DEVELOPING A LOCAL INSTRUCTION THEORY ON THE CARTESIAN COORDINATE SYSTEM

A THESIS

Submitted in Partial Fulfillment of the Requirement for
the Degree of Master of Science (M. Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education, Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
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This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang, July 2015

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ABSTRACT

This study is aimed at contributing the development of a local instructional theory that can foster students' understanding of the positive quadrant of the Cartesian coordinate system. To achieve the intended aim, design research is chosen as a study approach by performing an iterative process of designing instructional activities, conducting teaching experiments, and accomplishing retrospective analysis. In this study, the instructional activities are designed based on realistic mathematics education (RME) theory and a hypothetical learning trajectory (HLT) is developed to know how the fifth graders' (aged 10 – 11 years old) learn and understand the Cartesian coordinate system. Data collection were conducted through recording the classroom activity or group works, colleting the students' written work and making field notes. The developed HLT was then compared to students' actual learning trajectory to investigate whether the designed-instructional sequence support students' understanding of the Cartesian coordinate or not. The retrospective analysis of the teaching experiments confirms that the students were able to make an alphanumeric grid system as the primary activity before learning the Cartesian coordinate. In addition, they were also able to make a system looked like the positive quadrant of the Cartesian coordinate by investigating the horizontal and the vertical distance as embedded in the taxicab distance. Their understanding about both distances, in further, helped them to use an ordered pair to locate and plot any points on the positive quadrant of the Cartesian plane.

Keywords: *Cartesian coordinate system, Local instruction theory, Realistic mathematics education, Design research, Hypothetical learning trajectory, Taxicab distance*

ABSTRAK

Penilitian ini bertujuan untuk memberikan konstribusi terhadap teori pembelajaran lokal (local instruction theory) yang dapat membantu pemahaman siswa mengenai sistem koordinat Cartesius untuk kuadran pertama. Untuk mencapai tujuan tersebut, design research dipilh sebagai pendekatan penilitian dengan melalui proses iteratif yang meliputi: merancang aktivitas pembelajaran, melaksanakan pembelajaran, dan melakukan analisis retrospektif. Aktivitas pembelajaran dalam penelitian ini di rancang bedasarkan pendekatan Pendidikan Matematika Realistik. Dugaan lintasan belajar (Hypothetical learning trajectory) juga dikembangkan untuk mengetahui bagaimana siswa kelas V (berusia 10 – 11 tahun) belajar dan memahami sistem koordinat Cartesius. Pengumpulan data dilaksanakan dengan merekam aktivitas belajar siswa, mengumpulkan pekerjaan tertulis siswa, serta membuat catatan hasil observasi. Dugaan lintasan belajar yang telah dikembangkan kemudian dibandingkan dengan lintasan belajar yang terjadi di kelas. Hal ini dilakukan untuk menginvestigasi apakah aktivitas pembelajaran yang telah didesain dapat membantu pemahaman siswa mengenai topik tersebut. Hasil analisis retrospektif dari pelaksaaan pembelajaran mengkonfirmasi bahwa siswa mampu membuat sistem grid alphanumeric sebagai dasar untuk mempelajari sistem koordinat Cartesius. Selanjutnya, mereka juga mampu menemukan sebuah sistem seperti kuadran pertama dari sistem Cartesius dengan cara melakukan investigasi temtang jarak horisontal dan vertikal yang terdapat pada jarak taxicab. Pemahaman mengenai jarak tersebut membantu mereka untuk menggunakan pasangan berurutan untuk melokasikan dan meletakkan sebarang titik pada kuadran pertama dari sistem koordinat Cartesius.

Kata kunci: Sistem koordinat Cartesius, Teori pembelaaran lokal, Pendidikan matematika realistic, *Design research***,** Jarak *taxicab*

SUMMARY

Cartesian coordinate system plays a fundamental role to the skills related map reading, reading graph as well as making graph (Somerville & Bryant, 1985; Blades & Spencer, 2001). Signifying that importance, however, the Cartesian coordinate system is commonly introduced as a ready-made system that leads to a lack of students' understanding because they are merely asked to remember the rule and the notation (Palupi, 2013; Kemendikbud, 2014). For this reason, a set of instructional activities are needed to have students encounter, reinvent and understand the mathematical ideas behind the Cartesian coordinate by themselves.

This study is aimed at contributing the development of local instructional theory on the Cartesian coordinate focused on the positive quadrant in fifth grades. To achieve the intended aim, we develop innovative educational materials based on realistic mathematics education (RME) approach by and iteratively conduct adjustment on it. It implies that we need both the design of instructional materials and the research on how these materials support students understanding the concept. Therefore, this study employs design research as the research approach.

Within the main two cycles of design research, namely micro and macro cycle, the hypothetical learning trajectory (HLT) plays an important role. We designed the HLT consisting of four main components: students' starting points, learning goals, learning activities, and the conjecture of students' thinking. This HLT was then implemented to thirty students of the fifth grade (i.e Pusri Primary School, Palembang) within pilot teaching experiment and teaching experiment.

Based on the findings of this study, it can be concluded that the students could make an alphanumeric grid system as a prior notion before learning the Cartesian coordinate. This study also revealed that they could reinvent a system looked like the positive quadrant by investigating the horizontal and the vertical distance as embedded in the taxicab distance. Their understanding about the horizontal and the vertical distances, supported them to use an ordered pair to locate and plot any points on the Cartesian plane. Accordingly, the students significantly improved their understanding of the Cartesian coordinate focused on the positive quadrant.

RINGKASAN

Sistem koordinat Cartesius memiliki peranan yang sangat penting terhadap keahlian yang berhubungan dengan membaca peta ataupun grafik serta membuat grafik (Somerville & Bryant, 1985; Blades & Spencer, 2001). Menyoroti pentingnya materi tersebut, sistem koordinat ini biasanya diperkenalkan sebagai sistem yang siap pakai kepada siswa sehingga berujung kepada kurangnya pemahaman siswa terhadap materi tersebut (Palupi, 2013; Kemendikbud, 2014). Menanggapi hal tersebut, seperangkat aktivitas pembelajaran sangat dibutuhkan untuk mengajak siswa menemukan kembali serta memahami konsep matematika dibalik materi sistem koordinat Cartesius.

Penelitian ini bertujuan untuk mengembangkan teori pembelajaran lokal mengenai sistem koordinat Cartesius yang difokuskan pada kuadran pertama. Untuk mencapai tujuan tersebut, seperangkat materi pembelajaran yang inovatif dikembangkan berdasarkan prinsip pendidikan matematika realistik (RME) dengan melakukan penyesuain secara iteratif. Hal ini menyiratkan bahwa dibutuhkan dua hal penting, yakni desain materi pembelajaran serta penelitian tentang bagaimana materi tersebut membantu siswa untuk memahami sistem koordinat Cartesius.

Dugaan lintasan belajar (HLT) memiliki peranan yang sangat penting dalam dua siklus utama dari design research, yakni siklus mikro dan makro. Dugaan lintasan belajar (HLT) tersebut didesain dengan menenkankan empat komponen utama, yakni pengetahuan awal siswa, tujuan pembelajaran, aktivitas pembelajaran, serta dugaan cara berpikir siswa. HLT tersebut kemudian diimplementasikan kepada 30 orang siswa kelas V (SD Pusri Palembang) dalam dua tahapan, yakni pembelajaran pada kelompok kecil serta pembelajaran di kelas besar.

Berdasarkan hasil temuan dari penelitian ini, dapat disimpulkan bahwa siswa mampu membuat sistem *grid* alfanumerik sebagai dasar untuk mempelajari sistem koordinat Cartesius. Penilitian ini juga menyingkap bahwa siswa mampu menemukan sebuah sistem seperti kuadran pertama dari sistem koordinat Cartesius dengan cara melakukan investigasi temtang jarak horisontal dan vertikal

yang terdapat pada konsep jarak *taxicab*. Pemahaman mengenai jarak tersebut membantu mereka untuk menggunakan pasangan berurutan untuk melokasikan dan meletakkan sebarang titik pada kuadran pertama dari sistem koordinat Cartesius. Sejalan dengan hal tersebut, siswa mengalami perubahan secara signifikan terkait dengan pemahaman mereka mengenai sistem koordinat Cartesius yang difokuskan pada kuadran pertama.

"The only way to do great work is to love what you do. If you have not found it yet, keep looking. Do not settle. As with all matters of the heart, you will know when you find it."

Steve Jobs

"Education is not the learning of fact, but the training of the mind to think"

Albert Einstein

For Sudjanarko and Suwaning Supriyatin

The most inspired parents in my universe

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CHAPTER I
INTRODUCTION

The coordinate system plays a fundamental role in many aspects that children have to learn either at school or in their daily lives. To have map reading as well as graph reading integrated in mathematics and science disciplines, it really counts on an understanding of the coordinate system (Somerville & Bryant, 1985; Blades & Spencer, 2001). Moreover, the coordinate system is also undoubtedly applied to organize things in people's daily live (Palupi, 2013). For instance, one needs to develop a classification system to arrange books and documents in a library in such a way that it helps someone to easily find the books or the documents. A system of organizing is also appears in the context of airplane seats in which to number the seat we need at least two parameters, for instance a combination of numbers and letters.

Signifying the importance of the coordinate system, this topic is taught in primary schools in many countries, including Indonesia. However, it is common that the coordinate system is introduced as a ready-made system that needs to be understood by remembering the rule, the procedure, and the notation as well (Palupi, 2013). In addition, Zulkardi (2002) also revealed that most of mathematics textbooks in Indonesia merely contain a set of rules and algorithms, which is already at the formal level and which lacks applications. Concerning the facts, van Galen & van Eerde (2013) claimed that if the rules or the procedures were not understood well then these would be vulnerable tricks for the children.

There are several types of the coordinate system, one of which is the Cartesian coordinate system. Plentiful experimental studies have been conducted to examine children's ability in using the Cartesian coordinate system, but they came to completely different results about the age at which children first have a grasp of coordinates. Piaget et al. (1960) and Carlson (1976) concluded that children after age 8 or 10 years old could use the coordinate system. On the other hand, Somerville & Bryant (1985) and Blades & Spencer (2001) revealed that children at age 4 until 6 years old have been able to use the coordinate system. This discrepancy is caused by the different complexity of the tasks used to examine children's performances (Blades & Spencer, 2001). Rather than giving a ready-made grid in the tasks, Piaget et al. (1960) and Carlson (1976) asked children to construct their own coordinate grid using measurement and drawing equipment. Consequently, the children may have trouble to get the solution that requires constructing a grid and that is why only relatively older children were able to do this (Blades & Spencer, 2001).

Regardless of children's difficulty in constructing the coordinate grid, there is also a lack of students' understanding about the Cartesian coordinate system. According to Clements & Sarama (2009), even young children can use the coordinate system if guidance is available, but they may not yet be able to use the coordinate system by themselves spontaneously when working with traditional tasks. Sarama et al. (2003), furthermore, claimed that children tend to reverse the value of *x* and *y* when they plot points that have zero as its coordinate (e.g. (0, 8)). Moreover, some children seemed to ignore the presence of the origin. For instance, they tried to plot a new point starting from the previous point, not from the origin point (Palupi, 2013; Sarama et al., 2003).

In contrast with plentiful experimental research on children's ability to use the coordinate system (Carlson, 1976; Piaget, et. al., 1960; Shantz & Smock, 1966; Somerville & Bryant, 1985), there is surprisingly little study on designing instructional units about the coordinate system that is applicable in a practical situation (Palupi, 2013). Therefore, it specifically demands more studies concerning the improvement of a developmental theory at the level of a local instructional theory. Regarding to this, this study proposed a set of instructional learning activities about the coordinate system that can be implemented by the teachers in a real classroom situation.

Rather than giving a ready-made coordinate system to children, it is preeminent to have the children encounter and reinvent the mathematical ideas behind the topic by themselves (Piaget et al., 1960; Palupi, 2013). One of the mathematical ideas that should be noticed is the use of distance because it becomes a prerequisite to understand the concept of the positive quadrant of the Cartesian coordinate (de Lange et al., 1997; Piaget & Inhelder, 1956). Comprehensively, the use of distance is essential to locate an object in a plane using the coordinate system (Huttenlocher, Newcombe & Sandberg, 1994). Although there is a similar study developing a local instructional theory about the coordinate system (Palupi, 2013), the mathematical idea of using distance is not covered in that study. Accordingly, this study specifically proposed the use of distance to support students understanding of the positive quadrant of the Cartesian system.

Considering those issues, we conduct a study aimed at contributing a local instructional theory that can foster students' understanding of the positive quadrant of the Cartesian coordinate system in primary schools. Therefore, we formulate the following research question. *"How can we support students in understanding the positive quadrant of the Cartesian coordinate system?"*

CHAPTER II
THEORETICAL BACKGROUND

This chapter draws attention to the theoretical framework that will be used to design an instructional sequence about the first quadrant of the Cartesian coordinate system. This chapter begins with the definitions of the coordinate system in general and the Cartesian coordinate system in specific. It gives a short insight that the Cartesian coordinate system is a coordinate system used to pinpoint any (geometric) object in a plane or in space. It continues to describe the importance of the Cartesian coordinate for children either in school or in their daily life. Afterwards, the students' difficulties in understanding this topic are also highlighted according to the previous studies. These issues show that the research areas about learning the Cartesian coordinate system is important yet problematic.

We also explain how realistic mathematics education (RME) is used as the grounded theory for designing the learning sequence. With instructional design in mind, we characterize the RME theory by the design heuristic of guided reinvention built on Freudenthal's idea. To support the (guided) reinvention of the postive quadrant of the Cartesian coordinate, three main sequential concepts are elaborated in this study: the alphanumeric grid systems, the taxicab distance on the grids, and the postive quadrant of the Cartesian coordinate. Since the study is conducted in Indonesia, this chapter also gives a general overview of teaching the Cartesian coordinate system based on the Indonesian curriculum. At the end of this chapter, the research questions as well as the research aims are also described.

2. 1. Cartesian coordinate system

Prior to the 15th century, geometry and algebra were regarded as distinct branches of mathematics. However, a change occurred when a study that integrated geometry and algebra called analytic geometry was developed (Aufmann et al., 2010). In the analytic geometry, the coordinate system plays an important role to describe the (geometric) objects in algebraic terms like real numbers and equations (Aufmann et al., 2010; Szecsei, 2006; Woods, 1922). For example, the distance between two points can be determined if given the coordinates of both

According to Woods (1922), a coordinate system is a system to fix the location of a (geometric) object in the plane or space by using a set of numbers, direction or angle. It means that the location of the (geometric) object including points in the plane or space is uniquely pinpointed by the coordinate system. To describe the precise location of a point in the plane, several sophisticated coordinate systems have been developed, one of which is Cartesian coordinate (Szecsei, 2006).

The Cartesian coordinate system (so called the rectangular coordinate system) is a coordinate system used to uniquely describe the location of any points in a plane or in space (Aufmann et al., 2010; Szecsei, 2006). In the plane, this system is formed by one horizontal axis (*x*-axis) and one vertical axis (*y*-axis), which intersect at the zero point called origin (Aufmann et al., 2010). In this system, an ordered pair of the form (*x*,*y*) indicates the location of a point, where *x* represents the horizontal distance between the point and the *y*-axis, while *y* represents the vertical distance between the point and the *x*-axis (Szecsei, 2006). The axes divide the plane into four regions called quadrants (see figure 2.1). Any point in quadrant I have both positive *x*- and *y*-coordinate, so that it also called as positive quadrant.

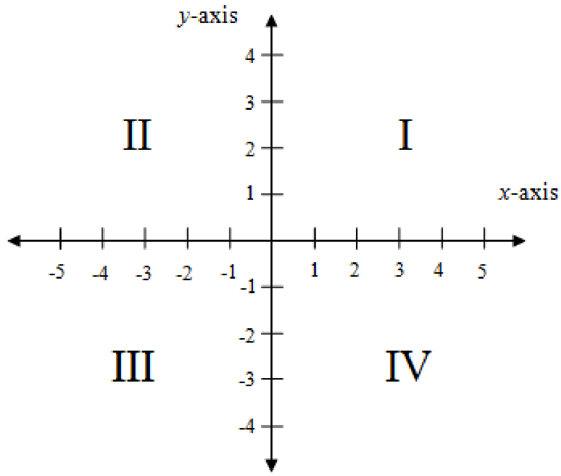


Figure 2. 1 The Cartesian coordinate with four quadrants numbered I, II, III, and IV

According to the explanation above, the Cartesian coordinate system becomes a fundamental concept used in higher-level mathematics. This concept is not as simple as we thought because there are many crucial components (e g. coordinate axes, origin, ordered pair, etc) that must be understood well. Apart from helping children to understand this system, this study pays close attention to the reinvention of the Cartesian coordinate by designing sequential learning activities. However, considering the complexity of the design, this study merely focuses on the positive quadrant of a Cartesian coordinate. It means that only positive numbers are on the axes such that the coordinate of a point is described as an ordered pair (x,y), where x and y are positive numbers.

2. 2. The Importance of the Coordinate System

The coordinate system plays a fundamental role in many aspects of a children's world at school and their daily lives. The skills of reading maps as well as working with graph simultaneously integrated in mathematics and science teaching really rely on children's understanding of the coordinate system (Somerville & Bryant, 1985; Blades & Spencer, 2001; Sarama et al., 2003). According to Somerville & Bryant (1985), maps and graphs typically use the concept of the coordinate system in which the location of any points in space is determined by the intersection of two extrapolating lines from positions on both vertical and horizontal axes. Suppose that children are asked to graph the relationship between the distances a car has driven represented in the vertical axis and the time represented in the horizontal axis. To plot each point, students need to envision lines extending horizontally from the vertical axis and vertically from the horizontal axis and figure out where they intersect. Similarly, in a map, a certain position in the map can be specified from the intersection of its longitude represented in the vertical axis and its latitude represented in the horizontal axis.

The embodiment of the coordinate system in people's daily live is also embedded in a system of organizing (Palupi, 2013). These systems are made to help us to locate an object easily and precisely. The seat arrangement in an airplane, for instance, uses a combination of letters represented in a column line and numbers represented in a row lines. This system helps both the flight attendant and the passengers to find the location of a particular seat easily and effectively. Another example is house numbering that is commonly used in neighborhoods in Indonesia. The idea is similar to that of airplane seats: every house is typically labeled with the combination of a letter and a number. The system is very helpful for people, especially the mail carrier, to find the location of a certain house.

Understanding the coordinate system also contributes to the development of spatial ability, in particular spatial orientation (Clements & Sarama, 2009). In people's live, spatial orientation is simply about knowing where we are and how to move around the world (Clements & Sarama, 2009). When this simple concept is defined further, it emphasizes the understanding of the relationships between different positions in space firstly the position with respect to our own position and our movement through it, and secondly different positions from an abstract viewpoint including a map and coordinate system (Clements & Sarama, 2009).

2. 3. Students' Difficulty in Understanding the Coordinate System

Children's difficulty in learning the coordinate system is inseparable from the debate on the age at which children first grasp coordinates (Blades & Spencer, 2001). Piaget et. al (1960) and Carlson (1976) affirmed that children are able to use the coordinate system after the age 8 or 10 years old. On the other hand, some studies revealed that children at age 4 until 6 years old initiated to use the coordinate system (Somerville & Bryant, 1985; Blades & Spencer, 2001). This discrepancy, according to Blades & Spencer (2001), is grounded on the complexity of the tasks used to examine children's performance. Rather than providing a ready-made grid in the coordinate tasks (Somerville & Bryant, 1985; Blades & Spencer, 2001), Piaget et al. (1960) and Carlson (1976) allowed the children to construct their own coordinate grid using measurement equipment when dealing with the tasks. Children may have trouble to get the solution that need the construction of the grids. Consequently, only relatively older children seemed to be able to do the tasks (Blades & Spencer, 2001).

Regardless of children's difficulty in constructing the coordinate grid, there is also lack in students' understanding of the coordinate system. To plot any points in the Cartesian coordinate system, children tended to repeatedly count by ones from each axis (Sarama & Clements, 2003). They mostly did not use the coordinate labels, which verifies other signs of not conceptualizing the axes as number line. For instance, if they had plotted a point at (10, 5) and the next point was at (13, 5), they would repeatedly count by ones from each axis. They did not immediately identified the second position as 3 units to the right of the first. This factual example demonstrates that those children could not use the numerals on the axes for determining the length (Carpenter & Moser, 1984; Stefie & Cobb, 1988 cited by Sarama & Clements, 2003).

A common misconception that children have is about plotting and describing any points in the form of ordered pairs (Sarama & Clements, 2003). In certain tasks, children initially tend to plot a point from the previous point they have, not from the origin. For instance, if they had plotted a point at (15, 10) and the next point was at (4, 0), they would place the points 4 units to the right of (15, 10), at (19, 10). In describing the points, they also had initial errors such as verbally reading (13, 17) as if it was 13.17 or 13/17. This mistake could happen because they have a tendency to label a point with a single numeral or understand coordinates as separate entities (Sarama & Clements, 2003). A more common mistake was reversing the value of x and y in ordered pairs when a certain point had zero as one of its coordinates (Sarama & Clements, 2003). For example, a point, which was at (5, 0), was described at (0,5) or vice versa.

The number zero within all aspects of the coordinate system causes particular problems for children (Sarama & Clements, 2003). For example, students have difficulties in understanding the meaning of zero in the coordinate system by asking, “What is zero for?” They may assume that label zero belong in a corner, even if the grid has four quadrants. Moreover, most children also argued that origin could be represented by one zero, instead of two numbers in the form of (0, 0) (Sarama & Clements, 2003). This misconception happened because of they do not understand that that the number zero actually labels the axes which is why the origin is described by two numbers in the form of (0, 0).

2. 4. Realistic Mathematics Education

To explain how the learning sequences we designed in this study help students to reinvent and understand the first quadrant of the Cartesian coordinate system, realistic mathematics education (RME) is used as the grounded theory. With instructional design in mind, we apply the five tenets of RME and characterize RME theory by the design heuristic of guided reinvention as a framework for the designing of the learning sequences.

2. 4. 1. Five Tenets of Realistic Mathematics Education

The process of designing a sequence of instructional activities was inspired by five tenets of realistic mathematics education defined by Treffers (1987) that are described as follows.

1. **Phenomenological exploration**

   The starting point of mathematics learning in RME is allowing students to immediately be engaged in the contextual problems. The contextual problem refers to situational problems that are experientially real to the students not only in their life situation, but also in their mind. According to Sarama & Clements (2003), contextual problems play an important role in teaching the coordinate system, but it must be anchored with well-articulated mathematical goals and instructions. In addition, the coordinate system related tasks embedded in contextual problems help children to understand (Blades & Spencer, 2001). Accordingly, the contextual problem of city blocks in this study is used to help students reinvent and understand about the first quadrant of the Cartesian coordinate system in the form of rectangular grids. Afterward, the context of finding the location of a sunken ship in a sea map allows students to understand the Cartesian coordinate system without the manifestation of the rectangular grids anymore. Even students in Indonesia probably not get familiar enough with the context of city blocks, they may easily engage with the context as long as it is introduced well such that the students can imagine it. In addition, the context related to city blocks in some aspects is also embedded in a certain electronic game such as *SimCity* or *City Block* that might be well known enough for children.

2. **The use of models or bridging by vertical instruments**

   The term of model in RME can be interpreted as the concretized of the formal abstract mathematics such that it can be accessible to the students. It acts as a bridge between the real-world situations or the concrete level and the intended mathematical concept or the formal level. Accordingly, students' knowledge about identifying the location of cells using the alphanumeric grid system needs to be developed into knowledge about identifying the location of points using the first quadrant of the Cartesian coordinate system. Consequently, the activity about a *taxicab* distance embedded in the rectangular grids in this study was drawn as the bridging activities to elicit students' understanding of the positive quadrant of the Cartesian coordinate.

3. Students' own constructions and productions

The freedom for students to construct their own strategies leads them to the productions of various solutions that can be used to consider the learning course. The students' own strategies or solutions in each learning activity are discussed in class to eventually support students' understanding of the first quadrant of the Cartesian coordinate. Accordingly, a student-made system for labeling city blocks, as the basic emergence of the alphanumeric grid system, is a prior activity before learning about the first quadrant of the Cartesian coordinate. Afterward, students' own strategies of locating of an object in the city blocks using a pair of numbers (related to the horizontal and vertical distances) will be a prior stage before being introduced to an ordered pair.

4. Interactivity

Similar to social interaction, the learning process requires extensive communication among individuals in the classroom to make it effective. It can be realized through group work or a classroom discussion since the students have the opportunity to share their opinions with the others. Moreover, it also evokes reflections of the students such that it can improve their understanding of particular concepts or ideas. For this reason, this study opens more space for students to have interactivity by working in groups or pairs and conducting classroom discussions for each activity.

5. Intertwinement of various learning strands

The intertwinement principle means that mathematical strands such as number, algebra, geometry, measurement, and data handling are not considered to be isolated units, but heavily integrated. It implies the importance of considering the integration of several mathematics topics. In this study, students are not only encouraged to learn about the first quadrant of the Cartesian coordinate system, but also about other topics such as science and geography. Here, the students will simultaneously learn a *taxicab distance* (the informal concept of vector), angle measurement, cardinal directions (north, east, south, and wet), and an artificial map.

2. 4. 2. Guided Reinvention

Freudenthal's idea about "mathematics as human activity" provide insight into a viewpoint that students should be given an opportunity to reinvent and develop their own mathematics and mathematical thinking and reasoning (Gravemeijer, 2010). Instead of giving mathematical concepts as ready-made product, Treffers (1987) declared that students should be facilitated to reinvent those concepts under the guidance of the teacher and the instructional design. This principle of RME is called guided reinvention. According to Gravemeijer (1994), the term of a guided reinvention implies that students need to experience mathematics learning as a process that is similar to the process of inventing mathematics itself.

According to Bakker (2004), mathematical instruction that supports the guided reinvention can be designed using three different methods. The first method is what Freudenthal called as a "thought experiment". Here, the designers need to think on how they could have reinvented the mathematical issue by themselves. Consequently, it can inspire them to design the intended mathematical instruction. The second method is studying the history of how the mathematical issue is invented, which is called historical phenomenology. The third method, elaborated by Streefland (1991), is using students' informal strategies as a source. Here, the teachers as well as the designers need to think on how they can support students' solutions in getting closer to the main goal. In this study, we elaborate the first and the third method, which mainly focus on three notions as elaborated below.

1. **Alphanumeric grid system**

   The introduction to the alphanumeric grid system becomes a primary activity before students learn about the first quadrant of the Cartesian coordinate system (Ministry of education, 2008; Booker et al., 2014). The alphanumeric system uses an alphanumeric grid with assorted numbers and letters started from the bottom left hand corner of the grid and locate the letters across the bottom of the grid (Booker et al., 2014). Using this system, we can identify the location in the form of cells as co-ordinates as shown in figure 2.2 below.

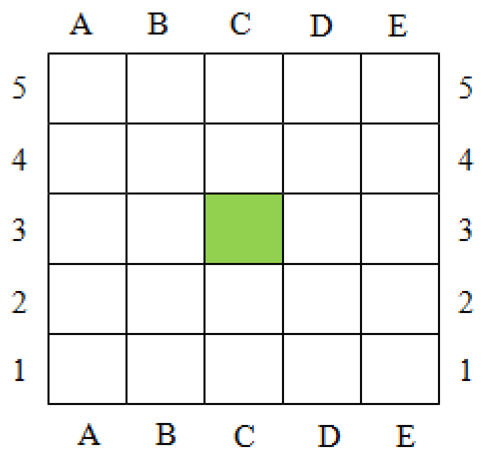


Figure 2. 2 Alphanumeric Grid System Showing C3

The ability to use this grid system turns out to be an important component of spatial structuring, which refers to the mental operation of constructing an organization for a set of objects in space (Ministry of education, 2008; Clements & Sarama, 2009). In this case, children may first perceive a grid as a collection of cells in the form of squares, rather than as sets of perpendicular lines. According to Booker et al. (2014), a similar system with the alphanumeric grid system is employed to identify a certain local area that includes several building such as the school, the local park, etc.

Based on the explanations above, the first instructional activity of this study is making a system of organizing city blocks and identifying the location of particular block in the city blocks, one of which is using the alphanumeric grid system. However, since the grid system identifies a region rather than a point, the precise location cannot be described (Ministry of education, 2008). Consequently, in the following activity, students will become aware that the system is not sufficient anymore to describe the location between two objects placed in the same cell (in this case, the same block). Consequently, they are challenged to find another system that can describe those locations using the idea of *taxicab* distance as described in the following section. This activity will bridge them to reinvent the positive quadrant of the Cartesian coordinate.

2. Reinventing the positive quadrant of the Cartesian coordinate system

In this study, the notion of reinventing the positive quadrant of the Cartesian coordinate can be associated with interpreting the grid lines in the rectangular grid as number lines only consisting of positive integers. To come up with this idea, we need to coordinate the idea of the distance relationship within the rectangular grid that is embodied in the concept of *taxicab* distance. Before working with this idea, however, students need to conceptualize the grid as a set of perpendicular lines, not as a collection of cells anymore as in the alphanumeric grid system (Clements & Sarama, 2009). Here, students are invited to interpret the street map of the city block as the rectangular grid that consists of a set of perpendicular lines (de Lange et al., 1997).

As mentioned before, to bridge the students up to interpret the grid lines as rulers, the concept of *taxicab* distance is employed this study. According to de Lange, van Reeuwijk, Feijs & van Galen (1997), the *taxicab* distance is the distance between two objects, which is calculated along a pathway on a rectangular grid as a taxi-cab would drive (see figure 2.3). With many possible *taxicab* routes from object A to B, the *taxicab* distance between A and B can be easily determined by adding the horizontal distance and the vertical distance (see the right part of figure 2.3).

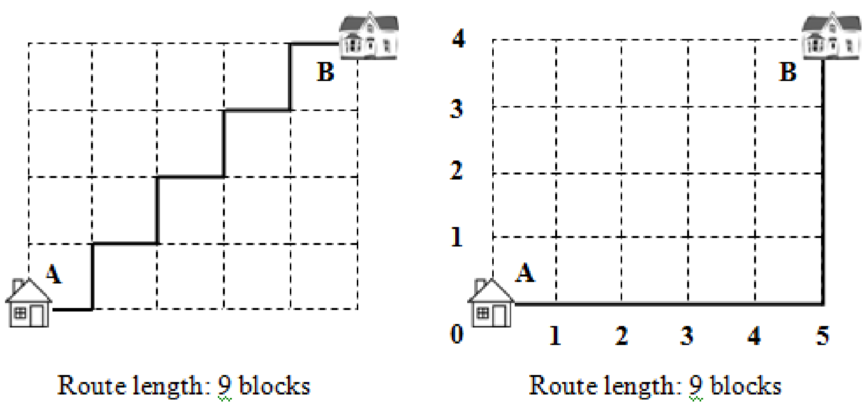


Figure 2. 3 The example of *taxicab* routes from point A to B

In other words, the taxicab distance between two (geometrical) objects can be calculated by adding the horizontal distance and the vertical distance as shown in the red path (see figure 2.3).

In the context of the city blocks, we design an activity in which students are asked to investigate that the shortest paths between two objects in the city blocks with the same starting and ending points cover the same *taxicab* distance, which involve the horizontal and the vertical distance. The role of the *taxicab* distance here can bridge them up to make a system that can help them to easily find the shortest distance between two any objects on the street map. When determining both the horizontal and the vertical distances, students are expected to interpret the grid lines as number lines or rules that consist of whole numbers. Consequently, the students are wittingly guided to reinvent the first quadrant of the Cartesian coordinate system

3. Locating and plotting any points in the first quadrant of the Cartesian coordinate system using ordered pairs

Locating and plotting points in the form of an ordered pair (x, y) become very important when students learn about the Cartesian coordinate system. At first, students are asked to describe the location of a certain object using a pair of numbers as the representation of the horizontal and the vertical distance. This will be a preliminary activity before students are introduced to the use of an ordered pair. To understand how this notation works for locating objects, students are challenged to an issue about which distance (the horizontal or the vertical) that should be considered as the first and the second coordinate of the ordered pair. This activity will lead them to an agreement about which distance that should be notated as the first coordinate.

To make it work reversibly, they are also required to be able plot a certain point on the Cartesian plane by using the ordered pair. According to Sarama et al. (2003), the most crucial issue when students plot points on the Cartesian plane is from which position students plot the points, whether from the origin or from a certain points. Children initially tend to plot a point from the previous point, not from the origin (Palupi, 2013; Sarama et al.,2003). Accordingly, a specific discussion about plotting points on the Cartesian plane is also required in the end of the series of instructional activities.

The understanding of the Cartesian coordinate plays an important role to be able to do graph work, one of which is making graphs (Somerville & Bryant, 1985; Blades & Spencer, 2001; Sarama et al., 2003). In this study, students start to make a simple graph (a special quadrilateral figure) through locating and plotting points on the first quadrant of the Cartesian plane.

