

mini-symposium Algebra & ICT

Introduction: Why Algebra & ICT?

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2011-12-13



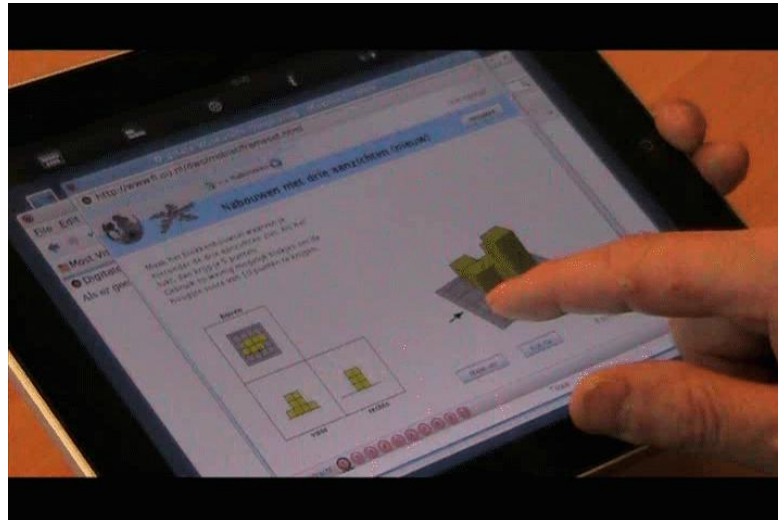
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Today's questions:

- What's the problem with secondary algebra education?
- Can ICT play a role in solving this issue?



Today's time schedule

13:30 – 14:00 Paul Drijvers (UU):
Introduction: why algebra & ICT?

14:00 – 15:00 **Johan Jeuring** (UU/OU):
Hints and feedback in learning environments for algebra -
technology for following and helping students

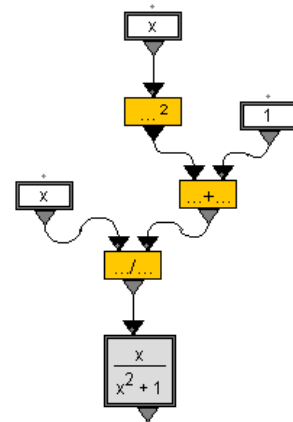
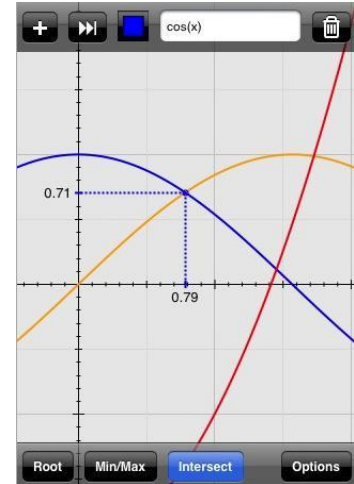
15:00 – 15:30 Break

15:30 – 16:30 **Abraham Arcavi** (Weizmann Institute):
Technology at the service of algebra learning

16:30 Drinks

Introduction: Why algebra & ICT?

1. What is algebra?
2. How are we doing with algebra?
3. What are difficulties in learning algebra?
4. Can ICT help here?



1. What is algebra?



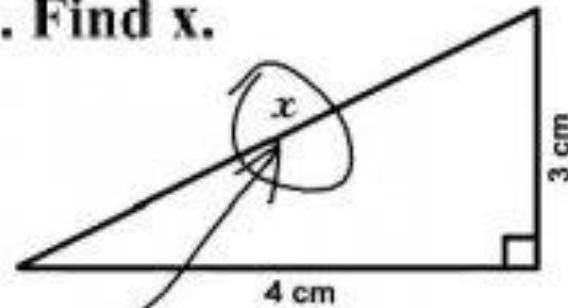
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1. What is algebra?

3. Find x.



Here it is

$$\lim_{x \downarrow 8} \frac{1}{x - 8} = \infty$$

$$\lim_{x \downarrow 5} \frac{1}{x - 5} = 5$$

1. What is algebra?

One use of the word 'algebra' is the **abstract study** of number systems and operations within them, including such advanced topics as groups, rings, invariant theory, and cohomology.

The word "algebra" can also refer to the "**school algebra**" generally taught in American middle and high schools. This includes the solution of polynomial equations in one or more variables and basic properties of functions and graphs.

