From Exotic Options to Exotic Underlyings: Electricity, Weather and Catastrophe Derivatives

Dr. Svetlana Borovkova

Vrije Universiteit Amsterdam
History of derivatives

**Derivative**: a financial contract whose value is derived from some other financial instrument (stock, index, commodity, exchange rate, bond, ...)  


Trading in derivatives started **May 16, 1972**, on Chicago Board of Trade (CBoT)  
- **1987**: 1.1 trillion USD ($10^{12}$)  
- **1994**: 20 trillion USD  
- **1998**: 33 trillion USD  
- **2000**: 98 trillion USD  
- **2006**: 270 (!) trillion USD

Financial Times, January 17, 2007: "... can (derivatives) market continue its monumental growth? Most (analysts) not only think it can, but believe it absolutely will".

NWD

February 2, 2007
Classic derivatives: plain vanilla options

**European option:** the right to buy (*call*) or sell (*put*) a financial instrument, e.g. a stock (*underlying asset*) on a specified maturity date $T$, at a specified strike price $X$.

Payoff of a call option:

$$c(T) = (S(T) - X)^+,$$

where $S(T)$ is the stock price on the date $T$.

An **American option** can be exercised anytime before the maturity date $T$.

These are the so-called **plain vanilla options**.
Black-Scholes option valuation

**Main assumption:** stock price $S(t)$ follows a Geometric Brownian motion:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t).$$

Discrete-time version:

$$\frac{S(t + \Delta t) - S(t)}{S(t)} = \mu \Delta t + \sigma \times N(0, \Delta t),$$

so stock returns are normally distributed, and the price itself is lognormally distributed.

**The key ingredient of Black-Scholes option valuation:** the risk-neutrality argument, used for construction of a replicating portfolio:

A portfolio that, at any time, consists of a call option and an appropriate amount of stocks is, on expiry date, exactly equal in value to the option’s payoff! (example)
Stock price paths

![Generated path of GBM](image1.png)

![Many generated paths of GBM](image2.png)
Risk-neutral valuation

Call option price can be expressed as the expected (discounted) payoff under the risk-neutral probability measure $Q$:

$$ c = e^{-rT} E_Q (S(T) - X)^+ $$

Under such risk-adjusted probability measure, the rate of return on a stock is equal to the risk-free interest rate $r$:

$$ \frac{dS(t)}{S(t)} = r dt + \sigma dW(t) $$

Mathematical tools: binomial trees, Itô calculus, martingale theory, change of measure, Radon-Nikodym derivative, Girsanov theorem.

NWD

February 2, 2007
Extensions of classical setup

Extensions of the celebrated Black-Scholes formula:

I. Replacing classic option payoff $(S(T) - X)^+$ by a more complicated ("exotic") payoff, which depends not only on the stock price at maturity date $T$ but the entire stock price path during the lifetime of the option $[0, T]$.

II. More sophisticated (and more realistic!) processes for the asset price, e.g. those incorporating price jumps. picture

III. Underlying asset is not a stock or index, but a commodity (gold, oil, agricultural products), electricity, credit, house, weather or insurance (against catastrophic events) $\longrightarrow$ "exotic underlying".
Stock price paths with jumps

Generated path of GBM plus jumps

Several generated path of GBM plus jumps

NWD

February 2, 2007
Exotic options: Asian options

The payoff is

\[(A(T) - X)^+,\]

where \(A(T)\) is the arithmetic average of daily stock prices during the lifetime of the option \([0, T]\) - very widely used options, especially in commodity markets!

Main difficulty: the main assumption of Black-Scholes model is that the stock price has a lognormal distribution, but the sum of lognormal random variables is not lognormal!

What to do?
- Replace arithmetic average by geometric average - the product of lognormal random variables is again lognormal!
- Assume the arithmetic average is lognormal, and match first few moments.
- Run a Monte Carlo simulation, in the risk-neutral world!
- An exact solution involves sophisticated mathematical tools: Laplace transform of the call price with respect to maturity.

NWD

February 2, 2007
Other exotic options

- **Barrier options**: provide the classical payoff \((S(T) - X)^+\) only if the asset price crossed (or not crossed) a pre-specified barrier \(B\) over the lifetime of the option. Can be: "up-and-in", "down-and-in", "up-and-out", "down-and-out" → clickfondsen. (picture)

- **Bermudan options**: can be exercised at any of the \(N\) given dates → ”between” American and European options

- **More exotic options**: Russian options, Parisian options, basket options, swaptions, quanto’s, volumetric (swing) options, ...
Barrier options

Upper, lower and corridor barriers

NWD

February 2, 2007
Difficulties and mathematical tools for exotic options

- Non-lognormality of the underlying value (Asian, basket options and quanto’s)
- Conditioning on some event(s) (barrier, double barrier)
- Optimization strategies and optimal stopping involved in American, Bermudan, swing and volumetric options

Tools available:

- **Risk-neutral valuation**: the option price = expected discounted payoff under the risk-neutral probability measure $Q$:

$$c(0) = e^{-rT} E_Q(\text{payoff})$$

- sometimes (rarely) the solution can be expressed in a closed form formula, most often it involves numerical evaluation of an integral.
Mathematical tools for exotic options

- **Monte Carlo simulations:**
  - a large number of price paths are generated under the risk-neutral probability measure $Q$
  - these are used to compute the option's payoffs $c_i(T)$
  - law of large numbers assures that the average payoff converges to the expected payoff under $Q$:
    \[
    \bar{c}(T) = \frac{1}{M} \sum_{i=1}^{M} c_i(T) \longrightarrow E_Q(\text{payoff})
    \]
  - discounted sample average gives the option price:
    \[
    \hat{c}(0) = e^{-rT} \bar{c}(T).
    \]
Commodity derivatives

Underlying asset: not stock or index but **metals** (gold, aluminium), **energy** (oil, gas) or **agricultural product** (wheat, soya, coffee, orange juice, pork bellies).

