Chaos and Complexity in Economics

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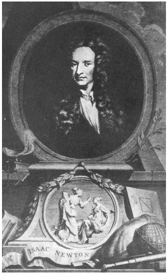
Main Themes

- nonlinear dynamics and chaos
- financial market model with heterogeneous traders
- estimation of nonlinear switching model
- laboratory experiments and nonlinear models

Why is economics so difficult?



Isaac Newton, about 1700
"*I can predict the motion of heavenly bodies, but not the madness of crowds*"

irrationality is difficult to predict and to model

expectations are difficult to predict and to model

The Traditional Rational View
Expectations are model-consistent

Milton Friedman, 1953: Irrational traders will be driven out of the market by **rational** traders, who will earn higher profits.

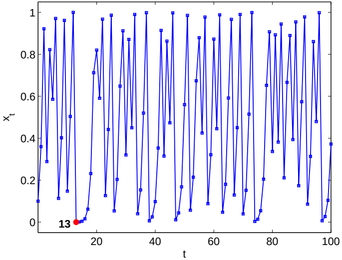
Robert Lucas, 1971: economic policy should be based on **rational expectations** models in macro-economics

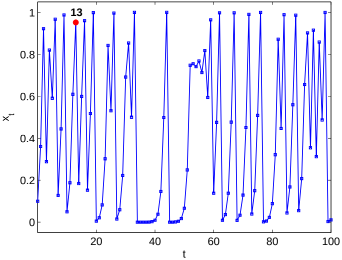
alternative, complexity, agent-based modeling approach in Economics

- **bounded rationality** and **behavioral economics** versus perfect rationality (Simon (1957) versus Lucas (1971)
- **heterogeneous** agents versus representative agent
- **market psychology**, **herding** behavior (Keynes (1936)) versus rationality
- markets as **complex adaptive, nonlinear evolutionary** systems versus representative agent model
- **computational** versus **analytical** approach

Chaos in Nonlinear Systems

quadratic example: $x_{t+1} = 4x_t(1 - x_t)$

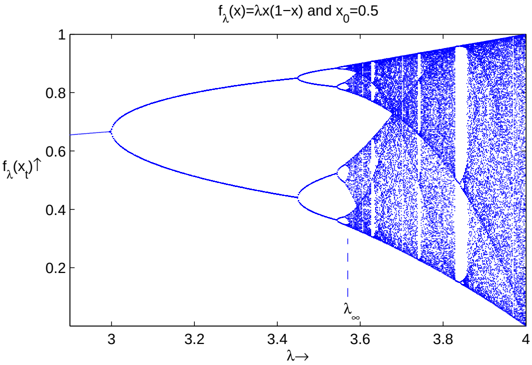




$x_0 = 0.1$ $\qquad$ $x_0 = 0.1001$

sensitive dependence on initial conditions

Period-doubling Bifurcation Route to Chaos for Quadratic Map $x_{t+1} = \lambda x_t(1 - x_t)$



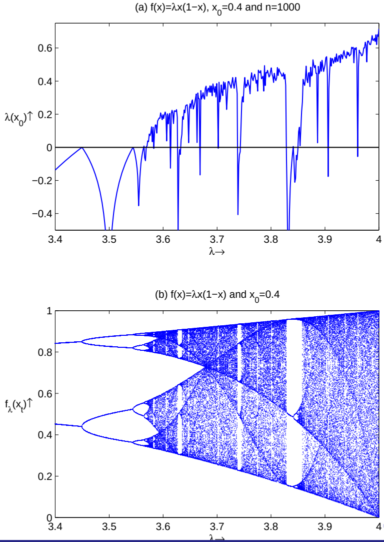
Lyapunov Exponent
measuring sensitive dependence

Lyapunov exponent $\equiv$
average rate of expansion/contraction along orbit

$$\lambda(x_0) \equiv \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln(|\, f'(f^i(x_0))\,|),$$

- $\lambda(x_0) < 0$: **stable** periodic behaviour
- $\lambda(x_0) > 0$: **chaos** and **sensitive dependence**

