 1000000 19.995=50	Hatice  50X20=1.000.000

Nezima and Hatice show that they are familiar with the characteristics of the numbers in this item. Both are sure that the answer should be 50. Zeki again shows a multiplication algorithm, but he chose the right number for his multiplication, so he must have known the correct answer upfront. However, he writes the wrong number in the answer box. Valentina also found the correct answer but she too shows a multiplication, even with a line with zeros in it. She writes the correct number in the answer box.

4.3.4.4 Preliminary conclusions at money

The results for the money items must be seen in the light of working with the guilder as the national currency. At the time this study was undertaken the new currency, euro, was hardly known. Now that the euro is about to become operational (January 2002) everyone in the Netherlands will have to get used to the euro. This may change the situation in the Netherlands for a while since most may have problems with payments in euros sometimes. However, looking at the type of tasks in this study, we may assume that it is not the currency itself that causes problems for the adults in this study but the underlying operations like pay and get change, estimation and rounding, and doing basic operations with money. The individual actions may change due to the fact that rounding is not necessary anymore in payment situations, but we may assume that similar problems will continue to consist.

In addition, although working with money is an everyday activity for most adults, it is not clear that they can also do money tasks on paper in a school setting. The results in this study show what people know about the money system and how they can work with it in a learning setting.

Conclusion 22
About 1/3 of the learners in this study do not show good insight into the Dutch currency system when they do tasks on paper. We do not know whether they would also have problems with the currency system in real payment situations.

B22
It is advised to start money tasks in learning settings with real money and simulation of real-life payment situations.

Conclusion 23
Doing estimations with money appears to be a problem for many learners in this study. They prefer to do exact computations and then round up or down to the next round number.

B23
The way most learners round up and down after first having done a precise computation, can be used as a start for learning to estimate. This can be combined with doing computations with round numbers. (see B7)

Conclusion 24
Only 8 adults (25%) were able to do computations with a large amount of money. We may wonder how all other adults in this study will deal with big amounts of money in real life situations, like doing computations regarding a loan for a car, or figuring the new rent after an increase.

B24
The ability of adults to do computations with round numbers and smaller amounts could be a start to work with larger rounded amounts of money, also in combination with a calculator.

4.3.5 Reading and Understanding Simple Data

The final section of this field analysis concerns reading and understanding data. Although this is not a basic skill to survive in everyday life, it is an essential skill nowadays to be able to participate in the information society. In many situations information (e.g. results of elections, population growth, statistics about nature life and environment problems) is presented in graphs or tables. In work situations it is becoming more important to be able to read and interpret information in graphs and tables that pertains to work instructions. People need to be able to process this kind of information to understand what is going on in society. All such activities are part of statistical literacy, which can be seen as part of numeracy and quantitative literacy. (Gal and Garfield, 1997, Gal et al, 1999, Forman and Steen, 1999, 2000, Steen, 2001)

The data items in the IB placement test are based on an experimental strand in the IB series because there is little experience in Dutch adult basic education in testing, learning and teaching about processing data. Therefore, it is interesting to see the results on these items and to discover the knowledge and skills learners in ABE have developed in this field. This may also provide information by which the strand can be evaluated and improved. Four items were created, one for each level. The first item concerns information about the sale of clothes in a street market (B1-9). The second one is a line graph about temperature (B2-9). The third item is about information required in an interview item on smoking and no smoking (B3-9) and the final item (B4-9) involves the interpretation of an annual overview of car sales. The Cito test has only one item in the second phase high test (C2-H-10). It asks for adding percentages, based on presentations in a bar graph. The results on these items are shown in table 4.27

Table 4.27 Results Reading and Understanding Graphs

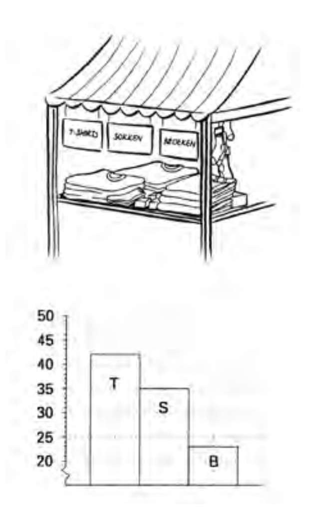
Level	Item	Topic	# correct	% correct
1	B1-9	reading bar graph, total market sales (to 100)	25	78
2	B2-9	reading line graph, temperature	23	72
3	B3-9 *	putting data in circle diagram - proportions	18	56
4	B4-9	reading bar graph, car sales (in thousands)	4	13
		Mean score	18	55
	C2H-10	add percentages visualized in a bar graph	3	9

The item at level 3 was already discussed at the section about proportions (4.3.2). In the total overview in the introduction this item was counted in “proportions”, not in “reading and understanding data”. This item is inserted here in this subsection as part of the data items to see the results on data interpretation. The results in table 4.27 do not affect the overall results. Without including this item here in this subsection, the mean score on #-correct items would still be 18 and the mean %-correct would be 54.

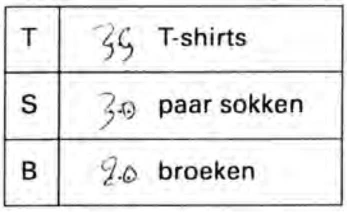
4.3.5.1 Item discussion

The context represents a street market salesman who has to keep a record of his sales per day of T-shirts, pairs of socks and pants (B1-9). The sales of each item on one day are shown in a bar graph. It was assumed that such a context would be understandable to most of the adults. Since this item was only attempted by seven learners (see section 4.2.), the rest of the correct scores on this item was gained by bonus points. The bonus points imply that all other people who took B-tests on a higher level, would have passed this item if they had taken it. From the seven adults who actually did this item, six responded incorrectly and one just skipped it. This means that, in fact, there was not one real correct answer in the results of this field research to this item. Despite that, a few interesting results can be shown.

Figure 4.44 B1-9 Street market

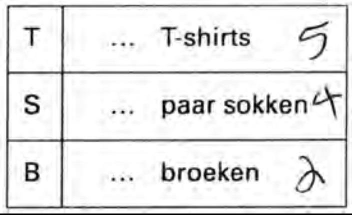


Zaara



T	35 T-shirts
S	30 paar sokken
B	20 broeken

Margaret



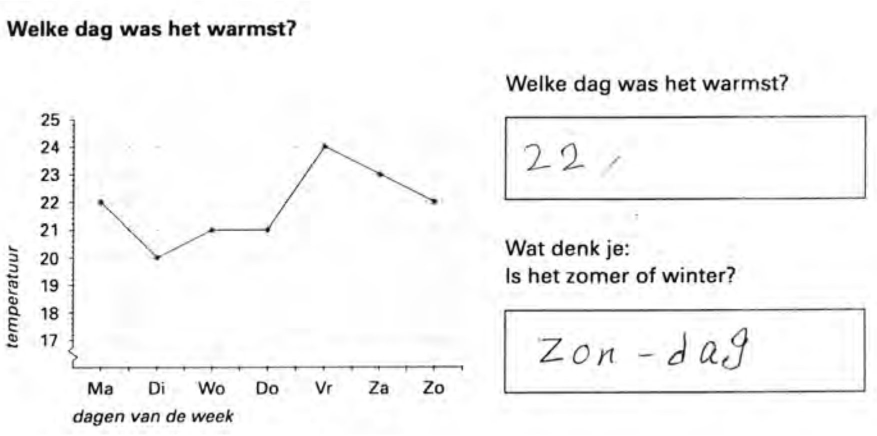
T	... T-shirts 5
S	... paar sokken 4
B	... broeken 2

A few learners show some understanding of graphs but do not exactly know what to count. Zaara determined the total number of items by counting them to the line below the letter that is in the bar (T, S, B). Mohamed did the same. They both counted 35, 30 and 20. Margaret counted the number of horizontal lines in the underlying grid of the graph and counted 5, 4 and 2. Fadumo put a check at the T in the bar and in the table. Farangis circled the B in the bar and the table.

From these examples we may learn that Zaara and Mohamed have a little understanding of a bar graph, but could be confused by the letters in the bars. The others at least tried to do something with the graph and created a link between the bars in the graph and the accompanying words in the table. It is not likely that these learners are used to interpreting such bar graphs.

The second item is a line graph about temperature (B2-9). It shows the temperature in one week from Monday till Sunday. The first question asks “Which day was the warmest day?”. The second question asks: “What do you think, is it summer or winter?”

Figure 4.45 B2-9 Temperature - Dina



Welke dag was het warmst?

Welke dag was het warmst?

22

Wat denk je:
Is het zomer of winter?

Zon - dag

Dina answered 22 and Sunday. It is possible he chose the first and the last number/mark on the horizontal axis. It could also be possible that he was confused by the words “zomer en winter” and interpreted “zomer” [summer] as “zondag” [Sunday]. This could be a language problem.

Mohamed indicated the warmest day was 25 degrees, the top of the vertical axis.

Enver and Nezira indicated the warmest and the coldest day (Friday and Tuesday)

Nadia asked to have the question read aloud. She said she had never done such a task before, but she understood the graph. After the questions were read out to her, she answered correctly.

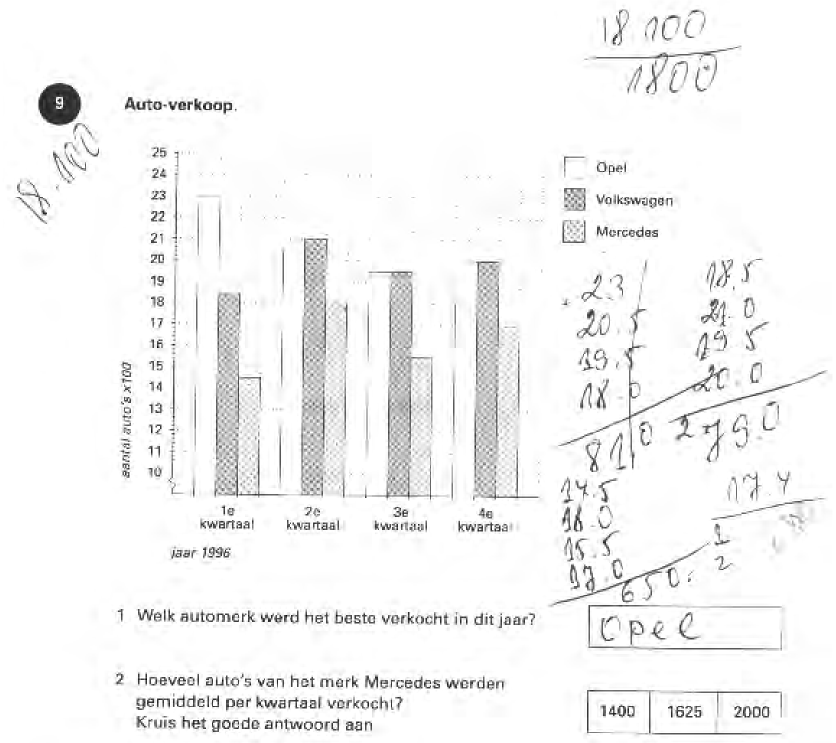
An interesting problem encountered here was a cultural one of a learner coming from Ethiopia who did not know the difference between summer and winter. He responded orally to this question afterwards: “It is summer in the morning and winter in the evening.” Only four learners did both tasks correctly.

The third item (B3-09), the interview item, was already discussed at section 4.3.2.2. There it was discussed that *about half of the learners don't actually have insight into the concept "a quarter". A second conclusion was that a circle could be a good mental model to show proportions. It could also be of help with gaining insight into data processing.*

The final item in this field is a bar graph about car sales (figure 4.45).
The first question here is: Which car brand sold best in this year?
The second question is: How many Mercedes cars were sold on average per quarter of a year? Check the right answer.

This item looks much more difficult than the other three items, but in fact it is not necessary to do any computations. For the first question a good reader can see at once that Mercedes was third on this list. One has to look more carefully to compare the sales of Opel and Volkswagen, but can look at the differences in height of the bars. Opel had the most sales.
The second question is a multiple choice item with three answers. The first answer shows the number 1400, which is below the minimum sales of Mercedes, so could never be the average. The third answer shows the number 2000, which is more than the highest sales of Mercedes, so could also not be the right answer. Therefore the second number must be the right answer. This can be easily checked by comparing the number 1625 with the sales in the four quarters.
Only Valentina showed her computations on paper (figure 4.45). She shows her understanding of the first question, but her blind ciphering shows she does not really "interpret" the data from the graph. She did not respond to the second question.

Figure 4.45 B4-9 - Car Sales - Valentina



1 Welk automerk werd het beste verkocht in dit jaar?

Opel

2 Hoeveel auto's van het merk Mercedes werden gemiddeld per kwartaal verkocht?
Kruis het goede antwoord aan

1400	1625	2000

From the 13 learners who took this item, 4 learners passed both tasks, 7 learners passed only one question and 2 failed on both tasks. Six out of the seven adults who passed only one question, scored correctly at the first item question (Opel). Only one person of this group passed the second item question.

A preliminary conclusion to this item: ten adults could analyze the first question, only 5 adults could determine the average. Four adults answered correctly to both questions. This result may indicate that the item in its own is quite understandable for learners on this level. It also indicates that the concept of 'average' is not very familiar to these adults.

A general conclusion to these graph items:
Though the learners in this study show some insights into reading and understanding graphical information, the results in this study indicate that this topic needs careful attention in ABE.

4.3.5.2 Preliminary Conclusions

In general it may be said that, though the results are low, most learners show some understanding of reading data in graphs. The results are low but not worrisome. Many learners at least tried to find out the information that is required and showed a little bit of understanding in reading graphs but are not used to interpreting information in graphs. The fact that they tried to read the graphs and showed some attempts is hopeful for adult education.

Conclusion 25

Most learners are able to analyze a simple bar or line graph (75% correct). About half of the learners are able to indicate 25% of a circle. This shows that they have a notion of proportions and simple statistics. When it comes to more complex graphs, like in the car sales item, only 4 adults (12.5%) are able to analyze this information and to compute an average.

B25

The results indicate that these adults can handle data in simple graphs. This knowledge can be used to learn to process data in more complex types of graphs and to acquire simple statistical notions. Careful attention must be paid to development of instructional materials in this area.

4.4 Conclusions

After the extensive analyses of the placement test results we will look back at three aspects:

1. the *results* and *characteristics* of the learners in this field study,
2. the *goals* of numeracy assessment as described in chapter 3,
3. the evaluation of the IB placement test in relation to the *criteria* for assessment tools, also described in chapter 3.

4.4.1 The results and characteristics of the learners

In general it can be said that the learners in this study have acquired only minimal mathematical knowledge and skills. Compared to the Cito levels, which are in line with the Dutch KSE levels, described in chapter 1, and compared to the numeracy levels, as described in chapter 2, we can conclude that most of them are still at a very elementary level of numeracy (KSE levels 1 and 2). Only two learners entered the lower region of functional numeracy (KSE-3). The question arising here is how their school knowledge and skills can be compared to their practical knowledge and skills, and how they can profit from these school skills in everyday life and work situations.

Most learners are able to count objects but about a quarter of them have problems with counting objects in a photo of a three-dimensional structure. Most of them can count with benchmark numbers such as 25. This may indicate that counting with benchmark numbers can be used to learn to do estimations and operations with large numbers. Most learners have insight into the place value of digits in small numbers, like 1436 or 3487, but when it comes to larger numbers, such as 1,357,328, about 60% of them have problems with the structure of the numbers. This may indicate that they also have problems with insight in large numbers.

In basic operations it is remarkable to see that most learners prefer to do algorithms. For example, for tasks where an estimation was suggested, they first did a precise computation by using an algorithm and then they made it an “approximation” by rounding up or down. Only a few algorithms were included in the tasks and these could also be done in more creative ways. We had hoped for more informal, self-invented ways of doing computations, based on practical knowledge and real life experiences, but most learners preferred to use algorithms. Wherever possible they applied algorithms. It seems as if they do not feel free to try to do computations in alternative, more creative ways. It is not clear here whether this is due to the school situation or that they really cannot do computations in different, more creative ways. It is interesting to see the variety of algorithms that were applied. This suggests that in multicultural learning settings people must be free to apply what they have learned in their native countries. The different procedures the learners bring could be a basis to start discussions about their own algorithms. In that way more creative ways of doing computations, based on insight and on mental math, could be introduced to substitute the traditional algorithms they often apply without any insight. Tasks in which they are invited to create their own computations and context problems that can be solved in different ways are good means to encourage alternative solving procedures.

A second important issue here is that these learners tend to only do exact computations, even when they are prompted to find an approximate answer. They are probably not used to doing estimations. To estimate they need to learn to work with round numbers as benchmark numbers for mental math. Rounding numbers up and down is necessary for estimation and could be a start here, since most of them master these skills due to the Dutch currency. In combination with new, flexible computation and notation systems this could be of help in developing functional math skills.

In context tasks they show some more of their individual way of thinking and apply different solving procedures. For instance, in the multiplication/division context in A4-6: “There are 40 glasses in a box and a restaurant owner wants to buy 3000 glasses; how many boxes should he buy?” Here we see variations in computations. This suggests that contexts, related to real life situations and offering more possibilities to find the answer, encourage adults to develop more creative and alternative solving procedures.

In proportions a few learners show some more informal strategies and individual mental models to solve proportion problems. The typical Dutch expressions used for proportions may cause confusion sometimes because they may be different from the way in which proportions are expressed in different languages may vary across countries. Working with proportions appears to be easy for many learners. This is a good sign, because proportions can be used as a basis for learning fractions, percent and decimals. These latter areas are a large concern for this group. About half of these learners don’t actually have insight into a “quarter”. They show very little insight into fractions. One advantage here is that most of the learners tried to do the pizza problem and showed their insights into proportions. This could help when learning benchmark fractions and the language of fractions.

In percent, only a quarter of the group could do computations with benchmark percentages and only one learner was able to do computations with other percentages, like computing a rent increase of 2%. This indicates that these learners cannot deal effectively with many everyday situations that require computations with percent, such as sales and discount offers. It may also be difficult for them to understand simple statistical information in which percentages are used.

In the field of measurement the learners can use elementary skills to measure with standard units in length, weight and volume, related to real life situations, but they fail at formal computations in the metric system.
Nearly all learners can read time and are able to do very simple computations with time, such as computing travel time. However, only half of the group succeeded at more complex computations like doing computations with time-tables for work and conversion of seconds into minutes. It is possible about half of the group does not even know that there are sixty seconds in a minute.
It is also disturbing that about one-third of these learners do not have good insights into the Dutch currency system. In paying an amount they make mistakes in their choices of bills and coins. In addition, these learners do not choose to do estimations with money. When they are asked to estimate they tend to first compute the exact amount and then round up or down. However, these skills can be used to learn to estimate.
Finally, about 75% of these learners could read and understand simple data in graphs and about 10% could interpret a more complex graph. This is hopeful because processing data by reading and understanding graphs is an important topic nowadays and must be part of numeracy education.

For better learning of mathematics it is desirable that these adults become more familiar with alternative, more creative solving procedures. Therefore, it is a good sign that these learners are sensitive to visual instruction. This could be very helpful and could also support their language skills. Working with representations of reality could help create links between their fragmentary knowledge in several topics, like proportions, fractions and percent. The use of real materials for instruction, good photos of real materials and carefully selected and created mental models that support their thinking, could be critical in learning and teaching mathematics.

The results show that the learners in this study have acquired only elementary skills in mathematics. Since this group is only one out of many learners' groups in ABE in the Netherlands, this may mean that there are many more adults who function on this low level of numeracy. This indicates that programs will have to focus on the development of good, flexible basic skills that lead to numerate behavior in general and also to easy access to further education and vocational courses. These adults first need elementary numeracy skills to be able to participate in society, for example: arrange family matters, do the shopping, talk with teachers about the progress of their children, help their kids with homework, etc. In addition, they will have to be prepared for work situations, for doing vocational courses or other educational programs.

Given these results we may argue that mathematics should at least be an obligatory part of the ABE program to improve the numeracy skills of learners in general and to help prevent dropping out in follow-up courses. We may also argue that programs in ABE should not exist of “teaching math” in the way these adults were taught in former school days, but should focus more on "*learning how to solve mathematical problems in real life*" which leads to numerate behavior. Real-life experiences and imaginable, recognizable real-life situations should be the core of such programs. These adults need math that is "functional" and directly applicable in actual situations.

4.4.2 Looking back at the goals of testing adults

In this subsection we look back at the goals of numeracy assessment as described in chapter 3 in relation to the results in this field study.

1. Learn more about the mathematical knowledge and skills adults have acquired when they enter ABE in order to determine what needs to be learned and to place them in the right courses.

The conclusions in the previous section, based on the results of the placement test, indicate what kind of mathematical knowledge and skills these learners have acquired when they enter ABE. From this information we can derive what kind of mathematics they may need. The IB test offers the possibility to analyze every individual test on specific mathematical problems, such as problems with three dimensions on paper or language problems, like the learner in the pizza problem (table-chair). The learners in this study show that it is important to start a mathematics course with a well-structured placement test. Based on the results of the Cito and IB test we may conclude that the more detailed the test, with items from all mathematical fields on different levels, the more qualitative information can be acquired. For these learners a diagnostic test is necessary to discover their individual competencies and needs. The IB test shows a finer-grained level indication and item pool than Cito. Whereas the Cito test, not intended to be a diagnostic test, measures a general proficiency level, the IB test indicates nuances of performance over more detailed levels. In a system where courses in learning routes are tuned and worked out in detail, such detailed information should make it possible to place these learners in the right courses and to plan what needs to be learned.

2. Learn more about adults' problem solving procedures in order to improve instruction.

The building blocks at each subsection provide ample information for teachers to adjust their ways of teaching to the incoming learners. Three main issues need further explanation.

1) In general, these learners have shown a tendency to keep to the traditional procedures they learned in school. Only a few adults show individual, idiosyncratic procedures. In context tasks they show some more their own thinking procedures. Because these learners come from different countries they

apply different algorithms. Discussing their algorithms and solving procedures while doing context problems could be a start for developing more creative problem solving procedures.

2) Many learners show fragmentary knowledge and skills. For example, they can compute 10% off by moving the decimal point one place to the left, but may not know that this is the same as 1/10 or divide by 10. This indicates that teachers should try to create links between topics in order to develop more complete insights into mathematical concepts.

3) Learners show they have problems linking school mathematics with real-life situations, in particular at doing computations and estimations with money. This indicates that teachers should encourage learners to work actively with real-life materials and representations of real-life situations. Everyday situations provide starting points for developing mathematical knowledge and skills but serve also to make mathematics applicable and to develop functional mathematics skills. The teacher should try to create a bridge between school mathematics and real life mathematics.

3. Learn more about adults' numeracy skills to be able to fine-tune educational programs to their needs and wishes.

The IB placement test indeed discloses much of the learners' knowledge, skills and needs. The building blocks listed in previous sections provide good ingredients for the development of educational programs in ABE. Such information may help curriculum developers to develop series of courses on different levels for elementary numeracy and to fine-tune programs within these courses to the learners' need and wishes.

Networks of courses could offer sufficient possibilities for learners to find their own way via individual learning routes in adult education. Support of learners by tutors will be necessary.

4. Monitor and document the individual learner's progress in the course of the numeracy program in order to guide the learners through their own learning routes and to prevent learning problems and drop-out.

The scores on the placement test provide profiles of incoming learners (see pages 177 and 178). In a well-developed curriculum formative tests should be an integral part of the program to monitor the progress of the learners, but also to create links across different courses. Also, in a well-organized learning setting individual support may help to prevent drop-outs and to adjust or change learning routes, if necessary.

5. Be able to evaluate numeracy programs in order to improve numeracy education in general.

The information acquired in this placement test could be of help for evaluating and improving existing numeracy programs in ABE and creating new programs. By recording such information nationally and internationally, also from formative and summative assessment to measure progress, a broad basis of knowledge could be acquired about adults' actual numeracy skills, in order to develop a scientific basis for learning and instruction in ABE.

6. Enable policymakers and program developers to adjust policy and numeracy programs to new demands and developments in the labor market and in personal and social life.

The results of this study show the starting situation for this group of adult learners. We must realize that this study concerns only one group of non-native ABE learners, but this group is only one of many of such ABE groups. We can assume that the national average of ABE learners will not differ much from this group, though we may wonder what other kinds of math problems can be encountered with native speakers in ABE.
The very low average mathematical level of these adults could be an indication for policy makers and employers to pay careful attention to these learners in ABE because these adults are their future employees. Curriculum developers should develop programs that focus on further development of the elementary competencies and skills these learners have acquired. The results of this study may provide qualitative information for this. These adults first need well-developed fundamentals to become numerate in general and after that they could go on to courses for vocational training. In current ABE programs the learning time is often too short for most of these learners. The current policy states that when second language learners have achieved sufficient mastery of the Dutch language, they can start doing vocational courses; mastery of mathematical skills is not taken into account. In vocational courses the mathematics programs focus on functional professional skills. It is becoming more evident that these programs do not link with ABE programs. There is still a big gap and there are still too many people who drop out (40-60%). Programs that offer more fundamental and functional mathematics, adjusted to the needs of these adults, do not exist in vocational programs. There may be math programs for individual support, but these can only be taken as part of a remedial program and depend on the setup of individual vocational programs. The point is that these adults do not need remedial math programs, but fundamental mathematics programs that focus on functional numeracy.

For policy makers it also means that these adults, after they have passed their ABE and vocational courses, will certainly need follow-up courses and training for further development and to keep up with technological developments. In the light of lifelong learning, policy makers and employers should facilitate courses for further specific training. The results of this study may provide qualitative information for developing such programs.

4.4.3 Looking back at the criteria for assessment tools

In this subsection we look back at the criteria for numeracy assessment tools in ABE as described in chapter 3, related to the instruments used in this study.

1. Numeracy assessment in ABE should be done in an appropriate way for learners in ABE.

When adults have mastered the language and have school experience, a paper-and-pencil test can be an effective test to gain a good view of the adults' skills in a short time, particularly when respondents are asked for their computations on paper. These computations reveal their procedures and thus can be used for better diagnostic analyses, if necessary and desirable. A precondition here is that the test is made up of a set of items that reflect the most important mathematical fields, with representations on different levels. However, for second language learners it must be taken into account that there may always be unforeseen language problems, for instance the pizza problem, where a learner answered "chair" when the word "table" was given in a previous answering box. (section 4.3.2.2). Visual support may help overcome such language problems, as was shown at the place value items. Learners should always have a chance to ask questions when they do not understand the item question. In specific situations, when people have not mastered the language sufficiently, the test could also be taken orally. For low-literate adults an oral interview, using the same test materials, is most appropriate.

Though not used in this study a few words must be spent on the use of computer adaptive tests for placement.
Given the results of the learners in this study we would argue that the use of computer adaptive tests for placement in ABE is still questionable since it is difficult to trace problems learners could have encountered during the test. It is also difficult to acquire diagnostic information about the learner's strengths and weaknesses. However, the developments in this field are worth to be examined. In general, there are two preconditions for the use of computer adaptive tests at placement:

- tests must provide detailed information about the learners' results per level and per sub-field.
- learners must have developed adequate computer skills to take such a test.

2. Assessment in ABE should enable adults to show what knowledge, skills and procedures they have acquired when they enter ABE, rather than what they do not have.

The results, in particular the computations learners show in this study, indeed provide information about what mathematical skills the learners have acquired.

In the context items, for example, they show clues to their thinking by the various ways in which they tried to solve the problems. They also show, for example, that they prefer doing algorithms and are not used to doing estimations. In particular the informal computations, as shown, for instance, by Margaret, Nadia and Zeki in figure 4.23, by Zeki in the work item in figure 4.28 and by Nadia in the paint item in figure 4.33, provide qualitative information for learning and teaching.

3. Numeracy assessment in ABE should provide insight into mathematical procedures and problem solving.

In general, in the IB test the learners are asked to write down their computations next to the tasks. This yields important information about their mathematical procedures. Also, many tasks in the IB test offer various possible methods of solution. Learners can apply their own best solving procedures to get at the right answers, like at the box items (A4-5 and A4-6) where they can apply counting, addition, multiplication and division strategies, or at the 'get at' items where they can choose combinations of numbers and construct their own computations. (section 4.3.1.3) Such items leave room for the learners to show their strengths. Information acquired from the placement test should be available for the teachers who are going to work with the learners.

4. Placement tests in ABE should reflect the goals, content and levels of the math curriculum so that adults know what they can expect during the course and can be placed in the right course.

In the IB placement test all fields are systematically represented on all levels. The type of items also reflects the content of the curriculum. Results and scores at subtests can be discussed with the individual learners afterwards and may provide insight into their own strengths and weaknesses. This may help determine where to start in the program and what to do. The following example shows how this could work for an individual learner. The test results of learner “S” are recorded in the tables 4.28 and 4.29. Together with the actual test, these results can be discussed with the learner. Knowing that the maximum at each subtest per level is 10 points, “S” has achieved the following results:

Table 4.28 Learner “S” - Scores per level and per subtest

learner S	level 1	level 2	level 3	level 4	totals
total subtests A	5	7	5	5	22
total subtests B		7	5		12
bonus points	(10)*				10
individual total	15	14	10	5	44
maximum total	20	20	20	20	80
starting level					3 (low)

* (10) are the bonus points the learner receives when he or she is assumed to have mastered that level, depending on the results at the A subtests. This learner received 22 points in the A-subtests in the first phase and therefore she did level B2 and B3 in the second phase and received 10 bonus points for level B1. (see IB scoring rules in chapter 3). Hence she achieved a total of 44 points out of a maximum score of 80 points. According to the level IB classification she can start at level 3. Results can even be recorded per sub-field per level. (table 4.29)

Table 4.29 Learner S - Results on sub-fields per level

learner S	level 1	level 2	level 3	level 4	ind total	max total
basic operations	3	5	4	5	17	26
proportions		1	-		1	11
meas/ dimension		3	4		7	17
money	2	3	1		6	16
time/calendar		1	1		2	7
data		1	-		1	3
bonus points	(10)*				10	
individual total	15	14	10	5	44	80
maximum total	20	20	20	20	80	
starting level					3 (low)	

Table 4.29 provides a profile of the individual learner by relating to the maximum score achievable in each sub-field through all levels. The overview shows the learner's strengths and weaknesses. Such overviews and individual items within subfields, can be discussed with the learner. This may help learners to gain insights into their own results. It may encourage learners to start a mathematics course and to decide what program would fit their profiles.
The same overview can be used for formative tests to measure the learner's progress.

5. Numeracy assessment in ABE should allow second language learners to apply their own mathematical procedures and algorithms they learned in their home countries.

Learners in this study were asked to write down their computations at the tasks in the test. They could apply their own procedures and algorithms, as shown by the Iranian learners and some Moroccan and Turkish learners. The Iranian learners showed clearly that they first did their computations in their own language and after that they wrote their answers down in the Arabic alphabet. (see figures 4.18 (Sausan) and 4.21 (Dina)). Sausan did all her computations in the test in her own notation system. Such notations may be very informative for teachers.

6. Text used in a paper-and-pencil math test should not hamper second language learners to take a mathematics test.

Texts in tests can always be a problem for second language learners. It can never be assumed that all test takers have mastered the language they need for doing the test. A dictionary is not always sufficient for help. It is often not a single word that can cause problems, but a combination of words or a different meaning of a word in a context, e.g. Whereas a 'chair' can mean a seat to sit on, it can also mean the chairperson of an organization. Visual support and sample items used as examples for test items may partially resolve these problems. At the place value items, for instance, there was a clear difference in results between the IB and the Cito items (see figures 4.17 and 4.18). Test leaders must have instructions as to how to respond to language problems in test situations.

In summary it can be said that assessment materials in ABE should provide an optimal test situation for adults to feel free to achieve with as much success as possible. Assessment should also yield quantitative and qualitative information that can help to arrange effective learning and instruction in ABE.
The extensive analysis of the mathematical knowledge and skills of the learners in this learning group may help acquire general insight into the actual competencies and needs of adults in ABE. Good placement assessment is the first step in arranging effective learning settings in ABE. The next step is to find information about learning and teaching in ABE.

5 Functional Numeracy Education in Adult Basic Education

5.1 Introduction

Since numeracy is a very young concept, only coming into being about twenty-five years ago, it is not yet clear what numeracy education should encompass. Early studies on this issue show clear differences between school mathematics and mathematics in real life situations, see e.g. Lave, Murtaugh and De la Roche, 1984, Resnick, 1987, Carraher, Carraher and Schliemann, 1985, Lave, 1988, Lave and Wenger 1991, Saxe, 1988, 1991. Recent studies focus more in depth on what and how to learn to become numerate for personal and societal life and for work, but are still explorative. See for example Bessot and Ridgway, (ed) (2000), Coben, O'Donoghue and Fitzsimons, (ed), (2000), Forman and Steen, (1999), Gal (ed), 2000, Steen (ed) (1997) Wedege (1999), Fitzsimons, O'Donoghue and Coben (ed), (2001).

Internationally numeracy programs for ABE have been developed since the late eighties. Initial programs published in Australia (Marr and Helme, 1987, 1991, 1995; Goddard, Marr and Martin, 1991; Marr, Anderson and Tout, 1994) and later in the USA (Goodridge et al, 1998; Huntington et al, 1998) show interesting experimental activities about how to *do* mathematics in real life situations, but when it comes to *learning* mathematics, the programs move to traditional ways of learning formal mathematics as used to happen in school, in particular concerning learning times tables, algorithms, fractions, decimals and percent. We may argue whether learners in ABE need and want such traditional mathematics. They may want it because they don't know alternative ways of learning mathematics. It is also possible that teachers don't know how to teach these subjects in a different way. These programs show that, despite the good activities, there is still not a theoretical ground for numeracy programs in ABE. They still rely on traditional school mathematics.

Compared to these programs the Dutch programs for learning mathematics in adult education may have an advantage, because they are based on RME principles that have already changed the traditional school math programs in the Netherlands in a radical way. However, as discussed in chapter 1, in the Netherlands too, a theoretical foundation for numeracy programs in ABE still has to be written. The purpose of this chapter is to set the basis for such a theoretical foundation.

In chapter 2 it was discussed that numeracy encompasses four components:
1) mathematical knowledge and skills, 2) management skills, 3) skills for processing new information, and 4) learning skills. It was also discussed that numeracy can roughly be divided into three levels: elementary, functional and optimal level. The results of the field study in chapter 4 show that about 25% of the adult learners in ABE are still at the elementary level and about 75% at the lower end of the functional numeracy. Hence the conclusion can be drawn that, given the reliability of the Cito test, numeracy education in ABE should mainly focus on learning functional knowledge and skills that lead to elementary and functional numeracy.

In this chapter we will first focus on theories about “adult learning” and “learning mathematics” in general. After that a theoretical basis will be established, integrating the four components of numeracy in a learning and instruction model for the development of functional numeracy in adult basic education. Ingredients for this study come from reflection on Paulo Freire’s pedagogy, studies about learning in practice, Action Theory, Constructivism and Realistic Mathematics Education (RME). The essence of these theories will be briefly summarized in section 5.2. These set the background information for the discussion about numeracy education in ABE in section 5.3.

Knowles (1990, p. 117) states that we can never create a fully new theory about learning. Every new theory is always an interpretation of existing theories in combination with our own (new) thoughts that fit best with the purposes of a new program in a certain situation. To save time and energy reinventing the wheel, he advises to take the best from existing theories in new situations. With these words in mind we start this literature study.

5.2 Adult Learning related to Learning Math in ABE

There is a gradual move in adult education from a pedagogical “teaching adults” into a more andragogical “helping adults learn” (Knowles, 1990). Dewey (1916) and Bruner (1968, 1996) already recognized that the human being is in principle a natural "learner". For Dewey, the human being is born with unlimited potential for growth and development, and education is one of the agents that facilitate growth (Jarvis, 1998, p.148). Bruner emphasizes that any didactical process, such as formalized instruction, is actually helping to create a sense of dependency in the learner rather than independency. (Jarvis, 1998, p. 146). Knowles (1990) in particular stresses people’s own perspectives and self-responsibility as determinants for the learning process. In adult education we should prevent any dependency of adults on teachers and should emphasize the adults’ own competencies and potential for growth and development. Teachers in adult education are only “facilitators” of learning and should help adults learn to teach themselves. (Jarvis, 1998, Knowles, 1990, Brookfield, 1986, Goffree and Stroomberg, 1989). Adults should take responsibility for their own learning in school as they do so in their everyday life situations. These thoughts lay the foundation for adult learning in ABE and open the way for lifelong learning.

5.2.1 Paulo Freire: Learning from Experiences

Freire's theory became known as "Learning from Experiences" and found its way into many countries all over the world, also into the Netherlands. His keyword for literacy and adult learning was *"dialogue"*. When people come into dialogue they become aware of their own situation and that is the basis for improving their own situation. The only ones who can help to improve the situation they are living in are the people (whether individuals or peoples) themselves. Freire's liberation education consists of acts of cognition, not of transfer of information. It is a learning situation in which the teacher becomes a learner and the learner is his own teacher.

> *"The teacher is no longer merely the-one-who-teaches, but the one who is himself taught in dialogue with the students, who in turn while being taught also teach" (Freire, 1970, pg.67).*

Though Freire's theory was developed in particular for people in oppressing countries, it has always been very popular in adult education in the Netherlands. However, we may wonder whether his ideas were elaborated in the right way in our western European country and even if it is applicable in every culture. (see also Coben, O'Donoghue and Fitzsimons, 2000). During the seventies and eighties "learning from experiences" was applied as a way of instruction for adults, especially in ABE and Open School learning centers. However, it never resulted in a real system of learning and teaching in the Netherlands. Despite the poor elaboration of Freire's pedagogy in the Netherlands, his theory has contributed a lot to the development of the early literacy programs between 1970 and 1980. Teachers became aware of their own authority as a teacher and of their ways of teaching. This awareness was a start for teachers and learners in adult education onto mutual equality, respect, acceptance and critical citizenship as co-learners. His theory resulted in four starting points for the Dutch ABE that started in 1987:

- learners' own decisions and self-management determining the actual program
- education and schooling in mutual relation
- learning from experiences
- mutual learning and teaching

(Ministerie van O en W, 1986, *Act on ABE*)

The gain of that time is that teachers and others in society started to look at their learners through different eyes. Illiteracy was something that could not exist here in the Netherlands because everybody has had a chance to go to school. It was, and still is, hard to acknowledge that there were, and still are, nearly illiterate or semi-literate people in the Netherlands. In principle everybody has the right to learn and is supposed to have had ten years of compulsory education. Reading, writing and arithmetic difficulties are often ascribed to the person's incapability or unwillingness to learn in former school years. The question whether the educational system or non-capable teachers might have caused learning problems in school is less emphasized, but perhaps not less true. In principle people are free and they can often manage their own lives in certain ways. They can come to classes for refresher courses, further development or for learning the Dutch language, whenever they want. This educational situation is not comparable with the oppressing situations Freire meant. However, the underlying thought of Freire's theory could be a good base for the development of an instruction model that would empower native and non-native illiterate and semi-literate citizens. Illiteracy and innumeracy still appear a hidden problem in western societies and should have much more attention and care. Freire's ideas could also be a good way of education for specific minority groups within western cultures, like women who still live in masculine subcultures. His starting points are still valid and directive in the new system for adult education in the Netherlands.

5.2.2 Learning in Practice

In this section we will describe a few specific elements that characterize adult learning in out-of-school situations which could have important impact on adult learning in school.

In practice adults have multiple tasks to fulfill. They have families, are parents, neighbors, citizens, customers, consumers, employers or employees, patients, members of sport clubs, volunteers in organizations, etc. In all those situations they have their own specific roles, tasks and responsibilities which ask for specific knowledge and skills and they learn from that.

"Learning in practice" is not a specific theory, but it includes all out-of-school studies about the learning of adults in real life situations. Many studies are done and have still been undertaken since the sixties in order to examine how adults develop knowledge and skills that are needed to function optimally in everyday life or to effectively manage all kinds of situations, in particular in work situations (among others: Lave, Murtaugh and De la Roche, 1984, Resnick 1987, Carraher, Carraher and Schliemann 1988, Lave 1988, Lave and Wenger 1991, Saxe 1991, Van der Kamp and Scheeren 1996, Noss and Hoyles 1996, Tuijnman, Kirsch and Wagner, 1997). Several conclusions emerge from these studies. The main points are discussed below.

1) *Adults are free to learn.* (Rogers, 1969).There is no compulsory education for adults. They learn because they need or want to be better informed, want to improve specific skills or to acquire more specific knowledge. They are not obliged to learn as they were in primary and secondary school. However, these

days the need for lifelong learning becomes more evident due to developments in technology and continuous changes in society. In that frame adults are encouraged to learn in order to be able to keep up with these developments.

2) *Learning happens in a functional situation.* (Resnick, 1987) There is a need for learning. All that is done in real life is embedded in specific situations and every situation asks for specific actions. Adults are continuously managing and solving problems and making decisions. Every situation is a source for learning but is also the context in which prior acquired knowledge and skills are applied. The advantage of learning in real life situations is that all new knowledge and skills are functional tools for managing and solving real life problems. One disadvantage may be that learning depends on the given situation. If there is no problem there is no need to learn. If there is a problem people have to solve that specific problem and this may be a source for learning, but the quality of learning and the result depend on the person in that particular situation. A second disadvantage may be that because of the development of knowledge and skills in very specific situations, people might not be able to see links with other situations or to transfer this new knowledge and these new skills to similar, but perhaps a bit different, new situations. In this way partial knowledge and skills are developed.

3) *Learning in practice is characterized by learning through authentic materials.*
Whereas in school situations learning often takes place through text books, photos, schemes and with the help of artificial hands-on materials, in practice this can be done in the actual situation with authentic materials. For example: computing the area of a floor and determining how many tiles are necessary to cover the floor can be done with the dimensions of the actual floor and tiles and by using professional tools like a measurement tape. Such materials may make it easier for adults to understand the mathematical situation and to analyze and solve the problem. Real materials also often offer possibilities to solve problems in different ways. A real tile, for example, can be used to determine how many tiles fit in the length and the width of the floor, even without computing the actual area. Computations can often be done in a creative, informal way. Such cannot be done with a photo of the tile and a map of the floor in a math book. Tasks in a text book often ask for application of formal computation procedures, like measuring the dimensions of both the tile in the photo and the map and computing the area of both and then dividing one by the other. Also, learning in a work situation is often related to learning how to work with specific tools, e.g. operating specific machines like a paint mixer, a sawing-machine, computer directed machines. This *"learning-by-doing"* leads to *"knowing-for-doing"* and is the base of functional numeracy. Boekaerts and Simons (1993) distinguish in this regard *"knowledge-as-a-tool"* and *"knowledge-for-knowledge"*. Whereas in school situations students often learn "content" because they should know it, in practice "content" is learned because people need or want to know it, to be able

do their job or other things. Knowledge acquired in practice is almost always functional and applicable.

4) *Every learning situation is a socio-cultural determined situation.* Saxe (1991) describes, referring to Vygotsky (1978), that social interactions are redirected by social and historical influences. These affect natural processes in cognitive development. In essence, learning is an *interactive and social act* in which everybody takes part. *Communication* by talking about problems which need to be solved and in what way, is an essential part of the learning process and starting point of developing reasoning skills and problem solving strategies and skills.

