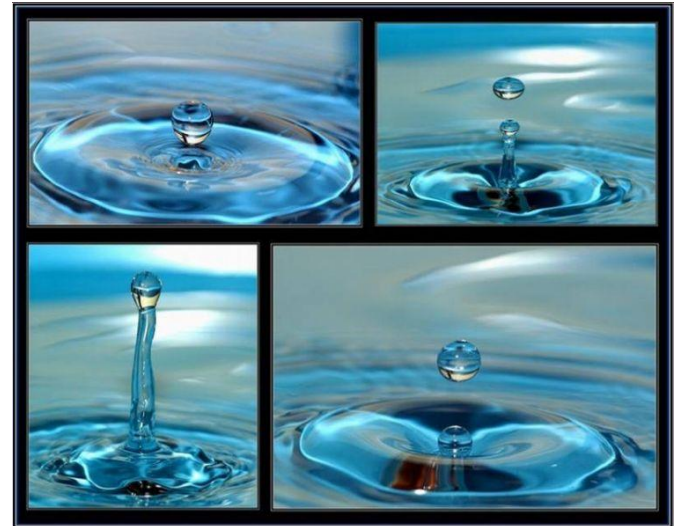
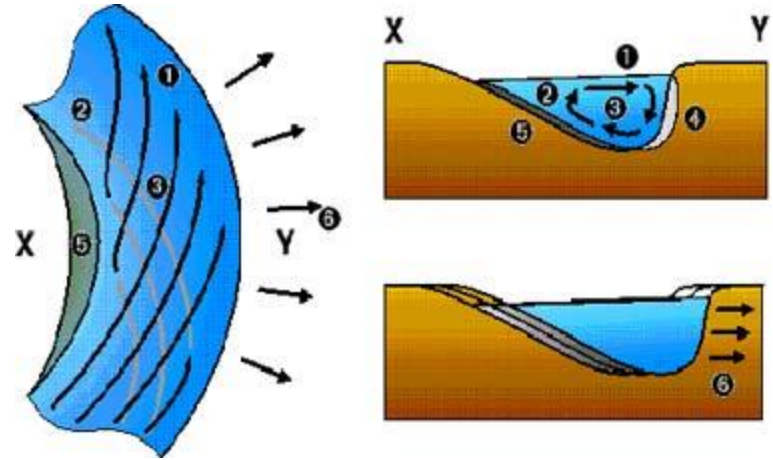


De wet van Bernoulli

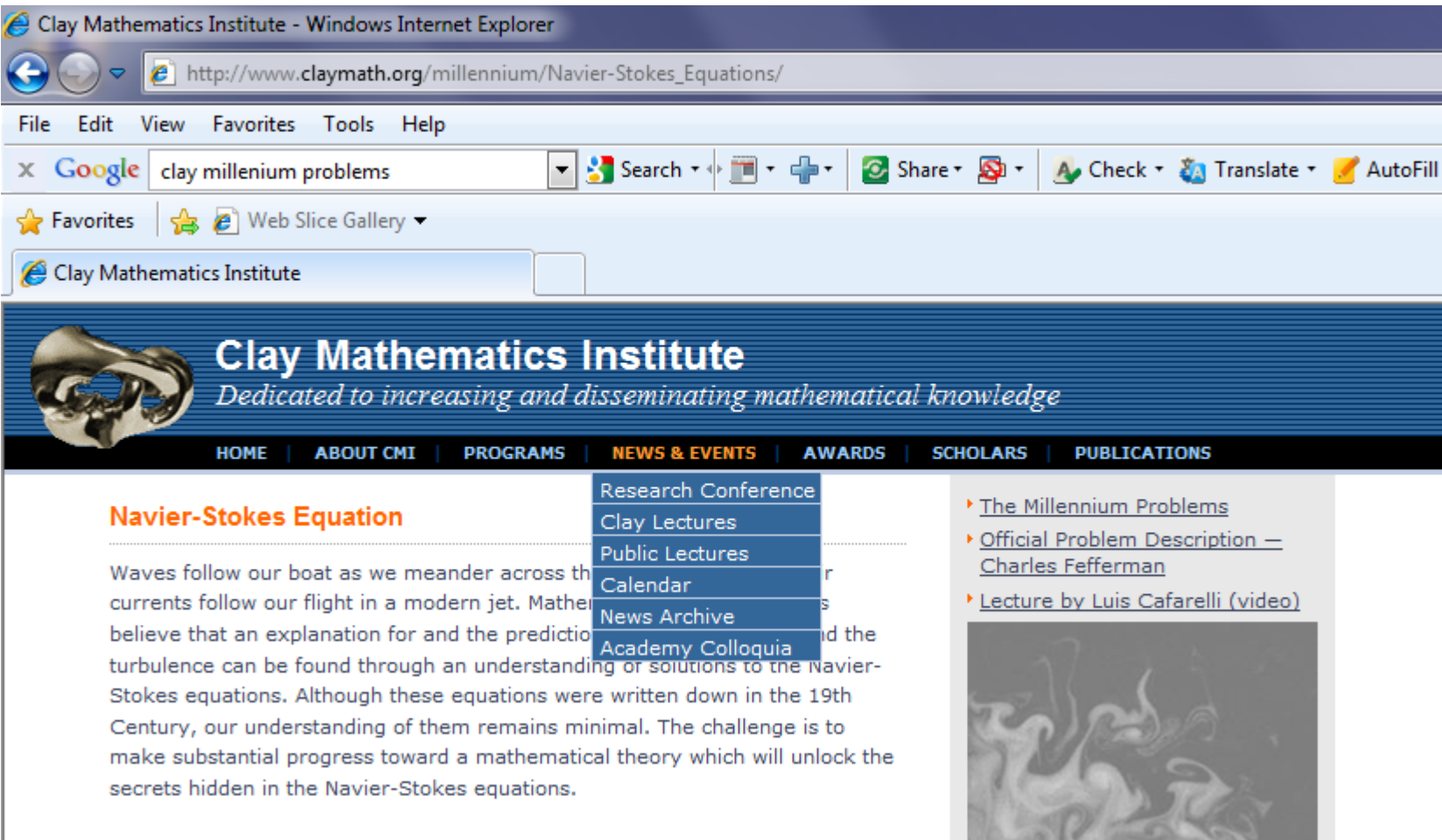


Bas van de Wiel

Stromingen en complexiteit



Millenium problems



The screenshot shows a Windows Internet Explorer browser window displaying the Clay Mathematics Institute website. The address bar shows the URL http://www.claymath.org/millennium/Navier-Stokes_Equations/. The browser's search bar contains the text "clay millenium problems". The website header features the Clay Mathematics Institute logo and the text "Clay Mathematics Institute" and "Dedicated to increasing and disseminating mathematical knowledge". A navigation menu includes links for HOME, ABOUT CMI, PROGRAMS, NEWS & EVENTS, AWARDS, SCHOLARS, and PUBLICATIONS. The main content area is titled "Navier-Stokes Equation" and contains a paragraph of text. A dropdown menu is open over the "NEWS & EVENTS" link, listing options such as "Research Conference", "Clay Lectures", "Public Lectures", "Calendar", "News Archive", and "Academy Colloquia". On the right side, there is a sidebar with links to "The Millennium Problems", "Official Problem Description — Charles Fefferman", and "Lecture by Luis Caffarelli (video)". Below these links is a small image showing a complex, swirling pattern, likely representing a turbulent flow.

