Utrecht University

Realistic Mathematics Education - An introduction

Paul Drijvers

Freudenthal Institute

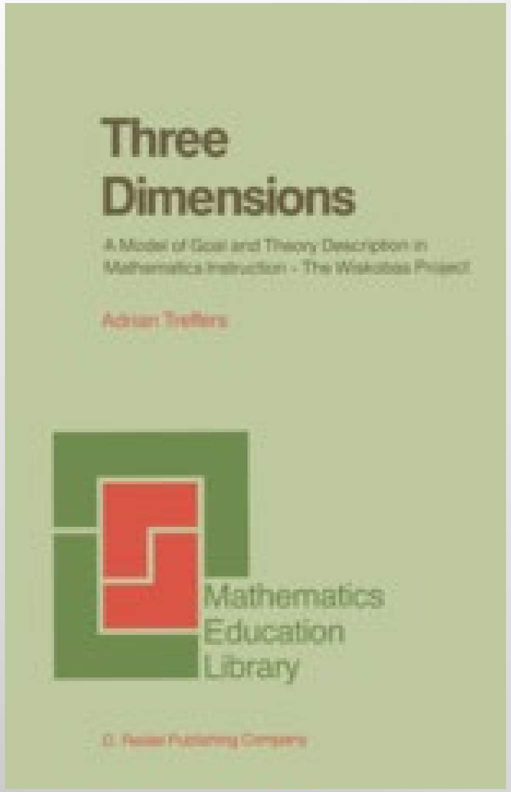
Utrecht University

p.drijvers@uu.nl

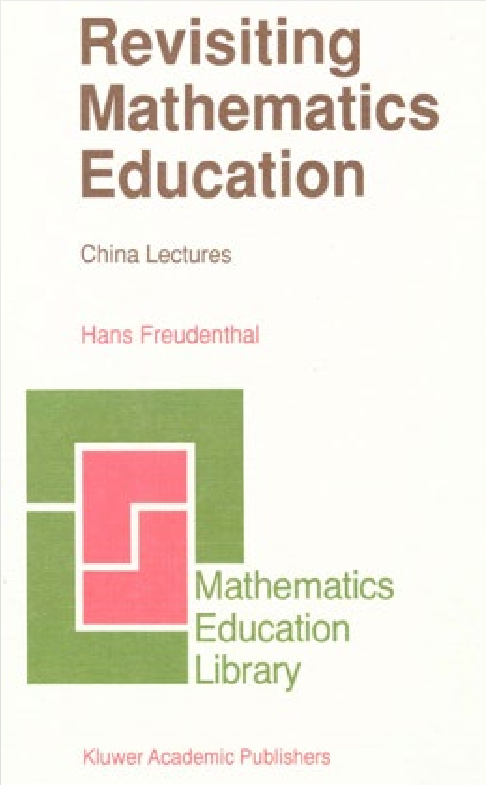
https://www.uu.nl/staff/PHMDrijvers/0

15-08-2022

RME: an "old" theory developed at FI .....

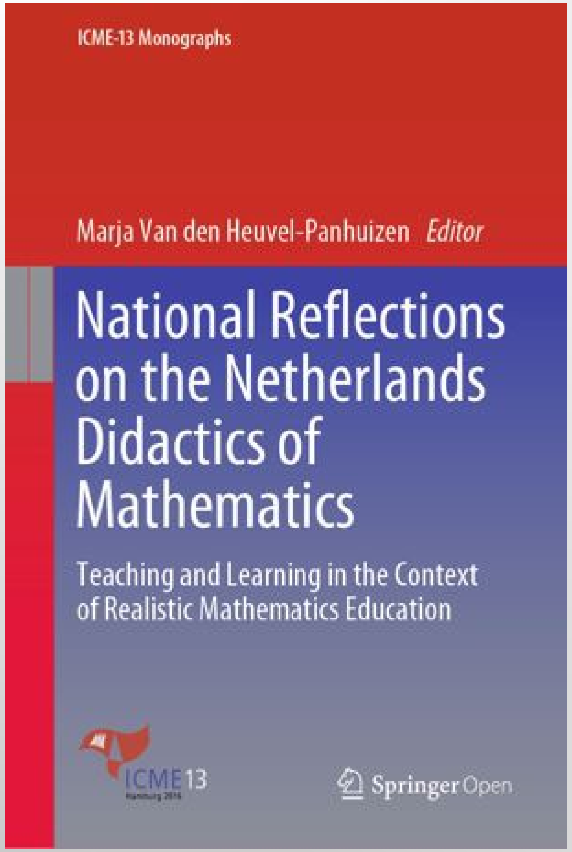


Treffers 1987

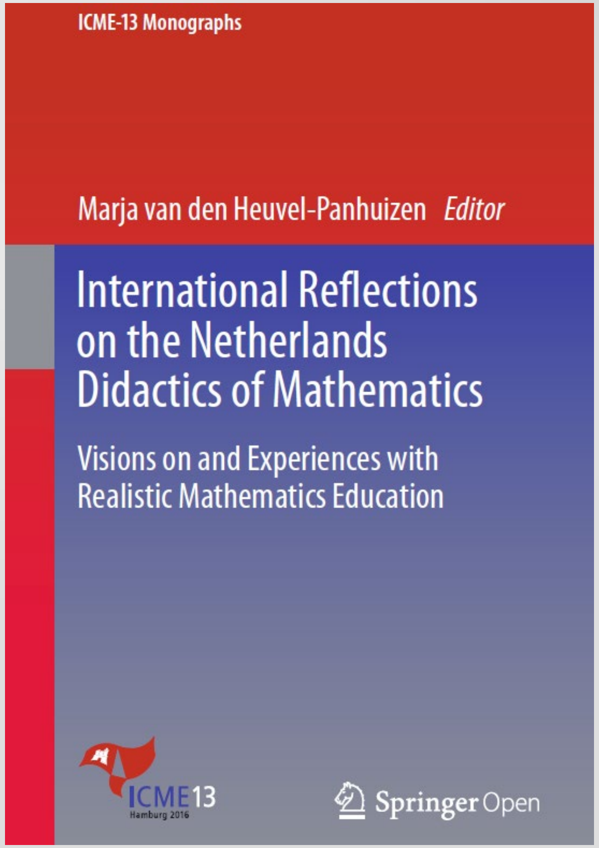


Freudenthal 1991

… but still alive and respected today!



Marja Van den Heuvel-Panhuizen (Ed.), 2020



Open access at https://link.springer.com/book/10.1007/978-3-030-20223-1 and https://link.springer.com/book/10.1007/978-3-030-33824-4

Aims of this presentation

- To introduce some key aspects of the theory of Realistic Mathematics Education (RME)
- To set up a shared vocabulary for this summer school
- To reflect on RME task design and the role of contexts

Utrecht University

Outline

- An introduction to RME
- Four RME key concepts
  - Mathematization
  - Didactical phenomenology
  - Use of models
  - Guided reinvention
- Hands-on task analysis
- Summary

What is Realistic Mathematics Education?

?

What is Realistic Mathematics Education?