2. 5. The Role of the Teacher

According to Gravemeijer (1994), realistic mathematics education (RME) positions the students different from the traditional learning approaches. Here, students are expected to be more independent, to not just keep asking for approval of their answer or guidance of a procedural solution. Indeed, the students are expected to be able to explain or justify their solution, understand the solution of others, and ask for explanation if needed. The change within the student's role significantly depends on the change of the teacher's role (Gravemeijer, 1994; Wanty et al., 2010; Cobb & Yackel, 1990). The responsibility of the teacher as the authority and the single source of knowledge have shifted towards being the facilitator of students' learning by initiating and guiding discussion (Gravemeijer, 1994; Wanty et al., 2010). According to Wanty et al. (2010), the transformation of the teacher's role can be described in terms of social and socio-mathematical norms in the classroom as elaborated below.

2. 6. Social Norms

The social norms in the classroom are defined as the expected manner of acting and explaining that is customized through a compromise between the teacher and students (Gravemeijer & Cobb, 2006). The role of the teacher in the traditional classroom is to explain and validate, while the social norms encompass the developing of a student's responsibility to understand and react to what the teacher has in mind (Gravemeijer & Cobb, 2006). As mentioned earlier, the students are also to explain and justify their answer. This instantaneously involves the students to understand the solution of the others and ask for explanation if necessary (Gravemeijer & Cobb, 2006; Wanty et al., 2010).

Accordingly, the role of the teacher in this study is asking probing questions that allow students to decide and reason whether an answer is right or wrong and whether their explanation is acknowledged or not in the class. Besides asking probing questions, the teacher is required to create a learning environment that can bring students' ideas and solutions as the focus of the classroom discussion. In this case, after having working groups for each learning activity, the teacher asks some representational groups to present their work, while the other groups are provoked to ask questions, or making remarks, or suggestions.

2. 7. Socio-mathematical Norms

Socio-mathematical norms can be differentiated from social norms as the expected ways of acting and explaining that are specific to mathematics. According to Yackel & Cobb (1996), socio-mathematical norms refer to normative aspects by which students within the classroom community discuss and justify their mathematical work. The socio-mathematical norms itself comprise what is considered a different, a sophisticated and an efficient mathematical solution, and an acceptable mathematical explanation and justification (Gravemeijer & Cobb, 2006; Yackel & Cobb, 1996). In this case, students need to develop their personal ways of judging their mathematical work that counts as different, sophisticated, efficient, or acceptable through negotiations within the classroom discourse (Gravemeijer & Cobb, 2006; Yackel & Cobb, 1996).

Accordingly, the role of the teacher in this study is asking alternatives solutions of given problems during the lesson such that the students can explore their personal ways of what counts as difference solutions. Moreover, the teacher is required to react with enthusiastic response to the student's solution that can be interpreted as an implicit indicator of what counts as a sophisticated or an efficient solution. For instance, regarding to the learning activity of making up a system for organizing city blocks, what counts as the sophisticated or efficient solution is a system that involves two parameters (row and column) such as the alphanumeric grid system. Furthermore, locating and plotting the object in a plane using the ordered pairs is also considered as a sophisticated or an efficient solution.

2. 8. The Coordinate System in the Indonesian Curriculum

Based on the Indonesian Curriculum 2013, the Cartesian coordinate system is taught in 6^{th} grades to approximately 11 -12 year-old students in the second semester (Depdiknas, 2006). The following table 2.1 shows the standard and basic competence of the Cartesian coordinate system in the Indonesian curriculum.

Table 2. 1 The Standard and the Basic Competence of the Cartesian coordinate

Standard Competence	**Basic Competence**
3. Using the coordinate system in solving mathematics problems	6.2 Knowing the coordinate of an object 6.3 Determining the location or the coordinate of a point in the Cartesian coordinate system

It is common in Indonesia that the coordinate system is introduced as a ready-made system that needs to be understood by remembering the rule, the procedure, and the notation as well (Palupi, 2013). For instance, to introduce a grid system as the prior knowledge to understand the Cartesian coordinate system, students are given a ready-made system that uses the combination of letters and numbers to describe the position of objects within the grid (Depdiknas, 2006). In this case, the students only need to remember the rule how to determine the object's position correctly. Moreover, the Cartesian coordinate system is also presented as a ready-made system formed by one horizontal axis (x-axis) and one vertical axis (y-axis), which intersects at the origin (Depdiknas, 2006). Similarly, the students only need to remember how an ordered pair works to describe and plot points in the Cartesian coordinate system.

Not only thick with sets of rules and algorithms, mathematical tasks in Indonesian textbook also lack applications that are important to make the concept become real for them (Zulkardi, 2002). For instance, to introduce the grid system as mentioned before, certain objects are illustratively placed on terrace floor tiles that look like a grid (Depdiknas, 2006). Students need to describe the location of the objects based on the given system. In a sense, this activity seems meaningless because the students will not grasp the idea why they need to determine that location. Moreover, after learning the Cartesian coordinate system directly at the formal level, students are not given any meaningful tasks to learn to apply the use of the coordinate system in other contexts (Depdiknas, 2006).

The common way of teaching the coordinate system topic as a ready-made system with a lack of applications seems to be an important reason to improve that way of teaching. To make it happen, this study focuses on designing sequential learning activities that can promote the reinvention of the topic of the Cartesian coordinate system in the 5^{th} grade of primary school. Rich contexts that are frequently encountered in Indonesia are also used for reinventing and at the same for applying the concept of the coordinate system in students' daily lives.

2. 9. Research Aims and Research Questions

This study is aimed at contributing the development of a local instructional theory in supporting students' understanding of the first quadrant of the Cartesian coordinate system. Based on the aim of the study, a set of instructional activities is designed based on realistic mathematics education (RME) theory with the research question formulated as follows. *How can we support 5^{th} grade students in understanding the first quadrant of the Cartesian coordinate system?*

CHAPTER III
METHODOLOGY

3. 1 Research Approach

The main concern of this study is to improve mathematics education in Indonesia by designing new innovative materials and a grounded theory on learning about the Cartesian coordinate. Accordingly, the aim of this study is to contribute a local instructional theory that supports students' conceptual understanding about the positive quadrant of the Cartesian coordinate in fifth grades. To achieve the aim, we develop innovative educational materials based on realistic mathematics education (RME) approach and conduct iteratively adjustments on it. It implies that we need the design of instructional materials and the research on how these materials support students understanding the concept. Therefore, the methodology of this research falls under design research approach. This approach provides a methodology to understand whether the developed instructional materials support students' understanding about the first quadrant of the coordinate system and how those materials work well in the natural setting of a classroom.

One of crucial characteristics of design research is its cyclic-iterative that is distinguished into macro cycles and micro cycles (van Eerde, 2013). In general, a macro cycle consists of three phases: preparation for the experiment, teaching experiment, and retrospective analysis (Gravemeijer & Cobb, 2006). Meanwhile, a micro cycle refers to a set of problems and activities that are conducted within one lesson. The three phases of the macro cycle will be described below.

3. 1. 1 Preparation and Design Phase

The preparation phase of a design study is mainly aimed at formulating a local instructional theory that can be elaborated and refined when simultaneously accomplishing the teaching experiment (Gravemeijer & Cobb, 2006). To reach this aim, we conduct three core steps, namely a literature review, formulating the research aim and the general research question, and developing the hypothetical learning trajectory (HLT) (van Eerde, 2013). In this study, the literature review about the topic of the first quadrant of the Cartesian coordinate system is performed to identify the disclosure of the relevant studies on the topic, the common way of the topic is taught, and the students' difficulties related to the topic. Consequently, we can define the knowledge gap and formulate the research aim and the general research question of our study.

For the development of a hypothetical learning trajectory (HLT) related to the topic of this current study, we employ a variety sources to design a set of instructional activities with specific goals for each activity. The specific tenets (Treffers, 1987) and design heuristic about the guided reinvention (Gravemeijer, 1994) for realistic mathematics education (RME) propose a basic framework to design the instructional activities related to our topic. Other sources such as the former studies, textbooks, and website also offer a general base for the design. During designing the activities, we make the conjectures of students' thinking and the teacher's actions on how to guide the learning process. All of this together will be the initial HLT. It will be tested during the teaching experiment and iteratively adjusted based on the students' actual learning process.

3. 1. 2 Teaching Experiment Phase

According to Gravemeijer & Cobb (2006), the purpose of the teaching experiment is to test and improve the HLT and to develop an understanding of how it works in the actual learning process. To achieve these aims, the teaching experiment in this study is only conducted within two cycles because we have very limited time for the data analysis. Consequently, it is wise not to collect excessively data during the teaching experiment.

In the first cycle, that is called pilot experiment, the researcher who acted as the teacher tests the educational design about the first quadrant of the Cartesian coordinate system with a small group of students as the participants. The pilot experiment is aimed to adjust and improve the initial HLT such that a refined HLT can be used for the next cycle. In the second cycle, conducted as the real teaching experiment, the refined HLT are tested in the classroom with the regular teacher and the whole students as the participants.

During the completely second cycle, the researcher and the teacher discuss about how upcoming activities should be conducted. The aim is to make clear the important aspects of the learning activity that the teacher should focus on. In addition, the reflections concerning the strong as well as the weakest points of each learning activity are also discussed. This discussion is conducted to get important information used to revise and adjust HLT.

3. 1. 3 Retrospective Analysis Phase

According to Gravemeijer & Cobb (2006), the aim of retrospective analysis depends on the theoretical intent of the design research. One of it is to contribute to the development of a local instructional theory (van den Akker et al., 2006). The data from the teaching experiment are prepared for analysis with HLT as the guideline for the focus of the analysis. During the analysis, we confront the actual learning that observed in the classroom with the learning conjectures described in the HLT. The analysis is not merely about the cases that verify the conjectures, but also the ones that contradict with it. Based on the result of the analysis, we can draw the conclusion of the study and answer the research questions.

In this study, the retrospective analysis falls under the constant comparative method. In this method, the researcher first listen video recordings or watch the videotapes and read the entire transcript chronologically (Bakker & van Eerde, 2013). After that, several interesting fragments are selected to propose assumptions. The assumptions are tested against the other episodes to get confirmation or counter-examples. To get the final assumptions, the researcher can reiterate the assumptions process several times and conduct peer-examination.

3. 2 Data Collection

3. 2. 1 Preparation and Design Phase

To connect the designed instructional activities with students' current knowledge, we need to determine the starting points during the preparation phase. Consequently, students should be assessed before the teaching experiment for instance by a performing a written-test, functioned as a pre-test. Later, after accomplishing the teaching experiment, we also conduct a post-test to obtain some data of the change in students' understanding about our topic.

The data collection in this phase is also aimed to gain insight into classroom conditions. To achieve this aim, we conduct an interview with the teacher (see Appendix A) and a classroom observation (see Appendix B). During the interview, we collect data about teacher's belief and experience, classroom management, classroom norms, students' prerequisite knowledge related to the given topic. The classroom observation is also conducted to compare in addition to what we have already obtained from teacher interview scheme. At this time, we draw the direct evidence what actually happens in the classroom. To get reliable data, we make an audio registration and field notes during both the interview and the classroom observation. All of the data gathered during this preparation phase will be used to adjust and revise the HLT if needed

3. 2. 2 Preliminary Teaching Experiment (cycle 1)

The preliminary teaching experiment, functioned as the pilot study, is mainly aimed to get an insight about students' thinking about the problems described in HLT and to test the conjectures about it. This preliminary teaching is conducted by implementing the initial HLT with a small group of five students as the participants and the researcher as the teacher. The students in the pilot study must be different from the students participating in the actual teaching experiment, but they should not differ too much in their knowledge. As such, they can follow the sequence of learning well and none of them dominates the lesson too much.

During the preliminary teaching, we collect video registration of classroom activities, make field notes about students' activities, and collect students' written work. All of these data are analyzed to compare the result with our initial HLT. The results are also used to improve the students' prediction in the initial HLT and to make necessary adjustments to the designed instructional activities before carrying out the real teaching experiment.

3. 2. 3 Teaching Experiment (cycle 2)

In the second cycle of teaching experiment, the revised HLT is implemented in the real classroom environment with the participants are students of fifth grade. The regular teacher conducts the whole lessons, meanwhile the researcher who acts as observers is assigned to conduct a video registrations of the classroom observation, makes field notes, and collects students' written work. The class observations are collected to analyze whether the conjectures in the revised HLT are certainly fulfilled or certain issues come out to revise the instructional activities in some extend. In practice, one dynamic video camera that can moves around is used to record all of the learning activities during the whole lessons.

To get more insight into students' thinking and reasoning, short discussions with a focus group is also performed. The focus group itself consists of 3-5 students who have average level of understanding based on the result of pre-test and teacher interview. It is essential to have a focus group here since we cannot capture and analyze all of students' thinking and reasoning in the class. By focusing only on one group, we have an inclusive students' thinking during the teaching experiment. However, the data of the whole group can be used merely as source for additional interpretation. In practice, the short discussions with the focus group will be recorded by using one static camera.

In the end, all of the data collected during this second cycle will be prepared to be analyzed in retrospective analysis. The result of the analysis, later on, will be used to final revise of the HLT. It is also used to answer the general research question about how we can support students' understanding of the first quadrant of the Cartesian coordinate system.

3. 2. 4 Pretest and Posttest

A pretest is performed to get information about the students' initial knowledge about the positive quadrant of the Cartesian coordinate. Since the pretest is conducted within the two cycles, the participants are students who join either in the first cycle or the second cycle. The first pretest is conducted before preliminary teaching, while the second one is before the teaching experiment. The items used in both pretest are the same and they are designed in such way we can examine students' strategies in solving coordinate system related problems. Some of the items in the pretest will be elaborated in the lesson during teaching experiment. The level of difficulties such as easy, intermediate, and difficult are also considered during designing the items of the pretest.

To assess students' acquisition of knowledge about rectangular coordinate system, we also conduct a posttest (see Appendix D) in the end of the instructional sequence in either the first cycle or the second cycle. The items used in the posttest are mostly similar to the items in the pretest. The modifying are only occurred in the parts of the context and the numerals on the given problems. The importance of conducting a pre-test and a post-test in the first cycle (preliminary teaching) is to ensure whether the items on both pretest and posttest are understandable or not. As such, we can make a necessary revision before we have them on the second cycle (teaching experiment). In addition, the pre-test in both cycles is also performed to assess the starting points of the students.

3. 2. 5 Validity and Reliability

According to Bakker & van Eerde (2013), the validity of design research concerns about the terms whether we really measured what we intended to measure. Meanwhile, reliability refers to the independence of the researcher. As explained before, in this study, we collect several data such as video registrations of classroom observation and of focus group discussion, field notes, students' written works, and teacher interview recording. The variety types of data collection above allow us to conduct data triangulation that contributes to validity of this study. We use electronic devices such as video cameras or tape recorder to get complete information during observations. It allows for greater trustworthiness and accuracy of the interpretation of the data such that it can contribute to the reliability of the data collection in this study.

3. 3 Data Analysis

3. 3. 1 Pretest

The analysis of the pretest is aimed to get information about students' current relevant knowledge and identify students' starting points about the first quadrant of the Cartesian coordinate system. The result of the pretest is analyzed based on the rubric we develop as a guideline (see Appendix G) without performing any statistical analysis at all. We also analyze the results of the students' written work qualitatively by considering their reasoning in solving the problems. The result of the analysis will be used to do some adjustment in the initial HLT so that it is appropriate with the students' starting points. Moreover, the result is also utilized to select the focus group consisting of students with various level of knowledge related the topic of the first quadrant of the Cartesian coordinate system.

3. 3. 2 Preliminary-Teaching (cycle 1)

The data collected from the preliminary teaching such as students' written work and the video registration is used to examine the students' learning process. The learning process is investigated by testing the conjectures stated in the initial HLT. At first, we watch all of the video registration and focus our intention on the interesting fragments. What we meant by interesting fragment is a video fragment in which we can observe the students' way of thinking in achieving the learning goals. However, it also can be a fragment that shows students' struggles attempted to find the solution of the given problem. After that, we need to make the transcript of the selected fragments and analyze these. In the end, the result of the observed learning based on the analysis of classroom observation supported with students' written work are compared to the conjectured learning in our initial HLT. In this sense, it can be noticed which conjectures is really occurred and which one is not.

From the whole analysis above, we can draw a conclusion whether our designed instructional sequence support students' understanding of the concept of the first quadrant of the Cartesian coordinate system or where it does not support it. In addition, considering the discussion and the interview with the students, we can investigate students' difficulties encountered during the lesson and evaluate the language and the instruction we used in the worksheets. All of these evaluations are employed to revise and improve our initial HLT before carrying it out in the real teaching experiment.

3. 3. 3 Teaching Experiment (cycle 2)

Similar to the analysis in the preliminary teaching, the video recording of students' activity in both the focus group and the class observation is used to investigate the learning process. Accordingly, the selected interesting fragments are transcribed to provide the interpretation of students' thinking. In addition, the selected fragments together with students' written work are analyzed and compared with the conjectures in the revised HLT from the first cycle. The result of the analysis, later on, is employed to answer the research questions, derive conclusions, and revise or improve the existing HLT.

3. 3. 4 Posttest

The method we analyze the result of the posttest is similar to what we do in the pretest. However, the results of the posttest are also compared with the result of pretest. This analysis is aimed to investigate whether the students have already achieved the learning goals for the lessons. Here, we analyze the way students, solve the given problem, and the strategy they use. The result of this analysis can be employed to provide additional information in deriving the conclusion and to answer the research questions of this study.

3. 4 Validity and Reliability

3. 4. 1 Internal validity

The internal validity concerns about the quality of the data that has led to the conclusion (Guba, 1981 cited in Bakker & van Eerde). In this study, we analyze several data from video registrations, field notes, students' written works, and teacher interview to answer research questions and draw the conclusion. The analysis of the variety types of data above allow us to perform data triangulation that contributes to the internal validity. Moreover, testing the provided data with the HLT both in the teaching experiment also increase the internal validity.

3. 4. 2 External and Ecological validity

The external validity, closely interpreted as the generalizabilty, refers to the question of how we can generalize the results from certain contexts to be functional for other contexts (Bakker & van Eerde, 2013). By giving the results of instructional theory, the HLT, and the educational activities as instances of something more general, others can adjust these to their local contingencies. In addition, in term of ecological validity, it refers to the extent to which the results of a study can be generalized to real condition. The implementation of our learning design in the real classroom situation, where the regular teacher of that class is teaching, contributes to the ecological validity.

3. 4. 3 Internal reliability

The internal reliability refers to the degree of independence of the researcher about how the data collected and analyzed (Bakker & van Eerde, 2013). The internal reliability in this study can be improved by discussing the critical fragments with colleagues, called inter-subjectivity, about the interpretations and conclusions during retrospective analysis.

3. 4. 4 External reliability

In design research, the external reliability is interpreted as “trackability” or “transparency”, which means that the readers must be able to track the whole process of the study (Bakker & van Eerde, 2013). In this sense, the clear explanation of how the study has been, how the data are analyzed, and how the conclusions have been derived can improve the external reliability. It is because the readers can easily track the whole process of this study.

3. 5 Research Subject

The research is conducted in a primary school named SD Pusri in Palembang. This school has been involved in *Pendidikan Matematika Realistik Indonesia* (PMRI) project. Accordingly, most teachers in this school have sufficient knowledge and experience about RME and design research. This study involves 5th grade students and their teacher as the research subject

3. 6 Timeline of the Research

The timeline of the study is summarized in the following table.

Table 3. 1 Timeline of the Research

	Date	Description
Preparation and Design Phase		
Preparation	September 2014 – January 2015	Conducting literature review and designing initial HLT
Discussion with the teacher	3 – 5 February 2015	Classroom observation, interview with the teacher and communicating the detail of the study with the teacher
Preliminary Teaching Experiment Phase (Cycle 1)		
1st meeting	16 February 2015	Pre-test (initial version)
2nd meeting	17 February 2015	Interview to clarify students' answer on the pre-test items
3rd meeting	18 February 2015	Activity 1: Labeling blocks of city blocks
4th meeting	20 February 2015	Activity 2: Chess notation
5th meeting	23 February 2015	Activity 3: *Taxicab* routes and distance
6th meeting	25 February 2015	Activity 4: How to use an ordered pair?
7th meeting	26 February 2015	Activity 5: Cartesian coordinate system (positive quadrant)
8th meeting	27 February 2015	Post-test (initial version)
9th meeting	28 February 2015	Interview to clarify students' answer on the pre-test items
Teaching Experiment (Cycle 2)		
1st meeting	12 March 2015	Pre-test (initial version)
2nd meeting	13 – 14 March 2015	Interview to clarify students' answer on the pre-test items
3rd meeting	24 March 2015	Activity 1: Labeling blocks of city blocks
4th meeting	25 March 2015	Activity 2: Chess notation
5th meeting	31 March 2015	Activity 3: *Taxicab* routes and distance
6th meeting	1 April 2015	Activity 4: How to use an ordered pair?
7th meeting	2 April 2015	Activity 5: Cartesian coordinate system (positive quadrant)
8th meeting	2 April 2015	Post-test (initial version)
9th meeting	6 – 7 April 2015	Interview to clarify students' answer on the pre-test items

CHAPTER IV
HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical Learning Trajectory (HLT), regarded as an elaboration of Freudenthal's thought experiment, becomes a main design and a research instrument in all phases of design research (Bakker & van Eerde, 2004). As described in the chapter 3, the HLT is developed in the preparation phase and it guides researchers in conducting the experiment such as what should be focused in teaching, interviewing, and observing. Later, during the retrospective analysis phase, it takes a role as a guideline for researchers to determine what should be focused on in the analysis (Bakker & van Eerde, 2004; van Eerde, 2013)

An elaborated HLT in this design study entails four main components: starting points, learning goals, mathematical problems and activities and hypotheses on students' thinking process (van Bakker & van Eerde, 2004; van Eerde, 2013). The starting points are established to connect the planned instructional activities with students' current or relevant knowledge. To orientate the design and redesign of mathematical problems and activities, the learning goals are also defined. To accomplish the criteria for accepting a mathematical problem, we propose a series of mathematical problems that provoke students' thinking and reasoning. In addition, several meaningful contexts are also assigned to emerge the indented concepts related to the positive quadrant of the Cartesian coordinate. The conjectures of students' thinking and reasoning are also described complete with the proposed actions of the teacher as the response to the conjectures. The proposed actions of the teacher are also elaborated in the teacher guide.

As already mentioned, this study is mainly aimed to support students understanding of the first quadrant of the Cartesian coordinate system. To achieve the aim, six sequential activities will be accomplished in five days in which the first and the second activity will be conducted in one lesson. The instructional activities for learning the first quadrant of the Cartesian coordinate system that are embedded in the hypothetical learning trajectory are described in detail as follow.

4. 1 Labeling Blocks of City Blocks

The introduction to the use of grid system, which identifies cells on a grid rather than points, becomes a primary activity before learning the Cartesian coordinate system in general (Ministry of education, 2008). In the beginning, students conceivably perceive a grid as a collection of cells in the form of squares, rather than as sets of perpendicular lines (Ministry of education, 2008; Clement & Sarama, 2009). To tell the location in the form of cells on a grid, students are stimulated to perceive the grid as rows and columns. Therefore, students need two parameters or a coordinate pair to identify that location. In regard to this concept, students in this activity are asked to make a system of labeling blocks that allows them to perceive the city blocks as a collection of cells (looks like a rectangular grid) consisting of rows and columns. Accordingly, students need to have sufficient understanding about rows and columns to deal with tasks embedded in this activity.

Learning goals

In this activity, we expect that students are able to make a system of organizing things, in this case a system of labeling the blocks, using a grid system that involves rows and columns. This main learning goal can be specified into some sub-goals as follow.

1. Students are able to understand that locating an object in the grid system by only using one parameter (rows or columns) is less efficient.
2. Students are able to understand that to locate an object in the grid system at least needs two parameters (rows and columns).
3. Students are able to understand that the grid system is used to identify locations in the form of region (cell).

Learning activities

The context that is embedded in the instructional activities is mostly about city blocks. Even Indonesian’s students may not get familiar enough with this context, they can easily engage with the context as long as it is introduced well such that the students can imagine it. To start with, the teacher tells a new about planning of adopting the system of city block for a new city in South Sumatera of Indonesia.

Recently, the local government of South Sumatera announces that they will develop a satellite city named Jakabaring. The city will be different with the other cities in Indonesia because it will adopt a system of City Blocks like in Barcelona.

After telling the story, the teacher shows the photograph and the street map of city blocks in Barcelona as in figure 4.1 below

Figure 4. 1 Photograph (*left*) and Street Map (*right*) of City Blocks in Barcelona

Based on the photograph of the city blocks (see figure 4.1 on the left part), the teacher tells that city blocks is a rectangular area in a city surrounded by streets and usually containing several buildings such as house, mall, apartment, etc. To get a deep depiction, the teacher can ask *have you ever heard about city blocks in a game?* In this case, students probably refer to Simcity or City Blocks game. However, if none of them knows, the teacher can tell the example of those games.

After introducing the context of city blocks, the teacher distributes the student's worksheet and asks students in a group of 4-5 people to discuss the following problem and make a poster for their work (20 minutes).

At the beginning of the project, the government does not have the names for the streets yet. Instead, they will label the blocks to start their project. Discuss with your group to make a system of labeling the blocks that can help people, who are getting around the city, find the location of a certain block quickly and easily!

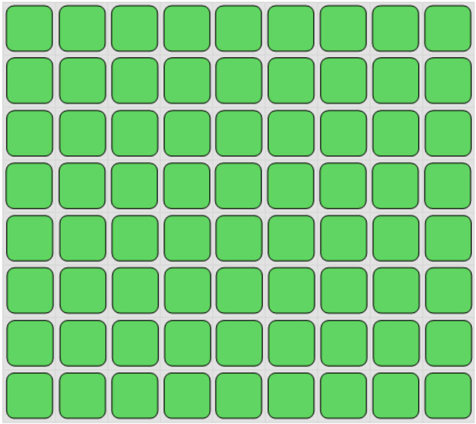


Figure 4. 2 The Street Map of Jakabaring Blocks City

According to the given problem above (see also Figure 4.2), students probably come up with a certain different system to label the blocks, either considering one parameter (rows or columns) or two parameters (rows and columns). React to those various solutions; the role of the teacher is certainly needed to support students in finding and reasoning about a better system for labeling the blocks.

Conjectures of students' solution and the proposed actions of the teacher

a. It is likely that some students will label the blocks using ordinal number starting from 1 to 72 horizontally or vertically. This implies that students only use one parameter to locate object in the grid system. This conjecture is based on Palupi's finding (2013), namely to number the plane seats arrangement, some students tend to label the seat only using the ordinal number. To provoke students in finding a better system, the teacher can ask whether the system they make really help people find a certain block, such as block 25, without passing through all of the blocks.
b. Even the students may not get familiar with the city blocks, some of them are likely know about block system that are widely used in neighborhoods in Palembang, Indonesia. Therefore, they may perceive the city blocks as rows and columns, which are labeled with letters and numbers or the other way around. To have students reason about their solution, the teacher can propose the similar question whether the system they make really help people find a certain block, such as block 7C or C7 quickly, without passing through all of the blocks.

c. If students perceive the city blocks as rows and columns, which are labeled with two numbers, the teacher can ask students to reason about their solution and propose the similar question like in the previous conjecture (part b).

After conducting the group discussion, two representational groups are invited to present their poster, meanwhile the other groups are asked to give questions or remarks related to the presentation. The first presenter (if possible) is a group who use ordinal number starting from 1 to 72 horizontally or vertically to label the blocks. About this solution, the teacher leads the discussion by asking the other groups whether the system really helps people, who are getting around the city, find a certain block without passing through all blocks. React to this question; students probably come up with the following arguments:

- It is difficult for people to find the location of certain pond using that system because someone need to walk along the blocks and count the blocks one by one in order to find a certain block.
- Since the blocks within each row or column is not multiple of landmark numbers (such as 5 and 10), it is quite difficult to find a certain block quickly because they need to count one by one.

In this manner, the teacher should not judge that the labeling system only using the ordinal number is less efficient. Instead, let the students to realize it themselves and ask the second presenter, who use other systems involving rows and columns, to present their work. By using the proposed system, the teacher can ask the same question whether the system really helps people to find a certain blocks easily and quickly without passing through all the blocks. In the end of the discussion, the teacher can ask to students which system that they like the most and why. This question is expected can help students reason that it is easier and quicker to locate a certain object by considering two parameters (rows and columns). This activity is continued with the activity about the chess notation in the same lesson as described after this section.

4. 2 Chess Notation

The alphanumeric grid system is is specifically used to identify the location in the form of cells as a coordinate pair of a letter and a number. In the current activity they are given an example of the alphanumeric grid system in the chess notation. Here, the students are asked to tell the location of some pawns on the chessboard using the chess notation, which is a unique coordinate pair of a letter and a number. For example, the white king is located on the square E1 or 1E. Since the activities of labeling the blocks and the chess notation are conducted within one lesson, the students have opportunity to reason and realize that the the chess notation is also applicable for labeling the blocks in the context of city blocks. Similar to the previous activity, students need to have sufficient understanding about rows and columns to deal with chess notation related tasks.

Learning goals

This activity is aimed to make students aware that the alphanumeric grid system is typically used in the conventional chess notation. This main learning goal can be specified into two sub-goals as follow.

1. Students are able to identify the location of particular pawns on the chessboard using the alphanumeric grid system.
2. Students are aware that the alphanumeric grid system can be used to label the blocks in the city blocks.

Learning activities

To start with, the teacher and students talks about their experience in playing chess. As this context is familiar with the students, the teacher can ask them whether they know about the conventional chess notation, which is the system developed to describe the position of pawns on a chessboard, or not. Let some students who know about the chess notation to explain how this notation works.

Conjectures of students' explanation

- The vertical columns of squares (either from the white's left or from the white's right) are labeled A through H, while the horizontal rows (either from the white's side or from the black's side) are numbered 1 to 8.

- The vertical columns of squares (either from the white's left or from the white's right) are numbered 1 to 8, while the horizontal rows (either from the white's side or from the black's side) are labeled A through H.

At this time, the teacher can accept all of possible explanation and clarify the conventional chess notation, which has been agreed among all chess organization. In the case that none of them knows about it, the teacher explains the system for the conventional chess notation, but does not tell how the system works on describing the location of certain pawns.

> *To describe the position of paws, we need to identify each square of the chessboard by a coordinate pair of a letter and a number. The vertical columns from the white's left are labeled A through H, while the horizontal rows from the white's side are numbered 1 to 8*

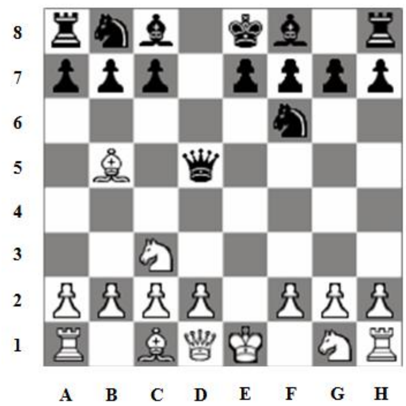


Figure 4. 3 The System for the Conventional Chess Notation.

Based on the figure 4.3 above, students in pairs are asked to determine the location of several pawns: white knight, black knight, black queen, white bishop, and black rook for about 10 minutes. The students' strategy of locating the pawans is probably different depending on which direction (the horizontal or the vertical) they notice first. Accordingly, the teacher can accept all possible solution, which will be discussed further in the class.

Conjectures of students' solution

a. A unique coordinate pair of a number and a letter respectively describes the location of a certain pawn. For instance, the white knights are located in the squares 3C and 1G, etc.

b. Conversely, a coordinate pair of a letter and a number respectively describes the location of a certain pawn. For instance, the white knights are located in the squares C3 and G1, etc.
c. For several two identical pawns such as white knight, black knight, white bishop, and black rook that have two locations, some students only tell one location. Conversely, some students probably have realized it and tell the two location of the identical pawns.

The teacher asks two different pairs, who have different ways of writing the pawns location as mentioned in the conjectures above, to present their work. Looking at these two ways of writing, the teacher ask the other pairs to argue whether these locate the pawns in the different squares or not. For instance, if student A move the white queen to F5 and student B move the black queen to 5F, teacher can ask where the locations of those two pawns now. Here, the students are expected to conclude that those two notations are same. If students only tell one location of a certain pawn (should be two location), it is not really matter because the focus of the discussion is about the way of telling the location of a certain pawns. Here, the teacher can remind that there are some identical pawns.

If none of students come up with the use of the alphanumeric grid system to label the blocks in the city blocks activity, the teacher can lead the last discussion whether the system used for the chess notation can be applied for the blocks or not. If students agree that the system can be applied for the blocks, the teacher can ask them to show how the system of chess notation works for the blocks. Similarly, if students do not agree about that, students are required to show why the system does not work. In end of the discussion, the teacher and students are expected to reason that the system is applicable since the city blocks and the chessboard problems are similar in term of its arrangement as row and columns.

4. 3 *Taxicab* Routes and Distance

Since the alphanumeric grid system identifies the locations in the form of cell rather than points, the precise location cannot be described. This activity is began with showing students to the fact that the alphanumeric grid system, embedded in the label of the city blocks, is less accurate to locate two objects or more that are laid on the same cell. Therefore, they need to put additional information such as the compass directions or the relative system to locate a certain object by using the alphanumeric grid system. Afterward, they are encouraged to make a system of locating an object from a certain origin position using the *taxicab* distance that bridge them to reinvent the positive quadrant of the Cartesian coordinate.