Clay Mathematics Institute
Dedicated to increasing and disseminating mathematical knowledge

HOME | ABOUT CMI | PROGRAMS | **NEWS & EVENTS** | AWARDS | SCHOLARS | PUBLICATIONS

Navier-Stokes Equation

Waves follow our boat as we meander across the currents follow our flight in a modern jet. Mathematicians believe that an explanation for and the prediction of the turbulence can be found through an understanding or solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

- Research Conference
- Clay Lectures
- Public Lectures
- Calendar
- News Archive
- Academy Colloquia

- ▶ [The Millennium Problems](#)
- ▶ [Official Problem Description — Charles Fefferman](#)
- ▶ [Lecture by Luis Caffarelli \(video\)](#)

“The challenge is to make substantial progress towards a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations”

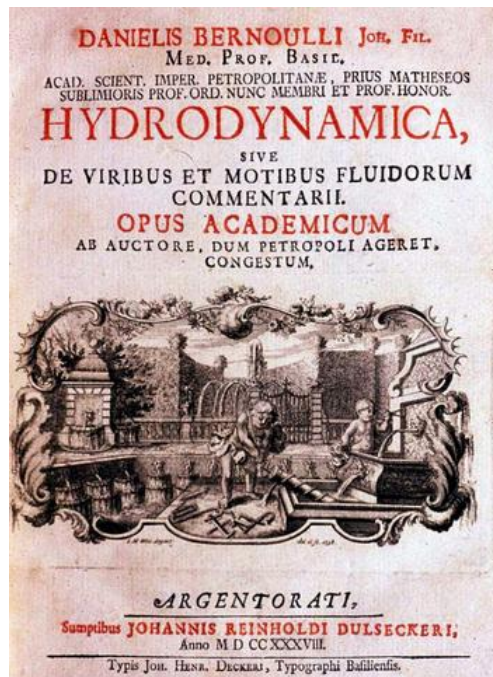
De wet van Bernoulli

- Onder bepaalde omstandigheden vereenvoudiging toegestaan
- De wet van Bernoulli
- Begrip alledaagse stromingsverschijnselen

Familie Bernoulli

- Wiskundigen: broers (1654-1705) Jacob en Johan (1667-1748) (Basel, later Groningen)
- Johan's leerling: Euler
- Zoon Johan: Daniel: natuurkundige (Groningen)

•



Daniel

*“Daar waar de snelheid hoog is,
is de druk laag....”*



Demonstratie

*“Daar waar de snelheid hoog is,
is de druk laag....”*

De bernoulli-vergelijking

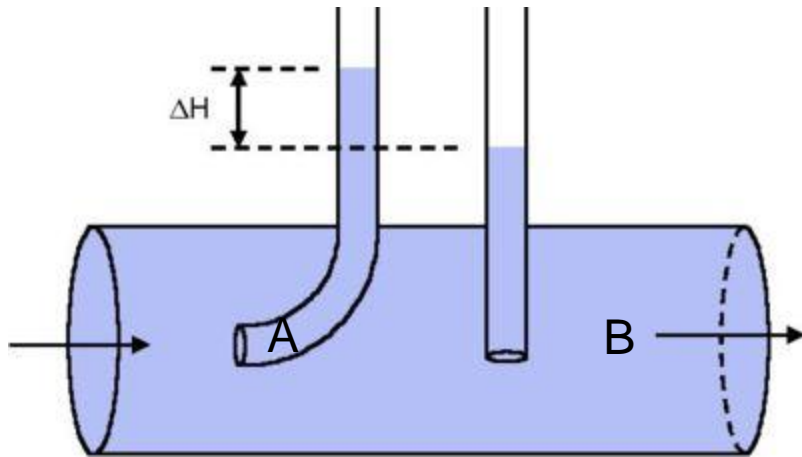
$$\underbrace{\frac{P}{\rho} + gz}_{\text{Pot. En.}} + \underbrace{\frac{1}{2} V^2}_{\text{Kin. En.}} = C$$

Behoud mechanische energie

(wrijvingsloos, stationair, rotatievrij)

De Pitot-buis

$$\frac{P}{\rho} + gz + \frac{1}{2}V^2 = C$$

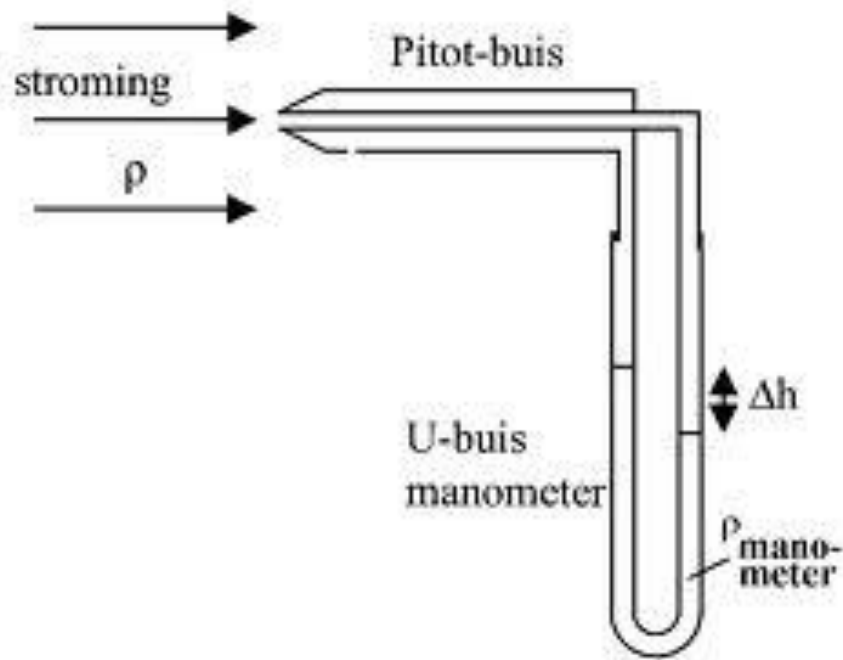


$$\frac{P_A}{\rho} + \frac{1}{2}V_A^2 = \frac{P_B}{\rho} + \frac{1}{2}V_B^2$$

0 (stagnatie)

