REPRESENTATIONS & PEER FEEDBACK

How many text messages are sent if four people all send messages to each other?

How many text messages are sent with different numbers of people?

https://www.primas.mathshell.org/pd/modules/7_Self_and_Peer_Assessment/html/index.htm

INQUIRY AND MODELING AS LEARNING FACILITATORS FOR MATHEMATICS

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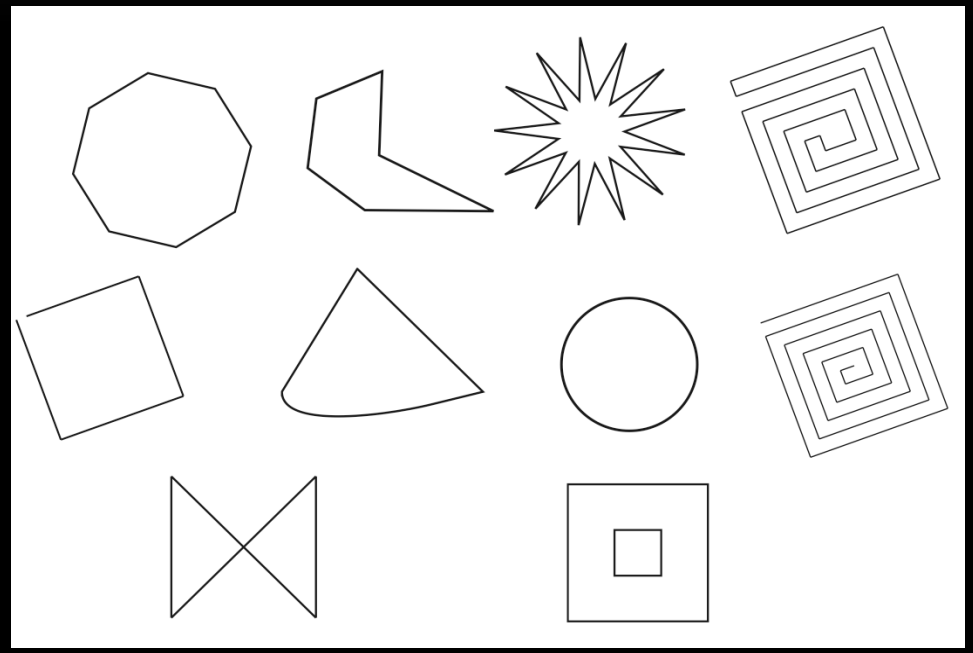
IBL IN MATHEMATICS
CLASSIFYING AND DEFINING

- Write down the definition of a polygon
- Exchange definitions and try to improve them
- Sort the objects: which are polygons according to your definition?
- Definitions need improvement?



IBL IN MATHEMATICS
PROVING AND ROLE OF EXAMPLES

Statements: never, sometimes or always true?

- A pentagon has fewer right angles than a rectangle
- If you add the same number to the top and bottom of a fraction, the fraction gets bigger in value
- Max gets a pay rise of 30% and Jim gets a pay rise of 25%. So Max gets the bigger pay rise.
- The integral of the derivative of function *f* equals the derivative of the integral of *f*

DRUG LEVEL – EXAMPLE TASK

DRUG LEVEL

A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient.

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- How much mg of the drug is in the blood of the patient after one day?
- Finish the table.

Day	Mg of drug in blood
0	0
1	1125
2	
3	

- Explain why you can calculate the amount of drug for the next day with the formula: new_amount = (old_amount + 1500) * 0,75
- After how many days has the patient more than 4 g medicine in the blood? And after how many days 5 g?
- What is the maximum of amount of the drug that can be reached?

DRUG LEVEL - QUESTIONING

A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient.

What (mathematical) question could you ask?

