Mathematics Education –
Procedures, Rituals and Man's Search for Meaning

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1. Some reflections about vision and practice in mathematics education.

A high priority recommendation of the mathematics education community, as well as the science education community, is reflection or metacognition. Our students are asked to reflect. They are asked to reflect about their solutions to given problems, about their solution strategies, about their mistakes, about their beliefs and about any possible aspect of thought which can be a candidate for reflection. However, as it happens quite often in education, we forget to use the advice we give to other people. I would like to use this opportunity to reflect on mathematics education from some angles which generally are ignored while we are so busy with investigating particular mathematical concepts, problem solving processes, the use of calculators, computers and the INTERNET. I would like to do my reflection by focusing on general features of human nature by means of which some specific mathematical behaviors can be explained.

The first aspect is the aspect of goals. It is almost banal since any reform document is full of declarations about goals and the importance of studying mathematics for the future of our children and for the future of our society.

The emphasis here is not so much on content, but on other aspects of learning mathematics, namely, how to think analytically, how to reason, how to solve problems and how to prove. The fact is that these goals have never been reached on any large scale. They can be considered as a vision. And, indeed, the Principles and Standards for School Mathematics, 2000, admit that *this vision of mathematics teaching and learning is not the reality in the majority of classrooms, schools, and districts. Today,* adds the document, *many students are not learning the mathematics they need* .

At this point it is worthwhile to remind us that the mathematical education community has never reached an agreement about the needs of mathematics students. Therefore, let us ignore the curricular questions and focus on the vision. It would be interesting to do a historical research about mathematical education documents and the visions they try to promote. It seems to me that, besides the *back to basics* call, all the visions include, perhaps with different emphasis, almost the same goals which I mentioned earlier. These documents have served the mathematics education rhetoric at least half a century. Has not the time arrived to examine the rhetoric, to reflect on it and to analyze it? If there is a huge gap between the rhetoric and the practice why don't we give up the rhetoric? The answer, I think, is very simple: We need the rhetoric. Thus, the next question is why do we need it? Here, there are some possible answers. My suggestion is: We need it because it makes our educational work meaningful. I will elaborate on this later on. In the meantime, let us discuss the practice.

The practice is that mathematics is considered by the majority of the students as a collection of procedures to be used in order to solve some typical questions given in some crucial exams (final course exams, psychometric exams, SAT etc.) This reflects also the way mathematics is taught by many teachers in many classes. Again, we may ask why it is so, and my answer, again, may look trivial. It is so because it satisfies the need of the majority of people involved in the process. I mean the real need, not the declared need. In the rhetorical documents there is a difference between two issues: 1. What do we want, as educators, to accomplish by teaching mathematics, and 2. Why it is advisable to a student to learn mathematics. This difference might be the result of the difference between educators' goals and students' goals. Our goal as educators, might be to promote analytical thinking, moral values and appreciation of science and art. The students' goal in life might be to enhance their chance to become "rich and famous." Therefore, the curricular documents should present some arguments to convince the students to learn mathematics. They should tell the students why it is desirable for them to study mathematics. In other words, they should explain to the students how mathematics might help them in their careers. *We live in a mathematical world*, says the above Principles and Standards document, *whenever we decide on a purchase, choose insurance or health plan, or use a spreadsheet, we rely on mathematical understanding…The level of mathematical thinking and problem solving needed in the workplace has increased dramatically…Mathematical competence opens doors to productive future. A lack of mathematical competence closes those doors.*

This is not the place to elaborate in length how misleading are these claims. In short I will say only the following: No doubt mathematical knowledge is crucial to produce and maintain the most important aspects of our present life. This does not imply that the majority of people should know mathematics. Farming is also crucial to at least one aspect of our life – the food aspect, and yet, in developed countries, about 1% of the population can supply the needs of the entire population. In addition to this argument, if you are not convinced, I recommend to you to look around and to examine the mathematical knowledge of some high rank professionals that you know - medical doctors, lawyers, business administrators, and many others, not to mention politicians and mass communication people. Thus, if the above claims about the need to study mathematics are misleading, why do our students study mathematics in spite of all? You might suggest that the students believe in these claims although these claims are misleading. I suggest that the students have very good reasons to study mathematics. It is not the necessity of mathematics in their future professional life or their everyday life. It is because of the selection role mathematics has in all stages of our educational system. Mathematical achievements are required if you want to study in a prestigious place (whether this is a junior high school, a senior high school or university). A prestigious school increases your chance to get a good job. This might help the vision of richness and fame become real. Jere Confrey formulated it clearly at a PME talk in 1995: *In the vast majority of countries around the world, mathematics acts as a draconian filter to the pursuit of further technical and quantitative studies...*

