

# Wavelets

Basics: algorithm

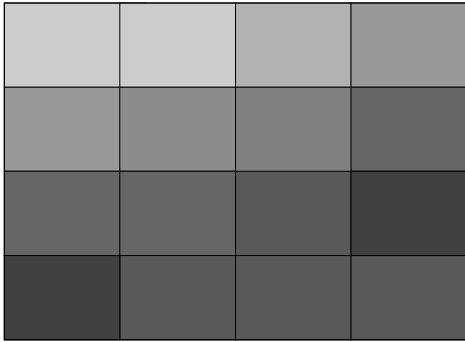
mathematical properties

Application to painting analysis

# Wavelets

illustrated via image analysis

# Digital images consist of pixels



Small squares, each with constant  
grey value

Typically: 256 different grey values  
(from pure white to  
pure black)

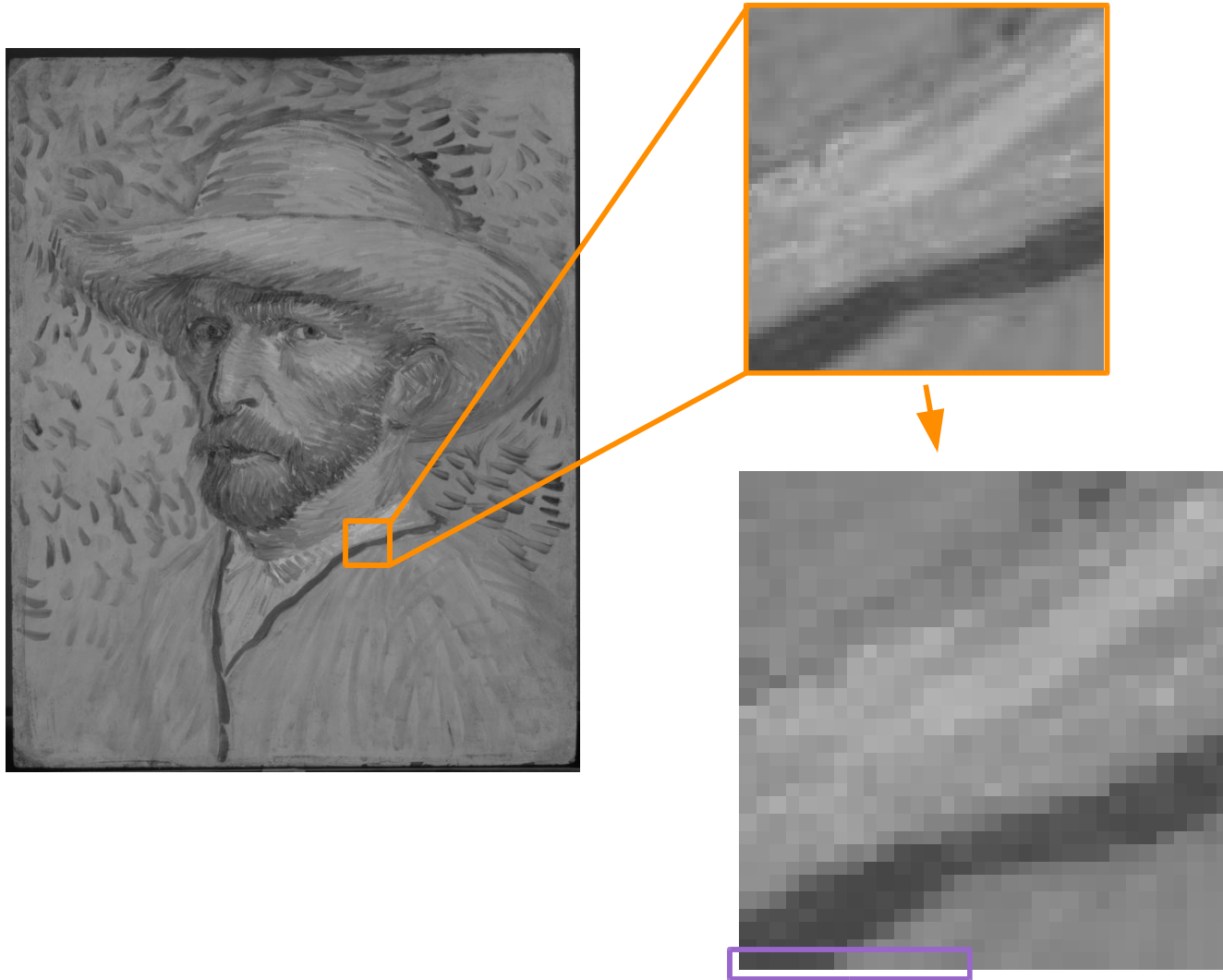
numbered from 0 to 255

# Example: a row in a self-portrait by Van Gogh



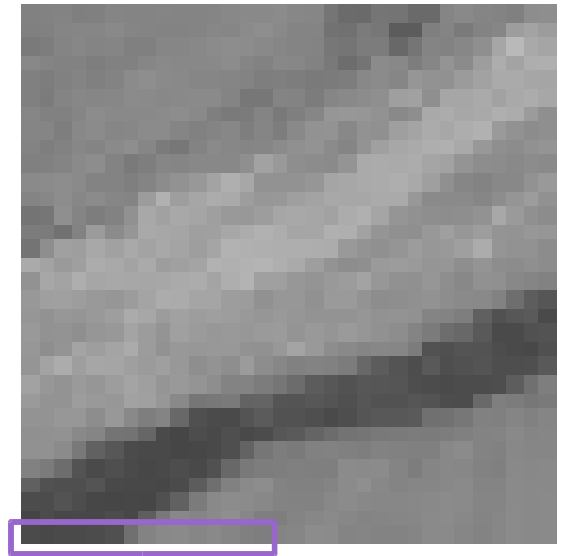
75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 134 →  
→ 140 139 132 133 131 131 133 138 138 134 131 135 139 137 138 ...

