Mathematics and the Groove in Music

Nationale Wiskunde Dagen
February 6, 2009

Anja Volk, ICS, Utrecht University

Institute of Information & Computing Sciences ICS

Universiteit Utrecht

Outline

- Introduction
  - Rhythm
  - Importance of Rhythm
- Inner Metric Analysis
  - Mathematical Model for Analyzing the Rhythm
  - Some Applications
- Dance Music Classification, Music Information Retrieval

Introduction

Mambo !

Picture from hcs.harvard.edu/~hma-bdc

Merengue !

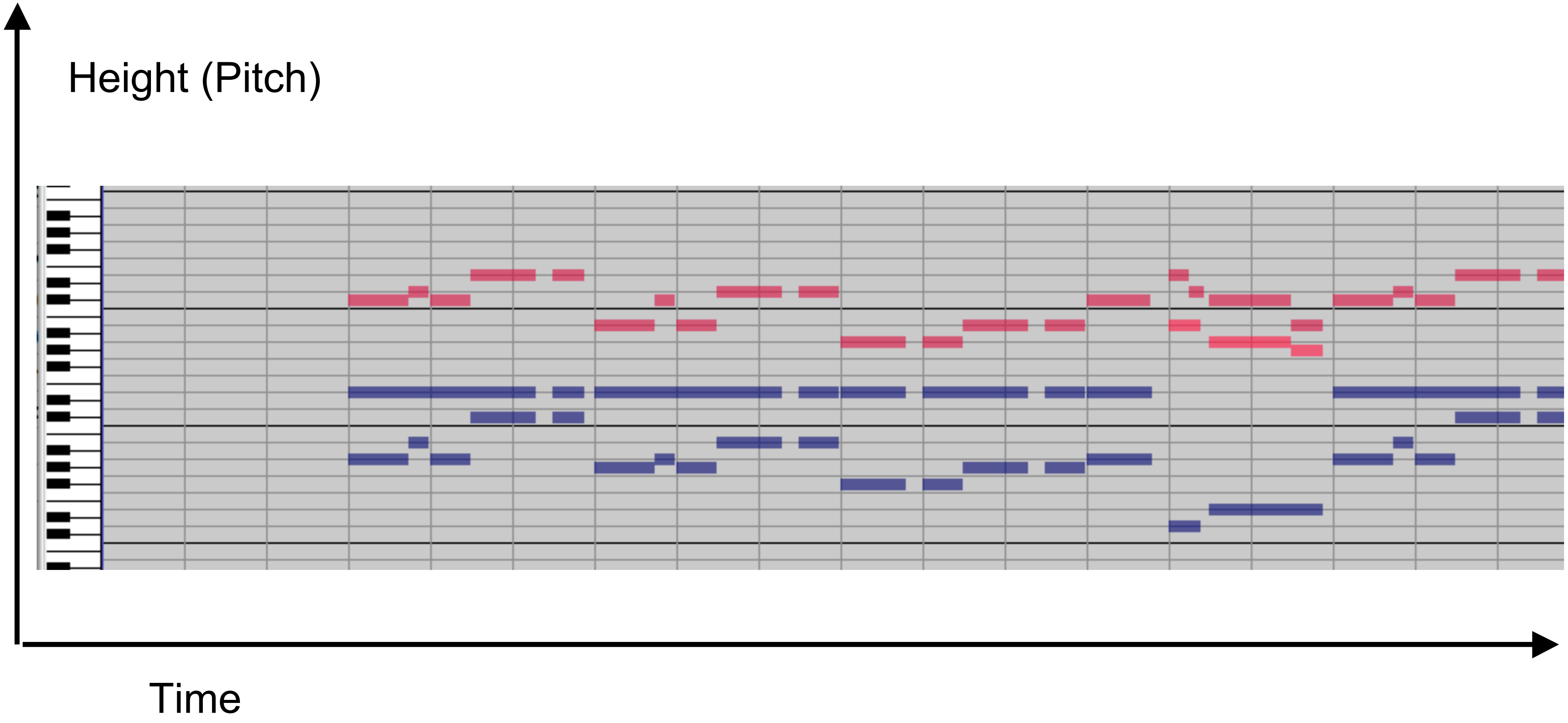
In how far are all mambos more similar to each other than to a merengue??

Jessica Phillips-Silver: Infant Rhythm and Music Research

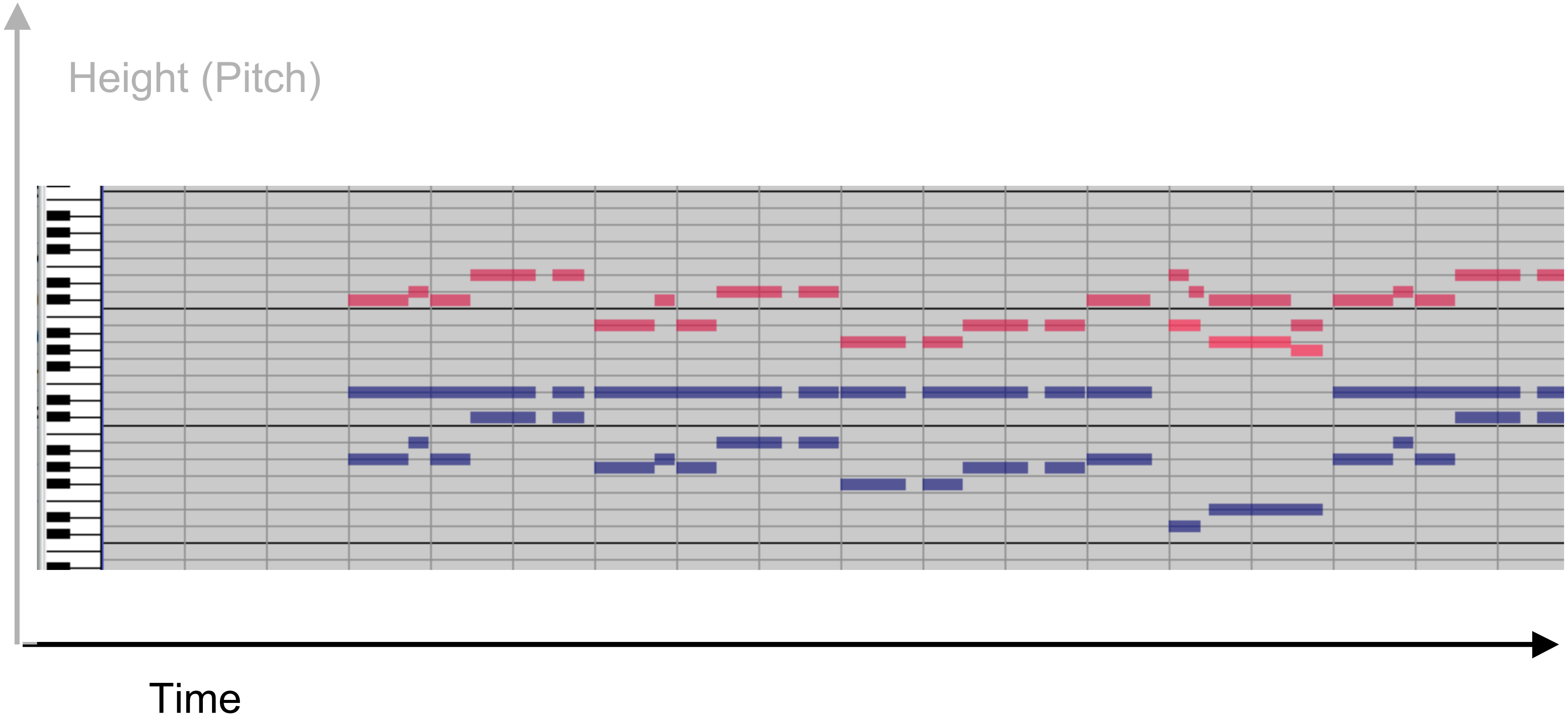
7 months old babies: triple meter bouncing, duple meter bouncing

Phillips-Silver/Trainor: Feeling the Beat: Movement Influences Infant Rhythm Perception, 2005