- **Realistic Mathematics Education (RME) is a domain-specific instruction theory on the teaching and learning of mathematics…**

- **… that has been elaborated into a number of local instruction theories for different mathematical topics, student ages, and achievement levels**

Utrecht University

Starting point

Hans Freudenthal (1905-1990):
Mathematics as human activity

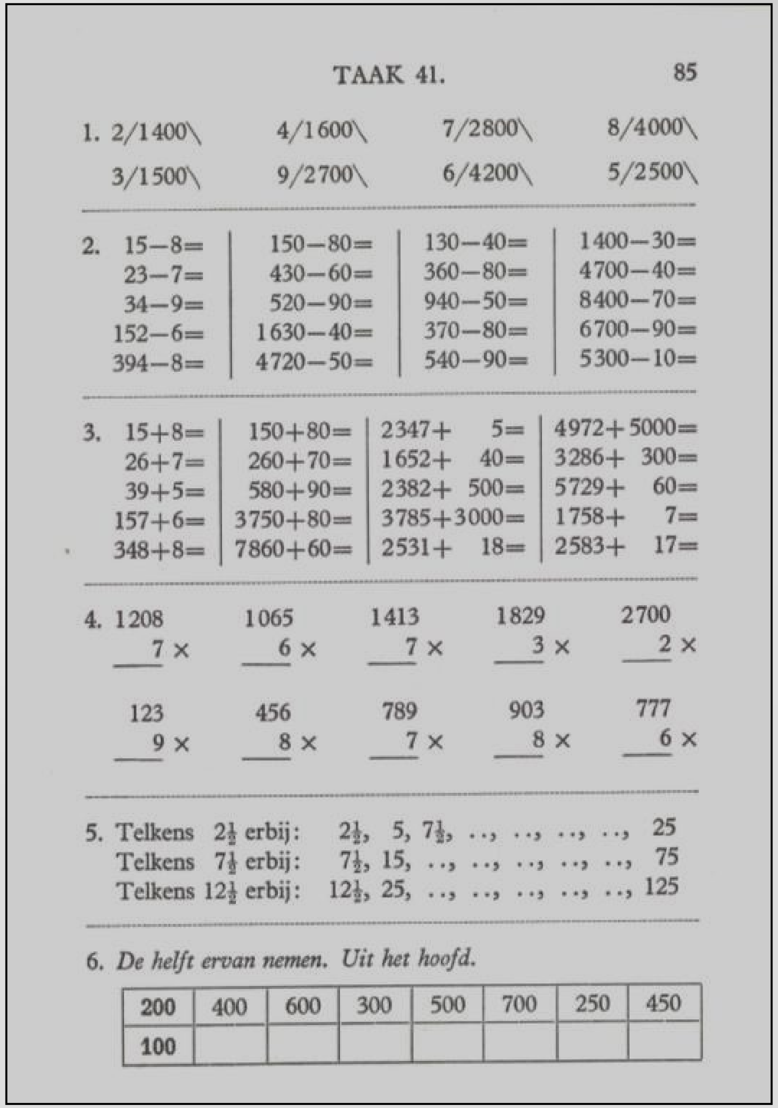
"What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics." (Freudenthal, 1968, p. 7)

Utrecht University

Why RME?

Freudenthal's opposition against "anti-didactical inversion": don't take the end point of the mathematician's work as a starting point for teaching!

As a reaction to the obvious limitations of mechanistic and structuralistic approaches to mathematics education



TAAK 41. 85

1. 2/1400\ 4/1600\ 7/2800\ 8/4000\
3/1500\ 9/2700\ 6/4200\ 5/2500\

2. 15—8= 150—80= 130—40= 1400—30=
23—7= 430—60= 360—80= 4700—40=
34—9= 520—90= 940—50= 8400—70=
152—6= 1630—40= 370—80= 6700—90=
394—8= 4720—50= 540—90= 5300—10=

3. 15+8= 150+80= 2347+ 5= 4972+5000=
26+7= 260+70= 1652+ 40= 3286+ 300=
39+5= 580+90= 2382+ 500= 5729+ 60=
157+6= 3750+80= 3785+3000= 1758+ 7=
348+8= 7860+60= 2531+ 18= 2583+ 17=

4. 1208 7 × 1065 6 × 1413 7 × 1829 3 × 2700 2 ×
123 9 × 456 8 × 789 7 × 903 8 × 777 6 ×

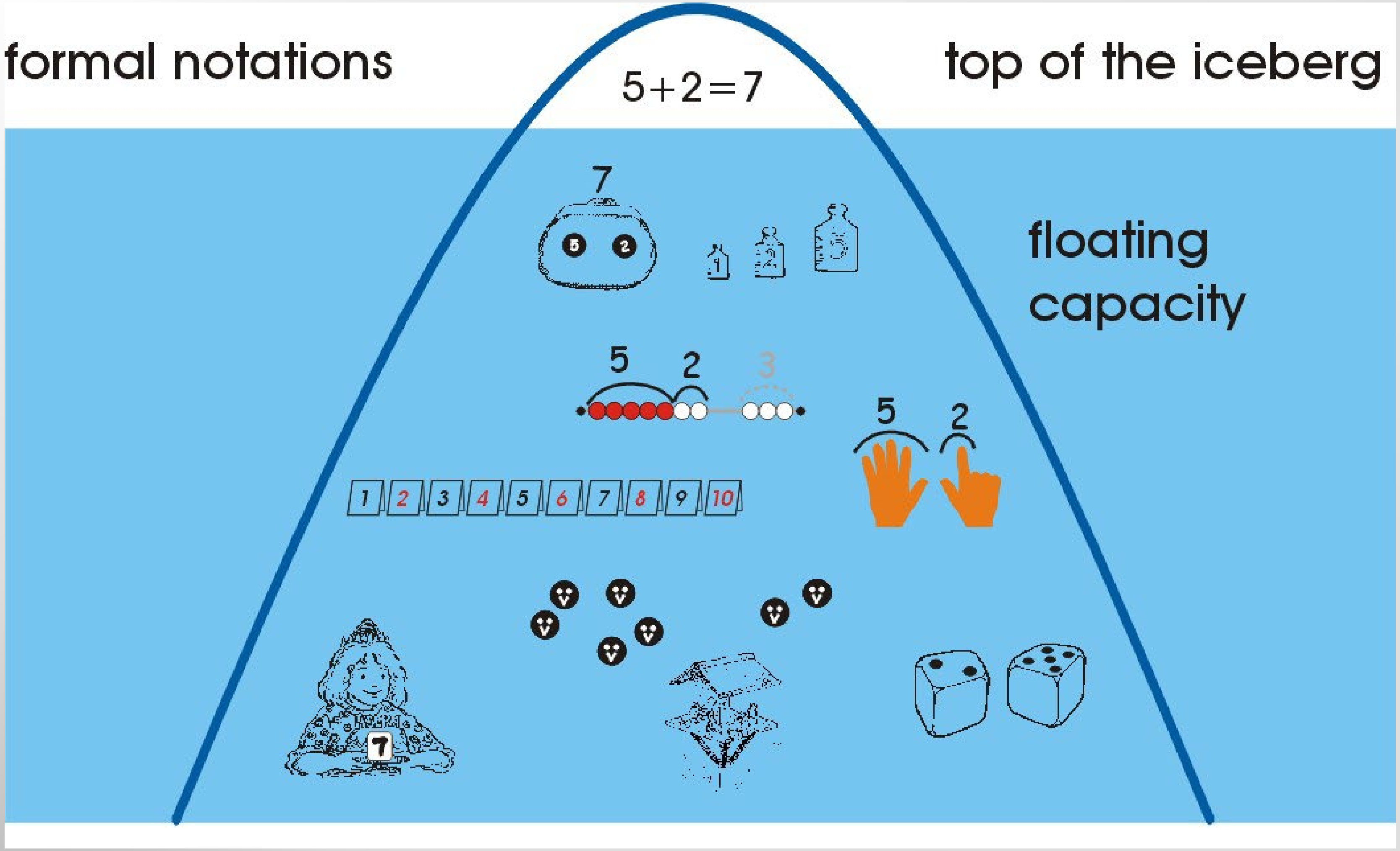
5. Telkens $2\frac{1}{2}$ erbij: $2\frac{1}{2}$, 5, $7\frac{1}{2}$, .., .., .., .., 25
Telkens $7\frac{1}{2}$ erbij: $7\frac{1}{2}$, 15, .., .., .., .., .., 75
Telkens $12\frac{1}{2}$ erbij: $12\frac{1}{2}$, 25, .., .., .., .., .., 125