$$V_B = \sqrt{2 \frac{P_A - P_B}{\rho}}$$

De Pitot-buis



F16

Op de fiets

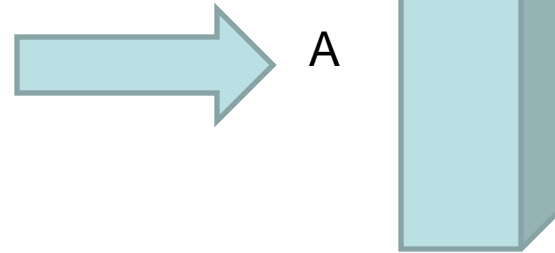
Herschrijf de Pitot formule:

$$V_B = \sqrt{2 \frac{P_A - P_B}{\rho}}$$

$$P_A - P_B = \frac{1}{2} \rho V_B^2$$

Luchtweerstand

$$F_w = A(P_A - P_B) = \frac{1}{2} \rho A V_B^2$$



Op de fiets

Vermogen

$$F_w V = \frac{1}{2} \rho A V^3$$

Volwassen recreant:

- vermogen 300 Watt
- dichtheid 1 kg/m³
- effectief oppervlak 0.6 m²

V =?

V = 10 m/s =
36 km/uur



25% harder?

25% harder,.... = 45 km/hr

Welk vermogen nodig?

$$F_w V = \frac{1}{2} \rho A V^3$$

Vermogen= 586 Watt

95% meer!!

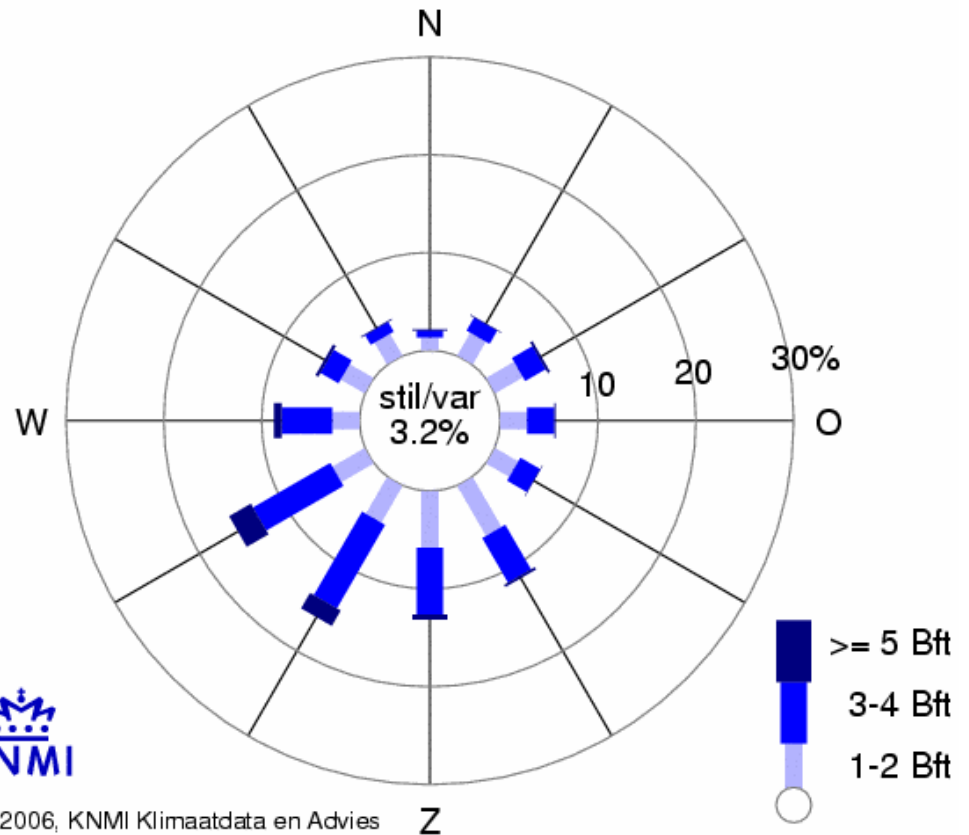


Windopbrengst



$$\text{Vermogen} \propto V^3$$

Windroos De Bilt, klimatologie januari



(c) 2006, KNMI Klimaatdata en Advies

Z

Bergafwaarts met Mart Smeets

Wie gaat sneller de lichte of de zware persoon?

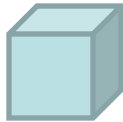


Bergaf

$$F_w = F_g \sin \alpha$$

$$\frac{1}{2} \rho_{lucht} \cdot L^2 \cdot V^2 = \rho_{fietser} \cdot L^3 \cdot g \sin \alpha$$