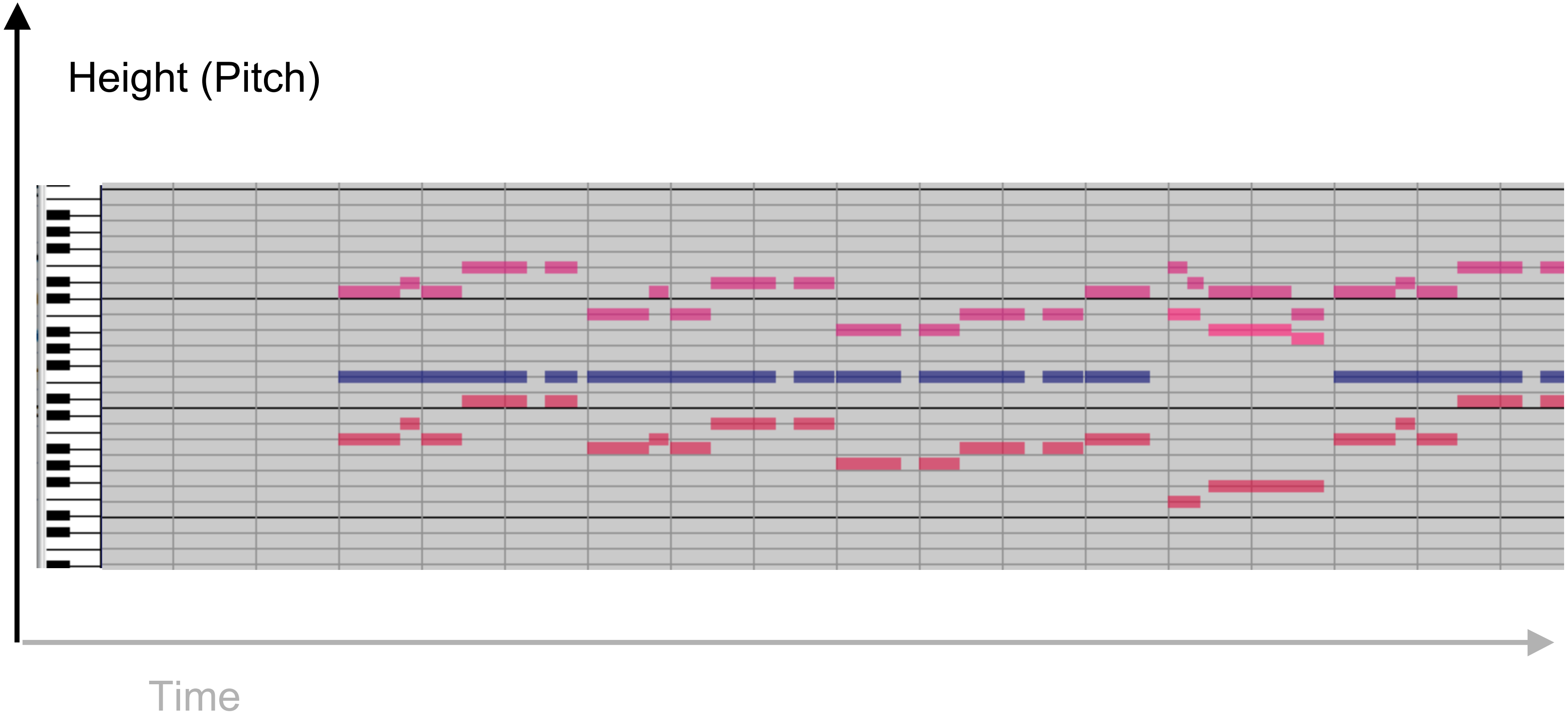
Music Features


Height (Pitch)
Time

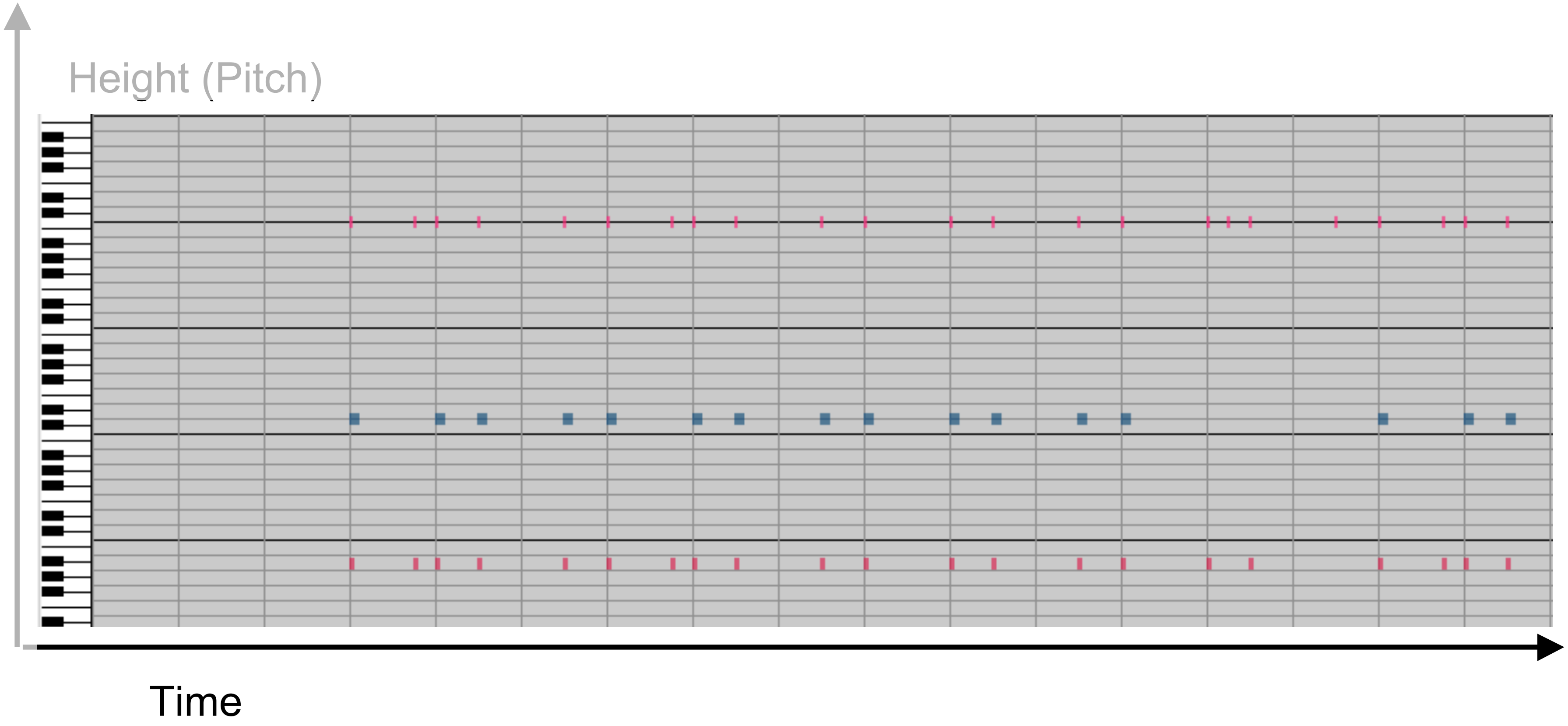
Music Features


Height (Pitch)
Time

Music Features

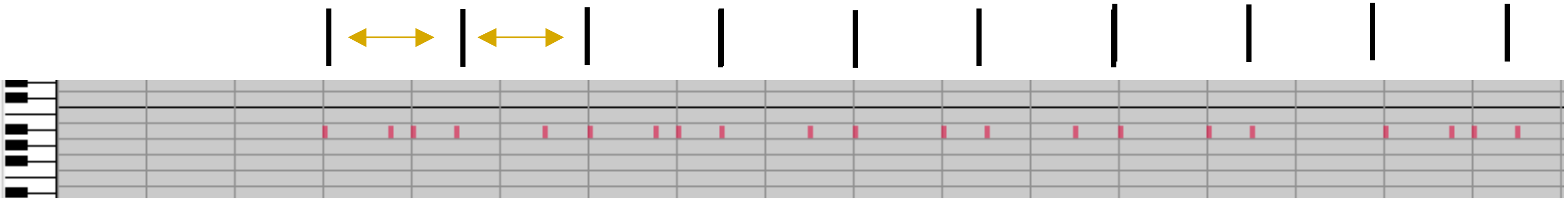


Time: Rhythm


Height (Pitch)
Time

Rhythm

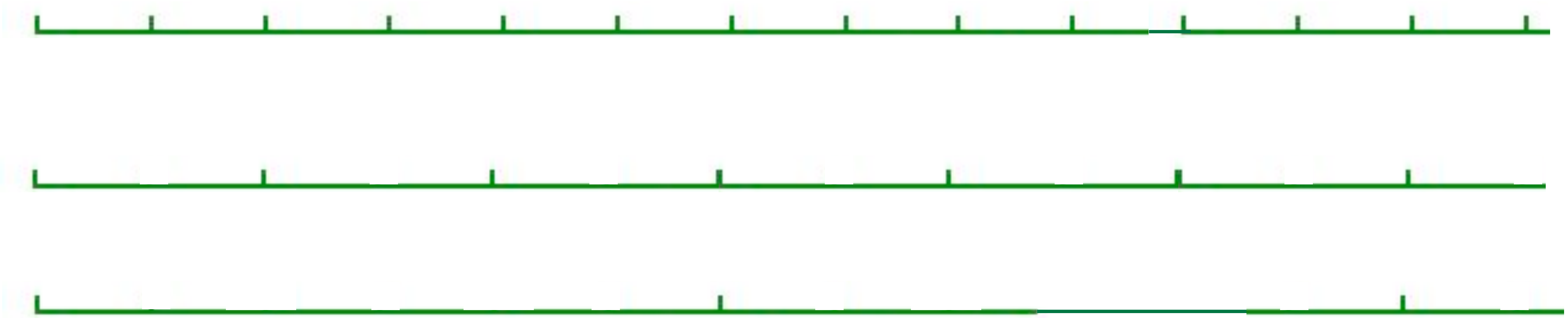
Spontaneous reaction: tapping



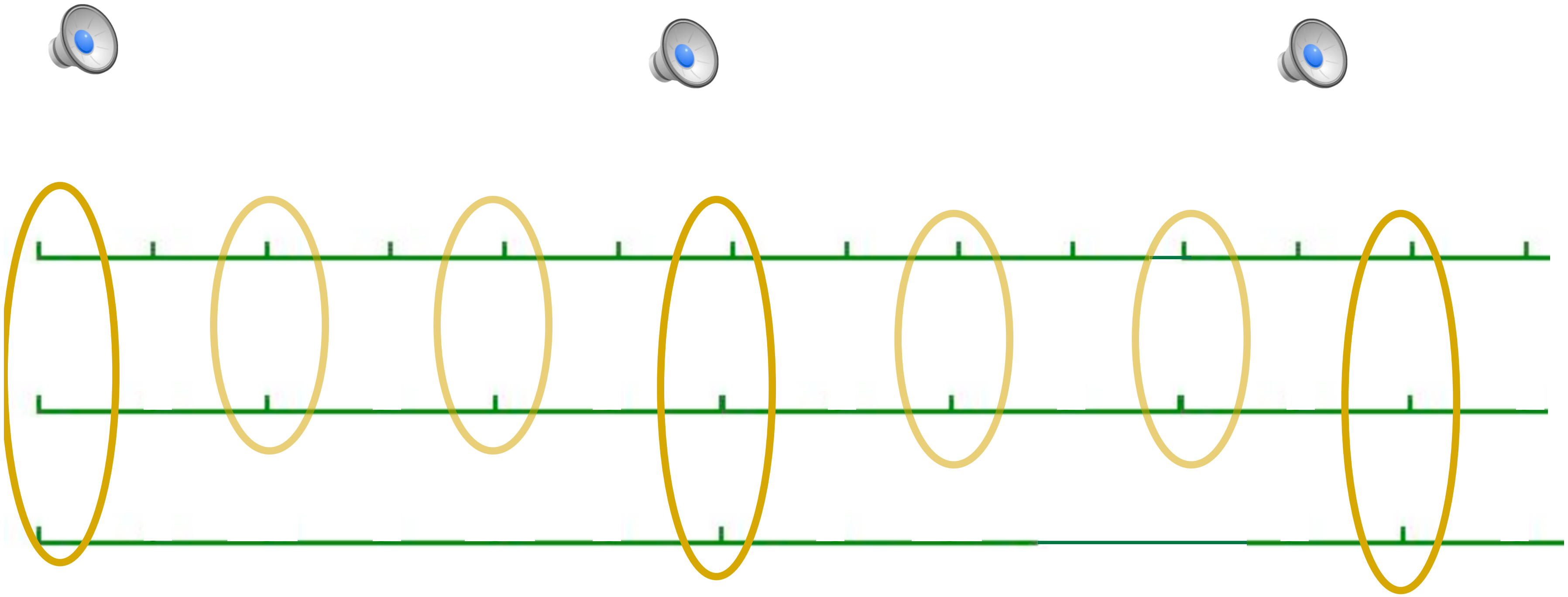
Rhythm

Rhythm: Meter

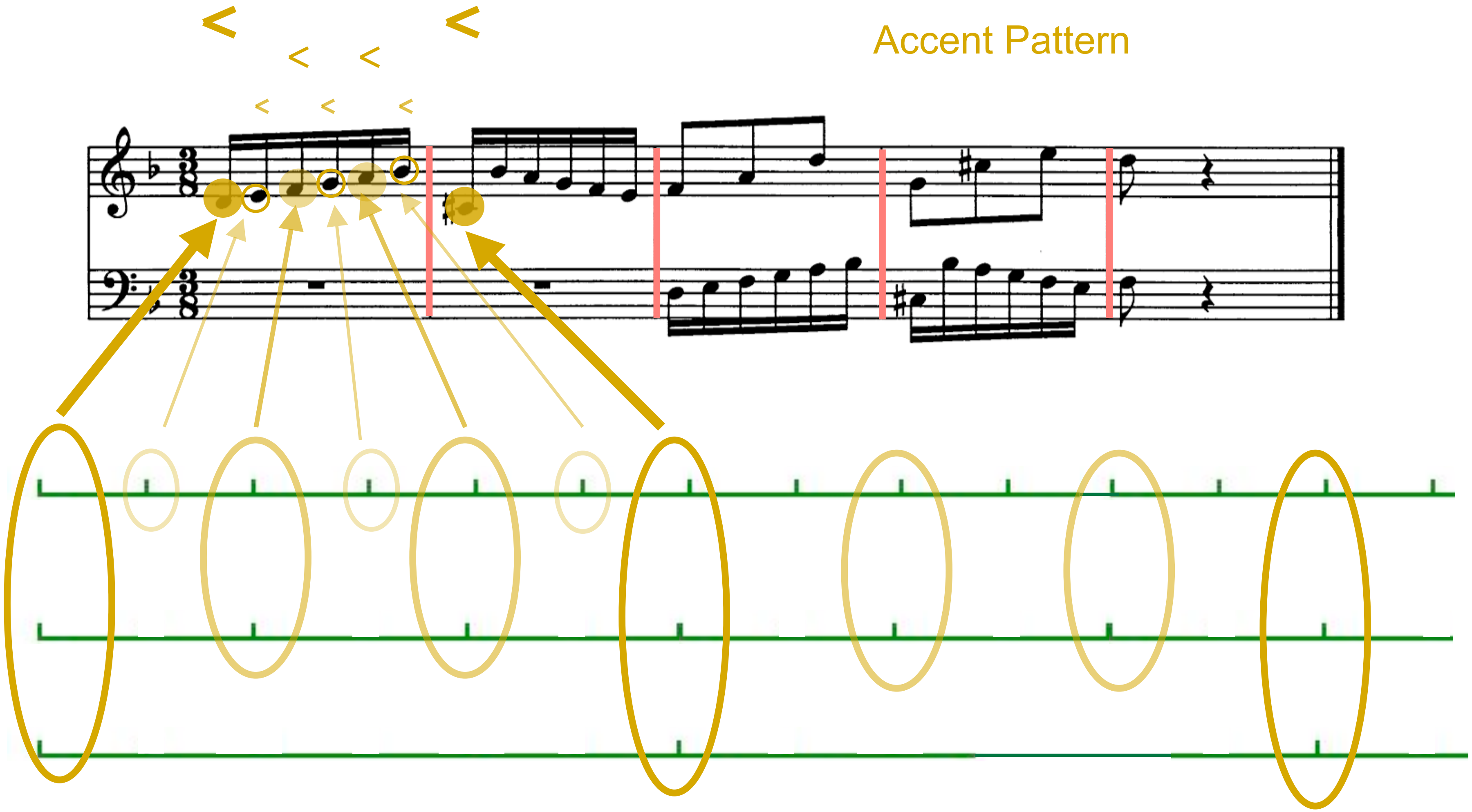
nested pulses



Rhythm: Meter

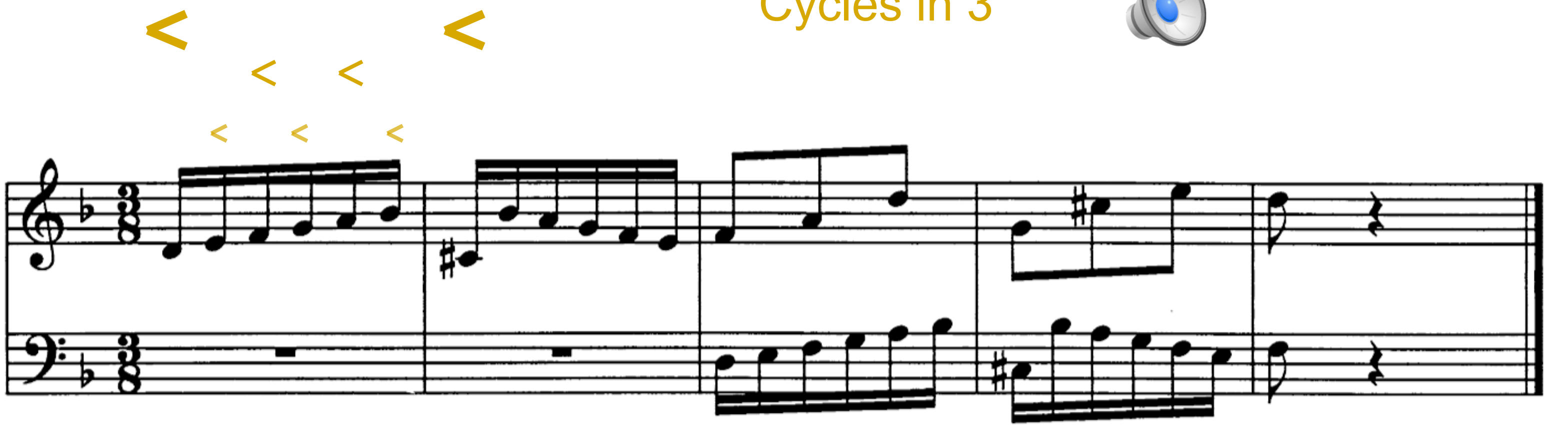


Rhythm: Meter


Accent Pattern

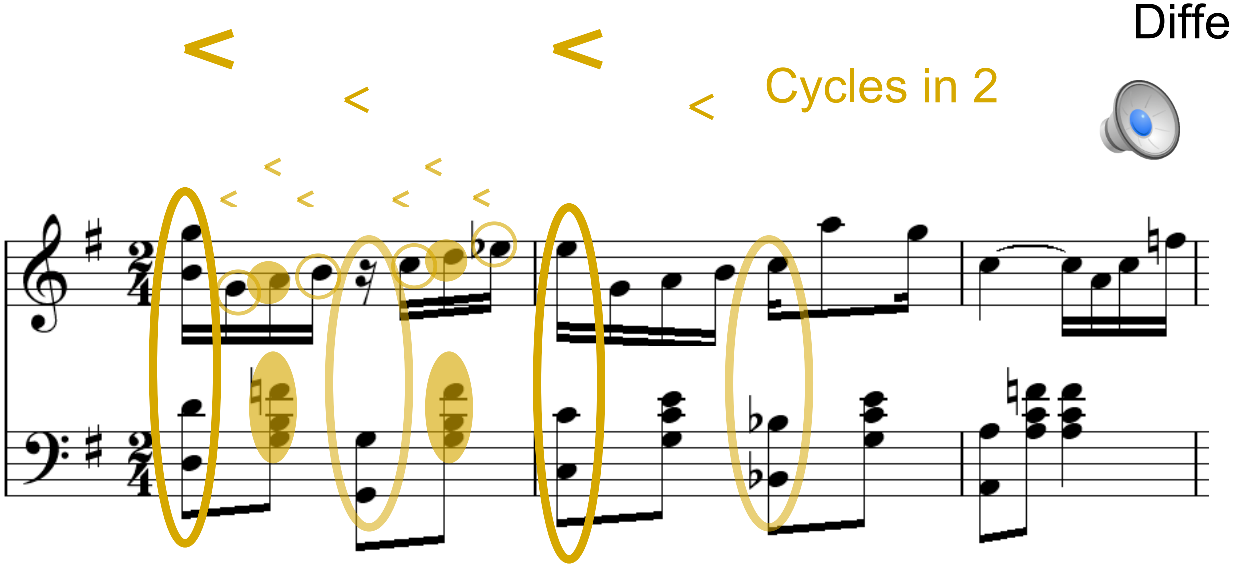
Rhythm: Meter

Cycles in 3



Different Accent Patterns

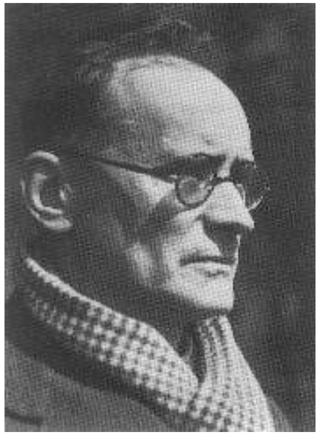
Cycles in 2



Rhythm: Meter

Anton Webern

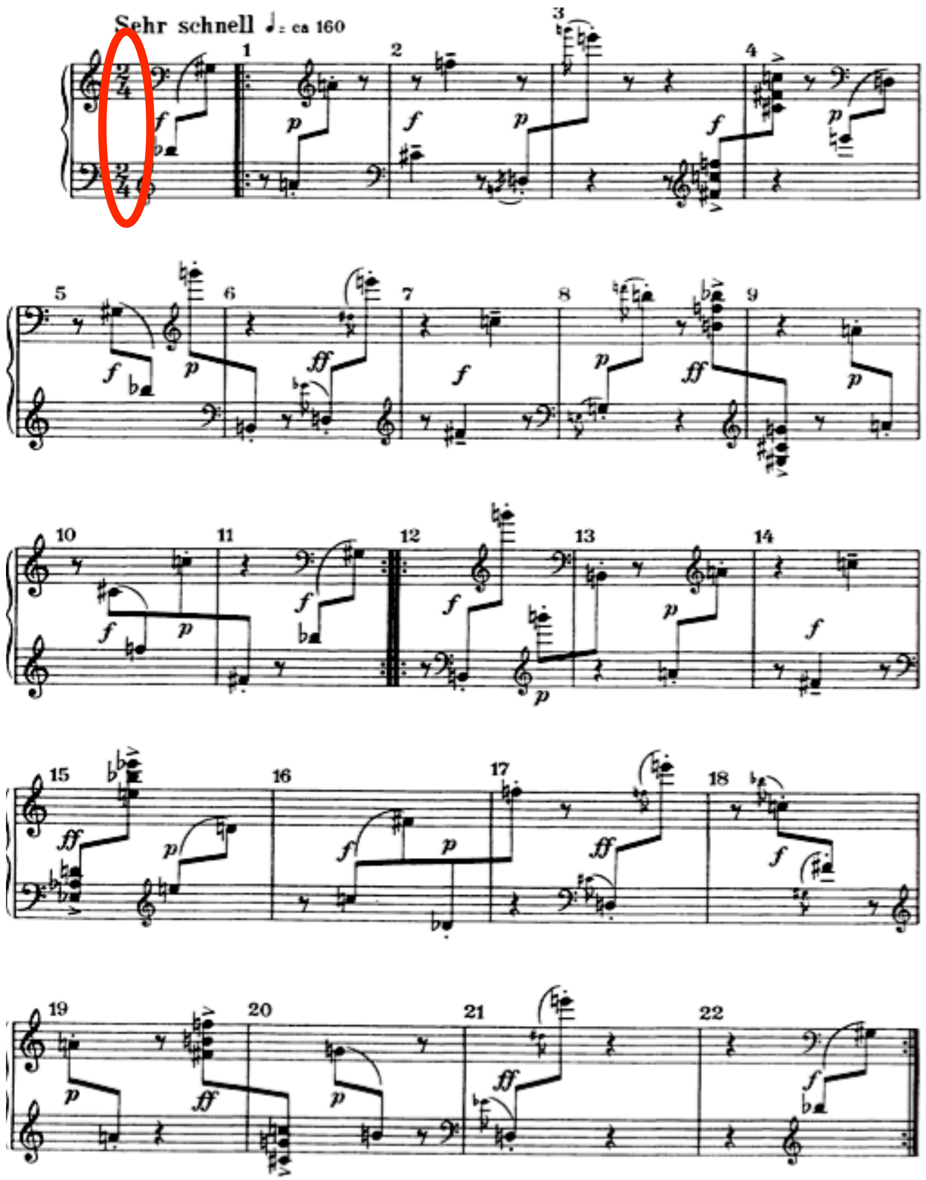
Piano Variations Op. 27, II

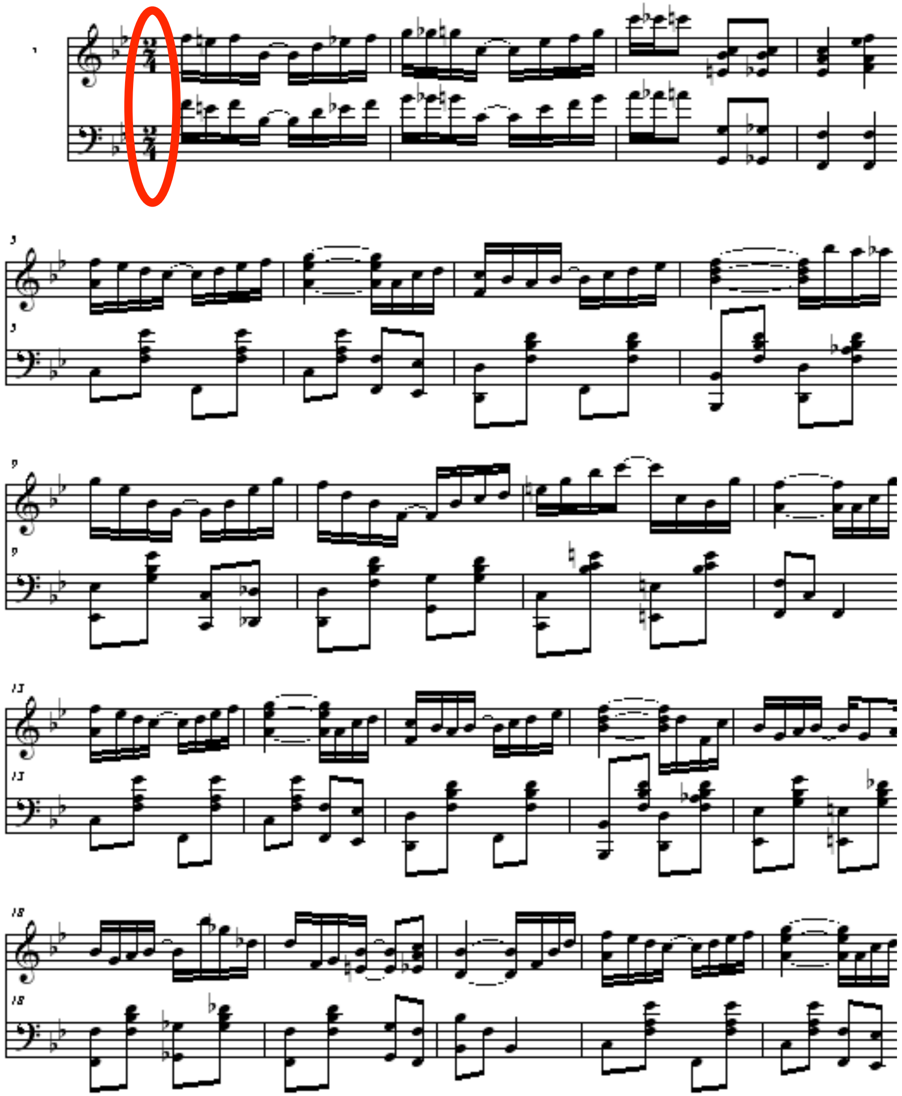


Scott Joplin

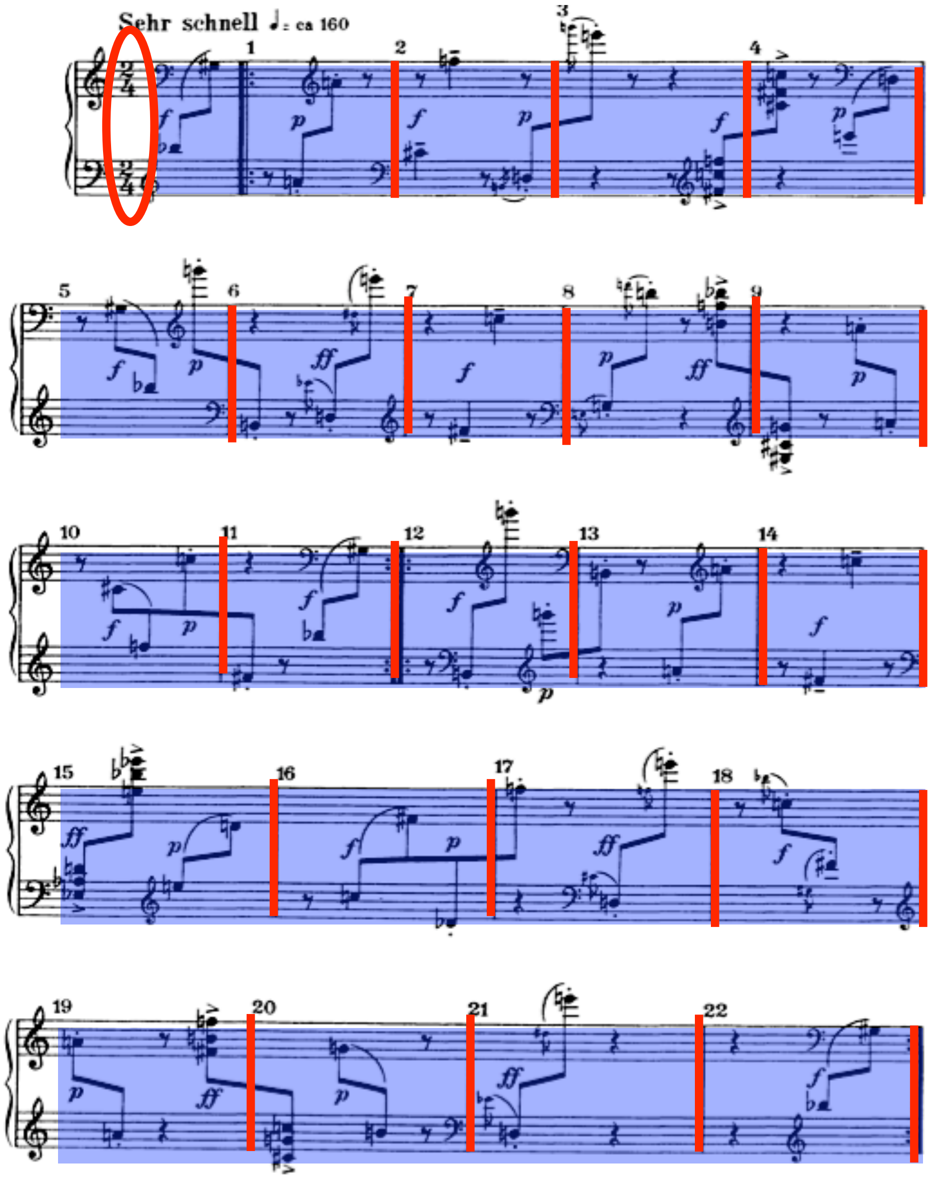
Nonpareil Rag

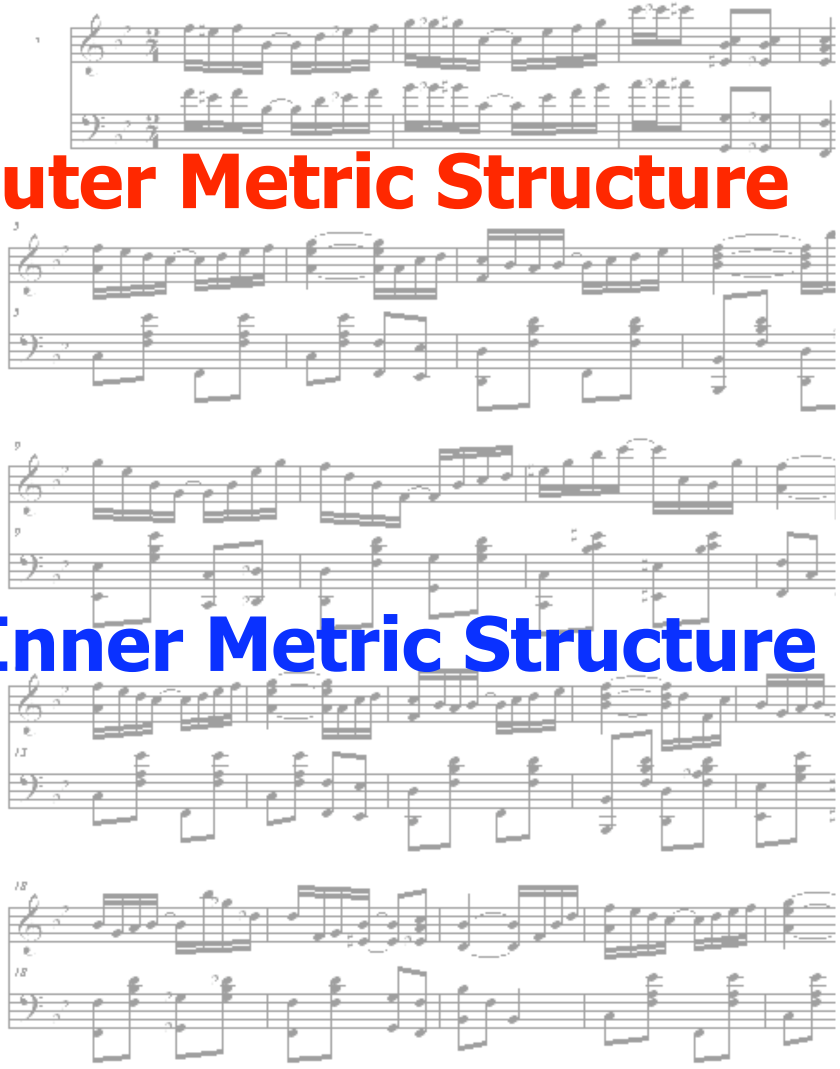
Rhythm: Meter


Sehr schnell ♩ = ca 160
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22



Rhythm: Meter


Sehr schnell ♩= ca 160
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22



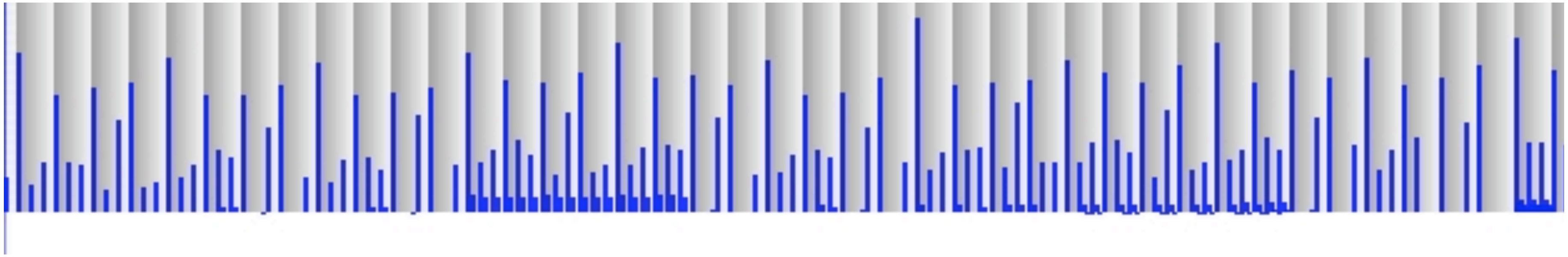
Outer Metric Structure

Inner Metric Structure

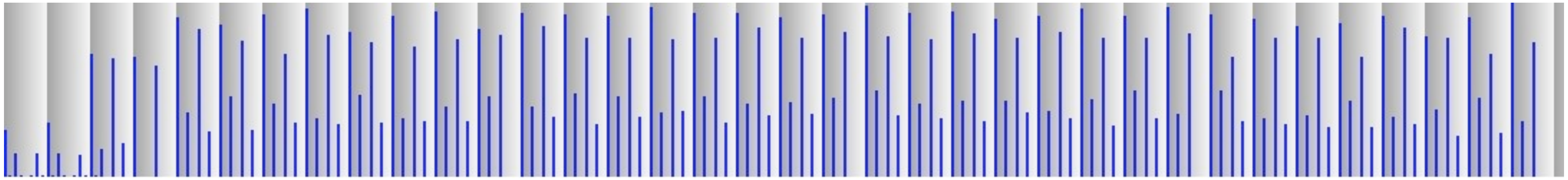
Inner Metric Analysis

Metric Weight

Cycles in 3



Cycles in 2

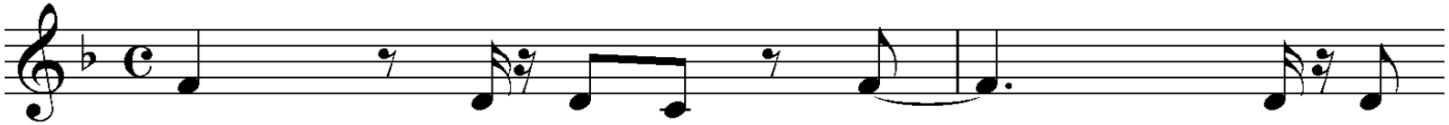


Outline

- Introduction
    - Rhythm
    - Importance of Rhythm
- Inner Metric Analysis

Inner Metric Analysis

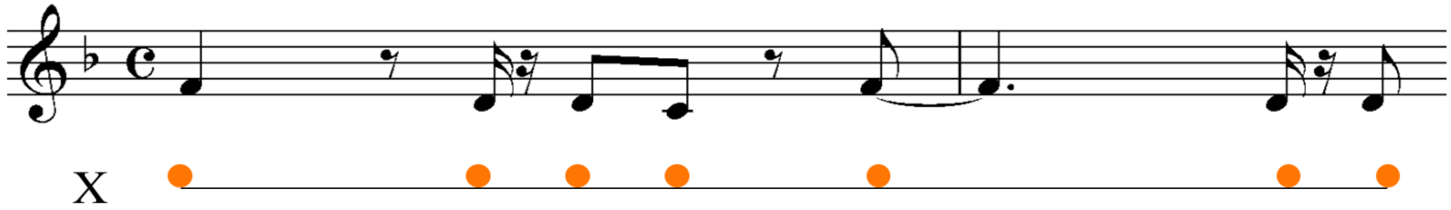
Metric Weight



Picture from www.ritzenhoff.de

Inner Metric Analysis

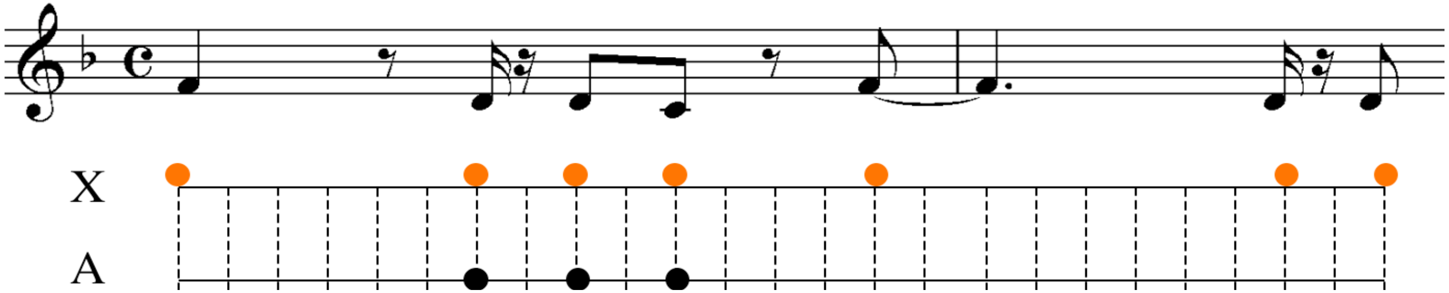
Metric Weight



Picture from www.ritzenhoff.de

Inner Metric Analysis

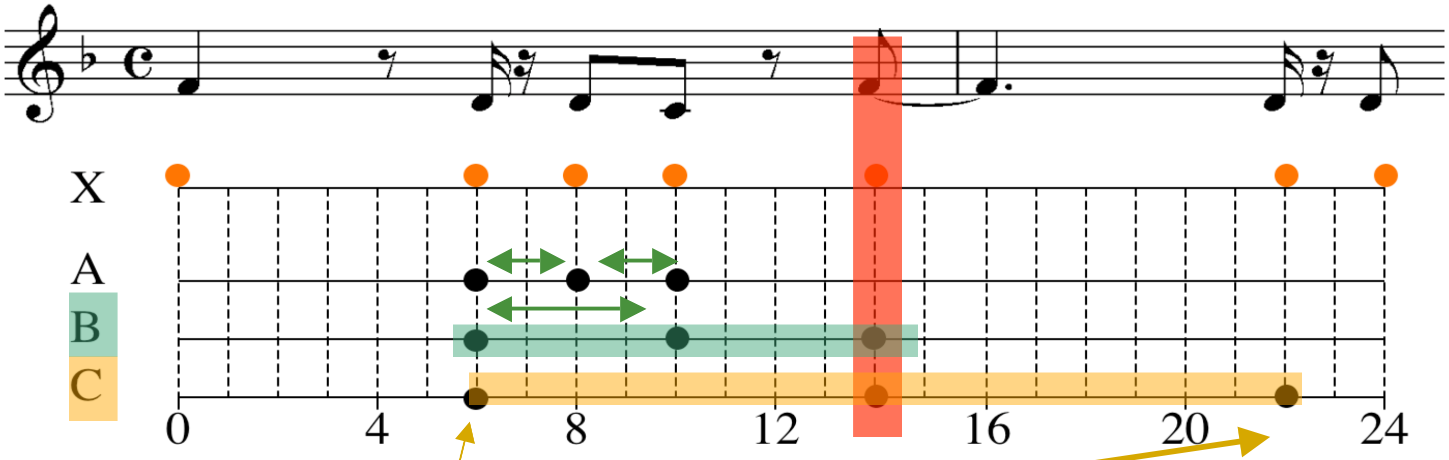
Metric Weight



Picture from www.ritzenhoff.de

Inner Metric Analysis

Metric Weight


X
A
B
C
0
4
8
12
16
20
24

$$m(s,d,k) = \{s + id, i = 0,\ldots,k\} \subset X$$

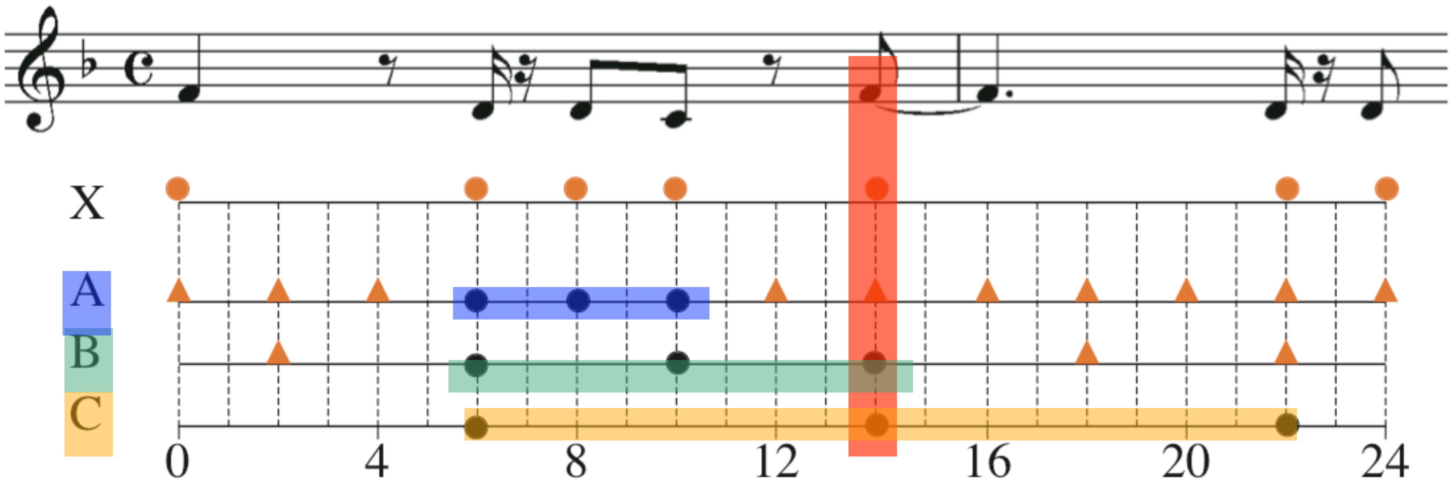
$$M(\ell) = \{m(s,d,k) : k \geq \ell\}$$

$$W_{\ell,p}(o) = \sum_{\{m\in M(\ell):o\in m\}} k^p$$

Picture from www.ritzenhoff.de

Inner Metric Analysis

Spectral Weight


X
A
B
C
0
4
8
12
16
20
24

$$m(s,d,k) = \{s + id, i = 0,\dots,k\} \subset X$$

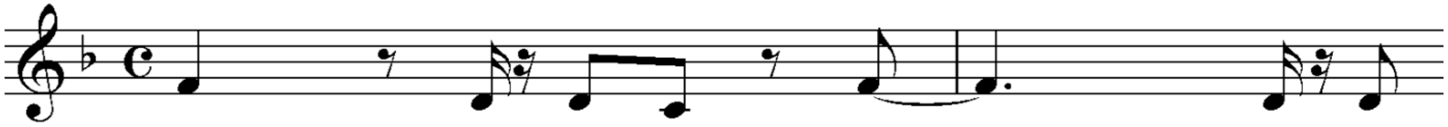
$$ext(m(s,d,k)) = \{s + id, \forall i\}$$