5) *Learning in practice focuses on "shared cognition", rather than on "individual cognition"* (Resnick, 1987). Though of course there are also situations in which people function individually, in a work setting employees may often be complementary to one another, like a chief and a secretary, a nurse and a doctor, a car salesman and a technician. In many work settings people only have very specific tasks without having insight in the full production process of the product to be made, e.g. in the automobile industry. There are only a few people who need to have the overall overview on the entire production process. In other situations, e.g. in a garage, employees can help each other with problems that cannot be solved individually. In such situations people learn to ask questions, to discuss the problems they meet, to look jointly for solutions and to work cooperatively.

Though students in school settings in recent years are more often expected to work together on problem-based and joint tasks, they are still assessed on what they can do individually. A student mostly passes or fails a test independent of the performances of other students. With the current changes in the adult education system, it should also be possible that students who work on joint tasks are assessed on cooperative learning aspects and study skills in addition to their individual competencies and their proficiencies in individual tasks.

6) The way in which learning in practice takes place is often *via showing - imitating - participating and applying.* There is no need to create specific instructional settings. People spontaneously work cooperatively when the situation asks to do so, like in work and family settings. In school settings we have to create such "practical" learning situations, based on instructional constructs in order to learn to work cooperatively. (Resnick, 1987)

7) For learning in practice people construct or re-construct their own *"rules-of-thumb"* and informal *"rules and laws"* for managing actions, situations, materials and the environment in which they work. For example to make concrete a rule-of-thumb is to use cement, gravel and sand in the ratio 1-2-3, but in certain situations this ratio may be adjusted. More informal rules and laws may emerge in working situations, e.g. after having completed a job, tools should be cleaned and put away and have their own places in the toolbox in order to prevent loss and damage. Such rules may develop as "generally accepted" rules, and by that they become part of common, general knowledge, but still *situation-based and situation- bound*. Such "rules and laws" are often developed in work settings to keep the work situation under control. General rules and laws learned in school will often deviate from those in practice. (Resnick, 1987). Situation-based rules and laws may affect the learning of adults in school situations as in ABE or in vocational education.

These key issues offer important findings for learning in adult education. In the current developments in adult and vocational education in the Netherlands we see that, gradually, new learning environments are created based on experiences and information from studies about learning in practice, e.g. problem based learning and cooperative learning. This is a good development, but we should realize that learning in school settings will never be the same as learning in practice. In his article *"Learning in and for Participation in Work and Society"*, Greeno (1999) challenges the education systems for adults. He argues that learning for participation in work and society can only take place in the workplace setting or in social communities in order to give meaning to the learning. Learning in informal ways in the course of activity in a meaningful setting has shown to be much more effective than learning in classroom settings. His thoughts indicate the need for a balanced system of learning in and out-of school situations for adults. It shows the importance of "learning in practice".

5.2.3 Action Theory

Action theory has its roots in the Culture Historical Theory from Vygotsky. It starts with the person as an entity who learns in dialogue with the culture in which he lives: interaction is the base for individual action and cognitive development. (Vygotsky, 1978). The starting point of Vygotsky's culture historical theory, as elaborated by Gal'perin and described by Van Parreren and Van Loon (1975), Van Parreren and Nelissen (1977,) Nelissen (1987), Van Parreren (1989), Van Oers (1987), and Van Eerde (1996), is that every person is an actively acting person, giving meaning to and constructing his own life by interacting with society. This is shown in his behavior and in the activities he undertakes in his own societal environment. Activities of men, and thereby their cognitive development, are always embedded in a culture historical environment (Saxe, 1991). The development of children is determined by transmission of culture historical information which takes place in the social interaction between adults and children. Children may adapt this information but not without smoothing and adjusting this to their own concepts and thoughts and completed with the creation of new conceptions based on this information. Therefore learning and teaching cannot consist of focusing on abstract matters separate from reality and culture. People act and learn from their actions in their own reality in interaction with others in their reality and in a joint cultural environment. On the one hand culture structures human activities and their thoughts, but on the other hand individual actions of people will change culture and history. In essence learning is a natural, active process. It is based on the humans' natural wish to give meaning to their own lives. Teaching does not consist of transmitting new knowledge and showing skills, but it mainly exists of asking questions, posing and discussing problems and provoking students by which students will become curious and feel challenged to act. Students' actions need to be based on initiatives of the students themselves. (Van Parreren, 1989)

In the Netherlands, Action Theory is informed in particular by Gal'perin's elaboration of "zone of proximal development". The keyword in Action Theory for cognitive development is "activity" (Van Oers, 1987, Van Eerde, 1996). Cognitive processes - such as manipulating, perceiving, thinking, and memorizing - are seen as actions which develop in communication with the world. Thinking is interpreted as mental action. Action and social interaction are inextricably bound to each other and are an essential part of people's lives and learning. Every thinking step is an activity: mathematics is a thinking process, thinking is an action process, mathematics is an action process (Ruijssenaars, 1992). In Action Theory as an instruction theory Gal'perin distinguishes five levels of conceptual development: manipulation, perception, verbalization, interiorization and abstraction (Van Parreren, 1989). In the original formulation of Gal'perin's theory manipulation in the actual society is the orientation level for the development of higher levels (see figure 5.1). Children need manipulation with objects in reality or with objects derived from reality (the

materialized reality, like working with Dienes blocks (MAB) and arithmetic rack, or using photos, schemes and maps of reality). Based on that they create "perceptions" of reality. Perceptive actions encompass perception and observation of manipulations and representation of manipulations through seeing, feeling and listening which in their turn lead to insight, in-feeling (thoughtfulness) and be-listening. At the level of verbalization the person can express what he thinks, what he is doing, plans to do or did orally or by words or other symbols on paper in his communication with others. The individual person can use the right concepts in the right situations and with the right words. At the interiorization level, new concepts are integrated in the individual's concepts and become part of his mental actions, which is shown in his actual actions in communication with others. Mental operations are thinking processes by which abstract concepts can be linked and are the basis for mental actions, but can also guide and support operations on the four lower levels when developing new concepts. In this way the cognitive development of children can be followed and directed. Programs and teaching can be developed based on these action levels to encourage and support this development. The assumption in action theory is that each level is a precondition for the following level and that conceptual development of children takes place in the proximate field of development. Each higher level of actions develops further based on actions at underlying levels. The quality of the actions at the higher levels depend on the completeness of actions developed at the underlying levels. This is an active process (see figure 5.1). Learning is in principle based on people's own activities but can be encouraged by offering the right materials in the right order and steps on the right moment and in the right situation.
Van Oers (1987) interpreted these levels in a slightly different way. (graph 5.2). He argued that conceptual development takes place through three action levels in sequence and in mutual relation: a manipulation, a representation and a symbolization level.

Figure 5.1
Gal'perin's model of Action Theory

Abstract Concepts Mental Operations
Interiorization
Verbalization
Representation
Manipulation

Figure 5.2
Van Oers' model of Action Theory

<table>
<tr><td rowspan="3">Mental Action</td><td rowspan="3">Verbalization</td><td>Symbolization</td></tr>
<tr><td>Representation</td></tr>
<tr><td>Manipulation</td></tr>
</table>

Actions on the manipulation and representation levels are preconditions for the next level. At the level of symbolization people are able to use mental concepts expressed by words and symbols, based on insight and developed by manipulations and perceptions. Verbalization is an activity that occurs in parallel to these three levels because actions on each level emerge in

communication with others, so on each level people must be able to express their actions in words or in written symbols. By discussion people build their own ideas and concepts. If people can tell what they do and why they do it in that way, then they understand what they do. Also, in principle, all human actions are based on goal-centered thinking processes, so all their activities are expressions of mental processes. Therefore mental action is also an activity, together with verbalization, that happens in parallel with the three levels. Cognitive development becomes visible in people's actions and communication.
There is a continuous up and down in this scheme. Young children develop concepts by starting at the lowest level. Based on experiences with real objects or materialized objects they build up their representations of real life things and learn to verbalize their thoughts. For example, the object chair is worded as "chair", but can also be addressed as a "seat". There are different types of chairs: a "stool", an "armchair" and other. The child develops representations of a chair, a seat, a stool and an armchair and this leads to insight in concepts and to the development of symbols. Communication with others, family, teachers and peers, and mental actions help them to link these concepts and to use the right words in the right situation. As children grow up they will function increasingly on the higher levels of representation and symbolization. They can acquire new knowledge and skills starting on these higher levels with the help of mental actions and communication, for example, when a new type of chair has been developed. They learn to refer to previous experiences, knowledge and skills and can process new information on a higher level based on mental operations. Only for really new information not based on previous experiences, will a child search for the best way of processing it by going back to the level of doing manipulations. That is, he needs to feel and see it and talk about it. After that he may build up a new concept based on his perceptions and feelings at the lower levels.

Action Theory found its way in particular into primary school and special education in the Netherlands. In the seventies and eighties it was concretized in curricula by offering instructional materials built up through these three main levels, in combination with group discussions and group work to encourage verbalization of actions. For example, the Dienes blocks and the abacus were used to materialize, visualize and verbalize the number system and arithmetical operations as addition and subtraction. The use of such manipulatives was discussed later by Gravemeijer (1994). (see also section 5.2.5)

5.2.4 Constructivism

From a *constructivist* point of view learning is seen as an active process of continuous self-organization and re-organization of one own's knowledge in combination with new experiences. Von Glasersfeld (1991) argues that there is no objective knowledge that can be transmitted from one individual to the other. Learning is an individual process of understanding meaning, reflection on experiences, adjusting and reorganizing the individual's own mind. Reflection on one's own thoughts and experiences is an important factor in creating new knowledge.

In *socio-constructivism* it is emphasized that mathematics is social and cultural in its nature. Mathematics as a science has been developed in interaction. Learners construct taken-as-shared mathematical knowledge through interaction. (Cobb, Yackel and Wood, 1992, Cob, 1994a, 1994b, Ernest, 1996, Gravemeijer, 1995).

Cobb (1994b) argues that the way in which this process takes place depends on beliefs and the mutual expectations students and teachers have concerning their roles in the learning process. Such *social norms* are intrinsic to socio-constructive mathematical development because the learning-teaching process is an *interpersonal process.*

In the traditional way of teaching and learning the teacher is the expert and "explains" mathematical subjects as if it is absolute knowledge, sometimes with the help of concrete materials or visual models. When students don't understand the subject matter, "knowledge" will be explained further in detail. This may lead to absorption and algorithmatization without reflection and real conceptualization (Cobb, Yackel and Wood, 1992, Gravemeijer, 1994)

In cooperative learning situations interaction is focused on peer discussions. In such discussions the roles of the teacher and the student will change. The teacher challenges his students to use their prior acquired knowledge in order to "construct" new knowledge to be achieved, based on their own insight and procedures, e.g. how to find out what is greater: 5/6 or 7/8 or 9/10 of an area? And what kind of conclusion can be drawn from this mathematical investigation? In such learning situations the teacher and the students iteratively constitute criteria for what a good or less good answer will be, *socio-math norms.* At what level solutions are elaborated and practiced depend on the knowledge students already have acquired. For instance: in a lower level group the students can describe their findings by drawing the division as a pizza-division problem (divide 5 pizzas with 6 persons, etc, at which table do you get more?), or by using a ratio table. In a higher level group the students and teacher could agree to describe their findings by reasoning in fractions, or in decimals, or both. There the pizza division could be a less appropriate answer. The teacher is supposed to ask questions about their reasoning instead of explaining the subject. He participates in the discussion and in this way the classroom community develops “taken-as-shared” meanings, interpretations and practices. Students develop beliefs and values about what counts as mathematical problems and what are acceptable explanations and solutions. Such *socio-math norms* determine the quality of the students’ mathematical actions. (Cobb, 1994b)

Individual learners develop mathematical knowledge and skills in interaction with other learners by negotiation of meaning. This results in taken-as-shared knowledge as a joint goal that can be accepted as general mathematical knowledge (Cobb, Yackel & Wood, 1992, p.16, Gravemeijer, 1996). Individual knowledge and skills may be developed as closely as possible in relation to taken-as-shared knowledge but may differ in some ways due to the individual’s thoughts, observations and interpretations. Taken-as-shared knowledge cannot be accepted as “absolute” knowledge. It is determined and affected by social, cultural and historical influences and may change when these factors change.
However, by taking the taken-as-shared meanings, interpretations and practices of the mathematics community it is possible for educators to construct objective curricula, because if there would not be general knowledge, learning goals in education would be hard to formulate and assessment could be nearly impossible. (Gravemeijer, 1996)

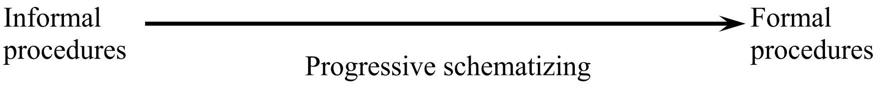
Accepting the starting points of socio-constructivism means that teachers and peer students may have more influence and control on developing the individual student’s mathematical conceptions, which may prevent or affect misconceptions, though in principle students construct their own knowledge. By sharing, discussing and comparing individual ideas and meanings with others, it would be easier to detect misconceptions. Based on these interactions students adjust their earlier acquired mathematical knowledge and build up new knowledge and procedures.

5.2.5 Realistic Mathematics Education

Freudenthal’s numerous examinations and reflections on the learning and teaching of mathematics have led to a total change in the Dutch mathematics education: Realistic Mathematics Education (RME). RME is based on Freudenthal’s assumption that mathematics has its roots in real life and constructs and reconstructs reality. It is a science and a learning theory in its own and is always further developing itself, because it evolves in and from real life situations and will never stop evolving. (Freudenthal, 1973, 1991, Treffers, 1991, Gravemeijer, 1994). Treffers wrote at the occasion of the opening of the Freudenthal Institute in Utrecht in 1990: “*His ideas emphasize rich thematic contexts, integration of mathematics with other subjects and areas of reality, differentiation within individual learning processes and the importance of working together in heterogeneous subgroups.*” (Treffers, 1991, p.19).

According to Treffers (1991) RME has five learning principles: learning is a constructive activity; learning moves through various levels of abstraction with the provision of models, schemes and symbols; learning takes place by reflection; learning is a social activity; and learning mathematics leads to a structured and interwoven entity of knowledge and skills. (Treffers, 1991)
RME starts from rich contexts that challenge children to develop and re-invent, to construct and re-construct mathematical conceptions, based on their own informal knowledge of experienced situations, by developing and applying informal mathematical procedures. These informal conceptions and procedures are gradually developed further through modeling and progressive schematizing into formal procedures without leaving the informal and creative problem solving strategies children may have. In other words, RME encourages creative informal solving strategies prior to formal procedures. That implies that before starting formal mathematical procedures and learning mathematical formulas, children must have had enough opportunities to experiment with their own invented informal procedures, based on their own real life experiences and mathematical notions, in order to develop or "re-invent" more general mathematical concepts and procedures. (Treffers, 1991). This iterative process is symbolized in figure 5.3.

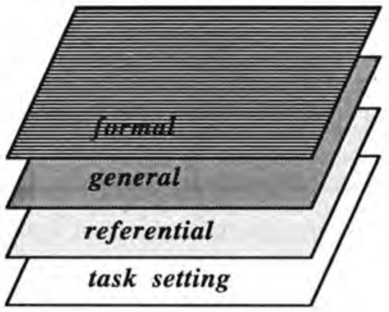
Figure 5.3: Progressive Schematizing



The process of mathematizing, generalizing and formalizing by progressive schematizing can be supported by children's own drawings, schemes, graphs, and symbols. As children develop their mathematical concepts and informal procedures further, drawings and schemes become more and more schematic and abstract and will finally result in formal procedures. This process of transforming informal procedures gradually into more formal is called "progressive schematizing".

Within this process of progressive schematizing Gravemeijer (1994, 1998) distinguishes four levels of activity in the development of mathematical procedures.

Figure 5.4: Gravemeijer's Levels of Activity



Mathematical activities start in specific task settings and models are initially tied to such specific settings. At the referential level the student creates a "model-of" acting on the problem situation. The focus shifts and general activity begins to emerge as the student's attention moves towards mathematical relations and the role of models change into serving as a means for mathematical reasoning rather than as a representation of a specific situation. Here the model becomes a "model-for" activities on the formal level. On the formal level this process has been completed.

Formal mathematical language and procedures can be used to solve mathematical problems. Emergent models may foster the transition from concrete informal to more formal mathematical activities, on a different level of abstraction. (Gravemeijer, 1998, p. 34-35). By going through this process children construct new mathematical concepts. See figure 5.4

Whereas in Action Theory the zone of proximal development is the field where learning takes place, symbolized in Gal'perin's original and in Van Oers' revised model, and can be directed and encouraged by others by having the students participate in adequate social practices, RME and Constructivism consider conceptual development as a result of constructive processes in the individual student. This is shown in Treffers' model of progressive schematizing and Gravemeijer's levels of activity. Though there are differences, in fact Action Theory, RME and Constructivism have a lot in common. The models used by Van Oers, Treffers and Gravemeijer all emphasize a gradual conceptual growth from informal mathematical activities into formal mathematical concepts and procedures through different levels of mental action. (Van Oers, 1987, p.360-366)

Another important issue in RME is that children are challenged to solve math problems in various creative ways. This creates insight in relations between mathematical operations. In this way they construct and reconstruct their own mathematical concepts and learn that mathematics does not exist of separate elements and single rules they have to learn, but everything links with everything, e.g. addition links with multiplication, subtraction and division; proportions, fractions, decimals and the metric system are interwoven, percent is part of proportions, etc. In this way mathematical knowledge and skills are being structured as interwoven entities that are flexibly applicable in real life situations. In the entire process interaction and reflection are important means for developing awareness of the students' own mathematical procedures. Discussing mathematical topics is the base of acquiring, implementing and integrating new knowledge and skills.

Finally, RME emphasizes the importance of the development of mental mathematics and estimation in combination with the use of a calculator and in combination with a new way of doing algorithms. New computation procedures, based on mental math, have been developed for the four basic operations addition, subtraction, multiplication and division, and have been gradually implemented in school materials and in learning and teaching.

5.3 Numeracy Education in ABE

The theories and studies described in the previous sections provide useful ingredients for numeracy education in ABE. These ingredients pertain adult learning in general, learning mathematics and the learning process itself.

The word "education" entails "learning" and "teaching". Since adult education eventually aims to lead to independent learning in the frame of lifelong learning, the emphasis in this study is more on learning than on teaching. Adults are supposed and expected to take responsibility for their own learning. The role of the teacher should be seen as supporting and facilitating the learning by adults and helping adults to get their learning processes clear.

For adult learning in general we will derive starting points from the ideas of Paulo Freire and key points of studies about learning in practice (section 5.3.1). For learning mathematics, the first component of numeracy, we will process the basic principles of Action Theory and RME (in 5.3.2). RME and Socio-Constructivism offer good possibilities for analysis of the learning process which includes the other three components of numeracy: management skills, skills for processing new information and learning skills. (5.3.3). Together they provide an instruction model for "*Functional Numeracy Education*" (FNE) in ABE (5.3.4) in the light of the four numeracy components as described in chapter 2.

5.3.1 Adult Learning

From Freire's pedagogy and studies about learning in practice we can derive seven general starting points for the learning of adults in ABE:

1) *Adults are free to learn.*
This is the most obvious difference from the learning of children. Adults are not obliged to learn nor to go back to school. Adults want to learn when they feel a need to learn. However, in recent years, the general need for lifelong learning becomes more and more clear due to the developments in society. This puts adult learning in a new light. Adults are more pressed to go back to school for recurrent education. As part of lifelong learning in the Netherlands, and in other countries, programs have been developed to provide learning opportunities for adults. Employers are supposed to facilitate and pay their employees for refresher courses and employees are often required to do such courses as part of their work. For those people who see the need of such programs the requirement will not cause problems. Others, often people who don't have good memories of school, may feel differently about this. But, thanks to these developments, numeracy is being seen as a field that calls for more systemic attention.

2) *Learners and teachers are equal partners in learning situations.*
Adults can be learners as well as teachers in learning situations. Adult learners have many real life experiences which may affect their own learning and that could be of help for the learning of others. If teachers and learners are aware of this, there can be a sharing of real life experiences as equal partners in the learning process, which may enrich the learning situation. This principle affects the teacher's role as teacher and the students' role as learners. It enables the adult learners to take responsibility for their own learning and causes teachers to become facilitators of learning. (Freire, 1970, Knowles, 1990)

3) *Adults' own experiences are the basis for learning.*
Adults always carry plenty of real-life based and school-based experiences when they are in a learning situation. These are indispensable learning ingredients in adult education. They are the starting point for learning: *learning starts when adults are aware of what they know and what they want to know.* New knowledge should be built on prior knowledge. Adults acquire new knowledge and skills in a learning situation by selecting new info which seems important or of interest to them. They compare new info to what they already know about that subject and integrate new knowledge by reorganizing their own conceptions about that subject. This leads to expansion and improvement of the person's knowledge and skills. (Freire, 1970, Knowles, 1990, Noss and Hoyles, 1996, van Groenestijn, 1992, 1993, 1996, 2000)

4) *Authentic materials should be used as instruction materials in school learning situations.*
Following the previous starting point instructional materials used in learning situations should be authentic or represent reality by means of photos, schemes, etc. in order to give meaning to the learning in school and to create the link between school knowledge and real life knowledge. Authentic materials may help to make school knowledge useful and applicable in real life situations.(Freire, 1970, van Groenestijn, 1996)

5) *Learning takes place by interaction and reflection.* By discussing new learning topics related to prior knowledge, adults become aware of and gain insight into their own knowledge and skills. This is the start of learning. Verbalizing thoughts and mathematical procedures in communication with others may also improve people's mathematical reasoning and communication skills, which are, in turn, preconditions for being able to manage everyday life situations and for cooperative learning situations. (Freire, 1970, Knowles, 1990, Cobb, 1994c, Gravemeijer, 1996)
In addition, for non-native adults communication has a twofold extra goal: it is an important means to become aware of cultural differences between their own mathematical background and this area in their new host country, and by talking about mathematical issues people improve their communication language as well as their language and knowledge of the subject.

6) *Learning in adult education aims to lead to functional knowledge and skills.* Based on studies about learning in practice, learning in adult education should focus on learning-for-doing rather than learning-for-knowing. Learning-for-doing leads to knowledge-for-doing, i.e. functional knowledge. Of course, we should distinguish here between people who need or have to learn and people who want to learn for their own pleasure and development. However, ABE should focus on acquiring functional knowledge and skills that are directly applicable in real life situations and that act as a basis for learning in vocational courses. (Boekaerts and Simons, 1993)

7) *Adults direct their own learning* by constructing and re-constructing, organizing and re-organizing their own knowledge, procedures and skills. This can be encouraged in learning situations by helping adults to create their own mathematical procedures based on their own insights and skills when solving mathematical problems. The results are visible in their own idiosyncratic procedures and mathematical notations in real life situations.
From an andragogical and a constructive point of view teachers cannot transmit knowledge, they are just facilitators of learning processes. The actual learning process is directed by the learners themselves. (Knowles, 1990, Von Glasersfeld, 1991, Cobb, Yackel and Wood, 1992b, Cobb, 1994b, Gravemeijer, 1995, 1996)

5.3.2 Learning Mathematics in ABE

The general starting points in the previous section set the basic principles for learning in ABE. Learning mathematics in ABE, described in this section, and the learning process itself, described in the next section (5.3.3) can only be realized in combination with these general points. In this section the discussion will focus on the first component of numeracy, the development of mathematical knowledge and skills. Though Action Theory, Constructivism and RME mainly concern the mathematical development of children, mathematics education in FE can profit from their starting points and basic thoughts. They offer good supporting learning models for acquiring mathematical knowledge and skills. This may help to get insight in the learning of mathematics by adults in ABE and to improve their learning.

It is obvious that there is a difference between the mathematical development of children and the learning of mathematics by adults. Adults have already acquired many mathematical conceptions in and out of school. We often see a mixture of informal and formal knowledge and skills, partly in combination with partial knowledge and misconceptions and bound to real life experiences. Having adults visualize and verbalize their mathematical thoughts and actions may give insight into the adults' thinking processes. Van Oers' action theory model and Gravemeijer's model of action levels may help the analysis of these processes. These models, together with Treffers theory about progressive schematizing, can also be useful in the development of new mathematical insight and actions of adults. However, we cannot just transfer these models to the learning of mathematics by adults in ABE. A few adjustments need to be made. This can be done through the following four steps:

1) Whereas mathematics in primary and secondary school focuses on general formal mathematics that prepares children for a wide range of possibilities in their future lives, mathematics in adult education aims to lead to *functional numeracy*. This means that the actual real life situations in which adults have to manage mathematical problems are the source as well as the focus of mathematics education. Functional mathematics and functional numeracy education start at the actual lived-in situation of the adult learners and aim to develop mathematical knowledge and skills that are usable and applicable in these situations but also to enable adults to broaden their perspectives. For this the meaning of the action models is placed in light of developing "*functional mathematics*". This may be seen as a restriction of mathematics education in general but is of importance for adult learners.

2) The three models discussed at Action Theory and RME are similar but differ in detail. These models can be synchronized with a little refinement of Van Oers' action scheme on the representation level. According to the starting point

of action theory mathematical development starts in actual real life situations. Based on these experiences adults develop representations of such real life situations which refer to their experiences. These can be of help with actions in other situations. In their turn these representations gradually develop further, as part of progressive schematizing, into more general schematic maps, patterns and other abstract models that can act as thinking models when solving similar problems in different situations. This is the basis for functional mathematical procedures. This is also the level where transfer is possible from mathematical actions developed in one situation to similar actions in a new situation. For example: if a person has experience in creating a book case with right angles and sides next to a straight wall, the next step can be to create a book case with slanting sides and angles that fits into a difficult corner in the loft. His representation of how a book case looks and schemes, drawings that show how to develop such a case may help him to build a new book case that fits in a different situation.

Figure 5.5 Van Oers' model of Action Theory refined

Mental Action	Verbalization	Symbolization and formal /functional operations	S
		using schematic representations	Ra
		using representations of real objects and situations	Rc
		Informal manipulations in actual real life situations	M

For this the representation level in the Action Theory model should be divided into two sublevels: a level that refers to actions of adults in actual situations (R-concrete, Rc), gradually moving into a more abstract level at which adults create more schematic mathematical representations like maps, schemes, charts, tables, number line, etc. (R-abstract, Ra). These serve functional mathematical operations on the formal level in new situations. The refinement of the Action Theory model is presented in figure 5.5.

The dotted lines in figure 5.5 indicate a gradual move from concrete into abstract and from informal manipulations (M) into formal/functional mathematical actions, using symbols (S). Actions based on these four levels lead to insight into mathematical concepts and operations in real life situations.

With this refinement Van Oers' level classification approaches Gravemeijer's four levels of activity. Also, the process can be seen as a reiterative process between informal and formal/functional mathematical actions that eventually leads to functional mathematical procedures. This is according to Treffers' model of progressive schematizing.

3) With adult learners in ABE we often notice two typical types of problems that can be traced back to partially developed mathematical conceptions. Based on the action theory model in figure 5.5 we can state that:

- Misconceptions may occur when people develop concepts not based on all four levels;
- Adults often apply formal mathematical operations like doing algorithms or computations with fractions, decimals or percent, without having insights into what actually is happening.

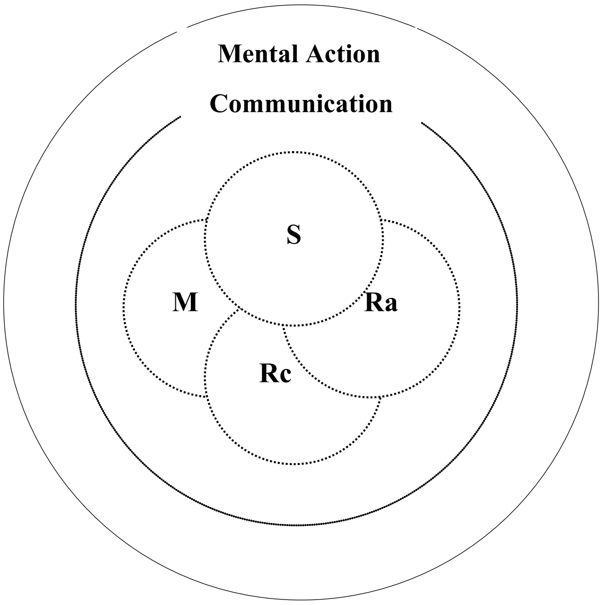
In Action Theory it is assumed that young children start developing mathematical concepts from the lowest level and grow gradually into the higher levels of concept building and doing formal mathematical operations, based on insight in well-developed lower levels. Adults are assumed to have achieved the higher levels. According to Van Parreren (1975) and Van Oers (1987) adults should be able to work with symbols on the highest level, based on insight, developed by perceptions and manipulations. Their actions are always mentally directed and they should be able to verbalize their actions in communication with others. For good conceptions all levels should be optimally developed. From this we can derive that misconceptions and application of formal mathematical operations without insight refer to no or little insight at the lower levels or not being able to create links between those levels, e.g. not being able to create links between informal manipulations in real life situations and formal mathematical operations learned in school.

When adults are confronted with their own misconceptions they may try to adjust their conceptions by going back from the symbolic level through the lower levels of representation to actual manipulation, informally, and build up from there. In real life situations this can often be done by using real objects or representations of it. The most common learning situation in actual real life situations is showing-and-doing in communication with others and accompanied by explaining in the course of the action (Resnick, 1987, Greeno, 1999).

4) Similar things can be said about situations in which adults meet new demands that require the development of new mathematical insights and concepts. In such situations the learning process starts from the symbolization level by talking with others about the new problem or action. When they don't understand it, they may want to go back to the representation and/or manipulation level. For example, when adults work with computers and have to or want to deal with new technological elements they are not yet familiar with, like working with "real play" or "chatting" or creating their own websites, they may first want to actually manipulate these "real" things on their computer and to talk about it with colleagues, friends or family in order to understand how it works. Based on that they develop representations and abstract concepts. In such learning

situations they work through all four action levels in parallel and are able to develop and acquire new concepts by moving from one level to the other, often at the same time. Communication with others in which they can verbalize their questions and thoughts, or reading instruction manuals and trying to find answers on problems they meet and questions they want to have answered, represent, in fact, the verbalization process. In this respect we may pose that the four action levels can better be seen as intertwined fields in four thin layers embedded in communication, in which adults, when they acquire new knowledge and skills, easily switch from one field to the other, in various ways, supported by verbalization through communication, and directed by mental action. Based on this we can change Van Oers' model to the chart presented in figure 5.6

Figure 5.6: Learning Mathematics by Adults through Actions



Explanation: In figure 5.6 representations on concrete and more abstract level (Rc and Ra) are the important links between the informal mathematical actions of adults in real life situations (M) and formal mathematical knowledge and procedures by using symbols (S). The process of moving from informal manipulations (M) through the representation levels (Rc and Ra) to doing operations with symbols (S) is a continuous iterative process in which adults can start from different fields but need to go through all fields to create a complete basis for mathematical concepts. Verbalization of this process and reflection on people's own knowledge through communication with others, are important supporting factors. The whole process is directed by mental action from the individual person. Mental action is the key to acquiring and organizing new knowledge in relation to prior knowledge.

The learning of mathematics by adults moves through these four intertwined circles in an iterative way. Adults in ABE often use a mix of informal and formal mathematical procedures. These should be transferred into "functional" procedures based on insight, along the lines of progressive schematizing. This transformation process should work in two directions:

- Adults who tend to use more informal procedures (M), often situation-bound but developed in the course of action and by that often based on insight, should be encouraged to create their own more formal but "functional" procedures

combined with notation systems that support their thinking and their computations, in order to make their procedures more general and flexibly applicable in new situations. They move through these circles from M through Rc and Ra to S, going back to M but on a higher level.
Eventually they will develop more general functional procedures which may be based on a combination of informal thinking strategies (M), in combination with their own idiosyncratic notation systems (S). Representation of their informal actions with the help of photos, drawings, schemes, etc. (R) may help to develop such functional procedures.

- Adults who are used to doing formal procedures (S), but lack insight into their own computations, should be encouraged to carry out their formal procedures in more informal situations in which they can show how their computations work (M) or explain their computations with the help of schemes, drawings, photos, etc (Rc and Ra). In doing this they will develop insight into their own formal computations such that their computations become more functional.

When learning mathematics adults will always move back and forth through these levels in various ways and quite often simultaneously. If adults in ABE are able to support their mathematical thinking and operations with functional notation systems, then they are showing that they understand the math they learned and that they can use it in a functional way. A precondition here is that mathematics in ABE programs focuses on real life problems. Communication and cooperation with other learners are important means to help adults clarify their thinking processes.

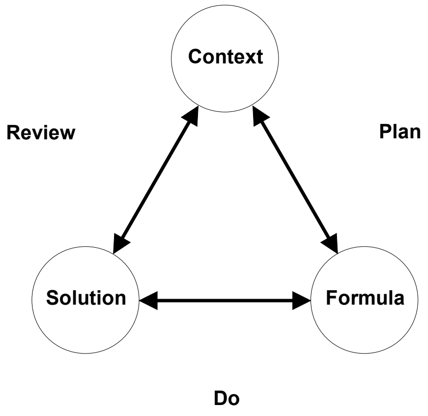
5.3.3 The Learning Process

The purpose of this section is to analyze the learning process itself alongside the other three components of numeracy: management and problem solving, processing information and development of learning skills for independent learning (see section 2.4). Based on this analysis a model will be developed that may help adult learners gain insights into their own learning processes.
RME and Socio-Constructivism offer good possibilities for such a model. Whereas RME provides learning through contexts, which asks for problem solving and leads to management of mathematical situations, Socio-Constructivism focuses on social norms and socio-math norms within this process of problem solving in order to arrange constructive learning situations and to encourage qualitative discussions, based on the learners' competencies. Both emphasize communication, reflection and interaction which lead to development of mathematical reasoning and learning skills. Analysis of adults' learning processes along the lines of RME and Socio-Constructivism may help adult learners to clarify their own learning processes and to develop skills for

independent learning. This is a metacognitive process in which the learner becomes aware of the cognitive process he is going through and by which he learns to reflect on this process, and to direct his learning in order to control and to improve his learning. (Boekaerts and Simons, 1993, Simons, 1993). The way in which this can be done is elaborated in this section at the *what-why-how* processes below, based on problem solving, socio-math norms and social norms.

RME implies *problem solving*. The basic pattern of problem solving can be described in six general steps. This process is visualized in figure 5.7. Within this process a distinction can be made between the process itself (*what*), the purpose and quality of the action (*why*) and the way in which the learning process is arranged (*how*) (van Groenestijn, 1998).

Figure 5.7 The Problem Solving Process



The actual problem solving process shows the "what"-process. To analyze this process with learners the three process steps plan-do-review should be discussed. Evaluation of these steps can be done by posing three questions:
- What are you going to do? (plan)
- What are you doing? (do)
- What did you do? (review)

These three *"What to do?"* - questions are the questions by which the teacher and the learners focus on the solving process itself. By asking and answering these questions learners are required to verbalize their actions. Hence they become aware of their thinking and actions and the steps in this solving-process, and by that they become conscious of their own learning process. The teacher can activate the learning process by asking follow-up questions to the three main questions. When doing this he can address the learners' prior knowledge and can refer to new knowledge and procedures discussed in learning situations. (van Groenestijn, 1996, 1998)

The actual process of problem solving adults go through can be described in six steps (the *what*-process).

Step 1:
RME in adult education starts and ends with functional contexts derived from real life situations. In ABE these contexts should always stay close to real life situations, though they may be broader than just everybody's individual everyday life or work situations. Contexts in ABE should anyway be recognizable as "real" for adults. In the setup of RME learners start the problem solving process at analyzing such a context problem and translating the context problem into mathematical and other information (horizontal mathematizing).

Step 2:
After having analyzed the problem, a *plan* can be made to solve the problem. This can be done individually or in cooperation with others. When learners do it jointly, they have to plan a joint solving procedure or different solving procedures that can be compared afterwards. In this way they can build in control and reflection. Interaction and discussion are part of this. This is where social norms come in (see the *how*-process, graphic 5.9) Individual learners will also have to plan their activities. During the phases of analyzing and planning they think about needed procedures and "formulas" to solve the problem. In this phase prior knowledge has been activated and the learners try to find out what they need more. They will choose procedures they are familiar with and may need "more". The "more" are the subjects they are supposed to learn in that particular learning situation and that the context refers to. To achieve this they may want to try new info. This is a reiterative process. Learners go back and forth from analyzing the context to coming to a suitable way of solving the problem and balancing between their own available knowledge and skills, and new knowledge and skills they are supposed to acquire in this learning situation.

Step 3:
The planning procedure results in the decision as to which kind of computations should be done with which formulas to solve the problem. A "formula" at this point may vary from a single basic operation like doing only addition and/or subtraction to a set of very complex formulas including more mathematical operations.

Step 4:
The way in which adults actually *do* their computations to solve the problem, depends on the purpose of their action and the quality of the mathematical skills they have developed, their prior knowledge, and new skills that are being developed during the process of problem solving. (vertical mathematizing). Problem solving is always a combination of applying knowledge and skills and constructing new knowledge and skills. It is also a combination of various skills somewhere on the scale of informal to formal procedures through progressive schematizing and vertical mathematizing according to Treffers (1991). These problem solving procedures may vary from informal, experimental actions, using hands-on materials and/or representations, to formal computations, using standard formulas, algorithms, calculators, etc. In this phase learners go through the different action fields (see figure 5.6). This is the phase where socio-math norms come in. The quality of the learners' actions may depend on the purpose they have and their level of mastery of mathematical actions.

Step 5:
At the end of the actual problem solving procedure the solution is there and learners will have to agree to that solution.

Step 6:
In the last part of the problem solving procedure learners are supposed to *review* their computations and solutions and reflect on it. Reflection is an important starting point in RME, because this is the actual learning moment. (Treffers, 1991, van Groenestijn, 1992,1993). By reflecting on their own steps and procedures and those of others, in the contextual framework, learners become aware of their own thoughts and actions and will adjust and reorganize their own conceptions, knowledge and procedures. New conceptions, knowledge and procedures are implemented, integrated and structured in the network of knowledge they already have. In this way adults "reconstruct" their conceptions and knowledge.

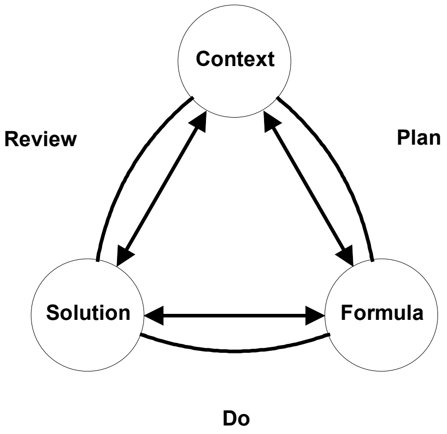
The *what*-process is the actual learning process learners go through. Discussing the six steps described above along the lines of *plan-do-review* in the chart (see figure 5.7) may help learners to become aware of their learning *process*. The three circles show the results, the "*products*". The process starts and ends at the context situation. The entire process is reiterative. People go back and forth continuously through all steps. They make decisions about how to solve a problem (analyze, plan and choice of "formulas"), about the answer (do computations and find solution) and about the correctness of the answer (review in relation to the context). The answer should fit into the framework of the context problem. At the start of a new problem they will start on a higher level in this process so that the learner can work more efficiently.

A second dimension within the what-process is the *why*-process.

Every functional mathematical action has a purpose. For the learner it must be clear why he works on mathematical problems. This may determine the *quality* of his mathematical procedures and reasoning. From a socio-math point of view the quality of mathematical actions depends on decisions learners take on what they see as a good solution (Gravemeijer, 1995). Here the quality of the solution itself and the accompanying mathematical procedures may be a goal of learning in order to improve mathematical actions. In adult numeracy education learning mathematics will not be a goal in its own. Results of a solved problem must always be seen in light of a purpose in real life situations since we work on learning-for-doing. That means that criteria for *quality norms,* the socio-math norms, are first and foremost related to the purpose of the mathematical action in a real life situation, the *functionality*. Adult learners may want to judge the quality of their actions and procedures of the resolved math problem to the extent to which they consider the result *usable* in (their) real life situations, rather than to the quality of the mathematical procedures and actions on their own. For example, in some situations an estimation will do whereas in another situation a very precise calculation should be made. Only in a second view may they also want to review the quality of their mathematical actions and procedures. At that point the norms for quality of their activities can be discussed in order to enhance the quality of the mathematical procedures. This may lead to new math norms for "taken-as-shared" knowledge and to broadening, adjusting and refining of individual knowledge and skills.

As an example, learners working in a subgroup could be doing the following task: "What is the average height of the learners you are working with in your subgroup?" Learners in one subgroup can exchange and review their ways of solving the problem and their answers may differ within their own subgroup from rough estimations to precise computations and also from those of other subgroups. When carrying out another task, e.g. compute the size of a window-pane that should exactly fit into a certain window frame, there will only be one correct answer. Mathematical computations should all lead to that answer and then the discussion may take place what would be the best mode of computation. Such problems may improve the quality of mathematical procedures and the learners' mathematical reasoning. Here we get at socio-math norms in light of functionality. The learners set their own *quality* norms.

Graph 5.8: Problem Solving and Quality Norms

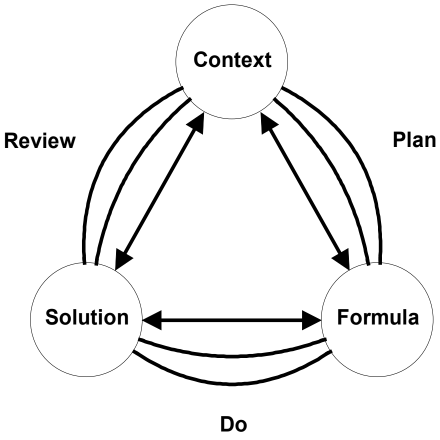


The *quality* of the mathematical process in the plan-do-review framework is symbolized in the arches around the triangle and can be analyzed by asking the following questions:
- Why are you going to do it in that way? (plan)
- Why are you doing it this way? (do)
- Why did you do it that way? (review)

These three *"Why?"* - questions may enhance the learners' thinking about the purpose of their mathematical actions and choices and should improve the quality of the learners mathematical concepts and procedures. This qualitative process, in fact, should lead to better functionality of their mathematical actions.