If this is true we have a convincing argument to study mathematics. We also have at least a partial explanation to the fact that mathematics is considered, and very often also taught, as a collection of procedures. This is a form which provides reasonable success to the majority of students in the crucial exams. Later on I will suggest an additional explanation. At this stage I would like to elaborate a little about the gap between vision and practice. It is quite clear that the vision is at the official documents and not in the classrooms. The educational system has not found a way to translate the rhetoric to practice. On one hand, the teachers are supposed to be the mediators between the rhetoric and practice. *Educational decisions made by teachers have important consequences for students and for society*, claim the Principles and Standards. On the other hand, the teachers cannot make any decision which might distract them from their presumably only task - to prepare their students for the crucial exams. In other words, their ultimate goal is to cover the syllabus. These circumstances eliminate the option to develop analytical thinking, reflection and problem solving strategies. If you want to develop these you should relate to students' ideas, mistakes, and misconceptions; you should give examples of failure in analytical thinking, you should analyze problem solving processes and you should discuss the principles of proof and justification. A teacher who has to cover an overloaded syllabus (and all syllabuses are, unfortunately, overloaded) does not have time to do anything besides the syllabus.

From the above analysis it follows that there isn't much hope for any vision to become real. It is true, as long as the educational system will stay as it is.

The title of an extremely important book about education by Neil Postman is The End of Education . The title is ambiguous because the word “end” is ambiguous. “End” means “goal” and it also means “termination.” The message is that if education does not challenge the students with meaningful goals it will die. Of course, it is pointless to suggest goals while the teachers do not have the means to translate these goals to the classroom practice. In order that suitable conditions for educational change will be formed, some major changes should occur in educational policy and in the attitude of society to education. I do not know how to cause such changes by organizational reforms. My rhetoric is that society should change. I know it is not practical. However, my presentation is essentially theoretical. On the other hand, many of us are teacher trainers. We have to tell our teacher-students the truth about the system in which they operate. Eventually, the teachers are the key. It is important for them to understand their role and the limits of their abilities to achieve some educational goals. Of course, it would be nicer if we could tell our student-teachers how meaningful and important is their work. However, if this is not the case, the result will be a cynical reaction. To understand what you are doing and the limits of your ability is extremely important. Of course, it is much better if you can improve the situation. But sometimes, there is nothing you can do about it. It is analogous, in a way, to cognitive psychotherapy: Life is not a picnic, as we all know, unfortunately. However, it is important for us to understand why it cannot be a picnic and hence to accept it without becoming bitter or desperate. At this point it will be suitable to quote Reinhold Niebuhr's famous prayer:

O Lord, grant me the serenity to accept the things I cannot change; the courage to change the things I can; and the wisdom to know the difference. Educational research is supposed to supply us with the wisdom by means of which we can see the difference

So, on what occasions people , really, use mathematics?

Sometimes, in order to find a support to claims about human nature, instead of relying on experimental research, it is helpful to use canonical texts or master pieces like the Bible, the Greek tragedies, the writing of Shakespeare, Moliere and other literary giants and also the operas of Mozart. Here is a famous aria from Don Giovanni which can illustrate the use of mathematics by ordinary and also by exceptional people. Leporello, Don Gionanni’s servant, presents the statistics of his maser’s conquests to a lady who happened to be a victim of Don Giovanni’s fatal charm. Here are some questions which can serve as an advanced organizer for you on one hand and, on the other hand, as an example of modern approaches in mathematical education to problem posing:

1. How do you say 1003 in Italian ?(ethno-mathematics)

2. What is the number in Turkey (authentic problem)
3. What is the number in Spain (also authentic)
4. Which one is greater? (low level mathematical thinking)
5. Why is it greater? (an open ended question)
6. How many times the number 1003 is mentioned in the text? (metacognitive)
7. What will be the result if you add them up? (drill)
8. Could you obtain this answer by using a different arithmetical operation? (control and reflection)

The words of Leporello's aria about Don Giovanni's conquests in different countries are the following:

Young lady, this is the catalogue made by me, of the beauties my Master has loved,
Look at it and read it with me.
In Italy – 640,
In Germany – 231,
In France – 100,
In Turkey – 91,
But in Spain – 1003.
Peasants, serving girls, countesses, baronesses, marchionesses and princesses,
Every class and shape and age.