$$SW_{\ell,p}(t) = \sum_{\{m \in M(\ell): t \in ext(m)\}} k^p$$

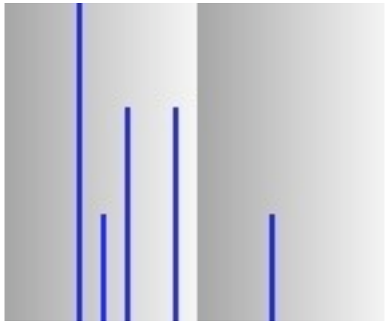
Picture from www.ritzenhoff.de

Inner Metric Analysis

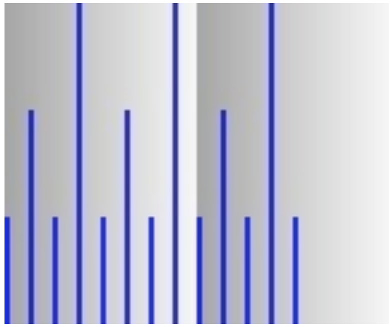
Metric Weight



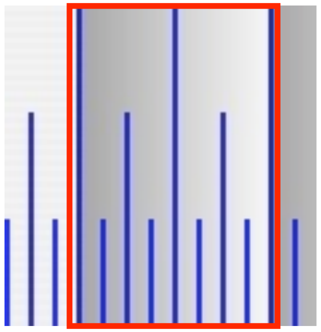
Picture from www.ritzenhoff.de

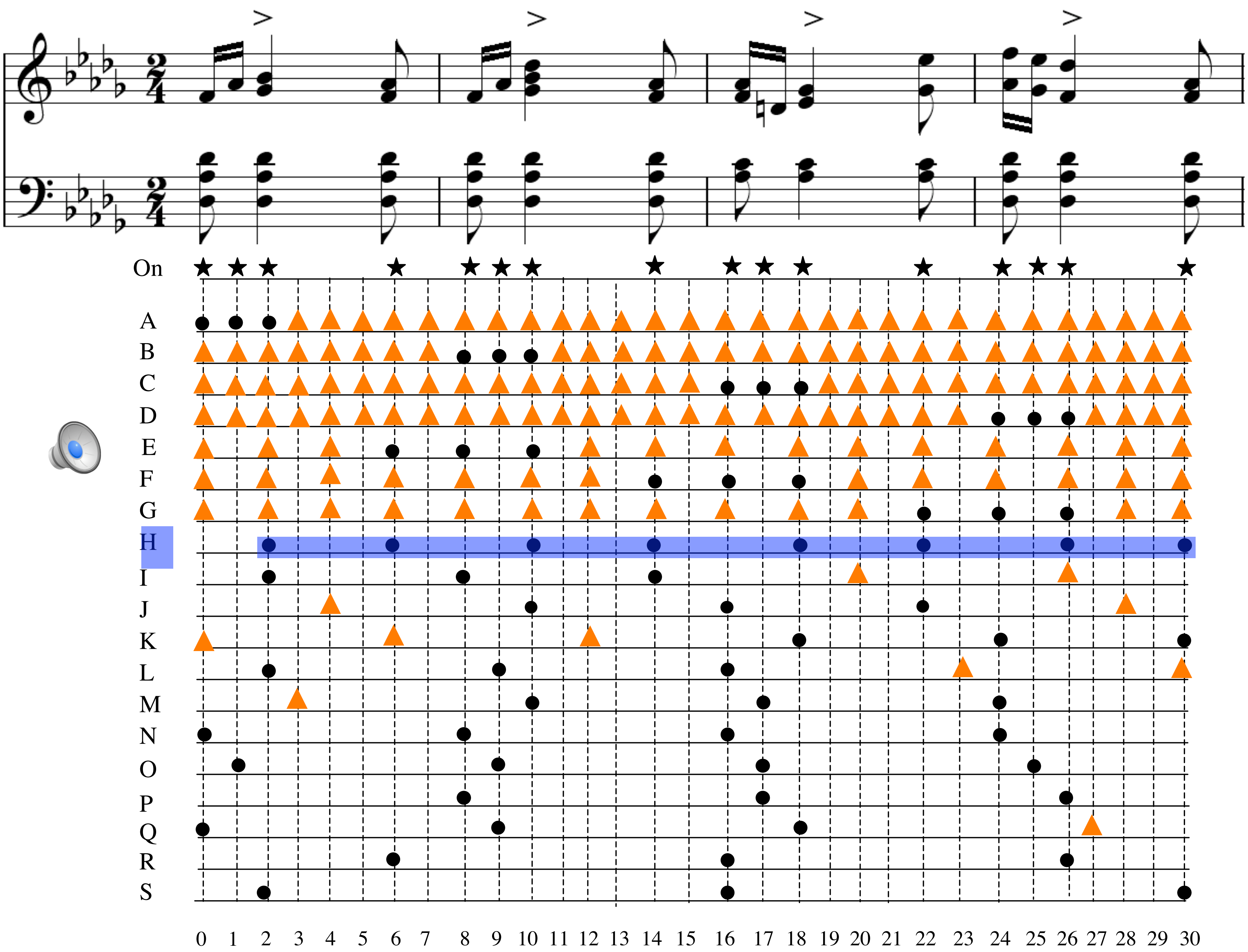


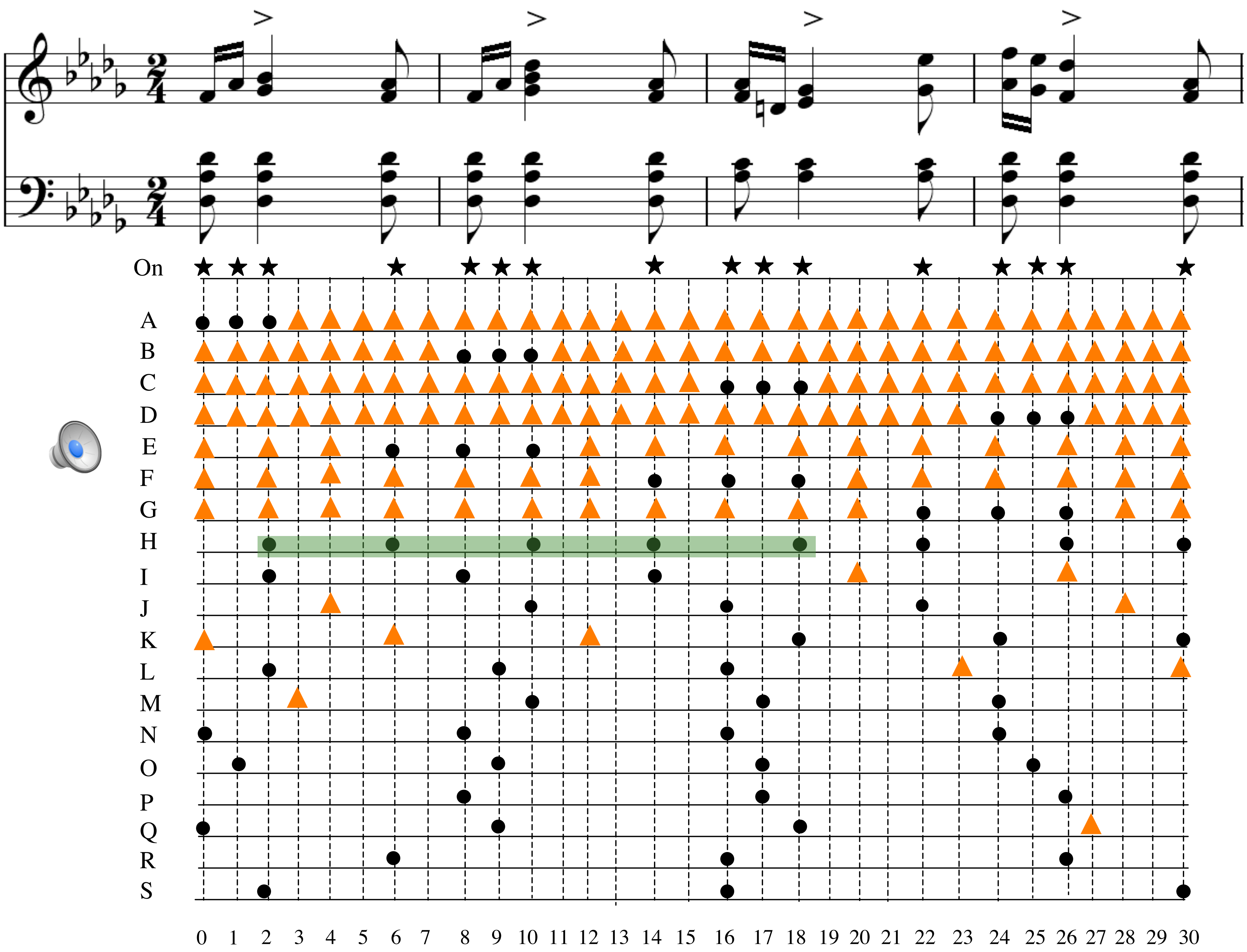
Metric Weight

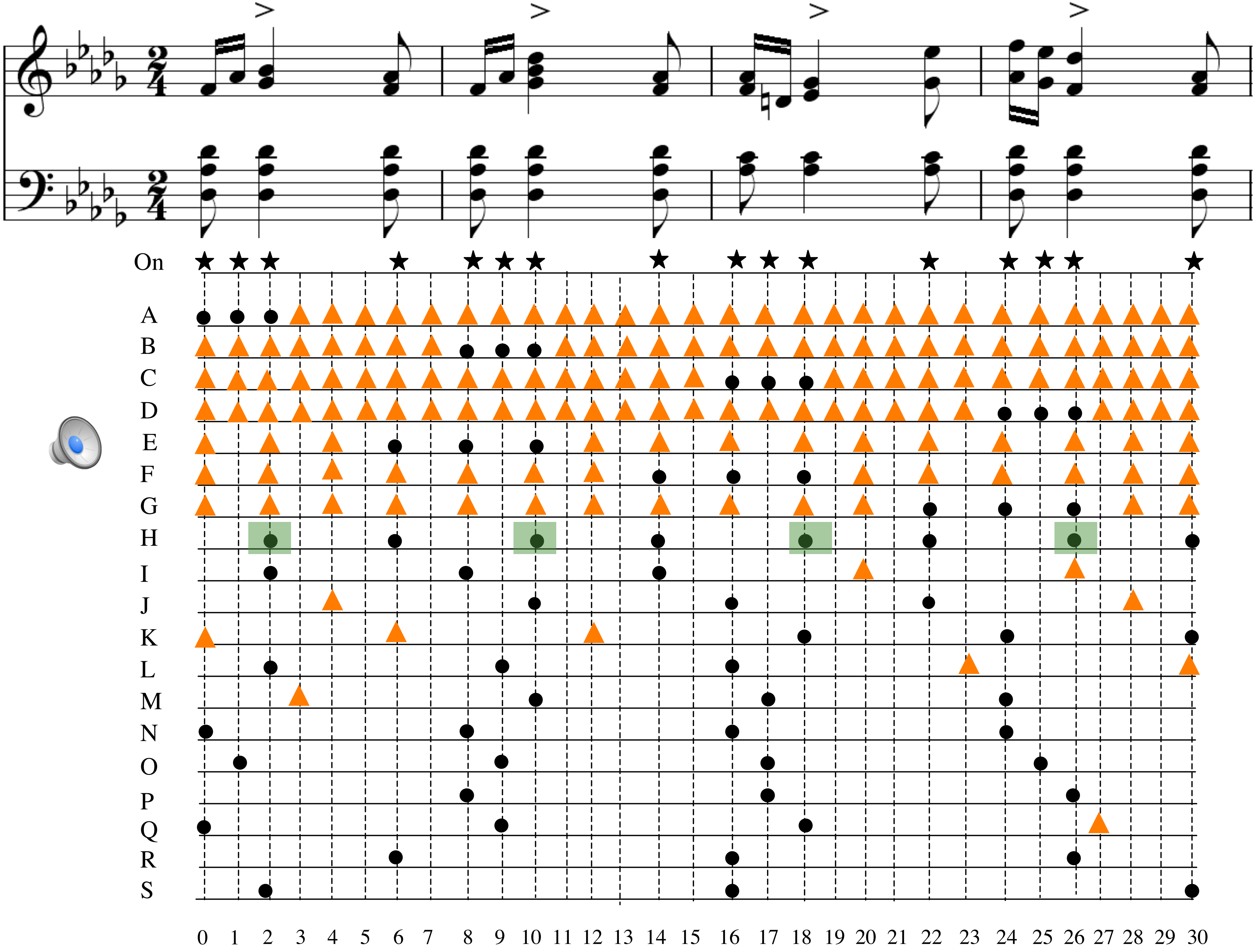


Spectral Weight




On
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

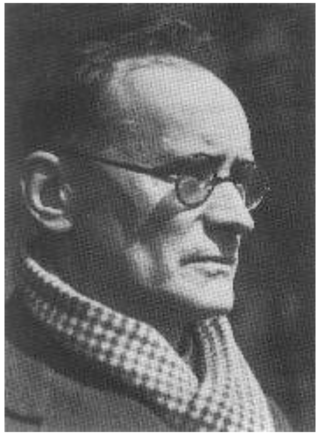

On
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30


On
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Webern versus Joplin

Anton Webern

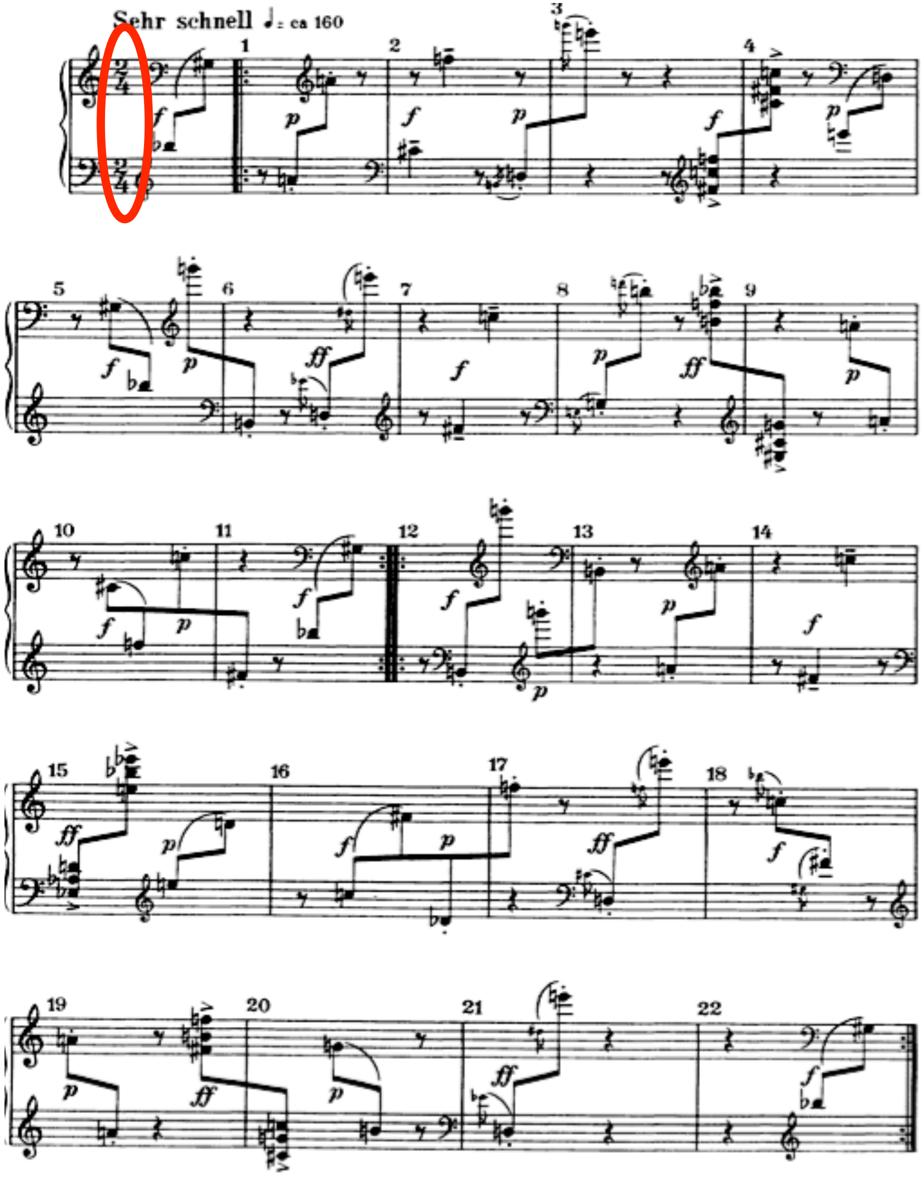
Piano Variations Op. 27, II

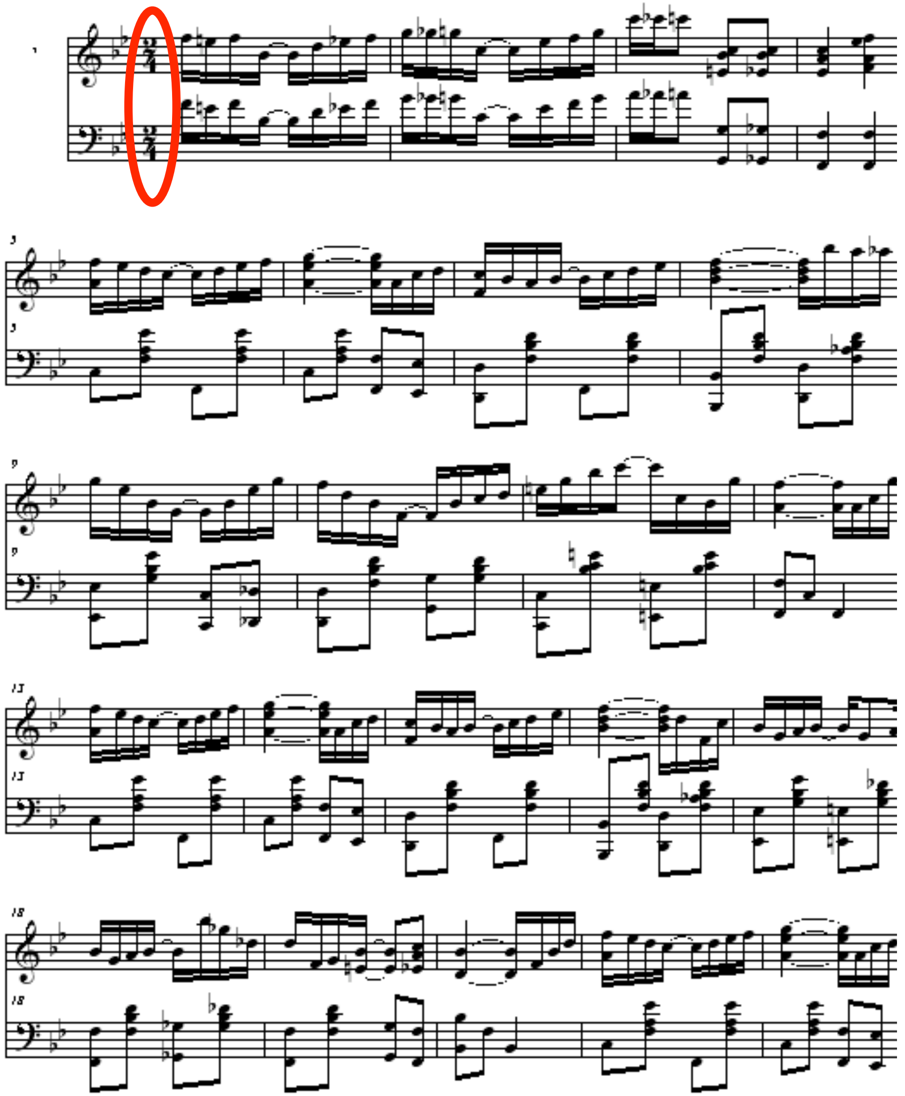


Scott Joplin

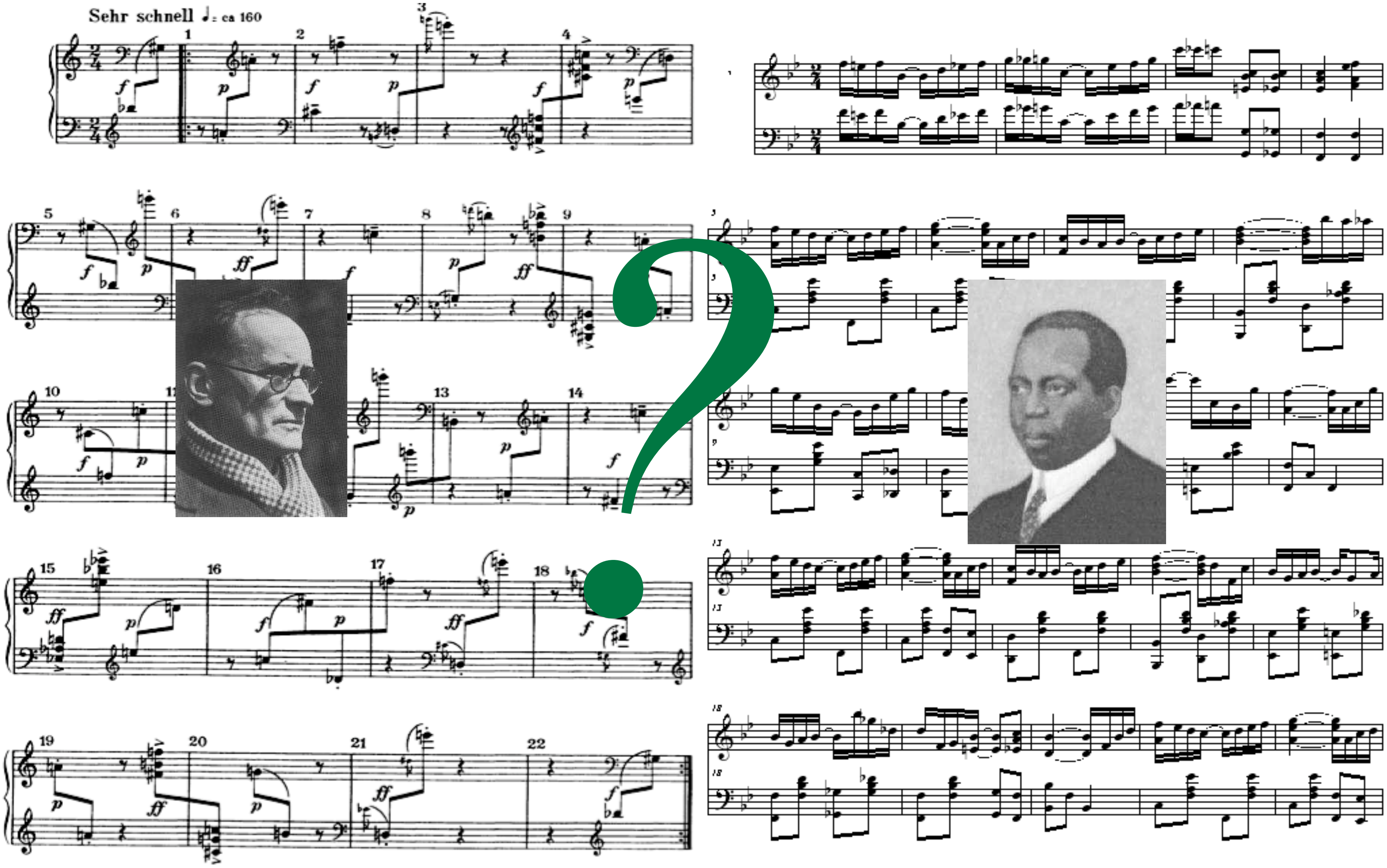
Nonpareil Rag

Webern versus Joplin

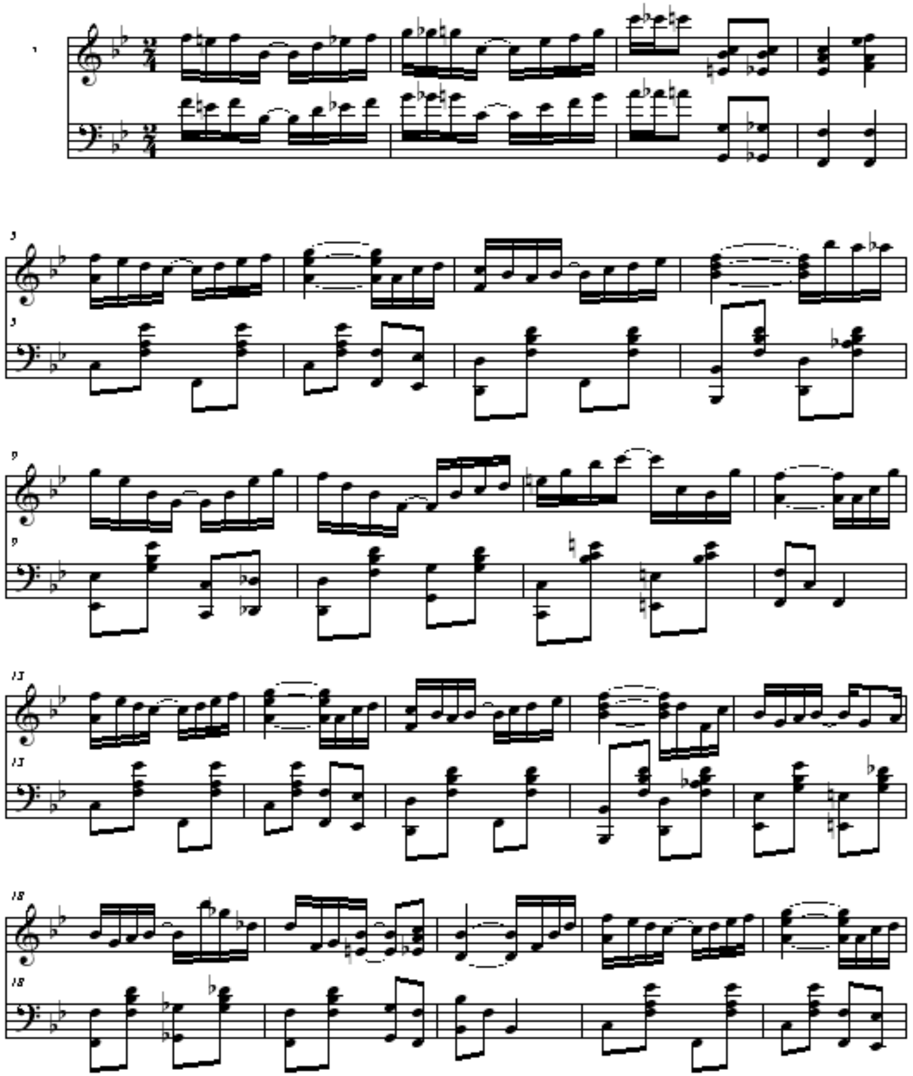

Sehr schnell ♩ = ca 160



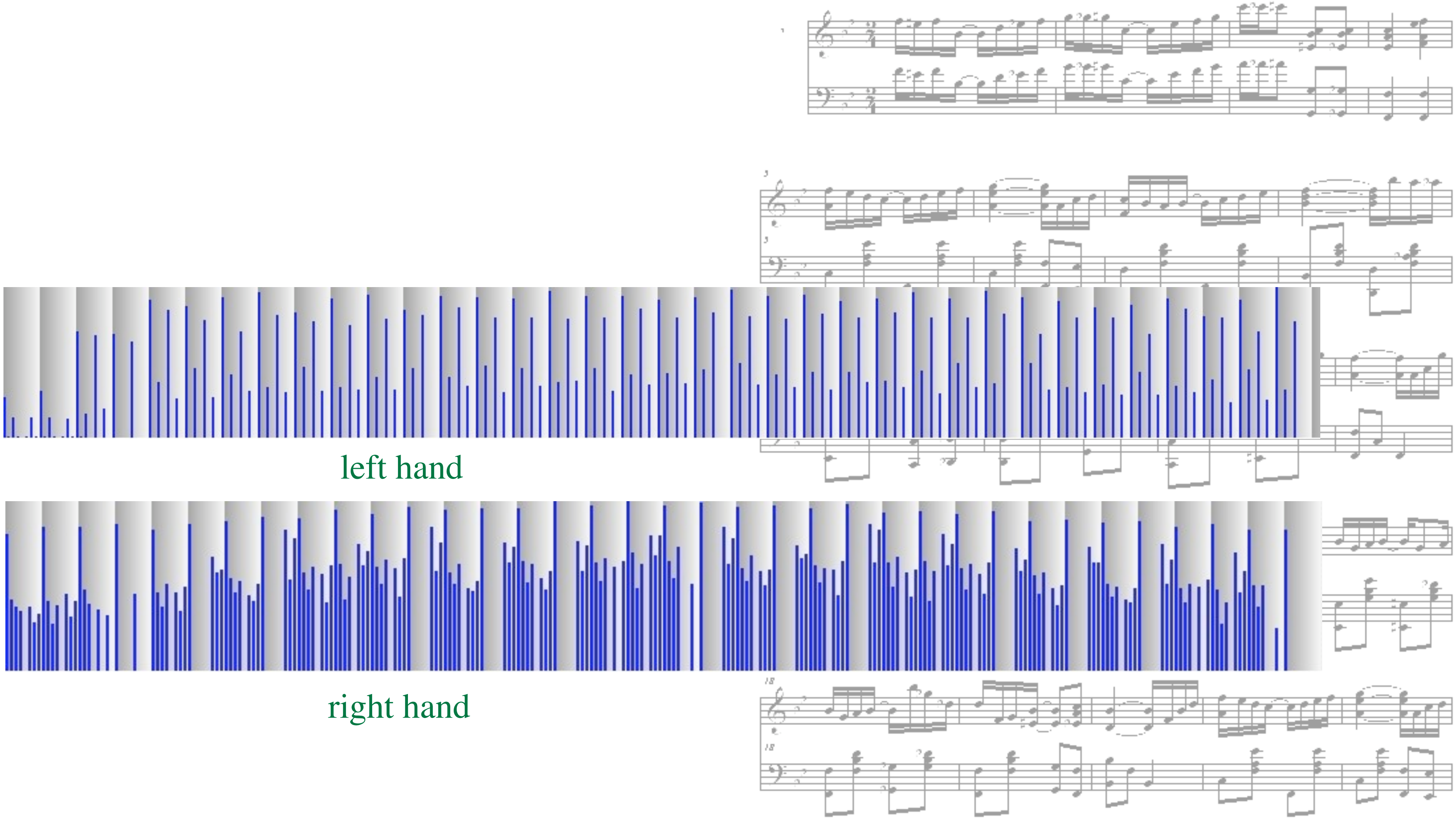
Webern versus Joplin


Sehr schnell ♩= ca 160

Webern versus Joplin



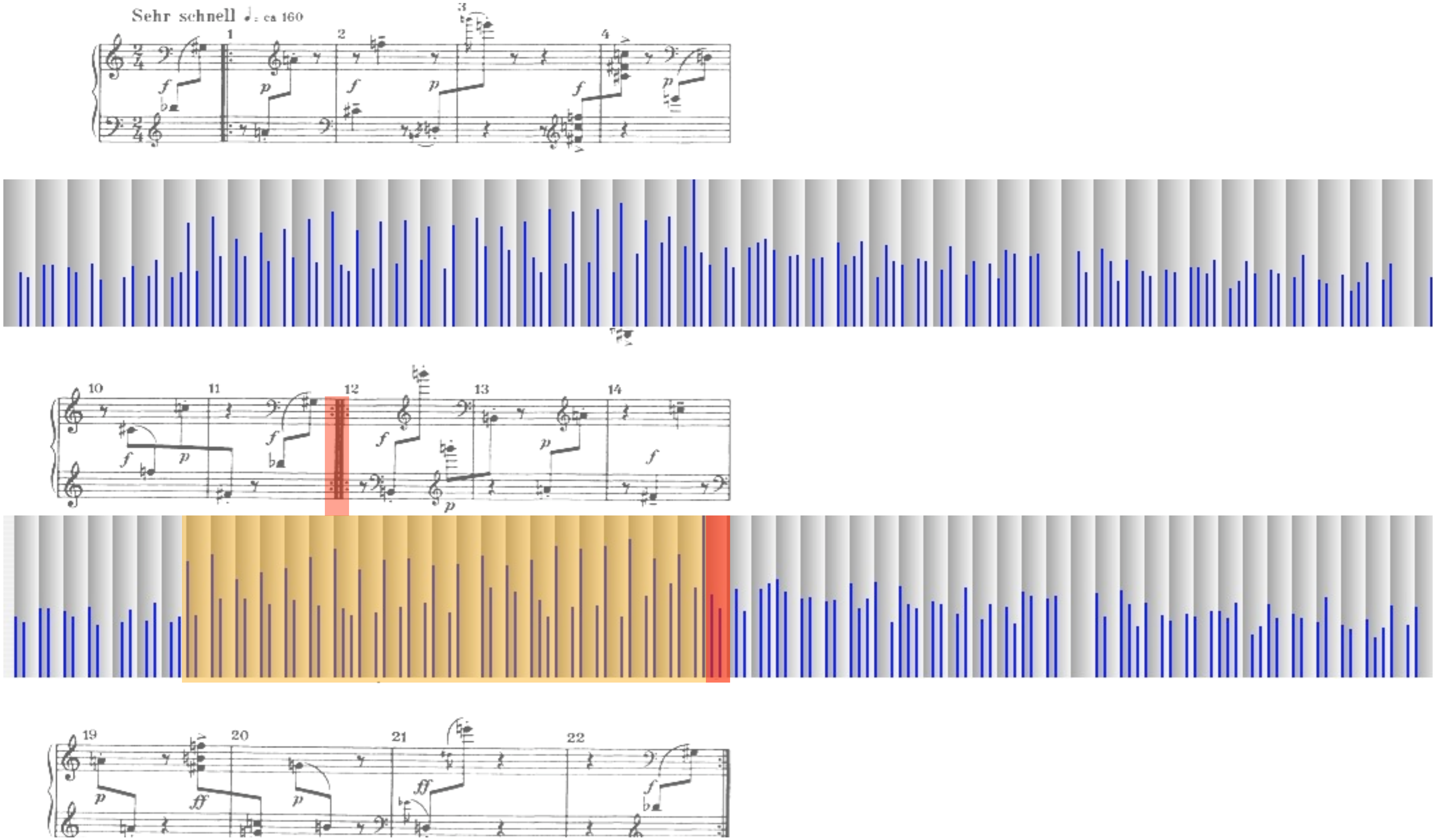
Webern versus Joplin



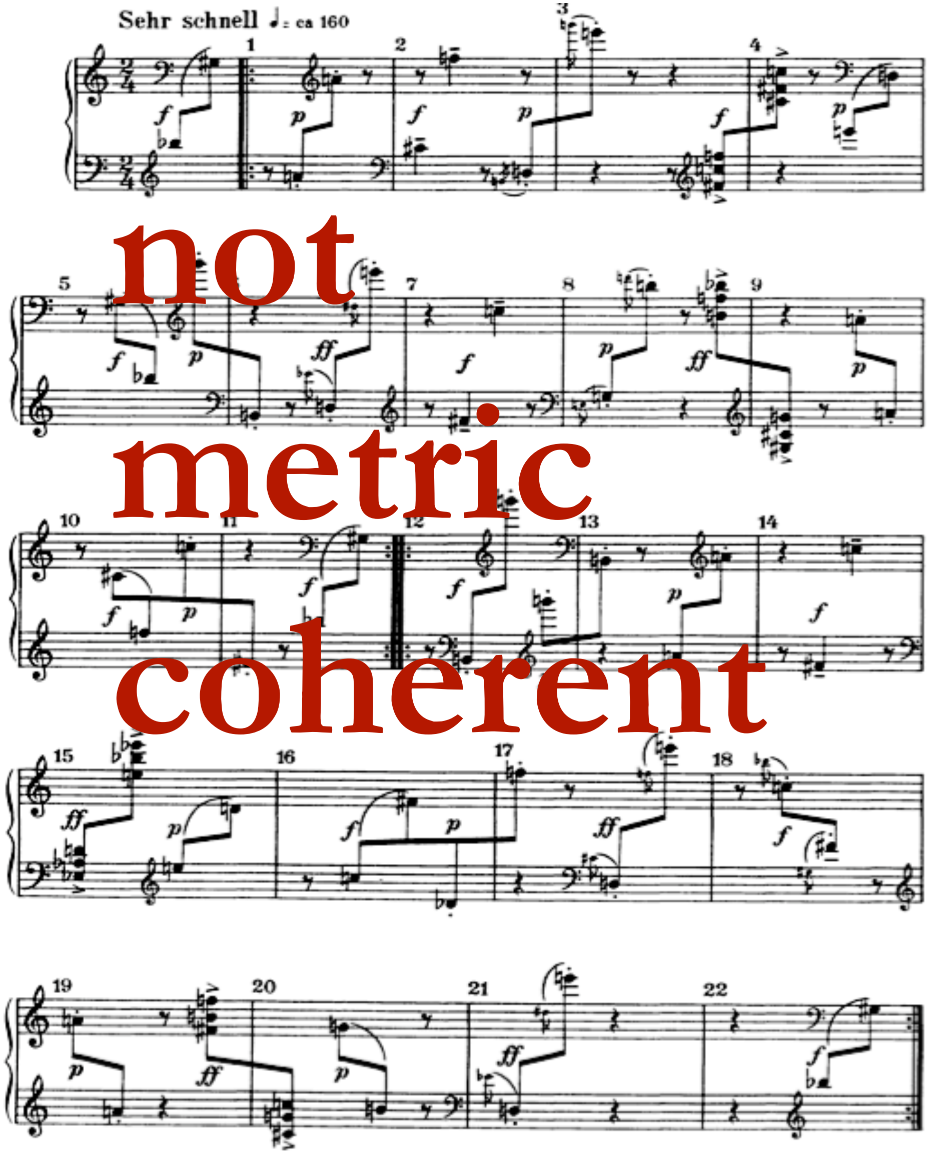
left hand

right hand

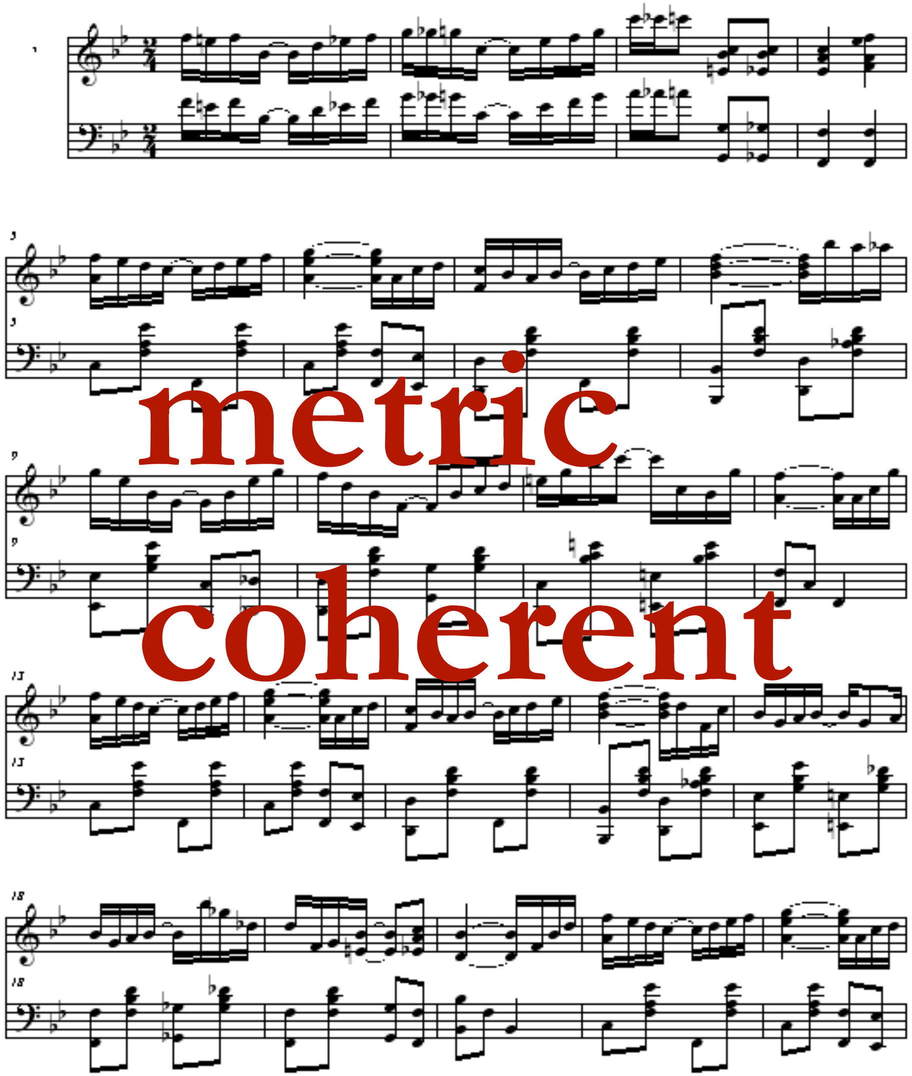
Webern versus Joplin


Sehr schnell ♩. = ca 160
1
2
3
4
10
11
12
13
14
19
20
21
22

Webern versus Joplin


Sehr schnell ♩= ca 160

not metric coherent

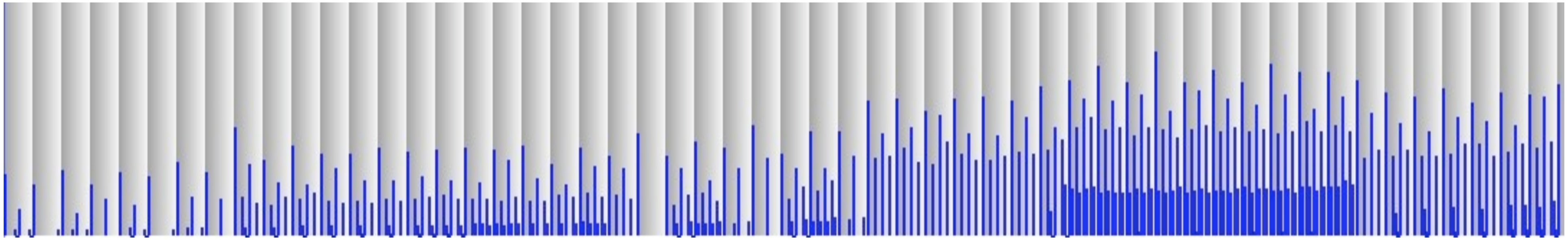


metric coherent

Symphony C Major K. 551

1. Movement (4/4)

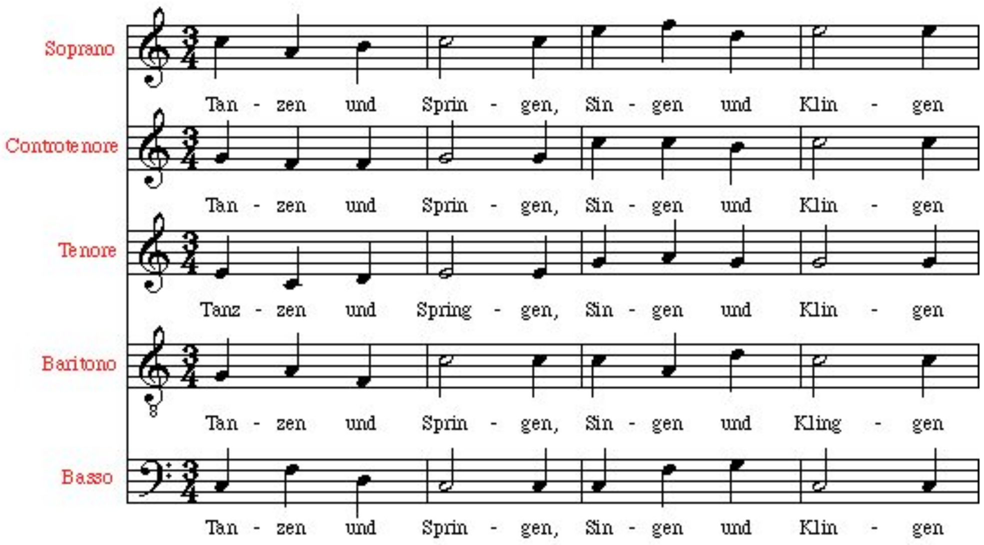
Wolfgang A. Mozart

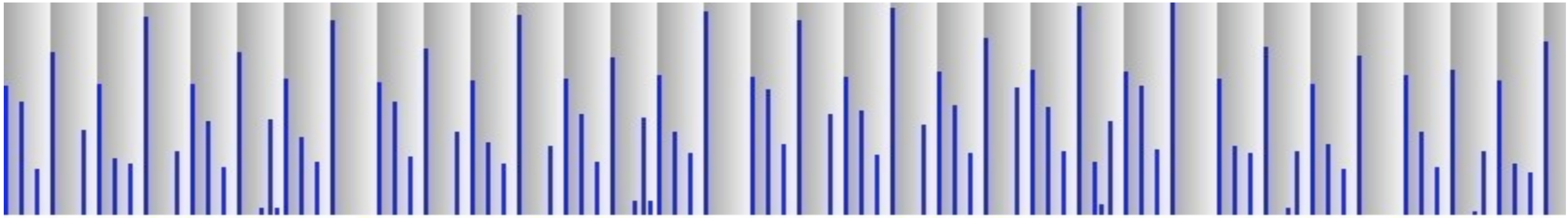


Excerpt from Metric weight $W_{2,2}$ of the exposition (bars 1-55)

Tanzen und Springen (3/4)

Hans Leo Haßler

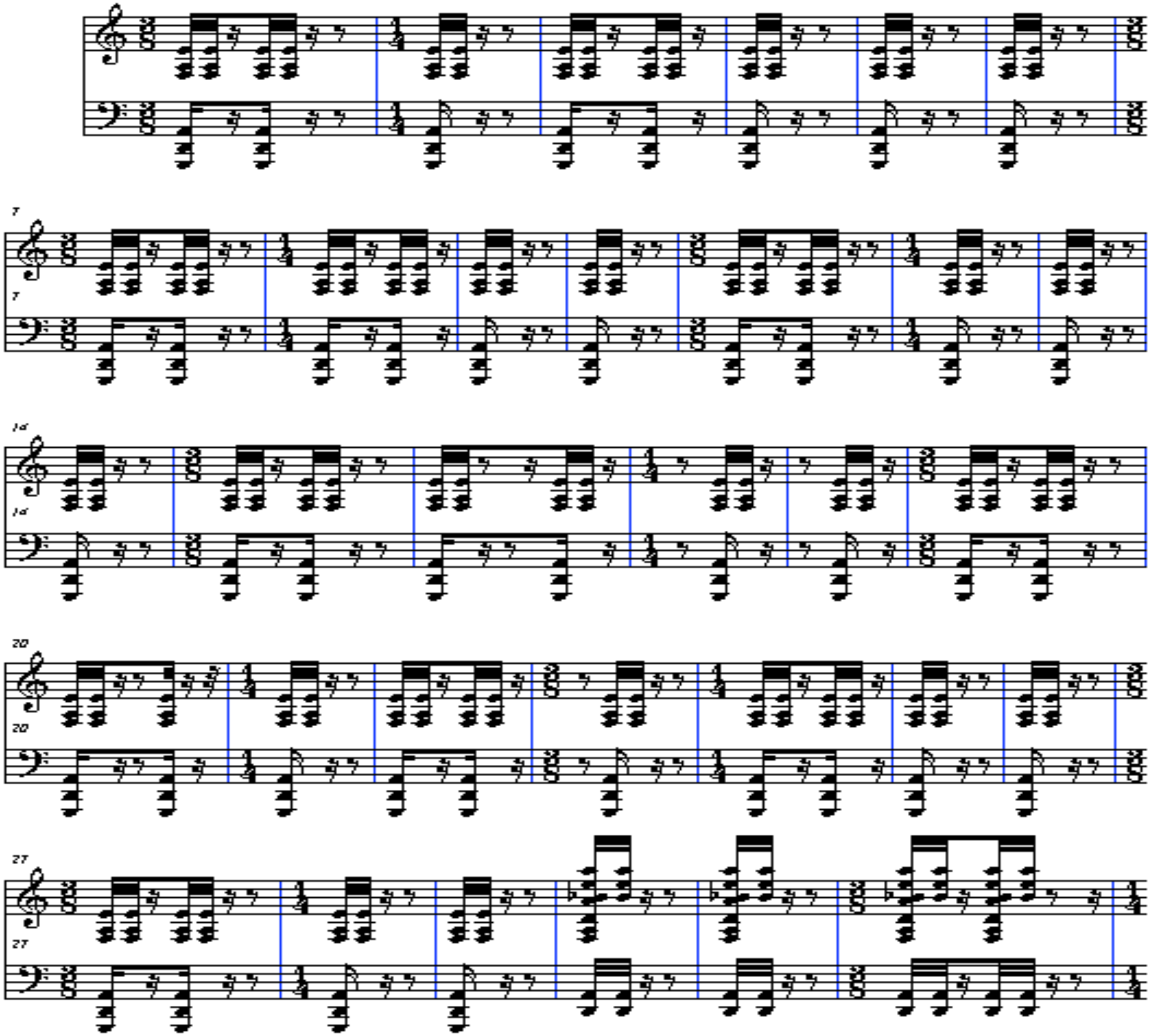

Soprano
Tan - zen und Sprin - gen, Sin - gen und Klin - gen
Controtenore
Tan - zen und Sprin - gen, Sin - gen und Klin - gen
Tenore
Tanz - zen und Spring - gen, Sin - gen und Klin - gen
Baritono
Tan - zen und Sprin - gen, Sin - gen und Kling - gen
Basso
Tan - zen und Sprin - gen, Sin - gen und Klin - gen

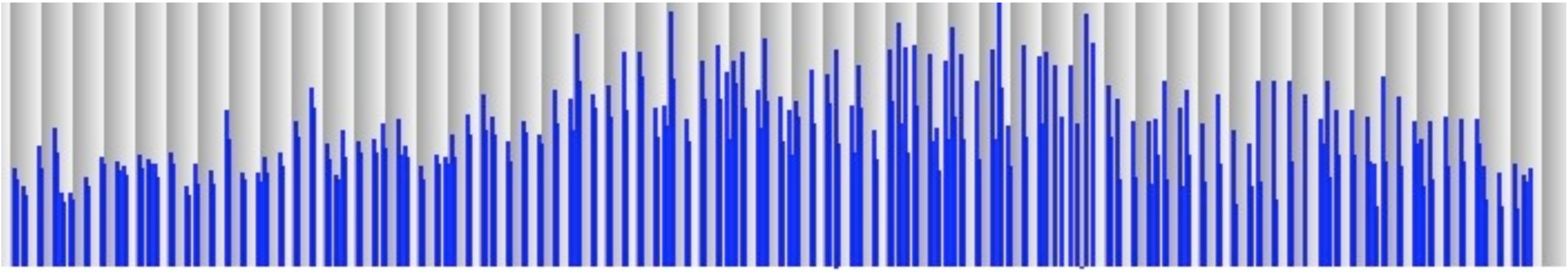


Igor Stravinsky: The Rite of Spring

Danse Sacrale

Part B: Measures 149-167 (rhythmical accompaniment)


7
7
14
14
20
20
27
27



Evaluation of the model: music theoretic knowledge

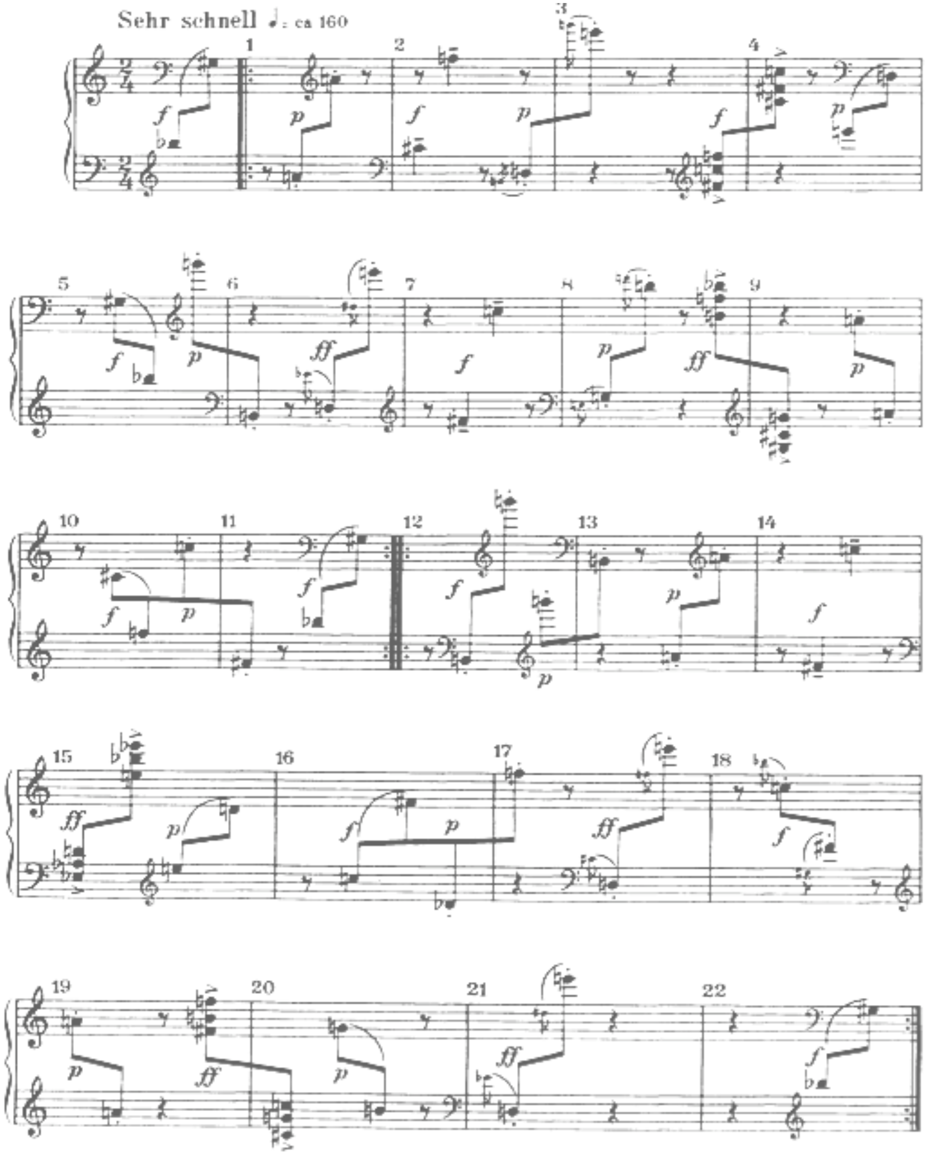
different epochs and styles:

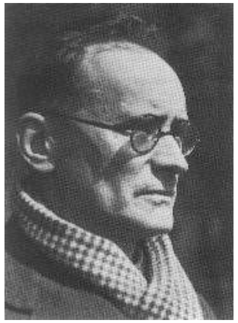
e.g. Ockeghem, Dufay, Morley, Wilbye, Händel, Bach, Mozart, Beethoven, Schumann, Schubert, Brahms, Stravinsky, Webern

corpus:

Brahms' 4 symphonies, Bach's B-Minor-Mass

Meter in Music as a Dynamic Process


Sehr schnell ♩. = ca 160
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22



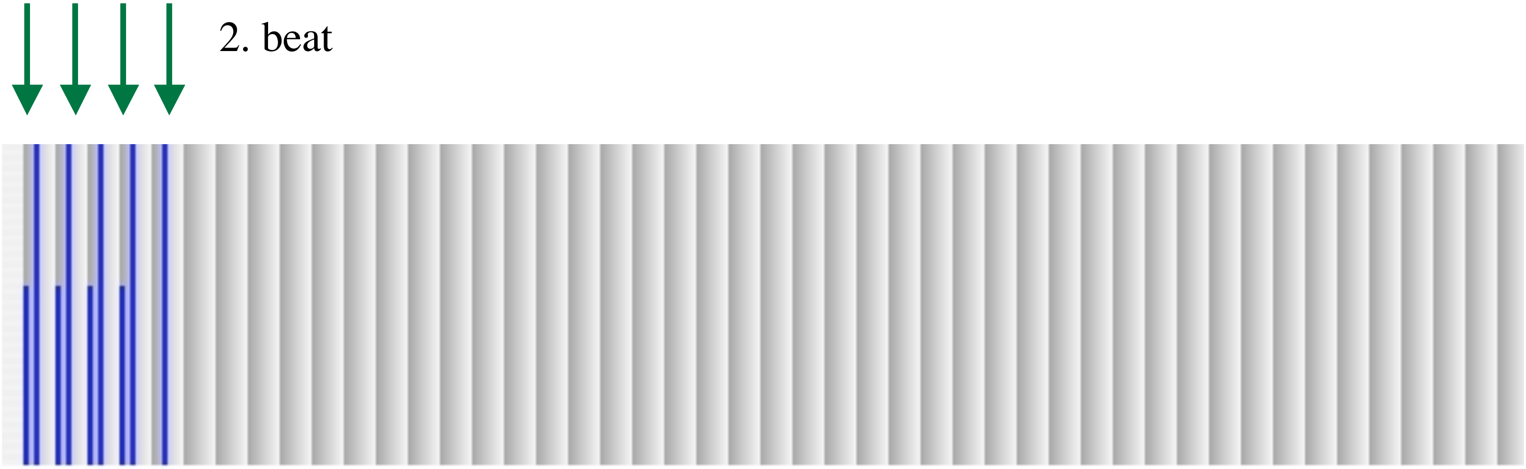
Meter in Music as a Dynamic Process

Meter in Music as a Dynamic Process

Meter in Music as a Dynamic Process

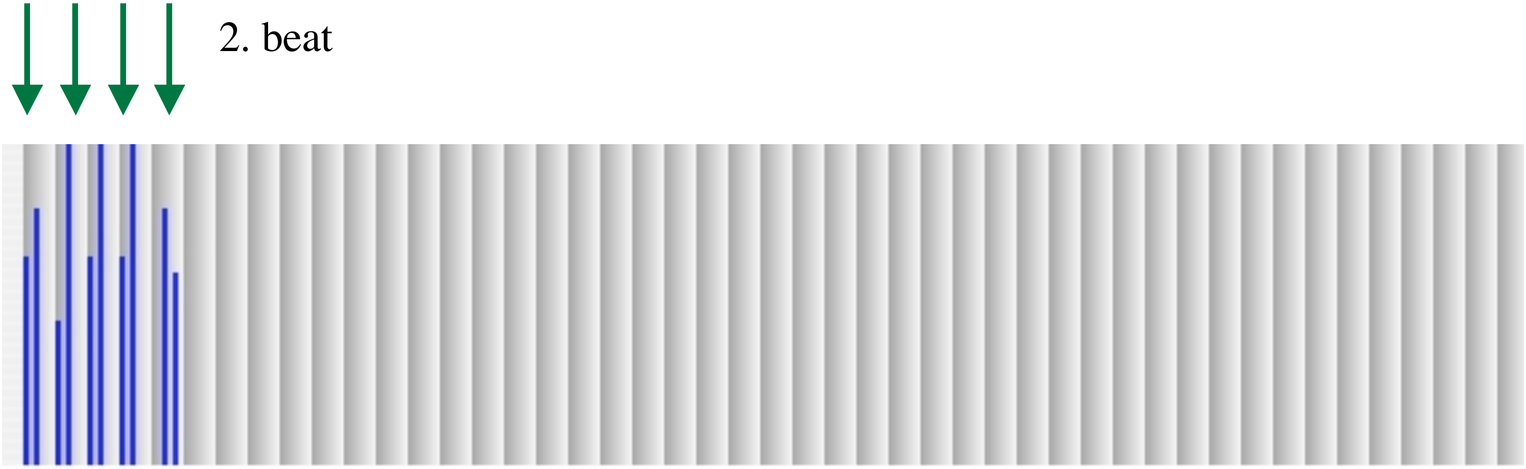
Meter in Music as a Dynamic Process

Meter in Music as a Dynamic Process



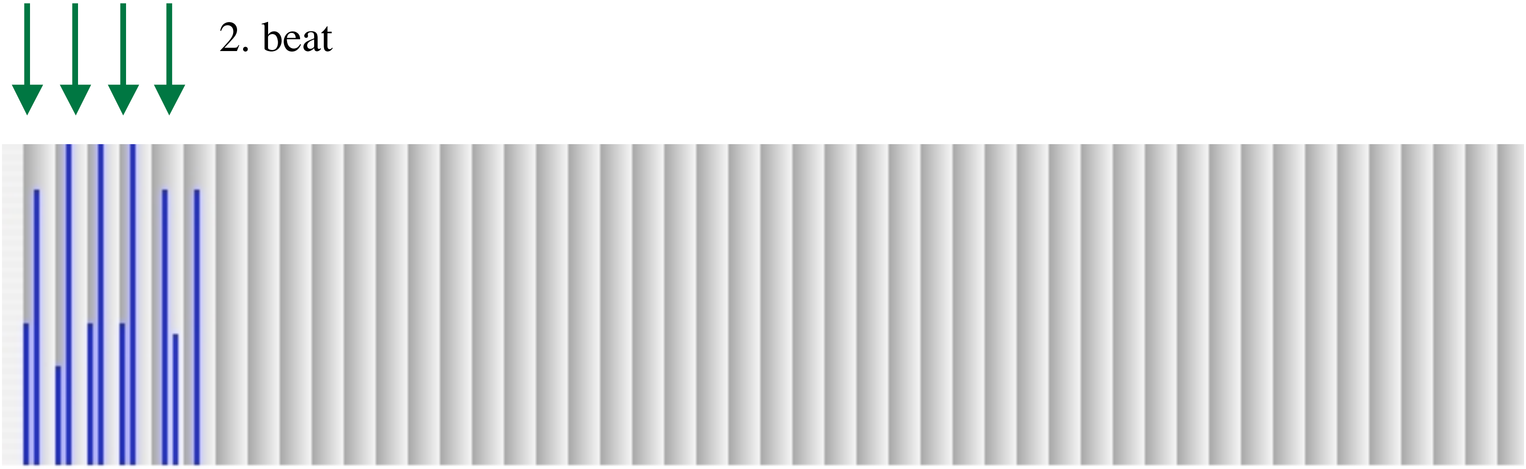
2. beat

Meter in Music as a Dynamic Process

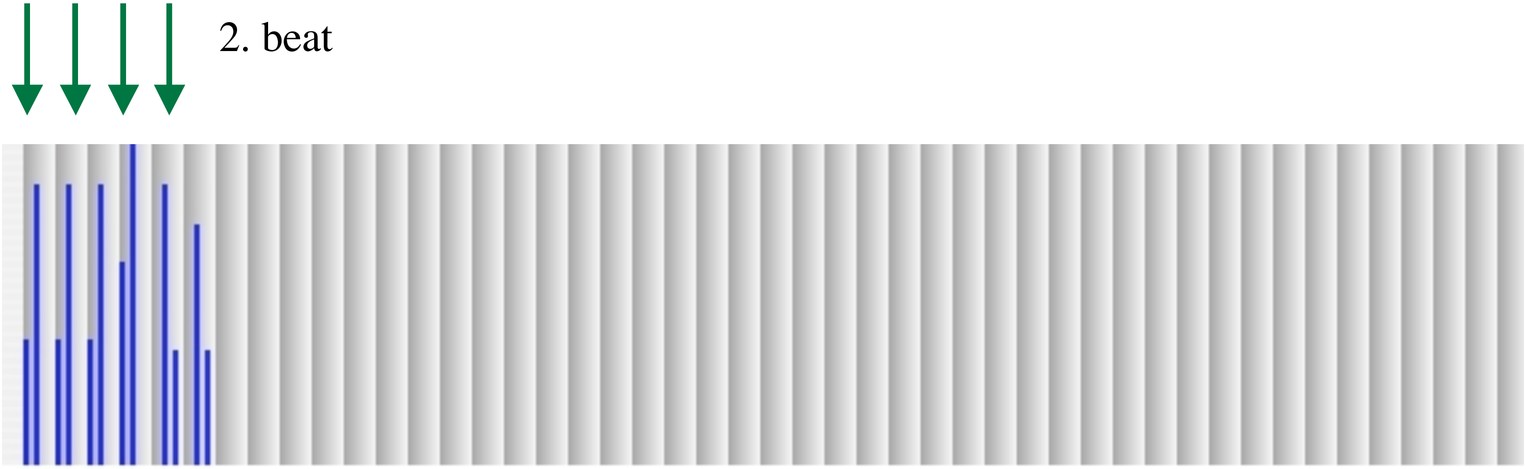


2. beat

Meter in Music as a Dynamic Process

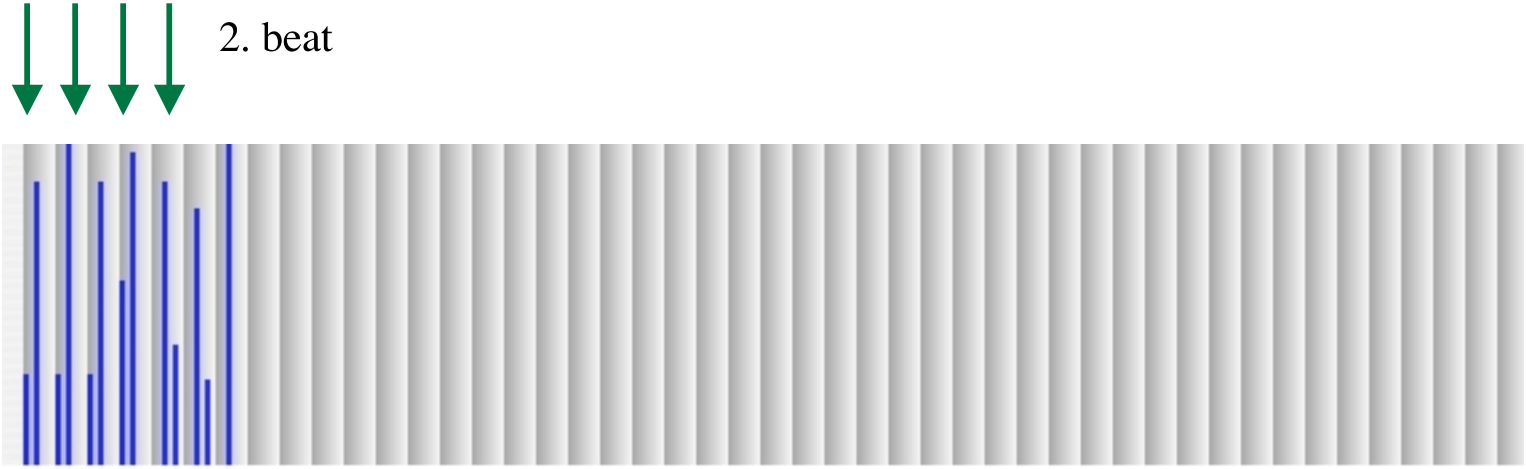


Meter in Music as a Dynamic Process



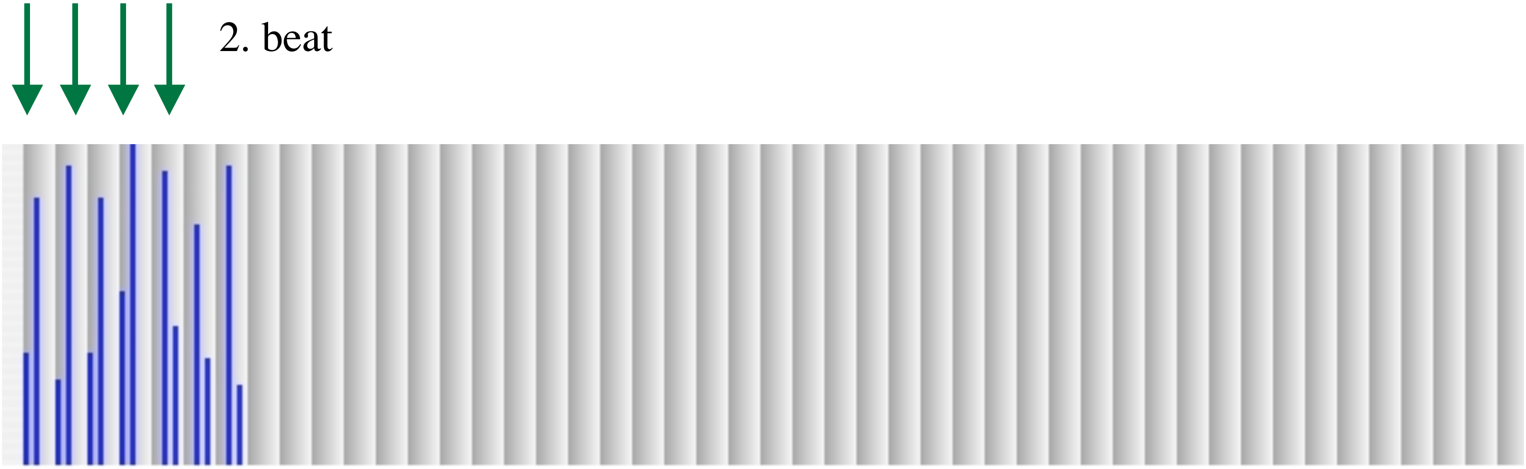
2. beat

Meter in Music as a Dynamic Process



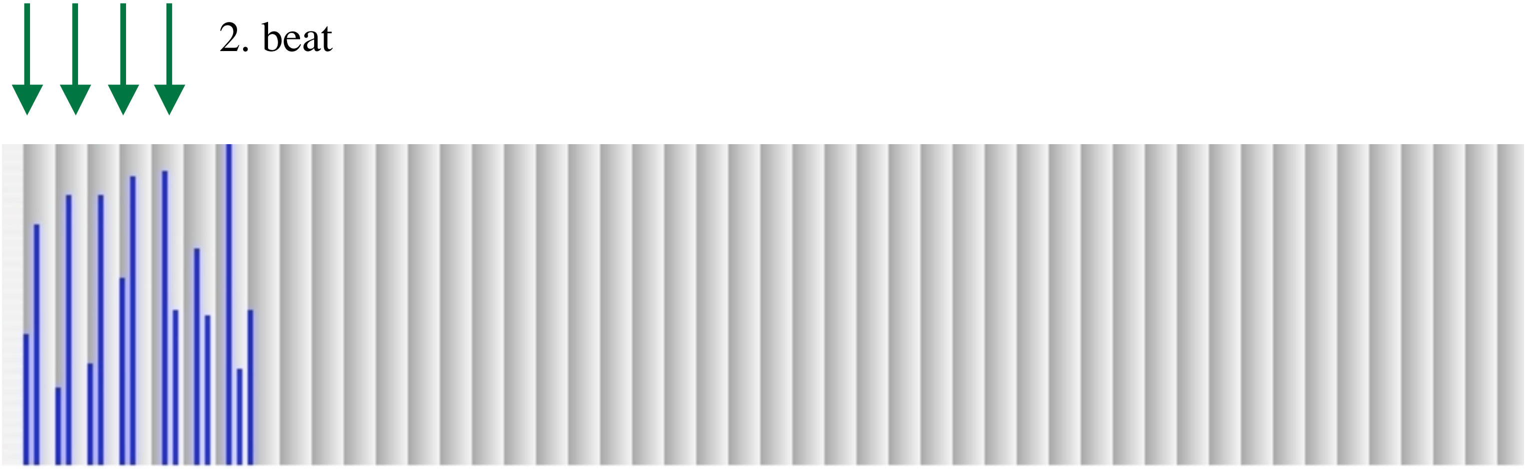
2. beat

Meter in Music as a Dynamic Process

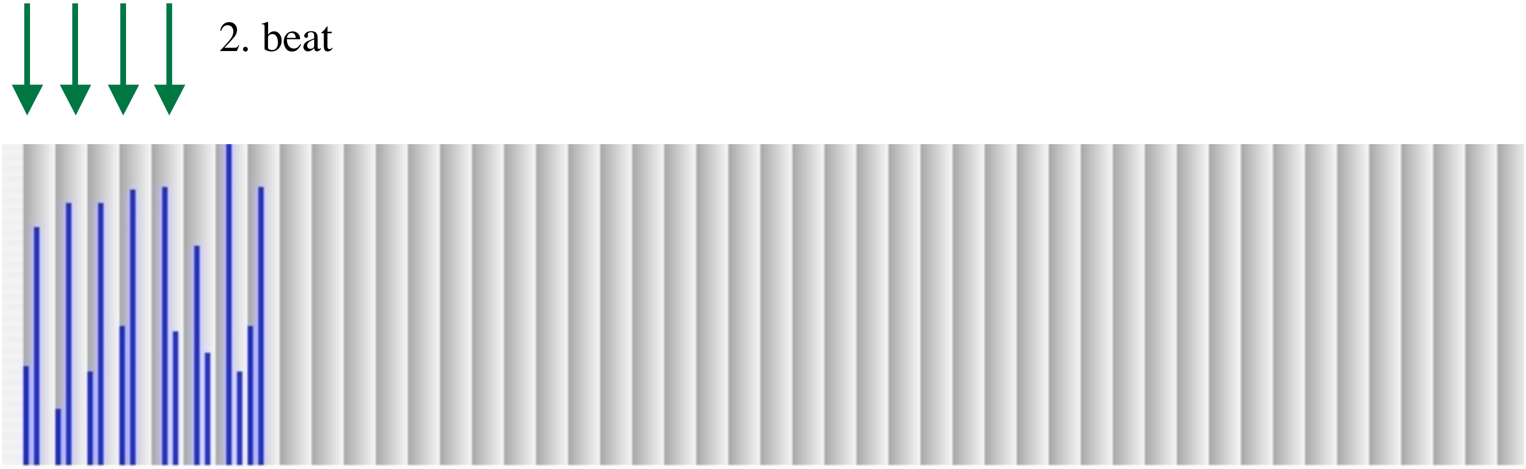


2. beat

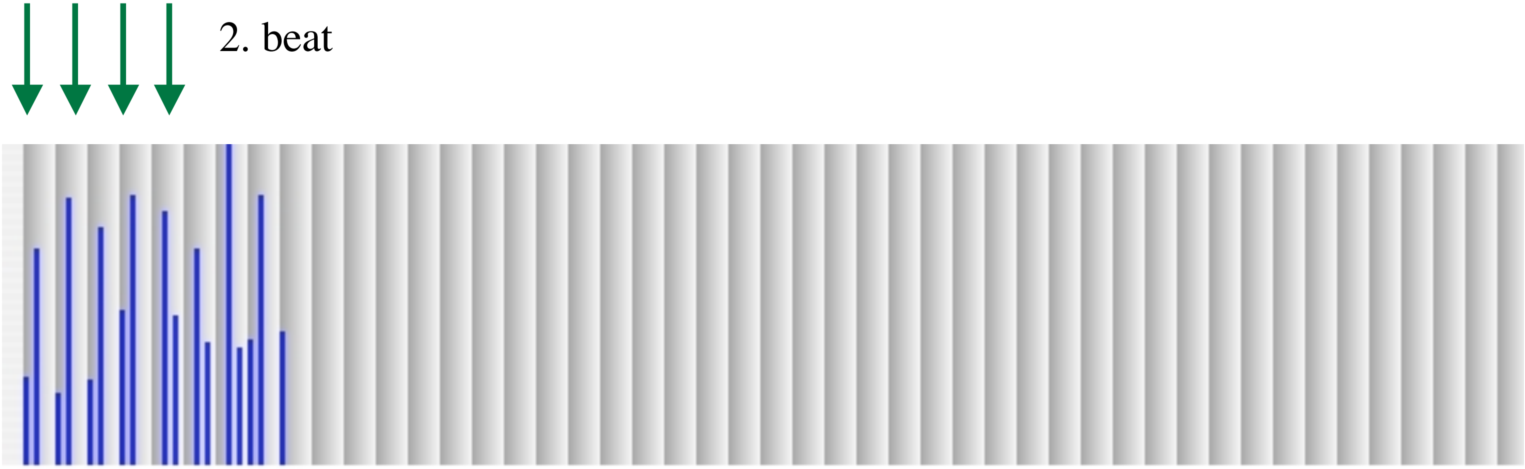
Meter in Music as a Dynamic Process



Meter in Music as a Dynamic Process

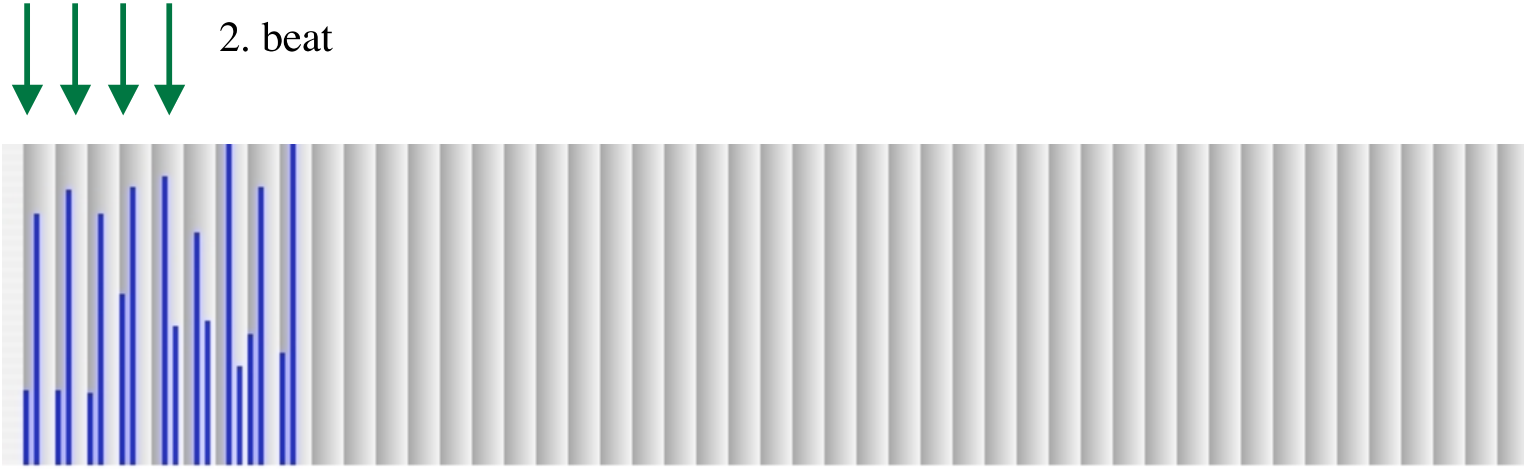

2. beat

Meter in Music as a Dynamic Process



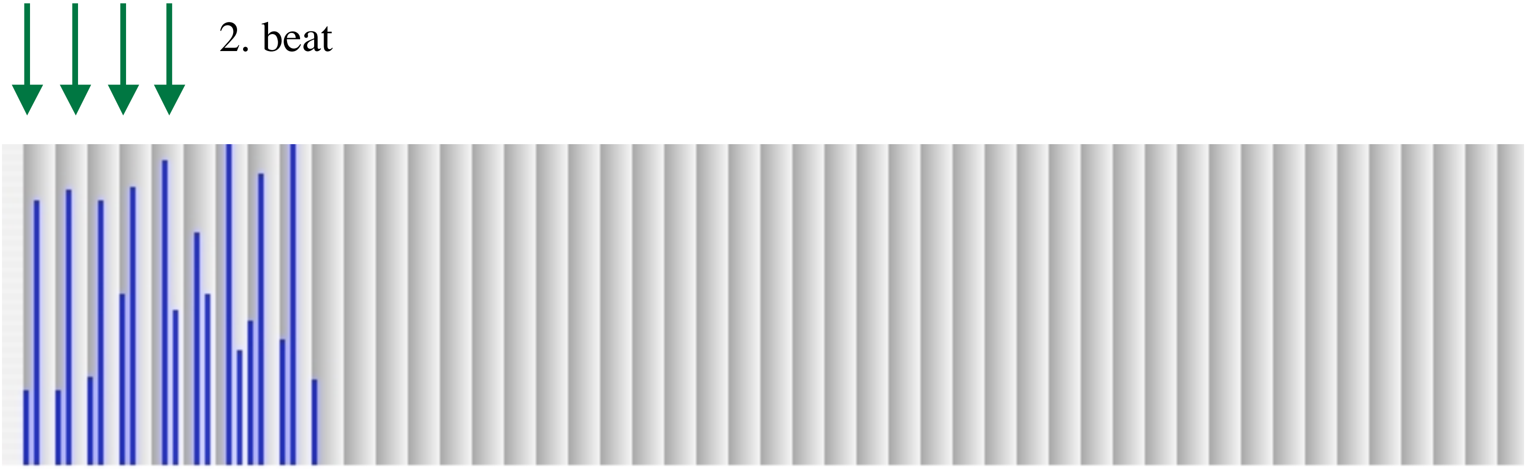
2. beat

Meter in Music as a Dynamic Process



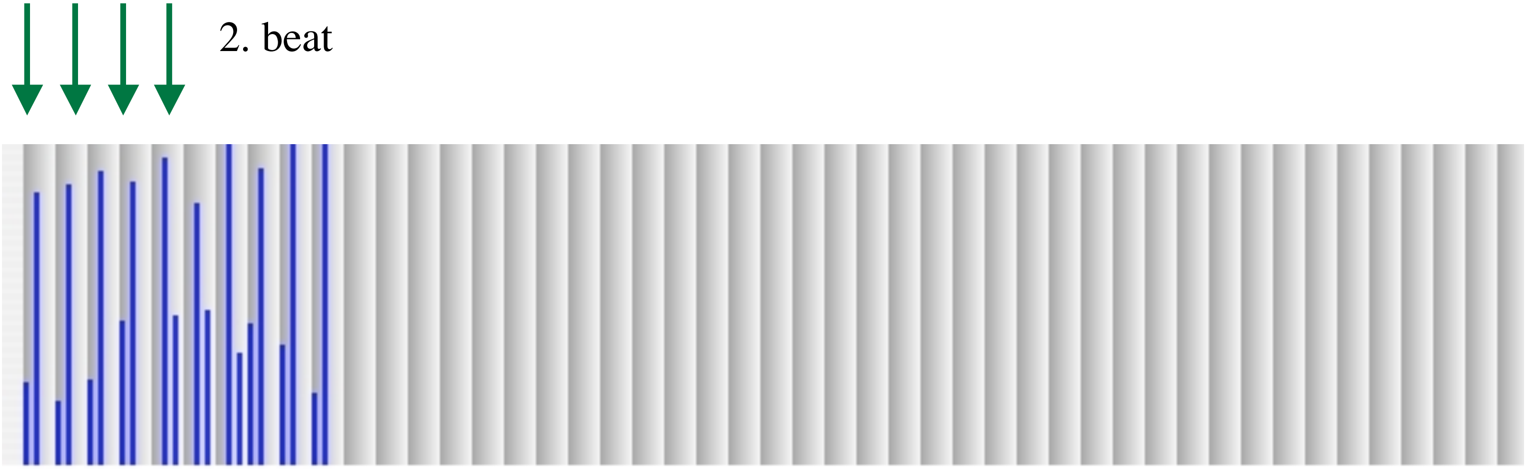
2. beat

Meter in Music as a Dynamic Process



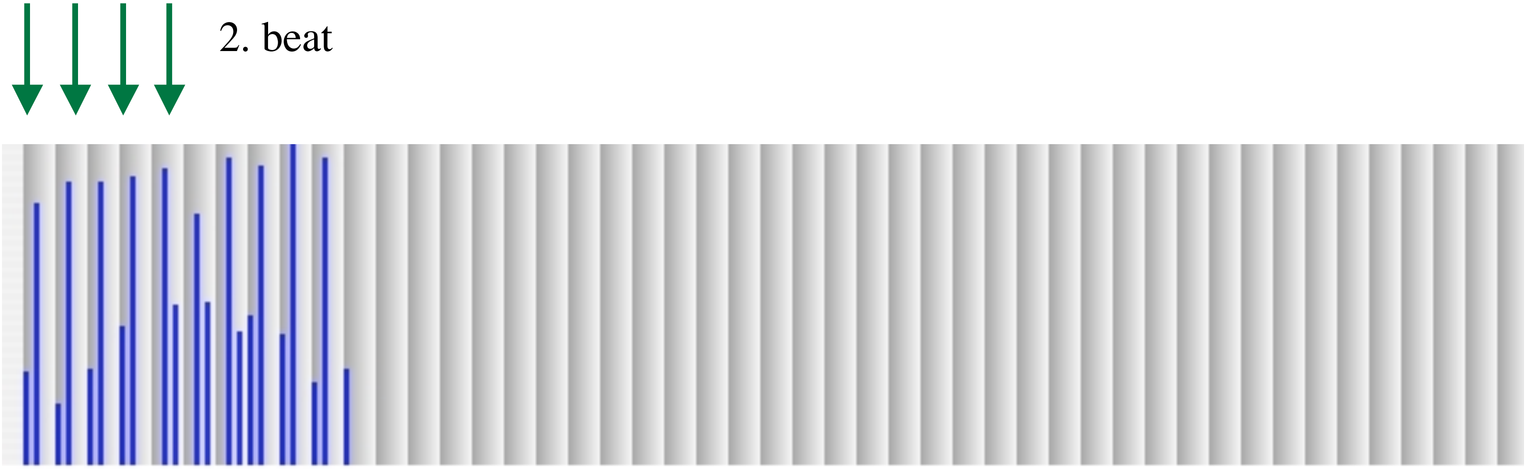
2. beat

Meter in Music as a Dynamic Process

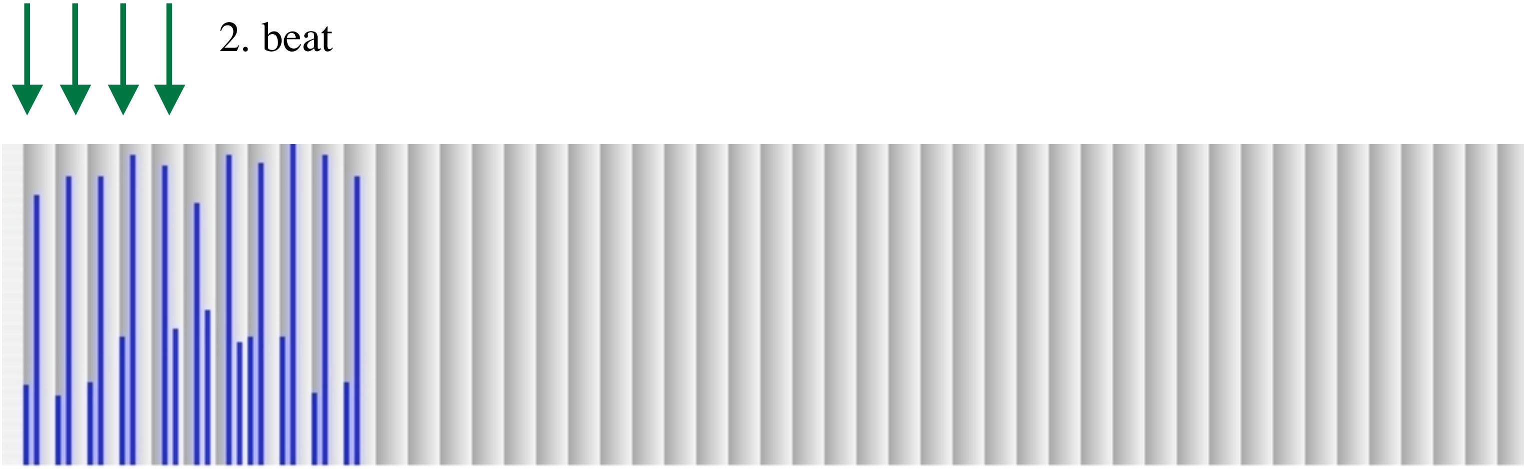


2. beat

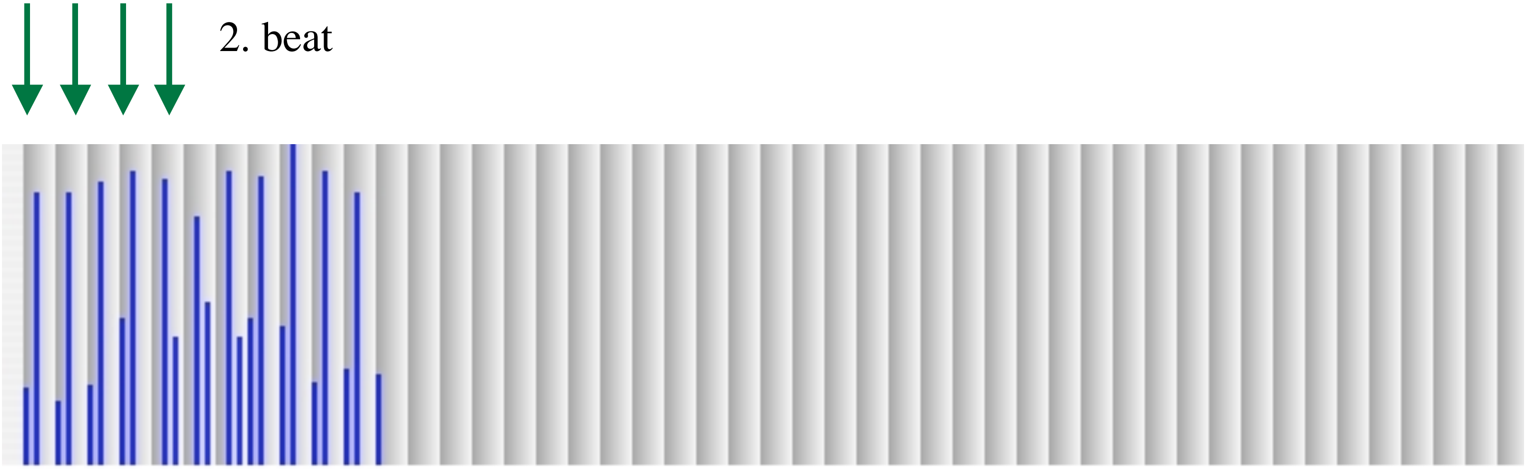
Meter in Music as a Dynamic Process



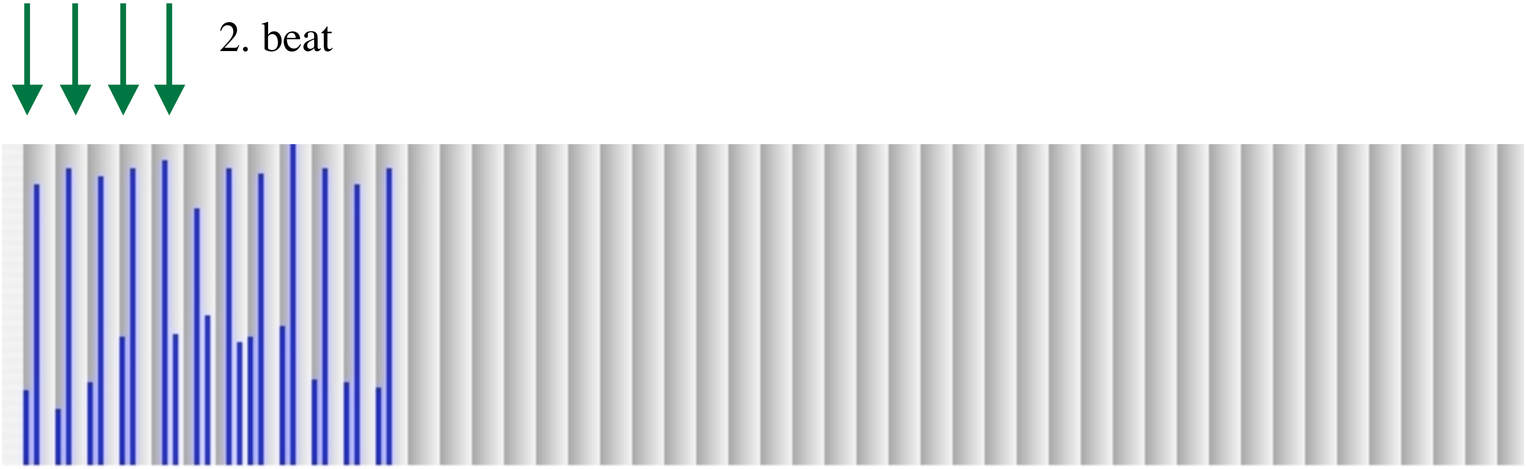
Meter in Music as a Dynamic Process



Meter in Music as a Dynamic Process

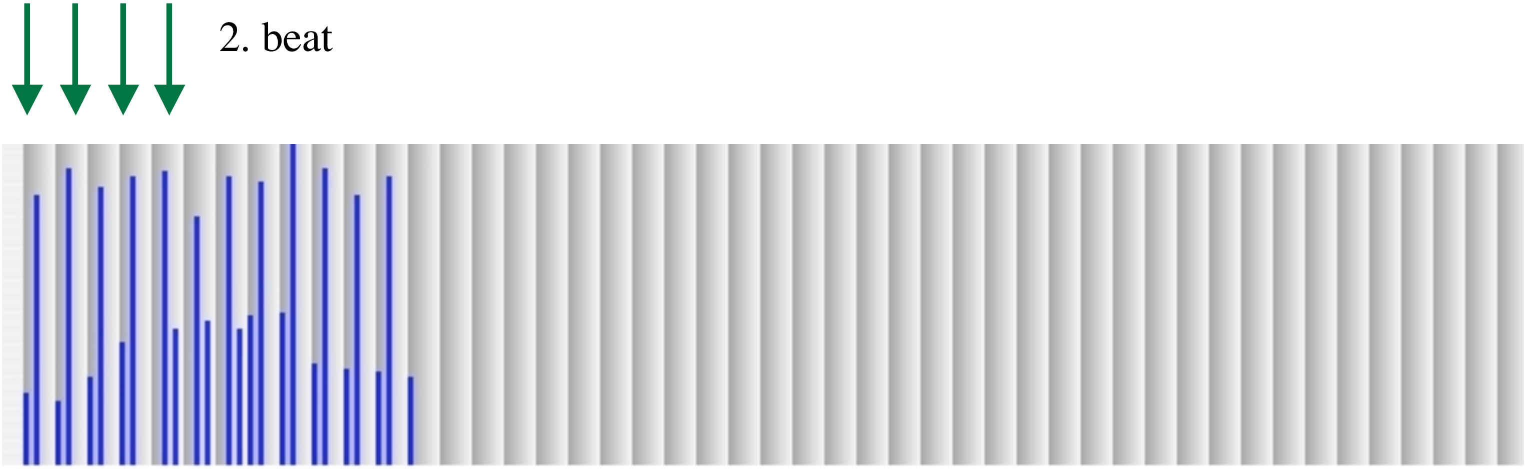


Meter in Music as a Dynamic Process

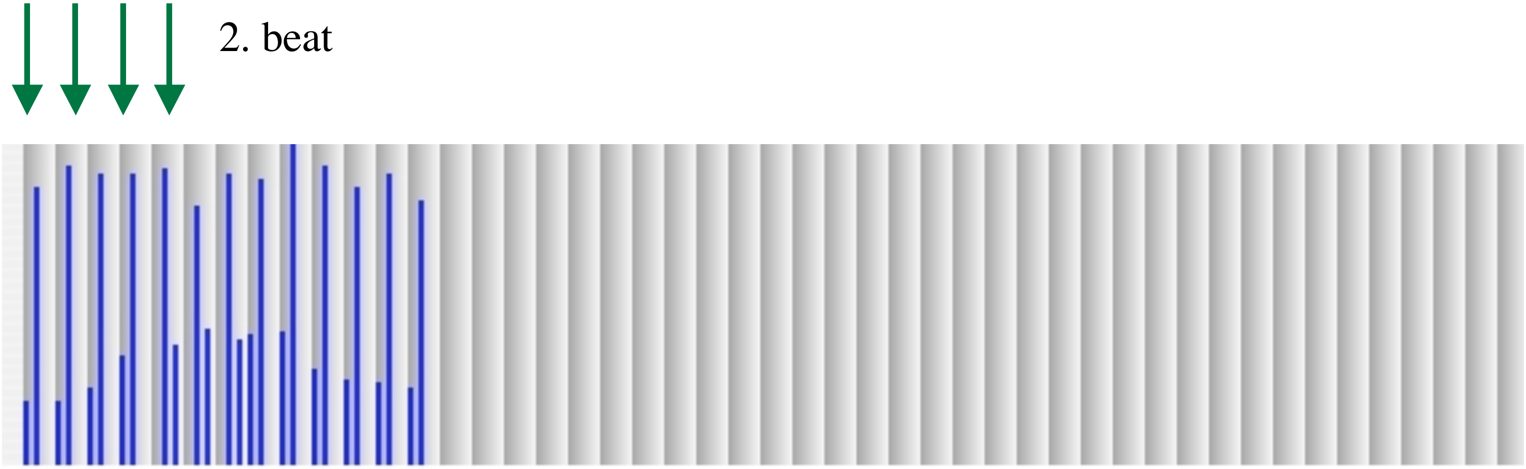


2. beat

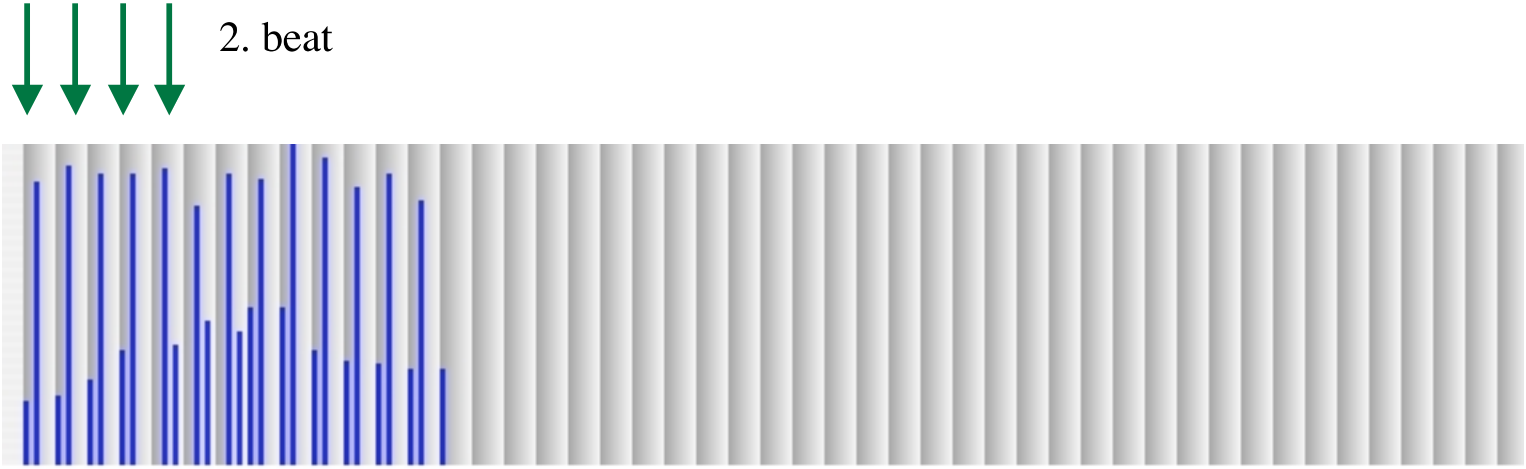
Meter in Music as a Dynamic Process



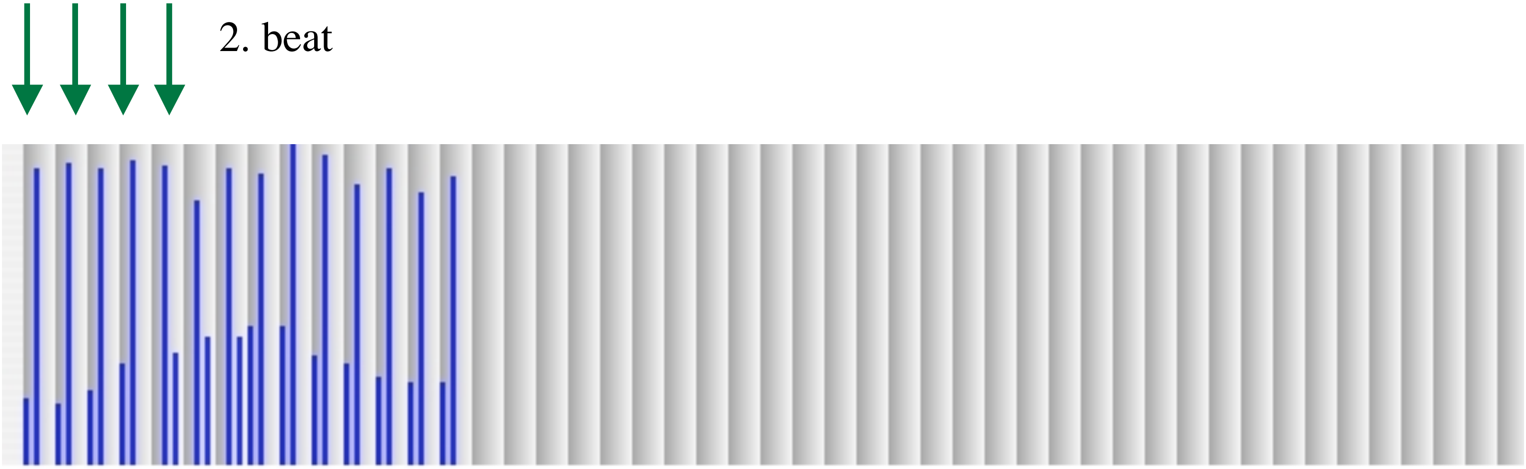
Meter in Music as a Dynamic Process



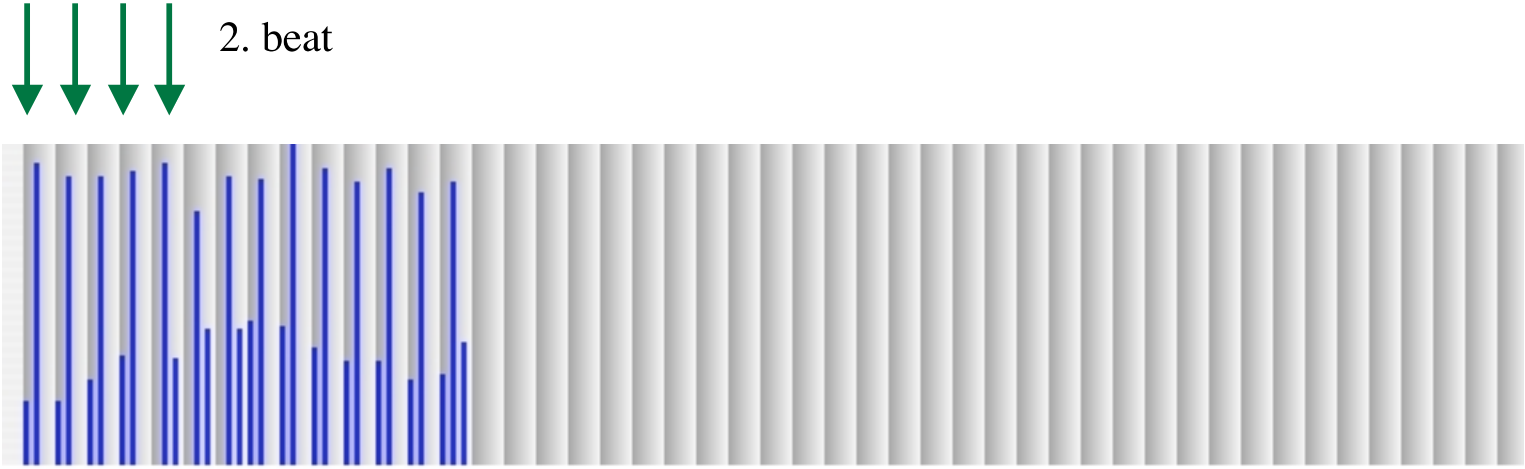
Meter in Music as a Dynamic Process



Meter in Music as a Dynamic Process

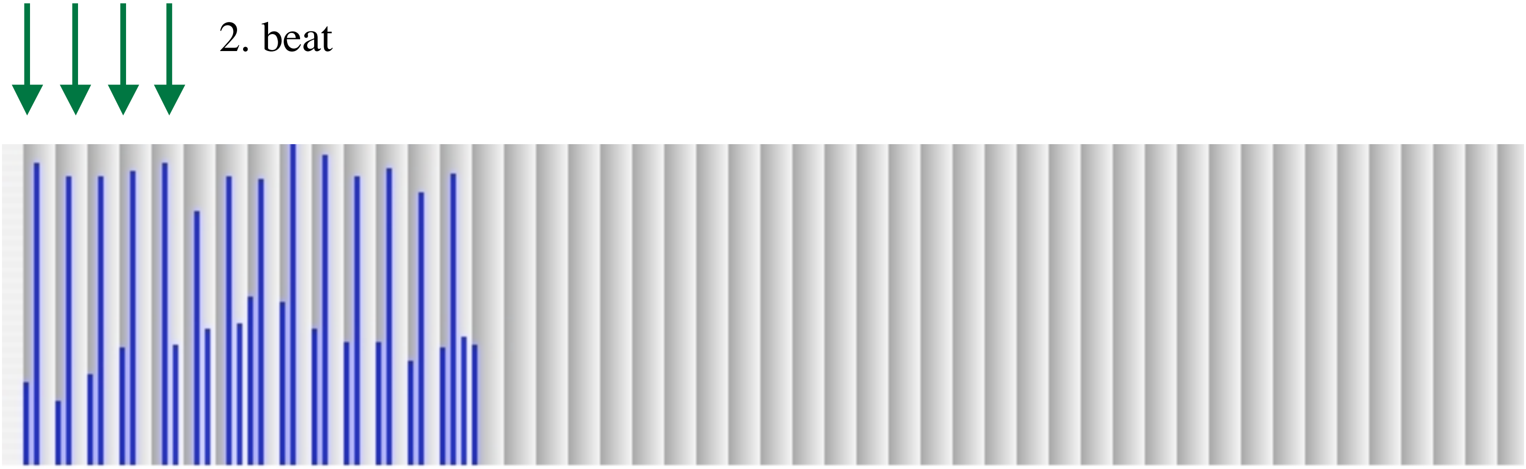


Meter in Music as a Dynamic Process



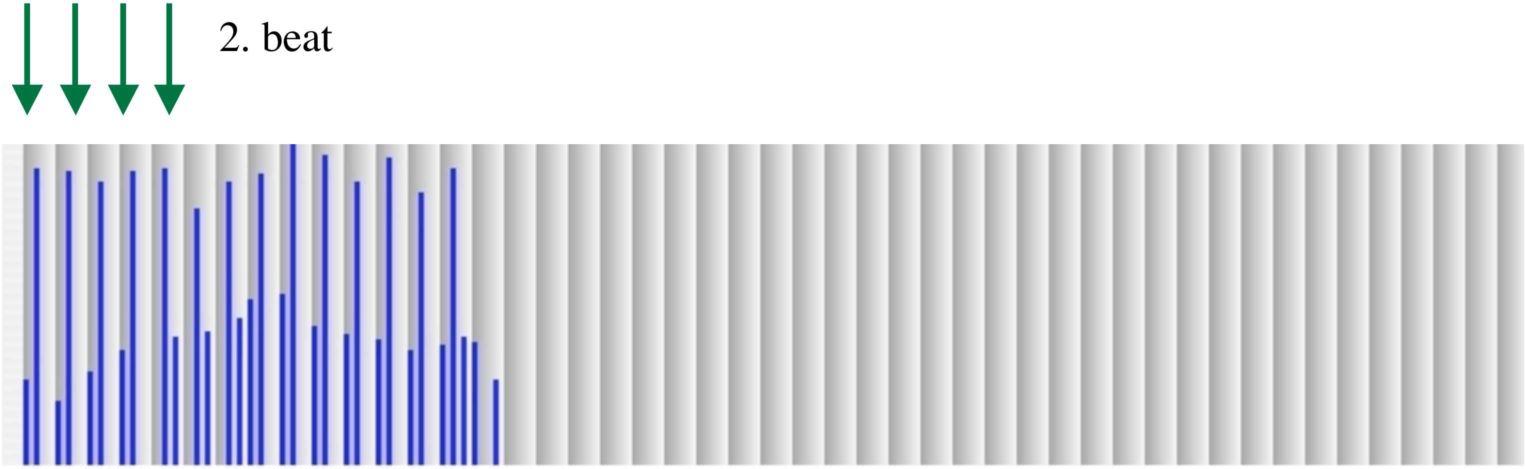
2. beat

Meter in Music as a Dynamic Process

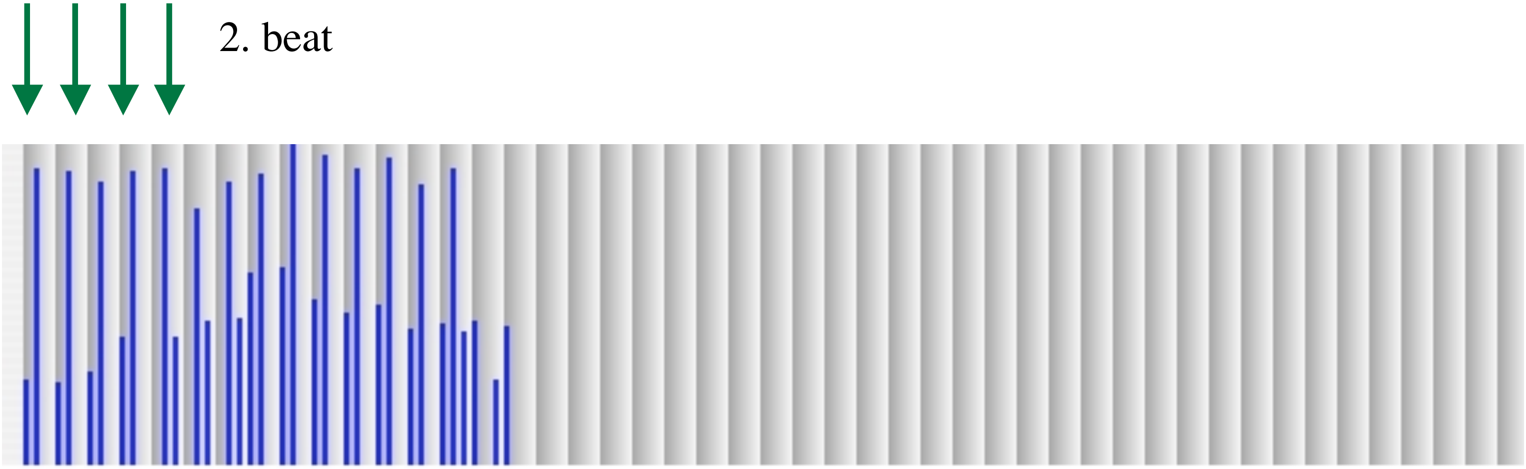


2. beat

Meter in Music as a Dynamic Process

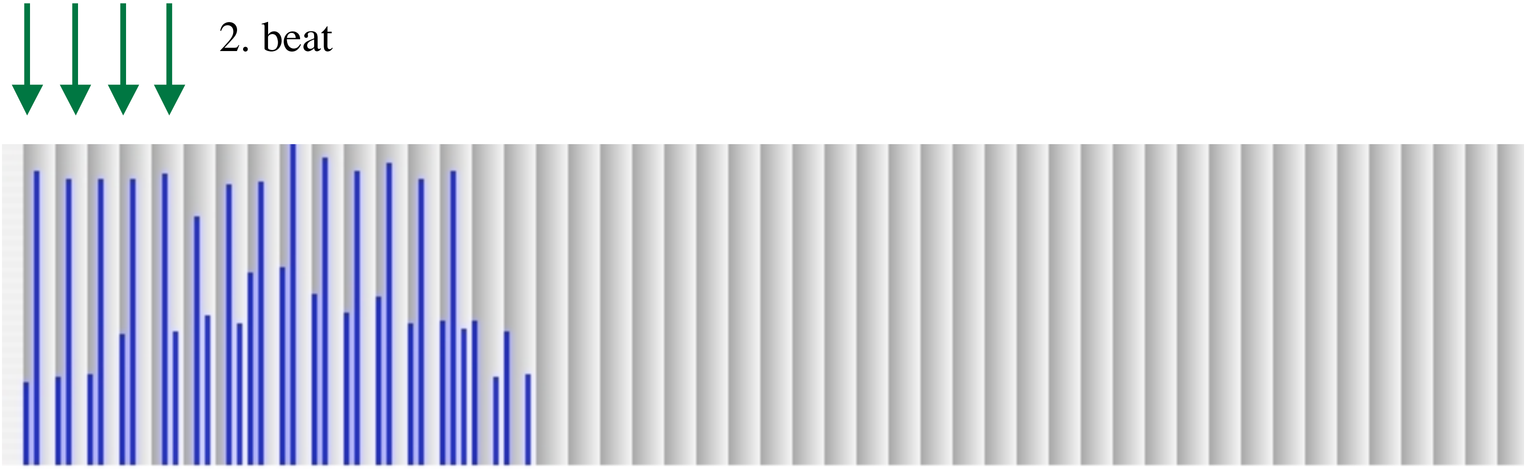

2. beat

Meter in Music as a Dynamic Process

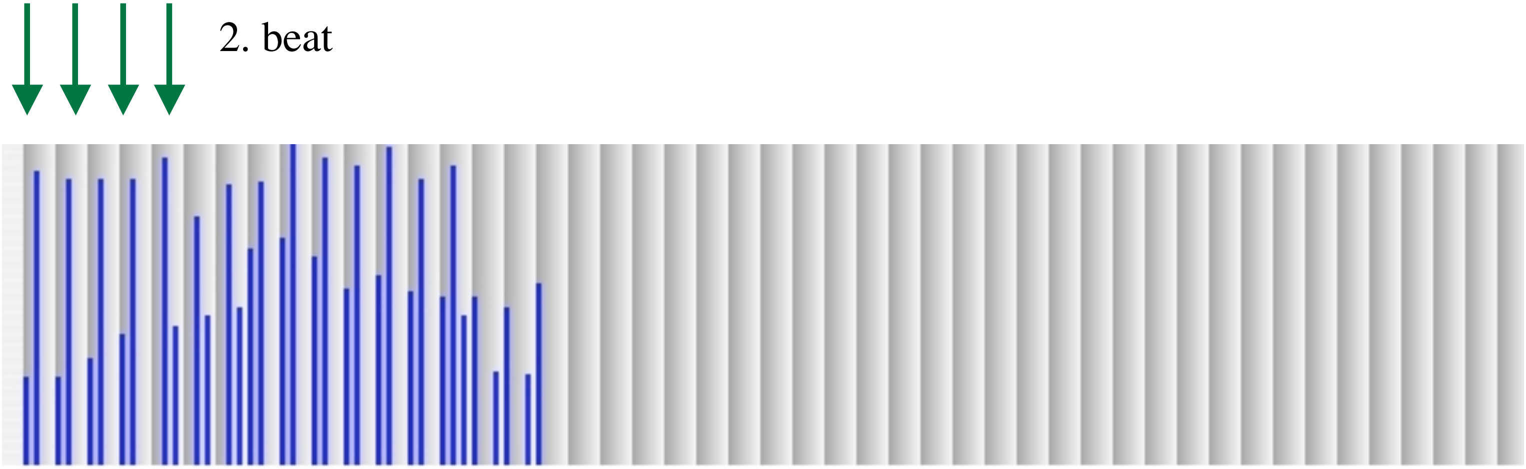


2. beat

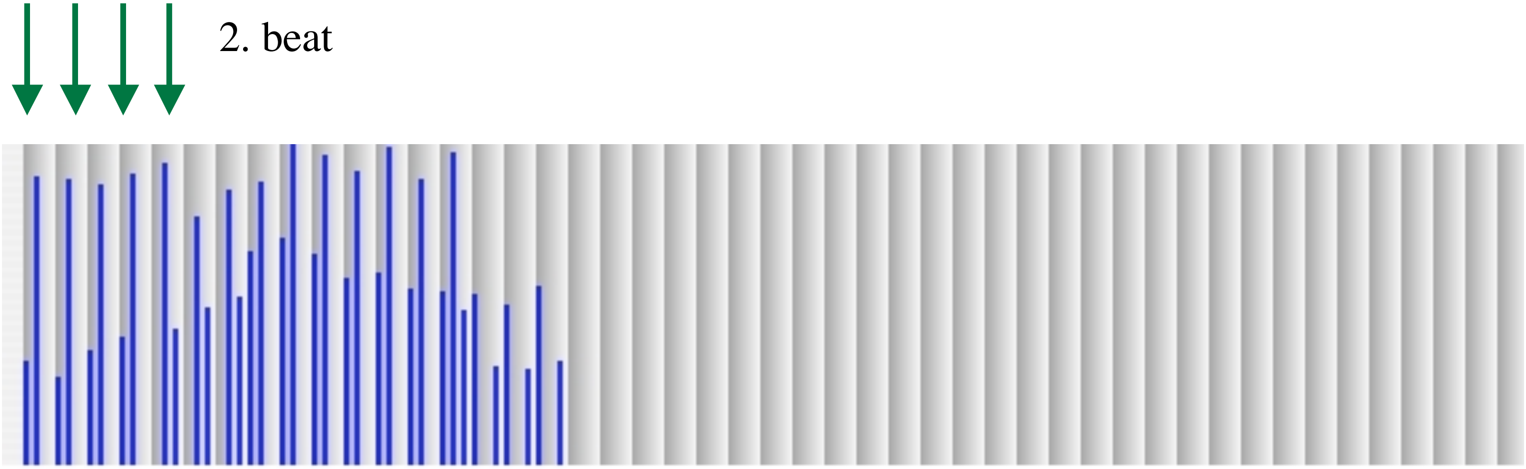
Meter in Music as a Dynamic Process



Meter in Music as a Dynamic Process

