

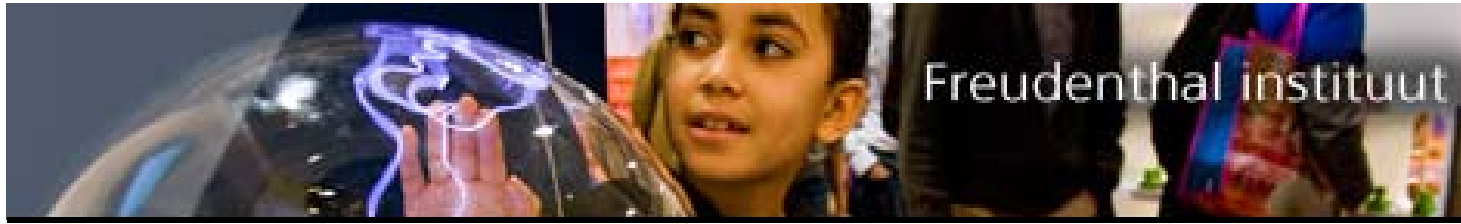
Goede middag

Ik ben zeer pleased hier te zijn

Jammer genoeg spreek ik het geen Nederlands,

So

Good afternoon!



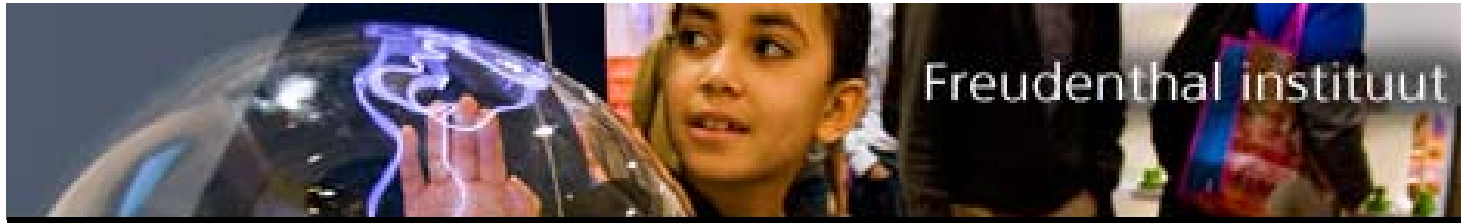
Universiteit
Utrecht

Mini-symposium Algebra & ICT

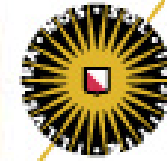
Technology at the service of learning algebra

Abraham Arcavi
Department of Science Teaching
Weizmann Institute of Science, Rehovot, Israel

December 13th, 2011



Freudenthal instituut



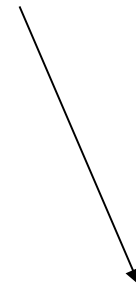
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*Technology at the service of
learning algebra*



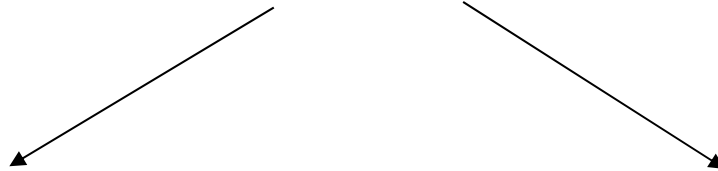
Basic skills



Symbol sense

What is **symbol sense**?

Focus: the search for **meaning**



Non-automatic actions

Uses, connections

Focus: the search for **meaning**

Starting point: symbols

End point: symbols

Starting point: symbols

The story of $\frac{2x + 3}{4x + 6} = 2$

$$\frac{2x + 3}{4x + 6} = 2 \quad \longrightarrow \quad 2x + 3 = 2(4x + 6)$$

$$2x + 3 = 8x + 12 \quad \longrightarrow \quad -9 = 6x$$

$$x = -1.5$$

**Starting point: symbols
and remaining within symbols**

**Starting point: symbols
and looking elsewhere...**

LEÇONS ÉLÉMENTAIRES
SUR LES MATHÉMATIQUES

DONNÉES A L'ÉCOLE NORMALE EN 1795.

Joseph-Louis Lagrange (1736-1813)



As long as algebra and geometry proceeded along separate paths, their progress have been slow and their applications limited.

But

when these two sciences joined company, they drew from each other fresh vitality and thenceforward marched on at a rapid pace towards perfection.

Geometry

- Space and its properties
- Graphic, diagrammatic representations
- Perceptual appeal
- Spatial intuition
- Rhetorical

Algebra

- Mathematical relations
- Structure
- Operational symbolism
- Manipulative
- Abstract, perception free

$$\frac{1}{9} + \frac{1}{8} \cdot \frac{1}{9} = \frac{1}{8}$$

$$\frac{1}{9} + \frac{1}{8 \cdot 9} = \frac{1}{9} + \frac{1}{72} = \frac{8+1}{72} = \frac{9}{72} = \frac{1}{8}$$