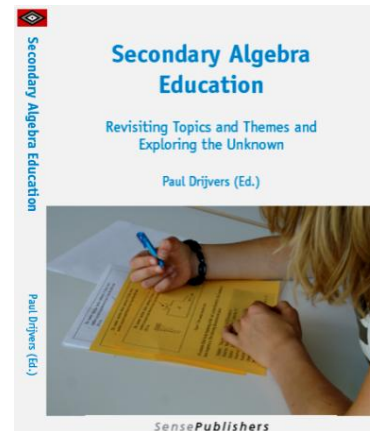
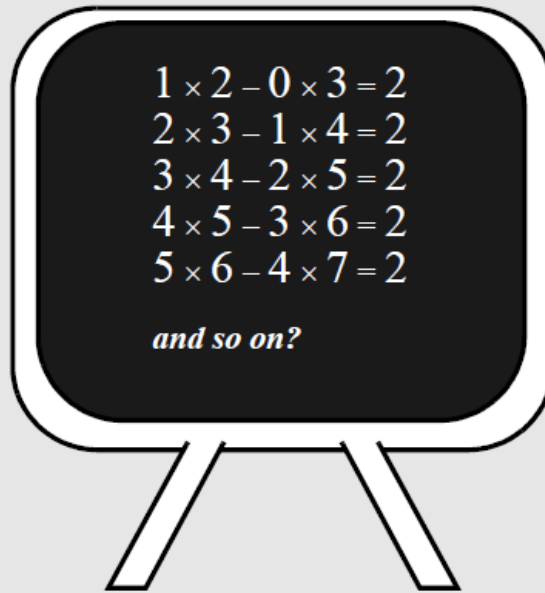
(<http://mathworld.wolfram.com>)

(School-)algebra and arithmetic

- ... are different...
- ... but numbers and numerical operations are a context for / may invite algebra.

- a. Check the sums on the board. and continue the sequence a bit further. Amazing, isn't it?*
- b. Think up a few more sums that would appear much further down in the sequence.*
- c. Can you be sure that the result will always be 2?*
- d. Now think of another regular sequence which also has the same result.*

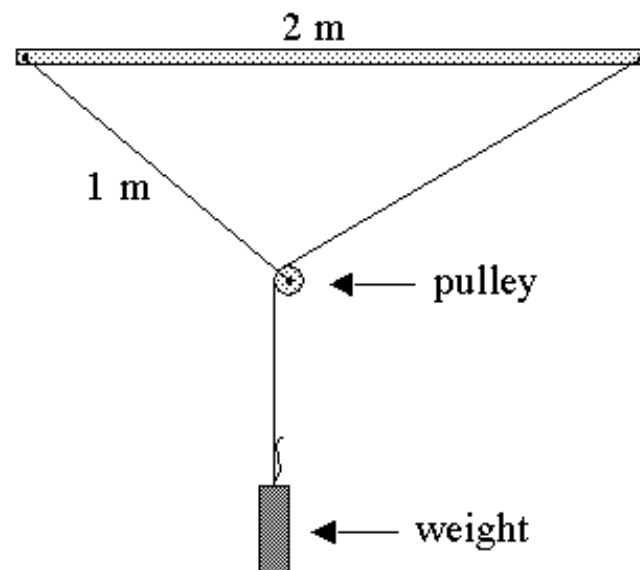
If yes, explain this with algebra.



Kindt, 2010, inspired by Sawyer 1959

Algebra and geometry

- ... are different...
- ... but (analytic) geometry can be a field of application for algebra.



$$y_1' + y_2' = 0 \quad (\text{max. oplossing})$$

$$y_1' = -y_2'$$

$$\frac{-(x-1)}{\sqrt{-(x^2-2x-a^2+1)}} = \frac{1}{\sqrt{2x+a^2-1}} \quad \text{kruislings verm. } \downarrow$$

$$-(x-1) \cdot (\sqrt{2x+a^2-1}) = \sqrt{-(x^2-2x-a^2+1)} \quad \text{kwadrateken } \downarrow$$

$$(x-1)^2 \cdot (2x+a^2-1) = -x^2+2x+a^2-1 \quad \text{solve } \downarrow$$

$$x = \frac{-(a \cdot \sqrt{a^2+8} + a^2 - 4)}{4}$$

och nog andere oplossing, maar klopt niet

Controle:

$$x = \frac{-(a \cdot \sqrt{a^2+8} + a^2 - 4)}{4} \quad | \quad a = 0,4$$

ar

$$x = 0,674342062058 \quad \text{klopt } \checkmark$$

2. How are we doing with algebra?



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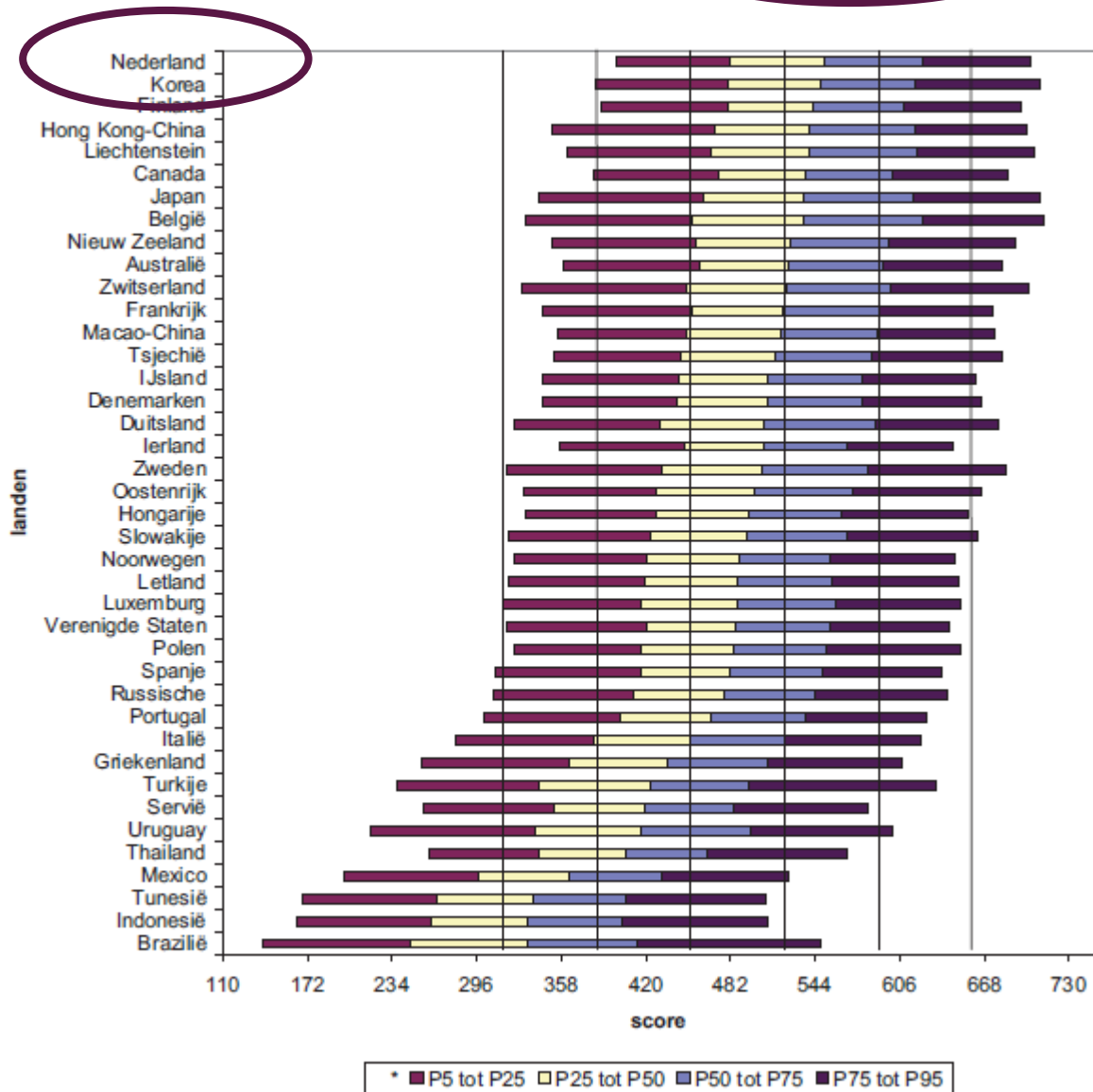
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2. How are we doing with algebra?

- PISA: mathematical literacy for 15 year olds
- PISA 2006 and 2009:
no specific information on algebra performance
- PISA 2003:
Good NL results on domain 'change and relationships'
- Let's wait for PISA 2012!