DRUG LEVEL – PLANNING

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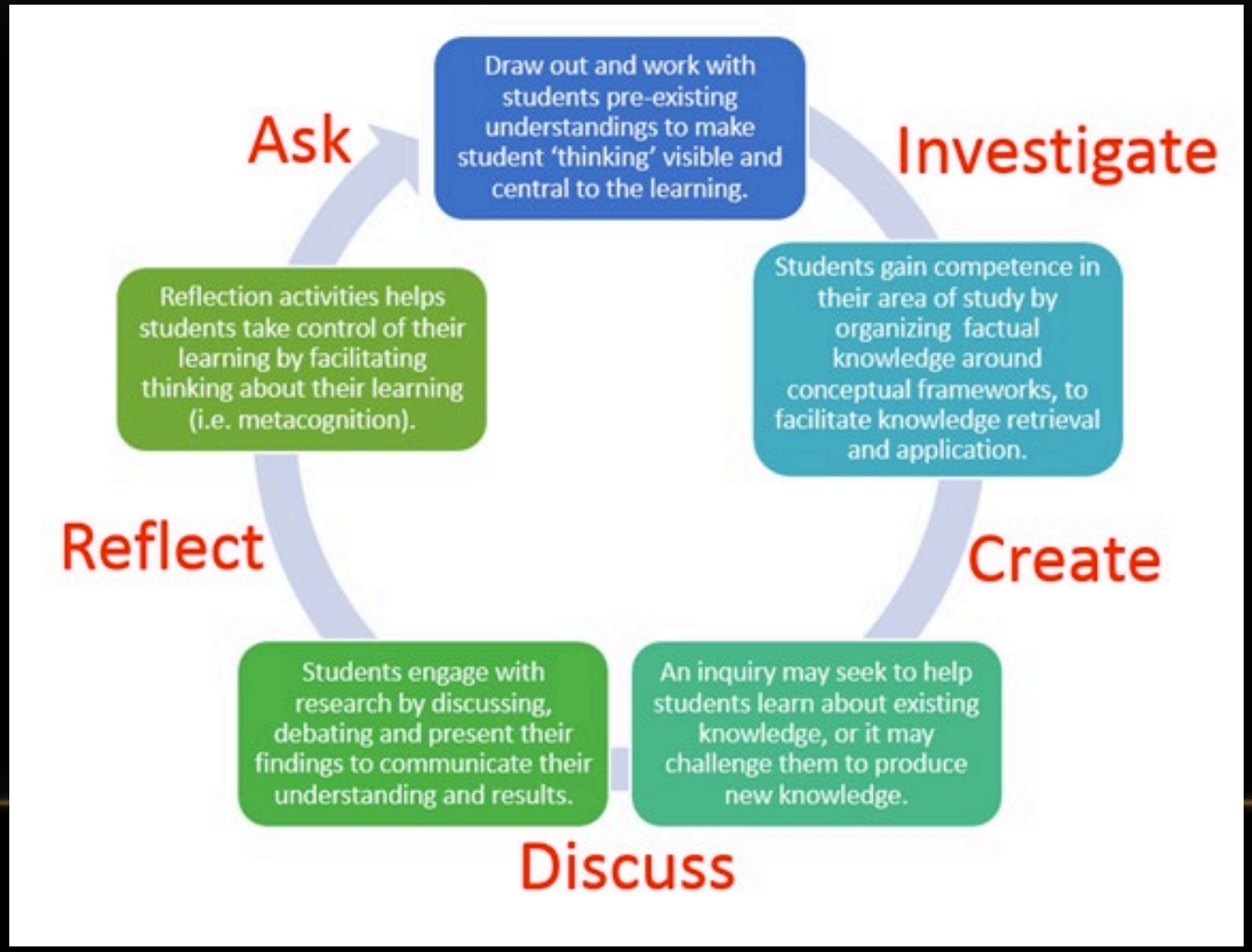
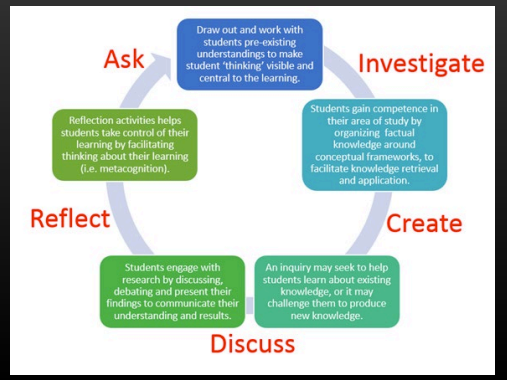
CHANGES IN THE DRUG LEVEL TASK INSPIRED BY INQUIRY-BASED LEARNING