6. *De helft ervan nemen. Uit het hoofd.*

200	400	600	300	500	700	250	450
100							

Utrecht University

The iceberg metaphor:



http://www.fisme.science.uu.nl/en/wiki/index.php/Iceberg_Metaphor

Realistic Mathematics Education

Marja Van den Heuvel-Panhuizen[1] and
Paul Drijvers[2]
[1]Freudenthal Institute for Science and Mathematics Education, Faculty of Science & Faculty of Social and Behavioural Sciences, Utrecht University, Utrecht, The Netherlands
[2]Freudenthal Institute, Utrecht University, Utrecht, The Netherlands

Keywords

Domain-specific teaching theory; Realistic contexts; Mathematics as a human activity; Mathematization

What is Realistic Mathematics Education?

Realistic Mathematics Education – hereafter abbreviated as RME – is a domain-specific instruction theory for mathematics, which has been developed in the Netherlands. Characteristic of RME is that rich, “realistic” situations are given a prominent position in the learning process. These situations serve as a source for initiating the development of mathematical concepts, tools, and procedures and as a context in which students can in a later stage apply their mathematical knowledge, which then gradually has become more formal and general and less context specific.

(Van den Heuvel-Panhuizen & Drijvers, 2020)

Six RME principles and key concepts

1. The activity principle
2. The reality principle
3. The level principle
4. The intertwinement principle
5. The interactivity principle
6. The guidance principle

(Van den Heuvel-Panhuizen & Drijvers, 2020)

Outline

✓ An introduction to RME
- Four RME key concepts
  - Mathematization
  - Didactical phenomenology
  - Use of models
  - Guided reinvention
- Hands-on task analysis
- Summary

Six RME principles and key concepts

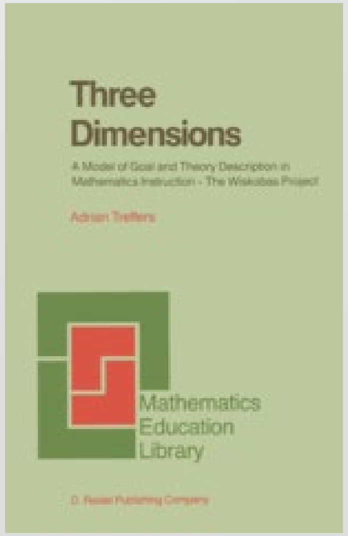
1. The activity principle → Mathematization
2. The reality principle
3. The level principle
4. The intertwinement principle
5. The interactivity principle
6. The guidance principle

(Van den Heuvel-Panhuizen & Drijvers, 2020)

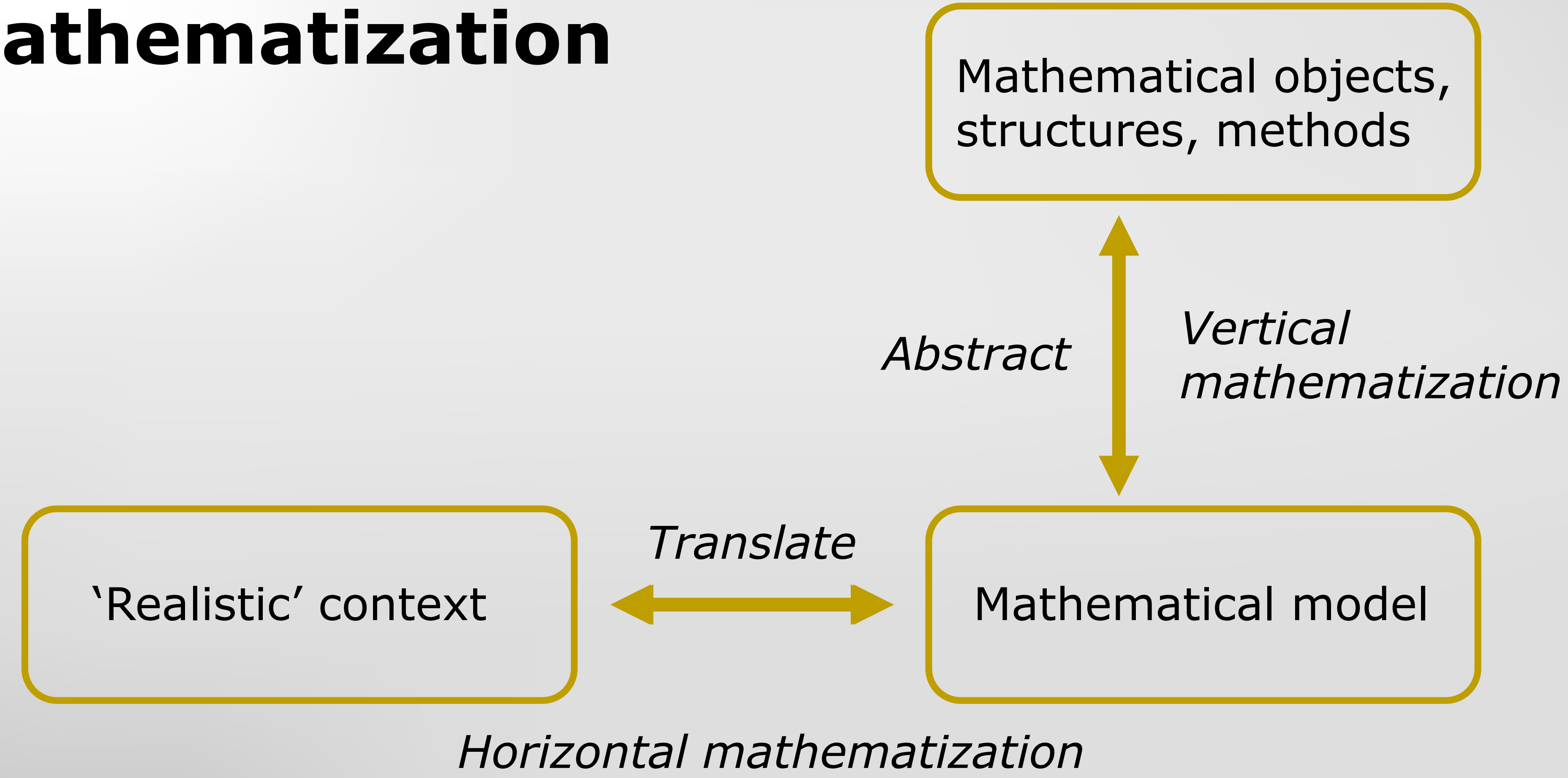
Mathematization

Mathematics as human activity:
Doing mathematics = mathematizing

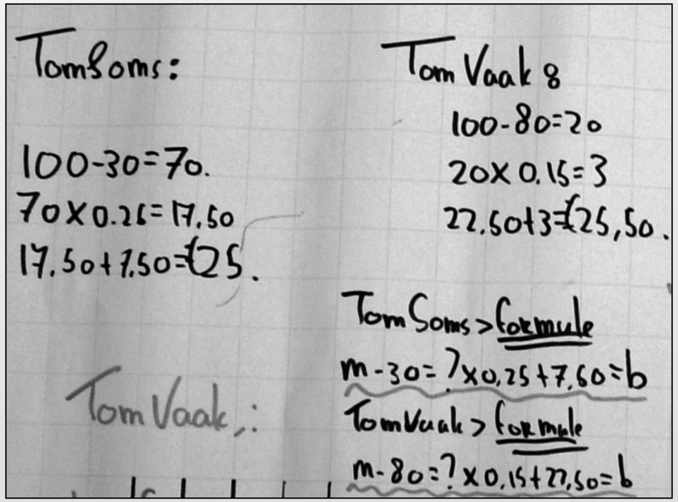
Treffers (1979): distinction between horizontal and vertical mathematization.