Scoreverdeling op de wiskundeschaal betreffende 'Veranderingen en Relaties' in de OESO-en partnerlanden



CITO, 2004

SIMMS and TIMSS

tabel 1 Scores van twaalf landen in SIMS en TIMSS

	SIMS 1982		TIMSS 1995		TIMSS 1999		TIMSS 2003	
	totaal- score (176 opg)	algebra- score (40 opg)	totaal- score (151 opg)	algebra- score (27 opg)	totaal- score (162 opg)	algebra- score (35 opg)	totaal- score (194 opg)	algebra- score (47 opg)
Japan	62	60	73	72	579	569	570	568
Vlaanderen	52	51	66	63	558	540	537	523
Nederland	58	52	60	53	540	522	536	514
Hongarije	57	51	62	63	532	536	529	534
Frankrijk	53	55	61	54	-	-	-	-
Finland	50	46	-	-	520	498	-	-
Duitsland	-	-	54	48	-	-	-	-
Engeland	47	39	53	49	496	498	498	492
USA	45	43	53	51	502	506	504	510
Zweden	44	34	56	44	-	-	498	480
Noorwegen	-	-	54	45	-	-	461	428

Noot:

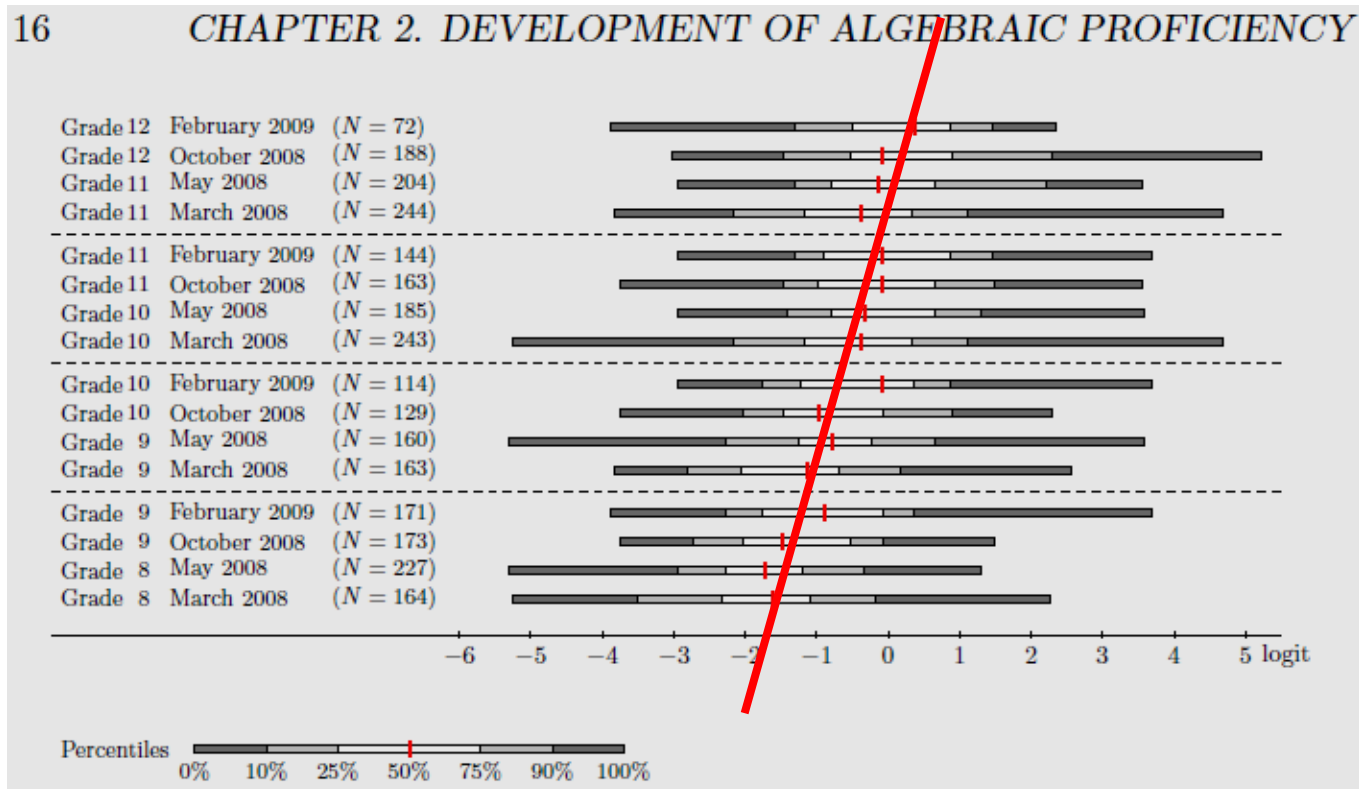
- betekent: geen deelname

De scores van 1982 en 1995 zijn een percentage (het gemiddelde percentage correcte antwoorden).

