PROMOTING THE DEVELOPMENT OF STUDENTS' INDIVIDUAL FRAME OF REFERENCE TO SUPPORT LENGTH APPROXIMATION/ESTIMATION SKILLS

A THESIS

Submitted in Partial Fulfillment of the Requirement for the Degree of Master of Science (M.Sc)

in

International Master Program on Mathematics Education (IMPoME)

Faculty of Teacher Training and Education, Sriwijaya University

in Collaboration with the Freudenthal Institute, Utrecht University

By

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ABSTRACT

Measurement approximation/estimation is a very important skill in literacy life. Through measurement approximating/estimating (length) activities, one could be stimulated to develop individual frame of reference which then be internalized to support sense of measurement. However, in Indonesia, this topic is not embedded yet in the curriculum of students' learning. In this study, we aim to contribute to the development of local instruction theory on mathematics education. We design, test, and evaluate a set of learning trajectory (5 lessons) and its hypothetical learning trajectory which is based on realistic mathematics education approach in two cycles of design-based research. Subject of this study for the first cycle is 4 fourth graders and for the second cycle is 34 fourth graders and one classroom teacher from MIN 2 Palembang. Data gained from students' written work, interviews and classroom observation are analyzed mostly in qualitative ways. Using triangulation method of data analysis we evaluate the learning trajectory and the hypothetical learning trajectory by comparing it to the actual learning which happen in the classroom for improvement of the design for the next implementation. The result of the data analysis shows that the development of individual frame of reference for length measurement can be promoted through the use of body parts as reference points then enhance the use to reason familiar objects as new reference points. Like a chain reaction, one can be promoted to reason new other objects using the body parts reference/external object reference for solving larger approximation/estimation tasks.

ABSTRAK

Approksimasi/estimasi pengukuran merupakan kemampuan yang sangat penting dalam kehidupan untuk literasi. Melalui aktivitas approksimasi/estimasi pengukuran siswa dirangsang untuk menumbuhkan acuan/referensi bantu individu pengukuran yang kemudian terinternalisasi menumbuhkan *sense of measurement* seseorang. Namun, kurikulum Indonesia secara eksplisit belum memuat materi approksimasi/estimasi pengukuran di dalam pembelajaran siswa. Di dalam study ini, kami bermaksud untuk turut berperan dalam memberi solusi terhadap masalah pendidikan ini dengan turut berkontribusi untuk teori instruksional local dalam pembelajaran matematika. Kami mendesain, mengetes dan mengevaluasi satu set alur pembelajaran (5 pembelajaran) berbasis Pendidikan Matematika Realistik Indonesia dan beserta hipotesisnya dalam 2 siklus riset berbasis design. Subjek penelitian pada tahap siklus pertama melibatkan 4 siswa kelas empat dan subjek penelitian pada tahap kedua melibatkan 34 siswa kelas empat dan seorang guru kelas di MIN 2 Palembang. Data yang kami peroleh dalam penelitian ini berupa pekerjaan tertulis siswa, observasi pembelajaran, serta wawancara dengan siswa dan guru. Data yang diperoleh dianalisis secara kualitatif dengan metode triangulasi data untuk mengetahui apakah desain alur pembelajaran dan hipotesisnya sesuai dengan realitas pembelajaran di kelas atau tidak guna perbaikan dan saran untuk implementasi berikutnya. Hasil analisis menunjukan bahwa acuan/referensi bantu individu pengukuran panjang dapat dikembangkan melalui penerapan anggota tubuh sebagai referensi, kemudian melatih siswa menalar objek yang familiar (referensi baru) menggunakan anggota tubuh. Layaknya sebuah reaksi berantai, siswa kemudian dilatih untuk menalar objek baru baik menggunakan acuan bantu berupa tubuh/benda eksternal dalam tugas yang lebih besar.

SUMMARY

Many studies reported low performances on length estimation tasks. Little is known how it can be embedded into instructional activities. Including in Indonesia, less attention is given to an activity of making sense of units for estimating. We therefore intent to provide and contribute designing instructional activities for students. The main research question of this study is *how can we support the development of students' reference points for length approximation/estimation?*

Measurement estimation does not relate to the development of general mathematical ability. Students should first experience a process of rough physical measuring (approximating) before mentally measuring the to-be-estimated objects (estimating). A skilled estimator employs educated strategies such as the use of reference points. Reference points develop through everyday experience but the development can be enhanced through appropriate approximating and estimating tasks. It serve as a critical point to develop students' sense and understanding of measurement.

In designing learning in approximation/estimation, realistic mathematics education is powerful to be employed. Approximating/estimating are context-bounded tasks, the instructions should be started and ended in a meaningful real world situations. Promoting the use of models may enhance students understanding how unit is iterated and used in efficient ways. Moreover, process of developing personal reference points is a students' own construction process, it cannot be forced to the students. Hence, an interactive instruction should be formulated in order to trigger discussion about students' personal reference points. In addition, it also important to note that in order to conduct a powerful instruction as proposed in RME tenets, change in class socio norms and socio mathematical norms should be promoted.

We employ two cycles of design-based research as research approach in this study. We design a learning trajectory and its hypothetical learning trajectory (HLT) describing a learning instruction and the possibilities of students' thinking to support students' development of reference points for estimating. Subject of this

study for the first cycle is 4 fourth graders and for the second cycle is 34 fourth graders and one classroom teacher from MIN 2 Palembang. The data were collected by semi-structured interview (teacher and students), classroom observation, and students' written tests. Data gained were analyzed using triangulation method. We compare interesting fragments of students' written works or registered video in which the learning takes places or not compare to the HLT. The analysis was mostly done in a qualitative ways and in modest quantitative ways.

The learning activities that we designed consist of 5 lessons. In the frog jumping and *Pocong* jumping activity the students shift from using rulers to use other reference points. In the second lesson, measure and use your body part, the students were able to use body parts as reference points for approximation. Meanwhile, in the third lesson, shoes-couch-and flood, the students were able to associate and reason using body parts to approximate the length of objects as new reference points. In the fourth lessons, the student were able to associate objects to other objects to gain new reference points. At the end, in the fifth lesson, students identified, used and reasoned using reference points/IFR for solving length approximation/estimation tasks in social arithmetic problem.

We conclude that the development of individual frame of references could be supported by exposing the students to use body parts as reference points then using this IFR to gain new reference points in form of external objects which then again can be trained to gain new other reference points. Meanwhile, developing students' estimation skills should follow the process approximating, internalization of IFR, and then estimating. Realizing that this study only contributes a very little to the development of local instructions theory in mathematics education, further study about how to boost the development of students' skills from approximating to estimating might be useful for learning theory in the domain of measurement.

RINGKASAN

Banyak studi melaporkan bahwa siswa memperoleh hasil yang rendah dalam mengerjakan soal estimasi pengukuran panjang. Sangat sedikit penelitian yang fokus dan tahu bagaimana cara menggabungkan topik ini ke dalam kurikulum. Seperti halnya di Indonesia, sedikit perhatian diberikan untuk mengajak siswa memahami secara bermakna satuan pengukuran panjang. Kami berniat untuk berkontribusi untuk menanggulangi masalah ini dengan cara mendesain aktivitas pembelajaran untuk siswa. Pertanyaan utama dalam penelitian ini adalah *bagaimana kita dapat mendukung perkembangan acuan bantu individu siswa untuk approksimasi/estimasi?*

Estiamasi pengukuran tidak terkait dengan perkembangan matematika secara umum. Siswa harus melalui tahap perkembangan dalam mengaproksimasi secara fisik sebelum bisa mengestimasi secara mental. Seorang yang terlatih dalam mengestimasi menggunakan metode yang bermakna seperti penggunaan acuan bantu. Acuan bantu berkembang melalui pengalaman sehari-hari namun dapat dirangsang melalui penugasan aproksimasi dan estimasi. Hal ini sangat penting dalam mengembangkan *sense* dan pemahaman siswa akan pengukuran.

Dalam mendesain pembelajaran aproksimasi /estimasi, pendekatan realistic mathematics education sangat tepat digunakan. Aproksimasi/estimasi merupakan tugas yang *terintegrasi dalam konteks*, pembelajaran bermula dan berakhir secara bermakna dalam situasi dunia nyata. Mendukung *penggunaan model* dapat pula merangsang pemahaman siswa tentang bagaimana iterasi satuan dan efisiensi pengukuran. Terlebih lagi, proses kembang acuan bantu individu merupakan *konstruksi siswa secara mandiri* yang mustahil dipaksakan antar individu. Sehingga, pembelajaran yang *interaktif* haruslah digalakan untuk memicu diskusi antar acuan bantu siswa yang mungkin bervariasi. Sebagai tambahan juga, dalam rangka melaksanakan pembelajaran yang efektif, perubahan dalam norma social dan norma social matematika harus turut digalakan.

Kami menggunakan dua siklus *design-based research* sebagai pendekatan penelitian dalam studi ini. Kami mendesain alur pembelajaran beserta hipotesis alur pembelajaran yang berisi tentang instruksi pembelajaran dan kemungkinan

pemikiran siswa. Subjek dalam siklus pertama penelitian ini adalah 4 orang siswa kelas empat dan dalam siklus kedua adalah 34 siswa kelas empat dan satu orang guru dari MIN 2 Palembang. Pengambilan data dikumpulkan melalui wawancara, observasi kelas, dan pekerjaan tertulis siswa. Data yang diperoleh dianalisis menggunakan metode triangulasi. Kami membandingkan fragment pembelajaran yang menarik baik dalam video observasi maupun pekerjaan tertulis siswa, apakah mendukung hipotesis alur pembelajaran atau tidak. Analisis dilakukan sebagian besar menggunakan metode kualitatif dan kuantitatif sederhana.

Aktivitas pembelajaran yang kami desain memuat 5 pembelajaran. Pada pembelajaran sambil bermain bernama lompat katak dan lompat pocong, siswa nampak beralih menggunakan acuan bantu lain untuk mengaproksimasi dibandingkan penggaris. Pada pembelajaran yang kedua, siswa mampu menggunakan anggota tubuhnya untuk melakukan aproksimasi. Sementara itu, dalam pembeljaran ketiga, siswa mampu menghubungkan dan menalar panjang benda familiar sebagai acuan bantu baru menggunakan acuan bantu ukuran anggota tubuh. Untuk pembelajaran keempat, siswa mampu kembali menghubungkan dan menalar acuan bantu baru berupa objek asing menggunakan acuan bantu objek familiar/bagian tubuh. Dan pada pembelajaran terakhir siswa mampu mengidentifikasi, menggunakan dan menalar menggunakan acuan bantu untuk menyelesaikan permasalahan aproksimasi/estimasi dalam permasalahan aritmatika sosial.

Kami menyimpulkan bahwa perkembangan acuan bantu individu dapat di dukung dengan mengekspos siswa dalam penggunaan anggota tubuh sebagai acuan bantu kemudian menggunakanya untuk menalar acuan bantu berupa objek eksternal yang nantinya dapat digunakan sebagai acuan bantu individu yang baru. Sementara itu, mengembangkan kemampuan estimasi haruslah melalui tahap kembang aproksimasi-internalisasi-estimasi menggunakan acuan bantu individu. Kami sadar bahwa penelitian ini hanya berkontribusi sedikit dalam perkembangan teori pembelajaran local dalam pendidikan matematika, sehingga penelitian lebih dalam mengenai bagaimana cara mendukung kemampuan siswa beralih dari proses aproksimasi ke estimasi akan sangat berarti guna perkembangan teori pembelajaran.

STATEMENT PAGE

I hereby:

Name : Rudi Hartono

Place of birth : Kuala Pembuang

Date of birth : November 2, 1991

Academic Major : Mathematics Education

State that:

- All the data, information, analyses and the statement in analyses and conclusions that presented in this thesis, except from references are the result of my observations, researches, analyses, and views with the guidance of my supervisors.
- The thesis that I had made is original of my thought and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get an academic sanctions such as cancelation of my degree that I have got through this thesis.

Palembang, June 2015

Rudi Hartono
NIM 06022681318078

CURRICULUM VITAE

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He was graduated in 2003 from SDN 1 Kuala Pembuang. Then, he continued his study at SMPN 1 Kuala Pembuang and SMAN 1 Kuala Pembuang, in the year 2009 he was graduated as the best students of the school. The author continued to pursue his bachelor degree (S.Pd) at the university of Lambung mangkurat on mathematics education study program. He graduated for 7 semesters with *cumlaude.* In 2012, he got a chance to attend an intensive English course held by Sriwijaya University Language Institute for 3 months. Then, he was passed as an awardee of International Master Program on Mathematics Education (IMPoME) of Sriwijaya University in collaboration with Utrecht University. At the beginning of 2014, he graduated with cumlaude from Utrecht University of the IMPoME program. And currently, he is pursuing his master degree (M.Sc) at Sriwijaya University.

When he was in high school he was chosen as the chief of Koperasi Sekolah. In college, he was actively involved in several religious organizations and movement inside the university. In the year 2012, he achieved the third winner from Pemilihan Mahasiswa Berprestas FKIP, Unlam. In the same year, the author was awarded as national finalist of ONMIPA PT on mathematics in Surabaya. At the end of the year, he was again awarded as the third winner of OSN Pertamina on mathematics in Jakarta.

PREFACE

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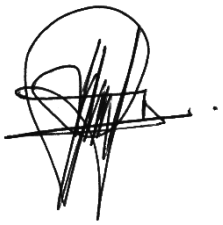
Palembang, June 2015



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CHAPTER 1
INTRODUCTION

Length measurement estimation ability is a very important skill in real life. In the absence of standard measurement tools, we often try to estimate objects by using our body as a tool for measurement such as hand span and fathom or using any other objects of which we know the size. For instance, when we measure the length of a rope usually we use hand span as measurement tools and when we estimate the height of a building, we multiply the height of one floor as an assumption. Further, technically speaking, this skill has two-sided advantages: first it facilitates the development of an awareness of the need of standard unit of measurement and understanding of unit iteration for elementary school students (Hildreth, 1983).

Many studies have reported low performances on estimation tasks and documented how students solve measurement estimation problems (e.g Hildreth, 1983; Joram, Subrahmanyam and Gelman, 1998). For instance in Hildreth (1983), he interviewed 24 students to solve 24 estimation tasks (length and area estimation), 40% of the tasks were categorized as using inappropriate strategies including wild guessing. In other case, Joram, Subrahmanyam and Gelman (1998) found that students often try to recall a standard unit of measurement such as meter or yard and use it mentally to estimate the length of an object. This strategy makes the students struggle to memorize how long one meter is. This way of solving length measurement estimation tasks is to be considered inflexible and inefficient also less meaningful for the students. Therefore, the learning and teaching of linear measurement estimation needs more attention.

Nevertheless, despite its importance, only few studies that focus on developing students skill in length estimation tasks (G. Jones, Taylor, & Broadwell, 2009). Most researchers tend to focus on investigations of the cognitive processes that students employed during the estimation tasks yet little is known how it can be embedded into instructional activities to support estimation skills (Joram,

Subrahmanyam, & Gelman, 1998). Therefore, it is not surprisingly that, estimation skills such as measurement estimation are not explicitly embedded in the Indonesian curriculum (BSNP, 2006). For the length measurement topic, the teaching and learning tendency is more focused on mastering to use standard measurement tool and how to convert standard units.

In order to integrate this topic into a curriculum, Hogan and Brezinski (2003) suggest that the measurement estimation should be addressed as a separate skill and different approach as part of spatial ability since it does not relate to the development of general mathematical ability. Moreover, Markovits and Hershkowitz (1997) argue that exposing students with concrete objects to-be-estimated can enrich their visual experience as fundamental knowledge for estimating. Other studies point out that the development of linear measurement estimation skill is significantly related to personal prior experience which form individual frame of reference or benchmark of specific length (Joram, Subrahmanyam, and Gelman, 1998; Gooya, Khosroshahi, and Teppo, 2011). Therefore, making meaning of unit through concrete visual objects is necessary for supporting individual skills of length estimation.

Based on these findings, we aim to design meaningful instructional activities to support students' length measurement estimation skill. Therefore, the research question we address is:

"How can we promote the development of students' individual frame of reference to support length estimation skills?"

CHAPTER 2
THEORETICAL FRAMEWORK

This chapter provides theories that serves as a framework of thinking in this study. First of all, we elaborate the concept of measurement estimation for teaching and learning. Second, we describe how an individual frame of reference develops. Third, we also address 5 tenets of realistic mathematics education for supporting instruction on developing students' individual frame of reference. Fourth, we will review curriculum material of length measurement estimation in the Indonesian curriculum 2006 and 2013. Finally, we will summarize the theories as a basis for this present study and formulate the research questions.

2.1. Measurement Estimation

To begin with, the readers must establish an understanding of differences between approximation and estimation in the measurement concept. Approximating is a process of obtaining the value of an attribute by assigning a certain value (units) using tools. The process of approximating requires more time to physically iterate a unit which tend to result in an exact answer with a certain degree of precision. Indeed, measuring can be categorized as approximating since the process of measuring requires tools (e.g. meter tape, rulers, etc.). Meanwhile, estimation relies on the ideas of producing an immediate and rough answer in mental ways that is sufficiently exact (Hall Jr, 1984). Hence, in estimating, there is no physical pacing is involved and no measuring tool is used except paper and pencils.

Measurement estimation can be defined as skills to make an educated guess of continuous magnitude in the absence of measuring tools (Bright, 1979; Smart, 1982). Skills for estimating length are important in daily life. James and Taylor (2010) stated that individuals in a variety of professions argue that estimating skills are essential for their careers. For instance, an architect estimates the space needed

for a body to pass a doorway, a butcher estimates the dimension of meat that she/he should cut to meet a certain weight, and a park ranger estimates the distances between landmarks. Apart from practicality, measurement estimation skills also support the understanding of basic properties of physical measurement (Hildreth, 1983; Joram et al., 1998). In this sense, measurement estimation it is important to incorporate measurement estimation in mathematics education.

Nevertheless, many studies have reported low performances of students in measurement estimation tasks especially in length estimation (Hildreth, 1983; G. M. Jones, Gardner, Taylor, Forrester, & Andre, 2012; Joram et al., 1998). Teachers and students tend to focus on procedural and exact solutions rather than on thinking hypothetically, roughly and inexactly as proposed by measurement estimation ideas (Forrester & Pike, 1998; Roth & Roychoudhury, 1993). Accordingly, learning instructions in length estimation should provide a classroom environment in which can accommodate flexible ways of thinking and differences of students' answers but prevent students to be in a confusing and too open situations. Therefore, openness of the tasks should be appropriate for students (Chang, Males, Mosier, & Gonulates, 2011).

Moreover, the learning instructions should explicitly address length estimation in spatial ability by practicing and enriching students' visual experience through interaction with physical objects (Hogan & Brezinski, 2003; Markovits & Hershkowitz, 1997; Smart, 1982). In other words, students should first experience a process of physical measuring (approximating) before mentally measuring attributes (estimating). The experiences are expected to support students to have mental images of units to minimalize wild guesses occurring in length estimation. This mental images are called as individual frame of reference (IFR) by which they can sense magnitude.

2.2. Developing Individual Frame of Reference (IFR)

When students try to solve linear measurement estimation, different strategies will be employed depending on their proficiency in estimating (Hildreth, 1983). Most studies have found that skilled estimators tend to use

benchmarks/mental images of something that they already know the length of (Crites, 1992; Gooya, Khosroshashi, & Teppo, 2011; G. Jones et al., 2009; Joram, Gabriele, Bertheau, Gelman, & Subrahmanyam, 2005). For instance in Joram et al.'s (2005) study, when a student was asked to estimate a 9-inch-long piece of rope, one student said he was thinking about a Pringles can and trying to measure it in his mind against the rope. Meanwhile, less skilled estimators tend to estimate lengths of something by wild guessing without further reasonable explanation.

The mental images/benchmarks also refers to other terms such as personal referents, reference points, mental rulers and or individual frames of reference (hereafter, IFR) (Clements, 1999; Crites, 1992; Gooya et al., 2011; Joram et al., 2005). An IFR helps estimators to estimate by imagining an object with known measurement and then compare it to an object to-be-estimated (Joram et al., 2005). This imaginable object develops through everyday experience and internalization of the standard units of measurement from a specific object by which one can feel or have sense for the size of a unit (Sowder, 1992). Each person has different personal frames of reference, for instance one might use their body height to estimate the height of a classroom, or even imagine a 20-feet-long crocodile to estimate the length of a classroom as found in Hildreth's (1983) study. It is not surprising that using IFR makes estimation meaningful, easier and more accurate (Joram et al., 2005).

Developing an IFR should become a primary goal of measurement estimation instructions (Bright, 1976; Lang, 2001). Clements (1999) also highlighted it as a critical point to develop students' measurement sense. Nevertheless, students often do not spontaneously use IFR as a strategy to estimate (Hildreth, 1983; Joram et al., 2005). It is caused by complex interaction among three aspects: students' preference, context of the tasks and the nature of estimation activity (Gooya et al., 2011; Joram et al., 1998). For instance, providing particular estimation cues such as tiles in the context of estimating the length of a blackboard may influence students' choice of IFR. On the other hand, estimation in the context of a marble game may stimulate students to use/imagine their hand-spans. Therefore, a learning instruction for promoting and developing IFR for length estimation should be designed by considering the aforementioned aspects.

One approach for developing the learning instructions developed by Joram et al. (2005) suggested three major elements as follows:

Developing accurate representations of measurement units (creating IFR). In order to create an IFR, it is suggested that the best application of IFR is a combination of body measurement models and external models (e.g. a familiar object in the classroom or at home) (Bright, 1976; G. Jones et al., 2009). In addition, with regards to the importance of a unit of meter, a meter tape can serve as an external model of IFR since it meaningfully facilitates the internalization of one meter unit into IFR (Clements, 1999; Smith & Darrin, 1978).

Iterating IFRs through physical and mental measurement activities. The practice of iterating IFR is considered helpful for students to improve accuracy and to maintain the IFRs in measurement estimation tasks. To iterate means that the estimator neither creates a gap nor an overlapping area when using a unit in a measuring activity. In other words, students should know where to begin and where to end when using IFR as a unit repeatedly (Joram et al., 1998). Tasks involving the use of IFR in physical measurement (approximation) should be introduced first before students shift using the IFRs in mental ways (estimation).

Promoting flexibility to use IFRs. The context of the tasks should engage students in both physical and mental activities in which they are stimulated to seek efficient but more accurate answers. Students should be facilitated to establish additive/multiplicative relations among IFRs and coordinate them (small, larger and larger units). For instance, a task which involve both using feet and arm spans can be discussed in a classroom for seeking their relation and efficiency. In addition, the length estimation tasks are not merely meant to ask students to estimate the length of named objects but also to challenge students to name objects with specific lengths (Bright, 1976). It is also important to provide a situation in which the students use IFR flexibly to estimate both horizontally (length) and vertically (height). A study by Jones (2012) suggests that the students could perform well both in the two orientations.

It should be noted that the three elements as described above do not work in progressive ways but in unison. This means that the three elements may occur in

the same instructional activity. For instance, when practicing of iterating the IFR, students may think flexibly and develop new IFR which might be more efficient than the previous ones. It suggests that, the development of IFR and flexibility of using them can be achieved through practice of physical approximation and mental estimation. Therefore we can summarize the mathematical development of IFR and estimation skills as shown in Figure 1.

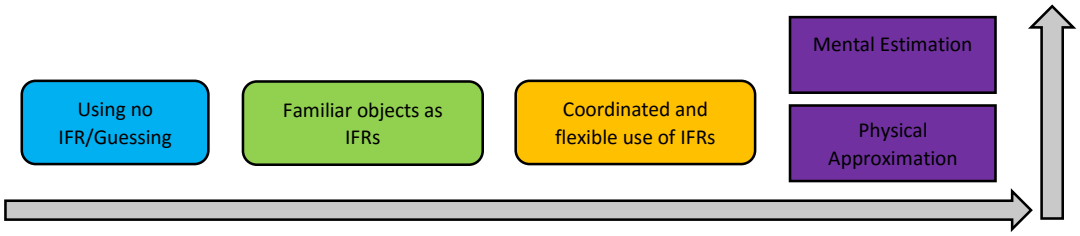


Figure 1. Two dimensional development of IFR and estimation skills

2.3. Realistic Mathematics Education Approach to Support Instructions for Development of students' IFR and Estimation Skills

Realistic mathematics education (RME, hereafter) is an educational approach rooted in Freudenthal's view of mathematics as a human activity (Freudenthal, 1973). He sees that an educational goal of mathematics education should facilitate students to be able to mathematize an everyday problem situation in mathematical terms and employ it within mathematics itself (K. P. E. Gravemeijer, 1994).

In case of the development of students' IFR, the process of mathematizing appears when students grasp a situation in which they interact and be familiar with objects then assign the standard unit of measurement to the objects (informal knowledge). Later on, within mathematics itself, students mathematize the use of IFRs into more formal numerical precision by means of iteration and accurateness using IFR.

RME provide a framework for designing an instructional task for the progressive mathematization which is determined by five tenets of realistic instructions by Treffers (1987) as follows:

2.3.1. Phenomenological exploration by means of context

Phenomenological exploration means one should do a thought experiment to seek contexts that are self-contained or can contain mathematical ideas, concepts or structures. The context might be in the form of a game or a story that is sufficiently real and meaningful in the students' mind for mathematizing. It serves as both a starting point and an end point of an instruction which makes mathematical knowledge and abilities applicable. Moreover, it also convey meanings of mathematical ideas within mathematics itself (formal operation).

Considering the fact that measuring, approximating and estimating are context-bounded tasks, one should bring out meaningful contexts for development of IFRs through making sense of units through daily objects as the starting point (Sowder, 1992) and estimating length in various application in the real world as the end point of the instruction (G. Jones et al., 2009). By this learning path, the students will experience a meaningful learning that connects their informal knowledge to real-life problems supporting mathematization of estimation problem.

2.3.2. Bridging by vertical instruments (modelling)

Models serves as a bridge between mathematical phenomena in the reality and a formal system as symbolic representations of the real-world situations in which the mathematical ideas are embedded. The process of modelling could be powerful for structuring, generalizing and reflection of mathematical ideas by conveying meaning to symbols, procedure and formula.

Modelling in measuring, approximating and estimating takes form as situational models of the contextual problems. For instance, representations of attributes (to-be-estimated objects) such as the length of a school yard takes the form of sketched drawings of the school in which iteration of IFR be performed. On a higher level, it could be represented as magnitudes in numbered line in which the IFR serve as the units. Shifting students from the exhausted representation of

measuring, approximating or estimation to the general representation such numbered line models may contribute to efficient and appropriate iteration of the IFR. The notion of the iteration may then trigger the notion of less error production and coordination of smaller and larger IFR. Therefore, promoting the use of models in estimating instructions could support more accurate and reasonable estimations.

2.3.3. Pupils' own constructions and productions

Supporting students to mathematize means to let the students grasp the mathematical idea, concepts and structures in meaningful tasks by their own actions (construction) and their reflections (productions). A set of instructions cannot be forced to the students if they are not ready yet to comprehend them. In other words, the instructions basically are constructed by the students as they achieve. In this case, the role of the teacher is not as the information transmitter rather as the supporter for the construction of knowledge of the students.

Accordingly, the process of developing IFR through measuring/approximating or estimating tasks cannot be forced to the students because IFRs relates to a mental perception of length through the process of internalization. The internalization process itself occurs individually depending what they perceive and experience. Therefore, to support the development of students' IFRs, one should let the students reflect by themselves about the choice of the IFR.

2.3.4. Interactive instruction

Interactive instructions provide an opportunity for students to participate, to negotiate and to cooperate about mathematical tasks to other students. This interactivity may support the process of constructing and reflection of the students. In this case, the teacher functions as the moderator and facilitator of the interactivity.

This tenet can be applied for instructions of developing students' IFR because a selection of IFR tends to dependent on the students' preference. This individuality may probably have some degree of error and misinterpretation. It then impede the development of IFRs. In order to minimize the error and

misinterpretation, the students should be engaged to listen, discuss and negotiate other students' perspectives, strategies and IFR used toward the tasks.

2.3.5. Intertwining of learning strands

It is very rare that a mathematical phenomenon consists of only one mathematical concept. Usually it manifests to form connected links among several mathematical ideas or concepts. For instance understanding estimation needs arithmetic fluency and proportional thinking such in case of estimating from a photograph and coordinating larger IFR. This suggests that instructions in estimation length for developing IFR cannot be isolated only in the measuring strands, instead it involves some portion of arithmetic and proportion.

2.4. The Role of the Teacher Based on RME to support Instructions for Development of students' IFR and Estimation Skills

As mentioned earlier, the role of the teacher in mathematics education should allow students to mathematize by themselves. The role of the teacher shifts from a transmitter of information or instructions and one who gives justification as right or wrong, to be the one who plan, organize, facilitate and guide students along their learning path (K. P. E. Gravemeijer, 1994). Accordingly, the role of the students is also changed from passive receivers of information to the ones who actively construct the learning route. They have opportunities to give explanation, justification and argumentation about their own work without relying on the judgments of the teacher.

As an example, to be able to understand the mathematical ideas of iteration (how to iterate correctly), students may be given an opportunity to pace units of measurement/IFR to objects being measured/estimated and discuss relations between the iterations and produced errors. The students should discover the big ideas of "the more you iterate the more chances you probably get bigger error" by themselves. Meanwhile, the teacher should hold back from telling this big idea. In other words, the teacher should be able to 'hear' all students' reasoning and give appropriate feedback without interruptions and judgments (Hattie & Timperley, 2007; Towers & Hunter, 2010).

Nevertheless, shifting both the teacher's and the students' role does not come spontaneously There is a need to establish and renegotiate socio-norms among the classroom community so that the students realize that they cannot always go to the teacher for clarifications (K. P. E. Gravemeijer, 1994; Yackel & Cobb, 1996). It implies that, either explicitly or implicitly, the teacher should be able to establish what Yackel & Cobb (1996) called socio-mathematical norms where students and the teacher agree on what count as mathematically different, sophisticated, efficient or elegant and what counts as acceptable mathematical explanations and justifications.

Accordingly, since estimation tasks require students to cope with rough thinking, inexactness and vagueness (Forrester & Pike, 1998) the teacher should be able to establish agreements what kind of answers/strategies is considered as "good guesses" and "efficient" (e.g. reasonably using IFRs to estimate) and what kind is not acceptable (e.g. wild guessing without using reference points) (Lang, 2001). On advance levels of estimation tasks in which measurement cannot be performed physically, the teacher should also be able to create socio-mathematical norms of how to judge which mental estimation is better based on the IFR used. In addition, there is a need to agree about the mathematical terms involved in the instruction such words like guessing, measuring, approximating and estimating (Towers & Hunter, 2010).

2.5. The Concept of Length Estimation in Indonesian Curriculum

Based on the Indonesian curriculum 2013 the concepts of measurement and length estimation are introduced in the first grade (6-7 years old), the second grade and the third grade (Kemendikbud, 2013). Table 1 shows basic competences for teaching and learning measurement and length estimation in those grades.