To support the reinvention of the positive quadrant of the Cartesian coordinate, students are asked to investigate some shortest paths between two object in the city blocks. This investigation will leads them to the conclusion that any paths with the same starting and ending points cover the same *taxicab* distance, which involve the horizontal and the vertical distance. The role of the *taxicab* distance here can bridge them up to make a system that can help them to easily find the shortest distance between two any objects on the map of the city blocks. In the end of this activity, students are expected to interpret the grid lines in the rectangular grid as number lines (rulers) only consisting of whole numbers. To deal with tasks, students are required to understand about zero, positive numbers, its position in the vertical and horizontal number lines.

Learning goals

In this activity, we expect that students to be able to make a system for determining the taxicab distance between an origin position and a certain object that bridge them to reinvent the first quadrant of the Cartesian coordinate system. This main mathematical goal can be specified into some sub-goals as follow.

1. Students are able to realize that the alphanumeric grid system needs refinement to locate two objects or more that are laid on the same cell.
2. Students are able to find the *taxicab* distance between two objects Students are able to look for different shortest paths on the map of city blocks with the same starting and ending points.
3. Students are able to understand that any paths with the same starting and ending points cover the same *taxicab* distance.
4. Students are able to find a shortest path that only involves one turn.
5. Students are able to interpret the grid lines in the rectangular grid as number lines (rulers) only consisting of whole numbers.

Learning activities

To start with, the teacher reminds students about the city blocks they have dealt with in the previous lesson and asks whether the alphanumeric grid system used in the chess notation system is applicable for the labeling the blocks. By using the provided street map, teacher asks a student to explain how the alphanumeric grid system works for the blocks (see figure 4.4).

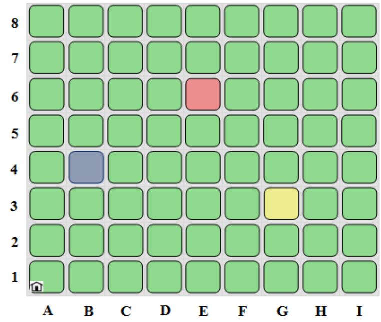


Figure 4. 4 The Alphanumeric Grid System in the City Blocks

To remind how the system work for locating a certain block, the teacher asks students to determine the location of the blue, the red, and the yellow blocks (see figure 4.4). Students are expected to locate the blocks using a coordinate pair of a letter and a number such as B4 or 4B as they do in the chess notation problem. To check their understanding about the sameness of those notations, the teacher ask whether B4 and 4B locate the different block or not.

After that, students in a group of 4-5 people are asked to solve several problems in the student's worksheets divided into two parts. For the first part, students are asked to determine the precise location of Bhayangkara hospital in a certain block that also encloses another hospital (Srwijaya Hospital) for about 10 minutes.

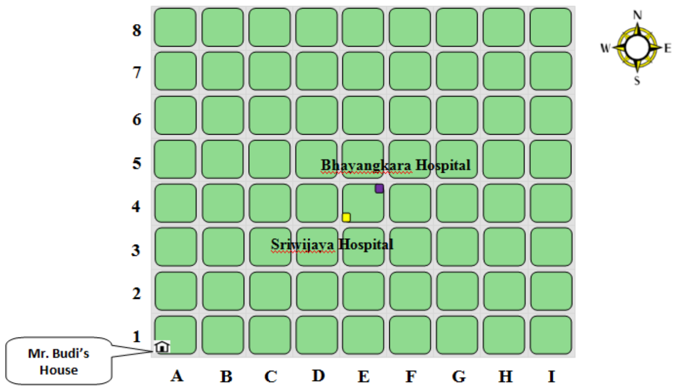


Figure 4. 5 The Street Map of Jakabaring CIty Blocks

According to the given problem, students' solution of describing the location of Bhayangkara hospital is perhaps mostly related to the provided information about the label of the blocks and the compass direction (see figure 4.5). During the group discussion, the roles of the teacher are certainly needed to provoke students that they need to tell the precise location.

Conjectures' of student's answer and the proposed actions of the teacher

a. Similar to the chess notation problem, students may tell that Bhayangkara hospital is located in block E4 or block 4E without giving any additional information. React to this, the teacher can ask about the location of Sriwijaya Hospital and argue "Is there any chance Mr. Budi will arrive at the wrong hospital (Sriwijaya Hospital) if you only tell the label of the blocks?" Afterwards, the teacher can let students to decide what they should do by themselves, either stick with their solution or find another solution.
b. Since there are two hospitals in the same block namely block E5 or 5E, some of students may put additional information such as the compass direction to. For instance, the hospital is located on the northeast corner or the north part of block E4 or 4E. At this time, the teacher only needs to clarify why the put such additional information.

A classroom discussion is focused on how to describe the precise location of a certain object laid on the same cell with the other object. To start with, the teacher asks two groups who have different solution to present their work in front the class. The first group (if possible) is a group who only use a coordinate pair of a letter and a number. In this case, the teacher can ask whether Mr. Budi will find Bhayangkara hospital precisely whereas there is another hospital in that block. The second presenter would be a group who put additional information such as compass directions. React to this solution, the teacher can provoke students' reasoning by asking "Can you be sure that Mr. Budi arrive at the right hospital based on your information?" Let students come to realize which solutions that leads to the precise location. In the end, let students to reason whether it is enough to tell the location of an object precisely only using the label of the block.

The lesson is continued with solving the second part of the student's worksheet. Students in groups of 4-5 people are asked to solve the five problems as mentioned below for about 15 minutes.

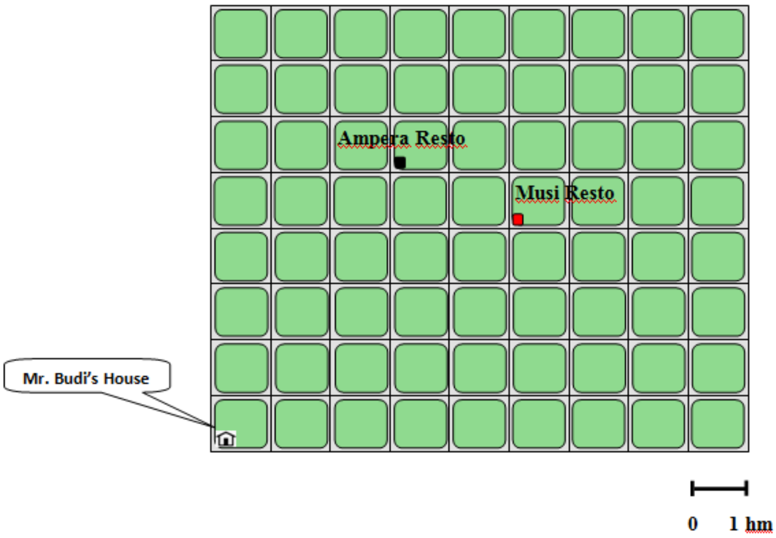


Figure 4. 6 The Street Map of Jakabaring City Blocks

1. *Which one is the nearest restaurant from Mr. Budi's house? Explain your answer!*
2. *Draw three shortest paths that Mr. Budi probably takes from his house to his selected restaurant!*
3. *Based on your drawing on the previous question, what can you say about the distance covered by those paths?*
4. *Draw two the shortest path from Mr. Budi's house to the nearest restaurant that need minimal turn! (Notes: the drawing can be the same or not with your previous drawing)*
5. *Draw two the shortest path from Mr. Budi's house to the other restaurant that also need minimal turn!*

Those five questions are mainly about investigating the shortest path between two objects, one of which is considered as the origin position. From this investigation, students are expected to cope with the conclusion that any paths with the same starting and ending points cover the same *taxicab* distance. Even though the teacher does not mention and explain explicitly about the *taxicab* distance, students are expected to realize about the horizontal and the vertical distance embedded in this concept. During the group discussion, the role of the teacher is needed to clarify students' answer and to minimize the misunderstanding or misinterpretation about the given problems.

Conjectures of students' thinking and the proposed actions of the teacher

a. For the first question, the students will conceivably compare the distance between the two restaurants to determine the nearest restaurant. Accordingly, the nearest restaurant is Ampera Resto since its distance is 8 hm, while for Musi Resto is 9 hm. For the clarification about their strategy of solving the given problem, the teacher can ask how they get 8 hm and 9 hm.

b. The students' drawing of the shortest paths in the second question will be varying depending on the chosen nearest restaurant and the paths that they choose. Based on the students' drawing, teacher needs to clarify about which the starting and the ending object. Consequently, it is very essential to have students give arrow(s) on their drawing to show the direction.

c. For the third question, most students are likely argued that all of paths in the previous answer should cover the same distance. Its distance is 8 hm for Ampera Resto or 9 hm for Musi Resto. To clarify, the teacher can ask students to show how they calculate the distance for the three different paths.

d. In the third question, it is also possible that some students argue that the paths do not cover the same distance. Based on the study conducted by Sarama et al. (2003), some students probably argue that the paths with more turns are longer than the paths with fewer turn. React to this, the teacher can suggest students to calculate the distance for their each shortest path that they make and ask them to see and compare the results.

e. For the forth and the fifth question, the students' answer will be varying, but the possible drawing could be shown in figures 4.7 and 4.8 below.

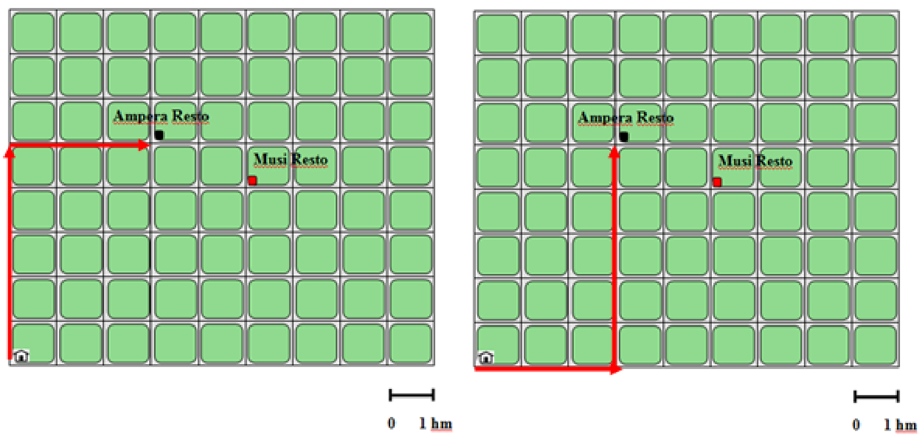


Figure 4. 7 The Shortest Path with One Turn from the House to Ampera Resto

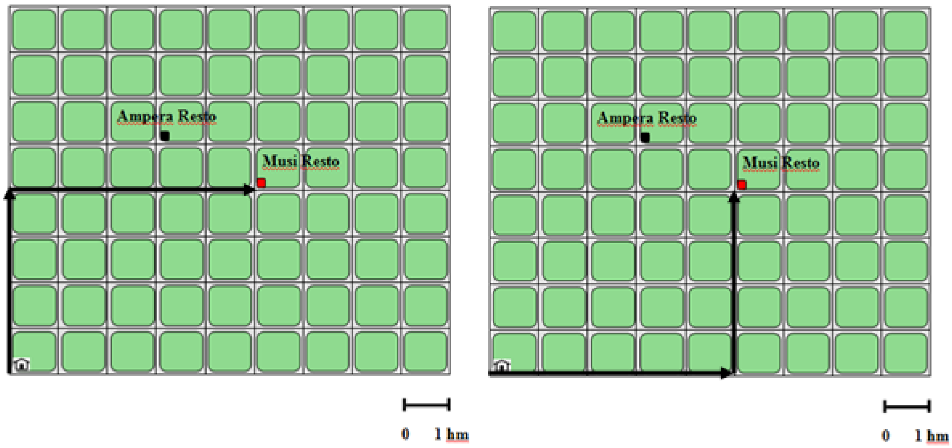


Figure 4. 8 The Shortest Path with One Turn from the House to Musi Resto

The second session of the discussion is focused on how any shortest paths between two points on the rectangular grid cover the same *taxicab* distance. To start with, the teacher can ask a group to present their work and simultaneously holds the class discussion. For the first problem, the discussion is focused on how students determine the nearest restaurant. The students perhaps give assumption that the word of nearest has relation with distance such that they need to compare the shortest distance between the two restaurants if each of them is calculated from Mr. Budi's house. For the second and third question, the discussion should be focused on how any shortest paths between two places on the city blocks cover the same distance. If there any students who argue that the paths with more turns are longer than the fewer one, the teacher can suggests them to calculate the distance for each shortest path that they make and see the results of their calculation. The discussion on the fourth and fifth problem is focused on how the shortest path that involves one turn help students to see the horizontal and the vertical distances. This will be a connecting activity to make a system for determining the shortest distance between the origin position (the house) to any places in the city bclocks, so-called the first quadrant of the Cartesian coordinate.

The last discussion is the most crucial part in this activity because it is the time for students to reinvent the first quadrant of the Cartesian coordinate system under the guidance of the teacher. The teacher presents the street map of the Jakabaring city blocks with some objects that are wittingly located near the crossroads as shown in figure 4.9 below.

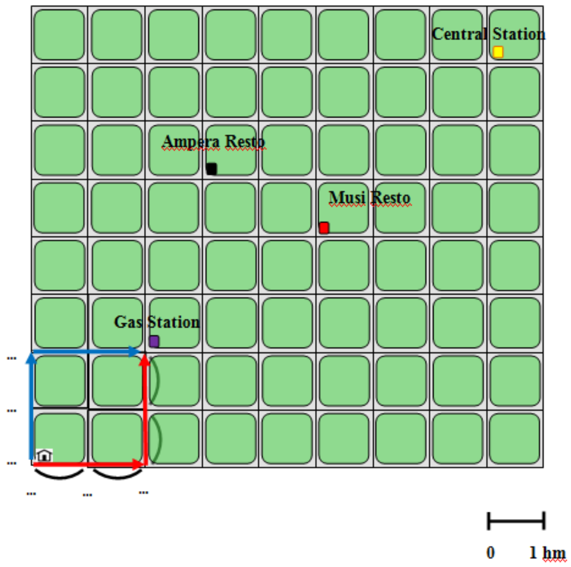


Figure 4. 9 The Street Map of Jakabaring City Blocks

To begin the discussion, the teacher shows two shortest paths, represented by the red and the blue lines, between the house and the gas station with only one turn (see figure 4.9). To determine the distance between the house and the gas station, teachers asks students to determine the horizontal and the vertical distances by simultaneously labeling the grid lines. Later on, by applying the same method, the teacher can ask some students to determine the distance between the house and the other places alternately. Students will unconsciously make a system for determining the *taxicab* distance between the origin position and any places, which looks like the positive quadrant as shown in figure 4.10 below.

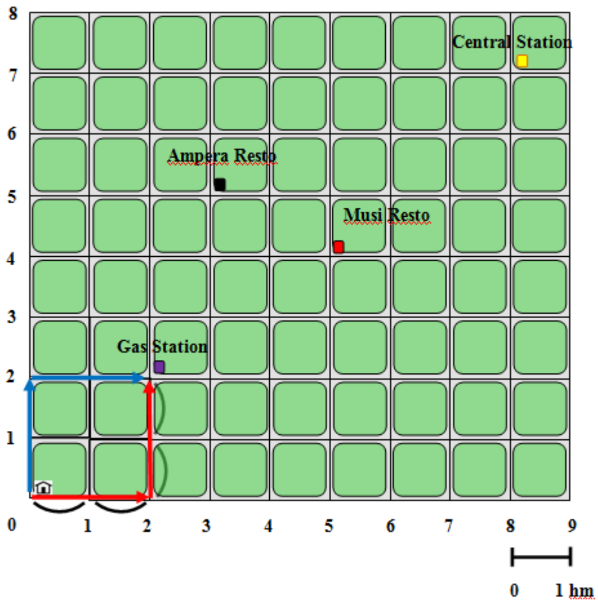


Figure 4. 10 A System for Determining the *Taxicab* Distance

It needs to be noticed that at this time the teacher does not tell that the system they found is called the first quadrant of the Cartesian coordinate system. Based on the system as shown in the figure 4.10, the teacher asks to the students whether the system can be used to easily determine the shortest distance between the house to any places or not and the reason behind it. In this case, the students are expected to be able to interpret the grid lines in the rectangular grid as number lines (rulers) only consisting of whole numbers.

4. 4 How to Use an Ordered Pair

Locating and plotting points in the form of an ordered pair (x, y) become very important when students learn about the Cartesian coordinate system. For a very start of this activity, students are challenged to describe the location of a certain object in the city blocks by using the horizontal and the vertical distance that they have learnt before. To some extent, students may put additional information to tell the location precisely such as using their knowledge about the cardinal direction or the relative system. This will be a preliminary activity before students are introduced to the use of an ordered pair. To understand how this notation works, students are faced to a problem about which distance (the horizontal or the vertical distance) that should be considered as the first and the second coordinate. This activity will lead them to an agreement about which distance that should be notated as the first and the second coordinate.

Learning goals

This activity is aimed to have students be able to locate a certain objects on a rectangular grid using ordered pairs. This main goal can be specified into some sub-goals as follow.

1. Students are able to locate an object on the rectangular grid by using the horizontal and the vertical distance.
2. Students are able to make agreement to locate an object on the rectangular grid.
3. Students are able to understand about an ordered pair.
4. Students are able to locate any object on the rectangular grid using the ordered pair.

Learning activities

To begin with, the teacher reminds students about locating hospital problem by using the information related to the label of the blocks. Afterwards, the teacher challenges students to tell the location of both hospitals using the system for determining the *taxicab* distance that they reinvent before. Students in groups are asked to determine the location of Bhayangkara and Sriwijaya Hospital

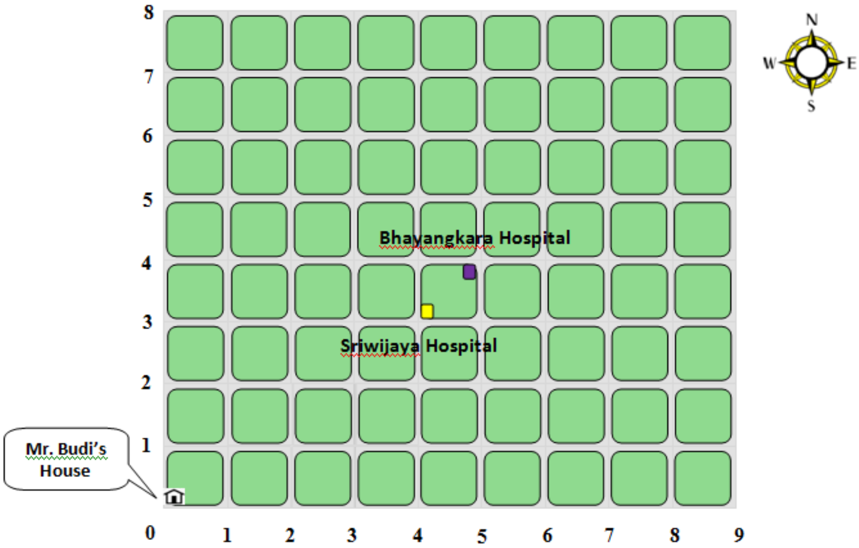


Figure 4. 11 System for Locating Objects in the City Blocks

(figure 4.11). Consequently, before starting the group discussion, the teacher needs to ask "What are the meaning of numbers notated on the given street map?" (see figure 4.11). After getting a clarification that the numbers notated on the given system represent the distance, students in groups can start to solve the given problem. During the group discussion, the teacher have a role to ask clarification related to the students' answer as described below

Conjectures of students' answer and the proposed actions of the teacher

a. Students probably combine the information related to the distance and the cardinal direction to locate the hospital. For instance, Bhayangkara Hospital is located 5 hm to the east and turn 4 hm to the north from the house, or the other way around. To get the clarification of students' answer, the teacher can ask "Why do you add the information about the cardinal direction such as north and south?"

b. It is also possible if students use words “go straight”, “turn up”, or “turn left” to describe the location of the hospitals. For instance, Bhayangkara hospital is located 5 hm go straight and turn 4 hm over up from the house or Sriwijaya hospital is located 3 hm go straight and turn 4 hm left from the house. Since students in this case use the relative system, the clarification about in which direction students start to move needs to be asked by the teacher.
c. Considering the chess notation as described in the second activity, students probably only put additional information such as using words “horizontally” and vertically. For example, Bhayangkara hospital is located 5 hm horizontally and 4 hm vertically. The reason why they put such kind of additional information must be revealed by the teacher.
d. To tell the location of both hospitals, students may only mention the distance without giving additional information related to the direction at all. For example, Sriwijaya hospital is located 4 hm and 3 hm or vice versa. Here, the information about in which direction students should go first must be clarified. In addition, the teacher can ask “If we start with the different direction, does it arrive at the same place?”

All of students’ possible answers are accepted and the teacher asks two groups to present their work. The first presenter would be a group who only considering the use of distance to locate the objects without giving the direction. React to this solution, the teacher can give probing question: “If we start with the different direction, does it arrive at the same place?” This question is expected to make students realize and reason to locate a certain object precisely, they need two parameters, namely the distance and the direction for instance. Afterward, the teacher can ask another group who has different answer with the previous one, which is a group who considers both the distance and the certain direction to locate the objects. During the discussion, the reason why students put additional information about the direction should be revealed. In the end of the discussion, the teacher can ask a reflection question: *is it enough to tell the location of a certain object only using the distance information*. Here, students are expected to realize that to locate a certain object precisely, they need two parameters, namely the distance and the direction for instance.

The teacher continues the lesson by telling a story about TV news reporting a fire accident happened in the Jakabaring city blocks. Before dealing with the main problem, students are asked to tell their experience about witnessing a fire disaster. Afterwards, students in a group of 4-5 people are asked to solve the following problem (see also Worksheet 3, Appendix E).

> *A fire accident happened last night in a neighborhood of Jakabaring city blocks due to short-circuit. The electricity powers from four electrical towers are turned off and the police line is installed around the danger zone. The four electrical powers are located on (1,1); (7,1); (7,5) and (1,5). Since the installed police-line connects those four towers, which figure that illustrate the danger zone. Explain!*

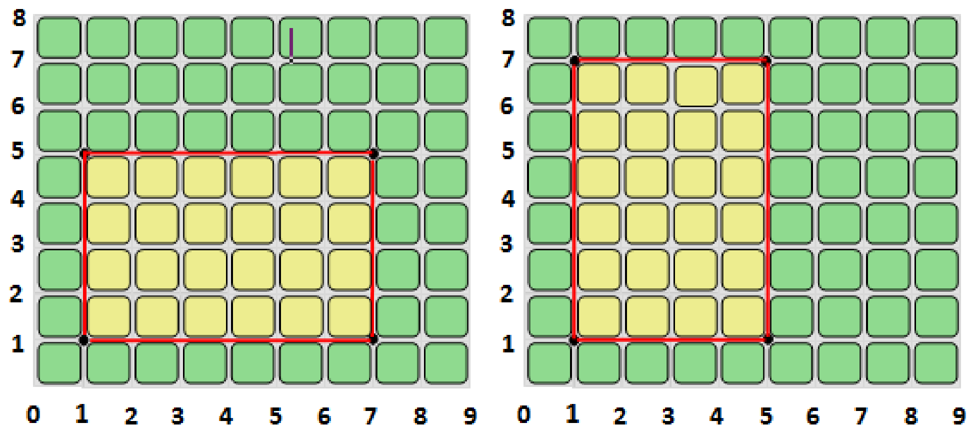


Figure 4. 12 The Danger Zone in Figure A *(left)* and in Figure B *(right)*

This problem will be the main activity in which students need to understand how the ordered pair works. Students are exposed to a problem about which distance (the horizontal or the vertical distance) that should be considered as the first and the second coordinate. During the group discussion, if students tend to stick with one figure as the answer, the teacher can ask students interpret the notation as the opposite way of the proposed students' interpretation.

Conjectures of students' answer and the proposed actions of the teacher

a. The danger zone is illustrated as in Figure A (see the left part of figure 4.12) because we (students) usually consider the horizontal first rather than the vertical one. It is also possible that students will argue that the answer is figure A because all of four points are plotted on the street map or the diagram (Palupi, 2013). Since in this case students interpreting the horizontal distance as the first coordinate of the ordered pair, the teacher ask them to interpret the first one as the vertical distance and visualize how it looks like.

b. Conversely, if students consider figure B as the representation of the danger zone by interpreting the vertical distance as the first coordinate of the ordered pair, the teacher can ask the similar question as mentioned in point b to students.

c. Since none of students know about the ordered pair *(x, y)* yet, most of them may argue that both figures illustrates the danger zone correctly because it depends on own interpretations. If the first coordinate is considered as the horizontal distance, then figure A is the answer. However, if the vertical distance is interpreted as the first coordinate, then figure B is the answer. React to this, the teacher can ask "What can we do if we want to use the notation as the conventional way of locating a certain object?"

The lesson is continued on the introducing the efficient of communicating the location of any points using an ordered pair (x,y). At this time, students have not known that x represents the horizontal distance between the point and the vertical axis and y represents the vertical distance between the point and the horizontal axis. The classroom discussion is started with the presentation of a group who choose the figure A or B as the answer. Other groups are asked to give remarks or question related to the presentation. Afterwards, the teacher can suggest them to plot the objects that is contrast with their interpretation. For instance, if they consider the horizontal distance as the first coordinate, then the teacher ask them to consider the vertical distance as the first coordinate. The discussion is continued with a group who consider the both figure as the answer. React to this, the teacher can ask "What can we do if we want to use the notation as the conventional way of locating a certain object?" From this question, students are expected to come up with the idea of making agreement about which distance that should be considered as the first coordinate.

As the reflection of the discussion, the teacher ask the students to tell the location of four objects, namely Bhayangkara hospital, Sriwijaya hospital, the gas station and the central station as shown in figure 4.13 below.

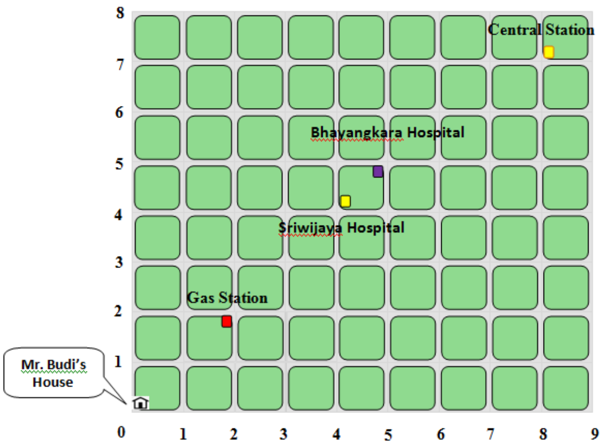


Figure 4. 13 The Street Map of Jakabaring City Blocks

Here, students are expected to locate those four objects using the ordered pairs that they have been discussed intensively before.

4. 5 Sunken Ship

Locating a certain object using the ordered pair without providing the coordinate grid will be the focus of this activity. Differ from the previous activity, in this activity, the students are expected to produce the coordinate grid and label the axes by themselves to be able to locate a certain object using the ordered pair. The ability of locating a certain point on the rectangular system embedded in a certain context turns to be important before students deal with the formal concept of the first quadrant of the Cartesian coordinate system. Within the context of the sea map, students are also challenged to tell the location a certain object, which is not located in the intersection of the grid lines. It will be the enrichment for students to locate a certain object by using fraction, one of which is involving "half" as its coordinate. The issue related to how to locate the origin is also discussed in the end of this activity. This will be an interesting topic discussion because most children argued that origin could be represented by one zero, instead of two numbers in the form of (0, 0) (Sarama et al., 2003). Based on these concepts, students are required to have sufficient knowledge about zero, positive numbers, fractions, and comprehensively about its position in the vertical and horizontal number lines to be able to work with tasks embedded in this activity.

Learning goals

This activity is aimed to have students be able to locate a certain object, represented as a point, in the form of the ordered pair by producing the coordinate grid by themselves. This main goal can be specified into sub-goals as follow.

1. Students are able to locate a certain object laid on the horizontal or the vertical axis.
2. Students are able to locate a certain object, which has “half” as its coordinate.
3. Students are able to locate the origin in the form of the ordered pair (0,0).

Learning activities

The lesson is started by talking about the experience of having trip by using a ship and telling a story about the sunken ship as described below.

> *One day, Srwijaya Ship has voyaged in the Java Sea and has trouble with the machine. Shortly, the backside of the ship began to sink such that the captain instructs his crew to to tell the coordinate of their ship to the officer of the lighthouse.*

Based on the story, the teacher ask what the officer will do to evacuate the sunken ship after getting the emergency call. Students are expected to argue that the officer needs to send rescue teams. Teacher continues the story that three rescue teams named “team A”, “team B”, and “team C” are sent to evacuate the sunken ship. Students in groups are asked to determine the coordinate of the sunken ship and the rescue teams from the lighthouse and make a poster of their work.

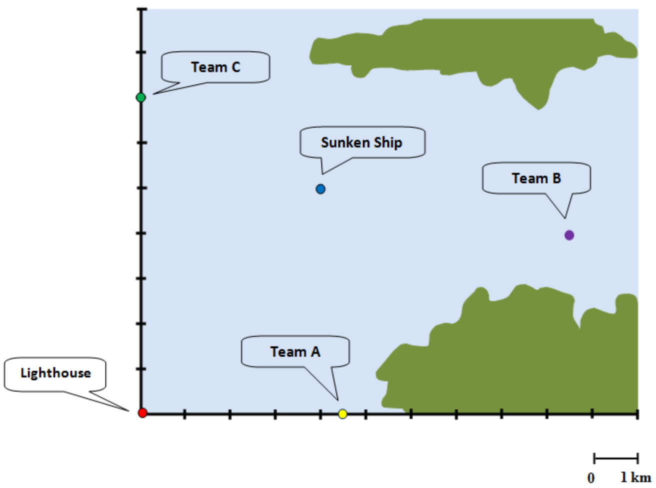


Figure 4. 14 The Sea Map

Before starting the group work, the teacher can give additional information that the ship's crew and the officer have the same map. This information is given to make the context real in students' mind so that they can start to work merely with the given map. The support of the teacher during the group discussion is certainly needed to monitor what kind of system that students use whether the Cartesian system or the other system.

Conjectures of students' thinking and the proposed actions of the teacher

- In the previous lesson, students have already learnt to locate objects in the street map of the city blocks, as the representation of a rectangular grid, by using distance. Considering this experience, most students probably have an idea of drawing the coordinate grid by themselves. By having the coordinate grid (either in their mind or not), students may begin to label the axes by using whole numbers and putting zero in the location of the lighthouse is appointed as shown in figure 4.14 below.

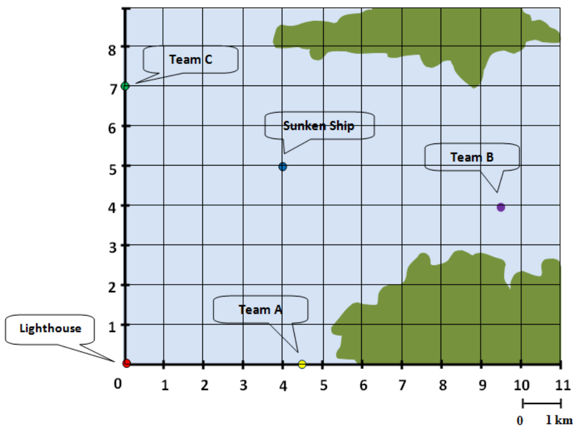


Figure 4. 15 The First Quadrant of the Cartesian System in the Sea Map

- Based on the system above (see figure 4.15), students may locate the sunken ship using an ordered pair with the lighthouse as the origin. If they interpret the ordered pair as (*x*, *y*), as the agreement in the previous lesson, the ship is located at (4, 5). Conversely, if they interpret it as (*y*, *x*), then the location is at (5, 4). To clarify students' answer, the teacher can ask what they mean by numbers notated on the map such as 0, 1, 2, etc. Students are expected to argue that the numbers are related to the distance. The teacher can ask for which distance (the horizontal and the vertical distance) students notate the first coordinate of the ordered pair.

- For the objects located on the horizontal or the vertical axis, namely team A and team C, students may have difficulty to interpret zero as one in its coordinate (Palupi, 2013). For instance, team C that should be at (0, 7) is located at (1, 7). If it is occurred, the teacher can ask them to determine the horizontal and vertical distance of the object and suggest them to reflect on their answer. It is also possible that students to reverse the value of *x* and *y* such that team C at (0, 7) is located at (7, 0) (Sarama et al., 2003). Here, the teacher can ask for which distance (the horizontal and the vertical distance) students notate the first coordinate of the ordered pair and suggest them to reflect on the agreement they made.
- For the objects that involve fraction as its coordinate, namely team A and B, students probably cope with the idea of fraction because the objects is not precisely located in the intersection of the grid lines. In this case, students may involve fraction "half" to locate the objects because they are wittingly placed on the middle of two numbers. At this time, the teacher can ask how they come up with the idea of using fraction.