An additional question (only for the mathematically gifted students): Why in the above presentation there is no information about the Netherlands?

I hope that you are quite convinced now that counting is important. But note the specialization phenomenon. The master seduces, the servant counts.

2. A suggestion for a theoretical framework.

Earlier I spoke about the needs of mathematical educators and also about the needs of students. The word "need" rose there naturally in the context, but I had an additional intention to use it. I want to use it also in a technical way as a term in some psychological theories which try to explain human behavior. Freud, at the beginning of the twentieth century, suggested the concept of *DRIVE* or *DRIVING FORCE* as an explanation for human behavior. He used the German word *TRIEB.* He spoke about the sex drive and the aggression drive and claimed that they had physiological origin. This was the beginning of biological psychology. Many years later, in the seventies, Maslow spoke about the *needs* of human beings. For instance, the need for social recognition or the need for self fulfillment. Maslow did not relate to the question whether all the human needs have physiological origins. However, although psychology and biology are quite far from being able to establish their claims about physiological origins of human needs, the common assumption is that everything is biochemical. In addition to this, a new direction started to develop from the theory of evolution, called evolutionary psychology. The idea here is that psychological schemas have physiological origins which during the course of evolution gave evolutionary advantage. For instance, the fact that human beings have a drive (or a need) for sweet food enable them to save time on eating. This is because sweet food has more calories than, for instance, vegetables. Thus we, human beings, do not have to spend all our time on eating, as cows do.

Therefore, in the course of evolution, human mutations, which had the ability to produce other activities than just eating, could develop. Taking all this into account, the analysis of human behavior may become a three stage project as follows:

1. Identifying typical activities.
2. Suggesting some needs that these activities are supposed to satisfy.
3. While assuming that these needs have physiological origins - trying to explain what evolutionary advantage these needs gave us in the course of evolution.

It is also possible that some needs do not have evolutionary advantage. It is not clear for instance that the need for self fulfillment, a major need in Malsow's psychology, has an evolutionary advantage. The need for social recognition, which drives us to do all kinds of things in order to obtain other people's respect and appreciation, clearly has an evolutionary advantage. It increases our chance to find a spouse and thus to guarantee the continuation of our species. So, as a little exercise, just think of some activities we are involved with , and try to suggest some needs that these activities are supposed to satisfy. When I say "need" , I mean an internal need, not external needs, like writing exams, waiting for your turn in a line or paying income tax. The list I am suggesting here is:

1. Dancing. 2. Listening to music. 3. Writing poetry. 4. Praying. 5. Watching football games. 6. Competing. 7. Gambling. 8. Giving orders. 9. Following orders. 10. Mountain climbing. 11. Going through procedures. 12. Taking part in rituals.

Note that the above activities are not universal. Not everybody has an irresistible need to listen to music, to watch football games or to solve mathematical problems.

I would like to use the above three stage approach to analyze also mathematical behavior. After all, mathematical behavior is part of human behavior.
All this is in general. Now I will be me more specific.

