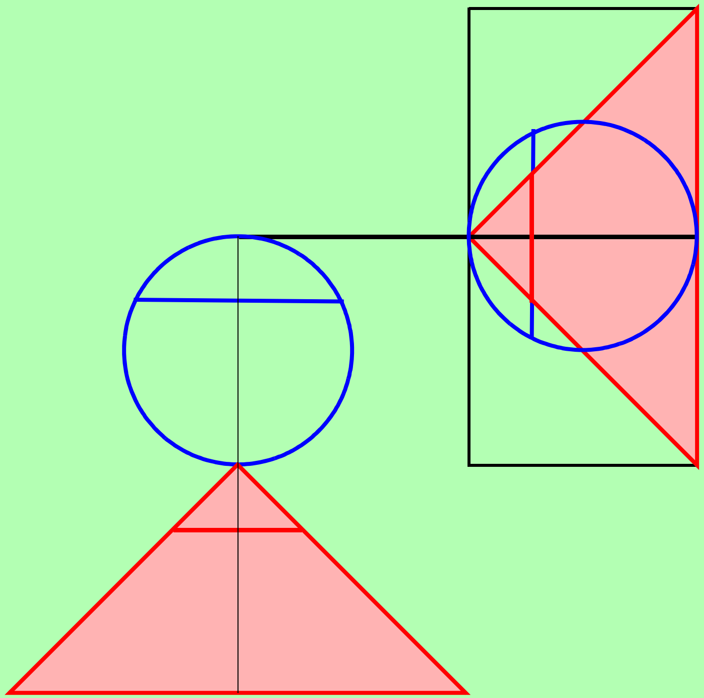
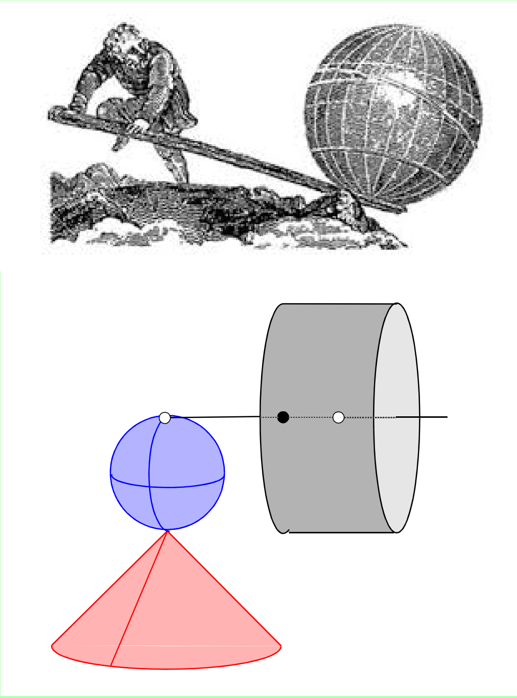
Evenwichtige Meetkunde

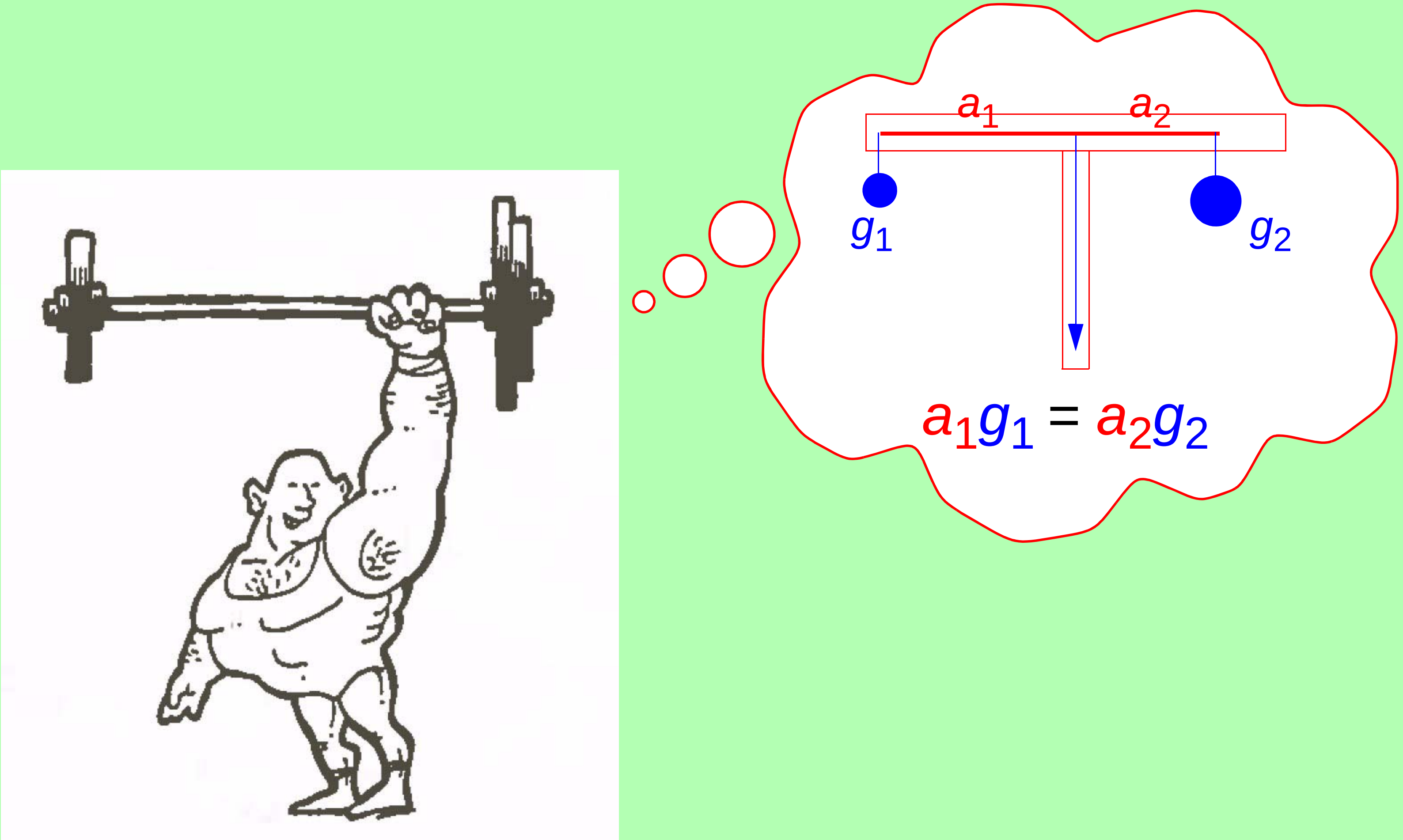


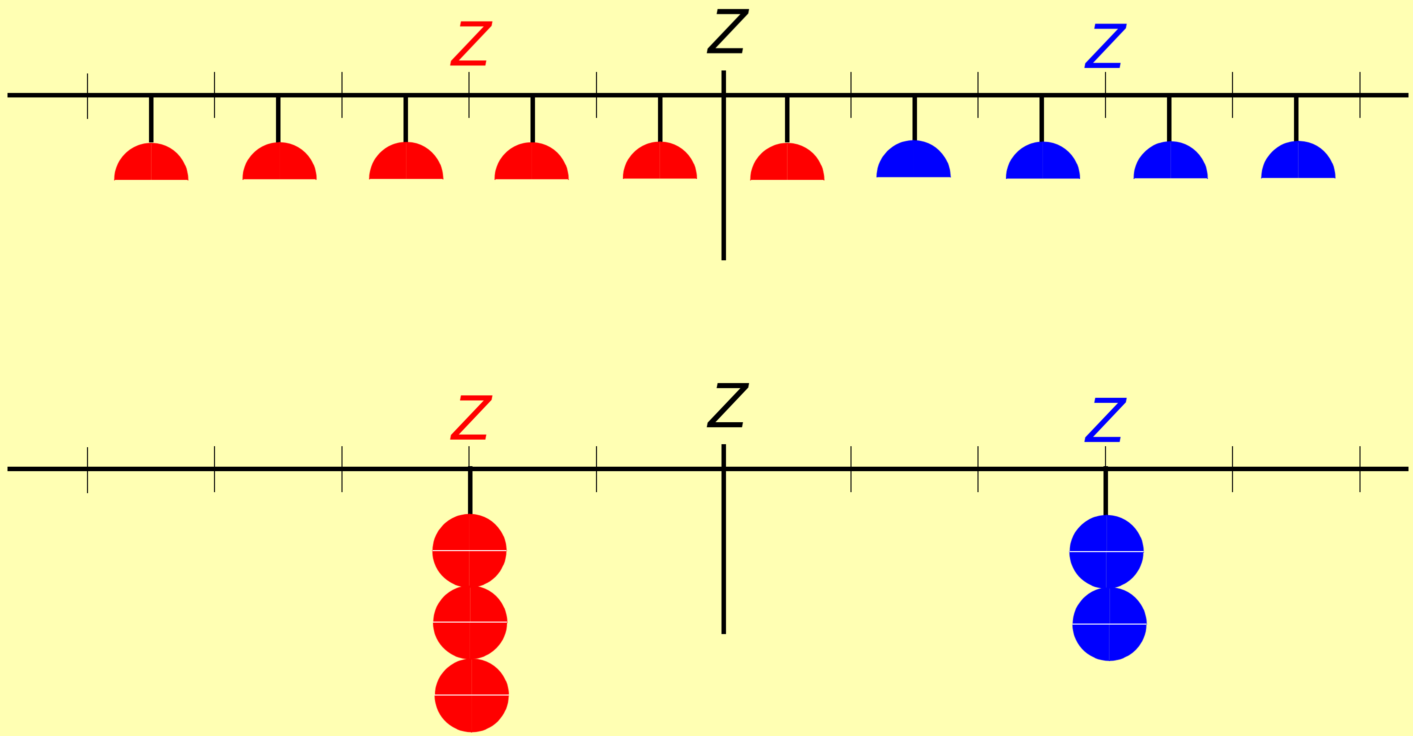


Nationale Wiskunde Dagen 2015

Archimedes aan Eratosthenes

... daar ik weet dat gij ijverig zijt, een voortreffelijk docent in de filosofie en vol belangstelling voor wiskundig onderzoek, heb ik gemeend goed te doen door voor U een bijzondere methode uiteen te zetten, met behulp waarvan het U mogelijk zal zijn om zekere wiskundige zaken in te zien met behulp van de mechanica ...