# Example: a row in a self-portrait by Van Gogh



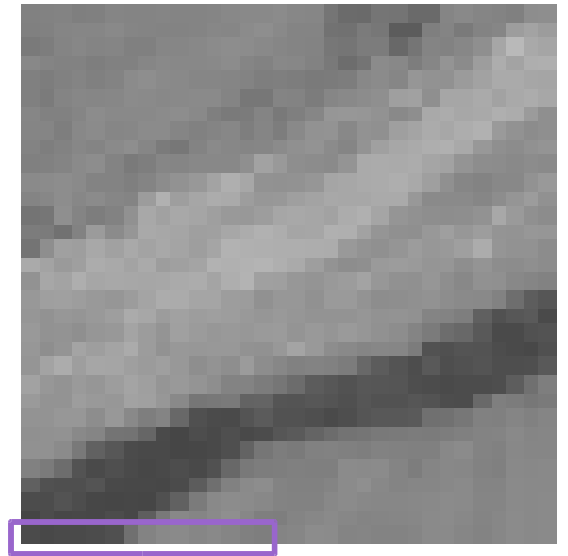
75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132

Example: a row in a self-portrait  
by Van Gogh



75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 ...

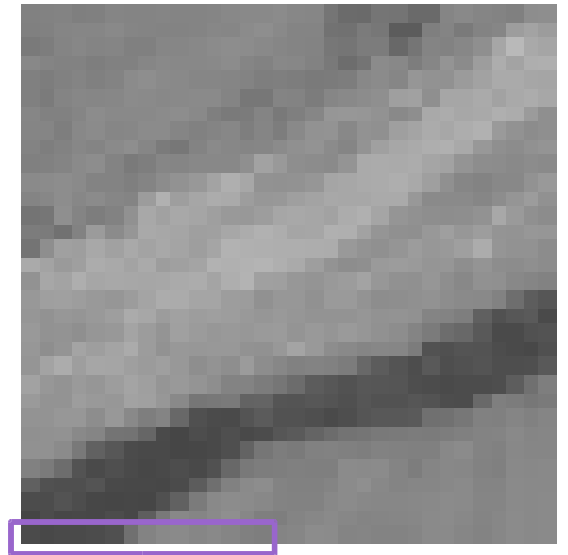
Example: a row in a self-portrait  
by Van Gogh



75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 ...

73.5	74	78	121.5	137.5	135.5	130.5	129
------	----	----	-------	-------	-------	-------	-----

# Example: a row in a self-portrait by Van Gogh



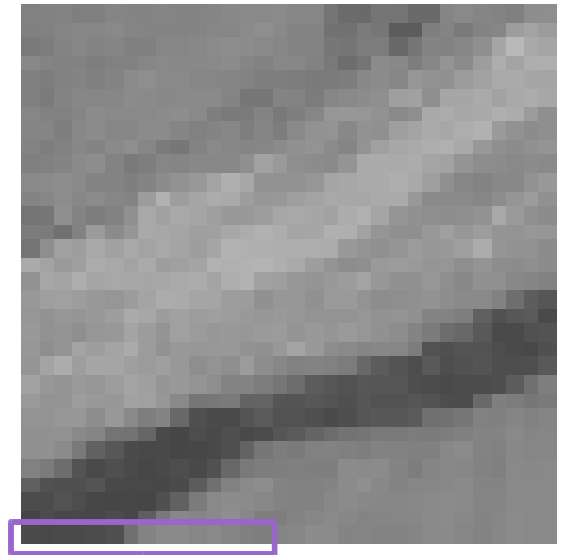
75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 ...

73.5	74	78	121.5	137.5	135.5	130.5	129
------	----	----	-------	-------	-------	-------	-----

73.75	99.75	136.5	129.75
-------	-------	-------	--------



# Example: a row in a self-portrait by Van Gogh



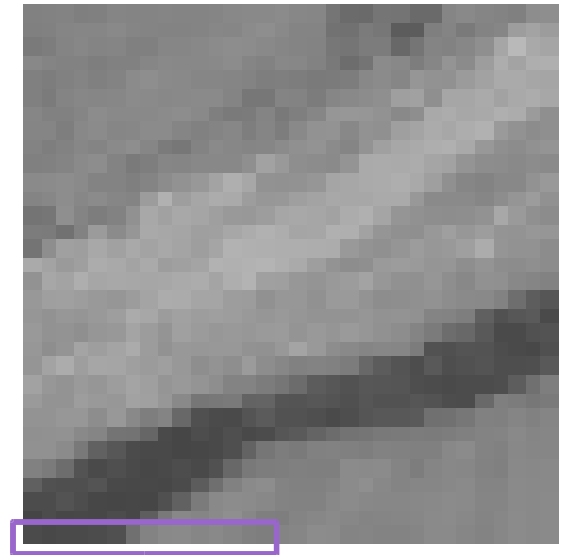
75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 ...

