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Algebraic reasoning in primary school

Summerschool workshop 18-8-2022

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Early algebra

- Short introduction of the research project
- Working with the hanging mobile
- Student work

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Beyond Flatland Project

Goal: Introduce more mathematical reasoning in primary school
→ Stimulate higher-orde thinking

Focus on grade 5

Three **mathematical domains**:
- Dynamic data modelling (graphs)
- Probability
- Algebra

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Algebraic reasoning

- In the Netherlands, teaching algebra starts in secondary school

- However, there is much evidence that it can successfully be implemented in the primary grades (Blanton et al., 2015; Kaput et al., 2008; Van den Heuvel-Panhuizen et al., 2013)

  e.g. 10-year olds solving linear equations with unknowns on both sides of the equal sign

  $2T + 7 = T + 20$ (Brizuela & Schliemann, 2004)

- EARLY algebra

- Does **not** mean: teach formal algebra only at younger ages

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Early algebra

Goal: develop a teaching sequence (for grade 5) consisting of six lessons about early algebra

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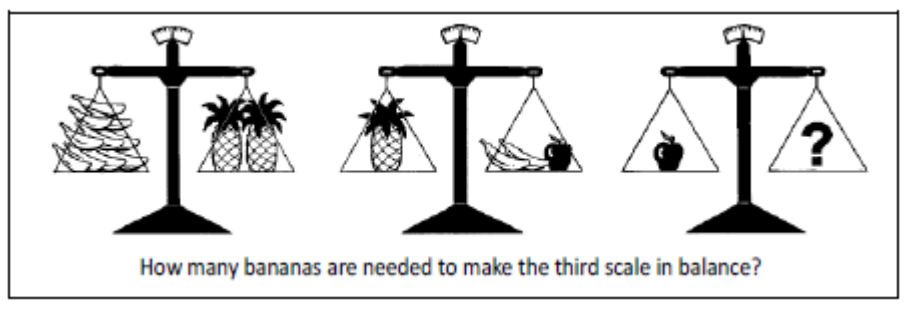
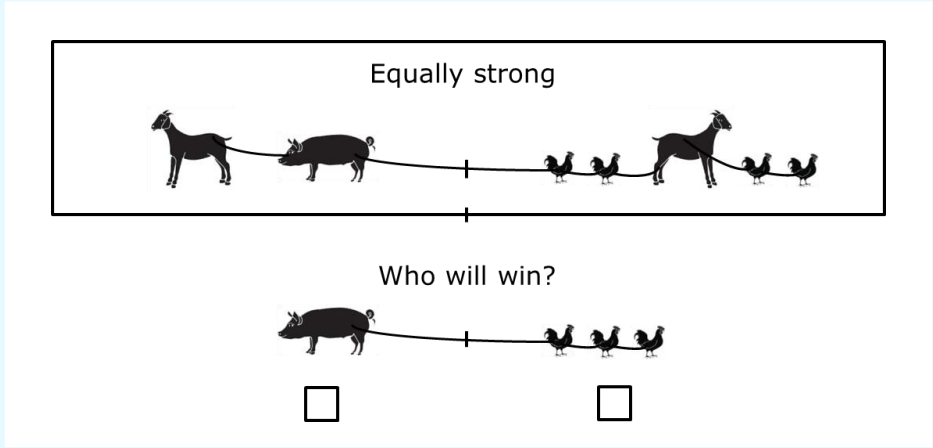
Development of Teaching Sequence: Focus on equations

- Broad domain of algebra, chose to focus on *equations*
- Informal algebra
- More specifically: use context-based equation-like problems as starting point instead of formal equations





~~10Y = 2X~~
~~X = 2Y + Z~~
~~Z = …. Y?~~

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Embodiment Theory

Offering students **bodily experiences** with a certain (mathematical) concept, can contribute to a deeper understanding of this concept.

E.g. offering perceptuo-motor experiences

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So far

Developing teaching sequence

- Informal linear equations
- Embodiment theory

I will now show let you work on the assignment which we developed for primary school students.

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Assignment

5 groups

Assignment: Discover what things can be done while keeping the hanging mobile straight

→ Make a poster of your findings

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Discussion of findings

What things can be done while keeping the hanging mobile straight?