De scores van 1999 en 2003 zijn berekend naar een schaal met internationaal gemiddelde 500 en $sd = 100$.

Vos, P. (2007). Algebra-prestaties van tweede klassers.
Euclides, 82(4). 129-132

Little improvement over the years?



“... the majority of items were too difficult for students of grade 8, and were still too difficult for students of grade 12.”
(Van Stiphout, 2011, p. 26)

3. Difficulties in learning algebra



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Reaction to student difficulties

Opgave 3 Vereenvoudig de volgende uitdrukking zover mogelijk:

$$\frac{(5ab^2)^2 \cdot \sqrt[3]{8b^{-6}}}{5b^2 \cdot \sqrt{a^4}}$$

antwoord	10
percentage	31

16 studenten wisten geen raad met $\sqrt[3]{8}$; 38 studenten dachten $\sqrt[3]{8b^{-6}} = 8b^{-2}$; 9 studenten gaven als antwoord: $5b^2\sqrt[3]{8b^{-6}}$; 1 student gaf als antwoord: $5\sqrt[3]{8}$.

Opgave 4 Vereenvoudig de volgende breuk zover mogelijk $\frac{x^2-1}{x^2+2x+1}$

antwoord	$\frac{x-1}{x+1}$
percentage	45

Een te lage score

Opgave 5 Als $\frac{1}{y} = x + \frac{1}{c}$, dan is y gelijk aan

- A. $\frac{c}{cx+1}$ B. $\frac{1}{x+c}$ C. $\frac{c}{x+1}$ D. $\frac{c+1}{x+1}$

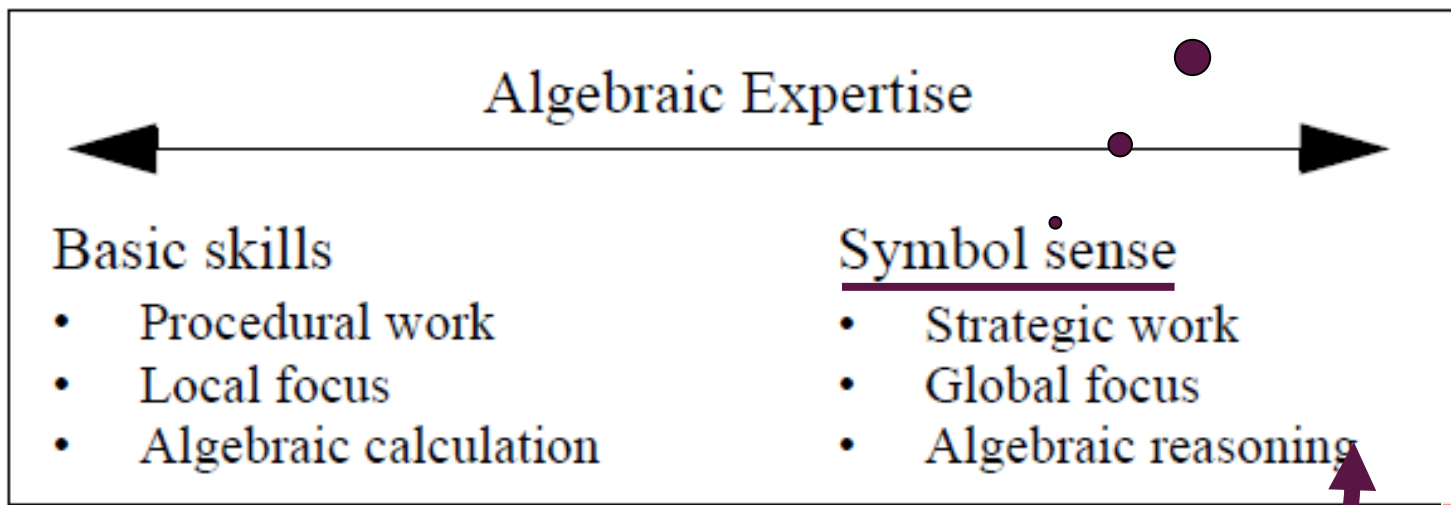
antwoord	A	B	C	D	anders
percentage	38	42	13	1	7

42% van de studenten neemt term voor term het omgekeerde! Het goede nieuws is dat in de eerste toets slechts 18% van de studenten een formule wist om te werken.