Mathematization



Example horizontal / vertical mathematization



Vertical:
The development of a method / theory for solving systems of two linear equations in general

Horizontal:
Translating a problem on fixed and variable costs (e.g., mobile phone offers) in two linear equations

Outline

- ✓ An introduction to RME
- Four RME key concepts
  - ✓ Mathematization
  - ○ Didactical phenomenology
  - ○ Use of models
  - ○ Guided reinvention
- Hands-on task analysis
- Summary

Six RME principles and key concepts

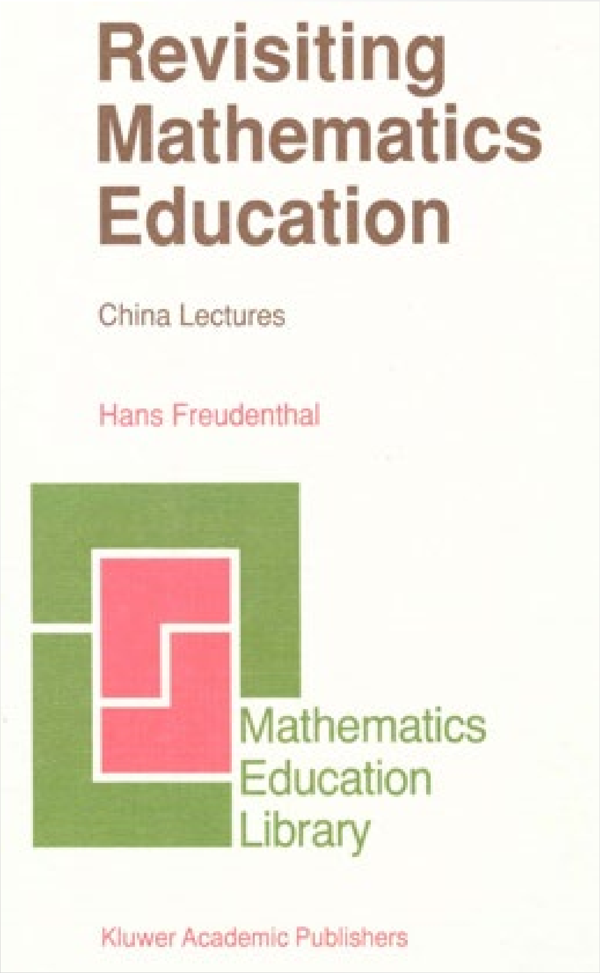
1. The activity principle
2. The reality principle → Didactical phenomenology
3. The level principle
4. The intertwinement principle
5. The interactivity principle
6. The guidance principle

(Van den Heuvel-Panhuizen & Drijvers, 2020)

What is Realistic?

?

Utrecht University



“I prefer to apply the term ‘reality’ to what at a certain stage common sense experiences as real.”

Freudenthal (1991, p. 17)

Treffers about realistic

The realistic view [..] takes the reality as a point of departure, i.e., the world of the child, which implies that it tries to identify the appearances of mathematical phenomena that fit the world of the child, so to which the child can attach meaning

Treffers (1979, p. 12-13, my translation)

What do we mean by "Realistic"?

"Realistic" may have different meanings:

- **Realistic in the sense of *feasible* in educational practice**
- **Realistic in the sense of related to *real life* (real world, phantasy world, math world)**
- **Realistic in the sense of *meaningful*, sense making for students**
- **Realistic in the sense of "*zich realiseren*" = to realize, to be aware of, to imagine**

Didactical phenomenology (1)

A didactical phenomenology…

… relates mathematical thought objects to phenomena in the (physical, social, mental,…) world

… as to inform us how these mathematical thought objects may help to organize and structure phenomena in reality.

Didactical phenomenology (2)

As such, it identifies phenomena that …
… beg to be organized by mathematical means
… invite students to develop the targeted mathematical concepts
… and help teachers and designers to decide which contexts to use

These phenomena can come from real life or can be 'experientially real'

Didactical phenomenology (3)

In Freudenthal's words (1983, p. ix), a didactical phenomenology of mathematics can "show the teacher the places where the learner might step into the learning process of mankind."

(Van den Heuvel-Panhuizen, 2020)

-> Didactical phenomenology guides task design
(cf. didactical engineering, Margolinas & Drijvers, 2015)

Utrecht University

1 Maak de keersommen.

×	7
1	
3	
5	
7	
9	

×	7
2	
4	
6	
8	
10	

×	6
5	
6	
7	
8	
9	

×	4
9	
7	
0	
6	
8	

2 Hoeveel komt er in elk laatje?

a Je verdeelt 18 losse spijkers over 3 laatjes.

b Je verdeelt 24 grote spijkers over 6 laatjes.

c Je verdeelt 28 losse schroeven over 4 laatjes.

3 Maak de sommen.

27 = 3 × …   24 = 6 × …   18 = 2 × …

Non-Example

Outline

✓ An introduction to RME
- Four RME key concepts
  - ✓ Mathematization
  - ✓ Didactical phenomenology
  - ○ Use of models
  - ○ Guided reinvention
- Hands-on task analysis
- Summary

Six RME principles and key concepts

1. The activity principle
2. The reality principle
3. The level principle → Use of models
4. The intertwinement principle
5. The interactivity principle
6. The guidance principle

(Van den Heuvel-Panhuizen & Drijvers, 2020)

Broad meaning and important role for models

Within RME, models are seen as representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have different manifestations. (Van den Heuvel-Panhuizen, 2003, p. 13)

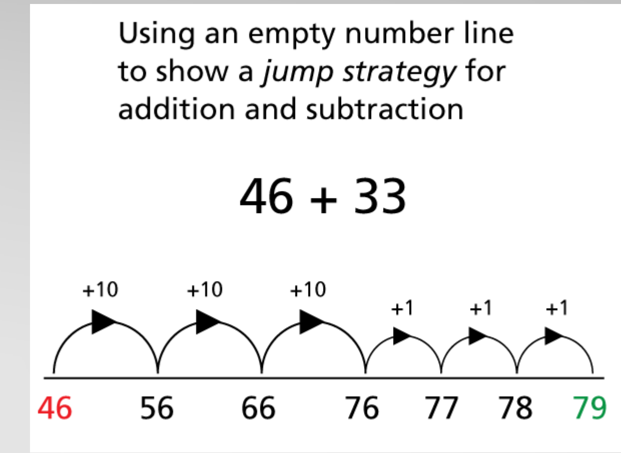
A model may be material, a situation, a sketch, a diagram, …

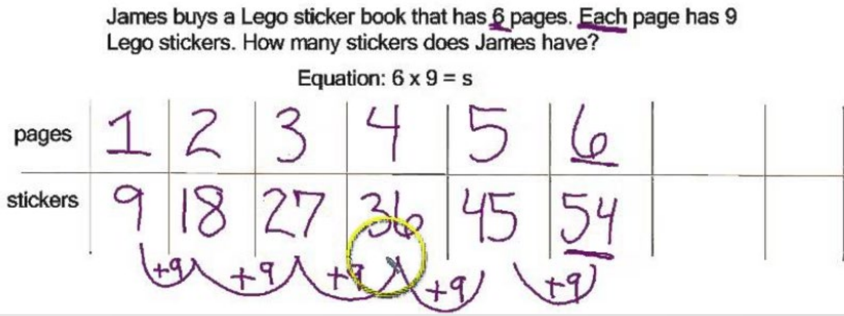
The meaning and role of these models may shift during the learning process, from being situation-related to becoming more general.