➢ Based on the findings of Palupi's study (2013) that employ the similar task with the provided coordinate grid, students may use the alphanumeric grid system. They identify the location of the objects within a cell rather than in the intersection of the grid lines. In this case, students may describe the location using a coordinate pair of a letter and a number with additional information of the compass direction as they do in the third activity (city blocks) of locating objects using the label of the blocks. For instance, the sunken ship is located at northeast part of D5. In this case, the teacher do not need to judge their answer is less efficient or not, but provoke them to find another system to locate the sunken ship more by using the given information about distance as notated under the sea map.

After the working group, the first discussion is focused what kind of system that students can use to locate the objects in the sea map by employing the information about distance as notated under the map. The teacher asks a group (if possible) who use the system that looks like the first quadrant of the Cartesian system to present their poster. At this time, the teacher can ask what students first idea to be able to locate those objects and what information that students can use to support that idea. Afterward, the discussion is focused on how students locate those objects using the ordered pair especially for the objects laid on the horizontal or the vertical axis and the objects that are not in the intersection of the gridlines.

The discussion about locating the origin using the ordered pair seems to be interesting because students sometimes have a common misconception that origin can be represented by one zero, instead of two numbers in the form of (0, 0) (Sarama et al., 2003). Considering this issue, after having discussion about locating the sunken ship and the rescue teams, the teacher can ask students to tell the location of the lighthouse. As mentioned before, students may use one zero (0) or two zero in the form of (0, 0) to describe that location. To overcome this issue, the teacher can ask "How many number lines (rulers) that you can see in the map?" By arguing that the notated numbers actually label the axes (the horizontal and the vertical axis), the teacher can ask "How many zero that actually you have?" From these probing questions, students are expected to realize that they actually have two numbers of zero so that they need to locate the origin (0, 0).

4. 6 Cartesian coordinate system (Positive Quadrant)

At the formal level, students are required to be able to locate and plot any points on the positive quadrant of the Cartesian coordinate using an ordered pair. In this activity, the main issue encountered by students when plotting points on Cartesian diagram is from which position students plot the points, whether from the origin or from a certain points. According to Sarama et al. (2003), children initially tend to plot a point from the previous point they have, not from the origin. The understanding of the Cartesian coordinate system plays an important role to be able to do graph work, one of which is making graph. Here, by making a special quadrilateral figure through locating and plotting points on the plane, students start to learn about graph making, but in the simple shape (not curve). Accordingly, students need to have sufficient understanding about the properties of special quadrilateral figure (rectangle, square, parallelogram, rhombus, kite, and trapezoid) to deal with this activity

Learning goals

1) Students are able to plot and locate any points on the Cartesian diagram from the origin using an ordered pair
2) Students are able to identify special quadrilateral figure formed by four points represented in the form of ordered pair

Learning activities

To start with, the teacher talks about a game named "Guess the Shape" in which someone needs to guess a certain special quadrilateral shape formed by four points (in the form of an ordered pair) and explains about the rules of the game.

- *First player choose four points that can be formed into a special quadrilateral.*
- *The first player mentions his/her four points to the second player*
- *The second player marks those points in the given diagram, guesses the quadrilateral shape, and show his/her drawing*
- *The first player checks whether the drawing same or not*

The teacher asks students to mention the example of the special quadrilateral they have been familiarized with, namely rectangle, square, parallelogram, trapezoid, rhombus, and kite. Students are also allowed to ask questions related to the rules. Afterward, the teacher tells a story about two children named Anton and Rosi play "Guess the Shape" game as shown in the conversation below and ask students in pairs to work with this problem for about 15 minutes.

Anton : I have four points, namely (2, 2); (7, 2); (9, 5); (4, 5). Guess the shape, please!
Rosi : Wait a minute (Rosi marks the points in the diagram)
Yeah, I can guess it. The shape is .....
Rosi : The four points that I have are (2, 0); ($6\frac{1}{2}$, 0); (2, $4\frac{1}{2}$); ($6\frac{1}{2}$, $4\frac{1}{2}$). Please, guess the shape!
Anton : Ok, let's try (Anton marks the points in the diagram)
It is so easy, the shape is ....

The most crucial thing that should be monitored by the teacher during the working groups is from which position students plot the points, either from the origin or from a certain points. Here, the roles of the teacher to clarify from which point students plot a certain point is certainly needed at this time. In addition, an issue about for which distance (the horizontal and the vertical distance) students interpret the first coordinate of the ordered form should be noticed by the teacher.

Conjectures of students' thinking and the proposed actions of the teacher

a. If the students know how to plot the points in the form of ordered pairs (considering the horizontal coordinate first and then the vertical coordinate), then the answer would be:

Rosi' s drawing:
(2, 2); (7, 2); (9, 5); (4, 5)

Anton's drawing:
(2, 0); $(6\frac{1}{2}, 0)$; $(2, 4\frac{1}{2})$; $(6\frac{1}{2}, 4\frac{1}{2})$

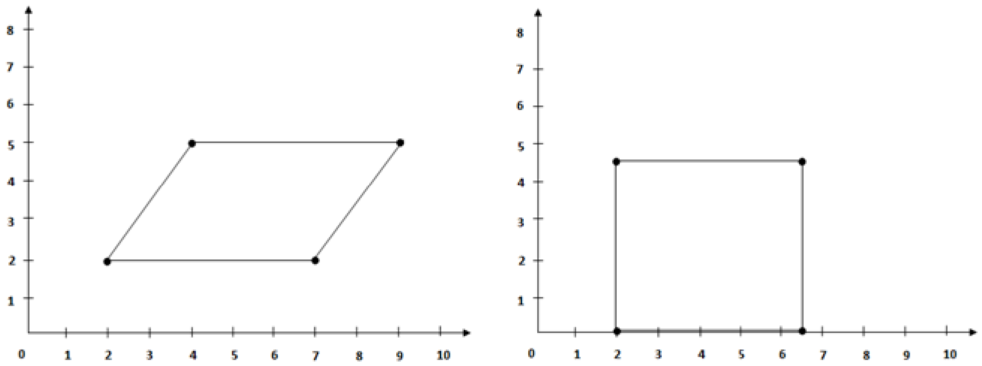


Figure 4. 16 The First Alternative Drawing

Conversely, if students consider the vertical coordinate and then the horizontal one, then the answers would be

Rosi' s drawing:
(2, 2); (7, 2); (9, 5); (4, 5)

Anton's drawing:
(2, 0); $(6\frac{1}{2}, 0)$; $(2, 4\frac{1}{2})$; $(6\frac{1}{2}, 4\frac{1}{2})$

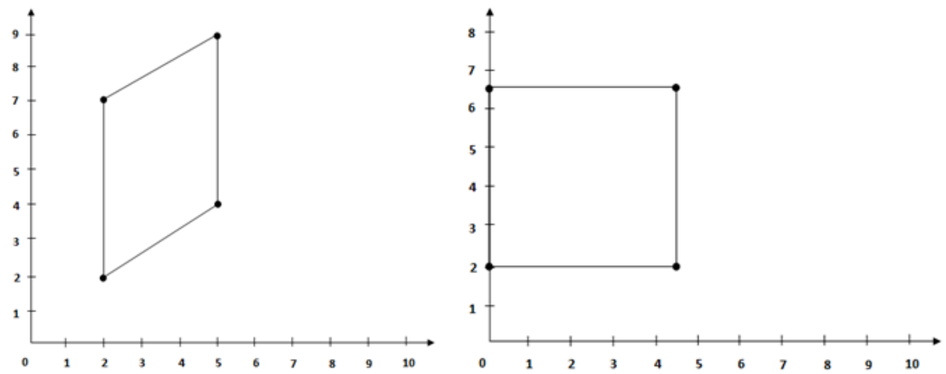


Figure 4. 17 The Second Alternatives Drawing.

The quadrilateral shape derived from both alternatives answer are the same, namely a parallelogram for the first question (Rosi's drawing) and a square for the second (Anton's drawing). The difference is merely about the orientation of the shape (see Figure 4.16 and 4.17). In the case that students interpret the vertical coordinate first, the teacher can recall them to think about the agreement they make during working with "the danger zone" problem in the context of city blocks.

c. After plotting the first point, it is also possible that students plot the other points from the first point. For example, if they had plotted a point at (2, 2) and the next point was at (7, 2), they would place the points 7 units to the right and 2 units to the above of (2, 2), at (9, 4). According to Sarama et al. (2003), children initially tend to plot a point from the previous point they have, not from the origin. To deal with this situation, the teacher can ask why they plot it from the previous point and suggest them to think again about the vertical and the horizontal distance represented by the coordinate pair.

After having group discussion of the first problem, the teacher continue about a game named "Guess the Last Point" in which someone needs to determine the forth point if given three points that can be formed into a certain special quadrilateral. The teacher tells that two children named Arga and Rossa play this game as shown in the conversation below and ask students in pairs to solve the problem for about 10 minutes.

Arga : I want to make a rhombus from three points (4, 2); (2,5) and (4,8). Let, guess the last point!
Rossa : It's so easy. Wait a minute. Ahaa, the last point is located in ……… Am I right?
Arga : Let's check
Few minutes later
Rossa : Now, it's my turn. I have three points (0, 7); ($5\frac{1}{2}$, 4); and ($1\frac{1}{2}$, 4) and I want to make an isosceles trapezoid. Guess the last point, please!
Arga : Wow. It is kind a be hard, but I can solve it for sure.
The last point is in ……. Come on, let's check together

Not only working with coordinates involving the whole numbers, in this problem, students are challenged to deal with coordinates involving "half". This will be a step for students to apply the their understanding about "half" coordinates in the sea map context to the formal situation.

Conjectures of students' thinking and the proposed actions of the teacher

To answer this question, the students need to understand about the properties of rhombus. Apart from that case, students interpretation about the ordered pair, which is in the form of *(x, y)* or *(y, x)* also affect to student's drawing led to a certain answer. In addition, the students' understanding to plot a certain points from the origin also need to be noticed by the teacher.

a. By interpreting the ordered pair as *(x, y)*, students will come up with this following drawing (see figure 4. 18) and answer that the forth point should be at (6, 5) for Rossa's drawing and at (0, 0) for Arga's drawing.

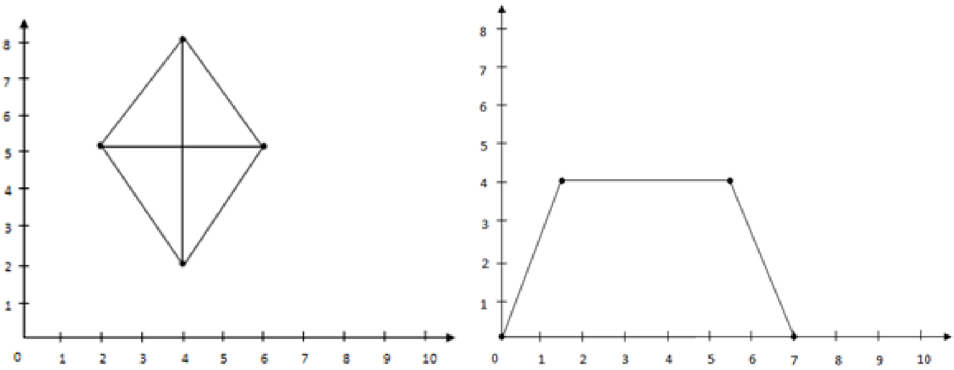


Figure 4. 18 Rossa's Drawing *(left)* and Arga's Drawing *(right)*

Conversely, if the ordered pair is considered as (*y*, *x*), students will come up with this following drawing (see figure 4. 19) and answer that the forth point should be at (5, 6) for Rossa's drawing and at (0, 0) for Arga's drawing.

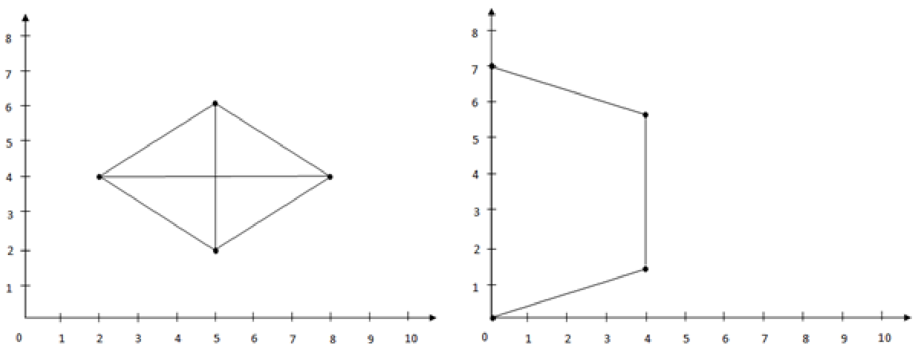


Figure 4. 19 Figure 4. 18 Rossa's Drawing *(left)* and Arga's Drawing *(right)*

In the case that students interpret the vertical coordinate first, the teacher can remind them to think about the agreement they make during working with "the danger zone" problem in the context of city blocks.

b. In this problem, it is also possible that students do not know the properties of the rhombus such that it will leads them to varying answers about the location of the last point. For this case, students can ask students about the properties of the rhombus that they know and suggest them to reflect with their answer.
c. The similar case of plotting a certain points not from the origin, but from the other point is also possible to be happened. To overcome with this situation, the teacher can ask a clarification why they plot the point from the previous point suggest them to think back again about the vertical and the horizontal distance represented by the coordinate pair.

The discussion is focused on how to plot points on the Cartesian diagram. It should be noticed whether students plot points from the origin or not. To begin with, the teacher can ask a pair (if possible) who plot a point not from the origin to present their work. The other groups are asked to give remarks or questions. The teacher leads the discussion by asking a clarification why they plot a new point from the previous point. Afterward, students are reminded about the vertical and the horizontal distance represented by each coordinates. Here, they are expected to realize that a new point is plotted from the origin, not from the previous point. In the end, students in pairs are allowed to play the both games to practice their capability of plotting and locating any points on the Cartesian diagram by using the ordered pair.

CHAPTER V
RETROSPECTIVE ANALYSIS

In this chapter, the hypothesized learning that comprises the predictions of students' learning are compared with the actual learning as observed. During this process, called as the retrospective analysis, the HLT serves as a guideline for the researcher to focus on in the analysis (Bakker & van Eerde, 2013). The retrospective analysis of the preliminary teaching is described by providing the analysis of the pre-test, the comparison of the initial HLT to the actual learning, and the analysis of the post-test. After being analyzed, the initial HLT is reformulated into the revised HLT that are used as a guideline for the follow-up cycle (teaching experiment). In the teaching experiment, we analyze the classroom observation and the pretest, compare the revised HLT with the actual learning, and analyze the post-test.

5. 1. Retrospective Analysis of the Preliminary Teaching (First Cycle)

During the preliminary teaching, the initial HLT were tested to a small group of fifth graders: Darren, Basith, Salsabila, and Siti in PUSRI primary school Palembang. The researcher acted as the teacher and the students were divided into two groups. According to the teacher's recommendation, they were grouped based on its gender because they will feel secure to work with. Within this retrospective analysis, the initial HLT is compared to the actual learning process. It is conducted to adjust and revise the substance of the initial HLT. Then, the revised HLT is implemented in the next teaching experiment that involves the real classroom environment. The following are the retrospective analysis of the completely preliminary teaching.

5. 1. 1 Pretest

The pretest was conducted on Monday, February 16th 2015 in PUSRI primary school in Palembang. The students were asked to solve five problems individually for about 25 minutes. These problems were given to know and assess students' prior knowledge related to the coordinate system. The analysis of the pre-test was then resulted in the adjustment of the initial HLT to be appropriate with students' prior knowledge. Since the result of the pre-test is analyzed qualitatively, we do an interview aimed at revealing the students' way of thinking. The following are the important points revealed from the result of the pre-test that will be used to adjust the initial HLT.

❖ **Making a System that Involves Rows and Columns**

The first problem about the bus seat arrangement is performed to know whether students can make a system involving rows and columns to specify the location of a seat. Darren and Salsabila made their system by dividing the seat arrangement into two columns (east-west or right-left) and numbering the seat either horizontally or vertically (see figure 5.1). Meanwhile, Basith and Siti, merely numbered the seat horizontally starting from one.

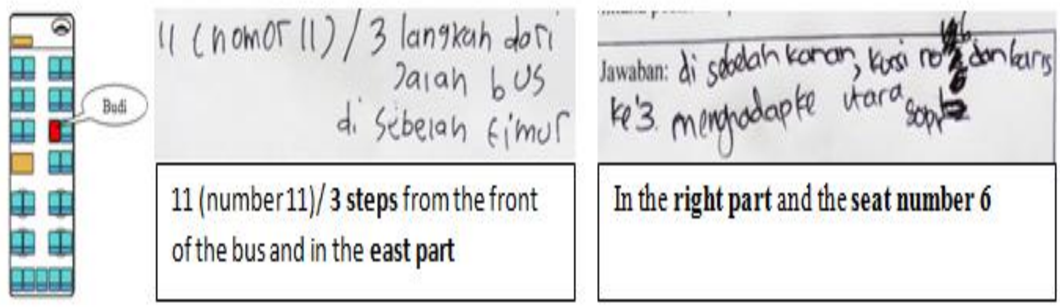


Figure 5.1 Darren's *(left)* and Salsabila's *(right)* answer of the first item of pre-test

Based on the analysis above, it showed that some students have not been able yet to make a system involving rows and columns. Consequently, they need to encounter an activity, which is about making a system involving rows and columns as embedded in the first learning activity about labeling blocks.

❖ **Understanding the Use of the Alphanumeric Grid System**

The alphanumeric grid system as embedded in the second problem is given to know students' ability in specifying the location of a pawn by using the chess notation. Darren, Salsabila, and Siti used a pair of a letter and a number to locate it (see figure 5.2). However, Basith merely used a number and ignored the letters at first (see figure 5.2). However, during the interview he corrected into a pair of a letter and a number.

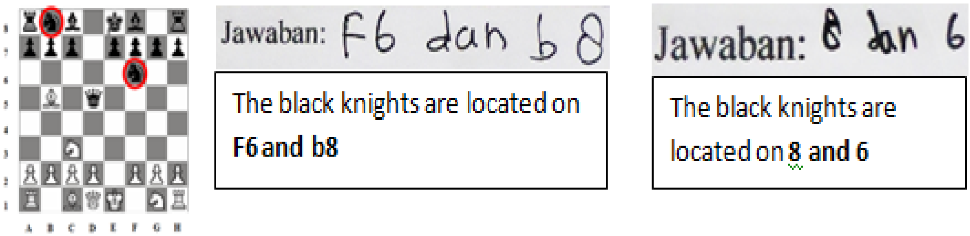


Figure 5.2 Darren's *(left)* and Basith's *(right)* answer of the second item of pre-test

If the alphanumeric grid system is provided, it can be sum up that students have been able to locate a certain object using a coordinate pair of a letter and a number. This finding reveals that students have a sufficient prior knowledge to do the first lesson, namely understanding the use of the alphanumeric system for labeling city blocks.

❖ Calculating the Distance between Two Points on the Number Line

It turns to be important to check students' ability in calculating the distance between two points on each vertical and horizontal number line as embedded in the third problem. To calculate the distance, all of them merely employed the strategy of counting one by one (see figure 5.3). They seemed to ignore the numbers given in the number line that actually can help them to determine the distance by subtracting strategy.

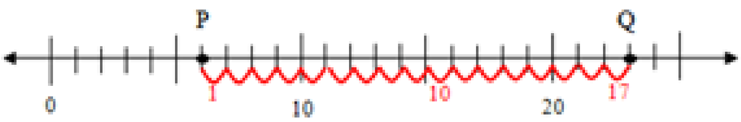


Figure 5.3 Calculating $\overline{PQ}$ using the strategy of counting one by one

It can be concluded that most of students were able to calculate the distance between two points by using the strategy of counting one by one. This result showed that students have a sufficient starting point to specify the location of an object on a plane using the idea of horizontal and vertical distance.

❖ Locating an Object on the Rectangular Plane

The forth problem was given to assess students' capability of specifying the location of an object on a rectangular plane. To precisely describe the location of a ship from a lighthouse on a sea map, it needs two parameters: the distance and the cardinal direction. Basith and Siti merely used one of the parameters, which lead to multiple possible locations. Meanwhile, Darren specified the location using both of them, but he misunderstood that the ship has two locations: 6 km to the north and 4 km to the east (see figure 5.4). Salsabila located it using a pair of coordinate “7,4” referring to 7 km to north and turn 4 km to west (see figure 5.4).

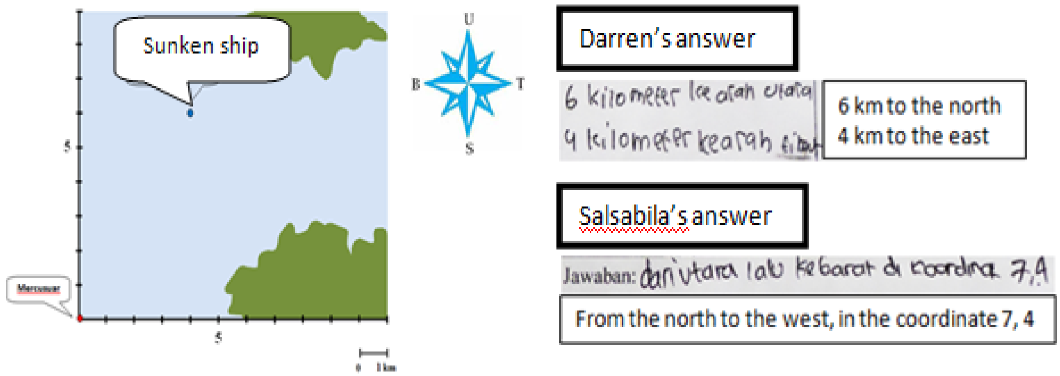


Figure 5.4 Darren's and Salsabila’s answer of the fourth item of the pre-test

Based on the analysis above, most of students still have difficulty to locate an object on a rectangular plane by using the distance and the cardinal direction. Consequently, they need a learning activity that can support them to be able to locate an object using the distance and the cardinal direction as embedded in the third learning activity.

❖ Locating and Plotting Points on the Positive Quadrant

In the formal task, students’ ability of locating a point on the positive quadrant of the Cartesian diagram will be assessed through the fifth problem. Similar to the forth problem, Darren located the point by using the distance and the cardinal direction, but in the wrong the location (see figure 5.5). Meanwhile, Siti was not able to locate it using the distance at all (see figure 5.5).

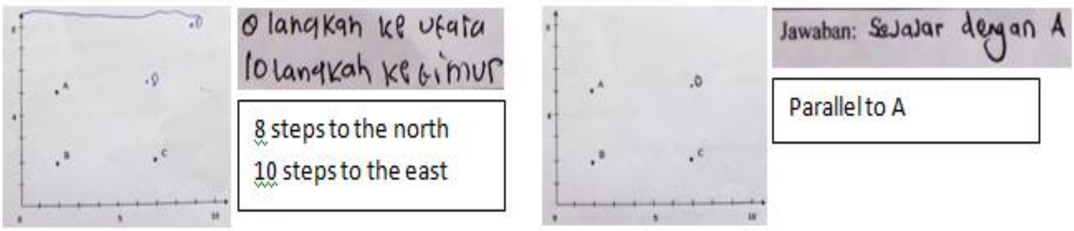


Figure 5.5 Darren's answer *(left)* and Siti’s answer *(right)* of the fifth problem

Surprisingly, Basith and Salsabila were able to locate it using a pair of coordinate by considering the first coordinate as the vertical distance (see figure 5.6).

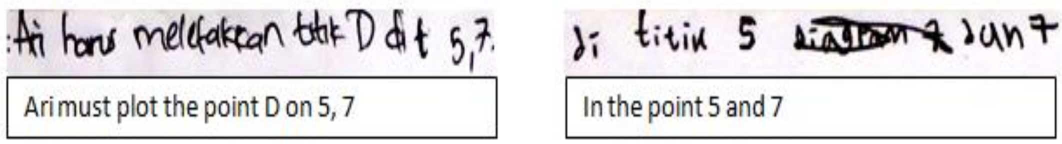


Figure 5.6 Salsabila's *(left)* and Basith’s *(right)* answer of the fifth item of pre-test

It can be concluded that some students were able to identify the coordinate of a point on the coordinate plane, while the others were not. However, none of them used the ordered pair (*x, y*) properly. It means that they need encounter a problem about which distance (the horizontal or the vertical distance) that should be considered as the first or the second coordinate of the ordered pair as embedded in the third learning activty.

The analysis of the pre-test was also resulted to the revision the pre-test items. Consider to our learning goal, it is also important to assess students’ ability in idizentifying the coordinates of a point that has “half” as one of its coordinate for the fifth problem. To know students’ familiarity of an ordered pairs (*x*, *y*), we also want ask students to plot an ordered pairs (*x, y*).

5. 1. 2 Activity 1: Labeling Blocks of City Blocks

The teacher started the lesson by telling a story about developing a new city in South Sumatera that will adopt the system of city blocks in Barcelona, Spain. Since this context is not familiar for Indonesian’s students, the teacher showed a video, an aerial photograph, and a street map of the city blocks. Afterwards, the teacher told the description of city blocks and asked students to make a system for labeling the blocks (see figure 5.7).

In the beginning of the project, the government do not have name for the streets in the Jakabaring City, instead they label the blocks. Make a system of labeling the blocks that can be used to determine the location of a certain object easily, precisely, and quickly!

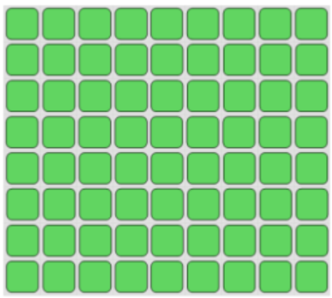


Figure 5.7 First task of making s system for labeling city blocks Jakabaring

The two groups made two different systems for labeling the blocks. The first group (Salsabila and Siti) perceived the city blocks as rows and columns, and labeled the blocks vertically with 2A – 2H, 3A – 3H, 4A – 4H, etc (see figure 5.8). They employed a coordinate pair of a number and a letter to label the blocks, but they did not any reason to start the label with number 2 when being clarified. On the other way, the second group (Basith and Darren) proposed a system that is not systematically based on rows and columns. For each four blocks forming a square, they labeled with A1 – A4, B1 – B4, etc (see figure 5.8).

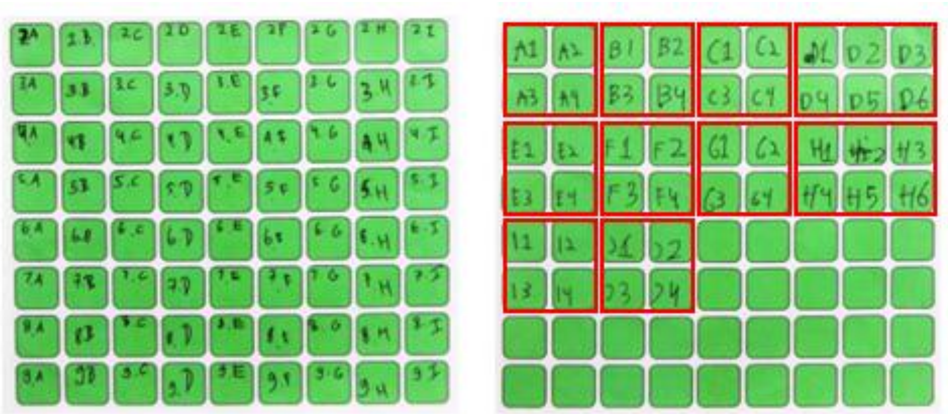


Figure 5.8 The Written Work of the First Group (*left*) and the Second Group (*right*)

The following fragment shows the reason why the second group made that kind of system, which can be related to their little misunderstanding about city blocks

1	Teacher	:	How did you make it (the system labeling the blocks)?
2	Students	:	The strategy is (label it with) A, B, C, D, and so on
3	Teacher	:	What do you mean by A, B, C?
4	Basith	:	The blocks
5	Teacher	:	Ok, you label it with A, B, C. But then, how is the rule for label it with A, B, C, and so on?
6	Basith	:	A have four numbers
7	Teacher	:	Why do you choose four numbers?
8	Students	:	Because it is similar to a square. And then, B is also a square, C is also a square, but D is made to be a rectangle
9	Teacher	:	Why do you make it looks like a rectangle?
10	Darren	:	Because there are three buildings (the remaining blocks)

Based on the fragment above, it can be identified that the second group miss-recognized one block as one building (line 3, 4, and 10). Therefore, the four blocks were perceived as four buildings and grouped them into one block that is looked like a square (line 5 - 8). If we traced back to the shown picture of the city blocks in the beginning of the lesson, the students might think that a block should be like a square. That is why they grouped the blocks into a square.

To sum up, the first group was able to make a system that involves rows and columns as we predicted in the HLT. The vertical columns was numbered 2 to 9, while the horizontal rows was labeled A – I. In a different way, the second group perceived the four blocks as four buildings, which is out of the prediction in the HLT. They grouped them into one block, labeled with the same letter, and numbered 1 to 4. Since there was a miss-perception about the context of city blocks, it would be nice to give a clearer illustration about the context for the next teaching experiment. It can be started with showing a video of city blocks in Barcelona in a more detail. Starting with a map of Spain zoomed in a city blocks in Barcelona, then is focused on monitoring one block with several buildings.

5. 1. 3 Activity 2: Chess Notation

The first lesson was continued with the chess notation related task. To begin with, the students were asked about their experience in playing chess and using the chess notation system. They initially confused with the term of the chess notation. However, after being explained, they tried to understand it. They knew that the letters A-H are listed along the left edge and the numbers 1 – 8 are along the bottom side. Based on this system, it was asked to identify the location of a pawn on the chessboard (see Chapter IV, Activity 2). All of the groups used a coordinate pair of a number and a letter to locate a certain pawns (see figure 5.9).

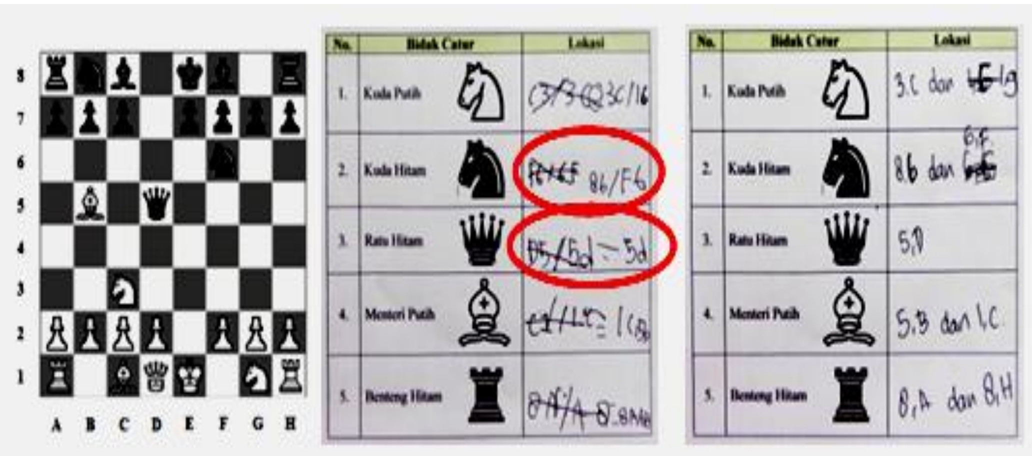


Figure 5.9 Groups' written works on locating pawns using the chess notation

Based on the written work above, one group initially was not sure whether the notations like 6F or F6 specify the same location or not (see figure 5.9). This finding led to a nice discussion as shown in the following fragment.

1	Teacher	:	6F and F6 are specified the same location or not?
2	Students	:	No, they were different
3	Teacher	:	Why do they different?
4	Students	:	*(silence)*
5	Teacher	:	If White Queen is moved to 6F, where the location is?
6	Students	:	Here *(point the square of 6F)*
7	Teacher	:	Now, if White Queen is moved to F6, where it is?
8	Students	:	Here *(point the same square with 6F)*
9	Teacher	:	So, 6F and F6 pinpoint the same location or not?
10	Students	:	The same (location)

All of the students initially argued that the notations 6F and F6 were different, but they could not explain the reason (line 1 - 4). However, after being asked to pinpoint the location shown by those notations, they came with the same square (line 5 - 8). Therefore, it can be concluded that the notations like 6F and F6 specify the same location on the chessboard (line 9 - 10). Relate to the city blocks problem, they are invited to reason whether the chess notation system can be applied for labeling the blocks. All of them claimed that it was applicable by showing how the system works and pinpointing a block based on that system.

To sum up, we can say that all of the groups were able to use the chess notation system to specify the location of pawns on the chessboard. Here, they employed a coordinate pair of a number and a letter to locate the pawns as we hypothesized in the HLT. Moreover, they were also able to apply the chess notation system for labeling the blocks as we predicted in the HLT. In regard to this result, the adjustment related to the content of the task is not needed.