Table 1. Basic Competence for Measurement and Estimation in the Indonesian Curriculum 2013 for Earlier Grades

Grade	Basic Competence
One	Comparing and estimating lengths of objects using daily-life terms (longer than, shorter than) Comprehending length, area, mass, capacity, time and temperature.
Two	Comprehending lengths and masses of objects, and distances of places in daily life, school and playground using nonstandard and standard units of measurement.
Three	Comprehending the relation between units of time, units of length, and units of weight which are usually used in daily life. Estimating and measuring length, perimeter, area, capacity, mass, time, and temperature using standard and nonstandard unit of measurement.

It can be seen from Table 1 that the basic competences are sequenced to promote integration of measurement and measurement estimation. In the first grade students are taught to estimate the relative size of objects. In the second grade, students are introduced to both standard and nonstandard units of measurement of familiar objects. Finally, in the third grade, students are engaged in formal measurement and estimation and length activities. However, the basic competences for the third grade will not yet have been implemented before June 2015 (Joewono, 2013). Therefore, no specific information can be evaluated from length estimations instructions in the curriculum 2013.

In addition, as a comparison, Table 2 shows standard competences and basic competences of geometry and measurement topic in the Indonesian Curriculum 2006 (BSNP, 2006).

Table 2. Standard Competences and Basic Competences of Geometry and Measurement in the Indonesian Curriculum 2006 in Earlier Grades

Grade	Standard Competence	Basic Competence
One	Using time and length measurement	Comprehending lengths of objects using daily-life terms (long, short) and comparing them.
Two	Using time, length, and weight measurement in problems solving.	Using standard and nonstandard measurement tools (cm, m) which are usually used.
Three	Using time, length, and weight in problem solving.	Using measurement tools in problem solving.
		Comprehending the relation between units of time, units of length and units of weight.

We can see from Table 2 that the notion of length estimation is not embedded in the curriculum 2006. Instead, the curriculum tends to focus on using units (standard and nonstandard) and 'exact' physical measurement by promoting the use of measurement tools. It is not surprising that the analysis of most used standardized books (Fajariyah & Triratnawati, 2008; Masitoch, Mukaromah, Abidin, & Julaeha, 2009; Mustoha, Buchori, Juliatun, & Hidayah, 2008; Purnomosidi, Wiyanto, & Supadminingsih, 2008) issued by the Indonesian ministry of education (Kemendikbud, 2014) shows no evidence of length estimation instructions. Hence, students have no chance to employ their informal knowledge about the lengths of familiar objects to be used as a reference for both measuring and estimating since the instructions are dominated by using measurement instruments (e.g. rulers).

In addition, the way of teaching and learning mathematics in Indonesia makes length estimation difficult to develop. This is because measurement estimation should involve problem solving and making sense of units. In reality, the mathematics classroom in Indonesia seems mostly to be mechanistic and only focused on abstract concepts like algorithms and memorizing procedures instead of understanding and application (Hadi, 2002; Zulkardi, 2002).

In the case of learning measurement, the teaching and learning tends to be formal and focused on memorizing procedures for converting standards units of measurement, with little emphasis on making sense of units. The mechanistic learning also causes students to have difficulties to cope with problem solving which involves the construction of mathematical symbols from contextual problems (Sembiring, Hadi, & Dolk, 2008). It implies that developing IFR seems to be difficult in the mechanistic teaching and learning instructions because length estimations tasks are very common in contextual problems. Therefore, designing a learning and teaching trajectory for length estimation should prevent students from a mechanistic way of memorizing and instead promote the process of making sense of units through contextual problems.

2.6. Present Study

This present study tried to design instructional activities to support students' development of IFRs for estimating length and to contribute to the development of a local instructional theory in mathematics education. Indeed, the development of IFR is strictly related to the process of making sense of units (internalization). It implies that the starting point of students learning is that students already learn about both standard and nonstandard units of measurement. Therefore, according to the Indonesian Curriculum 2006 it is appropriate for instructions for the fourth graders.

The activities focused on aspects of the two dimensional development of IFR and estimation skills (see Figure 1) and will be based on the five tenets of RME. Nevertheless, we realized that such development is too big for this small series of lessons. Hence, we thought the aspect of mental estimating would not become the primary goal of this lesson. Instead, we facilitated students to be able to start to mental estimate by first experience processes of approximating and gaining new vocabulary of individual frame of references.

To begin with, we designed activities in which rough and quick result more desired with a purpose to shift students to not rely on a ruler but using a rough approximation. Secondly, students were more engaged in activities in which they

come up with the idea of using body part as reference points. It is also meant to promote students to the process of connecting and reason using the body references to external to-be reference objects. Later on, they also were challenged to connect and reason using their IFR (objects) from photographs to learn new references points and they were expected to discuss the efficiency. Finally, they were given chance to apply their new vocabulary of IFR to be used for approximating/estimating objects in an experience based learning.

Therefore, based on explanation above, we formulated following specific research questions:

1. *What strategies used by students to approximate/estimate lengths?*
2. *How could the use of body part/familiar objects for approximation and estimation facilitate the development of individual frame of reference?*

CHAPTER 3
METHODOLOGY

In this chapter, we describe methodology of this research into five main points. First of all we will elaborate selection of the research approach. Second, in data collection, there will be descriptions of what methods and instruments we use to collect the data, we also describe validity and reliability issues of the data collection. Third, in the data analysis, we will describe what method we employ to analyze the data, we also describe validity and reliability issues of the data analysis. Finally, we describe the research subjects of the research.

3.1. Research Approach

Choosing a research approach is a matter of selecting an approach that is most possible to be successful to answer the research aims (Denscombe, 2010). Considering our research question, *how can we promote the development of students' individual frame of reference to support length estimation skills?* We aim to improve the theory of teaching and learning in mathematics education. It seems to be logic that we employ an approach that directly speaks how to design a learning trajectory such as classroom activities and the teaching and learning materials. Therefore, design-based research (DBR, hereafter) was chosen as the most suitable research approach of this study.

DBR is characterized by its cyclical or iterative process of designing-revising the educational materials specifically the learning trajectory (Bakker & van Eerde, 2013; Barab & Squire, 2004; Edelson, 2002). The learning trajectory is designed and tested in 3 phases (preparation and design, teaching experiment, and retrospective analysis) and revised in several cycles (one cycle consists of the three phases) (Bakker & van Eerde, 2013; K. Gravemeijer, 2004). Further, DBR does not only also speaks how to design, but also how to describe the students' learning development which is used to advice for better teaching and learning action.

According to the topic of this study, using DBR we would design a learning trajectory describing a learning instruction to develop IFR for supporting length estimation skills. Moreover, we would also explain how the students' learning and give practical advice about it.

Hypothetical Learning Trajectory (HLT)

We designed the learning activity based on rationale that knowledge and the learning path should be constructed by the learner itself (the third tenet of RME) and teacher only functions as a facilitator of that journey. Students are put in learning situations where the learning is neither easy nor difficult but in the edge of the students' knowledge (Dolk & Fosnot, 2001). In this regard, the learning should facilitate openness for students' constructions but in goal-oriented activities.

Thought experiments about the possibilities of students' responses for the open tasks should be formulated. This thought experiment was used to construct the instructions in such a way it could render the gap between the goals and the possibilities of students' responses. To accommodate this, we employed hypothetical learning trajectory (HLT) as a design and a guide about how and when certain development in learning of students should be emerged (Simon & Tzur, 2004). HLT consists of three main elements: goals of the learning, learning activities and prediction of students' learning.

In this study, HLT acts as the main artifacts that we designed, tested, and improved approached by the design-based research. Accordingly, we formulated a learning trajectory of developing students' IFR to support length estimation skills consisting of 5 HLTs in 5 lessons (cycle one). All of the HLTs was designed, tested and revised through the following DBR phases in 2 cycles.

a. Preparation and Design Phase

In this phase, the designer used all relevant knowledge including the previous local instructional theories and teaching experience to construct an initial HLT. Here, the role of the HLT is as the design itself. First of all, the designer reviewed journals, learning instructions, and related curriculum material. Using this information, the designer conducted a thought experiment to design the initial HLT.

In this study, things already mentioned would be conducted for the preparation phase in the both two cycles. For instance, reviewing theories and

curriculum materials related to strategies for length estimations, the use of individual frame of reference in length estimations and teaching process in length estimations class. We also conducted a pretest for the students and interview with the teacher and classroom observation.

b. Teaching Experiment Phases

The aim of teaching experiments is to test the design. Data of students' learning such as written work and classroom observation are collected during this phase. It is then used to adjust the next day of the teaching experiment and to be analyzed for the third phase of DBR (all the lessons). In this phase, HLT takes role as a guidance for the teacher to conduct the learning.

In this study, in the first cycle, the designer as the teacher, conducted a pilot teaching experiment, implemented the initial HLT to selected participants. Meanwhile, in the second cycle, a regular teacher and students from another class attended the teaching and learning experiments. The students' written work and data of classroom observation were collected as the basis for analyzing and improving the HLT.

c. Retrospective Analysis Phase

In this phase, collected data (students' written work and observation of the classroom) from series of teaching experiments are analyzed. Conjectured students' thinking formulated in the HLT are compared with the actual learning in the teaching experiment to see whether they are confirmed, rejected or not predicted beforehand. This time, the HLT functions as a guideline for evaluation.

In this study, the results of retrospective analysis of the first cycle were used to revise the initial HLT for the next cycle. Meanwhile, in the second cycle the results were used to answer the research questions and contribute to development of local instructions theory on developing students' IFR to support length estimation skills. In addition, to strengthen the analysis the result of the pretest were compared to the post-test conducted after all lessons.

3.2. Data Collection

In this study, data were collected from the preparation phases and the teaching experiments phases of design-based research and posttests afterward. The data were collected by semi-structured interview, observation, and questionnaire (written tests) (Denscombe, 2010).

3.2.1. Preparation Phase

The main aims of data collection in preparation phase are to study relevant present knowledge of students and get insight about the learning environment. In order to study students´ present knowledge, a pre-test was held for each cycle. The pretest assessed the students whether they had and used IFR on estimation or approximating tasks (see booklet). Students´ written work were collected afterward and by considering the teacher's advices, four students were chosen. The chosen students were then interviewed to reveal their reasoning and understanding about the topic which was about the use of IFR for length estimations. The interview was audio-visually registered using a video recorder.

On the other hands, to study the learning environment, observation of teaching and learning in a classroom was conducted by taking field notes focusing on both social norms and socio-mathematical norms and other aspects related to process of students learning mathematics in classroom (see Appendix 1. Classroom observation scheme). Afterward, the teacher was interviewed focusing on aspects described in Appendix 2. The interview also was registered using a video recorder.

3.2.2. First Teaching experiment (first cycle)

The participants in this pilot teaching experiment were the selected students from the preparation phase, the researcher as the teacher accompanied by his coworker as the cameraman. Kinds of data collected during this phase were data of classroom observation recorded by video and data from students' written work during all the 5 lessons of cycle one.

3.2.3. Second teaching experiment (second cycle)

The participant in this phase were all students from a selected class including a focus group (4 students) selected in the preparation phase of cycle 2 and a regular teacher. In this natural setting, kinds of data collected were data of

classroom observation recorded by video and data from students' written work during the series of lessons.

3.2.4. Posttest

After the series of lessons, both in the first cycle and the second cycle, a posttest was conducted to assess the students' development of IFR for estimating length. Items of the posttest were constructed the same to the pretest (see booklet). Students' written work of the post-test were collected. In addition, the focus students or the focus group were also interviewed focusing on their reasoning and thinking to solve the problems.

3.2.5. Validity and Reliability of Data collection

Validity issue of data collection is related to the credibility/trustworthiness of collected data, whether the instruments/constructs really measure as we want to measure. The trustworthiness of the collected data is really important in this study to show the outsiders that the data gained really reflect the conditions of the students or the teachers. Indeed they are as ground of our reasoning to answer the research questions.

The instruments/constructs such as pretest/posttest, students' written test, interview guide, classroom observation guide, and the HLT were consulted to experts before being used to collect the data. The expert were a team of supervisors of the researcher who have a lot of experiences in research on realistic mathematics educations and design-based research. In addition, we also conducted what Denzin in (Bryman, 2003) called, methodological triangulation, gathering data using more than one methods to reduce the uncertainty of the data. For instance, along with the classroom observation or the pretest and posttest, we conducted interview with the teacher or the students. The consultation to the expert and the triangulation method were expected to be able to enhance internal validity of data collection.

In addition, in the second cycle, the data were collected from a natural classroom setting where most learning actually occurs and complex social interaction among the participants happens. This was expected could contribute to the ecological validity of the data collection.

In addition, we used a video camera to record both the interview with the students and the teacher and the preliminary classroom observations. For the teaching experiment we used two video camera: one camera was a static camera that was still in one place to record all learning process and the other one was a moveable camera to record some crucial moments in learning of the focus students. These tools were expected to be able to give authentic and objective information/data and contributed to the internal reliability of the data collection.

3.3. Data Analysis

In this section, we describe how data gained from interviews, classroom observation and students' written work are analyzed. We elaborate the discussion of data analysis in the preparation phase, teaching experiment phase of DBR and the posttest.

3.3.1. Preparation Phase

Data collected in the preparation phase both in the first cycle and the second cycle are analyzed to know students' present knowledge, to select focus students and to prepare teaching experiments.

Students' written test of the pretest were graded according to an assessment rubric (see booklet). The students' grades along with the teacher advice were used to select a small group of students (4 students) with considerably heterogenic as participants in the first cycle and as the focus group in the second cycle. Afterward, the data from the students' interview of the pretest were watched chronologically and crucial fragments consisting students' strategies, reasoning or arguments about the pre-test, were interpreted. The analysis focused on determining the starting points of the students. The starting point functioned for adjustment for the first teaching experiment.

Meanwhile, the registered video of the interview with the teacher and the registered video of the preliminary classroom observations which focus on the classroom environment were also watched chronologically and analyzed. The crucial fragments which gave clues about socio norms and socio mathematics norms of the classroom were interpreted. The analysis were used as grounds to

establish or adjust appropriate socio norms and socio mathematical norms for the teaching experiments on the topic of developing students' IFR to support students' length estimation skills.

3.3.2. First Teaching Experiment (cycle 1)

Data from the students' written work and classroom observation were analyzed and interpreted. The video were watched chronologically. Crucial fragments which reflect particular learning goals and students' development of IFR were chosen to be interpreted along with the corresponding students' written work. These interpretations of the actual learning of the students were tested to other episodes of the lesson. The final interpretation were compared to the HLT whether they confirmed the hypothesized students' learning or show crucial learning processes that were not predicted beforehand. All of these data were used to adjust and improve the initial HLT for the teaching experiment in the second cycle.

3.3.3. Second Teaching Experiment (cycle 2)

Data from the focus students' written work and observation of the lessons in the second teaching experiment were interpreted by the researcher and his colleague. The registered video of the classroom observations from the two cameras (one for whole class and one for the focus group) were watched chronologically. Similar with the first cycle, crucial moments of learning of the students of the focus group which reflected particular learning goals were fragmented and transcribed.

The fragments along with the written work were interpreted and tested against other episodes of the lessons. The final interpretation of the fragments were compared to the present HLT whether the hypothesized students' learning were confirmed or rejected. Moreover, other crucial fragments of students' learning that were not conjectured in the HLT beforehand were also interpreted and analyzed why they occurred.

In addition, data from other students outside the focus students were also analyzed. Due to time limitation of the study, it seemed to be impossible to analyze the data as deep as focus students. The analysis would be in a more general level especially the information about their interaction in the class discussion.

The result of the data analysis from the first cycle and the second cycle were elaborated in the retrospective analysis and used to answer the research questions of this study and to make recommendation for next implementations.

3.3.4. Post test

The results of the posttest of all the students in the first cycle or the focus students in the second cycle were graded according to an assessment rubric (see booklet). In addition, registered data (video) of interview of the post-test with the selected students were interpreted. Crucial moments that shows students' reasoning, strategies and arguments especially the use of IFR for estimating length were fragmented and transcribed. The fragments were interpreted then the results were compared to the results of the corresponding pre-test. Analysis of the comparison were done in qualitative ways and in a modest quantitative analysis (no inferential statistical methods were used). The conclusion of this comparison were used to investigate the learning achievement during the whole learning process for each student and to strengthen the analysis for answering the research questions in the second cycle.

3.3.5. Validity and Reliability of Data Analysis

The following explanation is aimed to describe how we see validity and reliability of data analysis and what we do in order to strengthen them for quality of this study.

a. Validity

Validity of data analysis is elaborated into internal validity and external validity. Internal validity in data analysis concern on the soundness of reasoning when analyzing data that is used to lead to the conclusions. Meanwhile, external validity concerns on a question, do the results can be generalized across situations?

Data triangulation was performed when analyzing the data gained from the several methods such as students' written work and classroom observations. It was expected that these independent data could contribute and clarify each other to reduce uncertainty of the data or assumptions made. This was expected to contribute to the internal reliability of the data analysis.

During the retrospective analysis we looked for confirmations and counter examples about the conjectures on the existing HLT. Episode by episode revealed

themselves, and be tested against each other to be used to improve the HLT for the next cycle. The way of our reasoning during this improvement process also strengthened the internal validity of data analysis.

Meanwhile, in order to enhance the external validity, we described detail our data analysis, framed the important issues and embedded the limitation of the learning process in the teaching experiments. We also created explicit educational material (HLT, teacher guide, and students' worksheet) which could be followed by outsiders. Therefore they could judge to what extent they should adopt the HLT for their own educational setting.

b. Reliability

Reliability of data analysis is elaborated into internal reliability and external reliability. Internal reliability concerns on repeatability or consistency of the analysis. Meanwhile the external reliability concerns on replicability of the study to a similar situation.

We conducted peer examinations for interpreting the data gained from the classroom observations and the students' written work. The data were also consulted to the team of supervisors of the researcher. Moreover, a conference about how to analyze data on DBR was held which was expected that the outsiders share common understanding when analyzing the data (intersubjectivity) (Anderson, 2008). This was expected to contribute to the internal reliability of this study.

The reporting format of this study was clearly as possible organized in trackable way. We transparently made thick descriptions of the failures and success, procedures, the conceptual framework used as ground of selection of the context of the lessons, type of students activities, and the formative assessments in this study as explicit as possible. This trackability could facilitate the outsiders for replication and reconstruction of this study. Therefore, by making the report of this study trackable and transparent, it is hopefully that the experience in this report could reveal itself to be justified and experienced by others (Freudenthal, 1991). This contributed to the external reliability of this study.

c. Ecological validity

Ecological validity concerns how far the results of data analysis is applicable to the real world setting without boundaries of certain situations including artificial elements. Indeed, the strong point of the design-based research is its natural setting when implementing the design, where most the learnings actually occurred and complex social interactions among participants happened (Barab & Squire, 2004). Moreover, since the researcher and the teacher were intensively involved in discussions about the design and implementation, feedbacks from the teacher were used to adjust the design to the classroom setting. This also contributed to the ecological validity of this study.

3.4. Research Subjects

The study was conducted in MIN 2 Palembang, one of Islamic elementary school in Palembang, Indonesia. The reason why the school was chosen because the school has involved in *Pendidikan Matematika Realistik Indonesia* development project which was the Indonesian version of RME. The participants of this study were fourth graders from two different classes, each for the first cycle and the second cycle and a regular teacher of the class for the second cycle.

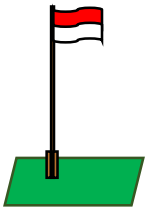
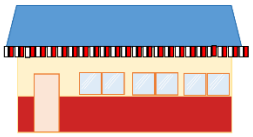
CHAPTER 4
HYPOTHETICAL LEARNING TRAJECTORY

HLT serves to minimize discrepancy between openness and goal-oriented elements of learning tasks. Hence, one should formulate conjectured students' thinking along with learning activities/tasks and goals of the learning. In this study, HLT functions as instructional design, guideline to conduct the teaching, and evaluation guideline of the design.

In this chapter we elaborated hypothetical learning trajectory (HLT) as the main instrument of this study into five lessons. In each lesson, we described the learning goals of the lessons, learning activities, and hypothesized students' thinking together with suggestion for the teacher to response to the students' thinking. HLT in this chapter was a prototype of the design as the result of thought experiment in preparation phase of design-based research. Therefore, this HLT has not been yet implemented in teaching experiments.

Following we give overview of the learning trajectory of this study to ease the reader to understand the development of students' learning in our design (see Table 3). Along with that, we also ask the reader to read the teacher guide, students' worksheet, and students' personal journal of reference in the booklet of this thesis.

Table 3. Overview of the Learning Trajectory

Illustrations	Activity	Main Goal
	Frog Jumping and *Pocong* Jumping	Shift students' from using rulers to use other reference such as body part for approximating
	Measure and Use Your Body Parts	Knowing accurate lengths of body parts to be used as references.
	Length, Width and Height of the Building	Develop other external references and ordering their efficiency. Visualizing iteration of references.
	17th of August Decoration: Balloons and the Flag Pole	Internalization of standard units of measurement. Shift students to do mental iteration (estimation) using references.
	17th of August Decoration: Plastic Flag and the Rope	To spot, use and reason using individual frame of reference to solve length estimation problems involving simple social arithmetic.

4.1. Frog Jumping and *Pocong* Jumping

4.1.1. Starting Point

Starting point describes what concepts or knowledge that students already learn and might be necessary as prerequisite knowledge for students to learn new concepts from the present activity. Since the development of students' IFR relates to a process of internalization to standard units of length measurement, the starting point is that students know both standard (metric systems) and nonstandard unit of length measurement. It is not necessarily that the students already understand the concept of unit iteration since this concept will be explored in this learning sequence.

4.1.2. Learning Goals

The main goal of this lesson is that students shift from relying on rulers to use other references for measuring. Through a game, students are expected:

- Students are aware that short rulers are tedious to use to measure long attribute.
- Students use external reference for measuring rather than rulers.
- Students understand the word 'efficient' in approximation/estimation.

4.1.3. Description of Learning Activities, Hypothesized Students' Thinking and Suggestion for the Teacher

Part A: Frog Jumping and Pocong Jumping

This activity is chosen to accommodate learning through a game, raise awareness to compare lengths, stimulate conflict about used reference and to shift students from using rulers to more efficient ways. In this part, the students in group of 4, measure a long distance through a game called "Frog Jumping and *Pocong* Jumping". The game is played on the school yard, each students in their group alternately jump like a frog 4 times, jump like a *pocong* (Indonesian ghost) 4 times, and mark their distance using a drinking straw, as shown on Figure 2.

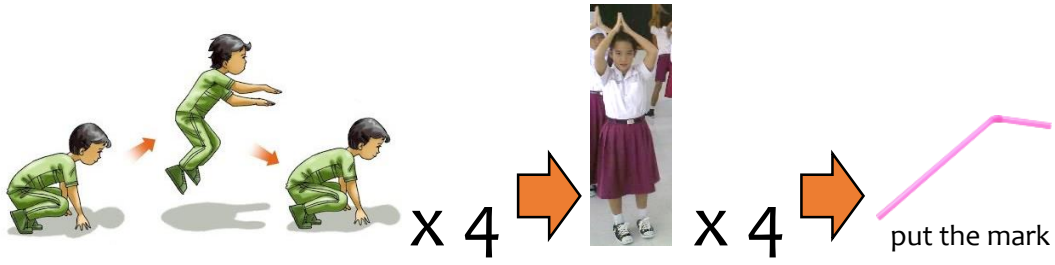


Figure 2. Illustration of Frog Jumping and Pocong Jumping Game

In order to be a winner of this game, a linear track made of the students' jump should reach more than 30 meter long. Students are expected to measure the track as quick as possible using rulers or other references before claiming their victory. A groups who announce that their track reaches 30 meter long will be discussed by other students to check the measurement and used strategies.

Hypothesized Students' Thinking and Suggestion for the Teacher

In the measuring phase, we conjecture that:

a. Some students use a ruler to measure the length of the track.

   If students do this way, the teacher should let them. There is no good reasons to directly prohibit them to use ruler (artificial) since students should realize by themselves why they have to use other references instead of rulers. However, the teacher can remind the students to work quickly and not in a very precise way otherwise they will be beaten by other groups.

   It is important that this strategy is compared to other strategies of measuring because it is a crucial point to naturally shift students from using rulers to other quick and rough strategy of measuring. The teacher may pose a questions such as "Who can explain which way is better to measure the track, using rulers or other reference (e.g. walk pace) in this situation?"

b. Some students might feel tedious to use a short ruler to measure the track and considering the fact that they need to be quick, they might use their walk pace,

feet, arms spans or other longer references spotted on the school yard for measuring.

If students come up with this strategy, ask them to explain to other students on questions such as "Why do you use this reference rather than rulers?", "How do you use this reference for measuring your track?" Afterward, the teacher may ask other students to tell and explain "Do any of you use other kinds of reference?" The discussion of used reference is expected to make the students aware that they can use other references rather than rulers and enrich their references for quick and rough measuring.

c. Some students might do mental estimation applied partially or in total of the track. (e.g. students imagine the length of their house garden or house).

   If students come up with this strategy, ask the students to explain "What are references that you imagine?" Since the references is in a person's mind, ask the student "Can you show/demonstrate us how long the reference?" then "How you use it to measure your track?" It is really important to discuss further about the strategy, ask other students questions such as "Do you think this strategy is acceptable in this situations?", "What is the advantages and disadvantages using this strategy?" By this questions, it is expected that students reason/mention the notion of "personal feel of length" that one can estimate by comparing a well-known objects in mind to the objects being estimated. Then ask other students, "Can someone tell us about their personal reference?", and ask other students, "Think about the reference mentioned by your friends, do you agree?"

d. Some students might just guess that their track reaches 30 meters. They might reason "I don't know, I just see it", "Because it is long"

   If this situation occurs, discuss it with other students "Do you think their track reaches 30 meter?" From the discussion, it is expected that the students realize that they need to reason about their measurement using a mental reference or a physical reference.

Part B: Journal Time

The aim of this activity is to introduce students to a personal journal that they will use in all the lessons. The notebook mainly is used to record and relate references that students notice during the lessons (see booklet).

In this part of the lesson 1, students individually are asked to record references (and its lengths) that they notice during their measurement activity or from class discussions. After that, they are asked to order them considering their efficiency when used in the Frog Jumping and *Pocong* Jumping Game. Then, there will be a short class discussion whether students write body parts as references and how long they are.

Hypothesized Students' Thinking and Suggestion for the Teacher

Students may mention: rulers, walk paces, arm spans, or other external reference such as the length of the classroom, the length of the school yard, etc. In one side, some of them will write the lengths in centimeter or meter based on what they already hear on class discussions. On the other side they will try to measure their own body parts for references such as walk paces or arm spans. In this sense, students might notice that each persons have different lengths of body parts. In this occasions, the teacher might ask the students "Which length should you write on your personal notebook?" Guide the discussion to reach an agreement that they should write on their personal notebook the length of their own body parts and give them a special note such as "Rudi's walk pace, or Rudi's arm spans"

On the next part of the tasks, it is also predicted that when students order the reference they might argue that

- Longer references will be more efficient (quickness)
- A ruler is more efficient because it has units (accuracy)
- Walk pace is the most efficient because it is easy to be employed (accessibility)

Discussion what "efficient" means is crucial in estimation/approximation activities. Hence, the teacher should encourage a discussion about this notion in this occasion. First of all, let students think in scope of determining 30 meter like in the game. Ask them "Do you need accuracy in this activity?" Secondly, discuss the references if they are used in other estimation/approximation contexts such as determining the length of the school yard, the length of a table, the length of a piece of paper etc. Then ask them "Should you be quick to measure the objects?" "Can you use all the references for the other contexts?" "Do you need accuracy in the new contexts?" Finally, the teacher can establish the meaning of the "efficient"

word in the context of estimation/approximation during the lessons as “You may need to consider three things when choosing a reference: think about its accessibility, think about its accuracy, and think about its quickness and pay attention to the context of the problems.”

4.2. Measure and Use Your Body Parts

4.1.4. Learning Goals

The main idea of these series of activities is to provide students experience to physically approximate objects in photographs. The goals of the learning are formulated as follow:

- Students are able to identify body parts for approximating length on photograph.
- Students are able to determine typical length of body parts of their age.
- Students are able to find additive/multiplicative relations between body parts.
- Student are able to identify situations where body parts can be used for approximating.

4.1.5. Description of Learning Activities, Hypothesized Students’ Thinking and Suggestion for the Teacher

Part A: Approximating on Photograph of Indonesian Traditional Games

This activity is chosen to provide students with familiar contexts in which they are stimulated to use for approximating objects. Students in group should identify objects/reference on 4 photographs of Indonesian traditional games to be used to approximate asked distances/heights/lengths on the photographs. Table 4 shows the 4 photographs and the questions.

Table 4. Photographs and questions of lesson 2 A

Photograph	Question
	1. If you play marble with one hole, a person whose marble is the closest to the hole has the first turn. a. What do you use to measure the distance of the marbles to the hole? b. Approximate the distance of each marbles from the photograph!
	2. *Bakiak* is a long-wooden sandals that is used for *bakiak* race. a. What do you use to measure the length of the *bakiak* sandals? b. Approximate the length of the *bakiak* !
	3. Jumping rope is a traditional game played by most of Indonesian children. a. What do you use to measure the height of the rubber rope above the ground? b. Approximate the height of the rubber rope above the ground!
	4. Playing the two fortresses game needs two poles or usually two trees as the fortress. a. What do you use to measure the distance? b. Approximate the distance between the two trees!

After working in group, there will be a discussion on part A. The discussion focuses on the students' selection of references primarily the use of body parts and

the lengths. Each group compare the answers to other groups which may stimulate a conflict of the lengths of the body parts.

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Some of the students state that they have to use a ruler to measure something. They need measurement tools that has number on it.

   If this case happens, first of all ask other group if they agree with it. If they disagree ask them to explain it "Do you think we can measure something without rulers?" The teacher may remind the students about the first lesson or nonstandard unit of measurement if necessary.

b. Several students may guess the measurement without mentioning the references on the photographs. They use either mental estimation or wild guesses.

   If this case happens the teacher may invite other group to explain whether these methods are acceptable or not. If the worst case happens (all group agree with this method), the teacher may ask the groups how to convince outsiders to accept their answers. For instance to convince others that guessing is a fair method to determine the order of peoples who play the marble.

c. Using body parts

   - For problem 1, they may come up with using the length hand spans for measuring since it is a common ways when playing the game.
   - For problem 2, they may come up with using the length of a feet. They might have also an idea to use the length of an arm.
   - For problem 3, they may come up with using the height of the girls in the photograph.
   - For problem 4, they may come up with using the length of arm spans (5 people arm spans).