a_1
a_2
g_1
g_2
$a_1 g_1 = a_2 g_2$

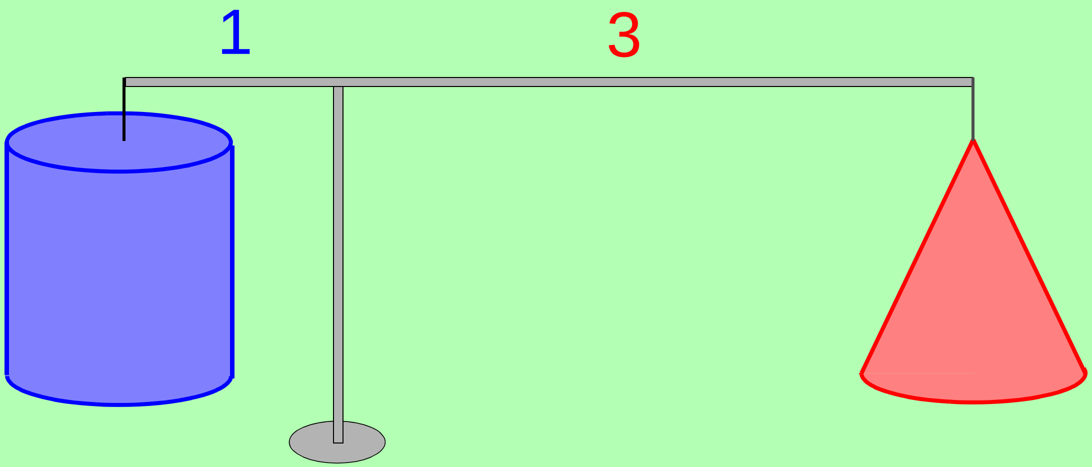


Archimedes: *Evenwichten van Vlakke Figuren (propositie 6)*

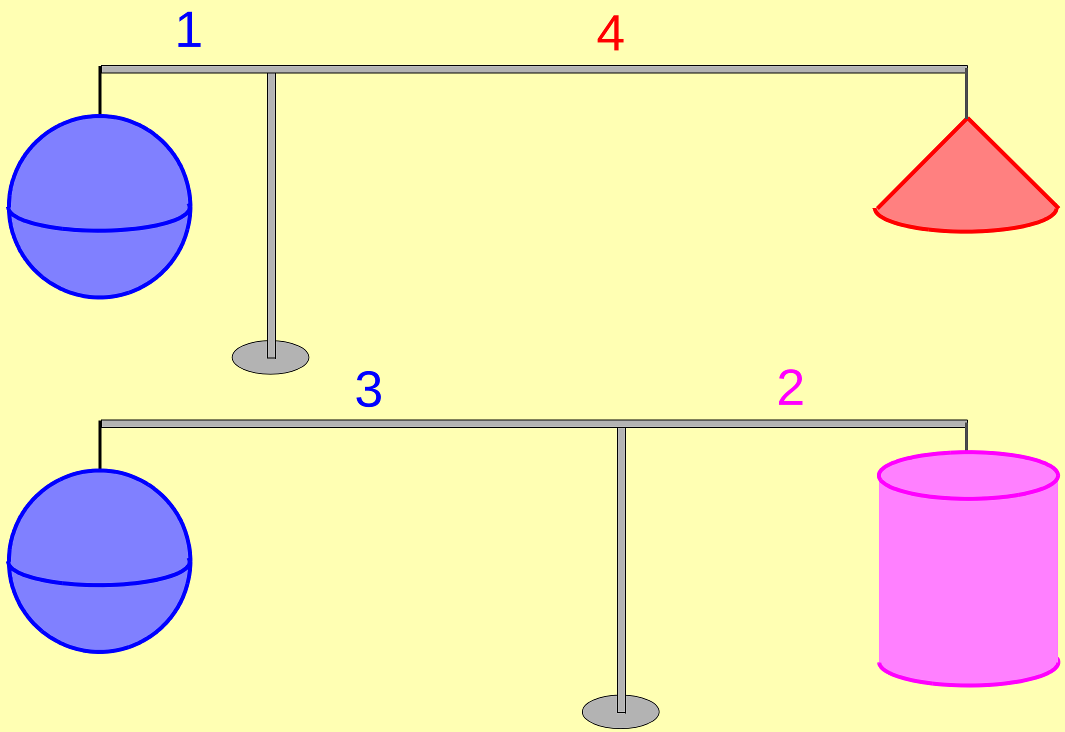
Democritos

Eudoxus

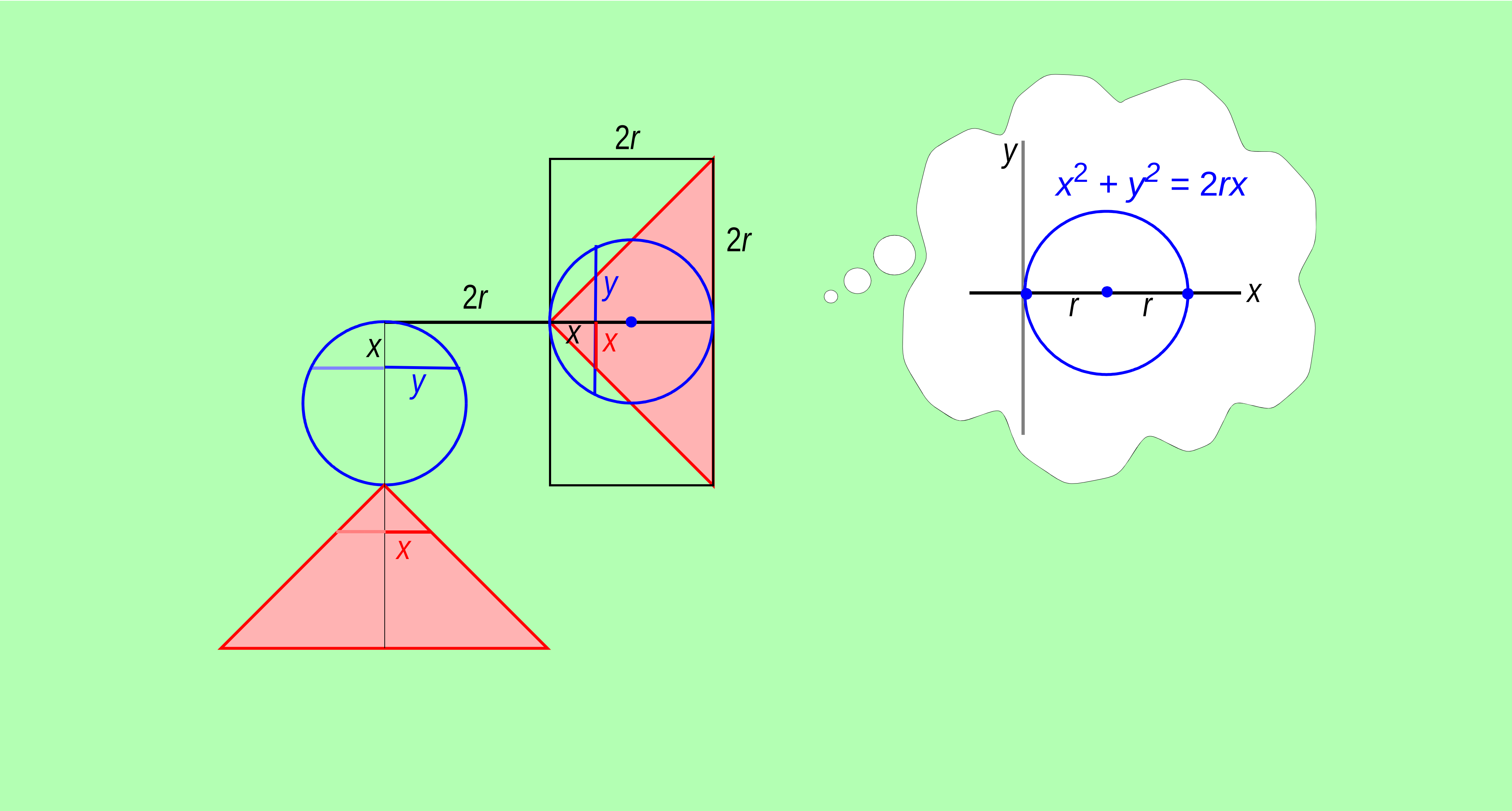
Volume cilinder = 3 × volume kegel



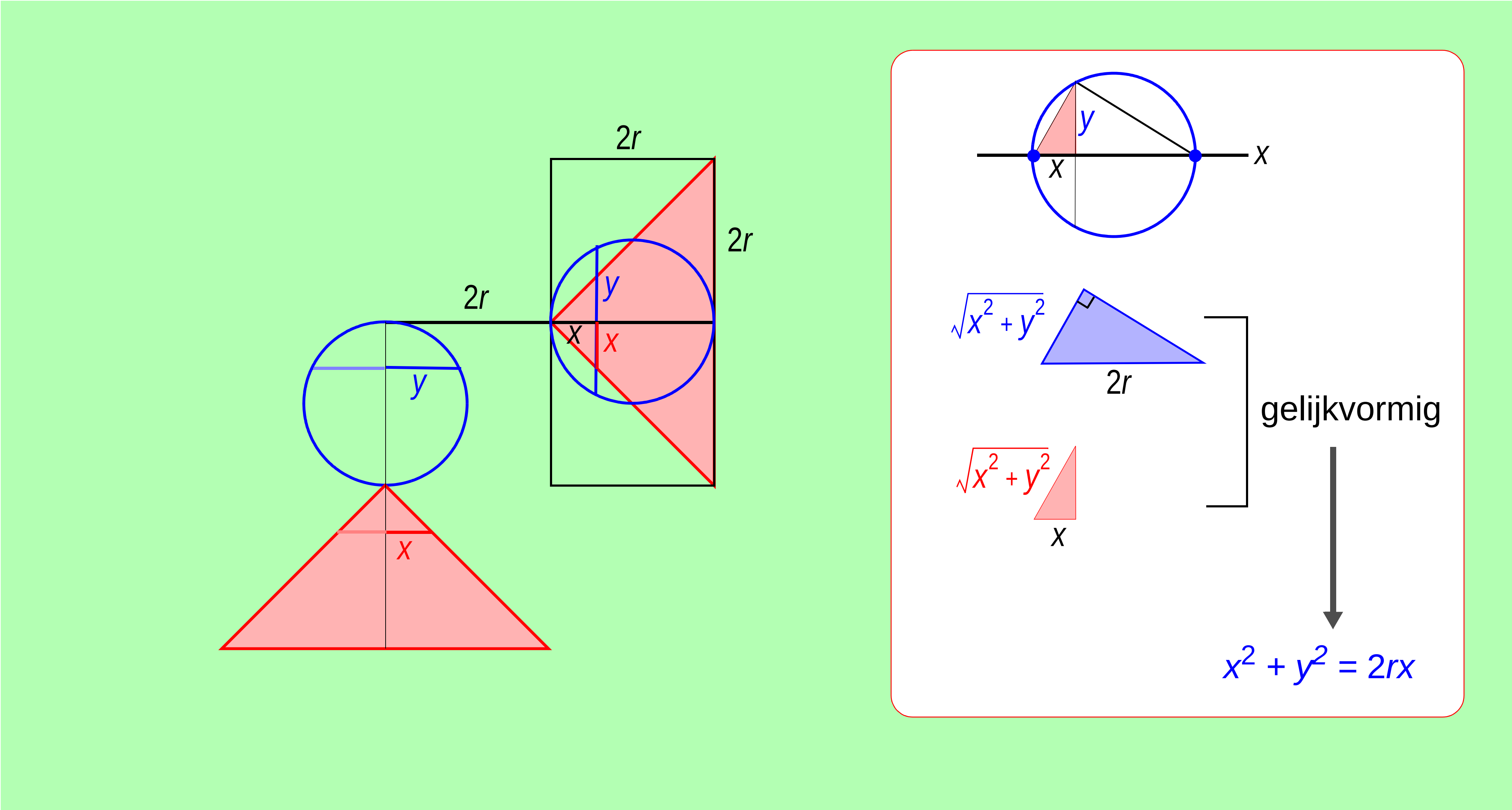
Archimedes ontdekt ......

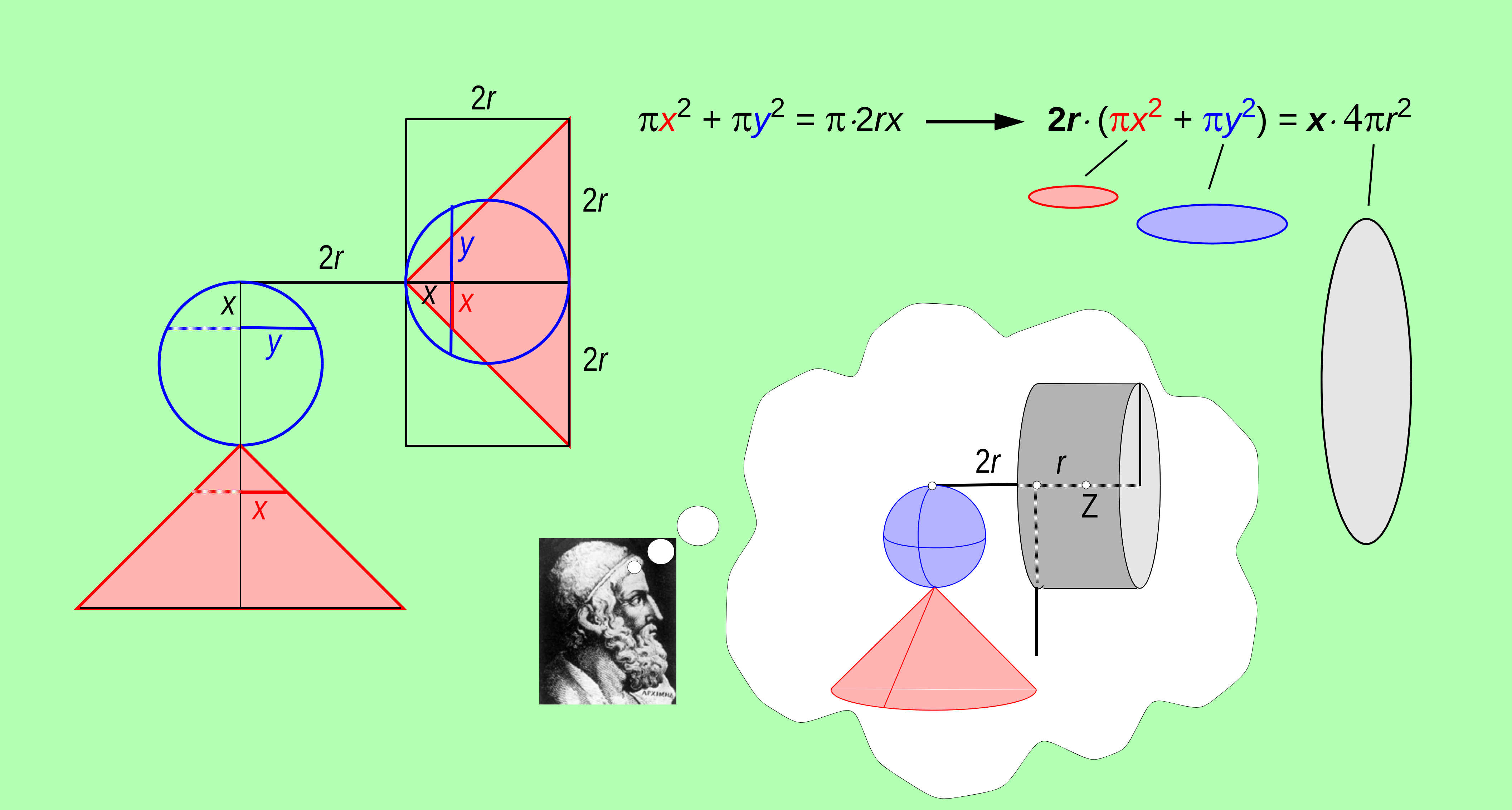


‘De Methode’ *propositie 2*

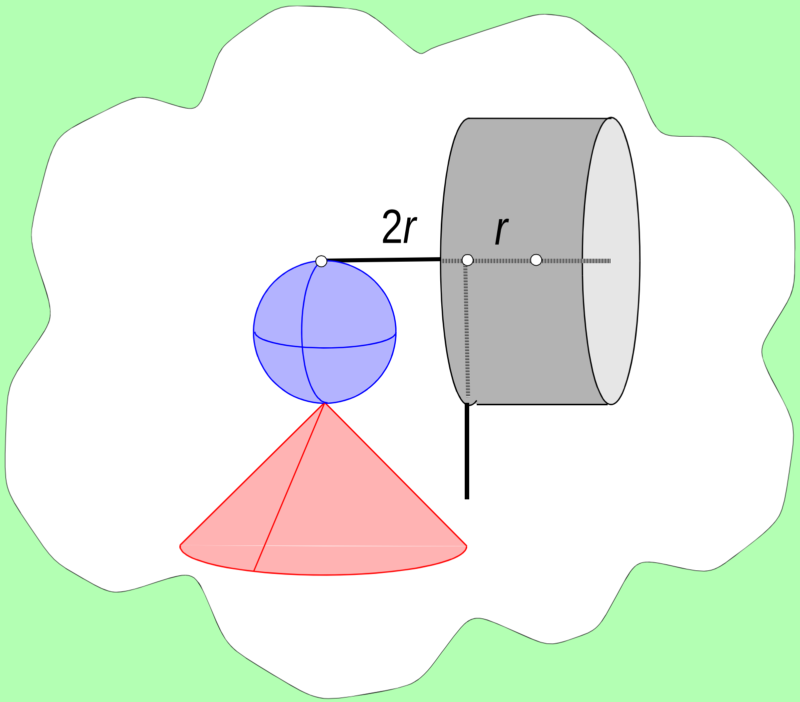


2r
2r
2r
x
y
x
y
x
x
y
x² + y² = 2rx
x
r
r


2r
2r
2r
y
x
x
y
x
y
x
x
√(x² + y²)
2r
√(x² + y²)
x
gelijkvormig
x² + y² = 2rx



$\pi x^2 + \pi y^2 = \pi \cdot 2rx \longrightarrow \mathbf{2r} \cdot (\pi x^2 + \pi y^2) = \boldsymbol{x} \cdot 4\pi r^2$
2r
2r
2r
2r
x
y
x
y
x
x
2r
r
Z

