DEVELOPING A LOCAL INSTRUCTION THEORY ON RATIO AND SCALE

Master Thesis

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SURABAYA STATE UNIVERSITY

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DEVELOPING A LOCAL INSTRUCTION THEORY ON RATIO AND SCALE

MASTER THESIS

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APPROVAL OF SUPERVISORS

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DEDICATION

Sungguh saya bukan siapa-siapa tanpa doa dan didikan orangtua

Untuk ibuku, Sunarni
Yang selalu mengajarkan dan menanamkan "maaf", "tolong", dan "terima kasih"

Untuk adekku, Anggra Nur Cahyo
Yang telah menunjukkan arti kegigihan dan kesabaran

Untuk suamiku, Ronggo Nurseto
Yang berjuang bersama mulai dari nol

Untuk bapakku, Prof. Dr. Sukadiyanto, M.Pd.
Yang telah mengajarkan kesederhanaan, hidup untuk menanam kebaikan, dan berpikir visioner

ABSTRACT

Wirani, W. 2015. *Developing a Local Instruction Theory on Ratio and Scale.* Thesis, Mathematics Education Study Programme, Postgraduate Programme of Surabaya State University. Supervisors: (I) Prof. Dr. Siti M. Amin, M.Pd. and (II) Dr. Agung Lukito, M.S.

Keywords: ratio, multiplicative comparison, double number line, RME, design research

For students in the elementary education, learning ratio is important as their base in learning and reasoning about proportion on a higher level of education. There are two key concepts of ratio, multiplicative comparison and composed unit. Even though ratio holds the notion of multiplicative comparison between numbers, students struggle to use multiplicative comparison in ratios. According to many studies, students use difference and addition to determine the relationship between number in ratio. Considering students' misconception in relating numbers in ratio and the lack of explanation about learning instruction on ratio and scale, there is a need to develop a local instruction theory to support students in learning ratio and scale. Therefore, the study aims at contributing a local instruction theory, which supports students in learning ratio and scale by using double number line. To emerge the model, the contexts in the study are about plans and maps. The study used design research as the research approach and consisted of two cycles of teaching experiment. The learning instructions are designed by implementing Realistic Mathematics Education (RME). The study was conducted in SDIT At Taqwa, Surabaya. The participants of the study were the mathematics teacher and students of grade five. In this study, data such as video registrations of the learning and students' written work were obtained. These data were confronted to the hypothetical learning trajectory. The analysis result indicates that some students were able to use multiplicative comparison to determine the relationship between numbers on ratios using double number line and some other still used difference and addition to relate the numbers.

ABSTRAK

Wirani, W. 2015. *Developing a Local Instruction Theory on Ratio and Scale*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Siti M. Amin, M.Pd. dan (II) Dr. Agung Lukito, M.S.

Kata kunci: rasio, *multiplicative comparison*, garis bilangan ganda, RME, *design research*

Mempelajari rasio merupakan hal yang penting bagi siswa di sekolah dasar sebagai dasar untuk mempelajari dan menalar mengenai proposi pada tingkat pendidikan yang lebih tinggi. Terdapat dua konsep penting pada rasio, yaitu *multiplicative comparison* dan *composed unit*. Meskipun rasio terkait dengan penggunaan *multiplicative comparison* untuk menyatakan hubungan antar bilangan, siswa mengalami kesulitan dalam menyatakan hubungan antar bilangan dalam suatu rasio atau antar rasio dengan menggunakan *multiplicative comparison.* Berdasarkan penelitian, siswa menggunakan selisih dan penjumlahan untuk menyatakan hubungan antar bilangan pada rasio. Menyadari miskonsepsi yang terjadi pada siswa dan kurangnya penjelasan mengenai instruksi pembelajaran pada perbandingan dan skala, diperlukan adanya pengembangan *local instruction theory* yang dapat mendukung siswa untuk memahami perbandingan dan skala. Oleh karena itu, penelitian ini bertujuan untuk memberikan kontribusi *local instruction theory,* yang dapat mendukung siswa untuk memahami rasio dan skala dengan menggunakan garis bilangan ganda. Untuk memunculkan model, konteks yang digunakan pada penelitian ini antara lain denah dan peta. *Design research* digunakan sebagai pendekatan penelitian yang terdiri dari dua siklus eksperimen pembelajaran. Instruksi pembelajaran pada penelitian ini disusun dengan mengimplementasikan *Realistic Mathematics Education* (RME). Pengambilan data dilakukan di SDIT At Taqwa, Surabaya. Partisipan penelitian ini adalah guru matematika dan siswa kelas lima. Pada penelitian ini, data yang diperoleh berupa fragmen video penting yang terkait dengan pembelajaran dan pekerjaan siswa. Data tersebut akan dibandingkan dengan *hypothetical learning trajectory*. Hasil analisis menunjukkan bahwa sebagian siswa menggunakan *multiplicative comparison* untuk menentukan hubungan antar bilangan pada rasio dengan menggunakan garis bilangan ganda, dan sebagian siswa lain masih menggunakan selisih dan penjumlahan.

PREFACE

Praise and thank to Allah for every blessing and joy. Finishing the thesis will be impossible without support and cooperation from many people. Therefore, I would like to give my appreciation to:

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This thesis is not perfect as the writer still learning from her point of view. To improve, the writer needs different points of view from the readers. Therefore, critics and feedback are welcomed.

Wisnuningtyas Wirani

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CHAPTER 1

INTRODUCTION

A. Research Background

Many things in life are related to mathematics. One of the example is proportion. According to Tourniare and Pulos (1985), proportion is two equivalent ratios. People might not recognize the use of proportion in daily life. To illustrate the use of proportion in daily life, we provide an example. There is promotion price of tissue in two different shops, namely shop A and shop B. From brochures, we know that in shop A, people can buy 3 packs of tissue for IDR 8.000. In shop B, the price for 4 packs of tissue is IDR 9.000. A woman wants to buy tissue and she decides to go to shop B because the price is cheaper. When the woman is able to reason that tissue in shop B is cheaper, she has the notion of proportional reasoning. Proportional reasoning is related to the idea of reasoning for the equality of two ratios.

Proportion is related to ratio. What is ratio and how it is related to proportion? According to Lamon (2012), ratio is a comparison between two quantities. Refer back to the definition of proportion, proportion consists of two ratios. Ratio is a common topic we can find in the daily life. For example, in shopping context, shops usually give promotion of a product with label “buy 3 get 1 free”. In this case, we compare the number of product we buy and the number of free product we get.

Ratio is also related to scale. We can say that scale is special case of ratio

which talks about distance. To be more specific, scale is comparison between distance on the map and real distance (Ben-Chaim et al., 2012). We may find scale in maps. However, there are different examples which contains the notion of scale such as plans, miniature, and printing/copying. Scale usually use to shrink or enlarge objects. In maps, scale can be represented into symbols or drawings, for example double scale line. As an example, when we open google maps, we can see two scale lines with different measurement. These scale lines represent distance on the map and real distance.

Several studies have been done to find out students' thinking related to proportion and ratio (Karplus et al., 1983; Hart, 1984; Tourniare & Pulos, 1985; Singh, 2000). These studies provide explanation on students' misconception in learning ratio and proportion. There are two key concepts of ratio, multiplicative comparison and composed unit (Lobato and Ellis, 2010, as cited in Rathouz et al., 2014). In working on ratio problems, students tend to use subtraction and addition rather than use multiplication and division to determine relationship of numbers on ratios (Hart, 1984; Singh, 2000; Riehl & Steinthorsdottir, 2014).

In Indonesia, the learning of proportion is under the name of learning ratio. Ratio becomes one of important topics in school mathematics in Indonesia. Curriculum in Indonesia has been changed several times. However, the topic of ratio remains in the curriculum, especially mathematics curriculum for grade five. If we observe mathematics textbooks in Indonesia, the topic involves

contexts such as maps and shopping. However, the instructional learning on the textbook is limited to determine the real distance, the distance on the map, and the scale using formal method. The textbook focuses on students' ability to calculate rather than understanding the concept of ratio. In addition, the context does not support students to investigate the concepts of ratio using students' informal knowledge. Therefore, when it comes to ratio problems, students usually stick to the formula to determine real distance and use cross-multiplication to solve problems.

To help students in learning ratio, we can use a certain context and a model to help students in visualizing information. One of models we can use to introduce ratio is double number line. Using the context of maps, double number line elicits in the double scale line, in which scale is involved. According to Küchemann et al., (2014), double number line is a proper model to introduce ratio to students. This argument is supported by Abels et al. (2006) who mention the benefit of using double number line as a model. Numbers on the double number line are well ordered. It supports students to see the aligned relationship along the lines.

B. Research Question

Based on research background, we formulate the research question as follows: *How can the double number line support students in learning ratio and scale?*

C. Research Aim

Study which explicates the learning instruction on ratio and scale is still rare. Therefore, the study aims at developing the local instruction theory on topic ratio and scale. In this study, we involve double number line as a model to help students understand multiplicative comparison. We also involve contexts which fit well and elicit the model of double number line.

D. Definition of Key Terms

We define the key terms from the title, the research question, and the research aim to avoid misunderstanding of meaning and to restrict the focus of study.

1. To support

   To facilitate, to engage, and to stimulate students to do investigations, observations, and discussions to achieve certain purposes.

2. Ratio

   Ratio is comparison between two quantities. Ratio is the subset of proportion. In this study, we focus on determine missing value of equivalent ratios.

3. Scale

   Scale is comparison between distance on the map and real distance. Scale is the subset of ratio. In this study, we use scale in the map context.

4. Double number line

   A double number line is a mathematical model which consists of two

number lines with two different scales. In this study, we use double number line to represent ratios and to express multiplicative comparison between ratios.

5. Local instruction theory

   The instruction theory in this study consists of envisioned learning route and mathematical activities on topic ratio and scale.

E. Significance of the Research

There are two significancies of the study. The first significancy is to contribute to the development of local instruction theory to support students in learning ratio and scale. The second significancy is to give mathematics teachers an insight about learning instructions to support students in learning ratio and scale by involving certain contexts and double number line as a model.

CHAPTER 2

THEORETICAL FRAMEWORK

In this chapter, we will present the elements involved in the study. Before we discuss about ratio and scale, firstly we will discuss about proportion and proportional reasoning. We provide literature review about the importance of learning ratio and scale, the knowledge gap, and students' misconceptions in learning ratio.

In this study, we will design a learning trajectory based on the design principles of Realistic Mathematics Education (RME). Considering the study will take place in Indonesia, the context will be modified so that it is familiar to Indonesian students. Besides, we will provide an overview of the topic according to the Indonesia mathematics curriculum.

A. The Concepts

1. Proportion and proportional reasoning

Tourniare and Pulos (1985) defined proportion as equality relationship between two ratios in the form of $\frac{a}{b} = \frac{c}{d}$. To say that two ratios are equivalent, one must be able to reason. According to Ellis (2013), reasoning related to the idea of equality of two ratios is adressed as proportional reasoning. One must be able to understand that to get an equivalent ratio, multiplication and division are needed. Therefore, proportional reasoning becomes an indicator for students who have good understanding of

multiplicative comparison (Lamon, 2012).

Proportion problems can be distinguished into two types, missing value problems and comparison problems (Tourniare & Pulos, 1985). Missing value problems usually provide three given numbers, in which children are asked to determine an unknown number to satisfy the equivalent ratios. As an example, given a car needs 36 minutes to travel 24 km. Children are asked to determine how many minutes needed for a car to travel 6 km. For comparison problems, children are asked to compare ratios. Given car brand A consumes 3 litres of fuel to travel 40 km while car brand B consumes 4 litres to travel 50 km. We can say that the fuel consumption of car brand B is more efficient.

2. Ratio and scale

Ratio is a part of proportion. In general, ratio is widely used in describing the relationship between two measurements. Lamon (2012) defined ratio as a comparison of two quantities. For example, if there are 3 boys and 7 girls in a classroom, then the ratio between boys and girls is 3:7. Ratio can be represented as fractional form. As mentioned before, to determine whether two ratios are equal, we can write ratios into fractions. Using fractional form, it is easier to perceive ratio as a unit (Lamon, 2012).

In ratio, using the example above, it is incorrect to say that the number of girls is four more than the number of boys. According to Lobato and Ellis (2010, as cited in Rathouz et al., 2014) there are two key concepts of ratio,

composed units and multiplicative comparisons. To interpret the ratio 3:7 in composed units, students must be able to perceive that for every 3 boys, there are 7 seven girls. In addition, if students are able to express that the number of girls is 7/3 times as many of the number of boys, it means students understand that there is a multiplicative comparison in ratio.

Even though there is not much literature that specifically discusses about scale, Freudenthal (1999) mentioned scale as the example of ratio. Ratio and scale are connected one another. In addition, Ben-Chaim et al., (2012) claimed that "scale (in measurement) can be defined as the ratio between a unit of measure on a map and the real distance (using the same unit of measurement)" (p. 25). Since scale also compares two quantities, we may derive a conclusion that scale is also ratio. There are different ways to express scale, for example a verbal scale, representative scale, and graphic scale (Roberge & Cooper, 2010). Scale and its representations are usually found in map and resizing context.

According to the literature review, we can conclude that proportion, ratio, and scale are related. The three key terms talk about comparison. Proportion denotes equal comparison between two ratios while ratio denotes comparison between two quantities. Since ratio is comparing two quantities, scale is a part of ratio which compares distance on the map and real distance. There are two key concepts of proportion which ratio and scale also have, multiplicative comparison and composed unit. In this study, we will focus on perceiving

multiplicative comparison between numbers on ratios. Students need to use multiplicative comparison to determine missing values problems to show that they understand the concept of ratio.

B. The Importance of Learning Ratio and Scale

Multiplicative reasoning, equivalence, and computational fluency become the focus of mathematical themes for students in grade 3-5 (NCTM, 2000). During these grades, children must be able to understand the idea and the relationship of multiplication and division. Moreover, children must be able to recognize, create, and use the idea of equivalence using numbers and representations. In equivalence, children will involve fractions, decimals, multiplications, and divisions as their prior knowledge. Regarding to multiplicative reasoning and equivalence, children should be fluent in understanding the relationship among numbers. As mentioned before, proportion, ratio, and scale are related to multiplicative comparison and equivalence. Therefore, children must be able to employ the three mathematical themes in confronting with proportion, ratio, and scale.

Children's prior knowledge will be used together with ratio to study other topics such as similarity (Streefland, 1984) and linear equations (Ellis, 2013). When children study the concept of similarity in geometry, they will utilize the concept of ratio. For example, to prove that a triangle is similar to another triangle, students must be able to see which sides are being compared and whether the three pairs of sides are proportional. Comparing corresponding

sides on triangle is in fact ratio. Moreover, comparing whether all pairs of corresponding sides holds the same ratio is related to proportion.

On the lower level of secondary education, children will encounter ratio as an integrated concept. Children's understanding about ratio will be used in the concept of slope in linear equations (Rathouz et al., 2014). As an example, to describe what slope is, students must be able to perceive the relationship between the value of x and y. The comparison between x and y can be written as m, in a linear equation it is called as gradient. Observing every pair of x and y, children find out that the value of all pairs is the same. Thus, children who have a strong ratio concept will have better understanding of slope (Ellis, 2013).

C. Knowledge Gap

Many studies have been done to explore students' understanding in learning ratio and proportion (Karplus et al., 1983; Hart, 1984; Tourniare & Pulos, 1985; Singh, 2000). However, the study about developing a local instruction theory on ratio and explanation on how the learning sequences help students to learn the concept of ratio are still rare. Therefore, as a researcher, we need to carry out the study which aims at developing learning sequences on ratio. We hope the result of the study can contribute to the development of local instruction theory on ratio.

Other than that, there must be a change in introducing ratio in school. As mentioned before, ratio and proportion can be written as fractional form.

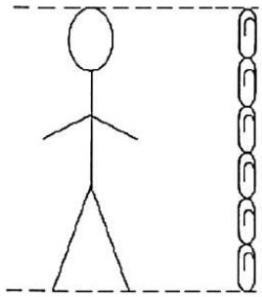
Teachers have tendency to instruct students to solve ratio and proportion problems in fractional form using cross-multiplication. Students who already understand the formal method of working with fractions might solve the problems using cross-multiplication. However, teachers need to be careful in introducing cross-multiplication. It may cause students to make mistakes, especially when students use incorrect proportions (Ellis, 2013).

D. Students' Misconceptions

There are different strategies children use to solve ratio problems. According to Tourniare & Pulos (1985), there are at least four strategies.

1. Constant difference (subtraction and addition)

   In this strategy, children determine the difference within the ratio. Later, children add the difference on another ratio. We provide the problem of Mr, Short and Mr. Tall as in figure 2.1.



In the picture, you can see the height of Mr. Short measured with paperclips. Mr. Short has a friend, Mr. Tall. When we measured their heights with matchsticks, Mr. Short's height is 4 matchsticks and Mr. Tall's height is 6 matchsticks. How many paperclips are needed for Mr. Tall?

Figure 2.1. A proportion problem about heights (Riehl & Steinthorsdottir, 2014)

From the problem, we know that the given ratio for the height of Mr. Short is 4 matchsticks:6 paperclips. Riehl and Steinthorsdottir (2014) found out that some children use constant difference to solve the problem as in figure

2.2.

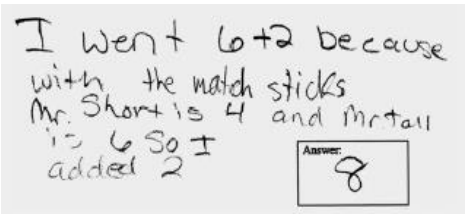


Figure 2.2. The child determined the difference within the given ratio to determine the number of paperclips for Mr. Tall (Riehl & Steinthorsdottir, 2014)

2. Multiplicative and constant difference

Children compare numerator or denominator between ratios using multiplicative comparison. However, when the relationship between ratios is non-integer, children use constant difference. As an example, we provide an example from a study (Singh, 2000). A child was asked to determine the width of a rectangle. The researcher provided another dimension of rectangle as the given ratio as in figure 2.3.

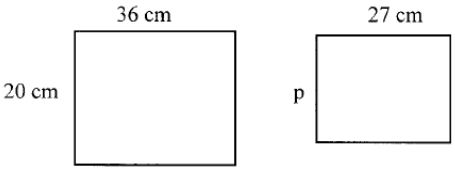


Figure 2.3. Determining the width *p* (Singh, 2000)

The child tried to determine the relationship between 36 and 27 using division. However, she could not obtain an integer as the relationship. Therefore, to determine *p*, she subtracted 36 by 27 and added the difference to 20.

3. Building up (repeated addition)

   In this strategy, children are able to perceive the ratio within numbers. However, instead of using multiplicative comparison to determine the relationship between ratio, children use addition. We provide an example to illustrate how children use this strategy (Tourniare & Pulos, 1985). "Given, 2 pieces of candy cost 8 cents. How much does 6 pieces of candy cost?" From the problem, we know that the given ratio is 2 candies:8 cents. To solve the problem, children may add 8 cents more to get 4 candies and add 8 cents more to get 6 candies. However, even though children are able to perceive the ratio, this strategy may leads students to an error when the numbers are non-integer (Steinthorsdottir, 2006).

4. Multiplicative

   Children are able to perceive the multiplicative comparison within and between ratios. Using the candies example, children understand that the relationship within ratio is the price is four times the number of candies. In addition, children understand that the number of candies in the second ratio is six times the number of candies in the first ratio. To determine the price of 6 candies, children multiply 6 candies by 4.

The first three strategies often happen to children in solving ratio problems (Hart, 1984; Karplus et al., 1974; Singh, 2000). There are two reasons children make errors in solving ratio and proportion problems (Steinthorsdottir, 2006).

The first, children do not pay attention to the given information on the problem. Ratio problems provide the given ratio of the problem. Rather than understanding the given ratio and the complete information, children tend to solve the problem using part of information (Tourniare & Pulos, 1985). It means that children do not fully understand the problem, especially the notion of ratio.

The second error is that children determine difference of numbers between ratios. As mentioned in previous studies, children are struggling to determine an integer relationship between numbers in ratios. Therefore, to obtain an integer, children are more comfortable to use addition and subtraction. We cannot separate the struggles and the fact that children are experiencing counting and whole numbers before multiplying, dividing, and working with non-integer numbers (Lamon, 2012). The second error is related to children's fluency in understanding the relationship among numbers. The fact that children use addition and subtraction in solving ratio problems proves that they are not able to grasp ratio as a multiplicative operation (Hart, 1984; Karplus et al., 1974; Singh, 2000).

Based on the literature review, we can conclude that students tend to determine difference and to do addition in order to find missing values. The cause of these misconceptions is that students try to obtain integers as the relationship within ratio and between ratios. To tackle students' misconception, we plan to design learning sequences that enable students to focus on the relationship between

ratios using multiplicative comparison.

E. Double Number Line

Students in grade 3-5 must be able to develop and to use models or representations in learning process to achieve three standards (NCTM, 2000): (1) to model problem situations, (2) to investigate mathematical relationships, and (3) to justify or to disprove conjectures. Using models or representations, students are able to select important information related to the problem.

There are different models to introduce ratio to students such as ratio table and double number line. In this study, we want to involve double number line in the learning process. On the double number line, we can display two quantities with different scales. In addition, the use of double number line supports students in understanding relationship within ratio and between ratios (Orrill & Brown, 2012). Given 2 mangos can be exchanged with 6 apples and the problem asks the number of apples for 5 mangos. Students can utilize the double number line to illustrate the ratio by putting the number of mangos on the lower line and the number of apples on the upper line as in figure 2.4.

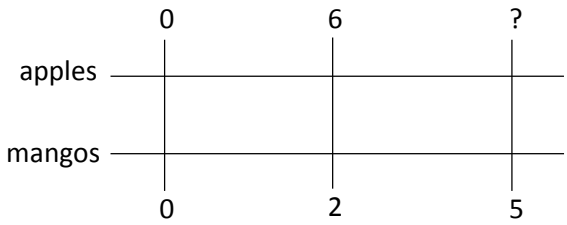


Figure 2.4. Using double number line in ratio problems

The double number line supports students to understand the relationship within ratio, the number of apples is three times the number of mangos. To

determine the number of apples in another ratio, students multiply 5 mangos by 3. Thus, students are able to determine the missing value and to understand that the numbers on the second ratio is 2,5 times the numbers on the first ratio. In addition, students are able to see the composed unit since the paired numbers are aligned and connected (Orrill & Brown, 2012).

Double number line is an appropriate model for students in grade five. As mentioned before, students in grade five do not only involved integers but also decimals and fractions. According to Abels et al. (2006), double number line serves many advantages such as it gives visual support and allows students to have an accurate calculation. Moreover, double number line has the same nature as number line, numbers on the lines are well ordered. It allows students to directly observe the relationship between numbers on the lines, especially to reason that numbers on the right side are greater than numbers on the left side. In addition, Küchemann et al. (2014) suggested to use double number line to learn about ratio. To be more specific, double number line support students in relating numbers within ratio and between ratios using multiplicative comparison.

F. Realistic Mathematics Education

Realistic Mathematics Education (RME) is an approach which has been implemented in the Netherlands since the 1970s. The development of RME was started by Freudenthal (1977, as cited in van den Heuvel-Panhuizen,

2003), who thought that mathematics should be connected to the reality. Freudenthal (1991) proposed that children should experience mathematics as a human activity. Mathematics will be more relevant and meaningful for children if they are allowed to investigate the solution of problems using their own strategy.

Even though mathematics should be meaningful for children, it does not mean that the context should be real and exists in real life. Since the word 'realistic' in Dutch means 'able to imagine', it is possible for the teacher to make up the context. At least, by using the made-up context, students are able to imagine (van den Heuvel-Panhuizen, 2003).

Choosing and designing contexts are important in developing learning sequences. To support students in building their own understanding, it is also important to design learning sequences according to the three key principles of RME which were proposed by Gravemeijer (1997).

1. Reinvention/mathematising

   The idea of the first key principle is to give students an opportunity to experience the same process as the process of how a specific concept of mathematics was invented. By having a discussion and informal talk, students will develop their informal knowledge to formal knowledge (Freudenthal, 1991).

   To emerge students' reinvention on the double number line, we provide problems in the beginning of the learning which ask students to use the

length of an object as a reference to measure the length of a room. The problems also ask students to express the length of a room using the length of an object. When students use their references to mark and measure the length of the room on the picture, they have their own illustration of the informal model. In another lessons, we also ask students to draw an illustration which represents the relationship between two quantities such as double scale line and double number line.

2. Didactical phenomenology

To define phenomenology more clearly, Freudenthal (1999) described mathematical objects as *nooumena*. In addition, experiencing a mathematical object can be called as *phainomenon*. Freudenthal (1983, as cited in Gravemeijer & Bakker, 2006) also distinguished phenomenologies into mathematical phenomenology, historical phenomenology, and didactical phenomenology. Here are the descriptions for each phenomenology.

> The mathematical phenomenology analyses how a mathematical thought object organizes mathematical phenomena, the historical phenomenology analyses how, and by what organizing activity, various concepts, procedures and tools are constituted over time, and the didactical phenomenology looks at both from a didactical perspective. (p. 2)

We involve contexts related to scaling as a didactical phenomena. For the context, Roberge and Cooper (2010) suggested to use a context which is familiar with students' surrounding and continue the context with a new situation. Students are familiar with environment around their home and school. In this study, we design the first activity with plan context and continue to larger area, for example travelling to different cities using maps. In plan and map contexts, students will use a reference.

According to Streefland (1991), mathematical activities involving the reference promotes the notion of comparison and multiplication. Students compare the reference to determine how many times the reference fits the length of the object or the path. In this case, the reference becomes the representation of fraction (Roberge & Cooper, 2010). It is also possible to include following problems involving real distance and travel time. Using the length of the path, students can determine the total travel time by multiplying the given time for a certain distance to the number of reference fits the path.

In this study, we will involve the context of map and travelling in which students are familiar with. We design problems about determining the real distance, the length of travel time, and fuel consumption to introduce the concept of ratio. Using these contexts, we expect students are able to understand the relationship within ratio and between ratios.

3. Self-developed models

Models may support students in learning mathematical concepts, especially to bridge informal and formal knowledge (Gravemeijer, 1997). There are different levels in which the informal *model of* shifts to the formal *model for*. In this study, we use double number line as a model to support students in solving ratio problems. We will describe each level of models as seen in figure 2.5 and its connection with the study.

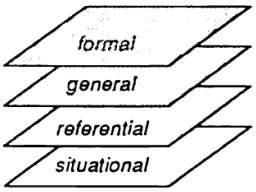


Figure 2.5. Levels of emergent model (Gravemeijer, 1997)

a. Situational level

In the first level, students use strategies which are within the context of the situation (mainly out-of-school situations) (Gravemeijer, 1997). Students' strategies in solving problems depend on their prior knowledge. In the study, we provide problems which involve students' prior knowledge in making and using a reference as a scale line. Students use the length of a reference to determine how many time it fits the length of an object. While comparing and iterating the reference on the object, students make their own illustration to represent the comparison between the length of the reference and the length of the object.

b. Referential level

Students will solve problems using strategies and models which are referring to the problems (Gravemeijer, 1997). In other words, the model

in this level becomes a *model of* for problems. In the study, the scale line develops to the double scale line. Students will be asked to illustrate the relationship between distance on the map and real distance into a double scale line. The double scale line considers the precise measurement. Therefore, the model will be more like a ruler, in which spaces between numbers are important.

c. General level

After some time, the focus of the model shifts towards the mathematical relations (Gravemeijer, 2004). In the general level, the model becomes a *model for* for the mathematical activity. Later in the study, it is unnecessary for students to preserve the precise mesurement between numbers on the model. As the context is not always about finding the real distance, students shift the model to the formal double number line.

d. Formal level

In the last level, students will work with conventional procedures and symbols (Gravemeijer, 1997). Students will not need the model to solve ratio problems. The focus is to elicit the multiplicative comparison using the formal method.

G. Indonesian Mathematics Curriculum

Before students are introduced to ratio, they have learned about addition, subtraction, multiplication, division, and measurement in the previous grades. Students also have prior knowledge about fractions, percentages, and decimals.

We provide the standard competence and the basic competence of learning ratio for students grade five in table 2.1.

Table 2.1. The concept of ratio and scale in the Indonesian mathematics curriculum (Kemendikbud, 2013)

The Standard and Basic Competence in Grade 5	
3. Understanding the factual and conceptual knowledge by observing and asking about their environment	3.4 Learning the concept of ratio and scale

H. Social Norms, Socio-mathematical Norms and the Role of Teacher

The notion of social norm comes from a situation where a person is involved in a specific environment (Tatsis & Koleza, 2008). To interact well, the person must agree to the written or unwritten rules set in the environment. A social norm is general and can be found in different situations (Yackel & Cobb, 1996), for example in a mathematics classroom. However, we cannot say that norms in the mathematics classroom are socio-mathematical norms. Socio-mathematical norms are unique and can be applied only in the mathematics classroom.

What makes socio-mathematical norms different compared to social norms? Socio-mathematical norms are about noticing mathematical difference, mathematical sophistication, and acceptable mathematical explanation and justification (Yackel & Cobb, 1996). The mentioned aspects are aiming to grow an intellectual autonomy in mathematics, which means that students are aware that they are able to reason and to draw conclusions based on their

mathematicals knowledge.

A certain classroom culture contributes to the mathematics reformation (Gravemeijer, 2004). One of affecting elements in the classroom is the teacher. Since it is important to raise students' awareness of what they are capable of in the mathematics classroom, the role of the teachers is important. Teachers support students by asking questions or making explicit statements on why a students' strategy in solving a problem counts as a good strategy. In addition, the teacher may ask students to reason how their answers are different compared to other students' answers.

In the study, the role of the teacher is to focus on supporting students to discuss about concepts related to ratio. For example, the teacher can conduct a discussion about a scale line, double scale line, and double number line during the lessons. During the discussion, the teacher may ask questions such as what students know about the term, what students can derive from the term, what information the model holds, and what differences and similarities among those models. In addition, the teacher should give an example to distinguish between difference and comparison. The teacher may ask students what they know about ratio. Later, using students' understanding about ratio, the teacher gives reinforcement that difference denotes the notion of subtraction and addition while comparison denotes the notion of multiplication and division.

Students might not able to understand the notion of ratio on problems and it may lead to errors. To support students, the teacher points out the given ratio.

According to literature review, students often use subtraction and addition to determine missing value. The teacher should suggest students to focus on multiplicative comparison between numbers on ratios. The teacher should discuss students' answers. For example, the teacher invites students to discuss what effect students will face if they are using subtraction and addition or multiplication and division to solve problems.

I. Local Instruction Trajectory

Gravemeijer (2004) defined local instructional theory as "the description of, and rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic" (p. 107). There are four elements of local instruction theory: (1) tools as the representation for students to solve the problem, (2) imagery as students' understanding on the tools, (3) activity, and (4) potential mathematical discourse topic. According to the definition, local instruction in this study provides the description of envisioned learning routes and mathematical activities on the topic of ratio and scale.

CHAPTER 3

METHODOLOGY

A. Research Approach

As mentioned before, the study and the development of local instruction theory on the teaching and learning of ratio and scale is rare, especially in Indonesia. According to the research question of the study: *How can the double number line support students in learning ratio and scale problems?*, we aim to design learning sequences on ratio and scale. Moreover, the study wants to find out how the learning sequences enable students to learn about ratio and scale. In other words, the study aims to innovate the current learning and to contribute to the development of local instruction theory on ratio and scale. The aim of the study fits with the design research approach because it gives insights about innovative ways of teaching (van Eerde, 2013).

According to Gravemeijer and Cobb (2006), there are three steps in carrying out design research. In each step, the designed HLT has its own function. In this section, we will mention and connect the steps to the study.

1. Preparing for the experiment

   The aim of this step is to formulate a local instruction theory, which can be revised and elaborated (Gravemeijer & Cobb, 2006). In this step, the researcher needs to do literature review to formulate the research aim, the research question, and to design hypothetical learning trajectory (HLT). The

HLT becomes a guideline for the researcher to develop the design instructional materials in the preparation phase (Bakker, 2004).

Other than that, we perform interview with the teacher and class observation to find out socio norms, socio-mathematical norms, and the teaching-learning process in the classroom. To find out whether students have a notion on a multiplicative comparison and on using double number line to solve ratio problems, we carry out a pretest. Problems in the pretest are based on the literature review on ratio and scale.