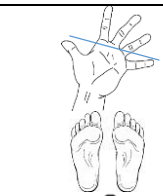
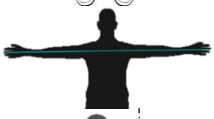
   If students come up with the idea of using body parts for approximating, encourage them to explain how to use them on the photographs, "Why do you think it can help you to approximate the length of this?" or "How do you use it to approximate the length?" We expect that they draw a line on the photographs since students should not involve in mental iteration yet.

The teacher may ask the groups that use the body parts (e.g. hand spans) to compare the length of their hand spans each other. This stage is very crucial before entering Part B.

Part B: Measure Your Body Parts

The aim of this activity is for students to get accurate measurement of their body parts as references for approximating. First of all, each group of the students choose one of their friend within the group who might be typical at their age. Secondly, they measure his/her body parts using rope and a ruler. Lastly, they measure fill the table as shown below and compare the result in front of the class.

Table 5. Body part and actual length

Body part	Actual Length
 One hand span	
One foot	
 One arm span	
 Body height	

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Students may choose a students who is in average body size (not too big/tall nor too small)

   In this sense, students are not necessarily involved in statistical reasoning such as mode, average, median.

b. Students may measure the body height/arm spans using rope then measure the rope using a ruler. There is a possibility that the students incorrectly iterate the ruler.

   If this case happens, ask them "Is it allowed to leave a gap/overlapping area between two iterations of the rulers?" then ask them to compare whether not

leaving a gap/overlapping area is more accurate for measurement. If necessary, the teacher may ask the students to demonstrate how to measure using hand spans in marble games "Is it fair to leave a gap or overlapping area when measure using hand spans when playing marble games?"

c. In the class discussion, students may select the most common numbers that appear on the blackboard as the typical lengths of body parts of students at their age.

   If this happens, ask the students to identify whether they can apply the result to the problems in part A. They might only identify the first and the second problems are played by children at their age. Meanwhile, they might think that the third and the fourth problems are played by older children. In this sense, the teacher may ask the students opinions about the length of the body parts of older children by asking question such as "Do you know the height of your older brothers or sisters?" or other similar questions.

Part C: Using Body Parts in Other Situations

The aim of this activity is for students to identify situations in which certain body parts can be employed for approximating. Students work in group to fill a table consisting 4 body parts (hand spans, feet, arm spans, and body height) and 6 situations/contexts. They are asked to determine whether or not a body parts can be used to approximate the given situations and give explanation about it. Afterward, there will be a discussion by the students to compare the answer. The table below shows how the questions are organized.

Table 6. Body parts to approximate given situations

Body parts	Situation						
	For measuring the height of a door	**For measuring the length of cable**	**For measuring the length of paper**	**For measuring the length of a table**	**For measuring the width of a path**	**For measuring the length of a volleyball court**	**For measuring the depth of a beginner swimming pool**
Hand spans	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation
Feet	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation
Arm spans	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation
Body height	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation	Yes/No Explanation

Hypothesized Students' Thinking and Suggestion for the Teacher

Students' answers will vary. It is predicted that the students' thinking will be based on the following aspects:

a. Students may answer by considering whether certain body parts are too long/too short to be used for the given conditions (e.g. arm spans is too long for measuring paper).
b. Students may answer by considering efficiency of the body parts compare to others in the same situation (e.g. it is easier and more efficient to measure the length of the volley ball court by using arm spans rather than feet).
c. Students may answer by considering whether the body parts are viable to be used in the given conditions or not. (e.g. you cannot measure the depth of the swimming pool by using your hand spans).

In this activity, the teacher may tell the students that multiple answers (yes or no) are possible. The teacher may also encourage the students to discuss about appropriateness of their answers "Which body part is more appropriate in this situations, why?" The teacher may also suggest them to imagine or demonstrate the measuring activity if they come up with different idea. For instance measuring the

door of the classroom or imagining they measure the depth of the swimming pool/imagine the class full of water, etc.

Part D: Journal Time

Journal time gives students an opportunity to keep track their vocabulary and development of individual frame of reference for all of the lessons. In this activity, students, individually, are given a chance to reflect and write references they noticed during the previous activities especially the body parts on their personal journal. Then, they are asked to describe situations in which the new references are suitable to be used. In addition, they are also asked to investigate by looking at the length of the new references (numerically) whether there are additive or multiplicative relations among them.

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Students may think and ask whether they should write or not the references that they already write on the first lesson.

   If this happens, suggest them to re-write the references to the meeting 2 also if there is a correction about the lengths of the references.

b. Students may think and ask, which one they should write: the lengths of body parts of common students or theirs?

   The teacher may suggest the students to write both and give them an extra description such as "the lengths of common body parts of 4 graders" and "the length of my body parts." Consequently, the teacher may let the students to measure their own body parts.

c. Students may think that there is no relation among the references/body parts.

   The teacher may suggest the students to not focus only to find an exact equal lengths but to consider whether their lengths are relatively equal. For instance, their measurement of their body height may be different from the length of their arm spans. However, theoretically speaking, body height and the length of arm spans are considerably equal. The teacher should guide the reflection so that students may come up with this notion.

4.3. Length, Width, and Height of the Building

4.3.1. Learning Goals

- Students are able to spot a reference that they know the length from a photograph and use it for approximating an object on the photograph.
- Students are able to order provided solutions of an approximation problem based on the efficiency of the used references.

4.3.2. Description of Learning Activities, Hypothesized Students' Thinking and Suggestion for the Teacher

Part A: How do you approximate?

In this part, the students work in pair to solve two problems. Working in pair is expected to be able to force the students to think since it is important to discuss a lot of strategy and for preparing a rich of discussion in part B. The first problem is about a greasy pole contest, the students are asked to approximate the height of the greasy pole from the photograph (see Figure 3 left). The idea is that the students are engaged to spot reference points for approximating in a relatively simple situation. Solving this problem is expected can give the students some insight to solve the next problem. The second problem is about approximating the length, height, and width of a building (see Figure 3 right). The students are given a photograph which consists of several references for approximating. After solving the two problems, the students are invited to discuss their strategy to solve the problems especially the choice of references and the corresponding lengths.

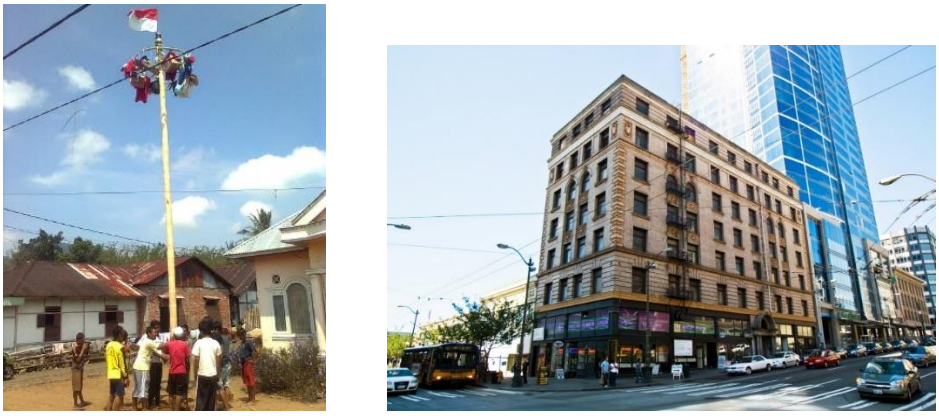


Figure 3. Greasy pole problem (left) and the building problem (right)

Hypothesized Students' Thinking and Suggestion for the Teacher

For the first problem, it is conjectured that the students:

a. Some of the students may guess and cannot give any reasonable explanations. If this happens, the teacher may invite this kind of answers as the very beginning of the classroom discussion. It is expected that it can stimulate a rich discussion from other students.

b. Some of the students may use their previous knowledge about the height of the greasy pole. For instance, it could be that they read or know the height of common poles for the contest.

   Similarly to the first hypothesis, bring this into the class discussion. It is suggested for the teacher to inform that the pole might be different from one contest to others so that we should use the photograph as the clue. The purpose is to prevent a single judgmental-solution from whom that know the height of the common pole.

c. Some of the students may imagine and assign other external references to approximate the pole. For instance, they may recall the height of a tree then reason that the height of the tree is equal or two times as etc. the height of the pole.

   If students come up with this idea, invite them to clearly as possible describe the imagined tree or other external references. The teacher may question "What kind of objects do you imagine?" "Could you tell your friend how you know the length of the objects and its relation to the pole?" "Can you show us how high/long is it?"

d. Some of the students may mentally imagine a standard unit and iterate it to the pole. For instance students may segment the pole by one meter in scale of the photograph.

   It is important for the teacher to bring the students to discuss it to the whole class. Since the ability to mentally feel "one meter" is a very crucial for either approximation or estimation. Ask the students to answer "How could you know that it is one meter?" "Can you demonstrate to make one meter?"

e. Some of the students may use a person's height on the photograph and iterate it for approximating the height of the pole.

   The teacher may invite the students to explain how the iteration is done. In addition, it is also important for the teacher to ask the height of the person that the students may assume then bring it to the classroom discussion.

f. Some of the students may use the height of the house on the photograph as a reference for approximating.

   The teacher may ask to the students "Why do you use the height of the house as the reference?" "How can it help you to approximate the height of the pole?" "How do you know the height of the house?"

For the second problem, it is conjectured that:

a. Some of the students may just guess and cannot give any reasonable explanations.

   If this happens, the teacher may invite this kind of answers as the very beginning of the classroom discussion. It is expected that it can stimulate a rich discussion from other students.

b. Some of the students may mentally imagine a standard unit and iterate it to the pole. For instance students may segment the building by one meter in scale of the photograph.

   Accordingly, the teacher may ask to the students to answer "How could you sure that is one meter?" "How do you know that?" "Can you demonstrate to make one meter?"

c. Some of the students may use objects on the photograph as references. For instance: the length/the height of the windows, the length of the car, the length of the bus, the height of one floor, the height of the road lamp etc.

If this case happens, it is suggested to the teacher to bring them into a class discussion and ask the students to explain how they are used to approximate. Moreover, the discussion should also discuss the lengths for each particular references.

In general, in part A, the teacher is suggested to focus to bring the students to discuss about the used references and their lengths. A discussion about the efficiency and comparison between the references are not necessarily conducted in this part since part B will accommodate the students to do the comparison.

Part B: Ordering the Strategies

In this part, the students work in groups to discuss on the second problem of part A (the building problem). They will be given three different sample of the solutions of that problem. They are asked to order the strategies (references) used based on its efficiency to solve each particular questions: for approximating the length, the height and the width. All of the given solutions do not have approximation in standard unit (meter). This is aimed to make the students to focus more on the references. For instance, the height of the building is only written as 8 floors height instead of 8 times 3 meters (each floor 3 meters). Following are the 3 solutions given to the students:

Figure 4. Example of solution (solution a)

Figure 5. Example of solution (solution b)

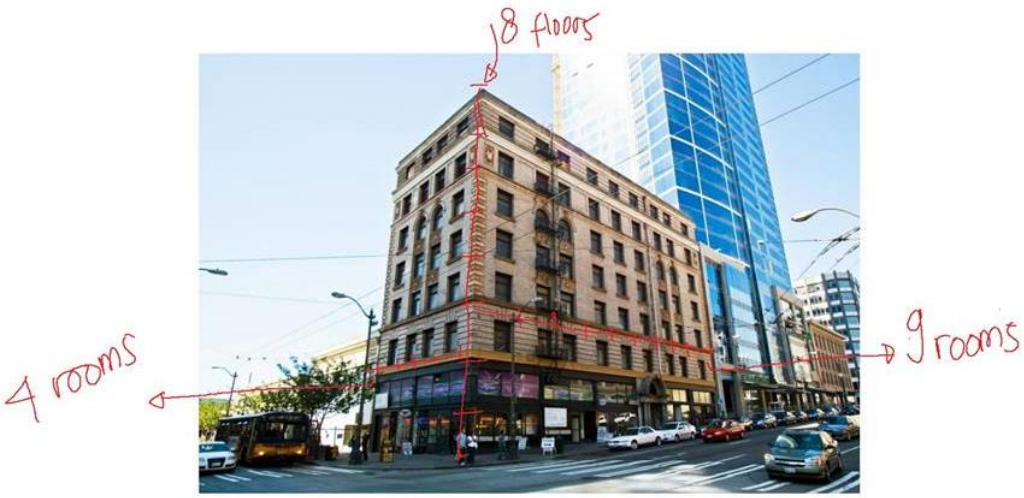


Figure 6. Example of solution (solution c)

After they discuss and analyze the three solutions, they are asked to order the used strategies/references based on its category (shown on the table below). For instance, the first row (the most efficient) is not necessarily filled by the same solution (solution a for instance) instead, the students free to write "approximating the length: (1st row) solution A: using the road lamp, approximating the width: (1st

row) solution B: using the doors, approximating the height: (1st row) solution B, using the doors etc.

Table 7. Table for students ordering the efficiency of the reference

No	Approximating length	Approximating Width	Approximating Height
1			
2			
3			

A class discussion focusing on the order are held afterward to discuss the several of possibility of the students' thinking.

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Some of the students might think that the most efficient references are the ones that are located at the building itself. For instance, they may prefer the door or the windows as the reference rather than the bus, the car and the road lamp.

   The teacher may ask the students in the discussion, why they prefer the embedded references rather than the others. Bring the discussion to the idea of error that may be resulted using the both references. Moreover, teacher may also ask the students to demonstrate a simple approximation in the classroom analogously, for instance asking the students to approximate the length of the classroom using windows (embedded objects) and using tables as the references.

b. Some of the students might prefer to choose the reference that they more familiar the lengths with. For instance, they may prefer the windows, the door or the car if they think they know the lengths.

   This time is the best time for the teacher to explore the students experience about length of external objects around them. The teacher may ask the students to tell the story behind their familiarity with the references and whether other students agree with the length.

Part C: Journal Time

In this activity, the students, individually, are given a chance to reflect and write references they notice during the previous activities on their personal journal. Moreover the students are also asked to write any other familiar objects that they know the lengths and potentially could be used as references for approximating/estimating.

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Some of the students may write references that are already discussed during the lesson.

   It is suggested for the teacher to make sure that the students write the most agreed lengths for the references.

b. Some of the students may write references that is personally known by her/himself.

   In this case, the teacher should explore his/her knowledge about the references, how they know the lengths. In case the written references are considerably not make sense, the teacher may ask other students to react.

4.4. 17th of August Decoration: Balloons and the Flag Pole

4.4.1. Learning Goals

- Students are able to internalize one meter unit as their reference.
- Students are able to estimate length of objects using reference points without physical iteration.
- Students are able to reflect and mention references for 1 cm and 1 km.

4.4.2. Description of Learning Activities, Hypothesized Students' Thinking and Suggestion for the Teacher

Part A: Rope for Eating Krupuk Contest

In this activity, students work in group of 4 to make a representation of one meter magnitude in the context of making rope for eating *krupuk* contest. The purpose of making the one-meter-long rope is to promote internalization of one meter unit as an individual frame of reference which is important for mental

estimating. In the activity, the students are asked to investigate how to cut rope into one meter long in at least two strategies without using a ruler.

Figure 7. Eating *krupuk* contest

After the hand-on activity of cutting the rope, the students are invited to show their rope by hang it in front of the class together with ropes from other groups. This is aimed for students to visually compare their ropes and to begin the discussion. The discussion will focus on what references that the students use to approximate one meter.

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Some of the students may guess in making the one meter rope.

   If the teacher finds this case, it is suggested to ask the students questions such "Are you sure you make one meter?" "Convince me that you make one meter?" "Do you use something to measure with?" It is recommended that all the students already shift from guessing to use a reference.

b. Some of the students may imitate the rope from the photograph.

   If this happens, the teacher may ask the students "Could you tell me about how long the rope in the photograph?" "What is your point of reference to say that?" It is expected that the students reason or mention a reference on the photograph.

c. Some of the students may use "mental reference" to make the rope into one meter.

   If the teacher notices this case during the group discussion, just see how far the students can make the one meter. If it is considered producing intolerable error,

the teacher may interrupt and ask the mental references that the students use. Moreover, the teacher may try to involve other students within the group to reason and to agree then to contribute to the correction of the reference. On the other hands, if the teacher finds the produced rope reasonably approximates one meter, the teacher may bring this strategy to be discussed in the class discussion.

d. Some of the students may open their personal journal and use body parts (hand spans or arm spans) to make the rope into one meter.

   This is absolutely allowed but the teacher is not suggested to command the students to open their personal journals in order to let various strategies occur. The teacher may pay attention that the students use the actual length of the body parts not the common length of body parts. If this case happens, the teacher may clarify to the students "What is meant by the common length of the body parts that we learned in meeting 2?" "What is the function?

Part B: Class Decoration and the Rope for the Flag Pole of the School

In this activity, students are engaged in activities in which they are asked to estimate the height of their classroom and the height of the school flag pole. This contexts are used to prevent the students to physically approximate the objects. Instead the students are shifted to begin imagine reference or mentally iterate the reference. In this sense, the skills of iterating from the photograph that they already learn in the previous lessons and the 'feel of size' of one meter magnitude might be useful for estimating the objects.

The students are asked to solve two problems in context of decorating the ceiling of the classroom and in context of rope for the flag pole of the school. The first problems asked the students to investigate and move along the classroom to find better perspective to spot references. Similarly, the later problem also encourages the students to dynamically observe the flag pole outside the classroom. Both of the problems ask the students to estimate the attributes using at least two strategies which mean to spot at least two reference.

Afterward, the students' discuss and compare their answers by writing it on the provided two tables on the blackboard. It is expected that the students could see numerically the differences of their answers and be stimulated to discuss the strategies they use.

Hypothesized Students' Thinking and Suggestion for the Teacher

a. Some of the students may guess the height of the classroom/the height of the flag pole without reasonable explanations. For instance, the students may respond "I just know that" when being asked "How could you know?"

   If this case happens, the teacher is suggested to guide the students to use point of reference. For instance the teacher may ask further questions such as "how could you know the length if you don't use anything?" "Are you familiar with the height of the classroom/flag pole related problem?" Moreover the teacher is also suggested to ask other students' opinions within the group to agree about the estimation.

b. Some of the students may use prior knowledge about the height of the classroom/the flag pole. For instance the students may claim that "I know the height of this classroom is about 3 m because I read it somewhere".

   The teacher may remind the students to estimate using at least two strategy as mentioned in the worksheet and using prior knowledge/guessing does not count as a strategy. Furthermore, the teacher may explain to the students that there is a possibility that the height of this classroom differ from the common classroom height that the student know.

c. Some of the students may use "mental meter" or reason using the magnitude of the one-meter rope.

   The teacher may suggest the students to give more concrete explanation that other students may understand easily. For instance, the teacher may ask question such as "How many meter you count?" "From where to where or what on that wall you think is one meter?" "Can you draw the sketch of your strategy?"

d. Some of the students may use the height of the door, windows or other classroom objects to estimate the height of the classroom. In case of the flag pole problem, the students may use the height of the classroom, trees or other objects that they can see on the school yard.

   Actually this is a good strategy that the students use a reference for estimating. However, it is important to be noted that the teacher might promote all the member of the group/ other group during the class discussion to talk whether length of the reference is considerably reasonable or not. Here there is also a

possibility that the students use ‘mental meter’ for estimating the length/height of the reference before estimating the height of the classroom/flag pole.

e. Some of the students may mentally use individual frame of references that she/he is personally familiar with.

   In this case, the teacher may ask the students to tell the story about the students’ individual frame of reference. If necessary, the teacher may also ask the students to demonstrate in nonverbal way the magnitude of the length/height of the reference since it is important for others to be able to understand what the students have in mind.

Part C: Journal Time

In this part, the students are given some time to think, reflect and write references that they noticed during the lesson on their personal journal of reference. Moreover, the students are asked to write objects that could be used as reference for one centimeter and one kilo meter then tell the story how they could know that the objects have those lengths. The purpose of this questions is to explore students’ informal knowledge about their internalized standard units.

Hypothesized Students’ Thinking and Suggestion for the Teacher

a. Some of the students may write the height of the windows, doors, trees, or even the height of the flag pole and the classroom as new references.

   Special attention should be addressed to the students who might not write reasonable length of the references. The students may forget the agreement of the classroom about the length of the references. In this case, the teacher may re-ask the students or other students to discuss/mention the length of the references.

b. Some of the students may draw a line that representing one centimeter and explain that she/he remember how long one centimeter is.

c. Some of the students may mention that the width of a thumb/pinky finger is one centimeter.

   It is recommended for the teacher to encourage the students to draw a line using their thumbs/finger as reference indicating one centimeter and tell the story whether they ever use it for approximating (also for b).

d. Some of the students may mention that the one kilometer is a distance from somewhere to somewhere that the students familiar with.

   If necessary, the teacher may suggest the students to write the name of the exact location meant by the students.

e. The students may not have any idea about references for one centimeter or one kilometer.

   If this happens, the teacher may suggest the students to pick a ruler and make one centimeter line then ask the students whether they know something that has length as long as the line. In case of one kilometer reference, the teacher may be only allowed to ask whether the students know the distance of something that is one kilometer or ask whether they know the distance from their home to the school/market. It is important to be noted that the students cannot be forced to memorize reference that they have no experience at all.

4.5. 17th of August Decoration: Plastic Flags and the Rope

4.3.3. Learning Goals

- Students are able to spot, use and reason using IFR to solve length estimation problem involving simple social arithmetic.

4.3.4. Description of Learning Activities, Hypothesized Students' Thinking and Suggestion for the Teacher

Part A: How many packs of plastic flag we need?

In this part, the students are involved in an activity to apply their skills in length estimation in a realistic situation. The context of the problem is about decorating the school with plastic flags and rope. The students are asked to estimate the total costs to buy rope and plastic flags needed for the decoration.

The students are given a worksheet with a photograph of the flag decorator. This photograph is expected be able to show how the flags are arranged. Moreover, it is also expected that the students can approximate the width of the plastic flag from the photograph (see Figure 8). It also functions as illustration of how the flag

decoration are hanged on the ceiling outside the classroom. Figure 9 is not intended as an authentic image that can be estimated from.

Figure 8. Flag decoration maker

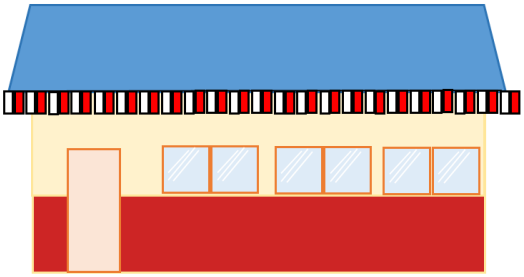


Figure 9. Illustration of the plastic flag decoration

The students need to calculate the total cost for the decoration, indeed the students are given two clues: the price of one pack of 100 plastic flags and the price of 30-meter-long rope which are respectively Rp 9000,- and Rp 3000,-. Using this information the students are encouraged to observe their school lively. Then, they are expected to make a poster consisting their strategy and calculation of the budget. The posters are presented in the presentation and class discussion session.

Hypothesized Students' Thinking

a. Length estimation and the use of reference

   Some of the students may think to approximate the width of the flag from the photograph. Then, they observe the school to estimate the length of the rope associated with the total length of the classrooms and the school offices by using references (e.g. mental meter, the length of the windows, the length of the

school yard, etc.). It could be that they might only estimate the length of one or several similar classrooms then do multiplication to the number of the rooms in order to know the total length. In the end, they might divide the total length of the rope to the width of one plastic flag to obtain the total plastic flag needed.

b. Mental numerosity estimation

Some of the students estimate the number of the flag as in numerosity estimation. They think the flag as discreet quantities. They may observe one or several classrooms then imagine the quantities of the plastic needed for that particular classrooms. Afterward, the number is used in multiplicative way to obtain the total number of the flags.

c. Guessing

Some of the students might guess and cannot explain in reasonable way how they get the length of the rope and the total number of the flags.

d. Budget Calculation

- Some of the students might do not realize that they can only buy the plastic flag in packs of 100 flags and the rope in rolls of 30 meter-long rope.
- Some of the students round up the total numbers of the flag into the next packs. For instance if they get 140 they might buy 2 packs. Similarly, they might also round up the total length of the rope to the next roll. At the end they multiply the number of the packs/rolls to the each price.
- Some of the students round down the total number of the flag into the present packs. Especially if the number is not really big. For instance, they may round up 112 to 1 packs since they can make a gap between the flag arrangement. At the end they multiply the number of the packs/rolls to the each of their price.

Suggestion for the Teacher

It is important to be noted that the teacher should be able to prevent chaotic when the students work outside of the class. The teacher may first discuss the problem with the student inside of the class. The teacher may ask them to plan their strategy and list what they want to know if they are outside of the class related to their strategy.

In general, teacher may let the students to solve the problem without interruption except if something very crucial happens such as they if they say “we need meter tape” , “we need to a ladder to measure the ceiling”. In this case, the teacher may inform them that they can perform estimation to get the length without rulers or meter tape. Moreover, the teacher may inform them that they not need to measure in exact ways since if they buy the flag and the rope more than needed they still can save them. Yet, they should be reminded that the most reasonable budget calculation is more desired.

When that students’ work on their poster, the teacher may observe their strategies to solve the problem. Based on rationale the most basic strategies should be presented first followed by more sophisticated ones. The teacher may be challenged to order the presentation and connect every students’ presentations.

CHAPTER 5
RESTROSPECTIVE ANALYSIS

We already described the hypothetical learning trajectory of the initial design of this study. In this occasion, we will elaborate the analysis of data from the first cycle and the second cycle including the pretest and the posttest of the second cycle. We also provide a part in which we summarized the result of the first cycle for improvement of the next cycle namely 'design refinement'.

5.1. Pilot Teaching Experiments (cycle 1)

Pilot teaching experiment consist of five sequence of lessons. Pretest and posttest were added to measure students' development during the teaching experiments. The pretest was conducted to 30 fourth graders meanwhile the 5 teaching experiments were conducted to the 4 selected students. The selection was based on students' score of pretest and advices from the teacher such as communication skills of the students.

5.1.1. Pretest

The pretest is aimed to get information about students' prior knowledge about the topic of length estimation. The pretest was held on 16 February 2015. Thirty students from class IV C became the participants. The students took about 20 minutes to solve 4 problems of the pretest followed by interviews with the 4 selected students to gain more information about students results. In particular, we wanted to know students' sense of units, individual frames of reference (IFR) and approximation/estimation strategies. During the analysis of the pretest and the interviews we found several critical points of students' prior knowledge that would become the starting points of the students for the teaching experiments.

a. Students' sense of units

In general, students had difficulties to use proper units of length. In addition, they also improperly represented a magnitude of objects. For instance, if they thought that the object is very long they would write a relatively very big

number. One student said that the length of a piece of A4-sized paper is 450 cm, and another student said it is 5 or 7 cm. Hence, we would say that the students had not yet develop good sense of units marked by either improper use of units and numbers.

b. *Students' use of individual frames of references and approximation/estimation strategies*

Most of the students did employed or recalled any personal referents related to the approximation and the estimation tasks. However, there are several students who recalled to use a half of hand span to represent one meter or use a hand span to represent one centimeter. Nonetheless, most of the students used guessing methods when they were asked how they know their references. Hence, in most cases, students did employed reference points but they have no idea about the length of the reference points.

5.1.2. Lesson 1: Frog Jumping and *Pocong* Jumping

Frog jumping and *pocong* jumping is aimed to accommodate learning through a game, raise awareness to compare lengths, stimulate conflict about used reference points and to shift students from using rulers to more efficient ways. Students in pair, were asked to measure/approximate a relatively long distance through a game called "Frog Jumping and *Pocong* Jumping". The game was played on the school yard, each students in their group alternately jumped like a frog 4 times, jumped like a *pocong* (Indonesian ghost) 4 times, marked their distance, and then approximated whether the distance is 30 meter or not.

We found a confirmation to our HLT that the students did not want to use a ruler since it would be tedious to use. Instead, the students used arm spans as one meter unit. Moreover, there was also a conflicting situation in which the students argue whether their arm span is one meter long or not. Following fragment shows their reasoning.

Fragment 1. Conflict about the length of an arm span

1	Teacher	:	Do you agree that they got 54 (meter)?
2	Sugi&Aziz	:	No!
3	Teacher	:	Why? Why?
4	Sugi	:	(pointing to Fitria) her arms are short.

From Fragment 1 we can see that the teacher asked the boys whether they agree or not with the result of the girls' approximation which was 54 meter. Sugi refused to believe that the girl group got 54 meter, since the girls using arm spans, Sugi thought that Fitria's arm are too short to represent one meter. At this point, we may say that the students' became aware of the differences and accuracy of using individual frames of reference.

Overall, lesson one provided a sufficient situation to raise students awareness to employ references points. What was need to be improved is that the game should facilitate the use of several reference points (not only arm spans) to stimulate a rich discussion.

5.1.3. Lesson 2: Measure and Use Your Body Parts

This lesson is aimed to provide students to physically approximate object in photographs by using reference points (body parts). The students were given a set of familiar photographs about Indonesian games. The photograph were selected in such a way that the photographs have clues of body parts that possibly could be used for the students for approximating. They were asked to find real lengths or distances of objects in the photographs. For instance, they were asked to approximate the length of a *bakiak* (long-wooden sandals) given clues such as feet, tiles, etc.

Figure 10. Bakiak Problem

We found that most of the students could figure out the lengths of the objects employing reference points such as hand spans, body height and arm spans as predicted on the HLT. For instance, the students first measured their hand spans using a ruler as a reference points to be used on the photograph. It is however, the students did not spontaneously perceive feet as a reference points for approximating especially for the *bakiak* problem (see Figure 10) The students tended to guess the length of the *bakiak* by imagine their own version of one meter (mental) in that photograph. It could be because the students are not familiar to use feet for measurement.

Overall, lesson 2 could trigger students' sensitivity to make body parts as reference points. A point to improve is that the photograph of the *bakiak* should be revised in order to stimulate students' awareness of the use of feet as reference points. Moreover, some part of the lesson should be deleted due to the time limitation and complexity matter. We would elaborate the reasons in next parts of this chapter.

5.1.4. Lesson 3: Length, Width, and Height of the Building

In this lesson, the students were asked to approximate two photographs (a greasy pole and a building). The first photograph acted as stimulus for the students to aware of the use of various reference points. Meanwhile, the second photograph was the main task in this lesson. The students were expected to be able to spot reference points that they already knew the lengths from the photograph and use it to approximate the length, width and the height of the building on the photograph.

Afterward, they were given three examples of solutions of the problem together with the reference points. They were expected to be able to order and reason about the solution based on the efficiency of the used reference points.

We found that the students used guessing for approximating at the very beginning. However it turned to be out that for the first problem, the students employed their thumb as a reference for one meter on that photograph. It was probably that the students first assumed the one meter, and then assigned it to their thumb.

Later on, we noted a student named Aziz used a person on the photograph as clue for approximating, the following fragment shows his reasoning.

Fragment 2. Aziz's reasoning of imagine people arm span.