Chaos in Quadratic Map $x_{t+1} = \lambda x_t(1 - x_t)$



Financial Market Model

Investors can choose between risky asset and a risk free asset

- $R = 1 + r > 1$: gross **return on risk free asset**
- risky asset pays stochastic dividends
  - y_t: stochastic **dividend** process for risky asset
  - p_t: **price** (ex div.) per share of risky asset
- price of risky asset determined by **market clearing**:

$$Rp_t = \sum_{h=1}^{H} n_{ht} E_{ht}(\mathbf{p}_{t+1} + \mathbf{y}_{t+1})$$

- n_{ht}: **fraction** of agents of type h

Rational Expectations (RE) fundamental benchmark

$$Rp_t = E_t(\mathbf{p}_{t+1} + \mathbf{y}_{t+1})$$

common beliefs on future earnings and prices
unique bounded RE **fundamental price** p_t^*:

$$p_t^* = \frac{E_t(y_{t+1})}{R} + \frac{E_t(y_{t+2})}{R^2} + ...$$

For special case of IID dividends, with $E(y_{t+1}) = \bar{y}$

$$p^* = \frac{\bar{y}}{R-1} = \frac{\bar{y}}{r}$$

pricing equation in **deviations** $x_t = p_t - p_t^*$ from fundamental:

$$Rx_t = E_t x_{t+1}$$

Behavioral asset pricing model (Adaptive Belief Systems)

standard asset pricing model **with heterogeneous beliefs**

one risky asset, one risk free asset

- price of risky asset determined by **market clearing**
- **beliefs** about future prices given by **simple**, linear rule
- forecasting strategies updated according to **discrete choice model** with **realized profits**

equilibrium price of risky asset

$$Rp_t = \sum_{h=1}^{H} n_{ht} E_{ht}(\mathbf{p}_{t+1} + \mathbf{y}_{t+1})$$

(in **deviations** $x_t = p_t - p_t^*$ from RE-fundamental)

$$Rx_t = \sum_{h=1}^{H} n_{ht} E_{ht} x_{t+1}$$

Fractions of Strategy Type h

fractions of belief types are updated in each period:
discrete choice model (BH 1997,1998) with **asynchronous updating**:

$$n_{ht} = (1-\delta)\frac{e^{\beta U_{h,t-1}}}{Z_{t-1}} + \delta n_{h,t-1},$$

where $Z_{t-1} = \sum e^{\beta U_{h,t-1}}$ is normalization factor,
$U_{h,t-1}$ **past strategy performance**, e.g. (weighted average) **past profits**

δ is **probability of not updating**
β is the **intensity of choice**.
$\beta = 0$: **all types equal weight** (in long run)
$\beta = \infty$: fraction $1-\delta$ **switches to best predictor**

Example with 4 belief types

(zero costs; memory one lag)

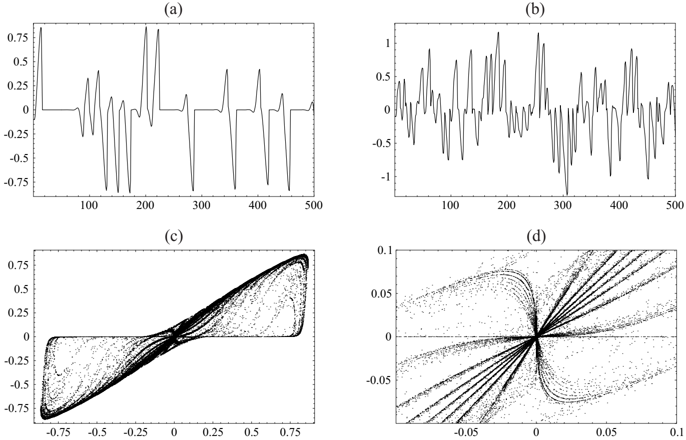
$x^e_{h,t+1} = g_h x_{t-1} + b_h$

$g_1 = 0$	$b_1 = 0$	fundamentalists
$g_2 = 1.1$	$b_2 = 0.2$	trend + upward bias
$g_3 = 0.9$	$b_3 = -0.2$	trend + downward bias
$g_4 = 1.21$	$b_4 = 0$	trend chaser