2. Experimenting in the classroom

According to Gravemeijer and Cobb (2006), the aims of experiment are to test conjectures on the HLT and to understand how conjectures work. In this step, the HLT becomes a guideline for the teacher and the researcher to focus on every activity in the learning sequences, especially the teaching-learning activity, interview, and observation (Bakker, 2004). There are three stages of cyclic process in experimenting: testing, understanding, and developing the learning trajectory. The cyclic process aims to improve the HLT for the next cycle.

In this study, the first cycle is conducted using the designed HLT. In every lesson, the researcher compares the HLT to the actual learning. Moreover, we reflect on what can be improved for the next lesson. The process is called micro cycles. The micro cycles form macro cycle, which shapes the local instruction theory (see figure 3.1).

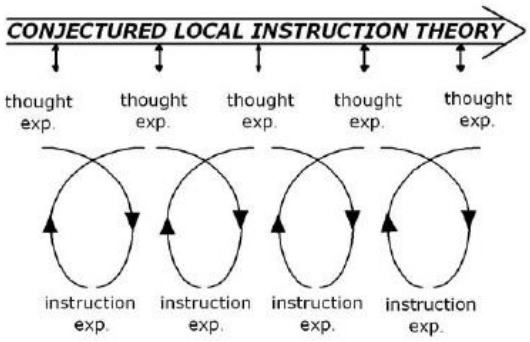


Figure 3.1. Micro cycles form and shape the local instruction theory (Gravemeijer & Cobb, 2006)

Once the first cycle has finished, the researcher analyzes all collected data and compares the HLT to the actual learning. The researcher finds out things to be improved or revised for the design. The improved HLT is carried out on the second cycle.

3. Conducting a retrospective analysis

In general, the aim of the retrospective analysis is to contribute to the development of a local instruction theory (Gravemeijer & Cobb, 2006). In this step, the HLT becomes a guideline for the researcher to determine the focus of the analysis in the retrospective analysis (Bakker, 2014).

In this study, the retrospective analysis is executed after the second cycle. Furthermore, the HLT is compared to the actual learning. From the analysis, the researcher confirms findings which are in line or contradict the HLT. In addition, the researcher derives conclusion from the analysis and use the conclusion to answer the research question of the study.

B. Data Collection

1. Observation

In order to gain an overview about the classroom and the participants, the researcher observes the learning in the classroom. The researcher pays attention to the way the teacher manages the class and conducts the lesson, the interaction among students, and the interaction between the teacher and students. In specific, the focus of the observation is socio norms and socio-mathematical norms in the classroom. The researcher makes field notes according to the observation scheme (see appendix A) and video registration during the observation.

2. Semi-structured interview

After the observation, the researcher interviews the teacher to ask questions according to the observation. The interview is conducted according to the interview scheme (see appendix B) to obtain the intended information. Through the interview, the researcher gains more information not only about what happens in the classroom but also things which the researcher cannot observe in the class, for example the background of the teacher, learning material and teaching method the teacher used on ratio and scale, and students' knowledge. The researcher makes recordings of the interview for data analysis.

3. Pretest

Before the researcher starts implementing the lesson, a pretest is conducted. The pretest consists of five problems on the topic of ratio and scale (see appendix C). In general, the goal of each item is to determine students' ability in using multiplicative comparison to solve ratio problems. Moreover, we want to find out whether students use representations to help them solve problems.

The researcher also conducts an interview and makes video registrations to gain information about how students understand the topic of ratio and scale and on how students solve problems. In the first cycle, the researcher interviews students who become the participant of the pilot study. In the second cycle, the researcher interviews the focus group students.

4. The pilot study (first cycle)

The first cycle becomes a pilot study which aims to try out the designed HLT and involves five students of class 5B who are intermediate achievers according to the teacher's recommendation. In the first cycle, the researcher becomes the teacher. With the help of observer, the researcher collects students' written work, and makes field notes and video registration on the teaching-learning process. Once the collected data is analyzed, the researcher revises and improves the HLT to be implemented in the next cycle.

5. The teaching experiment (second cycle)

In the second cycle, the researcher implements the improved HLT. The teaching experiment involves 34 students from class 5C and the regular mathematics teacher. There is one focus group consists of four students with the intermediate level of mathematics according to the pretest and the teacher's recommendation. In the second cycle, the teacher is the regular mathematics teacher in that class while the researcher acts as the observer.

The researcher collects students' written work, makes field notes and video registrations. There are two video recorders to record the activity and discussion in the focus group and the activity of the whole class.

6. Posttest

Once the researcher has finished implementing all the five lessons, the researcher conducts a posttest in each cycle. The posttest aims to find out the development of students' understanding about ratio, especially the notion of multiplicative comparison. Furthermore, we want to find out whether students use representations such as double number line, to solve problems. The content of the posttest is as the same as in the pretest (see appendix C). In addition, the researcher interviews students and makes video registrations for the interview in each cycle to find out students' development in understanding the topic and solving problems. In the pilot study, the researcher interviews the five students while in the teaching experiment, the researcher interviews the focus group students.

7. Validity and reliability

Validity and reliability are important in the data collection to derive valid and reliable conclusion. In general, validity means whether the instruments truly measure what it intends to measure while reliability means the independency of the researcher (Bakker & van Eerde, 2014).

To improve the validity, the researcher consults the design materials to experts. In data collection, the researcher collects data using different methods. The collected data is triangulated in the analysis. The data triangulation also contributes to the internal validity of data collection. Furthermore, the natural learning environment, in which the teacher conducts the lessons and the learning happens in a regular classroom, promotes the ecological validity of the study. To increase the reliability of the data collection, the researcher obtains video registrations by using video recorders.

C. Data Analysis

1. Observation

In the study, the researcher chooses important and relevant fragments such as socio and socio-mathematical norms in the class. The researcher describes and interprets the fragment. The result of the analysis is used to improve the HLT and to support the conclusion of the study.

2. Semi-structured interview

This study analyzes the interview recording as the same as in analyzing the fragments in observation. Together with the result of observation analysis, the result of analyzing transcripts and fragment of interview is aiming to improve the HLT and to support the conclusion of the study.

3. Pretest

The researcher analyzes the pretest in both cycles qualitatively by looking at students' reasoning in solving ratio problems. The pretest analysis aims to determine students' knowledge about ratio and scale. Important fragments from the interview reveal students explanation on solving problems. The researcher describes and use fragments to support students' written work.

The result of the pretest can be used to improve the HLT about students' prior knowledge. Along with the recommendation from the teacher, the result of the pretest in the second cycle is used to determine the focus group students.

4. The pilot study (first cycle)

In this study, the researcher analyzes the collected data by comparing the designed HLT to the actual learning. The researcher chooses important fragments from video registrations about students' thinking, especially in the notion of multiplicative comparison and the notion of using double

number line to solve the problems. These fragments give information about the occurrence of the conjectures. Together with the fragments, the researcher uses relevant field notes and students' work to illustrate and describe students' thinking. The result of the analysis of each lesson is used to improve the HLT for the next lesson while the result of the whole analysis is used to improve the HLT for the next cycle.

5. The teaching experiment (second cycle)

The researcher analyzes the collected data in the second cycle by selecting and describing important fragments from video registrations. In general, the researcher chooses important fragments to confirm or to refute conjectures in the HLT. Since the result of the retrospective analysis is to answer the research question, it is important for the researcher to choose a fragment which shows students solving problems using the double number line and multiplicative comparison. The researcher interprets the fragments together with relevant field notes and students' written work as an illustration of students' thinking. To analyze students' written work, the researcher uses conjectures to compare it to the actual students' work.

6. Posttest

The posttest is analyzed in the same way as in analyzing the pretest. From the video registrations on interviewing students after the posttest, the researcher chooses important segments to be described and transcribed. The

example of important segments is the fragment shows students explaining their method in solving problems. From the posttest, the researcher also derives supporting arguments about the development of students' understanding in learning ratio and scale, especially in using a double number line and a multiplicative comparison to solve ratio problems.

7. Validity and reliability

Bakker and van Eerde (2014) classified validity and reliability into internal and external aspects. In the study, the researcher tests the data with the HLT. From the analysis, the researcher adjusts and improves the HLT for the next lesson and cycle. This process contributes to the internal validity.

External validity is related to the generalizability of the result which can be used in different contexts (Bakker & van Eerde, 2014). To improve the generalizability of the study, thick description is needed. In the study, the researcher provides the description and interpretation on the actual and observed learning. In addition, the researcher considers the diversity of environment conditions such that others can adjust the design according to their own condition.

A study has a high external reliability if it can be replicated. Therefore the documentation of the study must be very clear that the readers are able to track how the experiment was going and how the researcher derives the conclusions (Bakker & van Eerde, 2014). In this study, the researcher

explains explicitly about the process and the result. Therefore, the readers can follow the line of this study.

D. Research Subject

The participants of the study are students in the fifth grade and their mathematics teacher in SDIT At Taqwa, Surabaya. In the first cycle, the pilot class is class 5B which consists of five students. For the real experiment, the chosen class is class 5C which consists of 34 students.

CHAPTER 4

HYPOTHETICAL LEARNING TRAJECTORY

Before carrying out a design research study, a researcher needs to design learning sequences. A researcher has to find out the prior knowledge of students to learn the topic and decide the end point of the learning. Every learning sequence contains planning about how the learning will be implemented, which is called hypothetical learning trajectory (HLT). It is important for a researcher to have HLT according to background theories. HLT provides an insight about the focus of each learning. According to Simon (1995), an HLT consists of three components: the learning goals, the learning activities, and conjectures of students' answer.

As mentioned in chapter 3, HLT has different functions in the design research phase (Bakker, 2004). In the design phase, the formulated HLT functions as a guide of the design of developed instructional material. Researchers will have a foundation as a focus of the study and still be able to improve the planning and the learning material during design phase. The HLT becomes a guideline not only for researchers, but also teachers in teaching experiment. It helps researchers and teachers to focus on teaching process, interview, and learning observation. In this phase, the HLT can be improved for the next lessons. In the retrospective analysis phase, the HLT functions as a guideline for researchers to analyze the collected data. Using the HLT, researchers compare conjectures of students' answer to the actual learning and produce instruction theories. The HLT can also be improved

before researchers implementing the next cycle.

In this study, we focus on developing a local instruction theory on ratio and scale. Using information on the preparation phase, such as reviewing literature and interviewing the teacher about students' prior knowledge, we design five lessons which aim to support students to learn about ratio using double number line. Each lesson has its own learning goal. We provide the instructional activities for learning ratio as follows.

A. Lesson 1

1. Starting points

In previous grades, students have learned about expressing inequality informally by comparing a pair of opposite characteristics, estimation, multiplication, and division. Before students learn about ratio, they have learned about fraction and decimal. Using the preliminary knowledge, we would like to allow students to develop their knowledge. Thus, we assume that students have starting points such as:

- Students are able to estimate the length of an object
- Students are able to compare two quantities
- Students are able to use multiplication and division
- Students are able to present numbers into fractions and decimals

2. The learning goals

Students are able to use a scale line.

- Students are able to use the length of a reference to describe the length of an object.
- Students are able to use the scale line to describe the real length.
- Students are able to make and to use a scale line in a plan.

3. Description of activity

Activity 1

Problem 1a

The teacher starts the lesson by displaying the picture of a studio plan. The purpose of displaying a picture of a studio plan is to trigger students to come up with the notion of a scale line. To support students, the teacher asks information students usually get from a plan and leads students to mention about scale. These are questions the teacher can ask students:

- *"What do you know about scale?"*
- *"Where do you find scale other than in a plan?"*
- *"What representation do you use to illustrate scale?"*

A scale can be in different forms. As in a plan, it can be in numeric or illustration such as a scale line. Some students might not familiar with a scale line. Thus, the teacher supports students by telling that a scale line is a line which contains a certain scale.

Later, the teacher asks students to measure the dimension of the bedroom. Some students might propose using a ruler since it is a plan. The

teacher asks students to imagine it is in a real condition, therefore it is impossible to use a ruler because it will be inefficient. Besides, the focus is not about being precise in measuring but describing how many times the length of an object fits the length of another object. The teacher suggests students to use a paper strip as a ruler. If students have no idea, the teacher may lead students to use furniture on the plan to measure the dimension of bedroom. An object which students use to measure is called reference. Once students are able to use the reference, the teacher proceeds to students' worksheet. Figure 4.1 illustrates the problem. The teacher narrates the context:

"An architect draws a studio plan. However, he does not write the measurement of the studio. If people who are interested in buying the studio want to know the measurement of the studio, what will they do?"

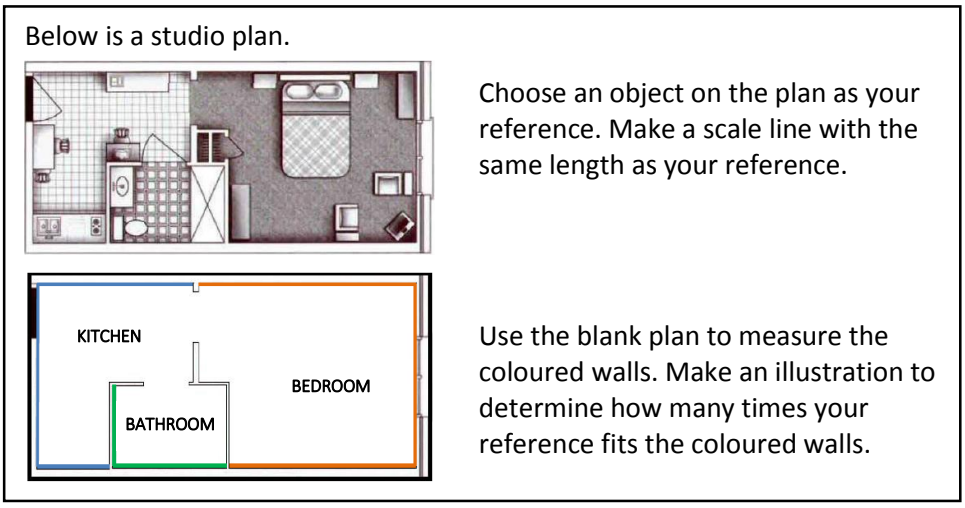


Figure 4.1. The studio plan and the blank studio plan

To emerge discussion, the teacher gives students some time to think and to

discuss with their friends. We provide conjectures of students' answer in table 4.1.

Table 4.1. Conjectures of students' answer for problem 1a in lesson 1

Conjectures of students' answer	**Suggestions for the teacher**
Students are not able to use the length of a reference as a scale line and not able to illustrate the relationship of a reference and a coloured wall	The teacher may encourage students to use the reference to determine how many times the reference can fit the coloured walls. To make an illustration, the teacher can encourage students to compare the reference and the object, then mark how many times the reference fits the length of the object
Students are able to use the length of a reference as a scale line, determine the length of walls, and illustrate the comparison between the reference and the coloured wall	If students use small furniture, the teacher may ask students to use large furniture

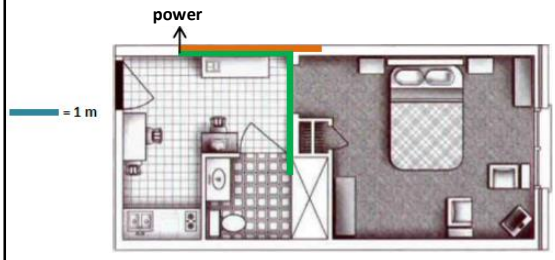
Once students finish working, the teacher conducts class discussion. The teacher can discuss about different references give different measurements. For example, to fit the orange wall using the length of a bed, we need two times of the bed's length. However, when we use the length of a couch, we need seven times of its length to fit the orange wall. The teacher needs to point out that different references influence different measurement, especially in determining how many times the reference fits the coloured

walls.

Problem 1b

The context of the problem is as the same as the previous problem. Students will encounter a problem which involves a scale line with a specific measurement as in figure 4.2. The problem promotes students to make their own model to compare the length of a scale line and the cables.

A technician will install cables from the power to bedroom and toilet.



The orange and green lines illustrate the installation of cables.

Make an illustration to determine how many times the scale line fits the other lines.

Determine the real length of both cables.

Here is my illustration.

The length of orange line is _____. So, the real length is _____.

Here is my illustration.

The length of green line is _____. So, the real length is _____.

Figure 4.2. Making illustration and determining the real length

Students will be given some time to think and to discuss with their friends. We provide conjectures of students' answer in table 4.2.

Table 4.2. Conjectures of students' answer for problem 1b in lesson 1

Conjectures of students' answer	**Suggestions for the teacher**
Students are not able to use the scale line and not able to draw an illustration to compare the length of a scale line and the cables	The teacher may encourage students to draw the scale line above the orange or green line. Ask students to determine how many times the scale line fits the other lines by marking the orange or green line
Students are able to use the scale line and able to draw an illustration to compare the length of a scale line and the cables	The teacher may ask students to find out how they use the scale line

In the class discussion, the teacher needs to point out the illustration students make to illustrate how many times the scale line fits the cables. The illustration will be students' own construction model. There are questions the teacher can ask to students in the class discussion:

- *"What do you know about the blue line on the problem?"*
- *"What the blue line represents?"*
- *"How do you use the blue line to measure the orange and green line?"*

Activity 2

In this activity, students will have to draw a plan for their classroom. Figure 4.3 shows the problem on the worksheet. We provide conjectures of students' answer in table 4.3.

Make a plan of your classroom. Choose an object as your reference. Use the reference as a scale line to draw the plan. Describe the dimension of your classroom.

I use _____ as a reference. I estimate its real length is _____. Below is my scale line:

Below is my classroom plan:

Figure 4.3. Instructions to draw a classroom plan

Table 4.3. Conjectures of students' answer for activity 2 in lesson 1

Conjectures of students' answer	**Suggestions for the teacher**
Students are not able to use a scale line to draw a classroom plan and to determine the real dimension of the classroom	The teacher may encourage students to imagine how many times their reference fits the classroom. Later, ask students to draw the dimension of classroom using the scale line. Since students already estimate the real length of the reference, ask students if for one reference the length is *x*, then what about, for instance, four times of reference
Students are able to use a scale line to draw a classroom plan and to determine the real dimension of the classroom	The teacher may ask students to think about what will happen if another student choose different reference with different length

In a class discussion, it is important to point out how students use a reference as a scale line to draw the classroom plan. Students might choose

different references and different estimations for the length. The teacher can discuss how different references and different lengths may affect students' classroom plan.

B. Lesson 2

1. The learning goals

Students are able to work with a double scale line.

- Students are able to use information about real distance and distance on the map to determine which map fits the given double scale line.
- Students are able to illustrate the relationship between the distance on the map and the real distance into a double scale line.
- Students are able to use the double scale line to determine real distances.

2. Description of activity

Problem a

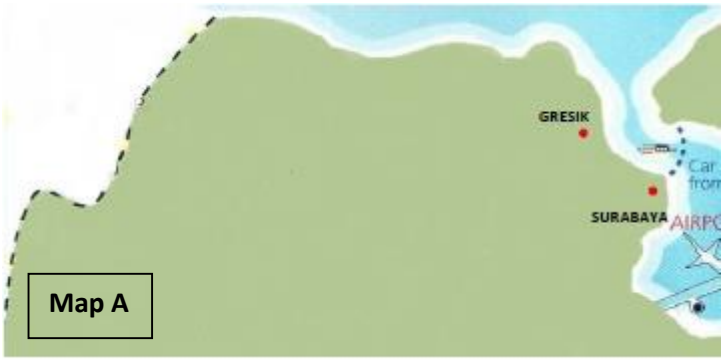
The teacher starts the lesson by asking following questions related to a map:

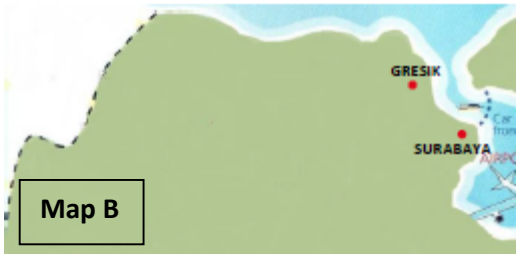
- *"What information you can get from a map?"*
- *"Have you ever seen a double scale line? What do you know about it?"*
- *"Why do you think it is called as a double scale line?"*
- *"Why there are two lines on a double scale line?"*

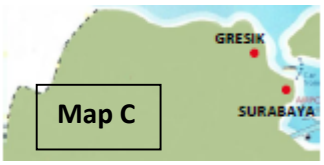
Later, the teacher proceeds to introduce the problem to students. The teacher may narrate the problem. Give students some time to understand

information given on the problem and to discuss with their friends. In this lesson, students are allowed to use a ruler to measure. Figure 4.4 illustrates the problem. We provide conjectures of students' answer in table 4.4.

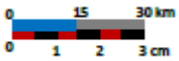
Here are three maps with different sizes.







A map usually has a double scale line. Below is the double scale line.



Given the distance from Gresik to Surabaya is 15 km, determine which map fits the double scale line. What will happen to other maps if those maps use the provided double scale line? Explain your answer.

Figure 4.4. Three different size maps and a double scale line

Table 4.4. Conjectures of students' answer for problem a in lesson 2

Conjectures of students' answer	**Suggestions for the teacher**
Students have no idea on working with the double scale line	The teacher may remind students about why there are two lines on a double scale line and what information it holds
Students are able to determine which map fits the double scale line but they are not able to reason for other maps	The teacher may encourage students to compare 15 km on the double scale line to the two other maps
Students are able to determine which map fits the double scale line and able to reason for other maps	The teacher may ask students to show his reason using different distances from the three maps and the double scale line

In a class discussion, the teacher points out what will happen to other maps if those maps use the provided double scale line. Moreover, the teacher leads students to discuss why a double scale line on a map should represent the precise measurement. Other than that, the teacher may direct the discussion to ratio. The teacher tells students that a double scale line is a way to illustrate ratio and that ratio is a comparison between two quantities. The teacher may discuss ratio in depth relating to the double scale line by asking following questions:

- *"What do you know about ratio?"*
- *"What can you tell about double scale line related to ratio?"*
- *"Do you think scale is a ratio? Why?"*

Problem b

Once students have determine the map which fits the double scale line, students will have to draw double scale lines for two other maps. Figure 4.5 shows the problem. We provide conjectures of students' answer in table 4.5.

> You have determine which map fits the double scale line. Draw double scale lines for the other maps.

Figure 4.5. Students have to draw double scale lines

Table 4.5. Conjectures of students' answer for problem b in lesson 2

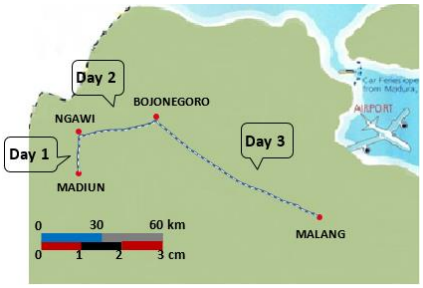
Conjectures of students' answer	Suggestions for the teacher
Students draw inappropriate double scale lines: • Students do not know how to draw a double scale line • Students do not write information on the double scale line • Information and students' drawing are not match	• The teacher may remind students about information they can get from a double scale line • The teacher may remind students to write information on the double scale line. Therefore, others understand what the double scale line tells • The teacher may lead students to have precise measurement in drawing the double scale line and make sure that the drawing represents the correct information
Students draw appropriate double scale lines	The teacher may ask students to think about how a double scale line can be used to determine real distance or distance on the map

In a class discussion, it is important for the teacher to emphasize the preciseness in drawing a double scale line. The teacher may relate to the previous problem, what will happen if a map does not have a proper double scale line. The teacher may use an example of travelling to a new place using GPS. If the double scale line on GPS does not show an accurate measurement, people might get lost while travelling.

Activity 2

In this activity, the teacher may ask whether students are able to use the double scale line to determine real distance. Later, the teacher narrates the problem as in figure 4.6. Since students do not know distance on the map, the teacher encourages students to use a ruler to measure distance between cities. We provide conjectures of students' answer in table 4.6.

Look at the map below.



Edi and his family lives in Madiun. They will travel to different cities by car on holiday. Each day, they will travel to another city.

Edi's father will choose the shortest route. Edi does not know how far they will travel, so his father makes an illustration. Help Edi's father to complete the line below by measuring distance between cities and determining real distance.

Figure 4.6. Using double scale line to determine real distance

Table 4.6. Conjectures of students' answer for activity 2 in lesson 2

Conjectures of students' answer	**Suggestions for the teacher**
Students do not use the model to determine real distances	The teacher may encourage students to show distances between cities on the map on the double scale line. The teacher may ask students to use the double scale line to determine real distances
Students use the model to determine the relationship between numbers to find out real distances • Students determine real distance for each cm • Students observe the relationship between distances on the map	The teacher may ask students to explain their work, especially how determining distance for each cm is different to observing relationship between distances on the map

It is possible that students might not consider the problem as ratio problem. In a class discussion, the teacher may ask students to show the ratio of the problem. Since ratio can be written in fractional form, the teacher may also relate the given ratio to fraction. Other than that, the teacher may conduct discussion on how students determine real distances. Some students might determine real distance for each cm, use formal method, or observe the relation among distances on the map. The teacher can discuss the drawback and the benefit of using different methods.

C. Lesson 3

1. The learning goals

- Students are able to represent the relationship between distance on map and real distance into an illustration.
- Students are able to order the given numbers on the stretched route to determine real distance.
- Students are able to use the stretched route to show the relationship between numbers.

2. Description of activity

Problem 1a

The teacher starts the lesson by reminding students about a double scale line. Given a Bali map as in figure 4.7, the teacher instructs students to draw the double scale line using given information. We provide conjectures of students' answer in table 4.7.

Pak Nyoman owns a tour business in Denpasar. He arranges tour for tourists. There is a bus to transport tourists in a journey to five different cities. Here is the route on the map.

Pak Nyoman knows 4 cm on the map represents 80 km.

Make a double scale line for the map.

Figure 4.7. A Bali map and its route

Table 4.7. Conjectures of students' answer for problem 1a in lesson 3

Conjectures of students' answer	Suggestions for the teacher
Students draw inappropriate double scale lines: • Students do not know how to draw a double scale line • Students do not write information on the double scale line • Information and students' drawing are not match	• The teacher may remind students about information they can get from a double scale line • The teacher may remind students to write information on the double scale line. Therefore, others understand what the double scale line tells • The teacher may lead students to have precise measurement in drawing the double scale line and make sure that the drawing represents the correct information
Students draw appropriate double scale lines	The teacher may ask students to think about how a double scale line can be used to determine real distance or distance on the map

In a class discussion, the teacher may remind students using class discussion in previous lesson about drawing double scale line. The teacher needs to make sure that students draw a precise double scale line and write appropriate information on the double scale line.

Problem 1b

The teacher proceeds to narrate the problem as in figure 4.8. While

narrating the problem, the teacher informs students that students do not have to work with a ruler since distances on the map have been given.

To inform the tourists how far the journey is, Pak Nyoman measures the distance of each distance from Denpasar. Here is the result.

Destination	Distance on the map (cm)
Ubud	1
Klungkung	2,5
Batur	5
SIngaraja	6,5
Negara	10

If the route is stretched into a line, it will be the illustration below. Use the line to determine the real distance from Denpasar to each destination. Complete the line with information.

Figure 4.8. Working with the stretched route

Other than that, students will work with a stretched route. Since the model has developed into more formal model, the teacher tells students that they may ignore the preciseness of measurement on the stretched route. Moreover, students need to focus on the given numbers to determine the relationship between numbers. We provide conjectures of students' answer on table 4.8.

Table 4.8. Conjectures of students' answer for problem 1b in lesson 3

Conjectures of students' answer	Suggestions for the teacher
Students are not able to order the given numbers on the stretched	The teacher may encourage students to think of the stretched

route	route as a number line or a scale line, in which numbers should be well ordered
Students determine real distances by relating given numbers using subtraction and addition	The teacher may encourage students to observe the relationship between numbers using multiplication or division rather than determine difference between numbers
Students determine real distances by relating numbers using multiplication and division	The teacher may ask students to help their friends, especially other students who determine difference to find out real distance

In a class discussion, it is important for the teacher to discuss students' work which use addition to determine real distance. The teacher may compare to other students' work who observe the relationship between numbers using multiplication or division. Using students' work, the teacher may conduct discussion what will be the drawback of the method and the benefit of the method. In addition, the teacher invites students to discuss the operation in finding difference and in comparing. The teacher underlines that difference is related to subtraction and addition while comparison is related to multiplication and division.

Referred back to the term ratio, in which students comparing two quantities, it means multiplication and division are needed. Therefore, the teacher supports students to observe the relationship between given numbers

using multiplication and division rather than determine the difference between numbers.

Problem 2

In this problem, students will have to draw the model by themselves. Students might struggle to decide what information should be on the lower or upper line. Therefore, the teacher decides that the given information is written on the lower line while the determined value is on the upper line. Figure 4.9 illustrates the given problem. The problem is related to the previous problem. It is using the determined real distances on the previous problem as the given information. We provide conjectures of students' answer in table 4.9.

> Pak Nyoman wants to inform the bus driver about the fuel consumption to reach each destination. He knows that 2 litres of fuel can be used to travel 25 km. Make an illustration as in 1b and use it to determine fuel consumption for each destination.

Figure 4.9. Problem 2 in lesson 3

Table 4.9. Conjectures of students' answer for problem 2 in lesson 3

Conjectures of students' answer	Suggestions for the teacher
Students are able to draw the model but not able to order the given numbers	The teacher may encourage students to think of the stretched route as a number line, in which numbers are should be well ordered
Students are able to draw the model and order the numbers. Students determine the relationship	The teacher may encourage students to observe the relationship between numbers using

between numbers using addition and subtraction	multiplication or division rather than determine difference between numbers
Students are able to draw the model and order the numbers. Students determine the relationship between numbers using multiplication and division	The teacher may ask students to help their friends, especially other students who determine difference to find out real distance

Since the problem has similar feature as the previous problem, it is important for the teacher to lead students to relate given numbers using multiplication and division rather than relate given numbers using subtraction or division.

D. Lesson 4

1. The learning goals

- Students are able to understand that numbers on the model must be well ordered.
- Students are able to express the multiplicative comparison on the model to determine equivalent ratios.

2. Description of activity

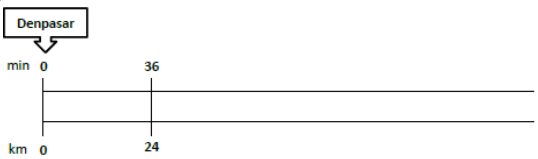
The teacher starts the lesson by reminding students about an illustration they draw in the previous lesson. The teacher informs students the name of the illustration, a double number line. Students and the teacher discuss the name of the model. The teacher may ask following questions:

- *"Why do you think the name is double number line?"*
- *"What is your opinion about double scale line and double number line?"*
- *"Can we use double number line to illustrate ratio which not only involve distance on the map and real distance?"*

The teacher tells students to work and to discuss the problems. Respectively, figure 4.10, figure 4.11, and figure 4.12 illustrates problem a, problem b, and problem c. Since the problems have the same feature, the class discussion is held before the class ends. In the discussion, the teacher has to point out that numbers on the model must be well order. The teacher can use the model to reason. For example, numbers on the right side are greater than numbers on the left side. Moreover, the teacher may ask whether all pairs of ratio have the same value. To prove that the pairs have the same value, the teacher can use fractional form. We provide conjectures of students' answer for the problems in table 4.10.

Do you remember Pak Nyoman and his tour business? Pak Nyoman is requested to arrange a trip for one destination. Pak Nyoman uses five cars for five different destinations.

He wants to include how long the trip will take for every destination. Assume that there is no traffic jam in Bali. It takes 36 minutes to travel 24 km. Pak Nyoman makes the drawing below.



Use the double number line to determine the travelling time from Denpasar to each destination.

Figure 4.10. Problem a in lesson 4

Pak Nyoman remembers, when he travelled to Gianyar (45 km away from Denpasar), his car consumed 3 litres of fuel. Determine the fuel consumption for each destination using the double number line.

Figure 4.11. Problem b in lesson 4

Pak Nyoman tells drivers that he needs to pay IDR 60.000 for 5 litres of fuel. Help drivers to determine how much money they will spend for fuel to each destination.