1	Aziz	:	This is five (meter)?
2	Sugi	:	Let me see.
3	Teacher	:	Just try to find clue on the photograph that can be used as references.
4	Aziz	:	Look at the person wearing red shirt, just imagine if he does this (Aziz spans his arm)

From Fragment 2, we may tell that Aziz imaged a person in the photograph does an arm span, which Aziz assumed one meter for each arm span. At this point, it seems students tended to imagine or iterate standard unit of one meter to approximate rather than using a ready-made point of references such as body height or the height of the house. This could happen since the students still need to be familiar shifting from using body parts to external objects.

For the second problem of lesson 3, we found that the students got difficulties to cope with the context of the problem. The following fragment showed the girls's conflict determining the length and the width of the building.

Fragment 3. Fitria and Dytha determing the length and the width of the building

1	Fitria	:	This is the length, this is the width.
2	Dytha	:	Hold on, hold on. This is length, width and height
		:	(pointing at words on the tasks instruction). This one is width.
3	Fitria	:	This one is length, and this one is width.

As we can see from Fragment 3, the students were confused determining the length and the width. Moreover, during the discussion we also found out that the students had little knowledge about 2 dimensional representation of a 3D object since they had not learnt about it. In addition, due to the structure of the building, the students confessed that they were not familiar with the type of building. Hence, refinement of the lesson would involve a very big change of this context.

Related to the second task, when they were asked to find the most efficient point of references, we found out that the students only focus on finding the longest objects. The students assumed that the longest object is the most efficient one to be used as a point of reference. They mentioned objects such as road lamp or a bus. Actually they were not really familiar with the lengths of the objects. At this point, we think that the students had not yet develop a good understanding of what is meant as reference points.

Based on this finding, we intended to revise the problem of lesson 3 as such we would involve more students' investigation on connecting body parts to external objects to develop students' reference point for approximating or estimating.

5.1.5. Lesson 4: 17th August Decoration: Balloons and the Flag Pole

In lesson 4, students were engaged in two activities. The first, students were asked to make a one-meter-long rope in the context of eating *kroepoek* contest. The aim was to promote internalization of one meter unit as reference point which is important for mental estimation. The second activity was the students were asked to estimate the height of their classroom and the height of the school flag pole. It

was aimed in order to shift the students to do mental estimation of their reference points instead of iterate them physically.

We found that in the first activity, the students again tended to use arm span to make the one-meter-long rope. It however, made them tediously shift to employ other reference points. For instance, in activity two, as predicted in HLT, they tried to approximate the height of the school flag pole and the height of the classroom using arm span as shown in the following photograph.

Figure 11. Aziz approximates the height of the wall using arm spans

From Figure 11 we may see that, Aziz used his one and a half arm span to approximate the height of the classroom. This situation was sufficient to make the student try to imagine the rest of the height of the wall. However, the students were still unsure about their answers. The students discussed and found out to use other references as shown in the following fragment.

Fragment 4. Discussion on the height of a bookshelf

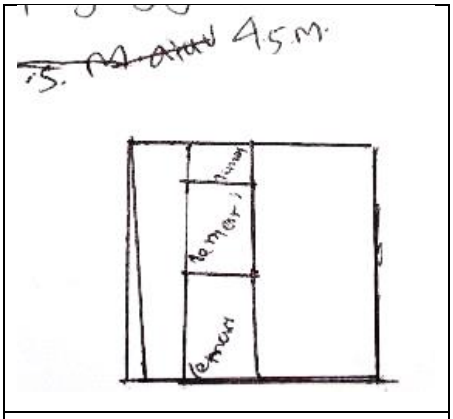
1.	Fitria	:	Ouu that is three meters high (pointing at a bookshelf).
2.	Teacher	:	Which one? The bookshelf? How high is the bookshelf?
3.	Sugi	:	Two, two, one meter and a half!
4.	Fitria	:	Two meters.
5.	Teacher	:	How do you know this is 2 meter high?
6.	Fitria	:	Because…..
7.	Aziz	:	It can’t be 2 meter high! You know that my body height is 151, don’t you?

From Fragment 4**,** we may see that Aziz connected his body height to make sense the height of the bookshelf being used to estimate the height of the classroom. The next fragment shows how they used the bookshelf for estimating.

Fragment 5. Discussion on the height of the classroom

1.	Teacher	:	Could you explain how to find out the height of this room?
2.	Sugi	:	Errrr, the bookshelf is two meter high. This one should be added by two meter and a half. Therefore, it becomes three meters and a half.

We may see from the Fragment 5 above that Sugi imagined the left over space between the top of the bookshelf and the ceiling was one and a half meter. It turned to be out that Fitria disagreed with Sugi's opinion. Fitria and Dytha as a group purposed the following answer.



Lemari: Bookshelf

Figure 12. Dytha and Fitria's Strategy

From Fragment 5, we see that Fitria and Dytha employed a mental estimation using the bookshelf as reference point. They imagined that there would be one and a part of bookshelf again that could be stacked to the ceiling. By this strategy they found out that the height of the classroom is 2 meter + 2 meter + 0.5

meter which is 4.5 meter. It shows that the students shifted to use external object as reference points and shift from physical approximation to mental estimation. Nevertheless, remark about lesson 4 would be discussed further in one of the part of this chapter.

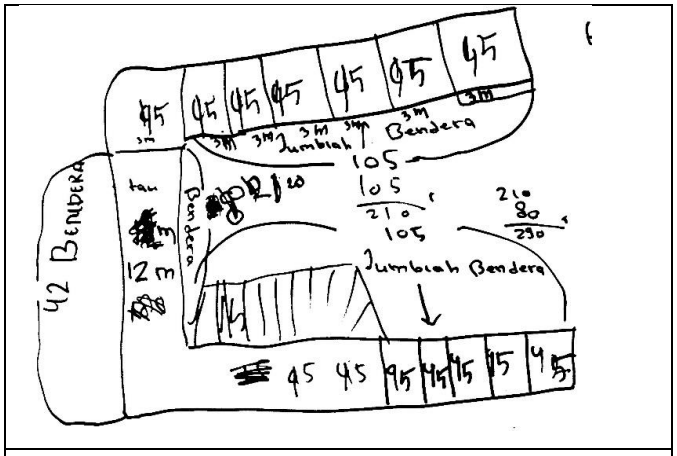
5.1.6. Lesson 5: 17th August Decoration: The plastic flags and rope

Students were engaged in a real situation in which they observed their school to determine how many plastic flags and how long the rope should be hanged on outer ceiling for 17 august decoration. The aim of the activity was to give students chance to spot and employ reference points at the school.

We found that the students again used arm spans as reference point for one meter to approximate (see Figure 13). They iterated their arm spans tediously, sometimes they got lost to cover the desired part of the school.

Figure 13. Aziz used arm spans to approximate the distance between pillars

As we predicted also in the HLT, one of the group (the boys group) creatively used distance between two pillars as aid for approximating as seen in the following written work.



Bendera: Flag , *Jumlah*: Total

Figure 14. The boys' written work

From Figure 14, we may see that the students segment the part of the school being observed based on the pillars. They approximated the distance of two pillars (3 meter) and estimated that there would be 45 flags in each two pillars (1 meter is 15 flags). By using this way, the students also could figure out the length of the rope needed. In other words, the students employed a new reference for the approximation but they still relied on arm spans. For refinement purpose, we see the need to emerge various strategies or reference points to be able to stimulate a rich discussion, not only the use of arm span.

5.2. Design Refinement

5.2.1. Refinement of lesson 1

Taken into account the analysis of lesson 1 of cycle 1, here we summarize crucial points of comparison between our HLT and the actual learning in the following table.

Table 8. Main points of comparison between HLT and actual learning of lesson 1 cycle 1

Hypothetical Learning Trajectory	Actual Learning
Various strategies/reference points of approximation would occur.	Only arm spans strategy was used to solve the problem.
Students would make an agreement about the definition of efficient in approximation/estimation	Students came up with 3 different definitions yet no single agreement was discussed.

Based on Table 8, we might see that the instruction was failed to stimulate various strategies/reference points for approximating/estimating. It was probably caused by the small number of the students group in this pilot study (only two group). Hence, to improve this condition, a point of instruction in students' worksheet would be revised to be *the first group who have the most unique/efficient/best strategy will be the winner of the game*.

From Table 8 also it is seen that the students had no clear definition about the word 'efficient'. To improve this situation, in part B we added a specific question to emphasize agreement between the students such as w*hat is the meaning of the word efficient for approximating/estimating in this lesson.*

In addition, taking into account changes in the number of the students for the second cycle, we did several adjustment in the instruction. The adjustment were made to prevent chaotic and disorganization of the students during the work field activity. At first, the teacher might have ask the students to make a group of 4-5 students, this was called class group. Secondly, two of the students from each group

were selected to join other selected students to form a group, this mixed group was called the play group. Hence, if in the first cycle all the four students played in the game, in the second cycle only 5 play groups (mixed) would play the game. Another member of the group who do not play were asked to observe and involve to approximate the distance of the track in the game.

5.2.2. Refinement of lesson 2

Taken into account the analysis of lesson 2 of cycle 1, here we summarized crucial points of comparison between our HLT and the actual learning in the following table.

Table 9. Main points of comparison between HLT and actual learning of lesson 2 cycle 1

Hypothetical Learning Trajectory	Actual Learning
Students will use feet as reference points to solve problem 2.	The students did not come up with the idea of using feet as reference
Students discuss part C	The discussion did not happen due to time limitation
Students describe reference points they used to be employed to other situations and find relation between them.	The students failed to comprehend this tasks due to the complexity of the tasks.

From Table 9, we may see that the students did not realize that feet could be used as reference points in photograph 2. We think provided photograph did not give explicit clues for the use of feet. Moreover, we also think that Indonesian students in general are not familiar with the use of feet as measurement tools. Hence, we replaced the *bakiak* problem photograph to be more explicit showing feet as reference points (see booklet lesson 2).

It was also revealed that part C of the task could not be held due to time limitation. Moreover, the task was also seems too complex/too soon for the students to comprehend since the tasks did not help the students visually, which was violated the theory described in the chapter 2. Hence, we decided to delete this part so that

the students became more focus on the task to gain knowledge of the use of body parts for approximating/estimating. Similarly, the task in Part D "finding the relation between used references" and "where this reference appropriately could be used" should also be deleted since it was related to the task part C. Meanwhile, part B of the tasks were no longer be a task for students. Instead, it was assembled in the whole class discussion after students solving task A.

In addition, we took the task from part A lesson 3, the greasy pole problem, to be added in part A of the lesson 2. This decision was made to enhance the use of body height as references for approximating/estimating objects. We think this photograph could make students realise that the reference could be iterated/stacked since photograph 3, the jumping rope, could not give a situation of the iteration of body height.

5.2.3. Refinement of lesson 3

Taken into account the analysis of lesson 3 of cycle 1, here we summarize crucial points of comparison between our HLT and the actual learning in the following table.

Table 10. Main points of comparison between HLT and actual learning of lesson 3 cycle 1

Hypothetical Learning Trajectory	Actual Learning
Students could approximate/estimate the length, width and the height of the building using reference points	The students had difficulty to comprehend with representation of 3D object. Some students did guessing to the length, width and the height of the building.
Students gain new reference points from the photograph and be able to use	The students merely listed objects they know from the photograph without any reasons whether they know the lengths.
Students may mentioned other reference points they know in part C	The students had difficulty to recall their individual frame of reference and tended to repeat their previously known references.

From Table 10, we may see that the students had difficulty to cope with the context of the building. Not only that the students were confused determining the length or the width but also they were not familiar with the structure of the building (Western typical). Moreover, the students tended to guess when approximating or estimating the building, imagining their own version of one meter in the photograph. The students mentioned that certain objects were more efficient than others even though they did not know the lengths. At this point, we think, the tasks were really force the students to give answer without giving sufficient clues. This also became the same case for the third point in Table 10. Therefore, to improve the learning, we decided to entirely replace the tasks including part A, the greasy pole. We made a set of problems in which they could connect/employ their previous knowledge in lesson 2 (body parts as reference). We focused more on how students could reason with their body references to know the length of external objects which were being used as reference points. We gave students, for example, tile and shoes problem to connect their references of feet to tile, and then use tile to solve the couch problem (see booklet lesson 3). Further, we provided flood context consists of 4 problems in which the use of body height was emphasized to reason and solve the problems.

5.2.4. Lesson 4: 17th August Decoration – Balloon and flag pole

Taken into account the analysis of lesson 4 of cycle 1, here we summarize crucial points of comparison between our HLT and the actual learning in the following table.

Table 11. Main points of comparison between HLT and actual learning of lesson 4 cycle 1

Hypothetical Learning Trajectory	Actual Learning
Students would give at least two distinct references for making rope for eating krupuk contest.	The students only used arm span for making the rope.

Students may use one meter rope they made to help them estimating the height of the school flag pole and the height of the classroom.	The students did not use the rope, the students used arm spans, a bookshelf as references. Moreover, at the beginning the students used guessing method.
Student may mention reasonable references for 1 cm and 1 km unit.	Students' reference points for 1 cm and 1 km were difficult to be confirmed the reasonableness.

From Table 11, we may find that the students again, used arm span to make the rope. Although they were asked to give at least two strategies/references, the students used the arm span to make a one meter leg span. We may say that the task seemed to repeat students' knowledge about one meter unit. Moreover, from the second point of Table 11, we may also see that the rope did not stimulate the students at all to imagine mental one meter. In reality, the arm span reference was more dominant for students' mental meter. At this point, we see a necessary to delete task A from lesson 4, since the task merely boost the students' use of arm spans instead of developing other reference points. Therefore, part B would be the main focus of lesson 4 which would be revised to be more external-objects-references-exploration.

Considering the third case in Table 11, we may notice that the task were hard to confirm, we were confused whether the students' answers of the references were reasonable or not (Part C number 2). For instance, the students mentioned an ant or a cand-wrapping plastic as one centimetre and one kilometre from their house to a certain place. The given examples made the other students were confused to percept because they also did not know what kind of ant or place that one students meant. Clearly, the task gives less clues for students to have a discussion. Therefore, the task should be deleted and focused on supporting the part B, gaining new visible objects as reference points.

5.2.5. Lesson 5: 17th August Decoration: Plastic flags and rope

Taken into account the analysis of lesson 5 of cycle 1, here we summarize crucial points of comparison between our HLT and the actual learning in the following table.

Table 12. Main points of comparison between HLT and actual learning of lesson 5 cycle 1

Hypothetical Learning Trajectory	Actual Learning
Students will use various strategies, including mental estimation	The students only used arm spans method and with the aid of distance between two pillars. No mental estimation employed.

From Table 12, we may see that the goal of the sequence of the learning to make student shift from approximating to estimating was not achieved yet. Instead of imagining individual frame of references mentally, the students still stuck with the idea of using arm span even though this method quite tedious to be used. Here, we might have to think that we should have revisit the theory of the students' development of estimation skills as shown in Figure 1, chapter 2. We may say that making IFR to be applied mentally for estimation was not enough in this sequence of lessons. Due to the fact that the students had less IFR and in development of learning measurement, this sequence of lessons should be focused on building the students IFR instead of force them to estimate mentally. Since, insufficient internalization/understanding of IFR could make estimation fall into wild guessing. Therefore, in lesson 3 or 4 we did not focus on shifting students from approximating to estimating.

Back to the lesson 5, we think there was a need to give an explicit instruction on the task that obliged the students *to have a unique reference point and strategy* and to have a 7 minutes discussion for planning their strategies and understanding the task. By these new instructions, we expected that various strategies of approximating/estimating could emerge hence it might have stimulate a rich discussion in the class to compare the strategies. Another point of improvement of the task was that we gave less text of the story problem and added illustration of how a pack of plastic flag and a roll of rope.

5.3. Teaching Experiments of the Second Cycle

5.3.1. Pretest

The pretest was aimed to get information about students' prior knowledge about the topic of this study. In particular, we wanted to know students' sense of units, individual frames of reference (IFR) and approximation/estimation strategies. The pretest was held on 4 March 2015. Thirty four students from class IVA of MIN 2 Palembang became the participants. The students took about 20 minutes to solve 4 problems of the pretest. Following are the goal of each pretest items (see the booklet for the pretest items).

Table 13. Goal of item of the pretest

Item	Goal
Mentioning reference for one meter and one centimeter. Drawing a line represent one meter or one centimeter on the paper.	To know whether the students have individual frame of reference for one meter and or one centimeter. To know students' perception of one centimeter and one meter magnitude.
Approximating the length of a paper	To know students' strategies for approximating
Approximating from a photograph	To know whether the students are able to spot references point from photograph to be used as reference for approximating
Estimating the length and the width of a whiteboard	To know whether the students are able to estimate (mental) objects using reasonable reference points.

The score of the pretest in general showed that the students mostly got zero (point) for each item. The students mostly, gave incomplete/not reasonable answers such as 'using hand' (without mention specific part of the hand) and using rope (too general). In other words, the students had not yet develop a good reference points or approximation/estimation strategies.

After the pretest, we conducted interviews with 8 selected students to gain more information about the students' reasoning toward the tasks. The result of analysis of the interviews showed relatively diverse students' performances. For

instance, some of the students showed a good sense of unit of length, they could represent one meter and one centimeter in a reasonable way. Moreover, they also mentioned a good IFR such as 30 cm-long ruler, arm spans as one meter, and finger. Nevertheless, some of the students tended to use guessing methods to approximate and even said for example that the length of a whiteboard is 28 cm.

In addition, the 8 students would become the focus students of this study. They would be separated into 2 groups of four students. The two group would be observed more during the teaching experiment about their learning. One focus group would be analyzed more in the retrospective analysis.

5.3.2. Lesson 1: Frog Jumping and *Pocong* Jumping

The activity is aimed to shift the use of rulers to rough approximation using individual frames of reference (IFR) that students know. We gave the students a situation where the use of rulers becomes tedious and inefficient. They worked in group playing a game called *Frog Jumping and Pocong Jumping* at the school yard (part A). Each group should jump 4 times like a frog and 4 times like a *pocong*, then they should confirm and convince whether their track reaches 30 meters or not. After that, they have a reflection session in which they discuss about their strategies and the meaning of word efficient for IFR (part B).

Indeed it was not easy by the teacher to organize field activity outside the classroom with relatively big number of students. At the first attempt, there was misunderstanding to the game instructions by some of the students. It made all other students who played do the same mistakes, they did jumping so many times without altering with their partner in their group. We think this happened due to the instruction that the teacher delivered. The teacher only told the students how to do the game not showed or gave example of how it was done. For the future design, we suggest to explicitly give instruction to the teacher to demonstrate or ask one of the students to demonstrate the game so the others can observe and understand the rule of the game. We also predict it can ease the teacher to cope with the big number of the students.

At the second attempt, the students played the game correctly and in order. Following is a photograph of the students played the game.

Figure 15. Students played the frog jumping and *pocong* jumping

After all the member of the groups finished their jumps, the teacher gathered all the students giving instruction to measure the track of their jumps. As we predicted in the HLT, the students did not use rulers to measure the length of the track. Instead, all the group used arm spans representing one meter to measure the track as shown in the following photograph.

Figure 16 Students used of arm span to measure the track

From Figure 16, we can see that all member of the group doing the approximation. They were in line using their full arm spans and iterating it to cover their track. The interesting part was that the students were not bothered by the use of different person's arm spans (unit of iteration). Instead, some of the students were doubt whether they should use a full arm span of a half of it. Following is the fragment of the students' thinking.

Fragment 6. Students' doubt using full an arm spans or half of an arm span

1	Student 1	:	Sister, that means we are doing wrong?!
2	Observer 2	:	Why do you think you are doing it wrong, honey?
3	Student 1	:	We measure it like this (demonstrating a full arm span)
4	Observer 2	:	How is your one meter that you use, honey?
5	Student 1	:	(demonstrating a full arm span)
6	Student 1	:	Oh, I see, you use it like that. So why it is wrong?
7	Student 2	:	Other say like this (demonstrating a half of arm span). How we represent one meter? Like this (demonstrating a full arm span), or like this (demonstrating a half of arm span)?

It is obvious from the transcript that the students also have no strong knowledge about the use of arm spans as one meter reference. Additionally, we found the following students' thinking during the discussion in the class.

Fragment 7. Students' confession about arm spans reference

1	Teacher	:	Are you sure that your hand (arm span) is one meter?
2	Retno	:	My mother said, this is one meter (a full arm span) and this is a half of one meter (a half of an arm span)

From Fragment 7, we see that it might be caused the use of arm spans is very common in Indonesian especially for adults. Children are told by the adult so that it becomes common knowledge and not surprising that the students encounter doubts.

We realized that we did not predict this matter would occur during the lesson. However, for the future implementation, we suggest to enrich the HLT and give the teacher suggestion to confirm the students' doubts such as using a ruler to compare both units (a full arm span and half of arm span). In addition, we also did not found evidences the use of other references mentally or physically even guessing by the students to confirm the length of the track. All the students employed an arm span as their IFR for one meter.

In part B, we found the students discussed about the meaning of efficient in approximating. The following fragment shows their thinking.

Fragment 8. Students discussion about the meaning of efficient

1	Teacher	:	Do you think it is efficient using arm spans?
2	Aulia	:	What is efficient mom?
			That is something that I would like to ask. What is efficient?
4	Students	:	(say I don't know)
5	Student 1	:	Correctly.
6	Teacher	:	What did you say?
7	Student 2	:	Correctly.
8	Teacher	:	Oh, efficient means correct. Does anybody have different opinion?
	…		
9	Student 3	:	Something which is fit.
10	Bagus	:	Something which is sufficient.

We see that the students still had no solid meaning for the word efficient. We predicted in the HLT that students might have come up with the idea that efficient meant something faster to be used, longer or easier. However, our predictions did not occur. Probably, due to the fact that the rest of discussion only discussed whether the use of arm spans is efficient or not. The chance that students would come up with the aforementioned predictions would increase if there was a discussion to compare efficiency at least two methods or two reference points.

To sum up, the instruction reached the goal of the learning that the students shift from using ruler to other way of approximating. We see that the students only employed one method/one IFR to approximate their tracks. The instruction should be improved so that the students could come up with various IFR or methods to approximate/estimate the track. In addition, comparing the efficiency of two or more IFR that are used, could also lead to a better discussion of the meaning of word efficient. Therefore, stimulating various IFR or method becomes crucial for the next implementation.

5.3.3. Lesson 2: Measure and use your body parts

The main aim of this activity was to provide students in a situation in which they figure out the length of objects in a set of Indonesian context-based photographs. The photographs were chosen in such a way give clues to the students to consider body parts to be used as reference point for approximating. Then, there would be a part in which the students have to reflect of their learning by listing their

IFR that they had been used during the lessons especially body parts. In addition, they also would be asked to discuss about the length of adult and children body parts for approximating.

As predicted in the HLT, some of the students employed body parts to solve the tasks such as hand spans, arm spans, and body height. For task number one, figuring out the distance of a marble and its hole, the focus group reasoned that the hand is an adult's. Another group asked permission to the adult in the class (in this case the observer) to let them to measure his hand by a ruler (see Figure 17).

Figure 17. Students measure the length of the observer's (adult) hand span

Similarly, the strategy occurred for task number four (figuring out the distance of a rubber rope to the ground). After the students had known the length of the observer's hand/body height, they used it as reference to approximate the asked distances. Moreover, for task number 2, some of the students employed arm spans shown in the photograph to measure the distance between the trees. However, students differed in determining the length of arm spans to be a reference point for one meter. The following transcripts and photograph shows the students' thinking.

Fragment 9. Students' strategy using arm spans to approximate the distance of the trees

1	Observer 1	:	Why it is 10 meter?
2	Retno	:	Because one of an arm is 1 meter.
	….		
3	Observer 1	:	Could you explain your answer?
4	Bagus	:	One meter, two meter, three meter, four meter, five meter (pointing at each person in the photograph

Explain your answer here

1 person is 2 meter, 5 persons are 10 meter

Figure 18. Students' answer to problem number 2

From Fragment 9 above we may see that, like in lesson 1, the student (Bagus) assumed that a full arm spans is one meter long. He assigned each person's arm span on the photograph by one meter. Meanwhile Retno (other group) thought a half of an arm spans is one meter long (supported by Figure 18) so she took 10 meters as her answer.

We also found out that the students in the focus group employed ruler directly to the photographs **(see** Figure 19).

Figure 19. Bagus measure the height of a girl on the photograph using a ruler

We found the focus groups did use ruler to measure the length in the photograph, but then multiplied it with a certain number as shown in the following fragment.

Fragment 10. Students used ruler to approximate

1	Bagus	:	This is eight! (Measuring by ruler)! Eight times two!
2	Observer 1	:	Why do you time it by two? Why do you time
3	Bagus	:	it by two?
4	Aulia	:	(laughing) Because Bagus said that.

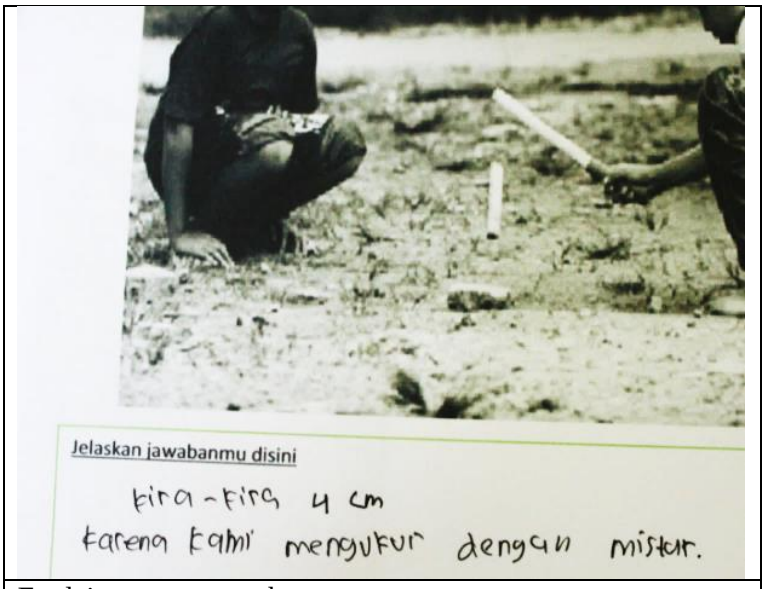
From the transcript above, we may see that Bagus measured the height of the girl (problem number 4) by a ruler and multiplied it with any random number to be used to make sense the height. Although Bagus did used the height of the girl to approximate, it seemed Bagus did a wild guessing. The group had not found any reference yet to be used for approximating the height. Moreover, as we predicted in the HLT, some of the other students only measured the length in the photographs shown by the following written work.

Explain your answer here

The length of the bakiak is about 8.5 cm
The method is by measuring by a ruler.

Figure 20. Students written work on problem number 3

We may think that the students were failed to understand the instruction of the tasks. We realized that it was a little bit tricky to ask the student to figure out the real length of an object from a photograph. Due to the fact that if one could not find any familiar references, the students' tendency would be on measuring by ruler and sometimes multiplied it by a random number to make sense the answer (Bagus' case). Another tricky thing that we did not expect before was that the students' measured and reasoned by using a photograph (problem number 5) even we only intended to make the photograph as an illustration (see figure X).



Explain your answer here

It is about 4 cm
Because we measure it by a ruler.

Figure 21. Students' misunderstanding of the photograph

What we can say about the above students' thinking is that we should carefully choose photograph and explicitly explain what the role of the photograph is. This way is expected can minimize students' confusion about the instruction of the task.

Back on problem number 3 (*bakiak* problem) as we have discussed in the lesson 2 of cycle 1 that the students did not spontaneously perceive a foot as

reference in the *bakiak* problem, here, in the lesson 2 of cycle 2, even though we had already revised the context (photograph) to be more explicit, yet the students also did not see a foot as reference at the first time. The following photograph of the students' activity shows how the students cope with the problem.

Figure 22. Students' activity to measure the length of the *bakiak*

We can see from the photograph that the students did not employ foot as references. Instead, they imitated the photograph and one of the student (Retno) approximate it by her arm span.

In other case, Retno confirmed our prediction in the HLT. For problem number 6 (approximating the height of a greasy pole) she reasoned using her IFR which was the height of a banana tree (four meter) to make sense the problem. The following fragment shows her thinking.

Fragment 11. Student used banana tree as IFR

1	Retno	:	This is 20 meters.
2	Observer 1	:	Why is it 20 meter?
3	Retno	:	Our predictions.
	…..		…
4	Retno	:	I think it is not. May I use my banana tree?
5	Observer	:	Please, explain it to me.
6	Retno	:	The banana tree in my backyard is about 4 meter
7	Observer	:	Is it (the greasy pole) as same as the banana tree?
8	Retno	:	No, absolutely no.

From Fragment 11, we may see that Retno did made sense the height of greasy pole by using her internalized IFR, the height of a banana tree. Retno knew that her banana tree is about 4 meter, she imagined it to the greasy pole and judged that the greasy pole was impossible 4 meter (it could be more).

We have shown to the reader that we found many evidences that support our HLT such as the use of body parts, using a ruler, wild guessing and mental reference. We also found out some elements for refinement for the next implementation such as making the instruction more explicit for approximating the real length and carefully choosing the role of the photographs.

From the overall teaching and learning process, still we were impossible to reach perfections. The teacher was lack of preparation to understand the teacher guide, the class became chaotic. Importantly, we could not conduct the part B (reflection/journal session) due to the time limitation and there was no explicit instruction from the teacher to remind the students again what was meant by 'reference point'. Hence, the part B became homework, the students got difficulty to list their reference point noticed during the lesson.

5.3.4. Lesson 3: Shoes, couch and Flood

By this activity students are expected to be able to identify references on photographs, connect and reason using body parts to approximate the asked lengths in the photographs. This activity consists of three part. In part A, students were given two implicitly related problems. The students were asked to approximate the length of a pair of shoes which is perfectly fit on a tile. Then, the students were asked to approximate the length of a couch on the tiled-floor. In part B, students were asked to approximate the height of water level in flood contexts which explicitly embed objects and body height as references. Meanwhile in part C, students were given a chance to reflect and list their preferred references that they had used during the part A and B. Part C was not held due to time limitation, instead the tasks became homework for the students.

In part A, we found no more guessing method employed. As predicted in the HLT, we saw students use feet and tile as references for solving problem number 1 shown by the following written work.