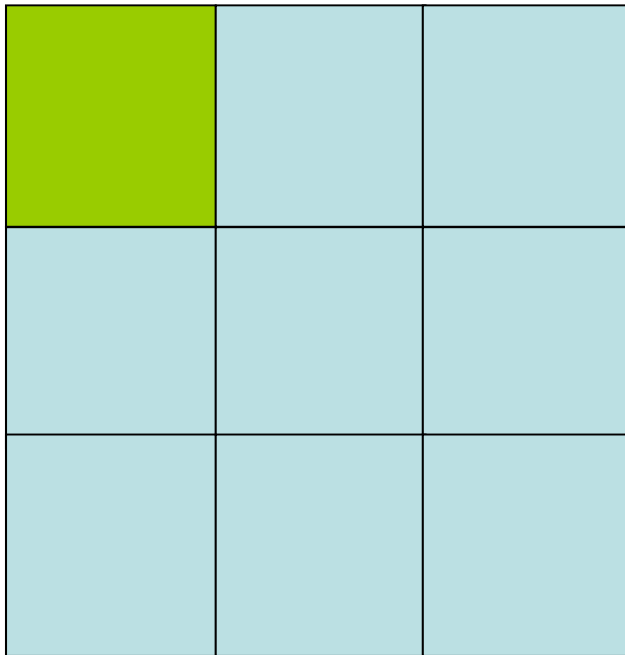
$$\frac{1}{9} + \frac{1}{8} \cdot \frac{1}{9} = \frac{1}{8}$$

$$\frac{1}{9} \cdot \left(1 + \frac{1}{8}\right) = \frac{1}{\cancel{9}} \cdot \frac{\cancel{9}}{8} =$$

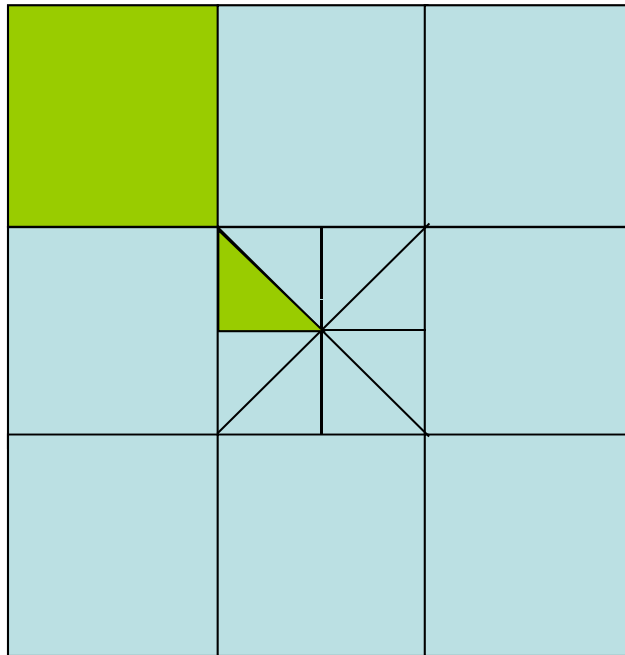
$$\frac{1}{9} + \frac{1}{8} \cdot \frac{1}{9} = \frac{1}{8}$$

$$\frac{\cancel{1}}{9} + \frac{1}{8} - \frac{\cancel{1}}{9} = \frac{1}{8}$$

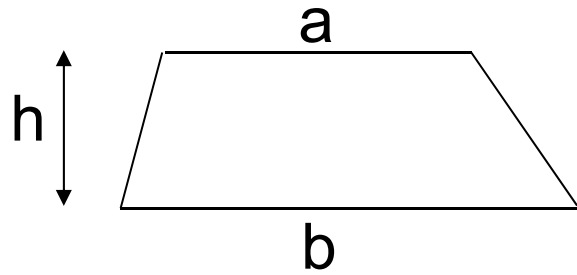
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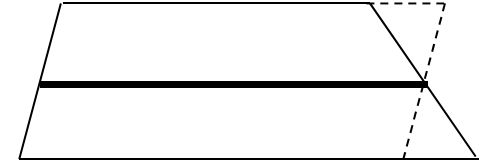
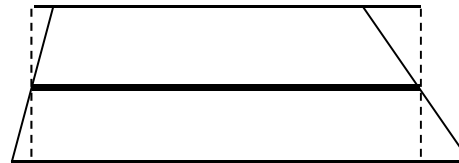
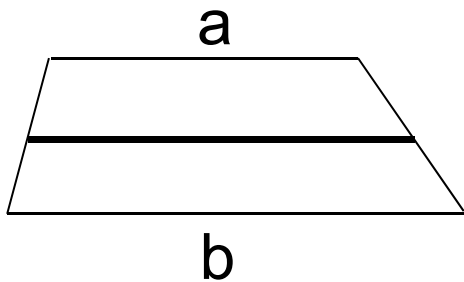


The area of a trapezoid

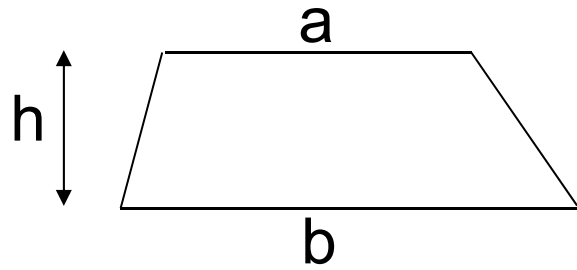


$$\frac{a + b}{2} \times h$$

What is the geometrical meaning of $\frac{a + b}{2}$?