Figure 4.12. Problem c in lesson 4

Table 4.10. Conjectures of students' answer for problems in lesson 4

Conjectures of students' answer	**Suggestions for the teacher**
Students do not understand that numbers on the model must be well ordered	The teacher may encourage students to think of the stretched route as a number line, in which numbers are should be well ordered. Smaller numbers should be put on the left side
Students are not able to express multiplicative comparison to determine equivalent ratios (students use subtraction and addition to determine equivalent ratios)	The teacher may remind students that ratio is related to multiplication and division. Encourage students to observe the relationship between numbers using multiplication or division rather than determine difference between numbers
Students are able to express multiplicative comparison on the model to determine equivalent ratios	The teacher may ask students to help their friends, especially other students who determine difference to find out real distance

E. Lesson 5

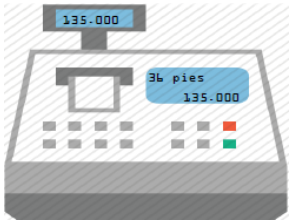
1. The learning goals

- Students are able to formulate information into a mathematical model.
- Students are able to understand that equivalent ratios can be determined by using multiplicative comparison.

2. Description of activity

During four lessons, students confront with map and travelling context. In this meeting, students will confront with another context such as shopping and food consumption. The teacher may ask whether students can use the model they usually use to solve problems with different contexts. Later, the teacher instructs students to work on problems and to discuss with their friends. Respectively, figure 4.13 and figure 4.14 show problem 1 and problem 2 in lesson 5. In the last lesson, the teacher ensures that students are able to determine relationship between numbers using multiplication or division to find out equivalent ratios. In the class discussion, the teacher may reinforce that ratio is related to multiplication and division while subtraction and addition related to difference. We provide conjectures of students' reaction for problems in lesson 5 in table 4.11.

Bali is famous with milk pie. Five tourists go to a gift shop. One of them has finished shopping. Below is the illustration on the cashier machine when he wants to pay for the pies.



a. If other tourists buy 20, 44, 4, and 12 pies, determine how much money each of them has to pay.

b. Once the tourists have finished their shopping, they head to a restaurant. The tourist who spent IDR 135.000 finds out that she can afford 9 portions of *nasi Bali* using her spent money.
Find out how many portion of *nasi Bali* each of the other four tourists can afford with the money they spent to buy pies.

Figure 4.13. Problems with shopping context

Near Tanah Lot, there is a village which is popular for its cow farms. A tourist takes a walk around the village. He meets the farmer and finds out that 8 cows consume 20 sacks of grass each day. The tourist continues his walk and finds out another farms. Here are the list of farms and the number of the cows.

Farm	**A**	**B**	**C**	**D**	**E**
Cows	10	5	13	2	4

Determine the number of sacks for each farm for one day.

Figure 4.14. A problem with food consumption context

Table 4.11. Conjectures of students' answer for problems in lesson 5

Conjectures of students' answer	**Suggestions for the teacher**
Students are not able to formulate the given information into a mathematical model • Students do not know what mathematical model they have to make	• The teacher may encourage students to think of an illustration they usually use during the lessons • The teacher may point out the given ratio and reminds students

• Students do not recognize the given information as ratio therefore they cannot formulate information into a model • Students are not able to order given numbers well	about the agreement to write the given numbers on the lower line and the determined value on the upper line • The teacher may remind students about a number line
Students are not able to understand that equivalent ratios can be determined by using multiplicative comparison (students determine equivalent ratios by determining difference between numbers)	The teacher may remind students that ratio is related to multiplication and division. Encourage students to observe the relationship between numbers using multiplication or division rather than determine difference between numbers
Students are able to understand that equivalent ratios can be determined by using multiplicative comparison	The teacher may ask students to help their friends, especially other students who determine difference to find out real distance

CHAPTER 5

RETROSPECTIVE ANALYSIS

A. The Timeline of the Study

SDIT At Taqwa is implementing curriculum 2013. In this curriculum, the teacher and students must be able to finish several themes in two semester. The vice principal of the school gave the researcher permission to do the study two weeks before theme seven ended. We carried out the study in two classes, 5B for the pilot study and 5C for the teaching experiment. According to the vice principal and the teacher, there is no significant difference in terms of mathematical ability in both classes.

Considering the time limitation of the study and the school availability, we conducted two cycles in the study. The pilot study was held when the students had their theme project. In this time, the students had more time to be involved in the research because the activity in the classroom was reviewing the study material. The teaching experiment was held when students had their mid-semester exam. The study was conducted after students finished the test. To fit with the given time, the pretest in the teaching experiment was conducted when the posttest in the pilot study was carried out. We provide the timeline of the study in table 5.1.

Table 5.1. The timeline of the study

Data Collection	Date	Participants
Classroom observation and the teacher's interview	17 February 2015	The regular teacher and students in class 5C
Pretest and the students' interview before the pilot study (cycle 1)	18 February 2015	Five students from class 5B
Cycle 1		
Lesson 1: Scale lines and plans	20 February 2015	Five students from class 5B
Lesson 2: Maps and double scale lines	23 February 2015	
Lesson 3: Maps and the stretched route	24 February 2015	
Lesson 4: Travelling around Bali	25 February 2015	
Lesson 5: Buying things and grass consumption	26 February 2015	
Posttest and the students' interview	27 February 2015	Five students from class 5B
Pretest	26 February 2015	Students in class 5C
The focus group students' interview	27 February 2015	Four students from class 5C
Cycle 2		
Lesson 1: Scale lines and plans	2 March 2015	The regular teacher and students in class 5C
Lesson 2: Maps and double scale lines	3 March 2015	
Lesson 3: Maps and the stretched route	4 March 2015	
Lesson 4: Travelling around Bali	5 March 2015	
Lesson 5: Buying things and grass consumption	9 March 2015	
Posttest	10 March 2015	Students in class 5C
The focus group students' interview	10-11 March 2015	Four students from class 5C

B. Data of Preparation Phase

Before we carried out the pilot study, we would like to have an insight about the characteristic of the teacher and the class for the teaching experiment. In this section, we will discuss about class observation in 5C and the regular teacher of 5C interview.

1. Class Observation

In this section, we will discuss two main concerns of the class observation, the classroom environment and the learning process. Observing the class environment, there were 34 students which were divided into nine groups. Even though students were grouped, the seat arrangement is classical. Most of the time, the teacher stood on the front of the class. When students worked on problems, the teacher walked around the class. The teacher asked students to get out of the class when they did not behave well.

During the learning, the teacher asked for students' participation. The topic of the learning was velocity. The teacher constantly asking and reminding students about the formula of velocity. Rather than giving students some time to think and to discuss, the teacher gave an example before students work on the real problem. The teacher also asked students to help each other. Later in a class discussion, the teacher gave students an opportunity to ask, to clarify, and to speak their mind. When students had different answers, the teacher asked students' opinion. We could not say that the teacher led students to use one strategy. However, the teacher kept on

reminding students about the formula of velocity. To close the learning, the teacher reinforced the formula of velocity and pointed out students' mistake in working on the problem.

2. Teacher Interview

We conducted an interview with the teacher once the class was over. In this section, we will discuss the teacher's background, the learning process, classroom norms, and RME or PMRI.

The teacher has been teaching in At Taqwa for 1,5 years. She is a mathematics tutor since in the bachelor programme. The teacher was assigned to teach grade three when she entered At Taqwa. She also teaches grade six to prepare students for the final exam. Starting in this year, she teaches grade five and grade six. The teacher does not have any specific favourite topic in teaching mathematics. She argued that she has to master all topics whether she likes it or not.

Students in grade five are using curriculum 2013. They have learned about ratio and scale. Since the learning is thematic, the context of ratio and scale depends on the theme. In addition, the teacher said that the thematic curriculum is not as deep as curriculum 2006. To introduce the concept of ratio, the teacher asked students to compare two things with opposite characteristics, such as good and bad, big and small, and short and tall.

Other than that, the teacher used tools such as marbles with different colours to compare the number of marbles for each colour. To connect with

scale topic, the teacher used the context of maps. Students had to investigate the real distance, the distance on the map, and the scale. The teacher added that students are not only familiar with map but also plan. They learned about plan and map in grade three. In this topic, students were able to draw a plan of their house and school and to understand the cardinal directions.

Curriculum 2013 provides word problems for students. However, the teacher acknowledged that it becomes drawback for students because they tend not to read problems and they struggle to understand problems. Even though the teacher thinks word problems is important, she preferred non-word problems as in the supplementary worksheet provided by the school. Talking about models, the teacher does not have any experience in using models in the learning. Considering the time limitation and the thematic curriculum, the teacher cannot and have not use the models.

To preserve a conducive condition, the class had some written rules. However, it was not effective. To create a conducive learning, the teacher usually asks students who do not behave well to quit the class. For the socio-mathematical norms, the teacher usually gives students an opportunity to speak in a class discussion. In addition, the teacher asks students' comment about their friends' work. The teacher has heard about PMRI but she does not know exactly about it.

C. Analysis of the Pilot Study

The pilot study aims at improving the hypothetical learning trajectory

(HLT). In this section, we will discuss about pretest before implementing the lessons, the learning process during the five lessons, and posttest after implementing the lessons. In the analysis, we will compare the hypothetical learning trajectory (HLT) to the actual learning. In the end of the analysis, we will provide conclusion and refinement of HLT and the learning materials.

In the pilot study, five students were involved. The students were divided into two groups, the boys group and the girls group. In pilot study, the researcher played a role as the teacher. The mathematical activities for each lesson are provided in chapter 4.

1. Pretest

We conducted pretest before the pilot study was implemented. The aim of conducting pretest is to find out the students' prior knowledge about ratio. The test contains five questions: (1) illustrating ratio by using sentences, figures, or symbols, (2) choosing a sentence which contains the notion of comparison, (3) working with a scale line, (4) working with a scale line with specific measurement, and (5) determining equivalent ratios.

For the first question, the students defined ratio as difference. Most students were not able to give an example other than bigger-smaller and shorter-taller. In the second problem, the students were not able to distinguish statements which contain the notion of difference and comparison. Thus, the students chose a sentence which contains the notion of difference. Even though the students chose one of statements, they were

not able to reason why the statement is comparison. Since the time was limited to improve the learning material, we use the findings of the pretest to make an improvement in the second cycle. We will add a problem to enable students to distinguish between difference and comparison in cycle two

The students were able to use a given scale line in the third problem. Instead of stating the length using how many times the scale line fits the line, the students tend to do a precise measurement using a ruler. However, the students were not able to work with a scale line with specific measurement in the fourth problem. They were able to find out how many times the scale line fits the line. Yet, they were not able to determine the real distance using the scale line. For the last problem, the students had difficulty in perceiving the given information as ratio. Even though the students tried to solve the problem, they used formal method and they were not able to solve the problem. There was no illustration made by the students to show how they understood the problem.

2. Lesson 1

In the first lesson, there are two activities. The first activity consists of two problems and the second activity consists of one problem. For the first activity, the students worked individually while in the second activity, they worked in group. The context in the first lesson is studio plan and classroom plan. The problems in the first lesson aim to enable the students in using the

scale line.

Activity 1 (Making and using scale line)

Problem 1a

The aim of the problem is to use the length of a reference to describe the length of an object. The students had to choose a reference on the plan to measure the length of coloured walls. In the end, the students were asked to draw an illustration as a form of comparison between the length of a reference and an object.

The students chose different references such as a table and a bed as in figure 5.1.

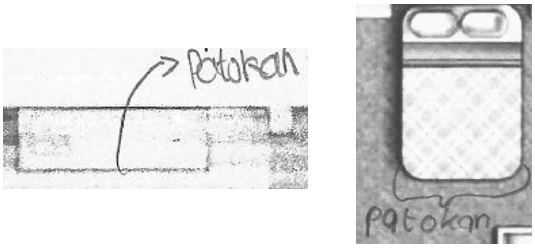


Figure 5.1. Choosing references

After choosing a reference, the students used the length of the reference to determine the length of the coloured walls. The students marked the coloured walls on the worksheet to know how many times the reference fits the coloured walls as in figure 5.2.

Figure 5.2. Using a reference to measure

The students used different tools to measure the reference, for instance using a ruler, fingers, a pencil, and a paper strip. The students were able to describe the length of the coloured walls using the length of the reference.

However, they were not able to illustrate the relationship.

Problem 1b

The aim of the problem is to use the scale line to describe the length of cables. The problem contains an illustration of installed cables in the studio and a scale line. The students had to measure the length of the cables and to determine the real length of both cables. In the end, the students were asked to draw an illustration as a form of comparison between the length of a scale line and the cables.

In accordance with the HLT, the students were able to use the scale line to describe and to determine the length of the cables. To find out the length of the cables, the students used the scale line and marked the cables on the plan as in figure 5.3. However, the students found difficulty to illustrate the length comparison between the scale line and the cables. The researcher helped the students to make an illustration.

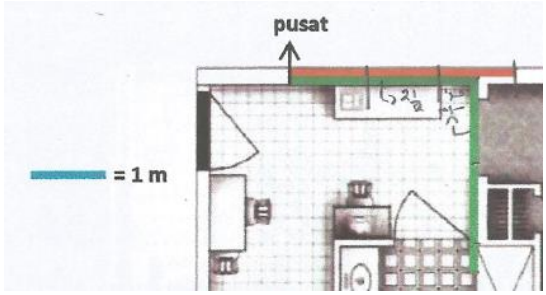


Figure 5.3. Using a scale line to measure

Activity 2 (Drawing classroom plan)

The aim of the activity is to enable the students to make and to use a scale line in a plan. The students were asked to draw a classroom plan using

a scale line and to determine the real measurement of the classroom. They had to choose a large object in the classroom as a reference and to represent the reference into a scale line.

Both groups chose the width of a door as a reference. However, the two groups had different estimation of a door's width and different steps in drawing the plan. The girls group estimated the width of a door is 0,5 meter while the boys group's estimation is 1 meter. The girls group drew the reference into a scale line and used the scale line to draw a classroom plan. In reverse, the boys group drew the classroom plan, then drew the scale line. From this activity, both groups had different real measurement of the classroom since they used different methods and different scale lines.

From the first lesson, we realize that the students took much time to read. In the next cycle, we will arrange the sentences on the problems using points. In addition, there is no problem which points out the difference between difference and comparison. Therefore, there should be a problem about difference and comparison on the next cycle.

Based on the analysis of the first lesson, we can conclude that the students were able to use a scale line to measure other objects on the plan. The students were also able to describe the length of an object on the plan by mentioning how many times the scale line fits the object. However, students needed support from the researcher to make the illustration. We summarize the conjectures and the students' actual answer in table 5.2.

Table 5.2. The students' actual answer on lesson 1 cycle 1

Activity	Conjectures of students' answer	Students' actual answer
Making and using a scale line to determine the length of coloured walls	Students are not able to use the length of a reference as a scale line and not able to illustrate the relationship of a reference and a coloured wall	
	Students are able to use the length of a reference as a scale line, determine the length of walls, and illustrate the comparison between the reference and the coloured wall	The students used different tools which have the same length as the reference to determine the length of walls. However, they were not able to make an illustration
Using the scale line to describe the length of cables and drawing an illustration as a form of comparison between the length of a scale line and the cables	Students are not able to use the scale line and not able to draw an illustration to compare the length of a scale line and the cables	The students were able to use the scale line but they were not able to draw an illustration to compare the length of a scale line and the cables
	Students are able to use the scale line and	

	able to draw an illustration to compare the length of a scale line and the cables	
Using a scale line to draw a classroom plan and to determine the real dimension of the classroom	Students are not able to use a scale line to draw a classroom plan and to determine the real dimension of the classroom	
	Students are able to use a scale line to draw a classroom plan and to determine the real dimension of the classroom	Rather than making a scale line first, the students drawing the plan, then deciding what the scale line will be

3. Lesson 2

In the second lesson, there are two activities. The first activity consists of two problems while the second activity consists of one problem. The students worked in groups in the first activity and individually in the second activity. The context in the second lesson is about map. The problems in the second lesson aim to enable the students in working with the double scale line. In the second lesson, Rangga did not come to school. Therefore, the boys group only consisted of Akbar and Ganang.

Activity 1 (Working with information on the map about distances)

Problem a (Matching the map to the double scale line)

The aim of the problem is to use information about the real distance and the distance on the map. The students were asked to decide which map fits the given double scale line if the distance between Gresik and Surabaya is 15 km. The researcher handed out the worksheet for lesson 2 and gave the students some time to read and to discuss.

The students had difficulty in reading and perceiving information, therefore the researcher narrated the problem. It turned out the students did not know how to work with double scale line. Before the researcher gave an example about working with double scale line, the researcher discussed about the name double scale line with the students. From the discussion, the students understood that a double scale line is scale lines which have two measurements, the distance on the map and the real distance.

Later, the researcher gave an example. The researcher gave an illustration of 3 cm on the map represents 30 km as in figure 5.4. 3 cm and 30 km are aligned because it corresponds to each other.

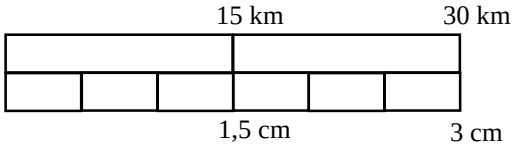
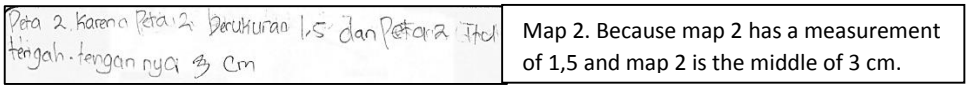


Figure 5.4. An illustration of ratio 3 cm:30 km

In line with the HLT, the students were able to determine that map B fits the double scale line. Using the provided double scale line, the students were able to find the measurement of the half part of the whole line. The students

stated that the distance between two cities in map B is a half of the double scale line as in figure 5.5.



Map 2. Because map 2 has a measurement of 1,5 and map 2 is the middle of 3 cm.

Figure 5.5. The students' reasoning

They found out that the distance on the map is 1,5 cm. It represents 15 km on the map. These two information becomes powerful tool for the students to determine which map fits the provided double scale line. The students understood that the double scale line should be represented precisely. Otherwise, it may create confusion to people who read the map and the double scale line.

Problem b (Drawing double scale lines)

The aim of the problem is to enable the students to illustrate the relationship between the distance on the map and the real distance into a double scale line. The students were asked to draw the correct double scale line for the two other maps.

When the students forgot to put information on the double scale line, the researcher reminded them to write information. Thus, other people will understand what the double scale line tells about. The students from the boys group struggled in drawing a proper double scale line as in figure 5.6.

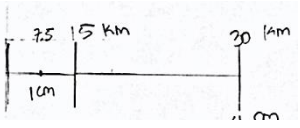


Figure 5.6. An inappropriate double scale line

As expected, even though they were able to draw the double scale line, it

does not represent the information given. The students were able to write information on the double scale line. However, observing the picture, the length of 4 cm is not twice as long as 2 cm. To support the students, the researcher reminded them about the previous problem, in which they had to be precise in drawing double scale line.

The students from the girls group extended their drawing as in figure 5.7. For example for map C, 1 cm represents 15 km. The students drew 4 cm but they struggled to determine the real distance that 4 cm represents.

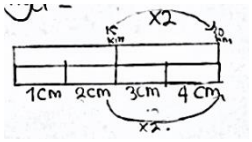


Figure 5.7. The students' double scale line

To help the students, the researcher used fractional form to find the equivalent ratio. Using the fractional form, the students were able to relate numbers between ratios and were able to determine the real distance. Since 4 cm is four times 1 cm, therefore the real distance of 15 km should be multiplied by 4 as well. Hence, the students got 60 km.

Even though some students were able to draw the double scale line and to extend the double scale line, students in the boys groups did not understand the relation between numbers on the model. Therefore, the researcher explained that each pair of ratio is aligned because it represents the given information.

Activity 2 (Working with the double scale line)

The aim of the problem is to enable the students to use the double scale

line to determine the real distances. The students did not know what to do. Thus, the researcher asked them to measure the distance on the map between two cities on each day.

To guide the students, the researcher told them to start from the given ratio and to write the given numbers on the lower scale line. The students must be able to decide which number has a relation to the given ratio on the lower line. However, the boys group did not have any idea about relation between numbers. When the researcher asked what the relationship is, they could not answer. Thus, the researcher used the model on the worksheet to support the students. If 3 cm on the lower line represents 60 km and it is divided into three parts, then how many km each part represents. It turned out the girl groups were able to determine that 1 cm represents 20 km. They used division to determine the number.

Other than having difficulty in determining relationship between numbers, the students struggled in working with double scale line. The students were not familiar with it. Thus, some of them misplaced the real distance that should be aligned to a corresponding distance on the map as in figure 5.7.

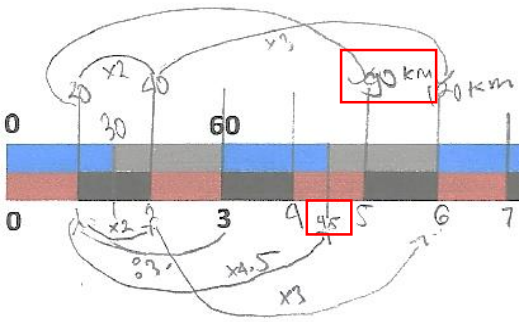


Figure 5.8. Misplacing 90 km

During two lessons, the students were not actively participating on the learning. There should be reward points to encourage the students to be more communicative and active.

Based on the analysis of the second lesson, we can conclude that the students were able to use information such as the distance on the map, the real distance, and the double scale line. However, the students still struggled in working with double scale line and understanding relationship between numbers. We summarize the conjectures and the students' actual answer in table 5.3.

Table 5.3. The students' actual answer on lesson 2 cycle 1

Activity	**Conjectures of students' answer**	**Students' actual answer**
Determining a map which fits the double scale line and reasoning	Students have no idea on working with the double scale line	The students did not know how the double line works
	Students are able to determine which map fits the double scale line but they are not able to reason for other maps	
	Students are able to determine which map fits the double scale line and able to reason for other maps	The students were able to determine map B as an appropriate map for the double scale line. The

		students understood that having inappropriate double scale line might create problem
Making double scale lines for other maps	Students draw inappropriate double scale lines	The students forgot to write information on the double scale line. Some of them also drew a double scale line which its length is not precise.
	Students draw appropriate double scale lines	The students drew the double scale line using precise measurement and wrote information on the double scale line
Using the double scale line to determine real distances	Students do not use the model to determine real distances	
	Students use the model to determine the relationship between numbers to find out real distances	The students struggled in using multiplication to determine relationship between numbers to find out real distances

4. Lesson 3

In the third lesson, there are three problems. The students worked in groups for the first problem. For other problems, they worked individually but they were allowed to discuss with their friends. The context in the third lesson is about map and travelling. In this lesson, Rangga joined the learning.

Problem 1a (Making Double Scale Line)

The aim of the problem is to represent the relationship between the distance on the map and the real distance into an illustration. The students were asked to draw the double scale line of a Bali map as a form of comparison between the real distance and the distance on the map.

The first problem contains information that 4 cm on the map represents 80 km. Using a ruler, the students started to measure and to draw a line with length of 4 cm. Later, they divided the line into four same parts, in which each part has the length of 1 cm. In making an illustration, some students did not put information on the double scale line. To make the double scale line more meaningful and understandable for the readers, the researcher reminded the students to put information on the double scale line.

Once the students finished their work, one representative of each group wrote their work on the board. The students understood that it was impossible to draw the real 4 cm on the board. Therefore, they divided the lower scale line into four same parts and assumed that the lengths are the

same. Figure 5.9 shows the example of the representatives' work.

Figure 5.9. The example of the students' work

The figure illustrates that both representatives from each group wrote inappropriate information about the relationship between the distance on the map and the real distance on the double scale line. Both students divided the lower scale line into four parts and wrote information on the end of the upper scale line that it represents 80 km. Both students and their groups could not reason why they put the information on the first or second small part of the lower scale line. The researcher decided to clear the students' confusion. After drew a double scale line, the researcher asked the students what the given information is. The researcher told the reason why they need to put information of 80 km and 4 cm aligned. It is because 4 cm on the map represents 80 km, therefore the information should be aligned.

To gain information whether the students were able to relate the distance on the map and the real distance, the researcher proposed another similar problem, 3 cm on the map represents 60 km. Aisyah volunteer to make and to explain her illustration. Aisyah drew double scale line and divided the lower scale line into three parts as shown in figure 5.10.

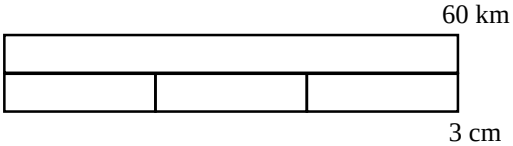


Figure 5.10. An illustration of Aisyah's double scale line

Aisyah aligned 3 cm with 60 km because she understands that 3 cm on the map represents 60 km. Therefore, the information should represent that those numbers are corresponding. In addition, Aisyah knew that 1 cm represents 20 km. Aisyah used division to determine the real distance for 1 cm on the map. It is possible that Aisyah focus on the relationship between numbers on the double scale line to determine the real distance for given distance on the map.

Problem 1b (Working with the stretched route)

The aim of the problem is to order the given numbers on the stretched route. The students were asked to use the stretched route to determine the real distance of each destination from Denpasar. Given, a list of distances on the map from Denpasar to several destinations. In addition, the students had to use the previous ratio information, 4 cm on the map represents 80 km.

The researcher told the students that they were not only dealing with problems about the distance on the map and the real distance. Moreover, the students was not working with the double scale line anymore. Therefore, the researcher told the students to ignore the exact measurement between numbers. The students had to order the given numbers on the stretched route.

Once the students ordered the numbers on the stretched route, they struggled to find the relationship between numbers as in figure 5.11.

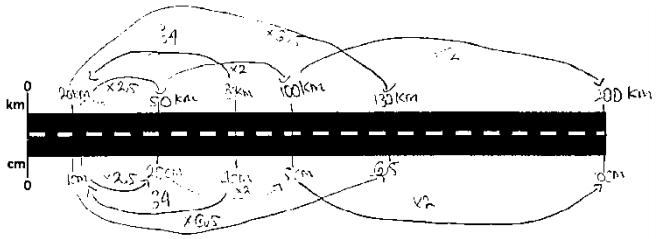


Figure 5.11. An example of the students' work

The researcher helped the students to start from information 4 cm represents 80 km. Since the numbers on the stretched route are well ordered, it enables the students to see the relationship between different numbers. The students were able to use division and multiplication in expressing the relationship between numbers. Moreover, the students multiplied or divided a pair of ratio to another with a scale factor.

Problem 2

The aim of the problem is to use the stretched route to show the relationship between numbers. The students were asked to draw an illustration similar to the stretched route and to use it to determine the fuel consumption to each destination. Given, 2 litres of gas are used to travel 25 km.

The students struggled in making and completing the illustration. The researcher and the students had agreed to put the given information on the lower line while the determined value on the upper line. The struggles continued when the students had to solve the problem. As expected, the students forgot how to start. The researcher reminded the students about the information they have to start solving the problem. Finding relationship

between numbers somehow difficult for the students. The researcher told the students to pay attention on the numbers. They had to relate one ratio to another to determine missing value. In general, the students were able to use the model to determine the fuel consumption.

Based on the analysis, we can conclude that the students struggled to represent the relationship between the distance on the map and the real distance into an appropriate double scale line. Furthermore, the students were also able to order the numbers on the stretched route and to determine the relationship between numbers using multiplication and division. We summarize the conjectures and the students' actual answer in table 5.4.

Table 5.4. The students' actual answer on lesson 3 cycle 1

Activity	**Conjectures of students' answer**	**Students' actual answer**
Representing the relationship between the distance on the map and the real distance into an illustration	Students draw inappropriate double scale lines	The students were able to draw a precise double number line. However, they wrote information inappropriately
	Students draw appropriate double scale lines	
Ordering the given numbers on the	Students are not able to order the given	

stretched route and using the model to determine real distances	numbers on the stretched route	
	Students determine real distances using subtraction and addition	
	Students determine real distances using multiplication and division	The students were able to relate numbers on the stretched route using multiplication and division
Drawing a model, ordering the given numbers, and using the model to show the relationship between numbers	Students are able to draw the model but not able to order the given numbers	
	Students are able to draw the model and order the numbers. Students determine the relationship between numbers using addition and subtraction	
	Students are able to draw the model and order the numbers. Students determine the relationship between numbers using	The students were able to draw the model and order the numbers. They used multiplication and division to relate

	multiplication and division	numbers

5. Lesson 4

There are three related problems as an exercise in the fourth lesson. The aim of the problems is to enable students to understand that numbers on the model must be well ordered. Using the double number line, the students must be able to express the multiplicative comparison to determine missing values on other ratios. In the fourth lesson, the students were active and in turn voluntarily showed how they use the double number line to solve the problems on the board.

Problem a

Given a Bali map with five different destinations and its distances. People spend 36 minutes to travel 24 km. The students were asked to use the double number line to determine how many minutes it takes to travel a certain distance.

In this problem, the researcher told the students that the illustration on the first problem is called a double number line. The researcher referred back to the double scale line. From the discussion, the students understood that a double number line is lines which consists of two quantities. The given information placed on the lower line while the determined value placed on the upper line.

Observing the given distances, the students struggled to find the relationship. The researcher decided to help by relating the given ratio to fraction and transforming the ratio into fractional form. The researcher asked the students to simplify the fraction and use it to find the relationship between numbers on the lower line.

Later, the researcher asked the students whether the paired numbers on the double number line have the same value. Aisyah argued that all pairs have the same value but she could not reason. To support the students, the researcher reminded the students to relate the pairs to fraction and simplify it. It turned out all pairs have the same form, that is $\frac{2}{3}$. Therefore, the students were sure that all pairs on the model have the same value. In the end, the students were able to use the model to show the multiplicative comparison between numbers.

Problem b

Problem b is related to problem a in the use of the distance. Given, every time the car travels for 45 km, it consumes 3 litres of fuel. The students were asked to determine the fuel consumption for each destination. The students transformed the given ratio into fractional form and simplified it. It turned out that the numbers were difficult and the students spent much time on calculating the fraction. In general, the students understood that they have to find the multiplicative comparison between numbers.

Problem c

The given information in problem c is derived from the fuel consumption for every destination in problem b. Given, Pak Nyoman spends IDR 60.000 for every 5 litres. The students were asked to determine the price the driver has to pay for each destination. Similar to previous problems, the students changed the given ratio into fractional form and simplified it to determine the relationship between given numbers. As expected, once the students determined the multiplicative comparison between numbers, they multiplied or divided the pair on the upper line with the scale factor.

There was an interesting answer from Ganang in determining the price of a certain litre of fuel. The student's writing on the board reveals that the student was not truly understand in working with fraction and decimal. As an illustration of the students' work on the board, we provide figure 5.12.

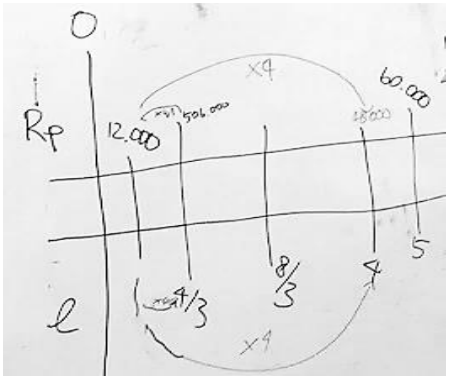


Figure 5.12. The students' work

Ganang tried to determine the cost for fuel consumption 4/3 litres from 1 litre. Ganang and other students had difficulty in working with fraction. In this case, Ganang perceived the fraction 4/3 as decimal 4,3. However,

Ganang multiplied the price of 1 litre fuel by 4 and divided the result by 3. Ganang also miscalculated the final result and it turned out the cost is larger than the cost of 5 litres fuel.

Using the double number line, Aisyah did not agree with Ganang's answer. She argued that 4/3 litres is located before 4 litres. If the cost of 4 litres fuel, which is greater than 4/3 litres, is only IDR 48.000, it is impossible for the fuel to cost IDR 516.000.

Based on the analysis, we can conclude that the students understood that numbers on the model should be well ordered. Therefore, the students were able to use it to reason that numbers on the left side cannot be greater than numbers on the right side. The students were also able to express the notion of multiplicative comparison between numbers and composed unit to find equivalent ratios on the double number line. We summarize the conjectures and the students' actual answer in table 5.5.