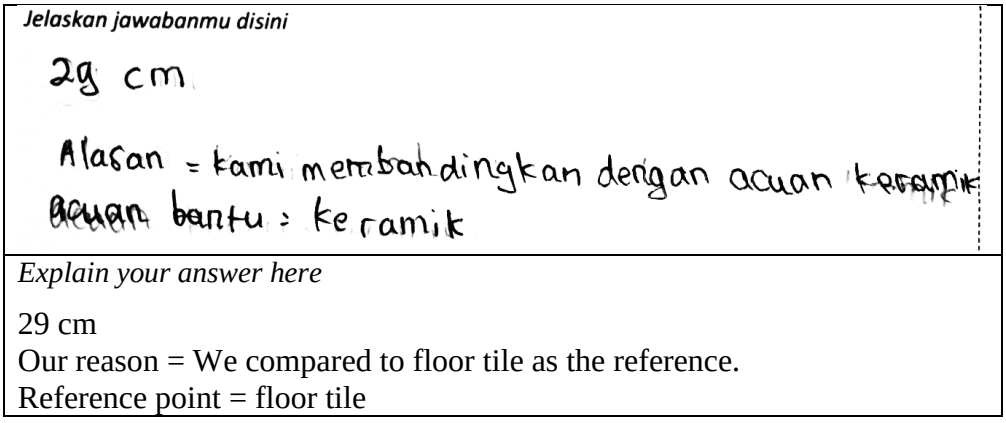


Figure 23. Students' written work using floor tile as reference

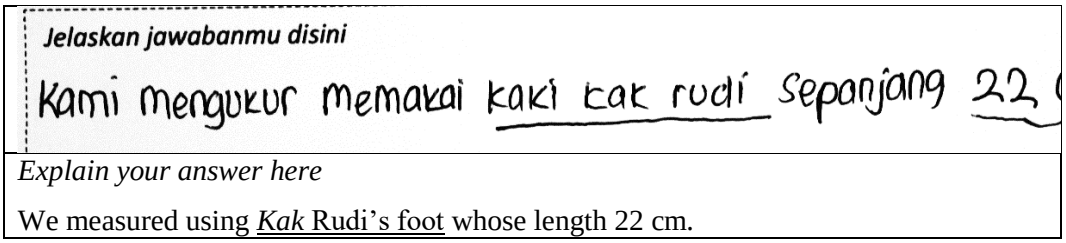


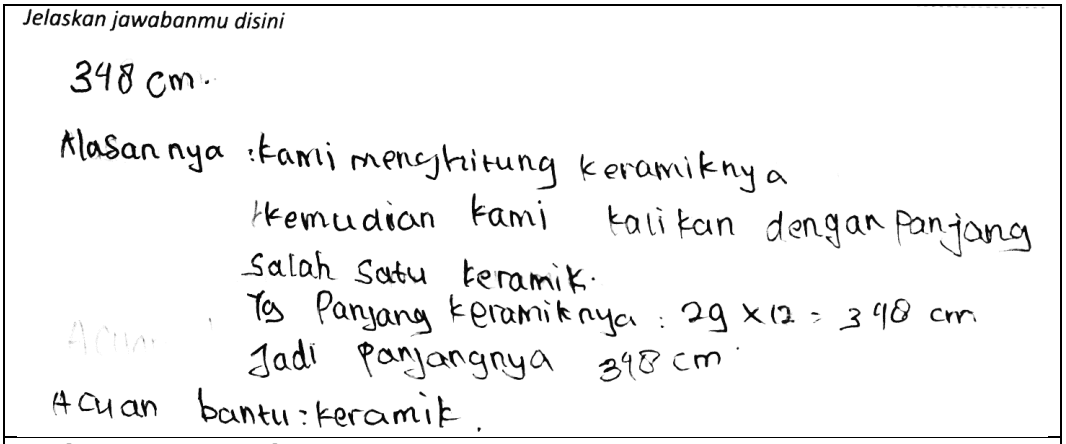
Figure 24. Students' written work using the observer's foot as reference

Figure 23 was given by the focus group, we may see that they employed floor tile as references. As soon as they noticed that the floor tile could be used to solve the problem, they measured the classroom tile using a ruler to know the length. Meanwhile, we might have also seen from the work of other students (Figure 24) that they use the observer's foot as references. It happened when they realized that Pak RT's foot (adult) probably nearly the same as the observer's foot then they measured the length of the observer foot. From both written work, the answers were quite differed, 29 cm and 22 cm. The teacher had a discussion about it during the discussion yet the teacher focused more on the variation of strategies rather than the range of the numbers.

We may also point out that for case number one (tile), the students invented a new 'vocabulary'/object as their references. Meanwhile for case number two (observer's feet), we may say that students' knowledge of adult feet as reference

(in lesson 2) was not internalized as individual frame of reference yet. Again, students' familiarity with feet as reference probably became the main factor as we found in lesson 2. The students needed more time to develop and internalized it as a reference then be able to reason using it without measuring by a ruler every time they wanted to approximate an object.

Similarly, for problem number two we saw some of the students employed the floor tile again as predicted in the HLT some others measuring their table/chair to be used as references. The following written works show students' strategies.



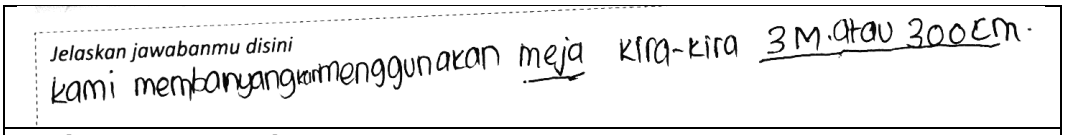
Explain your answer here

348 cm

Our reasons : we counted the tiles then we multiply with the length of one tile which length $29 \times 12 = 348\ cm$. Hence the length is 348 cm.

Reference point: Floor tile

Figure 25. Students' written work using floor tile as reference



Explain your answer here

We imagined using a table, which is about 3 m or 300 cm.

Figure 26. Students' written work using table as reference

We may see from Figure 25 that our focus group first counted all the tiles covered the length of the couch. After they figured out the number of the tile (12 tiles), they multiplied it with the length of a single tile (29 cm) so that they knew the length of the couch is 348 cm. Meanwhile, Figure 26 shows other group strategies, the students imagined that the length of one couch was about as the same as the length of one of their table in the class.

Here, we may see that both problems in part A could facilitate students to develop students the use of specific objects as references. The problems gave sufficient clues for students to prevent the use of guessing method.

Nevertheless, in part B, we noticed a significant unexpected thinking of the students. When solving the flood problems especially photograph RT 22 and RT 23, almost all the groups went to the parking lot seeking an ideal motorcycle shown in the photographs (figure 27).

Figure 27. Students measure the tire of motorcycles a ruler

As we can see from Figure 27, the students directly measured the tire of the motorcycle using a ruler to get clue for solving photograph RT 22 and RT 23. Similarly, for photograph RT 24, the students also directly employed rulers to measure the height of the observers or the teacher (see Figure 28).

Figure 28. A student measure the height of the observer using ruler

Based on the description above, we may say that the three photographs in part B gave less support for the development of students' IFR. Instead of trying to make sense in approximating/estimating the photographs using their body references connected to external objects as new references, the context of the problems led the students to directly measure the objects using a ruler. Hence, the students showed lack of reasoning to gain other references from references they already had known (e.g. half of body height) the learning tend to be a measuring activity and listing. Here, at best we can say that for the next implementation, improvement should be made for the three photographs. The photographs (context) should not give chance for students to employ a ruler to measure similar objects or the instruction should be more explicit that the students are not allowed to use rulers.

Meanwhile, the fourth problem (RT 25 photograph) made the students quite struggling. We found out most of the students assumed the house in the photograph as the same as the height of the classroom. Accordingly, the students tried to estimate the height of the classroom. Following transcript shows our focus students' way of thinking about the problem.

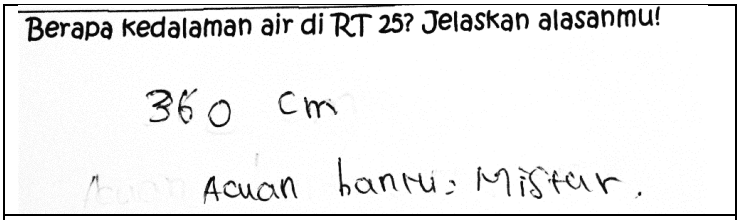
Fragment 12 Student's thinking to stack table for estimating

1	Fasli	:	The table can be stacked!
2	Teacher	:	Yes, the table can be stacked, how many table do you think?
	…		…
3	Aulia	:	One table is about 60 cm (measuring by a ruler)

As we can see from Fragment 12, the student (Fasli) noticed that they could estimate the height of the classroom using stacked tables. Nuri, another member of the group figured out that the height of one table was about 60 cm. Although the teacher asked the question “how many table can be stacked?” the students did not answer it and continue using ruler as shown in following figure.

Figure 29. The focus group stacked 3 rulers as one table height

Figure 29 shows that the students stacked three rulers which they assumed ‘another 60 cm (table)’ rather than imagine stacked tables. At this point, we noticed that the students were still thinking in the context of approximating, in concrete physical method rather than estimating. Then, for the next tables they shift to imagine the tables. The following figure is the focus group’s written work related to their strategies explained before.



How deep the level of water at RT 25? Explain your answer

360 cm

Reference point: ruler

Figure 30. Focus group's written work for photograph RT 25

From their written work, they answered that the height of the classroom was 360 cm using ruler as reference. However, we may think that the students imagine 4 other tables to be stacked so that there were 6 tables equal to the height of the classroom. Due to difficulty for the students to imagine 30cm-long ruler to be stacked, they converted the number of the tables to the number of the rulers. Hence, they answered ruler as their reference rather than table.

We also noticed another students employed arm span, and imagine it to be iterated along the height of the wall. At this point, we may conclude that the context of the photograph RT 25 was sufficiently be able to shift students to 'think' rather than direct measuring the object.

Nevertheless, we also noticed that the whole class discussions were still lack of students' involvement. The teacher focused on only asking every group's strategies and answers but lack of discussion whether other students agreed or not about the offered answers and strategies. Hence, the notion of socio mathematical norms in this approximation and estimation class were not yet established well. For the next implementation, due to the importance of socio mathematical norms on approximation and estimation topic, it is suggested for having discussion between the researcher and the teacher to have same understanding about the notion of the socio-mathematical norms in elaboration of various students thinking for the whole class discussion.

5.3.5. Lesson 4: 17th August - Balloon Decoration

The aim of this lesson is to have students to identify, select and reason about a set of references then discuss about its efficiency. Student were exposed to many external objects such as cupboards, a whiteboard, a door, tables, posters, photo frames, etc. from a photographs. The lesson consisted of two activities, the main activity (part A) and the reflective session (part B). In part A, the students were given three photographs of the walls of the students' school health care unit. The students were asked to approximate the height of the room using available spotted references as many as possible. Note that, they were not allowed to use ruler to measure similar objects in their classroom. Then, the students should have discuss the efficiency of their references in a whole class discussion. Meanwhile in part B, students were given a chance to reflect and list their preferred references that they had used during part A. Part B was not held due to the time limitation, instead it became homework for the students.

First of all, we would like to discuss our finding observed from our focus group. We found the students' thinking developed during the discussion. In the very beginning, the students used an inappropriate strategy to find the height of the wall as shown in the following fragment.

Fragment 13. Inappropriate strategy for approximating the wall

1	Observer	:	How is going on? Do you have any clue?
2	Bagus	:	(Spanning his hand covered the photograph)
3	Observer	:	Could you tell me what are you doing?
4	Fasli	:	How many (the result) (asking Bagus's method)?
5	Bagus	:	One, two, three, four, five, six, seven, eight (counting simultaneously across his handspan)

From Fragment 13 we may see that Bagus tried to approximate the photograph using his hand span. He wildly counted number on his hand. Yet, we were not really convinced that the student meant 8 meter or not. Then, we got answer from Fasli even though he was not able to explain what reference he had used.

Fragment 14. Fasli's answer of the height of the wall

1	Fasli	:	It is 360.
2	Bagus	:	360 again?
3	Fasli	:	We can reduce it if you want.
4	Observer	:	What length? Please explain, do not be shy.
5	Fasli	:	(Laughing)

We may see that Fasli mentioned 360 cm as his answer but he failed to explain what kind of reference he had use. At very best, we may think that the student was still influenced by the group answer on lesson 3 photograph RT 24, in which they had estimated the height of their class as 360 cm. That could be the reason why he got difficulty to explain what reference he had used.

Following the group discussion, Bagus came up with an idea to use white board as reference, shown in the following transcript.

Fragment 15. The use of white board as reference point

1	Observer	:	Please find as many as possible reference you may get.
	…		
2	Bagus	:	Whiteboard as reference point.
3	Observer	:	Then, how to use the whiteboard.
4	Bagus	:	Nah (laughing), it can be stacked. (pointing at the whiteboard of the class)
5	Observer	:	So, do you want to have the whiteboard being stacked? How many?
6	Bagus	:	Four meter
7	Observer	:	What is four meter?
8	Bagus	:	(staring at wall) three point eight five.

What we can say is that the student realized that the classroom door could be used as reference point for approximating the height of the class. The boy reasonably did educated-guess for 3.85 meter. In the next transcript of discussion they revised their answer and gave more explicit explanation about their strategy.

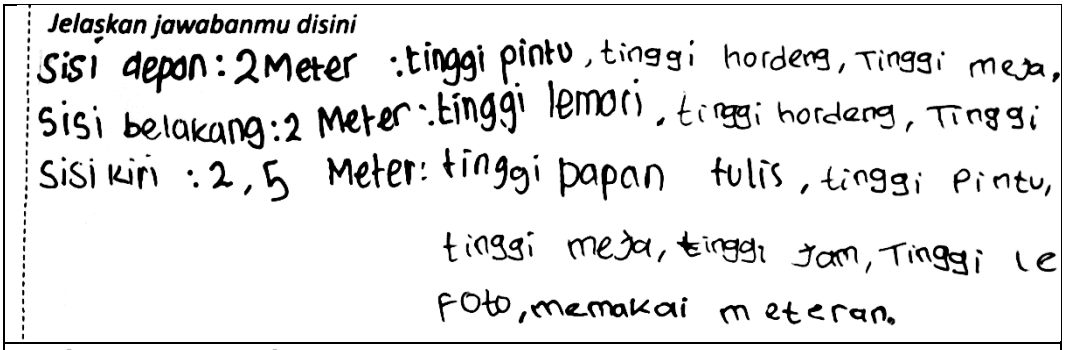
Fragment 16. The use of the door as reference point

1	Observer	:	From which you get 340?
2	Bagus		Guessing
3	Observer		What did you use for helping you guessing?
4	Bagus	:	Door
5	Observer	:	What? How high is one door?
6	Fasli	:	It is uncertain
7	Bagus	:	It is more or less 2 meter.

From Fragment 16 we see that the group led by Bagus got answer 340 cm and employed the door of the class as their reference point. We also pointed out that Fasli was still in doubt whether the door of the health care unit was as the same as the door of the class during the group discussion.

We also figured out from other students that similar development also occurred. One group told that the height of the class was about 3 meter. One of the students said she imagined she spans her arm to a door. Meanwhile, another group employed a classroom table as their reference point. Since they knew that the height of a table was about 70 cm then they said it could be stacked up to 4 tables. Hence their answer was 280 cm. In overall, the students' written work showed not too diverge numbers, the final answers were ranging from 260 cm up to 340 cm. Since the answers made sense and the use of reference point were emerged, we may say that the problem could support the students to develop a good skill in approximating or estimating.

Nevertheless, we found out that there were couple of groups who incorrectly understanding the instruction of the problems. The following students' written work shows how the students response to the three provided photographs of the school health unit.



Explain your answer here

Front side wall: 2 meter: the height of the door, the curtain, the table, the fan.
Back side wall: 2 meter: the height of the door, the curtain, the table.
Left side wall: 2.5 meter: the height of the whiteboard, door, fan, table, a clock wall, cupboard, photo frame, using meter device.

Figure 31. Students give different answers for the height of the wall

From Figure 31, we see that the students have different heights of the health care unit which were the left side was 2.5 meter and the front/back side wall was 2 meter. We understood this situation happened since the students do not well informed about the underpinning assumptions of the problem, that all the wall had the same high. Due to this matter, the students assumed the three provided photographs as separated similar tasks rather than one tasks with three clues. Hence, we think for the next implementation, the task should explicitly mention that the three walls are the same. Moreover, the teacher guide should also be improved so that the teacher have an opportunity to boost this underlying assumptions at the beginning of the lesson.

From Figure 31 also we may see that the students tended to list all the objects they saw from the photograph as reference points even though they had no idea about the length of the objects. Unexpectedly, the focus group, and other groups reacted to the tasks the same. It was revealed during the whole class discussion. The following fragment shows how our focus group thinking.

Fragment 17. Students tended to only list the reference points without knowing the lengths

1	Teacher	:	What are reference points you have employed?
2	Bagus	:	Door, the body height device
3	Teacher	:	The body height device? Okay, it is exist in the photograph. What is the height of that?
4	Bagus	:	(Scratching his head)
5	Teacher	:	We should know the height of the reference, shouldn't we?

From Fragment 17, we may see that the group wrote body height device as a reference point for approximating the height of the wall. However, they failed to explain why they did not know the length/height of this reference. We think it happened because, again, the students still had no yet comprehend well the definition of the references points. This word translated to Bahasa as 'acuan bantu' still needs to be improved and delivered to the students in much more understandable ways. We might think we could have a special mini lesson in which the definition of references point should be discussed in an explicit and meaningful way.

Besides, we might also think that the instruction of the task could also contribute to the students' misunderstanding. Remember that the students already discuss about the height of their classroom during lesson 3. Now, the task/instruction is kind of overlap to the lesson 3 since the students again discussed the height of a wall as the main focus. Hence, when the students already figured out the height influenced by their experience in lesson 3, the task for using other references to figure out known height seems did not make sense. Instead, for the next implementation the task could be changed for having students focusing to figure out the length/height of the objects explicitly by the use of the same three photographs. By this way, it is expected that the student could employ their knowledge of the height of the wall as reference point to reason and approximate the other objects.

Nevertheless, we found that our focus students' did reason using known individual frame of references to make sense and know the length of a new reference. The following fragment shows that the students' thinking.

Fragment 18. Students connects the body parts to approximate the shoeshelf

1	Teacher	:	Others?
2	Bagus	:	Shoe shelf.
3	Teacher	:	How high is the shoe shelf?
4	Bagus	:	Eh (rubbing his head).
	…	:	
5	Teacher	:	How high?
6	Bagus	:	Forty eight
7	Teacher	:	Forty eight? If it is compared to your foot, how might be
8	Bagus	:	it looks like?
9	Teacher	:	(touching a part of his table leg)
10	Bagus&Aulia	:	Oh like those, how high do you think it is? You can compare it to the tile or your body probably.
11	Bagus	:	(Measuring the tile and the leg of the table by handspans) It is as the same as 2 pieces of paper.

We may see from Fragment 18 that Bagus and Aulia employed their hand spans to figure out the length of tiles compared to the leg of their table. It was revealed that Bagus answered that the height of the shoe shelf was about 2 pieces of paper which he knew from his measurement to the tiles. At this point we may think that Bagus connected his individual frames of reference such as hand spans, paper, and tiles to know the height of a book shelf. This development of learning confirm our hypothesis that the lesson could support students' development of individual of reference.

At the class discussion, we also observed that a change of teacher style in leading the discussion. At the beginning the teacher tended to only focus asking the students' references point then the teacher tried to bring other students to confirm and discuss whether they agreed or not toward the references. The following fragment shows the teacher questioning style to involve the students.

Fragment 19. The teacher questioning style to engage the students

1	Teacher	:	One of your friend say that they use whiteboard as reference. They said the height of one whiteboard is about 140 cm. Do you agree with them? (silent for a moment), that it is about 140 cm. Do you agree? (silent)

We may see from Fragment 19 that the teacher was trying to involve more students for the discussion yet the teacher was still dominant. Unfortunately, the students were silent or too shy to talk. We think, for the next implementation it could be better if the teacher could make question to other students to repeat on their own sentences what have been said by a student. By this way, it is expected that the students have a reason to talk and could also stimulate the discussion.

Nevertheless, we figured out that during the whole class discussion about the efficiency of the references, we did not expect that the agreement '*which is the most efficient*' would be based on 'voting' instead of sophisticated reasons/arguments. At that time, there were 8 groups who agreed that using door was the most efficient for approximating the height of the health care unit. At this point we saw the need for the teacher to dig more students' reason behind their agreement of the efficiency. For the next implementation it will be better to have explicit guideline for discussion for the teacher specifically for discussing efficiency of reference points.

Overall, we may summarize that lesson 4 could facilitate and support the students to reason and connect about IFR to new references point. In some points, there are some crucial improvement should be done related to the instruction of the tasks such as figuring out the length/height of other objects by using the height of the wall as references, and that the height of the three walls on the photographs are the same. Moreover, explicit guideline for the teacher to discuss to compare efficiency of references and questioning during the whole class discussion should be embedded in the teacher guide due to the fact that the school/the class way of teaching and learning is still 'traditional'.

5.3.6. Lesson 5: 17th August Decoration – The rope and the plastic flag

The aim of this lesson is that students are able to identify, use and reason using reference points for solving length estimation tasks in social arithmetic problem. Students were engaged in field activity to estimate how many rolls of rope and packs of plastic flag that they might be needed for decorating their school. At first, the students were given a chance in the classroom to plan their action and possible strategies. Secondly, they were asked to observe the location/part of the school that should be decorated. Here, students worked by group to approximate/estimate the school part. At the end, they should make a poster of their work and present it in the whole class discussion. Unfortunately, due to the time limitation the poster session was not held in this study. Instead, the teacher conducted a discussion of the students' answers and strategies.

For our focus students, at the planning session, they discussed for a while and directly proposed their answers of the problem. The following transcript shows the students' answer and their reason.

Fragment 20. Students reason during the planning session

1	Observer	:	What are you writing?
2	Aulia&Nuri	:	Three packs of flag and …
3	Observer	:	So, you have already got the answer? How could you get the answer meanwhile you have not observed or measure it
4	Fasli	:	yet.
5	Bagus	:	I do not know
6	Observer	:	Just guessing
7	Fasli	:	How guessing? Does it make sense?
8	Observer	:	Yes, it does
9	Fasli	:	Why? Could you convince me that it makes sense? Hmmmm (thinking)

We may see from Fragment 20 that the group proposed three packs of flag. However, when they were asked to explain, Bagus admitted that he guessed. Yet, Bagus and Fasli were failed to give a reasonable explanation of their guessing. From this fragment, we could reflect that the students were influenced by the working style of lesson 2, 3 and 4 in which they approximated from photographs. In this

case, the photographs only function as illustrations of the context. Once again, it is important for the next implementation to give explicit instruction of the role of the photographs.

During the field work, we found the focus groups employed arm spans to approximate the length, as shown in the following figure.

Figure 32. the Focus group employed arm spans

As we can see from Figure 32, the students were in line, and iterate their arm spans alternately. Although in the lesson 2 the students showed awareness of possible differences of arm spans between one people to others, in this case they seems did not care about it.

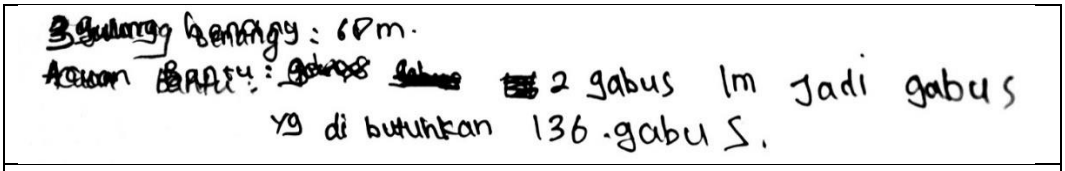
Nevertheless, since other group already used arm spans as reference, we asked the group to find and use different kind of references. The students came up with the idea to make a model of plastic flag (demonstrated by the teacher) using a paper. Then, they iterated the model to a distance between two pillars as shown in the following photograph and fragment.

Figure 33. Bagus iterate flag model between two pillars

Fragment 21. Students' strategy using the distance between two pillars

1	Bagus	:	From that pillar to that pillar
2	Observer	:	So, you count the flag one by one?
3	Bagus&Fasli	:	(nodding)
4	Observer	:	So, why you have a multiplication? What does it mean?
5	Fasli	:	How many in one (distance) of (two) pillars, then you should add, add and add.
6	Observer	:	So, you only need to count one (distance) of (two)
7	Fasli&Bagus	:	pillars?
8	Observer	:	(nodding)
9	Fasli&Bagus	:	And the rest be multiplied? (nodding)

From Fragment 21, we can see that the group used the distance between two pillars to count the number of the flags that could fit. Then, they multiplied the number of the flag for one distance to the number of the pillars. At this point, we can say that the students became more efficient in approximating yet they solved the first task without approximating the length of the part of the school. Here, as we predicted in the HLT, the power of the second task (finding how many rolls of rope) that it made students felt the need to approximate the length. With some uncertainty, the students were indirectly forced to use other references. The students came up with the idea of using styrofoam board as shown in the following written work.



Translation:
3 rolls of rope : 68 m
Reference point: 2 styrofoam board is 1 m, so the number of the stryrofoam board needed is 136.

Figure 34. Students use styrofoam board as reference point

From the written work Figure 34***Figure 34.*** *Students use styrofoam board as reference point* , the students knew that they needed 68 m rope hence they said they need 3 rolls of rope. Further, they also wrote that they needed 136 styrofoam because each one meter they needed 2 styrofoam. What we can say is that the students tried to involve 'other reference' to fulfil the task instruction even though it was irrelevant for the task since we did not ask the student to estimate how many styrofoam needed. We may conclude that a specific instruction to have students to use other strategies/reference points might burden the students themselves. On the other hand, if we do not command to do so, all the students tended to employ arm spans. In this case, a rich discussion is difficult to emerge. Therefore, at best for the next implementation, we suggest to still apply the instruction but the teacher must proactive to scaffold the students. The teacher can also use the irrelevant references/answer to trigger a rich discussion.

Furthermore, due to the instruction to use a unique reference, most of other group who used arm spans strategy claimed that they were the first who came up with the idea. The following photograph shows how group 2 used a model of plastic flag to an arm spans for approximating.

Figure 35. Students tracing the flag model along a student's arm span

As we can see from Figure 35, two of the students were iterating the flag model along the boy arm span during the planning session. It was revealed in the field work that the students approximate the part of the school by doing arm spans iteration then multiplied the number of arm spans to the number of flag for one arm span. Meanwhile, other group came up with the idea of using tiles to approximate the length as shown on the following figure.

Figure 36. Students counted the number of tile

From Figure 36, we can see that two students stepped on the tile to count the tile. It was revealed that the student made assumption that one tile could fit two flags (see the written work of Figure 36). At this point, we may say that the

instruction could stimulate the student to think about to use other strategies as we predicted in the HLT.

During the whole class discussion, we found the teacher's role was still too dominant shown in the following fragments.

Fragment 22. The teacher's way of questioning on the most efficient reference point

1	Teacher	:	Let's talk about the reference points, the first is tile, arm spans, wall, and windows. Do you know which one is probably the best, the simplest or the most efficient?
2	Students	:	(yelling: windows, pillars, arm spans, etc.)
3	Teacher	:	What is the longest one which is the easiest one to count and we do not need to count till hundreds?
4	Students	:	(yelling: the wall)
5	Teacher	:	The wall, or we may say it as the distance between two pillars. So, which one is the most efficient you think?
6	Students	:	The wall.

From Fragment 22, we can see that the teacher and the students were discussing the most efficient reference points. Line (3) indicates that the teacher gave too much clue for the students. It could limit the students' reasoning. Hence, at line (6) we see that the students made agreement about that. Nevertheless, this discussion session perhaps could dig up the students' reasoning and communication if the teacher limited herself to give one and only possible answer.

Unexpectedly, we found that most of the group of the students did not answer the main question of the task which was to count the total budget of the decoration (only one group remember this). The students, however, focused on finding the number packs of the plastic flag and the number of rolls of the rope (and the reference points). At the discussion also, the teacher did not talk about it at all. We might think that it was because of the time limitation and the difficulties of the tasks itself. For the future implementation, we suggest that this instruction should be repeated during the field work by the teacher when giving scaffolding to the students.

Overall, with specific instruction to find a unique way of solving the problem, the tasks could stimulate various strategies from the students. Some improvement for a better learning should focus on ways the teacher scaffold the students to come up with original ideas. Importantly, the teacher may limit her/himself to not too much giving direct clues which lead to judge a single answer.

5.3.7. Posttest

Posttest was held after the students completed the 5 sequence of lessons. The aim of the posttest is to gain information about students' performance of this topic. The information/the data would be used as secondary data for analyzing students' development. Students' written work and the score from the posttest would be compared to the result of the pretest by modest quantitative way. Indeed, this comparison would not tell much about the performance of the design itself. Yet, the role of the pretest and posttest here is limited to only support the result of retrospective analysis of the HLT.

The posttest was conducted on 31 March 2015 to 34 fourth graders of MIN 2 Palembang. We applied the same item as in pretest in this occasion (see the booklet). Shortly, the goal of the items are to know students' IFR and strategies to solve approximation and estimation problems.

Based on the analysis of students' written work, we see a development of the way students solved the problems and of the score of the test. In average, the pretest score increases 6.21 points to the pretest. Moreover, we also see that the students gave complete answer of the task. Almost in the students' written work the students wrote "the reference point is…." Despite its reasonableness, the students became more aware that in solving approximating/estimating problems one should give convincing arguments of the 'guessing', one of the ways is to reason using reference points or individual frame of references.

After the pretest, we also conducted interviews to eight selected students (focus students). The aim of the interview was to dig deeply about the students' written work and their reasoning toward the tasks in the pretest. Based on analysis of the interview we see that the students developed IFR and be able to employ it to the task.

The most basic example was that the students could use the arm spans to reason about objects. Fasli said that the height of a table cannot be one meter because the table did not fit with his arm span. Bagus on the other hands, could reasonably judge that the height of one floor in *Pagoda Pulau Kemaro* problem could not be two meters. He reasoned using the height of people in the photograph. Moreover, reference points such as a tile, hand span, thumb, window and whiteboard were also internalized well to the students' understanding. In Aulia's case, she used tile to approximate the length of a A4-sized paper meanwhile Nuri used her hand span as reference for 19 cm and thumb for 1 cm. In other words, we may say that the students developed IFR during the lessons.

In case of how and why the students experience the development in relation to the activities of the design in this study we would elaborate this in the next chapter by answering our research questions.

CHAPTER 6
CONCLUSION

6.1. Conclusion

The main research question of this study is *"How can we support the development of students' individual frame of reference for length estimation?"* Before we answer the main research question, we first would answer the two of sub research questions in the end of the second chapter.

Answering the first sub research question

The first sub research question is w*hat strategies used by students to approximate/estimate lengths?* To answer this question, we will summarize our finding from students' written work, interview with the students, and observation during the learning and including field note.

We noted that the most basic strategy that the students used is using arm span for one meter length. They were in line, iterate their arm spans alternately. They just assumed that each person's arm span is one meter (in case of lesson 1). The students also employed hand spans and body height as physical reference points by first measure it from a certain benchmark (their own/adult's) by ruler and apply the number to the tasks. Note that students were not spontaneously using feet as a reference point. Similarly, we also found out that the students also measured similar objects (e.g. wheel of motorcycles, tiles, chairs, etc.) using ruler to solve approximation problems which embed these objects/similar objects in its photographs. In addition, some of the students used estimation strategies by imagine IFR such as arm span as one meter, door (2 meter), body height (1.5 m) and cupboard (1.5 m). They iterated these object in their mind (mentally) to judge the magnitude of the to-be-estimated objects.