73.5	74	78	121.5	137.5	135.5	130.5	129
------	----	----	-------	-------	-------	-------	-----

-3	0	4	19	1	3	-5	6
----	---	---	----	---	---	----	---

73.75	99.75	136.5	129.75
-------	-------	-------	--------

# Example: a row in a self-portrait by Van Gogh



75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 ...

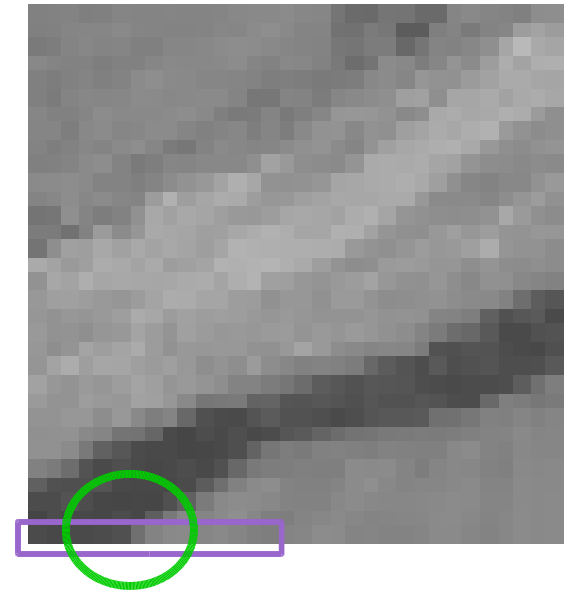
73.5	74	78	121.5	137.5	135.5	130.5	129
------	----	----	-------	-------	-------	-------	-----

-3	0	4	19	1	3	-5	6
----	---	---	----	---	---	----	---

73.75	99.75	136.5	129.75
-------	-------	-------	--------

.5	43.5	-2	-1.5
----	------	----	------

# Example: a row in a self-portrait by Van Gogh



75 72 74 74 76 80 112 131 137 138 134 137 133 128 126 132 ...

73.5	74	78	121.5	137.5	135.5	130.5	129
------	----	----	-------	-------	-------	-------	-----

-3	0	4	19	1	3	-5	6
----	---	---	----	---	---	----	---

73.75	99.75	136.5	129.75
-------	-------	-------	--------

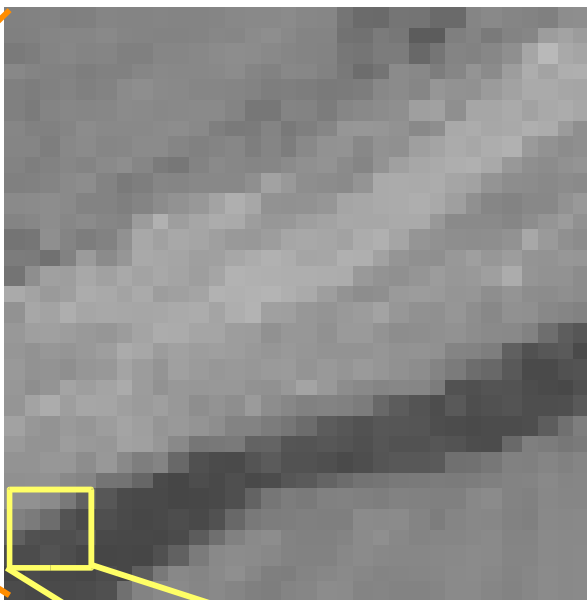
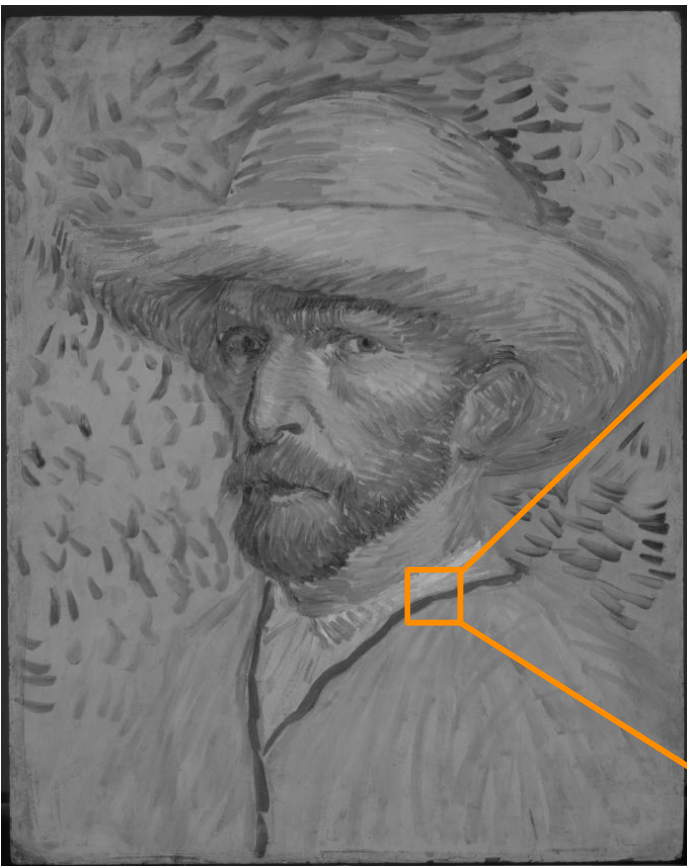
.5	43.5	-2	-1.5
----	------	----	------

large differences point to sudden transitions, such as edges

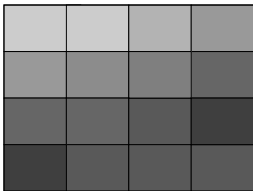
# The 2-dimensional wavelet transform.