5. 1. 4 Activity 3: *Taxicab* Routes and Distance

The activity was started with asking students to determine the label of a certain block using the alphanumeric grid system. They labeled the block with a coordinate pair of a number and a letter, such as block 4B, 6E, and 3G. This was a preliminary task to locate an object (Bhayangkara Hospital) placed on the same blocks with the other object (Sriwijaya Hospital). The first group used the distance and the cardinal direction to locate it (see figure 5.10). The second group described that it is at the northeast of the block 4E. To locate the object precisely, they argued that use the label of the block (block 4E) was not enough because there were two objects in the same block.

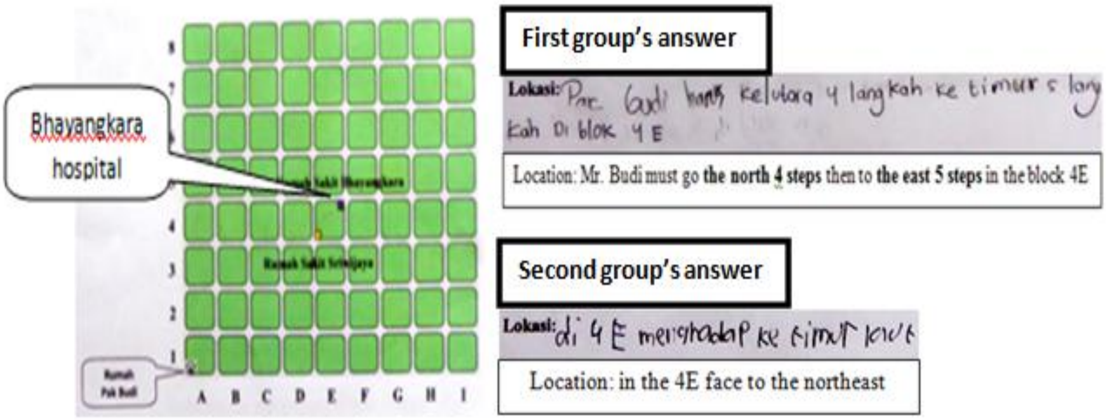


Figure 5.10 Students' written work of locating objects on the city blocks precisely

The activity was continued with asking each group to do a set of tasks related to the *taxicab.* distance that were focused on two main goals (see Chapter IV, Activity 3). First, any shortest routes between two points on the rectangular grid cover the same *taxicab* distance. Second, finding the taxicab distance leads to the idea of the horizontal and the vertical distance.

To achieve the first goal, the students were asked to make three possible *taxicab* routes between two objects, Mr. Budi's house and Ampera Resto (see Chapter IV, Activity 3). Then, they are required to conclude about the distance covered by those three routes. After making the routes, the first group concluded that their three routes have the different taxicab distance.

1	Teacher	:	So, what is your conclusion of the second question?
2	Students	:	The shortest (*taxicab*) distance is shown by the first (taxicab) route
3	Teacher	:	Why?
4	Students	:	Because there is no *(many)* turns

Based on the fragment, they argued the routes with fewer turns are shorter than the routes with more turns (line 2 - 4). React to this fact, the teacher suggested them to recalculate the distance for each routes and compare the results. On the other hand, the second group ended with the conclusion that their three routes covered the same taxicab distance (see figure 5.11).

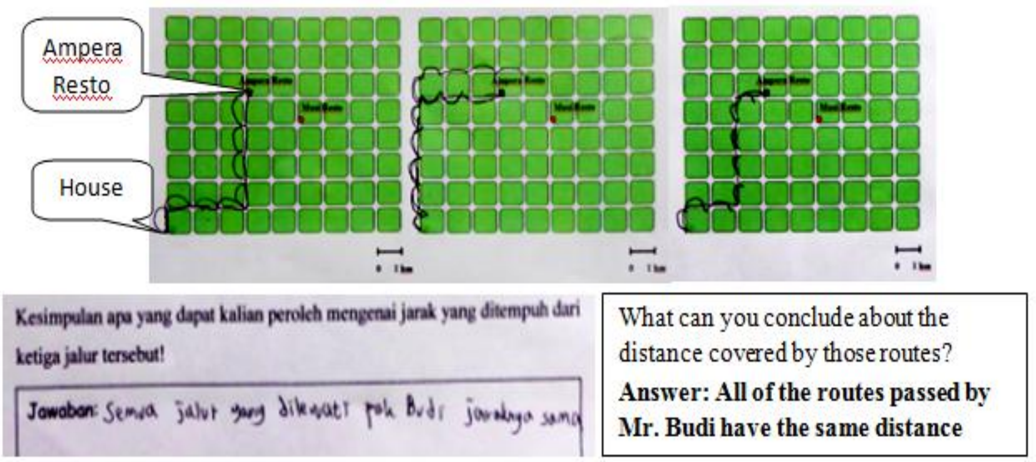


Figure 5.11 Second group's written work of making three taxicab routes and conclusion about the distance covered by those routes

In regards to the second goal, the task was continued with determining two different *taxicab* routes that merely involves one turn. None of the groups had difficulty to make the routes and determine the horizontal and the vertical distance for each route. Under the teacher's guidance, it was likely unproblematic for them to determine the *taxicab* distance between the house and a certain place by adding its horizontal and vertical distance. For each horizontal and vertical distance, were guided to label the grid lines (as the representation of the street in the city blocks) with whole numbers corresponded to its distance (see figure 5.12).

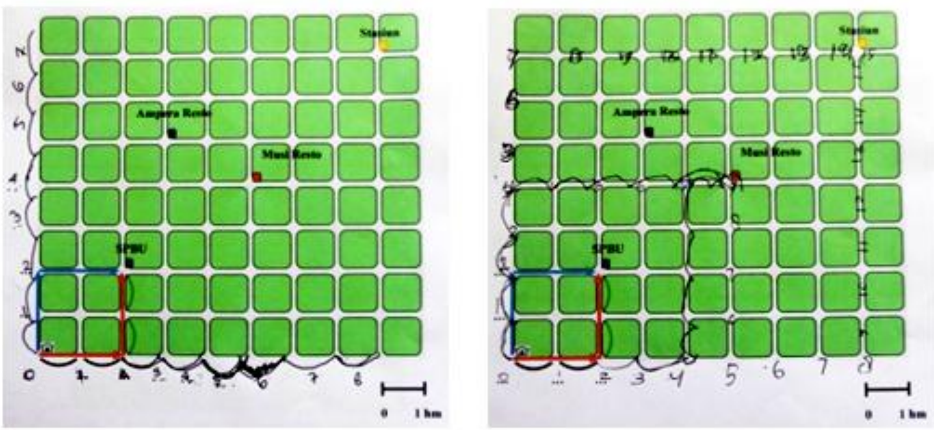


Figure 5.12 The written work on determining a system for *taxicab* distance in city blocks

Some students mistakenly notated the numbers on the side of the blocks, not on the street (see figure 5.12). Therefore, the teacher guided them to discuss whether the numbers should be label the street or the side of the block. The discussion ended up with a result that the numbers should be notated the street. Based on the students' written work (see figure 5.12), it can be noticed that they were eventually able to reinvent a system that looks like the positive quadrant of the Cartesian system under the guidance of the teacher.

According to the analysis above, we can say that all of the groups located a certain object on the city blocks using the label of the blocks, the distance, and/ or the cardinal direction as we expected in the HLT. They were also capable to reason that three different *taxicab* routes with the same starting and ending point covered the same *taxicab* as we predicted in the HLT. The use of the horizontal and the vertical distance embedded in the *taxicab* distance led them to the reinvention of a system looked like the positive quadrant of the Cartesian system.

Some adjustments related to the content of the tasks are conducted for the next teaching experiment by reflecting on our findings. The students' confusion about the use of hectometer as the unit distance causes us to replace it with the word of blocks as the unit. It is taken into account since one of them used "steps" as the unit distance with no troubles. Reducing the number of tasks related to the taxicab distance is also performed. Students in groups will be merely required to determine one *taxicab* route from the house to Ampera Resto including its *taxicab* distance. Hereafter, three groups with different routes will present their work and all of students need to conclude about the distance covered by those three routes.

5. 1. 5 Activity 4: How to Use an Ordered pair?

It was started by reminding the students about the location Bhayangkara hospital, namely the northeast of block 4E. By using another system, they were challenged to specify the location of Bahyangkara and Sriwijaya hospital. Even both groups could locate them precisely; miscalculation was happened in the middle of the work. For instance, the *taxicab* routes with two turns led the second group to the miscalculation of the vertical distance. It was said that Sriwijaya is at 5 hm east and 2 hm north. When being asked whether 1 hm of the first turn face to the east or north, they realized the mistake and corrected into 4 hm east and 3 hm north (see figure 5.13).

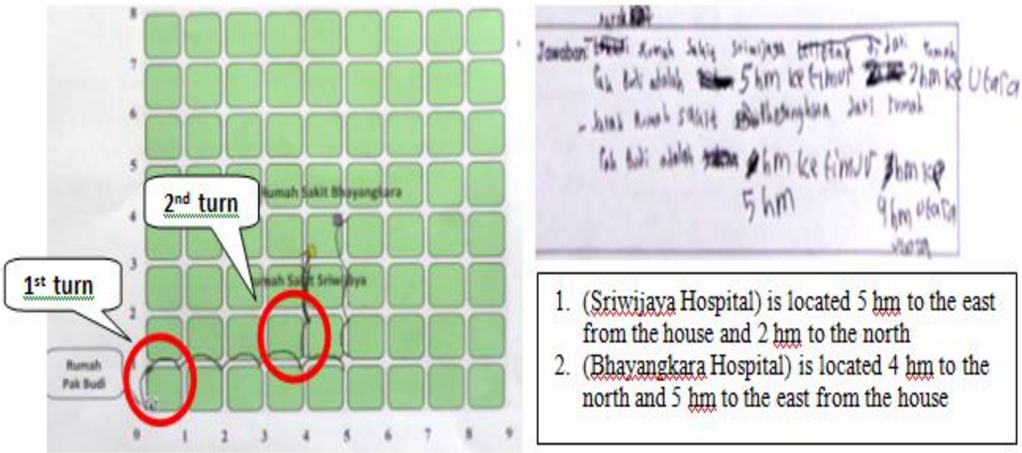


Figure 5.13 Second group's work on locating object using a system involving the distance

That was a preliminary task before students began to understand how an ordered pairs (*x*,*y*) works to specify the coordinate of a point on the rectangular plane. They were challenged to plot objects with the coordinates (1,1), (7,1), (7,5), and (1,5) on the street map of city blocks and to determine which figure that is formed by those objects (see Chapter IV, Activity 4). This task asked them to reason about which distance (the horizontal or the vertical distance) that should be considered as *x*-coordinate and *y*-coordinate of the ordered pairs (*x, y*).

Even though the first group could plot the four coordinates, they confused to determine which figure that is formed by those coordinates. At that time, Salsabila rotated the paper and claimed that the both figures are actually the same because the given four points are plotted (see figure 5.14). On the other way, the second group initially claimed that the answer should be figure B because all of the given points were plotted as shown in the fragment below.

Teacher : Which figure does illustrate the danger zone?
Darren : Figure B
Teacher : Why?
Darren : Because in here *(look at figure B)* you can find (1, 1), (1, 5), (7, 1), and (7, 5)
Teacher : How about this *(look at figure A)*? You also can find…..
Basith : (1, 1), (7, 1), (7, 5), and (1, 5)
Teacher : So, which figure is the answer?
Basith : Both of the figures (figure A and figure B)
Teacher : Why?
Darren : Because for figure A, we consider the horizontal coordinate as the first coordinate. And for figure B, we consider the vertical distance first

The second group appeared to realize that the four coordinates were also plotted as shown in figure A (line 6 - 7). It led them to the inference that the figure A and B are the answer because it depends on how we consider the *x*- and *y*-coordinate of ordered pairs (*x, y*) as the horizontal or the vertical distance (line 8 - 13). Since the *x*-coordinate could represent the horizontal or the vertical distance, during the class discussion, they were asked to make an agreement about it. They agreed that the horizontal distance is considered as the *x*-coordinate.

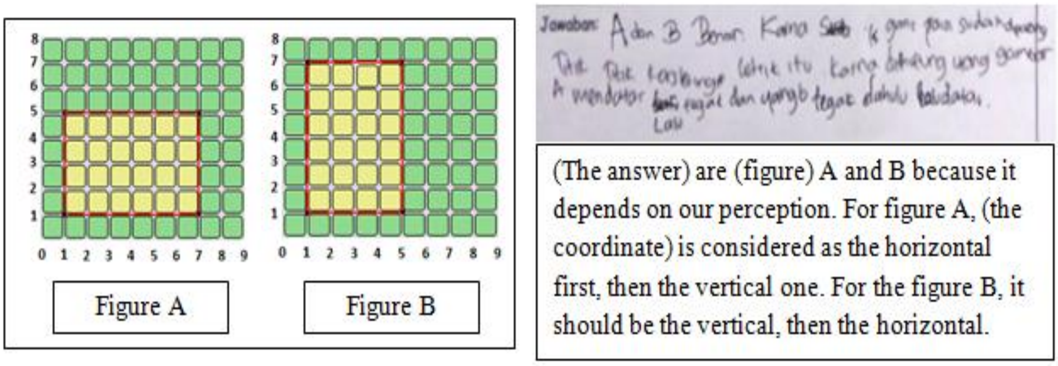


Figure 5.14 First group's written work on plotting four coordinates on the street map

Based on the result, we can say that all of the groups were able to locate an object on a plane using the horizontal and the vertical distance as we predicted in the HLT. The agreement about how the ordered pairs (*x, y*) was also made in the end of the lesson. Here, *x*-coordinate and *y*-coordinate are considered as the horizontal and the vertical distance respectively as we conjectured in the HLT. Providing those two figures sometimes made the students get lost in their argument. As such, for the next teaching, students in groups will be merely asked plot four coordinates. The groups who plot the coordinates as (*x, y*) and (*y, x*) will present their work and all of students will be asked to reason about the correct answer.

5. 1. 6 Activity 5: Sunken Ship

The teacher started the lesson by reminding the students about the agreement of how to use the ordered pairs (*x,y*) to tell the coordinate. Armed with this knowledge, the students located a sunken ship on the sea map (see Chapter IV, Activity 5) before working with the formal task of positive quadrant. In this task, none of them drew the coordinate grid or notated whole numbers on the axes as the representation of the distance. The first group located the ship using the ordered pairs (*x*, *y*), meanwhile the second groups was (*y*, *x*). However, after being reminded, the second group corrected it into (*x*, *y*) (see figure 5.15).

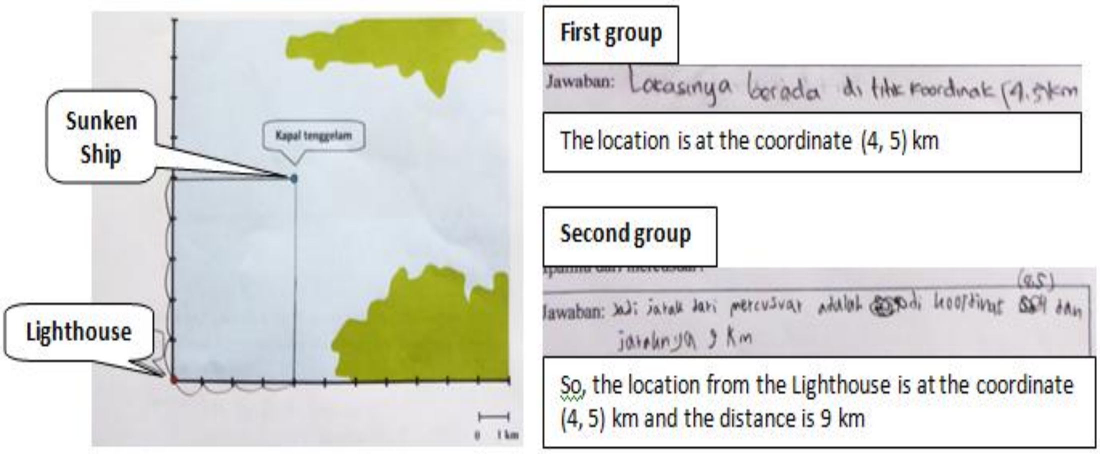


Figure 5.15 The groups' written work on locating a sunken ship using the ordered pairs

The task was continued with determining the coordinate of the rescue ships placed along the *x*-axis or *y*-axis. They also worked on the rescue ships that one of its coordinate has value “half”. To begin with, all of the groups tried to notate the whole numbers along the horizontal and the vertical axis, which are corresponded with the distance (see figure 5.16 and 5.17).

The first group tried to find both the horizontal and the vertical distance first to help them specify the coordinate of each object. With the similar strategy, they were also able to identify that a point, located along the x-axis or y-axis, has zero distance (either horizontal or vertical). For example, team SAR B is at (0, 7) since the horizontal and the vertical distance is 0 hm and 4 hm respectively (see figure 5.16). In addition, notating “half” coordinate using fraction seemed help them to easily specify the coordinates (see figure 5.16).

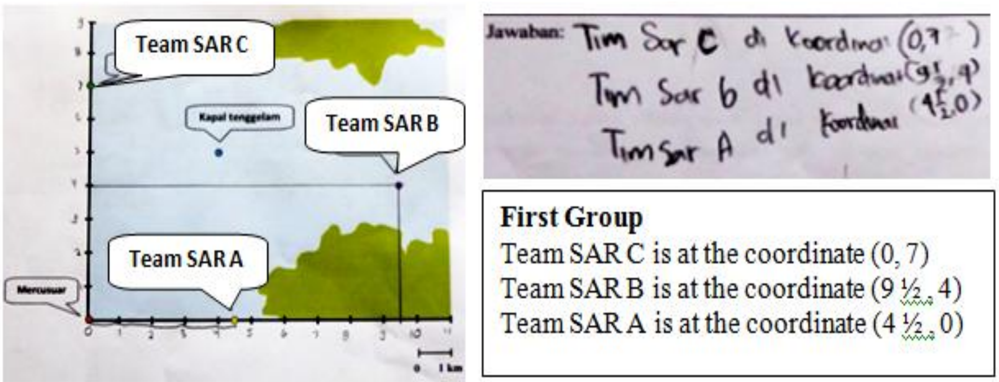


Figure 5.16 First groups' work on locating the rescue teams using the ordered pairs

The condition went slightly different for the second group when they had difficulty to determine the coordinate of objects placed along the axis and/or has “half” as one of its coordinate as shown in the following fragment.

1	Teacher	:	How about (the coordinate) of the team SAR A?
2	Student	:	4,5
3	Teacher	:	It is for the horizontal (distance), isn’t it?
4	Student	:	Yes
5	Teacher	:	How about the vertical (distance)?
6	Student	:	Zero
7	Teacher	:	And then, how did you write down (the coordinate)?
8	Student	:	(0, 45)
9	Teacher	:	(The agreement) is the horizontal or the vertical first?
10	Student	:	The horizontal, that is (45, 0)
11	Teacher	:	45 or 4,5?
12	Student	:	4,5 so that (4,5 , 0)

According to the fragment above, the students only considered one parameter (the horizontal distance) to locate team SAR A (line 1-4). However, after being asked about the vertical distance, they knew that the distance is equal to zero because it is located along the *x*-axis (line 5-6). Under the guidance of the teacher, in addition, they could express the coordinate using the ordered pair (*x, y*) (line 12).

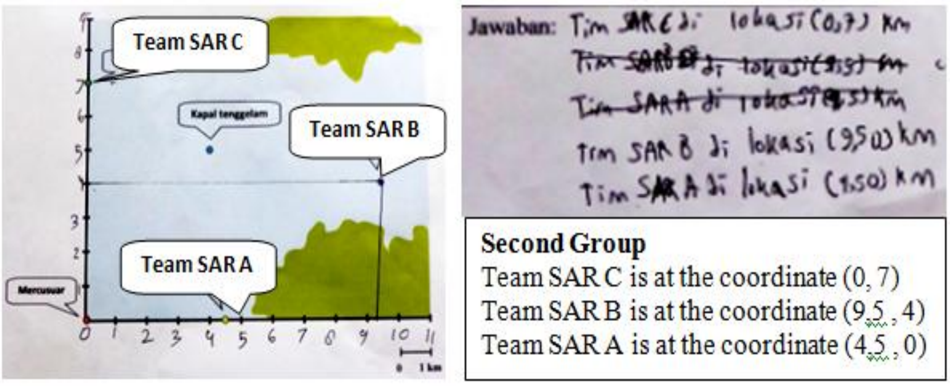


Figure 5.17 Second groups' work on locating the rescue teams using the ordered pairs

For the objects that have “half” as one of its coordinate, the first group wrote it using fractions (see figure 5.17), while the second group used decimal (see figure 5.17). Both of the answers were correct, however, the teacher suggested the second group to use “point” rather than “comma” to express the decimal.

In conclusion, both of the groups were able to determine the coordinate of objects on the map looked like the positive quadrant. For the objects placed along the x-axis or the y-axis, they knew that one of its coordinate should be equal to zero. They were also able to locate an object that has "half" as one of its coordinate using the ordered pairs (x, y). They expressed the coordinate either using fractions or decimal. For next teaching, there will be no significant adjustment for this activity. We merely plan to provide the sea map with the coordinate grid. It was related to the result of the next activity in which students have difficulty to plot points on without the provided grids.

5. 1. 7 Activity 6: Locating and Plotting Points on the Positive Quadrant

The teacher started the lesson by telling about the games: "guess the shape" and "guess the last point". For the first task of the first game, the students were required to plot four points, namely A (2, 2); B (7, 2), C (9, 5); D (4, 5) on the Cartesian diagram. If those points formed a quadrilateral ABCD, they needed to guess the quadrilateral figure. Even all of the groups were able to determine the figure correctly, the second group did not plot the points accurately (see figure 4.18). Similarly, for the second task, the first group did not plot the four points, K (2, 0); L ($6\frac{1}{2}$, 0); M (2, $4\frac{1}{2}$); N ($6\frac{1}{2}$, $4\frac{1}{2}$), accurately (see figure 5.18).

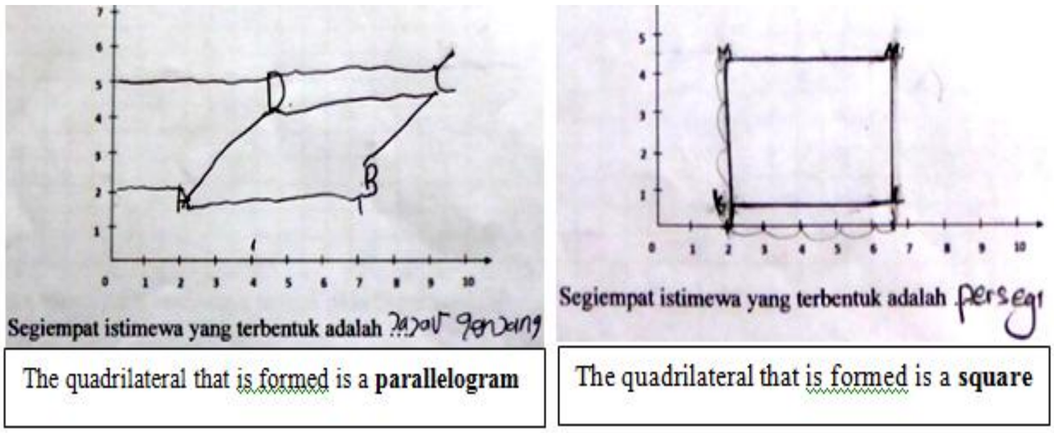


Figure 5.18 First (right) and second group's (left) written work on "guess the shape"

The location of the points that students plotted are less accurate since we did not provide a ready-made grid in the task (see figure 5.18). Consequently, the students constructed their own grid using a ruler or not at all (see figure 5.18). Apart from that case, however, students are able to locate those points in the form of the ordered pairs (*x*, *y*). The *x*-coordinate and the *y*-coordinate are considered as the horizontal and the vertical distance respectively.

For the second game, the students were challenged to determine the forth ordered pairs if given three points that can be formed into a certain special quadrilateral. For the first task, it was given three points, namely P (4, 0); Q (2,3) and R (4,6). From those points, it can be made a rhombus PQRS if the location of point S can be determined. Similarly, for the second task, there were three points, namely K (0, 1); L ($5\frac{1}{2}$, 4); and M ($1\frac{1}{2}$, 4). Those three points are used to make an isosceles trapezoid, but we need to determine the location of point N.

Differ from the first game, to determine the forth point, the students need to make their own Cartesian diagram on the grid paper. Based on the students' written work, all of them seemed not having difficulty to make the Cartesian diagram by themselves (see figure 5.19). In addition, they were also able to plot those given points on the diagram accurately since the coordinate grids were embedded (see figure 5.19). For the last, the location of the questioned-points was also specified using the ordered pairs (*x*, *y*) correctly (see figure 5.19).

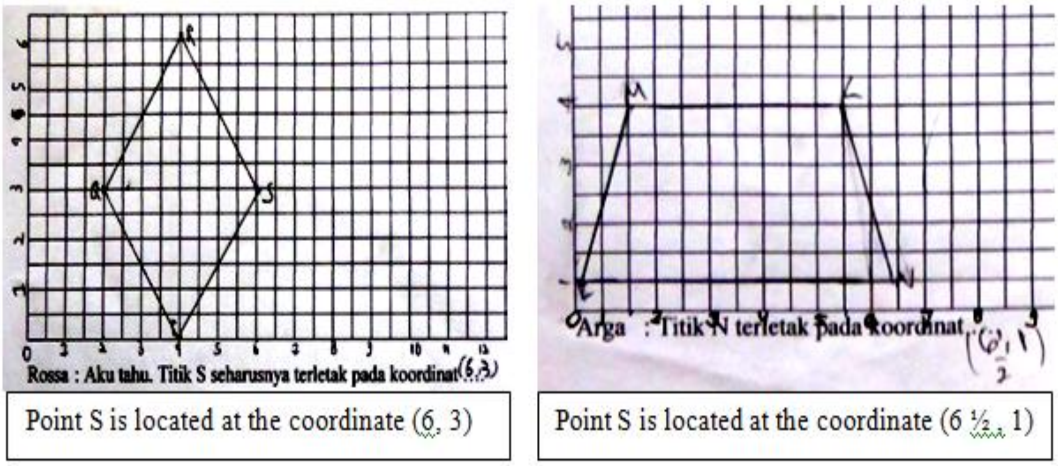


Figure 5.19 First group's (right) and second group's (left) work on "guess the shape"

Based on the analysis, it can be concluded that all of the groups were able to plot any points on the first quadrant of the Cartesian system as we predicted in HLT. They were also able to specify the coordinate of any points on the positive quadrant using the ordered pairs (*x*, *y*) as we hypothesized in HLT. When plotting the points, however, the second group seemed have a trouble to construct the coordinate grids by themselves. Therefore, in the next teaching, we plan to provide the coordinate grids for the given coordinate tasks.

5. 1. 8 Posttest

The post-test was conducted on Friday, 27th February 2015 in Pusri Primary School Palembang. The students were asked to solve 5 problems individually for approximately 20 minutes in total. The result of the post-test analysis is employed to assess students' acquisition of the knowledge related to the positive quadrant of the Cartesian coordinate after they joined a set of learning activities. In addition, the result is also used to ensure whether the items are understandable or not. Therefore, we can make a necessary revision before we have them on the next teaching experiment. The following are the important points revealed from the result of the pre-test that will be used to assess the students' acquisition.

➢ **Using an Alphanumeric Grid System**

The first item was about specifying the location of Arya's seat in the seat map of a cinema using the alphanumeric grid system (see figure 4.20). However, we do not provide complete numbers 1 – 5 for the vertical rows and letters A through J for the horizontal columns.

Based on the results, none of them located the seat using a coordinate pair of a number and a letter. Similar to the result of the pre-test, Salsabila, Basith, and Darren numbered the seat horizontally either from the upper left or right (see figure 5.20). However, Siti position the seat using the distance and the cardinal direction (see figure 5.20). Even being interviewed, they did not aware with the system and went on with their answers.

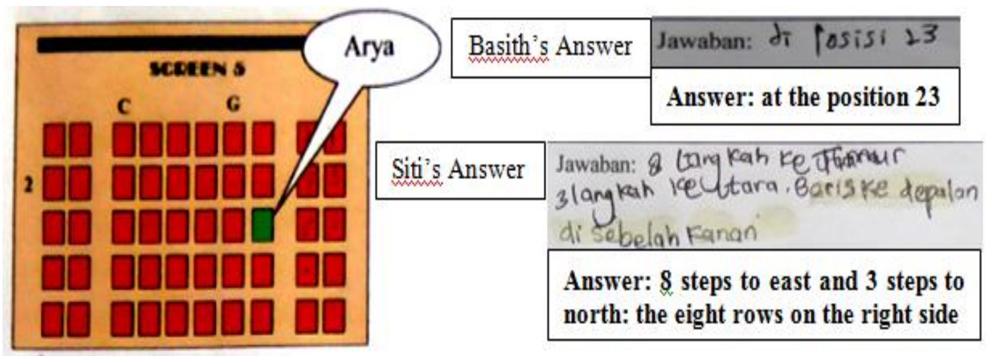


Figure 5.20 The students' work on locating the seat in the first item of post-test

It can be concluded that none of the students used a pair coordinate of a number and a letter to locate an object in the alphanumeric grid system. It was because the complete numbers (1 - 5) and letters (A - J) are not notated on the system. Thus, for the next cycle, we will notate the complete numbers and letters in the alphanumeric system.

- **Understanding Chess Notation System**

As the instance of the alphanumeric system, students' ability to locate a pawn on the using the chess notation was assessed through the second item. Basith and Siti used the chess notation to locate the pawns (see figure 4.21).On the other way, Salsabila and Darren numbered both the columns and the rows from 0 to 8, which is looked like the positive quadrant of the Cartesian system (see figure 5.21). Therefore, they located a pawn using a pair coordinate of two numbers. When being interviewed, they completely forgot about the chess notation system and went on with their system.

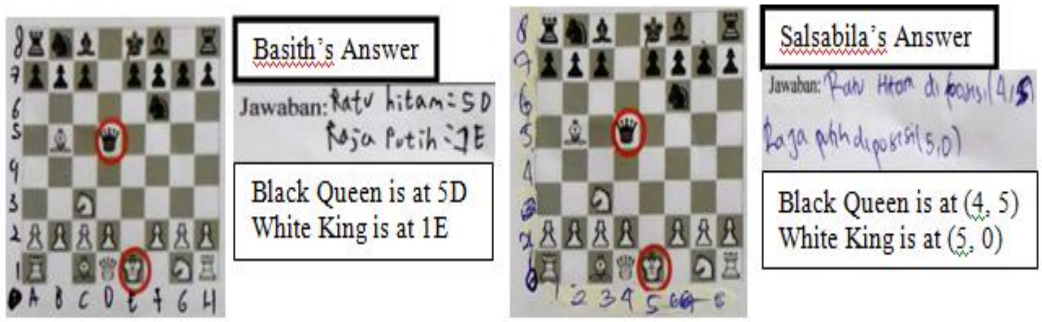


Figure 5.21 The students' work on locating pawns in the second item of post-test

We can say that to locate a pawn two of the students used the chess notation, while the others employed a system like the positive quadrant. It means that they had difficulty to differentiate the alphanumeric grid system with the Cartesian system (positive quadrant). These findings revealed that we needs emphasize that the chess notation system and the Cartesian system are two different systems.

- **Locating an Object a the Rectangular Plane**

Similar to the pre-test, this third item was given to assess students' capability of locating an object on a rectangular plane. To specify the location of a ship from a lighthouse on a sea map precisely, most of the students used two parameters: the distance and the cardinal direction (see figure 5.22). However, one of the students located it using a coordinate pair of two numbers (x, y) (see figure 5.22).

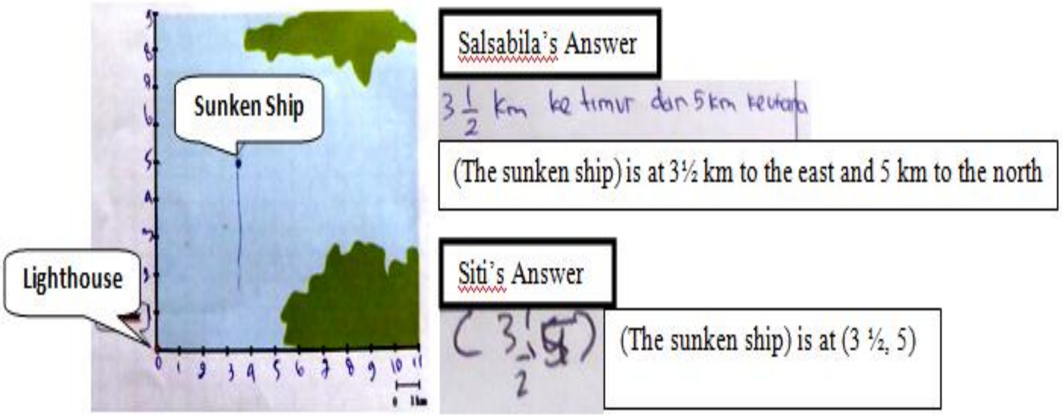


Figure 5.22 The students' work on locating the ship in the third item of post-test

It can be sum up that all of the students were able to specify the location of an object on the rectangular precisely either using two parameters (the distance and the cardinal) or ordered pairs (x, y). It was different with the result of the pre-test in which some of them located the object merely using one parameter, the distance of the cardinal direction. In regards to these findings, we will not adjust the content of this item for the next cycle.