Nevertheless, we also found strategies which we categorized as inappropriate such as guessing. It was revealed when the students fail to explain what kind of reference points that they used to approximate/estimate. Sometimes the students mentioned a specific reference point yet the approximation/estimation

went unreasonable due to insufficient internalization of the reference point. In case of approximating from photographs. We found that the students directly measured the photograph using ruler and use the indicated number as the answers. Even more, some of the student multiplied the number with some random number to make the answer more sound reasonable.

Answering the second research question

The second sub research question is *how could the use of body part/familiar objects for approximation/estimation facilitate the development of individual frame of reference?* To answer this question we would elaborate from the students' development of learning during the lessons.

The use of body parts such as arm spans was very powerful as reference for one meter unit of the students. In such away, the students have internalized this one meter unit in their perception. Further, their prior knowledge of one meter unit helps them to reason about new objects such as the height of a door, the length of a table, the height of a classroom, the height of a cupboard, the length of a whiteboard, etc. and also new body reference such body height. Then, as the result of their approximation/estimation toward the objects, the objects themselves become new reference points for the students to solve larger tasks. Therefore, we would say that, the development of individual frame of references occurred like chain reactions from known reference points to new reference points.

Answering the main research question

To answer this main research question, "How can we support the development of students' individual frame of reference for length estimation?" we would recall some elements from the sub research questions and elaborate it to some theoretical framework.

To support the development of students' individual frame of reference for length estimation, one may facilitate the students to be aware of the use body references for the first time. In case of Indonesian students and refer to the answer of the second research question, arm span is prior knowledge of the students. Hence, this advantage should be boosted by having the students to reason about other objects/body parts using this reference. Then, to the next level the students may be asked to reason using their new objects (new reference points) to approximate/estimate new objects which will become new references.

It should also be understood that skill in length estimation itself develop in two dimensional aspect as shown in chapter 2 Figure 1. This argument was strengthen by the evidence of the strategies that the students used during the lessons (referring to the answer of the first sub research question). In other words, Students cannot be forced to shift from approximation mode to estimation mode in relatively short learning. Instead, the shift develops by the experience of the students.

To sum up, we conclude that the development of individual frame of references could be supported by exposing the students to use body parts as reference points then using this IFR to gain new reference points in form of external objects which then again can be trained to gain new other reference points. Meanwhile, developing students' estimation skills should follow the process approximating, internalization of IFR, and then estimating.

As the result of the conclusion, we provide the readers what so-called local instruction theory in length estimation synthesized from our finding in this design-based research, see the following table.

Table 14. Local Instruction Theory on Length Estimation

Title	Goal	Activity
Frog Jumping and *Pocong* Jumping	To raise awareness of using references for approximation rather than ruler	Students approximate and confirm whether the distance of the track they made in the game reach 30 meter or not.
Measure and use your body part	To raise awareness for students to use body parts as reference points for approximation	Students approximate a set of Indonesian-game photographs in such a way that body parts become clues for the approximation
Shoes, couch and Flood	To associate and reason using body parts to approximate the length of objects as new reference points	Students approximate from photograph of shoes-tiles problem, and flood problems in which the use of body parts could be useful to reason other objects.
!7th August – Balloon Decoration	To associate objects to other objects to gain new reference points	Students are asked to find as many as possible references from the UKS room photographs and order their efficiency.
17 August – The rope and the plastic flag decoration	To identify use and reason using reference points/IFR for solving length approximation/estimation tasks in social arithmetic problem.	Students are asked to determine budget for the decoration by how many packs of plastic flag and rolls of rope needed to cover some part of the school.

6.2. Limitation

During the preliminary study, we had done some research about the appropriate context for students' tasks. However, due to the difference culture from one region to other regions in Indonesia and the development of the region particularly rural and urban area, we found out that some students had no experience playing game like in the context of lesson 2. The context seems to be outdated for the students. Unfortunately, still the students could imagine what the context tell about.

Another important point is that the teacher role in this study. Primarily in the second cycle, that the teacher could not establish expected socio norms and

socio mathematical norms. Although in approximation/estimation tasks more than one possible answers could emerge, the students got little chance to elaborate their thinking across group of work to discuss differences and sophistication of students' reasoning. In addition, the students who did not present or discuss in the whole class discussion seemed to be not listening to the presenter/the teacher. All of this socio/socio-math norms matters would influence learning in the whole class discussions.

6.3. Suggestion

Before implementing the lessons of this study one should carefully adjust the context appropriate to the new learning condition. Moreover, one should also take into consideration the improvement points of the second cycle discussed in the retrospective analysis. In addition, improvement of the teacher role could be facilitated by having an extensive discussion with the teacher about establishing appropriate socio norms and socio-mathematics norms before implementing the lessons (for extension of this study) including having a clear descriptive activity in the teacher guide.

Realizing that this study only contributes a very little to the development of local instructions theory in mathematics education, further study about how to boost the development of students' skills from approximating to estimating might be useful for learning theory in the domain of measurement.

REFERENCE

Anderson, K. T. (2008). Intersubjectivity. In L. M. Given (Ed.), *The SAGE Encylopedia of Qualitative Research Methods* (pp. 468-469). Thousand Oaks: SAGE Publications, Inc.

Bakker, A., & van Eerde, D. (2013). An introduction to design-based research with an example from statistics education. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Doing Qualitative Research: Methodology and Methods in Mathematics Education*. Utrecht.

Barab, S., & Squire, K. (2004). Design-based research: putting a stake in the ground. *The Journal of the Learning Sciences, 13*(1), 1-14.

Bright, G. W. (1976). Estimation as part of learning to measure. In D. Nelson & R. E. Reys (Eds.), *Measurement in School Mathematics* (pp. 87-104). Washington: National Council of Teacher Mathematics.

Bryman, A. (2003). Triangulation. In A. B. Michael Lewis-Beck, Tim Futing Liao (Ed.), *Encyclopedia of Social Science Research Methods* (pp. 1142-1143): SAGE Publications.

BSNP. (2006). *Standar Isi: Untuk Satuan Pendidikan Dasar dan Menengah*. Jakarta: BSNP.

Chang, K.-L., Males, L. M., Mosier, A., & Gonulates, F. (2011). Exploring US textbooks' treatment of the estimation of linear measurements. *ZDM Mathematics Education, 43*, 697-708.

Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics, 99*(1), 5-11.

Crites, T. (1992). Skilled and less skilled estimators' strategies for estimating discrete quantities. *The Elementary School Journal, 92*(5), 601-619.

Denscombe, M. (2010). *The Good Research Guide for small-scale social research projects*. Meidenhead: Open University Press.

Dolk, M., & Fosnot, C. T. (2001). *Young Mathematicians at Work: Constructing multiplication and division*. Portsmouth: Heinemann.

Edelson, D. C. (2002). Design Research: What we learn when we engage in design. *The journal of the learning Sciences, 11*(1), 105-121.

Fajariyah, N., & Triratnawati, D. (2008). *Cerdas Berhitung Matematika 3: Untuk SD/MI kelas III*. Jakarta: Depdiknas.

Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education, 29*(3), 334-356.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Reidel.

Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures*. Doordrecht: Kluwer Academic Publishers.

Gooya, Z., Khosroshashi, L. G., & Teppo, A. R. (2011). Iranian students' measurement estimation performance involving linear and area attributes of real-world objects. *ZDM Mathematics Education, 43*, 709-722.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105-128.

Gravemeijer, K. P. E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β Press.

Hadi, S. (2002). *Effective Teacher Professional Development for the Implementation of Realistic Mathematics Education in Indonesia.* (Doctoral), University of Twente, Enschede.

Hall Jr, L. T. (1984). Estimation and Approximation-Not Synonyms. *The Mathematics Teacher, 77*(7), 516-517.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81-112. doi:10.3102/003465430298487

Hildreth, D. J. (1983). The use of strategies in estimating measurement. *The Arithmetic Teacher, 30*(5), 50-54.

Hogan, T. P., & Brezinski, K. L. (2003). Quantitative Estimation: One, Two, or Three Abilities? *Mathematical Thinking and Learning, 5*(4), 259-280.

Joewono, B. N. (2013). Kurikulum 2013 diberlakukan secara bertahap. *Kompas*. Retrieved from http://edukasi.kompas.com/read/2013/02/13/16430169/Kurikulum.2013.Diberlakukan.secara.Bertahap

Jones, G., Taylor, A., & Broadwell, B. (2009). Estimating linear size and scale: Body rulers. *International Journal of Science Education, 31*(11), 1495-1509. doi:10.1080/09500690802101976

Jones, G. M., Gardner, G. E., Taylor, A. R., Forrester, J. H., & Andre, T. (2012). Students' accuracy of measurement estimation: context, units, and logical thinking. *School Science and Mathematics, 112*(3), 171-178.

Joram, E., Gabriele, A. J., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education, 36*(1), 4-23.

Joram, E., Subrahmanyam, K., & Gelman, R. (1998). Measurement Estimation: Learning to map the route from number to quantity and back. *Review of Educational Research, 68*(4), 413-449.

Kemendikbud. (2013). *Kurikulum 2013*. Jakarta: Balitbang Kemendikbud.

Kemendikbud. (2014). Buku Sekolah Elektronik: Bermutu & terjangkau untuk semua. Retrieved from bse.kemdikbud.go.id

Lang, F. K. (2001). What is a good guess anyway? Estimation in early childhood. *Teaching Children Mathematics, 7*(8), 462-466.

Markovits, Z., & Hershkowitz, R. (1997). Relative and absolute thinking in visual estimation processes. *Educational Studies in Mathematics, 32*, 29-47.

Masitoch, N., Mukaromah, S., Abidin, Z., & Julaeha, S. (2009). *Gemar Matematika 3*. Jakarta: Pusat Perbukuan Depdiknas.

Mustoha, A., Buchori, Juliatun, E., & Hidayah, I. (2008). *Senang Matematika 2: Untuk SD/MI kelas 2*. Jakarta: Pusat Perbukuan Depdiknas.

Purnomosidi, Wiyanto, & Supadminingsih, E. (2008). *Matematika 2: untuk SD/MI kelas 2*. Jakarta: Pusat Perbukuan Depdiknas.

Roth, W.-M., & Roychoudhury, A. (1993). The development of science process skills in authentic contexts. *Journal on Research in Science Teaching, 30*(2), 127-152.

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming Mathematics Learning in Indonesian Classrooms through RME. *ZDM Mathematics Education, 40*, 927-939.

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning, 6*(2), 91-104.

Smart, J. (1982). Estimation skills in Mathematics. *School Science and Mathematics, 82*(8), 642-649.

Smith, W. D., & Darrin, G. L. (1978). Make a meter tape. *School Science and Mathematics, 78*(5), 365-370.

Sowder, J. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York (etc.): Macmillan.

Towers, J., & Hunter, K. (2010). An ecological reading of mathematics language in a grade 3 classroom: A case of learning and teaching measurement estimation. *Journal of Mathematical Behavior, 29*, 25-40.

Treffers, A. (1987). *Three Dimensions: A model of goal and theory description in mathematics instruction-The Wiskobas Project*. Dordrecht: D. Reidel.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Zulkardi. (2002). *Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers.* (Doctoral), University of Twente, Enschede.

NOTULEN

THESIS DEFENSE

Student Name : Rudi Hartono
Student Number : 06022681318078
Study Program : Mathematics Education
Research Title : Promoting the Development of Students' Individual Frame of Reference to Support Length Approximation/Estimation Skills
Place of Defense : Kampus Bukit Besar, Sriwijaya University, Palembang, Indonesia
Date of Defense : 30 June 2015 / 12.00-13.00

No	Examiner	Questions or Suggestions	Reaction
1	Dr. Somakim, M.Pd	• Did you find students were interested with your lessons? • What are students' most obvious reaction when you implement this kind of lessons?	Have been explained in the thesis defense
2	Prof. Dr. Zulkardi, M.I.Komp, M.Sc	• If you design a lesson for students to develop their reference point for right angle, what objects will you promote? • What is your plan about this topic of research in your future career?	Have been explained in the thesis defense

Palembang, July 2015

Supervisor I,

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APPENDICES

Appendix 1. Classroom observation scheme

- Teaching and Learning Process
  - What topic the teacher teach?
  - How the teacher open the lesson?
  - Is there any context given?
  - How the teacher deliver the topic? (Style: explanatory, demonstrating, etc.)
  - Is there a discussion between the students? (in pairs, group or individually) How the teacher make the group?
  - How the teacher manage the group discussion?
  - What is the role of the teacher during the discussion?
  - Is there a presentation of students' work in the class?
  - How the teacher appreciate different opinion/different solutions of the students?
  - Is there a discussion about elegant, efficient or sophisticated solution? (Socio mathematics norms)
  - Is there any students who dominate or silent?
  - Are the students listening to the teacher when teaching?
  - What difficulties students encounter during the learning process?
  - Is there any irrelevant behavior of students during the lesson?
  - How the teacher ends the lesson?
  - How the teacher deal with time management?
  - How the teacher responses students' answers, opinions, or questions?
  - How does the teacher lead the discussion?
  - Does the teacher give the students thinking time after a question?
  - How the teacher manage chances for the students to talk their opinions? (mostly the same students or a variety of students?)
- Classroom Environment
  - How many students in the classroom? How many boys and girls?
  - How students sit in the classroom?
  - How the position of the teacher during the lesson or discussion?
  - Are the students mostly active or passive during the lesson? Who the active students? How the passive ones?
  - Is there any tools the teacher uses during the lesson?

Appendix 2. Teacher interview scheme

- Teacher Background
  - How long have you been teaching primary school?
  - Do you teach only mathematics or several subjects?
  - In which grade(s) you have experienced in teaching mathematics?
  - How long have you been teaching fourth grade?

- Teaching process
  - What do you think about your class?
  - Have you ever given your students open problems?
  - How you give your students feedback?
  - How you compare your students' answers?
  - How you accept your students' criteria (based on)?
  - Do you do formative assessment during the lesson?
  - Do you have specific rules for rewarding, punishing or anything else?
  - Do you make grouping of your classroom?
  - Do you use tools (computers, software, projectors) when teaching?
  - Do you follow certain book when teaching? How flexible are you?
  - How you manage your students when they are discussing?
  - How do you usually guide the students in learning measurement topic?

- Experience on PMRI approach
  - What do you know about PMRI?
  - Have you ever attended a PMRI seminar, workshop or involved in PMRI-based approach project (participant, research collaboration)?
  - Do you implement PMRI approach in your class?
  - What do you think about PMRI approach? Strong points? Weaknesses?
  - Do you think it is possible to implement PMRI in your class?

- About students
  - How many students in your class?
  - How active they are?
  - Do you know who the high achievers and the low achievers?
  - What difficulties you have encountered when teaching them?
  - Do they actively involved in class discussion?
  - Are they manageable if learning in experience-based activities?
  - Do they usually give presentation of their work?
  - Based on your experience, how you think the students will solve the problem on length measurement estimation?

Appendix 3. Classroom observation field note (example)

OBSERVATION SHEET

(Field Notes for Teaching Experiment)

Name of observer : Time Start :

Day/Date : Time Finish :

Lesson/Activity : 1/Frog Jumping and *Pocong* Jumping

Guidelines for completing the observation sheet

a. Write down your name and the lesson you observed.
b. Watch the teacher guide while you are completing.
c. Thick the aspects you observed, and give some explanations about your concern

1. Notes about students

Aspect		Description
Strategy used	○ Iterating rulers ○ Using body parts ○ Using external references specify: …… ○ Guessing ○ Others specify:…….	
Sense of Unit	○ Appropriateness of units used ○ Appropriateness of number used ○ Appropriateness of magnitude presented	
Reasoning and Ordering Efficiency of references	○ Considering Quickness ○ Considering Accuracy ○ Considering Feasibility	

Other concerns	

2. **Teacher**

Aspect	Description
Teaching flow (suitability with teacher guide)	
Classroom management	
Other concerns	

3. **Classroom conditions**

Aspect	Description
Social norms	
Socio-mathematical norms	
Teacher-students interactions	
Other concerns	

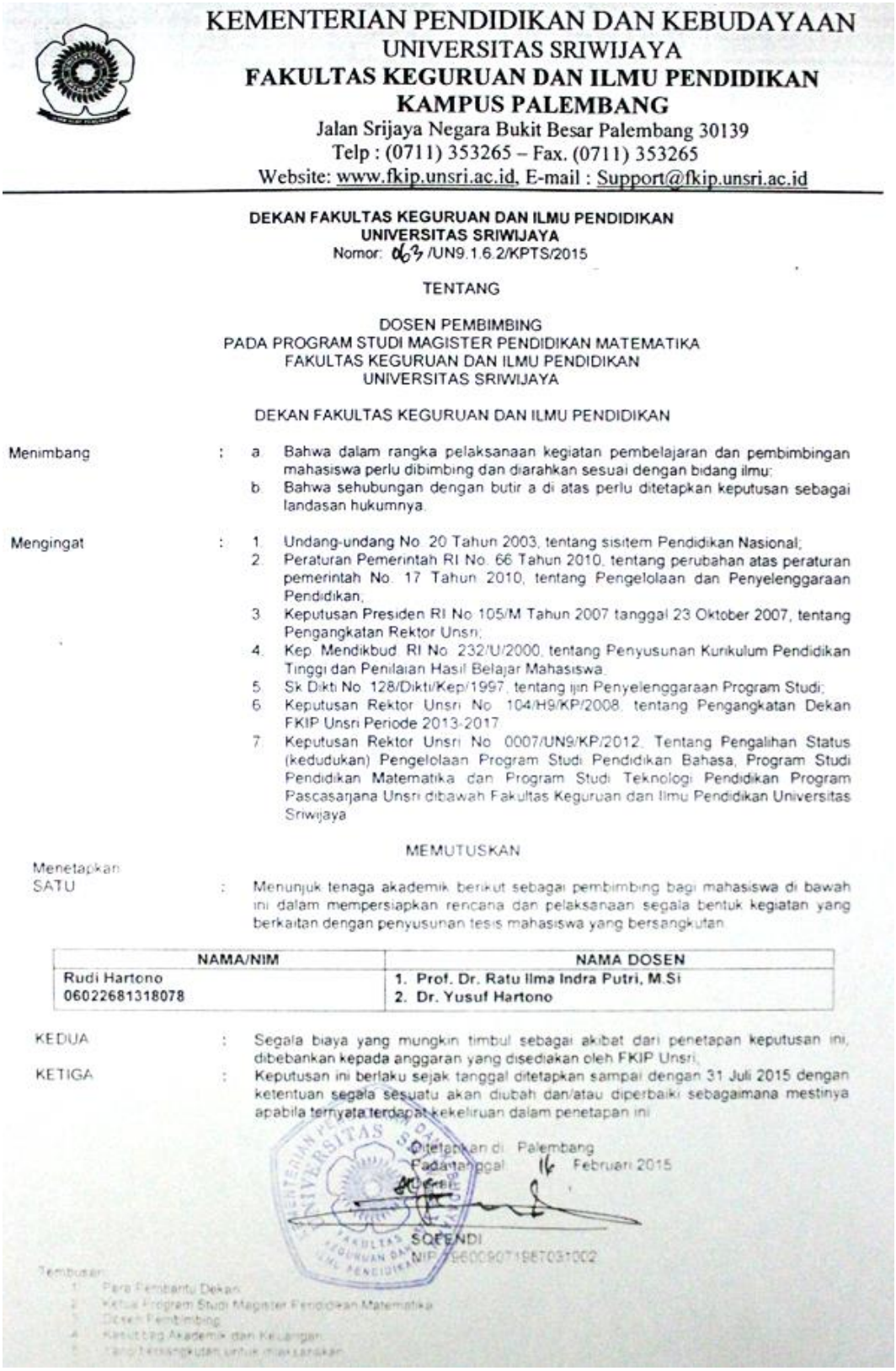
4. **Technical matters**

Aspect	Description
Time	
Learning materials	
Other concerns	

General Remarks

Suggestions

Appendix 4. Surat keputusan dosen pembimbing



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DEKAN FAKULTAS KEGURUAN DAN ILMU PENDIDIKAN
UNIVERSITAS SRIWIJAYA
Nomor: 063 /UN9.1.6.2/KPTS/2015

TENTANG

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FAKULTAS KEGURUAN DAN ILMU PENDIDIKAN
UNIVERSITAS SRIWIJAYA

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b. Bahwa sehubungan dengan butir a di atas perlu ditetapkan keputusan sebagai landasan hukumnya.

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2. Peraturan Pemerintah RI No. 66 Tahun 2010, tentang perubahan atas peraturan pemerintah No. 17 Tahun 2010, tentang Pengelolaan dan Penyelenggaraan Pendidikan;
3. Keputusan Presiden RI No 105/M Tahun 2007 tanggal 23 Oktober 2007, tentang Pengangkatan Rektor Unsri;
4. Kep. Mendikbud RI No. 232/U/2000, tentang Penyusunan Kurikulum Pendidikan Tinggi dan Penilaian Hasil Belajar Mahasiswa.
5. Sk Dikti No. 128/Dikti/Kep/1997, tentang Ijin Penyelenggaraan Program Studi;
6. Keputusan Rektor Unsri No. 104/H9/KP/2008, tentang Pengangkatan Dekan FKIP Unsri Periode 2013-2017.
7. Keputusan Rektor Unsri No. 0007/UN9/KP/2012, Tentang Pengalihan Status (kedudukan) Pengelolaan Program Studi Pendidikan Bahasa, Program Studi Pendidikan Matematika dan Program Studi Teknologi Pendidikan Program Pascasarjana Unsri dibawah Fakultas Keguruan dan Ilmu Pendidikan Universitas Sriwijaya

MEMUTUSKAN

Menetapkan:
SATU : Menunjuk tenaga akademik berikut sebagai pembimbing bagi mahasiswa di bawah ini dalam mempersiapkan rencana dan pelaksanaan segala bentuk kegiatan yang berkaitan dengan penyusunan tesis mahasiswa yang bersangkutan

NAMA/NIM	NAMA DOSEN
Rudi Hartono 06022681318078	1. Prof. Dr. Ratu Ilma Indra Putri, M.Si 2. Dr. Yusuf Hartono

KEDUA : Segala biaya yang mungkin timbul sebagai akibat dari penetapan keputusan ini, dibebankan kepada anggaran yang disediakan oleh FKIP Unsri.

KETIGA : Keputusan ini berlaku sejak tanggal ditetapkan sampai dengan 31 Juli 2015 dengan ketentuan segala sesuatu akan diubah dan/atau diperbaiki sebagaimana mestinya apabila ternyata terdapat kekeliruan dalam penetapan ini.

Ditetapkan di Palembang
Pada tanggal 16 Februari 2015

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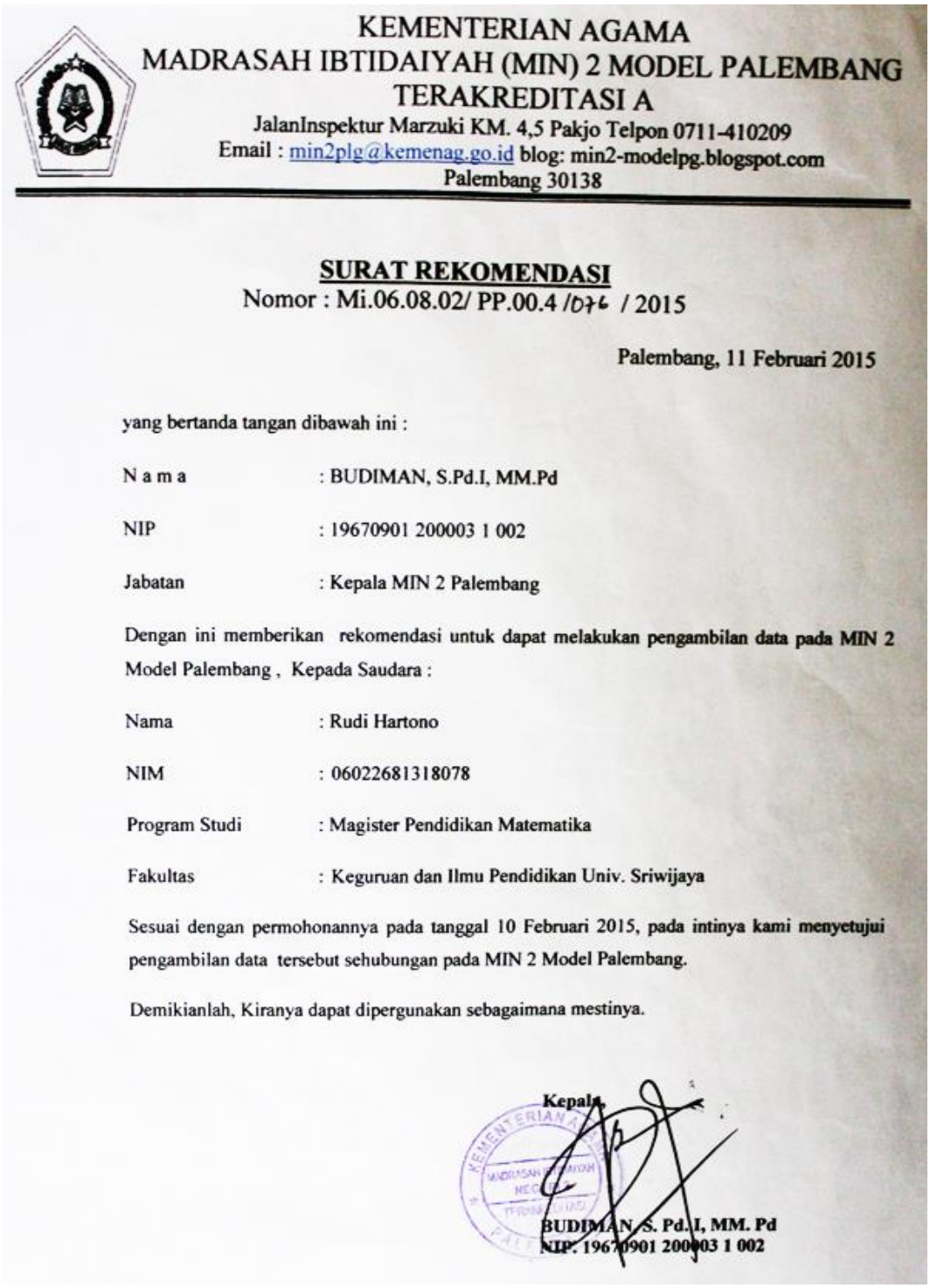
Tembusan:
1. Para Pembantu Dekan
2. Ketua Program Studi Magister Pendidikan Matematika
3. Dosen Pembimbing
4. Kasubbag Akademik dan Keuangan
5. Yang bersangkutan untuk dilaksanakan

Appendix 5. Surat Rekomendasi Penelitian



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Palembang, 11 Februari 2015

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Sesuai dengan permohonannya pada tanggal 10 Februari 2015, pada intinya kami menyetujui pengambilan data tersebut sehubungan pada MIN 2 Model Palembang.

Demikianlah, Kiranya dapat dipergunakan sebagaimana mestinya.

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Appendix 6. Surat keterangan telah melakukan penelitian

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Di-
Palembang

Menindaklanjuti surat Dekan Nomor : 356/UN9.1.6.2/PL.5/2015 tanggal 10 Maret 2015 Perihal Izin Penelitian, Kepada Saudara :

Nama : Rudi Hartono

NIM : 06022681318078

Program Studi : Magister Pendidikan Matematika

Bku : IMPOME

Judul Tesis : Developing Students' Individual Frames of Reference to Support Lenght Estimation Skills

Dengan ini yang bersangkutan telah selesai melaksanakan Penelitian guna keperluan Penyusunan tesis tersebut dari tanggal 02 Maret 2015 sampai dengan 02 April 2015 di MIN 2 Model Palembang.

Demikianlah, Kiranya dapat dipergunakan sebagaimana mestinya.

Kepala

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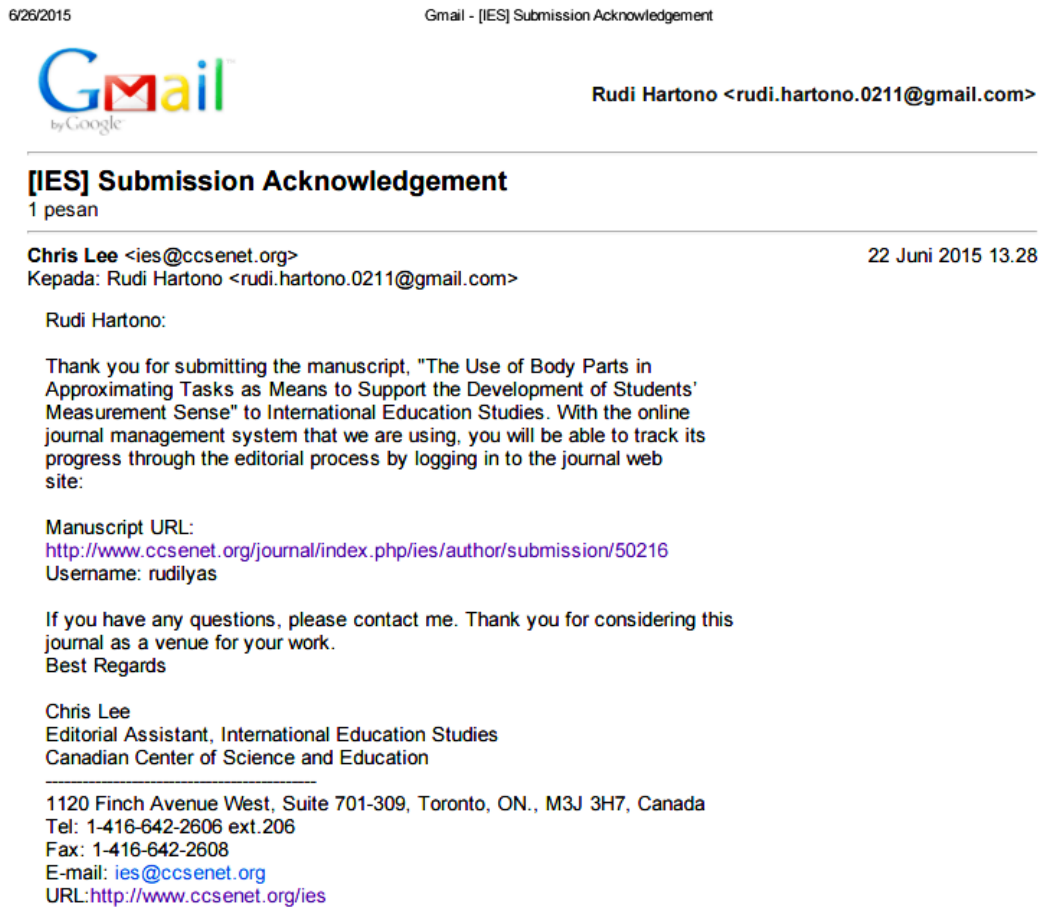
Appendix 7. Submission acknowledgement of journal article (the second cycle)



6/26/2015 Gmail - [IES] Submission Acknowledgement

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[IES] Submission Acknowledgement

1 pesan

Chris Lee <ies@ccsenet.org> 22 Juni 2015 13.28
Kepada: Rudi Hartono <rudi.hartono.0211@gmail.com>

Rudi Hartono:

Thank you for submitting the manuscript, "The Use of Body Parts in Approximating Tasks as Means to Support the Development of Students' Measurement Sense" to International Education Studies. With the online journal management system that we are using, you will be able to track its progress through the editorial process by logging in to the journal web site:

Manuscript URL:
http://www.ccsenet.org/journal/index.php/ies/author/submission/50216
Username: rudilyas

If you have any questions, please contact me. Thank you for considering this journal as a venue for your work.
Best Regards

Chris Lee
Editorial Assistant, International Education Studies
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LETTER OF CONSENT

We hereby give consent:

Student Name	:	Rudi Hartono
Student Number	:	06022681318078
Study Program/Class	:	Mathematics Education/IMPoME
Article Title	:	Promoting the Development of Students' Individual Frame of Reference to Support Length Approximation/Estimation Skills

To publish the article to International Journal namely International Educational Studies (IES).