$$V = \sqrt{\frac{2 \rho_{fietser} g \sin \alpha L}{\rho_{lucht}}}$$

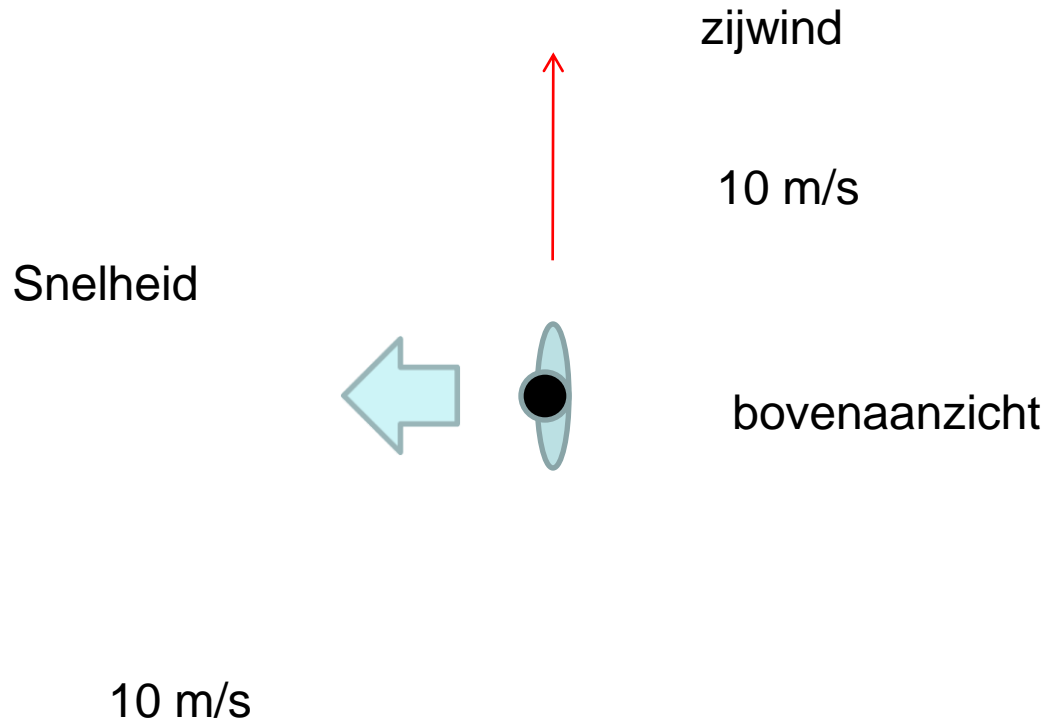


α

De grote is in het voordeel.....bergop is een ander verhaal.....

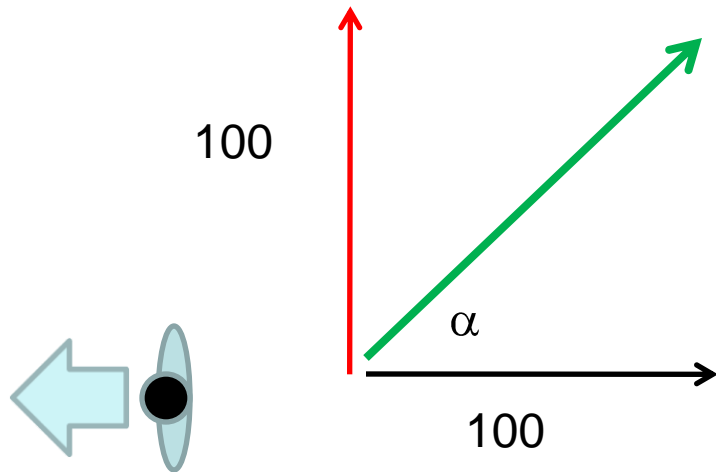
zijwind

“Ik heb altijd wind tegen!”



zijwind

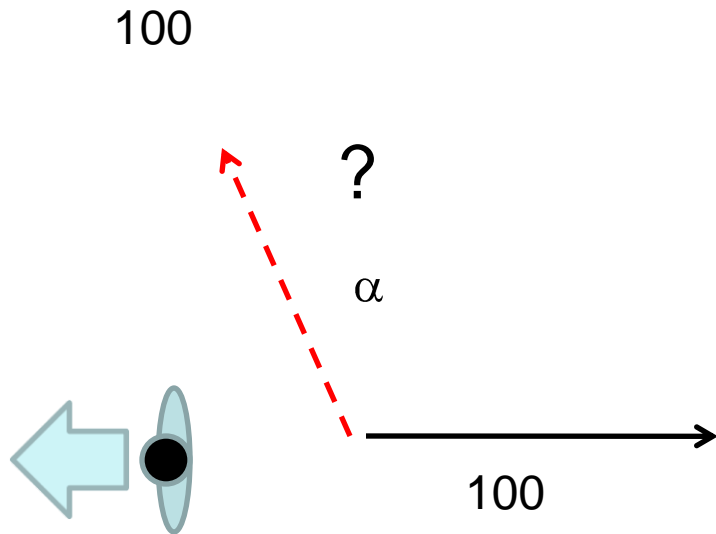
Voor 't gemak: $|\vec{F}| = |\vec{V}|^2$



$$F_x = \cos \alpha |\vec{V}|^2 = \frac{\textit{Fiets}}{\sqrt{\textit{zij}^2 + \textit{Fiets}^2}} \cdot (\textit{zij}^2 + \textit{Fiets}^2)$$

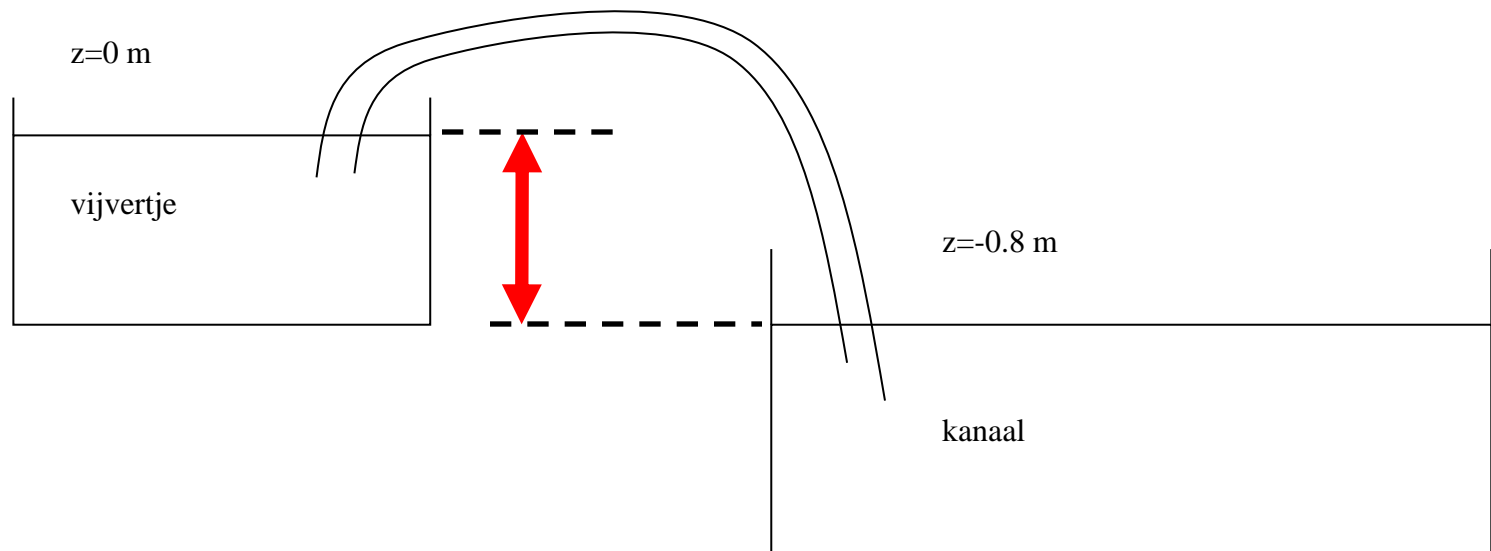
Het **kwadraat** veroorzaakt de netto zijwind bijdrage!