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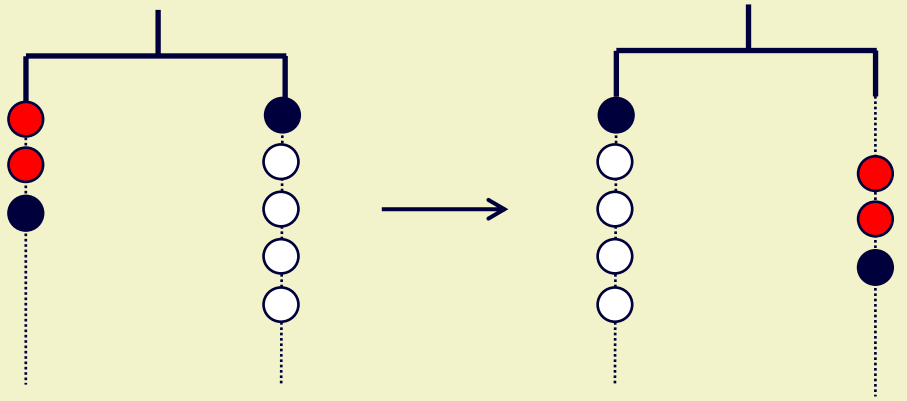
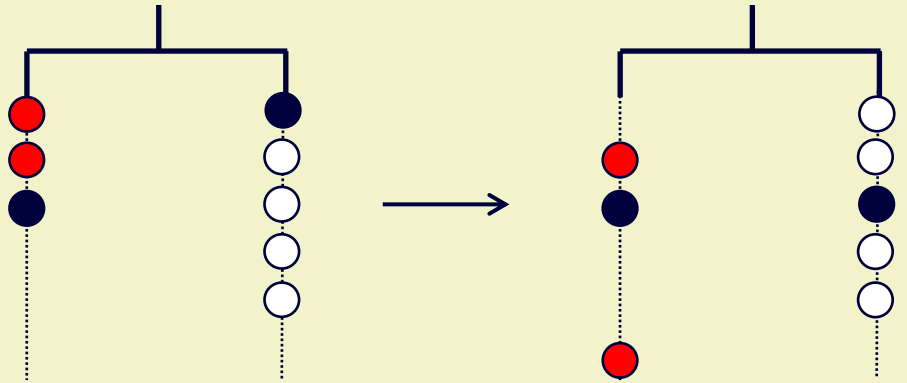
Some options

- Change L/R



- Change order of bags on one side



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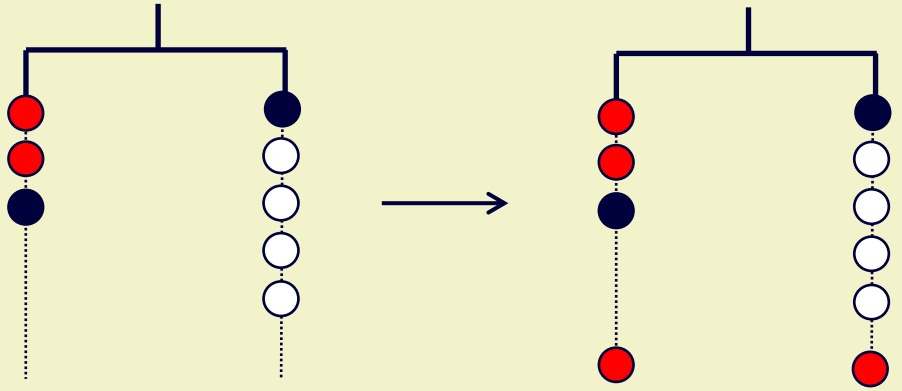
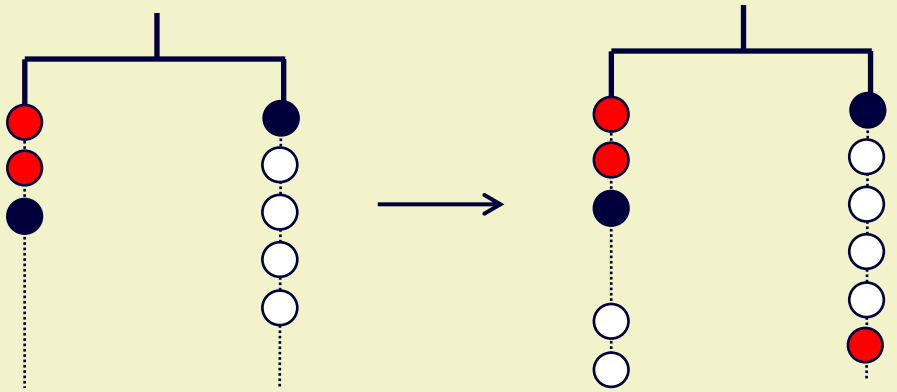
Some options

