Chaos and Complexity in Economics

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Main Themes

- nonlinear dynamics and chaos
- financial market model with heterogeneous traders
- estimation of nonlinear switching model
- laboratory experiments and nonlinear models
Why is economics so difficult?

Isaac Newton, about 1700
“I can predict the motion of heavenly bodies, but not the madness of crowds”

irrationality is difficult to predict and to model

expectations are difficult to predict and to model
The Traditional Rational View
Expectations are model-consistent

Milton Friedman, 1953: Irrational traders will be driven out of the market by rational traders, who will earn higher profits.

Robert Lucas, 1971: economic policy should be based on rational expectations models in macro-economics
alternative, complexity, agent-based modeling approach in Economics

- bounded rationality and behavioral economics versus perfect rationality (Simon (1957) versus Lucas (1971))
- heterogeneous agents versus representative agent
- market psychology, herding behavior (Keynes (1936)) versus rationality
- markets as complex adaptive, nonlinear evolutionary systems versus representative agent model
- computational versus analytical approach
Chaos in Nonlinear Systems

quadratic example: \( x_{t+1} = 4x_t(1 - x_t) \)

\[
\begin{align*}
  x_0 &= 0.1 \\
  x_0 &= 0.1001
\end{align*}
\]

sensitive dependence on initial conditions
Period-doubling Bifurcation Route to Chaos for Quadratic Map $x_{t+1} = \lambda x_t(1 - x_t)$
Lyapunov Exponent
measuring sensitive dependence

Lyapunov exponent \( \equiv \)
average rate of expansion/contraction along orbit

\[
\lambda(x_0) \equiv \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln(|f'(f^i(x_0))|),
\]

- \( \lambda(x_0) < 0: \) stable periodic behaviour
- \( \lambda(x_0) > 0: \) chaos and sensitive dependence
Chaos in Quadratic Map $x_{t+1} = \lambda x_t (1 - x_t)$

(a) $f(x) = \lambda x (1-x)$, $x_0 = 0.4$ and $n=1000$

(b) $f(x) = \lambda x (1-x)$ and $x_0 = 0.4$
Financial Market Model

Investors can choose between risky asset and a risk free asset

- $R = 1 + r > 1$: gross return on risk free asset
- risky asset pays stochastic dividends
  - $y_t$: stochastic dividend process for risky asset
  - $p_t$: price (ex div.) per share of risky asset
- price of risky asset determined by market clearing:

$$Rp_t = \sum_{h=1}^{H} n_{ht} E_{ht} (p_{t+1} + y_{t+1})$$

- $n_{ht}$: fraction of agents of type $h$
Rational Expectations (RE) fundamental benchmark

\[ Rp_t = E_t(p_{t+1} + y_{t+1}) \]

**common beliefs** on future earnings and prices
unique bounded RE **fundamental price** \( p_t^* \):

\[ p_t^* = \frac{E_t(y_{t+1})}{R} + \frac{E_t(y_{t+2})}{R^2} + \ldots \]

For special case of IID dividends, with \( E(y_{t+1}) = \bar{y} \)

\[ p^* = \frac{\bar{y}}{R - 1} = \frac{\bar{y}}{r} \]

pricing equation in **deviations** \( x_t = p_t - p_t^* \) from fundamental:

\[ Rx_t = E_t x_{t+1} \]
Behavioral asset pricing model (Adaptive Belief Systems)

standard asset pricing model *with heterogeneous beliefs*  
one risky asset, one risk free asset
  - price of risky asset determined by **market clearing**
  - **beliefs** about future prices given by **simple**, linear rule
  - forecasting strategies updated according to **discrete choice** model with **realized profits**

equilibrium price of risky asset

\[
Rp_t = \sum_{h=1}^{H} n_{ht} E_{ht} (p_{t+1} + y_{t+1})
\]

(in **deviations** \(x_t = p_t - p_t^*\) from RE-fundamental)

\[
Rx_t = \sum_{h=1}^{H} n_{ht} E_{ht} x_{t+1}
\]
Fractions of Strategy Type $h$

fractions of belief types are updated in each period:
discrete choice model (BH 1997,1998) with asynchronous updating:

$$n_{ht} = (1 - \delta) \frac{e^{\beta U_{h,t-1}}}{Z_{t-1}} + \delta n_{h,t-1},$$

where $Z_{t-1} = \sum e^{\beta U_{h,t-1}}$ is normalization factor,
$U_{h,t-1}$ past strategy performance, e.g. (weighted average) past profits

$\delta$ is probability of not updating
$\beta$ is the intensity of choice.
$\beta = 0$: all types equal weight (in long run)
$\beta = \infty$: fraction $1 - \delta$ switches to best predictor
Example with 4 belief types

(zero costs; memory one lag)

\[ x_{h,t+1}^e = g_h x_{t-1} + b_h \]