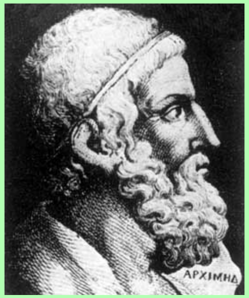


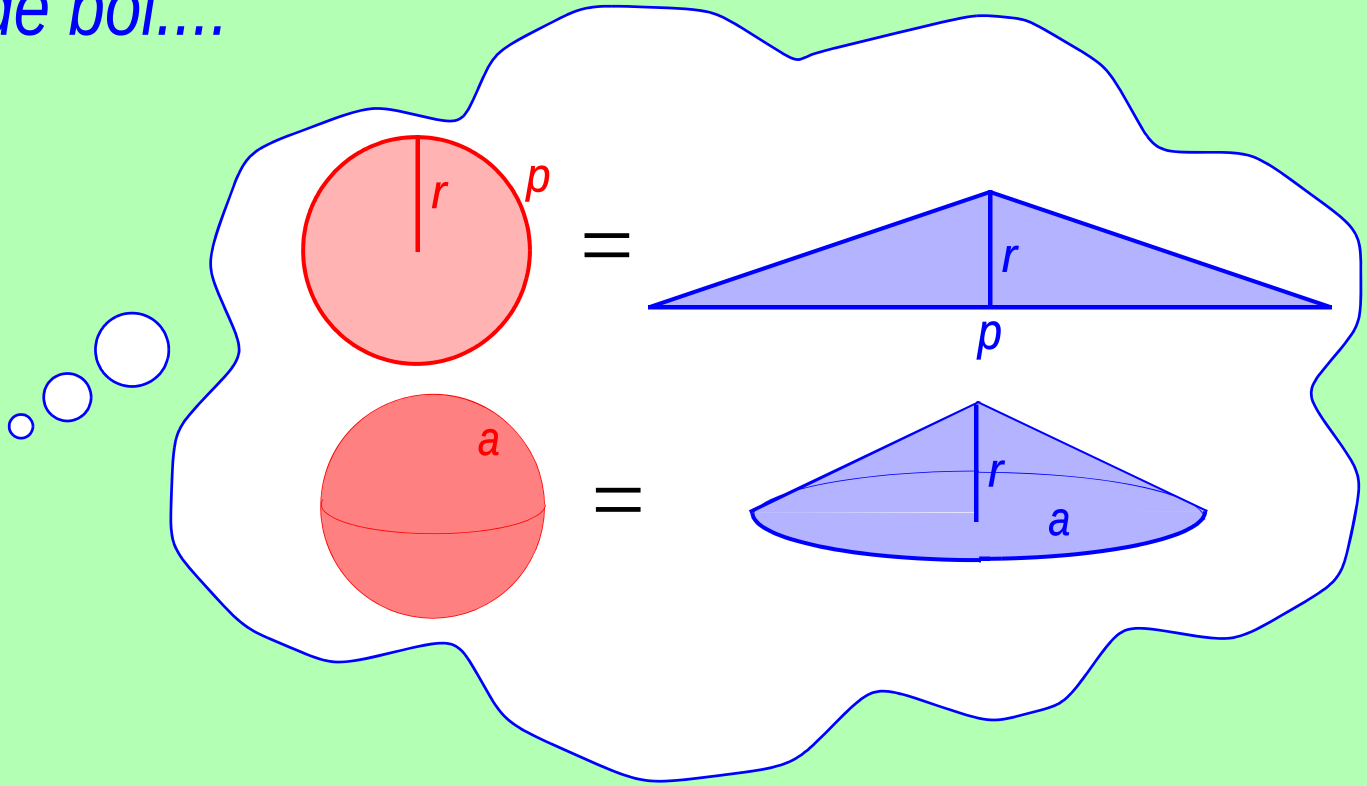
$$2r \cdot \left(? + \frac{8}{3}\pi r^3 \right) = r \cdot 8\pi r^3$$

↓

$$\frac{4}{3}\pi r^3$$

Toen dit was ingezien, ontstond het vermoeden dat de oppervlakte van elke bol het viervoud is van de grootste cirkel op de bol....

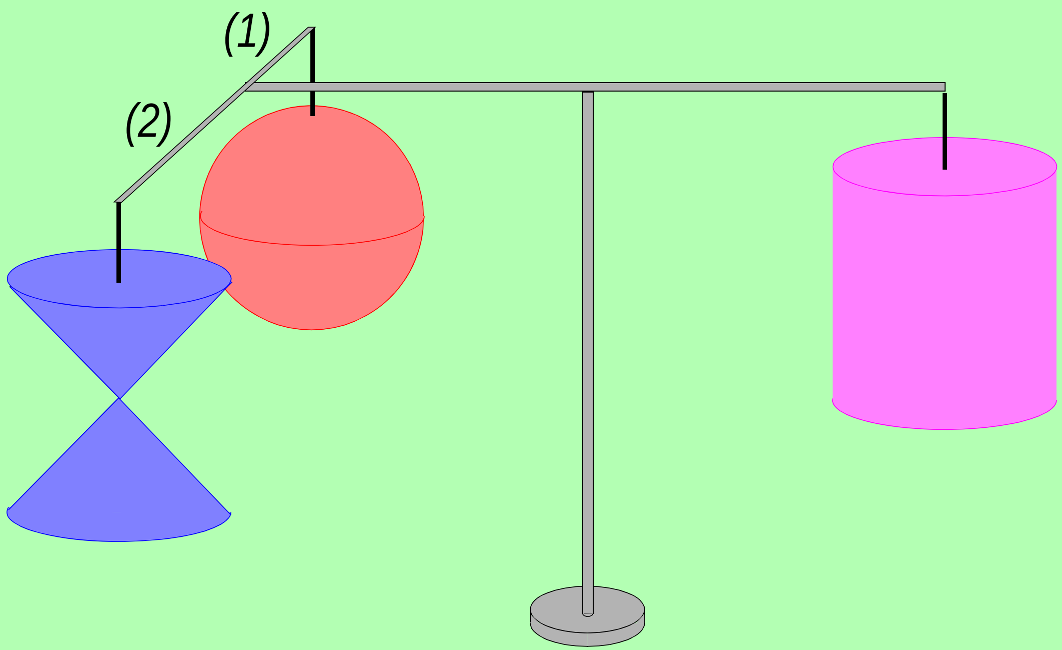


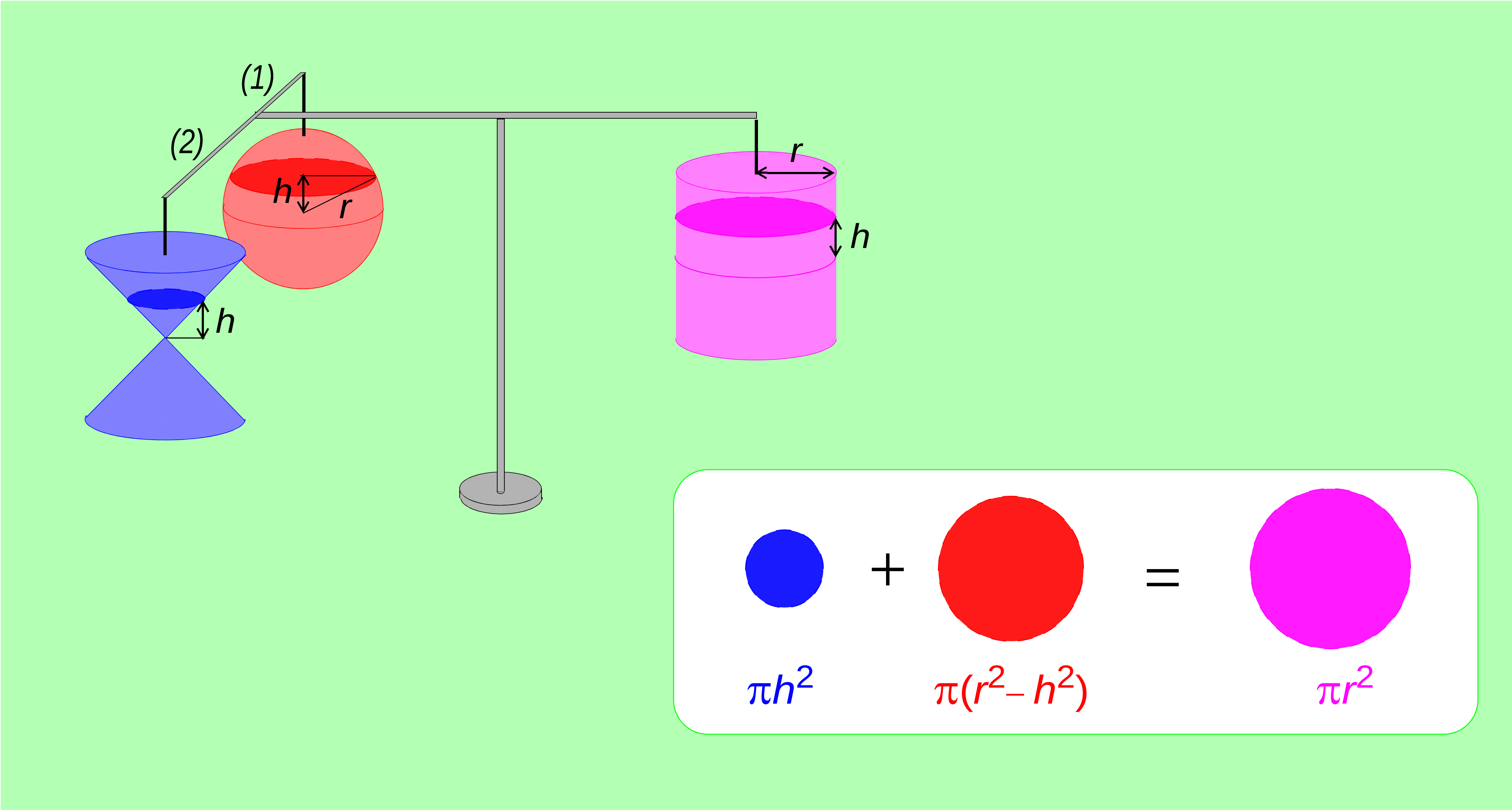


De onderstelling was namelijk dat zoals elke cirkel gelijk is aan een driehoek die de omtrek van de cirkel tot basis heeft en de hoogte gelijk aan de straal van de cirkel, zo ook elke bol gelijk is aan een kegel die de de oppervlakte van de bol tot basis heeft en de hoogte gelijk aan de straal van de bol.....

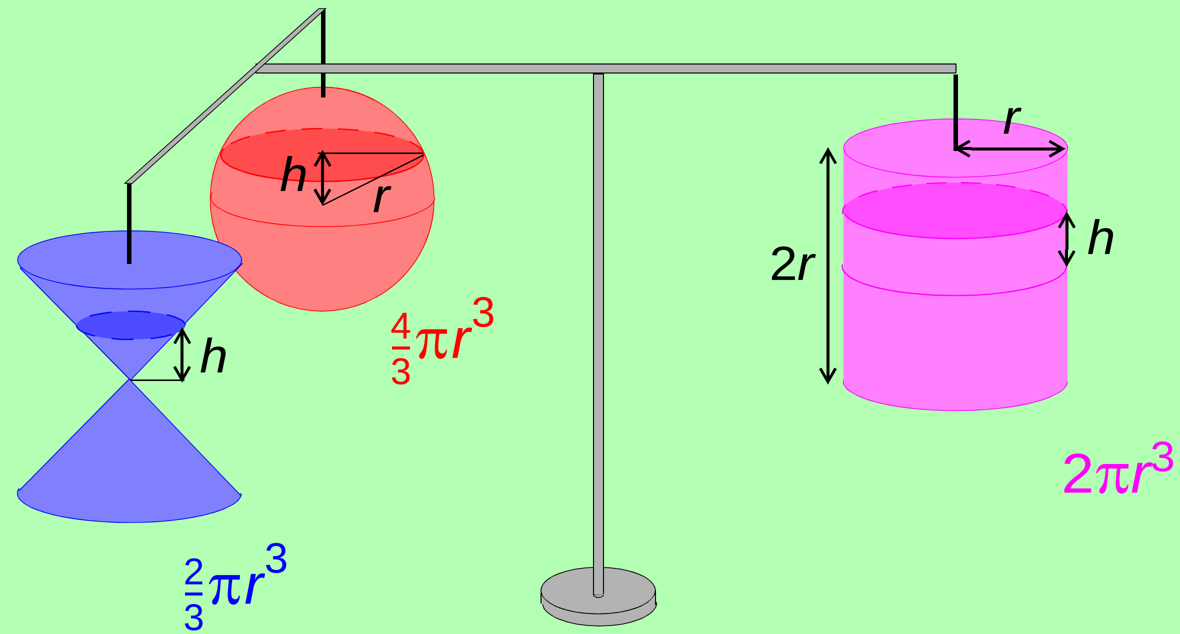
Door Archimedes bewezen in ‘Over bol en cilinder I’ (proposities 33/34)

Alternatief 'evenwichtsbewijs' .....