huiswerk



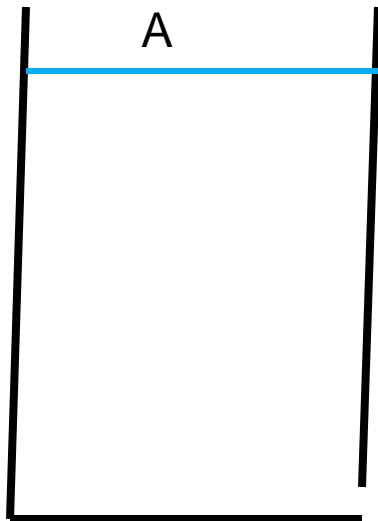
Bij welke hoek kan ik (eindelijk) spreken van een windvoordeel in de rug?

Leegloop vijver



Analoog model

$$\frac{\cancel{P_A}}{\cancel{\rho}} + g z_A + \frac{1}{2} \cancel{V_A}^2 = \frac{\cancel{P_B}}{\cancel{\rho}} + \cancel{g z_B} + \frac{1}{2} V_B^2$$



$$V_B = \sqrt{2 g z_A}$$

(val-snelheid!)

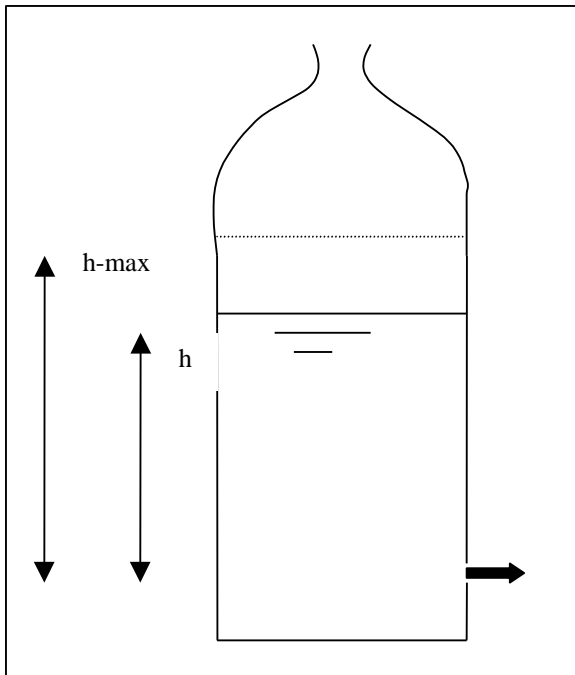
B

Referentie: z=0

Analoog model: PET fles

Massabalans

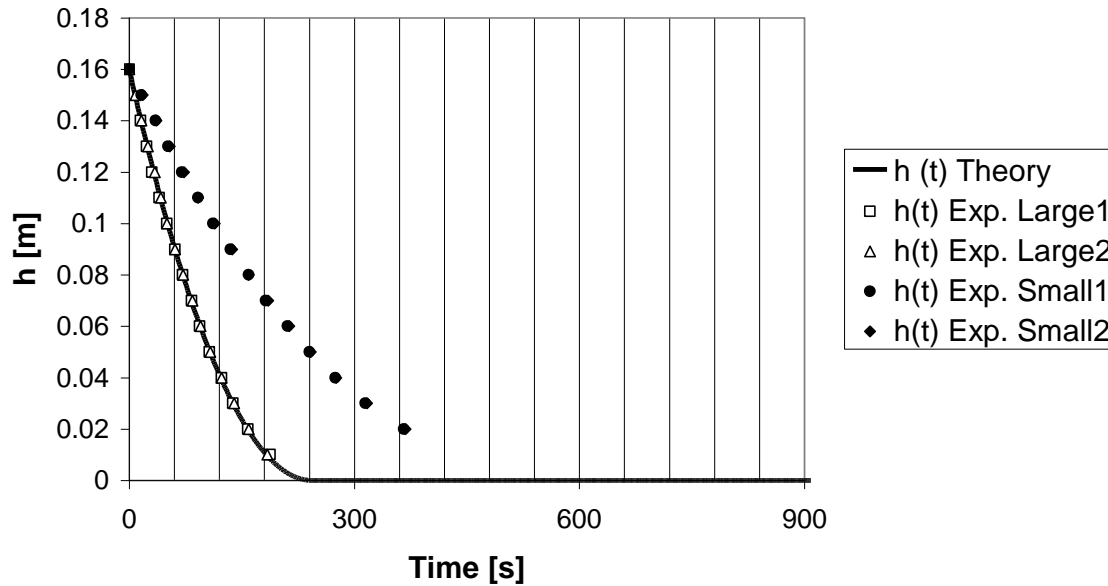
$$-A_{fles} \frac{dh}{dt} = \text{uitstroom} = -A_{\text{gaatje}} \sqrt{2gh}$$



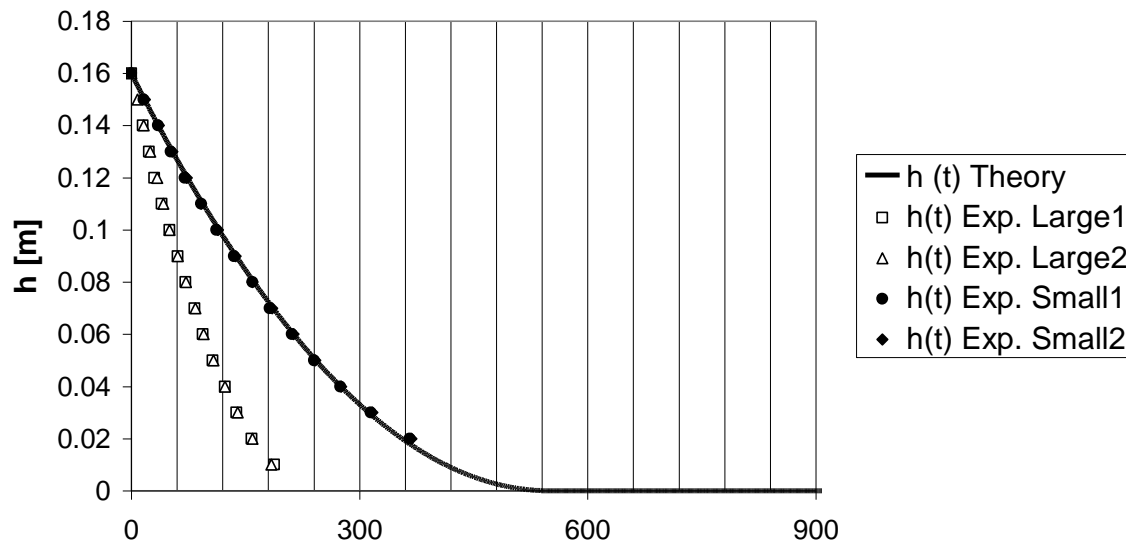
$$h = \left(\sqrt{h_{\max}} - \frac{1}{2} \alpha t \right)^2$$