INQUIRY BASED LEARNING (IBL) IS ……



- Education with an active role for students by involving them in (a selection of) phases of inquiry
- Goals: Raise interest, create ownership, deeper learning, develop inquiry skills
- Conditions: teaching materials and methods, classroom culture, …

- Inquiry based instruction is an intentional student-centered pedagogy that challenges the learner to explore concepts, ideas, and/or phenomena before formal explanations are provided (Marshall e.a., 2017)

FINDINGS

- A positive and large effect on students engaging in the epistemic domain of inquiry (Furtak et al. 2012)
- IBL (can) result in better understanding of science (Minner et al. 2010)

  "Fifty-one percent of the 138 studies showed positive impacts of some level of inquiry on student content learning and retention (…) there was no statistically significant association between amount of inquiry and increased student science conceptual learning"

- *G*reater improvements in students' science literacy and research skills, but lower gain in self-confidence in scientific abilities … (Gormally et al. 2009)

  "maybe due to experiencing complexity and frustrations, and over-estimation of traditional taught students"

- Direct instruction, worked examples and repeated practice have proven to be effective (Kirschner e.a., 2006)

REACTION ON KIRSCHNER E.A. (2006) (HMELO-SILVER E.A., 2007)

- IBL ≠ ‘free discovery’
- IBL works, when sufficiently ‘guided’

  Guidance is needed to

  - Make content knowledge and skills explicit
  - Structure complex and open tasks

  Offer support (scaffolds) with lesson plans, worksheets, …
- Structure out of your task → Structure into your lesson!

IBL ≠ 'FREE DISCOVERY'

- Various interpretations of IBL (Bruder & Prescott, 2013)
  - Structured Inquiry: The teacher gives the students a problem or question to be solved as well as the appropriate strategy and materials
  - Guided Inquiry: The teacher provides the students with the problems or questions and the necessary materials. Students have to find strategies
  - Open Inquiry: Students take the initiative to find problems or questions they would like to solve and answer
- Guided inquiry in mathematics education appeared most effective for motivation (and results) providing opportunities for students…
  - to generate strategies and solutions
  - to discuss and compare strategies and solutions
  - to make decisions and justify their decisions

IBL – A HOLISTIC VIEW (FOCUS IN PRIMAS)

Valued outcomes

- Inquiring minds
- Applying mathematics in real life
- Preparing for citizenship and lifelong learning
- Understanding the nature of science

What students do

- Inquire, pose questions
- Engage in problem solving
- Reflect on results and processes
- Make sense for themselves

Classroom culture

- Shared sense of purpose and justification
- Dialogic
- Collaborative

Teaching methods

- Evoke and structure inquiry processes
- Motivate and guide students
- Value and connect to students' reasoning

Teaching materials / IBL tasks

- Meaningful contexts / learning situations
- Involve students in inquiry processes
- Tasks ask for collaboration and communication

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EU IBL-PD-EVALUATION: PRIMAS