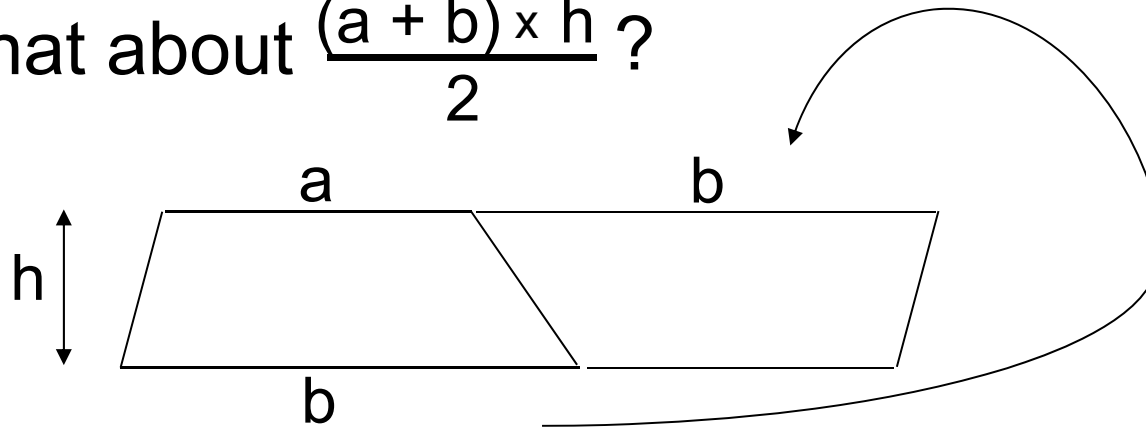


The area of a trapezoid

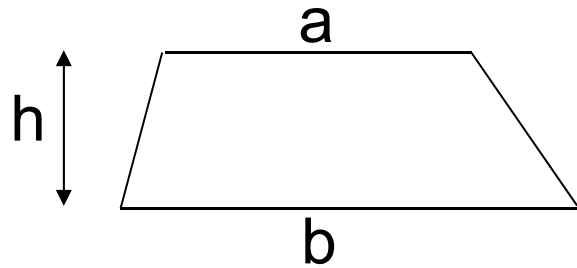


$$\frac{a + b}{2} \times h$$

What about $\frac{(a + b) \times h}{2}$?

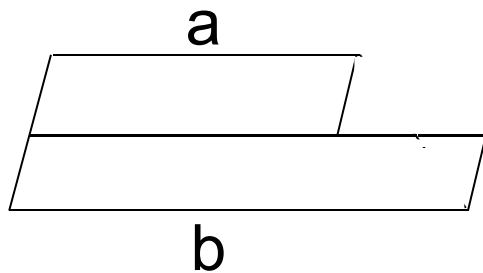


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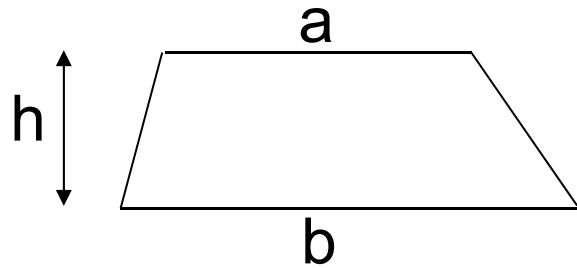


$$\frac{a + b}{2} \times h$$

What about $a \times \frac{h}{2} + b \times \frac{h}{2}$?

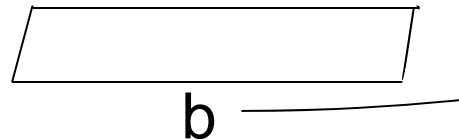
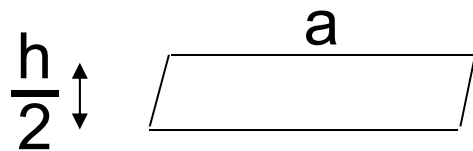


The area of a trapezoid

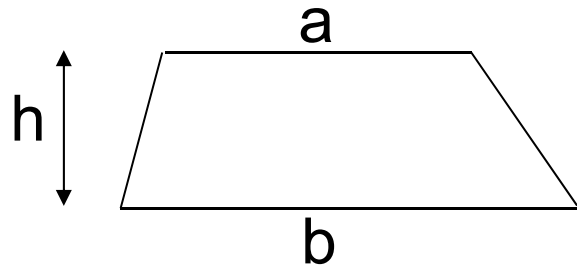


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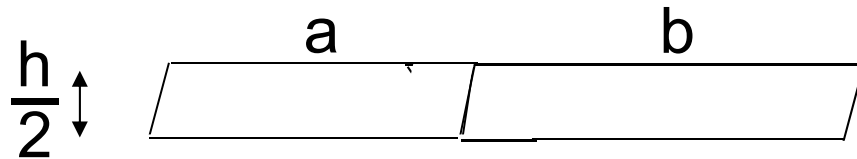


The area of a trapezoid



$$\frac{a + b}{2} \times h$$

What about $a \times \frac{h}{2} + b \times \frac{h}{2}$?