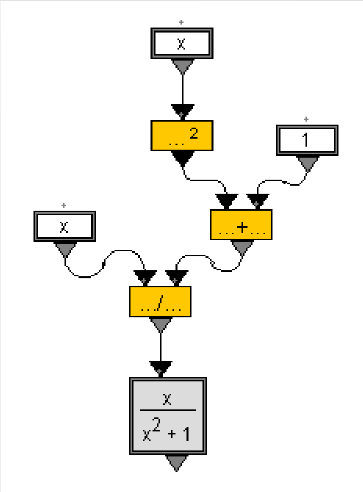
Examples of didactical models

- Empty number line (for arithmetic operations)
- Chocolate bar (for ratios)
- Ratio table (for operations with ratios)
- Pizza model (for fractions)
- Arrow chains (for functions)
- Tree model (for expressions)
- Abacus (for calculations)

…



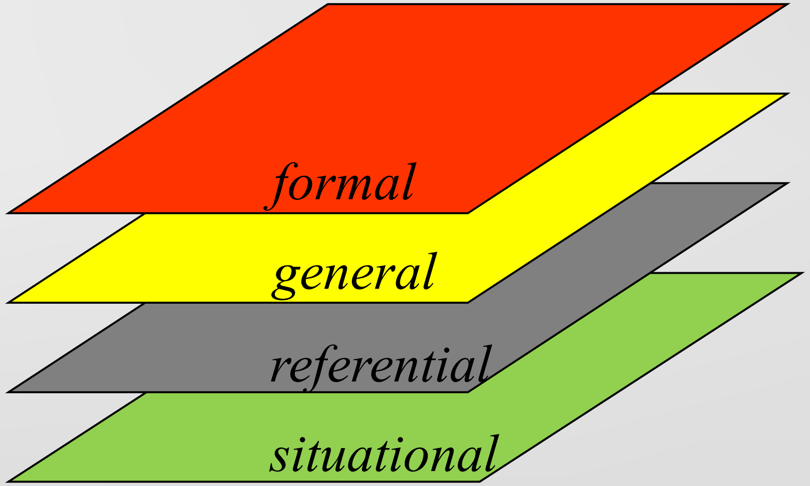




Model of – model for: emergent modeling

Models *of* informal mathematical activity develop into models *for* mathematical reasoning

(Streefland, 1985; Gravemeijer et al., 2000; Van den Heuvel-Panhuizen, 2003)

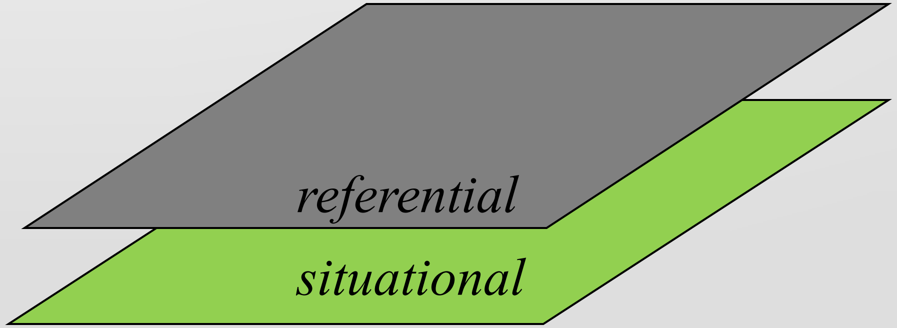


Emergent modelling

Situational level
Activity in the task setting. Interpretations and solutions depend on understanding of how to act in the (often out of school) settings

Emergent modelling (ctnd)

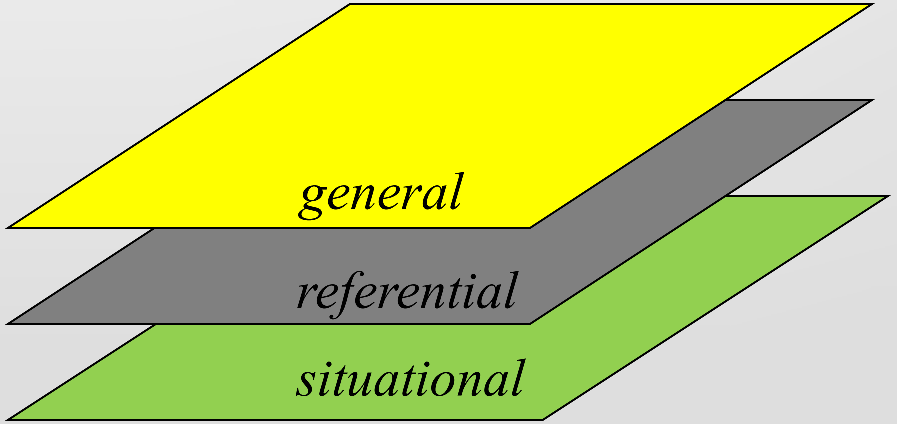
Referential level
Referential activity, in which models refer to activity in the setting of instructional activities (posed mostly in school)



Emergent modelling (ctnd)

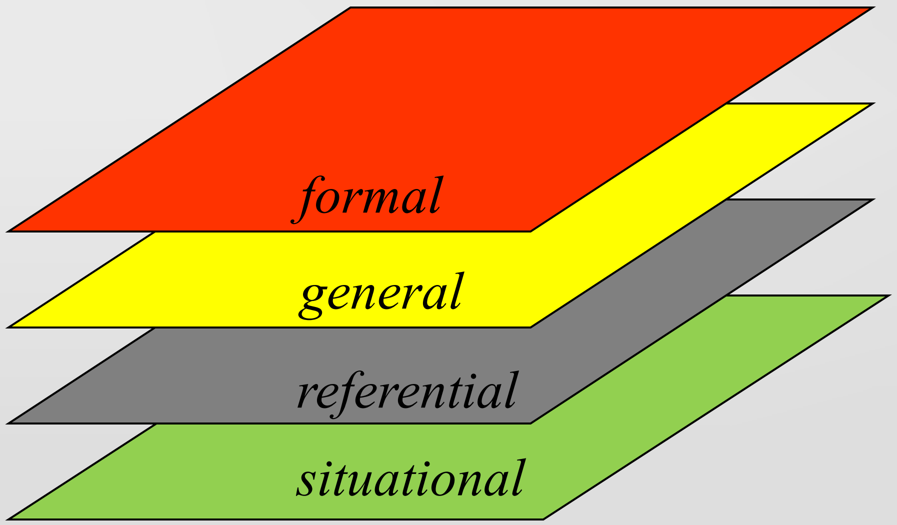
General level

General activity, in which models focus on situation–independent interpretations and solutions



Emergent modelling (final)

Formal level
Reasoning with conventional symbolizations, which is no longer dependent on the support of models



Utrecht University

Outline

- ✓ An introduction to RME
- Four RME key concepts
  - ✓ Mathematization
  - ✓ Didactical phenomenology
  - ✓ Use of models
  - ○ Guided reinvention
- Hands-on task analysis
- Summary

Six RME principles and key concepts

1. The activity principle
2. The reality principle
3. The level principle
4. The intertwinement principle
5. The interactivity principle
6. The guidance principle → Guided reinvention

(Van den Heuvel-Panhuizen & Drijvers, 2020)

Guided reinvention

Reinvention:

Reconstructing and developing a mathematical concept in a natural way in a given problem situation.