- Add (similar) bags on both sides



- Add bags (based on ratio) on both sides



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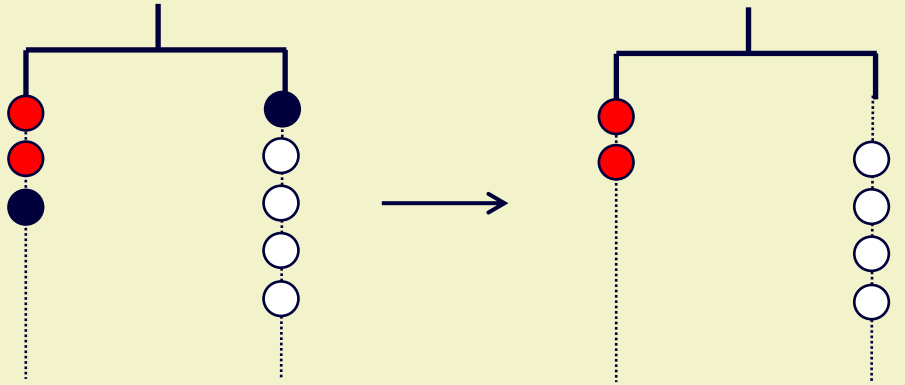
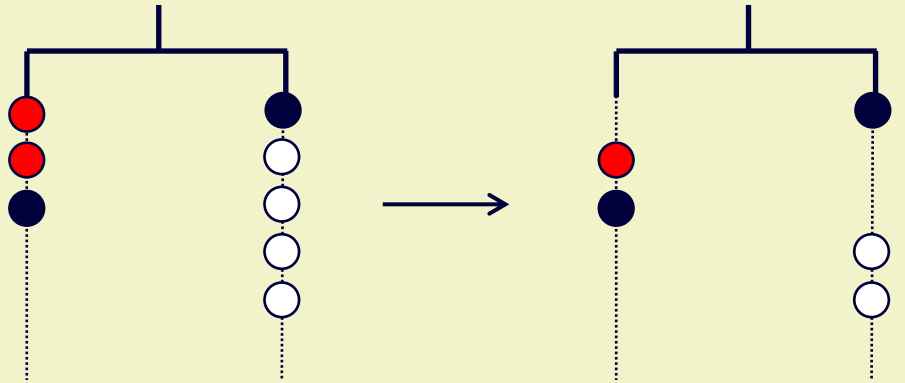
Some options

- Take away (similar) bags on both sides



- Take away bags (based on ratio) on both sides



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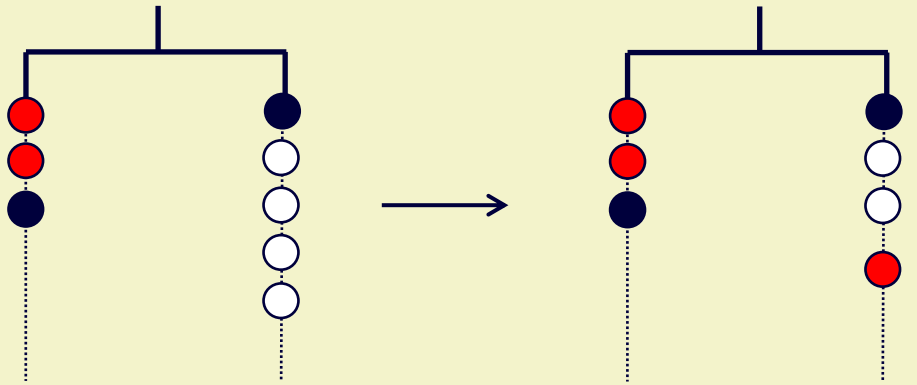
Some options

- Replace bags of certain color by another color (based on the ratio)



- ...