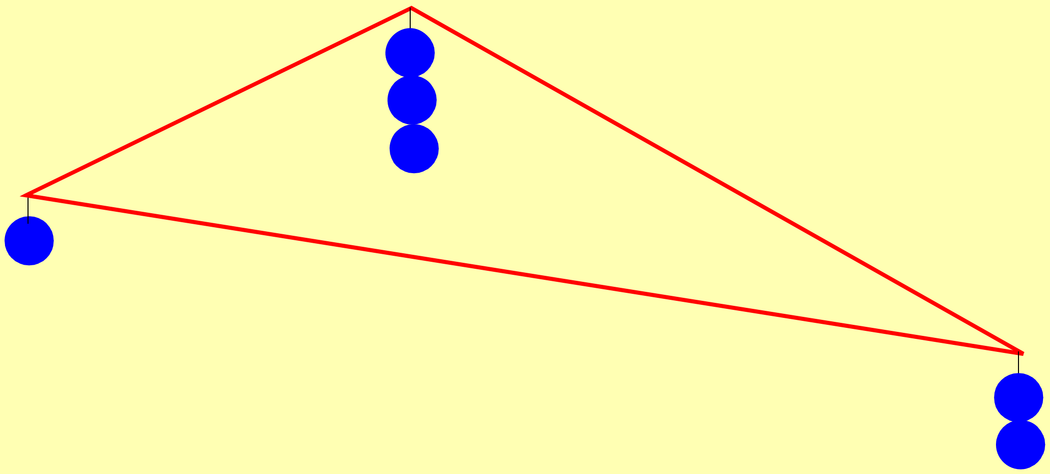


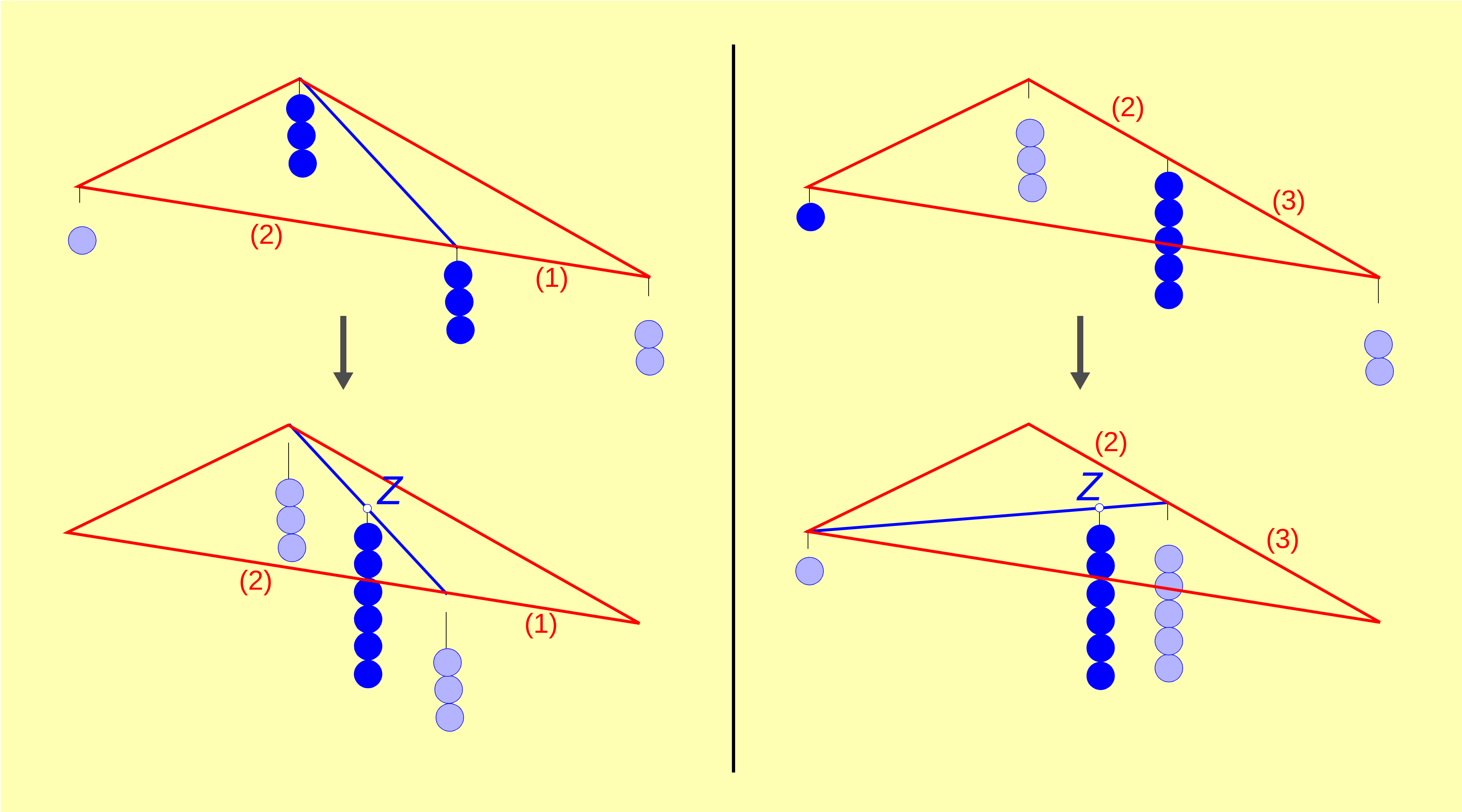
(1)
(2)
h
r
h
r
h
+
=
πh2
π(r2 – h2)
πr2

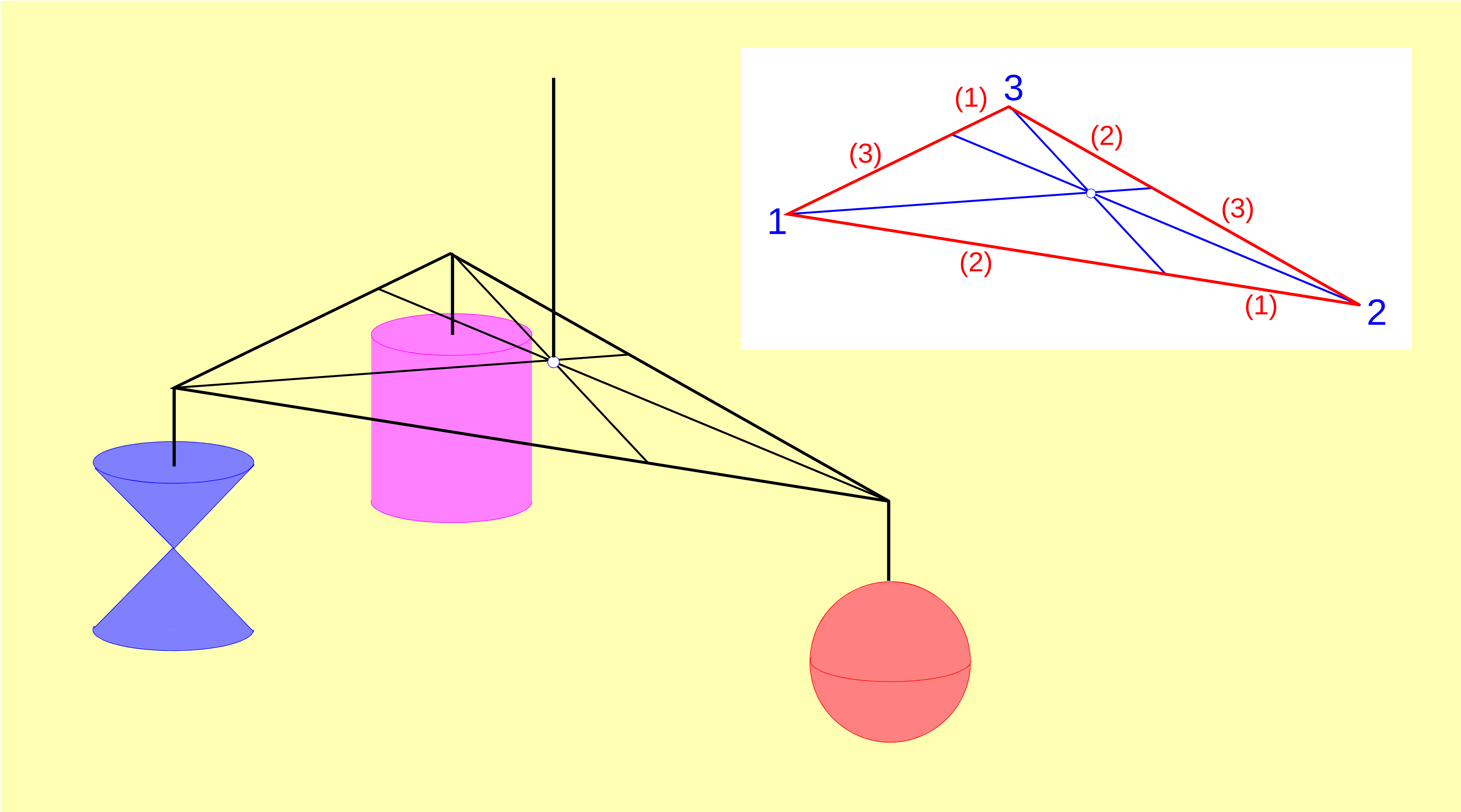


h
r
4/3 πr3
r
h
2r
h
2πr3
2/3 πr3

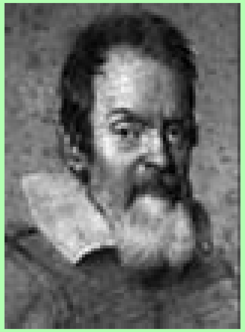
plaats zwaartepunt?




(2)
(1)
(2)
Z
(1)
(2)
(3)
(2)
Z
(3)



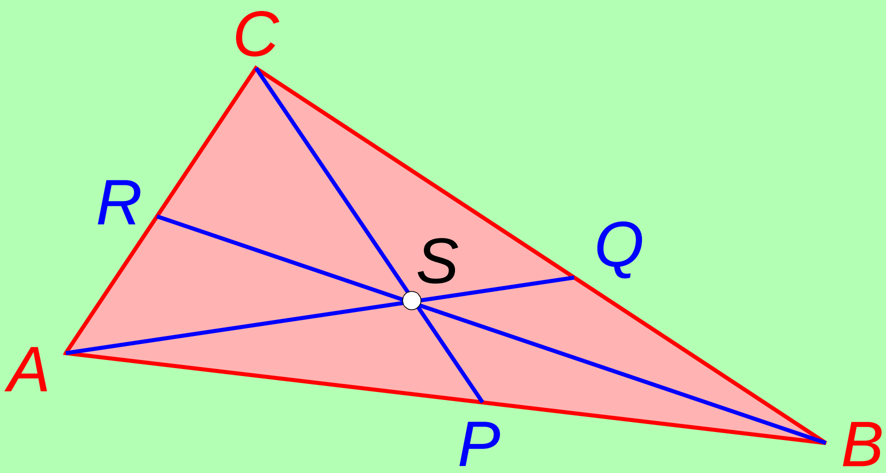
(1)
3
(2)
(3)
1
(3)
(2)
(1)
2



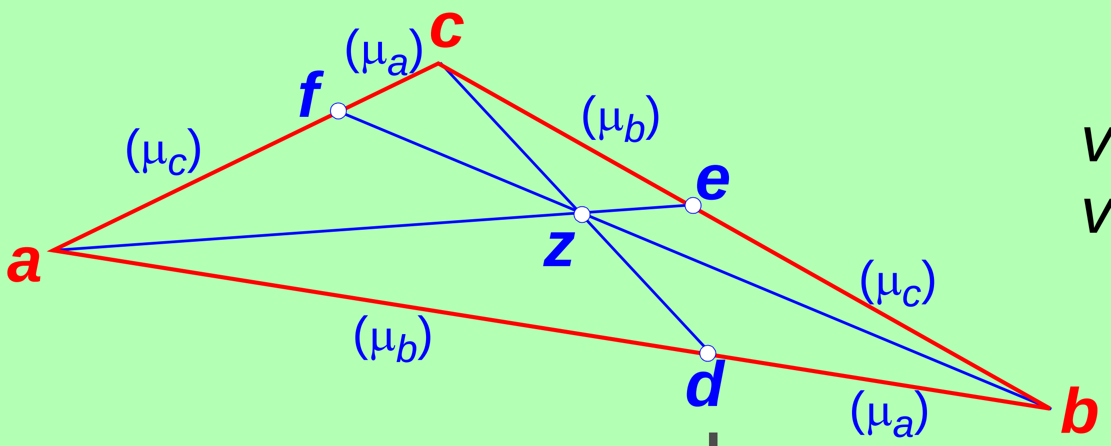
ca. 1734

Stelling van Ceva / Al Mu’taman

ca. 1083



AQ, *BR*, *CP* *concurrent* ⟷ $\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = 1$

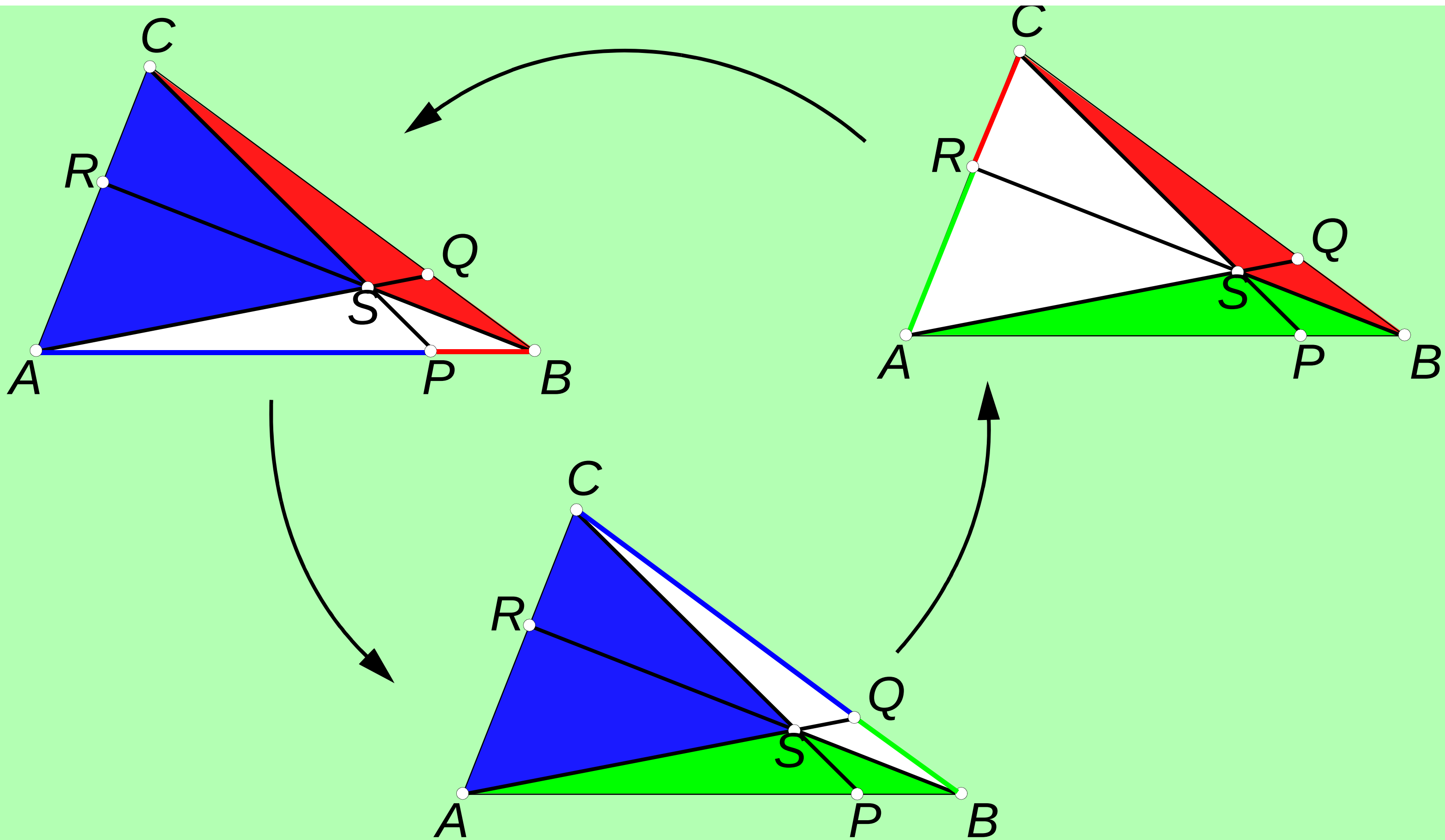


via gewogen gemiddelde
van vectoren ....