Guidance:

Students need guidance (from books, peers, teacher) to ascertain convergence towards common mathematical standards

Tension between reinvention and guidance?

Guided reinvention heuristics

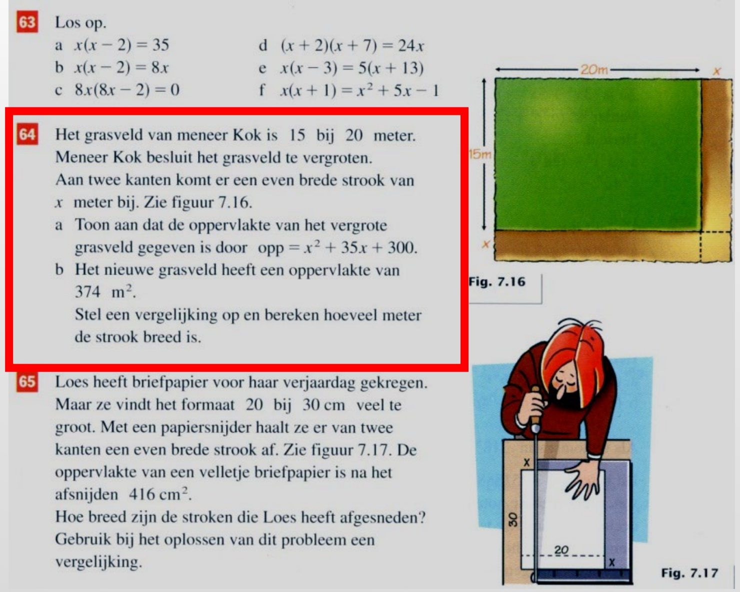
Think how you would approach a problem situation if it were new to you, ‘think how you might have figured it out yourself’ (Gravemeijer 1994, p. 179)

See what you can learn from the historical development of a mathematical concept for educational design

Outline

- ✓ An introduction to RME
- ✓ Four RME key concepts
  - ✓ Mathematization
  - ✓ Didactical phenomenology
  - ✓ Use of models
  - ✓ Guided reinvention
- Hands-on task analysis
- Summary

Task A: Extending the lawn



63 Los op.

a $x(x-2)=35$

b $x(x-2)=8x$

c $8x(8x-2)=0$

d $(x+2)(x+7)=24x$

e $x(x-3)=5(x+13)$

f $x(x+1)=x^2+5x-1$

64 Het grasveld van meneer Kok is 15 bij 20 meter. Meneer Kok besluit het grasveld te vergroten. Aan twee kanten komt er een even brede strook van x meter bij. Zie figuur 7.16.

a Toon aan dat de oppervlakte van het vergrote grasveld gegeven is door opp $= x^2+35x+300$.

b Het nieuwe grasveld heeft een oppervlakte van 374 m^2.
Stel een vergelijking op en bereken hoeveel meter de strook breed is.

65 Loes heeft briefpapier voor haar verjaardag gekregen. Maar ze vindt het formaat 20 bij 30 cm veel te groot. Met een papiersnijder haalt ze er van twee kanten een even brede strook af. Zie figuur 7.17. De oppervlakte van een velletje briefpapier is na het afsnijden 416 cm^2.
Hoe breed zijn de stroken die Loes heeft afgesneden? Gebruik bij het oplossen van dit probleem een vergelijking.

Fig. 7.16

Fig. 7.17

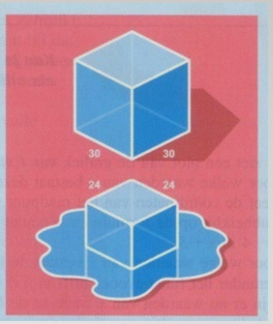
The lawn in Mr. Jones' garden measures 15 by 20 meters. Mr. Jones decides to extend the lawn. To two sides he adds a strip of equal width of *x* meters. See Figure 7.16.

a. Show that the area of the enlarged lawn is represented by
   Area $= x^2+35x+300$

b. The new lawn has an area of 374 m^2.
   Set up an equation and calculate the width of the strip.

T_4 Een ijsblokje met ribben van 30 mm begint langzaam te smelten. Elke minuut worden de ribben 1,5 mm korter. Het volume van het ijsblokje wordt beschreven door de formule $V = (30 - 1{,}5t)^3$. Hierin is V het volume in kubieke millimeter en t de tijd in minuten.

- **a** Bereken het volume van het ijsblokje op $t = 0$.
- **b** Wat zijn zinvolle waarden voor t? En voor V?
- **c** Plot en schets dat gedeelte van de grafiek waar beide variabelen betekenis hebben.
- **d** Volg met de cursor de grafiek en onderzoek na hoeveel minuten het volume kleiner dan 10 000 mm^3 is. Geef je antwoord in 1 decimaal nauwkeurig.



Task B: Melting ice

An ice cube with edges of 30 mm long starts to melt down slowly. Every minute, the edges get 1.5 mm shorter. The volume of the ice cube is described by the formula $V = (30 - 1{,}5\ t)^3$, where V stands for the volume in mm^3 and t for the time in minutes.

a. Calculate the volume of the ice cube when $t=0$.

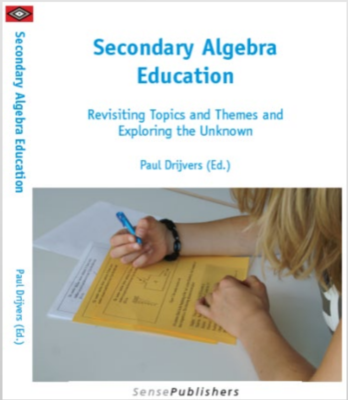
b. What are meaningful values for t? And for V?