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Discussion in groups

Which algebraic principles/concepts/strategies can be found in this task?

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Algebraic concepts/principles: what we thought of

- Equality / equivalence
- Restructuring principle:
  - Change L/R side
  - Change order of bags on one side
- Isolation principle:
  - Take away similar bags
  - Take away bags based on ratio
- Substitution principle:
  - Replace bags of certain color by another color
- Context-based notations
- Language
- …?

Algebraic strategies

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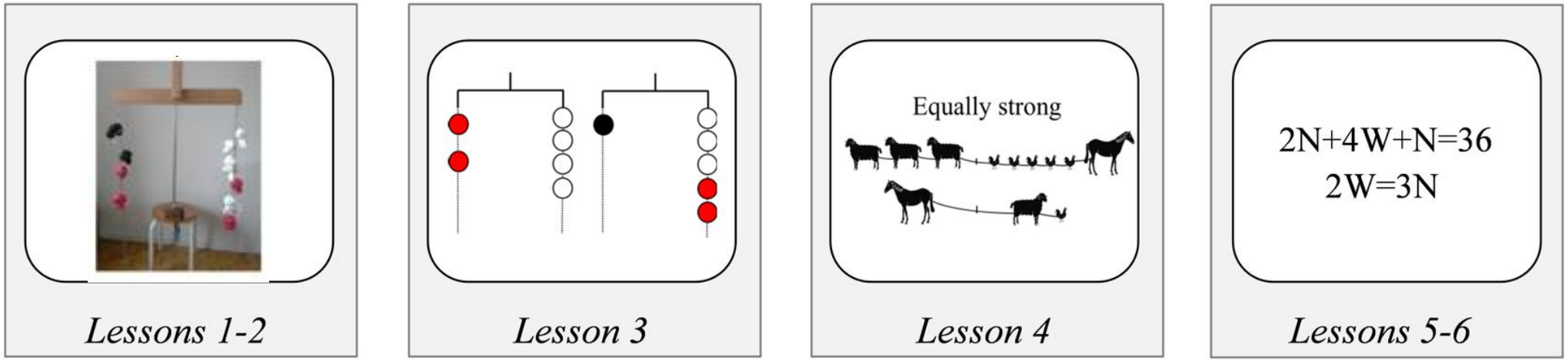
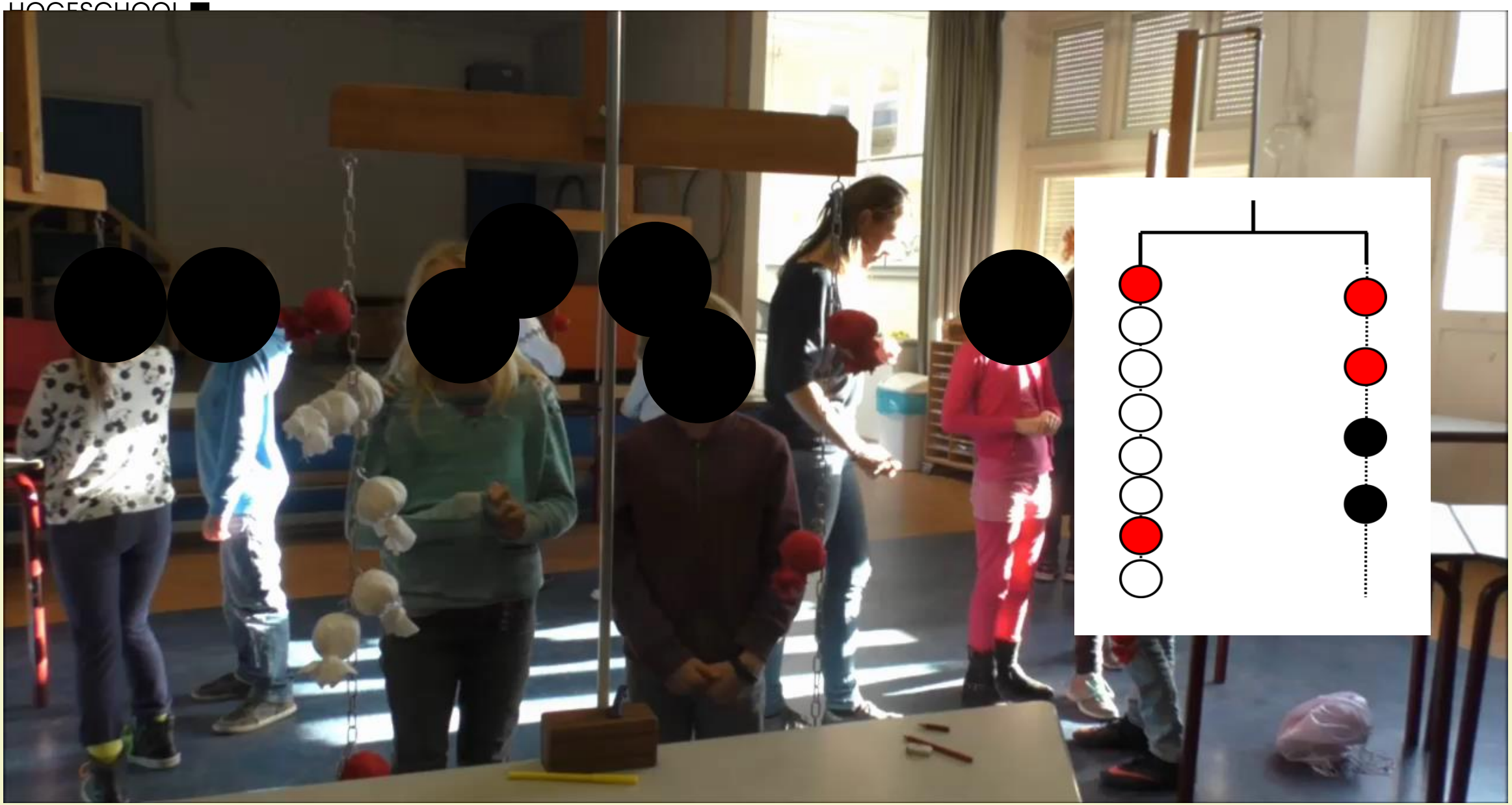
The teaching sequence