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INTERNATIONAI

EDUCATION STUDIE

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TITLE AND ABSTRACT

Title The Use of Body Parts in Approximating Tasks as Means to Support the Development of Students' Measurement Sense

Abstract

This study reports a part of a full study aimed to improve local instructions theory on mathematics education for supporting the development of students' measurement sense. The research approach in this study is design-based research in which we design, test and revise hypothetical learning trajectory. This study focuses on two of five classroom activities designed using Realistic Mathematics Education (RME) principles. We developed a set of approximating problems using Indonesian context in which the use of body parts are promoted. The participants of this study were thirty four 4^{th} graders (9-10 year old) and the regular teacher from one elementary school in Palembang, Indonesia. We collected data from students' work, classroom observation and interviews with the participants then we analyzed the data mostly in qualitative ways. The result suggests that the use of body parts as initial reference points for approximating may aid students to reason about other external objects which may support the development of new reference points.

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CONTRIBUTORS AND SUPPORTING AGENCIES

Agencies DIKTI, NUFFIC NESO, P4MRI, Monica Wijers; Freudenthal Institute

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Appendix 8. Article of the first cycle

SUPPORTING THE DEVELOPMENT OF STUDENTS' REFERENCE POINTS FOR LENGTH ESTIMATION

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Abstract

This study reports a part of a full study aimed to improve local instructions theory on mathematics education for supporting to develop students' reference points for length estimation. We design 5 classroom activities using Realistic Mathematics Education (RME) approach with Indonesian contexts through approximating and estimating tasks. Measuring relatively long object in lompat katak and lompat pocong game and approximating from Indonesian traditional games photographs build understanding of physical unit iteration and naturally shift from the use of rulers to use reference points.. Meanwhile, estimating the height of a classroom, and the height of a flag pole push the use of mental reference points and connecting body parts to external object as new reference points. The participants of this study are four 4th graders of MIN 2 Palembang. We collect data from students' work, classroom observation and interviews with the participants then we analyze the data mostly in qualitative ways. The results suggest that the students use guessing, employ reference points in form of body parts (arm spans, body height, etc) or external objects (height of a bookshelf, height of a room, etc.) and also mentally iterate IFR especially one meter unit by imagine the length of an arm spans to the to-be-estimated objects.

Keywords: *Reference points*, *Estimation*, *Approximation*, *PMRI*, *RME*, *local instructions theory*, *design-based research*, *hypothetical learning trajectory*

INTRODUCTION

Measurement estimation is a very important skill in life. Jones and Taylor (2009) stated that individual in a variety of professions argue that estimating skills are essential for their careers. For instance, an architect estimates the space needed for a body to pass a doorway, a butcher estimates the dimension of meat being cut off to meet certain weight, and a park ranger estimates the distances between landmarks. The action in the activities come naturally without the use of standard measurement tools.

However, many studies reported low performances on length estimation tasks (Hildreth, 1983; Joram, Subrahmanyam, & Gelman, 1998). For instance in Hildreth (1983), he interviewed 24 students to solve 24 estimation tasks (length and area estimation), 40% of the students were categorized as using inappropriate strategies including wild guessing. Moreover, only a few studies that focus on developing students' skill in length estimation tasks (G. Jones, Taylor, & Broadwell, 2009). Yet little is known how it can be

embedded into instructional activities (Joram et al., 1998). Furthermore, Markovitz and Hershkowitz (1997) followed by Hogan and Brezinski (2003) stated that a different approach is needed to teach and support students developing their skill in length estimation such as enriching visual or spatial experiences and developing personal point of references. Hence, teaching and learning about length estimation need more attention.

Including in Indonesia, length estimation is not explicitly taught in the Indonesian curriculum (BSNP, 2006). The curriculum tends to focus on how to convert and use standard units of length (metric units) and the standard measurement tools. Less attention is given to an activity of making sense of units for estimating. It seems that there is a need to improve this educational gap by designing a series of mathematics lessons/activities that can support students in developing their skills in estimating.

The goals of the present study are to investigate what strategy used by students use in estimating length and students' sense of length (point of references). In general, we want to contribute to local instructions theory in mathematics education about how to support students' skills in length estimating especially in the measurement strand. Therefore our intentions can be summarized as a general research question: *How can we support the development of students' reference points for length estimation?*

THEORETICAL FRAMEWORK

Measurement Estimation

There are three type of quantitative estimations: computational estimation, numerosity estimation, and measurement estimation. The first, computational estimation is related how ones can compute flexible and creatively (number sense) to find certain calculation of number. The second, numerosity estimation is how ones can estimate number of discrete objects. The last, measurement estimation (length) is related how ones can make an educated guessing about how long a continuous object without the use of measurement tools (Bright, 1979; Smart, 1982).

Measurement estimation does not relate to the development of general mathematical ability as the computational estimation. Instead, it should be addressed as part of spatial ability (Hogan & Brezinski, 2003). The learning instructions should enrich students' visual experience through interaction with physical objects (Markovits & Hershkowitz, 1997; Smart, 1982). In other words, students should first experience a process of rough physical measuring (approximating) before mentally measuring the to-be-estimated objects (estimating). It is expected that the use of concrete objects can enhance students' feel of length internalized into known objects called reference points.

Point of References

When students are given length estimation tasks, various strategies will be employed depending on their proficiency in estimating (Hildreth, 1983). A less skilled estimator tends to wild guess, mention inappropriate units or numbers without a reasonable

explanation. Meanwhile, a skilled estimator employs educated strategies such as the use of prior knowledge (recall identic objects), mental meter, and or reference points. Following is a summary of the development of students' strategies in approximating and estimating.

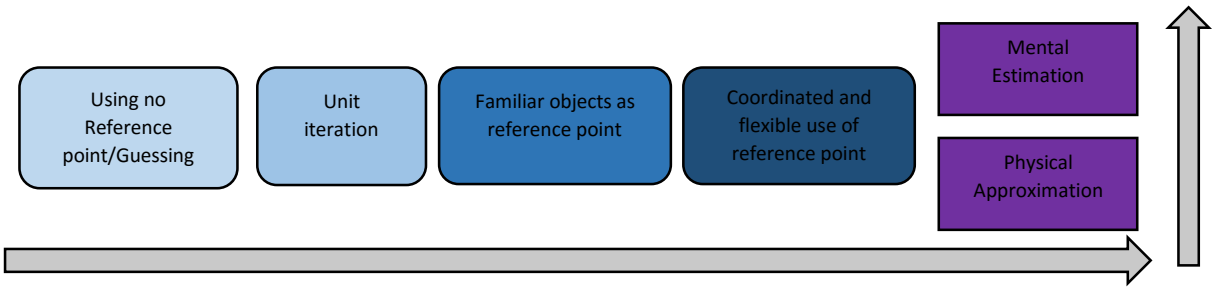


Figure 1: Two dimensional development of reference points and estimation skills

Personal point of references are object whose lengths are known physically or mentally (in mind) to be used for approximating or estimating. Reference points help estimator to approximate or estimate by imagining a specific object and compare it to the to-be-estimated (Joram, et al., 2005). This imaginable objects develop through everyday experience but the development can be enhanced through appropriate approximating and estimating tasks. For instance, one might use their body height to estimate the height of a classroom or even imagine a 20-feet-long crocodile to estimate the length of a classroom as found in Hildreth's (1983) study.

Furthermore, developing students' point of reference should be the primary goal of length estimation instructions, it serve as a critical point to develop students' sense and understanding of measurement (Bright, 1976; Clements, 1999; Lang, 2001). However, students often do not spontaneously employ reference points to approximate or estimate length caused by complex interaction among students' preferences, context of the tasks and the nature of estimation activity (Gooya, Khosroshashi, & Teppo, 2011; Hildreth, 1983; Joram, Gabriele, Bertheau, Gelman, & Subrahmanyam, 2005). Therefore a learning instruction in length estimation should carefully be designed by considering the aforementioned aspects. In this study, we employ Realistic Mathematics Education approach to design the activities.

Realistic Mathematics Education (RME)

RME promotes mathematizing, a process in which students be able to mathematize everyday problem situation in mathematical terms and employ it within mathematics itself (Freudenthal, 1991; K. Gravemeijer, 1994; K. P. E. Gravemeijer, 1994). RME provides a framework for designing an instructional task for the progressive mathematization (Treffers, 1987) determined by its five tenets. In Indonesia RME is adapted into PMRI (*Pendidikan Matematika Realistik Indonesia)* (Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002) which focus on supporting students with Indonesian local contexts

for learning mathematics (Zulkardi & Ilma, 2006). We briefly describe how the five tenets employed to the topic of our study.

Approximating/estimating are *context*-bounded tasks, the instructions should be started and ended in a meaningful real world situations. Promoting *the use of models* such as drawing, sketch, as proposed in RME, may enhance students understanding how unit is iterated and used in efficient ways. Moreover, process of developing personal reference points is a *students' own construction* process, it cannot be forced to the students because it relates to a mental perception depending what they perceive and experience. Hence, an *interactive instruction* should be formulated in order to trigger discussion about students' personal reference points. It is important because from listening and negotiating other perceptions and perceptive of length, one may build a good personal reference points.

In addition, it also important to note that in order to conduct a powerful instruction as proposed in RME tenets, change in class socio norms and socio mathematical norms should be promoted (Yackel & Cobb, 1996). The students should be realized that they cannot always go to the teacher for clarification of their answers. Moreover, the teacher should be able to establish agreement what kind of answers/strategies are considered as good guesses or efficient and how to judge which estimation is better in vague and inexactness of estimating tasks (Forrester & Pike, 1998). Therefore, the role of the teacher is changed from the information transmitter and the one who always give judgement to a facilitator and guide of students' own discovery.

METHOD

Research Approach

Considering our research question, it seems to be logic that we employ an approach that directly speaks how to design a learning trajectory such as classroom activities and the teaching and learning materials. Hence, we employ design-based research as research approach of this study.

DBR is characterized by its cyclical or iterative process of designing-revising the educational materials specifically the learning trajectory (Bakker & van Eerde, 2013; Barab & Squire, 2004; Edelson, 2002). The learning trajectory is designed and tested in 3 phases (preparation and design, teaching experiment, and retrospective analysis) and be revised in several cycle (one cycle consists of the three phases) (Bakker & van Eerde, 2013; Gravemeijer, 2004). Further, DBR does not only also speaks how to design, but also how to describe the students' learning development which is used to advice for better teaching and learning action. Therefore, we will design a learning trajectory and its hypothetical learning trajectory (HLT) describing a learning instruction and the possibilities of students' thinking to support students' development of reference points for estimating. In addition, we will also explain how the students' learnings take place and give practical advice about it.

Subject, Data Collection and Data Analysis

This study is the first cycle of two cycle DBR that we conducted. The subject of this pilot experiment is 4 fourth graders (two boys and two girls) of an elementary school in Palembang, Indonesia.

Data were collected from the preparation phases and the teaching experiment phases of design-based research and a posttest afterward. The data were collected by semi-structured interview (teacher and students), classroom observation, and students' written tests.

Data gained were analyzed using triangulation system, we analyzed students' written work, registered video of the learning and field note of the observer. We compare interesting fragments of students' written works or registered video in which the learning takes places or not compare to the HLT. The analysis was mostly done in a qualitative ways and in modest quantitative ways.

RESULT AND DISCUSSION

The learning activities that we design consist of 5 lessons. Before implementing the design, we conduct a pretest and interview to the students then a posttest afterward. It is important to note that both pretest and postest are not aimed to measure how far the design work. Instead, we analyse each lesson to know how the design and how the students' learning occurs. Following is the overview of the 5 activities.

Table 1
Overview of Activities and Main goals on supporting the development of students' reference points for length estimation

Activity	Main Goal
Frog Jumping and *Pocong* Jumping	Shift students' from using rulers to use other reference points such as body part for approximating.
Measure and Use Your Body Parts	Knowing accurate lengths of body parts to be used as reference points.
Length, Width and Height of the Building	Develop external reference points and ordering their efficiency. Visualizing iteration of reference points.
17th of August Decoration: Balloons and the Flag Pole	Internalization of standard units of measurement. Shift students to do mental iteration (estimation) using reference points.
17th of August Decoration: Plastic Flag and the Rope	To spot, use and reason using reference points for solving length estimation problems involving social arithmetic.

We may see from table 1, the lessons are sequenced to support students' development of using references point in a physical ways (approximation) to the mental one (estimation). We will analyse and discuss each of the lessons on the next paragraphs.

First of all, the prestest, it is aimed to get information about students' prior knowledge about the topic of references points for approximation/estimation. The result of the prestest and the interview afterward indicated that students have difficulties to use a proper unit of length. In addition, they also improperly represented a magnitude of objects. For instance, if they think the object is very long they will write a relatively very big number. One student said that the length of a A4-sized paper is 450 cm, and another student said it is 5 or 7 cm. The students had not yet developed good sense of units marked by either improper use of units and numbers. In addition, most of the students use guessing methods when they were asked how they know their reference points. Hence, in most cases, students did employed reference points but they have no idea about the length of the reference points.

Lesson 1, frog jumping and *pocong* jumping is aimed to accommodate learning through a game, raise awareness to compare lengths, stimulate conflict about used reference points and to shift students from using rulers to more efficient ways. Students in pair, were asked to measure/approximate a relatively long distance through a game called "Frog Jumping and *Pocong* Jumping". The game was played on the school yard, each students in their group alternately jump like a frog 4 times, jump like a *pocong* (Indonesian ghost) 4 times, mark their distance and then approximate whether the distance is 30 meter or not.

We found a confirmation to our HLT that the students did not want to use a ruler since it would be tedious to use. Instead, the students used arm spans as one meter unit. Indeed, there was also a conflicting situation in which the students argue whether their arm span is one meter long or not. Following fragment shows their reasoning.

Fragment 1: Conflict of arm span length

1. Teacher : Do you agree that they got 54 (meter)?"
2. Sugi and Aziz : No!
3. Teacher : Why? Why?
4. Sugi : (pointing to Fitria) her arms are short.

What we can say from fragment 1 is that Sugi refused to believe that the girl group got 54 meter because the Fitria's arms are too short to represent one meter.

Overall, lesson one provides a sufficient situation to raise students awareness to employ references points. What is need to be improve is that the game should facilitate the use of several reference points (not only arm spans) to emerge a rich discussion.

Lesson 2, measure and use your body parts, aimed to provide students to physically approximate object in photographs using reference points on the photographs. The students were given a set of familiar photographs about Indonesian games. The photograph were selected in such a way we embeded clues of body parts that possibly could be used for the students to approximate. They were asked to find a real length or distance of an object in the photograph. For instance, they are asked to approximate the length of a *bakiak* (long-wooden sandals) given clues such as feet, tiles, etc.

Figure 2: *Bakiak* Problem

We found that most of the students could figure out the lengths of the objects employing reference points such as hand spans, body height and arm spans as predicted on the HLT. For instance, the students first measure their hand spans using a ruler as a reference points to be used on the photograph. It is however, the students did not spontaneously perceive feet as a reference points for approximating especially for the *bakiak* problem. The students tend to guess the length of the *bakiak* by imagine their own version of one meter (mental) in that photograph. It could be probably because the students are not familiar to use feet for measuring something. Overall, lesson 2 could trigger students' sensitivity to make body parts as reference points. A point to improve is that the photograph of the *bakiak* should be revised in order to stimulate students' awareness of the use of feet as reference points.

In lesson 3, the students were asked to approximate two photographs (a greasy pole and a building). Still, we found that the students used guessing to approximate at the very beginning. However it turned to be out that for the first problem, the students employed their thumb as a reference for one meter on that photograph. It is probably that the students first assume the one meter, then assign it to the thumb. Moreover, one student named Aziz use a person on the photograph as clue for imagining a person's arm span. Hence, it seems students tend to imagine or iterate standard unit of one meter to approximate the problem rather than using a ready-made point of reference such as body height or the height of the house.

For the second problem of lesson 3, we found that the students got difficulties to cope with the context of the problem. The students were not familiar with the building and got confused about 2 dimensional representation of a 3D object. It was not surprising also that when they were asked to find the most efficient point of references, they only focus on finding the longest objects as point of references such as a road lamp and a bus even though they are not familiar with the length rather than using objects which they know the lengths. Based on this finding, we intend to revise the problem of lesson 3 as such we will involve more students' investigation on connecting body parts to external objects to develop students' reference point for approximating or estimating.

In lesson 4, students were engaged in two activities. The first, students were asked to make a one-meter-long rope in the context of eating *kroepoek* contest. The aim is to

promote internalization of one meter unit as reference point which is important for mental estimation. The second activity is the students were asked to estimate the height of their classroom and the height of the school flag pole. It is aimed so that the students shift to begin imagine mentally their reference points instead of iterate them physically.

We found that in the first activity, the students again tended to use arm span to make the one meter long rope. It is however, make them tediously shift to employ other reference points. For instance, in activity two, as predicted in HLT, they try to approximate the height of the school flag pole and the height of the classroom using arm span as shown in the following photograph.

Figure 3: Aziz approximates the height of the wall using arm spans

From figure 3, we may see that, Aziz used his one and a half arm spa to approximate the height of the classroom. Realizing that it was difficult, the students discussed and found out to use other references as shown in the following fragment.

Fragment 2: Discussion on the height of a bookshelf

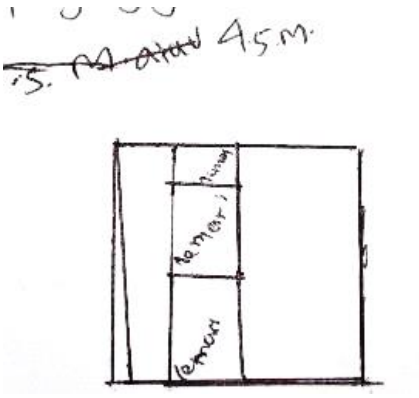
1.	Fitria	:	Ouu that is three meters high (pointing at a bookshelf).
2.	Teacher	:	Which one? The bookshelf? How high is the bookshelf?
3.	Sugi	:	Two, two, one meter and a half!
4.	Fitria	:	Two meters.
5.	Teacher	:	How do you know this is 2 meter high?
6.	Fitria	:	Because…..
7.	Aziz	:	It can't be 2 meter high! You know that my body height is 151, don't you?

From fragment 2, we may see that Aziz connected his body height to make sense the height of the bookshelf to be used to estimate the height of the classroom. The next fragment shows how they use the bookshelf for estimating.

Fragment 3: Discussion on the height of the classroom

1.	Teacher	:	Could you explain how to find out the height of this room?
2.	Sugi	:	Errrr, the bookshelf is two meter high. This one should be added by two meter and a half. Therefore, it becomes three meters and a half.

We may see from the fragment 3 above that Sugi imagined the left over space between the top of the bookshelf and the ceiling is one and a half meter. It turned that FItria disagreed with Sugi's opinion. Fitria and Dytha as a group purposed the following answer:



Lemari: Bookshelf

Figure 4: Dytha and Fitria's Strategy

From figure 4, we see that Fitria and Dytha employed a mental estimation using the bookshelf as reference point. They imagined that there will be one and a part of bookshelf again that could be stacked to the ceiling. By this strategy they found out that the height of the classroom is 2 meter + 2 meter + 0.5 meter which is 4.5 meter. It shows that the students shift to use external object as reference points and shift from physical approximation to mental estimation. Nevertheless, there are several remarks for improvement for lesson 4. Activity 1 of lesson 4 would be deleted since it makes students too rely on arm spans and make it difficult for students to use other reference points.

In lesson 5, students were engaged in a real situation in which they observed their school to determine how many plastic flags and how long the rope should be hanged on outer ceiling for 17 august decoration. The aim of the activity is to give students chance to spot and employ reference points at the school. We found that the students again use arm spans as reference point for one meter. They iterated their arm spans tediously, sometimes they got lost to cover the desired part of the school. As we predicted in the HLT, one of the group (the boys group) creatively used distance between two pillars which they approximate using arm spans first and then count the number of the pillars to figure out the number of the plastic flag and the length of the rope needed. In other words, the students employed a new reference for the approximation but they still rely on arm spans. For refinement purpose, we see the need to emerge various strategies or reference points to be able to stimulate a rich discussion, not only the use of arm span.

CONCLUSION

The research question of this study is how can we support the development of students' reference points for length estimation? In this part we will show how our design support the development of the students' from guessing to use reference points and from approximating to estimating. After summary of the result we discuss the limitation of this study and suggestion for further study.

In the teaching experiment, lesson 1 we see that students naturally use their very basic-familiar reference point which is arm span instead of using a ruler. This implies that the condition creates by the frog jumping and *pocong* jumping game could shift student into approximating mode from what they usually do in class, measuring. Meanwhile in lesson 2, the students observed, listed and reasoned using body parts on the photograph. Giving body parts as clues on the photograph could support the students to gain new reference points and for approximating. However in lesson 3, we found that students got a little problem to cope with unfamiliar context of building and 2 dimensional of the 3d objects. We also get insight that the students tend to claim several objects as references point even they do not familiar with the lengths. In this case, we think we need to simplify the problems and focus more on connecting body parts to external objects as new reference points. We see in the lesson 4 that the students try to estimate by making sense of objects with their body height. The students imagine a stack of bookshelf to estimate the height of the classroom. We see students grasp with the idea of mental estimation and shift from physical approximation. In the last lesson, the students use the distance between two pillars of the school as a new reference points which they obtain by using their arm spans. In general, our design activities could answer the research question, students which are used to use ruler shift to approximate using arm spans and then make sense object by their body parts for estimating.

Nevertheless, we realize that our study is only a preliminary study of two cycles of design-based research. We only use 4 students as the subject of this study. Hence this learning situation is far from the real situation of classroom in Indonesia. It is certainly needed to revise and reformulate some points of the design for the next implementation. We point out that for bigger classroom, the designer should facilitate the teacher to sufficiently understand to establish well socio norms and socio-mathematical norms in inexactness of approximation and estimation. Moreover, the designer should also be able to predict and give suggestion for the teacher to manage the dynamic classroom (e.g. field activity).

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REFERENCES

Bakker, A., & van Eerde, D. (2013). An introduction to design-based research with an example from statistics education. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Doing Qualitative Research: Methodology and Methods in Mathematics Education*. Utrecht.

Barab, S., & Squire, K. (2004). Design-based research: putting a stake in the ground. *The Journal of the Learning Sciences, 13*(1), 1-14.

Bright, G. W. (1976). Estimation as part of learning to measure. In D. Nelson & R. E. Reys (Eds.), *Measurement in School Mathematics* (pp. 87-104). Washington: National Council of Teacher Mathematics.

Bright, G. W. (1979). Estimating physical measurements. *School Science and Mathematics, 79*(7), 581-586.

BSNP. (2006). *Standar Isi: Untuk Satuan Pendidikan Dasar dan Menengah*. Jakarta: BSNP.

Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics, 99*(1), 5-11.

Edelson, D. C. (2002). Design Research: What we learn when we engage in design. *The journal of the learning Sciences, 11*(1), 105-121.

Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education, 29*(3), 334-356.

Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures*. Doordrecht: Kluwer Academic Publishers.

Gooya, Z., Khosroshashi, L. G., & Teppo, A. R. (2011). Iranian students' measurement estimation performance involving linear and area attributes of real-world objects. *ZDM Mathematics Education, 43*, 709-722.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β Press.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105-128.

Gravemeijer, K. P. E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β Press.

Hildreth, D. J. (1983). The use of strategies in estimating measurement. *The Arithmetic Teacher, 30*(5), 50-54.

Hogan, T. P., & Brezinski, K. L. (2003). Quantitative Estimation: One, Two, or Three Abilities? *Mathematical Thinking and Learning, 5*(4), 259-280.

Jones, G., Taylor, A., & Broadwell, B. (2009). Estimating linear size and scale: Body rulers. *International Journal of Science Education, 31*(11), 1495-1509. doi: 10.1080/09500690802101976

Jones, M. G., & Taylor, A. R. (2009). Developing a sense of scale: Looking backward. *Journal of Research In Science Teaching, 46*(4), 460-475.

Joram, E., Gabriele, A. J., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education, 36*(1), 4-23.

Joram, E., Subrahmanyam, K., & Gelman, R. (1998). Measurement Estimation: Learning to map the route from number to quantity and back. *Review of Educational Research, 68*(4), 413-449.

Lang, F. K. (2001). What is a good guess anyway? Estimation in early childhood. *Teaching Children Mathematics, 7*(8), 462-466.

Markovits, Z., & Hershkowitz, R. (1997). Relative and absolute thinking in visual estimation processes. *Educational Studies in Mathematics, 32*, 29-47.

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming Mathematics Learning in Indonesian Classrooms through RME. *ZDM Mathematics Education, 40*, 927-939.

Smart, J. (1982). Estimation skills in Mathematics. *School Science and Mathematics, 82*(8), 642-649.

Treffers, A. (1987). *Three Dimensions: A model of goal and theory description in mathematics instruction-The Wiskobas Project*. Dordrecht: D. Reidel.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Zulkardi. (2002). *Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers.* (Doctoral), University of Twente, Enschede.

Zulkardi, & Ilma, R. (2006). *Mendesain sendiri soal kontekstual matematika.* Paper presented at the Konferensi Nasional Matematika XIII, Semarang.

Appendix 9. Article of the second cycle (a part)

The Use of Body Parts in Approximating Tasks as Means to Support the Development of Students' Measurement Sense

Abstract

This study reports a part of a full study aimed to improve local instructions theory on mathematics education for supporting the development of students' measurement sense. The research approach in this study is design-based research in which we design, test and revise hypothetical learning trajectory. This study focuses on two of five classroom activities designed using Realistic Mathematics Education (RME) principles. We developed a set of approximating problems using Indonesian context in which the use of body parts are promoted. The participants of this study were thirty four 4th graders (9-10 year old) and the regular teacher from one elementary school in Palembang, Indonesia. We collected data from students' work, classroom observation and interviews with the participants then we analyzed the data mostly in qualitative ways. The result suggests that the use of body parts as initial reference points for approximating may aid students to reason about other external objects which may support the development of new reference points.

Keywords: approximation, estimation, design-based research, hypothetical learning trajectory, measurement, realistic mathematics education

1. Introduction

1.1 Low Performances in Approximation/Estimation Tasks

The feeling of length or sense to percept magnitude of lengths is an important skills in life (M. G. Jones & Taylor, 2009). In the absence of measurement tools, sense of length takes important role for approximating or estimating lengths of objects. For instance, a butcher should cut meat in certain dimension to meet a certain weight, farmers may estimate the length of their farm to know the amount of seed should be planted. Indeed, skills in estimating/approximating develops through daily experience when one encounters with measurement activities (Gooya, Khosroshashi, & Teppo, 2011; Joram, Subrahmanyam, & Gelman, 1998). In the field of mathematics education, learning to approximate/estimate may support students to develop the sense of measurement and a meaningful learning of measurement (Clements, 1999).

Nevertheless, several studies found low performances of students on length estimation tasks (Hildreth, 1983; Joram et al., 1998). In Hildreth (1983) students categorized using inappropriate strategies by doing wild guessing to solve the tasks. Despite its importance, it is reported that only few studies that focus on embedding approximation/estimation into instructional activities (G. Jones, Taylor, & Broadwell, 2009; Joram et al., 1998). It might be even harder for students in Indonesia who are taught in a mechanistic way to cope with vagueness and inexactness on learning approximating/estimating (Forrester & Pike, 1998; Hadi, 2002; Sembiring, Hadi, & Dolk, 2008; Zulkardi, 2002). In the Indonesian curriculum, converting among standard units of length is a common in classroom instructions. The students have their own song to memorize the hierarchy of the standard units. In other words, explicit instruction for length estimation is not embedded in the Indonesian curriculum (BSNP, 2006).

1.2 Measurement Estimation

There are three kinds of estimations in the field of mathematics education: computational, numerosity and measurement estimation (Hogan & Brezinski, 2003). Computational estimation may refer to skills for fast computation to find relatively rough number such as determining the first number of 10 000:1.675. Numerosity estimation is related to estimating the number of discrete quantity such as dots in an array. A practical example for instance is estimating the number of candies in a jar. Meanwhile, measurement estimation refers to skills making an educated guess of continuous magnitudes of objects such as length, height, weight, area, volume, etc. in the absence of measuring tools (Bright, 1979; Smart, 1982).

In case of linear measurement estimation (length and height), the learning instructions should explicitly address length estimation in spatial ability by practicing and enriching students' visual experience through interaction with physical objects (Hogan & Brezinski, 2003; Markovits & Hershkowitz, 1997; Smart, 1982). In other words, the learning instructions should be able to facilitate a shift and development of students from the approximation mode to estimation mode.

Indeed, the two terms have different meaning and process of thinking. Approximating is a process of obtaining the

value of an attribute by assigning a certain value (units) using tools. The process of approximating requires more time to physically iterate a unit which tend to result in an exact answer with a certain degree of precision. Meanwhile, estimation relies on the ideas of producing an immediate and rough answer in mental ways that is sufficiently exact (Hall Jr, 1984). Hence, in estimating, there is no physical pacing involved and no measuring tool used except paper and pencils. In other words, students should first experience a process of physical measuring (approximating) before mentally measuring attributes (estimating). The experiences are expected supporting students to have mental images of units to minimalize wild guessing when dealing with the tasks. This mental images are called individual frame of reference (hereafter IFR).