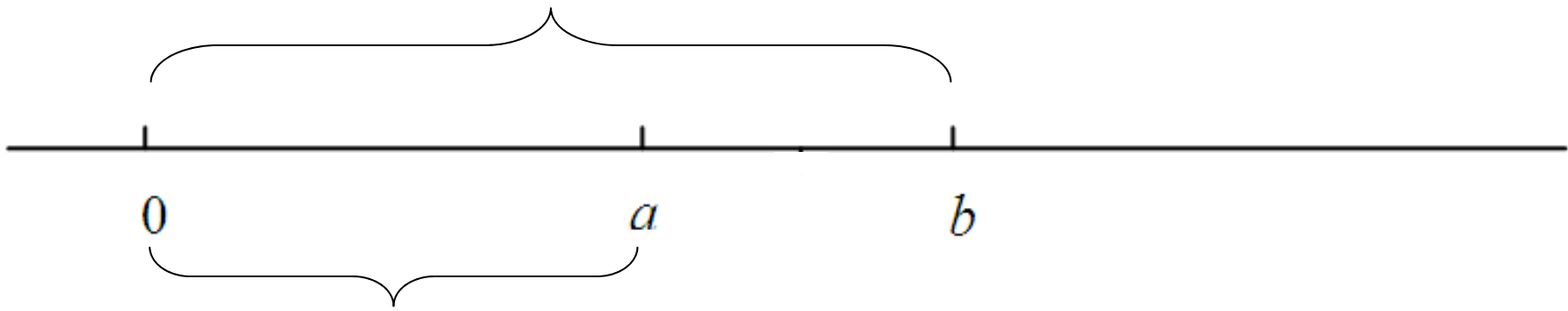


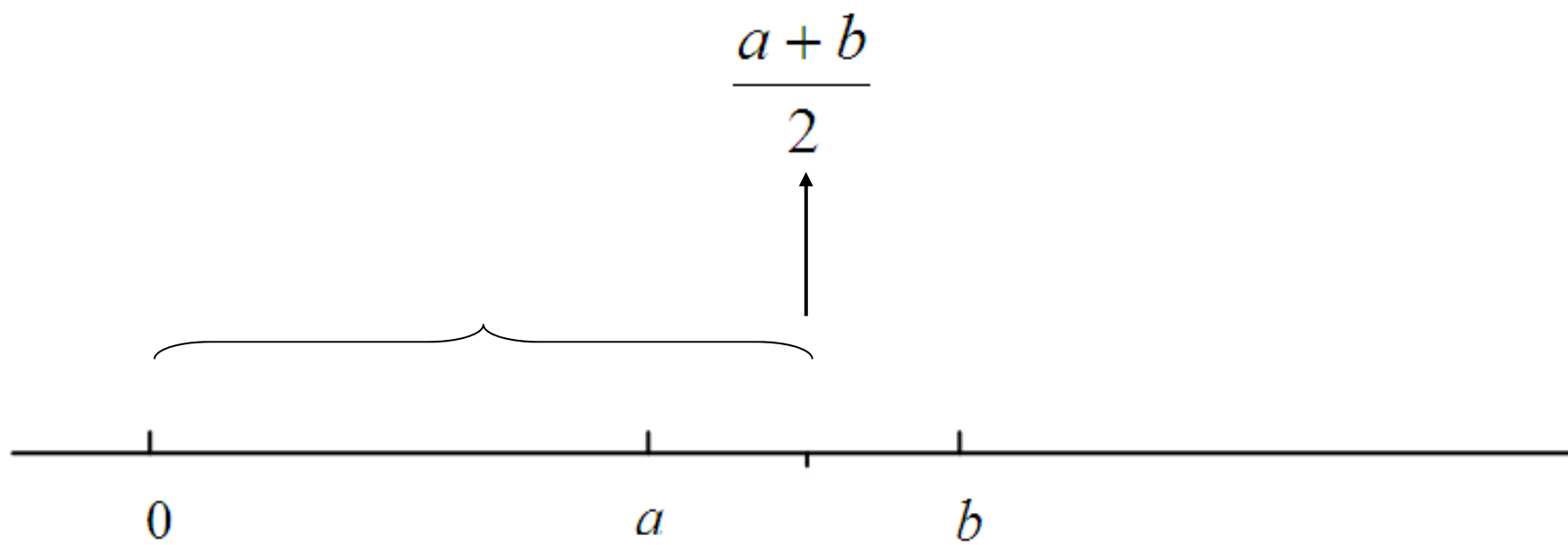
Note that sometimes the average of two positive numbers is the same as their (positive) difference.

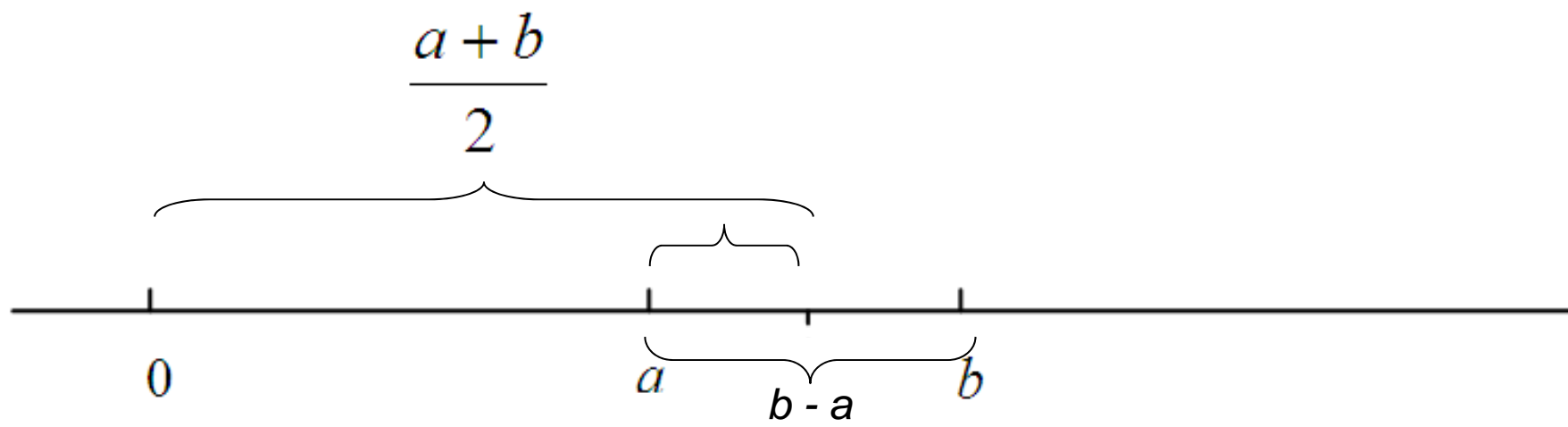
For example, the average between 10 and 30 is 20, and also $30-20=10$, but the average between 10 and 16 is 13, but $16-13 \neq 13$).