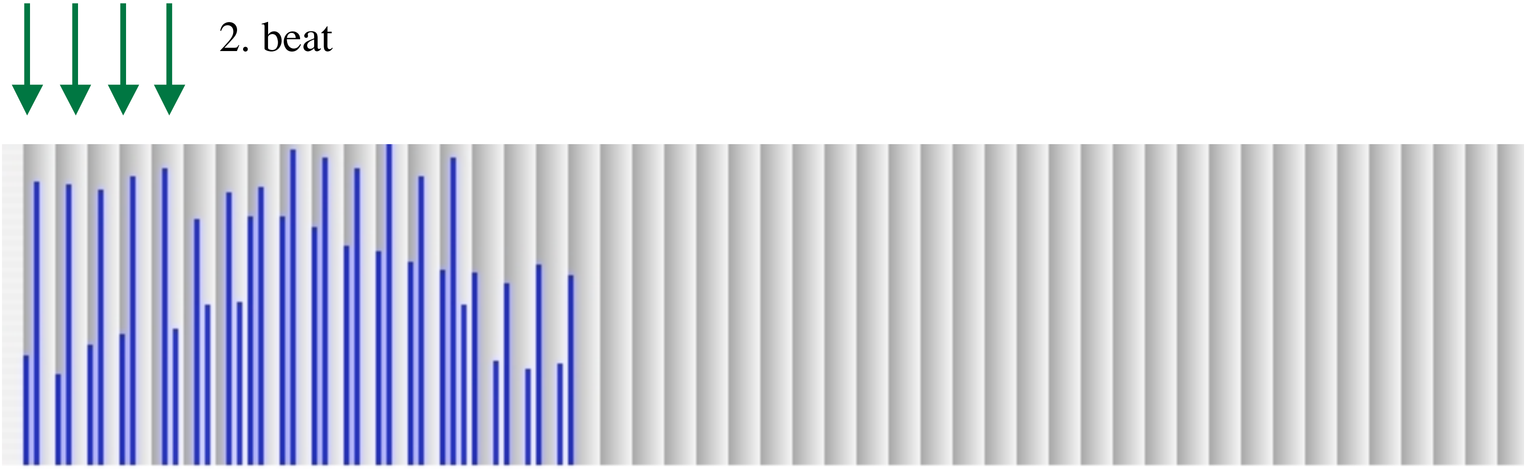


Meter in Music as a Dynamic Process



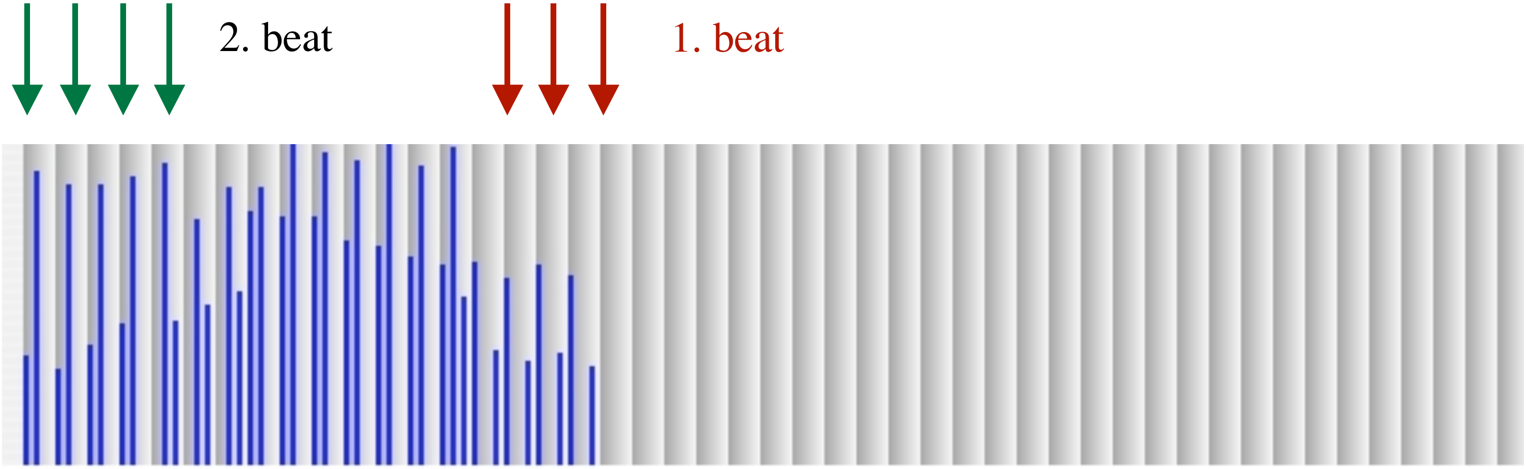
2. beat

Meter in Music as a Dynamic Process



2. beat

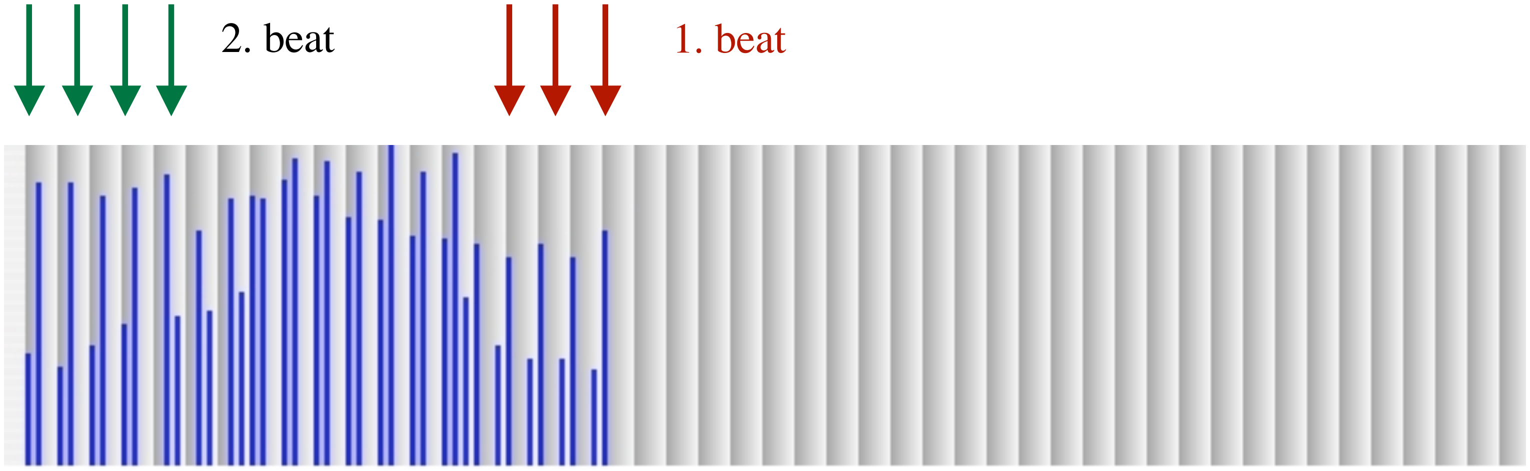
Meter in Music as a Dynamic Process



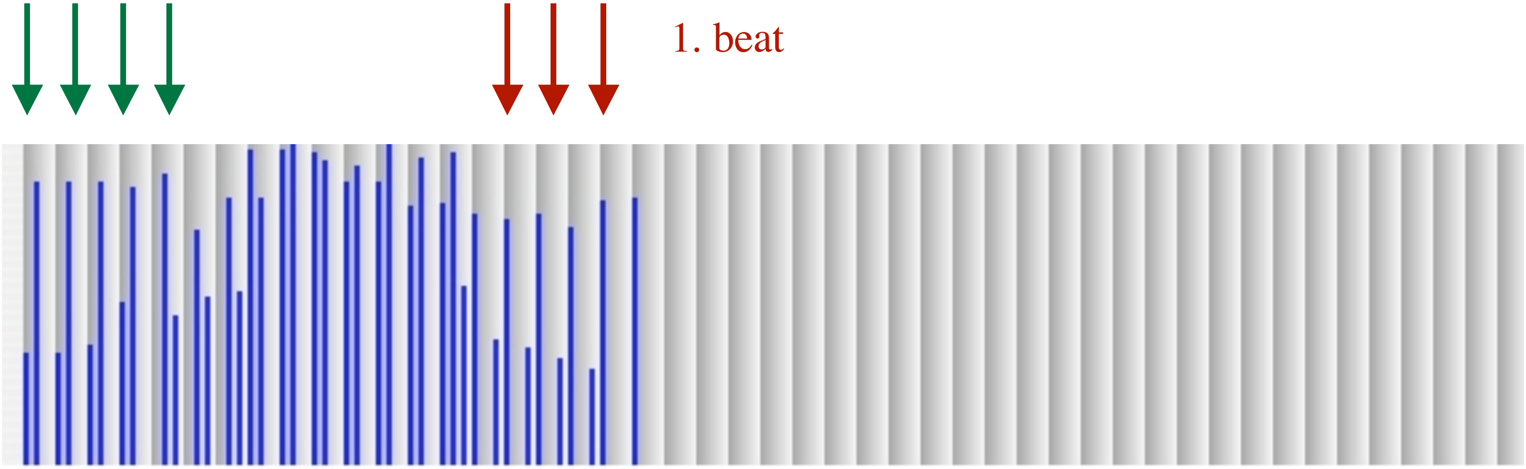
2. beat

1. beat

Meter in Music as a Dynamic Process

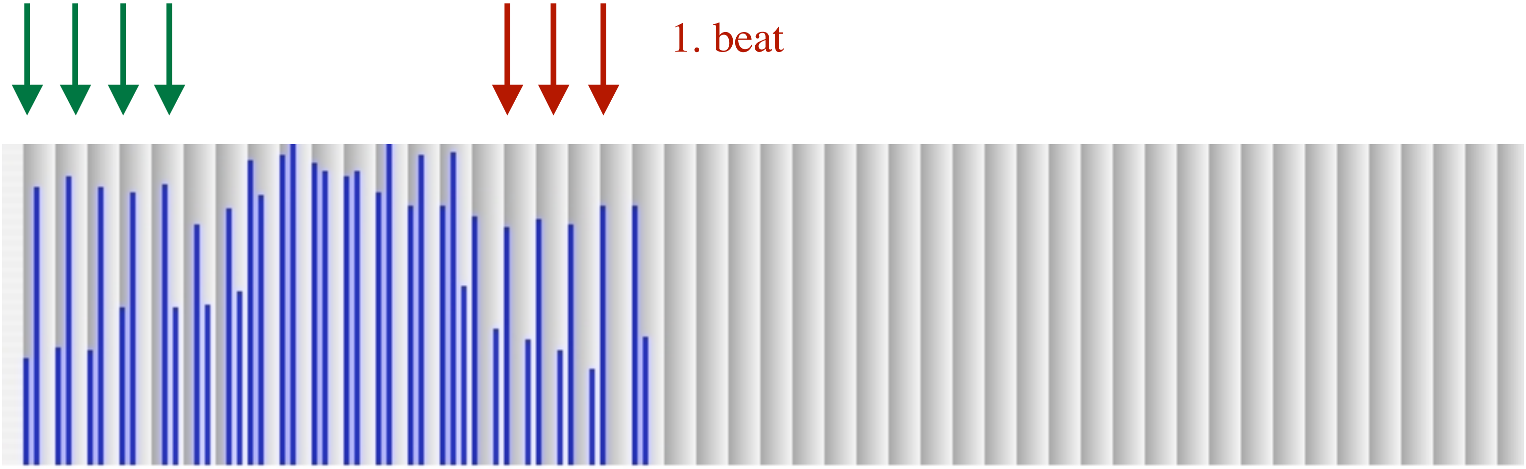

2. beat
1. beat

Meter in Music as a Dynamic Process



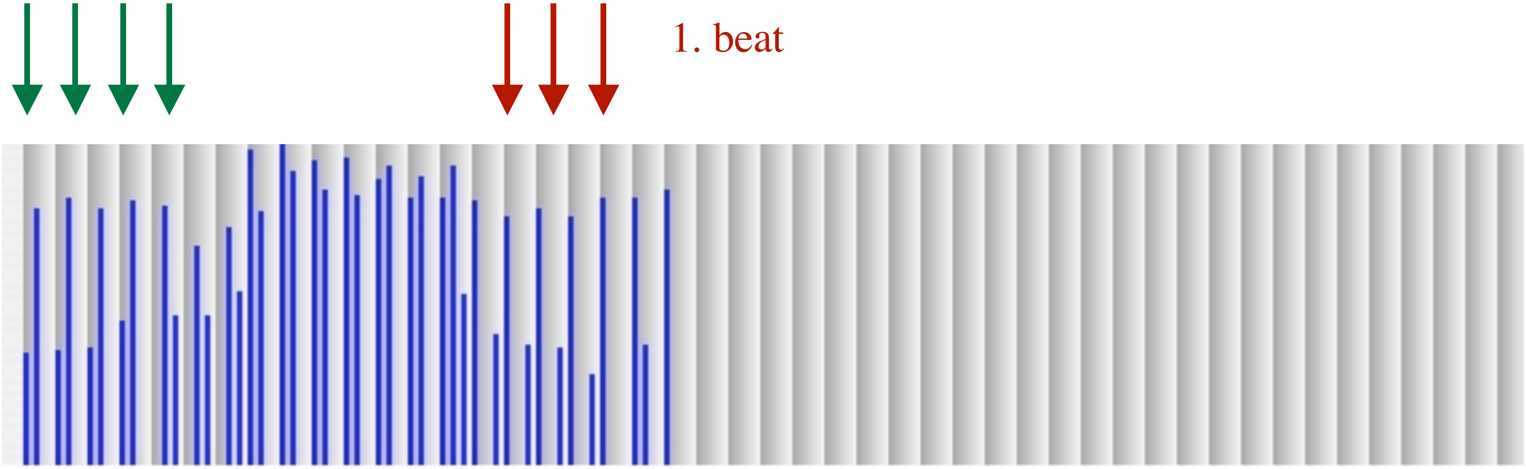
1. beat

Meter in Music as a Dynamic Process

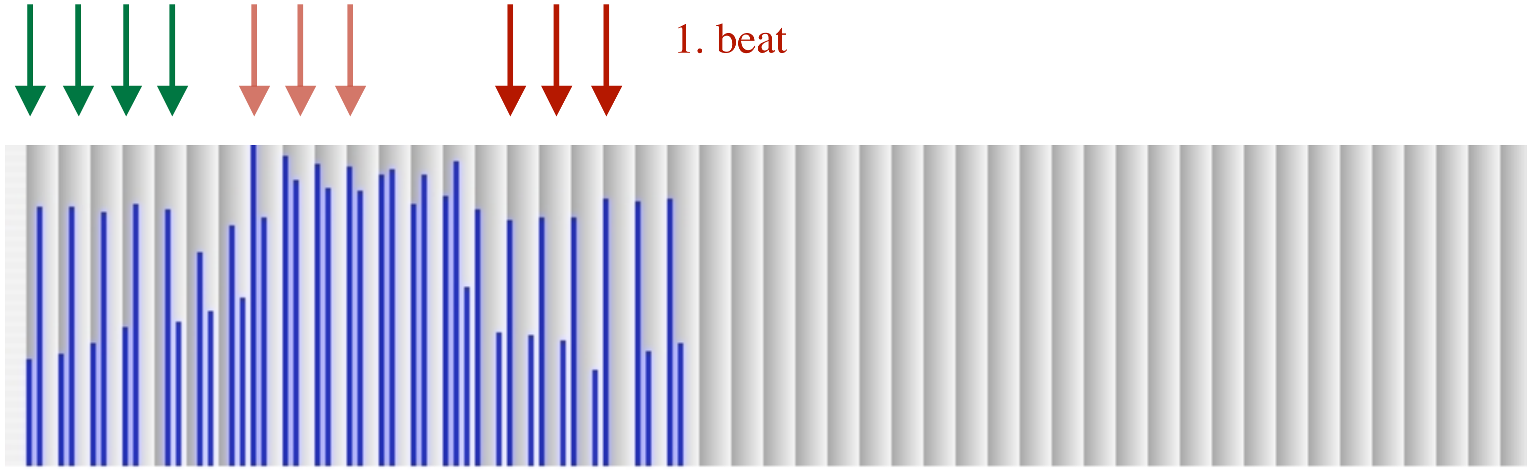


1. beat

Meter in Music as a Dynamic Process

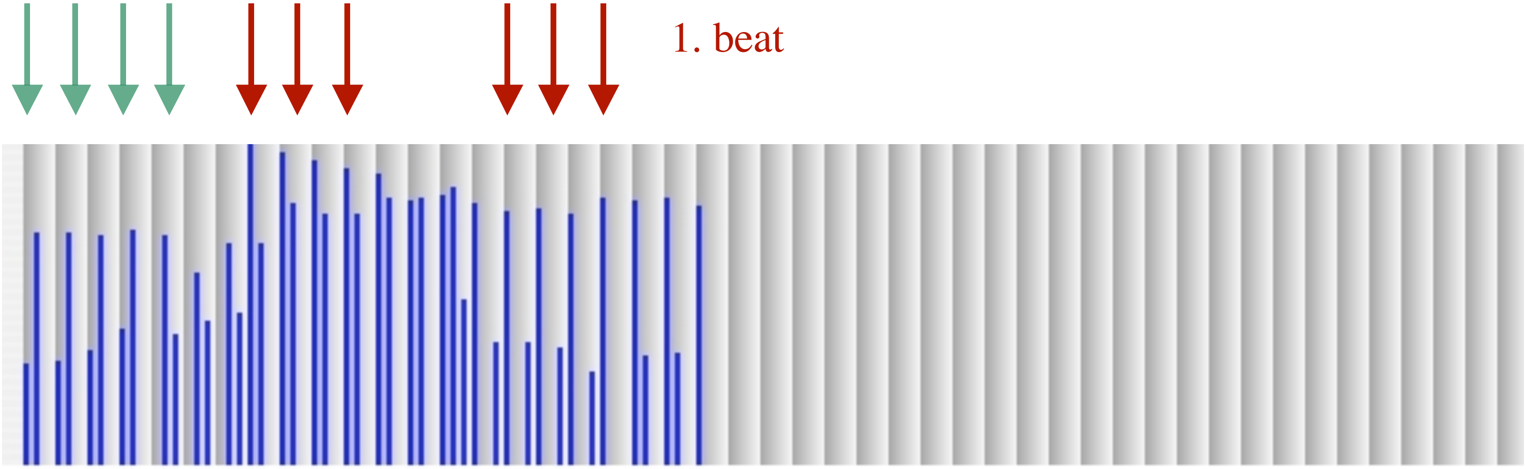


Meter in Music as a Dynamic Process

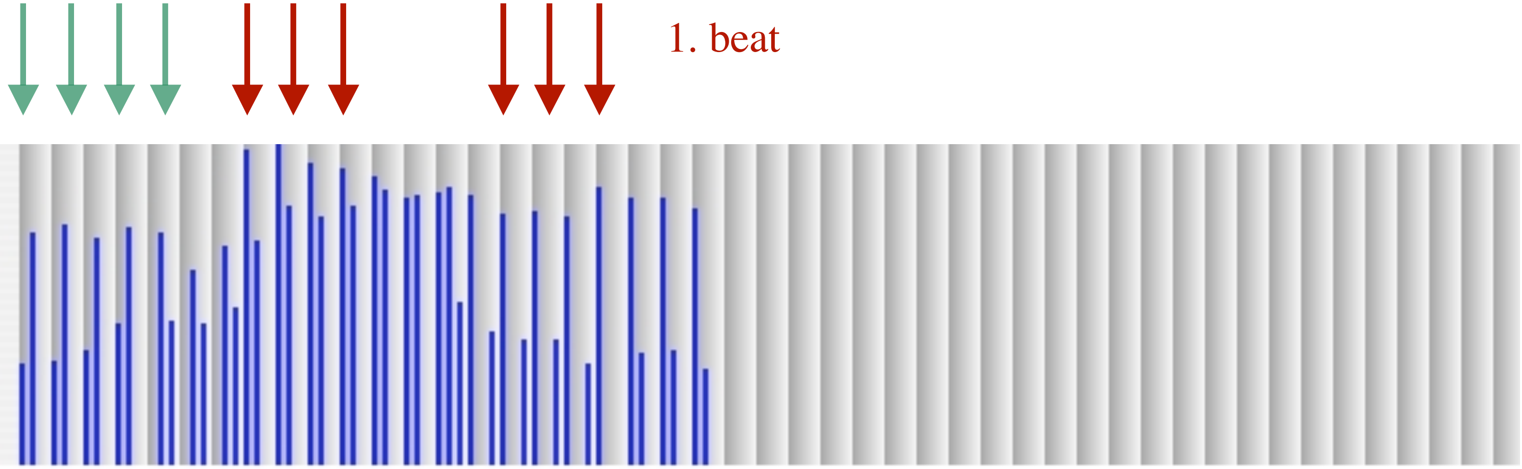


1. beat

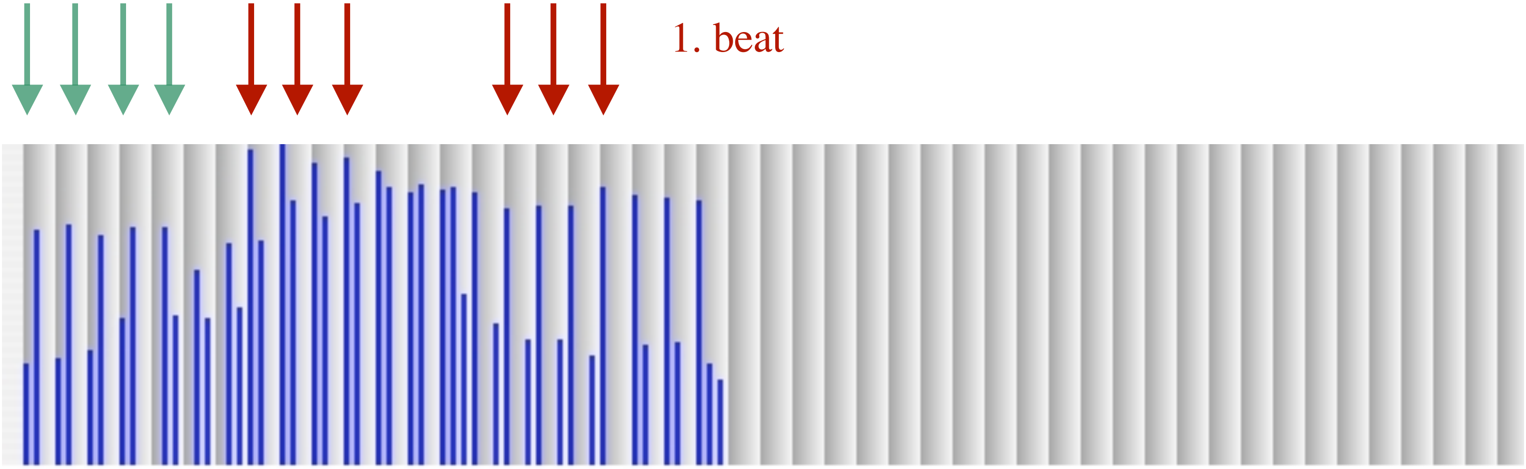
Meter in Music as a Dynamic Process



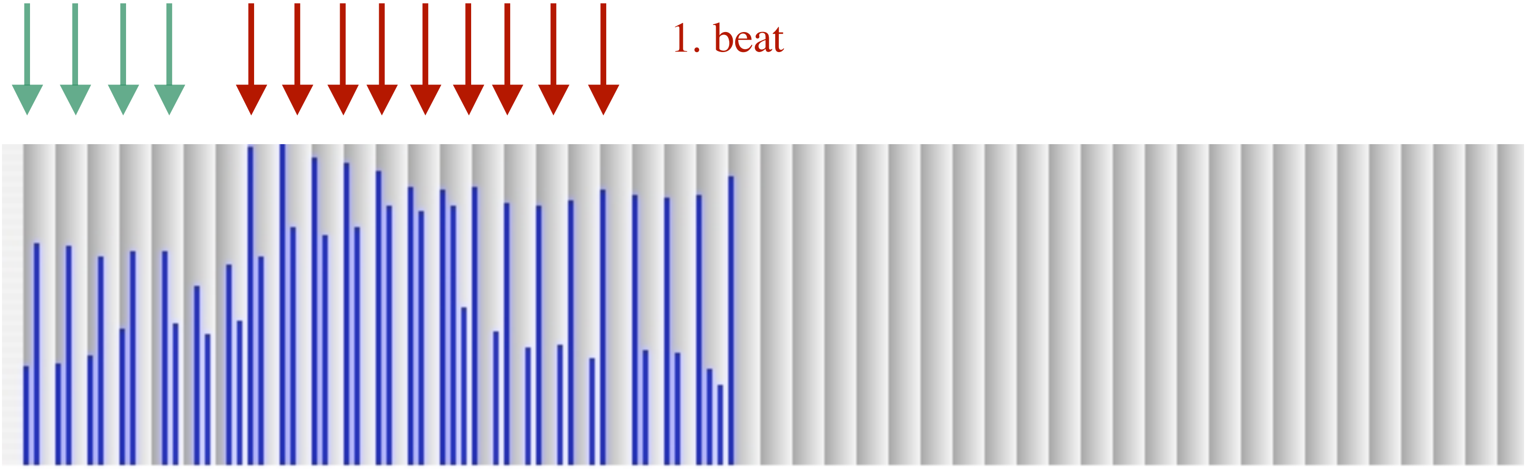
Meter in Music as a Dynamic Process



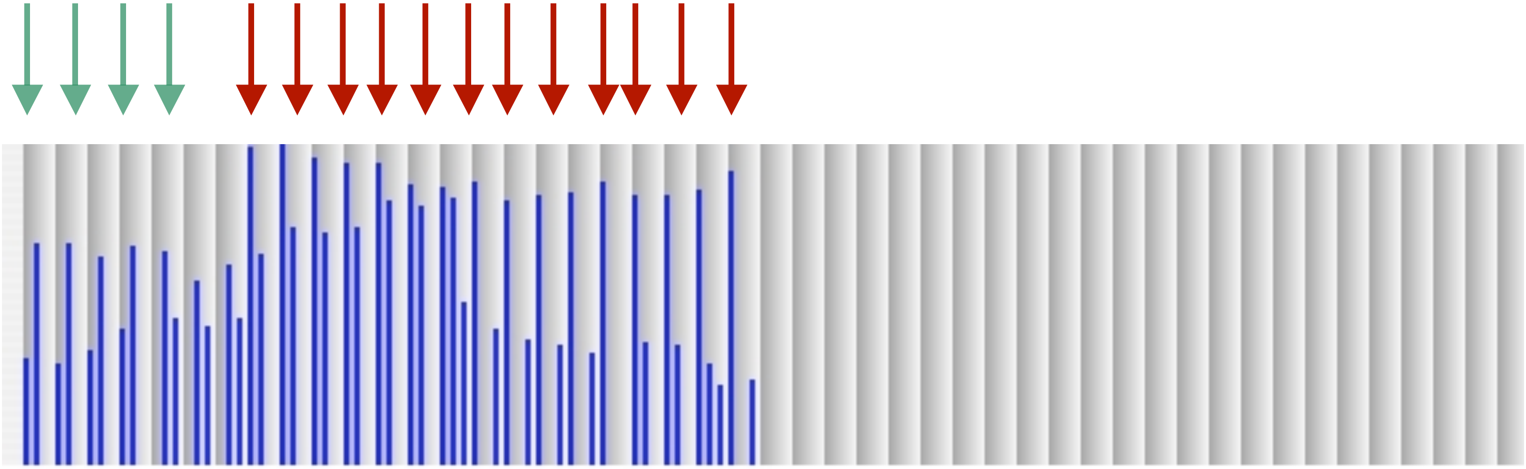
Meter in Music as a Dynamic Process



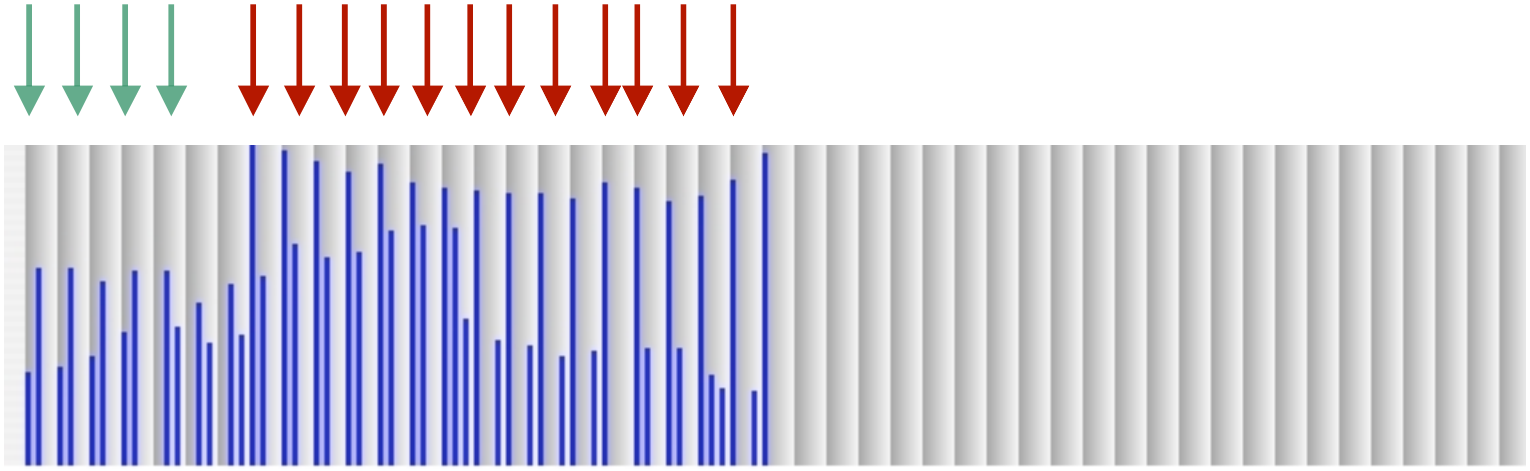
Meter in Music as a Dynamic Process



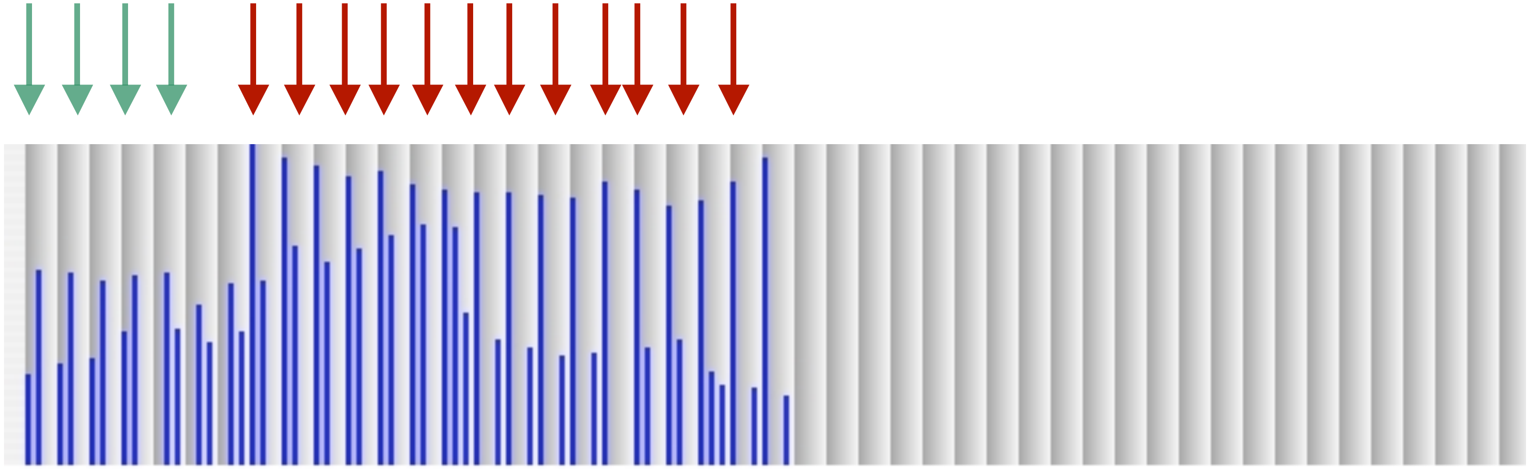
Meter in Music as a Dynamic Process



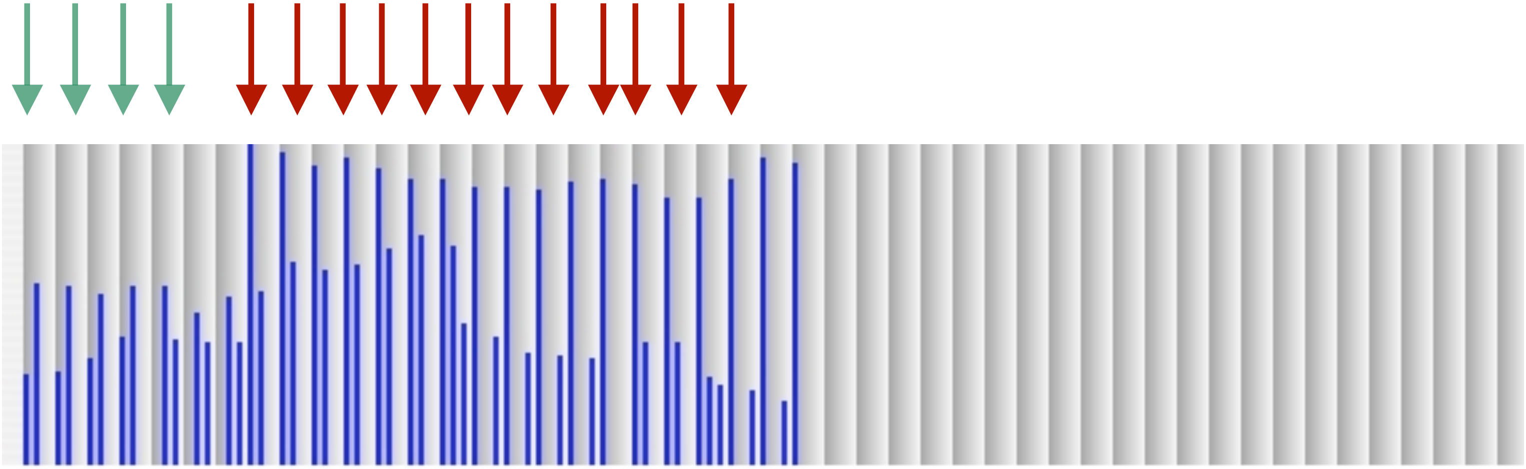
Meter in Music as a Dynamic Process



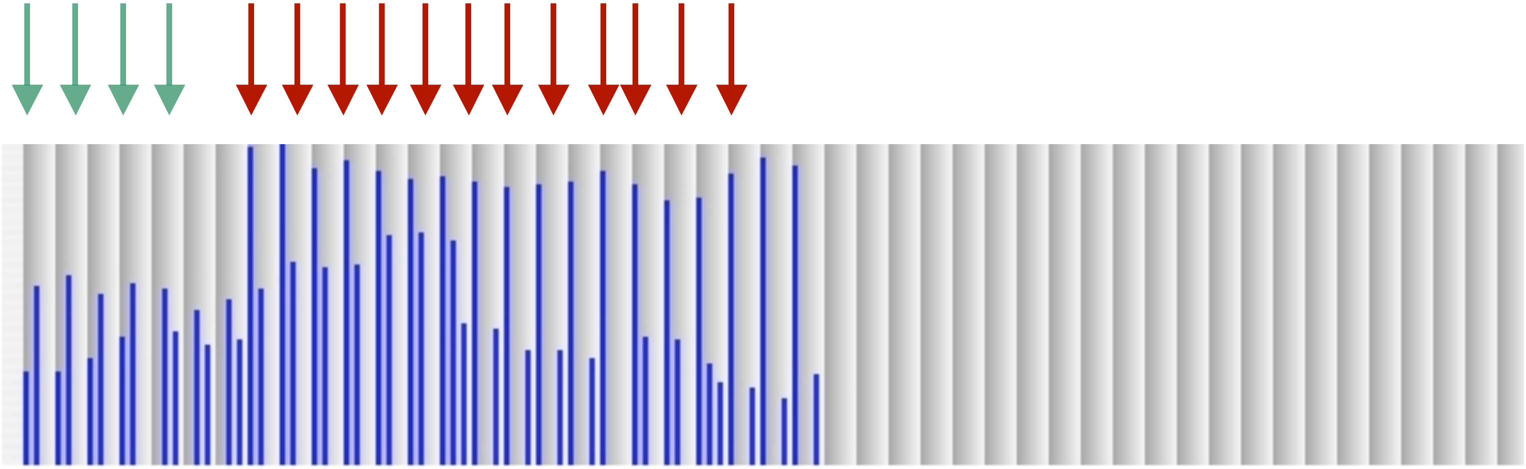
Meter in Music as a Dynamic Process



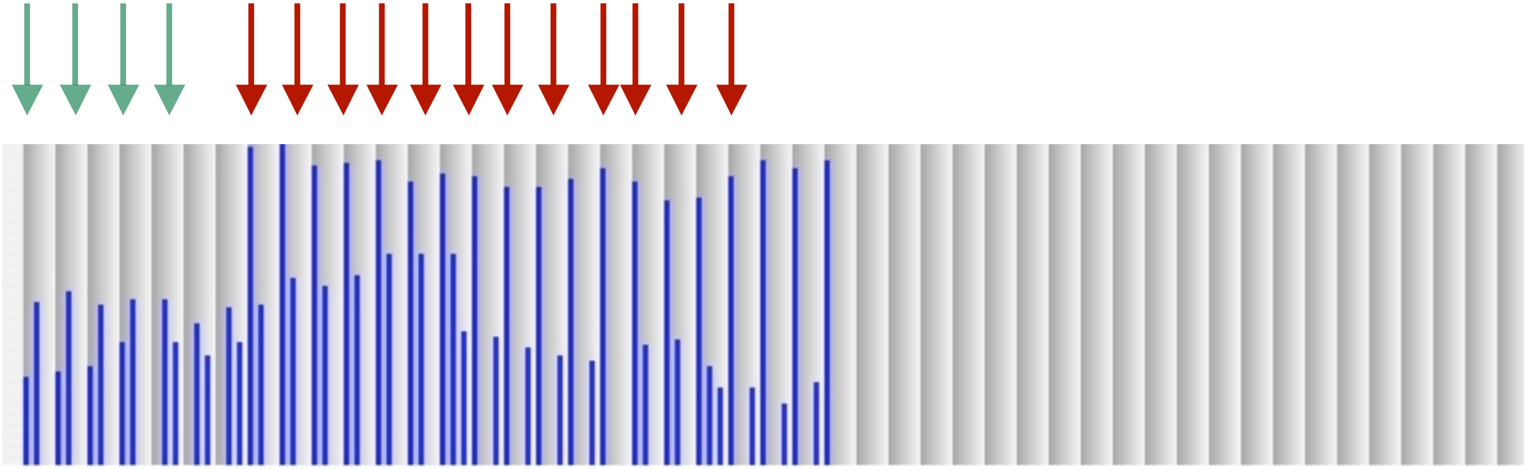
Meter in Music as a Dynamic Process



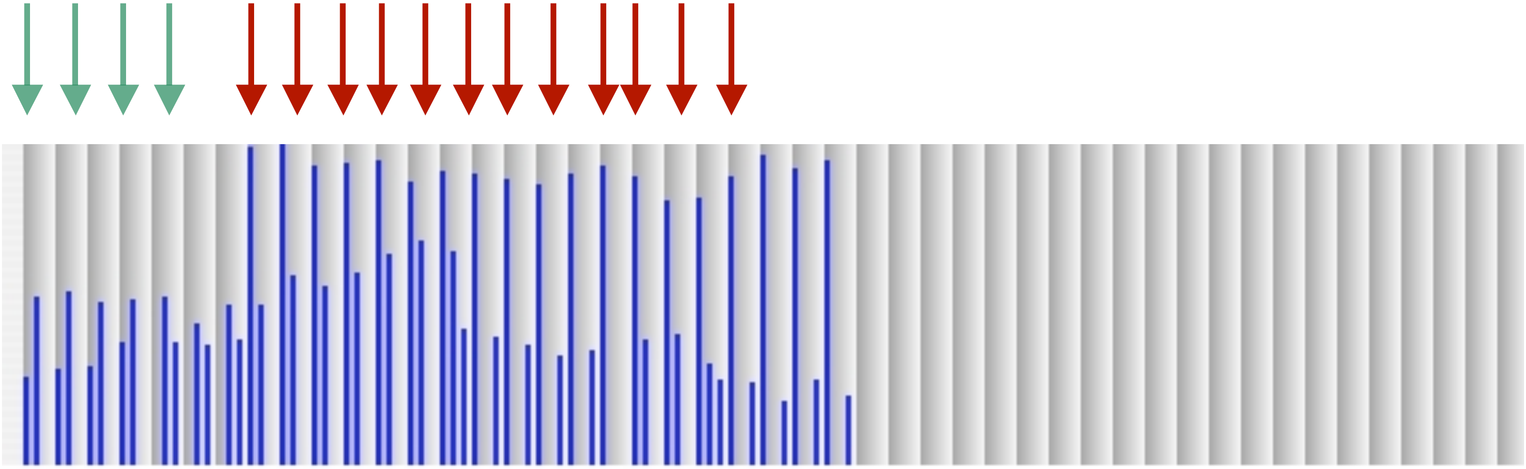
Meter in Music as a Dynamic Process



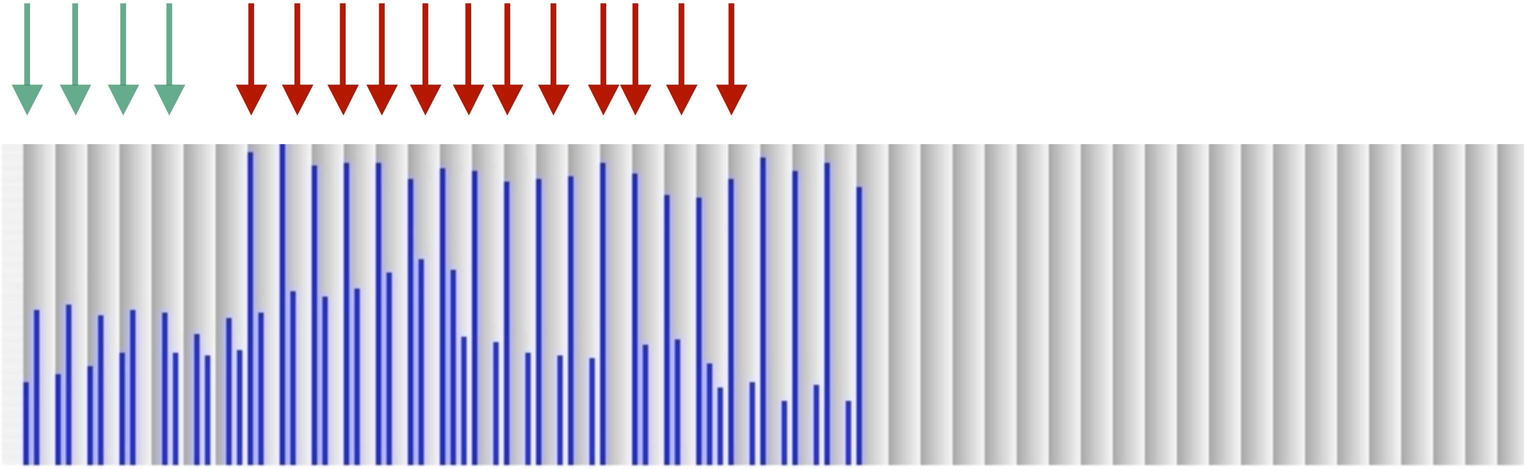
Meter in Music as a Dynamic Process



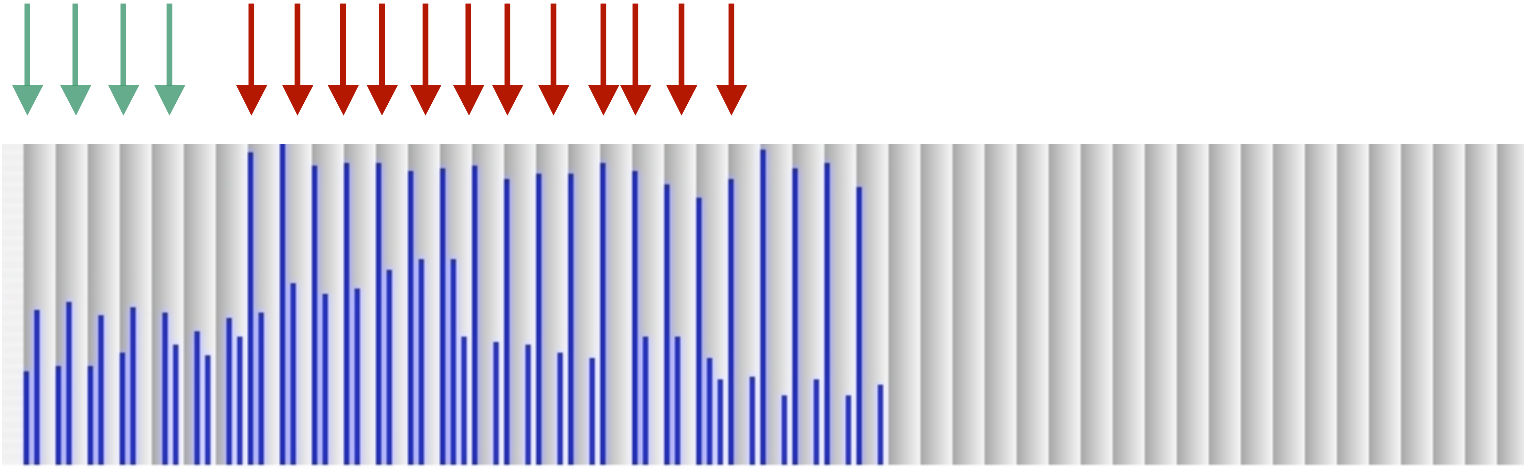
Meter in Music as a Dynamic Process



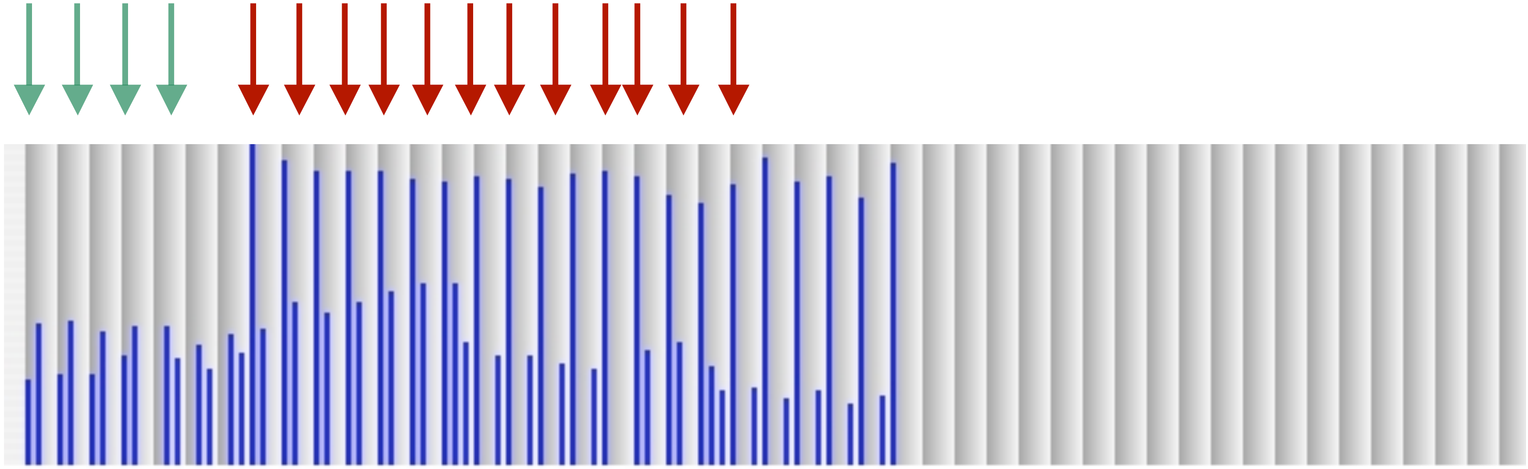
Meter in Music as a Dynamic Process



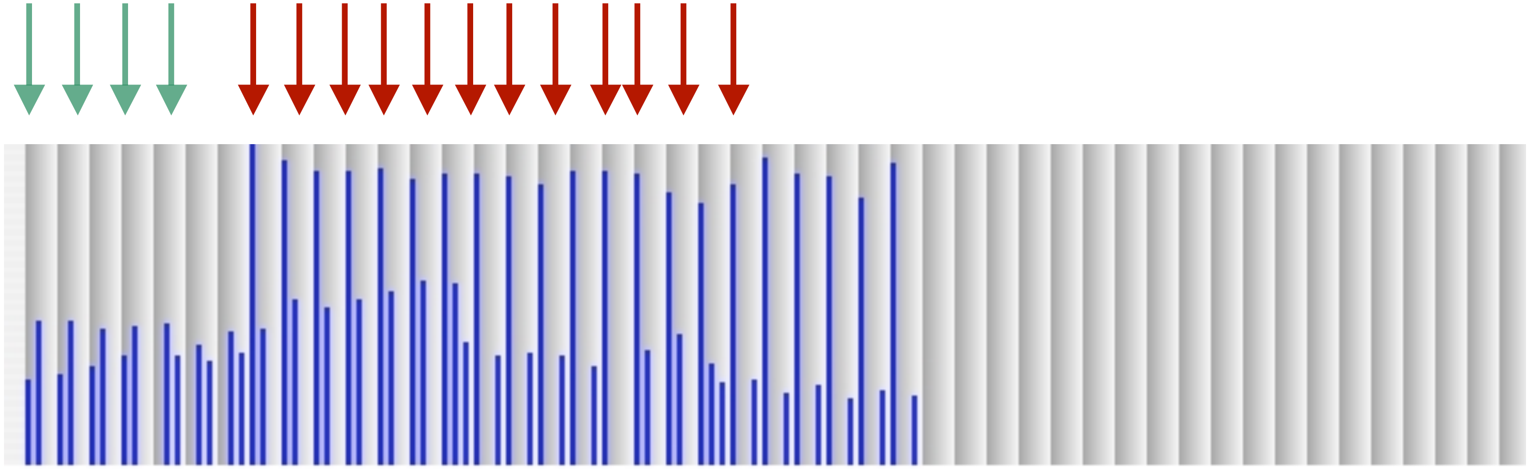
Meter in Music as a Dynamic Process



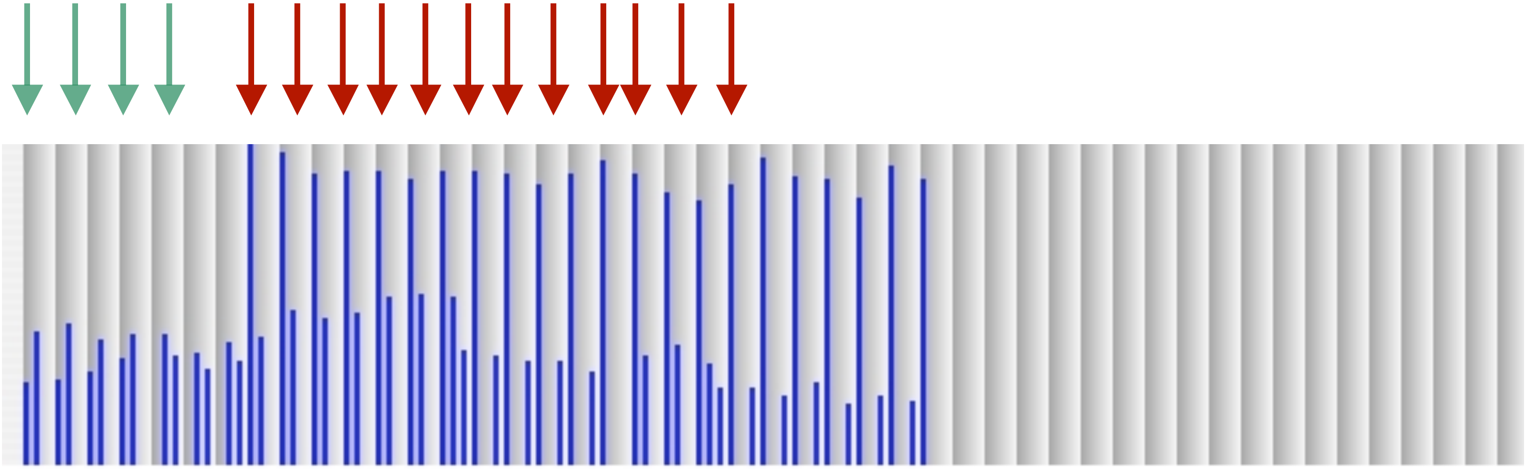
Meter in Music as a Dynamic Process



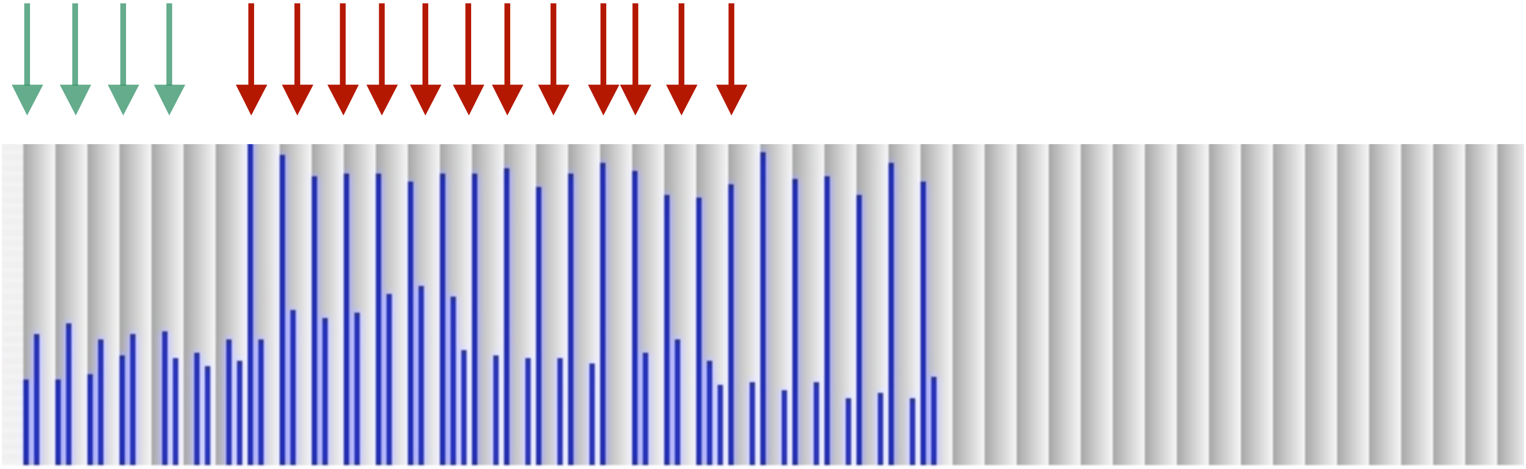
Meter in Music as a Dynamic Process



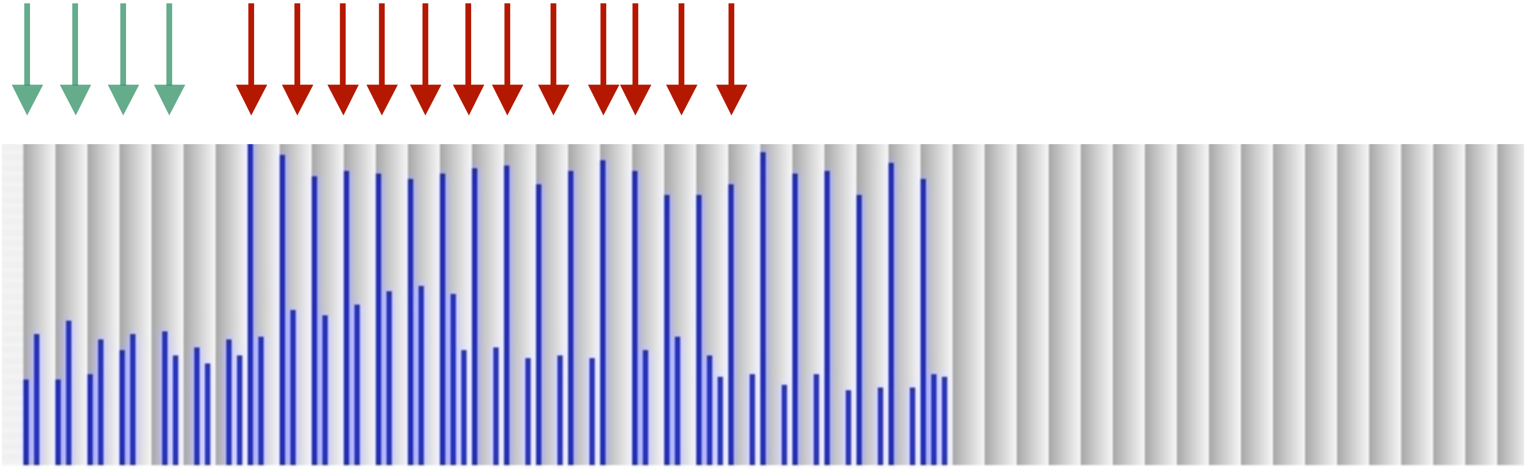
Meter in Music as a Dynamic Process



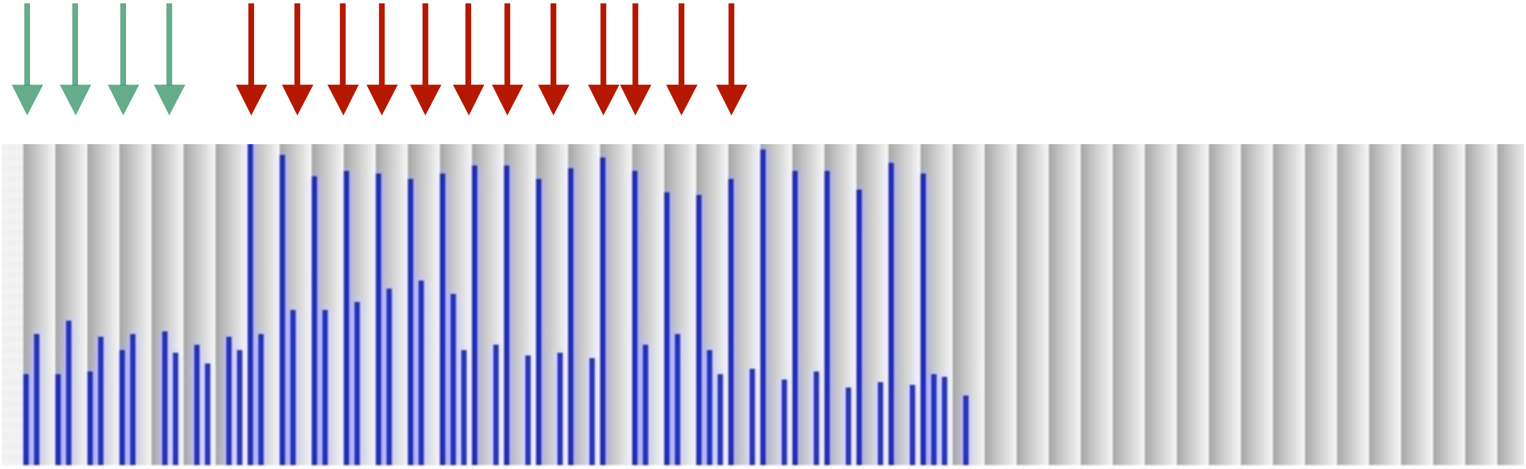
Meter in Music as a Dynamic Process



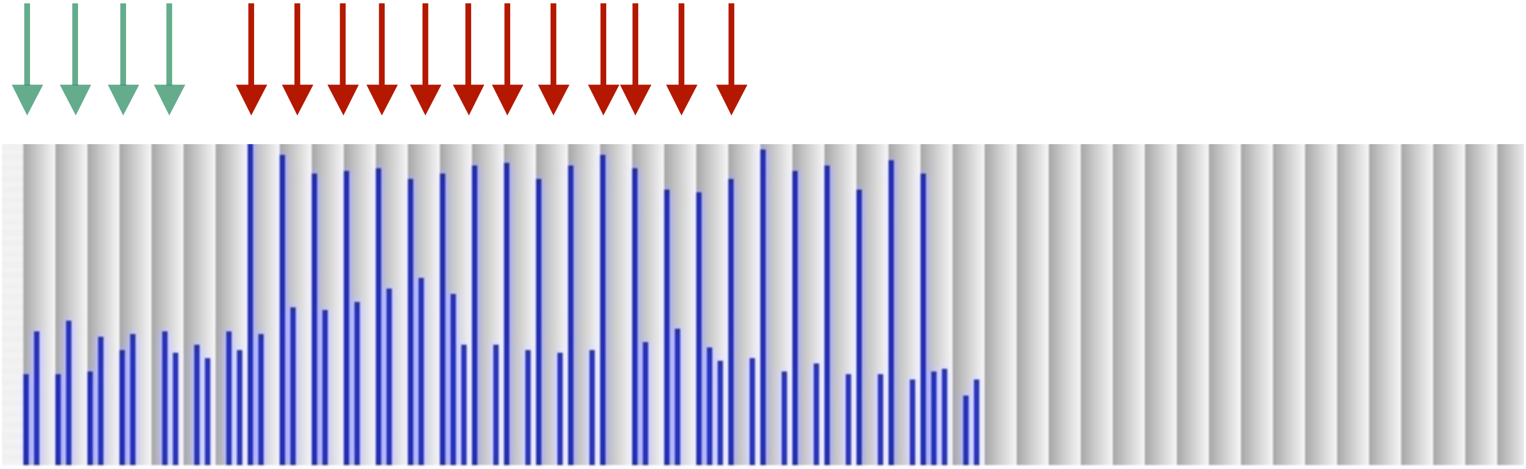
Meter in Music as a Dynamic Process



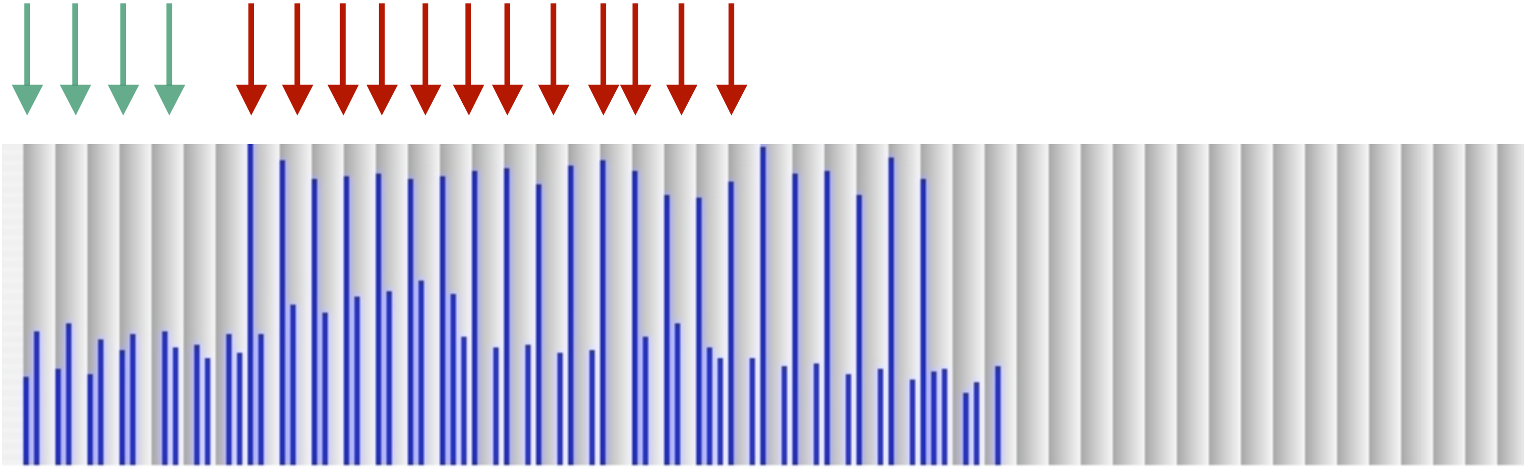
Meter in Music as a Dynamic Process



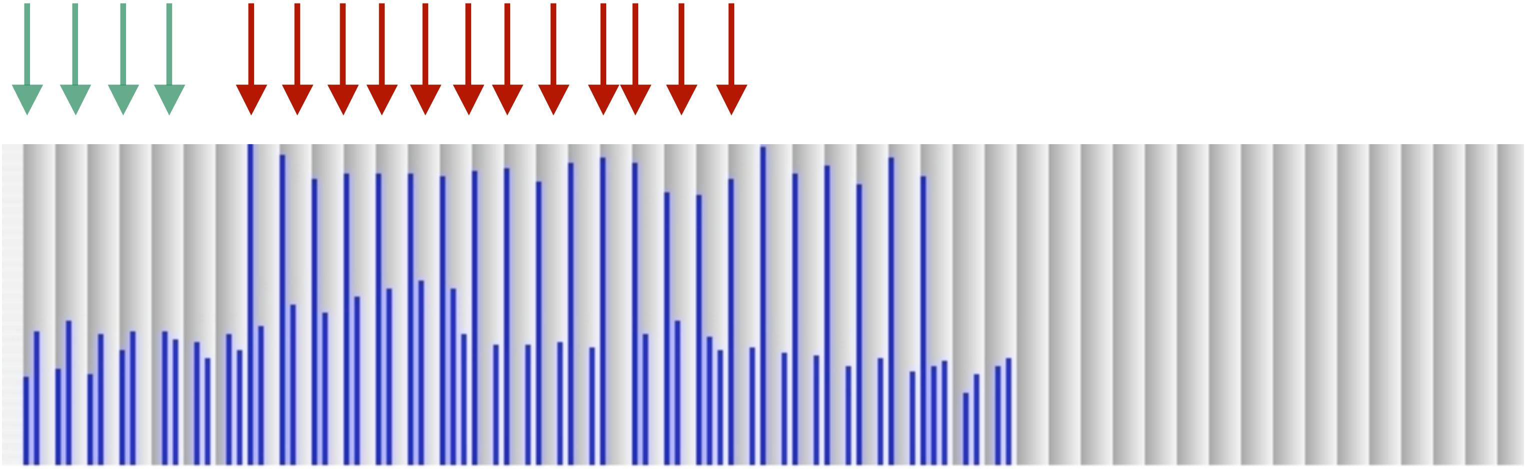
Meter in Music as a Dynamic Process



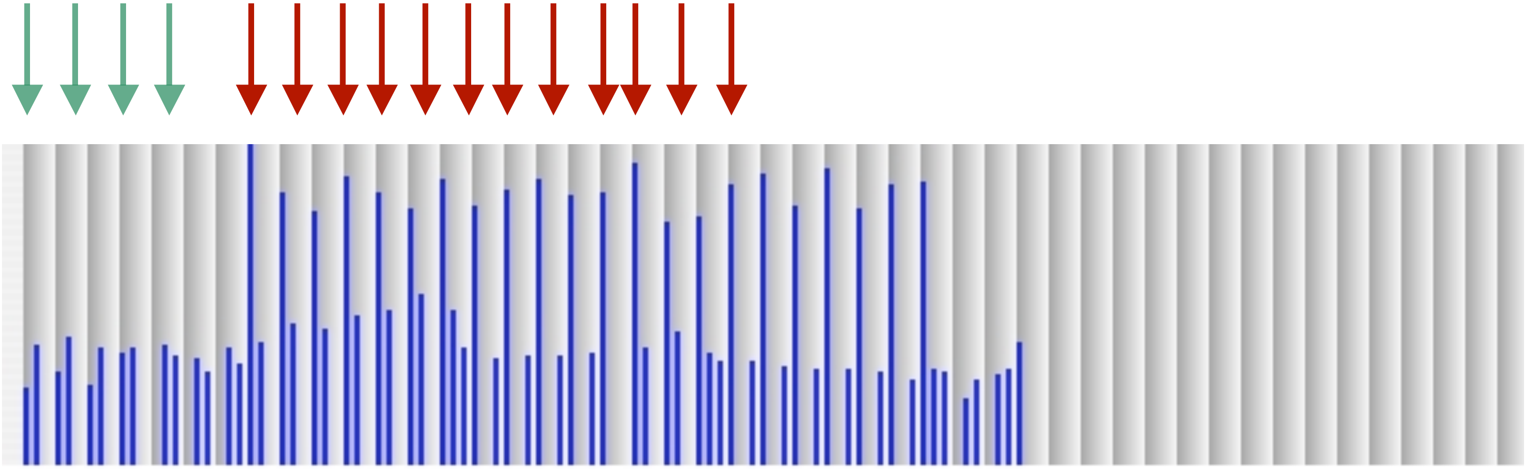
Meter in Music as a Dynamic Process



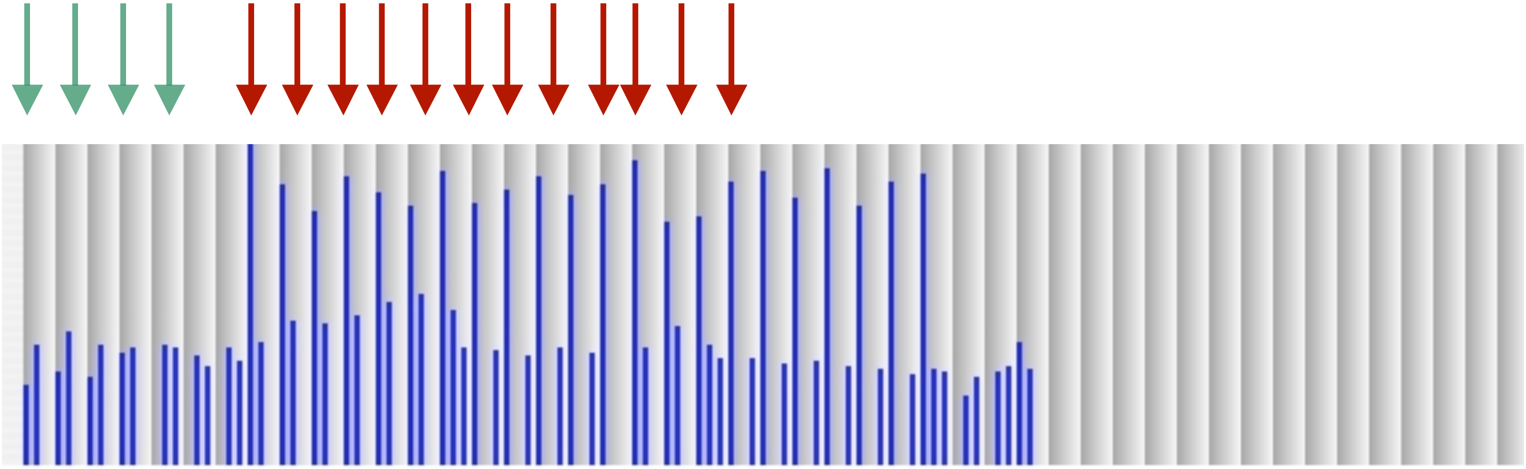
Meter in Music as a Dynamic Process



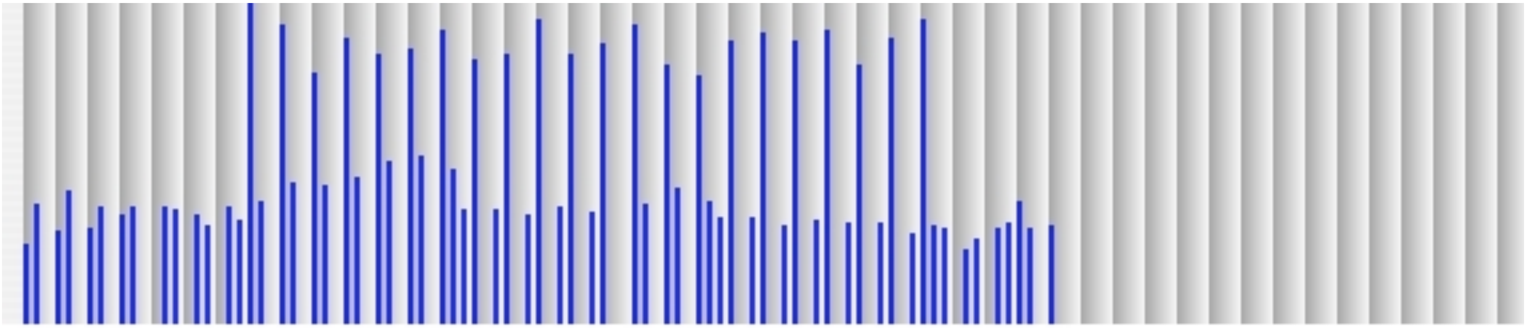
Meter in Music as a Dynamic Process