3. Beliefs and rituals.

A possible way to explain people's behavior at a given situation is to describe their belief system about the situation. When I say "beliefs about a situation" I also mean how people view the situation, what they think the situation is, how they should act at the situation, what the expectations are and so on. We have beliefs about every situation we take part in. See, for instance, Schoenfeld's theory about problem solving behavior. He considers the belief system people have about mathematical problem solving as a crucial factor in their behavior. Fischbein's theory about intuition presents intuitions as beliefs we have about given situations. He suggests that intuitions or beliefs are formed by the need for certitude. I would, therefore, like to suggest that there is a need for all kinds of beliefs in the human race. However, if we want to investigate beliefs it might be fruitful to look at the domain where beliefs are most dominant; namely, the domain of religion. Religion is not only a belief in God. It is a collection of many beliefs about desired behaviors. It includes instructions how to act with other human beings and instructions how to act with God.
The last set of instructions produces religious rituals (which probably were the starting point of all religions). Religious rituals are an important topic in anthropological, social and psychological research which is beyond the scope of this talk. I will mention only two references related to this topic. Freud (1912), for instance, deals with them in his classical paper Totem and Taboo, drawing analogy between rituals in primitive societies and habits of neurotic people (and all human beings are, at least a little bit, neurotic). Geertz, many years later (1973), discuss them from a different perspective, but also in the context of primitive cultures (primitive in the eyes of Western researchers). I am not aware of extensive studies which investigate rituals in everyday secular cultures. I think it is an important and extremely interesting topic for psychological research.
Here are some typical aspects of religious rituals as I see them:
1. You believe that accomplishing a ritual will please God, will make you accepted by God, will open for you certain gates (the gates of heaven).
2. You do not necessarily understand why you have to follow a ritual and sometimes you are even told that you are not supposed to understand, you are not supposed to ask, you are supposed to do it and that is all. (Many Jews who do not know Hebrew and many Catholics

who do not know Latin, say prayers the words of which they do not understand. However, they are sure that their prayers have important impact on their life and will give them credit and some good points in crucial moments.)

3. You feel that by accomplishing a ritual an approval is obtained to certain aspects involved in the ritual: relationships between human beings or between human beings and God. (Think for instance of wedding ceremonies, divorce ceremonies, baptizing ceremonies, circumcision, funerals, memorial services and more).

In addition to the religious rituals there are also secular rituals (We shake hands, we entertain birthday parties, we sign contracts, we take part in all kinds of celebrations, we sit at a dinner table according to a certain order and so on and so forth). Thus, human life is full of rituals.

If we accept this we may assume three theoretical assumptions:

a) *Rituals have some central functions in human life.*

(The analysis of these functions is beyond the scope of this talk. I would like to suggest two claims which relate rituals to general human needs: 1. Rituals give security when accomplished. 2. Rituals help people to develop their social identity, national identity and many other identities. Identity is also an essential need people have).

b) *Rituals are a psychological need.*

c) *The human mind has developed special mental schemas to deal with rituals.* These schemas identify rituals and they produce ritual behavior. I will call these schemas the "ritual schemas." This is a point where beliefs and rituals can be connected in an additional way. Namely, somebody believes that he or she are at a ritual situation and, therefore, they react accordingly, guided by suitable ritual schemas.

From the above it can be predicted that rituals will be formed and will be accomplished in almost every domain of our life including the domain of education in general and mathematics education in particular.

A ritual, in fact, is a procedure: a sequence of words, symbols, or actions. Thus, when examining a given procedure, in the general sense of the word, we can ask about it whether it is also a ritual. And in the domain of education or mathematical education the question is under what conditions we can claim that a given behavior is a ritual behavior? From my above characterization of rituals it follows that we should look for sequences of words (also mathematical symbols in our case) and actions (mathematical actions in our case) that fulfil the following (which are a suitable adaptation of 1, 2 and 3, above):

1*. You believe that accomplishing a given sequence of words, mathematical symbols, or mathematical actions will please somebody (the system, the teachers, the parents, etc.).