So, if $a > 0$ and $b > 0$, $a < b$, for which pairs of numbers their average is the same as their (positive) difference?

$$\frac{a + b}{2} = b - a \quad \Rightarrow \quad a + b = 2b - 2a$$



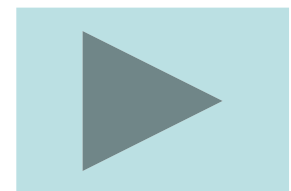




Focus: the search for **meaning**

Starting point: symbols

End point: symbols

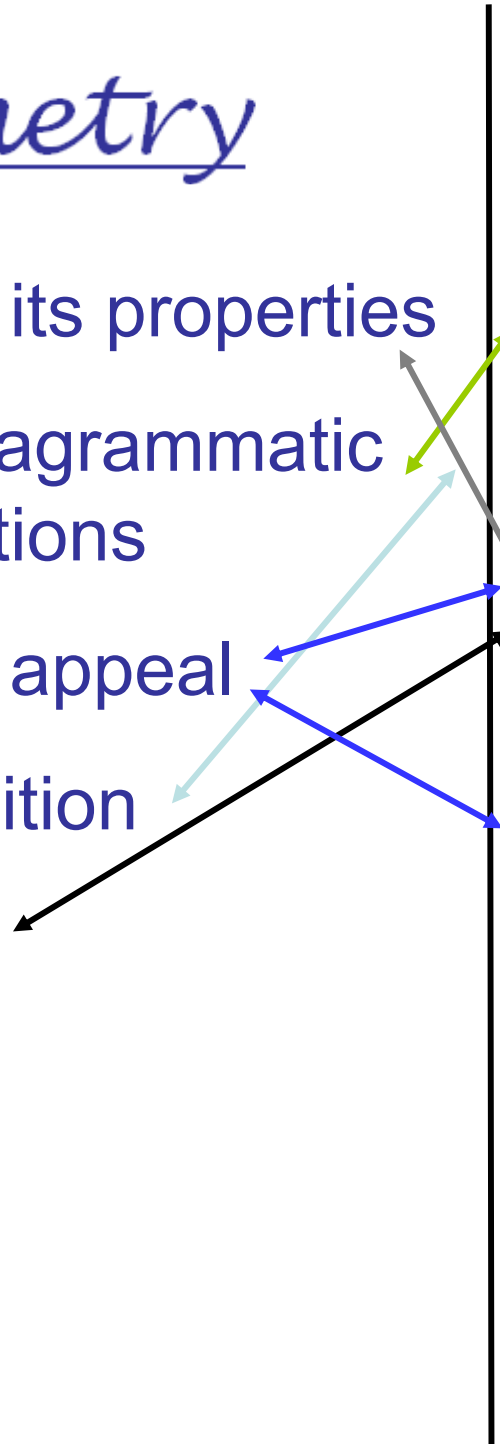


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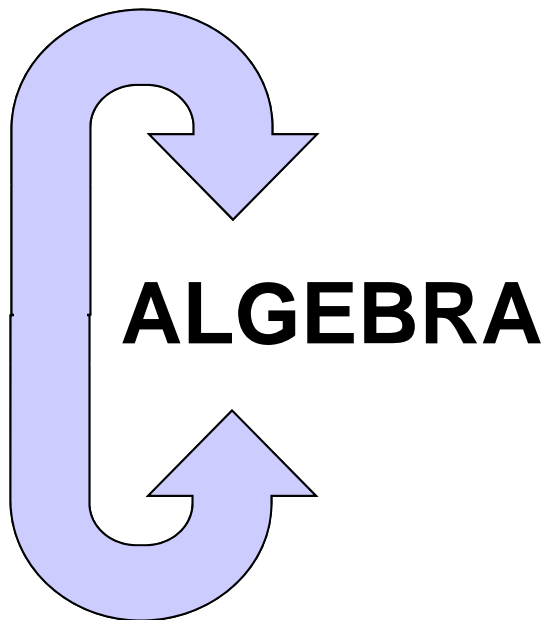
Symbol sense

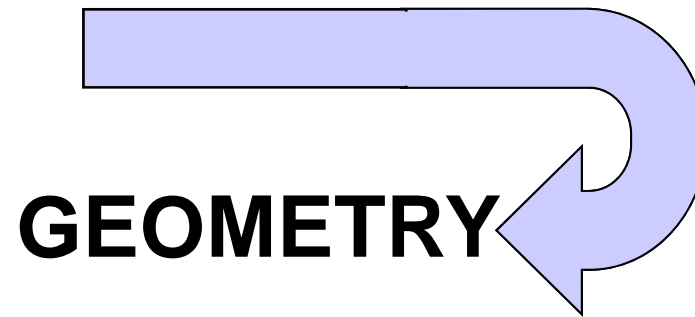
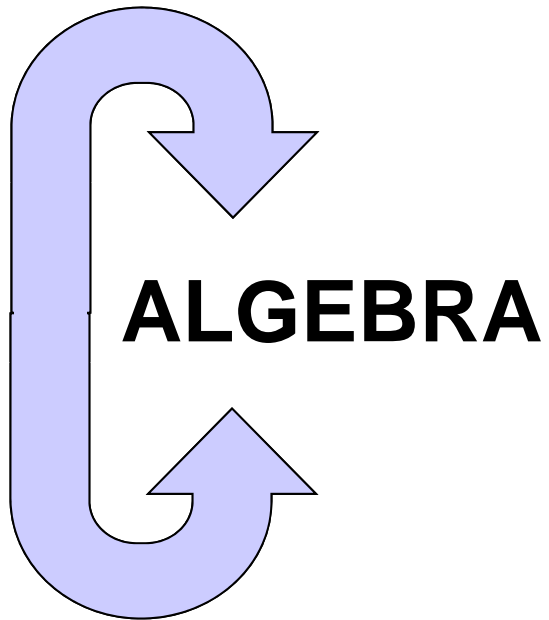
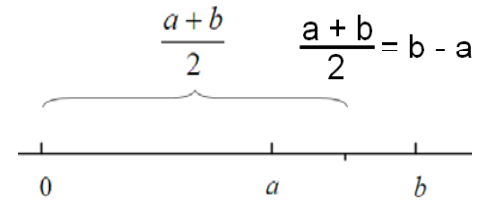
Focus: the search for **meaning**