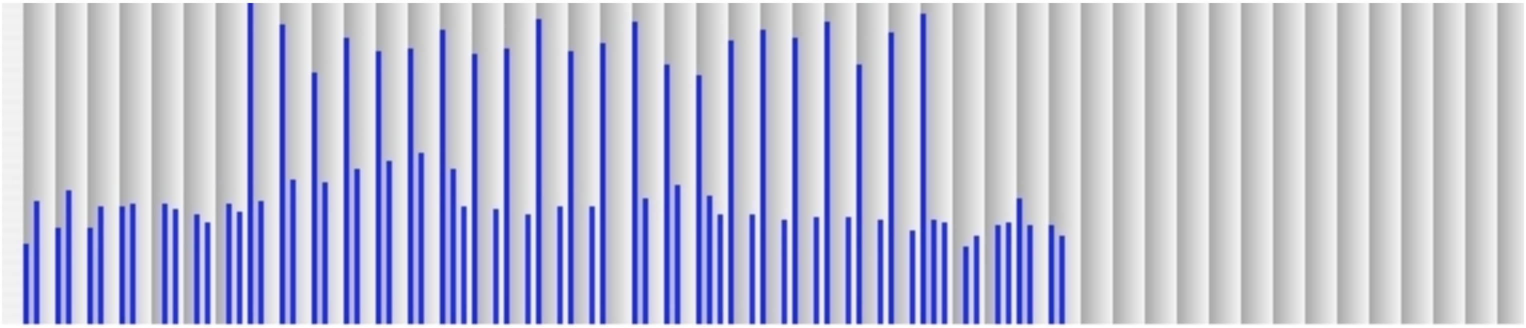
Meter in Music as a Dynamic Process



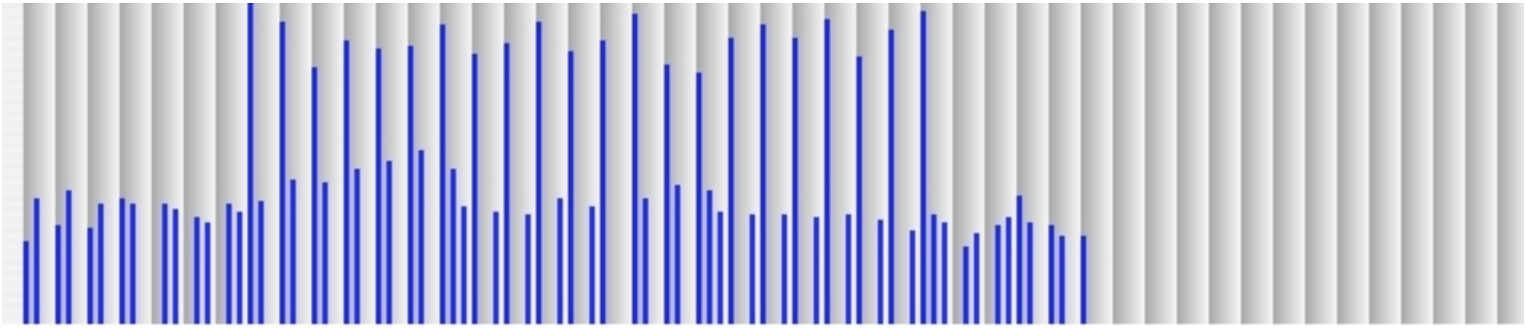
Meter in Music as a Dynamic Process



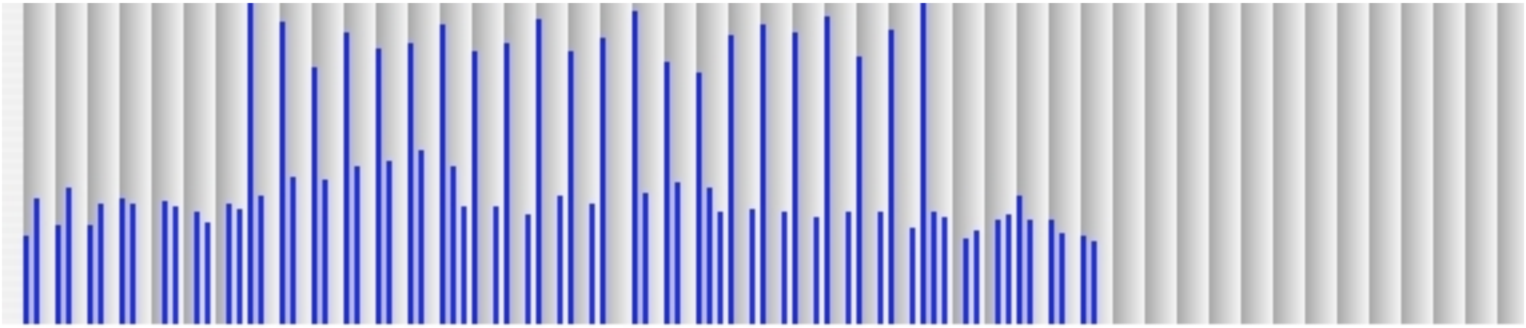
Meter in Music as a Dynamic Process



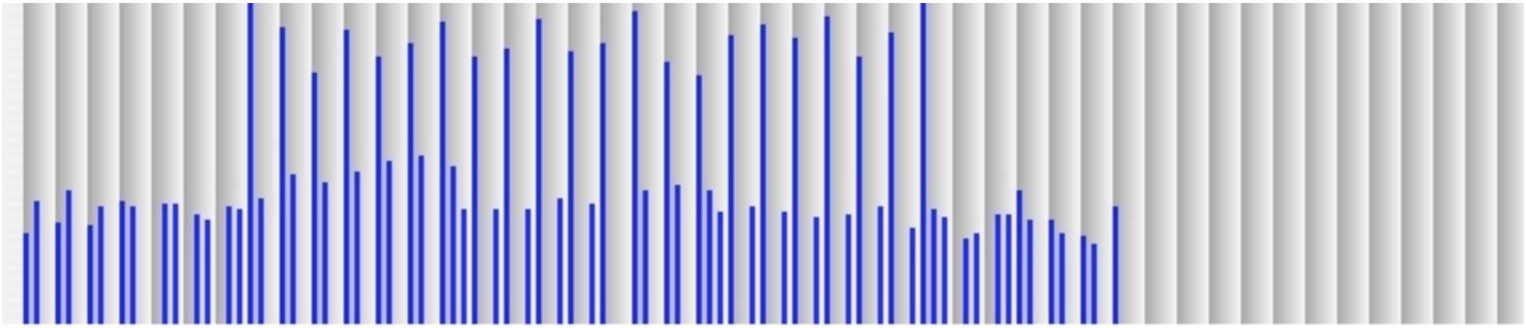
Meter in Music as a Dynamic Process



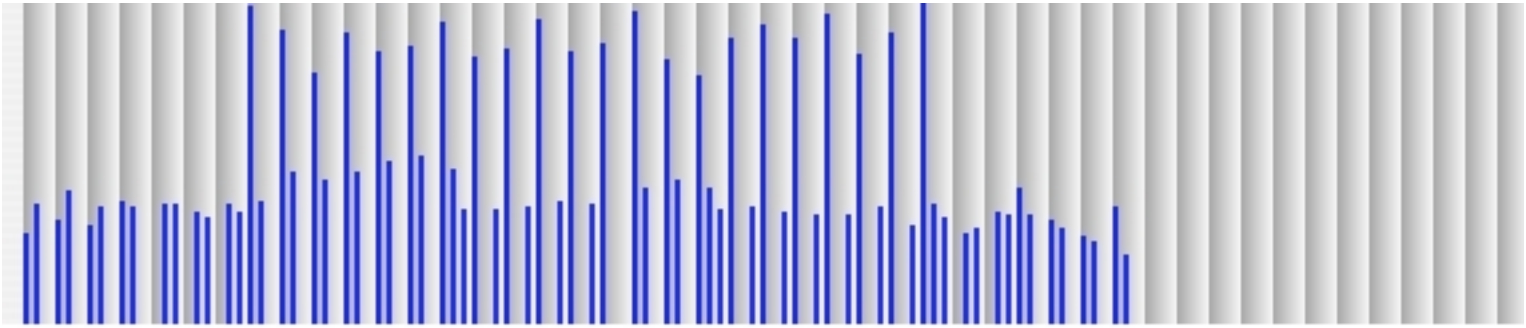
Meter in Music as a Dynamic Process



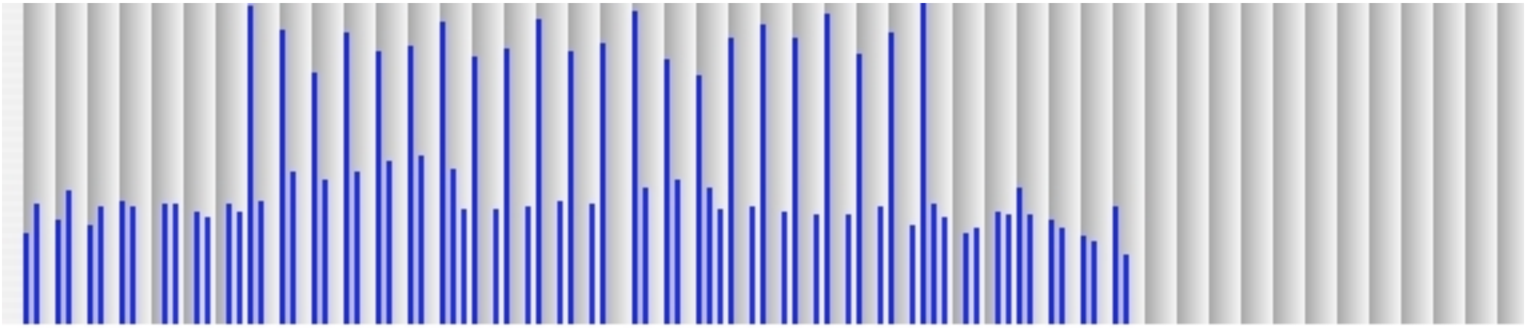
Meter in Music as a Dynamic Process



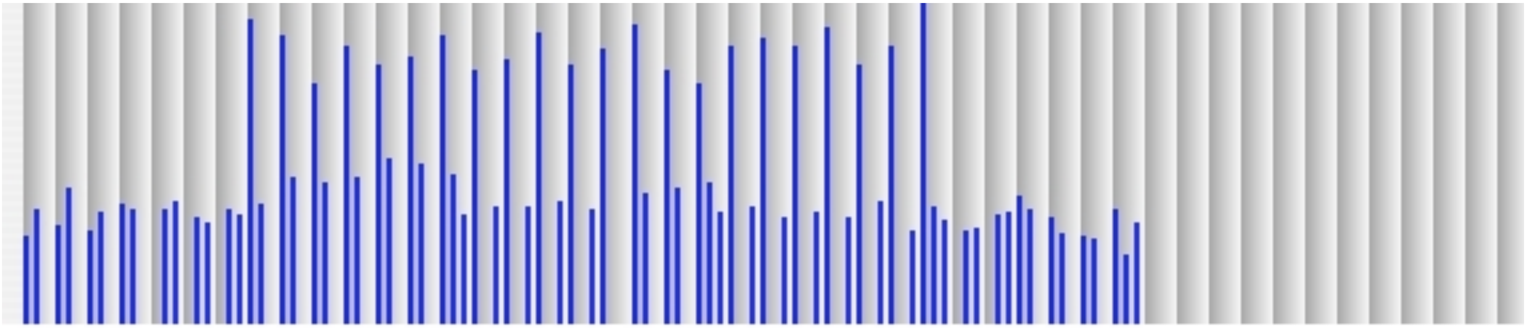
Meter in Music as a Dynamic Process



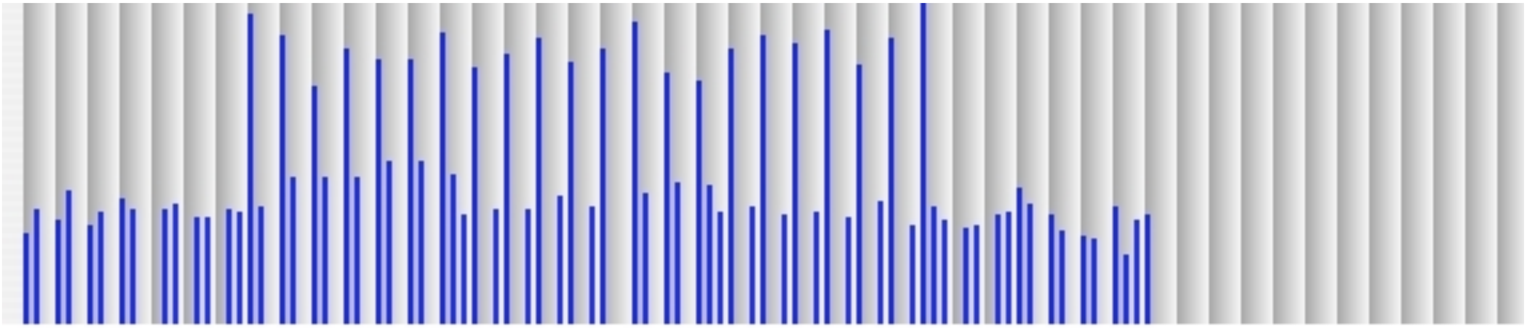
Meter in Music as a Dynamic Process



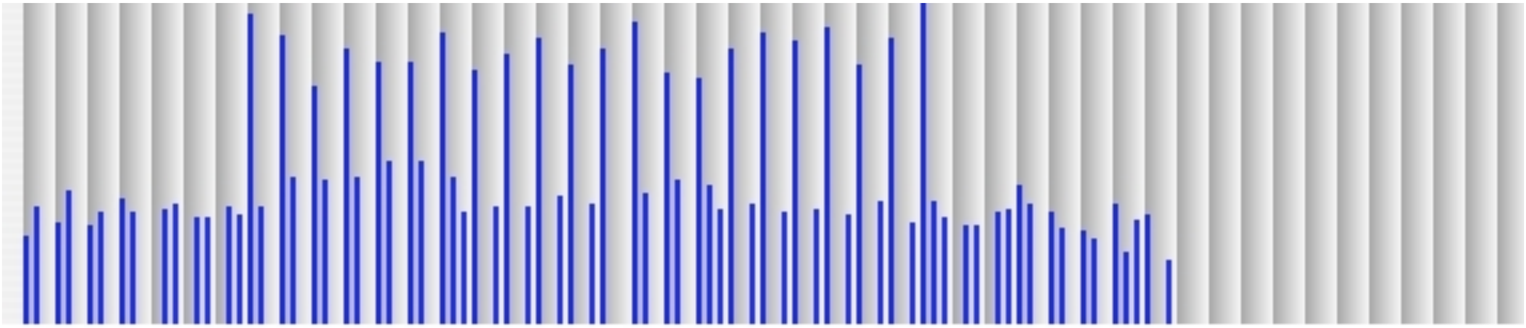
Meter in Music as a Dynamic Process



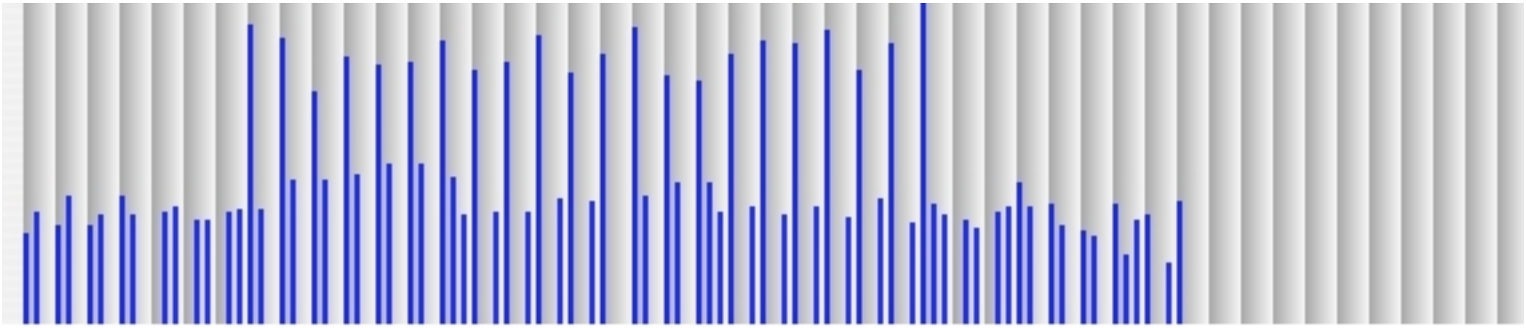
Meter in Music as a Dynamic Process



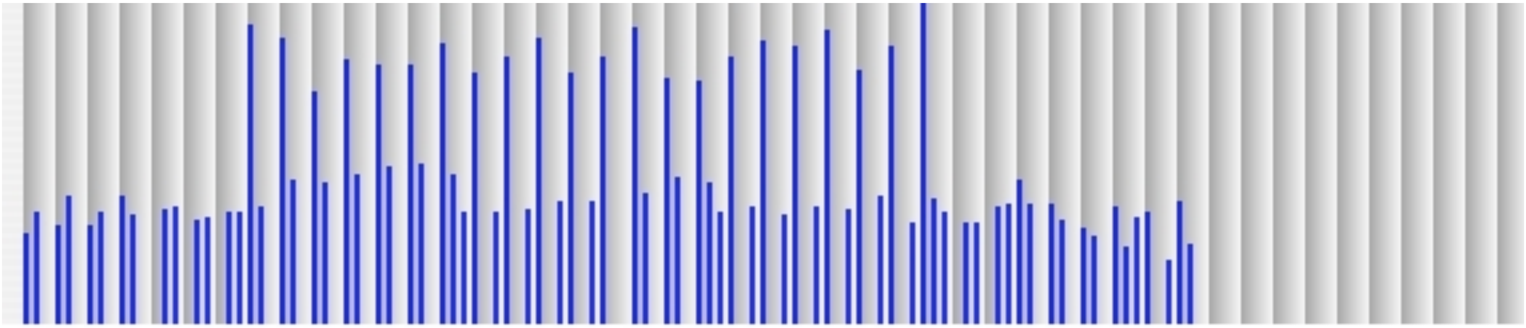
Meter in Music as a Dynamic Process



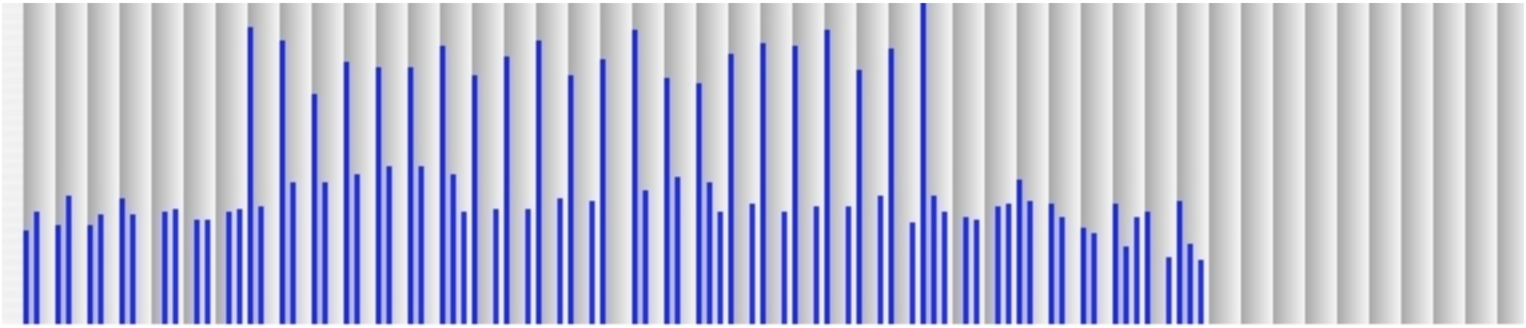
Meter in Music as a Dynamic Process



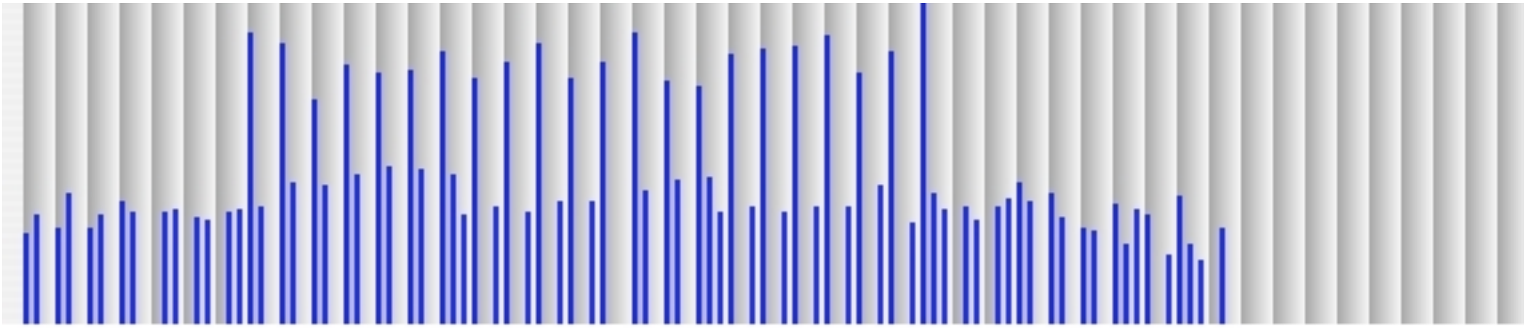
Meter in Music as a Dynamic Process



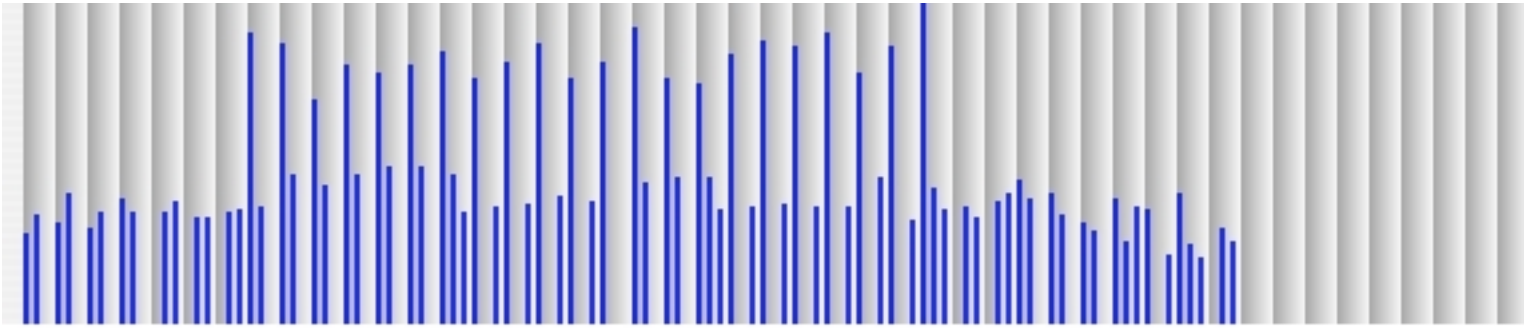
Meter in Music as a Dynamic Process



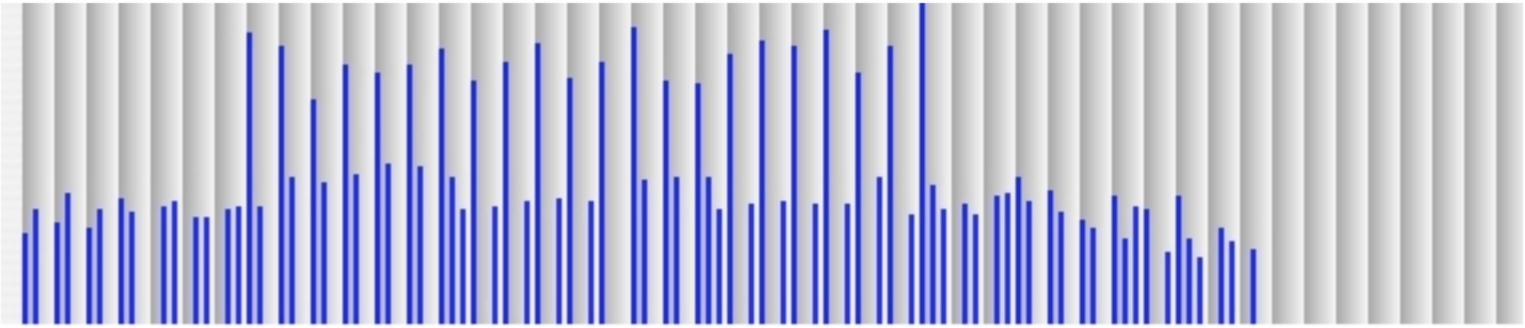
Meter in Music as a Dynamic Process



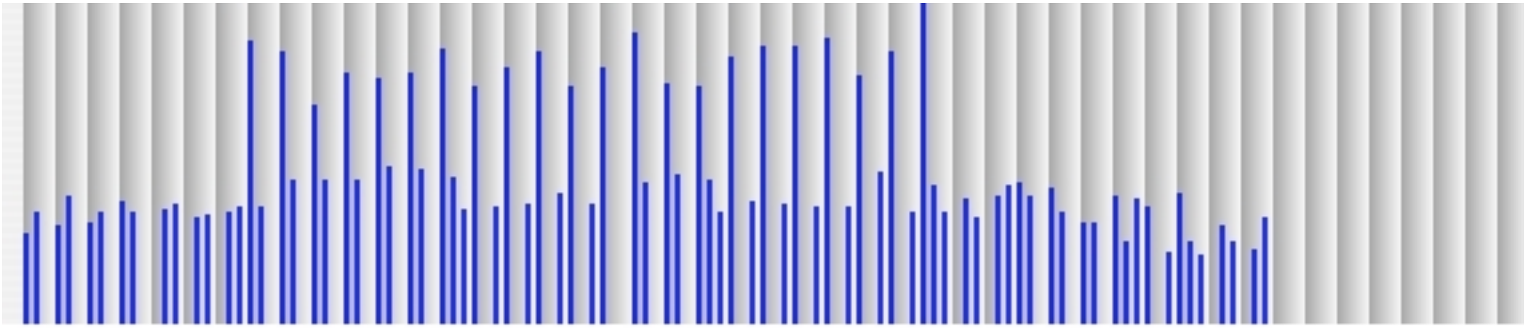
Meter in Music as a Dynamic Process



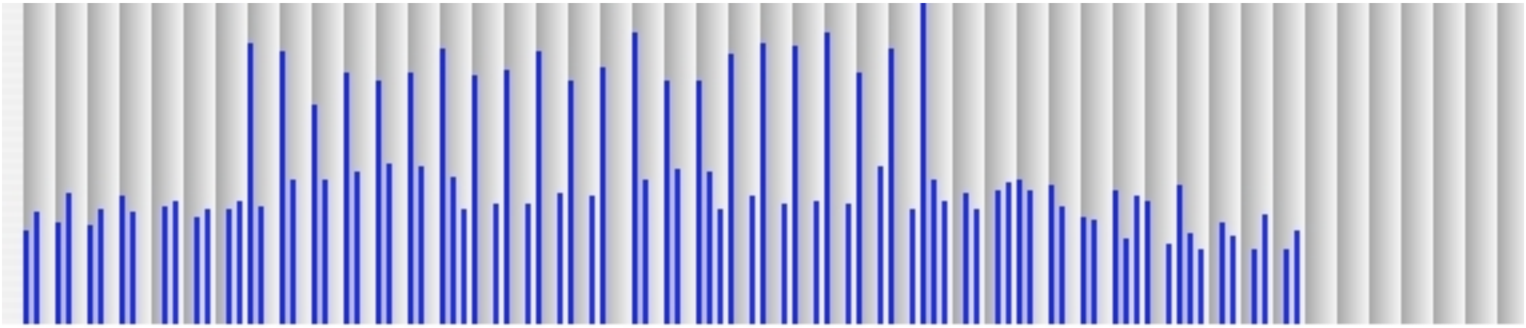
Meter in Music as a Dynamic Process



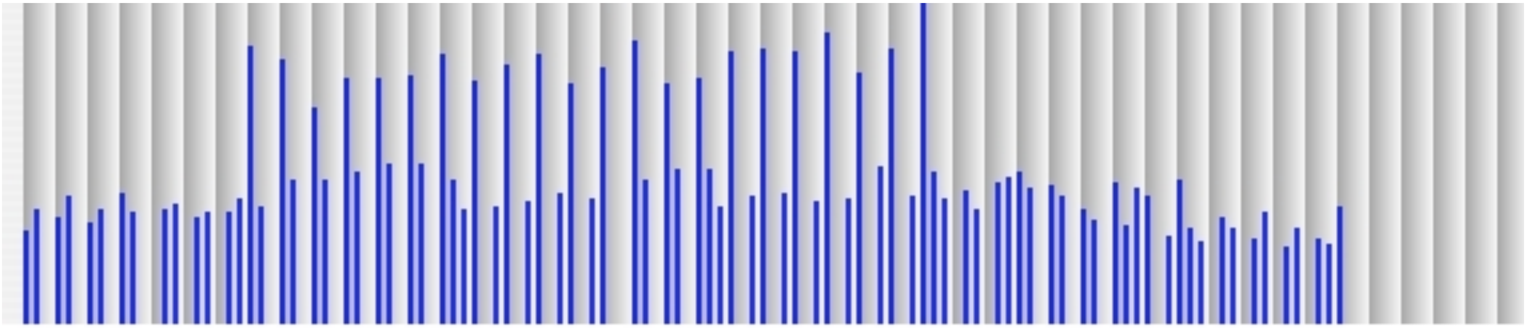
Meter in Music as a Dynamic Process



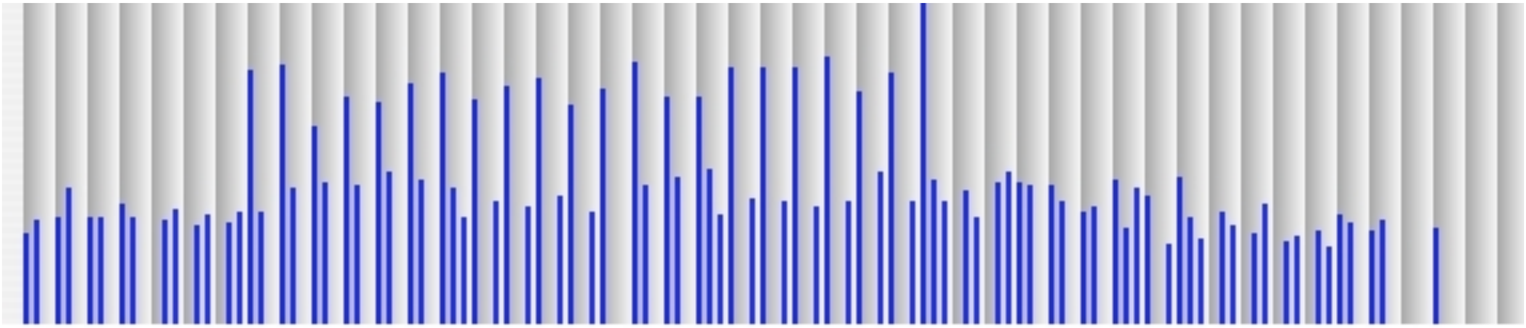
Meter in Music as a Dynamic Process



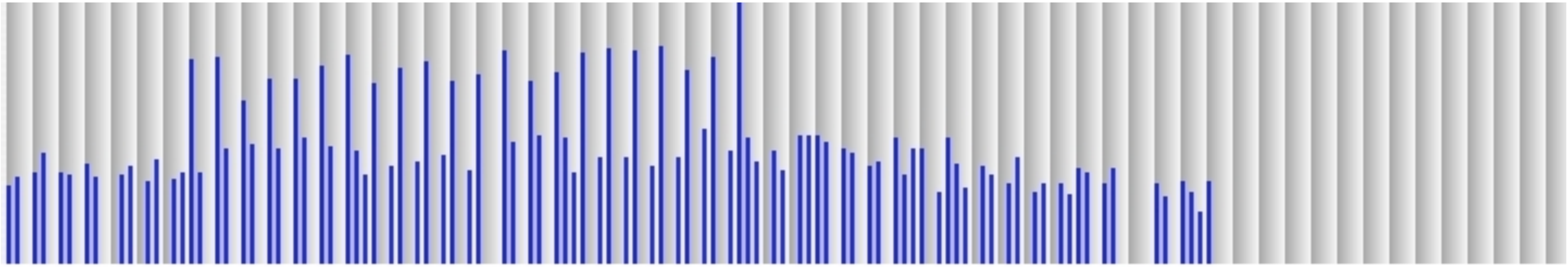
Meter in Music as a Dynamic Process



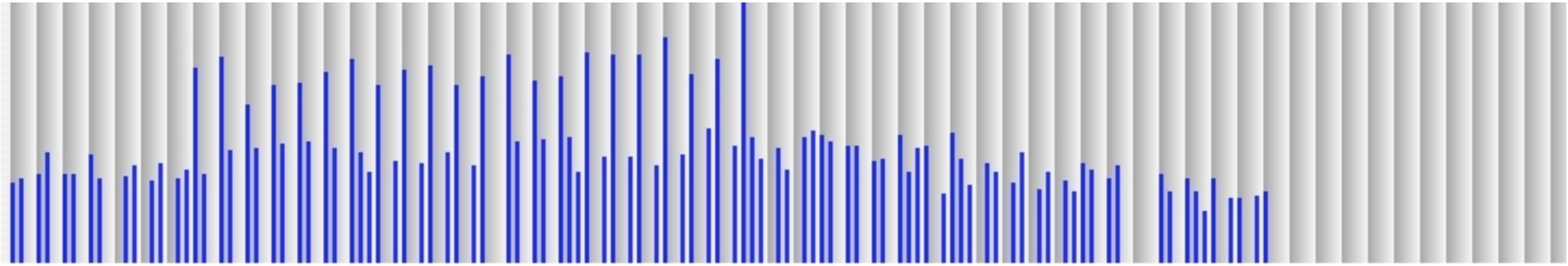
Meter in Music as a Dynamic Process



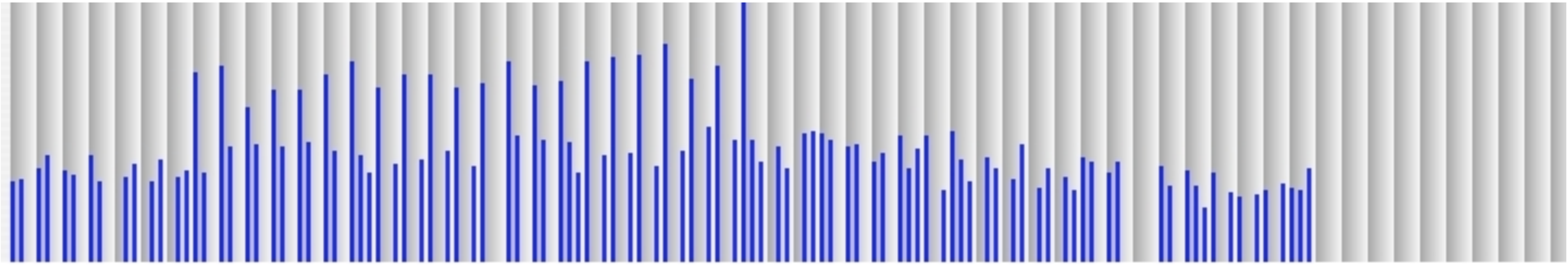
Meter in Music as a Dynamic Process



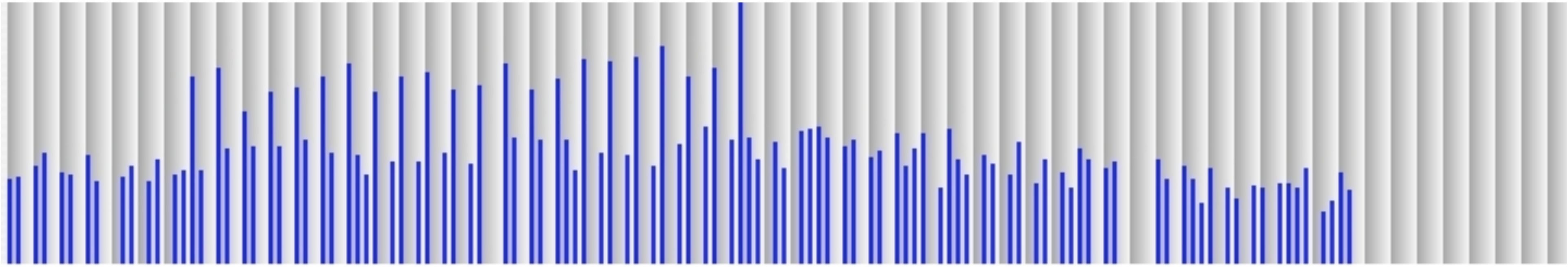
Meter in Music as a Dynamic Process



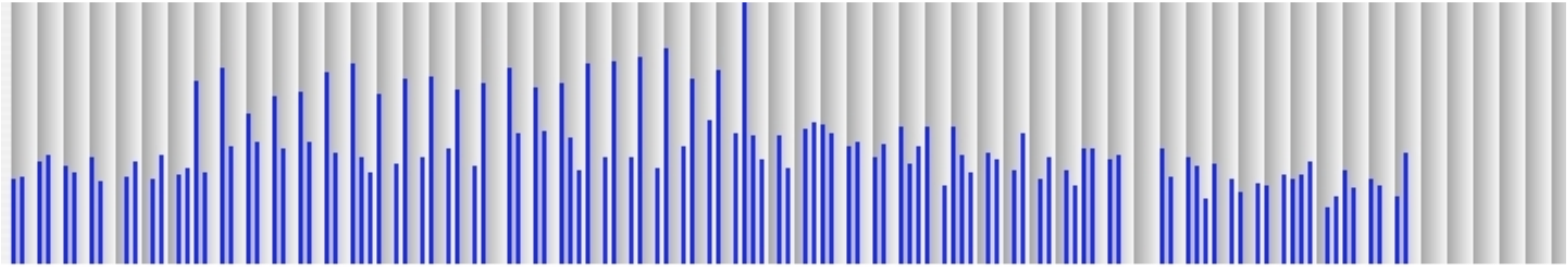
Meter in Music as a Dynamic Process



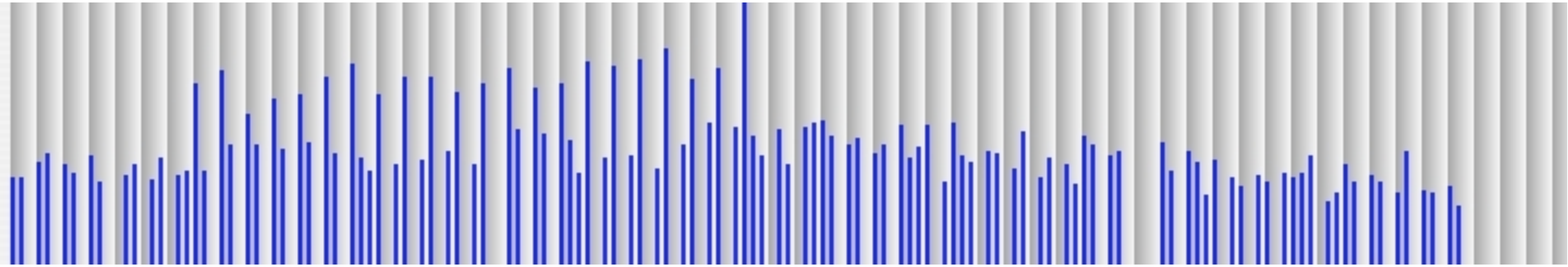
Meter in Music as a Dynamic Process



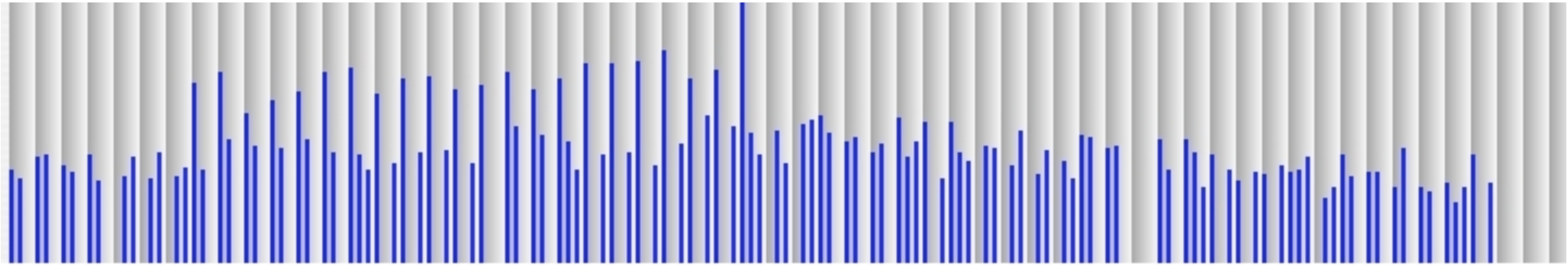
Meter in Music as a Dynamic Process



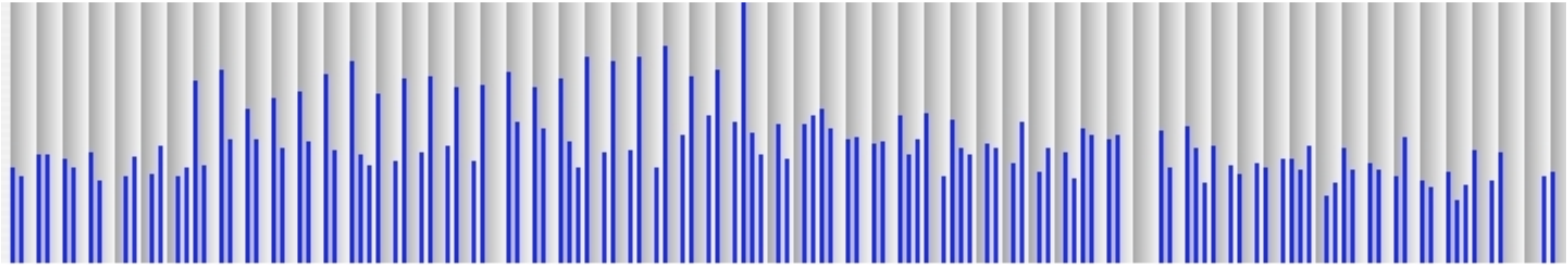
Meter in Music as a Dynamic Process



Meter in Music as a Dynamic Process



Meter in Music as a Dynamic Process

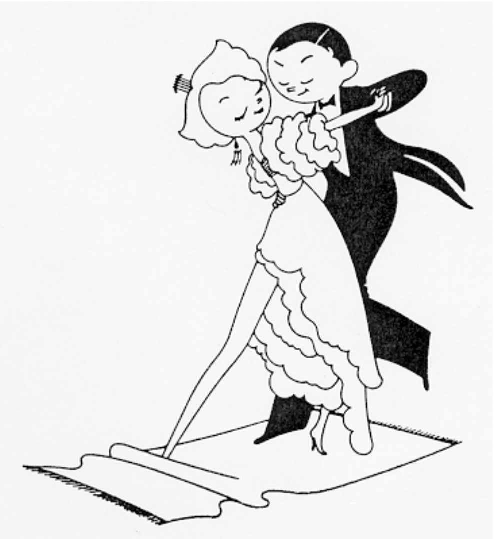


Outline

- Introduction
    - Rhythm
    - Importance of Rhythm
- Inner Metric Analysis
    - Mathematical Model for Analyzing the Rhythm
    - Some Applications
- Dance Music Classification, Music Information Retrieval

Classification

Genre classification: Latin American Dances in 4/4