2*. You do not have to understand why you and other people have to follow this sequence of words or actions.
3*. You believe that following a certain sequence of words or actions will give approval to some aspects involved in the ritual and which are not necessary clear to you while performing the ritual.
In the domain of mathematical behavior there are many procedures, as we all know. Is it possible that some students are going through mathematical procedures the same way people go through rituals (either religious or secular)?
Just think of some central activities in mathematical education as simplifying algebraic terms, solving equations, differentiating, solving word problems - going through them may look to many people as going through rituals. As already mentioned, since rituals are so common in human behavior, I suggested to assume that there exist in us a certain psychological schemas associated with rituals which I called the ritual schemas. My claim is that in many students the ritual schemas are activated when they do mathematics. When a student is activated by a ritual schema, usually, he or she are unaware of it. By choosing, perhaps unconsciously, a ritual reaction to a given stimulus, a person might exclude or reject, perhaps also unconsciously, another kind of reaction, the meaningful reaction. At this point, I would like to make it official: As well as the need for rituals we can speak about the need for meaning. It expresses itself in many ways. Within the circle of education, and especially within the smaller circle of mathematics education, people speak about meaningful learning, not always using the word “meaningful” (for instance: Ausubel, 1968; Skemp, 1976, 1979; Robert Davis, 1984). Within psychology and psychological therapy it is common to speak about meaningful actions and the meaning of life (Bruner, 1990; Frankl, 1978). To them we can add the Nobel prize laureate, the physicist Richard Feynman (1998). Although meaningful learning and meaningful life seem to be different concepts, it is quite possible that they have the same origin, but again, this is not the forum to elaborate on it.
The need for rituals and the need for meaning are, in a way, two conflicting needs. This is one case, out of many, which illustrates the dialectic nature of human beings. This dialectic nature causes a great difficulty to predict the human behavior. However, it might help us to explain a behavior already observed.

3. Some illustrations

Because of time limitations I will present here only three illustrations.

My first illustration is taken from Schor & Alston (1999). A group of elementary teachers was asked to *create meaningful situations or stories that would make sense* for some numerical expressions. One of the expressions was :

3 - (-4).

Here are two stories:

Sandy got squares for positive and negative numbers.

-1 = a square in red color.

1 = a square in blue color.

- *(-1) = a square in blue color.*

She took 3 red squares, and then subtracted 4 in blue. How many squares in what color did she have?

(It seems that the red and the blue got confused here but even if there were not, it would still be a meaningless story.)

Sharifa had $3 negative (out of pocket) and she gave Maria negative one times minus $4. How much did they have together?

If you look carefully at the examples you may admit that mathematical education at the elementary level formed a ritual which can be called the *create a story* ritual. In this ritual students, as well as elementary teachers, are supposed to form sequences of words which can be related to given numerical expressions. There are some key words and some rules by means of which the "story" is created. It is not so crucial whether the "story" makes or does not make sense. The task is to accomplish the ritual.

The second illustration is taken from a paper by Rogers (2000). He quotes a 1780 English book by Dilworth in which the author chose the form of catechism to introduce new topics (catechism is a summary of religious doctrine in the form of questions and answers). Here we have the *question and answer* ritual. Dilworth argues that *children can better judge of the force of an answer than follow reason through a chain of consequences. Hence the text also provides a very good examining book; for at any time in what place the scholar appears to be defective, he can immediately be put back to that place again.* The quotation illustrates the well known approach to learning – the rote learning approach. Rote learning is presented today in a contrast to meaningful learning. It is labeled as a meaningless procedure. This is in spite of the fact that when we rote learn we by all means have the option of doing it meaningfully (in Dilwoth's words, we can try to "follow reason through a chain of consequences."

The problem is that very often, since the teacher or the textbook author do not believe in the child's ability to "follow reason" they present the topic in such a way that the child is not expected to understand it and therefore he has no chance to understand it. The result is that the child acquires a question and answer ritual, a ritual which is widely practiced all over the world (in 1780 as well as in 2000) in common exams. Let us have a look at one excerpt from the book:

The introduction of arithmetic in general:

Q. What is Arithmetic?

A. Arithmetic is the art of science of computing by numbers, either whole or in fractions.

Q. What is number?

A. Number is one or more quantities, answering to the question, how many? …

Q. What is theoretical arithmetic?

A. Theoretical arithmetic considers the nature and quality of numbers, and demonstrates the reason for practical operations. And in this sense arithmetic is a science.

Q. What is practical arithmetic?

A. Practical arithmetic is that which shows that method of working numbers, so as may be most useful and expeditious for business. And in this sense arithmetic is an art.

Even without analyzing each sentence in the above paragraph, it is quite clear that any student at any age cannot understand the above text as an introductory text. Such a text can be understood after somebody has studied a great deal of arithmetic and also has acquired some reflective notions (like science, art, operation, reason, theoretical, practical and so on). Hence, studying the above sequence of questions and answers is studying a meaningless ritual the performance of which will allow the students to pass to the next stage of their studies. The fact that the author himself used the term "catechism" is an ample evidence that the analogy to religion clearly crossed his mind as an educational model for the majority of the students. I claim that similar approaches still exist today in many schools. Of course, different vocabulary is used and the topics are different.