Table 5.5. The students' actual answer on lesson 4 cycle 1

Activity	**Conjectures of students' answer**	**Students' actual answer**
Understanding that numbers on the model must be well ordered and expressing multiplicative comparison on the	Students do not understand that numbers on the model must be well ordered	The students were able to arrange numbers on the lower line. However, one student wrote incorrect determined

model		number, in which the number is larger than the number on its right side
	Students are not able to express multiplicative comparison to determine equivalent ratios (students use subtraction and addition to determine equivalent ratios)	
	Students are able to express multiplicative comparison on the model to determine equivalent ratios	The students were able to determine equivalent ratios by relating numbers using multiplication and division

6. Lesson 5

In the fifth lesson, the students worked on three problems. The first two problems are related. The aims of the problems are to formulate the given information into a mathematical model and to enable students to understand that equivalent ratios can be determined by using multiplicative comparison. In this lesson, the students will not deal with map and travelling context.

The students will have to solve the problem from different contexts using the double number line. In turn, the students voluntarily showed how they solve the problem on the board.

Problem 1a

The context of problem 1 is about buying milk pie, a famous snack from Bali. One tourist goes to the cashier and the machine shows that he needs to pay IDR 135.000 for 36 pies. There are four tourists in the shop buy different numbers of pie. The students were asked to determine the money each tourist spends to buy pies.

To guide the students, the researcher asked what information they had and what the question is. The students voluntarily came to the front to show their work. They did not use fraction anymore as a supportive form to see the relationship between numbers. As expected, the students directly drew a line with an arrow and wrote the multiplicative comparison between numbers to determine the money each tourist should pay for numbers of pie.

Problem 1b

The students had determined the amount of money each tourist should pay for pies. Using the spent money for each tourist, the students were asked to determine how many *nasi Bali* portion each tourist can buy. Given, the ratio information is a tourist who spent IDR 135.000 to buy pies, he can buy 9 portions of *nasi Bali* using the same amount of the spent money.

The students started to formulate the given information into a double

scale line. In turn, they ordered the amount of money each tourist spent on the lower double number line. In accordance with the HLT, the students used multiplicative comparison to express the relationship between numbers.

Problem 2

The context of problem 2 is about the grass consumption on a farm. Each day, 8 cows consume 20 sacks of grass. The students were asked to determine the number of sacks for five different farms.

As in previous problems, the students scanned what information given and what the question is. They used the information to be formulated onto the double number line. Sometimes the students mistakenly placed the given numbers not in the right order as in figure 5.13. Other students commented that the numbers on the model should be in order. In general, the students were able to determine the multiplicative comparison between numbers on the double number line.

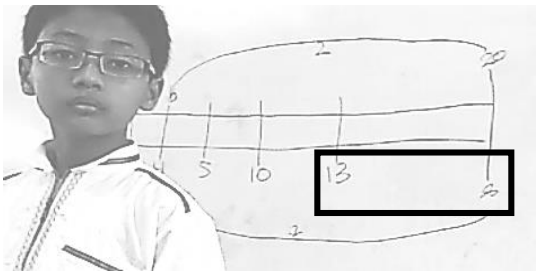


Figure 5.13. The student ordered numbers inappropriately

Based on the analysis, we can conclude that the students were able to formulate the given information onto a double number line. In addition, the students understood that equivalent ratios can be determined by using

multiplicative comparison. We summarize the conjectures and the students' actual answer in table 5.6.

Table 5.6. The students' actual answer on lesson 5 cycle 1

Activity	**Conjectures of students' answer**	**Students' actual answer**
Formulating the given information into a mathematical model and understanding that equivalent ratios can be determined by multiplicative comparison	Students are not able to formulate the given information into a mathematical model	The students were able to formulate the given information into a double number line. However, sometimes the students forgot the order of numbers on the model
	Students are not able to understand that equivalent ratios can be determined by using multiplicative comparison (students determine equivalent ratios by determining difference between numbers)	
	Students are able to understand that equivalent ratios can be determined by using multiplicative comparison	The students were able to formulate the given information into a double number line and determined equivalent ratios by relating

		numbers using multiplicative comparison

7. Posttest

The problems in the posttest are the same as in the pretest. By conducting a posttest, we want to find out and to clarify the students' development in the learning. Therefore, we will not use the posttest to be compared to the pretest.

For the first problem, the students defined ratio as a comparison between two things. However, not all students were able to give an example which illustrates ratio. The students who were able to give an example, mentioned the shopping context which is related to ratio. It is possible that the students were more familiar with the context and it is easier to recognize the ratio on the context.

In the second problem, two students thought the statement which contains the notion of difference as a ratio. The students used the term "longer" to reason their statement. The other two students thought that the two statements are the same while one student chose a statement which contains the notion of comparison as a ratio. The student who chose comparison statement tried to reason using double number line and the context of grass consumption to illustrate the multiplicative comparison. In the learning, we did not provide any problem which asks the students to

choose and to reason which statement contains the notion of ratio or comparison. Therefore, the students were not able to reason which statement contains the notion of ratio. To emerge the difference between difference and comparison, we will provide a problem which asks about difference and comparison in the next cycle.

The students were able to use the scale line in the third problem. They also marked the picture on the problem to find out how many times the scale line fits the line. Even though the students were able to use the scale line in the second problem, some students still did not know on using the scale line which contains a specific measurement. Two students did not use the scale line to measure the distance on the picture. Three students used the scale line to measure the distance on the picture. However, only two students knew on how to use the scale line to measure and to determine the real distance.

For the fifth problem, the students made a double number as their tool to solve the problem. However, two students could not use the model appropriately. One student did not perceive the given information as a ratio. He used another information as the ratio. Another student did not how to start to solve the problem using the double number line. As for three other students, they were able to use the given ratio and another information to help them to solve the problem. All of them used multiplicative comparison to solve the problem.

8. Discussion

According to the analysis on the pilot study, the learning materials need to be improved. Since there are not many difference between HLT and the actual learning, we preserve the designed HLT. In general, the problems should be structured into opening, given information, steps, and question. We provide the general refinement of the learning material and the additional HLT in table 5.7.

Table 5.7. Refinement of learning material and additional HLT

In Pilot Study	Refinement for Teaching Experiment
Lesson 1 1. No exercise which points out the difference between difference and comparison.	**Lesson 1** **Activity 1** **1. Problem 1** We provide one problem which points out the difference between difference and comparison regarding to the students' perception that difference is comparison. Dea compares the length of a laptop to the length of a table. She says that the length of the table is two units longer than the length of the laptop. **Question:** Does Dea's statement is a comparison? Give your reason. Dea's statement is _____. The reason is: HLT for the additional problem: • Students cannot recognize the notion of difference.

Activity 1

2. Problem 1a

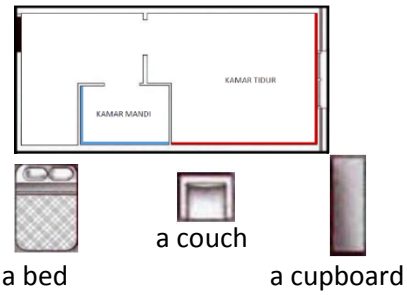
- We provided a studio plan containing different objects as references and many coloured walls.

- There was no leading sentence on the answer box.

- Students can recognize the notion of difference but not able to reason.
- Students recognize the notion of difference and able to reason why the statement is not comparison.

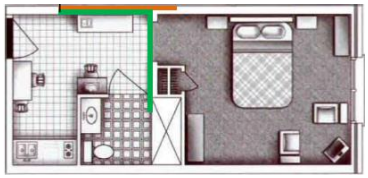
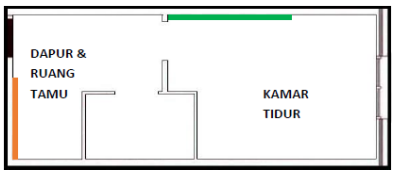
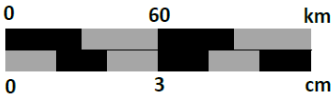
2. Problem 2

- We revise the coloured walls and provide three references from the top view to ease students to choose a reference.



- We provide the answer box with leading sentences.

I use _____ as a reference.
The length of the toilet is _____.
The width of the toilet is _____.
The length of the bedroom is _____.
The width of the bedroom is _____.

3. Problem 1b We used the studio plan and there is an L-shape line.  **4. Activity 2** There was no instruction to make the scale line first.	**3. Problem 3** We use a blank plan and provide straight lines to support students in using the scale line to measure the walls.  **4. Activity 2** We provide an instruction to make the scale line first, then use the scale line to draw a classroom plan.
Lesson 2 **Activity 2** • There was ratio information in the form of double scale line and sentence. **Remember:** The shortest route between Gresik and Surabaya is 15 km. • The provided double scale line was long and in colour. 	**Lesson 2** **Activity 2** • We omit the sentence which provides ratio information. Therefore, the students will focus on using the double scale line. • We shorten the provided double scale line and change the colour into black and grey. 
Lesson 3 **Problem 1 and problem 2** There were five different destinations after Denpasar.	**Lesson 3** **Problem 1 and problem 2** We reduce the number of destinations into three destinations considering the time limitation. However, we preserve the route of the bus.

Lesson 4	**Lesson 4**
1. Problem a There were five different destinations and five different distances from Denpasar. It affects the number of determined values on the next problems.	**1. Problem a** We reduce the number of destinations into three destinations regarding to the time limitation. It affects the number of determined values on the next problems.
2. Problem b The given ratio is 45 km consumes 3 litres of fuel.	**2. Problem b** We change the given ratio to easier number. Thus, the students focus on using the model not on the calculation. When the bus travels to Ubud which is 20 km away from Denpasar, it consumes 2 litres of fuel.
3. Problem c The given ratio is 5 litres fuel costs IDR 60.000.	**3. Problem c** We change the given ratio to easier number. Thus, the students focus on using the model not on the calculation. Mr. Nyoman informs all drivers that they have to pay IDR 60.000 for every 10 litres of fuel.

Lesson 5 **1. Problem 1** • There were five tourists (including one tourist in information) on the problem. • There was no name for each tourist. **2. Problem 2** The students need to determine the grass consumption of five farms.	**Lesson 5** **1. Problem 1** • We reduce the number of tourists into four tourists (including one tourist in information). • We give name for each tourist to avoid confusion. **2. Problem 2** We reduce the number of farms into three farms.

D. Analysis of the Teaching Experiment

The teaching experiment was conducted in class 5C of SDIT At Taqwa. One teacher and 34 students were involved in the teaching experiment. The regular teacher named Ustadzah Kartika, who her background is mathematics education, played the role as the teacher. The teacher carried out the improved HLT from the pilot study. For the teaching experiment, the teacher recommended four students named Diva, Ocha, Rizqi, and Sisi as the focus group students. According to the teacher, these students were able to communicate and their mathematics ability is on medium level. The mathematical activities in the teaching experiment are provided in the worksheets on appendix D.

1. Pretest

We conducted pretest before the teaching experiment was implemented. The aim of conducting pretest is to find out the students' prior knowledge

about ratio. The test contains five questions: (1) illustrating ratio by using sentences, figures, or symbols, (2) choosing a sentence which contains the notion of comparison, (3) working with a scale line, (4) working with a scale line with specific measurement, and (5) determining equivalent ratios.

There were only four students who answered the first problem. They mentioned words “difference”, “comparing different things”, and “longer than” to express ratio. As for the second problem, all students chose one of statements randomly. In addition, they were not able to reason why a statement has the notion of ratio. From 28 students who answered the third problem, only five students used the scale line and marked the figure on the problem to determine how many times the scale line fits the figure. The others only wrote numbers such as 4 cm and 6 cm without using the scale line to measure the figure.

None of students were able to determine real distances on the fourth problem. There were only five students who used the scale line to measure and to mark the route on the problem. For the fifth problem, none of students drew an illustration and most of them did not perceived the given information as a ratio. Even though 27 students answered the problem, only one student was able to solve the problem correctly using formal method. The other 26 students added the given information or multiplied the given information with a certain number.

2. Lesson 1

There are two activities in the first lesson. The context is about a room plan. The first activity consists of three problems while the second activity consists of one problem. The general aim of the first lesson is to enable students to use the scale line. Students worked and discussed the problems in groups.

Activity 1

Problem 1

The aim of the problem is to enable students to distinguish difference and comparison. A girl named Dea, compares the length of a laptop to the length of a table. She says that the length of a table is two units longer than the length of a laptop. The students were asked to decide whether Dea's statement is a comparison or not. In addition, they had to reason about their answer.

The teacher gave students time to discuss the problem in groups. The focus group students started to discuss the problem. Transcript 5.1 illustrates their discussion.

Transcript 5.1

1 Ocha : What do you think? (*talking to her group*) I think the
2 sentences "two units" is… This is laptop (*expressing two*
3 *laptops using her hands*)
4 Diva : Yea, two (laptops).
5 Ocha : It means 1:2.

The short transcript informs us that the students perceived "two units longer than" as ratio. In our interpretation, the students had no idea that difference

and comparison are different. Rather than perceiving “two units longer than” as difference, the students said that the ratio between the length of the table and the length of the laptop is 1:2.

When the teacher asked the class, they thought that the statement on the problem is comparison. The reason is because it compares two objects. Similar to the focus group students, other students did not pay attention on the word “longer than”. They did not realize that “longer than” contains the notion of difference. Therefore, the teacher supported students by giving an example.

Given, a laptop with the length of 30 cm and a table with the length of 90 cm. The teacher asked students to choose one of two statements which has the notion of comparison. The first sentence states “the length of a table is 60 cm longer than the length of a laptop”. The second statement is “the length of a table is three times the length of a laptop”. Students knew that the length difference between two object is 60 cm. They also understood that difference is related to subtraction as in figure 5.14. For the second statement, students were able to state the length comparison between a laptop and a table by using fractional form.

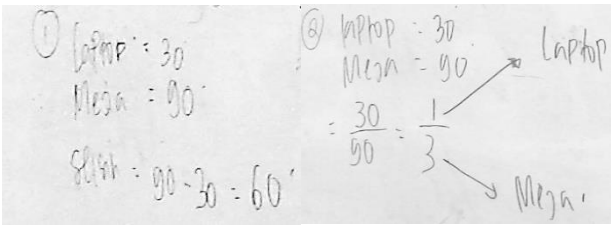


Figure 5.14. The teacher writings about difference and comparison

However, when the teacher asked students which statement is comparison, none of them answered. The teacher reminded students about stating comparison using fractional form but students still did not answer. Even though there was an example, it seems not all students had understood that the statement is not about comparison. The teacher continued to problem 2.

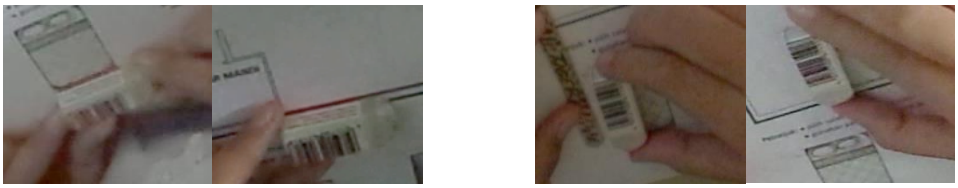
Problem 2

The aim of the problem is to use the length of a reference to describe the length coloured walls on a plan. There are three references to measure the coloured walls on a plan. The students were asked to choose one of references as a ruler to determine the length of the coloured walls.

Students were eager to have an exact measurement of the given references and the coloured walls. Since the problem does not ask for an exact measurement, the teacher forbade students to use a ruler to measure. Later, the teacher asked what kind of tools they can use to measure if there is no ruler. However, none of students answered the question. To help, the teacher demonstrated to measure the length of a table using the length of a pencil case and a pencil. Next, the teacher instructed students to discuss problem 2 with their groups. From the demonstration, students had an idea to measure the length of the reference using fingers, an ice stick and a paper strip.

The focus group students used the cover of an eraser to measure the bed

as the reference, and used it to measure the coloured walls. Interestingly, since the picture of the bed have a width and a length, the students used both dimensions to measure. When the students measured the horizontal red and horizontal blue line, they used the width of the bed. However, when the students measured the vertical red and vertical blue line, they used the length of the bed. Figure 5.15 illustrates how the students measured the coloured walls.



(a) (b)

Figure 5.15. Using the width and the length to measure the coloured lines

The students also had a discussion about estimating the part of the lines which is less than one unit of reference. Transcript 5.2 illustrates the discussion between the students in the focus group.

Transcript 5.2

1	Ocha	: If we put it like this (*putting the eraser on the bed's width*),
2		we can measure it can't we? (*the cover of the eraser fits the*
3		*width of the bed*)
4	Diva	: Oh yeah, we can.
5	Ocha	: (*putting the eraser on the coloured walls. Marking the red*
6		*line to find out how many times the reference fits the red line*)
7	Ocha	: So, it is 2 and a half.
	…	…
8	Ocha	: Is it 2 and a half?
9	Rizqi	: Wait. (*observing the reference on the red line*) No, it is not.
10	Diva	: One over three, I think.
11	Rizqi	: Yea, over three.
12	Ocha	: So, two and?
13	Diva	: Two and one over three.

From the transcript we found out that the students used fraction to express

the part which is less or more than one unit of reference. Students in other groups also used fraction to express the measurement of the coloured walls. Moreover, students struggled in estimating the part, whether it is closer to a half or a third. In general, students were able to use the reference to measure the coloured walls and they were also able to describe the length of the coloured walls using the length of the reference.

Problem 3

The aim of the problem is to use the scale line to describe the length of cables. The problem contains an illustration of installed cables in the studio and a scale line. The students had to measure the length of the cables and to determine the real length of both cables. In the end, the students were asked to draw an illustration as a form of comparison between the length of a scale line and the cables.

Before students started to work on the problem, the teacher gave an example. She demonstrated to use the scale line to measure the cable on the plan and drew an illustration as a form of comparison. The focus group students used an ice stick to determine how long the scale line is (see figure 5.16). As expected, they used it to determine how many times the scale line fits the orange and green lines.

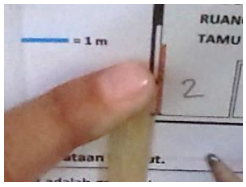


Figure 5.16. The students used an ice stick to measure the scale line and the other lines

Even though the teacher had given students an example of making the illustration, the students still struggled. Ocha used the length of the scale line on the stick to draw the illustration. She drew the scale line on the upper side and the green line on the lower side as in figure 5.17. In general, students were able to use the scale line to determine the real length of two cables. Moreover, students were able to illustrate the comparison between the scale line and the cables.

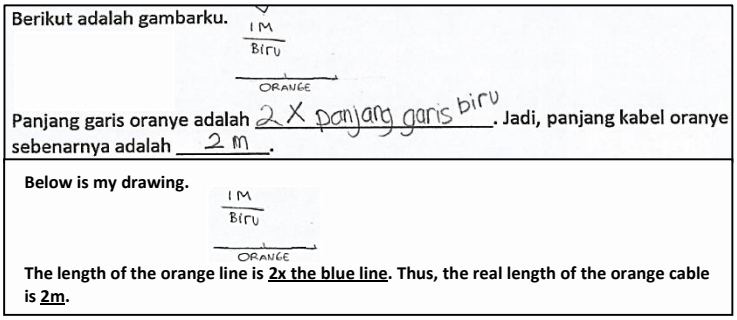


Figure 5.17. The student's drawing in comparing a scale line to the orange line

Activity 2

The aim of the activity is to enable students to make and to use a scale line in a plan. Students were asked to draw a classroom plan using a scale line and to determine the real measurement of the classroom. They had to choose a large object in the classroom as a reference and to represent the reference into a scale line.

As expected, the focus group students drew the plan first rather than drew the scale line. The students did not read the steps on making the plan on the worksheet, they only read the instruction. Even though the researcher asked them to read the steps, they did not revise their drawing. It happens in

other groups as well. When students drew the plan first rather than drew the scale line, they will get confuse especially when they have to determine the real measurement of the plan. Moreover, the estimation of the real dimension of the classroom would be too far. Figure 5.18 shows the drawing of the students in focus group.

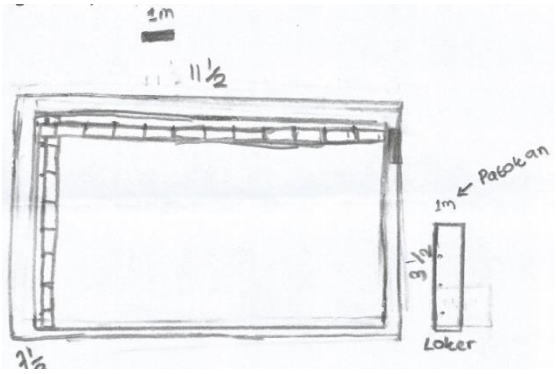


Figure 5.18. The students drew the plan first

Students did not get to discuss the second activity in a big discussion because the limited time. Even though students were able to use a scale line to measure, they struggled in drawing and using a scale line to draw a classroom plan. In addition, they did not realize the different result from making the plan first and in deciding the scale line first.

Based on the analysis, we can conclude that students were able to use a reference as a scale line to measure other objects. In addition, students were able to describe the length of an object using the length of a reference in multiplicative comparison. Students struggled in illustrating the length comparison between a scale line and an object on the plan.

We summarize the conjectures and the students' actual answer in table

5.8.

Table 5.8. The students' actual answer on lesson 1 cycle 2

Activity	**Conjectures of students' answer**	**Students' actual answer**
Distinguishing difference and comparison	Students cannot recognize the notion of difference	Students did not realize the word "longer than" as difference
	Students can recognize the notion of difference but not able to reason	
	Students recognize the notion of difference and able to reason why the statement is not comparison	
Making and using a scale line to determine the length of walls	Students are not able to use the length of a reference as a scale line	
	Students are able to use the length of a reference as a scale line and determine the length of walls	Students used different tools which have the same length as the reference to determine the length of walls
Using the scale line to	Students are not able	Students were able to

describe the length of cables and drawing an illustration as a form of comparison between the length of a scale line and the cables	to use the scale line and not able to draw an illustration to compare the length of a scale line and the cables	use the scale line but some of them were not able to draw an illustration to compare the length of a scale line and the cables
	Students are able to use the scale line and able to draw an illustration to compare the length of a scale line and the cables	Some students were able to draw an illustration to compare the length of a scale line and the cables
Using a scale line to draw a classroom plan and to determine the real dimension of the classroom	Students are not able to use a scale line to draw a classroom plan and to determine the real dimension of the classroom	
	Students are able to use a scale line to draw a classroom plan and to determine the real dimension of the classroom	Rather than making a scale line first, the students drawing the plan, then deciding what the scale line will be

3. Lesson 2

There are two activities in the second lesson. The first activity consists of two problems and the second activity consists of one problem. The

context in this lesson is about map. Students worked in groups for the first activity. For the second activity, they worked individually but they were allowed to discuss the problem with their friends. In general, the activities and problems in the second lesson aim to enable students in working with double scale line.

Activity 1

Problem 1

The aim of the problem is to use information about the real distance and the distance on the map. Students were asked to decide which map fits the provided double scale line if the distance between Gresik and Surabaya is 15 km. Before students started to work, the teacher informed students that the use of scale is to shrink the drawing of the earth. In introducing the double scale line, the teacher told students that the scale has two measurements, the measurement on the map and the real measurement. To continue, the teacher asked students to find out whether the distances between Gresik and Surabaya on all provided maps are the same. Figure 5.19 and transcript 5.3 illustrates students' struggle in determining which map fits the double scale line.

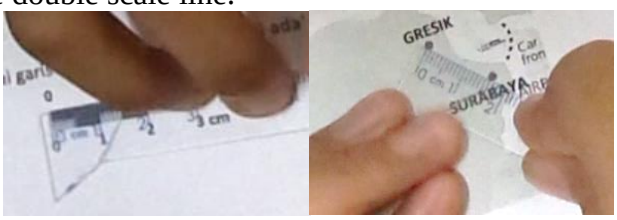


Figure 5.19. Students comparing the double scale line and the distance on maps

Transcript 5.3

1	Diva	: (*helping Ocha measured the upper scale line*) 1,5.
2	Ocha	: (*back and forth measuring the upper scale line and the*
3		*distance between Gresik and Surabaya on map B*)
4	Ocha	: (*measuring the distance between Gresik and Surabaya*
5		*on map C*)
6	Rizqi	: That's 1. (*looking at the measurement on map C*)
7	Ocha	: (*measuring the distance between Gresik and Surabaya*
8		*on map C*)
9	Diva	: Where is 15 km exactly (on the double scale line)? B
10		is closer. (*the distance on the map B is almost the same*
11		*as the upper scale line when it is 15 km*)
12	Ocha	: It seems the answer is B.
13	Researcher	: Why B?
14	Ocha	: Because…
15	Diva	: There is information 15 km.
16	Ocha	: Because there is information that the distance between
17		Gresik and Surabaya is 15 km.
18	Diva	: (*measuring the upper scale line and the distance*
19		*between Gresik and Surabaya on map B*)
20	Researcher	: Don't forget to write the reason. And what happen if
21		map A and map C use the provided scale line? Discuss
22		it together.

From the transcript we knew how the students struggled to determine which map fits the provided double scale line. The students compared the double scale line to the distance between Gresik and Surabaya on all maps. They thought the distance between Gresik and Surabaya on map B is almost the same as the length of 15 km on the double scale line. Even though the students were able to determine which map fits the double scale line, they did not have a further discussion about what will happen if other maps use the provided double scale line.

Once students had finished the group discussion, the teacher explained to students on how to determine which map fits the double scale line. She

divided the lower scale line into six same parts. Students were asked to determine the position of 1 cm and 2 cm if there are six smaller parts and the total length is 3 cm. One student showed the position of 1 cm and 2 cm on the board. Later, the teacher asked students the sum of three smaller parts on the lower scale line. Some students were able to answer her question but some were not. To help students, the teacher explained again about dividing the scale line into several same parts as in figure 5.20.

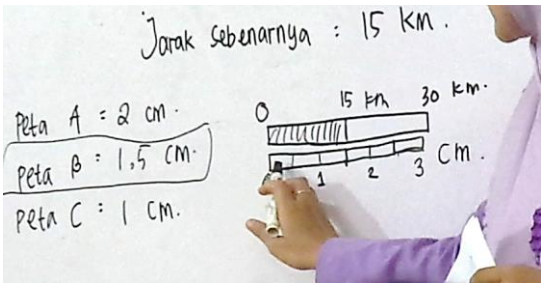


Figure 5.20. The teacher divided the lower scale line and asked a student to write information for every 1 cm

There should have been discussion about what happen if other maps used the provided double scale line. However, there was no further class discussion. It affects students' answer on the worksheet. They only answered which map fits the double scale line without knowing the impact of using inappropriate double scale line on a map.

Problem 2

The aim of the problem is to enable students to illustrate the relationship between distance on the map and real distance into a double scale line. The teacher instructed students to draw the double scale line for map A and map C. Some students were able to draw the double number line for other maps.

However, other students struggled in drawing the double scale line. Especially in dividing the double scale line. Transcript 5.4 illustrates the conversation between the students about dividing the double scale line for map A. Figure 5.21 shows the student's double scale line for map A.

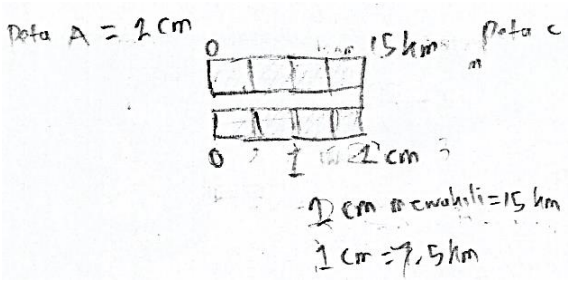


Figure 5.21. Tegar's double scale line for map A

Transcript 5.4

1	Tegar	: (*talking to Dinda*) Two parts. Each part is 1 cm.
2		And the middle (of 1 cm) is a half.
3	Dinda and Danis	: (inaudible) (*Dinda told Danis Tegar's*
4		*explanation*)
5	Tegar	: (*adding his explanation*) 2 cm. Because 2 cm
6		equals to 15 km.
	…	…
7	Tegar	: (*explaining to Dinda*) You divide it into four.
8		This is one, a half, and two. We know the real
9		distance is 15 km. So, what is 2 cm?
10	Tegar	: So, it represents 15 km. If we divide it (the lower
11		line), 1 cm represents 7,5 km. What is the result
12		of 7,5 + 7,5? 15 isn't it? So, here is 15 km.
13		(*pointing the end of upper scale line*)

From figure 5.21 and transcript 5.4, we understood that Tegar divided 2 cm into four parts. Tegar could be following how the teacher divided the lower line for each part on the previous problem. Therefore, the length of each part on the lower line is 0,5 cm. Since 2 cm represents 15 km, he knew 1 cm represents 7,5. It is possible that Tegar divided 15 km and 2 cm by 2.

When students were discussing with their groups, the teacher walked

around the class to find out students' struggles. The teacher had discussion with two students who also struggled in dividing the lower line. Realized that not only the two students who struggled in dividing the lower line, the teacher told the class to determine the real distance for each cm. Students were free to divide the lower line into several parts. However, the teacher did not continue the class discussion. She gave students more time to finish the double scale line and continued to the next activity.

Activity 2

The aim of the activity is to enable students to use the double scale line to determine real distances. The teacher narrated the problem and gave students instruction to use the double scale line on the map and to measure the travelled distance for each day. Students wrote every cm on the double scale line. Some of them also tried to determine the distance for 0,5 cm. Transcript 5.5 illustrates how Diva determine the real distance for travelling in the first day and figure 5.22 shows Diva's work on the double scale line.

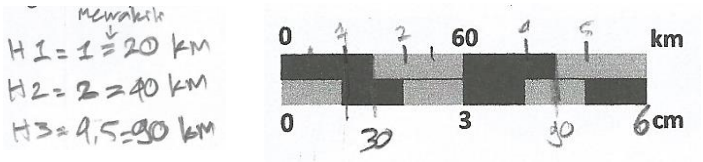


Figure 5.22. Information on Diva's double scale line

Transcript 5.5

1 Researcher : Where is 1 cm on this line?
2 Diva : Here. (*pointing at 1 cm*)
3 Researcher : So, how many (km)?
4 Diva : 20.
5 Researcher : How do you get 20?
6 Diva : From… I don't know.
7 Researcher : How long is it? (*pointing on the middle part of 1 cm*

8		*on the upper line*)
9	Diva	: 10.
10	Researcher	: How do you know it is 10?
11	Diva	: Because the difference is 10. Uhm, 10, 10, 10 km.
12	Researcher	: What do you mean?
13	Diva	: From 1 cm to another 1 cm is 10.

If we observe Diva's writing on the double scale line, he marked the half of 1 cm. Moreover, Diva said that he determined the difference for a certain measurement although he mistakenly said from 1 cm to another is 10 km. It should be from 0,5 cm to another is 10 km. From the transcript and the figure, we interpret that Diva used repeated addition to determine the real distance. Diva's method will not be efficient if the difference between numbers is large. By finding difference between numbers, he might answer the problem incorrectly if he asked to determine only one travelled distance. In addition, Diva did not pay attention on putting information. He wrote the distance on the map on the upper line while the real distance on the lower line.

The teacher walked around the class and talked to some groups. However, the class did not have a discussion for this problem. If there was a class discussion, the teacher could point out students' struggles, for example in determining the relationship between given numbers and writing appropriate information on the double scale line. Therefore, students can discuss and support each other to learn about working with the double scale line.

Based on the analysis, we can conclude that students were struggling in working with double scale line, especially in illustrating the relationship between the real distance and the distance on the map to a double scale line. Students struggled on dividing the double scale line into several same parts. Instead of determining the relationship between numbers, students used difference between numbers and found a pattern in working with the double scale line. The pattern is similar to repeated addition.

We summarize the conjectures and the students' actual answer in table 5.9.