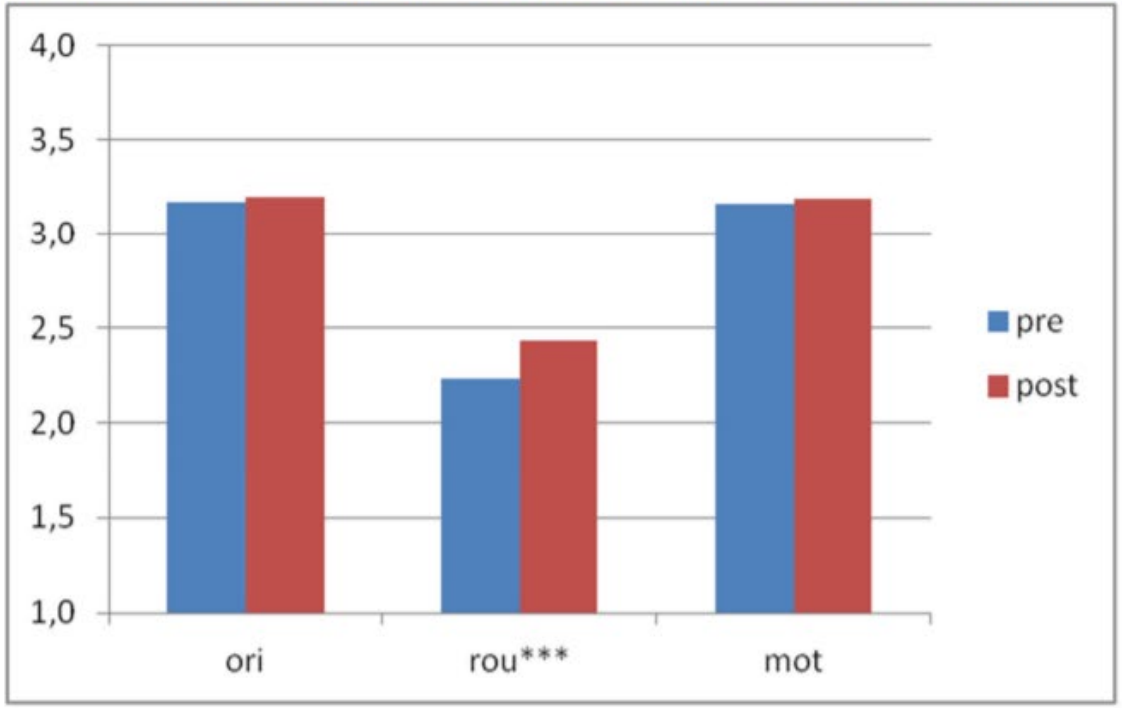
Table 11

Overview of the paired-samples of the teacher (pre-post study)

	Sample size (proportion)		science	maths
Cyprus	68	(.12)	27	40
Germany	14	(.03)	3	10
Hungary	72	(.13)	40	27
Malta	21	(.04)	8	13
Netherlands	16	(.03)	13	2
Norway	10	(.02)	0	10
Romania	100	(.02)	24	72
Slovakia	83	(.15)	53	30
Spain	49	(.09)	7	33
Switzerland	65	(.12)	14	50
UK	65	(.12)	0	64
Total	563	1.00	189	351