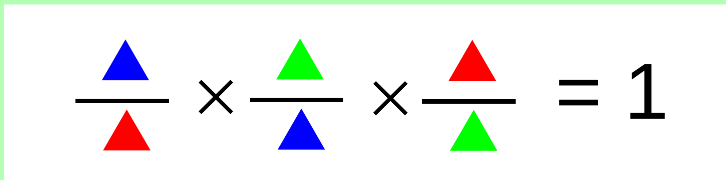
$$\mathbf{d} = \frac{\mu_a}{\mu_a + \mu_b}\mathbf{a} + \frac{\mu_b}{\mu_a + \mu_b}\mathbf{b}$$

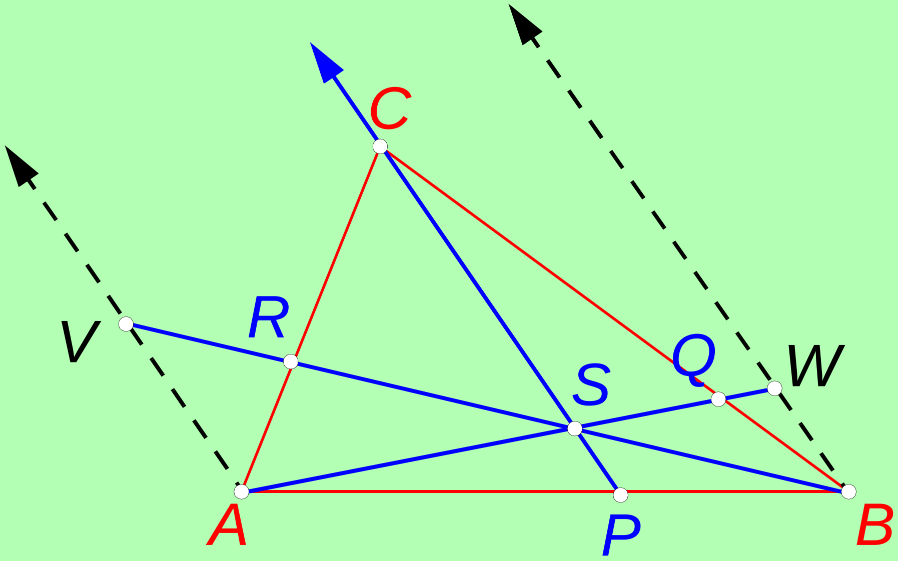
$$\mathbf{z} = \frac{\mu_a + \mu_b}{\mu_a + \mu_b + \mu_c}\mathbf{d} + \frac{\mu_c}{\mu_a + \mu_b + \mu_c}\mathbf{c} =$$

$$\frac{\mu_a}{\mu_a + \mu_b + \mu_c}\mathbf{a} + \frac{\mu_b}{\mu_a + \mu_b + \mu_c}\mathbf{b} + \frac{\mu_c}{\mu_a + \mu_b + \mu_c}\mathbf{c}$$



via oppervlakten ....





$$\frac{AP}{PB} = \frac{AV}{BW}$$

$$\frac{BQ}{QC} = \frac{BW}{CS}$$

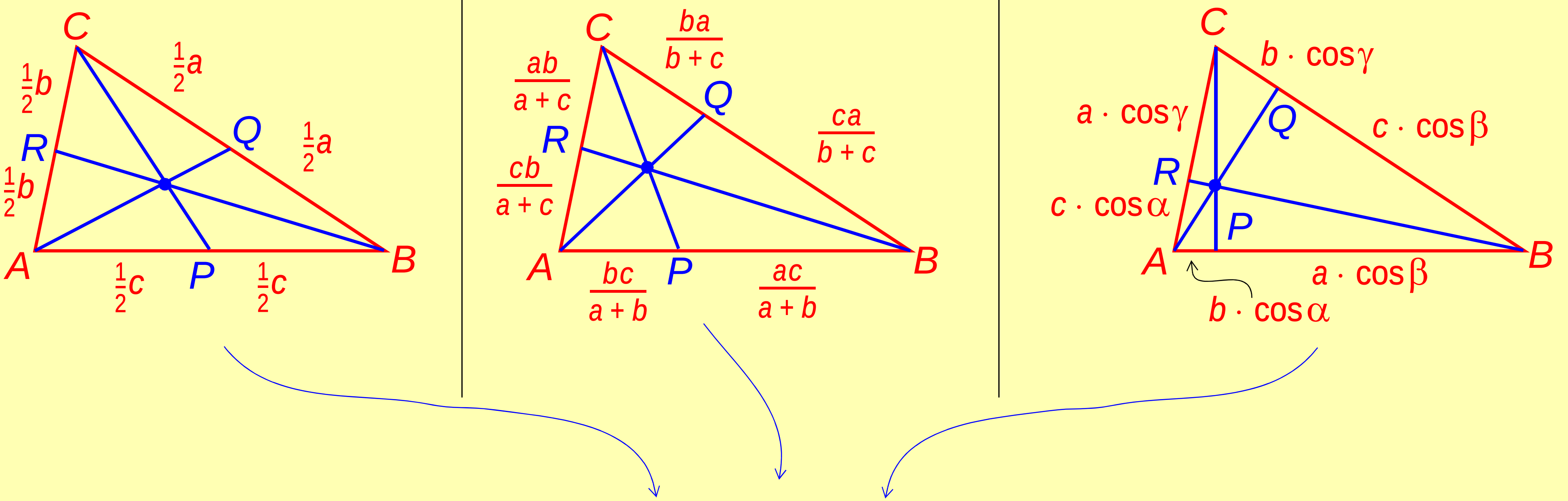
$$\frac{CR}{RA} = \frac{CS}{AV}$$

$$\times$$

$$\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = 1$$

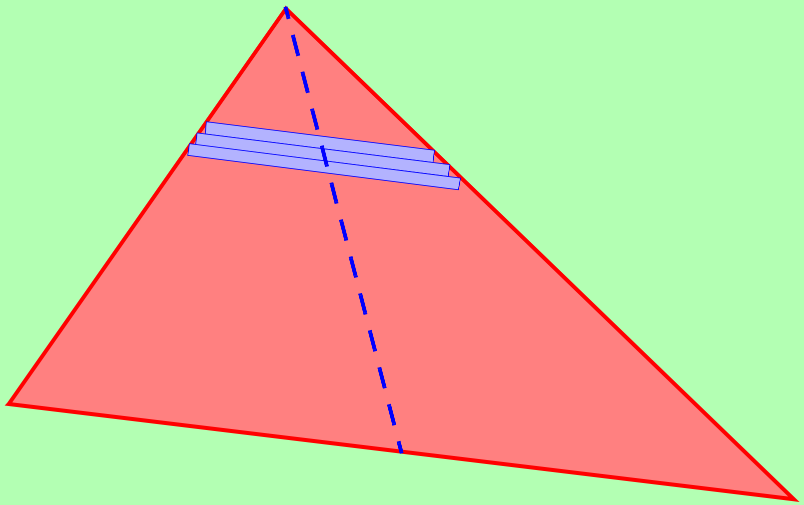
Bewijs van Al Mu'taman

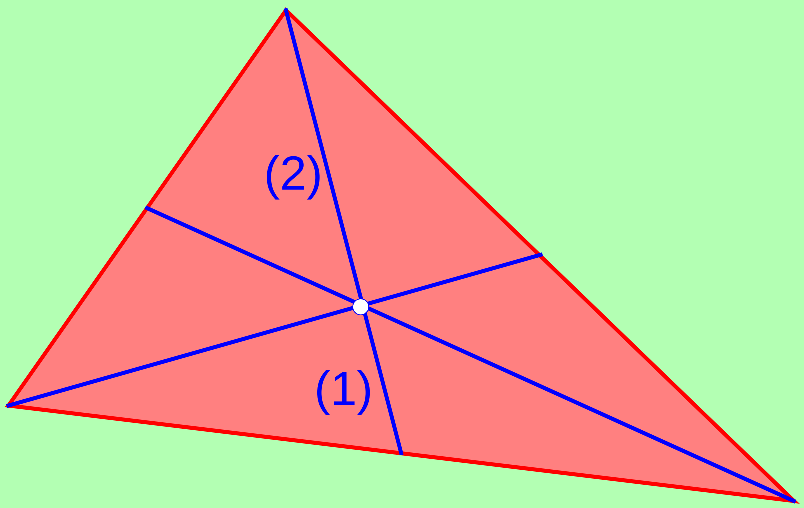
3 zwaartelijnen | 3 bissectrices | 3 hoogtelijnen



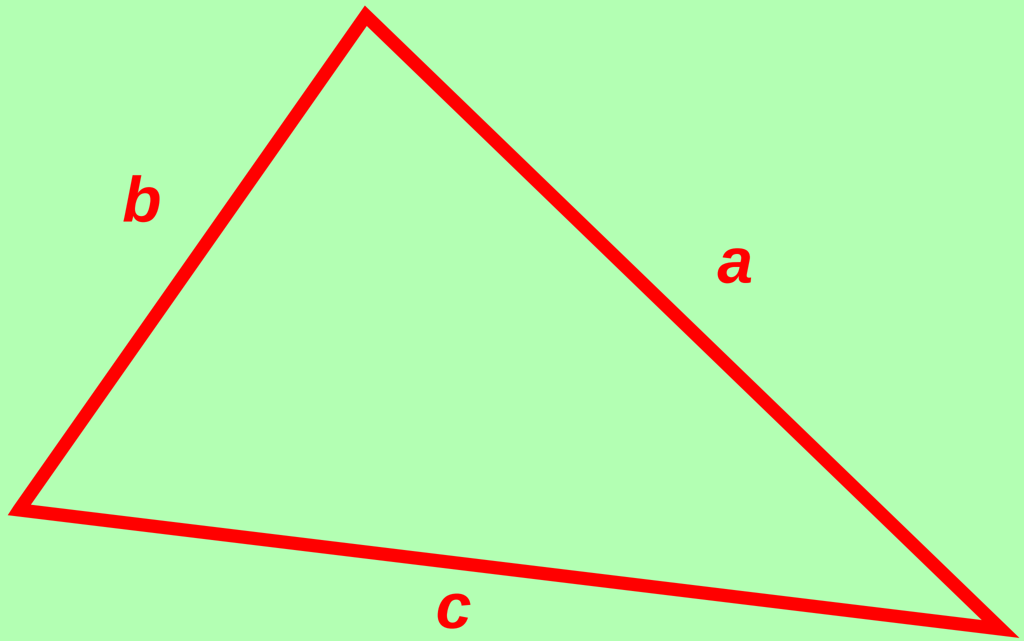
$$\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = 1$$

zwaartepunt massieve driehoek

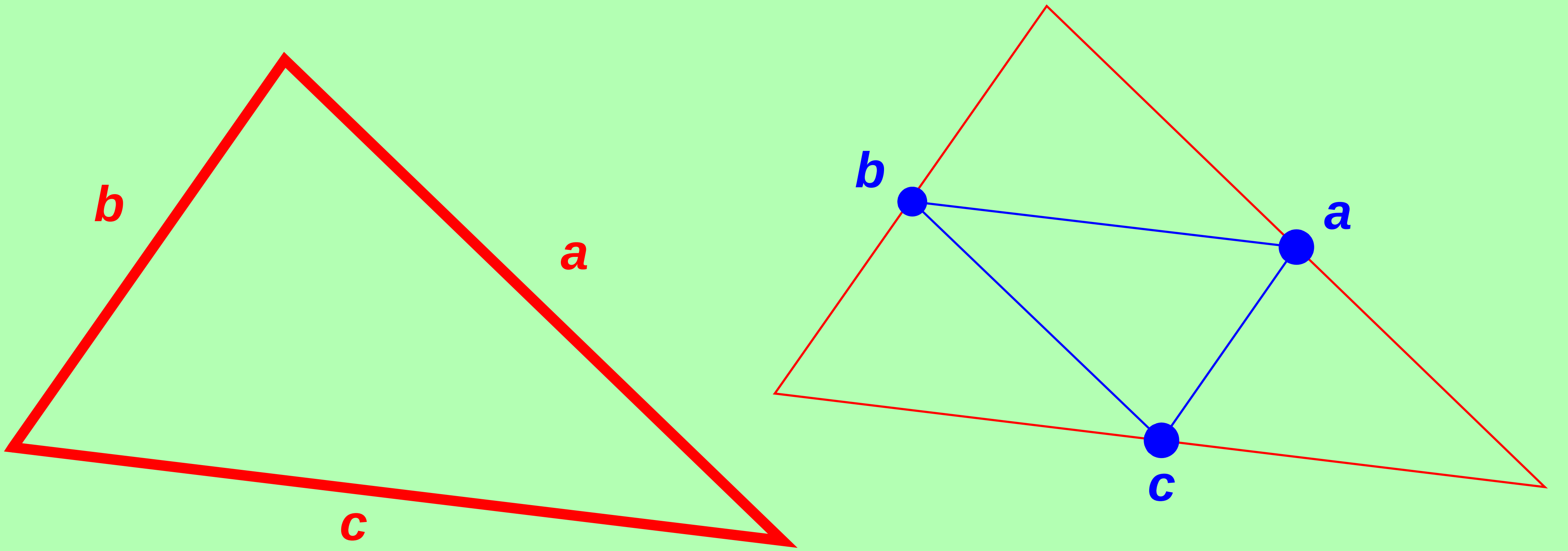




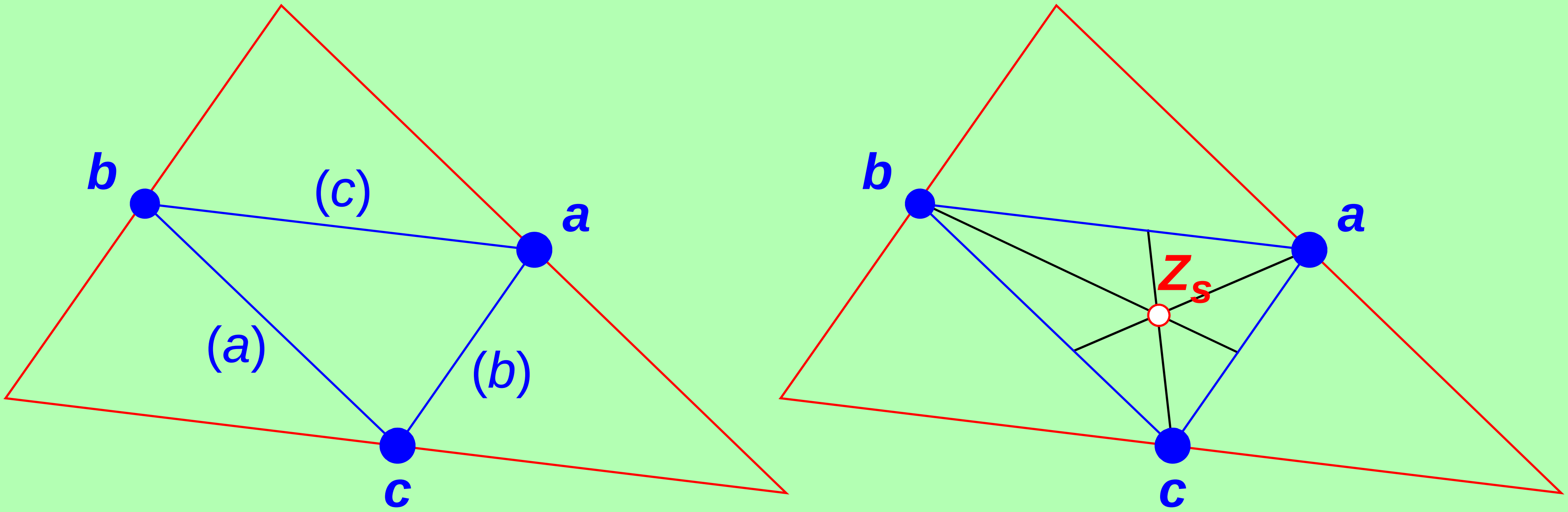
zwaartepunt stangendriehoek ?