Table 5.9. The students' actual answer on lesson 2 cycle 2

Activity	**Conjectures of students' answer**	**Students' actual answer**
Determining a map which fits the double scale line and reasoning	Students have no idea on working with the double scale line	
	Students are able to determine which map fits the double scale line but they are not able to reason for other maps	Students did not give reason what will happen if other maps use the provided double scale line
	Students are able to determine which map fits the double scale line and able to reason for other maps	

Making double scale lines for other maps	Students draw inappropriate double scale lines	Some students forgot to write information on the double scale line and did not draw a precise double scale line
	Students draw appropriate double scale lines	Students drew the double scale line using precise measurement and wrote information on the double scale line
Using the double scale line to determine real distances	Students do not use the model to determine real distances	
	Students use the model to determine the relationship between numbers to find out real distances	Students used the model by dividing the double scale line for every 0,5 cm. Most students used division to divide the lower line and addition to determine the real distance for every 0,5 cm

4. Lesson 3

There are three problems in the third lesson. The context of the third lesson is about map and travelling. In this lesson, students will not only work

with problems which ask about distances. Therefore, students should have ignored the exact measurement between numbers as long as they were able to order given numbers on the model. Students worked individually but they were allowed to discuss problems with their friends.

Problem 1a

The aim of the problem is to represent the relationship between the distance on the map and the real distance into an illustration. The students were asked to draw the double scale line of a Bali map as a form of comparison between the real distance and the distance on the map. Before students worked on the problem, the teacher asked students what they have learned during two lessons. Students answered they learned about comparison and double scale line. The teacher proceed to narrate the first problem and asked students to draw the double scale line for a map, given 4 cm on the map represents 80 km.

While walked around the class, the teacher was busy arranging the students who did not behave well to switch seats. The teacher also reminded students to write information on the double scale line. Students who had finished their work approached the teacher to ask her to confirm their work.

In line with the HLT, some students did not use a ruler to measure and to draw the double scale line. The researcher asked students whether their drawing represents the information and students changed their drawing. Transcript 5.6 illustrates conversation between students in drawing the

double scale line. We provide figure 5.23 as an illustration of the student in drawing the double scale line. Her drawing was incorrect because the measurement is not exact and precise.

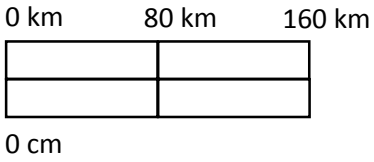


Figure 5.23. An illustration of Sisi's drawing

Transcript 5.6

1	Researcher	: How is it Sisi? Where is 4 cm, Sisi?
2	Sisi	: (*pointing the lower scale line*).
3	Researcher	: Where is 4 cm? Measure it using a ruler.
4	Ocha	: Is it correct? Is it 4 cm?
5	Researcher	: Please help Sisi, Ocha.
6	Ocha	: Is it 4 cm? (*measuring the line using a ruler*)
7	Sisi	: Yes 4 cm.
8	Ocha	: Wrong. (*the line is not precisely 4 cm*) 4 cm means…
9		Mr. Nyoman knows that 4 cm on the map shows 80…
10		4 cm shows 80 km. It means… It has to be… (*showing*
11		*the measurement using a ruler*) It supposed to be from
12		here (0 cm) to here (the middle of the double scale line)
13		is 4 cm. It has to be on the same vertical line, 80. 80 is
14		4 cm.

From the transcript, we may interpret that Ocha understood the measurement on a double scale line should be precise. Ocha kept asking Sisi whether the double scale line has a precise measurement. Moreover, Ocha knew that she had to align 4 cm and 80 km because 4 cm represents 80 km. It is possible that Ocha perceived the given information as a composed unit. For Sisi, it seems she still struggled in drawing the double scale line. She knew that 4 cm is on the lower line but she did not know where exactly 4 cm is. In the end, Sisi was able to draw a precise double scale line for the

map as shown in figure 5.24.

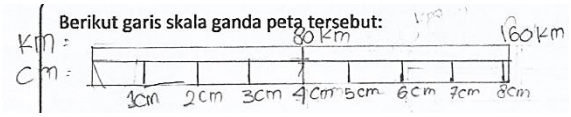


Figure 5.24. Sisi's double scale line becomes more precise compared to the previous

Even though some students were able to draw the double scale line, not all of them understood the importance of precise measurement on it. Some students did not measure the line precisely in drawing the double scale line. They got confuse in writing information about the relationship between the distance on the map and the real distance on the double scale line. Furthermore, instead of asking students how they drew the double scale line, the teacher asked students whether they were able to draw the double scale line and whether they had finished drawing it. It would be a good opportunity for the teacher and students if they discussed how to draw a double scale line by pointing out the ratio information. Moreover, a discussion about the importance of being precise in drawing a double scale line is also needed. However, there was no discussion for the problem and the teacher proceeded to the next problem.

Problem 1b

The aim of the problem is to order the given numbers on the stretched route. The students were asked to use the stretched route to determine the real distance of each destination from Denpasar. Before the teacher read the problem for students, she was busy in disciplining and asking students to be

quiet. She also asked students who did not pay attention on the lesson to switch seats.

Once the teacher had finished reading the problem and steps for students, she asked them to solve and discuss the problem with their friends. Students struggled in placing numbers on the stretched route, especially in dividing the route into several parts. Since the teacher did not inform students in the beginning that they were allowed to not have a precise measurement between numbers, students thought that they had to use the model as they use the double scale line. To help students, the teacher decided to tell the class that the model is similar to a number line in which the numbers must be well ordered.

Students had the same method in ordering the given numbers. They measured the stretched route and put information for every cm rather than writing only the given numbers. Figure 5.25 illustrates students' writing in completing the stretched route and determining the real distances.

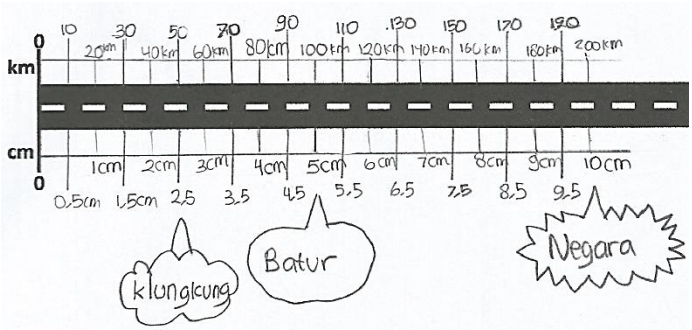


Figure 5.25. Students determined the real distance for every 0,5 cm

In our interpretation, students determined the real distances by doing repeated addition. There is a pattern between numbers, on the lower line the

difference is 0,5 cm and on the upper line the difference is 10 km. Students add 0,5 cm on the lower line and add 10 km on the upper line. Students' method is not efficient because they were determining numbers which are not asked.

Students also used their double scale line on the previous problem to help them in determining every 1 cm to solve the problem. Transcript 5.7 illustrates the discussion between Diva and other students about determining the real distance using the previous problem and repeated addition for the distance on map 2,5 cm.

Transcript 5.7

Diva : See the previous problem. (*referring to the double scale line)*
Rafi : 35, 35. (*guessing the answer*)
Diva : 2,5 is here isn't it? (*showing where 2,5 is using the double scale line*)
Rafi : 45, 45. (*guessing*)
Diva : Why is it 45? You add 5 to the ratio, how many is it?
Rafi : 50.
Diva : So that's it.
Tegar : It's 0,5 (cm). For 0,5, you add 10 (km). (*writing on the stretched route)*

As students worked on the problem, the teacher walked around the class and had a discussion with a student about dividing the stretched route. The student thought the stretched route a as a double scale line and the student was confused in dividing the route. Transcript 5.8 illustrates the conversation between the teacher and the student.

Transcript 5.8

Teacher : The important thing in dividing is it has to be equal. If you divide, 10 in here, then where is 5? Does 5 in here have the same spaces? Are these two parts equal?

Gladys : (inaudible)
Teacher : (inaudible) … If it is the same, the measurement should be precise.
Gladys : (inaudible)
Teacher : It has to be appropriate. If the distance is 10, let's say we look for 5. It's (5) in the middle. The length of this side (left side of 5) and this side (right side of 5) is divided equally.

Even though the model has developed, it is possible that the teacher and students still thought the stretched route as a double scale line. In addition, since the problem is about the distance on the map and the real distance, it is difficult to ignore the precise measurement. We interpret that chosen problems and context are affecting the use of model.

After some time, the teacher asked one student to draw his stretched route on the board and order the given numbers. When the student wrote his work on the board, the teacher asked other students the real distances for the given distances on the map. There was a student who had different answer. Instead of discussing her answer, the teacher directly pointed out the student's mistake was being not precise in measuring. In lesson 3, the important thing is not about measuring but ordering numbers and determining the relationship between numbers. However, the teacher did not point these out. In addition, the teacher told students to find each cm rather than to focus on the given numbers and determining the relationship. It could be an interesting discussion if the teacher asked students how they could get those answers rather than asked for the right answers.

Even though students were able to determine the distances and ordered

the numbers on the stretched route, students still depended on non-given numbers to order. The model has developed but students' ability in ordering given numbers did not go as expected because they still thought the model as a double scale line. In the end, the work of the student on the board was not being discussed in depth. The teacher and the class only checked whether the answer is correct. Later, the teacher proceed to the next problem.

Problem 2

The aim of the problem is to use the stretched route to show the relationship between numbers. The teacher read the problem for students and they were asked to draw an illustration similar to the stretched route and to use it to determine the fuel consumption to each destination. Given, 2 litres of gas are used to travel 25 km. Using the stretched route on the board from the previous problem, the teacher asked one student to show where 25 km is. The teacher allowed students to join the stretched route on previous problem to this problem. In formal, the model is triple number line. In addition, since the determined distances on the previous problem become the given numbers on this problem, the teacher instructed students to have the same measurement as the previous problem and to divide the lines on the model precisely.

As in the previous problem, the students in the focus group directly drew a line and marked the line for every 1 cm. However, there was an

interesting discussion between Diva and Rizqi. Diva marked the model for every 20 km and used repeated addition. The researcher tried to lead Diva to shift his method towards multiplication. During the discussion, Rizqi came up with an idea to multiply the given numbers. Transcript 5.9 provides information on how these students solved the problem.

Transcript 5.9

1	Researcher	: We want to find out, how many litres we need to travel
2		50 km. What you have to do to get 50 from 25?
3	Diva	: 25 to 50?
4	Researcher	: What you need to do? Add, subtract, multiply or divide
5		so that you get 50.
6	Rizqi	: You need to multiply it.
7	Researcher	: Multiply by what?
8	Rizqi	: By 2.
9	Researcher	: Don't you need to subtract it? I mean add it?
10	Rizqi	: No, because it is a comparison.
11	Researcher	: Now try to determine (the fuel consumption) for 50
12		(km). 25 to 50, what you need to do?
13	Rizqi	: Multiply by 2.
14	Researcher	: Multiply by 2, if you multiply 25 by 2 to get 50, then…
15	Diva	: (*writing 4 on the upper line which is on the same vertical*
16		*line as 50 km*)
17	Researcher	: How do you get 4?
18	Rizqi	: You multiply it (the upper line) as well by 2.

From the transcript, we know that Rizqi had the idea of doing multiplication to solve the problem. When he said that he multiplied both lines by 2, it indicates he knew that ratio is a composed unit. To get another equivalent ratio, he had to multiply both sides by a certain number. We provide transcript 5.10 as a proof that Rizqi could determine the relationship between given numbers by using multiplication.

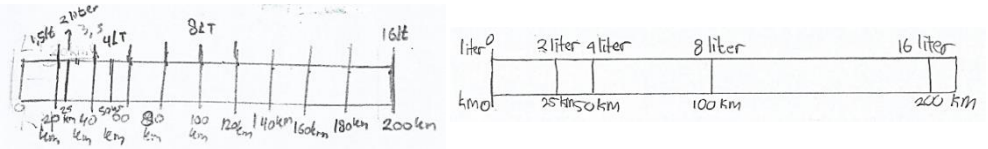
Transcript 5.10

1	Researcher	: What is the distance to Batur?

2	Rizqi	: To Batur is 100 (km).
3	Researcher	: Where is 100 (on the model)?
4	Rizqi	: Use this (multiplication), it is simpler.
5	Researcher	: How is it simple?
6	Rizqi	: (*talking to Diva*) 100 is what times what? Eh, 25 is what
7		times what to get 100?
8	Diva	: 4. (*showing 4 using his fingers*)
9	Rizqi	: Yes, times 4. You also multiply 2 by 4. (2 in the upper
10		line)
11	Diva	: 2 times 4 is 8.
12	Researcher	: Is there another way to get 100? We have determine 50,
13		can we use it?
14	Rizqi	: We can.
15	Diva	: 25 times 2.
16	Rizqi	: Hey! 50 times 2 is 100. So, we need to multiply the upper
17		by 2. That's another way.

Rizqi kept on saying that he had to multiply lower and upper line with the same number. In addition, Rizqi was able to use determined value to relate the given numbers using multiplication. In this case, Rizqi's method is more sophisticated than Diva's method, in which Diva had to determine the fuel consumption for every 20 km.

The researched moved to another group to find out whether students also still used repeated addition. The researcher interviewed one student named Danis. Danis has the same idea as Rizqi, using multiplication. Even though Danis and Rizqi shared the same idea, they drew different representation. Figure 5.26 shows illustrations for both students.



(a) (b)

Figure 5.26. Rizqi's (a) and Danis' (b) illustration

From the illustrations, Rizqi did not directly order the given numbers on the line. He ordered every 20 and inserted the given numbers in between. However, Danis directly ordered the given numbers on the line without using help of other numbers. In this case, we perceive that Danis' illustration is more sophisticated and suit the model.

In general, most students still perceived the stretched route as a double scale line. It influences students to draw and to mark for every 1 cm rather than directly order the given numbers on the line. Even though most students used repeated addition, there were students who shift their method by using multiplication. The different methods could be pointed out in the class discussion. However, there was no class discussion for the problem. To close the lesson, the teacher asked students to hand in their worksheet.

Based on the analysis, we can conclude that students were able to order the given numbers on the model. However, students had not ignore the exact measurement between numbers. In other words, students used the more formal model as a double scale line. Even though at first they used repeated addition to determine relationship between numbers, some students had shifted their method by perceiving the relationship as multiplicative comparison.

We summarize the conjectures and the students' actual answer in table 5.10.

Table 5.10. The students' actual answer on lesson 3 cycle 2

Activity	Conjectures of students' answer	Students' actual answer
Representing the relationship between the distance on the map and the real distance into an illustration	Students draw inappropriate double scale lines	Some students did not write information on the double scale line. Some others did not draw a precise double scale line
	Students draw appropriate double scale lines	Some students drew a precise double scale line
Ordering the given numbers on the stretched route and using the model to determine real distances	Students are not able to order the given numbers on the stretched route	
	Students determine real distances using subtraction and addition	Some students still thought the model as a double scale line. They determined real distances for every 0,5 cm
	Students determine real distances using multiplication and division	
Drawing a model, ordering the given numbers, and using	Students are able to draw the model but not able to order the	

the model to show the relationship between numbers	given numbers	
	Students are able to draw the model and order the numbers. Students determine the relationship between numbers using addition and subtraction	Some students drew the model as in double scale line. Some students used addition to determine fuel consumption for every 20 km
	Students are able to draw the model and order the numbers. Students determine the relationship between numbers using multiplication and division	Some students were able to determine relationship between numbers using multiplication

5. Lesson 4

There are three problems for this lesson. These three problems are related, the determined value on the problem becomes the given information for the next problem. The context of problems is travelling. The general aim of problems is to enable students to understand that numbers on the model must be well ordered. Moreover, students must be able to express the multiplicative comparison to determine equivalent ratios.

Problem a

Before the teacher read the problem for students, she asked what

students know about double scale line. However, the discussion did not go well as most students did not answer the question. The teacher proceed to read the problem and drew a double number line on the board. As an example for the next problems, the teacher showed students how to use the double number line. The teacher involved students to compare and to arrange given numbers on the double number line. Transcript 5.11 illustrates their discussion.

Transcript 5.11

1	Teacher	: We spend 36 minutes to travel 24 km. I estimate the
2		location of 24 km (on the model). Let's say 24 is here
3		(*drawing a vertical line on the model*). So, this is 24 km
4		and the upper line is 36.
5	Teacher	: Can you do it from 24 km to Singaraja which is 80 km?
6		Where is 80 km? (*pointing on the model*)
7	Students	: On the (right) corner. (*some students pointing the right*
8		*side of model*)
9	Teacher	: Let's say 80 is here. How many minutes it represents? We
10		don't know yet. Let's name it, Singaraja. Then, Batur 60
11		km. Where is 60? Around here, okay? (*drawing a vertical*
12		*line before 80 km*) 60 is Batur. You don't need to use a
13		ruler to determine the distance, just use an estimation.
14		Last, Klungkung is here isn't it? (*pointing location before*
15		*60*). It's impossible to be here (*pointing location after 60*).
16		It has to be ordered.

Even though we involve distance in the problem, the teacher shifted her method to place numbers on the model. Rather than started from 1 km and divided the line on the model into several same parts as in previous lessons, the teacher directly asked students to arrange the given numbers on the model. Moreover, students were able to arrange the given numbers on the model in order.

To support students in determining the travelled time for each destination, the teacher related the ratio information to fraction, 24 km:36 minutes to $\frac{24}{36}$. The teacher also led students to observe the relationship between numbers. Since it would be difficult for students to do 40 from 24, the teacher told students to simplify the fraction and they got $\frac{2}{3}$. We provide figure 5.27 as an illustration of the teacher's drawing.

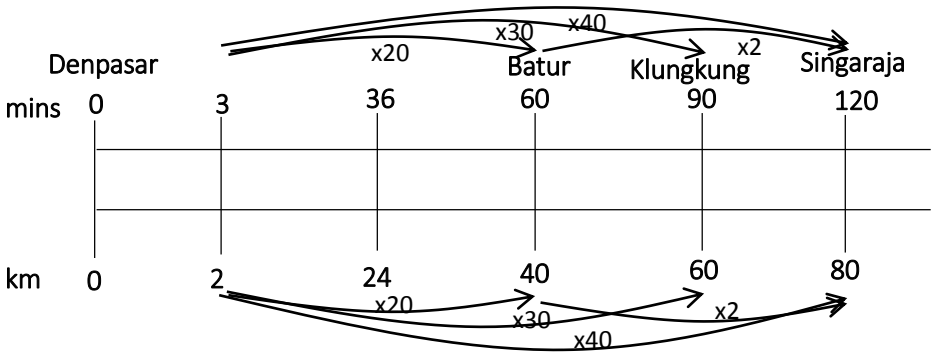


Figure 5.27. An illustration of the teacher's drawing on the board

From the simplified fraction, the teacher asked students to point out the relationship between numbers on the lower line. Students answered the relationship using multiplication. Since they multiplied a number on the lower line with a scale factor, they had to multiply the number on the upper line with the scale factor as well. The teacher also asked students whether all ratio have the same value. However, students were in doubt. To show that all pairs have the same value, the teacher reminded students to use fractional form and to simplify the fractions. In the end, they got $\frac{2}{3}$ for the pairs. Hence, students were sure that all pairs on the model have the same value. The teacher also told students that those pairs are called equivalent

ratios. Later, the teacher told students to work on the next problems.

Problem b

Given, the ratio information is to travel 20 km, the car consumes 2 litres of fuel. Using the determined time on the previous problem, students were asked to determine the fuel consumption for each destination. The teacher provided an empty double number line on the board in which the lower line should be for the distance and the upper line for the fuel consumption.

The focus group students directly drew the model and listed the given information. Once they finished drawing the model, they observed the given numbers for the lower line and arranged the order of the given numbers. To relate one number to another, they used multiplication. We provide an example of the students' answer as in figure 5.28.

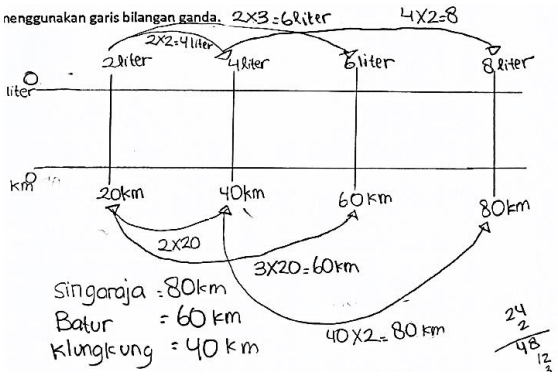


Figure 5.28. The students used multiplication to express the relationship between numbers

From the figure above, the students did not simplify the given ratio. She directly related one number to another using multiplication. The students did not only relate the number from the given ratio. The students used

another number to relate. For example, from 20 km, the students were able to determine the fuel consumption for 80 km. However, the students already figured out the fuel consumption of 40 km. Thus, the students started from 40 km to determine the fuel consumption. In addition, the students used a line with an arrow. It benefits other students and the teacher to follow the students' steps. By using multiplication to express the relationship between numbers, we interpret that the students have the notion of multiplicative comparison. Furthermore, the students multiplied the pair with a scale factor. It is possible that the students also have the notion of ratio as a composed unit to determine another equivalent ratio.

For some time, the teacher did not walk around the class. Since the students of focus group did not struggle as previous lessons, the researcher moved to another group to find out how they solve the problem. The researcher approached a student named Tegar who usually used repeated addition to solve ratio problem in previous lessons (see transcript 5.7). The researcher asked the student how he solved the problem since the model has developed into a double number line. Transcript 5.12 illustrates the discussion. We also provide figure 5.29 which shows Tegar's work to illustrate how Tegar worked with the double number line. The taken figure is Tegar's work after the discussion. In his work, Tegar simplified the given ratio first. Therefore, there is a pair of ratio 10 km:1 litre on the double number line.

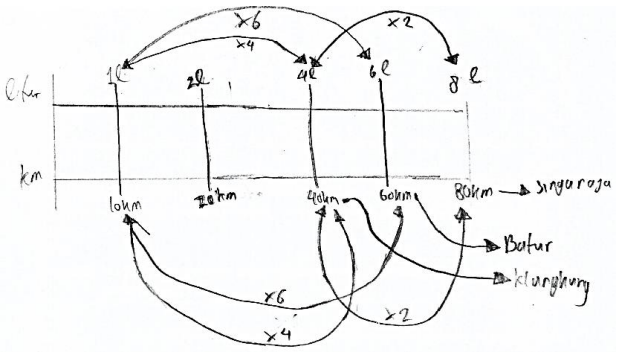


Figure 5.29. Tegar was using the double number line to solve the problem

Transcript 5.12

1	Researcher	: How did you get that, Tegar?
2	Tegar	: 20 km equals 2 litres doesn't it? Then, 10 km equals 1
3		litre.
4	Researcher	: Then?
5	Tegar	: Then, we need to determine (the fuel consumption for)
6		40 km. It means, you multiply. Just multiply.
7	Researcher	: From what? From what to what?
8	Tegar	: This one is 30 (*drawing an additional vertical line*
9		*between 20 km and 40 km*).
10	Researcher	: If it is from here, from here (20 km) to there (40 km),
11		can we do that? What do you think?
12	Tegar	: You continue it. Add. Every kilo… Every 10 km, add 1
13		litre, 1 litre.

From the transcript, we interpret that Tegar still used repeated addition to explain how he solved the problem. The researcher wanted to find out whether Tegar was able to shift his strategy and whether he was able to relate the numbers using multiplicative comparison. The researcher asked questions as in transcript 5.13.

Transcript 5.13

1	Researcher	: For example, we don't know it yet. (*covering numbers on*
2		*the middle part of the model*) You want to find out 60 km.
3		Can you find it from here (10 km) to here (60 km)?
4	Tegar	: Multiply it.
5	Researcher	: Multiply by what?
6	Tegar	: By 6.

7 Researcher : If the number on the lower line is multiplied by 6, what
8 will you do for the pair on the upper line?
9 Tegar : You multiply it by 6 as well.

From the following transcript, Tegar was able to relate numbers using multiplicative comparison after the researcher covered the middle numbers on the double number line. It is possible that the numbers on the problem lead Tegar to determine and to explain his work using repeated addition. Observing the given distance on the problem, the numbers on the lower line are multiples of 10. It makes sense if Tegar tried to explain his method using repeated addition. When the difference between numbers on the lower line is big, it pushes the student to relate the numbers using multiplicative comparison. From this case we can say that the numbers string influences students in solving the problem. It will be better if the numbers have a large difference and the multiples are not a landmark number such as 2, 5, and 10.

In the class discussion, the teacher asked a student to write his work on the board. We provide figure 5.30 as an illustration of the student's work.

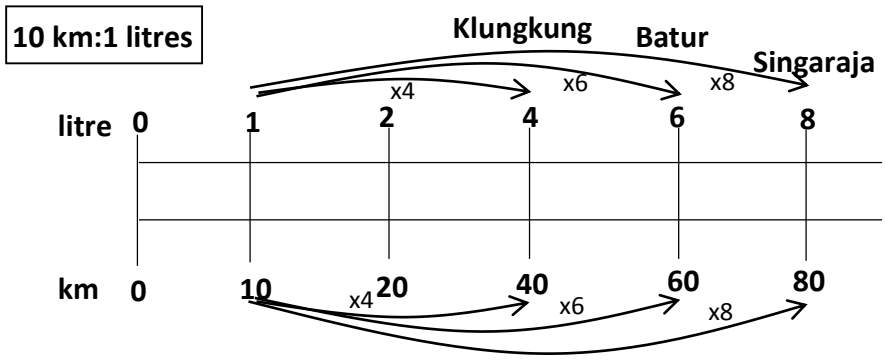


Figure 5.30. The student only related the numbers from 10

From the student's work, we know that the student only used 10 to relate to

other numbers. Even though the student was able to arrange the numbers and to use multiplicative comparison on the model, the student's ability in determining relationship between numbers is limited. In addition, the student simplified the given ratio. It is unnecessary for the student to use fraction and to simplify it if he observed the numbers.

The teacher told students that it is allowed to not simplify the given ratio. Some students did not simplified the given ratio. Students who did not simplify the given ratio were able to observe the relationship between numbers better than students who simplified. As in the analysis of figure 5.28, students were able to relate numbers with more combination.

Problem c

Given, the ratio information is 6 litres of fuel cost IDR 60.000. Using the determined fuel consumption on the previous problem, students were asked to determine the price of the fuel consumption for each destination. Figure 5.31 shows the focus group students' work.

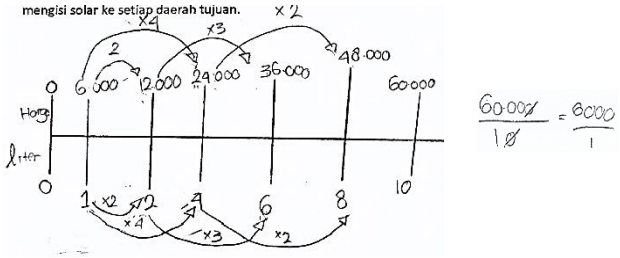


Figure 5.31. The students simplified the given ratio

From the figure, the students inserted 2 on the lower line as information from the previous problem. In fact, the problem does not contain a car consumption of 2 litres. The students mistakenly used 2 as a problem. From

the students' work, we know that they transformed the given ratio into a fractional form and simplified it. In this case, the students tend to choose integer rather than to have a decimal or a fraction as the scale factor. Our interpretation according to the students' work is that the students were attached to multiplication and integer as the scale factor. Therefore, rather than observing and determining the relationship between given numbers, the students tend to simplify the fraction and started to determine the relationship between numbers using multiplication and integer scale factors.

To find out other students' struggle, the researcher moved on another group. In another group, the students struggled in starting to solve the problem. The researcher asked the students to order the given information on the model. Once the students ordered the numbers, the researcher asked them to determine the relationship between the given numbers on the lower line. However, the students struggled to determine the multiplicative relationship. In fact, the students should have been able to relate that 10 is 2,5 times 4. Yet, they simplified the ratio to determine the multiplicative comparison.

As we mentioned before, the students wanted to obtain a whole number as the relationship between numbers. If the students were flexible in working with numbers, they would not have a problem in having decimal as the scale factor. The students struggled in using decimals not only in this lesson. In the second lesson, in which students had to determine the division

which resulted in decimals, they tend to do rounding to get integers. In this problem, rather than simplifying the given ratio, students may relate 10 with 4. The relation between two numbers is decimal. However, it turned out that most students did not try to determine the relationship between 10 and 4 directly.

The teacher decided to discuss the problem by asking one student to write his work on the board. Figure 5.32 shows the student's work.

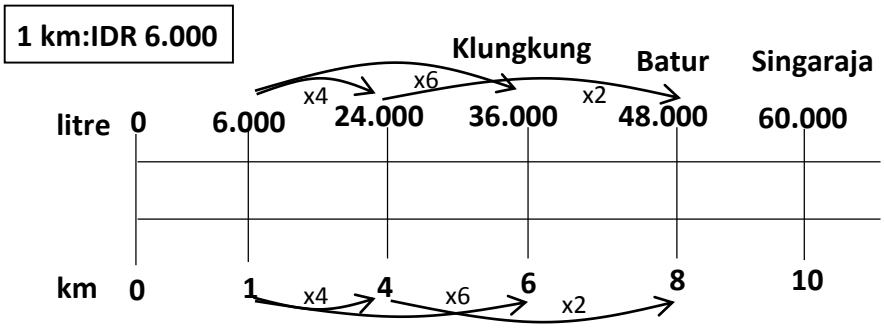


Figure 5.32. The student used multiplication to express relationship between numbers

Using the student's work, the teacher emphasized on simplifying the given ratio regarding to students' struggle in perceiving the relationship between numbers on the lower line. The teacher did not discuss what if they did not have to simplify the fraction. Students might come up with an idea to use decimals as the scale factor if the teacher asks. The student's work on the board is similar to figure 5.31, which becomes the representation of general students' answer.

Based on the analysis, we can conclude that students were able to

understand that numbers on the model should be well ordered. Moreover, they were able to determine the multiplicative comparison between numbers. Students used fractional form and simplified the fraction when the numbers are difficult. However, students tend to use multiplication in determining the relationship between numbers rather than using division. In addition, instead of involving decimals as the scale factor, students tried to involve only integers as the scale factor.

We summarize the conjectures and the students' actual answer in table 5.11.

Table 5.11. The students' actual answer on lesson 4 cycle 2

Activity	**Conjectures of students' answer**	**Students' actual answer**
Understanding that numbers on the model must be well ordered and expressing multiplicative comparison on the model	Students do not understand that numbers on the model must be well ordered	
	Students are not able to express multiplicative comparison to determine equivalent ratios (students use subtraction and addition to determine equivalent ratios)	

	Students are able to express multiplicative comparison on the model to determine equivalent ratios	Students were able to determine equivalent ratios by relating numbers using fractional form, multiplication, and division

6. Lesson 5

There are three problems in the fifth lesson. The first two problems are related. The contexts of problems are shopping and food consumption. The general aim of the problems are to enable students to formulate the given information into a mathematical model and to enable students to understand that equivalent ratios can be determined by using multiplicative comparison.

Problem 1a

The problem is about buying milk pie, a famous snack in Bali. Given, Adam bought 36 pies and he has to pay IDR 135.000. Adam's friends also buy pies with different amounts. Students were asked to determine how much many each person should pay. Before students started to work, the teacher read the problem and drew the model on the board as in figure 5.32.

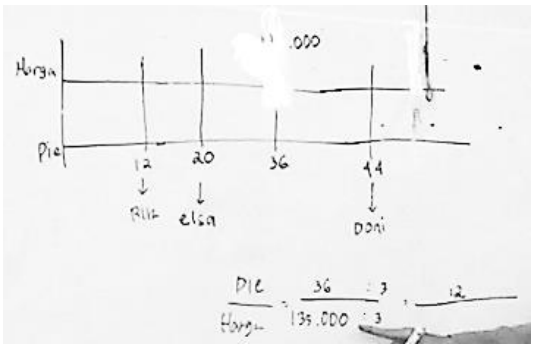


Figure 5.33. The teacher used fractional form and simplified it

According to the previous lesson, the class agreed to put the given information on the lower line while the determined value on the upper line. Once the teacher had ordered the given numbers, the teacher asked students whether they were able to determine relationship between numbers. It turned out that students struggled in finding relationship between the given numbers. Instead of asking students to relate 36 to other given numbers, the teacher led students to transform the given ratio into a fractional form and simplified it. Students also struggled in multiplying and dividing in which large numbers are involved. To continue, the teacher instructed students to do the problems.