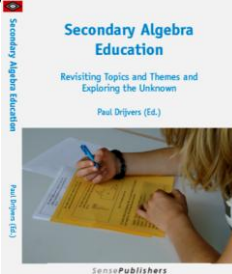
Werkgroep 3TU (2006). Aansluiting vwo en technische universiteiten. *Euclides*, 81(5), 242-247

The basic skill–symbol sense dimension

Arcavi, 1994,
2005



Drijvers, Kindt, Goddijn (2010), inspired by ...



4. Can ICT help here?



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4. Can ICT help here?

- Thesis Bokhove 2011
- Feedback questions
- ICT and concept development



Christian Bokhove

Use of ICT for acquiring, practicing and assessing algebraic expertise

Thesis Bokhove (2011)

www.algebrametinzicht.nl

$(x^2 - 7x + 12) \cdot (8x - 11) = (x^2 - 7x + 12) \cdot (3x + 14)$
 $8x^3 - 11x^2 - 56x^2 + 77x + 96x - 132 = 3x^3 + 14x^2 - 21x^2 - 98x + 3$
 $8x^3 - 67x^2 + 173x - 132 = 3x^3 - 7x^2 - 62x + 168$
 $5x^3 - 60x^2 + 235x = 300$
 $5x(x^2 - 12x + 47) = 300$

Δt	Technique	Interpretation
3	Expand	The student does not recognize the common factors and expands to remove the brackets
2	Aggregate common Terms	
3	Rewrite	
3	Factor	The student recognizes the common factor $5x$
		Student realizes he cannot solve a third degree equation, and gives up.



Christian Bokhove

Use of ICT for acquiring, practicing and assessing algebraic expertise

Strategic skills missing

$v \cdot \sqrt{u} = 1 + 2v \cdot \sqrt{1+u}$
 $v = (1 + 2v\sqrt{1+u}) \cdot \frac{1}{\sqrt{u}}$
 $v = \frac{1 + 2v\sqrt{1+u}}{\sqrt{u}}$
 $v = \frac{\sqrt{1+u} \left(\frac{1}{\sqrt{1+u}} + 2v \right)}{\sqrt{u}}$

Δt	Technique	Interpretation
0		The student isolates v by dividing by \sqrt{u}
3	Rewrite	
4	Rewrite	
5	Factor	The student realizes $2v$ poses a problem and tries to lose this term by factoring
9		The student tries a next step, backtracks, tries again and finally gives up

Quantitative analysis

Table 5: The multilevel model

	Null model	Model 1a	Model 1b	Model 2a	Model 2b	Model 4 ⁽¹⁾	Model 5
<i>Fixed</i>							
Intercept	78.893 (2.230)***	79.023 (2.046)***	79.023 (2.046)***	79.069 (1.960)***	79.069 (1.960)***	79.014 (1.941)***	79.528 (1.994)***
(pre-test-gm)		0.290 (0.038)***	0.290 (0.037)***	0.308 (0.037)***	0.308 (0.037)***	0.271 (0.038)***	0.248 (0.049)***
(time manual hr-gm)		-0.969 (10.878)	---	---	---	---	---
(time d14 hr-gm)				1.389 (1.246)	---	---	---
(time d5 hr-gm)				-0.264 (0.314)	---	---	---
(time d6 hr-gm)				1.931 (0.515)***	1.931 (0.515)***	1.189 (0.539)*	1.189 (0.539)*
(MATH-gm)				7.814 (4.986)*	7.143 (4.482)*	7.321 (4.757)*	7.282 (4.631)*
						5.034 (1.562)**	5.257 (1.414)***
<i>Random</i>							
Level 2 class	65.150 (27.235)**	55.996(22.857)**	55.040 (22.909)**	48.310 (20.965)**	50.538 (21.093)**	49.794 (20.718)*	50.270 (11.999)***
(pre-test-gm)/cons							-1.233 (0.500)*
(pre-test-gm)/(pre-test-gm)							0.027 (0.014)*
Level 1 student	172.197 (14.631)***	138.676 (11.914)***	138.674 (11.913)***	130.317 (11.195)***	129.258 (11.104)***	124.603 (10.704)***	120.113 (10.342)***
<i>Model fit</i>							
Deviance (-2 loglikelihood)	2363.508	2254.038	2254.046	2235.904	2233.737	2223.535	2203.204
χ^2		109.470	109.462	18.142	20.309	10.202	20.331
Df		2	1	4	2	1	2
P		$p < 0.001$	$p < 0.001$	$p < 0.01$	$p < 0.001$	$p < 0.001$	$p < 0.001$
Reference		M0	M0	M1b	M1b	M2b	M4
N	286	286	286	286	286	286	286