- One equation → System of two equations
- Hanging mobile context → Other contexts
- Find relationships between unknowns → Find values of unknowns
- Informal notations → Gradually towards more formal notations



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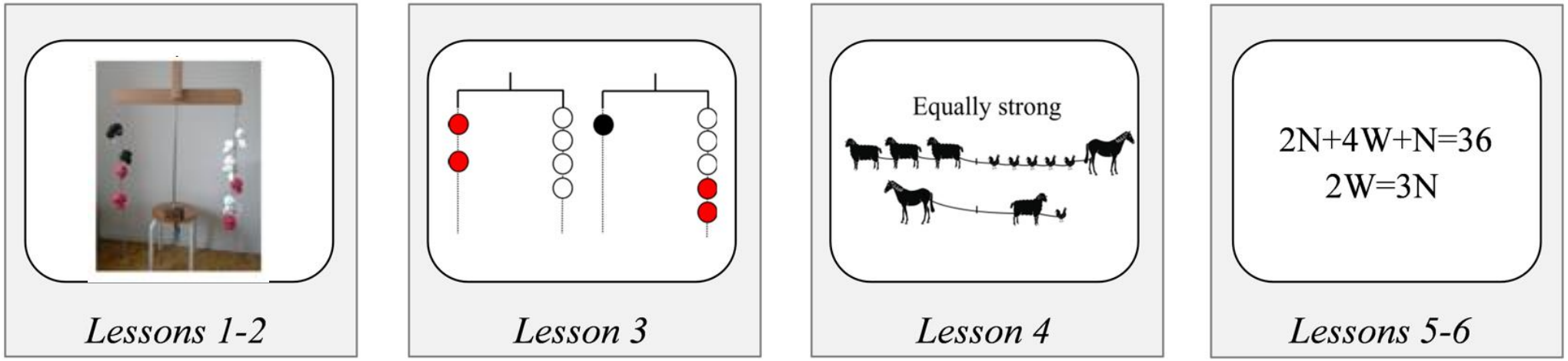
The teaching sequence