My next example is related to the most scaring procedure in mathematical behavior - the rigorous proof. Again, in order to elaborate on it I will use Mozart as an inspiring resource. This time it is his Magic Flute. It is quite obscure opera. It requires hermeneutics. Some people understand it as an opera about conflicts between good and evil, wisdom and stupidity. Tamino, the opera's hero, decides to join the order of wisdom. According to the regulations he is required to take some tests (the equivalence of our university candidacy exam). He has to go through some demanding rituals, part of it is facing the examining staff. Look at the man with the white hair and eye glasses. If I were not bald he could be myself.

Aren't we typical representatives of mathematics teachers in the eyes of the victims of mathematics? Mathematics purifies, I would have added to Mozart's rhetoric if I were allowed to do so. But even, without this addition the text can serve any curriculum document which advocates rational thinking and wisdom as a way of life:

He who would reach the light (that's wisdom) the dark of night (that's stupidity) must dare.
There to be purified by water fire and air (and also by mathematics)
There will he be tempered (become moderate and gentle); his soul made firm and strong,
Then will his way lead on toward a higher goal (that's living in a mathematical world).
His sight has hitherto been overcast and dim (till now everything was cloudy and gloomy),
But all life's mystery will now be clear to him (by means of mathematics you will understand the world).

Pay attention to the unbearable tension, to the step by step procedure (the same as in formal proof), marching is a crucial element in many rituals and looking back – that's reflecting on the starting point, the *given* in mathematical theorem.

After observing an illustration of *general proof rituals* in Mozat's opera, let us consider an illustration of *proof ritual* in the domain of mathematics education. It is taken from a questionnaire I made long time ago. It was created in order to investigate high school students beliefs about the notion of proof but since then I have used it with mathematics teachers in various occasions. It is the following:

In an Algebra class the teacher proved that every whole number of the form $n^3 - n$ is divisible by 6. The proof was: $n^3 - n = n(n^2 - 1)$. Using the formula

$a^2 - b^2 = (a + b)(a - b)$ we can write: $n^2 - 1 = n^2 - 1^2 = (n + 1)(n - 1)$. Thus, $n^3 - 1 = n(n + 1)(n - 1) = (n - 1)n(n + 1)$. But $(n - 1)n(n + 1)$ is a product of three consecutive whole numbers. Therefore, one of them should be divisible by 2 and one of them (not necessarily a different one) should be divisible by 3.

A day after that, the following exercise was given to the class as a homework assignment:

Prove that $59^3 - 59$ is divisible by 6.

Here are three answers which were given by three students:

1. I computed $59^3 - 59$ and found out that it is equal to 205,320. I divided it by 6 and I got 34, 220 (the remainder was 0). Therefore, the number is divisible by 6.

2, One can write $59^3 - 59 = 59(59^2 - 1)$. But

$59^2 - 1 = 59^2 - 1^2 = (59 + 1)(59 - 1)$ according to a well known formula. Therefore,

$59^3 - 59 = 59(59 + 1)(59 - 1) = (59 - 1)59(59 + 1)$.

*We got a number which is a product of three consecutive numbers. One of them is divisible by 2 and one of them is divisible by 3. Therefore, the product is divisible by 2*3, namely, by 6.*

3. Yesterday, we proved that every whole number of the form $n^3 - n$ is divisible by 6.

$59^3 - 59$ has this form. Therefore it is divisible by 6.

Which answer out of the three do you prefer and why?

In the student sample that I investigated (10th and 11th graders, N = 365), 35% preferred answer 2; 14% preferred answer 1; 43% preferred answer 3 and 8% did not have any preference. If you think of it from the perspective of meaning then going through the general proof in order to establish the particular case is pointless. However, if you look at a mathematical proof as a ritual then answer 2 is preferable. As a matter of fact, if we look at the form of traditional proofs in mathematics they do have some ritual elements (the form, the vocabulary, and the ultimate mantra, Q.E.D., at the end).