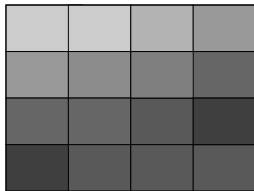
So far, we have worked in only 1 dimension

Pictures have pixels in TWO directions!



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76





121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



averaging pairwise



differencing pairwise

in each row

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5

1

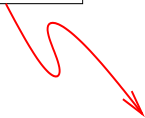
121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
-------	-----

1	-14
---	-----

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	

1	-14
-4	

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	85

1	-14
-4	-8

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	85
79.5	

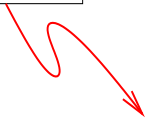


1	-14
-4	-8
-1	

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	85
79.5	74



1	-14
-4	-8
-1	-2

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	85
79.5	74
75	

1	-14
-4	-8
-1	-2
-2	



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	85
79.5	74
75	75.5

1	-14
-4	-8
-1	-2
-2	-1

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76



120.5	106
98	85
79.5	74
75	75.5

1	-14
-4	-8
-1	-2
-2	-1

121	122	113	99
-----	-----	-----	----

100	96	89	81
-----	----	----	----

80	79	75	73
----	----	----	----

74	76	75	76
----	----	----	----

120.5	106
-------	-----

98	85
----	----

79.5	74
------	----

75	75.5
----	------

1	-14
---	-----

-4	-8
----	----

-1	-2
----	----

-2	-1
----	----

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

120.5	106
98	85
79.5	74
75	75.5

1	-14
-4	-8
-1	-2
-2	-1

109.25
--------

-22.5
-------

-1.5
------

-5
----

averaging and differencing

averaging and differencing

vertically, in each of the two tables

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

120.5	106
98	85
79.5	74
75	75.5

1	-14
-4	-8
-1	-2
-2	-1

109.25	95.5
77.25	

-22.5	-21
-4.5	

-1.5	-11
-1.5	

-5	6
-1	

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

120.5	106
98	85
79.5	74
75	75.5

1	-14
-4	-8
-1	-2
-2	-1

109.25	95.5
77.25	74.75

-22.5	-21
-4.5	1.5

-1.5	-11
-1.5	-1.5

-5	6
-1	1

121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

120.5	106
98	85
79.5	74
75	75.5

1	-14
-4	-8
-1	-2
-2	-1

109.25	95.5
77.25	74.75

-22.5	-21
-4.5	1.5

-1.5	-11
-1.5	-1.5

-5	6
-1	1



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

109.25	95.5
77.25	74.75

-22.5	-21
-4.5	1.5



-1.5	-11
-1.5	-1.5



-5	6
-1	1





121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

109.25	95.5
77.25	74.75

-1.5	-11
-1.5	-1.5



-22.5	-21
-4.5	1.5



-5	6
-1	1



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

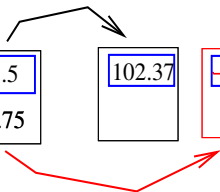
-1.5	-11
-1.5	-1.5



109.25	95.5
77.25	74.75

102.37
--------

-13.75
--------



-5	6
-1	1



-22.5	-21
-4.5	1.5



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

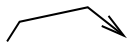
-1.5	-11
-1.5	-1.5



109.25	95.5
77.25	74.75

102.37
76

-13.75
-2.5



-5	6
-1	1



-22.5	-29
-4.5	1.5



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

-1.5	-11
-1.5	-1.5



109.25	95.5
77.25	74.75

102.37
76

-13.75
-2.5

-5	6
-1	1



-22.5	-21
-4.5	1.5



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

-1.5	-11
-1.5	-1.5



109.25	95.5
77.25	74.75

102.37
76

-13.75
-2.5

89.19

-26.37

-5	6
-1	1



-22.5	-21
-4.5	1.5



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

-1.5	-11
-1.5	-1.5



109.25	95.5
77.25	74.75

102.37
76

-13.75
-2.5

11.25
-------

89.19
-------

-26.37
--------

-8.12
-------

-5	6
-1	1



-22.5	-21
-4.5	1.5



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

-1.5	-11
-1.5	-1.5



109.25	95.5
77.25	74.75

102.37
76

-13.75
-2.5

11.25
-------



89.19
-------

-26.37
--------



-8.12
-------



-5	6
-1	1



-22.5	-21
-4.5	1.5



121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

109.25	95.5
77.25	74.75

89.19

-22.5	-21
-4.5	1.5

-1.5	-11
-1.5	-1.5



11.25



-8.12



-26.37



-5	6
-1	1





121	122	113	99
100	96	89	81
80	79	75	73
74	76	75	76

109.25	95.5
77.25	74.75

89.19

-8.12



11.25



-26.37



-22.5	-21
-4.5	1.5

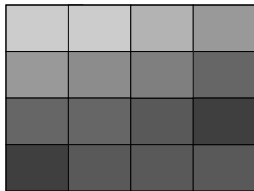


-1.5	-11
-1.5	-1.5



-5	6
-1	1





109.25	95.5
77.25	74.75

89.19

-8.12

-1.5	-11
-1.5	-1.5

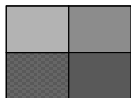
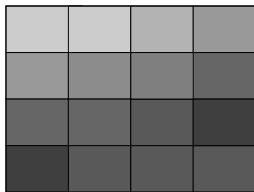
11.25