Starting point: symbols

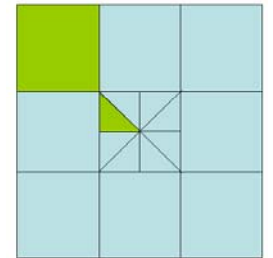
End point: symbols

The story of $\frac{2x+3}{4x+6} = 2$





$$\frac{1}{9} + \frac{1}{8} \cdot \frac{1}{9} =$$



The area of a trapezoid

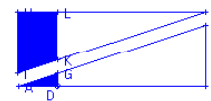
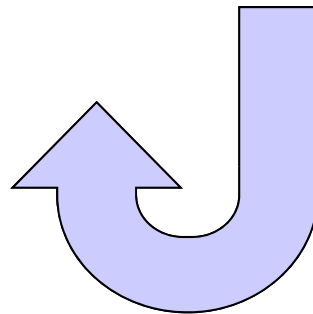
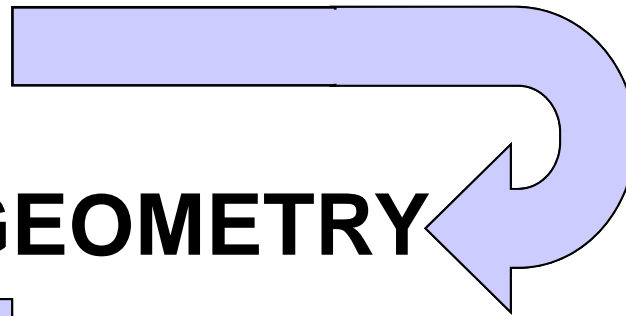
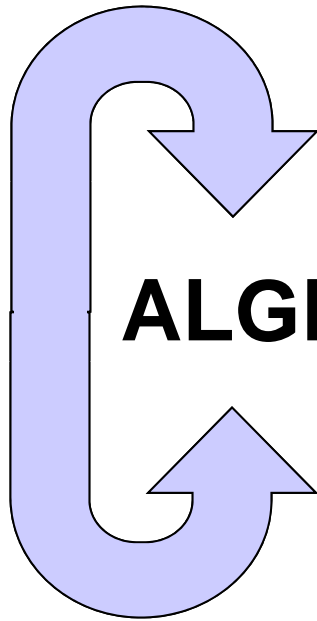


What about $\frac{(a+b) \times h}{2}$?