- **Plotting Points on the First Quadrant of the Cartesian System**

Students' ability of plotting points on the Cartesian diagram is one of important aspects in learning coordinate system. In this item, the students were asked to plot four ordered pairs {A(0, 2); B(4½, 2); C(6½, 5); D(2, 5)} and determine the formed quadrilateral figure. All of the students were able to plot the points, but some of them plot it in less precise way because they need to construct the coordinate grid by themselves (see figure 5.23).

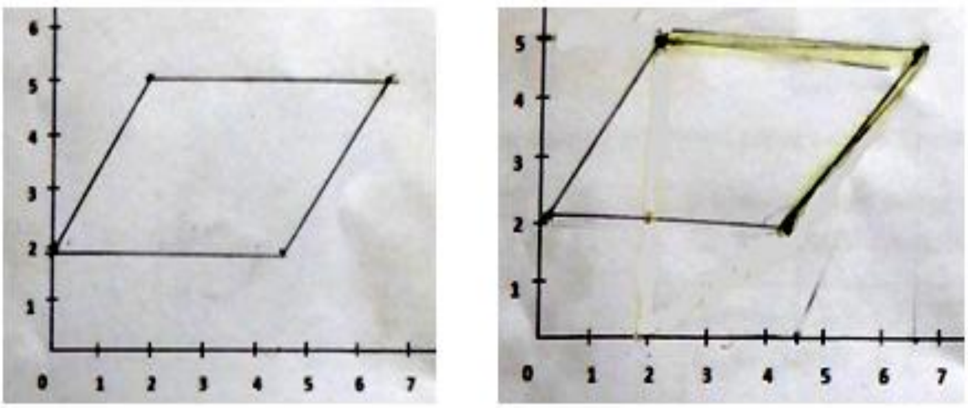
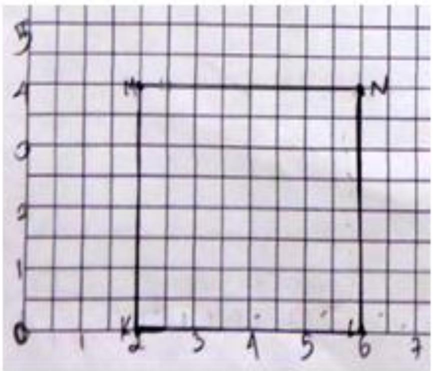


Figure 5.23 Students' work on plotting ordered pairs in the fourth item of post-test

Throughout the analysis it can be concluded that all of the students had difficulty to plot ordered pairs in precise way if the coordinate grids are not provided in the task. However, they were able to do that since they can determine the quadrilateral figure formed by those plotted points. Consequently, for the next cycle, it might be better to provide the coordinate grids in the given task/ item.

➢ **Plotting and Locating Points on the Positive Quadrant**

Two important aspects, locating and plotting points on Cartesian diagram, are assessed through the fifth item. In the task, it is given three points {K (2, 0); L (6, 0); M (2, 4)}. If those points are plotted, then it can form square KLMN by determining the coordinate of point S. Differ from the fourth item, to determine the coordinate of point S, all of the students made Cartesian diagram on the grid paper. Since the coordinate grids are provided in the task, all of them were able to plot the three ordered pairs precisely on and determine the coordinate of point S at (6,4) (see figure 5.24).



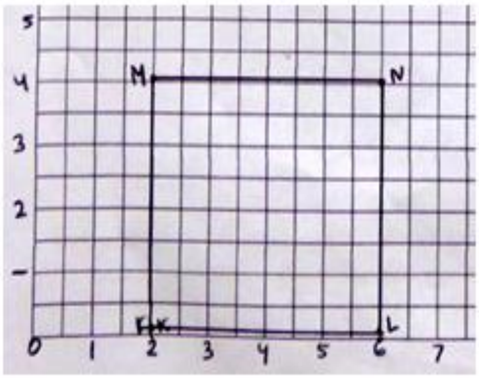


Figure 5.24 Locating and plotting points in the fifth item of post-test

It is revealed from the analysis above that all of the students were able to plot and locate points in the form of ordered pairs (*x*, *y*) on the positive quadrant of the Cartesian coordinate system. This result can be related to the task itself in which the coordinate grid were simultaneously embedded in the Cartesian diagram made by the students. In regards to this finding, we not adjust the content of this item for the next cycle.

5. 1. 9 Conclusion of the Preliminary Teaching Experiment (Cycle 1)

Based on the result of the retrospective analysis of cycle 1, it can be concluded that set of instructional activity helps students to understand the Cartesian coordinate system focused on the positive quadrant. In relate to the alphanumeric grid system, some of the students still cannot differentiate it with the Cartesian coordinate system. Here, they specify the location of a certain square of the grid system using the Cartesian coordinate that leads to imprecise location. This situation implies that the students need a clarification that the alphanumeric grid system as they dealt with in the first lesson has completely different use with the Cartesian coordinate. This finding can be a consideration for the next teaching experiment such that those two concepts are not overlapping.

Social norms become a big issue during this preliminary teaching experiment. Some of the students initially seemed less talk active such that it becomes trouble for the teacher to explore students' way of thinking. It also happened when they were asked to present their answer. Regardless this fact, however, they were very cooperative when being asked to listen their fellow students' answer explanation or opinion. After conducting several lessons, however, they began to be brave enough to ask question or give remarks in a condition that the teacher ask for it many times to them.

5. 2. Overview of Classroom Condition

Before conducting the next teaching experiment (cycle 2), we conduct a classroom observation with the participant is 30 students of the fifth grade in SD Pusri Palembang. About the seat arrangement, it was possible for the researcher to arrange it into groups consisting 4 – 5 students for each group. Since there are 30 students, it means that they can grouped into 7 groups with mixed composition of boys and girls.

Based on the observation, we find out that the students participant of cycle 2 are active students. They seemed familiar with conducting group work or class presentation. Most of them are brave enough to ask questions either to the teacher or the other students. However, there are also students who are passive and quiet.

About the way of teaching, the portion of the teacher's explanation and letting students to do task in groups or individual is in balance. However, the teacher still depends on the textbook when giving a certain mathematical problem to the students. About the socio-mathematical norms, the teacher seemed less provoke the students to propose the different of sophisticated solutions. Concerning this fact, we determined to improve those norms through intensive discussion with the teacher before conducting the learning design in the classroom.

5. 3. Overview of the Teacher

The teacher who participated during the teaching experiment is a bachelor majoring mathematics education. She has experience to be primary school teacher for about three years. She had taught the topic of the Cartesian coordinate system once time. Even this topic is considered as an easy topic, she declared that the students often make mistakes when being asked to specify the coordinate of a point using ordered pair. They tend to reverse the x- and y-coordinate and plot a new point not from the origin. Indeed, the students had difficulty to work with the coordinate involving the negative numbers since it was not experientially real for them. These findings become a reference for us to adjust our educational materials such that it helps student be able to work with the Cartesian coordinate tasks.

About *Pendidikan Matematika Realistik Indonesia* approach (Indonesian version of RME), the teacher have no experience to implement this approach in her class even she knew about it a little bit. In addition, she has not attended any conference or workshop about PMRI approach yet. It implies that she had less knowledge or experience related to PMRI. This fact becomes a challenge for us to introduce the superiority of PMRI approach through the learning of the Cartesian coordinate.

5. 4. Teaching Experiment (second cycle)

The revised HLT was tested out to a real classroom environment. Thirty students of 5^{the} grade from Pusri Primary School participated in five lessons and they were divided into seven groups. The researcher acted as the observer, while the regular teacher carried out the learning design that has been revised. Since it was truthfully complicated to analyze the whole students' thinking and reasoning, in this part, we merely focus the analysis on one group. However, the analysis of the students' thinking from the other groups is also conducted as long as it is important and interesting to be discussed. The focus group was deliberately chosen based on the result of the pre-test and the teacher's recommendation. The result of the analysis was then used to answer the research question, derive conclusions, and revise the existing HLT as described below.

5. 4. 1 Pre-Test

The pretest was conducted on Thursday, March 12th 2015 in Pusri primary school Palembang. The students were asked to solve five problems individually for about 25 minutes. These problems were given to know students' prior knowledge related to the coordinate system. The analysis of the pre-test result was employed to adjust the instructional activities that will be implemented. After the pre-test, the students' interview was conducted to clarify their strategy of solving the problems, which simultaneously reveal their way of thinking. The following are the important points revealed from the pre-test used to adjust the revised HLT.

✓ Making a System that Involves Rows and Columns

The bus seat arrangement problem is given to know how the students make a system to locate a seat involving rows and/or columns. Most of them numbered the seat horizontally starting from one (see figure 5.25). However, 5 out of 30 students divided the seta arrangement into two columns (right-left) and numbered the seat (see figure 5.25). However, one student located the seat using a coordinate pair of two numbers (see figure 5.25).

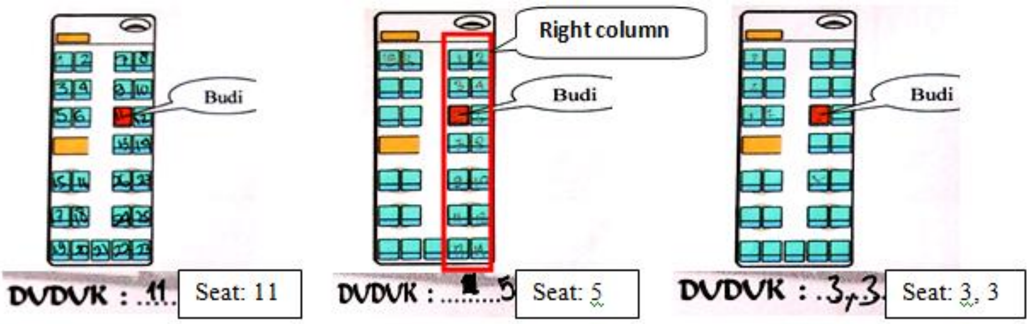


Figure 5.25 The students' work on locating Budi's seat in the first item of pre-test

The analysis above shows that most of the students made the system for the bus seat merely based on its rows, such as numbering it horizontally or vertically.

However, some of them made the system based on two parameters: rows and columns. These findings reveal that the students need to encounter an activity about making a system involving rows and columns as embedded in the first activity about labeling system for city blocks.

- ✓ **Understanding the Use of the Alphanumeric Grid System**

The second item is given to know students' ability in specifying the location of a pawn by using the chess notation as the representation of the alphanumeric grid system. More than a half students used a pair of a letter and a number to locate it (see figure 5.26). Yet, 8 out of 30 students merely considered the horizontal row or the vertical column, which means they used a number or a letter to specify the location of the pawns. (see figure 5.26).

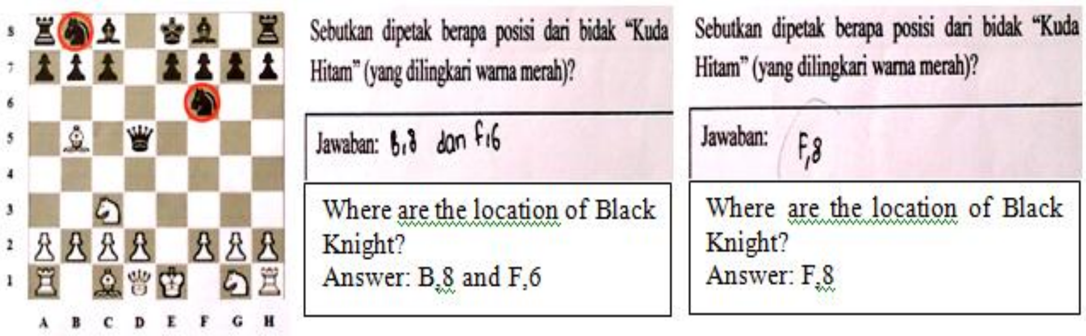


Figure 5.26 The students' work on locating pawns in the second item of pre-test

If the alphanumeric grid system is provided, it can be sum up that students have been able to locate a certain object using a coordinate pair of a letter and a number. This finding reveals that students have a sufficient prior knowledge to deal with the first lesson, namely understanding the use of the alphanumeric grid system for labeling city blocks.

- ✓ **Calculating the Distance between Two Points on the Number Line**

Before learning the coordinate system, it turns to be important to check students' ability in calculating the distance between two points on (vertical or horizontal) number line. To calculate the distance, 10 out of 30 students used a ruler as the tool such that the unit distance that they got is in centimeter. It means that the given number line was not utilized as tool to measure the distance. Almost a half of the students, however, determined the distance by counting one by one. They seemed ignore the numbers on the number line to help them determine the distance by subtracting strategy.

Even though some of students have not able yet to calculate the distance using the given number line, most of them were able to do it using the strategy of counting one by one. This result showed that students have a sufficient starting point to specify the location of an object on a plane using the idea of horizontal and vertical distance.

✓ **Locating an Object on the Rectangular Plane**

The forth problem was given to assess students' ability of locating an object on a rectangular plane. To locate a sunken ship precisely, it needs to use the two parameters: the distance and the cardinal direction. A half of the students used the cardinal direction to locate it, which lead to multiple possible locations (see figure 5.27). However, 2 out of 30 students specified the location using both parameters and the other eight students used a coordinate pair (8, 6) or (6, 8) to locate it (see figure 5.27). When being clarified, some of them explained that (8, 6) means 8 to east and 6 to north.

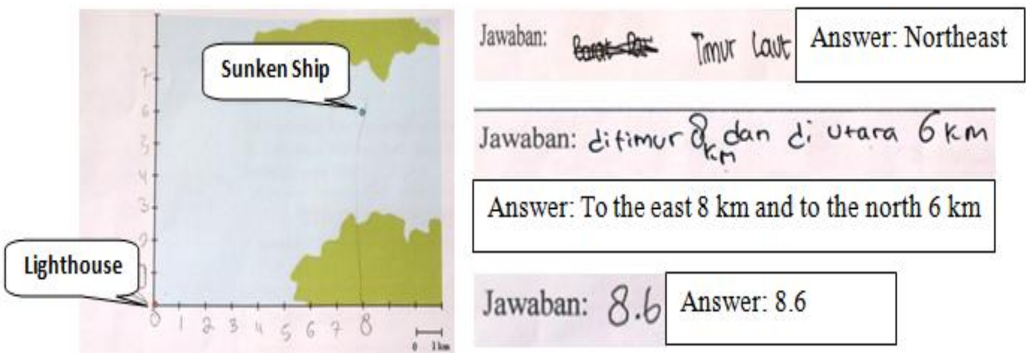


Figure 5.27 The students' work on locating the ship in the fourth item of pre-test

Based on the analysis above, even some of students were able to locate an object on a rectangular plane using both the distance and the cardinal direction or a pair of coordinate, most of them was not. Consequently, they need encounter an activity about locating an object using the distance and the cardinal direction.

✓ **Locating and Plotting Points on the Positive Quadrant**

At the formal level, students' ability of locating points on the positive quadrant will be assessed in the fifth problem. The task asked the students to specify the coordinate of a point that has "half" as one of its coordinate. Three out of 30 students were able to write the coordinate at (7½, 6), while nine of them as (7, 6) or (6, 7) (see figure 5.28). However, more than a half of the students were not able to specify the coordinate of the point at all (see figure 5.28). For the second question, they were asked to plot a an ordered pair (9, 7) on the Cartesian diagram. More than a half of the students plot the points in the wrong position, while the others were able to do so correctly.

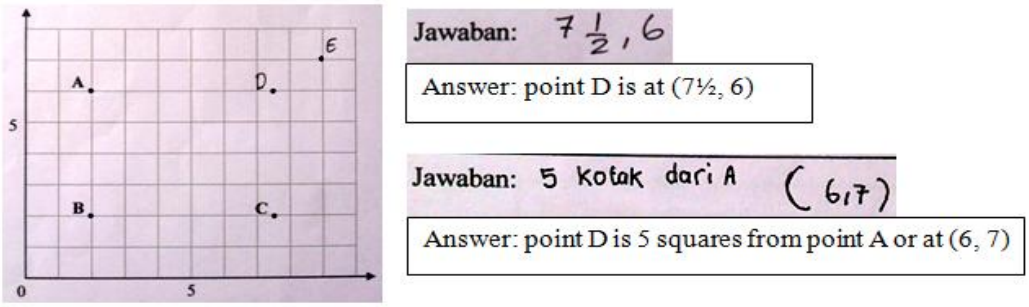


Figure 5.28 The students' work on locating point D in the fifth item of pre-test

It can be sum up that some students were able to locate points on a plane using a coordinate pair that is looked like the ordered pairs, while the others were not. It means that the students need to encounter a problem about which distance (the horizontal or the vertical distance) that should be considered as the first or the second coordinate of the ordered pair as embedded in the third learning activity.

5. 4. 2 Activity 1: Labeling Blocks of City Blocks

The teacher started the lesson by telling a story that the government of South Sumatera will develop a new city, named Jakabaring, which adopts the system of city blocks in Barcelona. To give the illustration of how it looked like, the students watched a video showing the city blocks from above and then it focused on scrutinizing merely one block. After watching it, the students were given an aerial photograph and a street map of the city blocks and asked to describe what they know about it. Afterwards, the teacher clarified the characteristic of the city blocks by emphasizing that one block contains several buildings. Based on the story (see figure 5.29), the students (in groups) were asked to make a system of labeling the blocks.

> In the beginning of the project, the government do not have name for the streets in the Jakabaring City, instead they label the blocks. Make a system of labeling the blocks that can be used to determine the location of a certain object easily, precisely, and quickly!

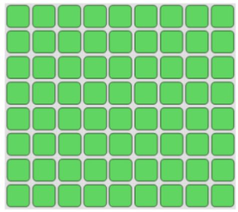


Figure 5.29 The task for labeling blocks in Jakabaring City Blocks

The typical systems that the students made can be categorized into three types. At first, 5 out of 7 groups made a system that we called as an alphanumeric grid system (see Chapter II). This kind of system uses assorted numbers and letters to label the blocks. Starting from the bottom left, for instance, group 1 and 4 labeled the blocks with assorted numbers and letters for the horizontal rows and the vertical columns for each (see figure 5.30). Meanwhile, group 3, 5, and 6 made the similar system, yet starting from the top left corner (see figure 5.30).

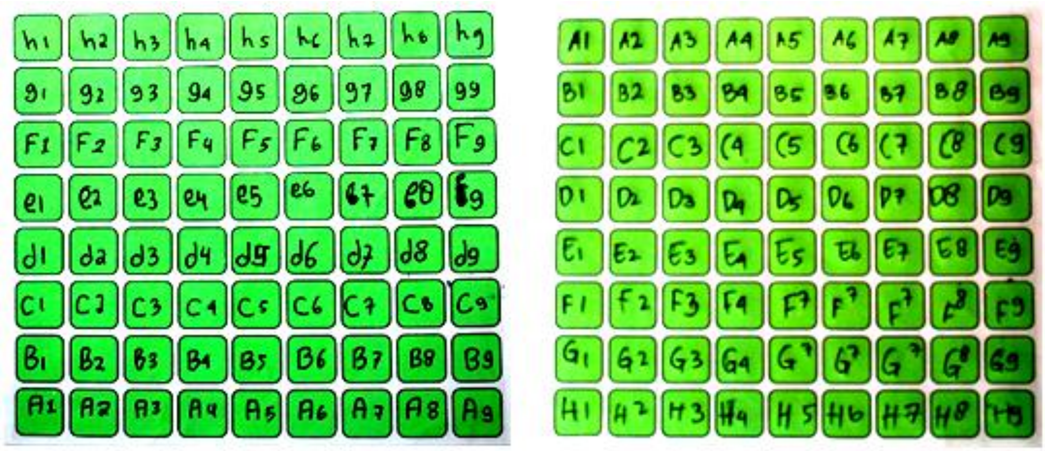


Figure 5.30 The groups work on making the alphanumeric grid system for labeling blocks

For the second type, group 7 labeled the blocks with assorted letters (A - Z) started form the top left corner, which means merely consider one parameter, the rows (see figure 5.31). Along with the weaknesses of this system, they were confused to determine the next label after block Z and preferred to use the assorted numbers after that. When being asked whether the system is helpful to find a block easily without passing through all of the blocks, they seemed realize the weakness of it and changed into the alphanumeric grid system.

The third type is characterized by a system looked like the alphanumeric grid system. Started from the top left corner, group 2 labeled the blocks horizontally with (A1 – A10), (B1 – B10), (C1 – C10), etc (see figure 5.31). Since the number of blocks in each horizontal row is 9 blocks and the group numbered the blocks from 1 to 10 horizontally, some blocks was not in the same row with the other blocks having the same letter as the label. For instance, block A10 was not located in the first row, consisting of blocks A1 to A9 (see figure 5.31).

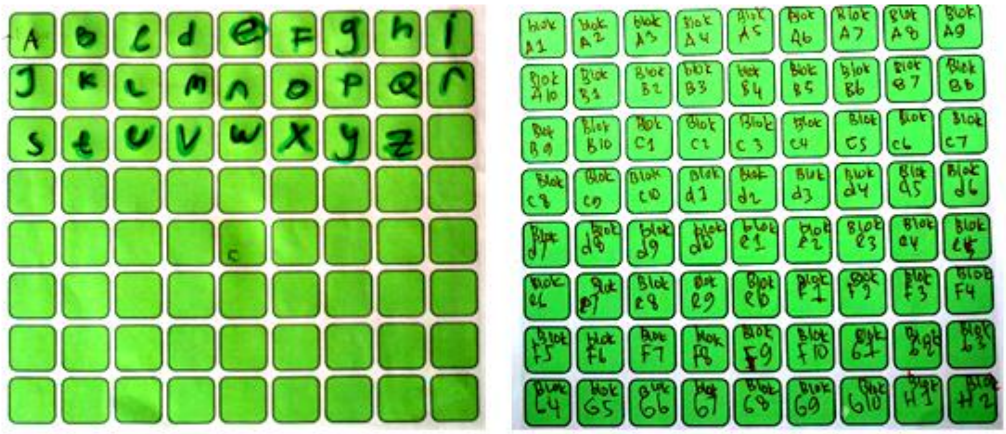


Figure 5.31 The written works of group 7 *(left)* and group 2 *(right)*

After having the working groups, the teacher asked group 2 and 5 to present their work alternately. Since they have different systems for each other, group 5 as the second presenter was challenged to argue why their system (the alphanumeric grid system) is superior to the system made by group 2 as shown in the fragment.

Teacher : What are the superiority of your system?
Galang : Perhaps, this is not confusing because if we want to find A9, it Ia still in one row. while, group 2 (notated) the A10 is in the bottom (of A1), so that it is confusing. It shows a different row
Teacher : So, do you mean it *(system made by group 2)* is mixed up?
Galang : No, mine is in one row
Teacher : Yes, this is one row. How about the previous group *(group 2)*?
Galang : Just now, (the blocks) A1 to A9 is in the upper (first row), but A10 is in the bottom (the second row). Therefore, the People who want to find it (block A10) get confused.

Group 5 argued that their system is more helpful because the blocks in the same row have the same letter as the label. For instance, the blocks in the first row have letter A as the label (line 2 – 3). Compare to their system, group 5 explained that the system made by group 2 is a little bit confusing because they put some blocks with the same letter as the label in the different row. For example, blocks A1 to A9 is in the first row, but block A10 is the next second row (line 3 – 4 and 8 – 10). In addition, group 5 also disputed that their system is helpful to find a certain block easily without walking around too many blocks as shown in the fragment.

Teacher : Ok. Does anyone have questions? Yes, Lucky, please
Lucky : We are from the second group. We have a question for them. How can you move from H1 to B6 without taking so many turns?
Teacher : Ok, without walking around too many blocks. From what?
Lucky : It is from H1 to B6
Teacher : It means that it starts from the bottom left, H1 to B6
Galang If we are from H1 heading to B6, we can count it (the blocks) from the map
Teacher : So, do you rely more on the map than the system that you made so that it is readable?
Galang : Umm, maybe we can sort the letters (ascending)
Teacher : After that, how many steps should we take to reach B6?
Galang : One, two, three, four, five/

Group 5 argued that people could find a certain block easily and quickly, without walking many blocks, by assorting the letter and the number of the block (line 22 - 23). Based on these entire arguments, the teacher led the students to have a conclusion, about which system is helpful to find a block easily and quickly.

To sum up, we can say most of the students were able to make the alphanumeric grid system for labeling blocks in the context of city blocks as we predicted in the HLT. This system allows them to label the blocks using assorted numbers and letters for the horizontal rows and the vertical columns. The label of a block is then identified as co-ordinates of a letter and a number, such as block A5 or 5A. In regards to the main goal of this activity, it can be concluded that the students were able to make a system of organizing things, a system of labeling blocks (city blocks), using a grid system involving rows and columns.

5. 4. 3 Activity 2: Chess Notation

At the same day, the lesson was continued with the activity of describing the location of pawns on the chessboard using chess notation system. The chess notation is actually the same with the alphanumeric grid system that allows us to use an alphanumeric grid with assorted numbers (1 – 8) started from the bottom left hand corner of the grid and locate the letters (A – H) across the bottom of the grid. To start with, the teacher asked students' experience about playing chess and their familiarity of using chess notation. One of the students was able to explain what the chess notation is, as we described before.

Using the chess notation, the students (in groups) were required to locate several pawns on the chessboard (see Appendix E, Worksheet 1). All of them specified the location of the pawns using a coordinate pair of a letter and a number (see figure 5.32). In their written work, 6 out of 7 groups wrote down the coordinate with the letter or the number first such as Black Queen is at 5D or 5D. Meanwhile, the another group wrote both kind coordinates (see figure 5.32).

Figure 5.32 The groups' written works on locating pawns using the chess notation system

The working groups were then continued with class discussion. To begin with, the teacher asked some representational students to write down the coordinate of those pawns, starting either with a letter or a number first. To check the students' reasoning whether the notated coordinate such as D1 and 1D point the same location or not, the teacher asked the students' opinion as shown in the fragment .

1	Students	:	For instance, I want to find the location of the white queen.
2	Teacher	:	Where is it?
3	Students	:	D1
4	Teacher	:	If I say 1D, is it ok?
5	Students	:	*Inaudible*
6	Teacher	:	If I start to walk towards 1 then towards D, can we find it?
7	Students	:	Yes, we can
8	Teacher	:	How about I start with D then to 1
9	Students	:	Yes it can
10	Teacher	:	So, (the symbol) of D1 and 1D specify the same location or not
11	Students	:	Yes (the same location)

The students initially got confused whether the coordinates D1 and 1D refer to the same location of White Queen or not (lines 1 - 5). It seemed that most of the students merely considered D1 as the single answer. However, after being showed the possibility of reading the coordinate from the vertical (number) to the horizontal (letter), it then made sense for them that the coordinates D1 and 1D specified the same square of the White Queen (lines 6 - 11). In relate to the first activity, the students were asked about the possibility of applying the chess notation system for the system of labeling blocks. All of them agreed that it was applicable by showing how the system works to find a block based on that system.

Based on the throughout analysis above, it can be concluded that all of the students were able to use the chess notation system to locate a certain pawn on the chessboard. Here, they employed a pair of coordinate a letter and a number to specify the location as we predicted in the HLT. Relating to the activity of labeling blocks, they were also able to apply and show how the chess notation system works on labeling the blocks as we conjectured in the HLT.

5. 4. 4 Activity 3: Taxicab *Routes* and Distance

The activity was started with finding out the label of a certain block (city blocks) using a coordinate pair of a letter and a number, such as block A4 or 4E. The teacher, afterwards, told a story about a man who wants to go to Bhayangkara hospital in Jakabaring city blocks. The students in groups were asked to tell the location of Bhayangkara hospital if it is placed on the same block with another hospital, namely Sriwijaya hospital (see Appendix E, Worksheet 2). Based on the written work, all of them realized that telling the location using merely the label of the block is not enough because another hospital is on the same block.

The students written work can be distinguished into three typical answers. First, 3 out of 7 groups used the label blocks and compass direction to locate it. Here, Bhayangkara hospital is at the northeast of block E4 (see figure 5.33). Second, three other groups used the cardinal direction and the distance with the term "step" as the unit distance. So that, Bhayangkara hospital is at 5 steps to the east and 4 steps to the north or the other way around (see figure 5.33). The third answer can be said as the combination of the first and the second answer. Rather than directly said block E4, the last group referred block E4 as 4 steps to the east and 4 steps to the north. Then the hospital is on the northeast (see figure 5.33).

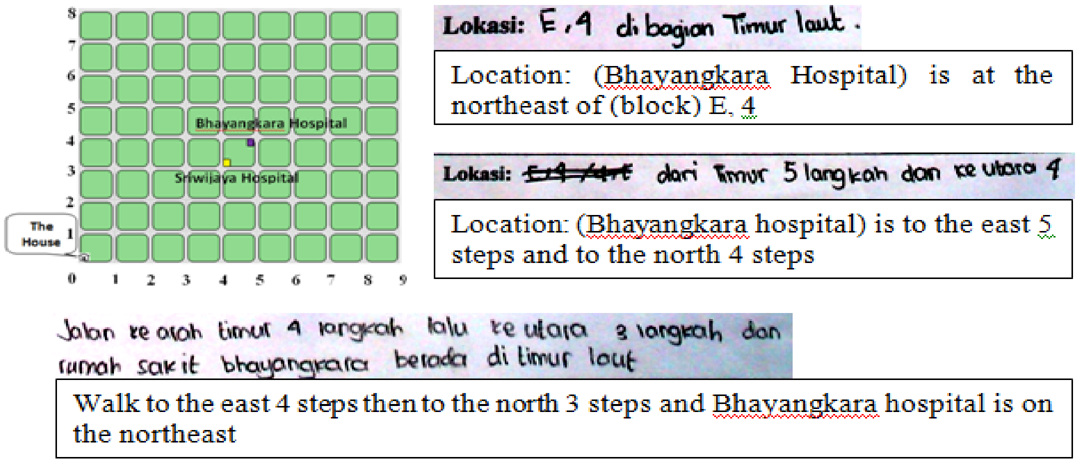


Figure 5.33 The groups' works on locating Bhayangkara hospital in the city blocks

That was a prior activity before they worked with the *taxicab* distance related tasks that lead to the reinvention of the positive quadrant of the Cartesian system. It was begun with making a shortest taxicab route between two objects, Mr. Budi's house as the origin and Bhayangkara hospital. This task was also simultaneously asked them to determine the distance covered by the route (see Appendix E, Worksheet 2). Five out of seven groups made a shortest route that involves merely one turn, while the other two groups were more than one turn (see figure 5.34). With different routes, all of the groups, however, had the same result for the distance, that is nine blocks. After the working groups, the teacher asked three representational groups who had different routes to show their routes including its *taxicab* distance (see figure 5.34).

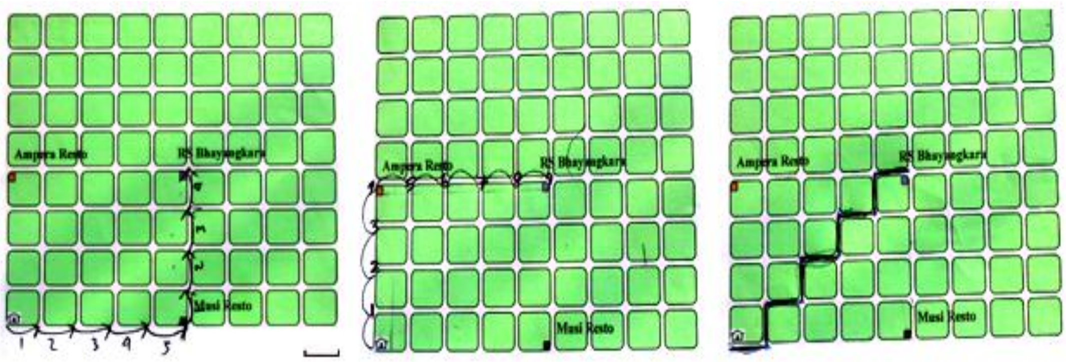
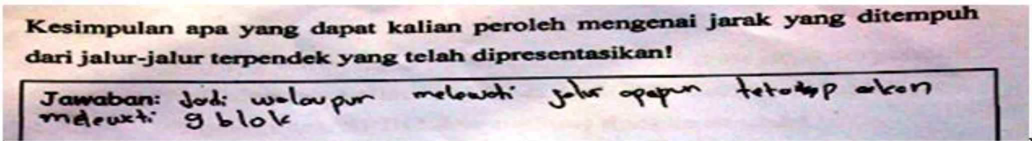


Figure 5.34 Three taxicab routes from Mr. Budi's house to Bhayangkara hospital

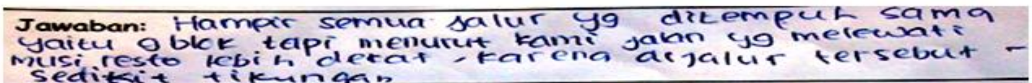
Based on those three routes, all of the groups were required to conclude about the distance covered by the routes. Even though this task was sound easy, more of the students had difficulty to get the main idea of how to conclude it. The following fragment shows how group 2 seemed struggle to get the intended conclusion.