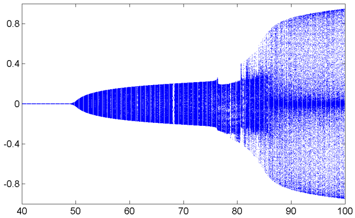
$$Rx_t = \sum_{h=1}^{4} n_{h,t}(g_h x_{t-1} + b_h)$$

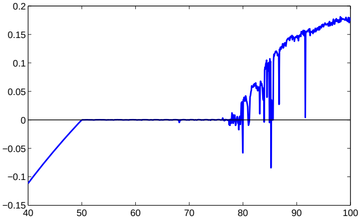
$$n_{h,t+1} = \frac{e^{\frac{\beta}{a\sigma^2}(g_h x_{t-2}+b_h-Rx_{t-1})(x_t-Rx_{t-1})}}{Z_t}, \quad h = 1, 2, 3, 4$$

Chaos in Financial Market Model with Fundamentalists versus Chartists

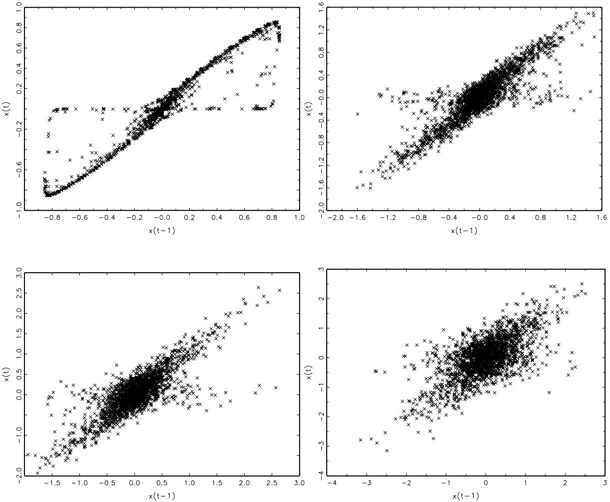


Chaos in Financial Market Model with Fundamentalists versus Chartists

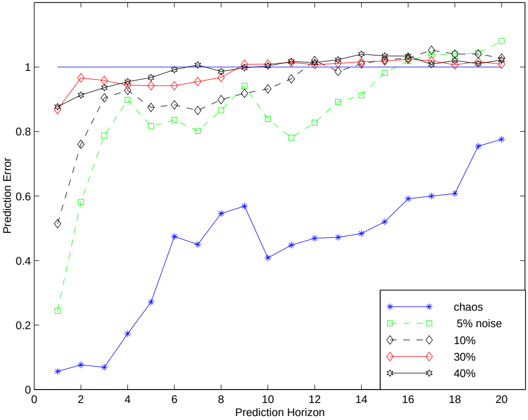




Model very sensitive to noise

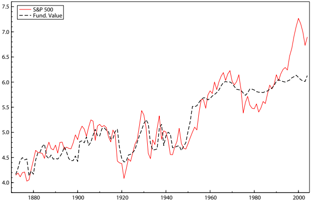


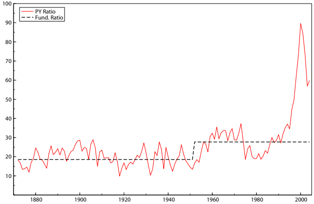
Nearest Neighbor Forecasting



S&P 500, 1871-2003 + benchmark fundamental $p_t^* = \frac{1+g}{1+r} y_t$

(g constant growth rate dividends)





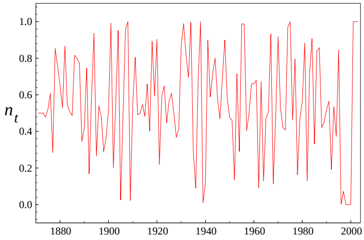
log S&P 500 and fundamental PD-ratio and PD-fundamental

Estimation 2-type model

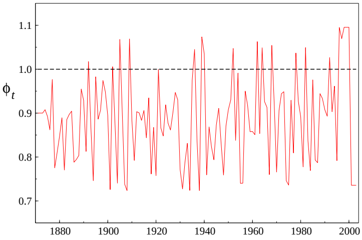
$$R^* x_t = n_t\{\underset{(0.056)}{\mathbf{0.762}}\, x_{t-1}\} + (1 - n_t)\{\underset{(0.036)}{\mathbf{1.135}}\, x_{t-1}\} + \widehat{\epsilon}_t \quad (1)$$

$$n_t = \{1 + \exp[\underset{(6.94)}{-10.29}(-0.373 x_{t-3})(x_{t-1} - R^* x_{t-2})]\}^{-1} \quad (2)$$

$R^2 = 0.82, AIC = 3.18, AIC_{AR(1)} = 3.24, \sigma_\epsilon = 4.77, Q_{LB}(4) = 0.44$



fraction fundamentalists



average market sentiment

How to model bounded rationality?