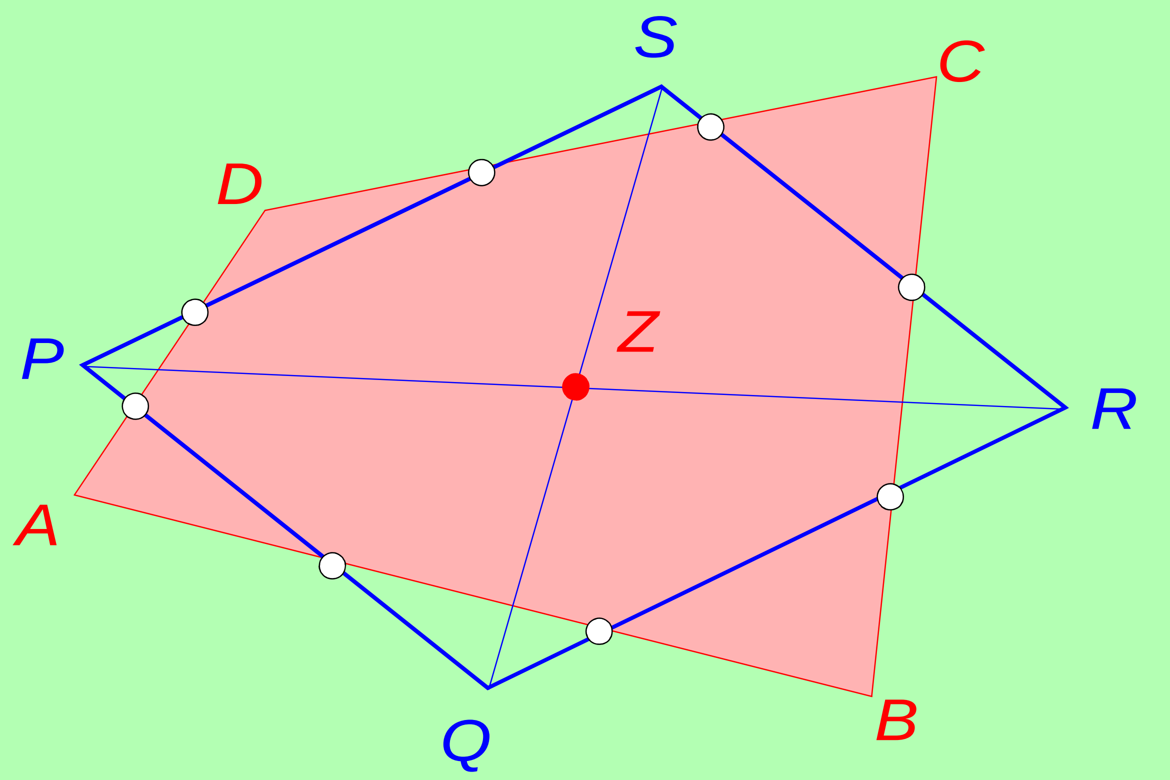
stangendriehoek

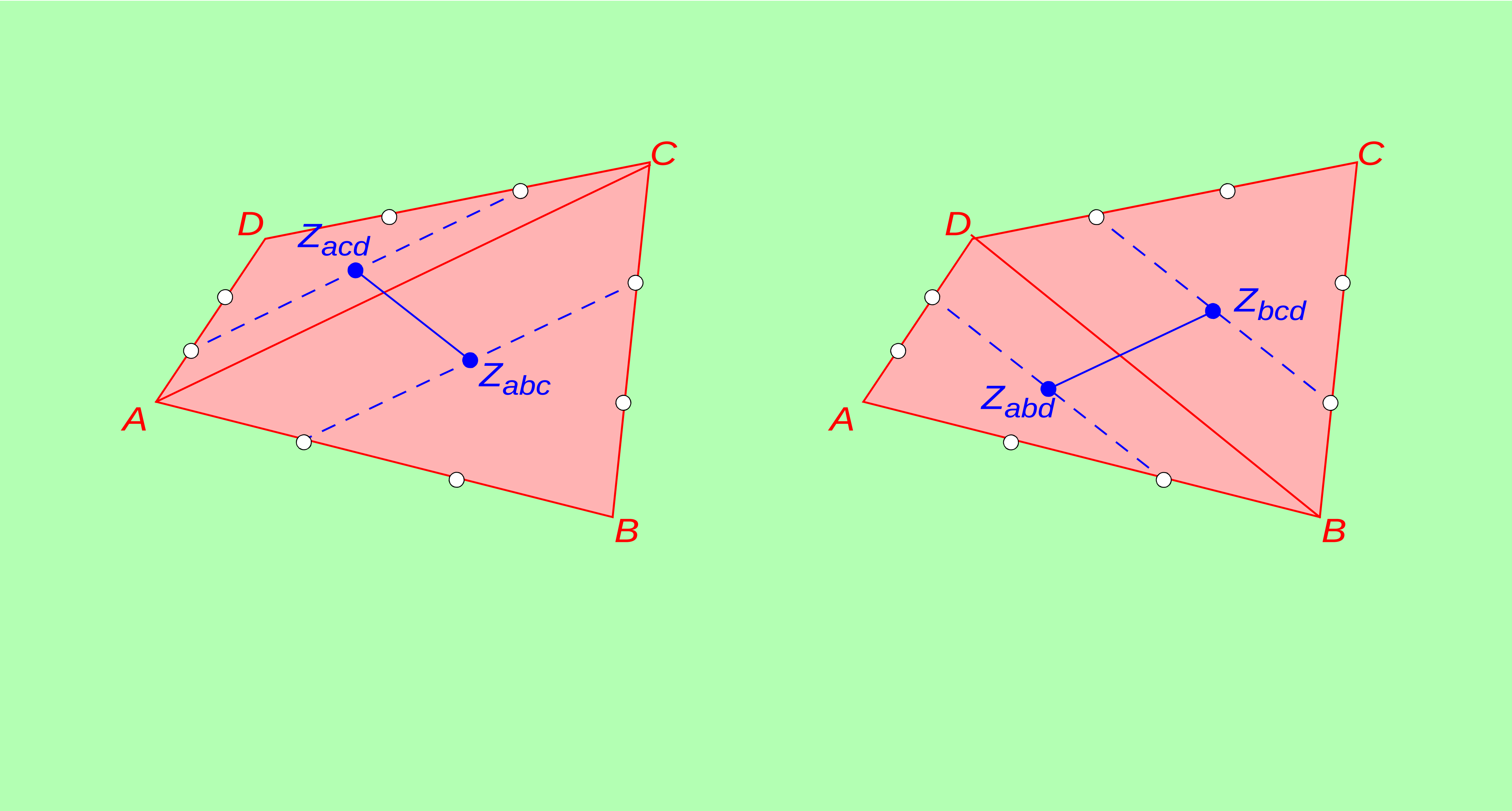


zwaartepunt stangendriehoek

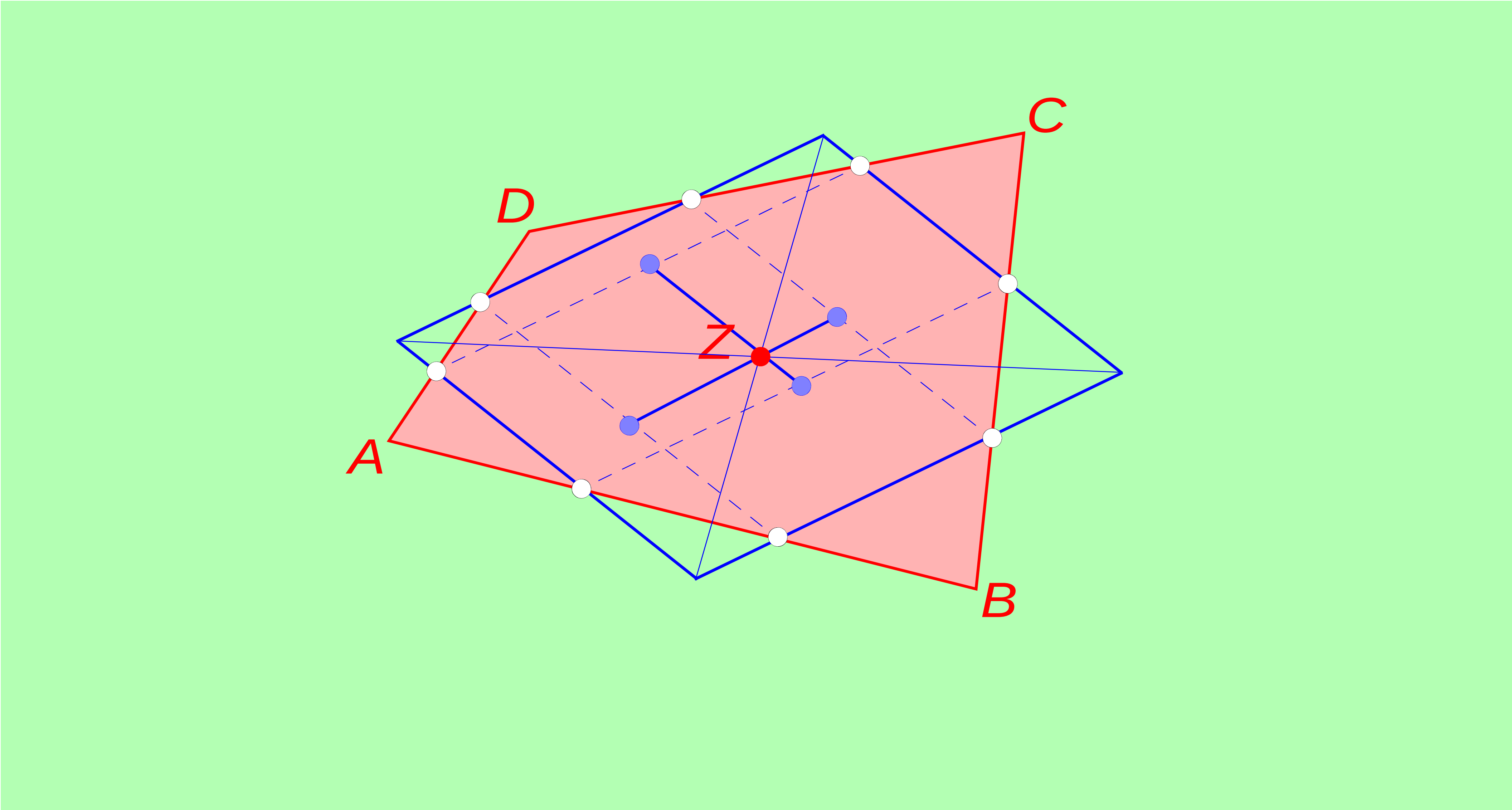


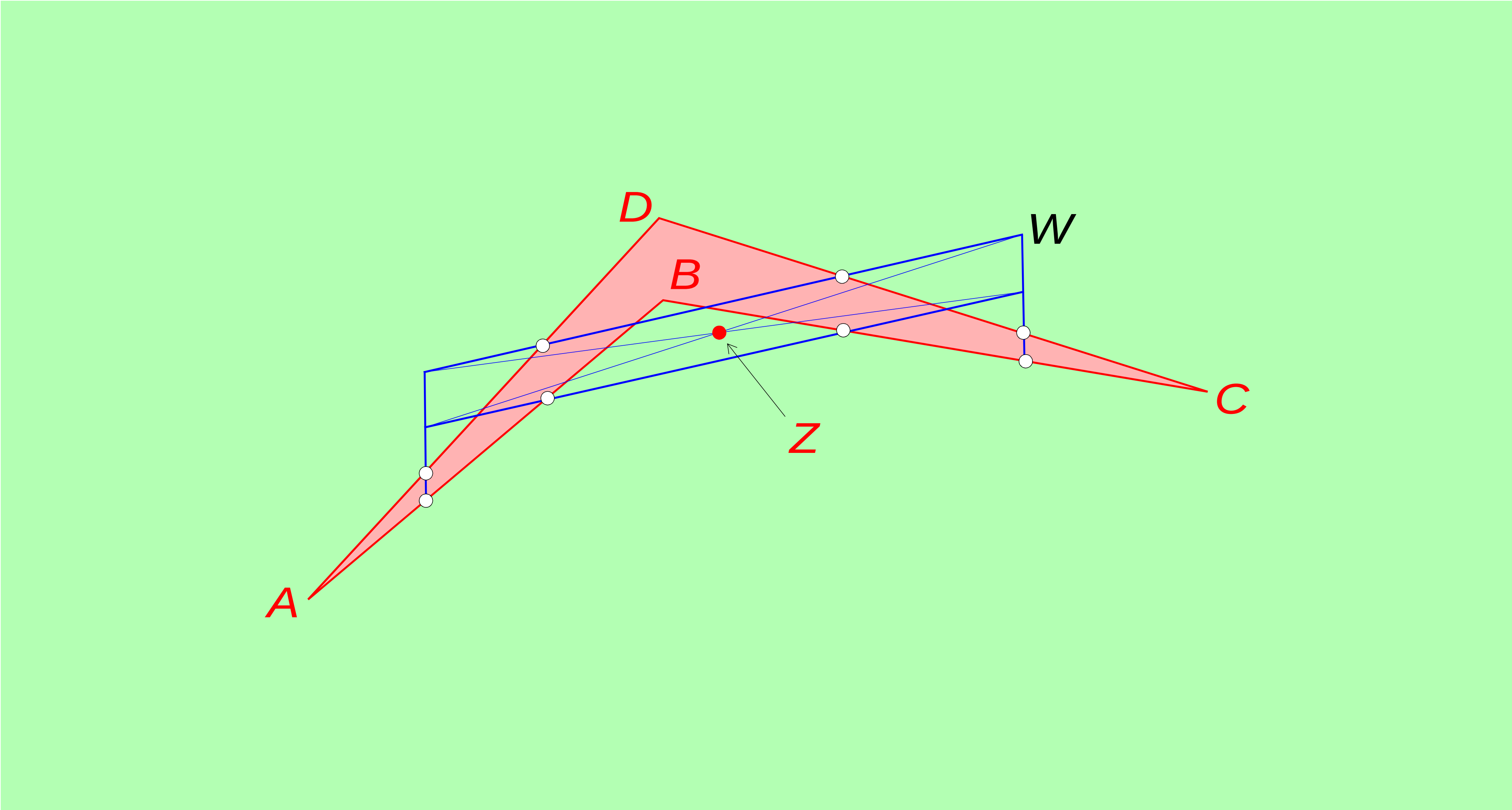
zwaartepunt massieve vierhoek

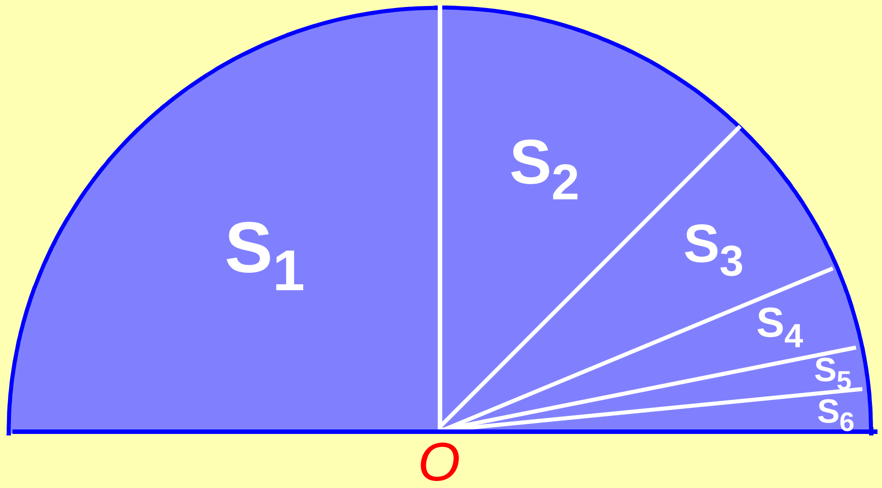


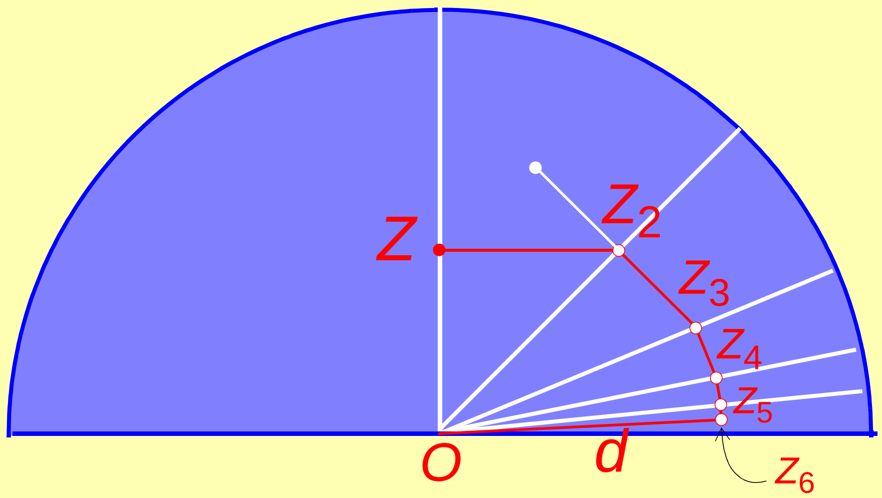


C
D
A
B
Z_acd
Z_abc
C
D
A
B
Z_bcd
Z_abd


C
D
Z
A
B


D
W
B
C
Z
A





Zwaartepunt halve cirkelschijf ?

Stel Z_6 *is zwaartepunt* S_6

↓ *projectie*

Z_5 *is zwaartepunt* $S_6 \cup S_5$

↓ *projectie*

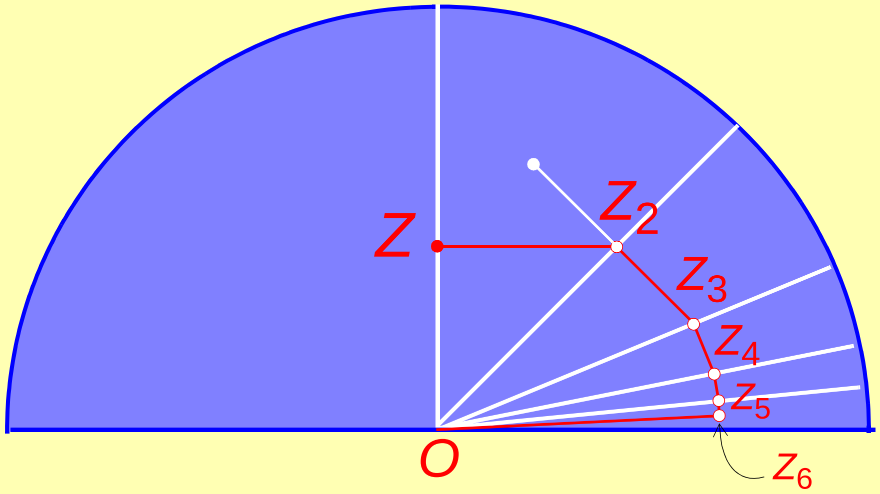
Z_4 *is zwaartepunt* $S_6 \cup S_5 \cup S_4$

↓ *projectie*

.....

↓ *projectie*

Z *is zwaartepunt halve cirkelschijf*



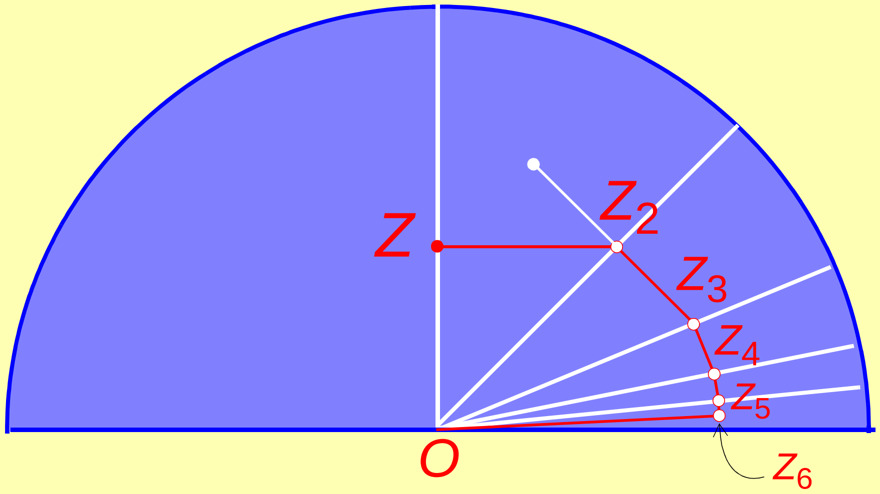
Stel $OZ_6 = z_6$

Dan $OZ_5 = z_6 \cdot \cos(\pi/64)$

Dan $OZ_4 = z_6 \cdot \cos(\pi/64) \cdot \cos(\pi/32)$

....

Dan $OZ = z_6 \cdot \cos(\pi/64) \cdot \cos(\pi/32) \cdot \ldots\ldots\ldots \cdot \cos(\pi/4)$