**Main differences:**

- Underlying asset price is **NO LONGER GBM**, but can have (picture)
  - seasonalities
  - mean-reversion
  - price jumps

- We **cannot costlessly hold a commodity** until option’s maturity (either must pay storage costs or completely impossible (agricultural commodities)).

NWD

February 2, 2007
Mean-reverting diffusion process (model for e.g. oil price)
Commodity prices

Crude oil, Natural Gas and Soybean
Crude oil: April 1994 - May 2004
Natural Gas: January 1997 - April 2004
Soybean: October 1998 - October 2001

NWD

February 2, 2007
Exotic underlyings: Electricity

Leap in difficulty: totally different CLASS of markets and derivatives: *exotic underlyings.*

Newly liberalized electricity markets, where electricity is traded as any other commodity.

**US:** PJM (Pennsylvania-New Jersey-Maryland), COB (California-Oregon Border)

**Europe:** Nordpool (Scandinavia), EEX (Germany), APX (Netherlands), UKPX (UK)

in the next few years also Italy, France, Belgium, ... .

**BUT:** Electricity is a totally new type of commodity! (*picture*)

- seasonality
- high volatility
- non-elasticity of demand → *price spikes*
- limited transportability
- non-storability!

NWD

February 2, 2007
Three major European power exchanges:

APX, UKPX and EEX

APX: Amsterdam Power Exchange
UKPX: UK Power Exchange, London
EEX: European Energy Exchange, Leipzig, Germany

All prices for 2001-2004:
Yearly seasonalities

The yearly seasonal component:

\[ f(t) = \sum_{k=1}^{2} (A_k \sin(2\pi kt) + B_k \cos(2\pi kt)) \]
Weekly pattern

Here we plotted price premia corresponding to a particular weekday, starting on Monday.
Electricity derivatives (cont’d)

**Main problems:**

- Realistic models for electricity price is needed.
- Option replication is impossible because electricity cannot be stored!
- Other, new types of options: volumetric options, swing options, flexible supply contracts...

**New tools:**

- Levy processes (pure jump processes), regime switching models, jump diffusions;
- Risk management with natural gas and weather derivatives;
- Power plants as real options.
Catastrophe (insurance) derivatives

Before 1993: reinsurance.
December 1993: introduction of catastrophe insurance futures and options (CAT) on Chicago Board of Trade.

- The payoff of a CAT derivative is paid if there was a large amount in insurance claims in a certain area, over a certain period.

- This happens in case of a catastrophic event, such as a hurricane, tornado or an earthquake.

- The payoff is based on the PCS (Property Claim Service) Index.
Use of CAT derivatives

- **Insurer** will buy CAT futures or CAT call options.

- **Sellers** of CAT derivatives: construction companies, reinsurance companies, speculators willing to take risk for profits.

CAT derivatives = perfect diversification instrument, the so-called zero-beta assets: low correlation to financial markets, investors willing to diversify their portfolios will buy/sell CAT futures and options.

Mathematical difficulties: seasonalities, spikes in case of a catastrophic event, no tradable underlying value, insurance versus financial valuation ...
Weather derivatives

Underlying value - any measurable weather factor: temperature, precipitation, snowfall, ...

Most popular: measures of temperature closely reflecting energy demand:
HDD (heating degree days) and CDD (cooling degree days):

\[ HDD(day \, t) = \max(18^\circ C - AVT(t), 0); \quad CDD(day \, t) = \max(AVT(t) - 18^\circ C, 0), \]

\( AVT(t) \) is the average temperature on the day \( t \).

- HDDs/CDDs are summed over a period
- The term of a contract may be a full year of a season:
  - ”Heating”: November-March
  - ”Cooling”: May-September
- The payoff depends on ”strike” and the number of HDD’s or CDD’s exceeding the strike times a nominal amount.
Using and valuing weather derivatives

**Users:** Energy, Agriculture, Construction, Tourism, Leisure, Transport, Retail, ...

**Fundamental difficulty:** the underlying asset (e.g. temperature) is NOT TRADED $\Rightarrow$ options cannot be hedged, i.e. replicated with the underlying asset.

Two existing approaches:

- **Actuarial**, or insurance method: uses historical statistical distributions of the weather variable $\Rightarrow$ requires a large diversified weather derivatives portfolio, plus extensive historical weather databases are needed.
- **Financial option theory**: more in line with financial markets, but the underlying asset is not traded, so option replication does not hold!

**Weather insurance**: low probability, high risk events (e.g. avalanche destroying a skiing resort).

**Weather derivatives**: high probability, lower risk events (e.g. no snow, such as this winter (2006-2007) $\Rightarrow$ low or no profits for a skiing resort).

NWD

February 2, 2007
Conclusions

- A single mathematical development (Black-Scholes option pricing theory) solely gave rise to an entire multi-trillion finance industry of derivatives!

- Sophisticated mathematical tools are needed to deal with exotic derivatives, realistic asset price models, exotic underlyings.

- New classes of derivatives are growing and establishing their importance in enterprize-wide risk management and in the financial marketplace.

- "Bermuda triangle" is formed by the energy, weather and insurance derivatives.