Picture from hcs.harvard.edu/~hma-bdc

How to distinguish between dances of same Tempo and Meter?

Classification

Picture from www.ritzenhoff.de

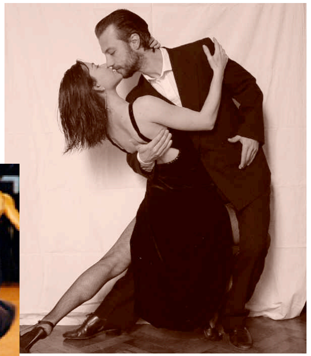
Classification

	B2	R3	B1	T1	R1	M2	M1	T2	R2	A
metric wt	0.62	0.55	0.49	0.41	0.36	0.34	0.34	0.31	0.31	0.26
spectral wt	0.62	0.61	0.44	0.33	0.37	0.45	0.54	0.31	0.31	0.30

Rumba

Picture from www.danceuniverse.co.kr

Tango



icture from www.accomodationbsas.co.ar

BossaNova

Picture from www.ritzenhoff.de

Merengue

Picture from www.mondolatino.it

March/Anthem

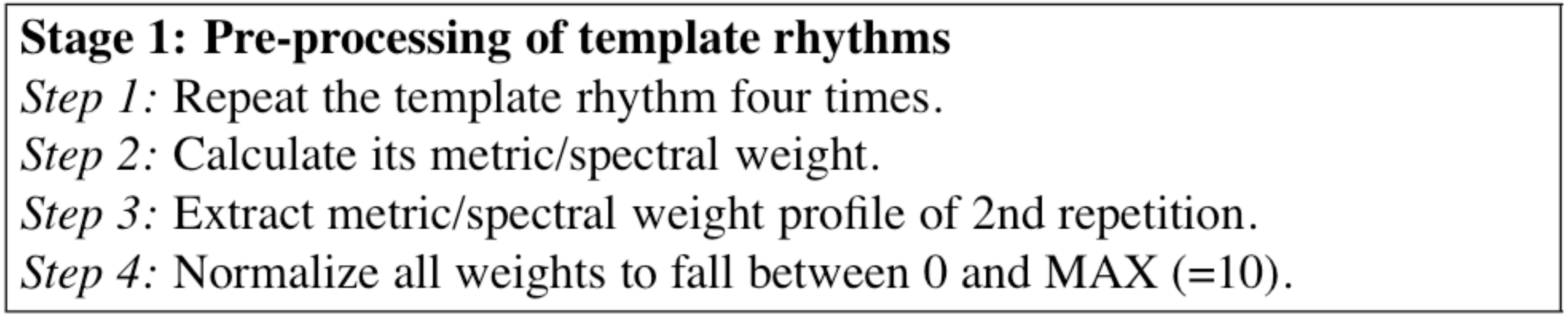
Picture from www.armygermany.com

Classification

CLASSIFICATION METHOD

STEP 1: Generate rhythm templates

- Complete Book of the Worlds’ Dance Rhythm



Stage 1: Pre-processing of template rhythms

Step 1: Repeat the template rhythm four times.

Step 2: Calculate its metric/spectral weight.

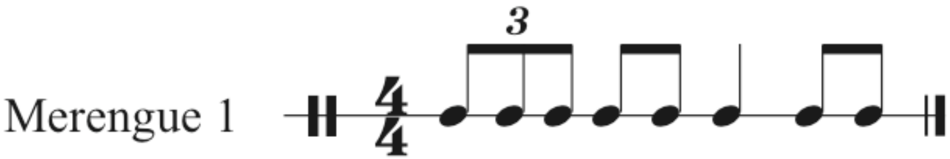
Step 3: Extract metric/spectral weight profile of 2nd repetition.

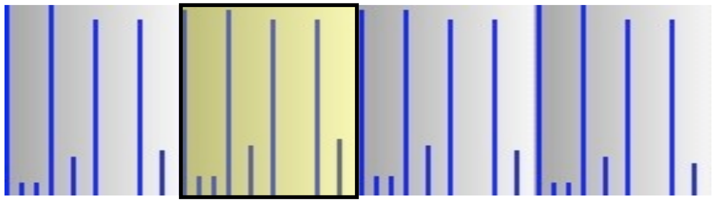
Step 4: Normalize all weights to fall between 0 and MAX (=10).

Classification

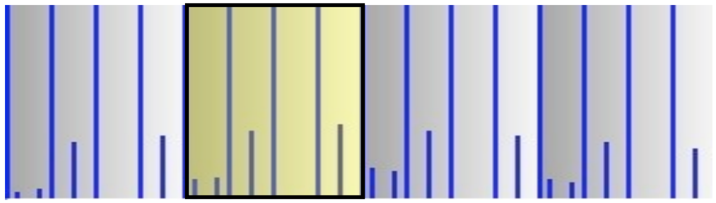
Picture from www.mondolatino.it

Example Template: Merengue


Merengue 1
3
4
4



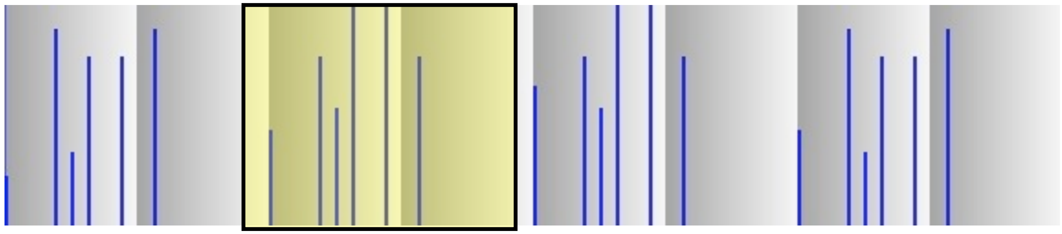
Metric Weight



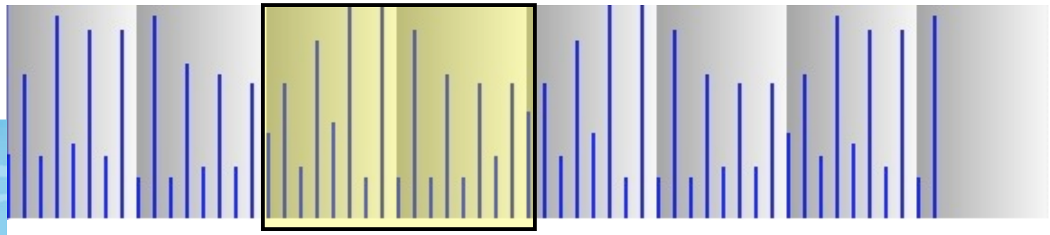
Spectral Weight

Classification

Example: Girl from Ipanema



Metric Weight



Spectral Weight

Picture from www.ritzenhoff.de

Classification

	B2	R3	B1	T1	R1	M2	M1	T2	R2	A
metric wt	0.62	0.55	0.49	0.41	0.36	0.34	0.34	0.31	0.31	0.26
spectral wt	0.62	0.61	0.44	0.33	0.37	0.45	0.54	0.31	0.31	0.30

Rumba

Picture from www.danceuniverse.co.kr

Tango

icture from www.accomodationbsas.co.ar

BossaNova

Picture from www.ritzenhoff.de

Merengue

Picture from www.mondolatino.it

March/Anthem

Picture from www.armygermany.com

correct genre: 80%

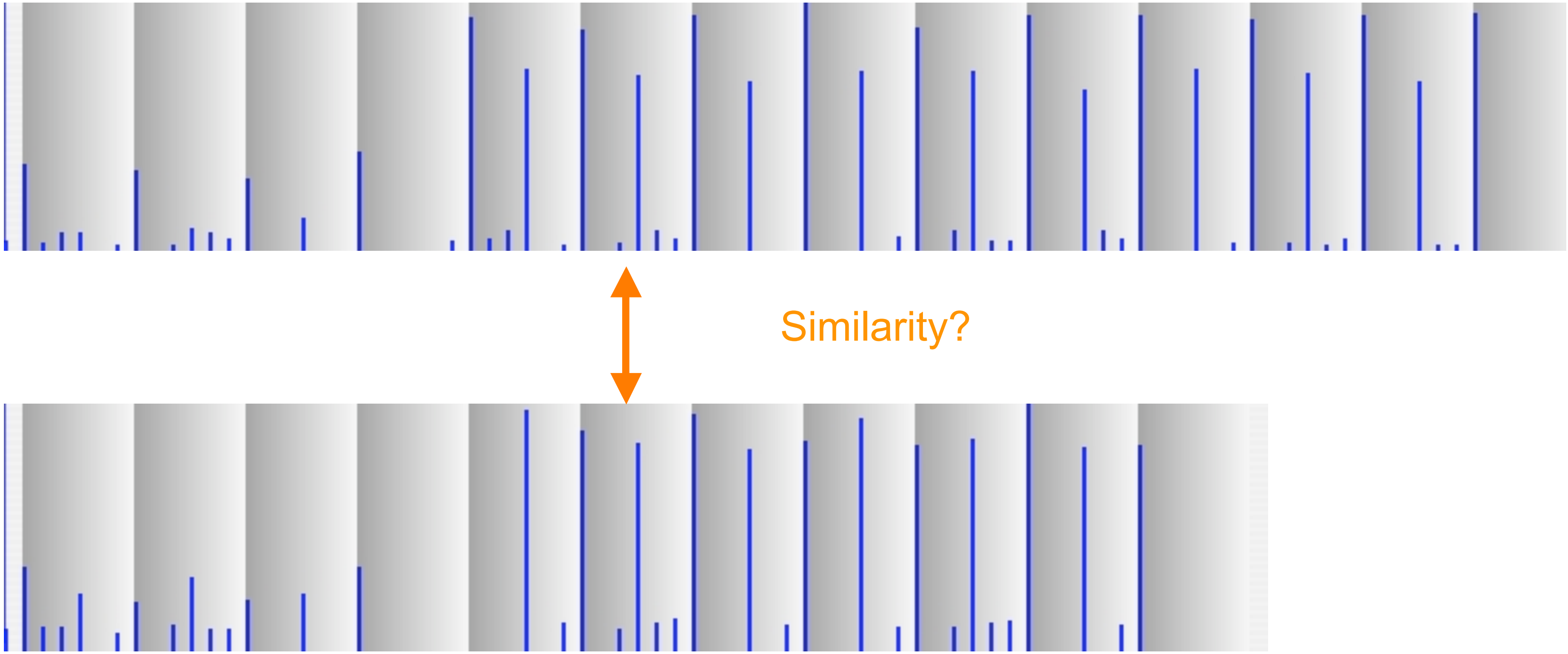
Rhythmic Similarity: Extension

What Is Topical in Cultural Heritage:
Content-based Retrieval Among Folk Tunes

WITCHCRAFT:

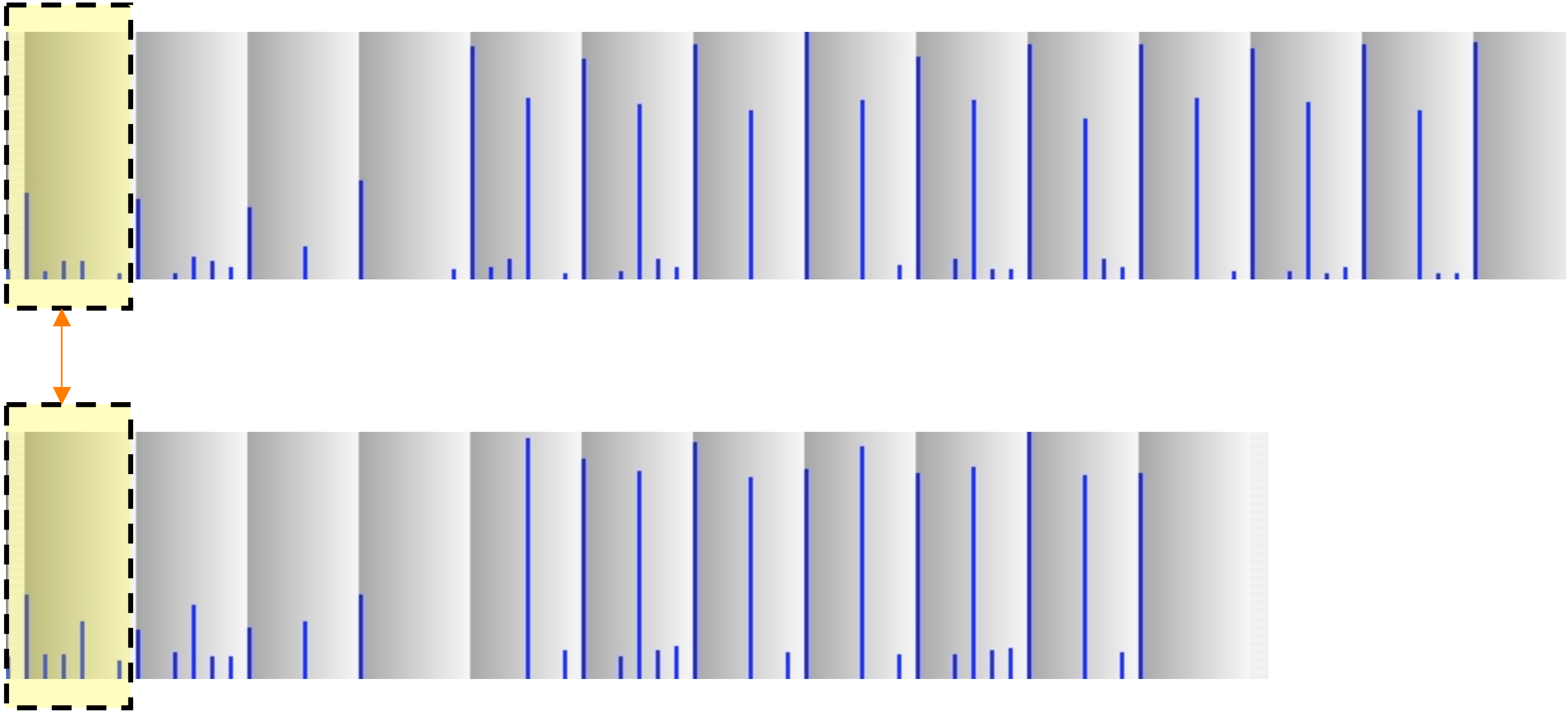
Retrieval of folk songs

Rhythmic Similarity: Extension



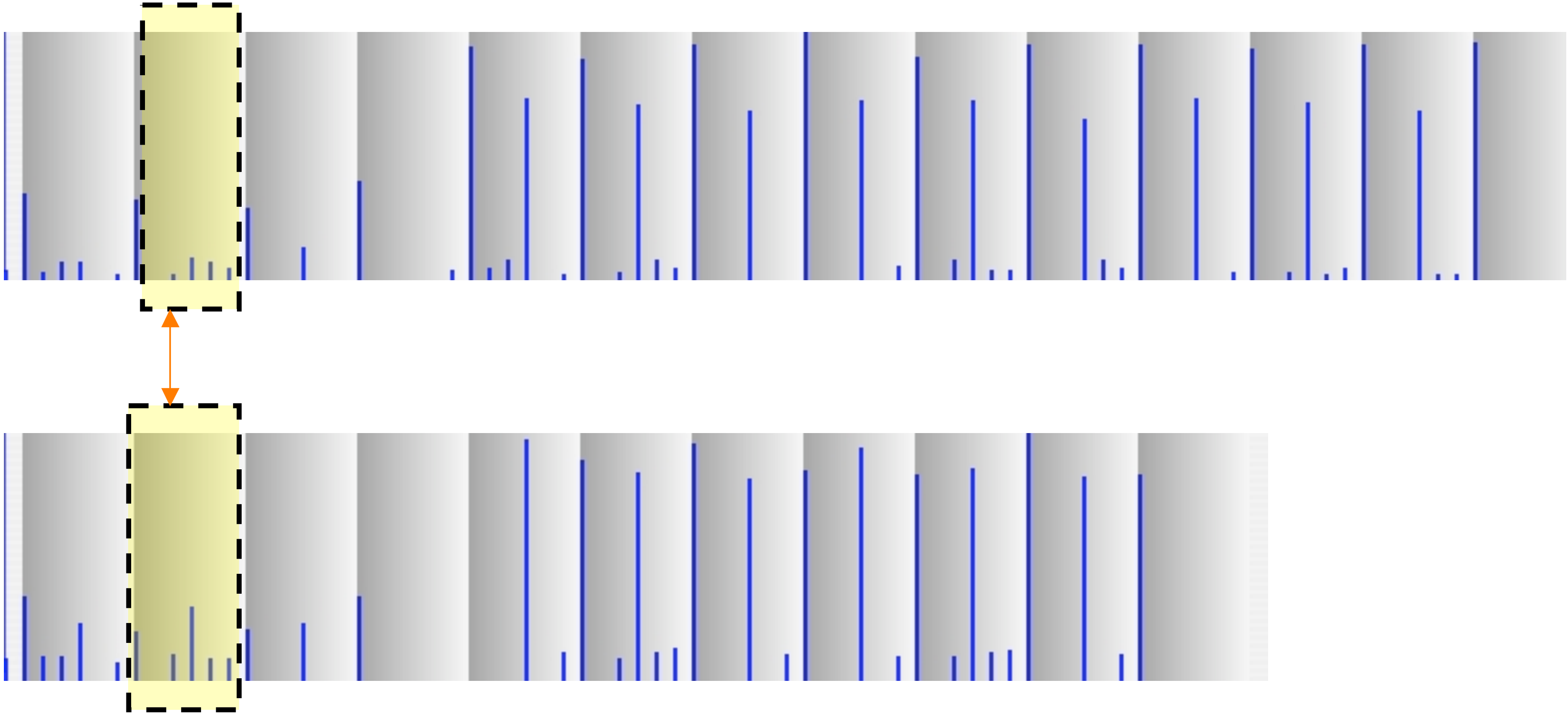
Similarity?

Rhythmic Similarity: Extension



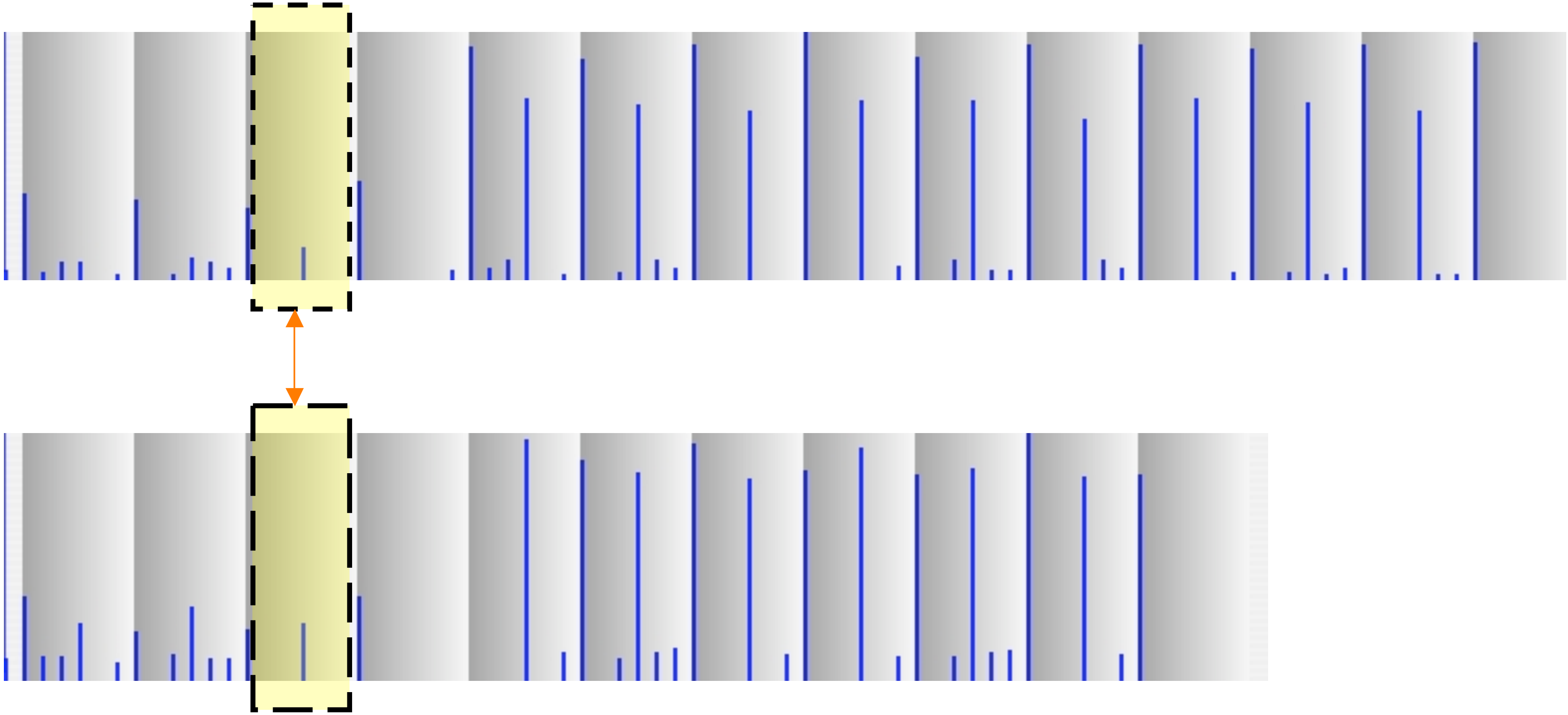
C$_1$

Rhythmic Similarity: Extension



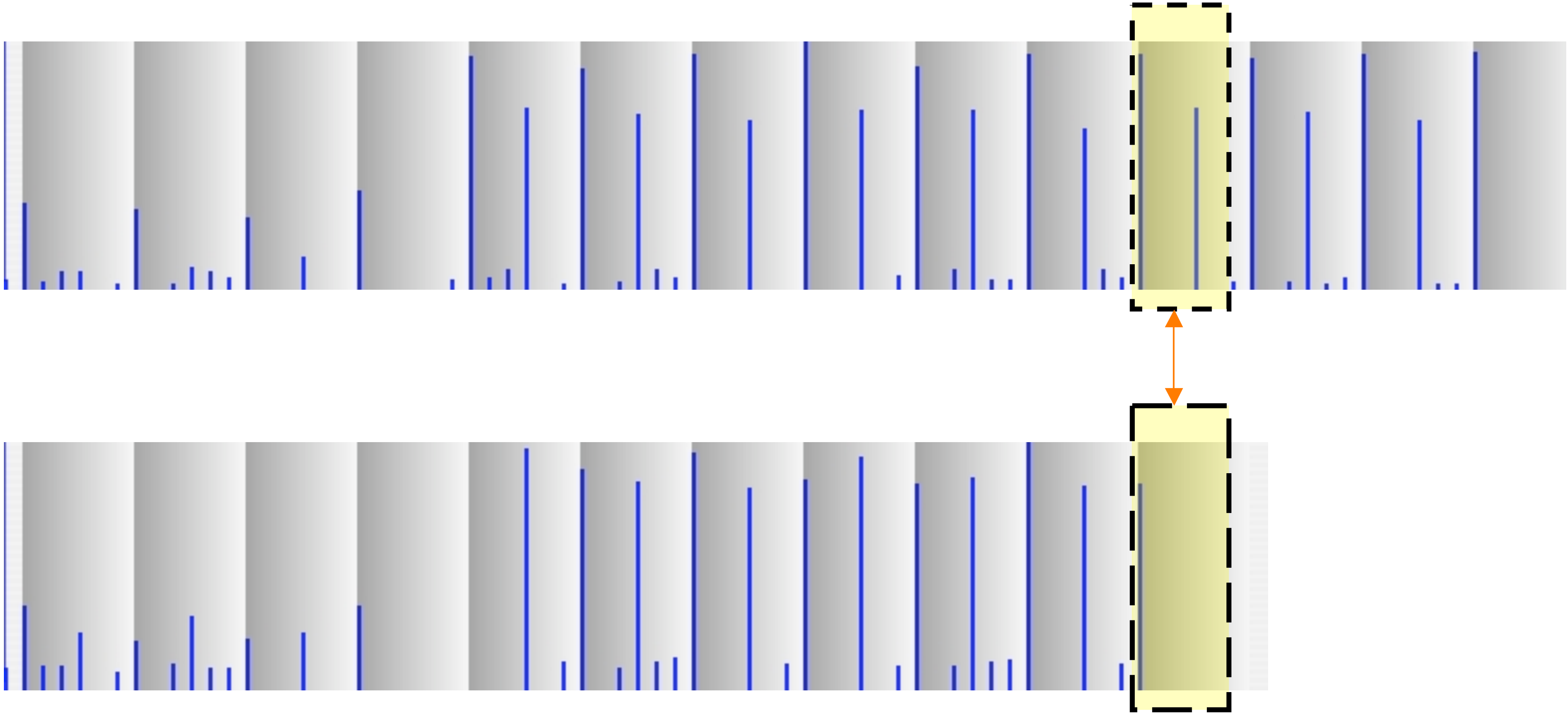
$C_1 + C_2$

Rhythmic Similarity: Extension



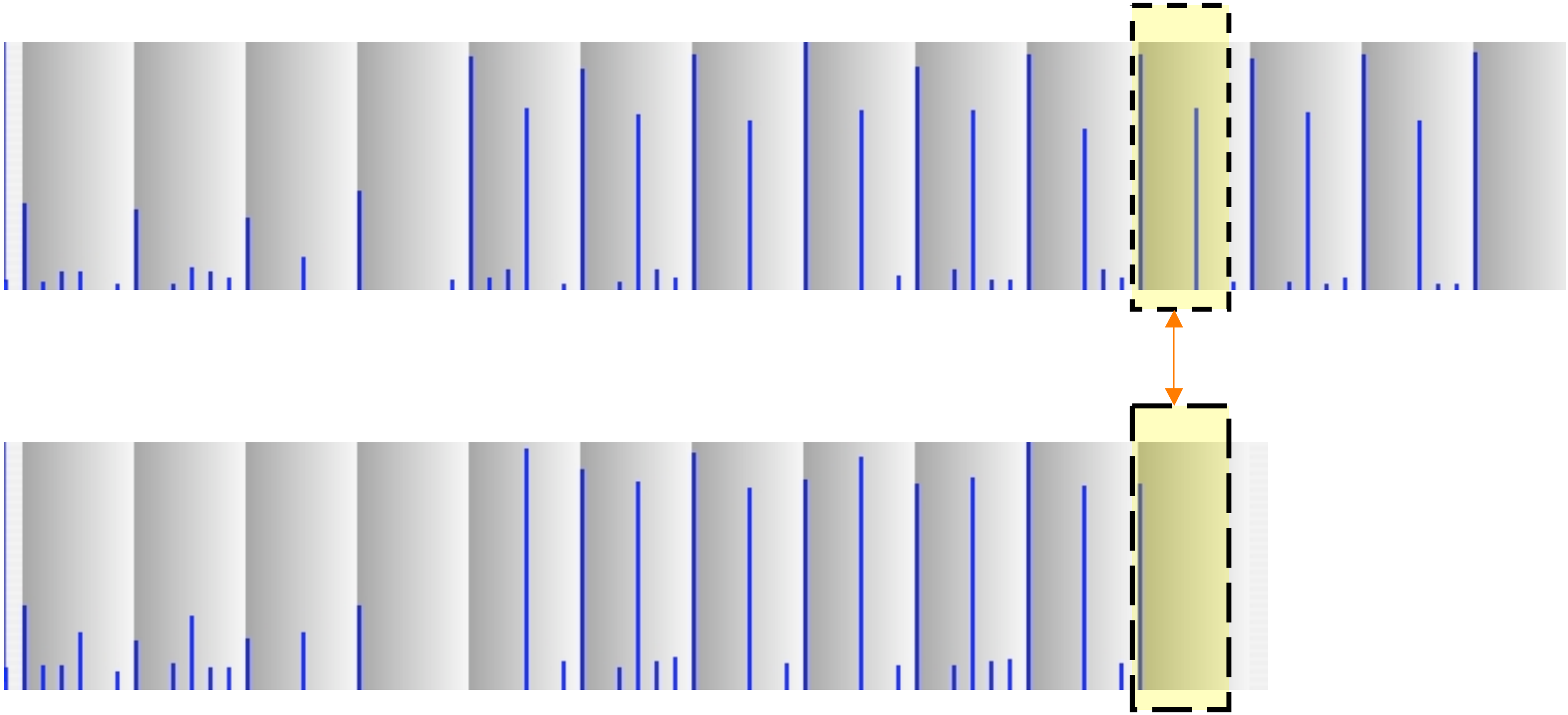
$C_1 + C_2 + C_3$

Rhythmic Similarity: Extension



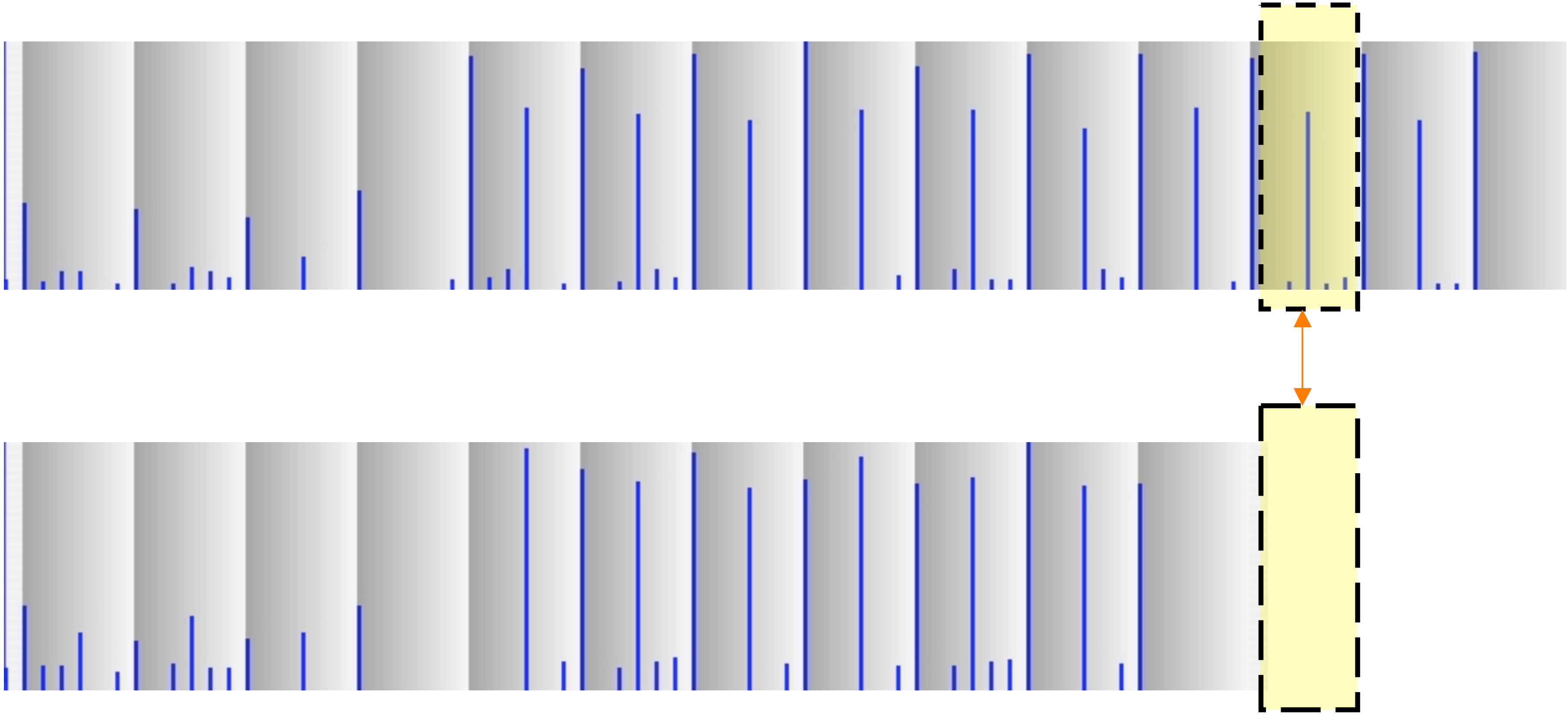
$C_1 + C_2 + C_3 + \ldots \qquad \ldots . \qquad \ldots + C_n$