The students in the focus group directly listed the given information and formulated the given information into a double number line. In addition, they were able to order the given numbers on the model. Some of the students transformed the given ratio into a fractional form and simplified it. The researcher talked to Rizqi who had not transform the ratio into a fractional form, to determine the relationship between numbers without simplifying the ratio. Transcript 5.13 illustrates the discussion and figure 5.34 shows the student's work.

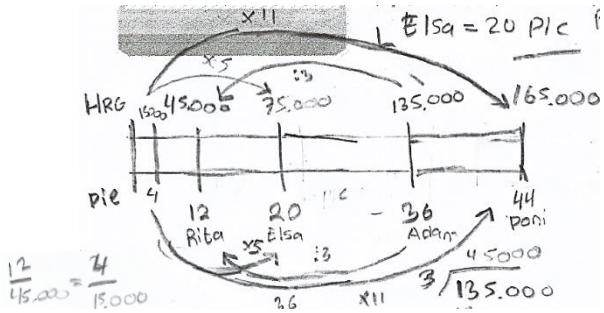


Figure 5.34. Rizqi determined the relationship between numbers

Transcript 5.14

1	Researcher	: Can we not simplify it? Are there any related numbers?
2	Rizqi	: 12 and 36.
3	Researcher	: What?
4	Rizqi	: 12 and 36.
5	Researcher	: 12 and 36. What is the relationship?
6	Rizqi	: 12 times 3 equals 36.
7	Researcher	: What about 36 to 12?
8	Rizqi	: Times 3.
9	Researcher	: Well, yea 12 to 36 you need to multiply it by 3. What if
10		36 to 12?
11	Rizqi	: Divided by 3.
12	Researcher	: Okay, now try it.

The transcript reveals that the student was able to determine the relationship between given numbers without simplifying the given ratio first. The student struggled when he wanted to relate 12 to 20 and to 44. The researcher suggested the student to simplify the given ratio. From the student's work, we know that instead of simplifying the fraction into the simplest form, the student used small factor as a divisor that is 3. From the new equivalent ratio, 4 pies:IDR 15.000, the student related 4 to 20 and 4 to 44. In our interpretation, Rizqi has a better ability in determining the relationship between numbers because he did not simplify the given ratio at first.

Similar to Rizqi, most of students did not simplify the fraction into the

simplest form. Their smallest fraction is $\frac{4}{15.000}$. However, there was a difference between how they determined the relationship between numbers. Even though they used multiplicative comparison, Rizqi's work shows that he has a better sense in determining the relationship between numbers compared to other students. Figure 5.35 represents the general students' work.

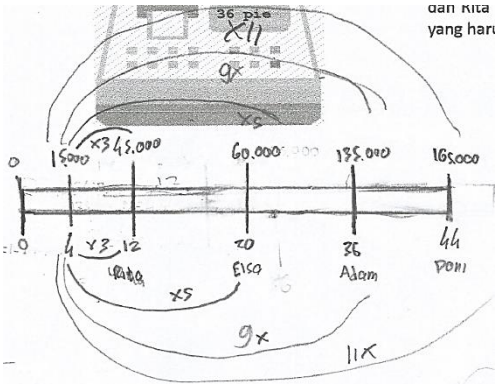


Figure 5.35. Students directly simplified the given ratio and started from 4

From the figure of students' work we may interpret that this answer is less sophisticated than Rizqi's. The reason is that students tend to do simplification at first rather than observing the numbers and trying to determine the relationship. In general, students were able to use the model and to express the multiplicative comparison on the model. However, it seems they did not pay attention at the numbers first. They directly simplified the given ratio.

When students were struggling to solve the problem, the teacher did not walk around the class. Students approached the teacher and the teacher told them to discuss the problem with their friends. However, among students,

they did not really discuss about the problem. Some of students had discussion but some others tend to look at their friends' work. Furthermore, there was no class discussion about the problem until the class ended.

Problem 1b

Using the determined value on the previous problem, students started to work on the problem. Most of students were able to formulate the given information into a model. In addition, they ordered the given numbers as in the previous problem order. However, some students struggled to understand the problem and to formulate the information into a model. One of the focus group students tried to help her friends in working on the problem. Transcript 5.15 illustrates the discussion between Sisi and her friends.

Transcript 5.15

1	Sisi	: What (information) do you know from the problem?
2	Aura	: 9 portions.
3	Sisi	: Write it.
4	Aura	: (*writing the given ratio*)
5	Sisi	: How much money do you need to buy 9 portions of *nasi*
6		*bali*?
7	Aura	: IDR 135.000.
8	Sisi	: Then write it.
9	Talitha	: Do we have to write it (the determined value in previous
10		problem) here? Do we need to make lines? (*drawing lines*)
11	Sisi	: Now, make like this (a double number line). You need to
12		make for the price.
13	Talitha	: (*ordering the given numbers on the line*)

The transcript reveals that students were unsure to formulate the given information into a model since the problem did not ask them to make a representation. Sisi helped her friends by asking questions about

information they had from the problem. Sisi also led her friends to do the problem using the model. She told her friends to make an illustration as in the previous problem even though the problem did not ask students to make a representation. We provide Sisi's work as in figure 5.36.

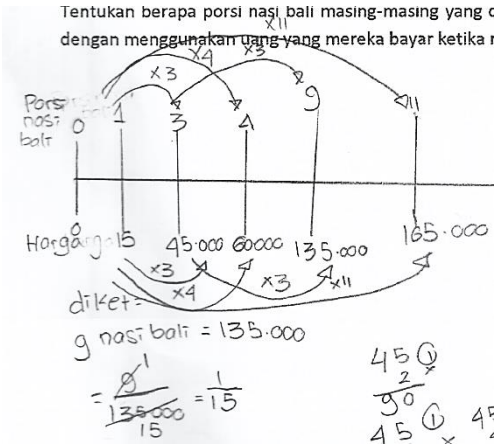


Figure 5.36. Sisi used multiplicative comparison to solve the problem

As expected, the figure shows that Sisi used multiplicative comparison to solve the problem. Sisi simplified the given ratio to the simplest form of fraction and related numbers mostly using multiplication. Even though Sisi was able to solve the problem, she struggled in working with large numbers. It did not happen to Sisi solely. Most students struggled to determine the number of *nasi bali* portion according to the large amount of money.

Even though it had been the fifth lesson for students, some students still used a model which pays attention to the spaces between numbers as in figure 5.37.

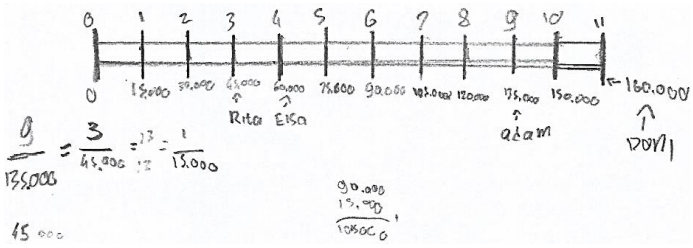


Figure 5.37. An example of the student's work

The student rejected to be recorded while working on the problem. Therefore, we could not find out the student's thinking. Observing the model, it is possible that the student used repeated addition to solve the problem. In addition, below the model, the student wrote an addition of 90.000 and 15.000. From the model, we know that every time we add 15.000 on the lower line, we also add 1 on the upper line. From the figure, the student determined not only the asked value but also the number of portion for every IDR 15.000. However, looking at the student's doodle, he simplified the given ratio. It is possible that the student tried to determine 3 portions and 1 portions using multiplicative comparison. However, in determining the asked value, the student did not show the notion of multiplicative comparison on his model.

In general, students were able to formulate the information into a double number line and to determine the number of *nasi bali* portion. Nevertheless, since the problem involves large numbers, students struggled in counting. Students were also able to express the multiplicative comparison. However, some students had not shift their method from repeated addition to multiplicative comparison. The teacher could have brought the topic in the

class discussion but there was no class discussion for the problem.

Problem 2

The problem contains information about cow grass consumption per day. Every day, 8 cows consume 20 sacks of grass. Students were asked to determine the number of sack for three cow farms. As the previous problems, students in the focus group were able to formulate the given information into a double number line. When the researcher observed the students, Diva had different model compared to other students in the focus group. The students drew a model as in figure 5.38.

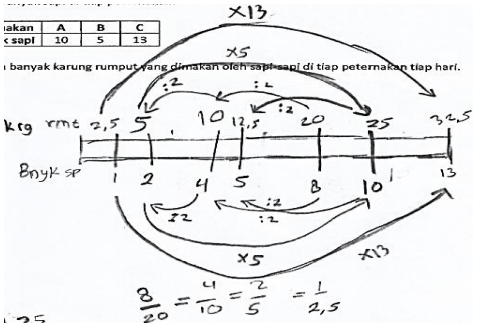
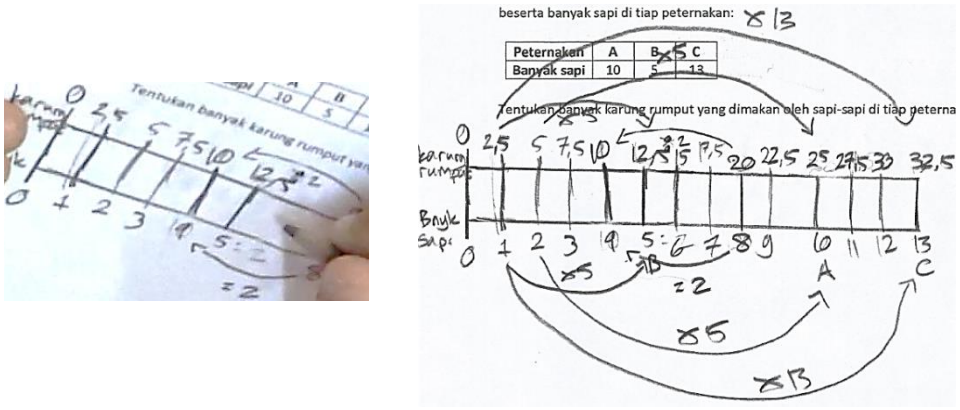


Figure 5.38. The students expressed multiplicative comparison using the model

As seen in the figure, the students determined the relationship between given numbers using multiplicative comparison. Following the arrowed lines, we are able to trace how the students started to solve the problem. The model and the student's thinking is more sophisticated compared to Diva's model as in figure 5.39. Transcript 5.16 illustrates the discussion to reveal Diva's thinking.



(a) (b)

Figure 5.39. Diva drew the model as in using a double scale line

Transcript 5.16

1	Researcher	: Diva, I want to ask. For 1, 2, 3, 4, how do you determine
2		the value?
3	Diva	: (*thinking*)
4	Researcher	: 1 you got 2,5. 2 you got 5. 3 you got 7,5. 4 you got 10.
5		How do you get that?
6	Diva	: Hmm. (*thinking*) I just do it.
7	Rizqi	: You just do it?
8	Researcher	: How is it Diva? I still do not get.
9	Diva	: I also don't know. (*drawing more lines for every 1 cow*)

From figure 5.39(a), Diva related 8 cows to 4 cows using multiplicative comparison. Based on the transcript, when the researcher asked him how he determined the value for every 1 cow, he could not explain. It is possible that Diva determined the number of sack for 1 cow using division. Later, to determine another sacks for 1 more cow, Diva added 2,5. Observing the video registration, it is possible that Diva determined the grass consumption for every number of cows. Once he had determined, he attempted to relate the numbers using multiplicative comparison. In our interpretation, even though Diva was able to determine the multiplicative comparison between

numbers, he started using repeated addition.

Similar to Diva, another student also struggled in relating the numbers using multiplicative comparison rather than addition. We obtain data in the form of students' written work as in figure 5.40.

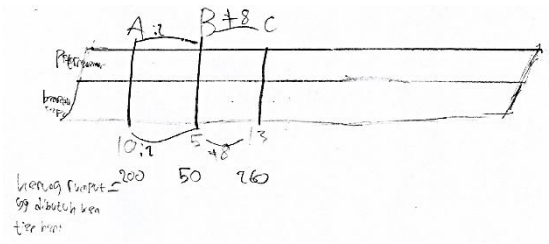


Figure 5.40. An example of students' work

Observing the figure, we know that the student only formulate the asked value into a model. The student did not write the given ratio on the model. Moreover, he was not able to order the numbers on the lower line well. Furthermore, since the student did not write the given ratio, he struggled to determine the number of sack. To determine the number of sack, the student paid attention on numbers on the lower line. The student struggled in determining the relationship between numbers on the lower line. There was an inconsistency, where the student used division and addition to determine the number of sack. The student was able to determine the multiplicative comparison between 10 and 5. Yet, he could not determine the number of sack for farm B since he did not know the grass consumption for farm A.

To determine the number of sack for farm C, the student used addition. It is possible that the student could not get an integer if he used multiplicative comparison in determining the relationship between B and C.

As mentioned before, students struggled in working with non-integer. Thus, the student chose to determine the difference between numbers, which produces integer. It was not clear how the student could get 200 sacks for farm A, 50 sacks for farm B, and 260 sacks for farm C.

Some students still struggled to shift their thinking, from using addition to using multiplicative comparison in determining the relationship between numbers. The teacher could have used students' work to discuss the key concepts of ratio. In addition, the teacher could have discussed the benefit of using multiplicative comparison and the drawback of using repeated addition or difference in determining the relationship between numbers. Similar to the previous problem, there was no class discussion for this problem as well. The teacher closed the lesson by asking students to hand in their worksheets.

Based on the analysis, we can conclude that students were able to formulate the given information into a double number line. However, some students still struggled in ordering the given numbers on the lower line. Not all students understood that they had to use multiplicative comparison to determine equivalent ratios.

We summarize the conjectures and the students' actual answer in table 5.12.

Table 5.12. The students' actual answer on lesson 5 cycle 2

Activity	Conjectures of students' answer	Students' actual answer
Formulating the given information into a mathematical model and understanding that equivalent ratios can be determined by multiplicative comparison	Students are not able to formulate the given information into a mathematical model	
	Students are not able to understand that equivalent ratios can be determined by using multiplicative comparison (students determine equivalent ratios by determining difference between numbers)	Some students made a model which seems like they were using difference and addition to determine equivalent ratios
	Students are able to understand that equivalent ratios can be determined by using multiplicative comparison	Some students determined equivalent ratios by relating numbers using multiplicative comparison

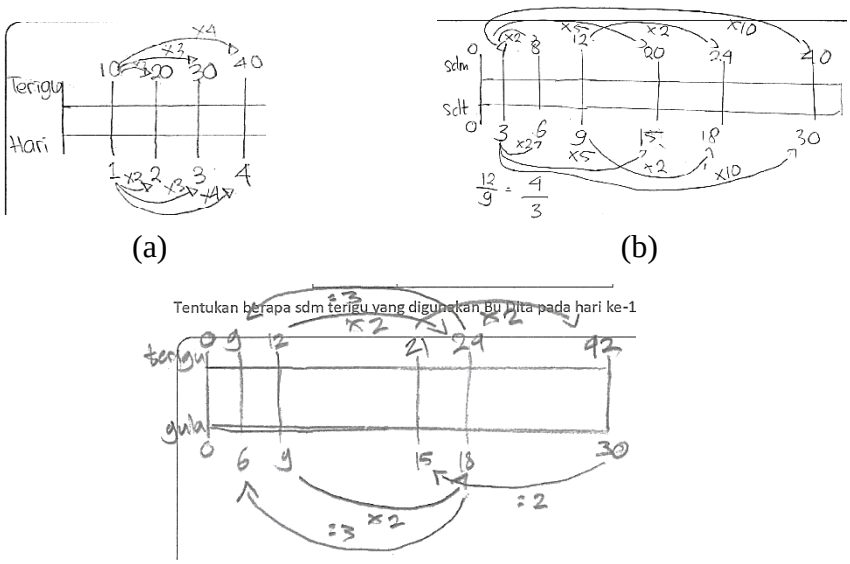
7. Posttest

We carried out the posttest after carrying out five lessons. Problems in the posttest are the same as in the pretest. By carrying out the posttest, we want to find out and to clarify students' development in the learning.

In the first problem, some students mentioned words which have the notion of difference such as “difference”, “shorter than”, and “longer than”. Other students also mentioned words which have the notion of comparison for instance “comparing two things”, “double number line”, “double scale line”, “multiplication and division”, and “fraction”. In the second problem, four students were able to choose which statement contains the notion of comparison and to reason correctly. These students reasoned that comparison is related to multiplication and division. One student was able to write the ratio of the two ribbons.

For the third problem, there are two kinds of students’ answers in measuring the figure. The first is some students used a ruler to determine the length of the figure. The students did not mark the figure and wrote cm on the paper. The second is some students used a scale line and marked the figure to determine how many times the scale line fits the figure. In problem four, 16 students were able to measure the route using the scale line. However, only nine students were able to determine real distances between ports by using the scale information.

There are two types of answer for problem five. Students used illustration namely double number line and no illustration to solve the problem. From 13 students who drew double number line, only four were able to use the model correctly (see figure 5.41(c)).

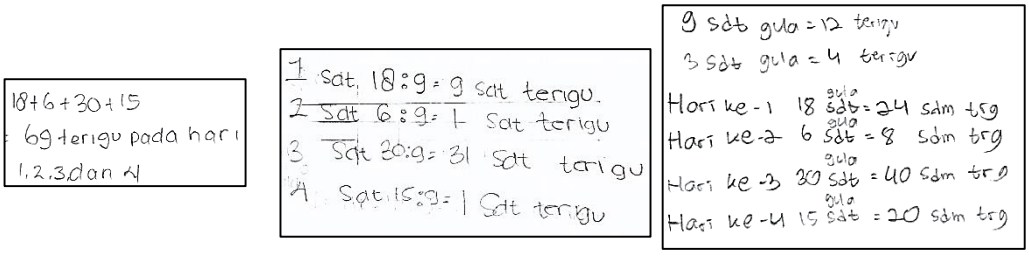


(a) (b)

(c)

Figure 5.41. Students used the double number line

Using the double number line, three students needed the simplest form of the equivalent ratio to relate the numbers while one student directly related the given numbers (see figure 5.41(b)). Students who did not use an illustration usually multiplied or divided the number of teaspoon of sugar by a scale factor or added the given number of teaspoon of sugar (see figure 5.42(a) and figure 5.42(b)). Only one student who used formal method was able to solve the problem correctly (see figure 5.42(c)). In general, the reason students were not able to solve the problem correctly because they were not able to realize the given ratio to determine other equivalent ratios.



(a) (b) (c)

Figure 5.42. Examples of students' answer with no illustration

8. Discussion

After analyzing the teaching experiment, we recognize the influence of context and numbers string on students' strategies to solve problems. For example, in the third lesson the teacher and students still used the model as using double scale line because the problem is about distance on the map and real distance. Other than that, students still used repeated addition because the numbers are multiples of landmark numbers (2, 5, 10).

Similar to the students in the pilot study, students in the teaching experiment were struggling in working with fractions and decimals. Moreover, students avoided working with non-integer numbers such as decimal. When students tried to determine relationship between numbers using multiplicative comparison and they could not obtain an integer as the relationship, they used addition and subtraction. It indicates that students were not flexible in working with numbers. Since the finding reveals that students in the fifth grade struggle in working with fraction in the topic of ratio, the next study may bridge the gap between using integer and fraction to support students in learning ratio using non-integer numbers.

The learning sequences in this study enable students up to general level. However, in the posttest, some students attempted to use formal method to solve the problem. One of students who used the formal method could solve the problem correctly. The student was able to underline the given ratio of the problem. Moreover, he simplified the given ratio and multiplied the

simplest ratio to solve the problem. In simplifying and multiplying, the student used division and multiplication, that is the multiplicative comparison in solving ratio problem.

The whole study was conducted in a short time. In addition, the researcher did not have much time to revise the learning instructions for the teaching experiment enormously. As a reflection, the researcher proposes major changes on the learning sequences for another study. We provide table 5.13 to illustrate the revision for the latter study.

Table 5.13. Reflection for the next study related to ratio

Activity	Concepts	Context	Model	Remark
Distinguishing difference and comparison	Ratio, multiplication, division	Comparing different lengths	-	This activity aims at enabling students to distinguish the notion of difference and comparison
Working with a scale line	Ratio	Using a reference to measure objects	Students' informal model	This activity aims at enabling students to compare two quantities and to understand multiplicative comparison within ratio
Working with double scale line	Scale, proportion, fraction	Maps (distance on map and real distance)	Double scale line	In this activity, provides two problems. For the first problem, use two ratios which consist of numbers string to enable students to understand multiplicative comparison within ratio and between ratios. In the second problem, use numbers string which allow students to determine missing value of equivalent ratios using strategies: (1) determining difference and using addition and (2) using multiplicative comparison
Working with double number line	Ratio and proportion	Travelling (time and distance, fuel and price)	Double number line	In this activity, provides two problems. For the first problem, use numbers string which allow students to use two strategies to determine equivalent ratios. In the second problem, use numbers string which allow students to use only multiplicative comparison to determine equivalent ratios
Formulating the given information into formal method	Ratio and proportion	Travelling (distance and fuel)	Formal method	In this activity, provides a problem which does not instruct students to draw a model. Support students to connect the concept of multiplicative comparison using the formal method

E. Validity and Reliability

In this section, we will discuss the validity and the reliability of the study. For the validity of the study, we did data triangulation. Using the collected data, the researcher triangulated the data in the analysis. The collected data were obtained by using various methods for instance classroom observation, students' written work, and students' interview. The researcher compared obtained data from different methods to find out whether the data are supporting each other in the analysis. The researcher also discussed how the data are supporting each other or not to produce a theory.

To improve the external validity of the research, we provide the hypothetical learning trajectory of the study. It allows the readers to follow the flow of the study and other researchers to adjust the design according to their own condition. In the real teaching experiment, we involved the regular mathematics teacher and the learning process went as they usually have. It contributes to the ecological validity of the study.

We provide the readers the thick description of the study which consists of the students' actual learning. We include the analysis of fragments from video registrations and students' written work to enable the readers to follow how we derive our interpretation and conclusion.

CHAPTER 6

CONCLUSION AND DISCUSSION

A. Conclusion

The aim of the study is to contribute to the development of local instruction theory on ratio. In addition, we involve double number line as a model to support students in learning ratio. To answer the research question: *"How can the double number line support students in learning ratio and scale?",* we use our findings in the analysis to derive conclusion.

Activities in the first meeting enable students to distinguish between comparison and difference. Students were able to make their own model to describe the length of an object. Students were able to understand multiplicative comparison within ratio. Working with double scale line enables students to draw and to use the double scale line to determine real distance. Students used repeated addition and determined real distance for every 0,5 cm.

Mathematical activities in the third meeting cannot fully promote the sense of ordering given numbers. Students ordered the number and marked for every 0,5 cm as in double scale line. Students used addition to solve problems. In the fourth lesson, the mathematical activities enable students to understand the order of numbers without writing numbers one by one. Moreover, students were able to express multiplicative comparison within and between ratios using the double number line.

Small numbers of students were not able to formulate information into an appropriate double number line. Even though in the previous lesson students were able to express multiplicative comparison to show the relationship between numbers in ratios, some students used repeated addition and an incorrect strategy. It happen because students were not flexible in understanding relationship between numbers, especially using multiplicative comparison. Furthermore, students were not fluent in working with non-integer number such as decimals. They tried to obtain a whole number as the product of relationship between numbers.

Some students were able to develop their understanding about multiplicative comparison using a double number line, some others were not. In the third lesson, the teacher should have led students to ignore the precise measurement between numbers. However, the teacher thought the model as a double scale line. In addition, instead of focusing on the relationship between numbers, the teacher told students to divide lines on the model into several same parts. Most of the time, there were no supporting class discussion. In the discussion, the teacher should have been able to focus on the relationship between numbers and to emerge the notion of multiplicative comparison in ratio.

According to the analysis and referring to the aim of the research, we provide table 6.1 about local instruction theory on ratio and scale.

Table 6.1. Local instruction theory on ratio and scale

Tools	Imagery	Activity	Concepts
Pictures or objects with a certain measurement	Signifies different measurements of two objects	Choosing and reasoning why a statement holds the notion of comparison	Distinguish comparison to difference
References on a plan	Signifies an informal ruler	Measuring objects on a plan using a reference on a plan	Recognize multiplicative comparison within ratio
References, scale line, paper strip	Signifies the relationship between measurement on the plan and real measurement	Drawing a plan and describing the measurement on the plan	Using a scale line to determine the real measurement
Double scale line and maps	Signifies an illustration of relationship between distance on the map and real distance	Determining a map fits the provided double scale line	Recognize scale as the relationship between distance on the map and real distance
Maps and scales	Signifies the connection between distance on the map and real distance	Making double scale line	Understand the multiplicative comparison within ratio and multiplicative comparison between ratios
Maps and a stretched route	Signifies the nature of number line and the relationship between two quantities	Ordering numbers on the model and determining relationship between numbers	Understand the multiplicative comparison between ratios and use fractions to express equivalent ratios
Double number	Signifies the relationship between	Determining missing values using	Apply multiplicative

line	two quantities	multiplicative comparison	comparison between ratios to determine missing values

B. Discussion

1. The weakness of the study

There are constraints and obstacles during the study. Below, we describe the weaknesses of the study.

a. Time limitation

The school gave the researcher two weeks for carrying out the study. The study was conducted during final theme project and mid-semester exam. Since we were given only two weeks, we did not have much time to reflect on the pilot study. When students on the pilot study were having posttest, students on the teaching experiment were having pretest. Therefore, we could not make major revision of the design for the teaching experiment.

The study was implemented every day for two weeks. The researcher and the teacher reflected on the lesson after the class ended. We discussed the next lesson one day before the implementation. Overall, the schedule did not provide much time for the researcher to discuss the learning instruction and learning materials with the teacher. Therefore, the teacher looked at and carried the teacher guide most of the time during the learning.

b. Role of the teacher

The researcher emphasized the role of the teacher during discussion with the teacher. It is important for the teacher to hold and to orchestrate class discussion. During the class discussion, the teacher can compare students' work and invite students about important concepts. Moreover, discussing students' strategy is also important to understand students' thinking. The teacher can ask questions to support students in learning ratio. However, the teacher did not hold a class discussion during three lessons. The class discussion held only to confirm students' work or to give instruction.

Other than discussion, it is important for the teacher to walk around the class to observe students' struggles and to bring it up on the class discussion. Rather than supporting students with questions, the teacher told students what went wrong with their work and what they had to do. It is possible that the teacher tried to finish all activities even though students had not finish or understand considering the time limitation.

c. Socio norms and socio-mathematical norms

From the interview, the teacher said that the class is used to discussion, especially to share opinions and to comment about one's work. However, it did not happen during the teaching experiment. In addition, students usually look at their friends' work rather than have a discussion. During the learning, often students approached the teacher to

ask for confirmation of their work.

As mentioned by the teacher in the interview, students did not like to read the problems first. They usually ask the teacher what the problem is about and complain that it is difficult. When the researcher asked whether students had read the problems, they answered ‘no’.

2. Suggestion

According to constraints and obstacles in the study, we propose several points of suggestion. For researchers who want to carry out design research study, it is important to have enough time to reflect and to improve the design. We also can use the time to talk to the teacher about our design and RME.

During the recent study, the teacher did not support students using questions as in the teacher guide. Yet, the teacher plays important role in the study. Researchers may approach and convince the teacher that supporting questions from the teacher influences students’ thinking. Confirmation questions will not trigger students to think.

Ratio can appear in different contexts. It is possible to use double number line and another contexts and concepts such as fraction for future research. Regarding to our finding that students in the fifth grade were not flexible in working with non-integer numbers, further research can be done to explore students’ flexibility in working with non-integer number.

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Learning Trajectory on Ratio and Scale

Situational

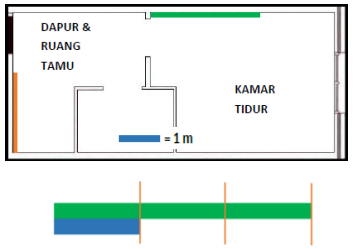
Referential

General

Formal

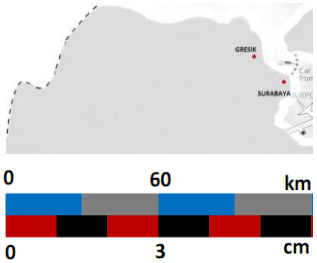
LESSON 1

Working with a scale line



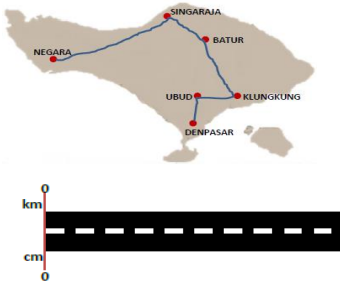
LESSON 2

Working with a double scale line



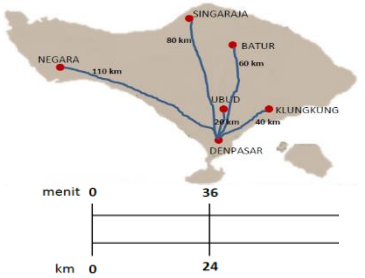
LESSON 3

Using the stretched route to show the relationship between numbers



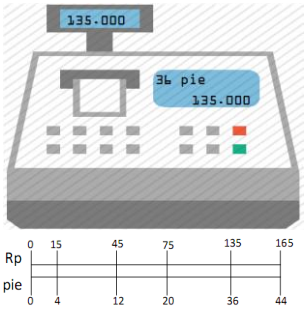
LESSON 4

Expressing multiplicative comparison to determine equivalent ratios



LESSON 5

Formulating the given ratio into a model and understanding



APPENDIX A

The Classroom Observation Scheme

Classroom environment

- Does the teacher wait students to be quiet before opening the lesson?
- How the seat arrangement in the classroom?
- Where the teacher positions him/herself in the teaching and learning process?
- How the teacher handles unwanted behaviour from students?
- How does the teaching and learning condition? (too quite or too noisy)

Teaching and learning process

- How does the teacher open the lesson?
- What kind of contexts the teacher uses in the lesson?
- Does the teacher encourage students to take part in the teaching and learning process? In what way?
- How is students' participation in the classroom?
- Does the teacher give students some time to think or discuss with their neighbours?
- Does the teacher instruct students to make groups to discuss?
- Does the teacher give students a chance to ask, clarify, or speak about their opinion in solving problems?
- How the teacher reacts to students answers or questions?
- How the teacher scaffolds students in discussion?
- Does the teacher lead students to use only one strategy?
- Does the teacher encourage students to reason with their answers/strategies?
- Does the teacher pinpoints and differs which answer is more efficient and sophisticated? Does she/he asks students why it is more efficient or sophisticated?
- How the teacher encourages students to conclude the lesson?
- How the teacher manages the time?
- How does the teacher end the lesson?

APPENDIX B

The Teacher's Interview Scheme

Background

- How long you have been teaching?
- From the first time you are teaching, are you only teaching grade five?
- Why do you choose this grade?
- What is your favourite topic in teaching mathematics?

Teaching and learning process

- How do you usually introduce ratio and scale?
- What preliminary knowledge students must have to learn ratio and scale?
- How many meetings do you need to finish the topic of ratio and scale?
- What kind of contexts you usually used to introduce the topic?
- What difficulties students usually have in learning ratio and scale?
- What difficulties you usually have in delivering the topic?
- What kind of books you usually use to support students learning ratio and scale?
- Are you always stick to the curriculum or sometimes expand the learning?
- How do you asses students' work?
- How to engage students in learning ratio and scale?
- Have you heard about mathematics models?
- Do you think it is important to use a model to bridge students' informal and formal knowledge?

Classroom norms

- How the teaching and learning usually go?
- How students participate in your class?
- Do you usually have discussion in the classroom?
- Do you give students a chance to speak or to ask?
- Do you compare students' solutions?
- Do you let students to comment other students' work and reflect on their work?

RME or PMRI

- Have you heard about RME or PMRI?
- Do you have any experience with PMRI?
- What is your opinion about implementing PMRI in a classroom?

APPENDIX C

PRETEST AND EXAMPLES OF EXPECTED ANSWER

- Waktu mengerjakan 30 menit
- Kerjakan sendiri
- Gunakan bagian yang masih kosong sebagai tempat untuk menghitung

1. Apa yang kamu ketahui tentang rasio? Gunakan gambar, kalimat, bagan, dll untuk menjelaskan.

   Dengan uang Rp 1.000,00, saya bisa membeli 3 permen

2. 