$$\alpha = \frac{A_{\text{gaatje}}}{A_{\text{fles}}} \sqrt{2g}$$

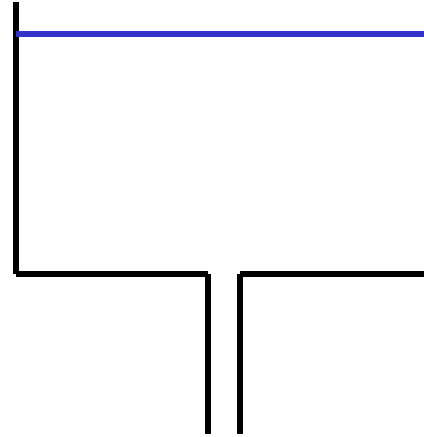
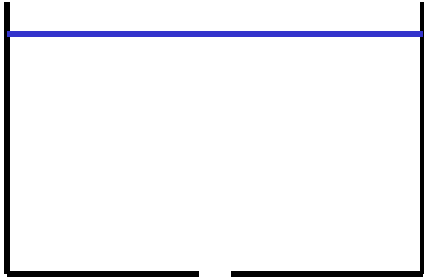
In de klas: PET fles



.....correctie voor
straalcontractie 80%



In de klas

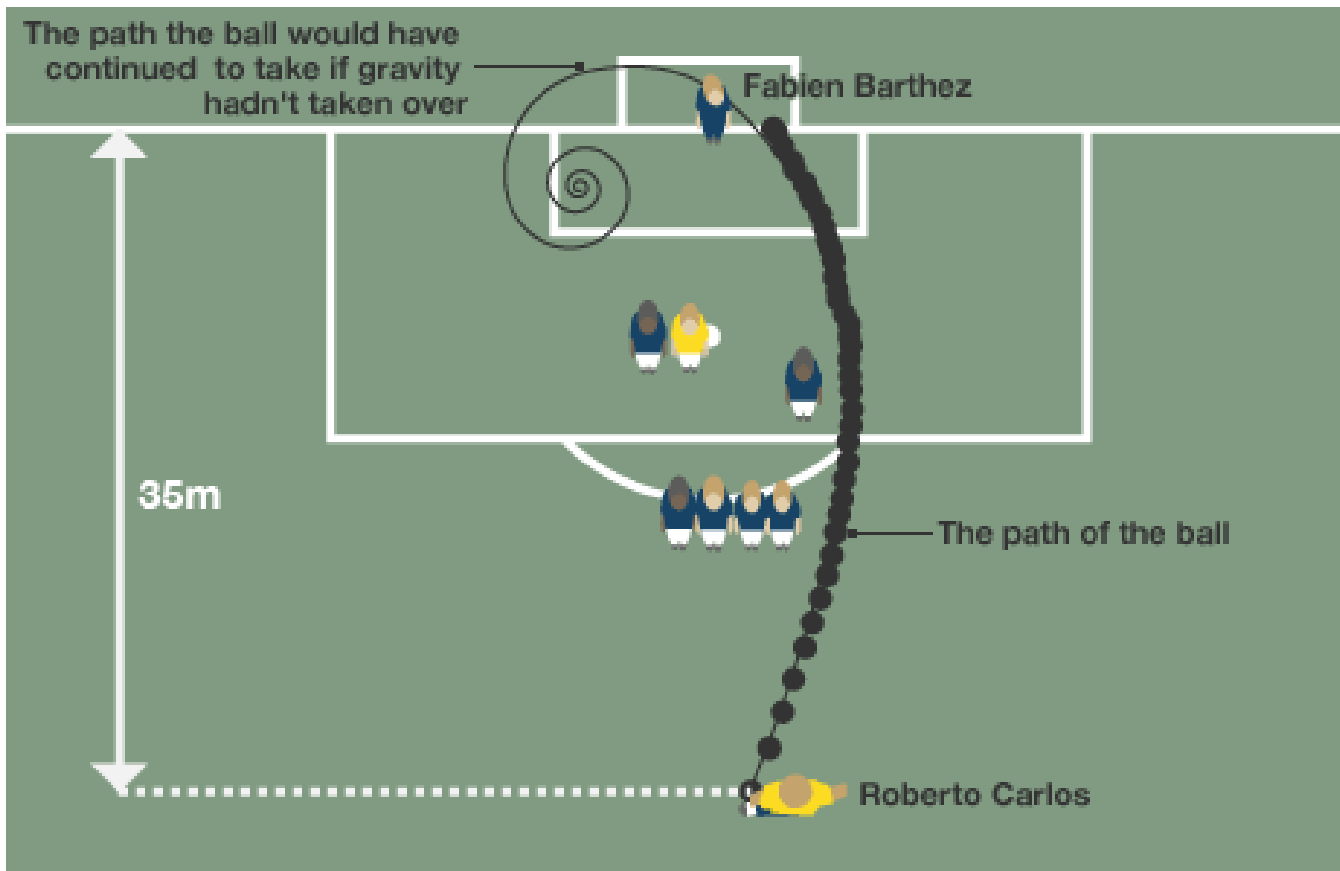


Welke loopt het snelst leeg?