The *way in which* learners solve math problems is a third dimension in this problem solving process and can be visualized by a second line of arches around the triangle, as shown in figure 5.9. It pertains to the organization of how to solve a problem. Here the roles of the teacher and the learners must be clear. Both, working alone and working in cooperation with others, ask for organization, discussion and planning of actions. For this learners need criteria for interaction and cooperative working and this comes close to the *social norms* as discussed by Cobb, Yackel and Wood (1992) and Gravemeijer (1996). It encompasses general skills for interaction and cooperative working, like: rules for reading the problem, looking for lacking information, linking with prior knowledge, analyzing and systematizing information, allocation of tasks, agreement on procedures to be followed, e.g. how to do computations, how to help each other and how to check the answer, agreement on the solution, watching the process of doing and reviewing the total process. Here we enter the process of cooperative learning and working. These skills are needed in many real life learning and learning situations. They enhance cooperative, self-directed and teacher-free learning. They include general communication rules and joint-working principles to enhance the interaction between learners. Learners can come to agreements on how to work cooperatively in learning situations. This is the basis for independent learning.

Figure 5.9: Problem Solving, Quality Norms and Social Norms



The outer arches represent the *social norms*.
For analyzing the social process of the problem solving situation the following three questions can be asked:
- How are you going to do it? (plan)
- How are you doing it? (do)
- How did you do it? (review)

These three *"How?"*- questions may help learners to analyze their way of working, individually and in cooperation with others. It may enhance cooperative working and that is the basis for teacher-free and independent learning.
This *"What-Why-How"* problem solving model, inspired by RME and Socio-Constructivism, determines the entire problem solving process and should be the basis for learning and teaching numeracy in ABE. With the help of the three charts and questions adult learners (and their teachers) can focus on details in this process and this may help to become aware of their own learning processes. This may enable them to gain insights into their ways of learning, to improve the quality of their learning and consequently the results regarding knowledge and skills, and to improve their ways of working in cooperation with others. It may help adult learners to take more responsibility for their own learning. That is what we aim to achieve in adult education. In this way RME and Socio-

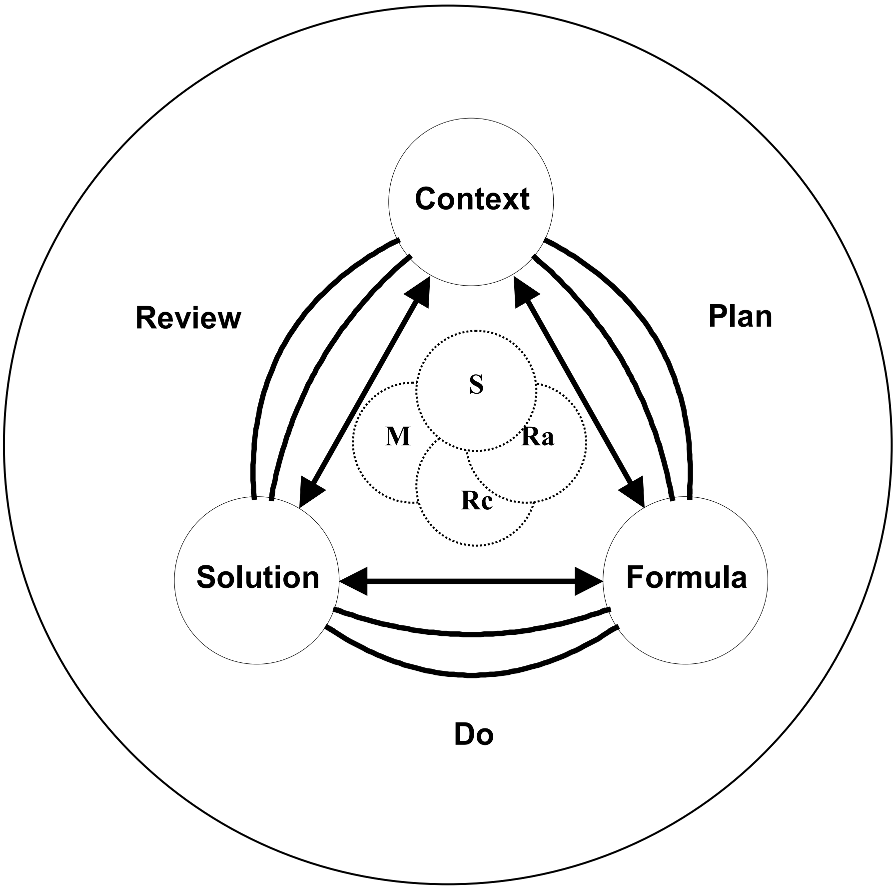
Constructivism can be a gateway to independent learning. (van Groenestijn, 1998)

5.3.4 An Instruction Model for Functional Numeracy Education

The actual learning of mathematics in ABE as described in 5.3.2 is, in fact, embedded in the what-why-how process of problem solving as described in section 5.3.3. For that, the final step in this process of developing a foundation for numeracy education in ABE is merging these two models into one metacognitive model. The four action fields are integrated in the process of problem solving. This creates, together with the andragogical starting points in 5.3.1, the foundation for *Functional Numeracy Education (FNE) in Adult Basic Education.* The result of this combination is visualized in the chart in figure 5.10.

This chart can be used as a learning and instruction model for FNE in order to help learners and teachers to get these processes clear. It shows the way how to develop functional numeracy and how to get learners gradually prepared for teacher-free learning. It is also the basis for independent learning in lifelong learning situations, directed by adults themselves and without any teachers. In this way the andragogical starting points for adult learning, Action Theory, RME and Socio-Constructivism go hand in hand and are a gateway to functional numeracy, to teacher-free learning and eventually to independent learning in real life learning situations. (van Groenestijn, 1998).

Figure 5.10: Functional Numeracy Education in ABE (FNE)



Explanation to figure 5.10
The underlying circle symbolizes the field in which the adult’s learning activities take place. Learning activities are directed by the thinking processes of the individual person, called “mental actions”. The four inner circles in the triangle symbolize the four action levels of manipulation (M), representation of concrete situations (Rc) and more abstraction of reality (Ra), and the symbolization level (S). These levels are interwoven in all individual learning activities. Depending on the conceptions the learner has concerning the subject to learn, he starts from one of the four levels in problem solving situations, but he needs all four levels to come to full and complete mathematical concepts and operations. Communication, including verbalization, is embedded in the what, why and how questions in the triangle plus the surrounding arches for quality and social norms. Together with the andragogical starting points, these components lead to functional numeracy and to teacher-free and independent learning. This instruction model can be used as the foundation of Functional Numeracy Education (FNE) in Adult Basic Education.

5.4 Conclusions

The pedagogy of Paulo Freire and studies about learning in practice have made clear that the essence of adult learning is that they want to learn when they feel a need to learn. A common way of learning by adults is informal learning in the course of real life situations. This, however, can also have disadvantages because learning in practice may yield partial and inflexible knowledge because it is often situation-based and situation-bound. Theories about learning in adult education should profit from the studies about learning in practice and should try to create learning environments that challenge adults to learn and make them feel the need to learn. Learning in adult education should focus on learning-for-doing, rather than learning-for-knowing in order to make school knowledge functional and applicable in real life situations.

Communication is an essential component in adult learning. Learning takes place through interaction and reflection. Communication helps the adult learner to verbalize his thoughts by which he may become conscious of his knowledge, conceptions and misconceptions. This is the start of new learning. Discussion about and reflection on the learner's actions are the basis for developing self-reflection and a better self-concept which may help the adult to function better in personal and societal life.

Action Theory and RME provide learning models that can be very helpful for the learning of mathematics by adults. Based on these models a new model was created for learning mathematics in ABE, composed of intertwined fields of manipulation, representation and symbolization. Adults go through all levels in various directions when learning mathematics. This instruction model can be a basis for the development of math instructional materials for ABE. It may help teachers to analyze their ways of instruction and to tune their methods to the adults' needs.

RME and Socio-Constructivism offer good starting points for creating learning situations in ABE based on problem solving. The what-why-how process may help adult learners to develop insights into their own learning processes and to improve their learning. The teacher can encourage this process by the plan-do-review questions. Getting a grip on their own learning processes is the basis for teacher-free and independent learning. Insight into the what-why-how process is essential for adults in lifelong learning.

RME, Socio-Constructivism and Action Theory can inform the development of a numeracy program in ABE that leads to functional numeracy and independent lifelong learning. Together with the andragogical starting points they help create the foundation for adults learning mathematics and for the development of numeracy programs in ABE. The final instruction model for *Functional Numeracy Education*, presented in figure 5.10, can help teachers in ABE when analyzing this learning process, in order to create learning situations in which adult learners can improve their learning and can take more responsibility for their own learning. In this way *a gateway to teacher-free and independent learning* can be created. (van Groenestijn, 1998)

6 Theory in Practice

6.1 Introduction

In chapter 5 we discussed the theoretical foundation for Functional Numeracy Education (FNE) in Adult Basic Education. This theoretical foundation should be seen as a result of a mutual, reiterative process between theory analysis and teaching experiences in the practice of ABE. Developmental research in actual learning-teaching situations in ABE, based on these theoretical considerations, started around 1987 and took shape in the experimental package *"Supermarket Strategy"* (van Groenestijn et al, 1992, van Groenestijn, 2000). This package provided an initial curriculum framework, assessment materials and instruction guidelines for teachers. These materials were used in several teacher training courses. (see chapter 1). In 1992 a first small-scale research based on these materials was done consisting of qualitative analysis of computation skills of adults in ABE in Utrecht, the Netherlands. (van Groenestijn, 1993). Twenty adults from literacy courses in ABE were interviewed about a selection of items of the *Supermarket Strategy.* The results offered new theoretical building blocks for numeracy education in ABE. These were processed into the "*In Balance"* instruction and assessment materials (*In Balance,* van Groenestijn et al, 1996-2000, van Groenestijn, 1996, 1998, 2000).
In the present study we analyzed computation skills of adults in ABE with the help of the *In Balance* assessment materials (see chapter 4) and will share some teaching experiences in this chapter. Together with the theoretical considerations in chapter 5 this will lead to revised and new building blocks for a numeracy program described in chapter 7.

Teaching experiences in ABE are almost always based on individual accidental events in classroom situations and are only in the mind of the teacher as part of the teacher's memories. In order to make our teaching experiences more explicit for others and to create a common base for discussion a learning program of six lessons was organized for the same group of learners who participated in the placement test, described in chapter 4. The general starting points for adult learning described in chapter 5 were used as guidelines for analysis of these lessons. The learning program had four goals:

1 Find empirical information about the actual learning of mathematics by learners in ABE that exemplifies the FNE theory.
2 Share our teaching experiences to make them more explicit for others and provide more insight into the actual mathematical knowledge of learners in ABE and their ways of learning.
3 Create a common base for a discussion about theory development of numeracy learning in ABE.

4 The information acquired is the next step to improving the quality of teaching in ABE and to further developing the FNE theory and ABE instruction materials.

The subject chosen was percent since this is a concept of which the understanding is crucial for adults. Also, the mean score on this subject was the lowest of all subfields in the In Balance and Cito placement tests: only about 17% of all questions was answered correctly. (see chapter 4.3.2). The performance indicates that this subject could be difficult or new to learners, which means it could yield interesting results. For these sessions the *In Balance* instruction materials were used.

In order to make our findings discussible and to structure the discussion, conjectures were worded which are, in fact, the original theoretical considerations behind the *In Balance* materials. These are now reworded and re-arranged to have them fit into the general FNE starting points listed in chapter 5, section 5.3.1.

In this discussion we will focus on the cognitive analysis of acquiring knowledge and skills by adults. For that reason we will not discuss the first two starting points, *"adults are free to learn"*, and *"learners and teachers are equal partners in a learning situation",* since these concern a general andragogical approach in adult learning.

Starting point 3, "Adults' own experiences are the basis for learning" leads to two conjectures. They represent the wish and the need to integrate adults' own experiences in the learning process in order to give meaning to new mathematical knowledge.

Conjecture 1: Prior knowledge on mathematical subjects is often patchy and based on misconceptions.

Conjecture 2: Real life experiences are an aid when building mathematical concepts and linking school math to real life situations.

Starting point 4, "Authentic materials should be used as instructional materials in school learning situations", is based on the assumption that hands-on materials in adult education should not be artifacts of reality but should be "real". In practice however, such materials can be problematic due to the fact that they were not designed as learning materials. This leads to *conjecture 3: Authentic materials in learning situations must be carefully selected.*

Starting point 5, "Learning takes place by interaction and reflection." refers to the need for creating learning situations in ABE in which learners become aware of their own knowledge by interaction and reflection and discover how they can learn from each other. This leads to *conjecture 4: Interaction and discussion are essential when acquiring new math knowledge.*

Starting point 6: “Learning in adult education should lead to functional knowledge and skills”, asks to be made operational. This leads to two conjectures which show the essence of a concise program for developing functional knowledge and skills: benchmark learning and the use of integrating visual models.
Conjecture 5: Learning through benchmarks helps to organize the learners’ way of thinking and to develop reference points for mental math and estimation.
Conjecture 6: Integrating visual models are useful means to link between informal and formal knowledge and to also cross-link mathematical concepts and operations.

Finally, starting point 7, “Adults direct their own learning by constructing and re-constructing, organizing and re-organizing their own knowledge, procedures and skills” sets a core idea in adult education. From this point we leave traditional school math education and this is where adults start to do math in a functional way. At this point adults are encouraged to create their own functional, mathematical procedures.
Conjecture 7: Functional notations created by adults themselves are preferable to traditional procedures, like algorithms, as learned in school.

Along the lines of these conjectures we will discuss our teaching experiences in the following sections. The learning program followed the placement test, described in chapter 4, and spanned six weeks, existing of one session per week. The set-up of the learning sessions is described in section 6.2, the explanation of the subject in 6.3 and the findings in 6.4. The quantitative results are presented in section 6.5

6.2 Setup of the Learning Program

The placement test was carried out in December 1997 and January 1998 and involved 32 learners. These learners were placed in two groups: a lower level and a higher level group. The lower level group was composed of learners who had achieved level 2 or 3 on the IB placement test. Learners on level 4 were placed in the higher level group. Twenty-six learners actually started the math course. Four teachers were involved.
As part of the regular program the learners were offered an experimental program on percent during the first six weeks of the math course. In each session one hour was spent on the topic of percent in each group. The rest of the session was used for the regular math program, but did not include tasks on percent. Both groups were offered the same program. All sessions were video-taped, documented and analyzed.
The program started with an introductory session structured to acquire more specific information about the learners’ prior knowledge. For this tasks from an NCAL research project (Ginsburg, Gal and Schuh,1995) were used. After that,

five instructional sessions were spent on learning percent with the help of the *In Balance* instruction materials. At the end a post test was done on percent, composed of the same percent tasks as administered in the placement test, 4 tasks coming from the *In Balance* test and 6 tasks coming from the *Cito* test. Afterwards, individual interviews were held with eight learners, four from each subgroup.

Table 6.1: Program planning March-June 1998

date	session	subject
Dec97/Jan98	0	placement test (pretest)
February-April 1998	1	introductory session
	2 - 6	instruction sessions
May	7	post test percent IB+Cito
May, June 1998		individual interviews

6.3 Subject of the learning sessions

6.3.1 The introductory session

To acquire more specific information about the learners' actual prior knowledge and skills on percent an introductory session was organized. Seven tasks were offered on benchmark percents, coming from the National Center on Adult Literacy (NCAL), Philadelphia, USA (Ginsburg, Gal and Schuh, 1995). These tasks were originally used in a study on informal knowledge and skills about benchmark percent with sixty American adults. (see English version in table 6.3). These tasks were chosen for two reasons:

- It was preferable to have external introduction tasks, unrelated to the IB materials.
- By using these tasks we created the possibility of comparing the results of these Dutch learners to the findings of the NCAL study in the U.S. in a separate study.

The tasks were presented as a paper-and-pencil task. The learners worked individually on these tasks for about 15 minutes and discussed it afterwards in the group setting.

Tasks 1-3 (conceptual tasks) examine knowledge about 100% as the basis of the percent system. Tasks 4 and 6 (shopping tasks) test computations on two benchmark percentages they may encounter in everyday shopping situations, 50% and 25%. Tasks 5 and 7 (school tasks) test insight in the same benchmark percent but in a different context. (see table 6.3) Twenty learners were present at the introduction session. The quantitative results are shown in table 6.2.

Table 6.2 The numerical results on NCAL tasks. (N=20)

Task		#-correct	%-correct
1. Blood test - 90%	correctness test	6	30
	reasoning 90%	5	25
2. Single parents-15%	answer 85%	10	50
	pie chart 15%	13	65
3.100% juice	reasoning 100%	11	55
4. Shopping 1 - 50%	reasoning 50% off	12	60
	answer 30 gld	15	75
5. School 1 - 50%	answer 50%	8	40
6. Shopping 2 - 25%	reasoning 25% off	9	45
	answer 60 gld	10	50
7. School 2 - 25%	circle 5 persons	6	30

Table 6.3 - NCAL Percent Tasks

1. New blood test for cancer
Stimulus: Circled header from a newspaper article stating "New blood test detects cancer correctly in 90% of all cases".
Questions asked: Do you think this is a good test? Why? What does the 90% tell you?

2. In 1970, 15% of all American children were living in single parent homes
Stimulus 1: Printed statement purportedly from a magazine or newspaper article.
Questions: Can you tell what percent of the children were not living in single parent homes?
Stimulus 2: Printed circle
Question: If this circle represents all the children in the United States during 1970, about how big a slice of it would be 15%?

3. 100% Juice
Stimulus: a photo/container of a orange juice that says "100% juice":
What does 100% juice mean?
(extra question for the discussion afterwards: Could it also be 200% juice?)

4. Shopping 1
Stimulus: photo of pants, priced *ƒ* 60.00. Text saying: 50% off
Questions: What does 50% off mean?
How much would you have to pay for this pants?

5. School 1
Stimulus: sketch of 20 stick figures persons in a diagram, representing students in a class.
Question: 10 persons of this group of students will do a math course. What percent is that?

6. Shopping 2
Stimulus: photo of a coat, priced *ƒ* 80.00. Text saying: 25% off
Questions: What does 25% mean?
How much would you have to pay for this coat?

7. School 2
Stimulus: sketch of 20 stick figures in a diagram, representing students in a class (same as in task 5)
Question: 25% out of this group of students did a math test. Draw a circle around the students who did the test.

The tasks were taken as a half-hour individual written test. After that the learners and the teacher discussed the tasks in the group setting.

At a first glance the results turned out better than expected. However, when we look more accurately we can spot some nuances. At the conceptual tasks 1, 2 and 3, the best result was achieved at task 3 and the lowest at task 1. At the computational shopping tasks 4 and 6 the best result was achieved at task 4, the 50% reduction. More difficult was task 6 about 25% reduction. In comparison to the shopping tasks the results go down drastically at the school tasks where the original number is smaller than the percentage. The lowest result was achieved at task 1, the blood test, where the concept “90% of all cases” was discussed. Tasks 3 and 4, which deal with 100% and 50% respectively, appeared to be the easiest.

6.3.2 The instruction materials

In sessions 2-6 the percent strand of the *In Balance* (IB) series was used (see appendix 6.2: In Balance units B3 and B4), which focuses on learning through benchmark percents (100%, 50%, 25% and 10%). Such benchmarks serve as reference points for mental math and estimation. Computations on paper or with a calculator can be can be developed for working with more difficult numbers and doing more complex computations. Learning through benchmark percent differs from traditional programs where learners often only learn about percent as a separate strand based on “100% is all”, how to compute 1% and then how to compute all other percentages by using a formula, like:

[(full amount ÷ 100) x percentage].

Learning through benchmark percents in the IB series goes in parallel with learning through benchmarks on decimals and fractions. Decimals and fractions are primarily embedded in the strand about measurement, focusing on learning the metric system, in order to give meaning to these numbers. By that the learning strand for the metric system has also been built up through benchmarks. The integrating link between these learning strands is proportions, visualized in proportion bars and the block-model (see below). These integrating visual models show proportions within a strand and cross-link between different strands. They encourage the adult learner to develop a broad insight into proportions, fractions, percent, decimals, the metric measurement system and their mutual relation. In all strands the four levels of mental action - manipulation (M), representation (Ra and Rc), and Symbolization (S), as described in chapter 5, section 5.3, are processed. Whereas RME encourages children to create their own models based on, for example, an empty number line and a proportion bar, the *In Balance* series offers adults a number of integrating structured models based on the number line and the proportion bar. The block-model (see figure 6.2), in fact, is a multiple proportion bar showing the benchmark points in mutual relation. Through this setup a concise but complete program was developed for the IB instruction materials. The choice for

learning through benchmarks based on integrating visual models was made for three reasons:
1) Learning through benchmarks may enable adults to develop reference points for doing computations and estimations, for instance computing 50% and 25% of an amount, and to gain insight into the relationship between strands, for instance, 50% is the same as 0.5 and as ½.
2) There is no time in adult education to have adults develop their own visual models in a spontaneous way. It is better to offer only a few visual models that support the benchmark learning and to use these models as integrating means through all strands. Such models can be an aid for adults to create their own mental models when acquiring new knowledge and skills.
3) Adults often know a lot but their knowledge can be patchy or based on misconceptions. Their computations are often based on merely applying rules, formulas and algorithms they learned in school, often without being based on insight. This inadequate knowledge has to be adjusted and developed further in a short time. By offering adults good integrating visual models it is hoped/assumed that they will develop good conceptions of percent, fractions and decimals as a basis for doing computations as well as for further learning.

While it contradicts the integrating IB approach, the focus in this learning program was only on the percent strand. The expectation from the benchmark approach is that new learners, who have never learned about percent in school situations, will develop a broad concept of percent, framed by proportions. Adults who have learned about percent in former school years, whose instruction was often only based on the 1%-rule and using the percent formula, may adjust and expand their original conceptions, or correct misconceptions on percent. Both groups of learners should develop broad(er) knowledge and skills for percent, which are flexibly applicable. The IB program has six main steps.

Step 1:

The percent strand starts in unit B3 with discussion tasks on the concept of 100 percent. Photos of advertisement leaflets show products saying, among other things, 100% cotton, satin, 100% pure juice, a coffee package saying “fifty-fifty”. After that a few statements follow, like: I don’t feel 100%, I am 99% sure, I have a job for only 60%. Learners are asked to discuss these statements and add a few more. These tasks are teacher-guided in the series. The teacher is encouraged in the teacher’s guide to also show hands-on materials pertaining to percent, to discuss the concept of percent in the context of proportions, using samples coming from the learners reactions, and to visualize percent as proportions in a bar or in a ratio table. These tasks are meant to create discussion and to activate prior knowledge. They call for informal and formal procedures learners have already acquired.

Step 2:

The second step is the benchmark 50%. This should be a relatively easy step because most learners know that 50% means "half". After a task about the number of male and female learners in their group, related to 100% and 50%, and visualized in a proportion bar, the following visual model is introduced:

Figure 6.1: 50%

<table>
<tr><td colspan="2">100%</td></tr>
<tr><td>50%</td><td>50%</td></tr>
</table>

Learners work with a blank model to compute a range of amounts for discount items like a TV, a radio, a microwave, a laundry machine, etc. This model is also used for whole numbers, fractions and decimals combined with the metric system. There it shows e.g. 1 kilo and half a kilo of sugar in fractions and decimals and 1000 grams of coffee in two containers of 500 grams.

Step 3:

At the next step the learners work on tasks pertaining to the benchmarks 50%, 25% and 10% with the help of the following visual "block"-model: (see IB-unit B3-3.8)

Figure 6.2: block-model benchmark percent

<table>
<tr><td colspan="20">100%</td></tr>
<tr><td colspan="10">50%</td><td colspan="10"></td></tr>
<tr><td colspan="5">25%</td><td colspan="5"></td><td colspan="5"></td><td colspan="5"></td></tr>
<tr><td colspan="2">10</td><td colspan="2">%</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
</table>

This "block" model is meant to support integrated thinking on proportions, percent, fractions and decimals, combined with the metric system. It is also used for multiplication and division for doubling-doubling and halving-halving computations (leaving out the bottom layer). It visualizes here the benchmark percentages 100%, 50%, 25% and 10% and shows their mutual relation, but it is also used at benchmark fractions, decimals, and their combinations, like 1/2 = 0.5 = 50% and 1/4= 0.25=25%. Accompanying blank work models on work sheets can be used by learners for doing their computations. With the help of this scheme learners can learn to start their computational thinking by using benchmarks in order to develop and improve mental computation and estimation.

Step 4:

At IB level 4 the 5% and 1% are added. From that point every percentage can be computed with the help of this model, e.g. 48% is the same as 50%-2%, or 37% is 25%+10%+2%, or 40%-3%, 30% is 25% plus 5% or three times 10%, etc. Blank work models are still used for doing such computations. In this way people develop insight and learn to profit by benchmark percentages. The calculator is introduced to do more complex computations.

Step 5:

The next step in the strand is doing computations with 10%, 25%, or 15% extra in shopping situations (e.g. a container of 400 grams rice plus 10% extra. How much does the container hold? see IB-B4- 3.6).

Step 6:

At IB level 5 more complex problems are offered, including computations with VAT, interest and application of percent in various situations. At this level a ratio table is introduced as a computation model. The learners are encouraged to do computations with the help of a calculator. The learners in the present learning program did not achieve this level.

The theme of the percent tasks in unit B3, level 3 step 1,2,3, focuses mainly on doing computations with money, and based only on 100%, 50%, 25% and 10%. In the final task at this level learners are asked to compute either the new price after the reduction, the amount of reduction, the percentage of reduction or the original price. The expectation here is that the learners can find this out by themselves, cooperatively, by using the visual model and the blank work models. (see IB-B3, 3.11).

In unit B4, level 4, the subject is set in a broader meaning. It focuses on processing simple data of a transportation investigation in four towns. The discussion in IB-unit B4-3.1 starts with a context involving a pre-investigation of 100 persons in each town. After that, the learners are told that the actual investigation was done with 2000 people in each town. Data are presented in a table and in pie charts. The learners are asked to sort out data spread over four modes of transportation: walking, by bike, by car, by public transport, related to accompanying pie charts. The benchmark model, with 5% and 1% added, is used to do computations. In this scheme the benchmark percents are presented in multiple ways: the “block”-model, the benchmark table and worded in fraction-language. The pie chart is introduced for all percentages. From then the pie chart, the “block”-model and the benchmark table go hand in hand as visual support in various types of tasks, also in parallel strands. The learners are encouraged to compute the right number of persons for each percentage mentioned in the transportation investigation, using the pie chart and the benchmark table.

Based on the results of the placement test it was decided that the higher level group would start with the basic program on level 3 of IB. It was assumed that they would need little time for that and could do more on level 4 tasks. The lower level group was expected to need more time for the level 3 tasks and might hardly achieve level 4.
Real-life materials used in this learning program were the same in all subgroups to keep conditions the same in the learning sessions, but in regular circumstances teachers are free to use leaflets, papers, objects, etc. preferably coming from the learners' actual lived-in situations.

6.4 Content Analysis of the Learning Program

The conjectures presented in the introduction are elaborated and examined in this section in light of the findings in the learning sessions. The results yield qualitative information for the discussion about the actual learning of the adults in these learning sessions and about the effects of the IB program. Though this information is limited due to the six sessions and studying only about the subject of percent, it may help ground the discussion on further development of the FNE theory.
The conjectures are listed as subtitles in this section as 6.4.1, 6.4.2, etc. Each conjecture will first be explained. Then the findings will be presented and results of the learners' work will be added. Finally, points for discussion will be summarized.
All sessions were video-taped, documented and analyzed. All learners' work sheets were kept for qualitative information. Transcriptions were made of all sessions. The most appropriate examples were selected to support the discussion to each conjecture. We focused on situations in which the teacher asked questions that conformed to the "what" and "why" aspects (see section 5.3.3) and did not "explain", in order to see what happened in the learners' mind when they answered the questions and/or responded to each other. These were situations in which learners "discovered" something or linked their knowledge to real life situations and their own real life experiences, and situations in which learners clearly showed their misconceptions or "Eureka"-moments. We tried to find situations in which learners applied their own idiosyncratic computations on paper, deviating from standard procedures like doing algorithms and situations in which learners helped each other and by that showed something about learning from each other. All verbatims in the examples in this section, originally in Dutch, were translated into English. We tried to keep the translations as close as possible to the words the learners used in Dutch and to their way of speaking Dutch. All learners are second language learners.

6.4.1 Conjecture 1: Prior knowledge on mathematical subjects is often patchy and based on misconceptions.

Explanation:

From an andragogical point of view learning starts when adults are aware of what they know and what they want to know. Discussions about mathematical subjects may start with a discussion about real life situations related to that subject to activate prior knowledge. The purpose of this is two-fold:
1) activate people's interest and investigate their real-life and school experiences on that subject. These can be used to encourage them to learn about the new topic and to link new knowledge to prior knowledge.
2) identify prior knowledge or possible gaps and misconceptions regarding the subject.
The latter is important to fine-tune new instruction in the best possible way to the learners' actual knowledge. Ginsburg and Gal (1996) worded this as follows: *Identifying people's partial understandings and intuitions is important because new learning will be filtered through and become integrated with prior knowledge. Each learner's informal knowledge should be identified so that new instruction can be designed to link with what already has meaning to the learner. At the same time, attention must also be paid to incorrect ideas or "patchy" knowledge so that these do not distort new learning or cause confusion.* (Ginsburg and Gal, 1996, p.3)
Learners in ABE have, in general, little school experience but a rich variety of real life experiences. Their mathematical knowledge often exists of a lot of informal, practical knowledge mixed with little formal school knowledge. At the start of ABE math courses it often appears that adults have only acquired partial conceptions and misconceptions about mathematical topics, e.g. the truck-driver who did not know the relationship between meters and kilometers (see chapter 1), and the woman who could read the analog clock but could not give meaning to her daughter's note: "I'm home at 15:30". (see also chapter 3)

Looking at percent, learners may know that 100% is "all" or "all good", or "perfect" and 50% is "half", but when it comes to more complex concepts or non-money-related issues they may get confused, for instance by "90% of all cases". Though learners may have basic conceptions like 100% and 50%, during computational tasks it often appears that they relate the percentage to a number, e.g. 10% is always 10. This becomes clear, for example, at money computations when it regards reduction, e.g. "10% off" means "10 off". Learners often become aware of such incorrect ideas when they discuss tasks in which the percentage is larger than the original amount, such as " 25% reduction of 20 guilders, what do you have to pay?", or "how much is 25% of 20 people?" It is important that such misconceptions are identified at the start of learning about percent.

Findings:

The NCAL tasks were meant to identify the learners' prior knowledge and possible gaps and misconceptions. Two tasks on the concept of 100% are discussed: the first one about 100% in a clear context without distractors, "100% juice", the second one in a more hidden context, the blood test for cancer. After that the concepts 50% and 25% are discussed.

Most learners understand the meaning of 100% in a clear, simple context without distractors, like "100% juice". The following example shows a part of the group discussion in the higher level group:

> **Example 6.1**
> Teacher, putting the juice container on the table: 100% juice, what does that mean?
> Learners were sure: no sugar, no water, only pure juice.
> Teacher: could it be 80% juice?
> Learners: yes,...with sugar.. and water.
> Teacher: could it be 120% juice?
> All, unanimously: No.

It appeared that almost all learners, also in the lower level group, could explain the 100% juice and also at other statements like "100% cotton" and "100% healthy", for example by mentioning the opposite, e.g. "no other things" in clothes, and "not ill", or by using the words "only" in e.g. "only cotton", or "all" in e.g. "all wool". However, the paper work at the NCAL introductory tasks 2 and 3, about the juice and the single-parents, shows only about 50% correct answers.

When it comes to more complex concepts as in the blood test: *"New blood test detects cancer correctly in 90% of all cases"*, learners could relate the 90% to 100% and tell that the test should be a good test, but their reasonings are not clear. We noted the following discussion in the lower level group:

> **Example 6.2**
> Teacher: The blood test for cancer. Is it a good test?
> Mohamed: 100% is very good.
> Teacher: yeh, others?
> Fadumo: Test is good. 90% many cases. For many people good, for some people not good.
> Teacher: Is 90% "good"? (stresses "good")
> Achmed: 100% is all good.
> Fadumo: 90% is good, but positive on test is not good. Good is not good, yes or no cancer.
> Farangis: a good test.
> Teacher: When would the test be perfect?
> Fadumo: 100%, yeh, that's perfect.

The learners show their knowledge about 90% related to 100%, but not clear here is whether they understand the context, namely “90% of all cases”. Mohamed may relate the 100% to the quality of the test. Achmed probably only responds to the teacher’s question “is 90% good?” Fadumo may relate the 90% to the outcomes of the test: “for many people good, for some people not good”. Her statement “positive on the test is not good” shows her real life experience, but is in fact confusing in this discussion. She probably relates the 90% to the number of people tested and are diagnosed positive on the test, but not to “90% of all cases”.
In the higher level group something similar happened:

> **Example 6.3**
> Sunita, hesitating: 90% cancer, that’s a lot. Many people.
> Azeb: 90% cancer, 10% no cancer.
> Sunita, showing her real life experience: but one blood test is not enough. Need more tests, more experts for diagnosing cancer.

Azeb also relates the 90% to the number of people tested. Sunita doubts, but not clear is whether she doubts about the number of people or about the quality of the test, or both. Both learners obviously cannot relate the percentage to the correct prediction of the test, “90% of all cases”.
Though the learners show their understanding of the concept 100% is “good” and the relationship of 90% with 100%, it is still doubtable whether the essence of the statement “detects cancer correctly in 90% of all cases” was clear to them. They probably thought of 90% of all tested people, not of correct predictions. These discussions show where learning starts.

In the next examples it appears that some learners relate the percentage to the actual number, e.g. 25% is 25 persons. Though almost all learners know that 50% means “half”, they get confused by computational tasks, e.g. for the following: *10 people out of 20 take a math course. How many percent is that?*

> **Example 6.4**
> In the lower-level group:
> Mohamed: you can’t do this. 20 people is 20%; 10 people is 10%, 1 person is 1%.
> Farangis transformed this problem to money: 20% is 20 guilders; 10% is 10 guilders.

These learners could not find out that 10 people should be 50%.
The same kind of responses happened to the task: 25% out of 20 learners took a math test. How many learners?

Example 6.5
Sahra: 20 people, 25% won't go. There are only 20 people; no 25%
Mohamed: 2 people, 100% is 2.
Teacher: then what is 100%?
Sahra, a bit hesitating: 100% is all, 25% is 25, so cannot take away from 20.

At the teacher's question: "could you circle a quarter of the group?", all learners could do that, and some understood that it should be 25%, but could not relate 100% to the number 20.
At money tasks we see the same happen at task 6, the raincoat: *price ƒ80.00, 25% off*:

Example 6.6
Sahra: 25% off is 25 guilders. Pay 55.
Teacher: You pay 55 guilders?
Fadumo: pay 60 guilders.
Teacher: are you sure?
Achmed: pay 20 guilders
Mohamed: quarter is 20 guilders off. Pay 60 guilders.
Teacher, looking at the group: 60 guilders? Is that correct?
The learners agreed.

Fadumo, Mohamed and Achmed could relate 25% to a quarter of the original amount, but Achmed forgot to subtract. Striking is that Mohamed could compute the correct amount at the 25% in this money context, but not in the previous context. For Sahra 25% is still 25.

Discussion:

The findings support the conjecture that prior knowledge on mathematical subjects is often patchy and based on misconceptions.
Almost all learners show a good understanding of 100% in a clear, simple context in the discussion, though not in the written tasks. However, in both subgroups learners got confused by the blood test for cancer, where the meaning of 100% was more hidden. They do know that 90% is close to 100% but in this context it was not clear whether the learners related the 90% to the correctness of the test, to the number of people tested or to the number of people tested positive. For computational tasks about three-quarters of the learners know that 50% is "half" and can apply that in money-related items. For the school context more than half of the learners could not compute that 10 people out of 20 should be 50%. The same difficulty at the benchmark 25%. More than half of the learners related the percentage to the actual number instead of the given percentage, e.g. 25% is 25 guilders or 25 persons. This means that most of these learners have only a partial conception of "percent" at the beginning of the course.

6.4.2 Conjecture 2: Real life experiences help at building mathematical concepts and linking school math to real life situations.

Explanation:

Freire's pedagogy about "Learning from experiences" in adult literacy programs taught us the value of real life experiences in school learning situations, in particular in adult education. It even became part of the Act on adult basic education in the Netherlands. (Freire, 1970, Ministerie van O en W, 1986)
With respect to numeracy and learning mathematics in school it means that real life experiences may help to develop insight in mathematical situations, e.g. in processing data about health matters, like the blood test for cancer, or knowledge about food issues like whole and low fat milk, data about elections or investigations like the transportation investigation, or composition of groups, like the percentage of men and women in the learners' own learning group. Such tasks appeal to learners' personal experiences and it is assumed that people will learn better if they feel personally involved. It may also help them to link school math to their personal situations.

Findings:

The blood test example in 6.4.1 shows that learners use their real life experiences in the learning situation spontaneously. During the group discussion about the meaning of 100% and 50% the learners were shown, among other things, two milk containers holding whole and low-fat milk. In Dutch this is indicated by "full" and "half-full" milk. The following discussion developed:

Example 6.7
Teacher: What does it mean here, 100% and 50%?
Sunita: full milk has more fat than half full milk. Half-full milk less fat.
Teacher: what is on the containers about fat?
Nezira reads on the low-fat container: fat, 1 comma 5
Hatice reads on the other one, the whole milk container: fat, 3 comma 6
Teacher: Is it right about the fat?
All at once: yes.
Teacher: Which milk is healthier?
Hatice: half-full milk
Teacher: why is the half-full milk healthier?
Hatice laughs and indicates a very fat person with her arms: I don't want to be fat.

Note here that in Dutch the decimal point is a decimal comma. Hatice shows that she understands the mathematical information and the difference between whole and low-fat milk. She can relate that to personal effects. Here the mathematical information has been put in a functional context which may help to bridge the gap between school math and functional real life math.

At the transportation investigation, IB - level 4, the higher level learners were first asked to visualize which countries they came from and how they came to school in a proportion bar and in a pie chart (see also elaboration at 6.4.6). After that it was not difficult for them to understand the tasks in the booklet and to analyze the transport survey data in the pie charts. They gradually moved from their own experiences into the mathematical context in the book.
However, a few learners, who missed this extra introduction here, had more problems. These learners started working in the booklets with the pie charts and could not find out what to do. After a while they skipped these tasks and continued with computational tasks
This example shows that the first few minutes of extra instruction, involving learners' own experiences, made it easier for them to understand the math in the booklets. Their own experiences were the link between real life math and the math in the booklet.

Discussion:

The findings support the conjecture that real life experiences help at building mathematical concepts and linking school math to real life situations.
The whole and low fat milk and the transportation investigation may show that real life experiences help at processing new mathematical information. In particular at the transportation investigation it was clear that the introduction about their own mode of transportation to school helped the learners to understand the pie chart and the accompanying tasks. The learners who missed this introduction had problems to understand that context.
The examples discussed above indicate that adults often inject their own real life experiences spontaneously. If the teacher creates a situation in which the learners feel involved, then mathematics becomes meaningful for them, making it easier to process mathematical information. Linking mathematics to the learners' own real life experiences is also the bridge to numeracy.

6.4.3 Conjecture 3: Authentic materials in learning situations must be carefully selected.

Explanation:

In general it is assumed that authentic real life instruction materials are essential for developing math skills which are applicable to the individuals' actual envisaged situations. They create the link between school mathematics and real life mathematics. This starting point is also accepted in the FNE theory. It strongly encourages the use of photos of real-life materials presented as tasks-in-context as well as actual real-life objects. However, authentic materials are not designed as instructional materials. If real-life materials are too complex or have many distractors, they may hamper learners when focusing on the actual mathematical topic and consequently at learning math.

Findings:

The learners in both subgroups were shown a few objects: a container holding 100% orange juice, containers of whole and low-fat milk, and the labels on their own clothes indicating the composition of the fabric, like 100% wool, or 70% wool and 30% viscose, 100% cotton. The learners compared and discussed the percentages in it. In the higher-level subgroup this all went well. They could answer questions like "which sweaters are made of pure wool or cotton?". They could also draw the percentages in proportion bars to show the composition of their own sweaters. The learners in the lower-level group had more problems with it. They stumbled on words like “viscose” and “acrylic” and “synthetic”. Discussion about these words distracted them from the intended task, comparing the percentages.

Another example concerned the whole and low-fat milk. The learners were asked to find out the difference between “full” and “half-full” milk. Two learners read the information on the actual containers, but the mathematical information was too much and too difficult, e.g. the use of decimal commas (Dutch notation):

Example 6.8

Teacher, showing two one-liter milk containers: What does that mean: full and half-full milk?

Sahra: full is fat, 100% ; half-full is good, 50% fat.

Teacher: How much fat is in the milk?

Sahra and Nezira start reading all the nutrition info on the containers, very slowly, with difficult
words like “carbohydrates”.

Teacher, focusing on the actual topic: how much fat is in the milk?

Sahra, reads on the containers: 36 gram fat, 15 gram fat

Nezira: no, no, not 15 gram.... it says one and half gram. How do you say that? One comma five

Sahra: yeah, and reads on the other container: three comma six.

Teacher: is that a lot? or a little?

Sahra: a lot, full milk, much more fat.

Sahra shows her practical knowledge about the difference between whole and low-fat milk in her first reaction, but at the same time she shows her difficulty with reading the text and analyzing the mathematical information on the containers. She also cannot imagine the actual amount of fat related to the 3,6 gram. She thinks it’s a lot. Perhaps the difference between the numbers 100% and 50% indicates to her that there should be a big difference, so “a lot”. She cannot explain the actual difference between whole and low-fat milk. So there were more problems. The following example about carpet tiles shows another problem when analyzing authentic materials. The learners were discussing the meaning of 100% to a few objects in photos. The question accompanying a photo of the carpet tiles was: “What does 100% synthetic mean?”

> **Example 6.9**
> Zaara reads the accompanying text in the photo of the carpet tiles advertisement:
> 100 percent synthetic, 50 plus 50 cee em......is half half.
> She confuses here "50x50 cm", the given dimensions of the tiles, with 50% (half half)

The learners in the lower-level group stumbled several times on such tasks.

Discussion:

The findings support the conjecture that authentic materials in learning situations must be carefully selected.
Though the learners showed in earlier tasks their concept of 100% is “all”, or “good”, or “nothing else”, some contexts appeared too complex and the text was too difficult to understand. When combined with some misunderstood mathematical information (50x50 cm), such instructional materials can be stumbling blocks. It indicates that we need to be selective with the choice of authentic materials and have to remove superfluous and distracting information if it appears too complex or too difficult. Only relevant information is helpful at concept-building.

6.4.4 Conjecture 4: Interaction and discussion are essential when acquiring new math knowledge.

Explanation:

In chapter 5 it was described that interaction and discussion are seen as a social and cultural base for learning in action theory, Freire’s theory and in andragogy. Also, in RME and constructivism interaction is seen as an essential part of acquiring and processing new math knowledge and building math conceptions. Interaction and discussion help learners to reflect on their own knowledge and skills and hence become aware of their own knowledge and skills and differences with others. In-context tasks that ask for problem solving are particularly appropriate for learners to discuss different strategies and to help each other when problems occur. It helps them to learn to communicate about mathematical problems, which is one of the facets of numeracy (see chapter 2). The ANN Massachussets Numeracy Standards (Leonelli and Schwendeman, 1994), the EFF Standards (Curry et al, 1996) the NCTM standards 2000 (NCTM, 2000) describe mathematical communication and reasoning as a precondition for work, social life and critical citizenship. Ginsburg and Gal (1996) indicate that learners may benefit from each other's observations and explanations "*because one student may be able to identify another student's point of confusion or explain a concept with examples that are especially helpful for that particular learner"* (Ginsburg and Gal, 1996, p. 11).