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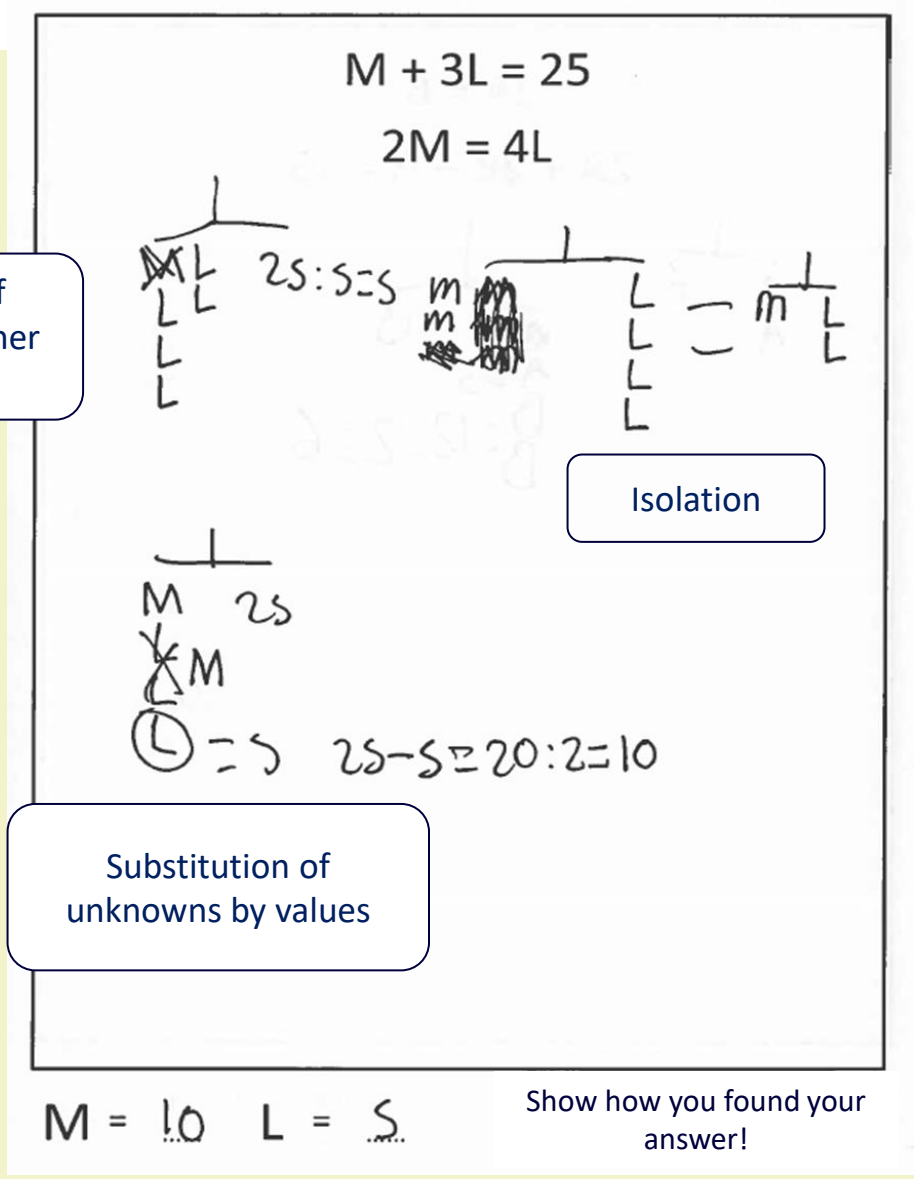

Substitution of unknowns by other unknowns

Isolation

Substitution of unknowns by values

Show how you found your answer!

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What do we see?