-26.37

-5	6
-1	1

-22.5	-21
-4.5	1.5





89.19

-26.37



-22.5 -21

-4.5 1.5



-8.12



11.25



-1.5 -11

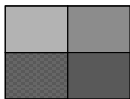
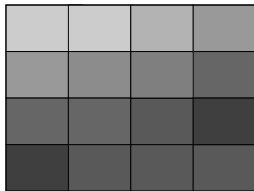
-1.5 -1.5



-5 6

-1 1





-22.5	-21
-4.5	1.5



-26.37



11.25



-8.12



-1.5	-11
-1.5	-1.5



-5	6
-1	1





**Wavelet decomposition:  
graphical illustration of  
the algorithm and its properties**





“Average”  
horizontally  
and  
vertically







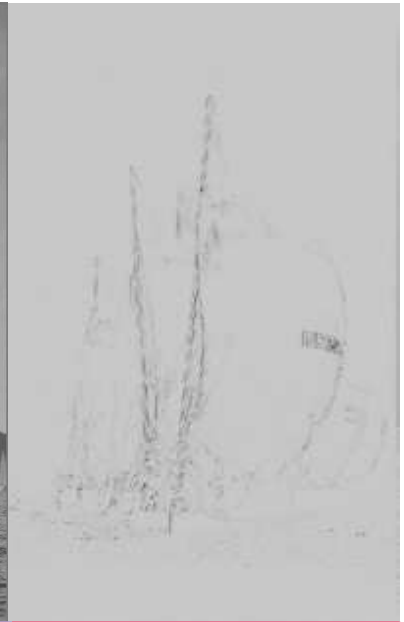
“Difference”  
horizontally

“Average”  
vertically





“Difference” vertically



“Average”  
horizontally

“Difference”  
horizontally

“Difference” vertically

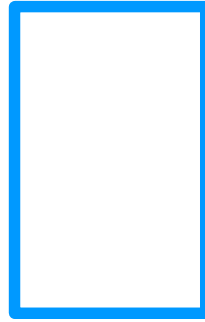




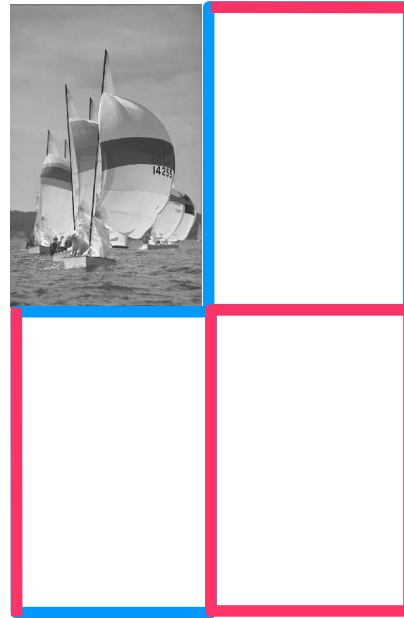
Repeat  
at the next  
scale











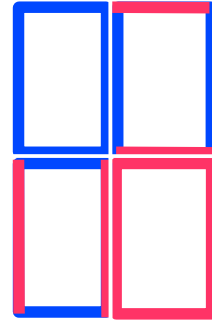






Repeat





























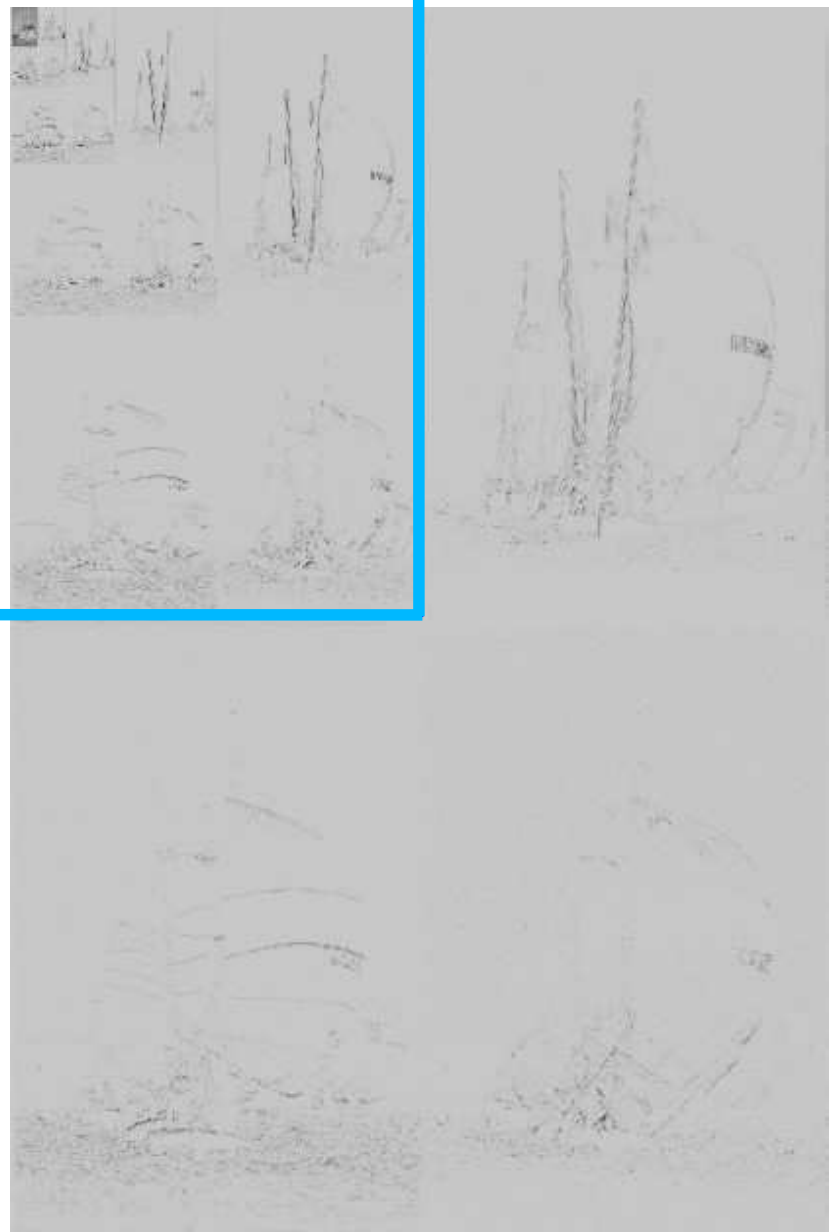
# Meaning of the successive approximations