From an andragogical point of view adults acquire new knowledge and process information by listening and talking to others and selecting that information that seems useful to them. Freire already encouraged such mutual learning and teaching in the seventies. The teacher only has to facilitate the process. When learners discuss their knowledge they become aware of it and reflect it. In that process they may also identify possible gaps and shortcomings. This is the start of learning: *adult learning starts when people are aware of what they know and what they want to know.* (see also the explanation at conjecture 1). The way in which this process develops depends on the quality of the interaction in the learning setting.

Findings:

In the higher level group we noticed the following discussion in a subgroup of four learners. They talked about the introductory task 50% of 20 people:

Example 6.10
Zeki: 20 people is 20%
Teacher: are you sure?
Zeki: 20 people is 20%, 100 people is 100%.
Azeb: no, the group is 100 percent, the entire group is 100 percent.
Teacher: is our group here 100%?
Azeb: yes.
Teacher: a group of 30 people, could that be 100%?
Azeb: yes.
Zeki, listening: group is 100%. Group of 30 people is 100%. 40 people also 100%?
Sunita: Yes, and half of it is 50%. So, half of 20 is 10, is 50%.

After that the discussion went on about 25% of 20 people:

Example 6.11
Zeki: pointing the other learners and himself: group of 4 people is 100%. I am leaving, is 25%.
You three is 75%.
Teacher: Now, 40 people in a bus; how many is 25%?
Zeki: eh..... 9.... or 8
Sunita, looking at Zeki: group is 40..... is 100%.
Zeki: 10.... of course.
Teacher: pointing at the drawing in the task: 20 people, how many is 25%?

> Zeki takes his work sheet, looks at the sketch and circles 5 people.
> Teacher: and what about a group of 60 people?
> Azeb: 15, ... 15, 30, 60
> Zeki: yeah.....that's right.....15, 30, 60
> It's time to go home and Zeki closes the session with: "Need to talk a lot"

In this example we see a real learning moment in the discussion. Zeki moves from a misconception to a good concept of 100%. He was helped by his co-learners and the teacher. The learners were able to listen and willing to explain. Here interaction and reflection led to qualitative improvement of knowledge. The learner realized that communication is a necessary means for learning. We also see that the learner understood the benchmark percent 50% and 25%.
There were a few more examples of helpful group discussions, e.g. at conjecture 3 where Sahra learned to read 3,6 (three comma six) and 1,5 (one comma five) gram at the whole and low fat milk with the help from Nezira, perhaps still without understanding the actual meaning of it. Yet she learned to look more closely at numbers, and discovered the decimal comma. She understood that the comma changed the meaning of the number. At conjecture 2 the learners who worked on the transportation survey first discussed the pie chart among themselves. By doing this they found out cooperatively what to do when attempting the further tasks. Fruitful discussions also occurred during the learning about functional notations. (see at conjecture 7, described in 6.4.7). The quality of the results depends on the quality of the discussion.

Conclusion:

The findings support the conjecture that interaction and discussion are essential when acquiring new math knowledge.
Zeki shows clearly how he learned from the interaction with his peer learners. In the first discussion he is open for the comments of his peers, but the peer learners were also willing to explain to him the concept of 50%. Zeki understood that his concept of 50% was a misconception. In the second discussion he shows that he had learned from the interaction and could apply the 25% computation in a new situation. Sahra showed she had learned something from Nezira. The learners who worked jointly on the transportation survey tasks were able to find out jointly how to interpret the pie charts in those tasks.

6.4.5 Conjecture 5: Learning through benchmarks helps to organize the learners' way of thinking and to develop reference points for mental math and estimation.

Explanation:

Math programs in ABE need to be concise due to time constraints. Learning through benchmarks enables learners to focus on reference points which may help them to develop insight into percent, fractions and decimals and their mutual relationship, and to organize their mathematical thinking. It may also help to create a bridge between informal and formal mathematical thinking and to improve estimation skills and mental computation.

In the present learning program the learners learned to work with the block model (see explanation in section 6.3.2). The focus is on the benchmarks 100%, 50%, 25% and 10%. From there 5% and 1% can be derived. Based on these benchmarks all computations with percent can be done. Benchmarks are also the start for learning to do mental computations and estimations. The block-model can also be used as a blank working model. After having worked with the extensive block-model (see section 6.2.3), the block model is simplified to only 100%, 50% and 10% as reference points, as a part of progressive schematizing, assuming that learners can derive other percentages from that.

Findings:

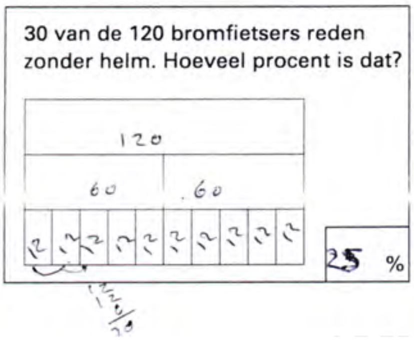
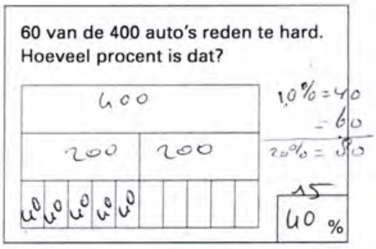
In the following examples three learners show their thinking through the benchmark model clearly. The three tasks were done after six weeks of training.

Mehmed shows he understands the benchmark model and can do the mental computation in the blank working model. He indicates 30% by combining three blocks of 10%. He writes 30% to it and the number 180. In addition he applies an algorithm, probably to check his answer: amount times percentage, divide by 100 (by leaving out the final two zeros).

Azeb shows she knows how to compute 25%: she takes the first and the second full box on the bottom line plus half of the third box (12+12+6). She could have taken half of 60, but perhaps she found it easier to start from 10%.

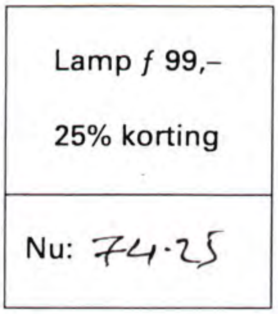
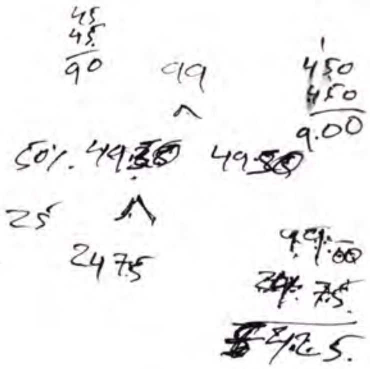
Hatice shows in her computation next to the benchmark model that she knows how to compute the percentage belonging to the number 60. She knows that 40 is 10% and 80 is 20%. She adds the correct answer "15" above the number "40" she perhaps first wrote in the answer box., which is 10%.

Figure 6.3

Mehmed:	Azeb:
Question: 30% of 600 biker riders rode too fast. How many biker riders are that?	Question: 30 out of 120 bike riders rode without a helmet. What percentage is that?
 **Hoeveel procent?** Teken de percentages in de schema's. **30% van de 600 fietsers reed door rood. Hoeveel fietsers zijn dat?** 	 **30 van de 120 bromfietsers reden zonder helm. Hoeveel procent is dat?**
Hatice: Question: 60 out of 400 cars drove too fast. What percentage is that?	
 **60 van de 400 auto's reden te hard. Hoeveel procent is dat?** 	

In the examples above we see that the structure of the block model indeed helps the learners organize their benchmark thinking. In the following three examples we see that the learners apply the benchmark thinking spontaneously, creating their own models.

Figure 6.4 Azeb:

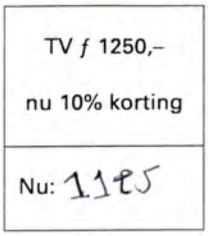
Task: a lamp costs ƒ 99.00 25% reduction. Now:	Her computations:
 Lamp ƒ 99,– 25% korting Nu: 74·25	

Azeb starts with dividing 99 into half. To be sure she applies addition algorithms to check that 45 and 45 makes 90 and that 4.50 plus 4.50 adds to 9.00. Her scheme shows a correction in the number 49.50. Not clear here is whether the correction was made before or after the two additions. Then she gets to the next step, 25%, by halving again. Her work does not show a computation for this. It only shows the answer 24.75. This is amazing because her work always shows algorithms at all her computations. She probably could do it mentally because we practiced rounding up and down and how to process the difference with the actual number. (see section 6.4.7) After that she applies a subtraction to compute the new amount to be paid. The many dots in it show her recounts to check her computation. The task in the left cell of the table shows the correct answer. In her scheme she applies her own idiosyncratic interpretation of the block model through repeated halving. Azeb shows that the benchmark model works as "model for" her mathematical thinking and her computations. Her notations are functional.

Nadia shows many algorithms, like at the placement test, but the benchmark model came also into her thinking process. Her work shows long divisions for dividing by 2, by 4 and by 10, but in addition she used the block model as a working model to compute the benchmarks. It is not clear whether she did the algorithms first to find the answers in the block model or whether she did the benchmark computations in the block model first mentally and checked her answers by doing long division. The answer is correct.

Figure 6.5: Nadia - posttest TV

Task:
TV *ƒ* 1250.00
10% reduction
Now:

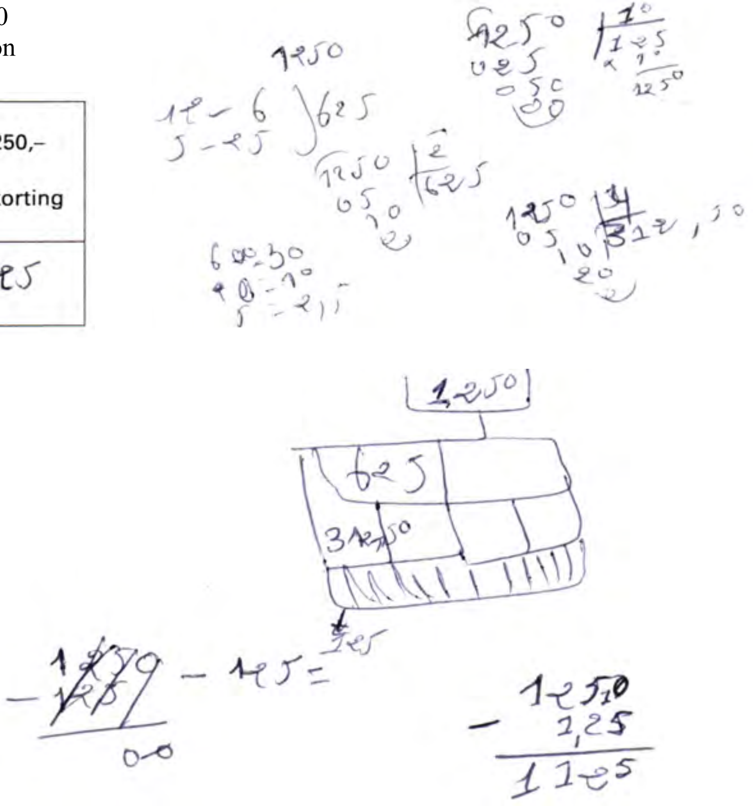


TV *ƒ* 1250,–

nu 10% korting

Nu: 1125

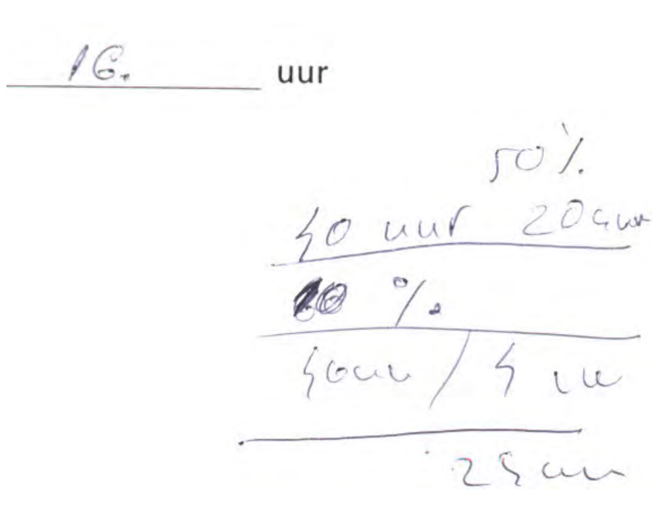
Her computations



At the interview she mentioned she does not feel confident when she only applies the benchmark model, because she is used to doing algorithms. It was the only thing she learned in school. Therefore she prefers to do both.

Figure 6.6 - Enver

Loes has a job of 40 hours per week. She was sick for a long time but can go back to work again for 60%. How many hours per week can she go to work?



16. uur

50%

40 uur 20 uur

10 %

4uur / 4 uur

24 uur

Another example was found in Enver's work on the posttest:
Enver uses the benchmark model to first compute 50% (20 hours). After that he computes 10% (is 4 hours). He adds this to 24, but in the final step he makes a mistake. He subtracts the 24 hours from 40 hours, and gets 16 hours. The notations show the answers of his mental computations.

Discussion:

The findings support the conjecture that learning through benchmarks helps to organize the learners' way of thinking and to develop reference points for mental math and estimation.
From the examples above and others found in the post test and in the interviews afterwards, we could derive that about half of the learners used the benchmark model for organizing their computations in some way, sometimes as the only computation, sometimes in combination with other computations, e.g. algorithms. Where the benchmark model only shows answers, we may conclude that the rest of the computations were done mentally, as for example at Enver's and Hatice's.
Although this study only involves a six-week program, these results are promising. The block model appears to be effective as a working model for doing computations with benchmarks as reference points. This is hopeful in that its success in a concise program for percents may mean that the benchmark model would also be effective for fractions, decimals and the metric system. Used in this way it may work as an integrating model.

6.4.6 Conjecture 6: Integrating visual models are useful means to link informal and formal knowledge and to cross-link mathematical concepts and operations.

Explanation:

Programs in ABE must be concise and effective. A lot has to be done in little time. Visual models can serve as important means of instruction in this, as described in chapter 5. In an ideal situation, as in primary school, RME learners are encouraged to develop their own models. In ABE it is difficult to have learners create their own models spontaneously, mainly due to time constraints. For that reason the IB instruction materials offer a few visual models as integrating models through all strands and levels, assuming and expecting, based on teaching experiences, that these can serve as models for developing and linking mathematical concepts, as described in chapter 5. Such models are the proportion bar, the block model and the pie chart. These models aim to enable the adult learners to model their thinking processes from informal to formal mathematical knowledge, but also show the links between mathematical subjects, e.g. the relationship between fractions, decimals and percent, e.g. “half” is 1/2 but also 50% and 0.5 and a “quarter” is 1/4 but also 25% and 0.25. (See also section 6.2.3.) By discussing topics and working in groups learners should be able to discover these links and to switch easily from fractions to decimals and percent when they are used to working with such integrating visual models.

Findings:

The use of visual models as integrating models is planned through all strands and levels in IB. In this learning program we only focused on percent which impedes showing cross links with other subjects. However, there are a few clear examples of such connections. At the transport survey (IB, level 4) the teacher presented a blank proportion bar to introduce a pie chart in a higher level sub-group. The learners were asked to show themselves in it in percent, in total 5 learners.

Figure 6.7: Percentages 1

Zeki 20%	Sunita 20%	Azeb 20%	Mehmed 20%	Hatice 20%

The percentage was related to 1/5 and “one out of five”. This went well. After that they were asked where they came from and to show it in the proportion bar: 3 from Turkey, 1 from Ethiopia and 1 from Bosnia. The teacher did it on the blackboard. So “three out of five” is 60%

Figure 6.8: Percentages 2

Turkey = 60%	E=20%	B=20%

Then Zeki was asked to do the same for their mode of transportation to school: by car, walking, by bike or by public transport. Data: 2 learners walked, 1 came by bike and 2 by public transport. He did it perfectly:

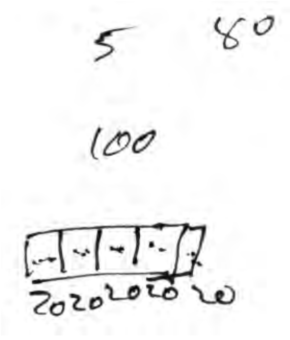
Figure 6.9: Percentages 3

40% walking	40% publ. transp.	20%bike

The next step was to transfer this information to a pie chart on the blackboard (not shown here) and to continue with the tasks on the transportation survey in the instructional materials (the level 4 IB materials, see section 6.2.3). Here the step from informal to formal math was made with the proportion bar as intermediary. The learners understood the task and went on.

At the post test Azeb, who was part of this subgroup, shows she understands this model and applied the same procedure in another context: "*4/5 of the learners passed their exams, what percent is that?*"(item C2-H08). She possibly understood that 4/5 is the same as "4 out of 5" and could transfer that into the proportion bar. The dots in the boxes show her counting. Her answer was 80%.

Figure 6.10: Azeb - Percentages

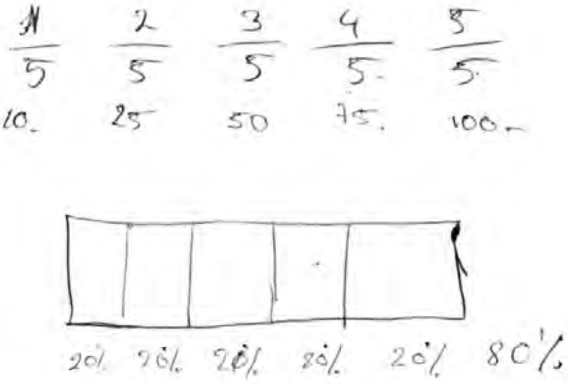


Azeb shows that this model fits her thinking. She is able to apply this visual model in a related situation but with a different notation system, fractions, as a mental model for doing her computations. It is the link between her informal and formal knowledge and between different concepts. Here the proportion bar becomes her integrating mental model.

Zeki skipped that item on the post test, but during the interview session afterwards he could do it with a little help.

He started writing down the fractions 1/5 to 5/5 (see figure 6.10). After a little thinking he wrote from right to left the numbers 100 - 75 - 50 - 25 - 10 underneath it.

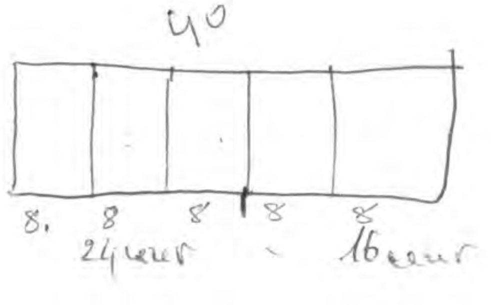
Figure 6.10 Zeki - relationship faractions-percentages



Example 6.12
Teacher: what are you writing?
Zeki, pointing: 100%, 75%, 50%, 25%, 10%
Teacher: And then?
Zeki: There is no number.
Teacher: You need a number?
Zeki: Yes.
Teacher: Well, let’s say 200?
Zeki: about 75%.
The teacher, drawing a blank bar-model: Could you show the one-fifth, two-fifth, and so on, in it?
Zeki draws four vertical lines in it, counting 1, 2, 3, 4, 5.
He looked at it and wrote below each cell: 20%. Then he knew: 80%
Teacher: 80%?
Zeki: yeah, 80%. That’s it, of course.

After that Zeki applied the same model at another item of the post test: *compute 60% of a full job of 40 hours per week* (C2-11):

Figure 6.11 Zeki - 60% of 40 hours



His answer: 24 hours

Discussion:

Though we explored only two examples these findings support the conjecture that integrating visual models are useful means to link informal and formal knowledge and to cross-link mathematical concepts and operations.

The bar-model appeared to work as an integrating visual model. It helped Azeb, for example, to do her informal computations, but at the same time she could use it to link fractions with percent. With a little help, Zeki could also apply the bar model as his mental model for different concepts: fractions and percent. Though these examples only regard two instances in this learning program, it may indicate that such integrative models can serve as links between different concepts.

6.4.7 Conjecture 7: Functional notations created by adults themselves are preferable to traditional procedures, like algorithms, as learned in school.

Explanation

In everyday life and work situations it is not necessary to apply standard algorithms and formulas as learned in school if adults can express their thoughts better in alternative procedures. Adults often create their own notations for their computations, whether or not in combination with the use of a calculator. Therefore we may question why it is necessary to teach adults algorithms if there are better and more functional ways to do computations that fit their thinking and to write them down in an alternative way, based on insight. As a starting point we could say that if people can explain what they do, show insight into their own computations and their answers are correct, then their computations are "functional". Functional notations show thinking processes or parts of it. Many such procedures are based partly on doing computations in one's head and partly on writing down results of mental math or subcomputations, possibly supported by computations on a calculator. The purpose in ABE should be to help adult learners develop their own functional notations as alternatives when they do not know or have partly forgotten traditional systems like algorithms. Such functional procedures may depend on people's own idiosyncratic computations, but can also be based on new ways of doing algorithms as developed in RME, if people don't have alternatives yet. Benchmarks and the benchmark model can serve developing such functional notations.

Findings:

In the lower level group Zaara, a women with no school education in her home country, showed her neighbor Jing how to divide an amount of 885 guilders into half and into a quarter to compute 50% and 25%. She used a procedure for this based on place value and addition.

Figure 6.12: Zaara - halving procedure

```
800  ➔  400      ➔ 200
 80  ➔   40      ➔  20
  5  ➔    2.50   ➔   1.25
-----   --------   ---------
885     442.50     221.25
```

She had created her own thinking model. The arrows show her thinking steps. She did all her computations mentally and noted only down the numbers and the answers. The teacher asked her to show it on the blackboard to also help other learners. From there a discussion started about how to do it in less steps, e.g. see figure 6.13

Figure 6.13

```
880  ➔  440      ➔ 220
  5  ➔    2.50   ➔   1.25
-----   --------   ---------
885     442.50     221.25
```

That worked, but she also used this way of working to compute 50% of more difficult amounts, like *f* 295.00 and *f*1199.00. That was more complex and took a long time. Here the teacher introduced rounding. That offered new possibilities.

In both computations, see figures 6.12 and 6.13, the learners' way of thinking was used to compute 50% for the price of the microwave and the radio (halving based on place value plus addition). In the computations below two important steps were taken. The first step was rounding to the nearest easy number. This was an eye-opener. The second step was subtraction. That was more difficult. Some learners are not used to doing subtraction and prefer addition. Most learners could write down the numbers in the way of an algorithm, but did the actual additions and subtraction in the head. They only noted their answers in the scheme.

Figure 6.14

100%	50%
microwave ƒ 295.00	
200 ➔ 90 ➔ 5 ➔	100 45 2.50 ------------ (+) 147.50
300 ➔ 5 ➔	150 2.50 ----------- (-) 147.50

100%	50%
radio ƒ 1199.00	
1000 100 90 9	500 50 45 4.50 ------------ (+) 599.50
1200 1	600 0.50 ----------- (-) 599.50

Such practice may help learners to organize and structure their computations based on their own insights. It can start in every new learning situation where learners try to find out new notation systems based on thinking processes. When numbers and computations become more complex, like in this situation, we may question when to start performing computations on a calculator. When learners understand the procedure and show insight into the underlying principles, like in this case, it may be easier to use a calculator. Then functional notations based on mental math and in combination with a calculator would be the most appropriate procedure. In fact, for the computations above adult learners don't need to learn the addition and subtraction algorithm on paper.

At the post test three learners showed their insights into benchmark thinking and expressed their computations in their own idiosyncratic functional notations at the following item:

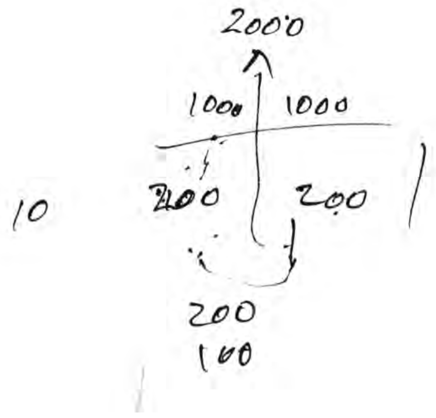
IB- B4-3:

At a speed check on the A2 freeway, past weekend, 300 out of 2000 drivers were driving too fast.
How many percent of the drivers were driving too fast? Cross the right box

10%	15%	20%	25%

Azeb shows the following computation:

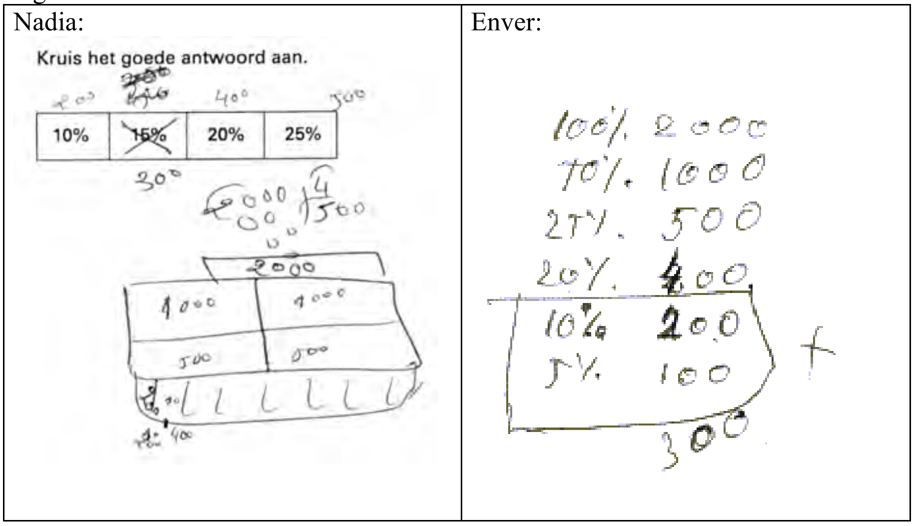
Figure 6.15 Azeb Halving procedure

	Explanation: She started with halving. Then at the next line she shows the 10% (200). Based on that she could derive 5% (100) and she checked the correct answer 15%.

Azeb shows her way of progressive schematizing of the block model.
It is her own idiosyncratic model for computing 10% and 5% of a number

Nadia and Enver express their understanding in the following computations at the same post test item.

Figure 6.16 Nadia and Enver


Nadia:
Enver:

Nadia was the person whose placement test was full of algorithms. At the post test she shows that she is now able to also use the benchmark models that were offered, but she still needs her algorithms. She first did a long division to find the 25% in the right box. Then she used the block model to find the 10% and 20% (200 and 400). She wrote those numbers above the check boxes and with the help of that she could find out that 15% is 300. She checked that box.

Enver, also used to only doing algorithms, shows his good understanding of the benchmark table. He did not check the right box (15%) but his answer is in the table.

Here we see three different functional computations at the same item. The learners show that this way of working fits their thinking. Benchmark thinking opened the way for Azeb, who could only do the addition algorithm at the placement test, to develop her own functional computations on paper in her own idiosyncratic way. Nadia and Enver changed their blind ciphering, shown in the placement test, into a mix of algorithms and functional notations based on benchmark thinking and mental math.

Discussion:

The findings support the conjecture that functional notations created by adults themselves are preferable to traditional procedures, like algorithms, as learned in school.

These learners show their ability to develop functional notation systems which support their computational mode of thinking, based on benchmark thinking after only six weeks of instruction. Some other lower level learners showed that they were able to develop procedures for halving based on their understanding of place value. Lower level learners in particular used these or similar informal but functional notations during the sessions. It could mean that people with little or no school education are more responsive to alternative procedures based on their own thinking processes. It is a more informal and a more natural way of doing computations and it fits the learners' ways of thinking. It shows their ways of problem solving.

Informal procedures can change into functional procedures instead of formal procedures along the lines of progressive schematizing and the four levels of mental action. This indicates that it is not necessary in adult education to teach traditional algorithms when learners did not learn this in former school days.

Insight into number, place value, the benchmark model, the proportion bar and other (self-created) mental models, are the basis and the ingredients for this. Functional notations have been developed based on insights, mental models and mental math. They express the learner's thinking process. A calculator can be of help for more complex computations. Functional notations may replace algorithms and may help learners to develop more creative and efficient ways of problem solving.

6.5 Quantitative results

The same percent items were used in the post test as were used in the placement test were used, so that the results could be compared directly. Therefore the test comprised four items from the IB test and six items from the Cito test. The list below shows the results of the learners who took both tests, the pretest and the posttest. More learners attended the group sessions but did only one of both tests. Seventeen learners out of twenty-six who actually participated, did both tests, which is 65%.

Table 6.4 Results placement test and posttest. N=17

Name	placement level IB	placement test	post test	increase in %
Level 2/3		%correct	% correct	
Dina	2	0.00	30.00	30
Enver	3	10.00	35.00	25
Zaara	2	10.00	47.50	37.5
Philomene	3	10.00	25.00	15
Mehmed	3	40.00	65.00	15
Nadia	3	20.00	67.50	47.5
Silva	3	17.50	65.00	47.5
Azeb	3	5.00	90.00	85
	mean difference pre- and post test			+37.81
Level 4				
Li	4	62.50	67.50	5
Sunita	4	57.50	77.50	20
Himzo	4	37.50	47.50	10
Nevzeta	4	30.00	47.50	17.5
Zeki	4	22.50	45.00	22.5
Nezima	4	32.50	32.50	0
Valentina	4	20.00	60.00	40
Hatice	4	17.50	57.50	40
Nuko	4	37.50	47.50	10
	mean difference pre- and posttest			+18.33

A few remarks can be made with this list.

1) Individually seven learners improved their skills only marginally (progress 0-19%), of which Nezima and Li attended classes only two times. Nine learners improved more (a progress of 20-47.5%) and Azeb shows that she came from nearly no knowledge about percent to a 90% correct score on the test. This makes clear that the benchmark approach can be very effective in only a short period of time.

2) Seven learners out of seventeen scored 60% correct or higher on the post test, compared to only one learner in the pretest. One learner performed close to 60% correct. This result is satisfying because the time spent on the topic percent was, in fact, only 6 hours (one hour per week).

3) In general the numerical results are satisfying for a six-week program. We may wonder what the results had shown if all learners would have attended the sessions for at least 80% and if all learners had taken both tests. However, these results do provide indications and ideas for further research into the topics discussed in this chapter.

6.6 Conclusions

The learning program followed the placement test and spanned only six weeks. Nevertheless, it was assumed that the program would yield good results because it was meant to be concise. Comparison of the quantitative results on the placement test and the post test provides more information about possibilities for such a concise mathematics program in ABE.

In general, though we only worked with 26 learners in an ordinary ABE learning center in a short period of time, the learning program yields interesting observations for further development of the FNE theory. While this study only concerned the subject percent, the general ideas behind the program as worded in the starting points in chapter 5, section 5.3.1, can be supported. Further research into more specific topics, like the issues of the role of integrating visual models and development of functional notations, could help expand this knowledge. In this way the learning of mathematics by adults in ABE becomes increasingly clear. Also, the *what-why-how* approach regarding the learning process itself, begs for a specific study. Some *what* and *why* situations emerged in this learning program but provided too little systematic information to elaborate here. For such a study a specific program is required and teachers first need specific training.
The qualitative analysis of what happened during the sessions offers insight into the actual level of education in ABE, the interaction in group sessions, and the

problems adults may encounter when learning mathematics, in this case on percent. The quantitative results on the post test are promising for the content and setup of such a concise program.

More specific to each conjecture we can conclude:

Conjecture 1: Prior knowledge on mathematical subjects is often patchy and based on misconceptions.
At the start of the course about half of the group had acquired the concept of 100% is “all” or “good” in a clear simple, context like “100% juice” at the start of the ABE math course. Regarding more complex or distracting contexts, like the blood test for cancer which involves “90% of all cases”, almost all learners are confused. They may still relate the 90% to 100% as “quite good” or “almost good” but they are confused by the context “all cases”. Regarding computations with percent about three-quarters of the learners are able to compute 50% in a money context, because that is related to “half” or “half price”. When 50% is used in a different, non-money-related context, the correct percentage decreases to 40% (school-1 context). With this item and with computations involving the benchmark 25% half of the group tends to relate the percentage to the actual number, e.g. 25% is 25 guilders or persons. They don’t know what to do when the original number in a percent context is smaller than the percentage. This means that the concept “percent” indeed is based on partial knowledge. While this study only considers the topic of percent, it is expected that similar nuances will occur in other subjects.

Conjecture 2: Real life experiences help at building mathematical concepts and linking school math to real life situations.
Real life experiences are very helpful for linking school math to real life math. In all sessions the learners spontaneously injected their own real life knowledge during the discussions, to give meaning to the math they were working on. Such information can be optimized to its essential parts in the learning session and indeed helps to better understand newly offered mathematical topics, like in the transportation survey. When people recognize the school math in their own situations it may be easier for them to learn more about such particular subjects and to apply this new knowledge when necessary in their own real life situations. In this way they create their own functional mathematics.

Conjecture 3: Authentic materials in learning situations must be carefully selected.
Authentic materials and representations of real life objects as learning materials in school situations may help to bridge the link between school math and real life math. This is a basic assumption in adult education in general and also in the FNE approach. However, authentic real life materials are not designed to be learning materials and the information contained in it may be too difficult and too complex. In a few situations in this learning program mathematical notations

and wording of information on hands-on materials distracted lower level learners. This suggests that we be selective with authentic materials and to offer only essential information for instruction on a specific topic, in particular for lower level learners.

Conjecture 4: Interaction and discussion are essential when acquiring new math knowledge.
Interaction and discussion appeared to be very helpful when acquiring new mathematical knowledge in almost all sessions. It helped the learners in improving partial conceptions and gaining new insights. The learners were able to listen to each other and to help each other with processing new knowledge. This offers possibilities for learning settings in which learners are supposed to work teacher-independently, yet more specific research will need to be done.

Conjecture 5: Learning through benchmarks helps with organizing the learners' way of thinking and developing reference points for mental math and estimation.
The concise setup of the percent program, from 100% to 1% through the benchmark steps of 50%, 25% and 10% appears to be a good approach for these learners, though, it must be said we did not try an alternate setup. The step to 10% was for some lower level learners a bit more difficult, because they need to know how to divide by 10 mentally. The block model and the benchmark table appeared to be good means to focus adults' thinking on benchmark operations and to develop such mental operations in addition to their computations on paper. Learners who learned to do computations with percent in a traditional way and tended to apply these procedures, showed a mix of operations at the end of the learning program and were better able to apply mental operations. This is also the basis of learning to estimate. The results here are promising.

Conjecture 6: Integrating visual models are useful means to link informal and formal knowledge and also to cross-link mathematical concepts and operations.
The integrating visual models that were introduced to the learners, in particular the proportion bar and the block model, enable learners to develop more formal knowledge based on their own informal conceptions. These models appeared to be useful for learning within the percent strand.
Since the learning program mainly focused on percent, there was not enough evidence that these models also function as integrating models between strands. Only a few learners showed that they were able to cross-link with fractions. Though the bar and the block-model show possibilities in this regard, further research is needed to provide more information about the cross-link function of these integrating models.

Conjecture 7: Functional notations created by adults themselves are preferable to traditional procedures, like algorithms, as learned in school.
Mathematical procedures in school programs should support the further development of adults' familiar strategies and procedures into more functional procedures and notation systems. The halving-halving procedure, for example, was in particular helpful for learners who prefer to do computations in more informal ways. Three other learners showed effective computations based on the benchmark table. The learning program shows two important results in this regard:

- Informal procedures of learners based on insights in combination with the benchmark model lead to functional procedures and notation systems.
- Learners who learned to do formal traditional computations, often based on little or no insight, started thinking based on benchmarks and used a mix of notations at the end in their post tests.

In summary:

The learning program yielded empirical data that lend credibility to the general starting points of the FNE theory as described in chapter 5. The results are promising and show that the basic ideas in this program match the learners' ways of thinking and learning mathematics. The concise buildup can be effective for a program on functional numeracy in ABE. This may lead to qualitative improvement of learning and teaching numeracy in ABE. The results also suggest topics for further research, in particular the role of integrating visual models and the *what-why-how* approach concerning the learning process itself. For these topics a different research design is required.

7 Developing a Program for Functional Numeracy Education in ABE

7.1 Introduction

The eventual purpose of this study is to build a framework for a program on *Functional Numeracy Education* (FNE) for adults in Adult Basic Education (ABE).

In adult numeracy communities like the Adult Numeracy Network in the USA (ANN) and the international organization Adults Learning Mathematics (ALM) teachers, curriculum developers, teachers and researchers are encouraged to develop materials for "good practice" in mutual cooperation by setting up networks, discussion lists through email listservs, newsletters and by organizing annual conferences. However, in practice numeracy programs are still based on fragmental sources and depend on initiatives of individual teachers. In general there is still little notion of how adults actually *learn* mathematics and what numeracy programs should include, though the first results of explorative research are coming (see for instance Coben, O'Donoghue and Fitzsimons, 2000, and Gal, 2000). Programs often focus on *what* mathematical content should be taught rather than *the way in which* it should be taught. The emphasis is still more on *teaching* than on *learning* mathematics and also still more on *mathematics* than on *numeracy*. We would argue that learning mathematics should be seen as part of developing numerate behavior, so *as part of* numeracy programs (Gal, 1997).

In this chapter we want to contribute to this discussion by constructing a framework for adult numeracy courses in ABE in which *learning* plays a central role. Here we return to the main question of this study: *What content should be offered in a numeracy program for learners in Adult Basic Education and how should such a program be organized?*

The objectives, starting points, content and the structure for a numeracy program described in this chapter are grounded on the concept of numeracy as elaborated in chapter 2, the theoretical considerations elaborated in chapter 5, analyses of the placement results in chapter 4 and learning experiments, examples of which are described in chapter 6. Finally, suggestions about assessment are based on the principles of assessment for ABE described in chapter 3. The basic ideas and the ingredients for this FNE program are provided by the "*In Balance*" materials of which the percent strand was used in chapter 6 (van Groenestijn et al, 1996-2000). The present chapter should be seen as an integration of all previous chapters.

The characteristics of the FNE program are pragmatic, idealistic, democratic and empowering.

Pragmatic, because adults want to learn math they can *use* in their everyday-life situations and the program should be *feasible* in the available course time.
Idealistic, because the FNE program aims to lead to *numerate behavior*, enabling people to better manage their own lives and to broaden their horizons. Idealistic also, because adults in ABE need to be equipped for a future which will require a lot of quantitative knowledge and flexibility to adjust to new technological developments. For that learning in ABE should create a gateway to further learning.
Democratic, because adults are responsible for their own learning process and take part in decisions regarding the content of their learning program. This creates a learning situation in which learners and teachers are supposed to be equal partners.
Empowering, such that adults will *feel* capable to indeed independently and effectively manage their own lives and that of their families, and to be critical citizens.

7.2 Entry Level of Adults in Adult Basic Education

In chapter 2 numeracy was described as: *the knowledge and skills required to effectively manage mathematical demands in personal and societal situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information technology.*

Within this definition we distinguish three levels of numeracy:
- *Elementary numeracy* encompasses minimal knowledge and skills necessary for managing personal everyday life situations and for functioning in simple work situations with perhaps a little help from others.
- *Functional numeracy* encompasses a broader set of functional mathematical knowledge and skills, including elementary numeracy skills, enabling people to manage their personal and their families' societal life and work situations effectively and autonomously and to be a critical citizen.
- *Optimal numeracy* encompasses all extra mathematical knowledge and skills, in addition to people's functional numeracy skills, for acting in broader societal and/or political communities, higher professions, and leisure activities.

In adult education only the levels of elementary and functional numeracy are addressed. These levels have to be operationalized in order to be able to design numeracy courses in adult education.
In chapter 1 the Dutch system of adult education was explained and placed in an international perspective. The system encompasses six levels of General Education (KSE) in combination with a system for Vocational Education (KSB). Only the three lower levels of KSE are relevant in this study. In short:

KSE-1: the *Elementary level* qualifies for elementary skills. It enables adults to perform simple and familiar tasks for their personal lives. This level leads to elementary numeracy.
KSE-2: the *Foundational level* qualifies for access to participation in vocational learning routes leading to assistance levels in vocations. At this level the basis is laid for functional numeracy.
KSE-3: the *Basic* level qualifies for access to a broad range of courses for further learning in general and for vocational training. At this level further aspects of functional numeracy have been developed. These are embedded in objectives for vocational courses and further education.

The general objective in ABE is mastering level KSE-2 in order to enable adults to participate successfully in the labor market or to enroll in courses for vocational training. The results in chapter 4 show that about a quarter of the adults in this research are still on level KSE-1, the elementary level, and about 75% at the beginning of level KSE-2, the foundational level. This means that all learners are still in the lower regions of elementary and functional numeracy. Though this research only regards a small group of 32 second language learners, we may assume, with some reservation, that this ratio mirrors the general ratio in ABE in the Netherlands, given the placement criteria for ABE and given that similar results were achieved during the construction of the Cito placement test in a representative sample of ABE learners in 1993. (Straetmans, 1994)

Speaking in detail it can be said that about 80% of the learners can solve simple counting, addition and multiplication problems. For more complex context tasks concerning basic operations the percentage of correct answers decreases to about 40%. In general the learners prefer doing algorithms to mental calculation, though the performance on the actual algorithm tasks are low (45% correct). Estimation and mental computation are the least favorite. Learners tend to do precise computations. This was in particular striking with money tasks. Though most money tasks were designed so as to encourage estimation as in real life situations, most learners tend to solve such tasks by doing precise computations through algorithms. The results for money tasks were about 50% correct. This is worrying because it indicates that these adults have only acquired minimal skills to handle money tasks in real life situations, or cannot transfer real life related solutions to tasks in school situations.

In the field of proportions, fractions and percent the learners show a 75% correct score on simple proportion tasks related to recognizable real life issues, e.g. comparing the length of a car with the length of a bus. The percentage correct decreased enormously at simple tasks pertaining to benchmark fractions (30% correct score) and percent (only 15% correct score). [1]
For measurement, including decimals involved in the metric system, the results on the placement test show that about 80% of the learners can do simple tasks related to applying measurement in real life situations. When comparing

measures within the metric system the percentage correct decreases to just over 50%. When doing formal computations with given measures on paper, this percentage decreases further to 20%. Only four learners (12.5%) are able to compute the area of a floor with the help of a map of the floor.[2]

For reading and understanding data in graphs we come to similar conclusions. Learners are able to read simple line and bar graphs related to everyday life issues, like reading a temperature graph (about a 75% correct score). Indicating a quarter in a pie chart leads to a 50% correct score and understanding a more complex bar graphic about average sales of cars in one year yields only about 10% correct answers.