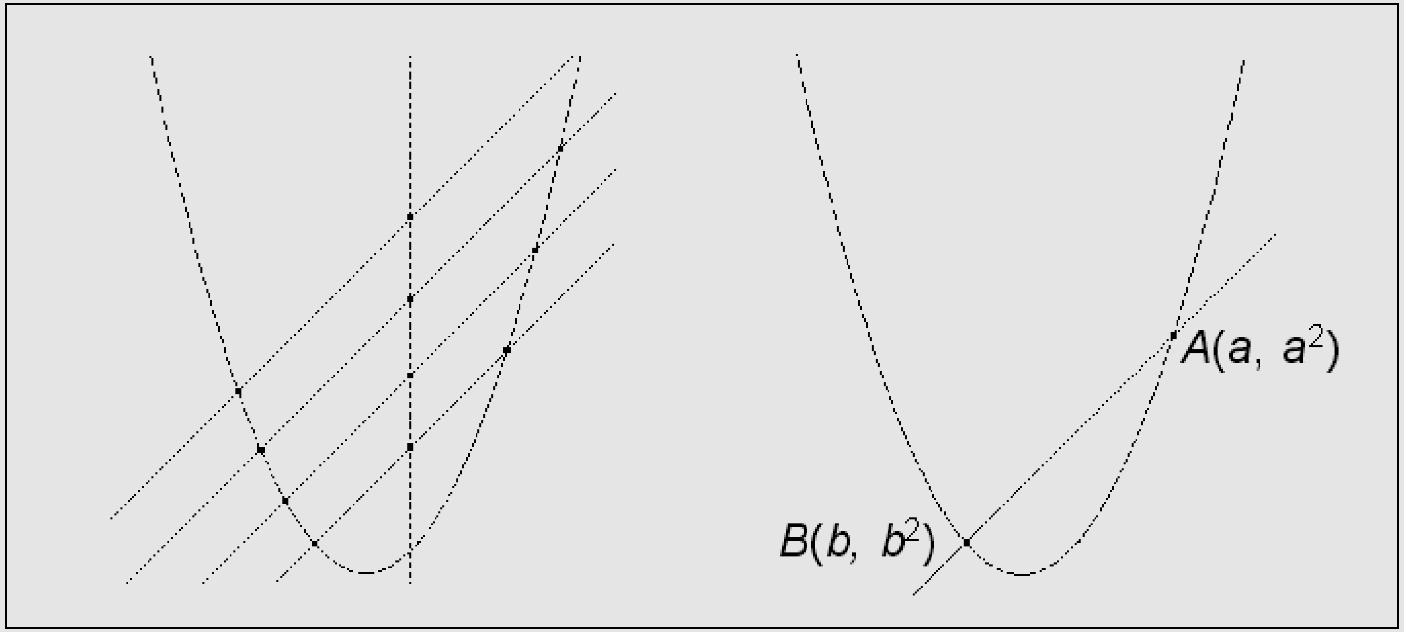
c. Plot and sketch that part of the graph for which the variables are meaningful.

d. Trace the graph with the cursor and investigate after how many minutes the volume is less than 10 000 mm^3. Provide your answer with a precision of one decimal.

Task C: Cutting a parabola

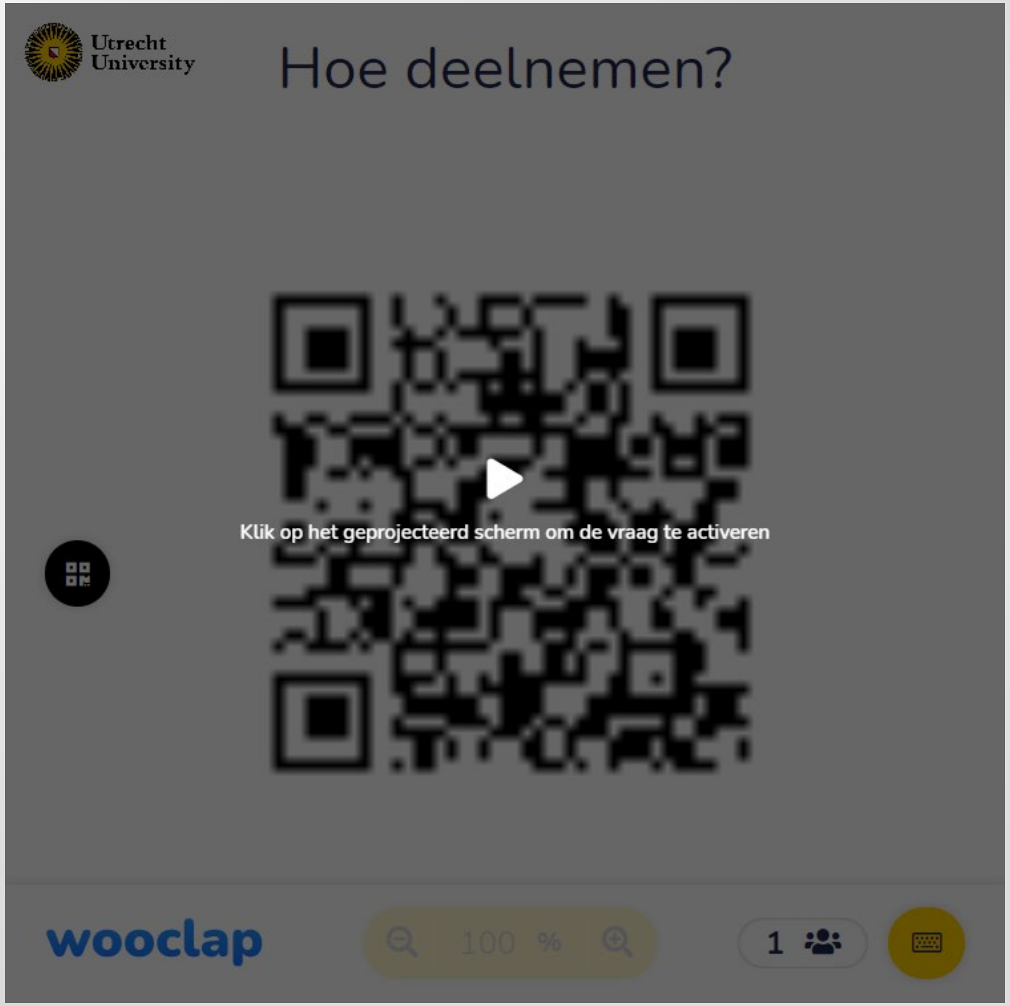
show





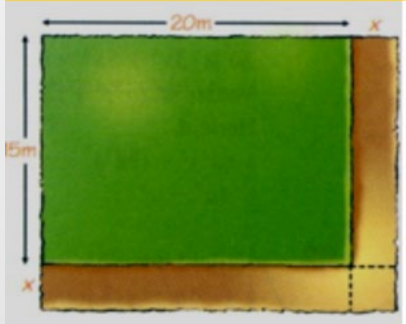
A parabola is intersected by a straight line. The line is moved upwards. The midpoint of the intersection points seems to move over a vertical line. Is this really the case?