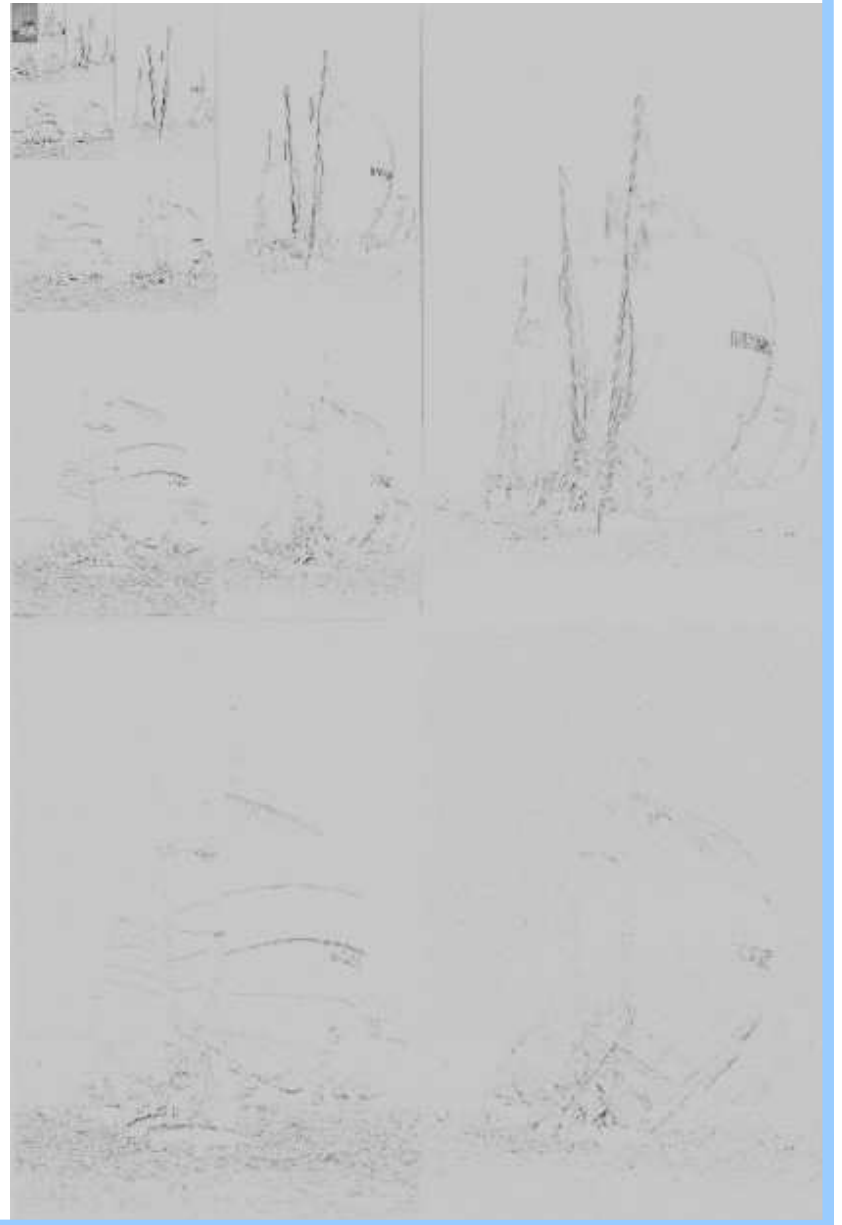




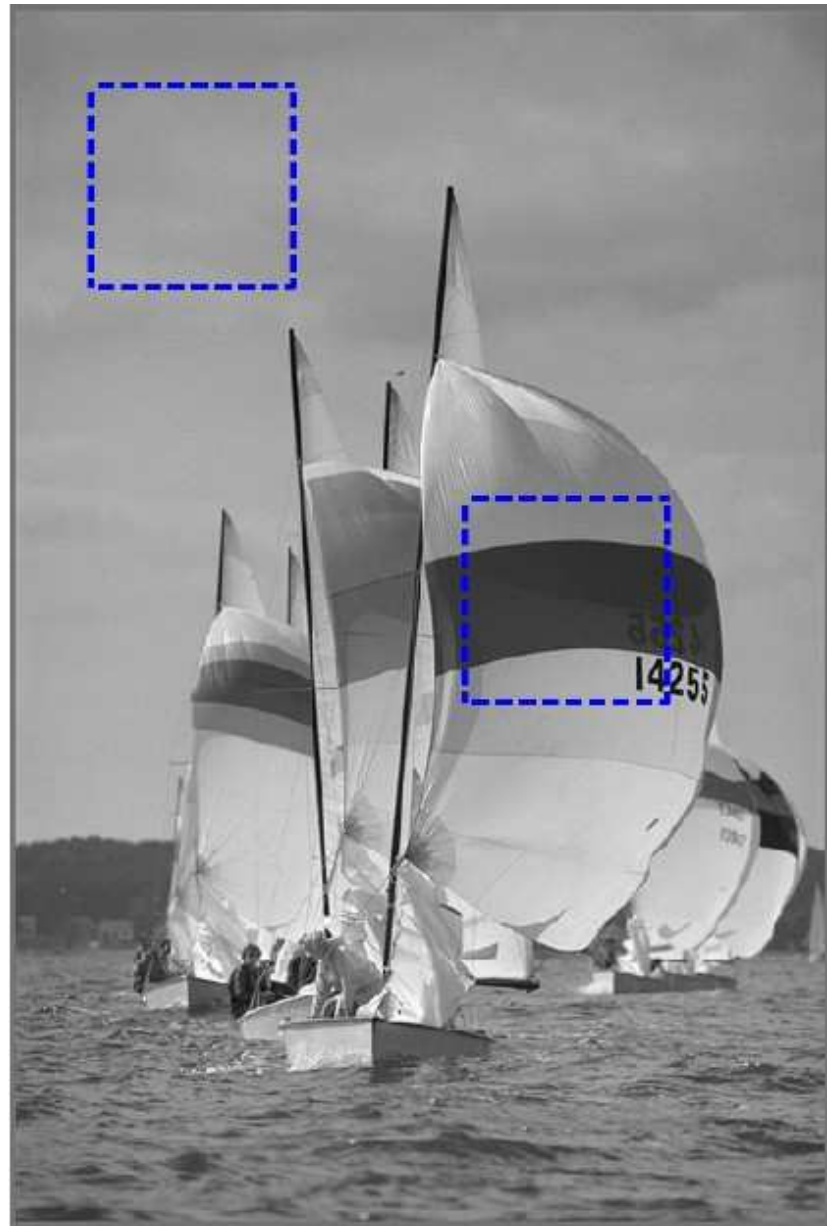


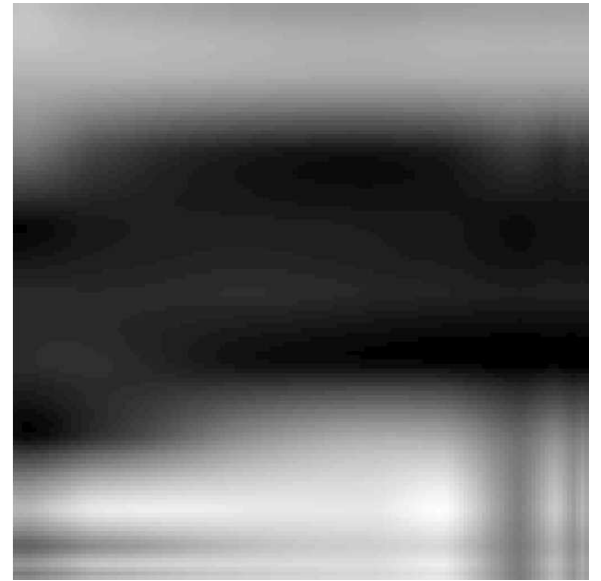


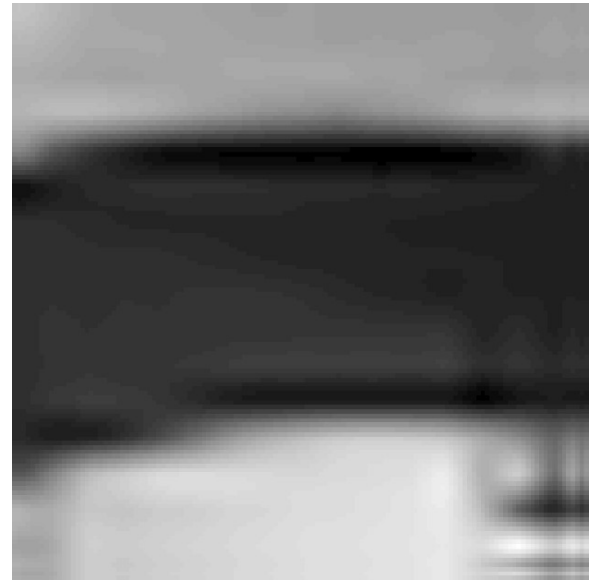