In general it can be said that these adults can do simple tasks in a clear context related to recognizable real life situations that ask for action and can be solved in multiple informal ways, but the percentage correct decreases drastically when they try more complex tasks and more formal mathematical procedures on paper. This may confirm the statement in chapter 5 that learning math in ABE should start with informal "do"- tasks in which people can apply the knowledge and skills they have already acquired, and should focus on practical, functional mathematics, leaving room for various solution procedures in order to develop functional numeracy knowledge and skills.

7.3 Program Design

The characteristics of adult learners and their entry levels provide essential information for setting up a numeracy program in ABE. Therefore a placement interview and an assessment procedure are indispensable at the start of every numeracy course. Though, a basic program for FNE can be designed, based on information and indications about the ABE population in general. The analysis of the numeracy concept, discussed in chapter 2, provides the basis for the objectives of an FNE program. Derived from the objectives and based on theoretical ideas, elaborated in chapter 5, starting points for the program can be formulated. The main structure of the program is determined by the four components of numeracy (mathematical content, management of mathematical situations, processing new information, and learning skills), accompanied by practical conditions.

Such practical conditions are often based on local facilities and constraints, such as local funds, buildings in which learning centers are housed, available child care, local employment, particular social situations, etc. They set the actual scene for every ABE course, so also for numeracy courses. Local arrangements and funds, for instance, may determine the actual course duration. In most situations it must be taken into account that ABE courses, in particular numeracy courses, are often very restricted as to time available. In general, numeracy courses may last about 100 hours per year. There may also be a difference in learning hours and teaching hours. In the Netherlands, for instance, the number of teaching hours in adult education is about half of the total number of learning hours for the learners. Hence teacher-free and cooperative learning must be organized in all courses. This means that the process of learning to learn takes on an important place in all courses. Such local conditions are decisive for every numeracy course. Therefore numeracy courses must be concise, feasible and effective.

After having described the objectives and the starting points, the four numeracy components determine the main structure of the FNE program (see table 7.1). They are distinguished but not separated in the course. In fact the mathematical content is embedded in the other three components of numeracy. In the FNE program the emphasis is on acquiring knowledge and skills to develop these four components. It may be clear that the first goal in a numeracy program is acquiring mathematical knowledge and skills. At the same time the other three components will also have to be developed.
Management skills are essential to adequately manage mathematical situations. In the FNE program it regards "*learning* how to manage" mathematical situations. This includes *application of* acquired knowledge and skills. This is distinct from the third component of numeracy: processing new information. In the FNE program it concerns "*learning* to process" new information and it focuses on *acquiring* new knowledge and skills in out-of-school situations. Processing new information cannot be seen separately from well-developed learning skills. Again, in the FNE program it regards "*learning* to learn" in order to develop learning skills. As may be clear, all four components go hand in hand.

The organization of the FNE program is based on the theoretical foundation discussed in chapter 5 and examined in chapter 6. The components described here organize the learning of mathematics by adults and the learning-teaching process itself.

Table 7.1 Program Design for Functional Numeracy Education

Principles	**Main Structure**	**Organization**	**Learning and Teaching**	**Assessment**
Objectives, Starting points	Mathematical content	parallel strands in horizontal and vertical coherence; concentric build-up; benchmarks; integrating visual models; action levels;	context-based tasks context-free tasks	Placement ass. Formative ass. Summative ass. Oral interview Paper-and-pencil CAT Ass. in action Learners' Progress Program evaluation Accountability
			precise computation estimation non-comp. tasks	
			mental computation computation on paper calculator/ computer	
	Management of mathematical situations	cooperative working, problem solving, functional procedures	interaction and discussion (focusing on *application* of knowledge and skills)	
	Processing new information	audio-visual information through communication, texts, graphs, charts, tables, etc.	interaction and discussion (focusing on *acquiring* new knowledge and skills)	
	Learning skills	what-why-how process	small group work work in pairs individual work	

To make the program operational for actual learning and teaching, different types of tasks must be planned. Each type of task serves a specific goal. Such tasks must be well-thought out, well-structured and well-planned to achieve the ultimate goal: a concise, effective numeracy program.

To be able to monitor the learners' progress, to evaluate the program and for accountability to funders, assessments must be applied at regular times. For this a coherent set of assessment tasks must be developed and executed using appropriate ways of testing. This part is built on the elaborations in chapter 3.

7.4 Objectives for an FNE Program

As described in chapter 2 the definition of numeracy implies that the content of numeracy may change over the years, depending on developments in society. Adults will have to be able to flexibly adjust their numeracy skills to new situations. Also, mathematical information is always embedded in real-life situations so that it is not always recognizable as a specific mathematical problem. Adults have to be able to identify and analyze mathematical information from a broader, almost always more complex context, to give meaning to it and to know what to do with it. This means that numeracy education in ABE should include four components:

- the actual mathematics component (learning functional mathematics)
- learning how to identify mathematical information/problems in everyday life situations and how to deal with such situations effectively (management and problem-solving)
- acquiring and processing new quantitative information by communication, interpretation and reflection, (processing information by interaction)
- acquiring learning skills which enable adults to keep up with developments in society and to brush up and renew their knowledge and skills. (learning to learn)

The following objectives are derived from these components:

1) FNE aims to help adults develop functional mathematical knowledge and skills.

The results of the field research in this study indicate a very practical, informal attitude of the participating adults, often based on personal experiences (see 7.2.3 and analyses in chapter 6) Formal school knowledge has been applied less adequately. This is in line with studies about learning in practice (see chapter 5, section 5.2.2) where it is often stated that practical knowledge and skills differ from school knowledge and skills. Lave, 1988, Nunes, Schliemann and Carraher, 1993, Hoyles and Noss, 1998, Boekaerts and Simons, 1993, distinguish learning-for-knowing from learning-for-doing and Greeno (1999) advocates learning-in-action.

The adult education community in the Netherlands has been learning from these studies. Learning outcomes in adult vocational education have been worded in terms of *competencies*. In adult vocational courses a minimum of 30% apprenticeship has been built in, together with another 30% of skills training in school. However, in ABE the courses are still very school-like. Though a start has been made for language apprenticeships for second language learners, mathematics courses are still “theoretical” and lead to learning for knowing. In the proposed FNE program learning *for* and *by* doing is more emphasized. Mathematical “skills” should be acquired by doing, e.g. by practicing measuring in an actual room for computing the area of a room or the number of tiles that is needed to cover the floor. Learning how to compute an area of a room should not only be learned from a book. A math book can only be a manual for how to do it in practice. According to Greeno (1999) this should be learned in an actual situation. In this frame FNE aims to activate *learning for doing* through *learning by doing,* or: *learning in action.*

2) FNE aims to help adults develop insights into their own mathematical procedures which enables adults to take responsibility for their own actions and to manage mathematical situations.
FNE cannot only exist of developing practical knowledge and skills. It also aims to develop insight into mathematical procedures and to develop mathematical reasoning, where adults learn to make decisions, to take responsibility for their own mathematical actions and to manage mathematical situations. The learning program in chapter 6 shows that if adults are encouraged to relate mathematical information to their own experiences they can process such information in a comprehensible way, e.g. the information about 100% juice or about the whole and low-fat milk. Such information makes sense and such classroom discussions may help adults to make decisions, e.g. about what to buy as a consumer, like Hatice did concerning the low-fat milk (see section 6.4.2). Adults look for reference points for themselves, like Zeki, (section 6.4.4) who found out that he was 25% of the learners group himself. Based on that he could do other computations, e.g. 25% of a group of twenty persons and 50% out of 40 persons. In this learning situation he learned that the percentage can be more than the actual number to do computations with (50% out of 40) and he could use this information in his own situation. Also, by discussing the “90% of all cases” at the blood test, adults learn to think critically about the meaning of such statements and the impact it may have on their own lives. (section 6.4.1) Here they learn to reason about mathematical topics and to make decisions. This is the basis of managing mathematical situations.

FNE aims to develop knowledge, comprehension and skills in mutual coherence so that adults learn to rely on their own insights and how to act adequately in everyday life situations. Therefore mathematics in an FNE program is offered in functional contexts that ask for problem solving.

3) FNE aims to help adults acquire and process new knowledge by communication, interpretation and reflection.
As elaborated in chapter 2 communication is key in a numeracy program for adults for the following three reasons:

- Communication is the basis for identifying, recognizing, analyzing, interpreting, reflecting and discussing mathematical problems in real life situations. By communicating about problems in contexts adults go through this process and learn how to discover the underlying mathematics embedded in broader situations and how to solve the actual problem. They also learn to give meaning to mathematical contexts. This leads to critical thinking and is the key to critical citizenship. (see also the “Equipped For the Future” statements, Curry, et al, 1996) During the discussion about the blood test, for example, Sunita states clearly that one should not accept the diagnosis of cancer based on only one test. (section 6.4.1) Such statements may help other learners to develop insights into such situations and to develop their own opinions.

- Communication is the basis for mathematical reasoning. For example: the learners could explain the difference between whole and low-fat milk (100% and 50% fat). However, it was not clear whether they realized that the difference between the actual amount of fat in the milk, only 3.6 and 1.5 gram per liter, was just a tiny bit. Also, though the learners showed their understanding of 90% is close to 100%, the statement “new blood test detects cancer correctly in 90% of all cases” was in fact not clear to them. (section 6.4.1). Discussions about such topics are the basis for giving meaning to such information and of mathematical understanding. This, in fact, is a key issue in numeracy.

- Communication is the basis for learning that mathematical problems can be solved in different ways. By reflecting their own solving procedures and by comparing these with those of others adults learn to approach mathematical problems from different angles. This may encourage them to develop more creative ways of solving problems through which they may discover more and better possibilities for managing mathematical situations efficiently.

4) FNE aims to help adults develop learning skills which enable them to acquire and process new information independently and to adjust their own mathematical knowledge and skills flexibly.

FNE cannot be a goal in itself. As indicated in the definition, numeracy is a dynamic concept and the content may change over the years due to developments in society. Learning a standard set of mathematical procedures might lead to rapidly outdated knowledge and skills and closes the door to new information. For example, there is an ongoing discussion by teachers about why we learn long division if we can use a calculator. To be equipped for the future (see UNESCO, 1997, Curry et al, 1996) the focus in ABE should be to develop literacy, numeracy and social skills which enable adults to learn cooperatively in order to be able to acquire and process new information independently from teachers. FNE can contribute to this process by offering real-life context problems and help them to solve these problems along the what-why-how questions described in chapter 5 (section 5.3.3).

7.5 Starting points for an FNE Program

Based on the objectives in previous section points for departure for an FNE program can be formulated. These starting points reflect the andragogical approach and the basic principles of FNE as described in chapter 5, section 5.3. This implies a strong relationship with the RME learning principles (Treffers, 1991, Gravemeijer, 1994) and constructivism (Gravemeijer, 1995, 1996)

1) *Adults' own experiences are part of learning mathematics.*
Following Freire's theory and the meaning of Vygotsky's Action Theory for adults, see chapter 5, section 5.2 and 5.3.1, learning through experiences is a main characteristic of learning in adult education and thus in FNE. The learners in the learning experiment in chapter 6 show continuously that their real-life experiences play a part in their thinking whilst learning. *Adults have many experiences with mathematical situations in their everyday lives which can help them to learn and understand the math in school better.* Hence real life situations are the source and focus of the FNE program. The FNE program offers enough room to include and process their experiential knowledge. Adults' prior knowledge and real life experiences are the starting points for learning about new topics and may give meaning to the math they learn in school. It is also the bridge between school math and numeracy, so indispensable in adult education programs.

2) *Functional and Context-based*
The essence of numeracy is that it is embedded in real life situations. This becomes clear in almost all definitions and content descriptions in chapter 2. For that, *adults need mathematics that is recognizable and applicable in real-life situations.* Therefore FNE includes actual real life problems or contexts derived from real life situations. Real life can be represented in contexts by creating action tasks, e.g. measurement tasks in the learners' own room, by using actual products and accompanied by photos in a book, schemes, drawings, sketches, maps, charts, tables, etc., so that people can "see" and "experience" what is meant. This may make the mathematics in books recognizable in real life situations and can help in doing action tasks. (see objective 1)

3) *Activating and Focusing on Problem-solving*
According to Resnick, 1987, and Greeno, 1999, amongst others, (see chapter 5) the focus in an FNE program is on learning-for-and-by-doing rather than learning-for-knowing. This can be confirmed with the practical comments learners made in the course of the learning experiment. (see chapter 6). *Learning-for-and-by-doing, or learning-in-action, implies problem solving in actual or simulated mathematical situations with hands-on materials where adults can actively participate.* This may challenge and encourage them to also apply the learned mathematical activities in their own lived-in situations. It may

strengthen their self-confidence and self-esteem if they feel able and capable to solve problems in actual real life situations they could not do before. It may also encourage them to ask for help in situations where they cannot solve a problem by themselves. This includes that they see the importance of working cooperatively and sharing their knowledge and skills with others. This is the basis for developing shared-cognition in work situations (see chapter 5).

4) *(Re-)constructive*
The phenomenological approach of Freudenthal (1973) and the principles of constructivism, encompass that developing mathematics is a never-ending process. (see chapter 5). *Adults construct and re-construct their own mathematics by experimenting in everyday life situations.* This leads to functional mathematics and numerate behavior and becomes apparent in people's organization of their lived-in situations. Constructing and re-constructing their own mathematical knowledge and skills by mathematizing real life situations enables adults to organize and re-organize their own lived-in worlds whereby the organization of their personal and work lives may become more manageable.

5) *Interactive and Reflective*
As described in chapter 5 interaction is an important means to informal learning in real life situations. *During interaction adults learn to "communicate" about mathematical problems and to reflect on their own mathematical knowledge and skills.* Adults become aware of their own knowledge and skills and learn to reflect on them in interaction with others. They may compare their own actual knowledge with new knowledge. This is the start for learning. This process was seen in the learning program described in chapter 6, for instance at the discussions about percents in particular where it concerned the percentage related to the number, e.g. 25% is a quarter and not 25 guilders, and where Sahra and Nezira discussed the "one point five" (1.5) gram fat. Learners were also able to help each other find new ways of doing computations, e.g. the halving procedure based on place value Zaara introduced. Interaction can also serve as the basis for developing mathematical reasoning skills, which may help them in their communication with and understanding of others.

6) *Integrating*
Mathematics in real life situations does not consist of only "addition" or "subtraction", or other isolated mathematical sub-topics. Most situations ask for application of integrated mathematical knowledge and skills. Therefore the mathematics in an FNE program is based on integrating strands that enables adults to acquire insight into the relation between mathematical topics. This may make their mathematical knowledge and skills more flexibly applicable. *This means that traditional strands, like e.g. "number", "addition and subtraction", and "multiplication and division", are seen as only one integrated strand "basic operations".* The same applies to other strands as measurement (e.g. learning

about length, volume, weight) and proportions (fractions, decimals and percent). Integrated strands may help to develop better insights into relations between different concepts, e.g. between fractions, decimals and percent. This way of working may help to work more flexibly at solving mathematical problems.
Within integrated strands the focus is not on learning separate procedures as a goal in its own, like learning different algorithms. Adults are free to construct their own mathematical computations and notations based on their own thinking, as Nadia shows in her computations to the paint item for computing the number of cans needed for 3 liter of paint (see figure 4.33) and as Azeb, Nadia and Enver show in their applications of the benchmark table (see section 6.4.1). Such informal procedures may help learners to organize their thinking and to overcome problems they may have with formal procedures, like the subtraction or division algorithms, often accompanied by negative feelings caused by former school experiences. It enables them to develop alternative skills and problem solving procedures based on their insight and competence.

7) *Challenging and Horizon-broadening*
A numeracy program based on FNE principles offers more than only learning math, solving everyday life problems and preparation for vocational training. It challenges adults to become curious about broader issues in society than only their own lived-in situations. Discussing more global topics e.g. the transportation investigation and the blood test for cancer, may encourage them to become critical citizens. Such tasks may help to decrease the gap between numeracy and innumeracy. (see chapter 2)

7.6 The Setup of an FNE program

The four components of the numeracy concept provide the basic structure of the FNE program. These components are elaborated further in this section based on the theoretical foundations described in chapter 5 and the empirical findings in the chapters 4 and 6. These serve as building blocks for the FNE program. It is again emphasized that these components cannot be seen separately but must be viewed as one whole.

7.6.1 Mathematical Content of an FNE program in Adult Basic Education

Based on our definition of numeracy, studies involving the need of numeracy to be equipped for the future (Curry et al, 1996), studies about actual levels of adult numeracy (as shown in IALS in 1996) and the results of this study presented in chapters 4 and 6, it becomes clear that a numeracy program for adults should include a broad range of mathematical activities. In countries like the USA, Canada, Australia, UK, Ireland, the Netherlands, Sweden and Denmark, teachers, researchers, program developers and national organizations are actively involved in developing such programs. The first results are visible in e.g. the Dutch KSE system, the Australian NRS, the GED program and the ANN Standards in the USA, and the GCSE program in the UK. However, these programs are very broad and focus on learning outcomes in general. Not described is how a program actually can be set up in practice and what kind of instruction is required. These topics are introduced in the following section.

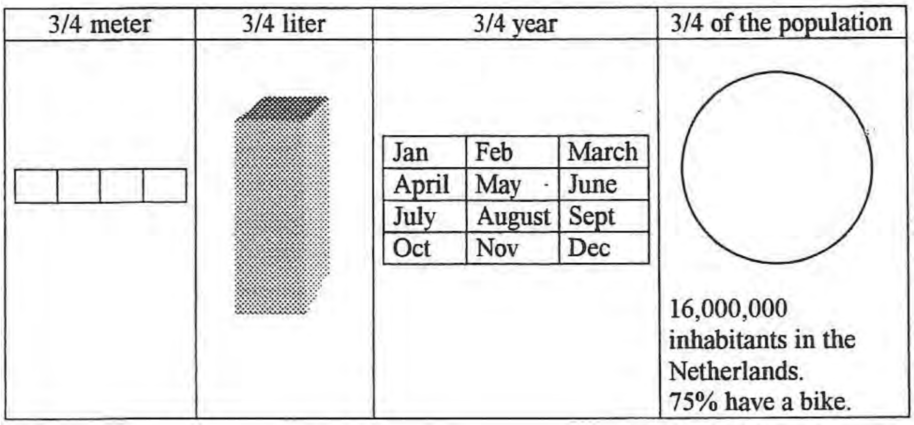
In this study we focus on a minimum program for elementary and functional numeracy that is feasible in ABE, but we keep further numeracy education in mind. ABE is meant to be foundational for further learning, at least in the Netherlands. The content of such a program should include mathematical activities that offer a solid foundation for functioning in everyday life and for getting well-prepared for courses in vocational education and other programs. The five “big ideas” of Steen (1990) offer a good basis for this (see chapter 2, figure 2.2). Within these ideas we will focus on basic operations and proportions as part of “Quantity and Number”, measurement, direction, shape and space, as part of “Dimension and Shape”, and simple data as part of “Data and Chance”, since these are the core of mathematics. (see figure 7.1). Additional topics that can be offered in ABE can be labeled “Miscellaneous” and may depend on local situations. More specific mathematics programs are embedded in follow up courses, depending on the specifics of the courses. “Pattern and Relationship”, “Chance” and “Change” are processed in such follow-up programs. Given the low level of the learners in (Dutch) ABE and the time constraints, we need to focus on general basic knowledge and skills that are precondition for all vocations.

Table 7.2 Overview of Mathematical Content for FNE in ABE (minimum program)

Big Ideas	Content	Required Skills
Quantity and Number	basic operations	counting, doing operations with addition, subtraction, multiplication, and division with whole numbers
	proportions	doing operations with ratio, fractions, decimals, percent
Dimension and Shape	measurement, (metric system)	measuring length/area, volume, weight, money (including decimals)
	direction/shape/space	reading time, calendar/ direction, working with shape, space
Data and Chance	data	reading and understanding simple tables, graphs, charts, diagrams
	miscellaneous	

In our approach to learning mathematics in ABE we consider the content within each of the sub-fields (basic operations, proportions, measurement and data) as unities, distinguished by their own specifics. They are also closely mutually related. In a program for FNE we want people to gain insights into underlying mathematical systems, e.g. the relation between measurement units in the metric system and proportions. That, for example, is also the basis for estimation. When somebody estimates, for example, that a stick is about “three-quarters of a meter”, people should be able to indicate what part of the standard “meter” the stick is and that this can be written as 0.75 m or 75 cm. (see figure 7.1) They should also know that the same applies to “three-quarters of a liter”, but the concept of “liter” and the visual interpretation or imagination of it and the notation is different, e.g. 0.75 liter, 7.5 dl., 75 cl., 750 ml.. The number 3/4 can also be combined with “three-quarters of a year” or “three quarters” and here it means 9 months. In the data strand three-quarters can be visualized in a bar or a pie chart as part of percent, e.g. 75% is three-quarters of the population. In fact the strand “quantity and number” is embedded in “dimension and shape” and in “data and chance”.

Figure 7.1: The concept 3/4 in different situations



In our view it is not sufficient to offer learners only nice real-life related (do-) tasks, like separate weight, length, or money tasks, often offered as “experience-based”. Only offering such tasks, without providing insight in the mathematical principles behind it, could cause partial knowledge and skills, as we saw at percents in chapter 6, section 6.4.1. Tasks must be planned as part of a well thought out framework. Numeracy courses should offer adults a program that helps them to develop insights into mathematical concepts within a short period of time. The latter is an important factor because adults don’t have the time to spend years on mathematics they did not learn in school when they were young. Also, due to the limited total number of course hours, mathematics in numeracy courses must be concise and clearly structured in order to offer adults a complete basic program that is feasible at the same time. Finally, learners must be enabled to take courses on different levels and to practice their skills to build proficiency.

To develop a program that incorporates the above described conditions (coherent, concise, focusing on insight, more starting levels, offering time for practicing skills) the choice was made for a horizontal and vertical structure in the program. Benchmarks and integrating visual models are the keystones in this program.

The mathematical content should be planned in *parallel strands*, conform Steen (1990), and in a *concentric set-up*. The latter is necessary to enable adults to do a coherent broad program through all levels. There is a *horizontal and vertical coherence* between strands. This can be realized by focusing on *benchmarks* in all strands, like working with “half”, a “quarter”, 25%, etc., visualized in *integrating visual models* which cross-link the strands (see next section). Visual models serve a dual function because they also link informal with formal, functional mathematical actions as elaborated in the four action levels in chapter 5, section 5.3.2 and below in section 7.6.1.2. These action levels shape the tasks in the program. In this way a concise program can be realized. Visualized, the mathematical content should be organized as shown in table 7.3.

Table 7.3 FNE Structure of Mathematical Content.

Content	Elementary Level KSE-1		Foundational Level KSE-2		Bridging Level to KSE-3	
Basic operations		Visual Models		Visual Models		Visual Models
Proportions						
Measurement						
Data						
Miscellaneous						

7.6.1.1 The Application of Benchmarks and Integrating Visual Models

Benchmarks are reference points in the mathematical system which help learners to organize their mathematical thinking and enable them to develop good mental computation and estimation strategies. These form the foundation for further development of doing precise and detailed computations, either by written notations or by the use of a calculator.

Benchmarks have shown to be very helpful at developing mental math, estimation and functional computation procedures on paper. (chapter 6, sections 6.4.5 and 6.4.6).

Learning through benchmarks starts at the elementary level and develops gradually into detailed thinking at the foundational and the bridging level. It can be compared to a tree that spreads along main branches into a more dense network of branches. The ramifications of the benchmarks grow as they applied in various contexts. This process can be supported by the use of integrative visual models. The bar-model, the block-model and the pie chart appear to be good means to visualize benchmarks and proportions through all strands. They can be used as working models. In combination with a number line they are sufficient to help learners to develop their own specific thinking models, as Enver, Azeb, Nadia, Mehmed, Zeki and Hatice show in their computations (chapter 6, section 6.4.5, 6.4.6 and 6.4.7).

Benchmarks and integrating visual models help learners to develop insight in mutual relations across strands. They also lay the foundation for flexible computational thinking, flexible procedures and estimation. The benchmark approach is elaborated in six steps in the IB materials through all strands. (see the percent strand in chapter 6, section 6.3.2). At the elementary level the foundation has been laid by doing informal doubling-halving procedures and working with round numbers. These are the point of departure for counting, addition and multiplication procedures (in combination) and for subtracting and division procedures (also in combination). These doubling and halving procedures are transferred to working with proportions within the metric system. Such doubling-halving procedures can be visualized with the help of visual models, e.g. the block model. This block model can also be used as a work model for the learners' notations.

In measurement real objects and photos of objects can be used to introduce the metric system in combination with the block model, using decimals and fractions, as shown in figure 7.2.

Figure 7.2: the use of actual objects and the block model at measurement

	
1 coffee container holding 500 gram 2 coffee containers holding 250 gram each	1 liter milk a half (1/2) liter milk, is equal to 0.50 liter a quarter (1/4) liter milk, is equal to 0.25 liter

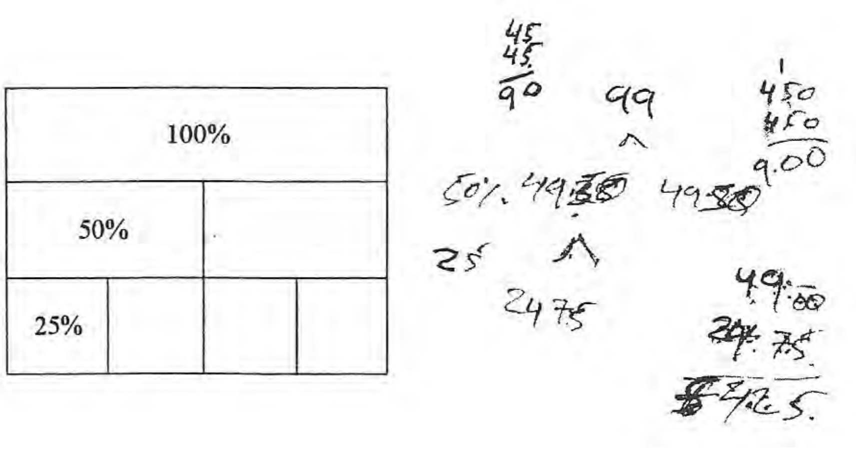
<table>
<tr><td colspan="4">1 kg = ...gram</td></tr>
<tr><td colspan="2">half kg (1/2)</td><td colspan="2">500 gram</td></tr>
<tr><td></td><td>1/4 kilo</td><td>?</td><td></td></tr>
</table>

<table>
<tr><td colspan="4">1 liter</td></tr>
<tr><td colspan="2">half liter (1/2)</td><td colspan="2">0.50 liter</td></tr>
<tr><td></td><td>?</td><td>?</td><td></td></tr>
</table>

Learners can practice with these models by manipulating actual objects through weighing and using a measuring cup, by examining photos and leaflets, and by completing the accompanying working models. The labels on the containers provide the numerical information about the content (including whole numbers, fractions, decimals expressed in kilograms, grams and liters). Words and mathematical notations can be written down on blank labels on the objects themselves and in blank working models. The metric relation between kilo and liter can be discussed. Similar things can be done with measuring tapes. Working cooperatively in small groups is strongly encouraged to create informal learning situations. Such tasks offer possibilities to learners to activate prior knowledge, to sort out accompanying conceptions and language usage, to build representations and mental models, to write down their notations and to learn in an informal way. Important in such learning situations is that learning materials offer the possibility and encourage learners to go through all four interwoven circles shown in figure 5.6 in an informal way. In this way adults can activate, adjust and reconstruct their knowledge and conceptions and acquire new knowledge. Moving through the levels of manipulation, representation and symbolizing insures that they develop a complete basis for insight into mathematical concepts and relations.

At the foundational level the same block model is used to introduce benchmark fractions, decimals and percent. Computations with benchmarks are done with the help of this model.
(see chapter 6, graphic 6.2 and 6.3). Here it works with a different concept at a higher abstract level. It encouraged Azeb, for example, to apply her notation system based on the halving procedure to compute 25% of 99 guilders. See figure 7.3 and also chapter 6, section 6.4.5.
In the data strand the bar model and the pie chart are good means to show proportions and to apply percent. The same setup through benchmarks can be applied, as shown in the transportation survey in chapter 6, sections 6.3.2 and 6.4.6.

Figure 7.3: Azeb, doubling-halving procedure at percent



The integrating visual models work in particularly well when set up with parallel strands showing which benchmarks cross at the same places in these strands. It enables learners to discover the equivalencies in these strands. In particular the bar and the block-model offer insight into the metric system. The basic principle here is thinking in proportions. Starting from doubling and halving in the frame of proportions the metric system evolves subsequently into a detailed system. In the examples we see that all four levels of mental action are practiced: experimenting with hands-on materials (M), using photos of real life materials (Rc), drawing visual models and schemes (Ra) and doing and writing down computations (S). (see chapter 5, graph 5.6) The tasks can gradually progress to a higher level of abstractness. Efficient use of actual objects or photos combined with the block model and their own computations and notations, may help adult learners at the process of (re-) constructing and (re-)organizing mathematical conceptions and procedures. It enables them to go through all four levels of mental action in their own way, to develop their own mental models, and to adjust and renew their knowledge.

In many learning situations common real life products can be very helpful, e.g. the rice package shown in figure 7.4, bought in a local supermarket.

Figure 7.4: a rice package

The package shows a transparent measuring bar from 0 to 1000 gram on the side of the container. It is a measuring cup of its own. The transparent plastic shows eight parts, each weighing 125 grams. Each part is sufficient for a meal for two persons.
Such actual real life items are indispensable to learners in ABE. They bridge the gap between school math and usable real life math. Visual representations of real life objects and integrating visual models like the block model are the bridge between informal and formal mathematical knowledge.

7.6.1.2 The Application of Action Levels

As discussed in chapter 5, section 5.3.2, in action theory learners are assumed to develop concepts and mathematical procedures by going through the four levels of activity in various order. Visual representations are seen as intermediates between informal acting in real life situations and doing formal mathematical operations through paper-based, or other, instructional materials and computations on paper or with the help of a calculator, using symbols. The integrating visual models, discussed in the previous section, are part of the representation level. By offering instructional materials on the three main levels of manipulation, visualization and symbolization, adult learners are invited to choose the instructional option that suits them best and to develop their own mathematical concepts and procedures to a higher action level with the help of these instructional materials. In chapter 6 we saw that learners, in particular Azeb, Enver, Zeki and Nadia, were able to develop mathematical procedures and notation systems based on visual instructional models that were offered. The expectation is that contexts, if set up in a good framework, will encourage adult learners to go through all levels of mental action through which they can develop their own new procedures.

In a context-based program we can refer to actual situations 1) by creating actual situations, 2) by creating representations of actual situations with the help of photos, or 3) creating schemes, charts, tables, etc. and 4) using symbols like formulas, words, mathematical notations, etc. Here we are back at the four levels of action theory (chapter 5, figure 5.6). The assumption is that by offering contexts on different levels, in coherence and at the same time, adults may see connections between these levels and may develop action skills on a higher mathematical action level by going through the four levels. These four types of contexts invite adults to act on a familiar level and may encourage them to think and work on a higher mathematical action level. This is also the way to help adults grow from informal to more formal and functional organized mathematical activities.

Adults in ABE who are used to acting three-dimensionally in their lived-in situations and who are not used to processing paper-based information, like learners in literacy courses, may want to start on the level of manipulation. These adults must “act” with real objects in a mathematics course, for example by counting structured packed objects, using measuring tapes and cups, weighing apples, working with coffee packs, milk containers, etc. Starting from their everyday life activities they can learn to work with representations of these real life situations and to write informal functional notations of their computations on paper. They come from informal mathematical knowledge and learning through action via representations of reality to more formal knowledge and functional notations. *(learning-by-doing)*

Other adults in ABE, often adults who have more school experience, may show patchy knowledge and skills, like, for example, Zeki, Nadia and Enver (chapter 6). They may be used to working with paper-based instructional materials but have “forgotten” former school knowledge. They may want to brush up their school knowledge and to give meaning to it by connecting school knowledge with their own real life experiences. Real life materials, like measuring tapes and cups, are good tools, but in many learning situations representations of real life situations provide sufficient information to be able to adjust and correct their knowledge. More abstract representations, like schemes and the bar and the block model, may help to gain insight into mathematical concepts and into computations, e.g. doing operations with decimals and percents, and to create cross-links between concepts. However, real life materials, like the rice package and coffee containers, may help to make mathematics recognizable in real life situations.

7.6.2 Management of Mathematical Situations

During the management of mathematical situations in actual real life situations, the actual numerate behavior of adults becomes visible. Here we come back to the ALL numeracy definition (chapter 2, 2.2.5) that says:
“Numerate behavior involves ***managing a situation or solving a problem in a real context*** (everyday life, work, societal, further learning) ***by responding*** (identifying, interpreting, acting upon, communicating about) ***to mathematical information*** (quantity & number, dimension & shape, pattern & relationships, data & chance, change) ***that is represented in a range of ways*** (objects & pictures, numbers & symbols, diagrams & maps, graphs, tables, texts, formulae) ***and requires the activation of a range of enabling processes and behaviors*** (mathematical knowledge and understanding, mathematical problem solving skills, literacy skills, beliefs and attitudes.)”. (Gal et al, 1999)

Also in chapter 2, section 2.4.2, we identified a broad set of skills necessary to solve complex mathematical situations:

- *generative mathematical understanding and insight* to give meaning to and to interpret numbers and to plan appropriate mathematical actions
- *literacy skills* to read and understand the problem and to reason about it;
- *communication skills* to be able to acquire information, to share the problem with others, if necessary, to learn from others how they would solve the problem and to work cooperatively;
- *problem solving skills* to identify, analyze and structure the problem, to plan steps, to select appropriate actions, to actually handle the problem and to take decisions.
- *reflection skills* to be able to control the situation, to check computations, to evaluate decisions and to come to contextual judgements.

Though all individual skills each require specific training on details, they must be considered as a unit when solving mathematical problems in actual real life situations. Adults will often make decisions by using a mix of these skills. Situations to be managed are characterized by *a "problem" that must be resolved* and by application of *functional procedures,* appropriate for the emerged situation.

7.6.2.1 Problem Solving

Context-based learning and learning-in-action, or learning-by-doing, include problem solving. Problem solving activities provide an ideal start for learning to manage mathematical problems individually, in pairs or in small groups, especially if they are organized as learning-in-action activities. However, learning how to solve problems cannot be done by only offering nice context problems and having adults determine how to do it. Teachers will have to help adults to identify problems, to analyze and structure these problems and what to do. In chapter 5, section 5.3.3, we made a distinction between the *what, why and how* of the problem solving process, focusing on the *process* itself, the *quality* of the mathematical action and the *organization* of the process. The basic structure of the problem solving process was described in six steps and may support adults when going through this process. These steps concern analysis of the problem, plan action, decide how to solve the problem, solve the problem, check the solution and review the solution in the frame of the context. When *learning* how to do problem solving we may think of the following detailed actions within these steps:

- locate a mathematical situation;
- identify the problem in it;
- analyze and structure the mathematical information in it
- interpret, give meaning to the mathematical information
- plan, discuss possible steps for solving the problem
- choose a solving procedure
- do computations, if needed, or act otherwise
- check the result
- apply contextual judgement, if necessary
- check possible consequences
- make decision
- reflect on the process.

The final step, reflecting the process, is important in the learning process because it helps adult learners to become aware of their own actions and to be better prepared in new situations. Such lists with guidelines can be used as check-list to learn how to solve problems in many real life situations, not only in numeracy situations. When learners are used to these what-to-do steps, the focus within this process may switch to the quality and the organization of their actions. (see chapter 5, section 5.3.3)

The key element in this process is *interaction*. By discussing numeracy problems in context adults become aware of their actual mathematical knowledge and skills. By reflecting on their knowledge and skills adults gain insight into what they know and what they want to know. This is the start of learning. By learning through contexts and problem solving a cyclical process can be created where learners gradually develop a higher level of knowledge, self-knowledge, self-esteem and self-confidence, and where they also learn to see how to manage problems themselves and in collaboration with others. Contexts that can be solved in multiple ways and that ask for action can create informal learning situations in which learners feel comfortable.

Analysis of the problem solving process may also help at learning-to-learn (see section 7.6.4). For learning how to do problem solving, working in pairs or small groups is strongly advised to encourage discussion and reflection.

7.6.2.2 Functional Procedures

Within this process of problem solving we want to emphasize the importance of developing *functional* mathematical procedures that fit adults' thinking, is based on their own insight in mathematics, and serve the process of problem solving. Such procedures are often a mix of formal and informal self-invented or work-related mathematical procedures which may deviate from school math procedures. They appear to be very effective. (see e.g. Lave, 1988, Nunes, Schliemann and Carraher, 1993, Hoyles and Noss, 1998).

As we saw in chapter 4, many adults in ABE have developed a mix of informal and formal mathematical procedures, but others tend to only apply algorithms without making sense of them. These people come together in classes and will have to acquire sufficient skills to start vocational or other follow-up courses within half a year, or a year at maximum. To help them achieve at least the minimal requirements to be successful in follow-up courses, we need to find ways to build on the learners' entering skills. At the same time they must acquire enough skills to be able to further develop themselves, to link with the high demands of society and to keep up with new developments in society. (see also chapter 2).

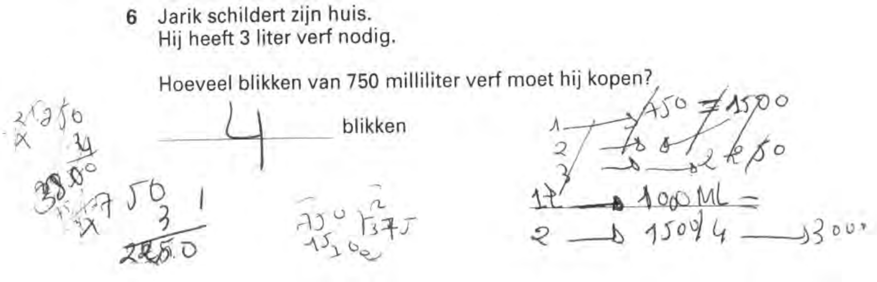
The practical situation in ABE, outlined above, is the first reason to look for "functional" mathematical procedures. A second reason for focusing on functional procedures rather than starting over with formal procedures like algorithms, is that viable alternatives exist, for instance, by using calculators. However, the problem is that many adults often still want to learn such algorithms because they recall from former schooldays that they could not do it. They want to overcome these frustrations now. They often do not believe that alternative procedures are allowed.

A third reason is the realization that many computation procedures that we learn in school are just part of "school math" and not functional at all in real life situations. In ABE we aim to focus on "learning-for-doing" rather than on "learning-for-knowing". Learning the long-division algorithm is "nice to know", but not necessary per se for doing division problems in real life situations, as we saw with the solutions for division problems in the placement test. What exactly do ABE learners "need to know"? For what purpose? Here we may get entangled in a more ethical discussion: who decides about what learners should learn? Teachers will often have discussions with their learners about decisions concerning what to learn, and, of course, if adults want to learn the long-division algorithm, this should be no problem, but they should be informed about the current developments in school education and why it doesn't matter that they don't know how to do the long-division algorithm if they have good alternatives.

The best evidence that functional procedures are effective was shown by the adult learners in this study at the placement test and at the learning program. (see chapters 4 and 6). Azeb and Zaara, for example, created their own halving procedures to compute 50% based on insight into place value. The learning program helped these learners to further develop such procedures. Mehmed, Azeb, Hatice and Nadia showed their new procedures based on mental math after they learned to work with the benchmark table for doing percent computations. Enver, used to blind ciphering, showed how he changed from formal procedures to more functional computations based on the benchmark table. And with the help of the bar model Zeki could relate fractions to percent. (see chapter 6, section 6.4) Also with Nadia. In chapter 4, section 4.3.3.1, her computation for the next item was explained (figure 7.5).

Repeated from chapter 4:
The item question is: *Jarik paints his house. For this 3 liters paint is required. How many 750 milliliter cans should he buy?*

Figure 7.5 Nadia's functional computations



6 Jarik schildert zijn huis.
Hij heeft 3 liter verf nodig.

Hoeveel blikken van 750 milliliter verf moet hij kopen?

4 blikken

Nadia shows a few formal computations at the left side and in the middle, and informal computations at the right side. Not clear is what came first.

At the left side we see a multiplication: 4 times 750, but she made errors and by that she came to 3800. We may wonder whether this was her first computation or a check at the end for the informal computation at the right side. In the second computation she did 3x750. Here she might have chosen to multiply the "3" from "3 liter" with the content of one container "750 ml". She also tried a long division. These attempts might be guesses to try "something". She apparently does not know which algorithm is the right one to use here.

On the right side we see two other attempts: right on top she tries to compute how many liters of paint she gets by adding 750 ml. three times (2250). Then she needs a side step: 1 liter ➔1000ML. Here she refers to her formal knowledge about liters and milliliters. Finally she computes in an informal way: 2 ➔ 1500 / 4 ➔ 3000. Here we see that she finds the correct answer by doubling the volume of the cans: 2 cans hold 1500 ml, 4 cans hold 3000 ml.

This informal computation is done mentally on the symbolization level, but refers to "paint cans" she possibly pictures in her mind. We may assume that her notations are based on her knowledge about actual paint containers. If this is the case, then she created her own mental "model-of" and based on that she created her own "model-for" and a functional notation system on the symbolization level. This computation is fully based on insight.

The essence of functional procedures is that they may depend on the content and the structure of the context. Nadia shows how she analyzes the context and solves the problem based on characteristics of the context itself. This analysis, her thinking process and her insight are shown in her computations. Since she also applied similar notations to more items, we may assume that she indeed creates her own idiosyncratic way of doing informal, but functional computations based on mental models.

Functional procedures are always based on insight and are more effective in problem solving situations than blind ciphering. Allowing such idiosyncratic procedures forces us to think about what kind of functional computations we may expect and which to allow as correct computations. It may be difficult for teachers sometimes, to understand what their adult learners write down. Teachers will have to be trained in reading such notations. A basic criterion here is that *if adult learners can tell what they do, understand what they do, can explain their notations, and their answers are correct, their notations should be accepted as functional.*

In addition to functional procedures on paper it is advised that adults learn to work with a calculator in combination with mental math and estimation. This is also recommended in RME and in the new NCTM 2000 Standards (NCTM, 2000). Learning algorithms is emphasized less in school programs in recent years. Despite these developments there is still a big difference between school math and functional math. For ABE it is strongly advised to spend the limited time learners have in a sensible and balanced way on doing mental operations, functional computations on paper and estimation in combination with sensible use of a calculator.