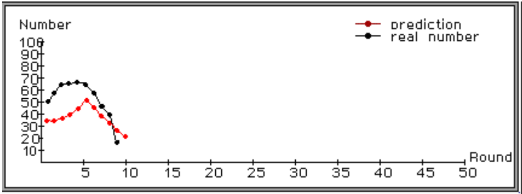
wilderness of bounded rationality:
"There is only one way you can be right, but there are many ways you can be wrong"

- use laboratory experiments with human subjects to test a **behavioral theory of heterogeneous expectations**
- fit simple complexity model to laboratory data
- test simple complexity model on real economic/financial data

Example: Laboratory Experiments wit Human subjects

psychology – behavioral economics – computer science

Computer Screen Learning to Forecast Experiment



Total earnings:	earnings this period:	Remaining time:
10357	1298	00

What is your prediction this period?
Your prediction must be between 0 and **100**

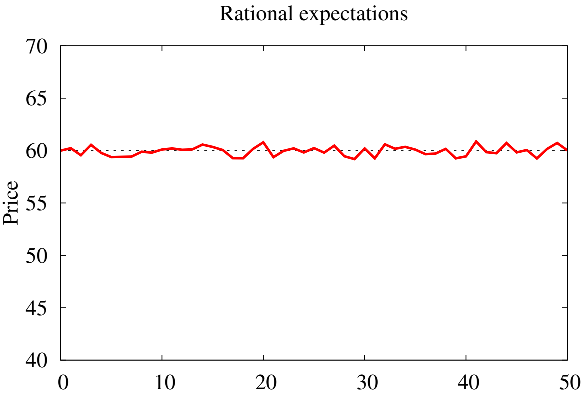
Prediction:

Round	Prediction	real value
1	33.70	50.23
2	33.70	56.63
3	37.00	65.32
4	40.10	65.00
5	43.50	66.12
6	50.00	64.53
7	48.35	58.35
8	38.70	42.35
9	30.10	40.01
10	28.25	