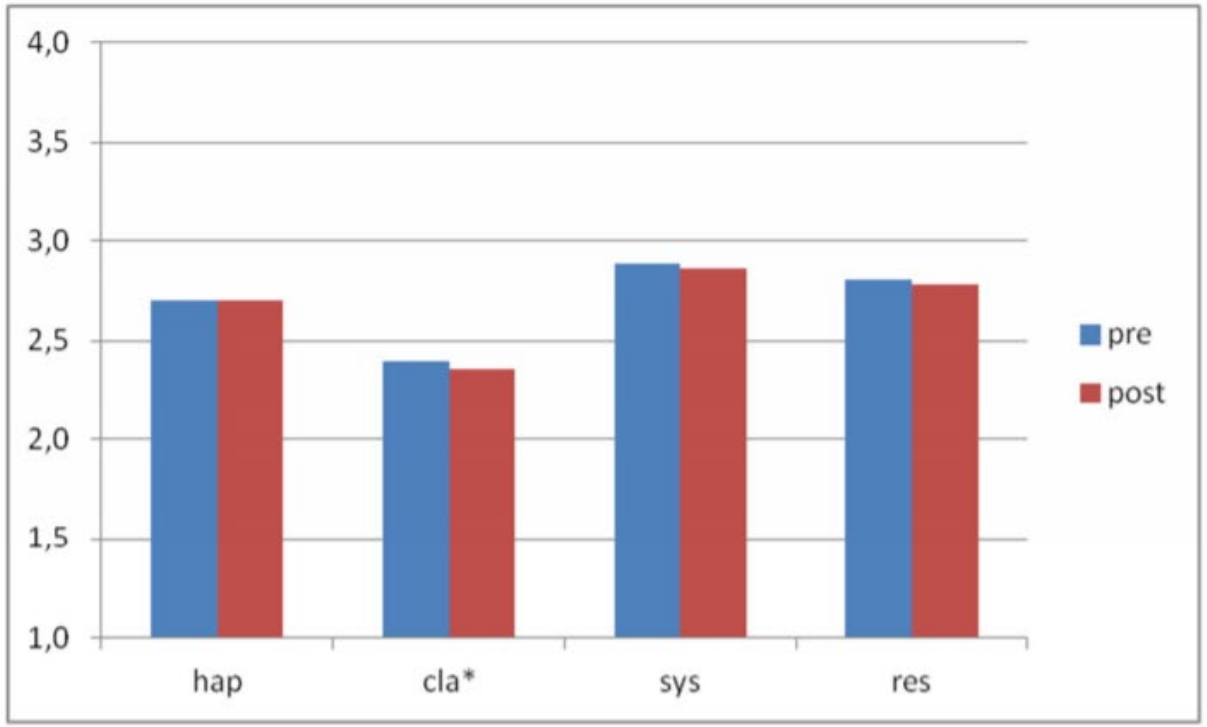
PRIMAS

Pre-post comparison of orientation towards IBL, routine use of IBL and belief that IBL motivates students (paired samples) (1: strongly disagree, 2: disagree, 3: agree, 4: strongly agree) (***difference is significant at the .01 level)



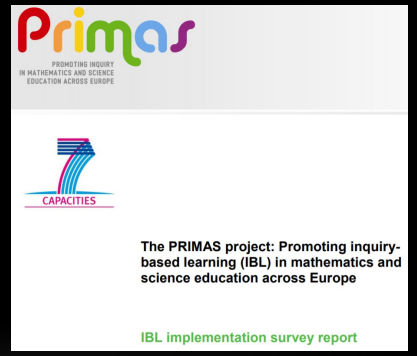
PRIMAS

Changes of happiness with the existing situation and of problems with implementation of IBL: *happiness (hap), classroom management (cla), resources (res)* and *system restrictions (syr)* (1: strongly disagree, 2: disagree, 3: agree, 4: strongly agree) (*difference is significant at the .05 level)



INQUIRY AS LEARNING FACILITATOR

- Helpful for enriching teaching repertoires
- Necessary for addressing and developing inquiry skills
- Can facilitate content learning (by eliciting and connecting to students' contributions)



$$a + 2 \cdot a + 7 = 3a + 7$$
$$a + 2 \cdot a + 7 = 3a + 14$$
$$a + 2 \cdot a + 7 = a^2 + 2a + 7$$
$$a + 2 \cdot a + 7 = a^2 + 9a + 14$$

INQUIRY AND MODELING AS LEARNING FACILITATORS FOR MATHEMATICS

REALISTIC MATHEMATICS EDUCATION [RME]

- Freudenthal (1905 – 1990):
  anti-didactical inversion = endpoint of the work of mathematicians is used as a starting point for instruction

REALISTIC MATHEMATICS EDUCATION [RME]

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REALISTIC MATHEMATICS EDUCATION [RME]

- Freudenthal (1905 – 1990):
  anti-didactical inversion = endpoint of
  the work of mathematicians is used
  as a starting point for instruction

- Alternative: learning mathematics as an activity

  - organizing subject matter from reality

  - organizing mathematical subject matter

- Result: mathematics that is more meaningful, relevant and applicable

DRUG LEVEL - IBL & MODELING

A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient.