1.3 Body Parts as Individual Frame of Reference

When students try to solve linear measurement estimation, different strategies will be employed depending on their proficiency in estimating (Hildreth, 1983). Several studies have found that skilled estimators tend to use benchmarks/mental images of something that they already know the length of (Crites, 1992; Gooya et al., 2011; G. Jones et al., 2009; Joram, Gabriele, Bertheau, Gelman, & Subrahmanyam, 2005). For instance, in Joram et al.'s (2005) study, when a student was asked to estimate a 9-inch-long piece of rope, one student said he was thinking about a Pringles can and trying to measure it in his mind against the rope. Meanwhile, less skilled estimators tend to estimate lengths of something by wild guessing without reasonable explanation.

The mental images/benchmarks also refer to other terms such as personal referents, reference points, mental rulers and or individual frames of reference (IFR) (Clements, 1999; Crites, 1992; Gooya et al., 2011; Joram et al., 2005). An IFR helps estimators to estimate by imagining an object with known measurement (length) and then compare it to to-be-estimated objects (Joram et al., 2005). This imaginable object develops through everyday experience and internalization of the standard units of measurement from a specific object. Each person has different personal frames of reference, for instance one might use body height to estimate the height of a classroom, or even imagine a 20-feet-long crocodile to estimate the length of a classroom as found in Hildreth's (1983) study.

As we have mentioned that the students may experience learning to approximate first, Jones (2009) suggested that using body parts as measurement tools/reference points for approximating significantly make the approximation more accurate. The body parts which might be employed such as hand spans, feet, arm spans (fathom), and or body height. The use of body parts gives advantages because this build-in tools since the history was used for measuring also. Accordingly, Bright (1976) suggested that the combination of body parts and external objects might be powerful to support the students' development. We may therefore summarize the mathematical development of IFR and estimation skills as shown in figure 1Figure .

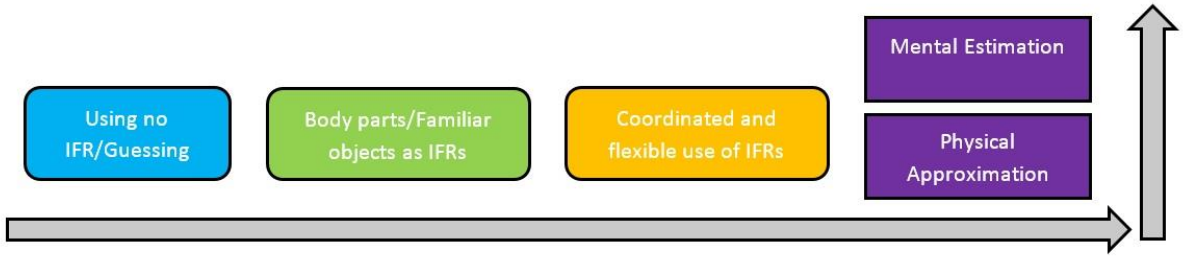


Figure 1 two dimensional development of IFR and estimation skills

Supporting the development of students' IFR should become a primary goal of approximation/estimation instructions (Bright, 1976; Lang, 2001). Clements (1999) also highlighted it as a critical point to develop students' measurement sense by which one can feel or have sense for the size of a unit (Sowder, 1992). Using IFR makes estimation not only meaningful but also easier and more accurate (Joram et al., 2005).

Nevertheless, students often do not spontaneously use IFR as a strategy to estimate (Hildreth, 1983; Joram et al., 2005). It is caused by complex interaction among three aspects: students' preference, context of the tasks and the nature of estimation activity (Gooya et al., 2011; Joram et al., 1998). For instance, providing particular estimation cues such as tiles in the context of estimating the length of a blackboard may influence students' choice of IFR. On the other hand, estimation in the context of a marble game from a photograph may stimulate students to use/imagine their hand-spans as reference points. Therefore, a learning instruction for promoting and developing IFR for length estimation should be designed by considering the aforementioned aspects.

1.4 Realistic Mathematics Education

Realistic mathematics education (RME, hereafter) is an educational approach rooted in Freudenthal's view of mathematics as a human activity (Freudenthal, 1973). He sees that an educational goal of mathematics education should facilitate students to be able to mathematize an everyday problem situation in mathematical terms and employ it within mathematics itself (Gravemeijer, 1994). RME provide a framework for designing an instructional task for the progressive mathematization which is determined by five tenets of realistic instructions by Treffers (1987). Following, we elaborate the five tenets of RME in relation with the topic of this study:

1.4.1 Phenomenological exploration by means of context

Phenomenological exploration means one should do a thought experiment to seek contexts that are self-contained or can contain mathematical ideas, concepts or structures. Considering the fact that measuring, approximating and estimating are context-bounded tasks, one should bring out meaningful contexts for development of IFR through making sense of units by exposing daily objects as the starting point (Sowder, 1992) and estimating length in various application in the real world as the end point of the instruction (G. Jones et al., 2009).

1.4.2 Bridging by vertical instruments (modelling)

Models serves as a bridge between mathematical phenomena in the reality and a formal system as symbolic representations of the real-world situations in which the mathematical ideas are embedded. Modelling in measuring, approximating and estimating takes form as situational models of the contextual problems. For instance, representations of objects such as the length of a school yard takes the form of sketched drawings of the school in which iteration of IFR can be performed. On a higher level, it could be represented as magnitudes in numbered line in which the IFR serve as the units.

1.4.3 Pupils' own constructions and productions

Supporting students to mathematize means to let the students grasp the mathematical idea, concepts and structures in meaningful tasks by their own actions (construction) and their reflections (productions). Accordingly, the process of developing IFR through measuring/approximating or estimating tasks cannot be forced to the students because IFRs relates to a mental perception of length through the process of internalization. The internalization process itself occurs individually depending what they perceive and experience.

1.4.4 Interactive Instruction

Interactive instructions provide an opportunity for students to participate, to negotiate and to cooperate about mathematical tasks with other students. This tenet can be applied for instructions of developing students' IFR because a selection of IFR tends to depend on the students' preference. This individuality may have some degree of error and misinterpretation. It then impedes the development of IFRs. In order to minimize the error and misinterpretation, the students should be engaged to listen, discuss and negotiate other students' perspectives, strategies and IFR used toward the tasks.

1.4.5 Intertwining of learning strands

A mathematical phenomenon usually manifests to form connected links among several mathematical ideas or concepts. For instance understanding estimation needs arithmetic fluency and proportional thinking such in case of estimating from a photograph and coordinating larger IFR. This suggests that instructions in estimation length for developing IFR cannot be isolated only in the measuring strands, instead it involves some portion of arithmetic and proportion.

1.5 Role of the Teacher in Approximation/Estimation Instructions

The role of the teacher should shifts from a transmitter of information or instructions and one who gives justification as right or wrong, to be the one who plan, organize, facilitate and guide students along their learning path (Gravemeijer, 1994). Accordingly, the role of the students is also changed from passive receivers of information to the ones who actively construct the learning route. They have opportunities to give explanation, justification and argumentation about their own work without relying on the judgments of the teacher. Moreover, the teacher should be able to 'hear' all students' reasoning and give appropriate feedback without interruptions and judgments (Hattie & Timperley, 2007; Towers & Hunter, 2010).

Nevertheless, shifting both the teacher's and the students' role is not an instant process. There is a need to establish and renegotiate socio-norms among the classroom community (Gravemeijer, 1994; Yackel & Cobb, 1996). It implies that, either explicitly or implicitly, the teacher should be able to establish what Yackel & Cobb (1996) called socio-mathematical norms where students and the teacher agree on acceptable sophistication mathematical

explanations and justifications. Since estimation tasks require students to cope with rough thinking, inexactness and vagueness (Forrester & Pike, 1998) the teacher should be able to establish agreements what kind of answers/strategies is considered as good guesses, efficient, and what kind is not acceptable (Lang, 2001). In addition, there is a need to agree about the mathematical terms involved in the instruction such words like guessing, measuring, approximating and estimating (Towers & Hunter, 2010).

1.6 Present Study

In order to integrate this topic into the curriculum, measurement estimation should be addressed into a part of spatial ability by exposing students with concrete objects for enriching students' visual experience (Hogan & Brezinski, 2003; Markovits & Hershkowitz, 1997). Promoting students to employ body parts as individual frames of reference could be a starting point for students to develop their approximation/estimation skills (Bright, 1976; G. Jones et al., 2009). Based on this rationale, we want to improve the educational gap by designing a series of meaningful activities that can support students to use body parts as individual frame of reference for approximating. We limit this study to only focus on approximation. Hence, we formulate the research question in this study as follow: *How could the use of body parts in approximating/estimating tasks support the development of students' measurement sense?*

2. Method

This study employ design-based research approach. We develop learning trajectory and its hypothetical learning trajectory. We mostly used qualitative analysis of the data gained from the students' written work, registered video of students learning, and interview with the students.

2.1 Research Approach

Two cycle of design-based research (DBR) is employed considering the aim of this study which want to design a series of activities for students in order to contribute/improve to a local instruction theory in mathematics education. DBR uses three phases of designing and several cyclical processes of revising toward the educational materials such as a learning trajectory (Bakker & van Eerde, 2013; Barab & Squire, 2004; Edelson, 2002).

We design the learning activity based on rationale that knowledge and the learning path should be constructed by the learner itself (the third tenet of RME) and teacher only functions as a facilitator of that journey. In this regard, the learning should facilitate openness for students' constructions but in goal-oriented activities. To accommodate this, we employ hypothetical learning trajectory (HLT) as a design and a guide about how and when certain development in learning of students should emerge (Simon & Tzur, 2004).

HLT consists of three main elements: goals of the learning, learning activities and prediction of students' learning. In this study, HLT acts as the main artifact that we design, test, and improve approached by the three phases of design-based research. In the first phase, preparation and design phase, we collected information such as reviewing theories and curriculum materials related to strategies for length approximation/estimations, the use of individual frame of reference in length and teaching process in length approximation/estimations class. We also conduct a pretest for the students, interview with the teacher and classroom observation. In the second phase of the DBR, teaching experiment phases, we implement the initial design in the classroom. The HLT takes role as a guidance for the teacher to conduct the learning. At last phase, retrospective analysis, collected data from several cycles are analyzed. The HLT are compared with the actual learning in the teaching experiment, this time the HLT functions as a guideline for evaluation.

2.2 Participant

On the first cycle (pilot study) of DBR, we had 4 students (2 boys and 2 girls) of fourth graders in one of primary school in Palembang, Indonesia. Meanwhile, in the second cycle, we had 34 fourth graders (with one focus group) and the classroom teacher from another classroom.

2.3 Data Collection and Analysis

Data were collected from the preparation phases and the teaching experiment phases of design-based research and a posttest afterward. We conduct what Denzin in (Bryman, 2003) called, methodological triangulation, gathering data using more than one methods to reduce the uncertainty of the data. For instance, along with the classroom observation or the pretest and posttest, we conduct interview with the teacher or the students. In addition, we also gathered students' written works along with the registered video of the students learning.

The data were mostly analyzed in qualitative ways. We employed method of HLT analysis like in Bakker and van Eerde (2013). We watched chronologically the registered video of students' learning and selected interesting

fragments which may consist of confirmation, rejection or unexpected behavior of the students toward the HLT. The fragment then were analyzed against to other episodes to reduce the bias. The result of this HLT-based analyzed supported by modest quantitative analysis of the pretest and the posttest were elaborated in the retrospective analyzes.

2.4 Learning Trajectory

Following is a table of the overview of the learning trajectory of this study.

Table 1 Learning trajectory on learning approximation/estimation

Title	Goal	Activity
Frog Jumping and *Pocong* Jumping	To raise awareness of using reference points for approximating rather than ruler	Students approximate and confirm whether the distance of the track they made in the game reaches 30 meter or not.
Measure and use your body part	To raise awareness for students to use body parts as reference points for approximating	Students approximate a set of Indonesian-game photographs in such a way that body parts become clues for the approximation
Shoes, couch and Flood	To associate and reason using body parts to approximate the length of objects as new reference points	Students approximate from photograph of shoes-tiles problem, couch and flood problems in which the use of body parts could be useful to reason new reference points (external objects).
!7th August – Balloon Decoration	To associate objects to other objects to gain new reference points	Students are asked to find as many as possible references from the UKS room photographs and order their efficiency.
17 August – The rope and the plastic flag decoration	To identify use and reason using reference points/IFR for solving length approximation/estimation tasks in social arithmetic problem.	Students are asked to determine budget for the decoration, estimate how many packs of plastic flag and rolls of rope needed to cover some part of the school.

3. Results and Discussion

We may see from table 1, the learning trajectories, of this study were designed to support the development of the students' individual frame of reference by initiate the use of body parts then be associated to external objects. In this occasion, we will primarily focus on discussing the result of the second and a part of the third lesson of cycle 2 which emphasize to support the initiation of using body parts.

3.1 Use and Measure Your Body Parts (lesson 2)

The main aim of this activity is to provide students in a situation in which they figure out the length of objects in a set of Indonesian context-based photographs. The photographs were chosen in such a way giving clues to the students to consider body parts as reference points for approximating. What makes this lessons different from other instructions is that, we designed the lessons carefully in such away the use of body parts as reference emerge without having the students to use it in explicit instructions. In the next paragraphs we describe our hypothesis of students' thinking for the problems (see appendix 1 for the photograph).

The first problem asks students to approximate the distance of a marble to the hole from the marble games. We predicted that students may measure their hand spans using ruler and reason with it to determine the distance. Meanwhile from the second problem, in context of playing *bentengan* students are asked to approximate the length between the two trees. In this case, we may predict that that students use arm span as reference points. For the third problem, we predict that the students may use feet to approximate the *bakiak*. Then, students are asked to approximate the height the rubber in rubber rope game. Once again, we predict that students use their body parts, in this case body height. Similar prediction the use of body height is also hypothesized for problem number 5 (greasy pole contest) in more advance way that the students iterate the body height.

Indeed the learning is not a linear process, along with the aforementioned hypothesized students' thinking we also predicted that the students may probably think other ways. At the very basic students may guess the length/height/distance. It could be that the students measure the photograph using ruler directly to get the length without scaling. In specific problems, students may come up using other references spotted on the photograph such as using the length of tiles (problem number 2). Moreover, students may also recall their previous knowledge

related the objects being asked such as the height of a for the greasy pole problem.

As predicted in the HLT, some of the students employed body parts to solve the tasks such as hand spans, arm spans, and body height. For instance, in task number one, figuring out the distance of a marble and its hole, one group reasoned that the hand is an adult's. They asked permission to the adult in the class (in this case the observer) to let them to measure his hand by a ruler as shown in figure 2.

Figure 2 Students measure the length of the observer's (adult) hand span

Similarly, the strategy occurred for task number four (figuring out the distance of a rubber rope to the ground). After the students had known the length of the observer's body height, they used it as reference points to approximate the asked height. Meanwhile, in task number 2, some of the students employed arm spans of people in the photograph to measure the distance between the trees. Since the students used arm spans as a one meter unit reference in the lesson 1, here the students differed in determining the length of arm spans. The following fragment and figure show the students' thinking.

Fragment 1

1	Observer 1	:	Why it is 10 meter?
2	Retno	:	Because one of an arm is 1 meter.
	….		
3	Observer 1	:	Could you explain your answer?
4	Bagus	:	One meter, two meter, three meter, four meter, five meter (pointing at each person in the photograph

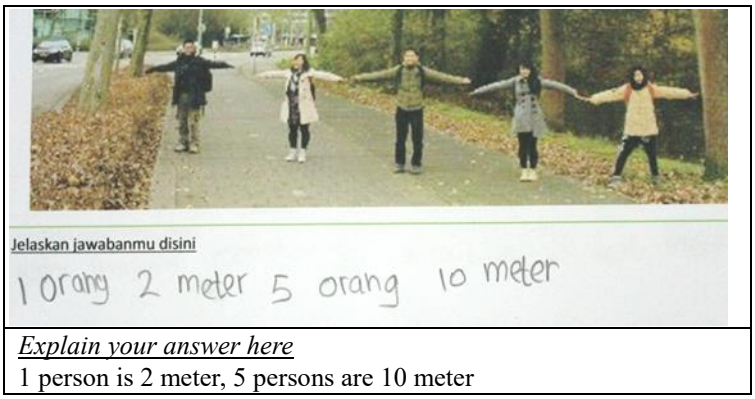
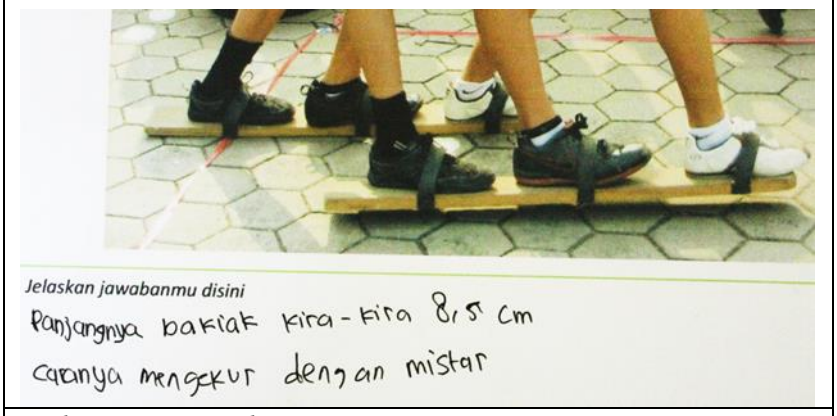


Figure 3 Students use of arm span as two meters

From the fragment above we may see that, the student (Bagus) assumed that a full arm spans is one meter long. He assigned each person's arm span on the photograph by one meter. Meanwhile Retno (other group) thought a half of an arm spans is one meter long (figure 3) so she took 10 meters as her answer. This different might happened since the students aware that people at different age have different length. Students mostly are given example from their parents that a half of the adult arm span is one meter. Meanwhile, students at their age mostly took it into account as a full arm span. This causes confusion for the students when solving the problem in which no information about the age of the people in the photograph given. This situation should be powerful for the students

to confirm the length of their own hand spans by measuring it using ruler.

It is also important to be noted that, some of the other students only measured the length of the *bakiak* using ruler as shown in the following written work.



Explain your answer here

The length of the bakiak is about 8.5 cm

The method is by measuring by a ruler.

Figure 4 Students measured the bakiak using ruler

Still in problem number 3 (*bakiak* problem), if in the lesson 2 of cycle 1, students did not spontaneously perceive foot as reference points. Here, in the lesson 2 of cycle 2, despite we had already revised the context (photograph) to be more explicit (see figure 4Figure), yet the students still did not see foot as reference point at the first time. The following figure of students' activity shows how the students cope with the problem.

Figure 5 Students using arm spans to approximate the bakiak

We can see from figure 5 that the students did not employ foot as reference point. Instead, they imitated the photograph then one of the student (Retno) approximated it by her arm span. We may say that, the use of foot as a reference point was not spontaneously emerged. Indeed, this is not a big deal in the instruction since we could not force the students to perceive it as a reference point. Noted that approximating/estimating also depends on the students' preferences. In this case, the students more likely employed arm spans.

In addition, we may also tell the reader that other students' thinking solving the problem also occurred. For instance, the students measure the photograph using ruler and even guessing. Further, there was also a group of the students who recall her IFR (the height of a banana tree) to reason about the greasy pole problem (number 6).

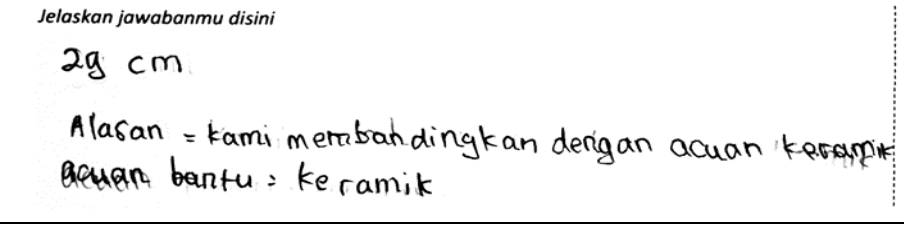
3.2 Shoes, Couch and Flood (lesson 3)

Figure 6 Shoes-tiles problem (left) and couch problem (right)

In this activity students are expected to be able to identify references on photographs, connect and reason using body parts to approximate the asked lengths in the photographs. At the first part, students were given two implicitly related problems. The students were asked to approximate the length of a pair of shoes which is perfectly fit on a tile. Then, the students were asked to approximate the length of a couch on the tiled-floor.

In the HLT, we hypothesized that students will use either feet or tile to reason about the shoes and the tiles. For the couch problem, reasonably we expect that the students use the length of the tile to approximate the length of the couch. Indeed, the learning will be not linear, students thinking may vary. For instance students may use IFR (chair, couch, etc.) to solve the problems. They may also probably guess the lengths or even try to find similar object to be measured as reference points such as adult height, the wheel of a motorcycle, etc.

In part A, we found no guessing method employed. As predicted in the HLT, we saw students used feet and tile as references for solving problem number 1 shown by the following written work.



Explain your answer here

29 cm

Our reason = We compared to floor tile as the reference.

Reference point = floor tile

Figure 7 Students' written work using floor tile as a reference point

Explain your answer here

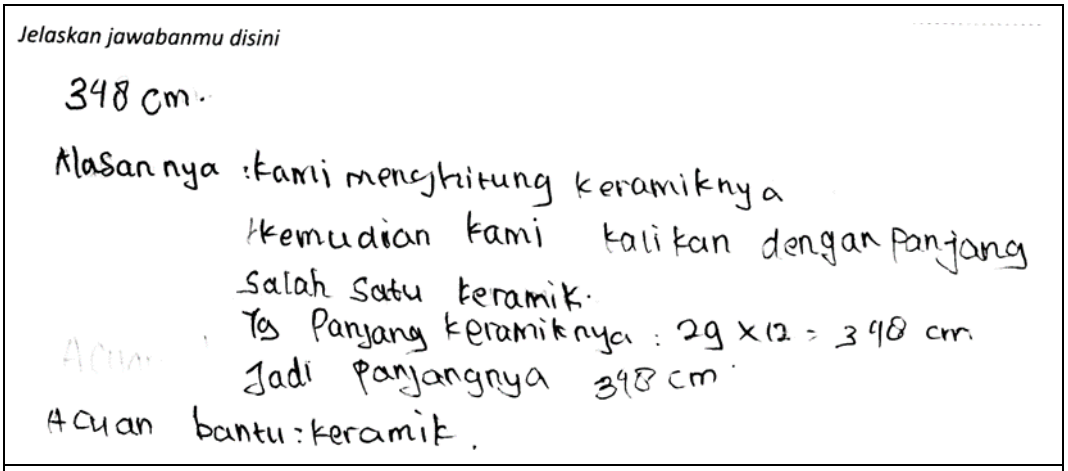
We measured using *Kak* Rudi's foot whose length 22 cm.

Figure 8 Students' written work using the observer's foot as a reference point

Figure 7 was given by the focus group, we may see that they employed floor tile as references. As soon as they noticed that the floor tile could be used to solve the problem, they measured the classroom tile using a ruler to know the length. Meanwhile, we may also see from the work of other students (Figure 8) that they used the observer's foot as references. It happened when they realized that Pak RT's foot (adult) probably nearly the same as the observer's foot. For case number two (observer's feet), we may say that students' knowledge of adult feet as reference (in lesson 2) was not internalized as individual frame of reference yet. Again, students' familiarity with feet as reference probably becomes the main factor as we found in the lesson 2. The students need more time

to develop and internalized it as a reference then be able to reason using it without measuring by a ruler every time they want to employ it.

Similarly, for problem number two we saw some of the students employed the floor tile again as predicted in the HLT. The following written works show students' strategies.



Explain your answer here

348 cm

Our reasons : we counted the tiles then we multiply with the length of one tile which length $29 \times 12 = 348\ cm$. Hence the length is 348 cm.

Reference point: Floor tile

Figure 9 Students' written work using floor as a reference point

We may see from figure 9 that our focus group first counted all the tiles covered the length of the couch. After they figured out the number of the tile (12 tiles), they multiplied it with the length of a single tile (29 cm) so that they knew the length of the couch is 348 cm.

In addition, other students' thinking to solve the problems were also observed. They employed IFR or recalled their previous knowledge about the length of a chair then associate it with the couch on the problem 2.

We may see that both problems in part A could facilitate students to develop the use of specific objects as references. The students started from the use of body part (foot) to external object (tile) as reference points. In this sense, one can support the students to develop new reference points (object) by engaging the students to reason with body parts as the initial reference points/IFR. Hence, it is reasonable to boost the students' knowledge by exposing them to more objects as we proposed in the next lessons.

We have shown to the reader that we found many evidences that support our HLT. However, we also noticed that the whole class discussions are still lack of students' involvement. The teacher focused only asking every group's strategies and answers but lack of discussion whether other students agree or not about the offered answers and strategies. Hence, the notion of socio mathematical norms in this approximation and estimation class were not yet established well. For the next implementation, due to this importance, it is suggested for having discussion between the researcher and the teacher to meet the same understanding about the notion of the socio-mathematical norms for elaborating various students thinking in a whole class discussion.

4. Conclusion

Measurement sense is very fundamental and useful skill in mathematics and science literacy. Indeed, this skill gained by accumulation of experience in which someone being exposed by measurement activities. One of which is having someone to approximate/estimate objects without explicitly relying on the use of the standard unit of measurement. The approximation/estimation tasks should be carefully designed for instructions in the classroom. For instance, one may support the development of students' individual frame of references for approximation before shift to estimation tasks. One of which is by engaging the students to use body parts as initial reference points/IFR in realistic contextual-problems and student-centered classroom environment.

The use of body parts as reference points may facilitated students to explore and find new objects as new reference points. In an appropriate context, the use of feet may aid to develop reference points such as tile which is useful

for approximating length of object on a floor. Hence, this build-in personal benchmark can support the development of measurement sense through developing individual frames of reference when students grasp in approximating tasks such as approximating from photographs.

Before implementing the lessons of this study one should carefully adjust the context appropriate to the new learning condition. In addition, improvement of the teacher role should be facilitated by having an extensive discussion with the teacher about establishing appropriate socio norms and socio mathematics norms before implementing the lessons (for extension of this study) including having a clear descriptive activity in the teacher guide.

Realizing that this study only contributes a little to the development of local instructions theory, further study about how to boost the development of students' skills from approximating to estimating might be interesting and useful for the development of learning theory in domain of measurement.

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References

Bakker, A., & van Eerde, D. (2013). An introduction to design-based research with an example from statistics education. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Doing Qualitative Research: Methodology and Methods in Mathematics Education*. Utrecht.

Barab, S., & Squire, K. (2004). Design-based research: putting a stake in the ground. *The Journal of the Learning Sciences, 13*(1), 1-14.

Bright, G. W. (1976). Estimation as part of learning to measure. In D. Nelson & R. E. Reys (Eds.), *Measurement in School Mathematics* (pp. 87-104). Washington: National Council of Teacher Mathematics.

Bright, G. W. (1979). Estimating physical measurements. *School Science and Mathematics, 79*(7), 581-586.

BSNP. (2006). *Standar Isi: Untuk Satuan Pendidikan Dasar dan Menengah*. Jakarta: BSNP.

Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics, 99*(1), 5-11.

Crites, T. (1992). Skilled and less skilled estimators' strategies for estimating discrete quantities. *The Elementary School Journal, 92*(5), 601-619.

Edelson, D. C. (2002). Design Research: What we learn when we engage in design. *The journal of the learning Sciences, 11*(1), 105-121.

Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education, 29*(3), 334-356.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Reidel.

Gooya, Z., Khosroshashi, L. G., & Teppo, A. R. (2011). Iranian students' measurement estimation performance involving linear and area attributes of real-world objects. *ZDM Mathematics Education, 43*, 709-722.

Gravemeijer, K. P. E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β Press.

Hadi, S. (2002). *Effective Teacher Professional Development for the Implementation of Realistic Mathematics Education in Indonesia.* (Doctoral), University of Twente, Enschede.

Hall Jr, L. T. (1984). Estimation and Approximation-Not Synonyms. *The Mathematics Teacher, 77*(7), 516-517.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81-112. doi:10.3102/003465430298487

Hildreth, D. J. (1983). The use of strategies in estimating measurement. *The Arithmetic Teacher, 30*(5), 50-54.

Hogan, T. P., & Brezinski, K. L. (2003). Quantitative Estimation: One, Two, or Three Abilities? *Mathematical Thinking and Learning, 5*(4), 259-280.

Jones, G., Taylor, A., & Broadwell, B. (2009). Estimating linear size and scale: Body rulers. *International Journal of Science Education, 31*(11), 1495-1509. doi:10.1080/09500690802101976

Jones, M. G., & Taylor, A. R. (2009). Developing a sense of scale: Looking backward. *Journal of Research In Science Teaching, 46*(4), 460-475.

Joram, E., Gabriele, A. J., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education, 36*(1), 4-23.

Joram, E., Subrahmanyam, K., & Gelman, R. (1998). Measurement Estimation: Learning to map the route from number to quantity and back. *Review of Educational Research, 68*(4), 413-449.

Lang, F. K. (2001). What is a good guess anyway? Estimation in early childhood. *Teaching Children Mathematics, 7*(8), 462-466.

Markovits, Z., & Hershkowitz, R. (1997). Relative and absolute thinking in visual estimation processes. *Educational Studies in Mathematics, 32*, 29-47.

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming Mathematics Learning in Indonesian Classrooms through RME. *ZDM Mathematics Education, 40*, 927-939.

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning, 6*(2), 91-104.

Smart, J. (1982). Estimation skills in Mathematics. *School Science and Mathematics, 82*(8), 642-649.

Sowder, J. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York: Macmillan.

Towers, J., & Hunter, K. (2010). An ecological reading of mathematics language in a grade 3 classroom: A case of learning and teaching measurement estimation. *Journal of Mathematical Behavior, 29*, 25-40.

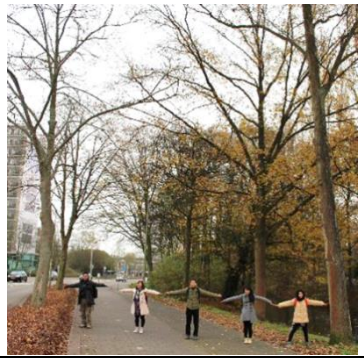
Treffers, A. (1987). *Three Dimensions: A model of goal and theory description in mathematics instruction-The Wiskobas Project*. Dordrecht: D. Reidel.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Zulkardi. (2002). *Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers.* (Doctoral), University of Twente, Enschede.

Appendix A

Problems and the Photographs in lesson 2

Photograph 1 Approximate the real distance of the marble to the hole?!	
Photograph 2 Approximate the real distance between the two trees in the following *benteng-bentengan* game.	

Photograph 3 Approximate the real length of the *bakiak* used in the photograph beside.	
Photograph 4 Approximate the real distance from the rope to the ground in the following photograph.	
Photograph 5 What is the real height of the following pole?	

Appendix 10 Thesis booklet

Download : https://goo.gl/H6iCiS