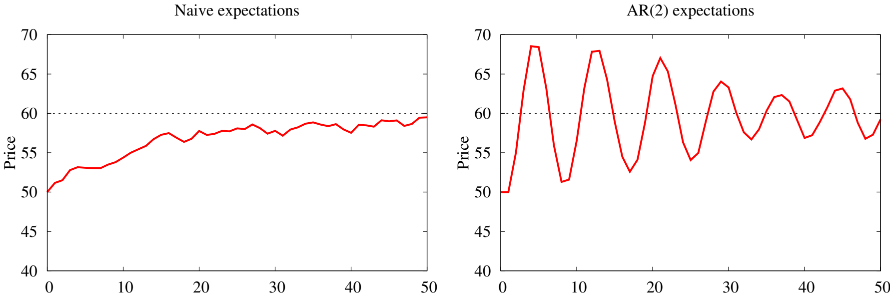
Rational Expectations Benchmark

If everybody predicts rationally fundamental price, then

$$p_t = p^f + \frac{\varepsilon_t}{1+r}$$

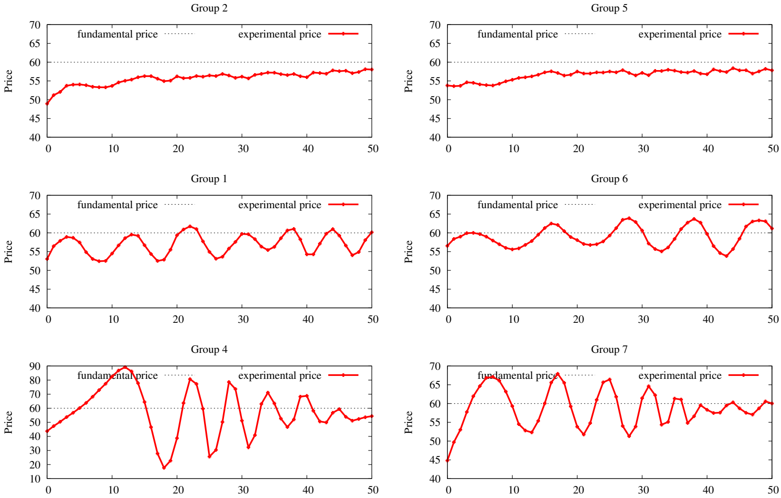


Naive and Trend-following Expectations Benchmarks

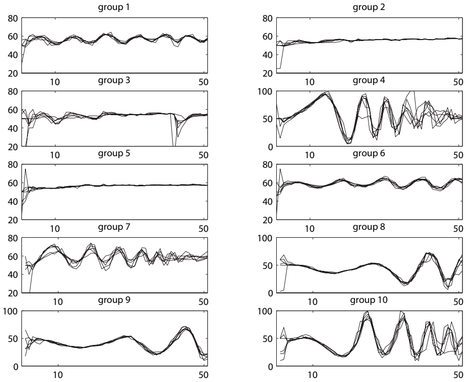


which expectations theory is correct?

Prices in the Experiment with Humans



Individual Forecasts in Experiments with Humans



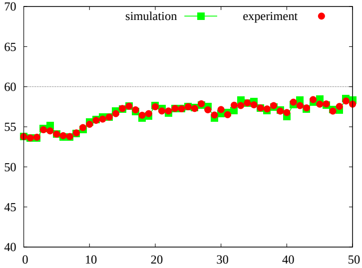
Heterogeneous Expectations Hypothesis

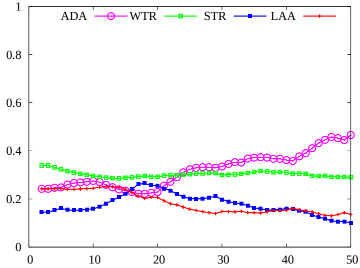
Heuristics Switching Model

- there are a few simple heuristics
  - adaptive expectations ADA
  - trend extrapolating rule STR, WTR
  - anchor and adjustment rule LAA
- impact of heuristics changes over time
- agents gradually switch to heuristics that have performed better in the recent past

Group 5 (Convergence)

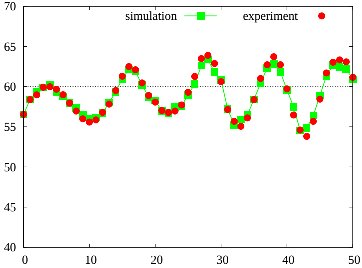
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$

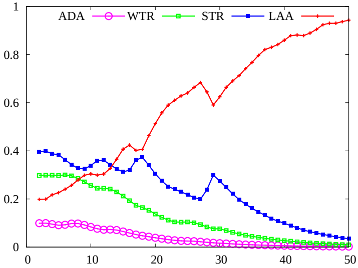




Group 6 (Constant Oscillations)

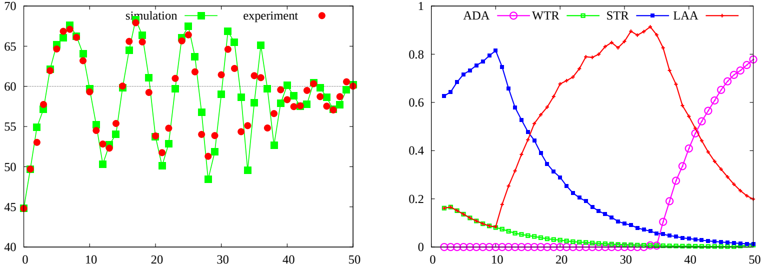
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$





Group 7 (Dampened Oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$



Concluding Remarks

- chaotic financial market model mimics bubble and crash dynamics
- simple 2-type model with fundamentalists versus chartists fits US stock market data
- nonlinear heuristic switching model with path dependence fits experimental data
- theory of evolutionary selection of heterogeneous expectations fits experimental data