$$\sin\alpha \cdot \cos\alpha = 2^{-1}\sin 2\alpha$$

Stel $OZ_6 = z_6$

Dan $OZ_5 = z_6 \cdot \cos(\pi/64)$

Dan $OZ_4 = z_6 \cdot \cos(\pi/64) \cdot \cos(\pi/32)$

....

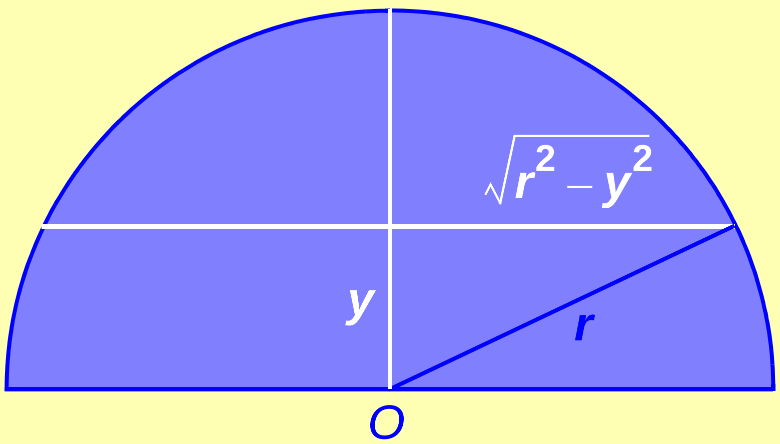
Dan $OZ = z_6 \cdot \cos(\pi/64) \cdot \cos(\pi/32) \cdot \ldots\ldots. \cdot \cos(\pi/4)$

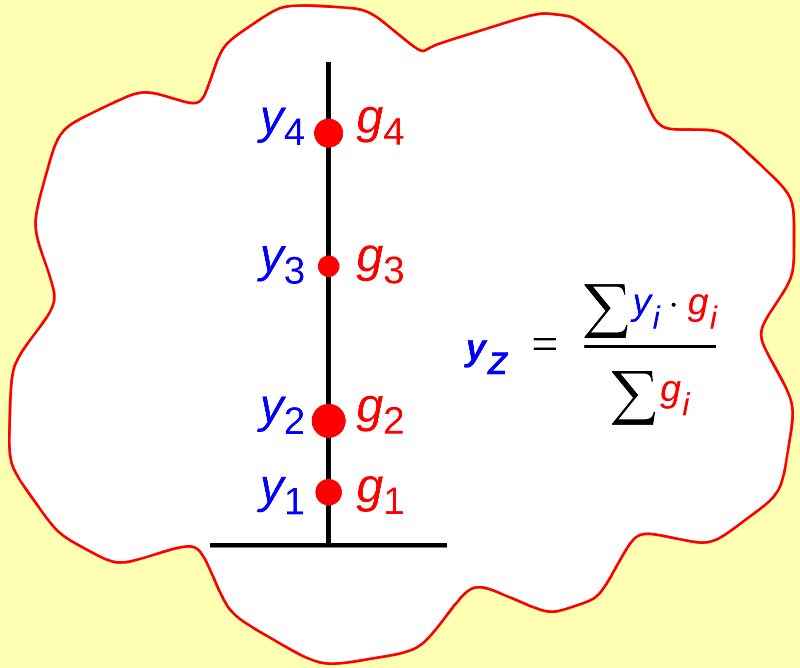
$$OZ \cdot \sin(\pi / 64) = z_6 \cdot 2^{-5}$$

$$\sin(\pi / 64) \approx \pi / 64$$

$$z_6 \approx \tfrac{2}{3} r$$

$$OZ \approx \frac{4}{3\pi} \cdot r$$





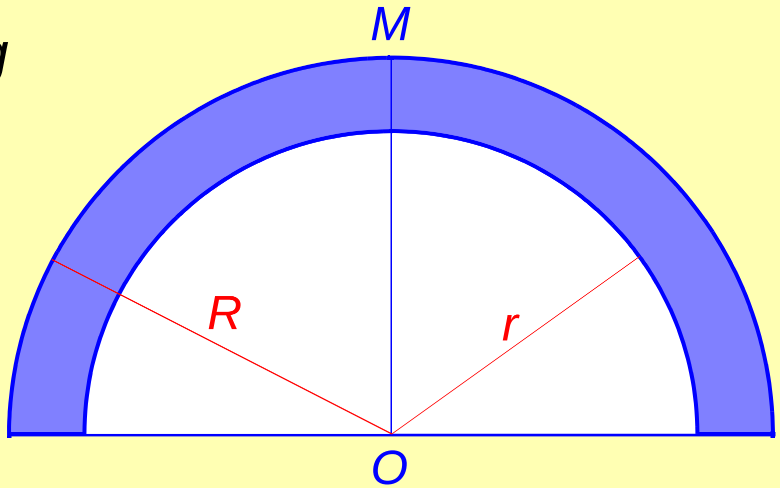
$$y_Z = \frac{\int_0^r 2y\sqrt{r^2-y^2}\,\mathrm{d}y}{\int_0^r 2\sqrt{r^2-y^2}\,\mathrm{d}y}$$

$$\int_0^r 2y\sqrt{r^2-y^2}\,\mathrm{d}y \longrightarrow \left[-\frac{2}{3}\left(r^2-y^2\right)^{3/2}\right]_0^r \longrightarrow \frac{2}{3}r^3$$

$$\int_0^r 2\sqrt{r^2-y^2}\,\mathrm{d}y \longrightarrow \frac{1}{2}\pi r^2$$

$$\longrightarrow \frac{4}{3\pi}r$$

zwaartepunt halve ring ligt op OM, zeg op hoogte y.