1	Teacher	:	What is the distance of the first route?
2	Revidya	:	9 blocks
3	Teacher	:	How about the second route?
4	Revidya	:	9 blocks
5	Teacher	:	And the third block?
6	Revidya	:	9 blocks
7	Teacher	:	So, what is your conclusion?
8	Revidya	:	By passing 9 blocks, we can arrive at Bhayangkara hospital
9	Teacher	:	Ok, do you have another conclusion?
10	Revidya	:	*Silent*
11	Teacher	:	For instance, I make a new route that is different
12			with the previous ones. How many distances is that?
13	Revidya	:	1, 2, 3, 4, 5, 6, 7, 8, 9 (blocks)
14	Teacher	:	So, what is your conclusion?
15	Revidya	:	By using any routes, it will have (the same) nine blocks

Even though her first conclusion was not completely wrong, she missed to mention the term related to three different routes (line 8). After giving an extra route that covers the same 9 blocks, however, she were eventually able to conclude appropriately (lines 11 – 15). Similarly, the other groups were also able to conclude that any *taxicab* routes with the same starting and ending point covers the same *taxicab* distance (see figure 5.35). Two of the seven groups, however, had additional conclusion that the routes with fewer turns are shorter than the routes with more turns (see figure 5.35). Since it did not discussed it further, the students did not aware that the number of turns is not related to its distance.



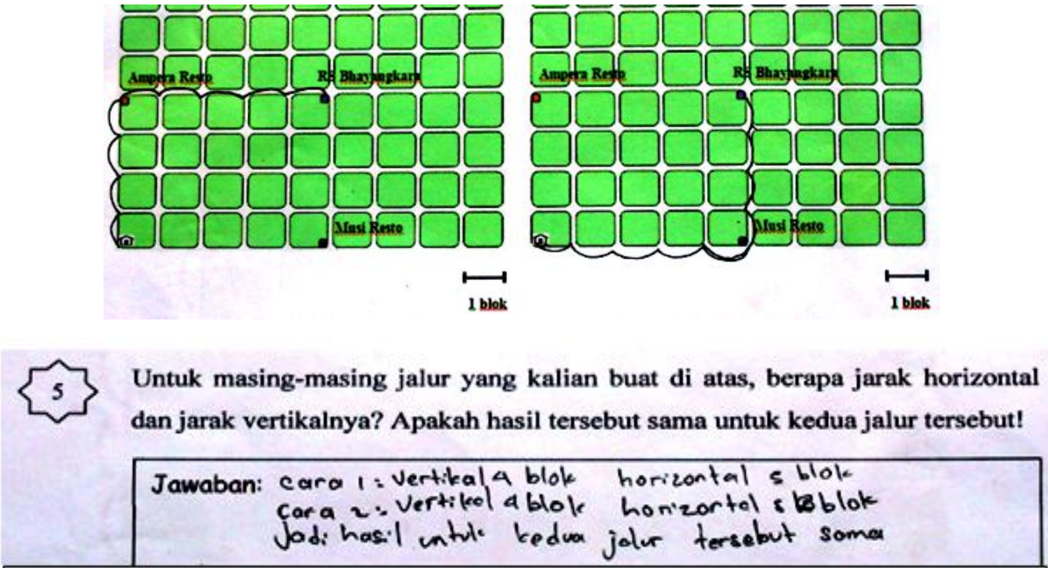
What can you conclude about the distance covered by those three routes that have been presented?
Answer: So, even though we pass any routes, (the distance) will be the same with 9 blocks



Answer: Almost all of the routes had the same (distance) 9 blocks. However, based on our opinion, the routes passing through Musi Resto is shorter because that route has fewer turn

Figure 5.35 The groups' conclusion about the distance covered by three different routes

Having two different *taxicab* routes that merely involves one turn, all of the groups had no significant problem to determine the horizontal and the vertical distance for each routes. In addition, the fact that those two routes covered the same horizontal and the vertical distance was also revealed (see figure 5.36). On the other hand, it seemed a big trouble when being asked about the relationship between the total (*taxicab*) distance with the horizontal and the vertical distance. Two out of seven groups noticed the *taxicab* distance as the sum of the horizontal and the vertical distance.



How many the horizontal and the vertical distance for each route? What the result are the same for both routes?

Answer: **Route 1:** vertical 4 blocks and horizontal 5 blocks
Route 1: vertical 4 blocks and horizontal 5 blocks
So, the result for both routes are the same

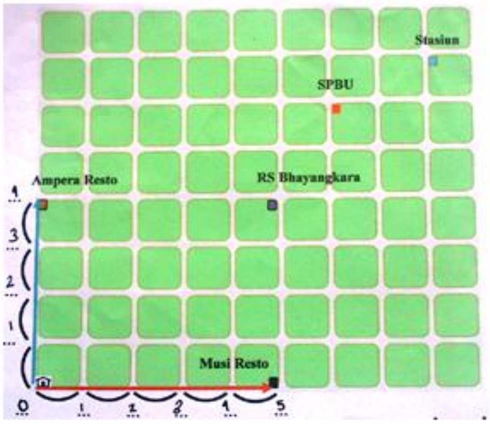
Figure 5.36 The student's work on determining the horizontal and the vertical distance

Under the teacher's guidance, all of the groups then tried to make a system for determining the taxicab distance between two objects, namely Mr. Budi's house (as the origin) and any places in the city blocks in a easy way (see Appendix E, Activity 2). The following fragment shows how group 1 was guided to make the intended distance by finding the distance for each Musi Resto and Ampera Resto first and simultaneously labeling the corresponding gridlines.

Teacher : You are asked to make a system to determine the shortest distance between Mr. Budi's house and every place in the city blocks. So, where is your starting point?

4	Tama	:	Here *(point Mr. Budi's house)*. Zero
5	Teacher	:	Now, the distance between the house and Musi Resto
6	Wawa	:	0, 1, 2, 3, 4, 5 *(notated the vertical gridlines)*
7	Teacher	:	What do you mean by the notation of 1, 2, 3, 4, 5 here?
8	Tama	:	Distance
9	Teacher	:	What is the unit distance?
10	Students	:	Block
11	Teacher	:	So, what is the distance between the house and Musi Resto?
12	Tama	:	5 blocks
13	Teacher	:	Now, let's try for the Ampera Resto
14	Tama	:	1, 2, 3, 4 blocks *(notated the horizontal gridlines)*
15	Teacher	:	Ok, now, can you guess the distance for Bhayangkara
16	Nafilah		9 blocks
17	Teacher		How do you know?
18	Nafilah		Because if we calculate it, 4 + 5 = 9

It can be noticed that the students determined the starting point (Mr. Budi's house) as the zero point and found the distance of Musi Resto by simultaneously numbering the vertical gridlines (1 - 5) across the bottom (lines 1 – 12 and see figure 5.37). It was continued with Ampera Resto in which the horizontal gridlines are numbered from 1 to 4 (lines 13 – 14 and see figure 5.37). For Bhayangkara hospital, its distance was easily found by adding the distance for Musi Resto and Ampera Resto (lines 15 – 18 and see figure 5.37). In this case, the distance for Musi and Ampera Resto are corresponded to the horizontal and the vertical distance of Bhayanghara hospital. By using the similar strategy, the students were guided to find both the horizontal and the vertical distance for each route between the origin (Mr. Budi's house) and other places (gas station and railway station). Consequently, the whole system that is looked like the positive quadrant could be found by numbering the horizontal and the vertical gridlines.



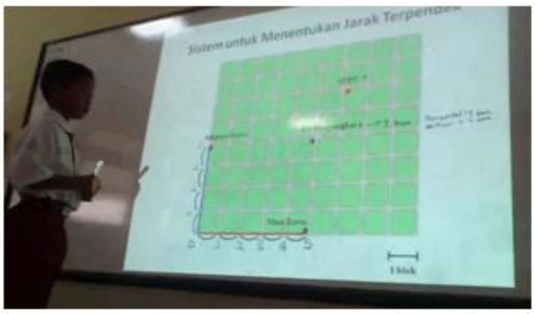


Figure 5.37 The student's work on making a system looked like the positive quadrant

The whole analysis revealed that some of the students were able to locate a certain object precisely in the city blocks using the information related the label of the block its direction as we expected in the HLT. However, the other students employed the distance and its direction to locate it as we not predicted in the HLT. These were preliminary results before the students to reinvent a system looked like the positive quadrant of the Cartesian coordinate system.

To reinvent the system, the students initially worked with the *taxicab* distance related tasks in which different *taxicab* routes with the same starting and ending point covered the same *taxicab* distance as we predicted in the HLT. To easily determine the *taxicab* distance between the origin and any places in the city blocks, they used the *taxicab* routes that merely have one turn for each. It helped them to easily find the horizontal and the vertical distance for each route by simultaneously labeling the horizontal and the vertical gridlines using whole numbers as we expected in the HLT. This activity, in the end, led them to reinvent a system for determining the *taxicab* distance that is looked like the positive quadrant of the Cartesian coordinate system as we predicted in the HLT.

5. 4. 5 Activity 4: How to Use an Ordered Pair

To begin with, the teacher reminded the students about locating Bhayangkara hospital using the label of the block and the cardinal direction, namely in the northeast of block E4. The task was then started with asking the students to locate a workshop and a railway station using another system used for determining the *taxicab* distance as in the previous activity (see Appendix E, Worksheet 3). None of them had troubles to specify the location using the horizontal and the vertical distance including its cardinal direction (see figure 5.38).

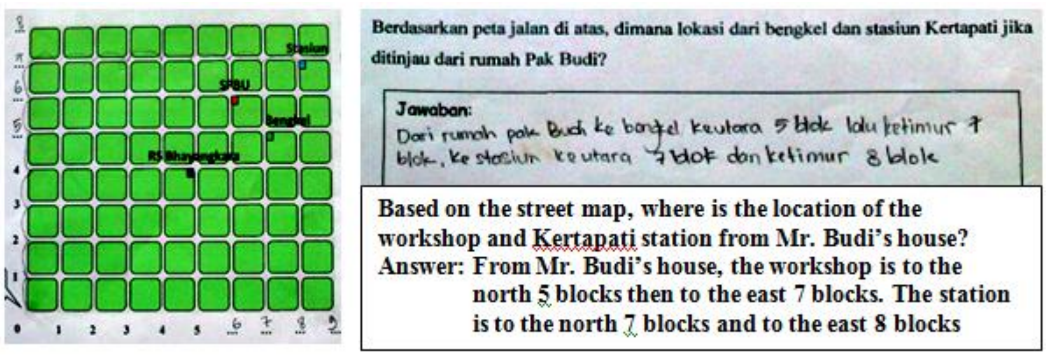


Figure 5.38 The written work on locating objects using the horizontal & vertical distance

In the other way around, a representational student of each group was asked to plot several objects on the street map of the city blocks if it was given its distance and cardinal direction (see figure 5.39). For instance, the students were required to plot Aryaduta Hotel located at 1 block to the north and 8 blocks to the east. All of them had no significant difficulty to deal with this kind of task (see figure 5.39).

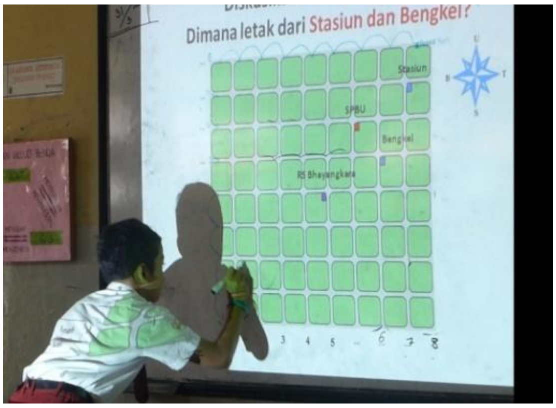


Figure 5.39 The student plotted an object on the street map of the city blocks

The teacher continued the lesson by telling TV news reporting a fire accident in Jakabaring city blocks. After putting out fires, a police line was installed on four electrical tower at the coordinates (1,1); (7,1); (7,5) and (1,5). The students in groups were asked to plot the towers and show the configuration of the police line (see Appendix E, Worksheet 3). The results show that 5 out of 7 groups considered the horizontal distance as the first coordinate (see figure 4.39). Meanwhile, the other two groups consider both the horizontal and the vertical distance as the first coordinate that leads to two answers (see figure 5.40).

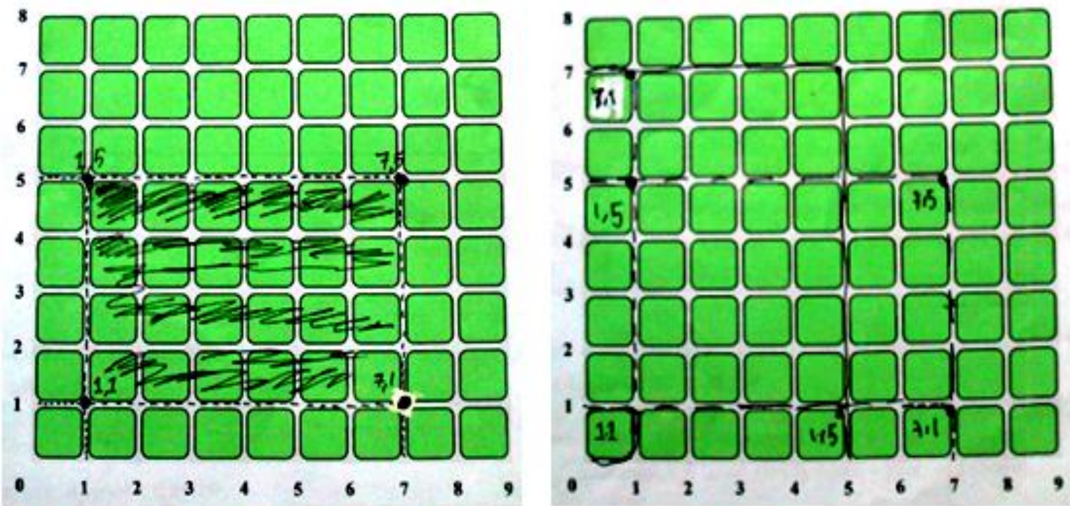


Figure 5.40 The example of written work on plotting the coordinates on the street map

The class discussion was started by asking group 4 to present their work on plotting the coordinates (*x*, *y*) with merely considering the horizontal distance as the *x*-coordinate. The presentation was then continued with the group 1 who interpret the vertical distance as the *x*-coordinate. Since there were two possible way of plotting the coordinates (*x*, *y*), the teacher asked which strategy that is acceptable and its reason. Most of them seemed confused until one of the students (Sari) came up with a good reasoning. She claimed that both of the strategies are acceptable depending on how we interpret the *x*-coordinate as the horizontal or the vertical distance. In respond to this answer, the teacher directly said the correctness of the argument and asked the students to make an agreement. Then, it was agreed that the first coordinate is interpreted as the horizontal distance.

To conclude, it can be said that all of the students were able to locate and plot a certain object in the city blocks using the information related to the horizontal and the vertical distance including its direction as we predicted in the HLT. Even the students did not come to the idea of making an agreement about interpreting the coordinate (x, y) as we expected in the HLT, however, they agreed to consider the horizontal and the vertical distance as the *x*- and *y*-coordinate respectively.

5. 4. 6 Activity 5: Sunken Ship

The lesson was started by talking about the trip experience using a ship and telling a story about a sunken ship in Java Sea. To ask for a help, the captain sent the coordinate of the ship to the officer in the lighthouse. Within this story, the students in groups were asked to tell the coordinate of the sunken ship if it was seen from the lighthouse (see Appendix E, Worksheet 4). This task is focused on how the students locate an object on a rectangular grid using the ordered pair (*x*,*y*) or the information related to the horizontal and the vertical distance.

The results shows that all of the groups could specify the coordinate using the ordered pair (*x*,*y*) (see figure 5.41). Indeed, they also notated the location of the ship using the distance and the cardinal direction (see figure 5.41). To begin with, some of the groups also notated the whole numbers along the horizontal and the vertical axis, which are corresponded with the distance. However, they did not notated zero at the origin (the lighthouse) (see figure 5.41).

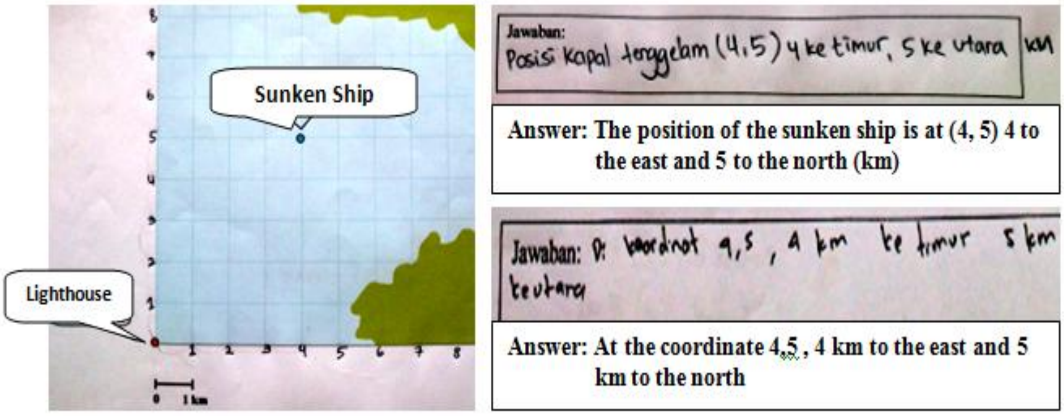


Figure 5.41 The example of written work on locating the sunken ship on the sea map

Two representational groups were asked to present their work in front of the class. The first presenter started to label the gridlines with whole numbers corresponding with the distance. The sunken ship was then located at 4,5. They probably want to use the ordered pairs, but they forgot to use the parentheses. As such, the second the presenter was asked to correct the written of the ordered pairs into (4, 5).

The task was continued with describing the location of the rescue ships that placed along the *x*-axis or *y*-axis, namely SAR team A and C (see Appendix E, Worksheet 4). Even some groups were able to see that the corresponded distance of the object placed along the axes is equal to zero, the other groups seemed get trouble. The following fragment shows how group 5 had trouble to find out the vertical distance of SAR team A placed on the *x*-axis.

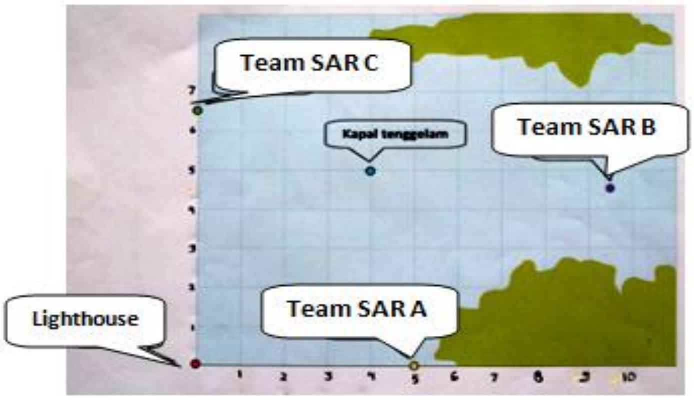


Figure 5.42 The task of locating SAR teams on the sea map from the lighthouse

1	Observer	:	The SAR is the yellow point here. What is the coordinate?
2	Diki	:	To the east 5 steps
3	Observer	:	Yes. Can you write down the coordinate? The coordinate

	:	that your friend wrote before. How does it look like?
Observer	:	Ok, let me ask a question. What is the horizontal distance?
Lutfia	:	Four (km)
Observer	:	Four or what? Let’s try to recount it
Lutfia	:	One, two, three, four, and five. It is five (km)
Observer	:	5 km for the horizontal. How about the vertical (distance)?
Lutfia	:	One, two
Observer	:	Here, the SAR team is in here. So, what is the vertical one?
Nabila	:	One

It was not a significant trouble to determine the horizontal distance between SAR team A and the lighthouse, even though they got miscalculation in the middle of their work (lines 1 – 9). However, it turned to be a problem when being asked about its vertical distance that is equal to one not zero (lines 11 – 12). Consequently, the observer presented a similar problem where the SAR team is placed one-step upper the SAR team A as shown in the fragment below.

Figure 5.43 The task of locating SAR team on the sea map from the lighthouse

Observer	:	Let’s use the other worksheet! Suppose the SAR team is in here (one step upper the x-axis) What is the horizontal distance?
Students	:	Five
Observer	:	What is the vertical distance? Look at here
Nabila	:	One
Observer	:	How about (the vertical distance) if the SAR team is in here (on the x-axis)?
Lutfia	:	Zero
Observer	:	So, what is the horizontal distance?
Lutfia	:	Five
Observer	:	The vertical one?
Lutfia	:	Zero
Observer	:	So, how do you write down the coordinate?
Nabila	:	(5,0)

Since the students eventually realized that the vertical distance of the (new) SAR team is one, then the vertical distance of SAR team A should be zero (lines 16 – 20). After determining each horizontal and the vertical distance, the coordinate of

SAR team is finally determined at (5, 0) (lines 21 – 26). Similar to this strategy, all of the students were able to locate SAR team A and C that are located along the *x*-axis or *y*-axis respectively (see figure 5.43).

Not only locating objects on the axes, they were also worked on locating SAR team B and C that one of its coordinate has value "half". The results show that all of the groups were able to locate those SAR teams using the ordered pairs (*x*,*y*) in which the *x*- and *y*-coordinate are corresponded with the horizontal and the vertical distance respectively. However, to notate the "half" as one of its coordinate, some groups expressed it using fractions, while the others employed decimal (see figure 5.44).

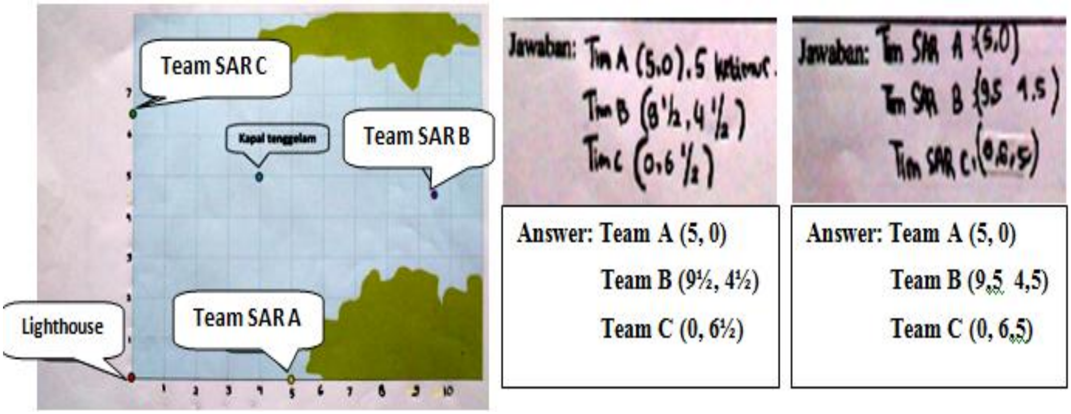
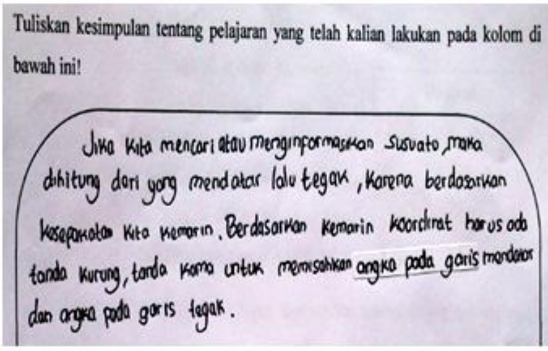


Figure 5.44 The examples of students' written work on locating SAR team A, B, and C

The groups work was then continued with the presentation of two groups who had different way of notating "half" coordinate either using fractions or decimals. React to this findings, the teacher emphasized that the students can use both those ways, but it was suggested to use "point" rather than "comma" to express the decimal as the worthwhile. It was conducted to avoid the misinterpretation of reading the coordinates itself.

In the end of the lesson, each group was asked to give conclusion about what they have been learnt during the day. The conclusion is typically about the use of ordered pairs to locate an object precisely by considering the horizontal distance first then the vertical one (see figure 5.45). The emphasize of using parentheses and comma in relate to the employment of the ordered pairs was also described in quite detail (see figure 5.45)



Write your conclusion about what you are being learned in the column below!

If we want to find or inform a thing (using ordered pairs), so it must be considered from the horizontal first then the vertical one based on our previous agreement. As the previous agreement, the coordinate (ordered pairs) must have parentheses and comma to differentiate the number on the horizontal axes and the number on the vertical one.

Figure 5.45 The students' conclusion related to the activity of locating ships on a sea map

In conclusion, all of the groups were able to determine the coordinate of objects on the map looked like the positive quadrant. As we predicted in the HLT, for the objects placed along the x-axis or the y-axis, they knew that one of its coordinate should be equal to zero. In addition, they were also able to locate an object that has "half" as one of its coordinate using the ordered pairs (x, y). They expressed the coordinate either using fractions or decimal.

5. 4. 7 Activity 6: Cartesian Coordinate System (Positive Quadrant)

The lesson was started by reminding the students about how to notate the ordered pairs (x, y) correctly. As they concluded in the previous lesson, the ordered pair is a pair of number written in a certain order, which is usually written in parenthesis like (x, y). The first number tells how far to move horizontally, while the second number for the vertical one. For the "half" coordinate, they can use fractions or decimals, but for expressing decimal they are suggested to employ "point" rather than "comma" as the worthwhile use.

The main activity can be differentiate into two tasks that is mainly about plotting and locating points on the positive quadrant of the Cartesian diagram using the ordered pairs. For the first task, the teacher told a story about playing "secret quadrilateral" game in which someone needs to guess the name of a quadrilateral formed by four ordered pairs. If given four ordered pairs {A (0, 2); B (7, 2), C (10, 6); D (3, 6)} and they formed a quadrilateral ABCD, the students are required to guess the name of the quadrilateral figure. To plot the given ordered pairs (x, y), the entire groups interpreted the horizontal as the x-coordinate and the vertical distance as the y-coordinate as shown by group 1 in the fragment below.

1	Observer	:	Why don't you locate (0, 2) here? *(pointed the x-axis)*
2	Nafilah	:	Umm… Based on the agreement before, it should be
3		:	the horizontal first then vertical one
4	Observer	:	For (0, 2), which one the vertical *(distance)* is?
5	Nafilah	:	2
6	Observer	:	Ok… Then, how about (10, 6)?
7	Students	:	10 *(he horizontal distance)* and 6 *(the vertical one)*
8	Observer	:	So, which one you interpret first?
9	Students	:	The horizontal first
10		:	So, what is the name of quadrilateral figure that is formed?
11	Students	:	Parallelogram

It can be noticed that the students frequently refer to the agreement they made to use the ordered pairs (*x*,*y*) (lines 1 – 2). Here, they always consider the *x*- and *y*-coordinate as the horizontal and the vertical distance respectively to plot the points (lines). Based on this knowledge, they could plot any points on the diagram correctly and indeed accurately since the coordinate grids are also embedded in the given task (see figure 5.46).

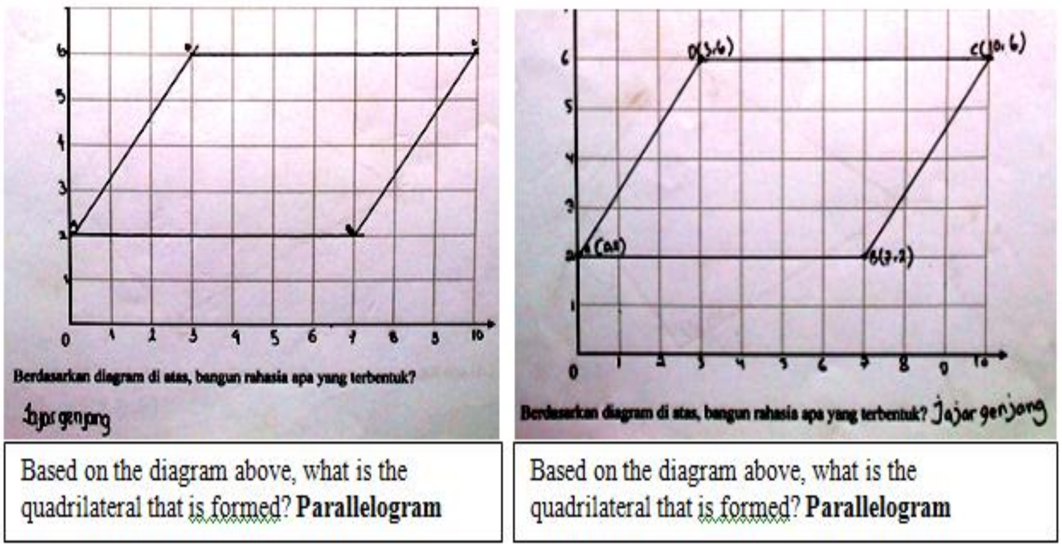


Figure 5.46 The students' work on plotting points A, B, C, D in *secret quadrilateral* task

It was then carried on the second task about "secret coordinate" game where we need to determine the forth ordered pairs if given three others ordered pairs that can be formed into a certain special quadrilateral. In this task, three given ordered pairs {K (0, 3); L (2½, 0); and M (8, 3)} can be formed into a kite if the forth ordered pair can be determined (point N). It might not really matter for most of the groups to plot K (0, 3) and M (8, 3) by merely considering the agreement that the *x*- and *y*-coordinate represent the horizontal and the vertical distance respectively. However, some of the groups seemed struggle to plot L (2½, 0) since it has both "half" and "zero" as one of its coordinate as shown by group 7 in the fragment below.

1	Observer	:	And then, where is the location of (2½, 0)?
2	Rif'at	:	Here *(point at (2, ½))*
3	Observer	:	Which one?
4	Students	:	No no no. It is wrong
5	Observer	:	Ok, now I asked: Which one is the horizontal distance?
6	Javier	:	2 ½
7	Observer	:	And the vertical *(distance)*?
8	Rif'at	:	0
9	Observer	:	So, how is it looked like?
10	Rif'at	:	Here *(point at (2 ½ , ½))*
11	Observer	:	Ok, let's mark the point here *(point at (2 ½ , ½))*
12		:	what is the horizontal *(distance)* of this *((2 ½ , ½))*?
13	Students	:	2 ½
14	Observer	:	The vertical *(distance)*?
15	Rif'at	:	0
16	Observer	:	The zero is in here *(point at the origin)*
17	Rif'at	:	Oh… ½
18	Observer	:	Instead, the coordinate being asked is (2½, 0)
19	Javier	:	Ohhhh… here *(point at (2½, 0))*
20	Observer	:	Ok, now connect those and determine where point N is
21	Javier	:	*(draw segments connected those three points)*
22	Students	:	Finish
23	Observer	:	Wait a minute. What is the coordinate? *(refer to point N)*
24	Akbar	:	(2 ½, 6)

The students wrongly plotted L (2½, 0) twice which are at *(2, ½)* and *(2 ½ , ½)* (lines 2 and 9 – 11). They seemed get confused to see "zero" as the vertical distance in their notated coordinates of *(2, ½)* and *(2 ½ , ½)* (lines 1 – 8 and 11 – 15). However, after being showed that zero (as the vertical distance) is along the *x*-axis, they came to plot (2½, 0) properly (lines 16 – 19). After connecting the three given points and determined the location of point N to make a kite, they were able to locate point N at the coordinate (2 ½, 6) (lines 20 – 24). For the other groups, they were able to plot the given ordered pairs accurately and determine the coordinate of point N precisely. Some groups used fractions to express the half, while the others employed decimals (see figure 5.47).

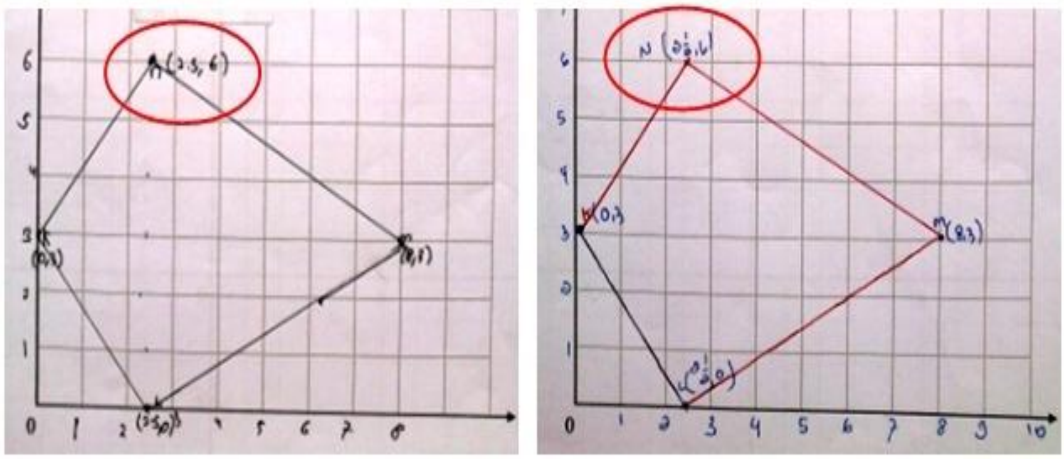


Figure 5.47 The students' work on plotting points K, L, M in *secret coordinate* task

By understanding the rule of how "secret quadrilateral" and "secret coordinate" work, each group had turn to challenge the other group either to guess the name of the quadrilateral figure or the forth point as embedded on both games. All of the students seemed enthusiastic to play the games such that it fruitfully supported the achievement of our main learning goals related to locating and plotting points on the positive quadrant using the ordered pairs (*x, y*).