ALGEBRA

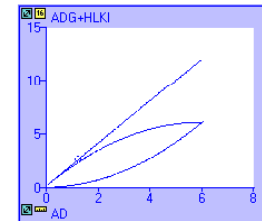
GEOMETRY



AD = 1.25

ADG = 0.26

HLKI = 2.30

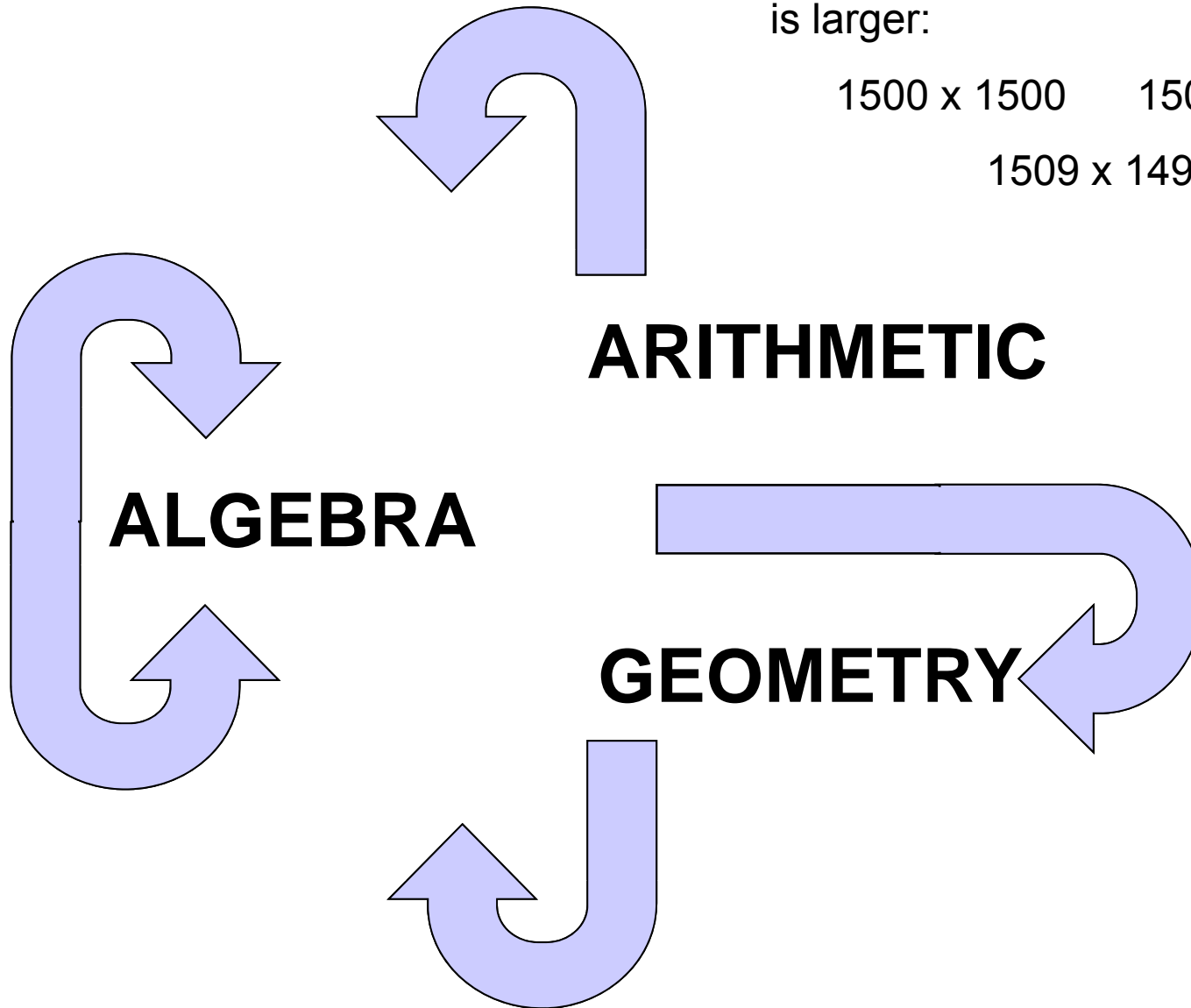


ADG+HLKI = 2.56

Without calculating, establish which is larger:

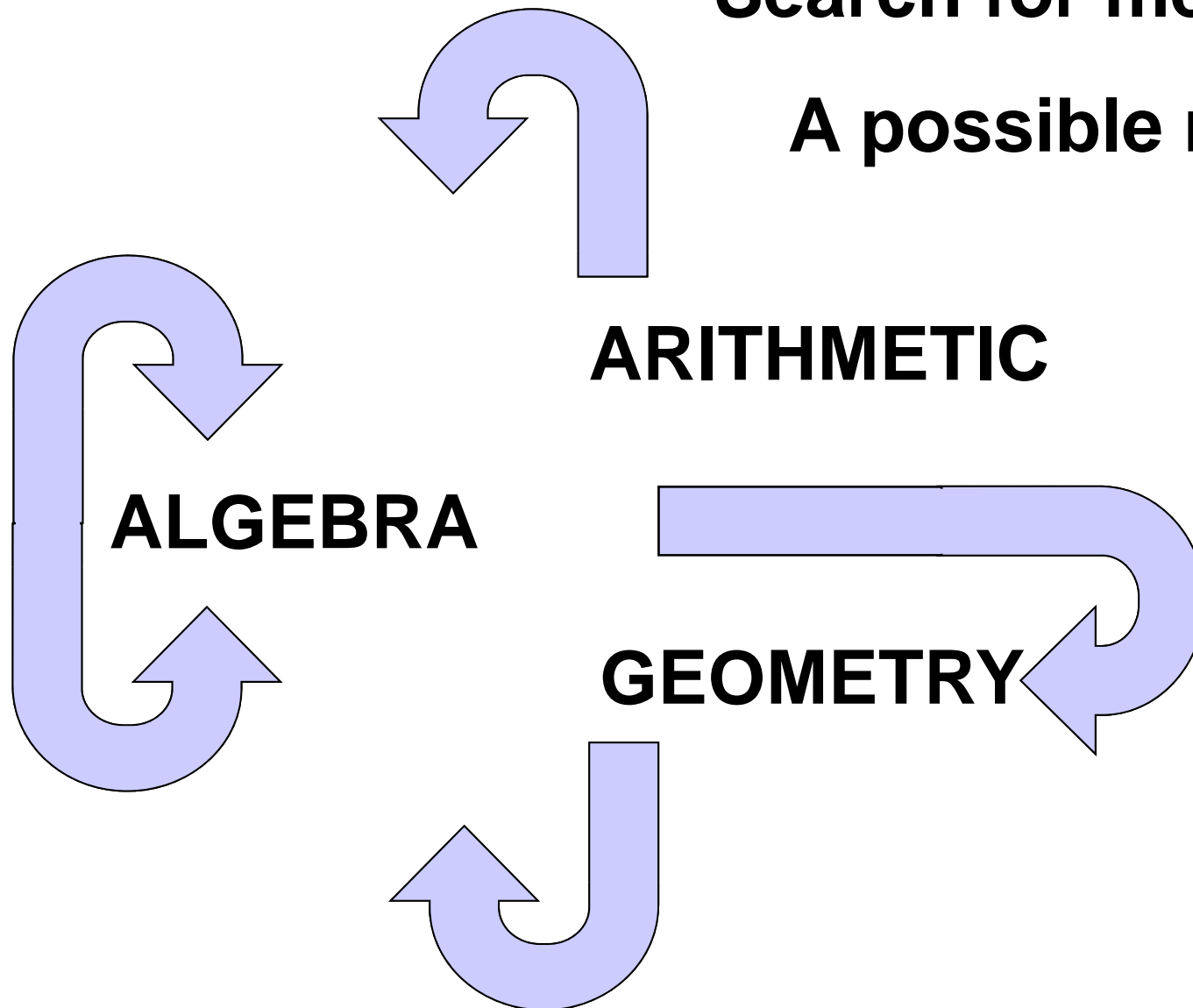
$$1500 \times 1500 \quad 1501 \times 1499$$