   Manakah di antara dua pernyataan berikut yang merupakan rasio? Jelaskan alasanmu.
   a. Panjang tali biru 3 meter lebih panjang dari tali merah.
   b. Panjang tali biru 4 kali panjang tali merah.

   b. Panjang tali biru 4 kali panjang tali merah

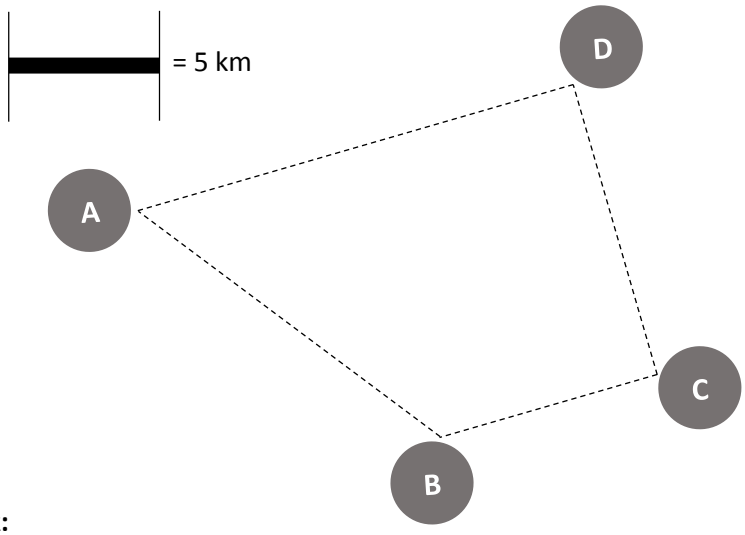
   Pernyataan kedua membandingkan dua ukuran dan menyatakannya dengan "kali". Sedangkan pernyataan pertama merupakan selisih.

3. Bantu tukang bangunan untuk mengukur panjang dan lebar suatu ruangan (persegi panjang biru) dengan menggunakan tali hitam pada gambar.

(menggunakan tali pada gambar untuk mengukur dan memberi tanda pada ruangan pada gambar)

Panjang ruangan pada gambar 4 kali panjang tali.
Lebar ruangan pada gambar 3 kali panjang tali.

4. Sebuah kapal berangkat dari Pelabuhan A untuk mengirimkan barang ke Pelabuhan B, C, D, lalu kembali ke Pelabuhan A. Garis putus-putus hitam adalah rute kapal tersebut. Panjang garis skala di bawah mewakili 5 km jarak sebenarnya.



Petunjuk:

a. ukur panjang rute satu per satu.
b. bandingkan panjang rute dengan garis skala yang tersedia.

Soal: Tentukan seberapa jauh kapal tersebut berlayar dari satu pelabuhan ke pelabuhan yang lain.

(menggunakan garis skala untuk mengukur dan memberi tanda pada setiap rute)

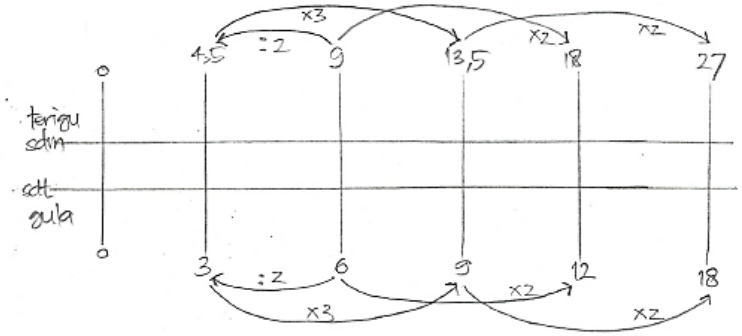
A ke B = 2,5 kali garis skala = 2,5 x 5 km = 12,5 km
B ke C = 1,5 kali garis skala = 1,5 x 5 km = 7,5 km
C ke D = 2 kali garis skala = 2 x 5 km = 10 km
D ke A = 3 kali garis skala 3 x 5 km = 15 km

5. Untuk membuat biskuit yang lezat, takaran yang pas adalah 6 sdt (sendok teh) gula untuk setiap 9 sdm (sendok makan) terigu. Dalam empat hari berturut-turut, Bu Dita membuat biskuit dengan takaran gula yang berbeda.

Hari ke-	Takaran gula (sdt)
1	12
2	3
3	18
4	9

Tentukan berapa sdm terigu yang digunakan Bu Dita pada hari ke-1, 2, 3, dan 4.

6 sdt gula untuk 9 sdm terigu



POSTTEST AND EXAMPLES OF EXPECTED ANSWER

- Waktu mengerjakan 30 menit
- Kerjakan sendiri
- Tulis jawaban pada kotak yang telah disediakan
- Gunakan bagian yang masih kosong sebagai tempat untuk menghitung

1. Apa yang kamu ketahui tentang perbandingan? Gunakan gambar, kalimat, simbol, dll untuk menjelaskan.

   Dengan uang Rp 1.000,00, saya bisa membeli 3 permen

2. 

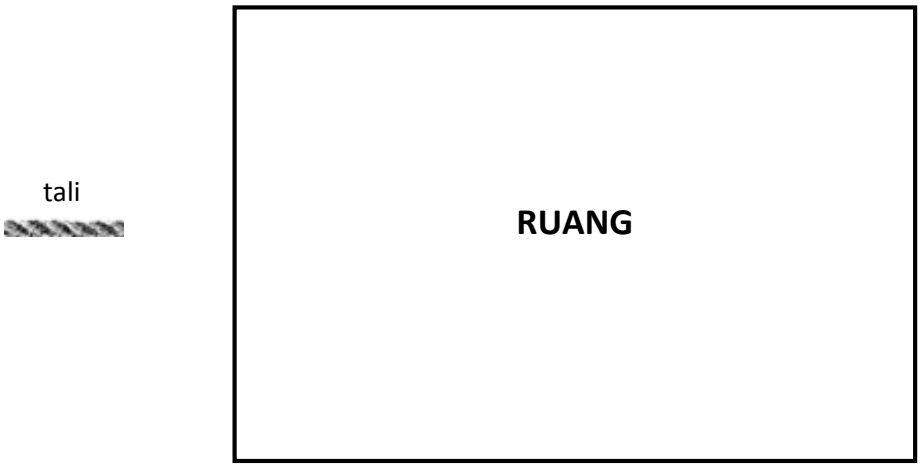
   Manakah di antara dua pernyataan berikut yang merupakan perbandingan? Jelaskan alasanmu.

   Pernyataan A: Panjang pita abu-abu 3 meter lebih panjang dari pita hitam.
   Pernyataan B: Panjang pita abu-abu 2 kali panjang pita hitam.

   Pernyataan B: Panjang tali biru 4 kali panjang tali merah

   Pernyataan kedua membandingkan dua ukuran dan menyatakannya dengan "kali". Sedangkan pernyataan pertama merupakan selisih.

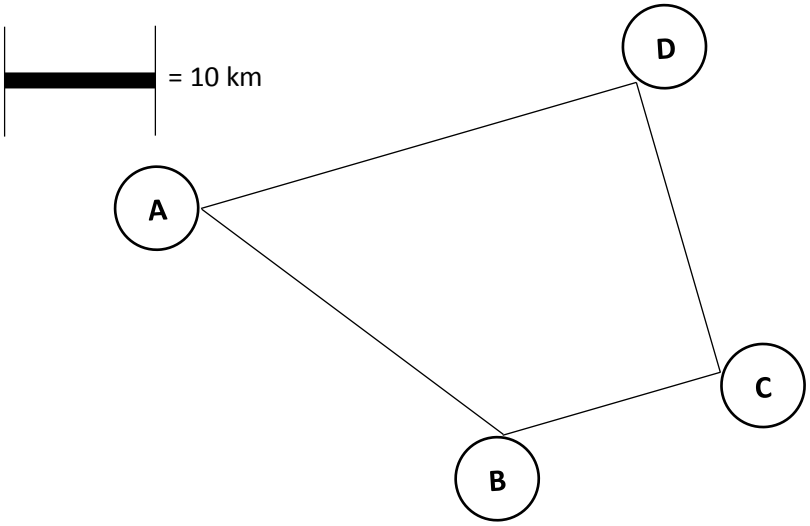
3. Bantu tukang bangunan untuk mengukur panjang dan lebar suatu ruangan (persegi panjang hitam) dengan menggunakan tali pada gambar.



(menggunakan tali pada gambar untuk mengukur dan memberi tanda pada ruangan pada gambar)

Panjang ruangan pada gambar 6 kali panjang tali.
Lebar ruangan pada gambar 4 kali panjang tali.

4. Sebuah kapal berangkat dari Pelabuhan A untuk mengirimkan barang ke Pelabuhan B, Pelabuhan C, Pelabuhan D, dan kembali ke Pelabuhan A. Garis hitam adalah rute kapal tersebut. Panjang garis skala di bawah mewakili 10 km jarak sebenarnya.



Soal: Tentukan jarak sebenarnya ketika kapal berlayar dari satu pelabuhan ke pelabuhan yang lain.

(menggunakan garis skala untuk mengukur dan memberi tanda pada setiap rute)

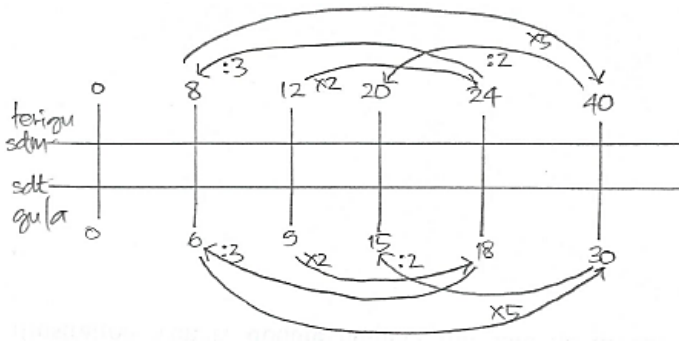
A ke B = 2,5 kali garis skala = 2,5 x 10 km = 25 km
B ke C = 1,5 kali garis skala = 1,5 x 10 km = 15 km
C ke D = 2 kali garis skala = 2 x 10 km = 20 km
D ke A = 3 kali garis skala 3 x 10 km = 30 km

5. Untuk membuat biskuit yang lezat, takaran yang pas adalah 9 sdt (sendok teh) gula untuk setiap 12 sdm (sendok makan) terigu. Dalam empat hari berturut-turut, Bu Dita membuat biskuit dengan takaran gula yang berbeda.

Hari ke-	Takaran gula (sdt)
1	18
2	6
3	30
4	15

Tentukan berapa sdm terigu yang digunakan Bu Dita pada hari ke-1, 2, 3, dan 4.

9 sdt gula untuk 12 sdm terigu



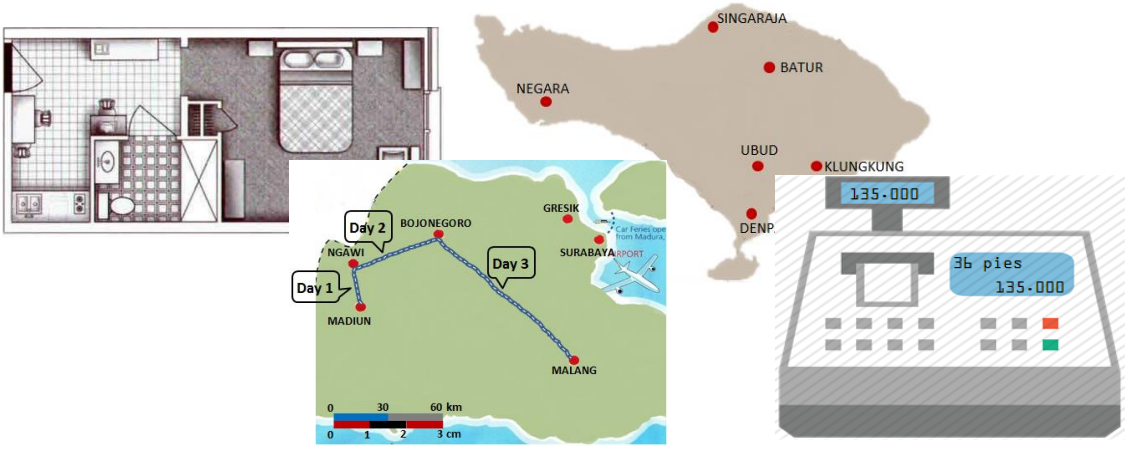
DESCRIPTION OF PRETEST AND POSTTEST

Item	Problem	Purpose
1	Expressing ratio into illustrations, sentences, symbols	To find out students' prior knowledge about ratio
2	Choosing one of statements and reasoning why it is comparison	To find out whether students are able to distinguish between comparison and difference
3	Measuring and determining the length and the width of the room using the provided rope	• To find out whether students are able to understand multiplicative comparison within ratio using a scale line • To find out whether students use representations to compare the scale line to the dimension
4	Determining how far the ship sails using a scale line which contains a certain measurement	• To find out whether students are able to understand multiplicative comparison within ratio using a scale line • To find out whether students are able to use scale factor to determine real distance • To find out whether students use representations to compare the scale line to the dimension
5	Determining the number of spoons of flour for different days	• To find out whether students make and use representations to solve the problem • To find out whether students are able to understand multiplicative comparison within ratio • To find out whether students are able to understand multiplicative comparison between ratios

KELAS 5

APPENDIX D

Perbandingan dan Skala



Panduan Guru

Bahan Ajar dan Catatan untuk Guru

DAFTAR ISI

Fokus Unit

Siswa menginvestigasi konsep yang berhubungan dengan perbandingan dan skala pada konteks peta dan liburan. Fokus unit ini adalah untuk:

- Menggunakan garis skala untuk memahami hubungan perkalian antar bilangan dalam suatu perbandingan
- Menggunakan garis bilangan ganda untuk memahami hubungan perkalian antar bilangan dalam suatu perbandingan dan antar perbandingan
- Mengenal perbandingan sebagai unit komposit
- Menggunakan model untuk menyelesaikan soal

Konsep Dasar yang Dimiliki

Pada unit ini, siswa diasumsikan telah memahami konsep sebagai berikut:

- Estimasi
- Perbandingan
- Bentuk pecahan dan desimal
- Perkalian dan pembagian

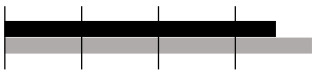
Tentang Model

Garis bilangan ganda adalah model yang digunakan untuk membantu siswa menyelesaikan permasalahan mengenai perbandingan. Bilangan pada garis bilangan ganda terurut, sehingga memudahkan siswa untuk mengetahui hubungan antar bilangan. Dengan menggunakan garis bilangan ganda, siswa diharapkan dapat menyadari hubungan perkalian antar bilangan dalam suatu perbandingan maupun antar perbandingan. Model garis bilangan ganda berupa ilustrasi yang tidak hanya dapat membantu siswa dalam memahami perbandingan, tetapi juga guru. Dengan menggunakan garis bilangan ganda, guru dapat membantu siswa dalam memahami dan menunjukkan hubungan perkalian antar bilangan pada suatu perbandingan maupun antar perbandingan.

Perkembangan Model

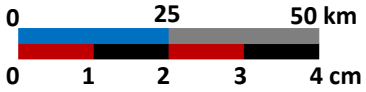
Pertemuan 1

Siswa membuat garis skala mereka sendiri untuk memahami hubungan perkalian antara panjang referensi dengan panjang benda



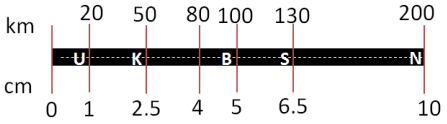
Pertemuan 2

Siswa bekerja dengan garis skala ganda yang memuat ukuran pasti jarak dalam peta (cm). Dalam pertemuan ini, siswa menggunakan model untuk memahami hubungan bilangan dalam suatu perbandingan dan antar perbandingan



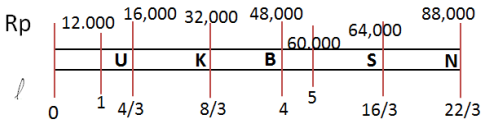
Pertemuan 3

Siswa dapat mengurutkan bilangan pada rute yang dibentangkan sebagai model. Pada pertemuan ini, ukuran dalam membuat model dapat diabaikan. Fokus permasalahan adalah mencari hubungan antar bilangan.



Pertemuan 4 dan 5

Siswa dapat memahami bahwa bilangan pada model harus terurut. Selain itu, siswa menggunakan garis bilangan ganda untuk memahami dan menunjukkan hubungan perkalian antar perbandingan. Siswa juga dapat mempelajari perbandingan senilai dengan menggunakan model.



Peran Guru

Diskusi antar siswa dan diskusi kelas menjadi perhatian pada pembelajaran ini. Untuk memancing terjadinya diskusi, peran guru sangat dibutuhkan. Guru diharapkan dapat memberikan pertanyaan yang memancing siswa untuk berpikir dan berdiskusi. Penting bagi guru untuk menarasikan soal pada LKS dan memberikan siswa waktu untuk berpikir dan berdiskusi dengan siswa yang lain.

Ketika siswa mengerjakan permasalahan pada LKS, guru dapat berkeliling untuk mengetahui kesulitan siswa. Selain itu, guru juga mempunyai kesempatan untuk memilih jawaban siswa yang menarik yang dapat dibahas ketika diskusi kelas. Dalam diskusi kelas, guru dapat mengundang siswa untuk membandingkan dan memberikan komentar pekerjaan siswa. Penting bagi guru dalam diskusi untuk mengajak siswa melihat hubungan antar bilangan dalam suatu rasio maupun antar rasio dengan menggunakan hubungan perkalian.

Perencanaan

Hari ke-	Alokasi Waktu	Konteks	Tujuan
1	2 x 35 menit	Denah	Siswa dapat menggunakan garis skala untuk memahami hubungan perkalian dalam suatu perbandingan
2		Peta	Siswa dapat menggunakan garis skala ganda untuk memahami hubungan dalam suatu perbandingan dan antar perbandingan
3		Peta dan perjalanan	Siswa dapat mengurutkan bilangan pada rute yang dibentangkan untuk memahami hubungan antar perbandingan
4		Liburan	Siswa dapat memahami bahwa bilangan pada garis bilangan ganda terurut dan dapat menyatakan hubungan perkalian antar perbandingan
5		Berbelanja	Siswa dapat merumuskan informasi pada permasalahan dalam model dan memahami bahwa perkalian dan pembagian digunakan untuk menentukan perbandingan senilai

Istilah Penting

garis skala (pertemuan 1) garis yang memuat skala tertentu

garis skala ganda (pertemuan 2) garis yang memuat dua skala berbeda namun kedua skala tersebut berhubungan. Garis skala ganda digunakan untuk membandingkan jarak pada peta dan jarak sebenarnya

garis bilangan ganda (pertemuan 4) garis bilangan yang memuat dua besaran dan menyatakan perbandingan tertentu

Konsep Penting

rasio perbandingan dua besaran

skala perbandingan antara jarak pada peta dengan jarak sebenarnya

perbandingan senilai beberapa perbandingan yang mempunyai nilai yang sama

unit komposit perbandingan A:B, untuk setiap A unit terdapat B unit dan untuk setiap B unit terdapat A unit

hubungan perkalian dua bilangan dapat dinyatakan sebagai perkalian satu sama lain

PERTEMUAN 1

Alokasi waktu: 2 x 35 menit

Tujuan pembelajaran: Siswa dapat menggunakan garis skala.

- Siswa dapat menggunakan panjang referensi untuk mendeskripsikan panjang suatu benda
- Siswa dapat menggunakan garis skala untuk mendeskripsikan panjang sebenarnya suatu benda
- Siswa dapat membuat dan menggunakan garis skala pada denah

Alat dan bahan:

- Secarik kertas
- LKS 1

Aktivitas 1

Soal 1 (15 menit)

1. Untuk membuka pertemuan, guru dapat menarasikan pertanyaan soal nomor 1 kepada siswa. Beri siswa waktu untuk berpikir dan berdiskusi dengan siswa lain. Minta siswa untuk memberikan alasan dalam memilih satu di antara dua pernyataan.

2. Bagi siswa yang bingung untuk memberikan alasan atau menyatakan bahwa pernyataan tersebut merupakan perbandingan, ajukan pertanyaan mengenai ukuran unit laptop dan meja. Lalu minta siswa untuk membandingkan ukuran kedua unit.

3. Dalam diskusi kelas, bantu siswa untuk melihat pernyataan tersebut sebagai selisih. Jelaskan bahwa selisih dan perbandingan berbeda. Selisih menyatakan perbedaan. Guru dapat memberikan contoh, misalnya:
   "*Panjang laptop 30 cm dan panjang meja 90 cm. Selisih panjang kedua benda tersebut adalah 60 cm. Jadi, panjang meja 60 cm lebih panjang dari panjang laptop.*"

Bantu siswa untuk memahami hubungan perkalian dalam suatu perbandingan dengan memberikan contoh.

"*Panjang laptop 30 cm dan panjang meja 90 cm. Perbandingan panjang kedua benda tersebut adalah 30 cm:90 cm atau 1:3. Jadi, panjang meja adalah 3 kali panjang laptop.*"

Contoh jawaban siswa:

> Pernyataan Dea bukan perbandingan.
>
> Alasannya: pernyataan tersebut menyatakan selisih. Dalam menyatakan perbandingan, seharusnya pernyatan Dea adalah "ukuran meja *x* kali panjang laptop".

Soal 2 (15 menit)

1. Guru membuka pembelajaran dengan menunjukkan gambar denah suatu ruangan yang bertujuan untuk memancing siswa berpikir tentang garis skala. Untuk membantu siswa, guru dapat bertanya beberapa pertanyaan pada siswa:
   - "Apa yang kamu ketahui tentang skala?"
   - "Dimana kamu menemukan skala selain pada denah?"
   - "Bagaimana mengilustrasikan skala menjadi gambar?"

   Guru dapat memberi tahu siswa bahwa skala dapat digambarkan dalam berbagai cara. Misalnya pada denah, skala dapat dituliskan dengan menggunakan bilangan atau digambarkan menjadi garis skala. Jika siswa tidak familiar dengan istilah garis skala, guru dapat memberi tahu bahwa garis skala adalah garis yang memuat ukuran tertentu.
2. Guru dapat bertanya bagaimana siswa dapat mengukur panjang dan lebar ruang tidur pada denah. Jika siswa mengajukan mengukur dengan menggunakan penggaris, guru dapat mengarahkan siswa untuk menggunakan perabot pada denah. Setelah siswa dapat menggunakan salah satu perabot pada denah untuk mengukur ruang tidur, guru dapat menarasikan soal 2.
3. Dalam diskusi kelas, guru dapat mengajak siswa untuk mendiskusikan bahwa perbandingan yang didapat berbeda karena menggunakan referensi yang

berbeda. Selain itu, guru juga dapat mengaitkan soal 1 dengan soal 2 jika siswa menyatakan panjang tembok dengan selisih.

Contoh jawaban siswa:

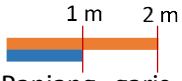
Saya menggunakan **panjang tempat tidur** untuk mengukur.
Panjang kamar mandi adalah **1 kali panjang tempat tidur**.
Lebar kamar mandi adalah **¾ panjang tempat tidur**.
Panjang kamar tidur adalah **1¼ panjang tempat tidur**.
Lebar kamar tidur adalah **1½ panjang tempat tidur**.

Soal 3 (15 menit)

1. Beri siswa waktu untuk membaca dan memahami soal. Guru dapat meminta siswa untuk berdiskusi tentang cara menggunakan garis skala untuk mengukur panjang kabel pada denah. Jika ada siswa yang menggunakan penggaris untuk mengukur garis skala dan kabel pada denah, minta siswa untuk menggunakan kertas dan menandai kertas tersebut dengan panjang yang sesuai dengan garis skala. Lalu, minta siswa untuk mengukur kabel pada denah dan menentukan panjang kabel sebenarnya.
2. Dalam diskusi kelas, guru dapat membahas bagaimana menggambar perbandingan garis skala dengan panjang kabel. Selain itu, guru dapat mengajukan pertanyaan sebagai berikut:
   - “Apa yang kamu ketahui tentang garis biru?”
   - “Garis skala menyatakan apa?”
   - “Mengapa kita memerlukan garis skala?”
   - “Bagaimana kamu dapat menggunakan garis skala?”

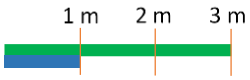
Contoh jawaban siswa:

1. Berikut adalah gambarku.



Panjang garis oranye adalah **2x garis skala**. Jadi, panjang kabel oranye sebenarnya adalah **2m**.

2. Berikut adalah gambarku.



Panjang garis hijau adalah **3x garis skala**. Jadi, panjang kabel hijau sebenarnya adalah **3m**.

Aktivitas 2 (25 menit)

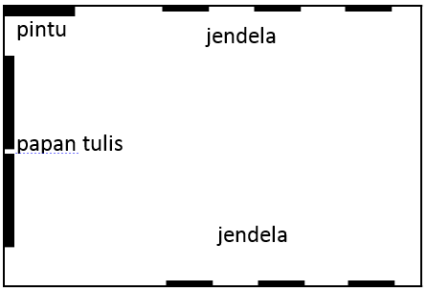
1. Guru dapat membimbing siswa untuk memilih benda yang berukuran besar sebagai referensi, misal papan tulis, penggaris papan tulis, jendela, pintu, dll dan minta siswa untuk memperkirakan panjangnya. Guru dapat mengarahkan siswa untuk menentukan referensi terdahulu, memperkirakan ukurannya, dan menggambar garis skala.
2. Dalam diskusi kelas, guru dapat membahas mengapa denah yang digambar siswa berbeda meskipun menggunakan referensi yang sama. Selain itu, apabila pekerjaan siswa yang menggambar denah terlebih dahulu dapat didiskusikan bersama. Apakah denah yang digambar siswa masuk akal jika dibandingkan dengan ukuran kelas sebenarnya.
3. Untuk menutup pertemuan, guru dapat memberikan pertanyaan terkait dengan pertanyaan pada kolom refleksi. Selain itu, guru dapat mengajak siswa untuk menyimpulkan perbedaan antara perbandingan dengan selisih. Guru dapat membantu siswa memahami hubungan perkalian dalam suatu perbandingan.

Contoh jawaban siswa:

Saya menggunakan **papan tulis** sebagai patokan. Saya perkirakan panjang sebenarnya adalah **2 meter**. Berikut garis skala saya:

= 2 m

Berikut denah ruang kelas saya.



Lebar kelas adalah 3 kali garis skala. Jadi, lebar kelasku adalah 6m.
Panjang kelas adalah 4,5 kali garis skala. Jadi, panjang kelasku adalah 9m.

PERTEMUAN 2

Alokasi waktu: 2 x 35 menit

Tujuan pembelajaran: Siswa dapat menggunakan garis skala ganda

- Siswa dapat menggunakan informasi mengenai jarak sebenarnya dan jarak pada peta untuk menentukan peta mana yang sesuai dengan garis skala ganda yang ada
- Siswa dapat menggambarkan hubungan antara jarak pada peta dan jarak sebenarnya dengan menggunakan garis skala ganda
- Siswa dapat menggunakan garis skala ganda untuk menentukan jarak sebenarnya

Alat dan bahan:

- Penggaris
- LKS 2

Aktivitas 1

Soal 1 (20 menit)

1. Untuk membuka pertemuan, guru dapat memberikan pertanyaan terkait dengan peta dan mendiskusikan pertanyaan berikut dengan siswa:
   - "Informasi apa yang kalian dapat dari peta?"
   - "Pernah kah kalian melihat garis skala ganda pada peta?"
   - "Mengapa namanya garis skala ganda?"
   - "Menurut kalian, apa hubungan garis skala dengan garis skala ganda?"

   Bagi siswa, istilah "garis skala ganda" mungkin adalah hal baru. Guru dapat menulis istilah tersebut di papan tulis. Dari diskusi pada pembukaan, guru dapat membantu siswa untuk menyimpulkan mengenai garis skala ganda.
2. Setelah mendiskusikan pertanyaan tersebut, guru dapat menarasikan soal 1 pada siswa. Guru dapat meminta siswa untuk saling berdiskusi dalam menyelesaikan soal.
3. Dalam diskusi kelas, guru dapat bertanya bagaimana siswa dapat menentukan

peta yang sesuai dengan garis skala ganda yang tersedia. Selain itu, guru dapat mengaitkan dengan bertanya mengenai perbandingan pada peta:

- "Apa yang kamu ketahui tentang perbandingan?"
- "Apakah menurutmu skala adalah perbandingan? Mengapa?"

Dalam diskusi kelas ini, guru dapat membantu siswa untuk memahami hubungan perkalian dalam suatu perbandingan dengan menggunakan garis skala ganda. Selain itu, penting bagi guru untuk membahas apa yang terjadi jika peta yang lain menggunakan garis skala ganda yang tersedia.

Contoh jawaban siswa:

> Yang sesuai dengan garis skala ganda adalah peta B.
>
> Jika kita ukur jarak antara kedua kota pada peta dan menggunakan garis skala ganda untuk menentukan jarak sebenarnya, peta B mendekati perkiraan.
>
> Jika garis skala ganda digunakan pada peta A, maka jarak sebenarnya antara Gresik dan Surabaya akan lebih dari 15 km.
>
> Jika garis skala ganda digunakan pada peta C, maka jarak sebenarnya antara Gresik dan Surabaya akan kurang dari 15 km.

Soal 2 (20 menit)

1. Soal 2 merupakan lanjutan soal 1. Guru meminta siswa untuk berdiskusi menggambar garis skala ganda untuk kedua peta yang lain. Guru perlu mengingatkan siswa mengenai informasi yang terkandung pada garis skala ganda, yaitu jarak pada peta dan jarak sebenarnya. Dengan menggunakan dua informasi ini, siswa dapat menggambar garis skala ganda. Dalam menggambar garis skala ganda, guru perlu menekankan kepada siswa untuk menggunakan penggaris. Garis skala ganda berhubungan dengan gambaran jarak. Oleh karena itu, dalam menggambarkan garis skala ganda, siswa harus melakukan pengukuran dengan tepat. Guru perlu mengingatkan siswa bahwa garis skala ganda harus memuat informasi yang sesuai. Jika tidak ada informasi pada garis skala ganda, maka garis skala ganda tidak akan bermakna.
2. Pada diskusi kelas, guru membahas hubungan perkalian dalam suatu

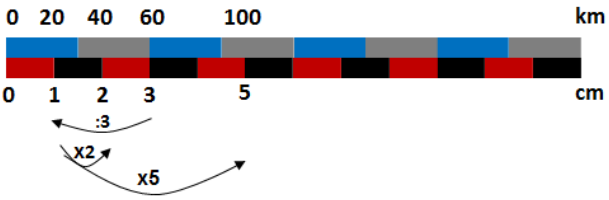
perbandingan dengan menggunakan garis skala ganda. Pada garis skala ganda, garis bagian bawah bisa dibagi menjadi beberapa bagian yang sama. Hal ini memudahkan siswa untuk membagi garis menjadi beberapa bagian yang sama besar. Guru dapat menggunakan garis skala ganda untuk mengenalkan hubungan perkalian antara perbandingan sebagai pengantar ke soal berikutnya.

Contoh jawaban siswa:

Aktivitas 2 (30 menit)

1. Guru dapat menarasikan konteks pada soal aktivitas 2. Jika perlu, guru dapat bertanya kepada siswa mengenai informasi yang terkait untuk menentukan jarak yang ditempuh oleh Edi dan keluarga tiap harinya. Dengan menggunakan persoalan sebelumnya, guru dapat bertanya apa yang siswa ketahui dari garis skala ganda. Selanjutnya, guru dapat meminta siswa untuk berdiskusi menggunakan garis skala ganda untuk menentukan jarak yang ditempuh Edi dan keluarganya.
2. Dalam diskusi kelas, guru dapat bertanya mengenai hubungan perkalian dalam suatu perbandingan dan hubungan bilangan yang menunjukkan jarak pada peta. Selain itu, guru dapat meminta siswa untuk menunjukkan hubungan tersebut pada garis skala ganda.
3. Sebelum guru menutup pertemuan kedua, guru dapat mengajukan pertanyaan sebagai berikut sebagai bahan refleksi:
   - “Apa perbedaan antara garis skala dengan garis skala ganda?”
   - “Apa kesamaan antara garis skala dengan garis skala ganda?”