<table>
<thead>
<tr>
<th>( g_h )</th>
<th>( b_h )</th>
<th>Belief Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>fundamentalists</td>
</tr>
<tr>
<td>1.1</td>
<td>0.2</td>
<td>trend + upward bias</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.2</td>
<td>trend + downward bias</td>
</tr>
<tr>
<td>1.21</td>
<td>0</td>
<td>trend chaser</td>
</tr>
</tbody>
</table>

\[ Rx_t = \sum_{h=1}^{4} n_{h,t}(g_h x_{t-1} + b_h) \]

\[ n_{h,t+1} = \frac{e^{\beta (g_h x_{t-2} + b_h - Rx_{t-1})(x_t - Rx_{t-1})}}{\sigma^2} Z_t, \quad h = 1, 2, 3, 4 \]
Chaos in Financial Market Model with Fundamentalists versus Chartists

(a)

(b)

(c)

(d)
Chaos in Financial Market Model with Fundamentalists versus Chartists

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Chaos and Complexity in Economics
Model very sensitive to noise
Nearest Neighbor Forecasting

The diagram illustrates the prediction error as a function of the prediction horizon, with different scenarios including chaos and various levels of noise (5%, 10%, 30%, 40%). The error increases with the prediction horizon and varies depending on the noise level, with chaos showing the highest error and 5% noise the lowest.
$S&P 500, 1871-2003 + benchmark fundamental$ $p_t^* = \frac{1+g}{1+r}y_t$

$(g \text{ constant growth rate dividends})$
Estimation 2-type model

\[ R^* x_t = n_t \{ 0.762 \, x_{t-1} \} + (1 - n_t) \{ 1.135 \, x_{t-1} \} + \widehat{\epsilon}_t \]

\( n_t = \{ 1 + \exp[-10.29(-0.373x_{t-3})(x_{t-1} - R^*x_{t-2})] \}^{-1} \)

\[ R^2 = 0.82, AIC = 3.18, AIC_{AR(1)} = 3.24, \sigma_\epsilon = 4.77, Q_{LB}(4) = 0.44 \]
How to model bounded rationality?

**wilderness of bounded rationality:**
"There is only one way you can be right, but there are many ways you can be wrong"

- use laboratory experiments with human subjects to test a **behavioral theory of heterogeneous expectations**
- fit simple complexity model to laboratory data
- test simple complexity model on real economic/financial data
Example: Laboratory Experiments with Human subjects
psychology – behavioral economics – computer science

Computer Screen Learning to Forecast Experiment

![Graph showing predicted vs. real numbers over rounds]

<table>
<thead>
<tr>
<th>Round</th>
<th>Prediction</th>
<th>Real Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.70</td>
<td>50.23</td>
</tr>
<tr>
<td>2</td>
<td>33.70</td>
<td>56.63</td>
</tr>
<tr>
<td>3</td>
<td>37.00</td>
<td>65.32</td>
</tr>
<tr>
<td>4</td>
<td>40.10</td>
<td>65.00</td>
</tr>
<tr>
<td>5</td>
<td>43.50</td>
<td>66.12</td>
</tr>
<tr>
<td>6</td>
<td>50.00</td>
<td>64.53</td>
</tr>
<tr>
<td>7</td>
<td>48.35</td>
<td>58.35</td>
</tr>
<tr>
<td>8</td>
<td>38.70</td>
<td>42.35</td>
</tr>
<tr>
<td>9</td>
<td>30.10</td>
<td>40.01</td>
</tr>
<tr>
<td>10</td>
<td>28.25</td>
<td></td>
</tr>
</tbody>
</table>

Total earnings: 10357
Earnings this period: 1298
Remaining time: 00

What is your prediction this period?
Your prediction must be between 0 and 100

Prediction: [Blank]
Rational Expectations Benchmark

If everybody predicts rationally fundamental price, then

\[ p_t = p^f + \frac{\varepsilon_t}{1 + r} \]
Naive and Trend-following Expectations Benchmarks

which expectations theory is correct?
Prices in the Experiment with Humans

Group 2
fundamental price
experimental price

Group 5
fundamental price
experimental price

Group 1
fundamental price
experimental price

Group 6
fundamental price
experimental price

Group 4
fundamental price
experimental price

Group 7
fundamental price
experimental price

Chaos and Complexity in Economics
Individual Forecasts in Experiments with Humans
Heterogeneous Expectations Hypothesis

**Heuristics Switching Model**

- there are a few simple heuristics
  - adaptive expectations ADA
  - trend extrapolating rule STR, WTR
  - anchor and adjustment rule LAA
- impact of heuristics changes over time
- agents gradually switch to heuristics that have performed better in the recent past
Group 5 (Convergence)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Group 6 (Constant Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Group 7 (Dampened Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Concluding Remarks

- Chaotic financial market model mimics bubble and crash dynamics
- Simple 2-type model with fundamentalists versus chartists fits US stock market data
- Nonlinear heuristic switching model with path dependence fits experimental data
- Theory of evolutionary selection of heterogeneous expectations fits experimental data