1e dag werkt 1500 mg
2e dag werkt 1125 mg + 1500 mg = 2625 mg
3e dag werkt 844 mg + 1125 mg + 1500 mg = 3469 mg
4e dag werkt 633 mg + 844 mg + 1125 mg + 1500 mg = 4102 mg
5e dag werkt 475 mg + 633 mg + 844 mg + 1125 mg + 1500 mg = 4576 mg
6e dag werkt 356 mg + 475 mg + 633 mg + 844 mg + 1125 mg + 1500 mg = 4932 mg

DAGEN	1	2	3	4	5	6
werkend medicijn (mg)	1500	2625	3469	4102	4576	4932
toename werkend medicijn (mg)	-	1125	844	633	475	356

dus:

DAGEN	
werkend medicijn (mg)	5
toename werkend medicijn (mg)	

met het slikken
eindpeil boco
bovenstaande t

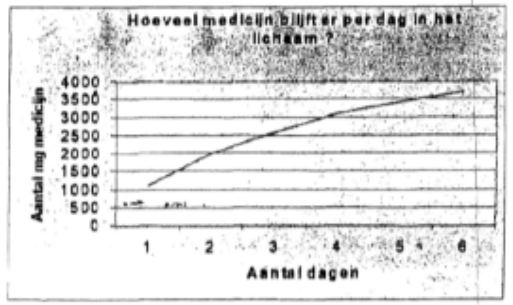
	3e x	totaal
	375	1125
0,75	(1789,0625)0,75	1341,8
	1341,8	
0,75	(1411,5+1500) 0,75	
	1433,26	1433,26
		1471,84
		1488,12
		1494,99

de halveringsregel

Als je eenmaal per dag naar het toilet gaat verlaat 25 % van de door jou ingenomen medicijnen je lichaam
Dat betekent dat als je eerste dag van je medicijngebruik 3 keer 500mg slikt er daarvan 1500 * 0,75 = 1125mg in je lichaam overblijft

Als je elke dag 3 keer 500mg van het medicijn zou slikken krijg je het volgende resultaat

Dag	*totaal (mg)*	*Via1*	*Via2*	*Via3*
1	1125			
2	1986,75	843,75		
3	2601,5625	632,8125	210,9375	
4	3076,2	474,6375	158,175	52,7625
5	3432,1	355,9	118,7375	39,437
6	3699,09	266,99	88,91	29,8275



De verschilrijen worden niet constant dus is het ook niet mogelijk bij deze rij een directe formule te geven. Wel is er een recusieve formule die luidt : $tn+1 = (1500 + tn) * 0,75$
Dit betekent dat het aantal medicijn in je lichaam geleik is aan het aantal van de vorige dag, daarbij komt 1500 mg en na het plassen blijft er nog 75 % van de totale hoeveelheid over in je lichaam

Het kan gebeuren dat je een dag vergeet je medicijnen in te nemen
Kun je dan zomaar de volgende dag de dubele dosis innemen en heeft dit gevolgen voor het eindpeil ?
Dat is in een tabel duidelijk weer te geven

Dag	*Constant*	*1 keer overslaan*
1	1125	1125
2	1986,75	843,75
3	2601,5625	2882,8125

Tussen de eindhoeveelheden zit niet zo een groot verschil, ongeveer 281,25 mg
Maar als je meerdere dagen overslaat en het later compenseert wordt het verschil steeds groter en krijgt het weldegelijk invloed op het eindpeil. Het is dan ook niet aan te raden dit te doen want hierdoor krijg je een veel te hoog eindpeil

Het kan natuurlijk ook voorkomen dat je een ander eindpeil hebt dan gewenst als je elke dag constant de medicijnen neemt. Dit kan komen doordat je gemiddeld meer of minder dan 25% uitscheid
Maar ook door hoe snel het lichaam de stoffen opneemt e d