$$1509 \times 1491$$



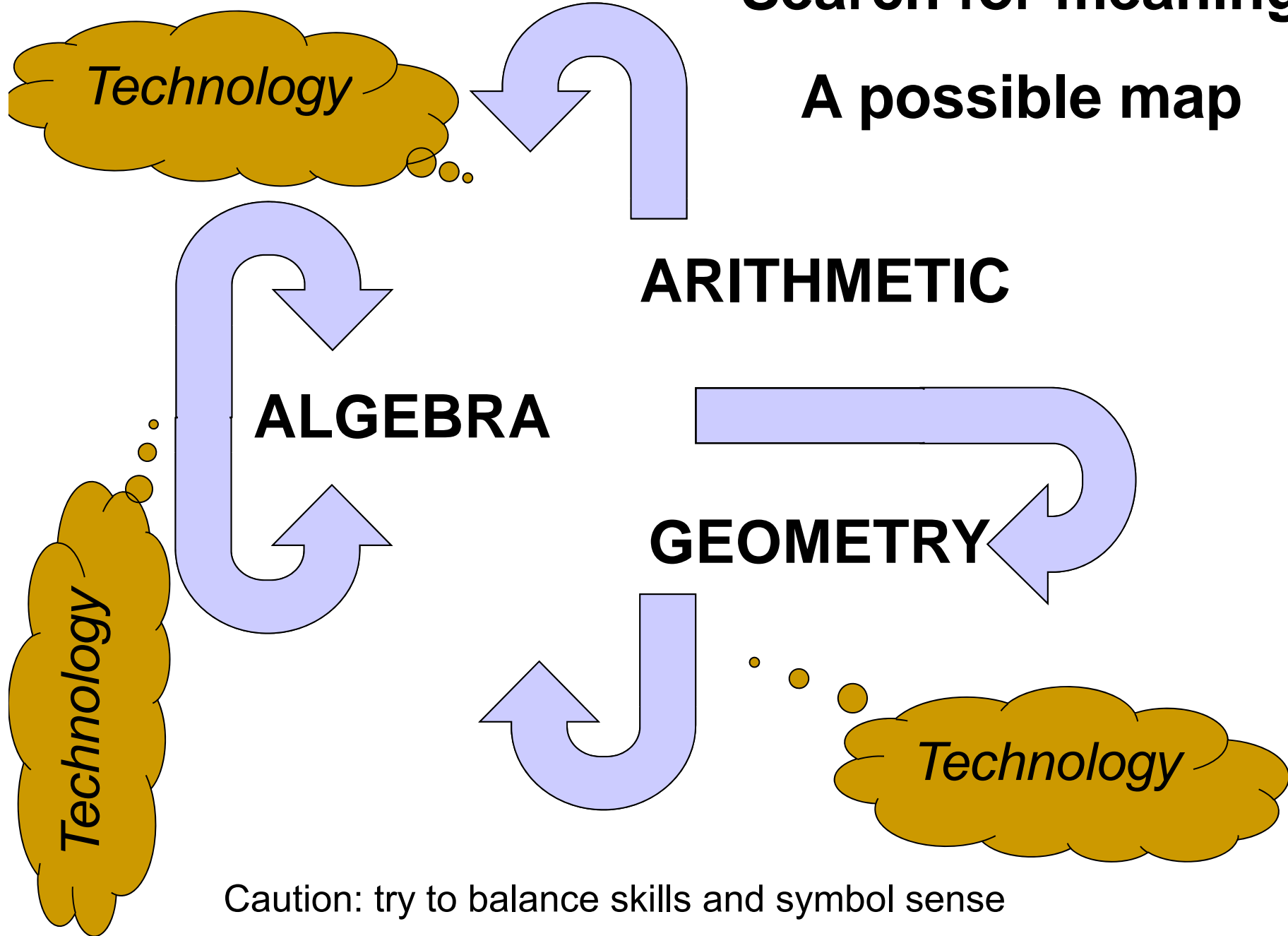
Search for meaning

A possible map

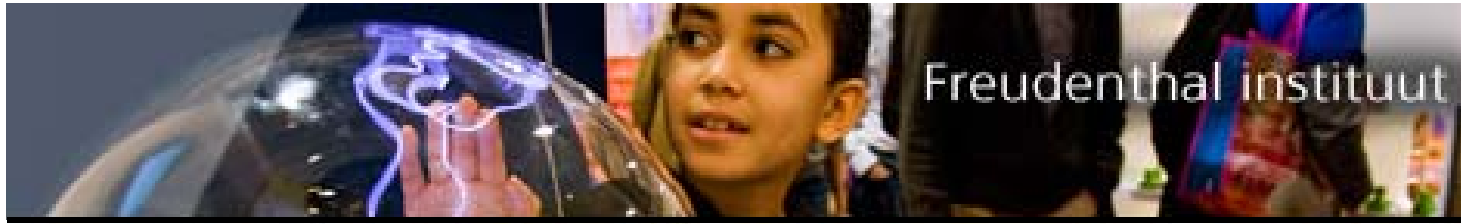


Search for meaning

A possible map



Caution: try to balance skills and symbol sense



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Hartelijk dank

December 13th, 2011