Contoh jawaban siswa:



PERTEMUAN 3

Alokasi waktu: 2 x 35 menit

Tujuan pembelajaran:

- Siswa dapat merepresentasikan hubungan antara jarak pada peta dan jarak sebenarnya
- Siswa dapat mengurutkan bilangan yang diketahui pada model untuk menentukan jarak sebenarnya
- Siswa dapat menggunakan model untuk menunjukkan hubungan antar bilangan

Alat dan bahan:

- LKS 3

Soal 1a (15 menit)

1. Untuk membuka pertemuan, guru dapat menarasikan soal dan mengingatkan siswa mengenai garis skala ganda.
2. Apabila siswa kesulitan merepresentasikan hubungan antara jarak pada peta dan jarak sebenarnya, guru dapat mengingatkan siswa tentang menggambar garis skala ganda pada pertemuan sebelumnya. Selain itu, ingatkan siswa untuk menggambar dan menuliskan informasi yang sesuai.
3. Dalam diskusi kelas, guru dapat membahas kesulitan siswa dalam menggambar garis skala ganda.

Contoh jawaban siswa:

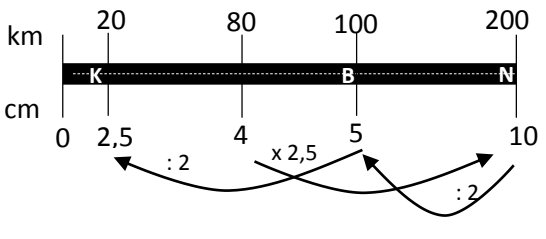
Soal 1b (25 menit)

1. Pada soal ini, guru memberitahu siswa untuk tidak menghiraukan jarak antar bilangan seperti ketika menggunakan garis skala ganda. Selain itu, meskipun

siswa berhadapan dengan soal yang melibatkan jarak pada peta dan jarak sebenarnya, model yang digunakan lebih formal dari garis skala ganda.

2. Apabila siswa tidak dapat mengurutkan bilangan yang diketahui pada model, guru dapat mengingatkan siswa akan garis bilangan ganda.
3. Siswa mungkin mencari selisih dan menggunakan penjumlahan untuk menentukan jarak sebenarnya. Guru dapat mendorong siswa untuk mengamati hubungan antar bilangan dengan menggunakan perkalian dan pembagian.
4. Dalam diskusi kelas, guru dapat mendiskusikan jawaban siswa yang menggunakan penjumlahan dan hubungan perkalian. Misalnya, guru dapat bertanya apa kelemahan dan keuntungan menggunakan cara tersebut. Guru dapat mengingatkan siswa bahwa perbandingan berkaitan dengan perkalian dan pembagian sedangkan selisih berkaitan dengan penjumlahan dan pengurangan.
5. Berikan siswa kesempatan untuk berpartisipasi dalam diskusi kelas. Guru dapat mengatur alur diskusi dengan memberikan tanggapan atas pemikiran siswa. Ajukan pertanyaan berikut:
   - "Bagaimana menurutmu tentang penjelasannya?"
   - "Bagaimana solusinya berbeda dengan solusimu?"
   - "Dapatkah kamu jelaskan apa yang telah ia katakan?"

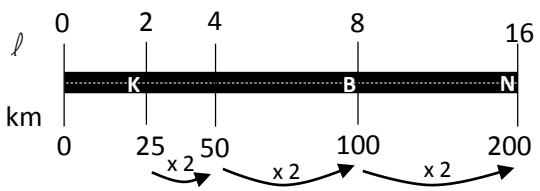
Contoh jawaban siswa:



Soal 2 (30 menit)

1. Pada soal ini, siswa menggambar model. Guru dapat memberikan petunjuk bahwa garis bawah pada model digunakan untuk menuliskan besaran yang diketahui sedangkan garis atas digunakan untuk menuliskan besaran yang ditanyakan.
2. Apabila siswa tidak dapat mengurutkan bilangan yang diketahui pada model, guru dapat mengingatkan siswa akan garis bilangan ganda.
3. Siswa mungkin mencari selisih dan menggunakan penjumlahan untuk menentukan jarak sebenarnya. Guru dapat mendorong siswa untuk mengamati hubungan antar bilangan dengan menggunakan perkalian dan pembagian.
4. Dalam diskusi kelas, guru dapat mendiskusikan jawaban siswa yang menggunakan penjumlahan dan hubungan perkalian. Misalnya, guru dapat bertanya apa kelemahan dan keuntungan menggunakan cara tersebut. Guru dapat mengingatkan siswa bahwa perbandingan berkaitan dengan perkalian dan pembagian sedangkan selisih berkaitan dengan penjumlahan dan pengurangan.
5. Guru dapat mengarahkan siswa untuk menggunakan panah dan keterangan dalam menghubungkan dua bilangan. Dengan menggunakan panah dan keterangan, siswa akan lebih mudah mengamati hubungan antar bilangan.
6. Untuk menutup pertemuan, guru dapat menegaskan bahwa perbandingan dua besaran yang berbeda juga merukapan rasio. Selain itu, penting bagi siswa untuk tidak memperhatikan jarak antar bilangan karena model yang digunakan berbeda dengan garis skala ganda.
7. Untuk memperkenalkan istilah “garis bilangan ganda” pada pertemuan selanjutnya, guru dapat bertanya mengenai perbedaan dan kesamaan model yang digunakan pada pertemuan 3 dengan garis skala ganda.

Contoh jawaban siswa:



PERTEMUAN 4

Alokasi waktu: 2 x 35 menit

Tujuan pembelajaran:

- Siswa memahami bahwa bilangan pada model harus terurut
- Siswa dapat menyatakan hubungan perkalian pada model untuk menentukan rasio senilai

Alat dan bahan:

- LKS 4

Pembelajaran

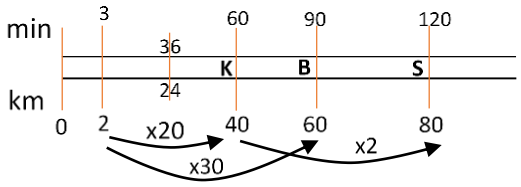
1. Untuk membuka pertemuan, guru dapat mengajukan pertanyaan mengenai model pada pertemuan sebelumnya. Selain itu, kenalkan garis bilangan ganda pada siswa. Berikut pertanyaan yang dapat diajukan:
   - "Pada pertemuan sebelumnya, ilustrasi apa yang biasa kalian buat?"
   - "Apa saja informasi yang kalian dapat dari ilustrasi tersebut?"
   - "Mengapa ilustrasi tersebut dinamakan garis bilangan ganda?"
   - "Apa kesamaan dan perbedaan antara garis bilangan ganda dan garis skala ganda?"
2. Ketika siswa berdiskusi dan menyelesaikan permasalahan pada LKS, guru dapat berkeliling untuk mengetahui kesulitan siswa. Kesulitan yang dihadapi siswa dapat dibahas dalam diskusi kelas.
3. Apabila siswa tidak paham bahwa bilangan pada model harus terurut, guru dapat mengingatkan siswa mengenai istilah garis bilangan.
4. Apabila siswa tidak menggunakan hubungan perkalian untuk menentukan rasio senilai, guru dapat mengingatkan bahwa rasio atau perbandingan terkait dengan perkalian dan pembagian.
5. Apabila kesulitan menentukan hubungan perkalian antar bilangan, guru dapat menggunakan pecahan untuk membantu siswa.

6. Dalam diskusi kelas, penting bagi guru untuk menekankan bahwa dalam menggunakan model, siswa tidak perlu memperhatikan jarak antar bilangan selama bilangan tersebut terurut. Selain itu, guru dapat bertanya apakah semua perbandingan pada model memiliki nilai yang sama. Pada diskusi ini, penting bagi guru untuk menggali kemampuan siswa dalam menentukan hubungan antar bilangan dengan menggunakan hubungan perkalian. Berikut pertanyaan yang dapat diajukan:

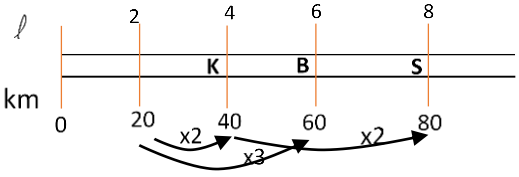
- "Bagaimana pekerjaan yang menggunakan penjumlahan dengan pekerjaan yang menggunakan perkalian berbeda?"
- "Di antara dua pekerjaan tersebut, manakah yang merupakan perbandingan? Mengapa?"
- "Dapat kah kalian menghubungkan bilangan satu dengan bilangan yang lain selain seperti pekerjaan ini?"

Contoh jawaban siswa:

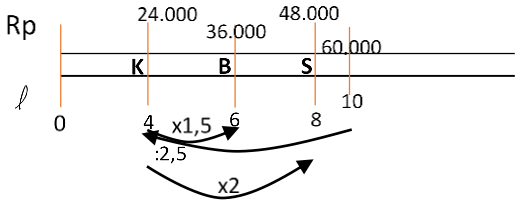
Soal a



Soal b



Soal c



PERTEMUAN 5

Alokasi waktu: 2 x 35 menit

Tujuan pembelajaran:

- Siswa dapat memformulasikan informasi menjadi model matematika
- Siswa memahami bahwa rasio senilai dapat ditentukan dengan menggunakan hubungan perkalian

Alat dan bahan:

- LKS 5

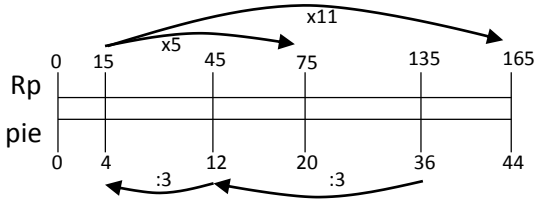
Pembelajaran

1. Ketika siswa berdiskusi dan menyelesaikan permasalahan pada LKS, guru dapat berkeliling untuk mengetahui kesulitan siswa. Kesulitan yang dihadapi siswa dapat dibahas dalam diskusi kelas.
2. Apabila siswa tidak tahu model yang harus digunakan, guru dapat mengingatkan siswa tentang garis bilangan ganda.
3. Apabila siswa kesulitan dalam memahami informasi yang diberikan sebagai perbandingan, guru dapat langsung menunjukkan perbandingan tersebut dan meminta siswa untuk memformulasikan informasi yang tersedia ke dalam garis skala ganda.
4. Apabila siswa tidak dapat memahami bahwa hubungan perkalian digunakan untuk menentukan rasio senilai, guru dapat mengingatkan bahwa rasio atau perbandingan terkait dengan perkalian dan pembagian.
5. Apabila kesulitan menentukan hubungan perkalian antar bilangan, guru dapat menggunakan pecahan untuk membantu siswa.
6. Dalam diskusi kelas, penting bagi guru untuk menggali kemampuan siswa dalam menentukan hubungan antar bilangan dengan menggunakan hubungan perkalian. Berikut beberapa pertanyaan yang dapat diajukan:
   - “Apa yang kalian ketahui tentang perbandingan?”

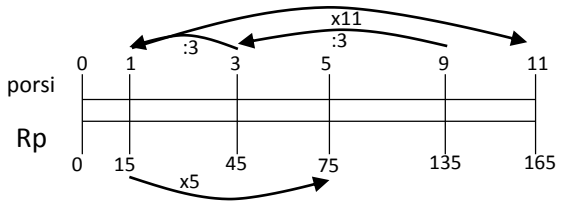
- “Dalam menggunakan garis bilangan ganda, hubungan perbandingan seperti apa yang dapat kalian ketahui?”
- “Bagaimana pekerjaan yang menggunakan penjumlahan dengan pekerjaan yang menggunakan perkalian berbeda?”
- “Di antara dua pekerjaan tersebut, manakah yang merupakan perbandingan? Mengapa?”
- “Dapat kah kalian menghubungkan bilangan satu dengan bilangan yang lain selain seperti pekerjaan ini?”

Contoh jawaban siswa:

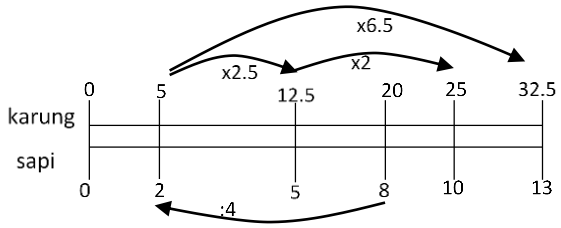
Soal 1a



Soal 1b



Soal 2



NAMA:

LKS 1

Aktivitas 1

1. Dea membandingkan panjang laptop dengan panjang meja. Menurut Dea, ukuran meja dua satuan lebih panjang dari panjang laptop.

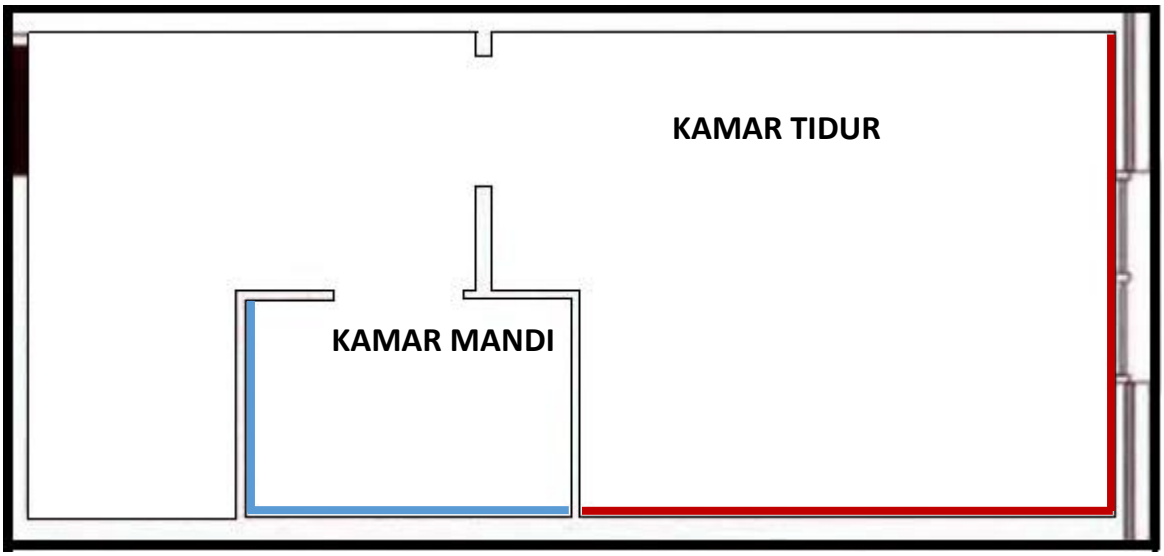
   Soal: Apakah pernyataan Dea merupakan perbandingan? Beri alasan.

   Pernyataan Dea _____.

   Alasannya:

2. Seorang arsitek membuat denah suatu apartemen namun ia tidak mencantumkan ukurannya.

   Soal: Bantu arsitek untuk mengukur panjang garis merah dan biru yang merupakan panjang dan lebar kamar mandi dan kamar tidur.



Petunjuk: ● pilih salah satu perabot rumah tangga di bawah ini
● gunakan panjang atau lebar perabot untuk mengukur


tempat tidur


sofa


lemari

Lengkapi pernyataan berikut.

Saya menggunakan _____ untuk mengukur.

Panjang kamar mandi adalah _____.

Lebar kamar mandi adalah _____.

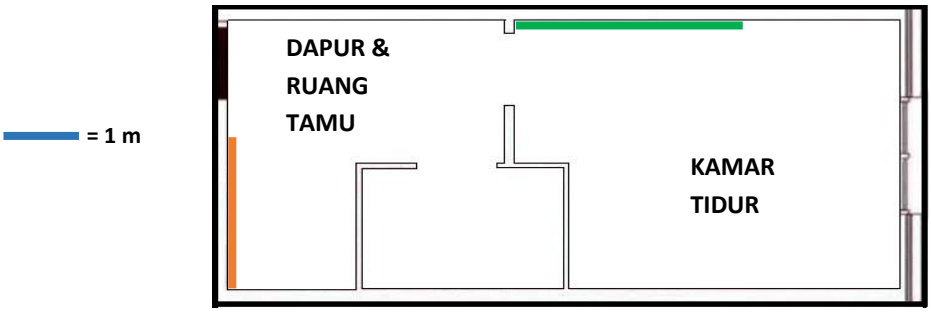
Panjang kamar tidur adalah _____.

Lebar kamar tidur adalah _____.

3. Garis oranye dan hijau menunjukkan letak kabel dalam kamar. Garis biru (garis skala) menunjukkan bahwa panjang sebenarnya adalah 1 m.

Soal: Tentukan panjang kabel sebenarnya.

Petunjuk: ● buat gambar yang membandingkan panjang garis biru (garis skala) dengan kabel pada gambar
● tentukan berapa kali garis biru sehingga panjangnya sama dengan panjang kabel pada gambar



Lengkapi pernyataan berikut.

1. Berikut adalah gambarku.

   Panjang garis oranye adalah _____. Jadi, panjang kabel oranye sebenarnya adalah _____.

2. Berikut adalah gambarku.

   Panjang garis hijau adalah _____. Jadi, panjang kabel hijau sebenarnya adalah _____.

Aktivitas 2

Buat denah ruang kelasmu dan lengkapi pernyataan yang tersedia.

Petunjuk: ● pilih satu benda di kelas yang dapat dijadikan patokan
● perkirakan ukuran sebenarnya
● buat garis skala yang mewakili ukuran patokan pada denah
● jelaskan ukuran ruang kelasmu

Saya menggunakan _____ sebagai patokan. Saya perkirakan panjang sebenarnya adalah _____. Berikut garis skala saya:

Berikut denah ruang kelas saya.

Gunakan kata-kata, gambar, simbol, dan lain-lain untuk menjelaskan apa yang kamu ketahui tentang garis skala.

Jelaskan dengan menggunakan kata-kata, gambar, simbol, dan lain-lain

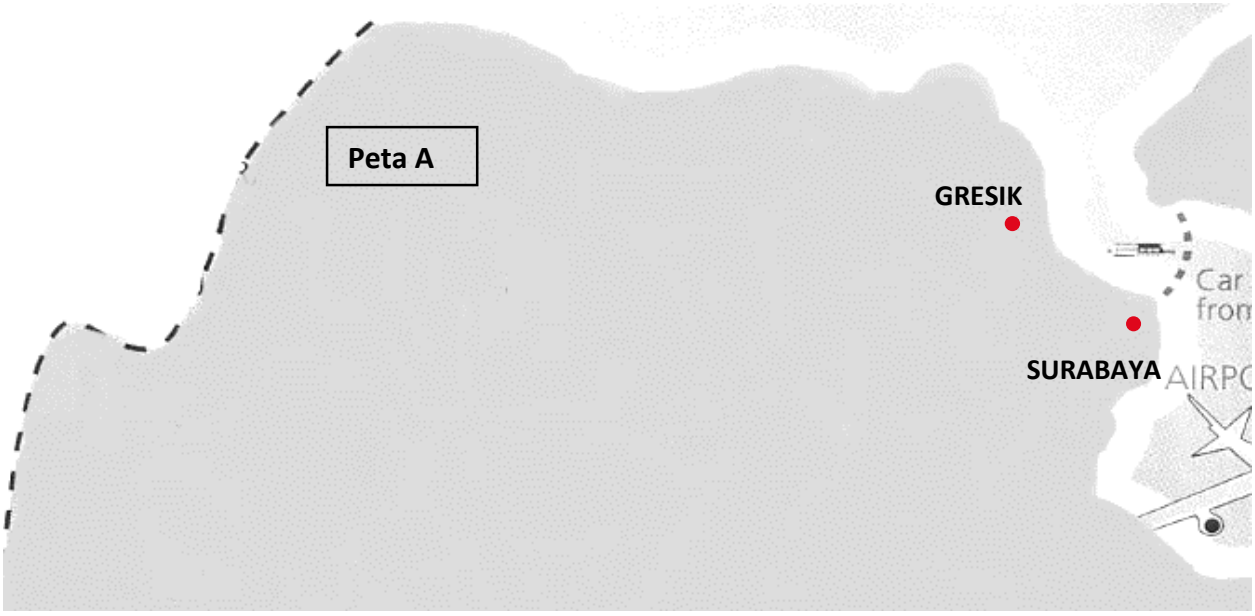
Hal baru dan hal penting yang saya pelajari:

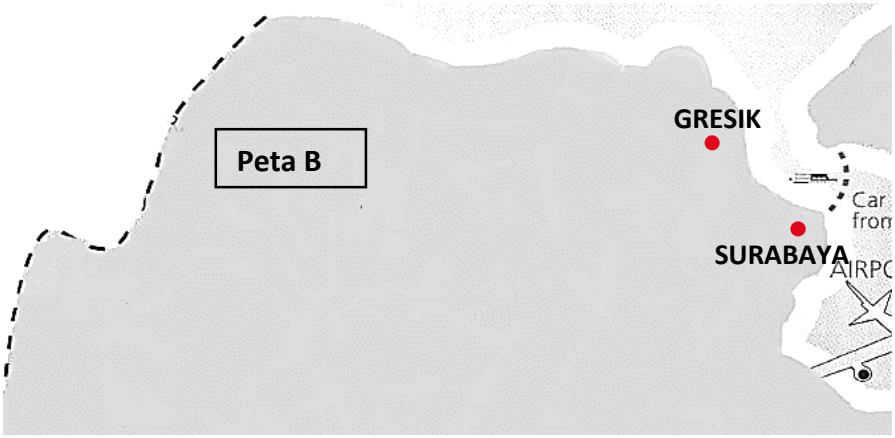
NAMA:

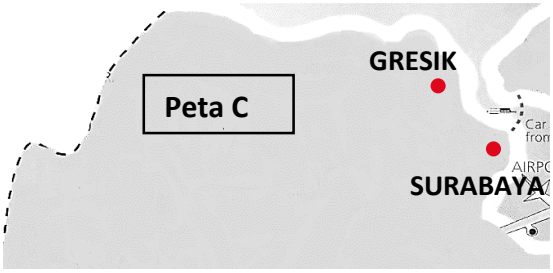
LKS 2

Aktivitas 1

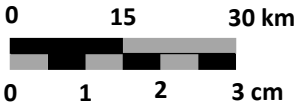
1. Berikut adalah tiga peta dengan ukuran yang berbeda.







Suatu peta biasanya mempunyai garis skala ganda. Di bawah ini adalah garis skala ganda.



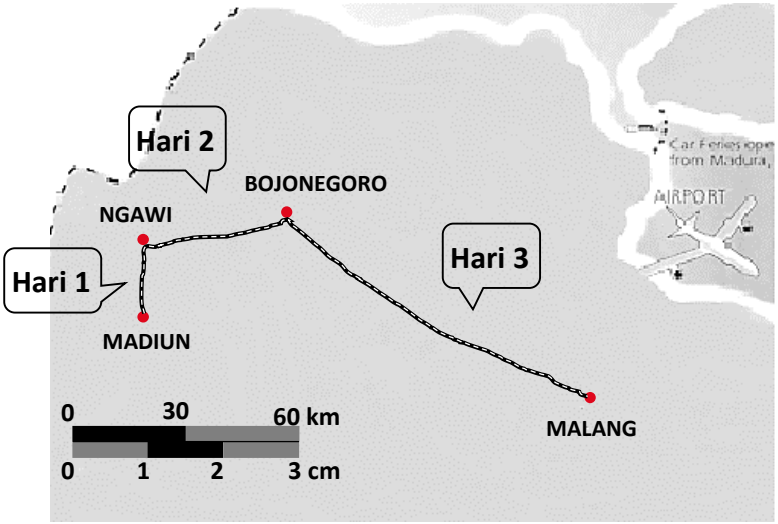
Diketahui: Jarak terpendek dari Gresik ke Surabaya adalah 15 km

Soal: Tentukan peta mana yang sesuai dengan garis skala ganda. Apa yang terjadi pada peta yang lain jika menggunakan garis skala ganda tersebut?

2. Buat garis skala ganda untuk peta yang lain.

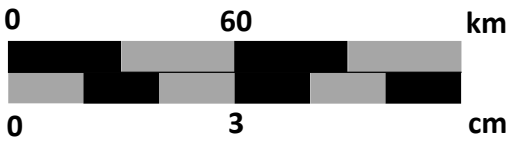
Aktivitas 2

Perhatikan peta di bawah.



Edi dan keluarga tinggal di Madiun. Mereka akan melakukan perjalanan ke beberapa kota dengan menggunakan mobil ketika liburan. Tiap hari, mereka akan pergi ke kota yang berbeda.

Edi ingin tahu berapa jarak yang akan mereka tempuh setiap harinya. Gunakan garis skala ganda di bawah untuk membantu Edi mengetahui jarak sebenarnya yang akan ditempuh tiap harinya.



Apa yang kamu ketahui tentang garis skala ganda?

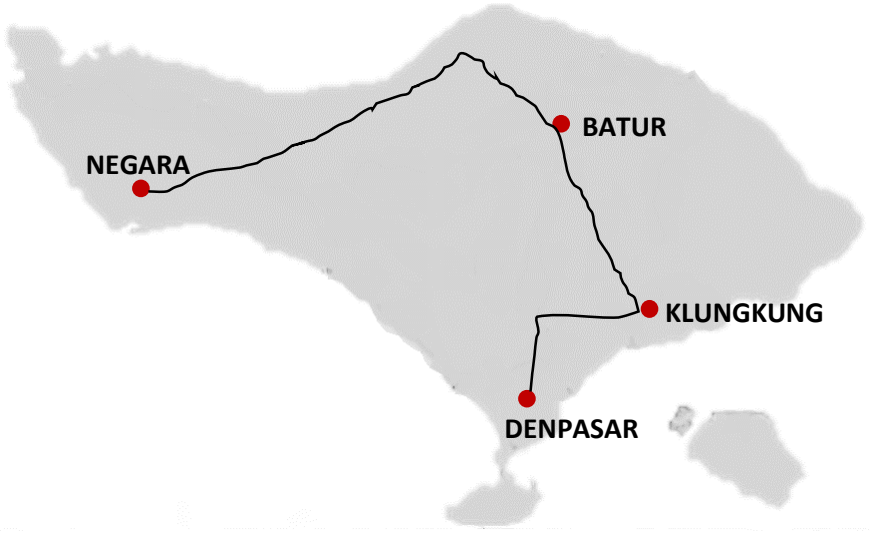
Jelaskan dengan menggunakan kata-kata, gambar, simbol, dan lain-lain

Hal baru dan hal penting yang saya pelajari:

NAMA:

LKS 3

1. Pak Nyoman punya bisnis tur di Denpasar. Ia menyusun jadwal tur untuk para turis ke 3 tempat berbeda. Berikut rutenya.



Diketahui: Pak Nyoman tahu bahwa 4 cm pada peta menunjukkan 80 km pada jarak sebenarnya.

a. Buat garis skala ganda untuk peta tersebut.

Berikut garis skala ganda peta tersebut:

b. Pak Nyoman mengukur panjang rute tiap tujuan dari Denpasar di peta. Berikut hasilnya.

Tujuan	**Panjang rute pada peta (cm)**
Klungkung	2,5
Batur	5
Negara	10

Jika rutenya dibentangkan menjadi suatu garis, maka akan menjadi seperti gambar di halaman selanjutnya.

Soal: Tentukan panjang rute sebenarnya dari Denpasar ke tiap tujuan.

Petunjuk: ● Tulis panjang rute pada peta untuk setiap tujuan pada garis yang tersedia di halaman selanjutnya
● Tunjukkan perhitunganmu dengan menggunakan garis tersebut
● Beri keterangan nama daerah pada garis tersebut

2. Pak Nyoman ingin memberi tahu sopir bus mengenai konsumsi bahan bakar untuk mencapai tujuan. Ia tahu bahwa 2 liter bahan bakar dapat digunakan untuk bepergian sejauh 25 km.

   Soal: Buat dan gunakan gambar seperti nomor 1b untuk mengetahui konsumsi bahan bakar ke setiap tujuan.

Beri contoh lain tentang perbandingan dua hal berbeda dalam topik perjalanan.

Jelaskan dengan menggunakan kata-kata, gambar, simbol, dan lain-lain

Hal baru dan hal penting yang saya pelajari:

NAMA:

LKS 4

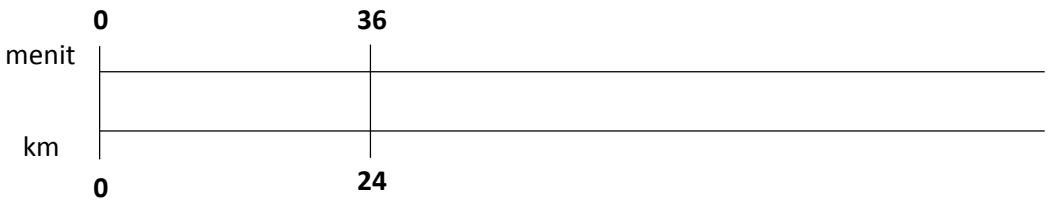
Pak Nyoman menyediakan 3 bus untuk 3 daerah wisata yang berbeda. Berikut rute tujuan wisata yang disusun Pak Nyoman dari Denpasar.

a. Jika tidak ada kemacetan di Bali, dibutuhkan 36 menit untuk bepergian sejauh 24 km. Tentukan waktu yang dibutuhkan untuk melakukan perjalanan dari Denpasar ke setiap daerah tujuan.

Petunjuk: Lengkapi dan gunakan garis bilangan ganda di bawah untuk menunjukkan perhitungan.

Denpasar

↓



b. Ketika bepergian ke Ubud yang jaraknya 20 km dari Denpasar, bus menghabiskan 2 liter solar. Tentukan konsumsi solar yang diperlukan untuk menuju setiap daerah tujuan dengan menggunakan garis bilangan ganda.

c. Pak Nyoman memberi tahu semua sopir bahwa ia membayar Rp 60.000,00 tiap kali membeli 10 liter solar. Bantu sopir untuk menghitung uang yang akan dibayar untuk mengisi solar ke setiap daerah tujuan.

Beri contoh perbandingan pada topik selain peta dan perjalanan.

Berikan pendapatmu ketika menggunakan garis bilangan ganda untuk menyelesaikan soal perbandingan.

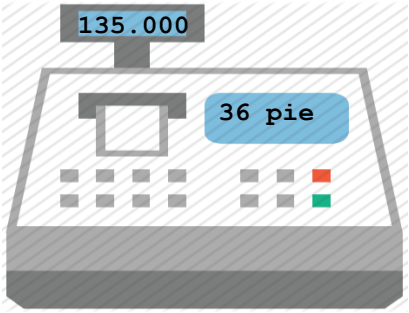
Jelaskan dengan menggunakan kata-kata, gambar, simbol, dan lain-lain

Hal baru dan hal penting yang saya pelajari:

NAMA:

LKS 5

1. Saat berlibur ke Bali, Adam dan teman-teman membeli pie susu. Berikut keterangan mesin kasir ketika Adam akan membayar pie susu yang dia beli.



a. Elsa membeli 20 pie, Doni membeli 44 pie, dan Rita membeli 12 pie. Tentukan uang yang harus dibayar tiap orang.

b. Setelah berbelanja, Adam dan teman-teman menuju warung makan. Setelah melihat menu, Adam yang menghabiskan Rp 135.000,00 untuk membeli pie susu mengetahui bahwa uang tersebut sama dengan harga 9 porsi nasi bali.

   Tentukan berapa porsi nasi bali masing-masing yang dapat dibeli Elsa, Doni, dan Rita dengan menggunakan uang yang mereka bayar ketika membeli pie susu.

2. Desta berjalan-jalan di desa peternakan sapi dan bertemu dengan peternak sapi. Peternak sapi bercerita bahwa 8 sapi dapat menghabiskan 20 karung rumput tiap hari. Lalu Desta melanjutkan perjalanan dan menemukan tiga peternakan lain. Berikut daftar peternakan beserta banyak sapi di tiap peternakan:

Peternakan	**A**	**B**	**C**
Banyak sapi	10	5	13

Tentukan banyak karung rumput yang dimakan oleh sapi-sapi di tiap peternakan tiap hari.