Of course, I do not expect anyone to tell me explicitly that they prefer answer 2 because it reproduces the entire proof ritual. I suggest this as an explanation. The nature of this suggestion is speculative. However, when we investigate people's beliefs we observe behavior and we speculate about the beliefs which might produce this behavior.

A more surprising distribution than the above I get when I distribute the questionnaire to mathematics teachers who attend my university courses. In my last sample (1999) I had 27 teachers. Eleven of them preferred answer 2, five preferred answer 1, nine preferred answer 3 and two had no preference. The arguments explaining why answer 2 is the best were like the following:

a) It was the teacher's intention. b) This answer indicates that the student really understands how to prove. c) The student reconstructed the procedure. d) The student did not substitute a number in a formula in a mechanical way. He related to the components of the task.

The answers rejecting answer 2 were similar to the following:

a) *The student reproduced the entire procedure which shows that he did not exactly understand what proof is for. b) The student imitated the general proof. He learned it by heart but did not show understanding.*

These results could raise in us some thoughts about the origins of the ritual approach to mathematics that so many students have.

Fortunately, not all procedures in mathematics are as serious and frightening as rigorous proof. For ordinary people the curriculum suggests simplifying algebraic terms, differentiating polynomials and solving quadratic equations. These procedures are sources of satisfaction and joy to many students.

A young lady with whom I spoke recently told me how much she hated going through mathematical procedures. *Cann't you think of any procedure which gave you even little satisfaction?* I asked . She reflected a few seconds and said: *As a matter of fact there was one. When I solved a quadratic equation and the square root happened to be a whole number.*

Not everybody wants, like Tamino in Mozart's Magic Flute, to acquire wisdom and to face intellectual challenges. Most human beings focus on more essential needs as survival and procreation (that's producing offsprings). We all know how this is done. However, not all of us are aware of the related rituals. There are billions of them. They include both rhetoric and actions (like most rich rituals). Here is one of them suggested to us by Mozart and Ingmar Bergman. It is the ritual of Papageno and Papagena. It is quite clear what these two have in mind. However, before getting to it they are going through a ritual. The rhetoric of this ritual is the following:

Papageno*: Can you be my very own?*
Papagena: *Yes, I am your very own*.
Papageno: *Then I'll be your old man*.
Papagena: *Yes, and I your little woman*.
Papageno and Papagena (in turns and together): *What fun we two will have.*
Yes, if Providence (one of the gods) *rewards us and in its good time provides us*
With a flock of little ones.
Papageno*: First a little Papageno*.
Papagena: *Then a little Papagena*.
Papageno: *And another Papageno*.
Papagena: *And another Papagena.*
Papagena and Papageno (in turns and together): *Life will be quite full of joy*
When many many many Papageni
Fill each corner of our house.

This has been the rhetoric. The actions can be seen in Ingmar Bergman's version of Mozart's Magic Flute.

5. A final comment.

The common tendency in the mathematical education community today is to move from meaningless procedures (rituals) to meaningful actions. This tendency itself is based on the belief that meaningful learning is one of the major goals of education. People may argue about it and we may find out that this belief is not necessarily shared by everybody. At least not all our students believe in it.

All mathematics educators whom I know believe in it but, of course, I do not know all mathematics educators. Do all mathematics teachers and all our students in teacher education programs believe in it? I doubt it.

I consider the tendency to move from the meaningless to the meaningful as a special case of a general tendency in human psychology which can be considered as *man's search for* *meaning (*Frankl, 1978). At this point the focus of the discussion can be turned to us, the mathematics educators. Assume we really direct all our efforts to make our students move from meaningless rituals to meaningful thought processes. What are our chances to succeed? Earlier I claimed that the human behavior (as well as human thought processes) are driven by the need for rituals and by ritual schemas. Therefore, when advocating the meaningful approach to our students we are telling them to act in some cases against their own nature. This is not a simple task. On the other hand, there is no reason to be discouraged. Education is supposed to be a recommendation to act against our nature; and if you wish - this is, perhaps, an essential feature of the entire human culture. Very often in mathematics education we are involved with questions about the meaning, for instance, of multiplication or division. Aren't we supposed to be involved also with questions about the meaning of it all?

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