Values between brackets are the standard errors. 'gm' refers to Grand Mean centering.

(1) Only the final model 4 was added to the table.

* $p < 0.05$.

** $p < 0.01$.

*** $p < 0.001$.

Bokhove, C. & Drijvers, P. (in press). Effects of a digital intervention on the development of algebraic expertise. *Computers & Education*.

Feedback questions:

Jeuring: Hints and feedback in learning environments for algebra

- Design?
- Availability?
- Timing?
- Fading?
- Adapting?

The screenshot shows a web browser window titled "DWO Math Enviroment - Mozilla Firefox". The page is titled "4. quadreq 3" and is part of the "Digitale Wiskunde Omgeving" by the "Freudenthal Instituut". The user is not logged in ("Niet ingelogd").

The main content area displays the equation $x(2x - 4) = 0$ and asks to solve it ("Los de vergelijking op."). The solution steps are shown:

$$x(2x - 4) = 0$$
$$x = 0 \text{ of } 2x - 4 = 0$$
$$x = 0 \text{ of } 2x = 4$$
$$x = 0 \text{ of } x = 2$$

On the right side, there are four hint boxes:

- de factoren op 0 stellen
- constante termen naar rechts brengen
- variabele vrijmaken door beide kanten te delen
- correct opgelost

At the bottom, there is a progress bar for the assignment ("Opdracht") with 10 steps. Step 5 is highlighted in green, indicating the current step. The score is 10 out of 10 ("Score: 10 totaal: 10").



Section C

Uitloggen

Taxi Fares

The Rainbow Cab Company new rates.

The base price is \$3.00.

The price per mile is \$1.30.

Siluh is a taxi driver. Now that the new prices are in effect, he wants to post a rate chart in his taxi. The rate chart will show the customers the new fares for typical distances.

9 Use the applet to make such a rate chart for Siluh.

miles	rate
.	...
...	...
...	...
...	...
...	...
...	...

After the company changed its rates, George slept through his alarm and had to take a taxi to work. He was surprised when it cost \$18.60!

- 10 a. Use the machine that you made in problem 8 to calculate the distance from George's home to work.
- b. It's also possible to find the answer to part a by making a calculation instead of using the machine. Make an arrow string for this calculation.

In-/Uitvoer

Bewerkingen

price

price

$x \rightarrow 1.3 \cdot x + 3$

Boon et al.

Pagina: 1 2 3 4 5 6 7 8

Score: 5 10

totaal: 15

ICT and concept development: the case of derivative in the DME

Digital Math Environment - Google Chrome
www.fi.uu.nl/dwo/sk/en/frameset.html

Digitale Wiskunde Omgeving
Departement Scheikunde uu >> E. Differentiation

Introduction Differentiating

1.3 Variations and slope in a graph

From the previous page we conclude that the **difference quotient** is a measure of how 'fast' the function value y changes compared with a given variation in x .

On the right, a function f and an interval are shown (this time the interval is given in more general notation, from x to $x + \Delta x$).

From the graph of f we see that the difference quotient is the same as the **slope** of the line through P and Q , the boundary points of the interval.

Problem 3

For an interval with starting point $x = 1$ we have $\frac{\Delta y}{\Delta x} = -0,5$. Find Δx as accurate as possible by adjusting the boundaries of the interval.

Answer: $\Delta x =$ ✓

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{0.82 - 1.63}{2.6 - 1} = \frac{-0.81}{1.6} = -0.5$$

Show values

Conclusion

- Algebraic expertise is versatile and complex
- ICT is promising but not straightforward to exploit
- We need to know more about the problem with secondary algebra education and the role ICT can play in solving this issue...
- ... so let's listen to our speakers!

Today's agenda

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