Rhythmic Similarity: Extension



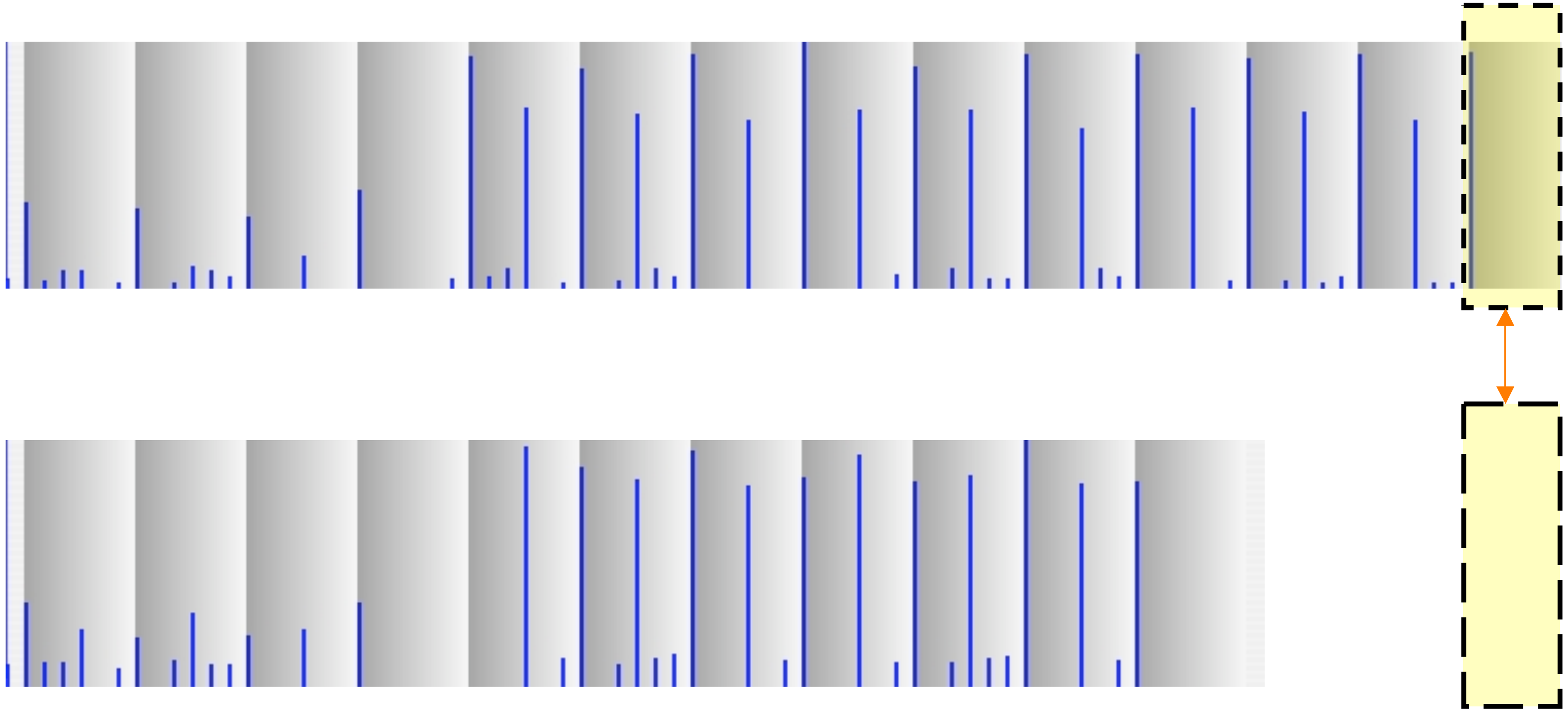
$(C_1 + C_2 + C_3 + \ldots \qquad \ldots . \qquad \ldots + C_n)/n$

Rhythmic Similarity: Extension



$(C_1 + C_2 + C_3 + \ldots \quad \ldots \quad \ldots + C_n)/n$

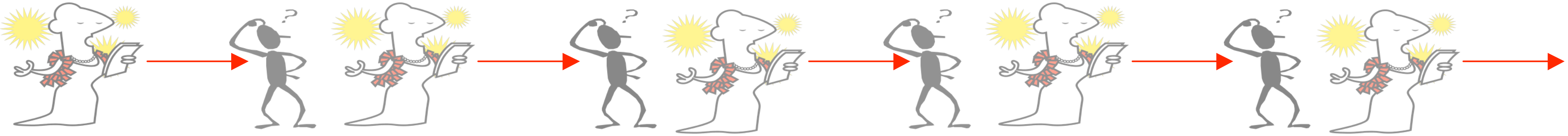
Rhythmic Similarity: Extension



$(C_1 + C_2 + C_3 + \ldots \qquad \ldots . \qquad \ldots \qquad + C_m)/m$

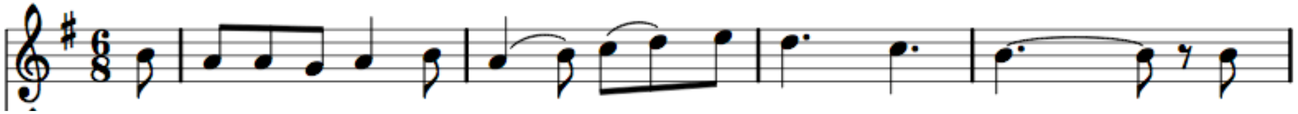
Application

Dutch Folk song collection: Onder de groene linde

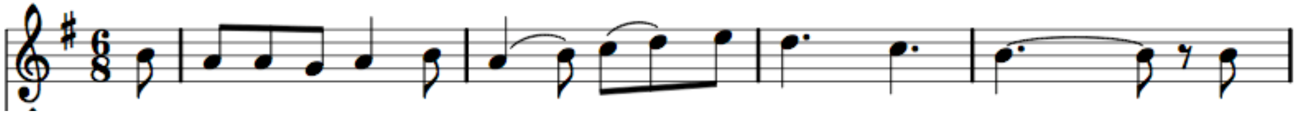


Oral Tradition

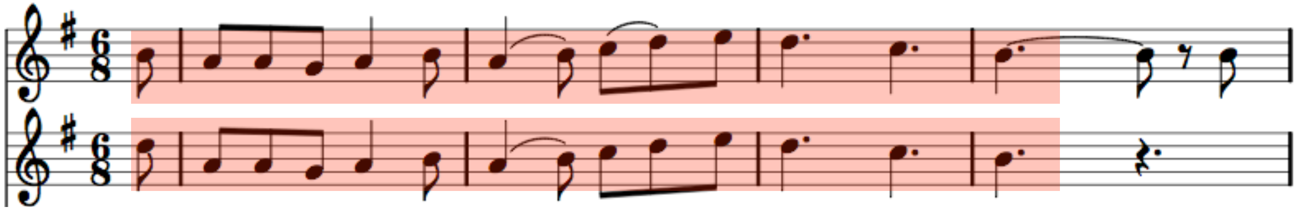
Melody Group



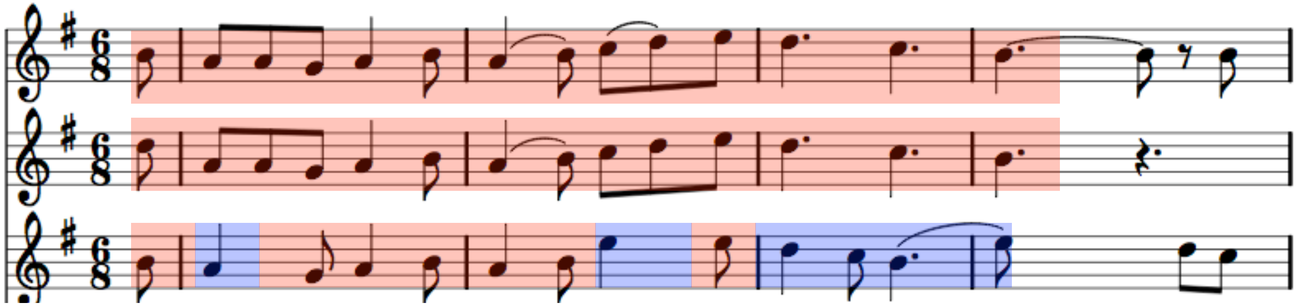
Melody Group



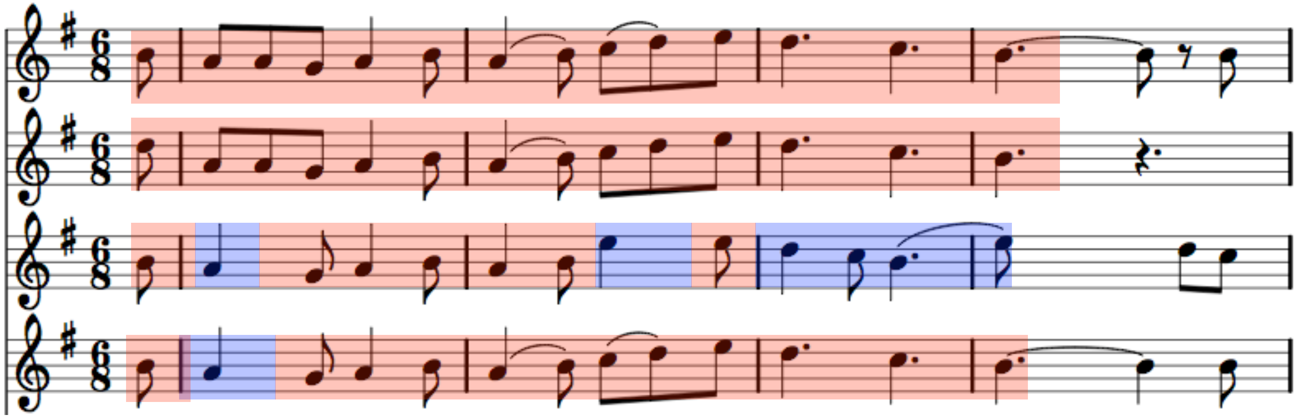
Melody Group



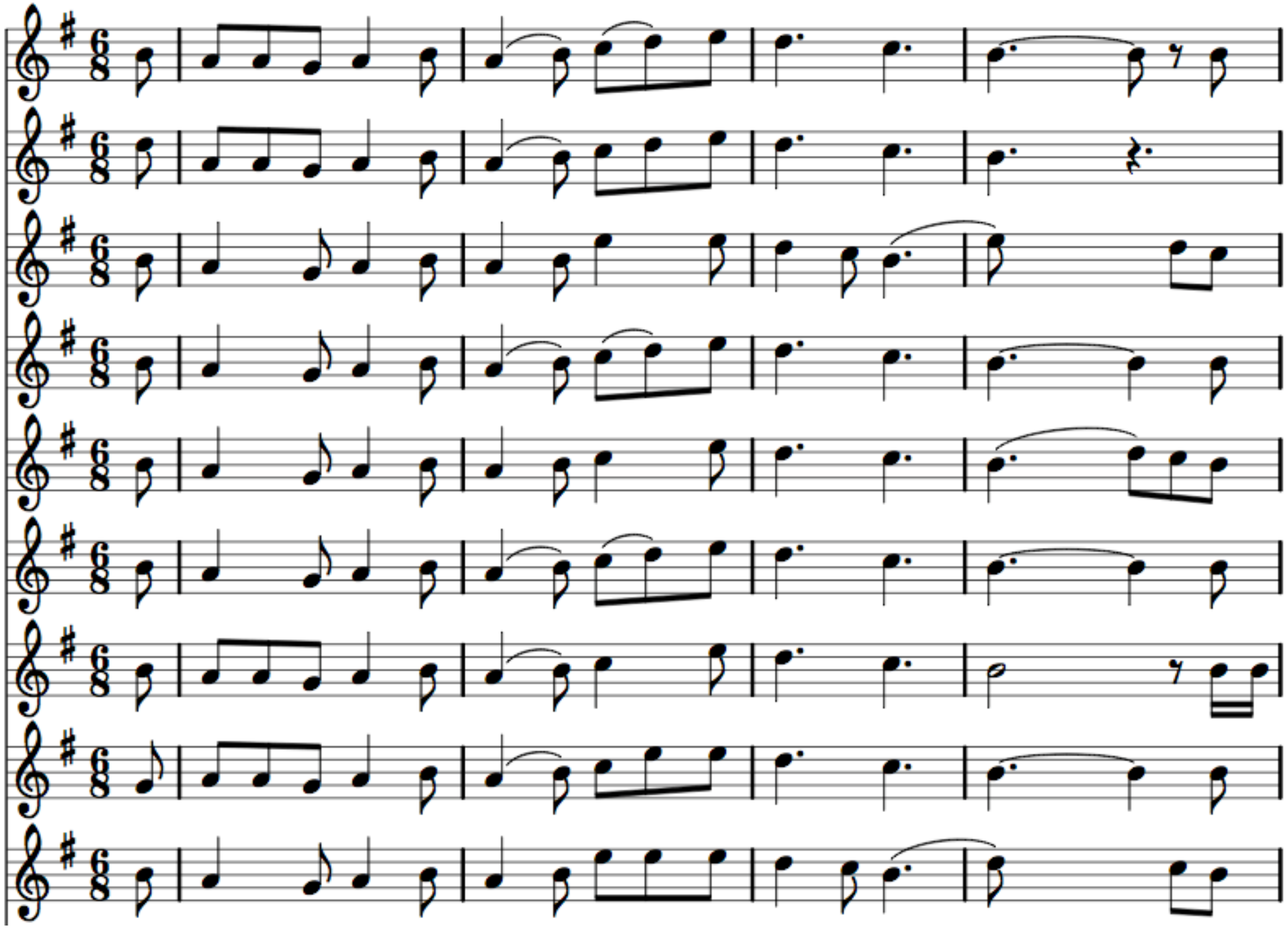
Melody Group



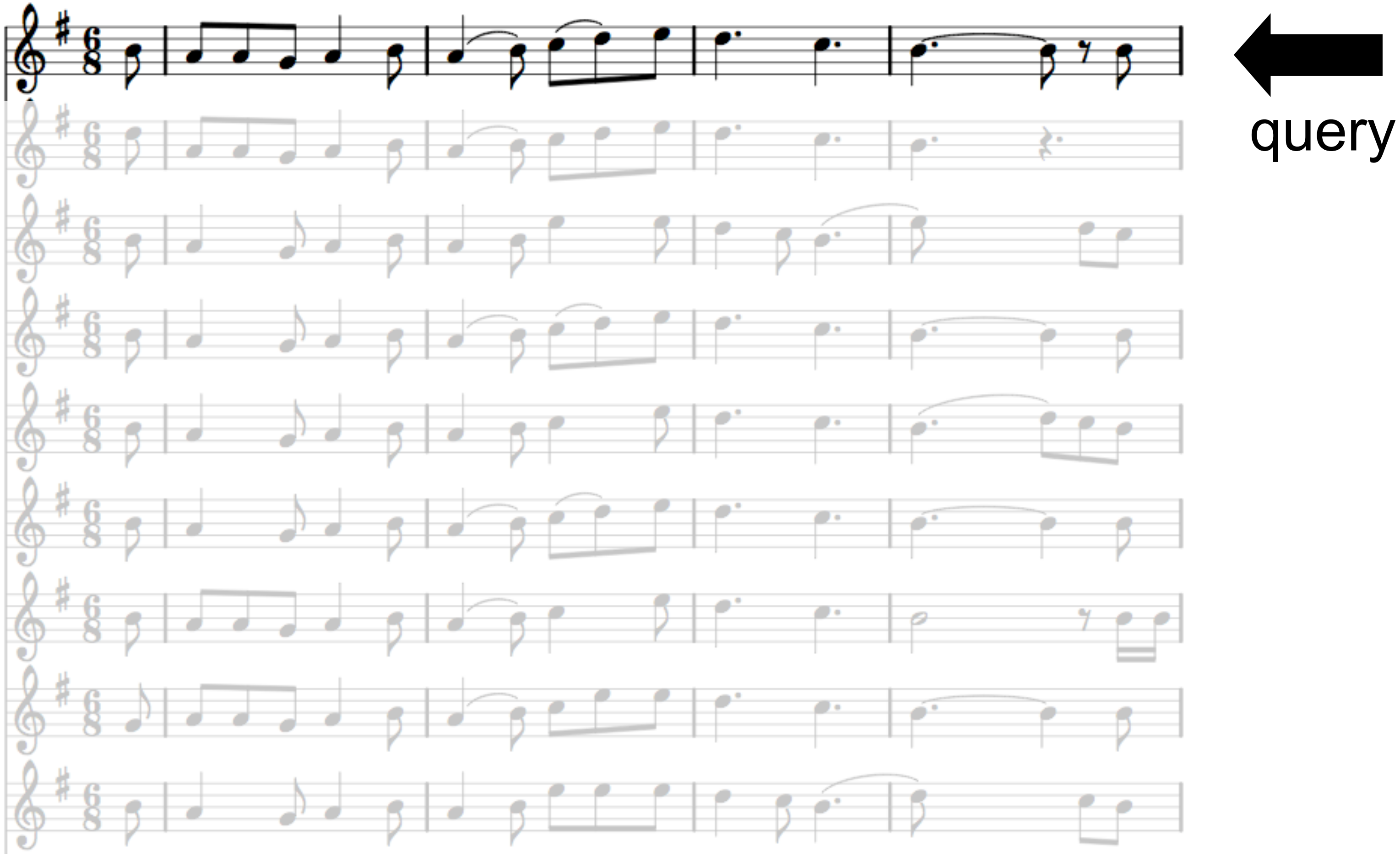
Melody Group



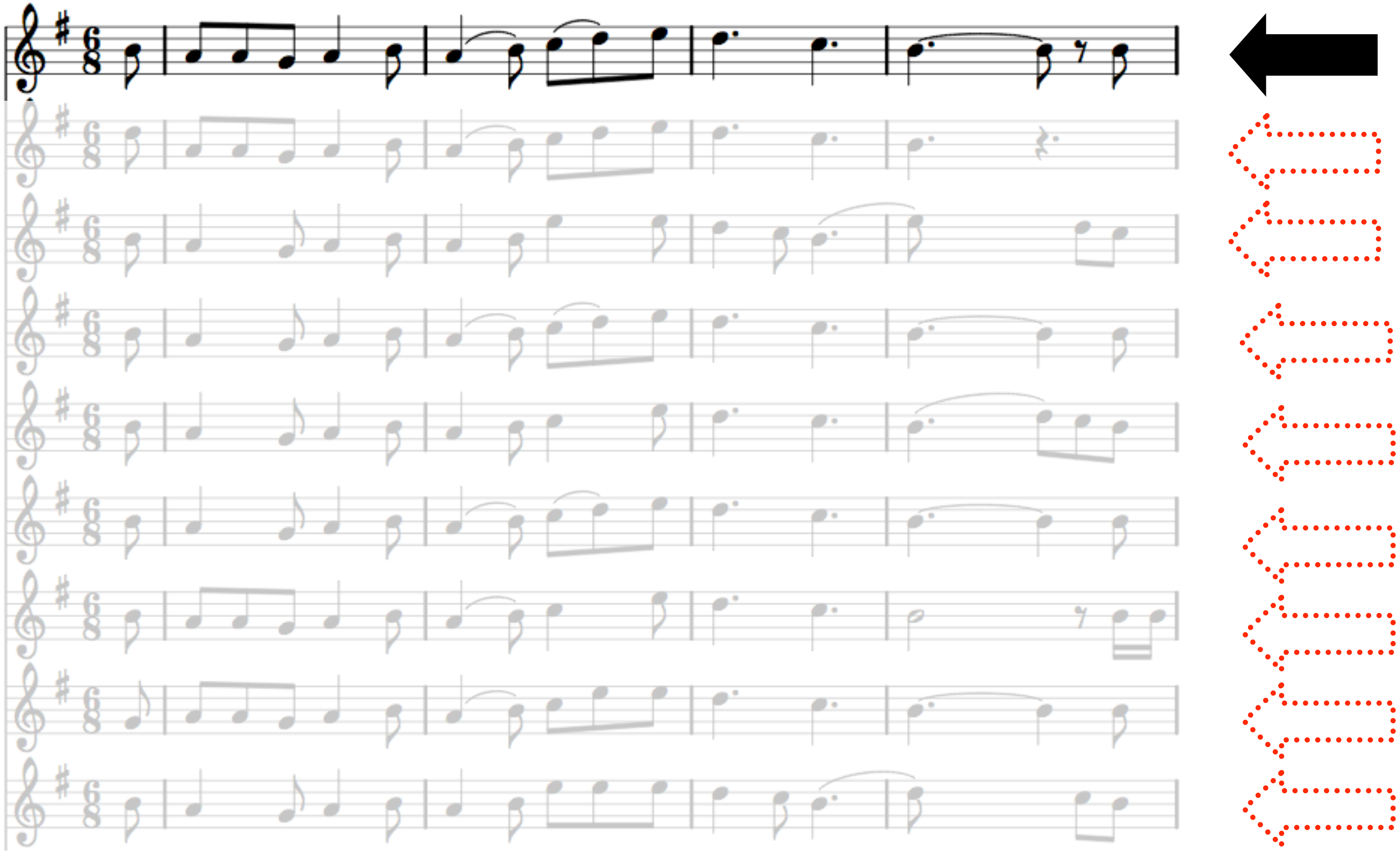
Melody Group



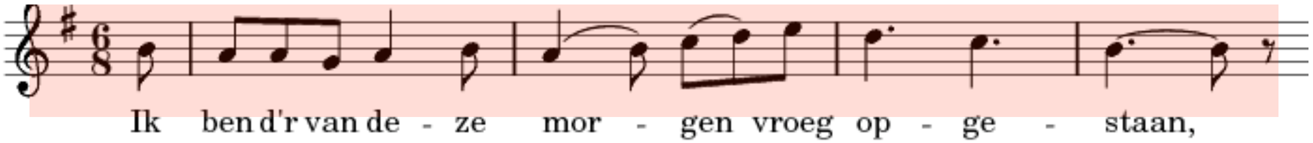
Melody Group



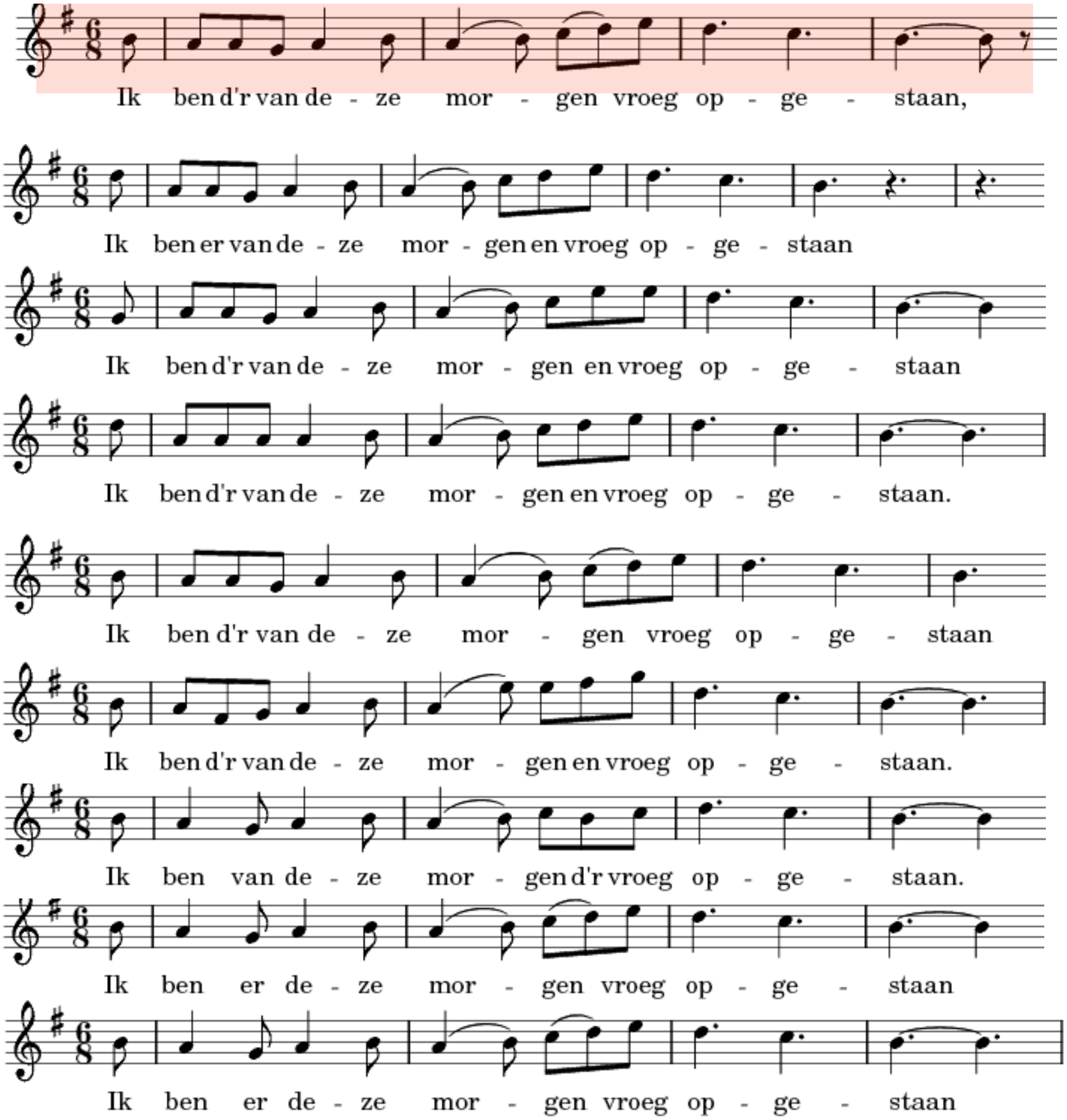
Melody Group

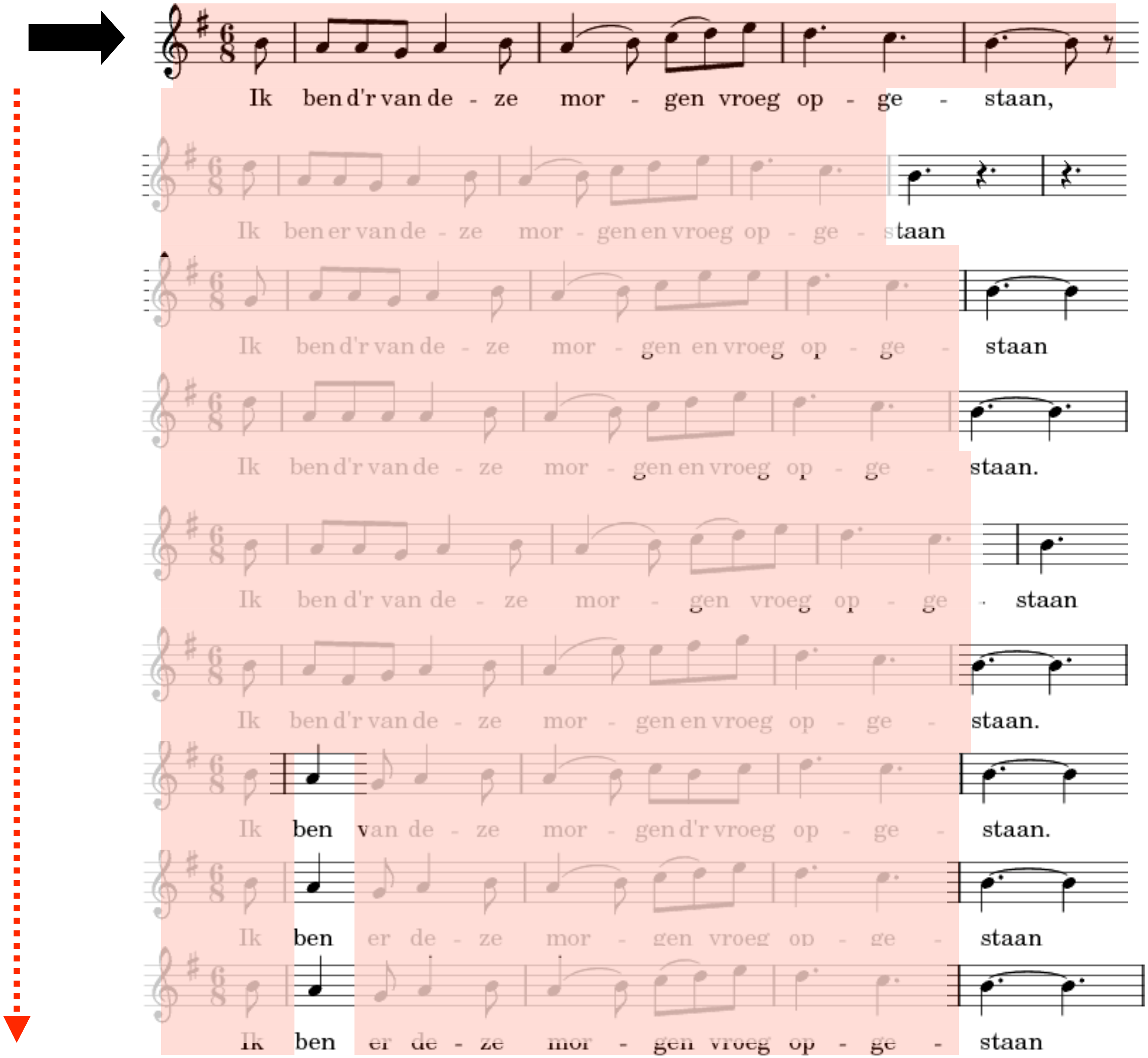


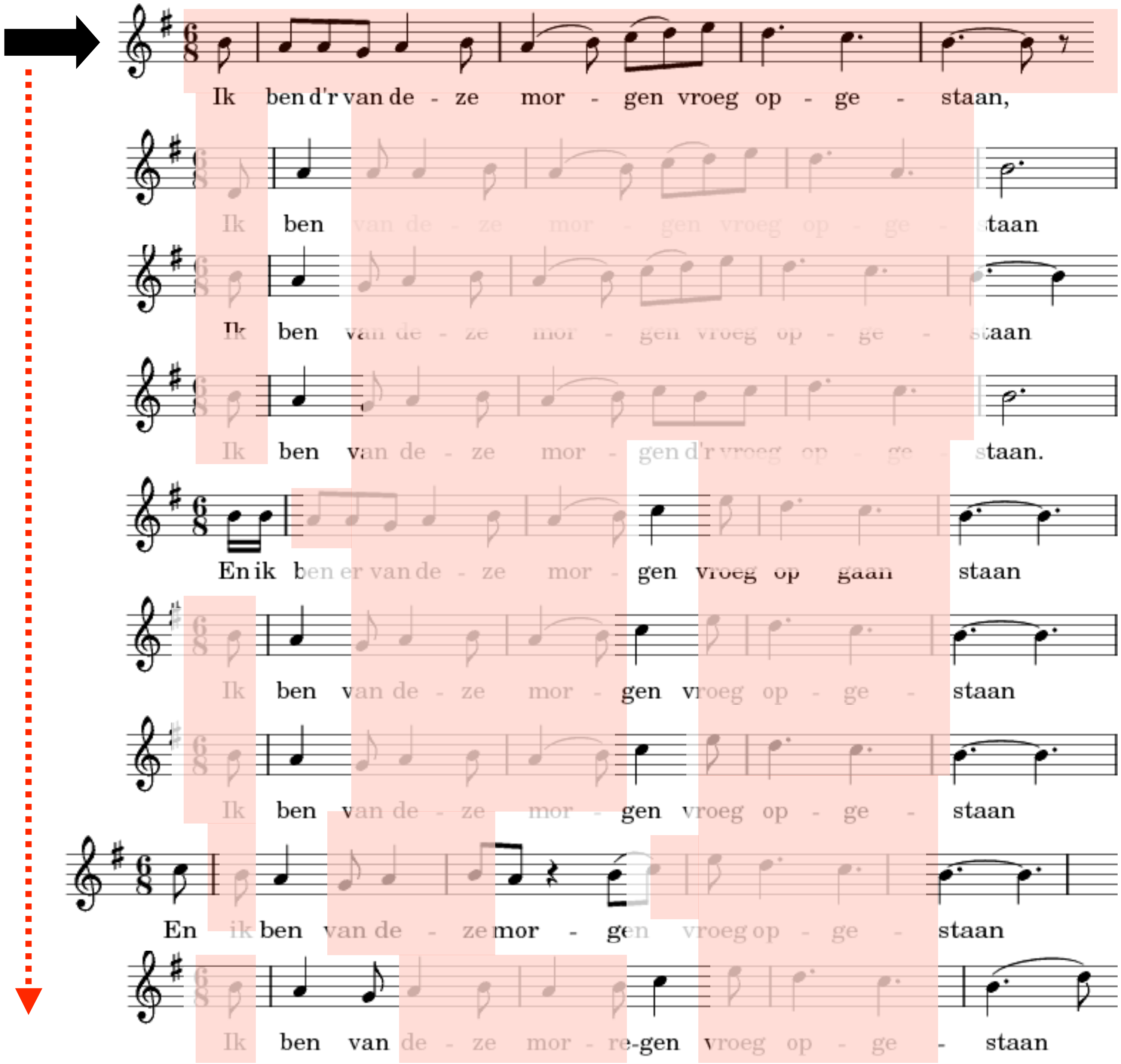
IMA →

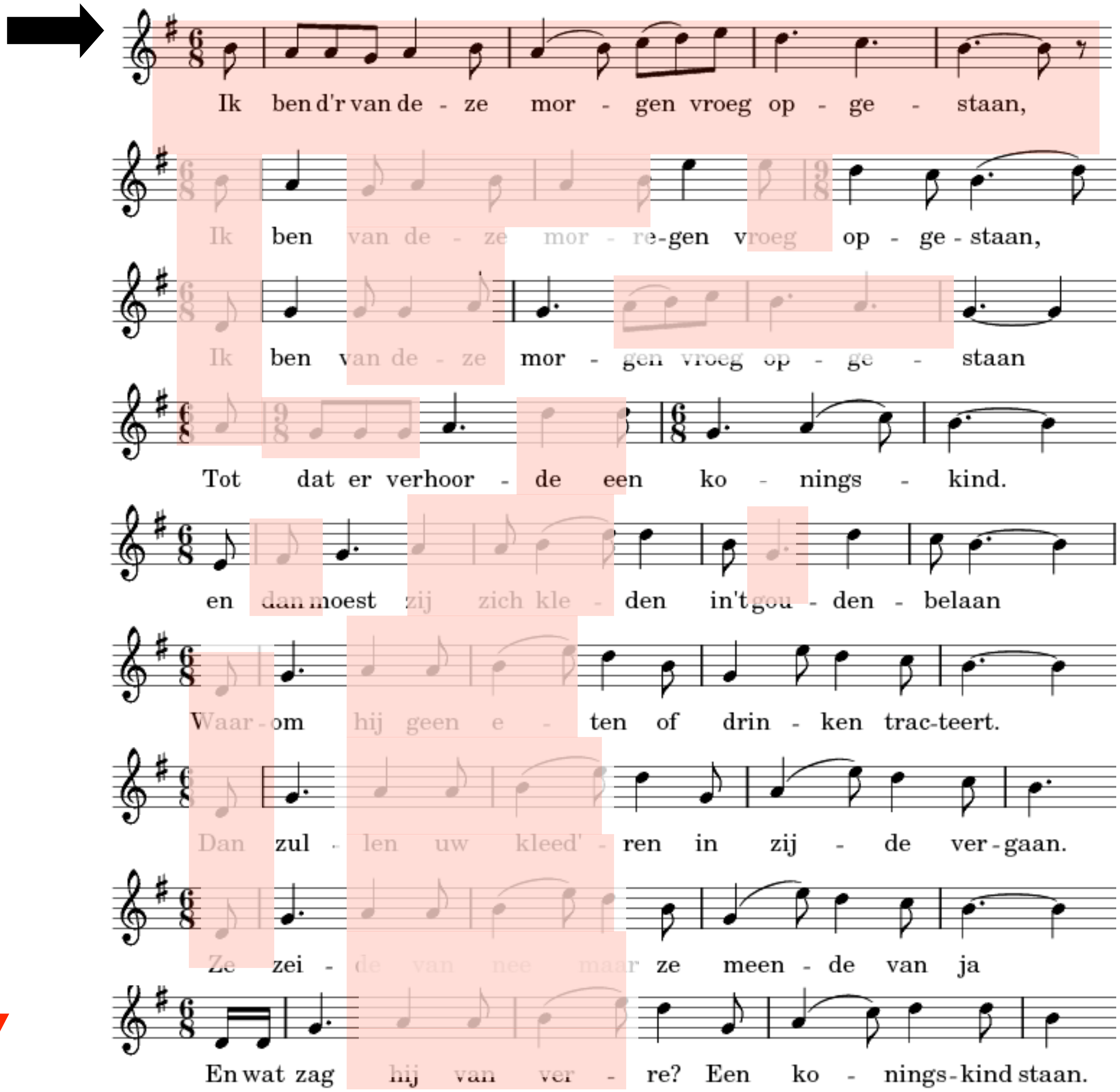

Ik ben d'r van de - ze mor - gen vroeg op - ge - staan,

IMA


Ik ben d'r van de - ze mor - gen vroeg op - ge - staan,
Ik ben er van de - ze mor - gen en vroeg op - ge - staan
Ik ben d'r van de - ze mor - gen en vroeg op - ge - staan
Ik ben d'r van de - ze mor - gen en vroeg op - ge - staan.
Ik ben d'r van de - ze mor - gen vroeg op - ge - staan
Ik ben d'r van de - ze mor - gen en vroeg op - ge - staan.
Ik ben van de - ze mor - gen d'r vroeg op - ge - staan.
Ik ben er de - ze mor - gen vroeg op - ge - staan
Ik ben er de - ze mor - gen vroeg op - ge - staan


Ik ben d'r van de - ze mor - gen vroeg op - ge - staan,
Ik ben er van de - ze mor - gen en vroeg op - ge - staan
Ik ben d'r van de - ze mor - gen en vroeg op - ge - staan
Ik ben d'r van de - ze mor - gen en vroeg op - ge - staan.
Ik ben d'r van de - ze mor - gen vroeg op - ge - staan
Ik ben d'r van de - ze mor - gen en vroeg op - ge - staan.
Ik ben van de - ze mor - gen d'r vroeg op - ge - staan.
Ik ben er de - ze mor - gen vroeg op - ge - staan
Ik ben er de - ze mor - gen vroeg op - ge - staan


Ik ben d'r van de - ze mor - gen vroeg op - ge - staan,
Ik ben van de - ze mor - gen vroeg op - ge - staan
Ik ben van de - ze mor - gen vroeg op - ge - staan
Ik ben van de - ze mor - gen d'r vroeg op - ge - staan.
En ik ben er van de - ze mor - gen vroeg op gaan staan
Ik ben van de - ze mor - gen vroeg op - ge - staan
Ik ben van de - ze mor - gen vroeg op - ge - staan
En ik ben van de - ze mor - gen vroeg op - ge - staan
Ik ben van de - ze mor - re-gen vroeg op - ge - staan


Ik ben d'r van de - ze mor - gen vroeg op - ge - staan,
Ik ben van de - ze mor - re-gen vroeg op - ge - staan,
Ik ben van de - ze mor - gen vroeg op - ge - staan
Tot dat er verhoor - de een ko - nings - kind.
en dan moest zij zich kle - den in't gou - den - belaan
Waar - om hij geen e - ten of drin - ken trac-teert.
Dan zul - len uw kleed' - ren in zij - de ver - gaan.
Ze zei - de van nee maar ze meen - de van ja
En wat zag hij van ver - re? Een ko - nings - kind staan.

Music Information Retrieval

- Find similar melodies based on their rhythmic structure!

More Information

- http://people.cs.uu.nl/volk

- Download JMetro-Tool:

http://pierement.zoo.cs.uu.nl/joerg/MaMuTh/Metro/

- Manual: email volk@cs.uu.nl