7.6.3 Processing new information

Quantitative information in real life is often packed in sources like TV bulletins, newspapers, journals, texts, graphs, charts, tables, etc. Analyzing and understanding such information requires quantitative literacy skills and notions of statistical concepts (Curry et al, 1996, Dossey, 1997, Gal, 1997, 2000). To be prepared for acquiring new knowledge in out-of-school situations adults should have an opportunity to acquire skills in how to process such information. In fact this numeracy component goes hand in hand with learning and management skills, and can be learned in combination. Discussion with other learners is again an essential part of this learning process. Topics for discussion could be provided by the learners themselves but should also be incorporated in the program. Topics can be discussed in small subgroups, but to be able to do so effectively, guidelines are desirable. It cannot just be assumed that learners themselves can develop guidelines for how to work in such situations. In addition to the guidelines for problem solving and learning how to learn, described in the previous and in the next section, a few key points, described in chapter 2, section 2.4.3, can be provided for learning how to process new information in out-of-school situations:

1. Read or listen to information
2 identify key points in the information
3. reflect on what is “new” *to me?*
4. communicate, discuss with others
5. reflect on possible implications for personal life. *What does it mean to me?*
6. reflect on possible implications for society or work

Central to such discussions are mathematical and statistical reasoning that finds the logic behind the information and helps explain information to others. This means that learners must be open for discussion and critical comments. Essential in acquiring new information is that learners learn to ask questions to get things clear, or go find more information about a certain detail. For that they need to know how to access the internet, libraries, dictionaries and other resources. Learning centers can help to arrange access to such resources.

It may be clear that topics discussed here will depend on interest of the adult learners and can be very broad. The task about the blood test, for instance, the introduction task described in chapter 6, could have been a topic for further discussion for the learners group in this research. A precondition for such discussions is that the learners can help each other to find out what they don’t know, in this case the “90% of all cases”, and how to learn more about that, without the help of the teacher. Another precondition is that such tasks are not assessed at the end of the course. Learners must feel free to work on this part of numeracy without feeling the pressure of tests afterwards. However, they must feel encouraged to study such topics and to report their results to other learners.

7.6.4 Learning to learn

As discussed in chapter 2 and 5, learning to learn is an important issue for learning in further and vocational education and for lifelong learning. Adults in adult education are increasingly expected to learn independently from teachers and to work in cooperative learning settings. However, we may not expect that adults in ABE can do so from themselves, since they have little or no school experience when they enter ABE. Hence, learning situations in ABE must provide guided learning environments in which they can practice how to learn to be prepared for follow-up courses where they are expected to do learn in more teacher-free learning settings. Problem-based learning may help the learners to reflect on their own learning process and to improve their learning. Particularly in mathematics/numeracy courses we see possibilities to create such learning situations, based on problem solving. In chapter 5, section 5.3.3 it was discussed that the what-why-how process eventually leads to independent learning. If we slightly change the wording to the three dimensions in this process, focusing on *learning*, then we come to the following expressions to each of them:

1- Through problem solving learners become aware of their learning *process (what)*
2- During problem solving learners may want to emphasize the *quality* of their learning *(why)*
3- When doing problem solving learners learn how to *organize* the learning process *(how)*

The six steps of the problem solving process were discussed in chapter 5, section 5.3.3 and earlier in section 7.6.2.1. The *what-why-how* process enables adults to take their own learning processes into their own hands in order to be able to manage numeracy problems in their own real life situations and to be equipped for future learning, which is according to our numeracy definition. Within this process the emphasis is on learning rather than on teaching though teachers play a crucial role in it.

Going through this process requires clear mutual expectations of learners and teachers and at this point conflict situations may occur. Adult learners often expect courses in a traditional way when they enter ABE. They are used to teachers who “teach” and learners sitting, listening, taking notes, writing, practicing, etc. They are less used to participating actively in courses by working in subgroups, discussing topics, problem solving activities, playing games, making decisions about the content of the program, etc. This is particularly the case with mathematics. In general, mathematics has been seen as a subject one has to learn and to do yourself, not in cooperation with others. People also see mathematics as a typical school subject and not as a social or real-life activity. Such discrepancies between expectations regarding school math and what adults really need and want to learn are clearly described by Curry et al (1996). This means that learners in FNE programs have to get used to 1) a different style of teaching (and not-teaching) and 2) a different content in the mathematics program. They may adapt only if they are well informed from the beginning and know about the purposes of this way of learning and teaching. Their former school experiences, social background, lack of self-confidence and self-esteem, their level of math anxiety and accompanying attitude may affect their participation in FNE programs. Learners often pose themselves as being dependent on teachers in learning situations, which could be a teachers' trap. It may influence the program so that teachers start "teaching" again. Such situations must be avoided. Learners need to be convinced of the need to participate actively and to take responsibility for their own learning. Teachers need to be convinced of their role of facilitating the learning process and not directing it (van Groenestijn, 1997, Arriola, 2000)

On a different level the what-why-how process is also the start for democratization and empowerment. It may help develop a sound critical attitude in learning situations and this may continue to work in out-of-school situations. Here we get at Freire's philosophy (Freire, 1970) and other political issues such as those discussed in Steen's most recent publication *Mathematics and Democracy. The Case for Quantitative Literacy.* (Steen, 2001).

Teachers need to be well-trained for their role as facilitator of learning. Many teachers in adult education are still used to teaching. A good example of how to set up, observe and guide such problem solving situations is described by Arriola (2000). Her classroom project eventually led to changes in her way of *teaching* but also in the learners' way of *learning*. Similar results were achieved with teachers in training sessions on the Supermarket Strategy package (van Groenestijn, 1992, 1993) where teachers commented that their way of teaching changed because they learned in the individual interviews how to ask questions. By applying this technique in their classrooms they were better able to activate the learners' thinking processes. They observed changes in their learners' way of learning. Their learners' attitude changed from "consuming" to "participating" and by that they became active learners. That is what we want to achieve in ABE: adults as independent learners. Learning to manage their own learning process depends ultimately on the way capable teachers guide this process.

7.7 Learning and Instruction

After having arranged and organized the program, program developers and teachers will have to develop tasks to make the program operational and to allow the learners to learn. This is where the actual learning (and teaching) process starts, i.e. the classroom sessions. To meet the goals of the program, different types of tasks have to be selected carefully. Roughly they can be divided into two main categories, which match the four numeracy components but emphasize learning:
- Learning functional mathematics (what to learn)
- Arranging the learning process (the way in which to learn)

The first component covers the mathematical content. The second component covers management of mathematical situations, processing new information and learning to learn.

Though these categories can be distinguished they cannot be seen as separate topics. They go hand in hand in numeracy courses: the mathematical content is embedded in the learning process.

7.7.1 Learning Functional Mathematics

Within learning mathematics other distinctions can be made between:
- context-based and context-free tasks
- estimation and precise computation
- mental computation, computations on paper, computations by using tools like a calculator or a computer
- tasks meant for exploration, for practicing and for application of mathematics.
- individual work, work in pairs and group work

Though these tasks are listed here as separate types of tasks they are often combined. Context-based tasks, for instance, focus on mathematics in functional situations and can be useful for group work and work in pairs to discuss differences in mathematical procedures. Context-free tasks can serve the development of estimation and mental computation in combination with a calculator. Such tasks are very suitable for individual work and work in pairs. Results can be discussed afterwards in the group. In all tasks a distinction can be made between explorative, practice and application situations. When learners start a new topic, group work and discussion may be important to uncover prior knowledge and partial knowledge, to develop new concepts and to help each other discover new ways of doing mathematical procedures.

In tasks for practicing mathematical procedures learners may want to work individually or in pairs. When they have mastered new mathematical procedures learners can be asked to use the mathematics they learned in new situations or in combination with other mathematical procedures in more complex tasks. Here they also enter the assessment stage.

In all assignments context-based and context-free tasks are the main ingredients. These will be discussed further in the following subsections.

7.7.1.1 Context-based Learning

Contexts in an FNE program are the main ingredients for giving meaning to mathematics in real life situations.[3]
Adult education settings have revealed the remarkable fact that many adults who received mathematics education in school often have problems giving meaning to this school mathematics in real-life situations (Verschaffel, 1997, 2000)[4], whereas adults who have little or no school mathematics education developed their own mathematics, which is meaningful to them. It is also noted that their personal procedures often have not been recognized as being "mathematics" by these adults themselves. (van Groenestijn, 1992).
Such experiences in adult education, the results in chapters 4 and 6 and our considerations in chapter 5, convince us that learning mathematics in ABE should focus on giving meaning to the mathematics in real-life situations. Hence, according to Greeno (1999), we consider learning-in-action as the best way of learning-for-doing, but, since such learning situations are often not feasible in ABE, contexts derived from and referring to real-life situations must be the base of learning-for-doing.[5] Eventually, *functional numeracy* is the goal of an FNE program. Hence contexts have a three-fold function:

- They represent math-in-reality and hence are the source and focus for giving meaning to mathematics.
- They open the way for meaningful learning and problem solving.
- They integrate all components in the FNE program (see table 7.1)

Though learning through contexts is the key to managing mathematical situations real life situations, it is not sufficient to only offer nice real-life related contexts in mathematics programs. Such contexts have to be situated in the framework of a well-thought out program. To do so, criteria for learning-through-contexts in the FNE program must be set:

1. Learning in an FNE program is context-based, not context-bound.
2. Contexts are set up based on the four action levels.
3. Contexts are set up based on complexity factors.
4. Contexts offer possibilities for multiple tasks and problem solving.

1. Learning in an FNE program is context-based, not context-bound

Context-based learning differs from context-bound learning. Context-bound learning is bound to particular situations and could yield partial knowledge, like Hans the truck driver who thought that kilometers have nothing to do with meters. In this regard money contexts, for example, could be an instructional trap. Money systems are often based on the decimal system. In practice it appears that teachers often introduce the decimal system by having learners do money computations. By doing this, they assume their learners will also understand the decimal system, but many adults don't see the link. Money computations are often bound to the characteristics of the money system, the specific bills and coins. Transfer from the money system to the decimal system is more difficult than we may think. Context-based learning implies that people are eventually able to work flexibly on different action levels and can easily apply knowledge and skills in different situations, even without references to actual real life situations. However, in such "context-free" situations people must be able to "imagine" appropriate context situations.

In order to make mathematical knowledge and skills developed in context-based learning be applicable flexibly and to prevent context-bound knowledge and skills, it is suggested to work along the next steps in an FNE program: (see *In Balance* teacher's guides, 1996)

1. Give context - give numbers
2. Keep context - change numbers
3. Change context - keep numbers
4. Keep context - have learners change the numbers
5. Have learners create a new context - keep numbers the same, etc.

By changing the numbers the difficulty level of the task can be varied. By changing the context learners learn to apply the same mathematical operation in a different situation, which may prevent context-bound knowledge and skills.

2. Contexts are set up based on the four action levels.

Special attention should be paid to the four action levels in context tasks. Working with these levels should be incorporated as described in section 7.6.1. This implies that many tasks, in particular at the elementary level, require working with hands-on materials to create manipulation situations.

In addition photos and visual models like schemes, proportion-bar, block-model, chart, and blank working models are necessary to create the bridge between actual manipulation and working with symbols. Adult learners are, of course, also encouraged to create their own visual models, like schemes, a drawing of a milk container to represent one liter, or a drawing of a number-line to indicate the length of a floor, etc. By creating such representations adults show their insight in the mathematical action. They can be encouraged to do so by hints in the context itself or by showing partially-elaborated schemes. This may help adults to create their own visual representations to support their computations on the symbolic level. To create well-developed concepts it is necessary that adults go through all four action levels in the way as described in 7.6.1 and shown in chapter 5, figure 5.6.

3. Contexts are set up based on complexity factors.

Working through the different strands from elementary level, through foundational level to the bridging level, contexts can be built up in difficulty levels along several complexity factors. Contexts in the “Supermarket Strategy” and in the “In Balance” materials are experimentally set up along the lines of following complexity factors (van Groenestijn, 1992, 1996, 2000):

1- from concrete to abstract
2- from informal to formal
3- from simple to complex problems

In addition all text is kept to short single sentences based on common words and text inherent to contexts. There are few or no distracting factors in the contexts. This was done to overcome reading problems for in particular second language learners. (see also chapter 3)

The numeracy team of the ALL-survey (Gal et al, 1999) developed over 100 items spread over all facets of the ALL definition of numeracy (see chapter 2, section 2.2.5, figure 2.1) These items are classified in five levels of difficulty based on the following complexity factors[6] (Manly & Tout, 2001):

1- *Problem transparency*: from obvious/explicit to embedded/hidden
   How difficult is it to identify and decide what action to take? How many literacy skills are required?
2- *Plausibility of distractors*: from no distractors to several distractors
   How many other pieces of mathematical information are present? Is all the necessary information there?
3- *Complexity of mathematical information/data*: from concrete simple to abstract complex
   How complex is the mathematical information that needs to be manipulated?
4- *Type of operation/skill*: from simple to complex
   How complex is the mathematical action that is required?
5- *Expected number of operations*: from one to many.
   How many steps and types of steps are required?

These complexity factors appeared to be very useful for the ALL item development. Furthermore they are also assumed to be a good means for developing contexts in numeracy programs and hence for the FNE program. The complexity factors enable contexts to be analyzed for complexity and level difficulty and new contexts to be developed. Considering the factors may lead, for instance, to developing of items for second language learners with a reduced text component in the contexts.

4. Contexts offer possibilities for multiple tasks and problem solving.

Within all criteria for contexts described above we can distinguish three main characteristics:

1- The information needed for the task is processed into the context itself. The problem to be solved is posed in a separate instruction task.
2- Contexts include the possibility to solve the posed problem in multiple ways.
3- Contexts can be changed in several ways to meet the needs of the learners. Starting from the actual or supposed level of learners, contexts can be adjusted to a higher level to challenge them to develop creative alternative solving procedures. Or, if necessary, contexts may be changed to an easier level to permit understanding of foundational principles.

Essential parts in a context are graphics, numbers and text which organize the mathematical information in a situation along the lines of the complexity factors described above. The problem or instruction question posed in the context exists mostly of text and numbers. Here all factors described in previous sections come together and challenge the learner to apply and reorganize knowledge and skills they already have acquired and still leaves room for their individual preferences.

The solving procedure can be started on each of the four action levels. A good context also challenges the learner to develop new knowledge and skills in the frame of the program. Brief descriptions of a few concrete examples may make this clear. (see appendix 5).

7.7.1.2 Context-free Learning

Whereas context-based tasks focus on mathematics embedded in real life situations, context-free tasks focus on characteristics of numbers in the tasks. Knowing such characteristics is essential for learning to work with round numbers and benchmark numbers. Such numbers occupy a prominent place for learning mental math and estimation strategies in combination with the use of a calculator for doing precise and more complex computations. Round numbers and benchmark numbers are also the start for learning to do functional computations on paper.

The decreasing importance of algorithms for doing basic operations in favor of mental math in combination with a calculator has been an ongoing discussion in RME since the beginning of the nineties. (Treffers, 1989, 1991). This discussion is now becoming more worldwide and is having an impact on development of new curricula. (See, for example, the new NCTM-2000 standards (NCTM, 2000)).

For adults the question of whether to learn algorithms in ABE is a pressing issue. Adults often have serious problems with algorithmic procedures. Though many adults learned to do algorithms in school, they often have forgotten most of it when they come back to school years later. In particular, long division and subtraction cause difficulties (van Groenestijn, 1992). Algorithms are seen as less important for adults to be equipped for the future since they can be replaced by a calculator. (See for example the teacher guide at the IB materials, van Groenestijn et al, 1996 and see also Curry et al, 1996). Mental math in combination with functional computations on paper and the use of a calculator can be much more effective. Therefore the latter has been emphasized in the FNE program.

If adults want to learn algorithms or to improve the ones they learned formerly, they are free to do so, but it is not the main purpose in FNE. Learning algorithms is done through the RME way of instruction, based on mental math and progressive schematizing (Treffers, 1989, 1991)

For good estimation and mental computations adults need to feel very familiar with numbers in order to flexibly “construct” the computations they have to do in practical situations. This familiarity can be achieved in several ways, e.g. by rounding money amounts, or practicing counting and multiplication strategies but also through playing number games, e.g. “get at” games, using *benchmark numbers,* like 100, 200, 50, 25, 75, 250, or, 10, 20, 30, etc. and applying basic operations like in the following examples (also elaborated in chapter 3 and 4):

Figure 7.9 Get at 1000 (1)

250	**175**	**150**	**100**
200	**225**	**400**	**300**

→ **1000**

The results on the placement test (chapter 4, section 4.3.1.3) showed that about a quarter of the learners could not do such computations. Another quarter solved it by trying several addition algorithms. Similar tasks can be offered in various ways, e.g. with multiplication:

Figure 7.10 Get at 1000 (2)

2	**4**	**5**
100	**125**	**250**

→ **1000**

Here there are many possible solutions: 4x250, 4x125 + 2x250, 5x100 + 2x250, 4x125 + 5x100 or 6x125 + 4x100, but also: 2x5x100, 2x4x125, and even: 5x250 - 2x125.

Discussing such problems may help adults understand characteristics of numbers, like: 4x250 makes 1000, so 8x250 makes 2000 and 40x250 makes 10,000. Such computations may help to improve their insights into number, to discover patterns in and relations between numbers and to work flexibly and in a constructive way. These games allow adults to construct and reconstruct their own knowledge and computations based on their own competencies and leave room for their individual notations. These types of tasks offer possibilities for adults to compare and discuss their computations with others and to discover that the computations can be done in many different ways. When adults feel familiar with benchmark numbers, they can learn to compute flexibly. Using easy, round numbers as benchmarks may help to keep the numbers in mind during their computations. Similar computations can also be done with benchmark fractions and decimals. This may improve mental math and serve as the basis for estimation. For more complex computations with difficult numbers a calculator can be used.

Experiences in ABE practice show that adults can often do addition and multiplication procedures better than subtraction and division and can use the former as alternatives for the latter. This was also shown in the placement test (see chapter 4, table 4.18). This indicates that, in fact, when people have mastered the first two, they actually don’t need the latter two if they know the relationships between all of them. Therefore, in the FNE program addition and multiplication are emphasized.

7.7.2 Arranging the Learning Process

Tasks for learning mathematics are embedded in the learning process. In fact, the teacher arranges the actual learning process. In current adult education in the Netherlands, however, the focus is more on teacher-free and cooperative learning. In such learning situations the teacher will be the "facilitator" of learning. (See also chapter 1 and 5.) In most learning-teaching situations it will depend on the teacher's capability to find a balance between "facilitating" and "teaching". At this point we may confront the learners' expectations regarding the role of the teacher and their own roles as learners. The learning-teaching process is very complex and the result is often a compromise of mutual expectations between learner and teacher. Interaction and discussion are essential components in this process. Carefully prepared tasks included in the program can help the teacher and the learners to gradually get used to their new roles as facilitator and active learners. Problem-based learning, which includes the six steps of the what-why-how process, may help to analyze the learning process in different stages. Prerequisites of the success of such tasks are interaction and discussion in pairs, small groups and with the entire learners' group.

Within the learning process three types of tasks can be distinguished that focus on:
1- the what-why-how components in the process of problem-based learning;
2- how to manage a mathematical problem (in real life situations);
3- how to acquire and process new knowledge (in real life situations).

For the first type of tasks the focus is threefold: *insight into the own learning process, into the purpose and quality of the mathematical action, and into the organization of the problem solving procedures*. This process was extensively elaborated in chapter 5. The six steps are summarized in table 7.5. As a rule-of-thumb for the learners, when analyzing the what-why-how process, learners can keep three key words in mind: plan-do-review. These three key words organize every task.

For the second type of tasks the focus is on *application of already acquired knowledge and skills in real life situations*. A list of relevant actions was discussed in section 7.6.2.1 and is shown in the middle column of table 7.5.

For the third type of tasks the focus is on *acquiring new knowledge and skills in real life situations.* This was discussed in section 7.6.3. The key words are shown in table 7.5, right column.

Summarized, tasks concerning the learning process itself can be analyzed with the help of table 7.5.

Table 7.5 help-list for learners to analyze the learning process.

what-why-how process	manage a math problem	process new info
▪ analyze the problem ▪ *plan* how the problem can be solved ▪ make decision about how to solve the problem ▪ *do* the necessary actions ▪ check the solution ▪ *review* the solution in the frame of the context	▪ locate ▪ identify ▪ analyze, structure ▪ interpret, give meaning ▪ plan, discuss ▪ do computations or act otherwise ▪ check result ▪ apply contextual judgment ▪ make decision ▪ reflect on the process	▪ read, listen, look at ▪ identify ▪ analyze, structure ▪ reflect on what is new ▪ discuss with others ▪ reflect on possible implications for yourself or others

Different types of tasks, either singly or in combination, offer a framework that can help to develop and classify a broad set of tasks that cover all facets of the FNE program. Topics for such tasks are numerous. A series of these tasks should be prepared and included in the program, for example the transportation survey, average car sales, public transport, the blood test. In addition there should also be room for individual interests of the learners and specific local, national and international topics to be discussed. Learners can contribute their own important topics.

A well-designed numeracy program must have a balanced series of mathematical tasks embedded in tasks for learning how to manage the learning process. The role of the teacher is mainly to revise and adapt a teacher-guided learning situation into a more teacher-free learning situation.

7.8 Assessment

To complete the program, assessment tasks must be developed to evaluate the learners' progress and the program.

In chapter 3 it was noted that the purpose of assessment in ABE is threefold:
- keep track on the learners' progress
- evaluation of the program concerning goals, content and planning
- evaluation of the program concerning reporting and accountability for funds

These goals are on three different levels but should operate in coherence. More precisely, 6 goals can be distinguished:

1- Learn more about the mathematical knowledge and skills adults have acquired when they enter ABE in order to determine what needs to be learned and to place them in the right courses.

2- Learn more about adults' problem solving procedures in order to improve instruction.
3- Learn more about adults' numeracy skills to be able to fine-tune educational programs to their needs and wishes.
4- Monitor and document the individual learner's progress in the course of the numeracy program in order to guide the learners through their own learning routes and to prevent learning problems and drop-out.
5- Be able to evaluate numeracy programs in order to improve numeracy education in general.
6- Enable policymakers and program developers to adjust policy and numeracy programs to new demands and developments in the labor market and in personal and societal life.

Consequently, criteria for assessment tools could be derived:
1- Numeracy assessment in ABE should be done in an appropriate way for learners in ABE.
2- Assessment should enable adults to show what knowledge, skills and procedures they have mastered when they enter ABE, rather than what they do not have.
3- Numeracy assessment in ABE should provide insight into mathematical procedures and problem solving.
4- Placement tests in ABE should reflect the goals, content and levels of the math curriculum so that adults know what they can expect during the course and can be placed in right course.
5- Numeracy assessment in ABE should allow second-language learners to apply the mathematical procedures and algorithms that they learned in their home countries.
6- Text used in a paper-and-pencil math test should not hamper second-language learners to take a mathematics test.

The first focus in assessment in ABE is on the learners. Results of assessment of individual learners should help to improve learning (and teaching) in the actual learning situation. As discussed in chapter 3, the way in which assessment is carried out should be appropriate for the learners in ABE, should reflect the content of the program on different levels and should also reflect the type of tasks learners will have to do in the course of the program. We discussed the use of oral, paper-and-pencil and computer adaptive tests (CAT). We also discussed a few types of tasks. In general it can be said that assessment should represent all types of tasks that are characteristic of the program. It should include context-based and context-free tasks, discussion tasks, action tasks, individual and group work. Assessment tasks should also be based on the four action levels and complexity factors.

Based on the set-up of the FNE program we can now suggest how to organize assessment for the FNE program. We will make a distinction between oral interviews, paper-and-pencil test, CAT tests and assessment-in-action.

Oral interviews are appropriate for semi-literate learners at the start of a numeracy course, for second language learners who haven't mastered the second language sufficiently. They are effective means to learn more about mental computation procedures and about computations on paper, and for diagnosis of possible learning problems.

Paper-and-pencil tests can be used for placement of learners who can read and have mastered the language, They can also be used as formative and summative tests. Paper-based tasks are particularly useful in learning more about the mastery of computation procedures when solving context-based and context-free tasks. Time should be taken afterwards for discussion to get more detailed information if necessary. The emphasis should be placed on the quality of the learners' productions rather than on correct-incorrect scores.

CAT can be very efficient for determining the learners' levels in a short time, particularly for formative and summative tests. The results of such tests are often only based on correct-incorrect scores. For placement purposes CAT should be used only if learners are familiar with computers and computer tests, and if they have achieved a math level high enough to be able to understand the type of tasks and the required type of operations that are directed by the computer.

Assessment-in-action is accomplished by tasks in which learners are required to show how they manage an actual mathematical situation in joint group work or individually. An example here would be the tile floor where learners can be asked to cover a certain area, to determine the number of tiles required based on the dimensions of the tiles, to compute the amount of necessary tile glue, the total price of the floor, the cost per hour for work, the total cost for the entire job, etc. Such tasks represent "real math" where the learners can show their actual numerate behavior. (see also Cumming and Gal, 2000)

Information acquired at the learners' assessment may also provide information for evaluation of the program:
- Results on the placement assessment may help to clarify the start position of the learners and their entry level. The program can be fine-tuned to the learners' needs and wishes.
- Results of formative assessment are very useful for evaluation of the goals of the program, the learning-teaching process, specific problems of learners, the mathematical content, the functionality of the tasks and the feasibility of the program (De Lange, 1987, 1995, v.d. Heuvel, 1996)

- Results of summative assessment can indicate the percentage of learners who pass the KSE-1 and KSE-2 levels and are placed in courses on the next KSE level or start courses in vocational education. The results may also indicate the number of drop-outs. Such information is often required for local arrangements and funds. It may also inform employers and employment offices about competencies of learners who pass the tests and move on to other courses or work. Annual reports for local authorities are often required for accountability and be the basis for funding.

In summary:
Assessment is indispensable for a variety of information about the learners and the program. It provides information about the learners' progress, the feasibility of the program and the stream-on possibilities of the learners to other courses. It should be organized in a systemic way, using all possibilities that value the different parts of the program. It can be done in various ways. It should be done using methods that are appropriate for the learners and that mirror the learning program. Assessment should be more than an obligatory standard procedure that is only based on traditional ways of testing. Summative assessment is necessary for evaluation of the program and for accountability to local authorities and funders.

7.9 Conclusions

Functional Numeracy Education in ABE is designed to prepare adults for further education and for lifelong learning. It also enables adults to learn to manage their personal lives and work situations. Adults in these courses have little or even no school experience. They learn the most from and in actual real life situations. The gap between their actual numeracy and literacy skills and the requirements for application for vocational courses is often very large. The problem in these courses is that there is little time to spend and a lot to do. That is also a challenge. Hence numeracy programs should be concise, complete and effective and should fit these adults' ways of learning. The FNE program aims to offer such a program.
Andragogical starting points, studies about learning in practice, Action Theory, RME and Socio-Constructivism form the underlying foundation for this program. Together they offer the building blocks for FNE: goals, starting points, parallel strands, a concentric setup, benchmarks and integrating visual models. The structure of the program and the content become visible in contexts and tasks. Contexts help learners to give meaning to mathematics and to develop problem solving skills. They are the basis for learning how to manage mathematical situations. The what-why-how processes are key elements in learning to manage their own learning processes. As part of problem solving they also learn to develop functional problem solving procedures and

computations. Such functional procedures can fulfill a key role in the learning process because procedures based on this are usually based on insight.

Learning in FNE programs focuses on practical knowledge and learning-for-doing. FNE tries to incorporate activities based on the learners' own real life experiences and to focus on functional mathematical knowledge and skills. Context-based learning, problem-solving and functional mathematical procedures are key in this process, in particular for the development of reflection, learning-for-doing, communication and interaction. It may also enhance cooperative learning. Which, in turn, is the key to teacher-free and independent learning.
In this process, adult learners learn to take responsibility for their own learning. Teachers emphasize learning rather than teaching. They are facilitators of the learning process. Eventually adults should be able to complete courses in vocational education and to better manage mathematical situations in their own lives.

The FNE program as described in this chapter offers a rich entity of mathematical activities, grounded in theoretical fundamentals and focused on the development of numerate behavior for the present and for the future. In this way Functional Numeracy Education offers *a Gateway to Numeracy*.

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[1] After the six-week training on percents in the learning program the results show an increase to a 53% correct score on the same percent tasks. This may show that the benchmark approach, described in chapter 6, could be effective for learning about percent. This way of learning could also be helpful for learning about fractions and decimals and to gain insights into their mutual relationship.

[2] The results of the field study in the present research may indicate that adults learn best when they can *do* measurement in recognizable situations. It may mean, for example, that they learn best how to compute the area of a room by working with carpet tiles in an actual room. An instruction book can be only a manual with suggestions how to do it in an actual situation. A map of the actual room can be a working model to move from informal to more formal math. The way in which adults do their actual computations should offer room for their own idiosyncratic computations.

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[3] Contexts in mathematics and numeracy education in general and in RME in particular have been subject of lively discussions. Freudenthal's awareness of the importance of real life as the source and as the field of application of mathematics led to the development of realistic mathematics education (RME) in the Netherlands in the eighties and nineties. His ideas got shape in learning through contexts in primary school. For him the "nature and richness of contexts" was the basis of learning mathematics (Treffers, 1991). In these years the discussion about the meaning of "realistic mathematics" started and may cause some confusion sometimes. The original label "realistic" refers to "*a foundation of mathematical knowledge in situations that are experientially real to the learners.*" This foundation can be in real-life situations but can also be in mathematics itself. Gravemeijer (1997, p.31) states that "*context problems in RME do not necessarily have to deal with authentic everyday-life situations.*" And he continues: "*The context in which a problem is situated can be in the mathematics itself but should be experientially real to learners in that way that they can act intelligently within this context. The goal is that eventually mathematics itself can constitute experientially real contexts for the learners. Still, real-life problems are an important feature of realistic mathematics education, but that does not only hold for RME. Any reformed mathematics instruction should incorporate real-life problems, for two reasons: (1) since mathematics is rooted in real-life problems, and (2) for reasons of mathematical literacy.*"

[4] In his study on the role of task and context variables at solving elementary arithmetical word problems Verschaffel (1997, 2000) concludes that "*The results of all these studies yield further evidence that by the end of elementary school many pupils have developed a tendency to approach school arithmetic problems in a superficial and mindless way by choosing the correct operation with numbers given the problem in the statement, without any (critical) consideration of the meaningfulness or appropriateness of their proposed solution in the relation to the realities of the problem context.*" (Verschaffel, 1997, p.126)

[5] Though learning-in-action offers the best possibilities for learning-for-doing we consider learning in an school-setting as a good alternative if learning is based on learning-through-contexts and if it offers possibilities for reflection on the learning process itself. A limitation of learning-in-action is that it may yield situation-bound knowledge and skills, which, consequently, may result in patchy knowledge.

[6] The underlying classification scheme that was necessary to determine the theoretical item difficulty level for these items appeared to be a very good predictor for item difficulty. There was a correlation of -0.8 with empirical difficulty of items found in the ALL feasibility study in 2000. The study of these item difficulty factors will be examined further in a new study in coming years in parallel to the ALL study itself, in order to find grounded criteria for item difficulty.

8 Conclusions and Discussion

In this chapter we look back to the research questions posed in chapter 1. The overall question was: *What content should be offered in a numeracy program for learners in Adult Basic Education and how should such a program be organized?* This question was elaborated along the lines of six research questions. The topics discussed under each question are briefly summarized and reviewed in this section to start the discussion and to identify issues for further research. At the end a few general points are posed for discussion.

Question 1: What is numeracy?

The literature study in chapter 2 made clear that numeracy is a dynamic concept that changed over the years and may also change in the future. The concept of numeracy includes an increasingly broader content. Where it originally covered *being familiar with numbers and doing operations with numbers in real life situations,* as part of mathematics, it now includes mathematics and focuses on *managing mathematical situations in real life*. The mathematical content of numeracy changed over the years, due to technological developments and new societal demands. Hence a working definition of numeracy must include the ability to adjust existing knowledge and skills flexibly to new future demands. Therefore, the working definition of numeracy in this study is a synthesis of what was found in the literature and focuses on future demands:
Numeracy encompasses the knowledge and skills required to effectively manage mathematical demands in personal, societal and work situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology.

Within this definition we distinguish three levels of numeracy:
- *Elementary numeracy* encompasses minimal knowledge and skills necessary for managing personal everyday life situations and for functioning in simple work situations with perhaps a little help from others.
- *Functional numeracy* encompasses a broader set of functional mathematical knowledge and skills, including elementary numeracy skills, enabling people to manage their personal and their families' societal life and work situations effectively and autonomously and to be a critical citizen.
- *Optimal numeracy* encompasses all extra mathematical knowledge and skills, in addition to people's functional numeracy skills, for acting in broader societal and/or political communities, higher professions, and leisure activities.

To make this definition operational for adult education four components are distinguished: mathematical knowledge and skills, management skills, skills for processing new information and learning skills. These skills form a unity but can be distinguished to be able to create learning situations in which all four components can be trained (see chapter 7).

Comparative studies of numeracy, for example YALS, NALS and IALS, showed that numeracy is of international concern in western societies. In IALS, on average, about 35% of the population in 12 participating countries achieved only at levels 1 and 2 (see appendix 1). This means that, assuming that level 3 should be the minimum to effectively participate in personal, societal and work situations, about one-third of the citizens in the participating western countries cannot fulfill this expectation. Contrary to common belief, technological tools like calculators and computers cannot help in reducing this problem. Numerate behavior is shown when people are able to think and communicate mathematically, to draw conclusions from mathematical reasoning and to manage mathematical situations. In many situations this cannot be done with a calculator. People need to know what to do in mathematical situations, how to take hold of mathematical problems, when and how to do computations, if at all, and in which situations a calculator can be useful. For example, to compute 10% discount, they need to know what to compute and how to do it with a calculator before they can start using the calculator effectively. Insight into mathematical concepts and computations cannot be replaced with a calculator. That is where most semi-numerate people fail.

Despite the developments in the field of numeracy itself, the concept of numeracy is hardly known in education across countries, in the Netherlands neither. In many countries a word for "numeracy" does not even exist in the local language. In the Netherlands the word numeracy ["gecijferdheid" in Dutch] is only used in academic discussions. In the Dutch adult and vocational education the word "mathematics" is used, which may recall memories of (negative) school experiences in earlier days. In Dutch adult and vocational education it would at least be better to use the words "functional mathematics" to indicate a difference with school mathematics. In some other countries, such as the USA, the UK and Australia, the word numeracy is more commonly used in adult education, often interchangeably with the concept of "quantitative literacy" and "mathematical literacy". In those countries numeracy courses attained an official place in curricula for adult education and indicate a difference with school math programs.

In general, however, the importance of numeracy for adults in contemporary society is still underestimated, especially in adult education. For instance, the fact that the number of dropouts in Dutch vocational courses is still about 50% is often ascribed to the lack of literacy and problem solving skills of learners, but the possible link with a lack of numeracy skills has not yet been made.

Question 2: What mathematical content should be offered to adults in ABE to be equipped for the future?

The answer to this question was elaborated in the chapters 2, 5 and 7 and is mainly based on Steen's broad ideas about quantitative literacy for the contemporary and future society (Steen, 1990). However, practical constraints force us to make choices and to restrict the mathematical content of a numeracy program in adult basic education to a minimum.

In chapter 2 it was discussed that numeracy skills may differ per person and depend on the individual's capacities, needs and wishes. The actual mathematical knowledge and skills people have acquired, as part of numeracy, are often determined by personal, societal and work-related demands. However, there should be a common set of basic skills that could serve as the minimum level from which adults can build on independently, depending on their individual goals, needs and wishes. In chapter 2 a distinction was made among elementary, functional and optimal numeracy. In general, the elementary level provides such a basis. To make this distinction operational for adult education it is necessary to determine what mathematical content would be the basis (the elementary level of numeracy) to build on in further education and vocational courses (functional numeracy). In some countries, for instance in Australia, the UK and the Netherlands, a start was made by the development of a coherent system of adult and vocational education. In the Netherlands this system is based on the KSE and KSB structure. The KSE level 3 would provide such a basis (see chapters 1 and 2). Adult Basic Education provides education on the KSE levels 1 and 2.

In chapters 2 and 7 the mathematical content for the elementary level was discussed. A precondition for composing a mathematical program is that the content consists of an integrated set of useful mathematical activities that focuses on functional mathematical knowledge and skills (learning-for-doing). Dossey (1997, p.46-47) describes such knowledge and skills as the "walking around quantitative skills we would expect of almost any citizen". For such "walking around skills" adults need to have acquired a coherent set of functional mathematics that can serve as a tool in their everyday life situations and as a basis for further learning. The basic idea for such a set was found in Steen's (1990) elaboration of quantity and number, dimension and shape, pattern and relationship, data and chance, and change. Steen describes the future world within these five broad fields of mathematics. His fundamental ideas set a broad range of mathematical concepts and activities for numeracy for the present and the future. world. He considers these fields as integrated fields but they may be distinguished because of educational purposes. His basic idea is not to separate them in sub-topics but to integrate sub-topics into these overarching ideas.
To make his five ideas operational for educational purposes and to develop

numeracy programs these ideas will have to be explicated in content and skills recognizable for teachers and learners. In a numeracy program for adult basic education we must restrict the mathematical content to a minimum because of time-constraints. This means that it at least should include familiarity with numbers and doing simple operations with addition, subtraction, multiplication and division, insight into simple ratio problems and proportion, insight into benchmark fractions, decimals and percents and doing simple operations with them, using measurement units, elementary notions of shape and dimension, and reading and understanding simple data.

In chapter 7 it was discussed that learning through context-based tasks may help adults to make their mathematical knowledge and skills functional and recognizable in their real life situations. The use of a calculator and computer is emphasized in addition to mental math and to doing functional computations on paper. Such tools may help learners to compensate for lack of algorithmic procedures, provided that their computations are based on insight.

In chapter 5 we argued that numeracy education for adults in adult basic education might want to focus on “math-as-a-tool” rather than on “math-for-knowledge”. Only functional mathematics can lead to functional numeracy. Numeracy courses should offer functional mathematics that is carefully structured, evaluated and adjusted to new developments at regular times. It should not only focus on mathematics that has already been developed but should also help to create an open mind for technological developments in the near future, which may include new mathematical demands. This is necessary to be equipped for the future.

Question 3: How do we assess the learners’ mathematical knowledge and skills when they enter ABE?

In chapter 3 we described the literature study concerning this question in order to set goals and criteria for assessment in adult basic education. Starting points were found in studies of assessment in school mathematics, particularly in De Lange’s five criteria for good assessment of school mathematics (De Lange, 1987), the NCTM standards (1985), in Romberg’s and Lajoie’s study of authentic assessment (Romberg, 1995, Lajoie, 1995), Van den Heuvels’ study of assessment in Realistic Mathematics Education (Van den Heuvel, 1996) and in Van Eerde’s study of diagnostic assessment (Van Eerde, 1996). For adult education, Cumming and Gal (2000) make a distinction among assessment goals for the learner, the curriculum and for program evaluation.

In chapter 3 it was noted that the purpose of assessment in ABE is threefold:
- keep track on the learners' progress
- evaluation of the program concerning goals, content and planning
- evaluation of the program concerning reporting and accountability for funds

Based on the studies mentioned above six goals could be set for assessment in adult basic education:
1- Learn more about the mathematical knowledge and skills adults have acquired when they enter ABE in order to determine what needs to be learned and to place them in the right courses.
2- Learn more about adults' problem solving procedures in order to improve instruction.
3- Learn more about adults' numeracy skills to be able to fine-tune educational programs to their needs and wishes.
4- Monitor and document the individual learner's progress in the course of the numeracy program in order to guide the learners through their own learning routes and to prevent learning problems and drop-out.
5- Be able to evaluate numeracy programs in order to improve numeracy education in general.
6- Enable policymakers and program developers to adjust policy and numeracy programs to new demands and developments in the labor market and in personal and societal life.

Consequently, criteria for assessment tools could be derived:
1- Numeracy assessment in ABE should be done in an appropriate way for learners in ABE.
2- Assessment should enable adults to show what knowledge, skills and procedures they have mastered when they enter ABE, rather than what they do not have.
3- Numeracy assessment in ABE should provide insight into mathematical procedures and problem solving.
4- Placement tests in ABE should reflect the goals, content and levels of the math curriculum so that adults know what they can expect during the course and can be placed in right course.
5- Numeracy assessment in ABE should allow second-language learners to apply the mathematical procedures and algorithms that they learned in their home countries.
6- Text used in a paper-and-pencil math test should not hamper second-language learners to take a mathematics test.

As argued in chapter 3, placement assessment for learners in ABE should be done in the way that is most appropriate for every adult in ABE and that yields qualitative information. Merely correct-incorrect scores and a proficiency score for level placement are not sufficient. Good assessment examines the learners' competencies. A good placement test offers a broad set of tasks on different levels that can identify the learners' mathematical knowledge and procedural abilities to prevent misplacement and dropout and to acquire information for learning and instruction. It enables learners to show their mathematical thinking and reasoning, procedures, computation and notation systems in a constructive way. The test can be organized in an adaptive way so that testing can be done as efficiently as possible and so that people will not have to do the entire test. An oral interview or a paper-and-pencil test, if carefully composed, often yields the best information. For learners in literacy courses oral interviews must be considered. Learners working on higher levels can take a paper-and-pencil test. The argument that such tests take too much time for teachers is offset by the qualitative information that can be acquired keeping in mind that the information that has not been acquired at the start of a numeracy course must necessarily be acquired later on during the course. Computer adaptive tests can be useful for quick scans in specific situations and for formative and summative assessment. More research is necessary to examine how to organize assessment so that it can be as efficient as possible yet result in the best qualitative information. The qualitative information acquired from the placement assessment is necessary to fine-tune numeracy courses to the actual wishes and needs of the learners and to provide information about learning and teaching for ABE teachers.

In general, a well-organized system of placement, formative and summative assessment is necessary to document the learners' progress but also for evaluation of numeracy courses and for accountability to stakeholders.

Question 4: What mathematical knowledge and skills have adults acquired when they enter ABE?

In chapter 4 we examined the mathematical knowledge and skills of learners in ABE at the start of a numeracy course. To acquire information, a group of 32 second language learners was tested in an ABE learning center near Utrecht in the Netherlands.

For this field study the In Balance placement test (IB) and the Cito placement test (Cito) were used. The Cito test is a standardized test, validated on a sample of about 1200 respondents in ABE in 1993 (Straetmans, 1994). The Cito test was used to be able to compare the IB quantitative results with an independent test. The IB test provided a lot of valuable qualitative information about the learners. The group of N=32 was comprised of 11 men and 21 women.

According to the Cito test results, about 75% of the learners achieved at the beginning of KSE level 2 and about 25% achieved at KSE level 1. Similar results were achieved on the IB test. Based on the numeracy level classification in chapter 2, it means that these learners mainly function on the elementary level of numeracy.

In general it can be said that these learners had acquired only very limited mathematical skills. The average score on all five main fields in the In Balance test (number and basic operations, proportions, measurement and dimension, money and data) was 56% correct. On the Cito test the average was even 50% correct. The highest average score on the IB subtests was on basic operations, 70% correct. The lowest average score was on proportions, 45% correct. The lowest average score of all subtests was on the percent items, within the field of proportion, only 15% correct.