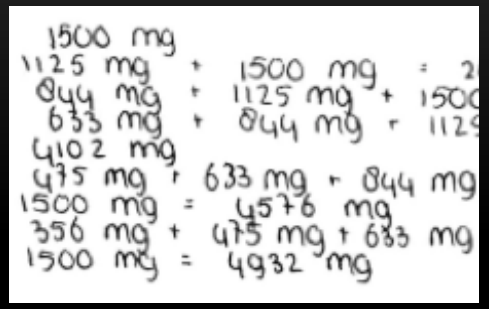
FROM *MODEL-OF* TO *MODEL-FOR*: 4 LEVELS

Situational level

Activity in the task setting, in which interpretations and solutions depend on understanding of how to act in the setting (often out of school settings)

Daily drug intake and % secretion

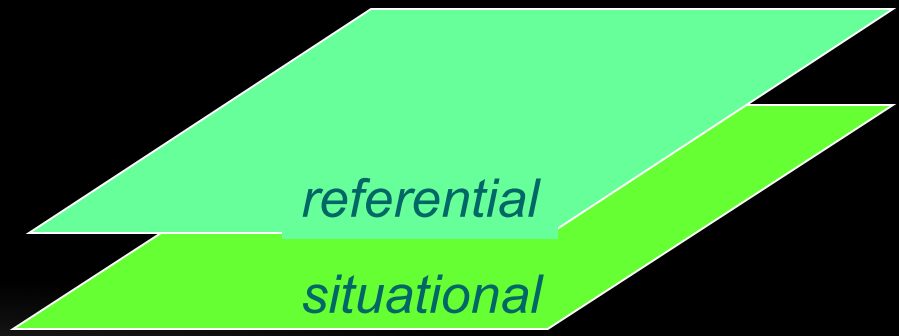
FROM *MODEL-OF* TO *MODEL-FOR*: 4 LEVELS



Referential level

Referential activity, in which an emerging model derives its meaning from the reference to activity in the task setting, and functions as a *model of* that activity.

Repeated calculations



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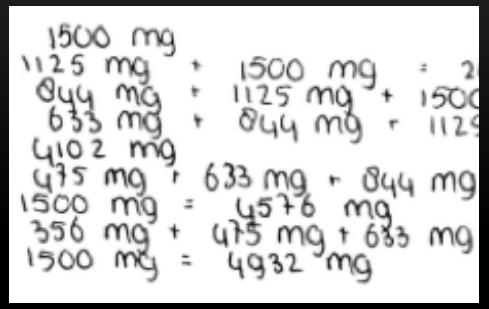
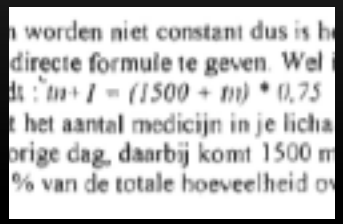


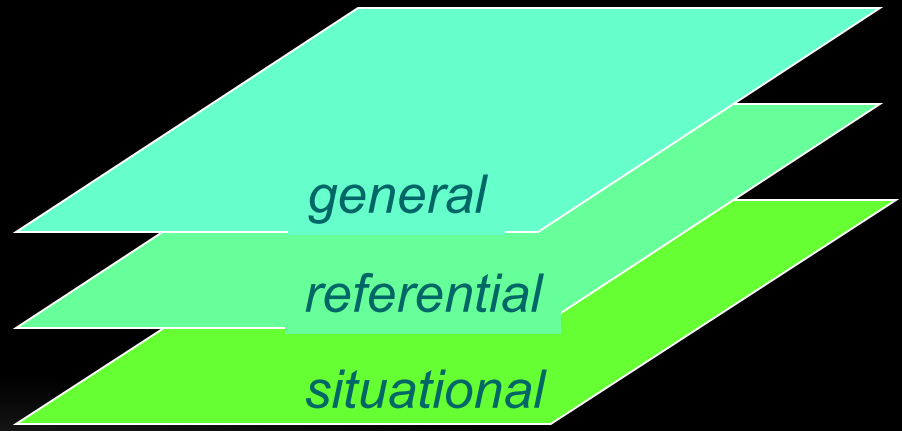


General level

Attention shifts towards mathematical relations, the model starts to derive its meaning from those mathematical relations, and becomes a *model for* mathematical reasoning