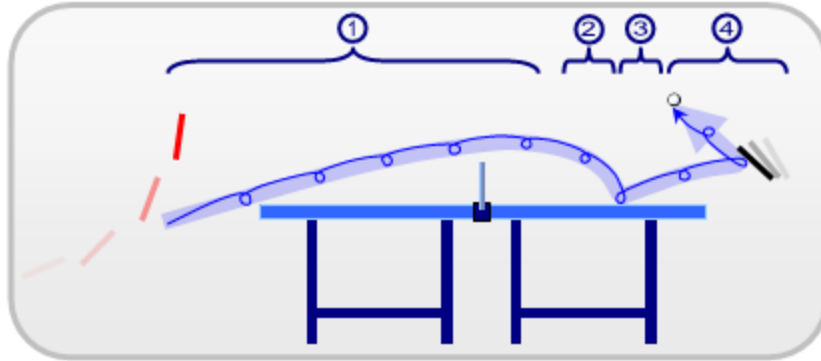
- a) Linker, natuurlijk
- b) Rechter, natuurlijk
- c) Even snel

Effectballen

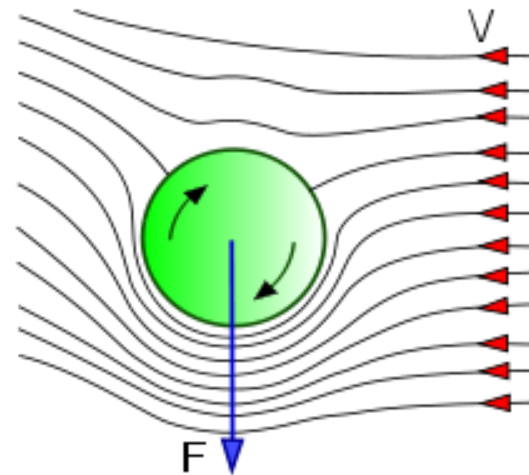
“the ball boy 10 yards to the right ducked instinctively”



Bernoulli-verklaring

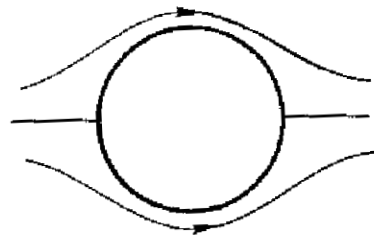


Het Magnus-effect

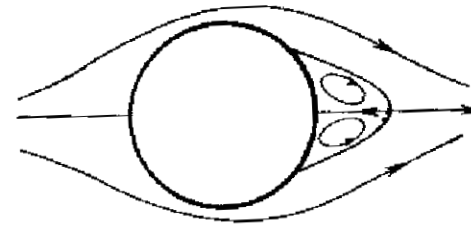


Hoge snelheid \sim lage druk

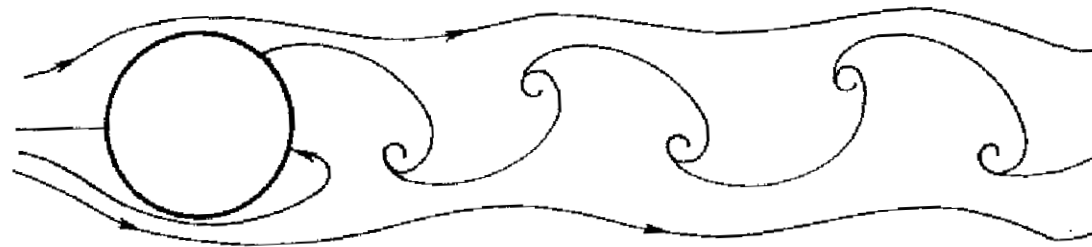
Grenslagen & loslaatpunten



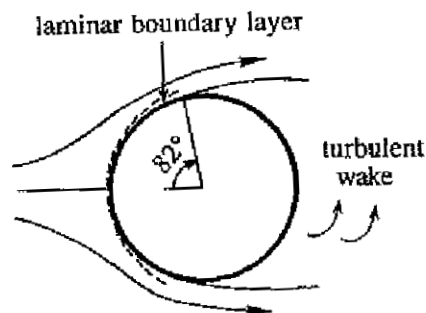
$Re < 4$



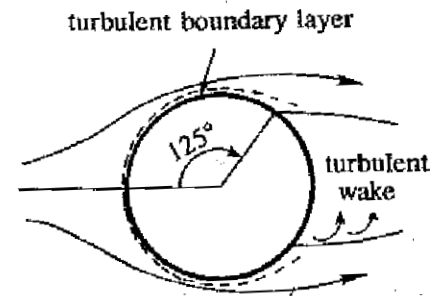
$4 < Re < 40$



$80 < Re < 200$



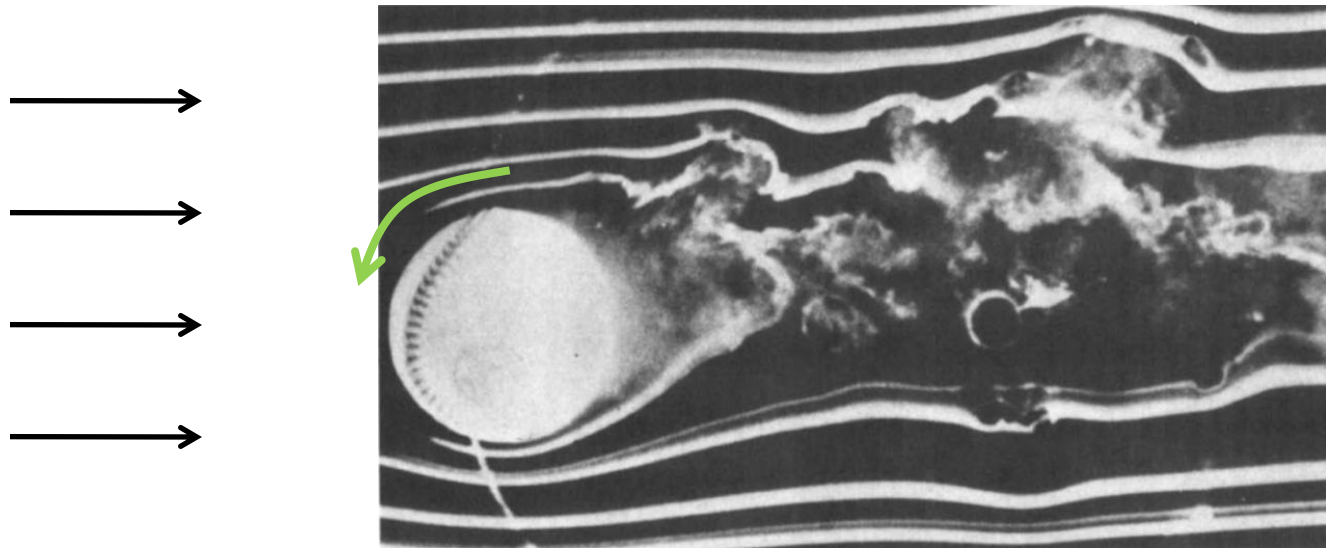
$Re < 3 \times 10^5$



$Re > 3 \times 10^5$

Turbulente zog

Actie=-reactie



Effectballen

Normaal Magnus-effect:

-Ruwe bal

-turbulent - zeer turbulent zog

-Net als Bernoulli verklaring

Omgekeerd Magnus-effect:

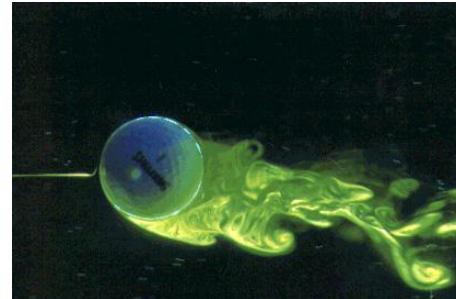
-zwabberende strandbal

-turbulent-Laminair zog:

-verkeerd-om Bernoulli

Baleigenschappen

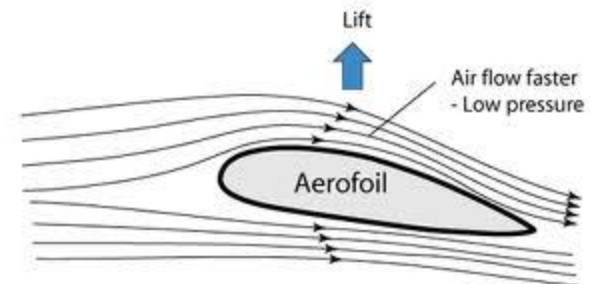
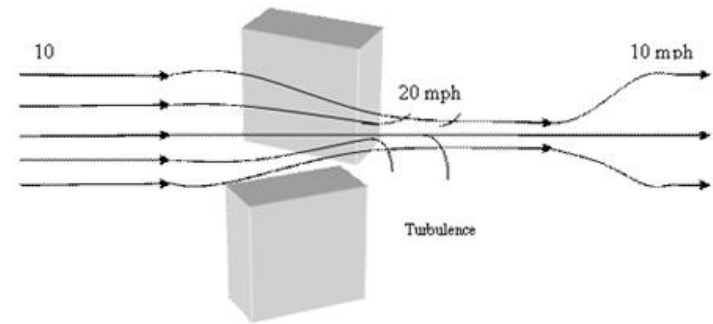
- Ruwheid
- Gewicht
- # omwentelingen (stabiliteit!)
& snelheid
-



Jabulani in windtunnel

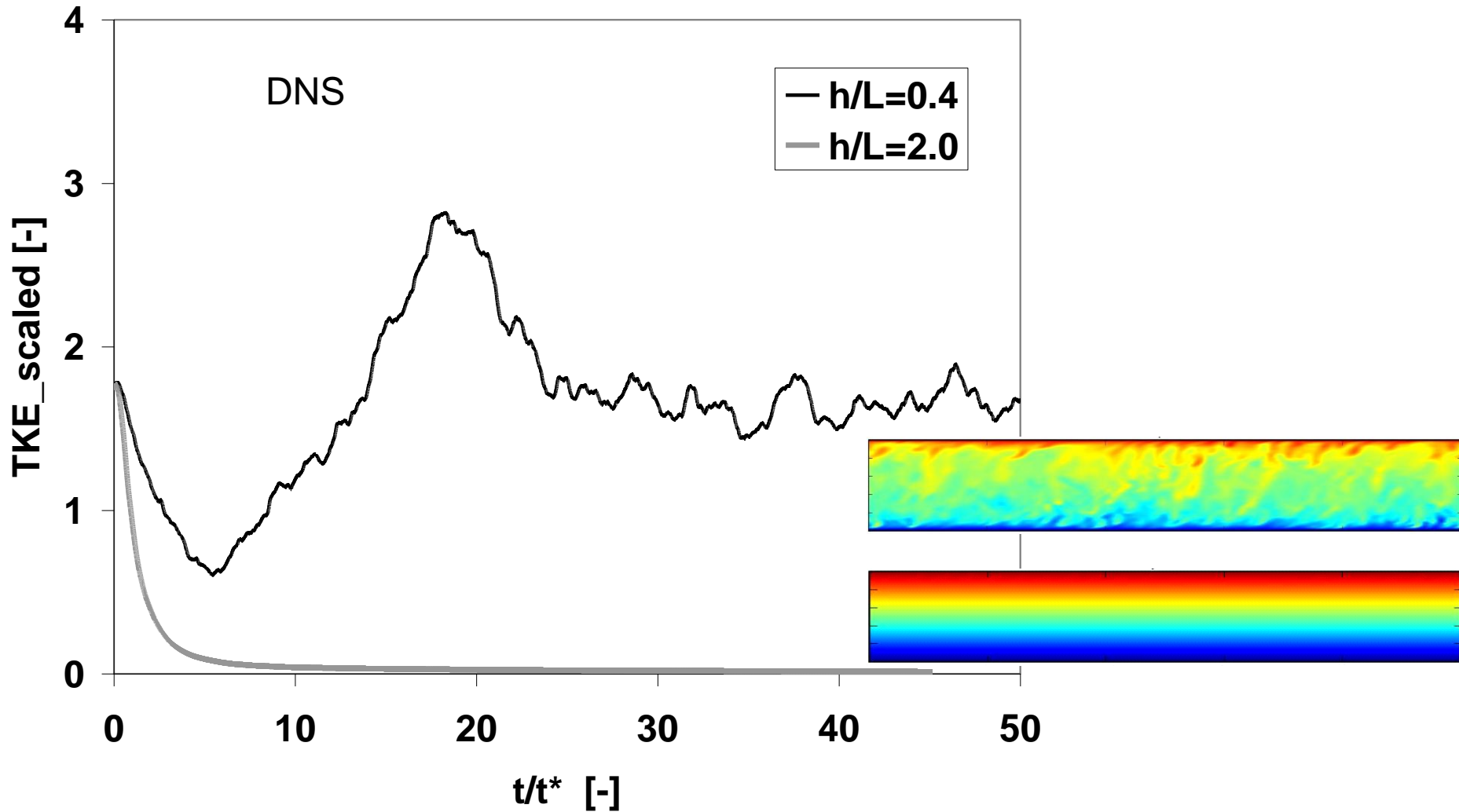


En meer....



Onderzoek





Afsluiting



Uitdaging voor toekomstige wis- en natuurkundigen!

Bedankt voor uw aandacht!