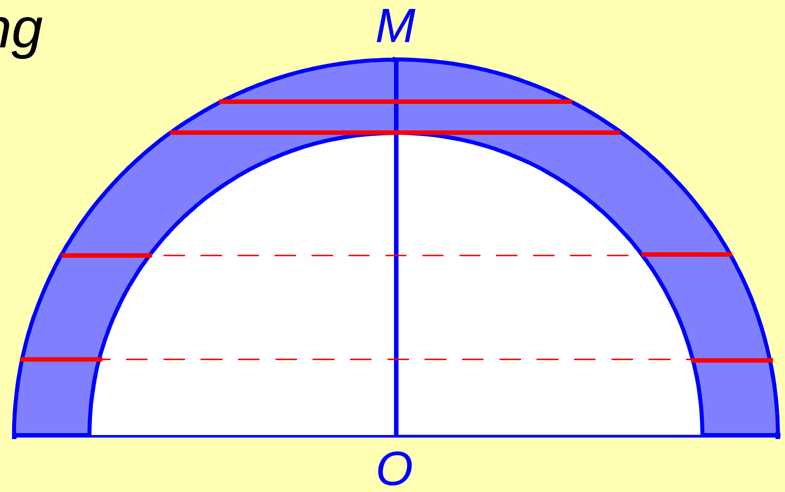


$$\frac{R^2-r^2}{R^2}\cdot y+\frac{r^2}{R^2}\cdot\frac{4}{3\pi}r=\frac{4}{3\pi}R$$

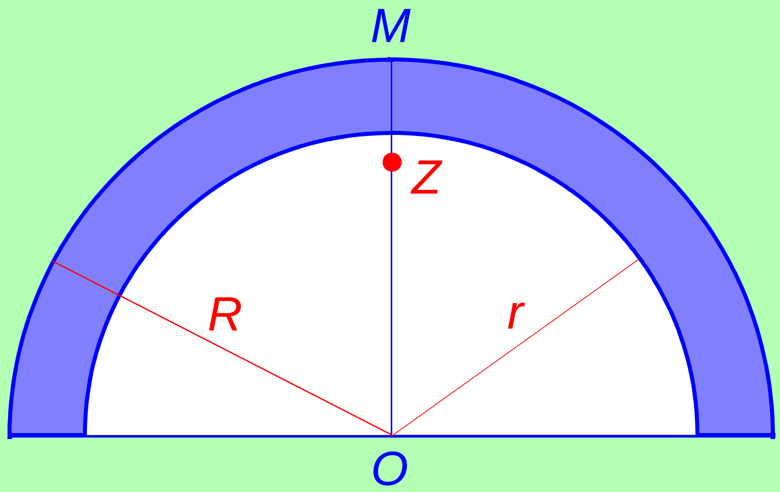
$$(R^2-r^2)\cdot y=\frac{4}{3\pi}(R^3-r^3)$$

$$y=\frac{4}{3\pi}\cdot\left(\frac{R^2+Rr+r^2}{R+r}\right)$$

of met integraalrekening



$$\frac{\int_0^r 2y\left(\sqrt{R^2-y^2}-\sqrt{r^2-y^2}\right)\,\mathrm{d}y \;+\; \int_r^R 2y\sqrt{R^2-y^2}\,\mathrm{d}y}{\frac{1}{2}\pi(R^2-r^2)} = \frac{4}{3\pi}\cdot\left(\frac{R^2+Rr+r^2}{R+r}\right)$$



$$y_Z = \frac{4}{3\pi} \cdot \left(\frac{R^2 + Rr + r^2}{R + r} \right)$$

$$r = 0{,}8R$$

$$y_Z \approx 0{,}72r$$

Bij welke verhouding van r en R ligt Z buiten de halve ring?

$$\frac{4}{3\pi}\cdot\left(\frac{R^2+Rr+r^2}{R+r}\right) < r$$

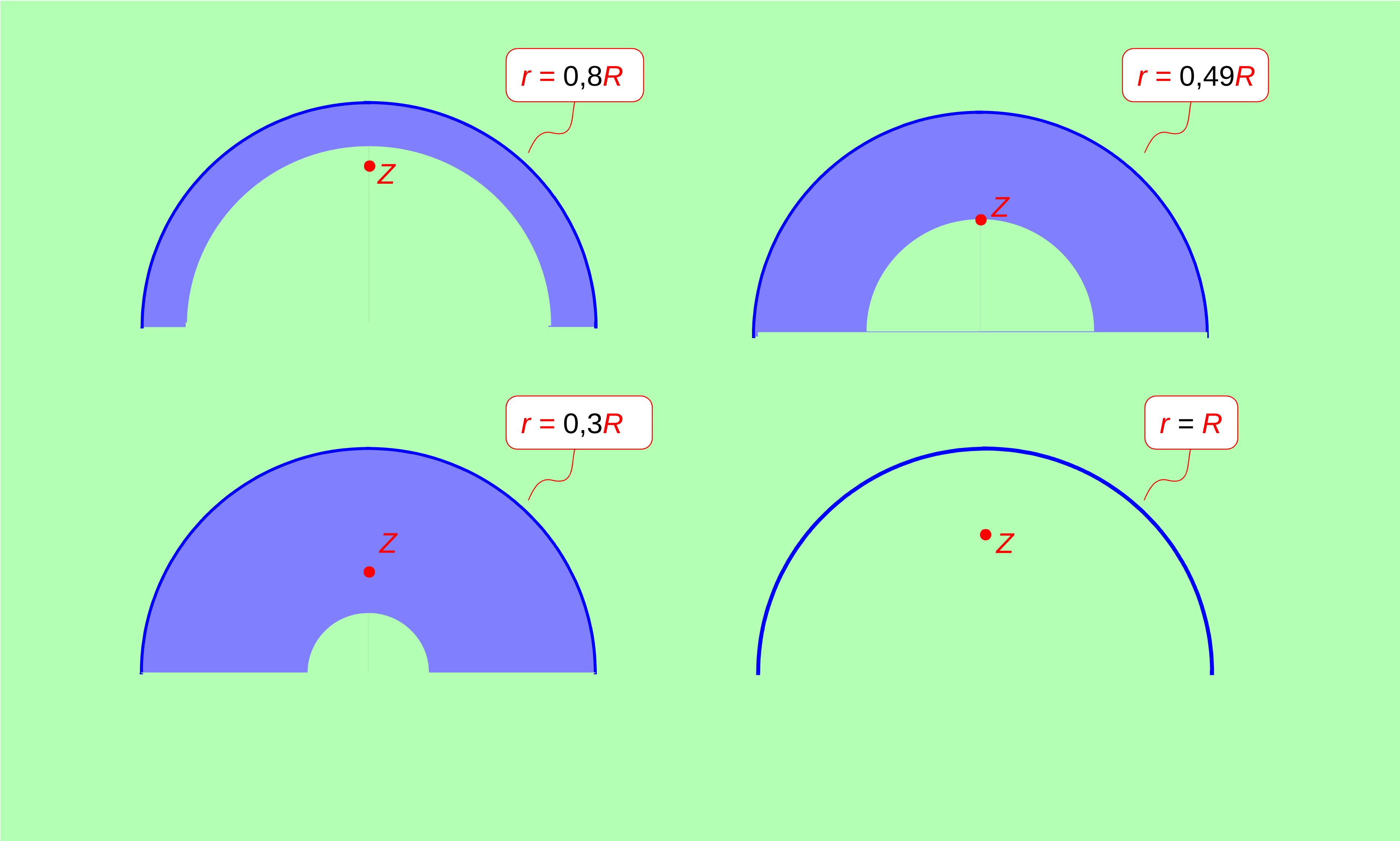
$$\Updownarrow$$

$$(3\pi-4)r^2+(3\pi-4)Rr-4R^2>0$$

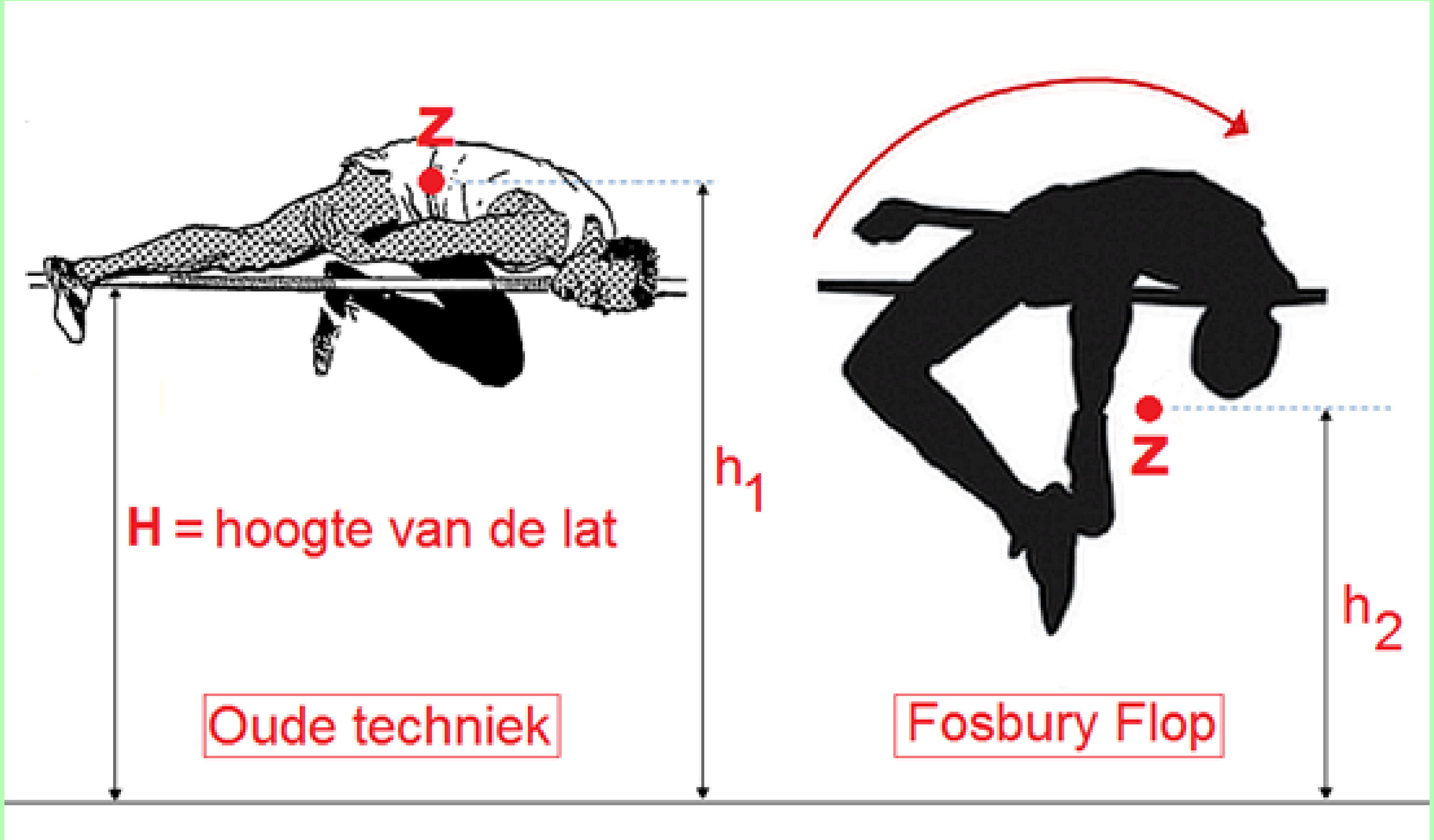
$$\Updownarrow$$

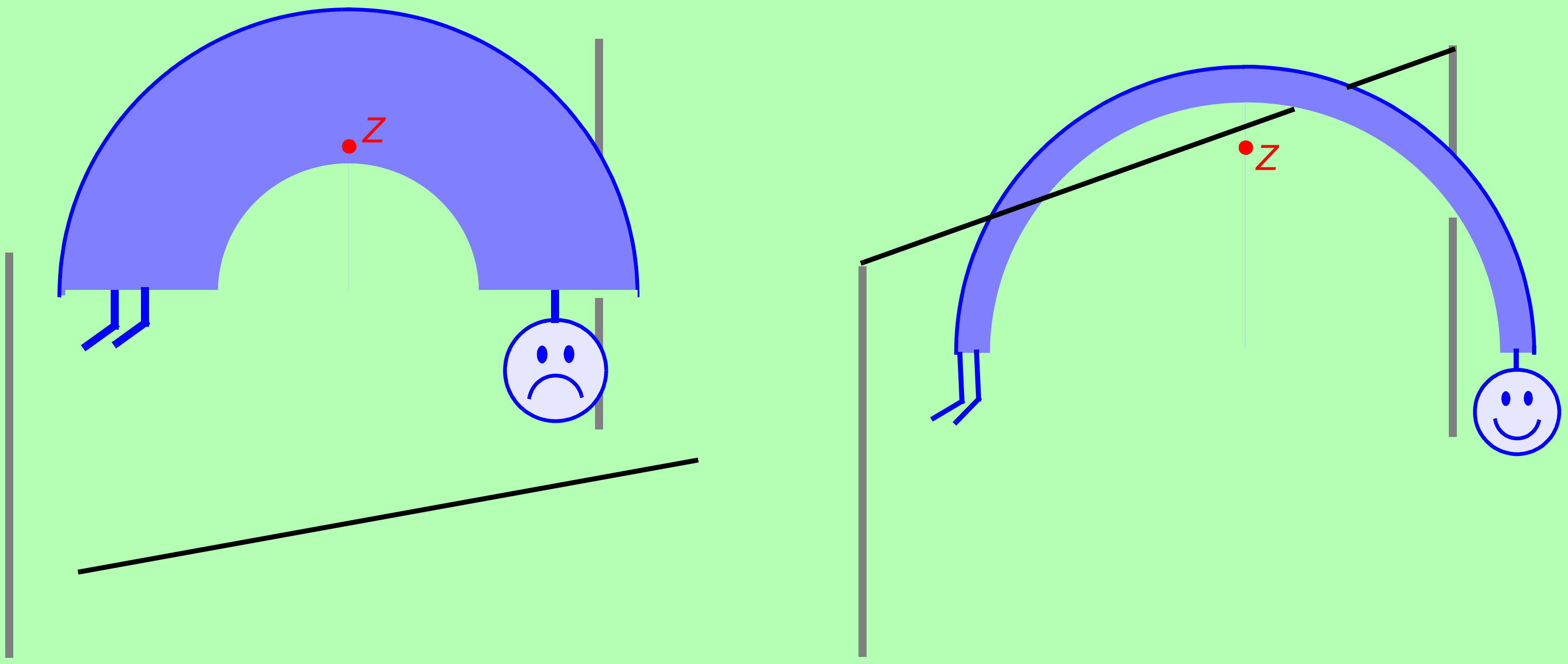
$$\left(\frac{r}{R}+\frac{1}{2}\right)^2>\frac{1}{4}+\frac{4}{3\pi-4}$$

grenswaarde 0.49365857....



r = 0,8R
Z
r = 0,49R
Z
r = 0,3R
Z
r = R
Z


Z
h1
H = hoogte van de lat
Z
h2
Oude techniek
Fosbury Flop



EINDE