General model for calculations

$x_n = a \cdot x_{n-1} + b$



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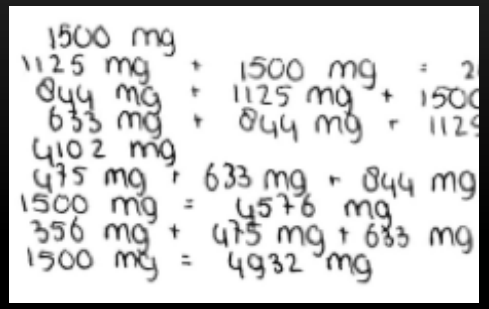
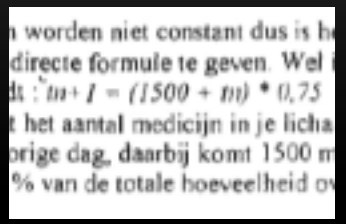
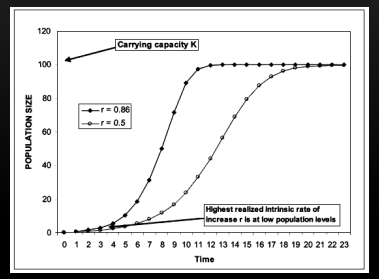


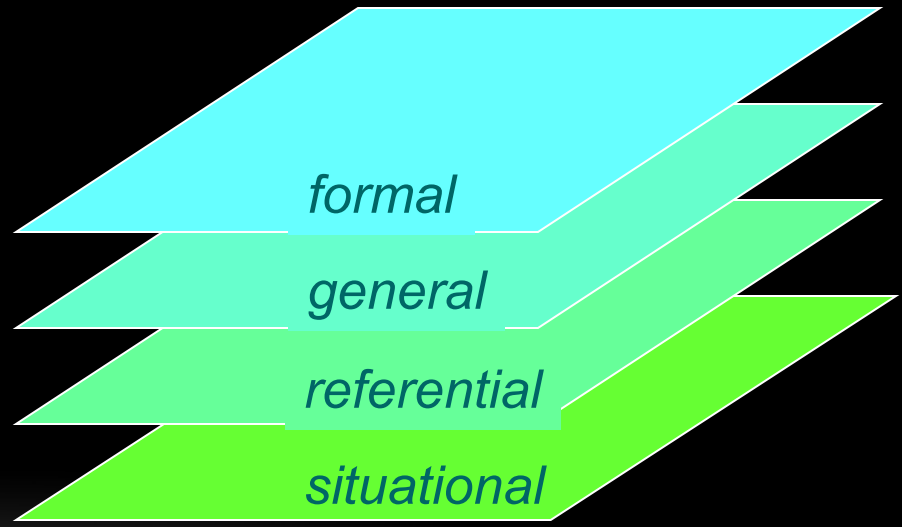




Formal level

More formal mathematical reasoning that is no longer dependent on the situational support

Reasoning about role of parameters on graphs and limits



DRUG LEVEL TASK – TWO PERSPECTIVES

IBL

- Who is owner of the question?
- Who is planning the solution procedure?
- What is the product?
- …

RME

- What are students' solutions and representations/calculations?
- Which models can connect to students' initial reasoning and how can they support the development of formal mathematics?
- How to ensure whole class progress?
- …

Role of the teacher?

LESSON STUDY AS INSTRUMENT FOR STUDYING AND IMPLEMENTING TEACHERS' PROFESSIONAL DEVELOPMENT

- https://time-project.eu/en/news/video-lesson-study
- https://www.youtube.com/watch?v=0eFJ2miTf1g