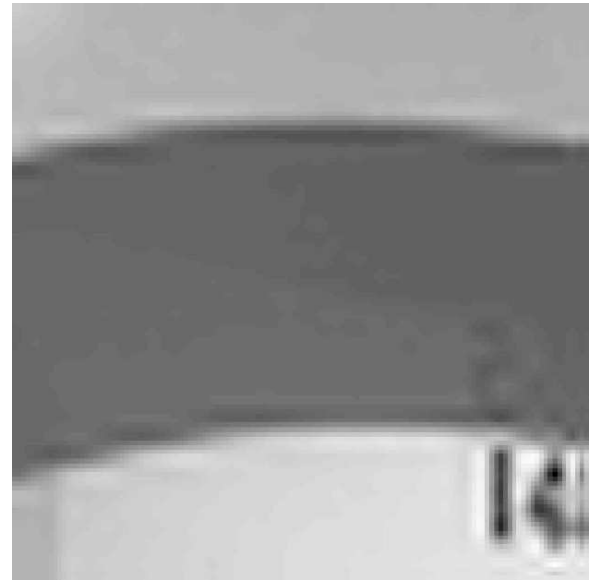


# **Local properties reflected by the successive approximations**













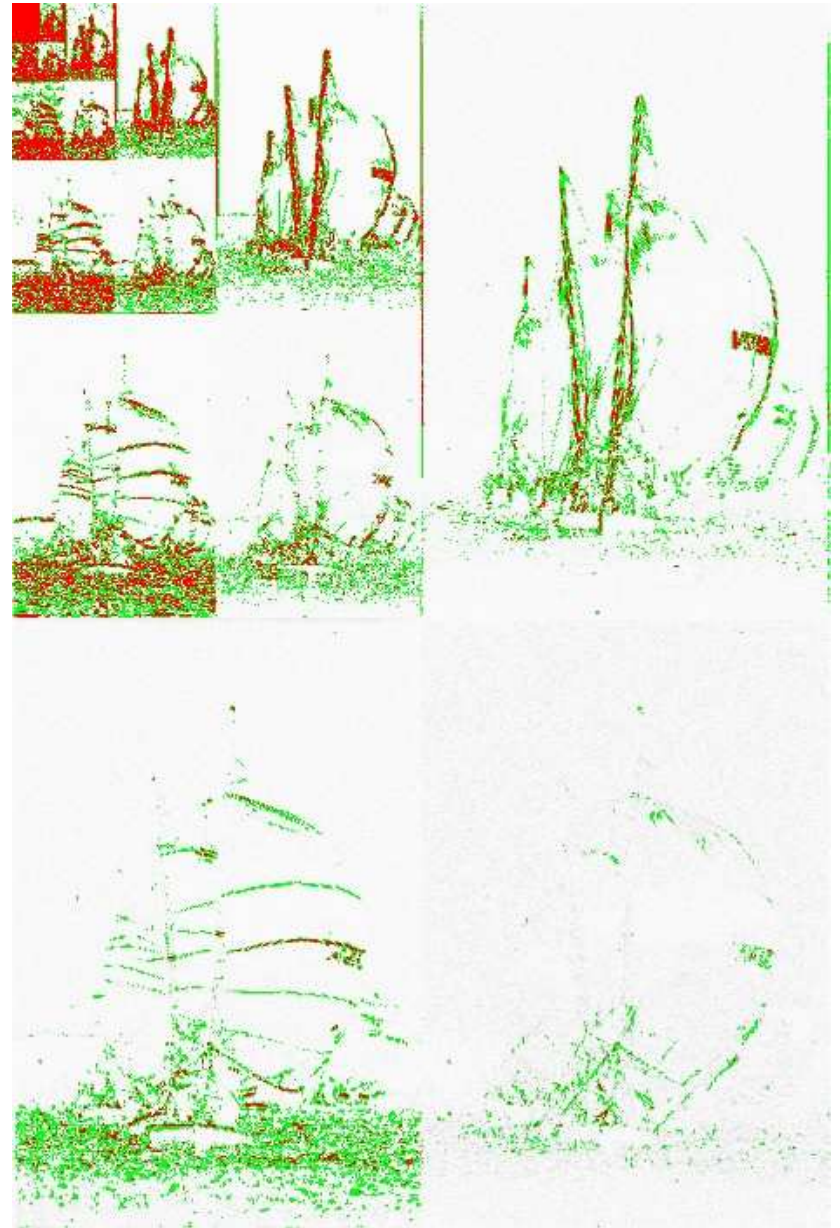


# Compression





Compression Ratio: 3.3%



Compression Ratio: 10%

**Localization:**  
**fast, interactive retrieval of data**