The qualitative analyses showed that most of the learners (about 80%) can solve simple counting, addition and multiplication problems. In general the learners prefer doing algorithms to mental calculation, though the performance on the actual algorithm tasks are low (45% correct). For more complex context tasks concerning basic operations the percentage of correct answers decreases to about 40%. Estimation and mental computation are the least favorite. Learners tend to do precise computations. This was in particular striking with money tasks. Though most money tasks were designed so as to encourage estimation as in real life situations, most learners tend to solve such tasks by doing precise computations through algorithms. The results for money tasks were about 50% correct. This is worrying because it indicates that these adults have only acquired minimal skills to handle money tasks in real life situations, or cannot transfer real life related solutions to tasks in school situations.

In the field of proportions, fractions and percent the learners show a 75% correct score on simple proportion tasks related to recognizable real life issues, e.g. comparing the length of a car with the length of a bus. The percentage correct decreased enormously at simple tasks pertaining to benchmark fractions (30% correct score) and percent (only 15% correct score).

For measurement, including decimals involved in the metric system, the results on the placement test show that about 80% of the learners can do simple tasks related to applying measurement in real life situations. When comparing measures within the metric system the percentage correct decreases to just over 50%. When doing formal computations with given measures on paper, this percentage decreases further to 20%. Only four learners are able to compute the area of a floor with the help of a map of the floor.

For reading and understanding data in graphs we come to similar conclusions. Learners are able to read simple line and bar graphs related to everyday life issues, like reading a temperature graph (about a 75% correct score). Indicating a quarter in a pie chart leads to a 50% correct score and understanding a more complex bar graphic about average sales of cars in one year yields only about 10% correct answers.

Given these results, we may wonder if these learners are able to benefit from their skills in everyday life. It may also be questioned how functional and how flexibly applicable their skills are with respect to the necessary skills that would enable them to solve mathematical problems in work and in vocational courses. Most learners showed that the skills they had acquired were limited to only using traditional computation procedures, i.e. algorithms, often completed without any insight. Such procedures were in particular striking with money computations and when doing estimation. Rounding amounts in money computations after having computed them precisely was seen as doing estimation. The percentage correct for rounding and estimation items was 20% on average. Only a few learners showed insight into their own creative, more informal, solution procedures, mainly in proportion and measurement tasks. These results are understandable if we consider that many learners only received very traditional mathematics education in their home countries.

The above supports the observation that second language learners need time, not only to learn a second language but also to get used to a different system of learning and teaching in adult education in a new country. In the Netherlands, particularly, they also need to adjust to new ideas concerning the learning of mathematics. In adult basic education it means that they have to overcome three main problems in a short period of time when preparing for vocational education and for work. These problems should be considered when numeracy programs are developed, but should not hinder the learners from taking a numeracy course. The observation may even emphasize the importance of numeracy courses alongside the literacy courses at the basic level. Contrary to the common assumption in vocational education, it takes more than just mastery of the second language to succeed in vocational courses.
If numeracy courses are organized in the way as described in chapter 5, based on the FNE instructional model, they can help learners to acquire skills necessary for learning in vocational courses.

Though the learners in this study are from only one common group in ABE they may be seen as representatives of the learners in ABE in general in the Netherlands (KSE levels 1 and 2) because of the placement criteria and because of their results on the Cito test. Since there are about 50,000 learners in ABE and at least 1,5 million people at this level in general in the Netherlands (given the results of the IALS study), this group of adults should be of more political concern.

Question 5: What way of learning and teaching mathematics could be appropriate for adults in ABE?

In chapter 5 we studied ways of learning that could be appropriate for learners in ABE. Answers to this question are based on literature about learning in general, the learning of adults specifically and on studies about mathematics education, in particular Realistic Mathematics Education. These theories were connected with teaching experiences in adult basic education concerning the learning of adults.

Andragogical starting points were formulated for adult education in general:
1- Adults are free to learn.
2- Learners and teachers are equal partners in learning situations.
3- Adults' own experiences are the basis for learning.
4- Authentic materials should be used as instruction materials in school learning situations.
5- Learning takes place by interaction and reflection.
6- Learning in adult education aims to lead to functional knowledge and skills.
7- Adults direct their own learning

Based on these starting points it can be concluded that learning mathematics in ABE must focus on math-as-a-tool and hence on learning-for-doing and learning-by-doing, i.e. learning-in-action. However, mathematics education is too often still a theoretical event that happens in classrooms and consists of learning from books. Greeno (1999) argued - and this can be confirmed by several other studies about learning at work and in other out-of-school situations - that informal learning in the course of action yields the best functional knowledge and skills. This indicates that mathematics education in ABE should focus more on real learning-by-doing with authentic materials. However, it should not only consist of learning-by-doing and learning-in-action because of the risk that such functional knowledge and skills may also yield context-bound and partial knowledge and skills. Learning in ABE must focus on context-based knowledge and skills, which means that it is better flexibly applicable in different situations.

Ways of learning mathematics were elaborated along the starting points of Action Theory, Constructivism and Realistic Mathematics Education (RME). Though these theories originally evolved as theories for the cognitive development of children, adult education can benefit from their results. From an adult perspective, they offer many possibilities to look at including to examine what adults could have missed in their youth concerning the development of mathematical concepts. Based on this perspective, strategies can be outlined for learning mathematics in adult education.

Action Theory and RME provided four action levels for learning mathematics in ABE:

- doing informal manipulations in actual real life situations,
- using representations of real objects and situations
- using schematic representations
- using symbols and formal/functional operations

Adults have often acquired a mix of mathematical knowledge and skills. To adjust, correct and expand their knowledge and skills they will need to go through all four levels, often at the same time. Hence, the action levels cannot be seen as real different *levels*, but more as *intertwined fields* where people go through from different angles and in different directions. However, they need to go through all fields to build complete concepts.

Constructivism and RME offer a frame of reference for the analysis of the learning process itself. In ABE problem based learning has been advised as a main way of learning. The six steps in problem solving offer possibilities for learners to learn manage mathematical situations in real life but they also enable them to analyze their own ways of learning. Within this process three elements can be distinguished:

1- Through problem solving learners become aware of *learning process* itself *(what)*
2- During problem solving learners may want to emphasize the *quality* of their learning *(why)*
3- When doing problem solving learners learn how to *organize* the learning process *(how)*

Through the *what-why-how* process learners learn to direct their own learning processes themselves and this is what they need for independent learning in real life situations.

Problem-based learning also offers possibilities to create learning situations in which mathematical concepts and actions are integrated. Mathematics in real life situations never occurs as separate mathematical tasks. Situations are often complex and the mathematics in it is hidden. Contexts in mathematics education should be real or derived from real life situations. They must offer possibilities to solve mathematical problems in multiple (informal) ways. Basic operations, for example, should be learned in a connected way. People need to know when they can use addition, subtraction, multiplication or division and what to do if they cannot do subtraction, for instance. Contexts are the main ingredients for learning mathematics in adult education. They must be structured carefully, using the complexity factors, as discussed in chapter 7, as the main factors and the four action levels as the second ones. In addition, context-free tasks can help familiarize learners with characteristics of numbers.
Knowing such characteristics is necessary for estimation and mental math.

When doing computations and mathematical operations adults are free to create their own ways of solving problems and to develop their own individual, functional notation systems. Such functional procedures and notations are almost always based on insight and support their individual thinking in specific situations. Hence they are often better usable than standard procedures like traditional algorithms.

The theoretical foundation, as developed in this study, is presented in an instructional model for *Functional Numeracy Education* (FNE).

Question 6: How to develop a program for functional numeracy education in Adult Basic Education that prepares adults to be equipped for the future?

The answer to this question is based on the answers to the previous questions and on experiences with the *In Balance* learning materials. Some of our experiences were showed in the exemplary teaching experiment in chapter 6.

In chapter 7 it was argued that numeracy courses should be built on the four components of numeracy. It is advised to build the first component - mathematical knowledge and skills - on integrated strands, benchmark learning, integrating visual models and action levels. The other three components - learning to manage a mathematical situation, acquiring new information in real life situations, and developing learning skills - can be addressed by problem solving.
In chapter 5 we discussed that Action theory and RME lay the foundation for learning functional mathematics. Constructivism and RME offer ingredients for the analysis of the learning process itself. Such analysis can be done by applying the *what-why-how* questions. Thus the focus within learning mathematics may switch from analyzing the learning process itself to focusing on how to manage a problem and on the quality of the problem solving procedure. In this way adults learn how to manage mathematical problems in real life situations.

Benchmark learning is the core for creating a concise but complete program in ABE. Learners are able to go through all important subjects in a short time and to develop basic computation skills based on benchmarks. Benchmarks may help adults to find the core relations between different mathematical concepts like proportion, fractions, decimals and percent and within the metric system. They may also help develop touchstones or points of reference for their computations and for mental math and estimation. The use of a calculator is strongly advised, especially at the lower levels, to compensate for lack of procedural skills. The benchmark approach is an effective way of learning in

ABE, because it offers a complete program on all levels and can be extended into more detailed procedures and computations at the higher levels.
The benchmark theory as elaborated in this study requires further research to extend and further develop these basic ideas.

Integrating visual models are the links between mathematical concepts and across strands. They, in combination with manipulations, help learners visualize their thoughts in real life situations. Manipulation and visualization in combination with communication enable them to develop complete concepts. It also enables second language learners to overcome language problems when learning mathematics. Though from a constructive point of view we strongly encourage teachers to have their learners develop their own visual models and to use models they already have developed in their own idiosyncratic way, a few core visual models, like a proportion bar, the block model as a multiple proportion bar, a number line and a pie chart, should be offered to help them getting used to this approach. Blank models of these can be used as computation models. Based on such core models learners can expand their own individual mental models. Further research into the basic ideas of integrating visual models is strongly advised. Specific research settings should be created to study the function of integrating visual models across strands and the benefits they could have for second-language learners.

Problem solving is recommended as an approach to help learners develop skills for managing mathematical situations and for cooperative and teacher-free learning. The six steps of problem solving and the *what-why-how* procedure, as discussed in chapter 5, can help to develop skills that are necessary for independent learning in real life situations and are the core of lifelong learning. More research is necessary to develop such programs in practice.

Time constraints are the main problem in adult education, and hence also in adult basic education. Many numeracy courses in adult education have a very short duration, often limited to about 100 hours, compared to about 600 hours for learning a second language and becoming familiar with the Dutch society. While this may not be a problem for learners on higher levels, but for learners in ABE the time for numeracy courses is often too short. They need a full basic program to acquire a set of adequate skills necessary for further education and work. The learners in this study are all on a very low mathematical level. We may question whether such short courses will enable these learners to move on to courses in further and vocational education. Besides, in most education programs numeracy (or mathematics) courses are not an obligatory part of the program. As long as this situation continues, under-schooled newcomers will have problems in vocational courses and in work. Mastery of the second language plus sufficient numeracy knowledge and skills, in combination with being familiar with the new society people live in, are preconditions for successful learning in vocational education.

In general:

In this study we have explored the field of learning mathematics by learners in adult basic education within the larger frame of developing numerate behavior. The study provides a lot of qualitative information about the mathematical knowledge and skills of adults in ABE.
Based on existing theories about the learning of adults and about learning mathematics we have tried to create a theoretical foundation for *Functional Numeracy Education* to adults that focuses on learning functional mathematics, applicable in real-life situations and managing mathematical real-life situations. As part of the learning process learners learn how to direct their own learning process to be able to acquire mathematical information in real-life situations.

Though a start has been made and a lot of information was acquired about numeracy in ABE, this study requires an immediate follow-up because we need more information about adults' actual numeracy skills and how they learn mathematics. A few selected topics are:

1- The proposed theory and program for *Functional Numeracy Education*, as elaborated in this study, are based on many years of teaching experience, studying theories and developing materials. The results of this study indicate that an FNE program based on these starting points can be very effective, but evaluation of such a program over a longer time frame is necessary.
Within this study two topics can be emphasized: benchmark learning and functional procedures.

2- In order to measure the effectiveness of the FNE program, research is necessary into how (under-schooled) adults actually manage mathematical situations in real life. Such could be set up based on topics in the FNE program in an ABE learning setting in combination with how similar topics are carried out in real-life situations, among which work situations.

3- Lack of numeracy skills of learners in the Dutch ABE (KSE-1 and KSE-2) who want to move on to further education and to vocational courses, may cause that these learners do not succeed in completing these follow-up courses. Evaluation of numeracy courses in ABE is necessary to measure the gap between the numeracy skills learners have acquired when they have completed KSE-2 programs with the required numeracy skills for programs in KSE and KSB on the basic level (KSE-3 and KSB-1).

4- Results of international comparative studies, like the IALS and the coming ALL survey, should be analyzed on possible consequences for adult education. For ABE it is desirable to compare numeracy programs in an international research setting.

The learners in this study represent only one common learners' group in ABE in the Netherlands (KSE levels 1 and 2). Despite this, their results on the placement tests and the learning experiment in chapters 4 and 6 may indicate on a wider level that numeracy in ABE needs careful attention. These adults are on such a low level of numeracy that they will have little chance of success in vocational courses and in work. In addition we must be concerned about the 36% of the entire Dutch adult population, native and foreign born, who achieved only level 1 or 2, out of five levels, on the quantitative literacy scale of the IALS survey. About 1.5 million people achieved only level 1. This means that innumeracy, and illiteracy as well, are still of great concern. Our current technological society demands a high level of numeracy skills. It continuously develops further and the gap between the top of a numerate society and the lower end of literacy and numeracy is increasing.

In addition to the above research topics we recommend that:

1- Adult learning centers work more actively on the acquisition of potential learners and on offering low-threshold programs and courses in which they can participate.

2- Teachers in ABE have acquired professionals skills to support and guide learners in their learning of functional mathematical knowledge and skills, e.g. based on the FNE theory.

3- Learning materials have been developed to effectively help these learners.

4- Literacy *and* Numeracy are topics on international policy agendas.

Concluding:

In the introduction this study was characterized as exploratory, descriptive, qualitatively analyzing, theory building and developing. We have tried to give the learners in adult basic education a face and to describe their competencies and their needs. We have also tried to show the importance of numeracy and numeracy courses for these learners, in particular in the frame of lifelong learning. This study shows that numeracy in ABE needs professional attention. We hope this study encourages program developers and teachers in ABE to develop effective numeracy programs that lead to functional numeracy. We also hope that this study encourages other researchers to take the thread and to further develop the theoretical ideas set out in this study.

In his book "Mathematics and Democracy, the Case for Quantitative Literacy" Steen (2001, page 108) argues in the epilogue,

> *"Counting people, counting dollars, and counting votes are part of the numeracy of life. Unlike the higher mathematics that is required to design bridges or create cell phones, counting appears to require only rudimentary arithmetic. To be sure, when large numbers, multiple components, and interacting factors are involved, the planning required to ensure accurate counts does become relatively sophisticated. So even though the underlying quantitative concepts are typically rather elementary - primarily topics such as multiplication, percentages, and ratios - the mental effort required to comprehend and solve realistic counting problems is far from simple."*

He ends the book with the words:

> *"Numeracy is not the same as mathematics, nor is it an alternative to mathematics. Rather it is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society. Whereas mathematics is a well-established discipline, numeracy is necessarily interdisciplinary. Like writing, numeracy must permeate the curriculum. When it does, also like writing, it will enhance students' understanding of all subjects and their capacity to lead informed lives."* (Steen, 2001, page 115).

With this study we have tried to pave a way to help adults in ABE learn to cope with numeracy demands in real life.

Samenvatting

Achtergrond

Deze studie vindt zijn oorsprong in de ontwikkeling van het reken-wiskunde-onderwijs voor volwassenen in de basiseducatie waar ik vanaf 1981 bij betrokken ben en in mijn interesse in gecijferdheid in het kader van lifelong learning. Jarenlang werken met laagopgeleide volwassenen, het ontwikkelen van materialen voor rekenen-wiskunde voor deze volwassenen, observaties in lesgroepen, vele diagnostische interviews met Nederlandstalige en anderstalige volwassenen, het lesgeven aan docenten in basiseducatie, en mijn betrokkenheid bij internationaal onderzoek naar gecijferdheid van volwassenen vormen de voedingsbodem van deze studie.
De werkelijke start van dit onderzoek werd gemaakt in december 1997 toen een groep van zevenendertig nieuwkomers startte met een cursus rekenen in het ROC Utrecht, afdeling Nieuwegein. Dit gebeurde in een tijd dat de basiseducatie in Nederland volop in ontwikkeling was en in een nieuwe structuur voor volwasseneneducatie opgenomen werd: de Kwalificatie Structuur Educatie (KSE). Dit betekende het einde van de basiseducatie als zelfstandige onderwijssoort en de start van funderende educatie als onderdeel van de nieuwe onderwijsstructuur. Dit had ook consequenties voor het rekenwiskunde-onderwijs aan laagopgeleide volwassenen.

Volwassenen in funderende educatie, de oorspronkelijke basiseducatie, vormen maar een kleine doelgroep in de volwasseneneducatie in Nederland. Daarbinnen bestaat deze groep voor het merendeel uit anderstaligen, voor een klein deel uit Nederlanders. Het onderwijs voor deze doelgroep focust daarom met name op het leren van Nederlands als tweede taal en de inburgering in de Nederlandse maatschappij om zo snel mogelijk te kunnen doorstromen naar beroepsonderwijs en de arbeidsmarkt. Dat daarvoor ook rekenvaardigheden noodzakelijk zijn staat veel minder in de belangstelling. Rekenen-wiskunde aan laagopgeleide en analfabete volwassenen heeft zich ontwikkeld in de schaduw van het tweedetaal onderwijs. De laatste jaren echter groeit de belangstelling voor gecijferdheid. Dit komt onder andere door de resultaten van twee nationale onderzoeken in de Verenigde Staten naar basisvaardigheden van volwassenen, de Young Adult Literacy Survey in 1986 (YALS) en de National Adult Literacy Survey in 1992 (NALS), en het daaropvolgende internationale onderzoek International Adult Literacy Survey (IALS) in 1996 (Van der Kamp en Scheeren, 1996, Dossey, 1997, NCES, 1998, Houtkoop, 1999). Bij deze onderzoeken werden resultaten verkregen en vergeleken op drie schalen: prozageletterdheid, documentgeletterdheid en kwantitatieve geletterdheid. Het IALS onderzoek werd gehouden in twaalf landen, onder andere in Nederland. De resultaten waren schokkend. In het YALS onderzoek, bijvoorbeeld, kon slechts 40% van de jongeren (21-25 jaar) uitrekenen hoeveel wisselgeld ze terug moesten krijgen na ontvangst van de rekening in een restaurant, het berekenen van de fooi (10%) en het aanbieden van een bepaald bedrag. In alle drie de onderzoeken scoorde een derde tot de helft van alle volwassenen laag tot zeer laag op de schaal van kwantitatieve geletterdheid (Van der Kamp en Scheeren, 1996, Dossey, 1997, Shillington, 1998, Houtkoop 1999). Dit betekent dat zij grote moeite kunnen hebben met het interpreteren van en betekenis geven aan kwantitatieve informatie en daardoor belemmerd kunnen worden in hun functioneren in de maatschappij. De resultaten van deze onderzoeken leidden tot het inzicht dat volwassenen naast geletterdheid ook moeten beschikken over een zekere mate van gecijferdheid, hoewel men door de technologische ontwikkelingen het tegendeel zou verwachten. Er worden immers steeds meer alternatieven ontworpen om ons van vele rekentaken te verlossen. De vraag echter naar invulling van het begrip gecijferdheid en de behoefte aan ontwikkeling van programma's voor gecijferdheid worden steeds groter.

Setting

Centraal in deze studie staat een groep laagopgeleide anderstalige volwassenen in het ROC Utrecht, afdeling Nieuwegein. Na ongeveer vier maanden intensieve taaltraining werd deze groep in december 1997 een cursus rekenen-wiskunde aangeboden om beter voorbereid te kunnen worden op een beroepsopleiding. De groep zou eerst worden getoetst. In overleg met het ROC werd besloten bij deze groep twee toetsen af te nemen, de *Cito* rekentoets voor volwassenen en de experimentele *In Balans* instaptoets, allebei gepubliceerd in 1996. Op basis hiervan kon kwantitatieve en kwalitatieve informatie verkregen worden over hun rekenvaardigheden. Aansluitend werd hen een programma geboden over procenten. Binnen dit programma werd bestudeerd wat deze volwassenen al aan (informele) voorkennis hadden verworven en op welke wijze zij de aangeboden leerstof het beste verwerken. De bijeenkomsten waren eenmaal per week. Het programma duurde zes weken (zes lessen van een uur) en werd afgesloten met een toets over procenten. Deze natoets was samengesteld uit de opgaven van de *Cito* en *In Balans* instaptoetsen. Hierdoor konden de resultaten van de instaptoets en de natoets worden vergeleken. De resultaten van de instaptoetsen en de leerervaringen van deze cursisten tijdens de lessen vormen de ingrediënten van deze studie. Zij schetsen een beeld van de actuele doelgroep in funderende educatie en helpen bij de ontwikkeling van theorie over het leren van rekenen-wiskunde door laagopgeleide volwassenen en bij de ontwikkeling van een programma voor deze doelgroep. Dit alles wordt geplaatst in het kader van functionele gecijferdheid en lifelong learning.

Hoofdthema

Rekenwiskunde-onderwijs aan volwassenen in basiseducatie/funderende educatie heeft nog maar een korte geschiedenis en is voornamelijk gebaseerd op ervaringen met het reken-wiskunde-onderwijs aan kinderen en jeugdigen. In

Engelstalige landen wordt in het volwassenenonderwijs gesproken over "numeracy courses", in plaats van "mathematics". Daarmee wordt in internationale discussies ook echt een verschil aangegeven tussen rekenwiskunde-onderwijs voor volwassenen en voor leerlingen in het basis- en voortgezet onderwijs. Wat dit verschil in de praktijk inhoudt is echter vaak niet duidelijk. Heel vaak bestaat een numeracy course nog uit het uitvoeren van traditionele rekenwiskunde activiteiten maar dan in een ander jasje. In Nederland wordt het woord "gecijferdheid" echter alleen gebruikt in wetenschappelijke discussies. Het wordt niet of nauwelijks gebruikt in de praktijk van het onderwijs aan volwassenen. Dit komt waarschijnlijk doordat in Nederland het rekenwiskunde-onderwijs aan volwassenen gebaseerd is op realistisch rekenen-wiskunde en daardoor een andere start heeft gemaakt dan cursussen voor volwassenen in het buitenland. Echter, ook in Nederland geldt, net als in het buitenland, dat de kwaliteit van het rekenwiskunde-onderwijs aan volwassenen in funderende educatie vaak afhangt van de bekwaamheid en ervaringen van individuele docenten. Internationaal is er nauwelijks een wetenschappelijke basis voor *wat* er zou moeten worden aangeboden in een cursus "gecijferdheid" (rekenen-wiskunde) voor volwassenen en *hoe* het zou moeten worden aangeboden. Het bestaan van standaarden, curricula en eindtermen is nog geen garantie voor de kwaliteit van lokaal georganiseerde cursussen. Er is nog weinig bekend over de effectiviteit van rekenwiskunde-programma's voor volwassenen. Vandaar dat de centrale vraag in deze studie luidt: *Welke inhoud moet een programma voor gecijferdheid bieden aan volwassenen in funderende educatie en op welke wijze moet zo'n programma worden opgezet?*

Vier onderwerpen staan centraal in deze studie:
1- een studie over het begrip gecijferdheid en een beschrijving van enkele onderzoeken die gedaan zijn naar gecijferdheid van volwassenen;
2- het verwerven van kwantitatieve en kwalitatieve informatie over rekenvaardigheid van laagopgeleide volwassenen in funderende educatie;
3- een theoretische onderbouwing van het leren van rekenen-wiskunde door deze volwassenen en van het leerproces zelf;
4- het ontwikkelen van een programma voor functionele gecijferdheid voor laagopgeleide volwassenen in funderende educatie.

Het doel van deze studie is een kwalitatieve bijdrage te leveren aan de ontwikkeling van kennis op het gebied van functionele gecijferdheid en het leren van rekenen-wiskunde door volwassenen in funderende educatie.

Opzet en samenvatting van de hoofdstukken

De studie is als volgt opgezet:

Deel 1: Gecijferdheid in funderende educatie
1. Ontwikkelingen in de basiseducatie in Nederland
2. Gecijferdheid, een dynamisch begrip

Deel 2: Rekenwiskundige vaardigheid van volwassenen in funderende educatie
3. Het toetsen van rekenwiskundige vaardigheid in funderende educatie
4. Analyse van rekenwiskundige vaardigheden bij een groep anderstalige volwassenen

Deel 3: Het leren van rekenen-wiskunde in funderende educatie
5. Het leren van rekenen-wiskunde door volwassenen in funderende educatie
6. De theorie in praktijk.

Deel 4: Ontwikkeling van een programma voor gecijferdheid in funderende educatie
7. De ontwikkeling van een programma voor functionele gecijferdheid.

Deel 5: Conclusies en discussie
8. Conclusies en discussie

Hoofdstuk 1 beschrijft in het kort de historie van basiseducatie in Nederland, de ontwikkeling van het huidige systeem voor volwasseneneducatie met daarbinnen de ontwikkeling van het reken-wiskunde-onderwijs voor volwassenen.

Basiseducatie werd gestart in 1987 en was opgebouwd uit projecten voor volwassenen uit de jaren '60 tot '80. Met ingang van augustus 1998 is basiseducatie onderdeel geworden van de algemene volwasseneneducatie en beroepsopleidingen, zoals is vastgelegd in de huidige wet op het beroepsonderwijs en de volwasseneneducatie (BVE) en is vormgegeven in Regionale Onderwijscentra (ROC's). Basiseducatie werd vervangen door de niveaus 1 en 2 van de Kwalificatie Structuur Educatie (KSE) en heet sindsdien funderende educatie (FE). Allochtone volwassenen nemen een bijzondere plaats hierin omdat zij in een overgrote meerderheid zijn.

De veranderingen in de afgelopen jaren leidden in Nederland onder andere tot discussies over het belang en de plaats van rekenen-wiskunde in de BVE. Sommigen willen rekenen-wiskunde helemaal laten opgaan in beroepsonderwijs, anderen pleiten nog steeds voor specifieke aandacht voor rekenen-wiskunde, met name in funderende educatie.

Rekenen met laagopgeleide volwassenen startte in Nederland in 1987 als een nieuw werkveld binnen de wereld van rekenen-wiskunde, maar is inmiddels de kinderschoenen ontgroeid. Realistisch rekenen-wiskunde, zoals ontwikkeld voor basis- en voortgezet onderwijs, vormde een goede voedingsbodem voor het rekenen met volwassenen. Dit was van de ene kant een gunstige situatie - er kwamen al vrij snel goede producten op de markt - van de andere kant kan het ook een belemmering zijn geweest om te kijken naar wat er in het buitenland gebeurde en nog steeds gebeurt. Ook daar wordt gewerkt aan het ontwikkelen van programma's voor rekenen-wiskunde voor volwassenen, ook al heeft men soms nog nooit gehoord van realistisch rekenwiskunde-onderwijs. Om het verschil aan te geven met de traditionele wiskunde zoals die nog in vele landen bestaat in basis- en voortgezet onderwijs, wordt met name het woord "numeracy" gebruikt in programma's voor volwassenen. Is dat alleen maar een verschil in woordgebruik of zijn er ook andere verschillen?

Aan het einde van dit hoofdstuk worden de onderzoeksvragen gespecificeerd. Afgeleid van de centrale vraag zoals geformuleerd bij het hoofdthema kunnen de volgende twee hoofdvragen worden geformuleerd:
1- W*elke inhoud moet worden aangeboden in een programma dat leidt tot functionele gecijferdheid van volwassenen?*
2- Hoe kan een programma voor gecijferdheid het beste worden opgezet?

Deze vragen kunnen worden opgesplitst in de volgende subvragen:
1- Wat is gecijferdheid?
2- Welke rekenwiskundige kennis en vaardigheden moet in funderende educatie aangeboden worden om uitgerust te zijn voor de eisen van de toekomstige maatschappij?
3- Wat is een juiste wijze van toetsen van rekenwiskundige vaardigheden van volwassenen als zij starten in funderende educatie?
4- Welke rekenwiskundige kennis en vaardigheden hebben volwassenen verworven als zij starten in funderende educatie?
5- Wat is een juiste wijze van leren en onderwijzen van rekenen-wiskunde voor volwassenen in funderende educatie?
6- Hoe kan een verantwoord programma voor rekenen-wiskunde worden opgebouwd voor funderende educatie dat leidt tot functionele gecijferdheid?

In *hoofdstuk 2* staat het begrip *gecijferdheid* centraal.
Zowel nationaal als internationaal heeft gecijferd van volwassenen groeiende belangstelling, onder andere doordat uit het International Adult Literacy Survey (IALS, 1995 en 1997) blijkt dat bijna de helft van de inwoners in twaalf OECD landen (Organization for Economic Co-operation & Development) functioneert op laag tot zeer laag niveau van kwantitatieve geletterdheid (niveaus 1 en 2 op een schaal van 5 niveaus). In Nederland is dat ruim 35%. (Houtkoop, 1999). Met "kwantitatieve geletterdheid" wordt bedoeld *de vaardigheid om rekenkundige bewerkingen te kunnen toepassen in dagelijkse activiteiten*. De groep die op de laagste twee niveaus functioneert bestaat voor bijna 94% uit laagopgeleide volwassenen en heeft de minste kansen op de arbeidsmarkt. Bijna de helft van

alle werkzoekenden functioneert op niveau 1 en 2 van kwantitatieve geletterdheid. Het onderdeel kwantitatieve geletterdheid in IALS wordt in een vervolgonderzoek, het internationale Adult Literacy and Life skills project (ALL) vervangen door een uitgebreider onderzoek naar gecijferdheid. Dit onderzoek is in ontwikkeling en zal uitgevoerd worden in 2002. Het begrip "numeracy" wordt in het ALL-project gedefinieerd als "*The knowledge and skills required to effectively manage the mathematical demands of diverse situations."* De resultaten van dit onderzoek kunnen bijdragen aan de ontwikkelingen in educatie op het gebied van functioneel reken-wiskunde-onderwijs aan volwassenen.

Gecijferdheid van volwassenen wordt al sinds het begin van de jaren tachtig uitvoerig bestudeerd, met name in Engeland, Australië en de Verenigde Staten. Er zijn vele definities in omloop. De meest gebruikte termen zijn "*quantitative literacy", "mathematical literacy"* en *"numeracy".* In Nederland bestaat het begrip "gecijferdheid" pas sinds het begin van de jaren negentig. Het begrip gecijferdheid werd in Nederland eigenlijk vooral bekend door beschrijvingen van het tegenovergestelde: ongecijferdheid. Van der Blij (1987) sprak in een lezing tijdens de Panama Conferentie in 1986 over het verschijnsel "wiskundige ongeletterdheid". Treffers (1990) beschrijft in zijn oratie in 1990 hoe "ongecijferdheid" op de basisschool kan worden voorkomen. Daarna beschrijft Goffree (1995, p. 19) dat mensen die "functioneel gebruik maken van het rekensysteem", zich gecijferd mogen noemen. De vertaling van het boek van Alan Paulos (1988), "*Ongecijferdheid",* heeft met name gezorgd voor bredere bekendheid van het begrip in Nederland.

Gecijferdheid is een dynamisch concept. De invulling kan veranderen als de maatschappelijke omstandigheden veranderen. In deze studie wordt gecijferdheid gedefinieerd als:

"De kennis en vaardigheden die nodig zijn om adequaat te kunnen omgaan met rekenwiskundige problemen in persoonlijke en maatschappelijke situaties, in combinatie met het vermogen om deze kennis en vaardigheden flexibel te kunnen aanpassen aan nieuwe eisen in een continu veranderende maatschappij die gedomineerd wordt door kwantitatieve informatie en technologie."

In het Engels vertaald is deze definitie als volgt:
"The knowledge and skills required to effectively manage mathematical demands in personal and societal situations, in combination with the ability to accommodate and adjust this knowledge and skills flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology"

Het eerste deel van de definitie is ongeveer hetzelfde als de ALL definitie. Het tweede deel geeft aan dat numeracy een dynamisch concept is. In een snel veranderende maatschappij zijn kennis en vaardigheden continu aan veranderingen onderhevig en men moet in staat zijn individuele kennis en vaardigheden aan te passen aan nieuwe omstandigheden. Dit is de essentie van lifelong learning. Dit vraagt echter meer dan alleen rekenwiskundige kennis en vaardigheden. Daarom bestaat gecijferdheid uit vier componenten:

1- rekenwiskundige kennis en vaardigheid
2- management van rekenwiskundige situaties
3- het verwerven en verwerken van nieuwe informatie
4- leervaardigheden.

Binnen het begrip gecijferdheid kunnen we vervolgens onderscheid maken in drie niveaus: elementaire gecijferdheid, functionele gecijferdheid en optimale gecijferdheid.

Elementaire gecijferdheid is nodig om minimaal te kunnen functioneren in werk en persoonlijk leven, bijvoorbeeld zelfstandig boodschappen doen. De persoon beschikt over minimale rekenwiskundige vaardigheden in combinatie met elementaire communicatieve vaardigheden. Hulp van anderen is nodig in bepaalde situaties.

Iemand is *functioneel gecijferd* als hij beschikt over rekenwiskundige kennis en vaardigheden die nodig zijn om zelfstandig en doelgericht te handelen in persoonlijk en maatschappelijk leven, adequaat functioneert in werksituaties en kan reflecteren op het maatschappelijk gebeuren. De persoon is in staat zijn handelingen flexibel aan te passen aan nieuwe ontwikkelingen in de maatschappij.

Iemand is *optimaal gecijferd* als hij beschikt over een breed scala van wiskundige kennis en vaardigheden, meer dan nodig om adequaat te kunnen handelen in persoonlijk en maatschappelijk leven en werk en om kritisch te kunnen reflecteren op maatschappelijke ontwikkelingen. We kunnen hierbij denken aan deelname in maatschappelijk en/of politiek georiënteerde belangenverenigingen, denksporten als schaken, bridgen, enz., maar ook aan hogere beroepen waarin veel wiskundig handelen voorkomt, zoals econoom, boekhouder, statisticus, wiskundedocent, natuurkundige, en aan vormen van kunst, zoals b.v. muziek, beeldhouwen, schilderen, tekenen, b.v. tekeningen van Escher. Nieuwe vormen van kunst, zoals webdesign, vallen hier ook onder.

In *hoofdstuk 3* wordt het toetsen van volwassenen bestudeerd.

Vanaf het begin van de basiseducatie was duidelijk dat begeleiding van volwassenen bij het reken-wiskundeonderwijs specifieke vaardigheden van docenten vereist. Volwassenen komen funderende educatie binnen met veel voorkennis, meestal gebaseerd op een mix van schoolse kennis en informele kennis uit het dagelijks leven. Van schoolse kennis is vaak veel “vergeten” of ooit maar half begrepen. Informele kennis is vaak gebaseerd op zeer specifieke, individuele ervaringen. Dat maakt het voor docenten moeilijk de beginsituatie te bepalen en de juiste begeleiding te bieden. Daar komt bij dat anderstalige volwassenen vaak andere rekenprocedures ontwikkeld hebben dan Nederlandse volwassenen. Verschillen in (informele) procedures, algoritmes en het anders benoemen van bewerkingen kunnen een probleem zijn voor docenten. Inzicht in rekenkennis en rekenvaardigheden van volwassenen is noodzakelijk om:

1- meer te leren over rekenwiskundige kennis en vaardigheden van potentiële deelnemers bij de start van een cursus en hun leerwensen en behoeften te leren kennen en om hen in de juiste groepen te kunnen plaatsen;
2- meer te weten te komen over hun probleemoplossende aanpak met het doel instructie beter op hen te kunnen afstemmen;
3- onderwijsprogramma's zo goed mogelijk op deelnemers te kunnen afstemmen;
4- de deelnemers zo goed mogelijk te kunnen begeleiden tijdens de cursus en hun vorderingen goed bij te kunnen houden, en om uitval te voorkomen;
5- onderwijsprogramma's te kunnen evalueren en te kunnen verbeteren
6- beleidsmakers, werkgevers en curriculumontwikkelaars in staat te stellen beleid en onderwijsprogramma's af te stemmen op nieuwe eisen van de arbeidsmarkt en ontwikkelingen in lifelong learning.

Over het ontwikkelen van goede toetsen en het stellen van de juiste vragen aan volwassenen tijdens mondeling toetsen kan veel geleerd worden van de ontwikkelingen binnen het realistisch rekenonderwijs en van diagnostische toetsen. Hierop gebaseerd worden criteria voor goede toetsinstrumenten voor volwassenen geformuleerd:

1- de wijze van toetsen moet afgestemd zijn op de deelnemer in funderende educatie;
2- toetsen moeten de deelnemers in staat stellen te laten zien wat ze kunnen en niet wat ze niet kunnen;
3- toetsen voor rekenen-wiskunde moeten inzicht verschaffen in reken-wiskunde-procedures en probleemoplossende vaardigheden van deelnemers;
4- plaatsingstoetsen moeten de doelen, de inhoud en de niveaus van het programma coveren, zodat deelnemers weten wat zij kunnen verwachten als zij aan de cursus beginnen en in de juiste cursus geplaatst kunnen worden;
5- toetsen voor rekenen-wiskunde moeten tweedetaal-leerders in staat stellen hun eigen procedures en algoritmes toe te passen zoals zij die geleerd hebben in hun thuisland;
6- taalgebruik in een schriftelijke toets voor rekenen-wiskunde mag tweedetaal-leerders niet belemmeren bij het maken van de toets.

Vervolgens worden drie toetsen besproken die ontwikkeld zijn voor basiseducatie, en de huidige funderende educatie: de Supermarktstrategie (1992), In Balans (1996) en Cito (1996). Ook wordt bediscussieerd wanneer mondelinge en schriftelijke toetsen het best toegepast kunnen worden en in welke situaties computer adaptief toetsen (CAT) gebruikt kan worden.

In *hoofdstuk 4* wordt het werk van deelnemers in funderende educatie op twee plaatsingstoetsen kwantitatief en kwalitatief geanalyseerd. In december 1997 en januari 1998 werd in het ROC Utrecht, afdeling Nieuwegein, een groep van 37 nieuwkomers in funderende educatie getoetst op hun rekenwiskundige vaardigheden bij de start van een rekencursus. Hiervoor werden de *In Balans* en de *Cito* plaatsingstoetsen gebruikt. De *Cito* toets, als onafhankelijke en gevalideerde toets, is gebruikt om scores op de experimentele *In Balans* toets te kunnen vergelijken. Naast kwantitatieve informatie geeft de *In Balans* toets veel kwalitatieve informatie. De resultaten van 32 deelnemers zijn opgenomen in de analyses. De resultaten van dit onderzoek worden uitgebreid besproken en geïllustreerd met voorbeelden van deelnemers.

In het algemeen kan gesteld worden dat ongeveer een kwart van de deelnemers in deze studie functioneert op niveau KSE-1. Ongeveer driekwart van de deelnemers functioneert op beginnend niveau van KSE-2. Dat betekent dat zij functioneren op het niveau van elementaire en beginnende functionele gecijferdheid.

Inhoudelijk kan gezegd worden dat het merendeel van de deelnemers eenvoudige basisbewerkingen zoals tellen en optellen beheerst (80% correct). Toch zijn er nog deelnemers, met name op niveau 1, die bijvoorbeeld problemen hebben met het tellen van voorwerpen in een tweedimensionale afbeelding. Dat betekent niet dat zij in werkelijkheid ook problemen hebben met tellen. Het betekent wel dat zij problemen kunnen hebben met het verwerken van visuele informatie op papier (of op een beeldscherm).
Bij wat meer complexere bewerkingen met basisvaardigheden (tellen, optellen, aftrekken, vermenigvuldigen en delen), daalt het percentage correcte antwoorden tot 40%. Dit is zorgelijk omdat hier al blijkt dat deze deelnemers problemen hebben met het uitvoeren van eenvoudige opdrachten zoals het berekenen van een totaal van 8x48 in een vermenigvuldig-context of het berekenen van het aantal dozen dat nodig is voor een totaal van 3000 glazen verpakt in dozen van 40 stuks.
In het algemeen geven de deelnemers de voorkeur aan traditionele rekenprocedures op papier, zoals cijferend optellen en vermenigvuldigen. Hoofdrekenen en schatten worden veel minder toegepast. Dit is met name opvallend bij geldrekenen. Hier wordt bij een aantal opgaven nadrukkelijk gevraagd om schattingen. Veel deelnemers rekenen de opgaven echter precies uit, ronden daarna af volgens het betalingssysteem (van de Nederlandse munt) en geven de afronding als schatting. De meeste van deze geldtaken werden uitgerekend met behulp van optel- en aftrekalgoritmen, iets dat in het dagelijks leven niet of nauwelijks wordt gedaan. De resultaten bij het geldrekenen zijn zorgelijk. Slechts de helft van de groep kan de vragen over rekenen met geld goed beantwoorden. De vraag rijst hier of deze deelnemers ook problemen hebben met geldrekenen in het dagelijkse betalingsverkeer of dat zij niet gewend zijn berekeningen over geld op papier uit te voeren.

In het gebied van verhoudingen, breuken en procenten scoren de deelnemers ongeveer 75% goed op eenvoudige verhoudingsproblemen, zoals het vergelijken van de lengte van een auto ten opzichte van een bus en het vergelijken van de sterkte van de koffie bij een gelijke hoeveelheid water en een verschillende hoeveelheid koffie in twee dezelfde koffiekannen. Het percentage goed daalt echter drastisch bij taken over kernbreuken als 1/2, 1/3 en 1/4 (ongeveer 30% goed). Bij het onderdeel procenten, waar het gaat om rekenen met mooie percentages als 10% korting, zijn in totaal maar 15% goede antwoorden. Slechts één deelnemer kan correct uitrekenen wat de nieuwe huurprijs zou zijn na een huurverhoging van 2%.

Bij het onderdeel meten, inclusief decimale getallen, kan ongeveer 80% van de deelnemers opdrachten goed uitvoeren die te maken hebben met praktische handelingen die herkenbaar zijn vanuit het dagelijks leven. Bij opdrachten over het vergelijken van maten binnen het metriek stelsel (bijvoorbeeld 6 flesjes koffiemelk van 200 ml., is dat meer of minder dan een liter? Hoeveel liter is het precies?), daalt het percentage correct tot ongeveer 50%. Het percentage correct daalt tot ongeveer 20% als het gaat om het uitvoeren van formele berekeningen met gegeven maten op papier. Slechts vier deelnemers (12,5%) kunnen de oppervlakte van een vloer berekenen aan de hand van een tekening van de vloer.

Bij het lezen en interpreteren van data komen we tot vergelijkbare resultaten. Ongeveer 75% van de deelnemers kan een eenvoudige weergrafiek lezen. Ongeveer de helft kan een kwart in een cirkel tekenen. Ongeveer eenderde van de deelnemers kan een complexere grafiek lezen over de totale verkoop van auto's in een jaar en slechts vijf deelnemers kunnen aangeven wat de gemiddelde verkoop van een bepaald automerk is per kwartaal.