Utrecht University



Utrecht University

www.wooclap.com/ITDJRT

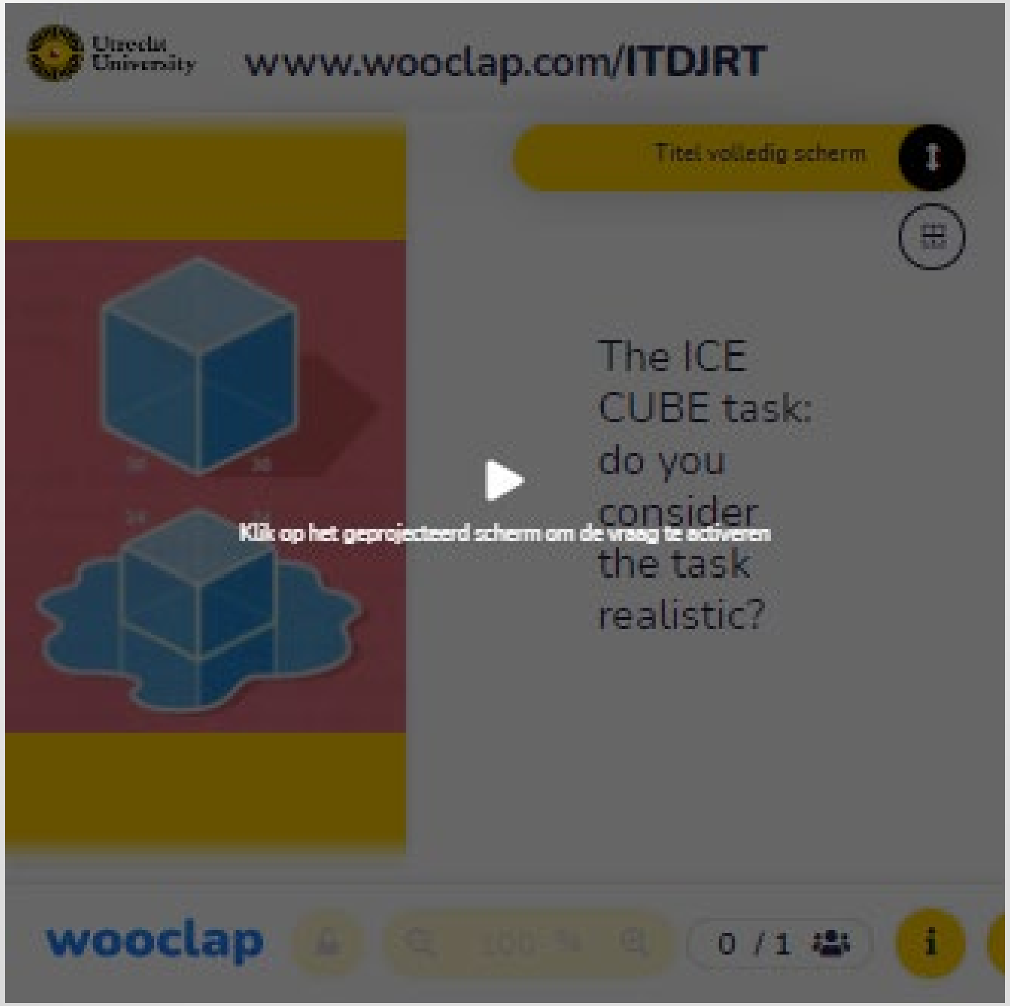


The LAWN task: do you consider the task realistic?

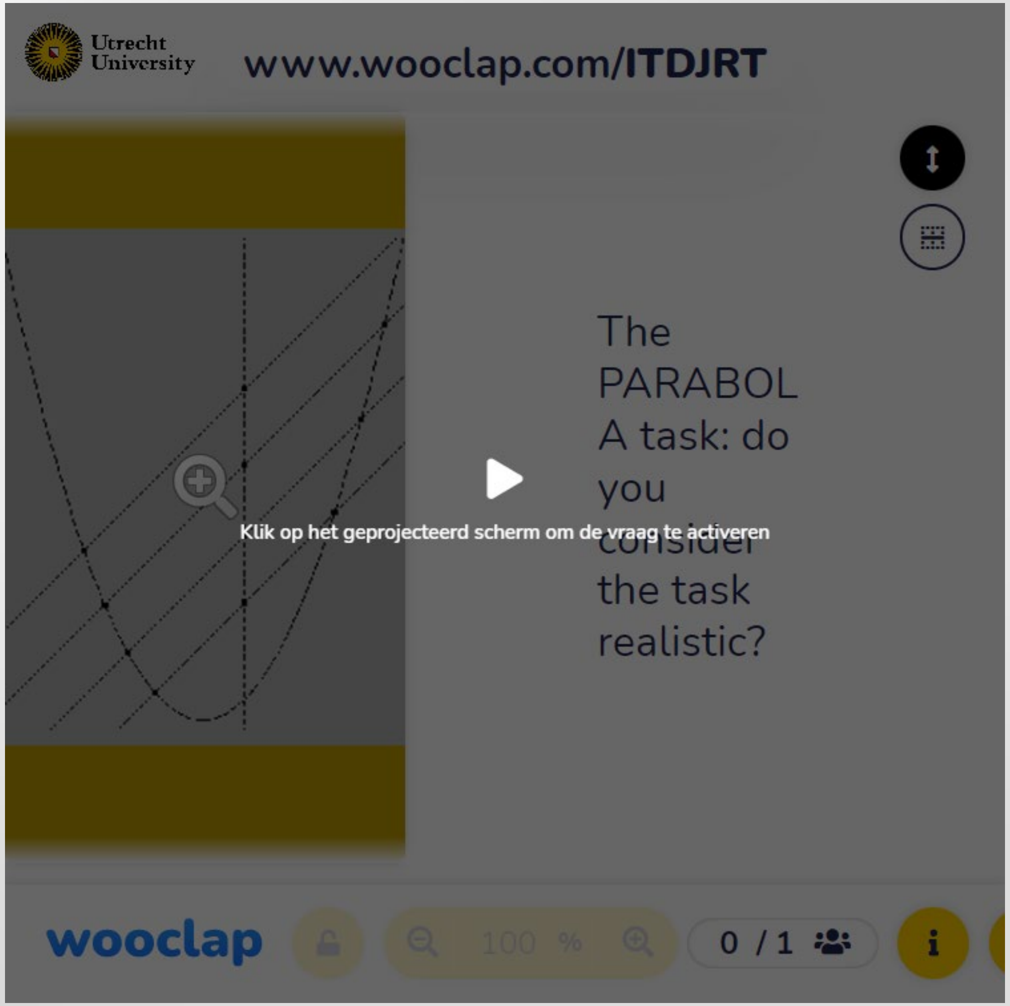
wooclap

100 %

0 / 1


Utrecht University
www.wooclap.com/ITDJRT
Titel volledig scherm
The ICE CUBE task: do you consider the task realistic?
Klik op het geprojecteerd scherm om de vraag te activeren
wooclap
0 / 1

Utrecht University



Discussion on the tasks

?

What is your opinion on the realistic qualities of the contexts and the tasks A, B and C?

Contexts in mathematics education …

- can be quite artificial
- can be quite confusing, for example from a science perspective
- may lack opportunities for mathematization
- should not necessarily be taken from dayly life

Misunderstanding: "RME means that tasks start with a real life story"

Realistic contexts in RME

An appropriate context or problem situation …

- **is meaningful for students**
- **can be a real-life situation, but can also emerge from the world of science or mathematics itself**
- **should take into account the skills, competences and interests of the students**

Utrecht University

Outline

- ✓ An introduction to RME
- ✓ Four RME key concepts
  - ✓ Mathematization
  - ✓ Didactical phenomenology
  - ✓ Use of models
  - ✓ Guided reinvention
- ✓ Hands-on task analysis
- Summary

Summary (1): What is RME?

- **RME is a domain specific instruction theory on the teaching and learning of mathematics**
- **'Reality' refers to what at a certain stage common sense experiences as real, in the sense of meaningful**
- **Mathematics is a human activity, you *do* mathematics through mathematization**

Summary (2): Four key words in RME

Students' learning of mathematics can be fostered through:

- **Mathematization**
- **Didactical phenomenology**
- **Use of models**
- **Guided reinvention**

Summary (3): Caution on contexts

- **Please mind not using artificial problem situations in textbooks and assessments that may puzzle students and don’t invite the mathematics at stake!**

- **Real life is not the main criterion; opportunities for meaning making is the challenge!**

Realistic Mathematics Education (RME) - An introduction

Thank you for your attention!

Paul Drijvers
Freudenthal Institute
Utrecht University
p.drijvers@uu.nl
https://www.uu.nl/staff/PHMDrijvers/0

15-08-2022

Some seminal past RME publications

De Lange, J. (1987). *Mathematics, Insight and Meaning*. OW & OC, Utrecht University.

Freudenthal, H. (1991). *Revisiting Mathematics Education*. China Lectures. Kluwer Academic Publishers.

Gravemeijer, K.P.E. (1994). *Developing Realistic Mathematics Education*. CD-ß Press / Freudenthal Institute.

Kindt, M. (2004). *Positive Algebra. A Collection of Productive Exercises*. Freudenthal Institute.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction – The Wiskobas project*. D. Reidel Publishing Company.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. CD-ß Press / Freudenthal Institute.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education (2nd Edition)*. Springer.