Indications for a relationship between the experiences with the hanging mobile and algebraic reasoning in other contexts

- Students' use of the model
- Use of strategies
- Descriptions in terms of actions

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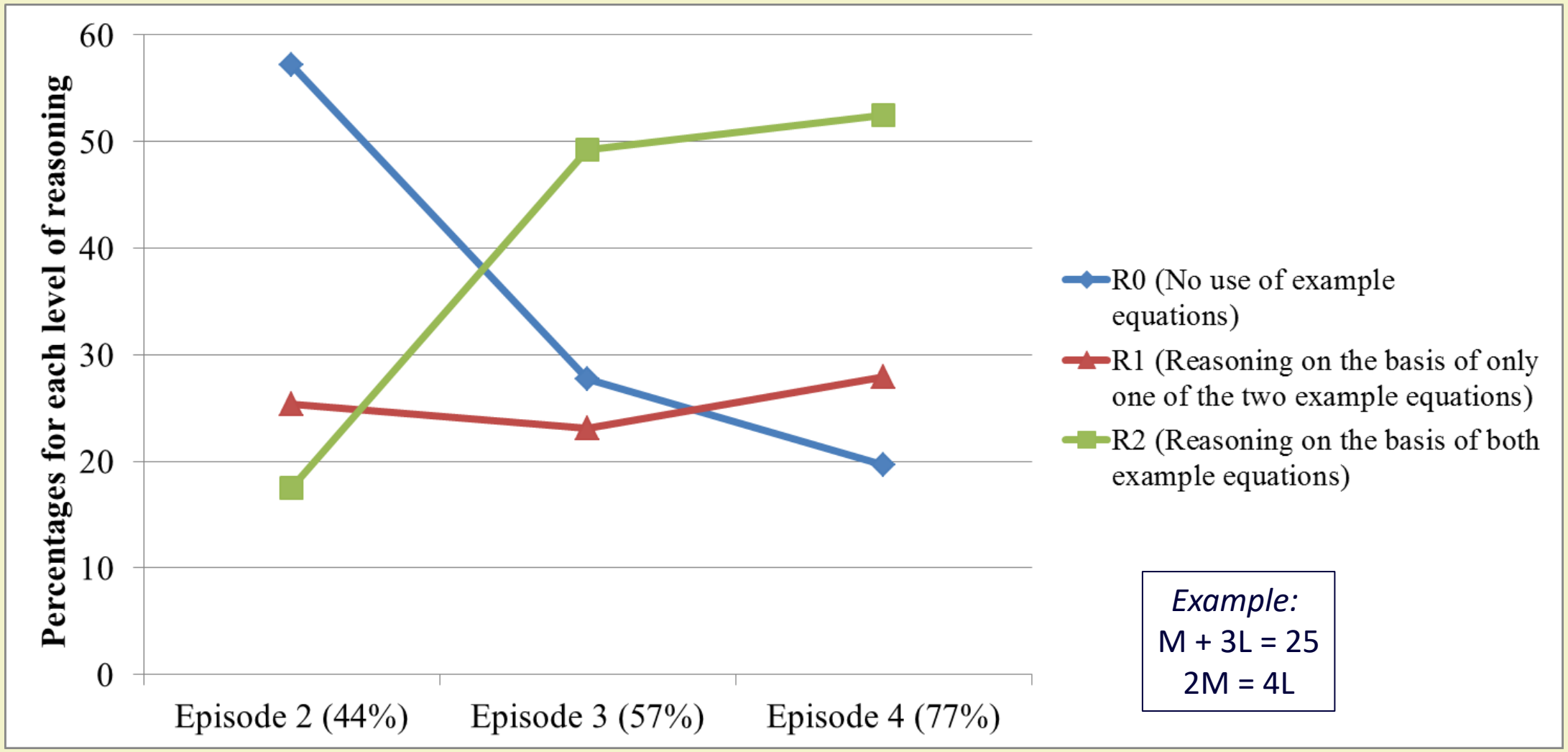
All students together



Example:
M + 3L = 25
2M = 4L

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Overall conclusions

Learning environment with the hanging mobile appears to be a suitable context for developing algebraic reasoning. Students' algebraic reasoning improved over the lessons.

→Continuous learning strand from primary to secondary edcation

→Higher-order thinking

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Thanks for your attention
Are there any questions?

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References related to dissertation

Articles:

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Whole dissertation: Otten, M. (2020). Algebraic reasoning in primary school: A balancing act [doctoral dissertation]. Utrecht University http://dspace.library.uu.nl/handle/1874/400332

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