In het algemeen kan gezegd worden dat ongeveer driekwart van de deelnemers simpele taken kan uitvoeren die herkenbaar zijn vanuit het dagelijkse leven en die op meer (informele) manieren opgelost kunnen worden. Een kwart van de deelnemers scoort minimaal op alle onderdelen. Bij het uitvoeren van formele rekenhandelingen, bijvoorbeeld het vergelijken van maten binnen het metriek stelsel, daalt het percentage correct tot ongeveer 50%. Slechts een enkeling kan wat moeilijkere en complexere formele rekenbewerkingen uitvoeren, zoals bijvoorbeeld het berekenen van een gemiddelde of 2% van de huur.

In *hoofdstuk 5* wordt vervolgens gezocht naar een theoretische onderbouwing van het leren van volwassenen en het leren van rekenen-wiskunde door volwassenen in funderende educatie.

Voor het leren van volwassenen in het algemeen is gezocht naar andragogische uitgangspunten. De eerste twee uitgangspunten hierbij zijn afkomstig van Freire (1970):

1) volwassenen zijn vrij om te leren, en 2) *onvoorwaardelijke acceptatie van gelijkheid tussen volwassenen in leersituaties*. Dat geldt ook voor de relatie tussen docenten en cursisten.

Docenten kunnen cursisten zijn in sommige situaties en cursisten kunnen docenten zijn in andere situaties. Dit principe bepaalt de basishouding van docenten en cursisten in leersituaties. Beide uitgangspunten zijn algemeen geaccepteerd in de wet op de basiseducatie in 1987 en in de nieuwe BVE structuur. Vandaar dat in de BVE gesproken wordt over "deelnemers" en niet over "cursisten". Internationaal is dit niet vastgelegd, maar de UNESCO spreekt over "adult learners". Hiermee wordt een essentieel verschil met "student" aangegeven.

Hierop aansluitend is het derde uitgangspunt geformuleerd: *De ervaringen van deelnemers zijn de start voor het leren.* Ervaringen van deelnemers zijn altijd belangrijke schakels in het leerproces. Zij helpen bij het verhelderen en activeren van voorkennis en kunnen verrijkend zijn tijdens het leerproces bij het uitleggen en begrijpen van nieuwe informatie. Door koppeling aan eigen ervaringen krijgen nieuwe kennis en vaardigheden betekenis en worden daardoor makkelijker toepasbaar in de leefwereld van de volwassene.

Een belangrijke element hierbij is het vierde uitgangspunt: *authentieke materialen moeten als instructiematerialen dienen.* Het leren met middelen uit de directe werkelijkheid geeft betekenis aan het leren en maakt het leren functioneel.

Het vijfde uitgangspunt, *leren vindt plaats door interactie en reflectie*, gaat in op het belang van samenwerken door volwassenen waardoor zij zich bewust worden van hun eigen kennis en leren reflecteren op hun eigen kennis. Dit is de basis voor leren in het werkelijke leven.

Het zesde uitgangspunt is gebaseerd op studies over het informele leren in en door de praktijk: *leren in volwasseneneducatie moet leiden tot functionele kennis en vaardigheden.*

Dit uitgangspunt houdt in dat kennis en vaardigheden direct bruikbaar dienen te zijn in het dagelijkse leven van volwassenen, of in ieder geval herkenbaar en voorstelbaar. Dat betekent dat al het leren zoveel mogelijk moet plaatsvinden in een contextrijke leeromgeving waardoor het leren direct betekenis krijgt. Het leren in funderende educatie focust op leren-om-te-doen en minder op leren-om-te-weten. Voor deze wijze van leren is learning-in-action (Greeno, 1998) de beste weg om te leren. Het creëert een informele situatie waarin volwassenen in een ongedwongen sfeer met elkaar kunnen leren.

Het laatste uitgangspunt is dat *volwassenen hun eigen leerproces aansturen* door kennis te construeren en te reconstrueren, te organiseren en te reorganiseren. Dit uitgangspunt is ook terug te vinden in de basisgedachte van het constructivistisch denken dat elk individu zijn eigen kennis (re-)construeert. Het geeft aan dat de volwassene verantwoordelijkheid op zich neemt voor zijn eigen leerproces en het resultaat daarvan.

Voor de studie naar het leren van rekenen-wiskunde door volwassenen worden aanknopingspunten gezocht en gevonden bij de handelingstheorie, het constructivisme en bij het realistisch rekenwiskunde-onderwijs zoals ontwikkeld in Nederland.

Het basismodel voor handelingstheorie wordt afgestemd op het leerproces zoals dat bij volwassenen plaats zou kunnen vinden. Hierbij gaan volwassenen door vier stadia van conceptontwikkeling: het actief manipuleren met materialen, het manipuleren op basis van directe representaties van materialen, het ontwikkelen van denkmodellen op basis van die afbeeldingen en uiteindelijk het ontwikkelen van abstracte concepten gebaseerd op de drie voorafgaande handelingen. Deze vier lagen worden gevisualiseerd als geïntegreerde cirkels waarin volwassenen zich willekeurig begeven tijdens het leren van rekenen-wiskunde. Het manipuleren met werkelijke materialen is de basis voor functioneel handelen in de leefwereld van de volwassene. Dit moet gekoppeld zijn aan goed ontwikkelde concepten. Dit laatste uit zich in het gebruik van de juiste taal- en rekensymbolen bij het uitvoeren van handelingen.

Volwassenen hebben echter al veel concepten ontwikkeld, al of niet op basis van juiste kennis en/of veronderstellingen. Het leren van rekenen-wiskunde richt zich met name op het bijstellen en uitbreiden van deze concepten en handelingen. De visuele voorstellingen zijn de verbindende schakels tussen het werkelijke handelen en conceptontwikkeling. Voor het ontwikkelen van functionele kennis en vaardigheden is het van belang dat volwassenen voortdurend door deze lagen heen en weer gaan om werkelijk handelen te koppelen aan de juiste concepten. Om dit proces te bevorderen wordt een keuze gemaakt voor visuele, integrerende denkmodellen en progressief schematiseren, zoals ontwikkeld in realistisch rekenwiskunde onderwijs, als de verbindende schakels tussen informeel en formeel/ functioneel handelen.

Constructivisme en realistisch rekenen-wiskunde leveren de bouwstenen voor analyse van het cognitieve leerproces. Hierbij wordt onderscheid gemaakt in drie facetten van het leerproces: het leerproces zelf (*what*), het doel en de kwaliteit van het leerproces (*why*) en de wijze waarop het leerproces uitgevoerd wordt (*how*). Analyse van dit *what-why-how* proces kan docenten en deelnemers in funderende educatie helpen bij het zich bewust worden van wat er werkelijk tijdens het leerproces gebeurt. Hierdoor kan de deelnemer leren zijn eigen leerproces aan te sturen en verantwoordelijkheid te nemen voor zijn eigen leerproces. Dit is de basis voor zelfstandig leren in het werkelijke leven, onafhankelijk van docenten. Het is tevens de basis voor lifelong learning.

De theoretische modellen voor het leren van rekenen-wiskunde door volwassenen in funderende educatie en voor het leerproces zelf worden uiteindelijk gevisualiseerd in een integrerend instructiemodel *(Functional Numeracy Education)*.

In *hoofdstuk 6* worden de uitgangspunten van hoofdstuk 5 getoetst aan het werkelijke leren van volwassenen in de praktijk van funderende educatie. Na de plaatsingstoets werd voor de deelnemers in ROC Utrecht, afdeling Nieuwegein, een programma opgezet van zes weken in de periode van februari tot april 1998. De lessen vonden een keer per week plaats en duurden een uur per les. Het onderwerp was procenten. Voor dit programma werd het lesmateriaal van de methode *In Balans* gebruikt. De lessen werden opgenomen op video en het deelnemersmateriaal werd bewaard. De doelen van dit programma waren:

1- Het vinden van empirische informatie over het werkelijke leren van volwassenen in funderende educatie dat voorbeeldmatig kan zijn voor de theoretische gedachten zoals beschreven in hoofdstuk 5;
2- Het delen van ervaringen van docenten in funderende educatie om deze meer expliciet te kunnen maken en om inzicht te geven in de werkelijke rekenwiskundige kennis van deelnemers in funderende educatie en hun wijze van leren;
3- Het creëren van een gemeenschappelijke basis voor discussie over de ontwikkeling van theorie voor het leren van rekenen-wiskunde in funderende educatie;
4- De verkregen informatie is de volgende stap in de verbetering van het leren en doceren in funderende educatie, de ontwikkeling van theorie, en van materialen voor funderende educatie.

De resultaten zijn geanalyseerd aan de hand van de theoretische uitgangspunten in hoofdstuk 5. Hiervoor worden in hoofdstuk 6 aannames geformuleerd die oorspronkelijk uitgangspunt waren bij de ontwikkeling van de 'In Balans' materialen. Deze aannames concretiseren de theoretische uitgangspunten en zijn bedoeld om de discussie te kunnen sturen. De aannames zijn:

1-. Aanwezige kennis van volwassenen over rekenen-wiskunde bestaat vaak uit gedeeltelijk ontwikkelde concepten en misconcepten.
2- Eigen ervaringen uit het dagelijkse leven helpen bij het ontwikkelen van rekenwiskundige kennis en vormen de link tussen schoolkennis en rekenwiskundige kennis voor het dagelijkse leven.
3- Authentieke materialen in leersituaties moeten zorgvuldig worden gekozen.
4- Interactie en discussie zijn essentieel bij het verwerven van nieuwe kennis over rekenen-wiskunde.
5- Het leren op basis van “steunpunten” helpt bij het organiseren van het wiskundig denken en bij het ontwikkelen van steunpunten voor hoofdrekenen en schatten.
6- Integrerende visuele modellen zijn zinvol bij het creëren van links tussen informele en formele kennis en tussen verschillende rekenwiskundige begrippen en handelingen.
7- Functionele notaties ontwikkeld door volwassenen zelf hebben de voorkeur boven traditionele procedures, zoals algoritmes, geleerd in school.

Deze aannames worden uitgebreid bediscussieerd en geïllustreerd met voorbeelden van de deelnemers.

In *hoofdstuk 7* komen alle componenten samen en wordt een samenhangend geheel opgezet voor een programma voor het ontwikkelen van functionele gecijferdheid. Dit programma steunt op de vier componenten van het begrip gecijferdheid, beschreven in hoofdstuk 2, en plaatst de theoretische componenten van hoofdstuk 5 in een onderwijskader. Hier wordt beschreven welke inhouden minimaal noodzakelijk zijn voor rekenwiskunde programma's in funderende educatie en op welke wijze rekenwiskundige activiteiten kunnen worden georganiseerd en uitgevoerd. Dit programma wordt gezien als een toegangsweg tot functionele gecijferdheid en is minimaal noodzakelijk voor deelnemers in funderende educatie om enige kans van slagen te hebben in beroepsonderwijs en op de arbeidsmarkt.

In *hoofdstuk 8*, tenslotte, wordt teruggeblikt op de onderzoeksvragen. Hierbij worden conclusies beschreven per onderzoeksvraag en worden suggesties gedaan voor vervolgonderzoek.

Karakteristieken van deze studie zijn:

Verkennend en beschrijvend
Een relatief onbekende doelgroep in funderende educatie wordt in beeld gebracht aan de hand van concrete resultaten.

Kwalitatief analyserend
De rekenvaardigheid van de volwassenen in dit onderzoek wordt kwalitatief geanalyseerd. Geprobeerd wordt een beeld te schetsen hoe zij rekenen en waar problemen zitten. Leerprocessen worden beschreven aan de hand van concrete voorbeelden uit de praktijk. Dit levert bouwstenen voor het opzetten van een programma voor functionele gecijferdheid.

Theorievormend
In bestaande literatuur en theorieën worden aanknopingspunten gezocht voor het leren van volwassenen in het algemeen en specifiek voor het leren van rekenen-wiskunde door laagopgeleide volwassenen. Op basis daarvan wordt een instructiemodel ontwikkeld voor het leren van rekenen-wiskunde in funderende educatie. Dit wordt gekoppeld aan het ontwikkelen van vaardigheden voor het managen van rekenwiskundige situaties in het dagelijks leven.

Ontwikkelend
De informatie verkregen bij de eerste drie punten wordt gebruikt voor het ontwikkelen van een programma voor rekenen-wiskunde in funderende educatie dat leidt tot functionele gecijferdheid.

Door deze studie hoop ik een bijdrage te leveren aan de ontwikkeling van kennis over een doelgroep en een type onderwijs die in Nederland, maar ook daarbuiten, inhoudelijk nauwelijks bekend zijn.

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Appendix 1: Results IALS Survey - Quantitative Literacy (1996)

Source: Houtkoop, W. (1999). *Basisvaardigheden in Nederland*
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QL Results IALS in percentages

Countries	Level 1	Level 2	Level 3	Level 4/5
Sweden	6.6	18.6	39	35.8
Germany	6.7	26.6	43.2	23.5
Netherlands	10.3	25.5	44.3	19.9
Switzerland (Fr)	12.9	24.5	42.2	20.4
Switzerland (G)	14.2	26.2	40.7	19
Belgium (Fl)	16.7	23	37.8	22.6
Australia	16.8	26.5	37.7	19.1
Canada	16.9	26.1	34.8	22.2
New Zealand	20.4	28.9	33.4	17.2
USA	21	25.3	31.3	22.5
UK	23.2	27.8	30.4	18.6
Ireland	24.8	28.3	30.7	16.2
Poland	39.1	30.1	23.9	6.8

Source: Houtkoop, W. (1999)

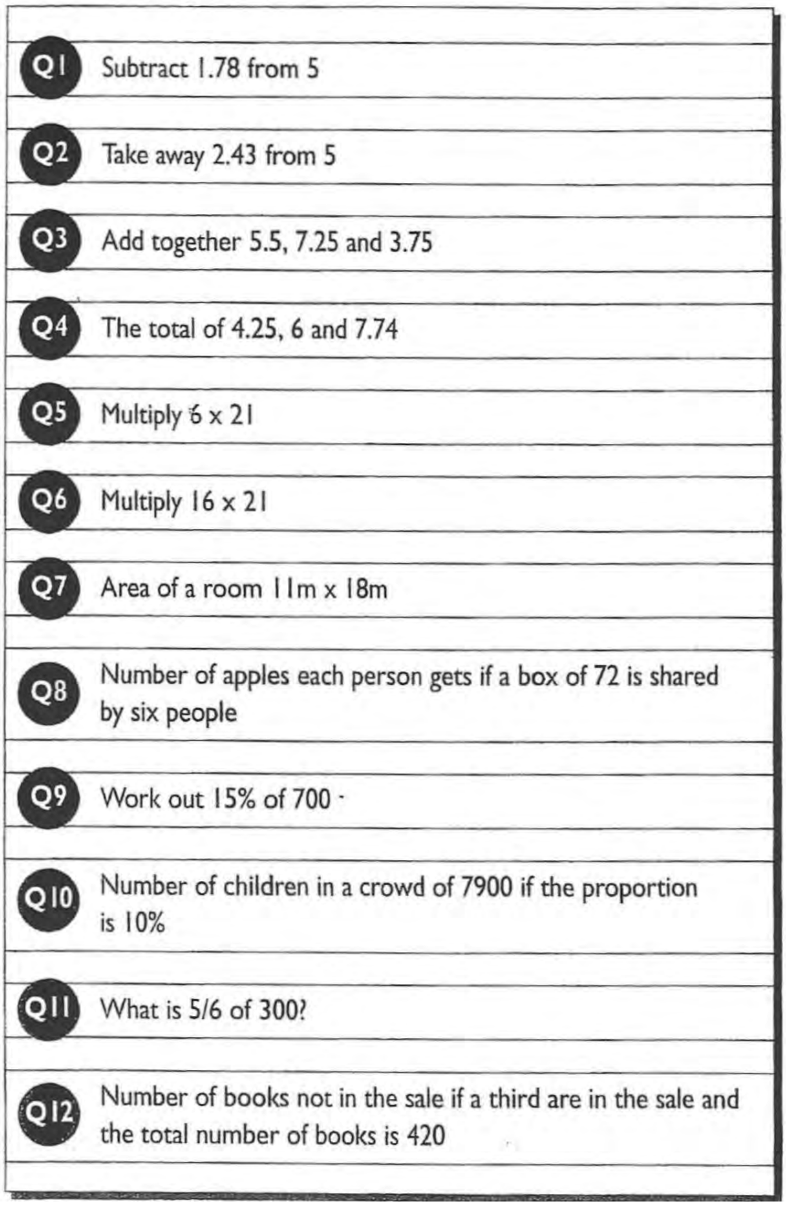
QL Results IALS in percentages

Countries	Level 1/2	Level 3	Level 4/5
Sweden	25.2	39	35.8
Germany	33.3	43.2	23.5
Netherlands	35.8	44.3	19.9
Switzerland (Fr)	37.4	42.2	20.4
Belgium (Fl)	39.7	37.8	22.6
Switzerland (G)	40.4	40.7	19
Canada	43	34.8	22.2
Australia	43.3	37.7	19.1
USA	46.3	31.3	22.5
New Zealand	49.3	33.4	17.2
UK	51	30.4	18.6
Ireland	53.1	30.7	16.2
Poland	69.2	23.9	6.8

Source: Houtkoop, W. (1999)

Appendix 2: International Numeracy Survey

Source: The Basic Skills Agency (1997). *International Numeracy Survey* Commonwealth House, London, UK



Q1 Subtract 1.78 from 5

Q2 Take away 2.43 from 5

Q3 Add together 5.5, 7.25 and 3.75

Q4 The total of 4.25, 6 and 7.74

Q5 Multiply 6 x 21

Q6 Multiply 16 x 21

Q7 Area of a room 11m x 18m

Q8 Number of apples each person gets if a box of 72 is shared by six people

Q9 Work out 15% of 700

Q10 Number of children in a crowd of 7900 if the proportion is 10%

Q11 What is 5/6 of 300?

Q12 Number of books not in the sale if a third are in the sale and the total number of books is 420

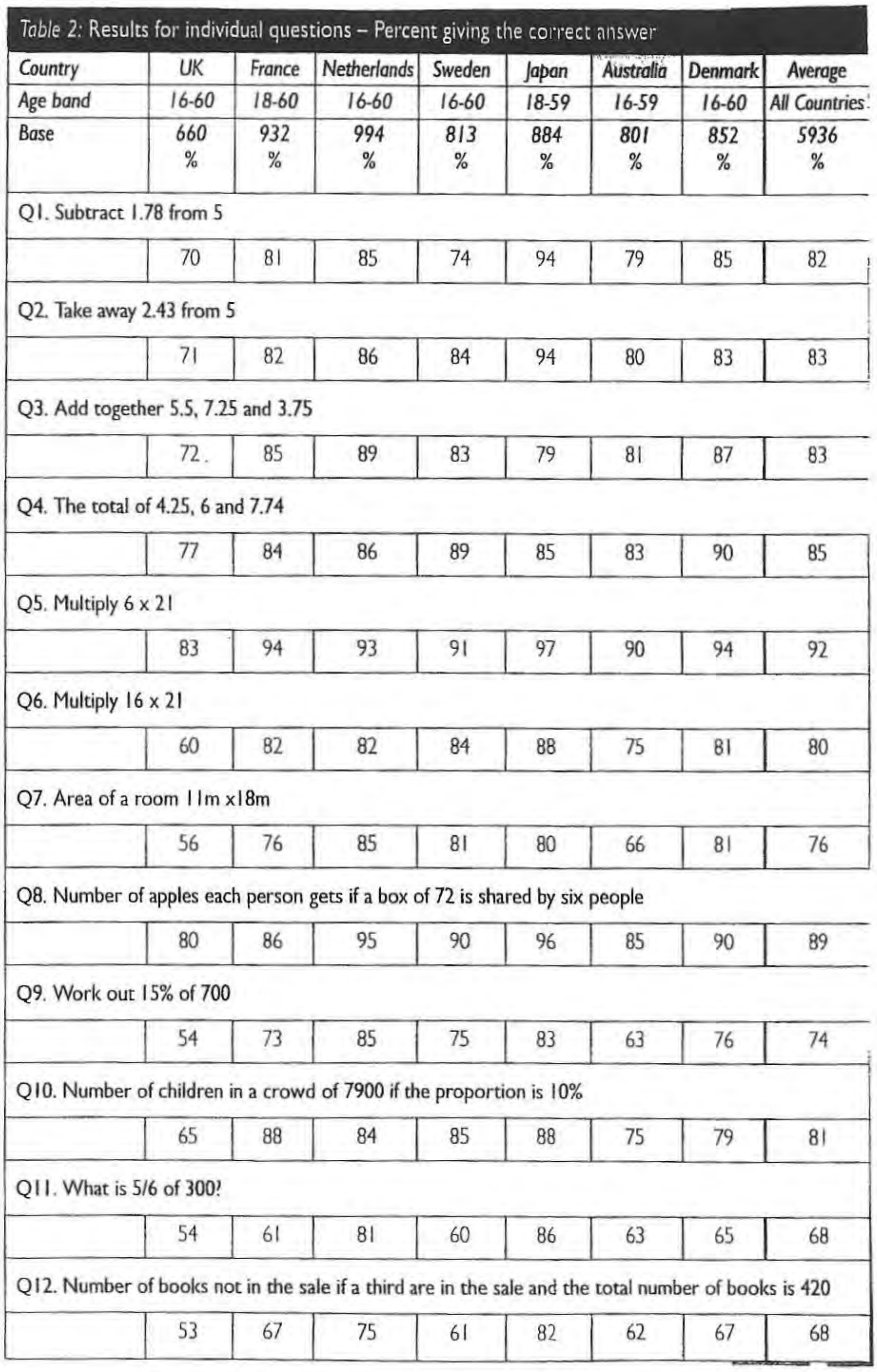


Table 2: Results for individual questions – Percent giving the correct answer

Country	*UK*	*France*	*Netherlands*	*Sweden*	*Japan*	*Australia*	*Denmark*	*Average*
Age band	*16-60*	*18-60*	*16-60*	*16-60*	*18-59*	*16-59*	*16-65*	*All Countries*
Base	*660* %	*932* %	*994* %	*813* %	*884* %	*801* %	*852* %	*5936* %
Q1. Subtract 1.78 from 5								
	70	81	85	74	94	79	85	82
Q2. Take away 2.43 from 5								
	71	82	86	84	94	80	83	83
Q3. Add together 5.5, 7.25 and 3.75								
	72	85	89	83	79	81	87	83
Q4. The total of 4.25, 6 and 7.74								
	77	84	86	89	85	83	90	85
Q5. Multiply 6 x 21								
	83	94	93	91	97	90	94	92
Q6. Multiply 16 x 21								
	60	82	82	84	88	75	81	80
Q7. Area of a room 11m x18m								
	56	76	85	81	80	66	81	76
Q8. Number of apples each person gets if a box of 72 is shared by six people								
	80	86	95	90	96	85	90	89
Q9. Work out 15% of 700								
	54	73	85	75	83	63	76	74
Q10. Number of children in a crowd of 7900 if the proportion is 10%								
	65	88	84	85	88	75	79	81
Q11. What is 5/6 of 300?								
	54	61	81	60	86	63	65	68
Q12. Number of books not in the sale if a third are in the sale and the total number of books is 420								
	53	67	75	61	82	62	67	68

Appendix 3: Overview Results on In Balance and Cito Placement Tests

ID	M	F	IBA	IBB	IBBONUS	IBTOTAL	IB-Level	C1	C2L	C2H	CTOTAL	CSkill	C-Level
001	0	1	18	1	0	19	1	1	0		1	1	1
002	1	0	13,5	11	0	24,5	2	6	9		15	79	2
003	0	1	14,75	10,5	0	25,25	2	4	8		12	67	2
004	0	1	14,5	7	0	21,5	2	5	9		14	75	2
005	1	0	20	15,25	0	35,25	2	4	13		17	83	2
006	1	0	30	13	10	53	3	10	3	0	13	71	2
007	0	1	28	12,25	10	50,25	3	7	3	0	10	59	1
008	1	0	35	8,75	20	63,75	4	5	10		15	79	2
009	0	1	25,5	13,25	10	48,75	3	7		0	7	98	2
010	0	1	10	5,5	0	15,5	1	0	2		2	1	1
011	0	1	14	14	0	28	2	2	7		9	55	1
012	0	1	29,5	10,75	10	50,25	3	3	10		13	75	2
013	1	0	27,25	15,5	10	52,75	3	7		5	12	98	2
014	0	1	23	11,75	10	44,75	3	0	11		11	63	2
016	1	0	18,5	15	10	43,5	3	6	12		18	96	2
017	0	1	24,5	12,75	10	47,25	3	6	11		17	92	2
019	0	1	28,5	8	10	46,5	3	8		0	8	101	2
021	0	1	23,75	13,75	10	47,5	3	11		2	13	94	2
024	0	1	37	11,25	20	68,25	4	8		5	13	101	2
025	0	1	35,5	13,75	20	69,25	4	7		6	13	101	2
026	1	0	36	15,5	20	71,5	4	5	11		16	83	2
027	0	1	32,5	13,5	20	66	4	5	11		16	83	2
028	1	0	31,5	12,5	20	64	4	7		8	15	108	3
029	0	1	34,5	12,75	20	67,25	4	4	11		15	79	2
030	0	1	33,25	14,5	20	67,75	4	6	12		18	96	2
031	0	1	35	19,75	20	65,75	4	4	12		16	83	2
032	1	0	35,75	11,5	20	67,25	4	7		5	12	98	2
033	0	1	30,5	17,25	10	57,75	3	4	8		12	67	2
034	1	0	33,5	12	20	65,5	4	3	12		15	79	2
035	0	1	35,5	10,75	20	66,25	4	11		9	20	124	3
036	1	0	35,5	11	20	66,5	4	9		4	13	101	2
037	0	1	29,75	13,25	10	53	3	5	11		16	83	2
N=32	11	21											

Appendix 4: Overview Results on all Items - Raw Scores

Number and Basic Operations (1)

Level	Item	Topic	partial credit	0-1	% correct
1	A1-1	Counting till 30	30	30	93,8
	A1-2	Number line - addition	28	26	81,3
	A1-3	addition to 25	30,5	30	93,8
	A1-4	addition to 50	30	30	93,8
	A1-5	addition to 50	31	31	96,9
	A1-9	repeated addition-multiplication (6x3)	26	26	81,3
2	A2-1	Counting till 100	21,5	21	65,6
	A2-2	Counting till 100	25	25	78,1
	A2-3	Addition to 100	29,5	29	90,6
	A2-4	addition/subtraction to 100 - rounding	22	18	56,3
	A2-5	repeated addition - multiplication	22	25	78,1
	A2-6	addition-subtraction	28,5	26	81,3
3	A3-1	place value	23,5	22	68,7
	A3-2	counting	28,5	28	87,5
	A3-3	addition to 1000	25,5	25	78,1
	A3-4	addition, subtraction to 1000 (rounding)	23,5	22	68,8
	A3-5	addition, subtraction to 1000	20,5	16	50
	A3-6	multiplication	27	27	84,4
	A3-7	multiplication	25	23	71,9
4	A4-1	place value	11	10	31,3
	A4-2	addition (rounding numbers)	18	18	56,3
	A4-3	addition, subtraction (algorithms)	18,5	17	53,1
	A4-4	addition, subtr. (smart calc., in the head)	14	12	37,5
	A4-5	multiplication (smart calc., algorithms)	17	17	53,1
	A4-6	multiplication - division	14	14	43,8
	A4-7	multiplication - division	17	16	50
		Mean scores		22,47	70,21

Number and Basic Operations (2)

Level	Item	Topic	0-1 scores	% correct
C1	C1-1	subtraction	14	43,8
	C1-2	place value	4	12,5
C2-L	C2L-1	division	29	90,6
	C2L-2	multiplication	27	84,4
	C2L-4	subtraction below 2000	22	68,8
	C2L-5	multiplication	24	69
	C2L-7	multiplication	29	90
	C2L-8	addition (algorithm)	26	81,3
	C2L-9	subtraction	30	93,8
	C2L-10	subtraction	23	71,9
C2-H	no items			
		Mean scores	22,8	70,61

Proportions

Level	Item	Topic	partial credit	0-1	% correct
1	B1-3	proportions (coffee)	30	30	93,8
	B1-4 (*)	proportions: What is the best buy (to 10 gld)	29	29	90,6
	B1-5 (**)	proportions (bus-car)	28	28	87,5
2	B2-4 (*)	proportions: What is the best buy (100 gld)	22	22	68,8
3	B3-3	find product, benchmark percents	13	2	6,3
	B3-4	proportions - fractions	3,5	1	3,1
	B3-9	proportions - fractions	19	18	56,3
4	A4-9	find product, 15% discount of ƒ 200,-	9	9	28,1
	A4-10	find product, 2% increase of rent	1	1	3,1
	B4-3	proportion, find percent, benchmark percents	7	7	21,9
	B4-10 (*)	proportion, exchange rate, multiplication	8	10	31,3
		Mean scores		14,27	44,62

Level	Item	Topic	0-1 scores	% correct
C1	C1-3	decimals - addition	21	65,6
	C1-4	decimals - addition	15	46,9
	C1-7	fractions - addition	15	46,9
	C1-8	fractions - compare and sort	12	37,5
	C1-10	proportions - ratio (1lt. gas 12 km)	11	34,4
	C1-11	proportion, find percent, 30%	11	34,4
	C1-12	find product, 96% of 5018	16	50
C2-H	C2H-1	decimals-subtraction	7	21,9
	C2H-6	proportions - scale	3	9,4
	C2H-7	fractions	3	9,4
	C2H-8	4/5 = ? % (compare fractions, percents)	2	6,3
	C2H-9	fractions - multiplication	2	6,3
	C2H-11	percents, proportions 60% of 40 hours	3	9,4
	C2H-12	find percent gain	3	9,4
	C2H-13(*)	proportion, exchange rate, mult/div	6	18,8
		Mean scores	8,67	27,16

Measurement and Dimension

Level	Item	Topic	partial credit	0-1	% correct
1	B1-2	Time, reading clock (hours, half hours)	30	21	65,6
	B1-6	weight	30	30	93,8
	B1-7	volume	28	28	87,5
	B1-8	space	27	27	84,4
	B1-10	Time, calculating with time (in hours)	26	26	81,3
2	B2-2	Time, reading clock (minutes)	24,5	17	53,1
	B2-3	volume	22	21	65,6
	B2-5	length, (m-cm)	29	29	90,6
	B2-6	weight	27,5	27	84,4
	B2-7	volume	17	16	50
	B2-8	space (multiplication boxes)	22	22	68,8
	B2-10	date (today's date-calendar)	27,5	27	84,4
3	B3-2	Time, calculating hours + min	11	10	31,3
	B3-5	length (dec., m, cm)	17	11	34,4
	B3-6	weight, reading labels	22,5	20	63,5
	B3-7	volume (l, cl)	19	19	59,4
	B3-8	shape	21	18	56,3
	B3-10	time, calculation with seconds + min	17	17	53,1
4	B4-2	time, calculation with seconds	4	4	12,5
	B4-4	volume, length, weight (fractions, dec)	11	10	31,3
	B4-5	area	6,5	4	12,5
	B4-6	weight (compare fractions, dec)	6	6	18,8
	B4-7	volume (calculations with ml)	8	6	18,8
	B4-8	shape (cube map)	2	1	3,1
		mean scores		17,37	54,35

Level	Item	Topic	0-1 scores	% correct
C1	C1-6	volume (liter, ml)	18	56,3
C2-L	C2L-6	Length (m-cm)	23	71,9
	C2L-12	reading and calculating time	27	84,4
C2-H	C2H-4	calculating time (min.-hours)	5	15,6
	C2H-5	calculating time (hours-min)	2	6,3
		mean scores	15	46,9

Money

Level	Item		Topic	partial credit	0-1scores	% correct
1	A1-6		pay and get change up to ƒ 10,00	26	25	78
	A1-7		paying an amount up to ƒ 10.00	21	17	53
	A1-8		estimate a total amount up to ƒ 20.00	24	14	43,8
	A1-10		estimate a total amount up to ƒ 25.00	26	26	81,3
	B1-1		rounding an amount to ƒ10,00	29	21	65,6
	A2-7		pay and get change	24,5	23	71,8
2	A2-8		getting change from ƒ 100.00	24,5	23	71,9
	A2-9		paying an amount up to ƒ 100.00	24	20	62,5
	A2-10		estimate an amount up to ƒ 25.00	16,5	11	34,4
	B2-1		Addition to 10 gld	28	25	78,1
3	A3-8		Addition to 1000 gld	25	25	78,1
	A3-9		pay and get change up to ƒ 200.00	18,5	14	43,8
	A3-10		estimate a total amount to ƒ 200.00	13	10	31,3
	B3-1	(*)	Price per kg. , estimate price of 2,5 kg	24	24	75
4	A4-8		Multiply an amount up to ƒ 1000.00	16	16	50
	B4-1		How many cars for one million?	8	8	25
			Mean score money IB		18,88	59,0

Level	Item	Topic	0-1 scores	% correct
	C1-5	addition to ƒ 40.000,--	15	47
C1	C1-9	Paying an amount up to ƒ 100.00	12	38
C2-L	C2L-3	addition to ƒ 20.000,--	28	87,5
	C2L-11	price per item	11	34,3
	C2L-13	Short of money: subtraction by addition	28	87,5
C2-H	C2H-2	Rounding numbers	2	6,25
	C2H-3	costs per month	3	9,4
		Mean scores Cito	14,14	44,28

Reading and Understanding Simple Data

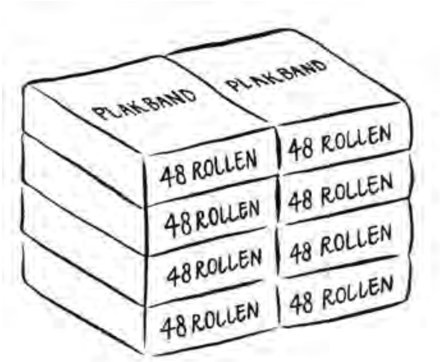
Level	Item	Topic	partial credit	0-1	% correct
1	B1-9	reading bar graph, total market sales (to 100)	25	25	78,1
2	B2-9	reading line graph, temperature	25,5	23	71,9
4	B4-9	reading bar graph, car sales (in thousands)	7,5	4	12,5
		mean scores		17,33	54,17

Level	Item	Topic	0-1 scores	% correct
C1	none			
C2-L	none			
C2-H	C2H-10	bar graph, addition of percents	3	9,4
		mean scores	3	9,4

Appendix 5 : Sample items to Chapter 7

1) Boxes

Figure 7.6: Boxes



How many tape rolls in total?

The context shows 8 boxes with 48 rolls of tape in each box. The sketch shows a schematic mathematical structure and can be used for various mathematical actions. It requires the integration of counting, addition and multiplication procedures. It shows a doubling-structure as discussed earlier in the block-model. It offers possibilities to do computations in informal as well as in formal ways, or combinations of both, as we saw see in chapter 4. The context is simple, clear and closed. There is little text and there are no distractors.

Such types of contexts are necessary, given that only 53% of the learners in this study could do this task correctly. This problem can be solved in several ways, as was showed in chapter 4, e.g. by smart counting strategies, repeated addition, doubling, smart mental computation using round figures (e.g. 8x50 minus 8x2, or 4x100 minus 4x4), using multiplication tables or multiplication algorithms. In interactive learning situations learners discover that they can do this in different ways. This may help them to develop more flexible procedures.

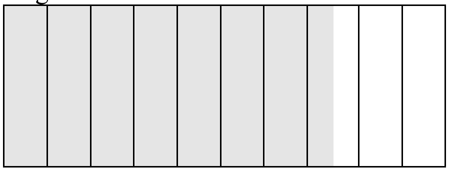
In a second step the same boxes can be used with easier or more difficult numbers, e.g. 3 piles of 4 boxes, or 3 piles of 5 boxes, in combination with the question: “What happens?” Also, the number of rolls of tape in a box can be changed, say to 24 or 52. Such changes can be chosen strategically in discussion with the group. At this point learners can adjust their computation strategies to the new context. For example for the first change, 3 piles of 4 boxes, people can start again with doing computations, but they can also add half of the total they

already had computed. When we change the context to 24 rolls per box, people should know that the total would be half of the total they had from the previous computation. When we change the total number to 52 per box, people might recognize that with smart computation they can do 50+2, instead of 50-2. The context can also be adjusted to lower or higher level tasks. At a lower level, numbers can be changed to round numbers, like 8x40, 8x20, or 8x25, or even to numbers less than 10, say 8x6, etc. At a higher level, numbers over 100 in a different context can be used. Then the learners may also want to use the calculator to check their answers.
The context can be changed by changing the content, the number of piles and number of pieces in the boxes. The possibilities are unlimited for these types of contexts.

2) The Fence model

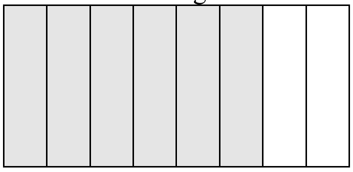
Another context is "the fence" which can be built up along the "fence model" as an integrating visual model. The context can serve to integrate proportions, fractions, decimals and percent, but also multiplication, division and measurement. The fence context starts with a photo of someone painting a real fence. The grey part in the sketches shows the part that already is painted. The fence is divided into parts helping the learners to estimate or compute the total length of the fence. A variety of tasks based on that fence can be developed. The length of the total fence, or of each individual part, may differ per context. Starting with a fence consisting of ten parts may make it easier to do computations related to meters and may help adults to see the similarity with the block model. Two examples are shown in figures 7.7.1 and 7.7.2.

Figure 7.7.1 Fence-model



Sample questions:
Which part of the fence has already been painted?
The length of each part is 1.5 meter.
How many meters have still to be done?

Figure 7.7.2



Sample questions:
Which part of the fence has already been painted?
Which part of the fence has still to be done?
The length of the fence is 12 meters.
How many meters have still to be done?

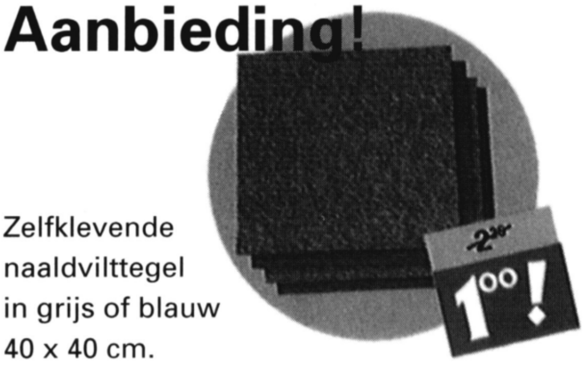
In the fence model all kinds of ratios and measures can be applied. The sketch of the fence itself shows the ratio between grey and white. Learners can link that

with fractions, decimals and percent. By asking different type of questions about the same model, different type of skills will be practiced. By this learners will discover the relation between fractions, decimals and percent.
For the second fence (figure 7.7.2) the learners have to convert their ten-part concepts of fractions, percent and decimals to eight parts. This makes it much more difficult, especially with the last question. The learner needs to figure out the length of one part, or he can go in steps from half of 12 to again taking a half, without computing the length of one part. Tasks can be offered on increasing levels of difficulty, for example, by changing the number of parts or the total length of the fence or by changing the item question.
By offering various possibilities the fence context is very effective at integrating skills. The fence model can easily be transferred to a bar model or to a number line which can be a step in the process of working on a higher action level.

3) Tiles

Carpet tiles are ideal materials for contexts. They can be used for learning multiplication strategies but also, of course, for learning about areas and perimeters. All kinds of exercises can be done with actual tiles. Carpet tiles are available in different sizes and prices and can often be purchased at local stores, often very cheaply. They are ideal instruction materials to practice learning for doing.

Figure 7.8 Carpet Tiles



The advertisement shows tiles of 40x40 cm at a reduced price. The problem posed is: *How many tiles would you need to cover the entire floor of the room you are in now?*
The context offers good possibilities for group activities in that. Learners working in pairs or small groups can solve the problem can in several ways. The

dimensions of the tiles and price in the advertisement can be adjusted. The learners should have a few actual carpet tiles to be used as a measure.

This context is very concrete but more complex in that it is a multi-step operation and an open task. The actual difficulty level depends on the dimensions of the room that the learners are in and the size of the tiles. There is still little text in the context and there are no distractors.

Learners can do this problem on different action levels. The activity may vary from actually measuring the room with the help of the tiles and then count the numbers of tiles they need to cover the floor, to measuring the length and the width of the room with a measuring tape and then applying multiplications to compute the area of the room and the tiles to determine the needed number of tiles. Activities like tiling a floor created in a school situation come close to real-life situations and are the basis for learning-for-doing. All four action levels can be practiced at the same time.

These three examples may indicate what we mean by learning through contexts based on benchmark learning, integrating visual models and the four action levels. The contexts are closely related to real life situations but they are selectively chosen because of the mathematical problems and possibilities in them. They show how to work with contexts and how to develop mathematics activities that make sense for adults. All context problems can be solved in multiple ways, which provides the opportunity for adults to use their most successful competencies. In discussion with other learners and teachers they may develop better adequate skills and strategies.

Series of such selected contexts, inserted in the right places in strands, set the core of a program for functional numeracy. Together with the integrating visual models and the principles of benchmark learning, the ingredients for a concise FNE program are set.

Also, using such context problems, in particular the more open and complex contexts like the tiles, adults learn to analyze problems, solve them and make decisions, which may help them learn to manage similar situations in their own everyday life situations. (see chapter 5, section 5.3.3)

curriculum vitae

Mieke van Groenestijn started as a teacher in special education in 1969. She worked with children with special needs for about ten years. She studied Theories of Education and Special Education and received her university degree in 1993. She is a co-worker at the Hogeschool van Utrecht where she works as a teacher trainer and advisor for teachers in special education in primary and secondary school and for teachers in adult education since 1981.
In parallel she started working as a volunteer teacher in a literacy course for adults in a local project in Woerden in 1981. She was coordinator of that project until this project merged with other projects for adults into a local learning center for adult basic education in Woerden. She was a co-director of that learning center until 1991.
From the start of the literacy courses in Woerden her interest was not only literacy but also mathematics for adults because many adults asked for help with their everyday math problems. She participated in the first working group in the Netherlands for the development of instruction materials for a teachers training course for mathematics in ABE.

Since 1989 she was project leader of two concessive projects for the development of mathematics materials for adults. The first project resulted in the publication of the “Supermarket Strategy”, an assessment tool for under-schooled and not-schooled adults, in 1992. This project was succeeded by a project for the development of learning units on mathematics for adults, the “In Balance” series. (1994-2000).

In 1993 the international organization Adults Learning Mathematics (ALM) was grounded in the UK. She is an ALM trustee and organized the ALM-5 Conference in Utrecht in 1998. She also an editor of the ALM-Newsletter that publishes short articles on adult numeracy three times a year.

Since 1998 she has been involved in the international Adult Literacy and Life skills survey (ALL). The ALL project is meant to compare adults’ skills on literacy, numeracy and problem solving across countries. She is a member of the numeracy team that developed a numeracy framework and a numeracy assessment tool for this survey. She is also a member of the Dutch ALL team that will execute the local ALL survey in the Netherlands.