In conclusion, we can say that all of the students had no difficulty to plot and locate any point on the positive quadrant using the ordered pairs (*x, y*). As we predicted in the HLT, they plotted the ordered pairs accurately on the Cartesian diagram since the coordinate grids were given in the task. To locate the ordered pairs that has "half" as one of its coordinate, they were able to express "the half" using fractions or decimals in such way it did not lead to multiple interpretation. Here, they employed point rather than comma to express the decimal numbers.

5. 4. 8 Post-Test

The post-test was performed on Thursday, 2nd April 2015 in Pusri Primary School Palembang. Five items were given to the students to be done individually for about 20 minutes in total. result of the post-test analysis is employed to assess students' acquisition of the knowledge related to the positive quadrant of the Cartesian coordinate system after they joined a set of learning activities. The following are the important points revealed from the result of the pre-test that will be used to assess the students' acquisition.

- **Using an Alphanumeric Grid System**

The first item is about specifying the location of a seat in a cinema using the alphanumeric grid system (see Appendix D). Since we provided the notating numbers 1 – 5 for the vertical rows and letters A – J for the horizontal columns, all of the students were able to locate it precisely using a coordinate pair of number and letter (see figure Figure 5.48). Similar to the first activity of locating a block and locating a pawn, they were able to specify the location of an object in arrangement of rows and columns using the alphanumeric grid system.

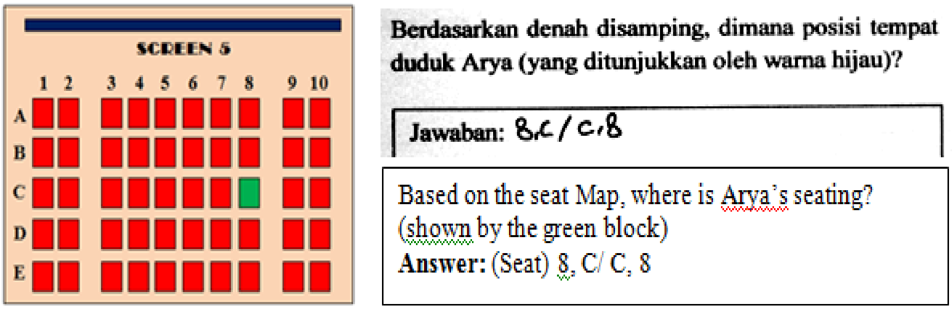


Figure 5.48 The students' answer on locating the seat in the first item of post-test

Based on the result above, it can be concluded that all of the students were able to specify the location of a certain object in arrangement of rows and columns using the alphanumeric grid system. A coordinate pair of number and letter was used to locate it. The reason why they were able to do so is that the corresponding numbers and letters of the alphanumeric system were provided in the task.

- **Understanding Chess Notation System**

Locating a certain chess pawn using the chess notation system, as the representation of the alphanumeric grid system, is embedded in the second item (see Appendix D). Differ from the first item, the corresponding numbers (1 – 8) for the vertical rows and letters (A – H) for the horizontal columns were not provided in this task. The typical of students' answer can categorized into three. First, 6 out of 30 students located the black queen and the white king using the the chess notation system (see figure 5.49). Some of them notated the numbers (1 – 8) for the vertical rows, while the others notated it for the horizontal column. A coordinate pair of number and letter is used to locate the pawn (see figure 5.49).

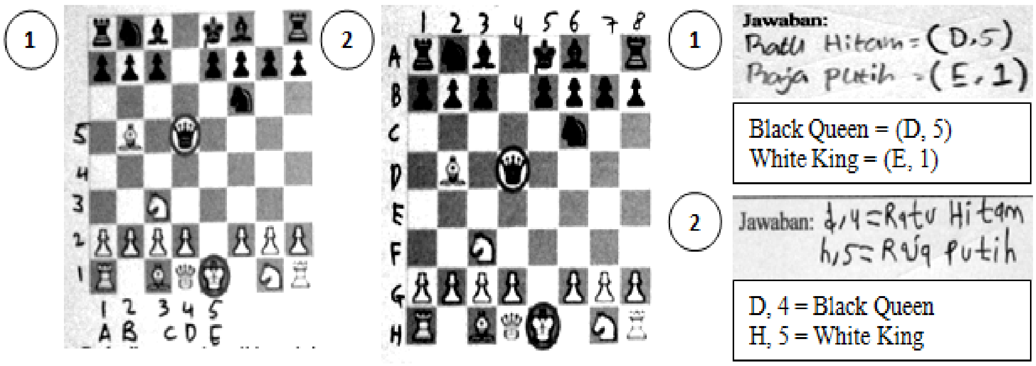


Figure 5.49 The students' answer of locating Black Queen and White King

Instead of using the chess notation system, 12 out of 30 students used the Cartesian coordinate of the positive quadrant. It means that the positive integers label the gridlines not the square region (see figure 5.50). Even a pair of coordinate number is employed to locate the pawns, it leads to imprecise location since the pawns are in the square region, not in the intersection of the gridlines.

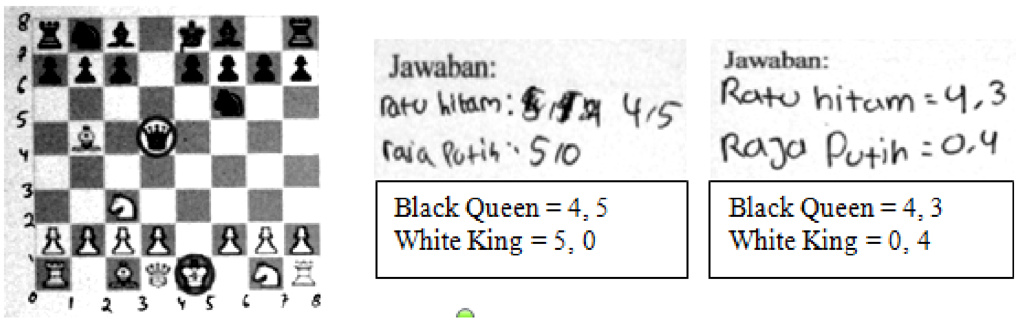


Figure 5.50 Two answers of locating pawns using the Cartesian coordinate

For the third typical answer, 11 out of 30 students label both the horizontal columns and the vertical rows with the positive integers either starting from one or zero (figure 5.51). Differ from the second type, a pair of coordinate numbers can specify the location of the pawns precisely (figure 5.51).

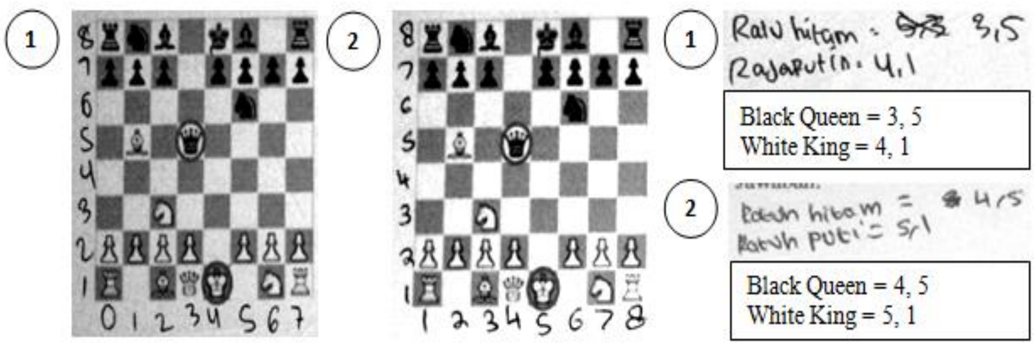


Figure 5.51 Locating pawns using a system looked like the Cartesian coordinate

In conclusion, we can say that merely few students were able to specify the location of a certain pawn using the chess notation, where the numbers (1 – 8) label the vertical rows and letters (A – H) label the horizontal rows. Most of the students had difficulty to differentiate the alphanumeric grid system that is used to identify a certain square region with the Cartesian coordinate that is employed to specify a certain point in the intersection of the gridlines. This implies that the students still get confused about use of both systems.

- **Locating an Object a the Rectangular Plane**

Similar to the pre-test and the fifth learning activity, this item is used to know the students' capability of locating an object on the rectangular object. To specify the coordinate of a sunken ship from a lighthouse (as the origin), most of them used an ordered pair (*x*, *y*), where the *x*- and *y*-coordinate represent the horizontal and the vertical distance respectively (see figure 5.52).

Not only using the ordered pair, some of them gave additional information involving the cardinal direction and its corresponding distance (see figure 5.52). Even they could express the coordinate using the ordered pair, however, some of them still forgot to use the parentheses (see figure 5.52).

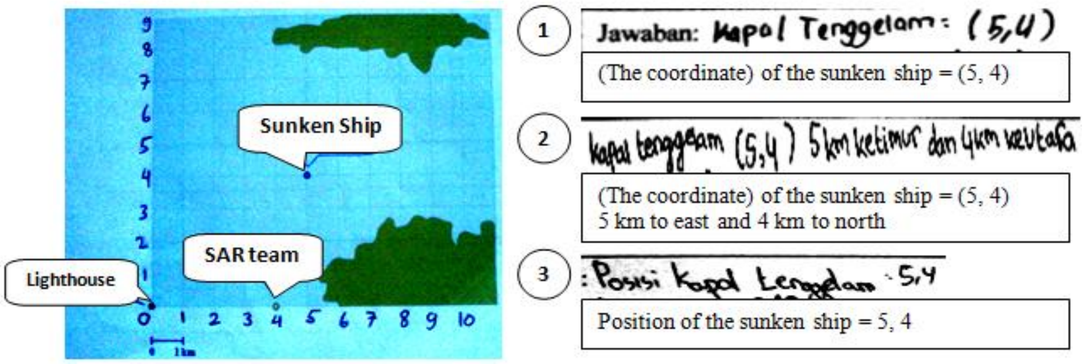


Figure 5.52 The students' answer of locating the sunken ship from the lighthouse

For the SAR team located on the x-axis (see figure 5.52), 16 out of 30 students located it at (4, 0), which means that the first coordinate correspondences to the horizontal distance (see figure 5.53). In contrast, 4 out of 30 students expressed the coordinate at (0, 4), which means the vertical distance represents the first coordinate (see figure 5.53). Two of them preferred to use the cardinal direction with its corresponding distance rather that the ordered pair (see figure 5.53).

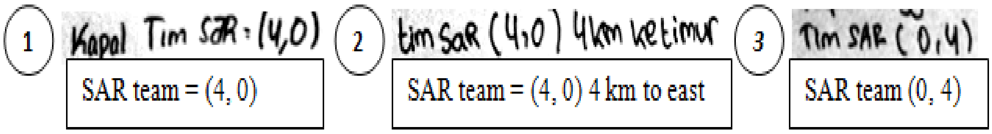


Figure 5.53 The students' answer of locating the SAR team from the lighthouse

For the coordinate of the lighthouse as the origin, more than a half of the students were able to specify it at (0, 0), even some of them sometime forgot to use parentheses (see figure 5.54). It implies they began to understand that the pair of zero represents both the horizontal and the vertical distance of the origin itself since it is located on the two-dimensional. Regardless to it, however, 2 out of 30 students located it at zero, which means they merely consider one parameter.

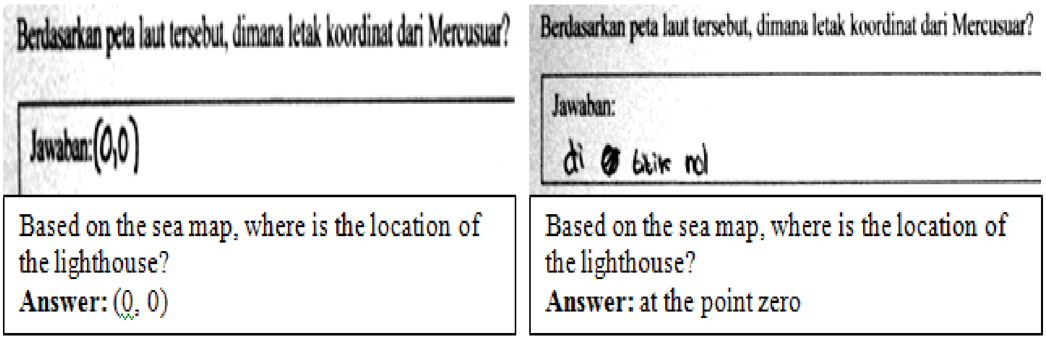
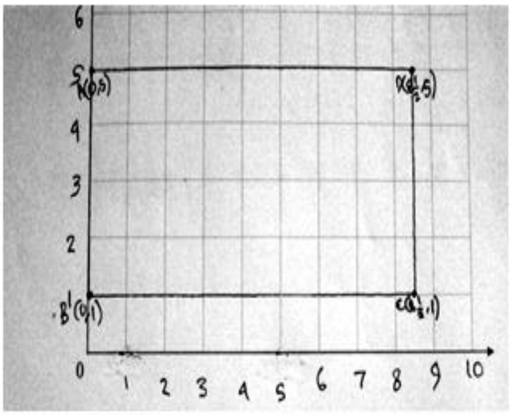


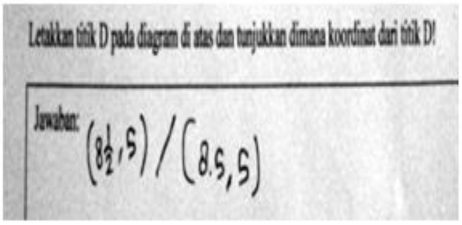
Figure 5.54 The students' answer of locating the lighthouse (as the origin)

Based on the analysis above, it can be concluded that most of the students were able to specify the coordinate of an object on the rectangular plane by using the ordered pair (x,y) including the object located on the x-axis or the object as the origin. However, the common mistake that happens for some students are the absence of the parentheses in writing the ordered pair. For the object on the x-axis, some of them specify the coordinate at (0, x), not at (x, 0), where the x-coordinate represents the horizontal distance.

- **Plotting and Locating Points on the Positive Quadrant**

With the provided coordinate grid, this item asked the students to plot ordered pairs {A (0, 5); B (0, 1); C(8 ½, 1)}. If those three points are connected, we can make rectangle ABCD by finding the coordinate of point D. Surprisingly, 23 out of 30 students plotted those three ordered pairs correctly by considering the first and the second coordinate as the horizontal and the vertical distance respectively (see figure 5.55). Consequently, they could easily find the coordinate of point D, which is at (8 ½, 5) or (8.5, 5) (see figure 5.55).



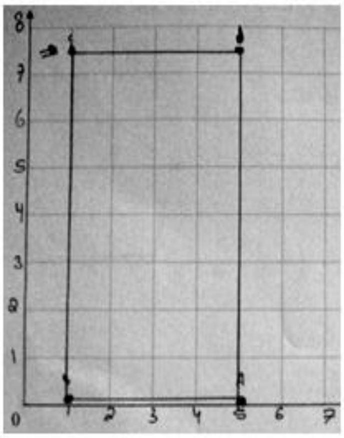


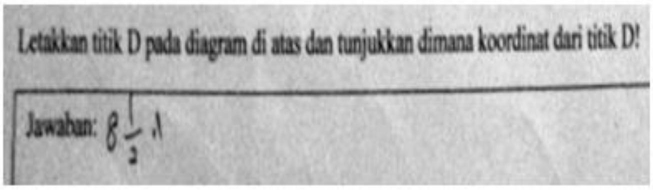
Plot point D on the diagram above and show what the coordinate of point D is!

Answer: (8 ½, 5) or (8.5, 5)

Figure 5.55 The students' answer of plotting A, B, C and determining point D

In contrast, 7 out of 30 students plotted the ordered pairs as (*y*, *x*). It means they considered the vertical distance as the first coordinate, not the horizontal one such that it leads to the incorrect coordinate of point D (see figure 5.56).





Plot point D on the diagram above and show what the coordinate of point D is!

Answer: 8 ½, 1

Figure 5.56 The students' answer of plotting A, B, C and specifying the coordinate of point D as the ordered pair (*y*, *x*)

To sum up, it can be said that most of the students were able to plot ordered pairs (*x*, *y*) on the positive quadrant of the Cartesian plane precisely and specify the coordinate of a point as the ordered pair (*x*, *y*). As a contrary, few students plot the ordered pairs as (*y*, *x*) and specify the coordinate of a certain point as (*y*, *x*).

5. 4. 9 Conclusion of the teaching experiment (cycle 2)

Based on the result of the retrospective analysis of cycle 2, it can be concluded that set of instructional activity based on RME principles helps students to understand the Cartesian coordinate system focused on the positive quadrant. However, the students need a clarification that the alphanumeric grid system as they dealt with in the first lesson has completely different use with the Cartesian coordinate. It was because during the post-test, most of the students employed the Cartesian coordinate instead of the alphanumeric grid system to locate chess pawns on the chessboard.

About the socio-mathematics norms, the teacher has begun to invite the students to propose other different solutions and did not stuck on merely one solution. Regardless to this improvement, however, the teacher sometimes was still used to ask true or false question. Consequently, it could diminish the students' reasoning about a certain problem.

CHAPTER VI
CONCLUSION AND SUGGESTION

This study is mainly aimed at developing a local instruction theory that can foster students' understanding of the positive quadrant of the Cartesian coordinate system. An instruction theory, in short, is a theory on how students can be supported to learn a certain topic such as the Cartesian coordinate system for our case. To contribute such a local instruction theory, this present study summarizes a reflective and prospective component as described further in this chapter. For the reflective component, we focus on answering our main research question. For the prospective component, we propose suggestions and recommendations for further teaching and learning as well as future study within the domain-specific of the Cartesian coordinate system.

6. 1 Conclusion

The general research question in this study is *"how can we support in understanding the positive quadrant of the Cartesian coordinate system?"* To answer this research question, we first describe on how the students use a grid system, particularly an alphanumeric grid system, to identify square region on the grids. Here, the introduction to the use of a grid system becomes a prior activity before learning the Cartesian coordinate. Afterwards, we focus on how the students' investigation of taxicab routes/ distance leads to the reinvention of positive quadrant of the Cartesian coordinate. The notion of horizontal and vertical distance as embedded in the concept of taxicab distance are then employed to know how an ordered pair (x, y) works to locate an object on a rectangular plane using that notion. In the end, we also described on how the students locate and plot the coordinate of a point on the positive quadrant of the Cartesian plane using the ordered pair.

The introduction of a grid system, in this case an alphanumeric grid system, is a primary activity before students learn about the Cartesian coordinate. The grid system itself allows students to conceivably perceive a grid as a collection of cells as rows and columns, rather than as sets of perpendicular lines. In regard to this notion, most students were able to make a system for labeling an arrangement of blocks (city blocks) by using the alphanumeric grid system. In this system, the horizontal rows of the blocks are numbered, while the vertical blocks are labeled with letters/ alphabet. Accordingly, the label of a certain block can be identified as co-ordinates of a letter and a number, such as block 5B or B5. The use of the alphanumeric grid system is also applied for the chess notation system. With the provided numbers and letters, the students were able to specify the location of a chess pawn using a coordinate pair of a letter and a number as they did in determining the label of a block in the previous task.

Since the grid system identifies the location of cells rather than points, the precise location cannot be specified. In the city blocks context, the use of the label blocks is less accurate for locating two objects in the same block. As such, the students put additional information, which are cardinal directions to locate those object precisely. Besides using the label of blocks, the students are guided to employ another system involving distance and it leads to the reinvention of the Cartesian coordinate in the positive quadrant. To reinvent this system, students investigated several *taxicab* routes with the same starting and ending points in the city blocks that result in the same *taxicab* distances. To easily determine the taxicab distance between the starting point (as the origin) and any places in the city blocks, it was used two *taxicab* routes that merely have one turn for each. These routes helped them to easily determine the horizontal and the vertical distance by labeling the horizontal and the vertical gridlines using positive integers. It eventually results in a system looked like the positive quadrant of the Cartesian plane.

Based on the reinvented system above, the students located an object in the city blocks using the cardinal direction (mainly east and north) including its corresponding distance. This result was apparently a good starting point before they deal with an ordered pair (x, y) to specify a certain location. Even the notion of *taxicab* distance is not directly involved, however, the related concept about horizontal and vertical distance are still used to understand how the ordered pair works. Since there were two possibility of interpreting the x- and y-coordinate as the horizontal or the vertical distance, they are guided to make an agreement that x- and y- coordinate represents the horizontal and vertical distance respectively.

The ability of locating and plotting the coordinate of points on the rectangular plane with a certain context turns to be important before working with the formal level of the Cartesian coordinate. Based on a sea map with provided coordinate grid, the students specified the location of the sunken ship by using the ordered pair (*x*, *y*), even the use of parentheses sometimes absence in their work. For the rescue ship located on either x-axis or y-axis, they were able to interpret that one of its coordinate should be zero since its horizontal or vertical distance equals to zero such as (4, 0), (0, 6), etc. In addition, for the location of the rescue ships that has "half" as one of its coordinate, they were also capable to express it into fraction or decimal such as (9 ½, 4), (4.5, 0), etc.

At the formal level of learning the Cartesian coordinate, the students were required to be able to specify and plot the coordinate of a point using the ordered pair (x, y). By providing the coordinate grids in the task, the students had no trouble to plot any ordered pair on the positive quadrant of the Cartesian diagram accurately. It also can be identified that they always plot any ordered pairs from the origin, not from the given previous point. It was because they interpreted the *x*- and *y*-coordinate of the ordered pair (*x*, *y*) as the horizontal and the vertical distance of a certain point from the origin. Similar to the previous result, they had no significant difficulty to specify the coordinate of a point located on the axes or a point that contains "half" as one of its coordinate. Here, to express the "half", they used fractions or decimals in such way it did not result in multiple interpretation. For instance, they used point rather than comma to express the decimal numbers.

6. 2 Suggestions

The starting point of mathematics learning in RME is allowing students to immediately engage with the contextual problems. In this study, contextual problems play important role in teaching either the grid system or the Cartesian coordinate system, but it has to be well anchored with the mathematical goals and instructions. Three out of six instructional activities in this study use the context of city blocks. Several literature reviews revealed that the context of city blocks, particularly in a certain neighborhood in USA, fruitfully helpful to introduce the informal level of learning the Cartesian coordinate to primary school students. Indeed, the activities related to the *taxicab* routes/ distances, which helps students to reinvent the positive quadrant of the Cartesian coordinate and to understand how the ordered pair (*x*, *y*) works, are also embedded in the context of city blocks.

Regardless the powerful role of the context city blocks in learning coordinate, this fact becomes a constraint for the researcher itself. It is because this context is not familiar for Indonesian student such that for some aspect, it leads to misunderstanding. For instance, in contrast with the definition of city blocks, some students in the pilot experiment perceived one block as one building. Even this finding can be resolved by showing a detail video and explanation of city blocks, the primary school students may need a context that experientially real in their life. However, if it sounds difficult to find the proper context for Indonesia students, we can still use this context as long as the illustration and the explanation about how city blocks look like is completely clear for students.

According to the teaching experiment, it can be identified that the students were able to locate and plot the coordinate of any points on the positive quadrant merely within five lessons. For further learning, perhaps teachers or designers can expand it into the complete four quadrants of the Cartesian coordinate. In relate to the designed-instructional activities in this study, the introduction to the four quadrants can be given in the sunken ship problem. It can be told that the sea map is expanded and someone needs to specify the coordinate of other ships spread out in the four quadrants. By merely giving the axes without the notated numbers, the teacher can provoke the students to notate an appropriate number in the left part of the *x*-axis and the lower part of the *y*-axis from the origin. Here, students are expected to come up with idea of negative integers. To deal with the ordered pair (*x*, *y*), the use of the horizontal and the vertical distance still can be used, but in a agreement that the distance with negative numbers has the opposite direction with the distance with positive numbers.

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APPENDIX A
Teacher Interview Scheme

1. Teacher's educational background and experience
   - What is the teacher educational background? Is he/she graduated bachelor or master? In what subject?
   - How long does the teacher teach in primary school?
   - How many times does the teacher teach 5^{th} grades?
   - What is the teacher's specialization or favourite subject?
   - What is the teacher's opinion about teaching primary students?

2. About PMRI
   - Have the teacher ever heard about PMRI?
   - What does the teacher know about PMRI?
   - What experiences that teacher has about PMRI?
   - Does the teacher have ever implemented PMRI approach?
   - How often does the teacher implement PMRI in her/ his class? What is the teacher opinions about her/his experience of implementing PMRI?
   - What are the immediate benefits or results that the teacher or students have after the implementation PMRI?
   - How often does the teacher use contextual problem to introduce a new mathematics topic?

3. Classroom management
   - Is there any certain rules used in the classroom related to how to ask questions, give opinions, and answer questions?
   - How often does the teacher conduct pair-discussion, group-discussion, or classroom-discussion in mathematics class?
   - How does the teacher make the group?
   - How does the teacher manage the discussion?
   - Have students ever made a poster after conducting group discussion?
   - Do the students are accustomed to explain their work/ ideas in front of the class?
   - What students do in response to her/his friends' explanation?

4. Teaching about coordinate system
   - Does the teacher have experience in teaching the Cartesian coordinate system?
   - How many meetings does the teacher need to teach that concept?
   - What difficulty does the teacher have in teaching that concept?
   - What difficulty do students have in learning that topic

APPENDIX B
Classroom Observation Scheme

1. Classroom condition
   - How many students are there in the class?
   - How do the seating arrangement of the students?
   - Is it possible to rearrange the seating arrangement into group?
   - Are there any tools that can be used to support the designed learning?

2. The culture in the classroom
   - How does the teacher open the lesson?
   - Is there any certain rules used in the classroom related to how to ask questions, give opinions, and answer questions?
   - How do students interact to each other?
   - How do students participate in the lesson?
   - Are the students can be categorized as active learners? Or passive one?
   - How does the teacher give guidance to students who have difficulty?
   - How does the teacher establish the closeness with his/her students?
   - What students do in response to teacher's explanation?
   - What students do while having discussion in pairs or in groups?
   - How does the teacher deal with any irrelevant behavior of the students?
   - How does the teacher close the lesson?

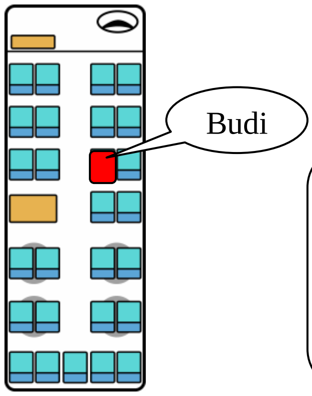
3. Teaching and learning process
   - How does the teacher teach mathematics in the classroom? (explaining/ demonstrating or promoting a discussion)
   - What is teaching approach that the teacher used in mathematics lesson?
   - How much are the portion of teacher explanation?
   - Does the teacher follow the textbook during the lesson?
   - Does the teacher give different students' worksheet from the textbook?
   - How does the teacher deal with time management?
   - How often do students work in groups or individually?
   - Is there any pair discussion, group discussion, or class discussion?
   - How does the teacher manage the discussion? Does the teacher guide the students or just sit and observe what the students do?
   - Does the teacher stimulate the students to propose different solution?
   - How does the teacher facilitate different solutions of the students?
   - Does the teacher discuss with the students about elegant, efficient, and sophisticated solutions?
   - How does the teacher give the formative assessment to the students?

APPENDIX C
PRE-TEST

Name: **Date:**

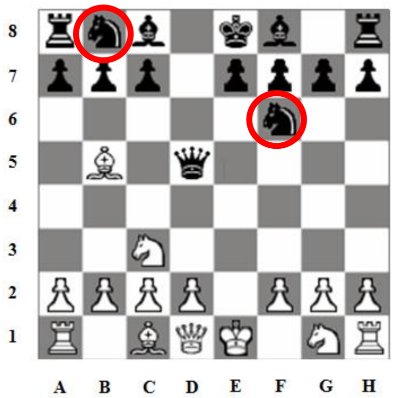
Answer the following questions!

1. Look at the seat map of a bus below!



Budi buys a ticket bus with destination from Palembang to Lampung. Where is Budi's seating written on the ticket below?

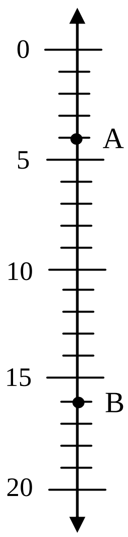
2. The figure below is a system for determining the location of a pawn

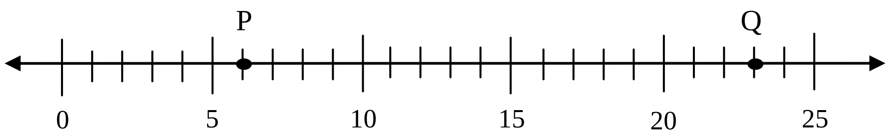


Where is the position of the "*black Knight*"?

Answer:

3. Look at the number lines below!

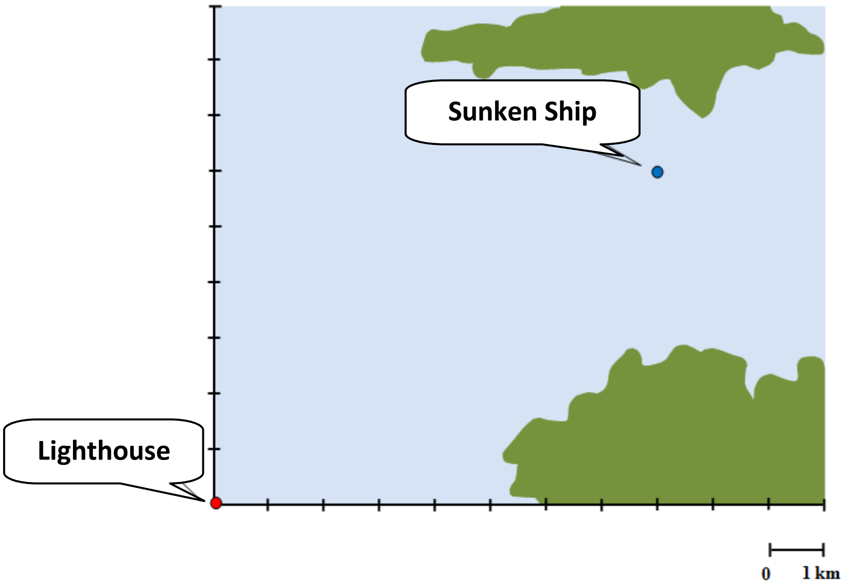




Dina will measure the distance between points A and B, while Anton will measure the distance between points P and Q. Who will get the longer distance? Explain your answer!

Answer:

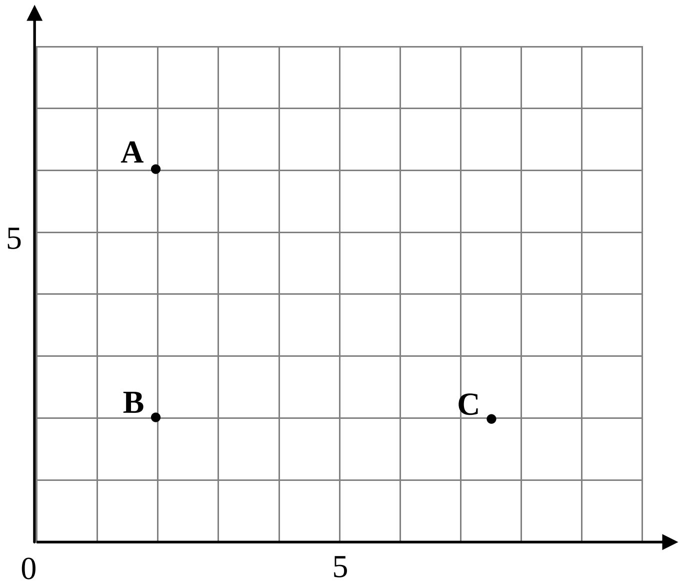
4. Look at the sea map below!



What is the location of the ship from the lighthouse based on the map above?

Answer:

5. Look at the diagram below!



a. Ari want to make rectangle ABCD. Plot point D on the diagram above and where the location of point D is.

Answer:

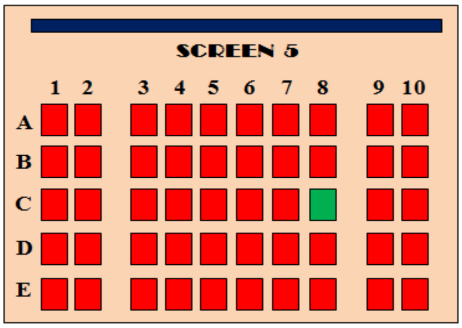
b. Ari has point E at (9,7). Plot point E at the diagram above!

APPENDIX D
POST-TEST

Name: **Date:**

Answer the following questions!

1. Look at the seat map of a cinema below!



Based on the seat map, where is Arya's seating (shown by the green block)?

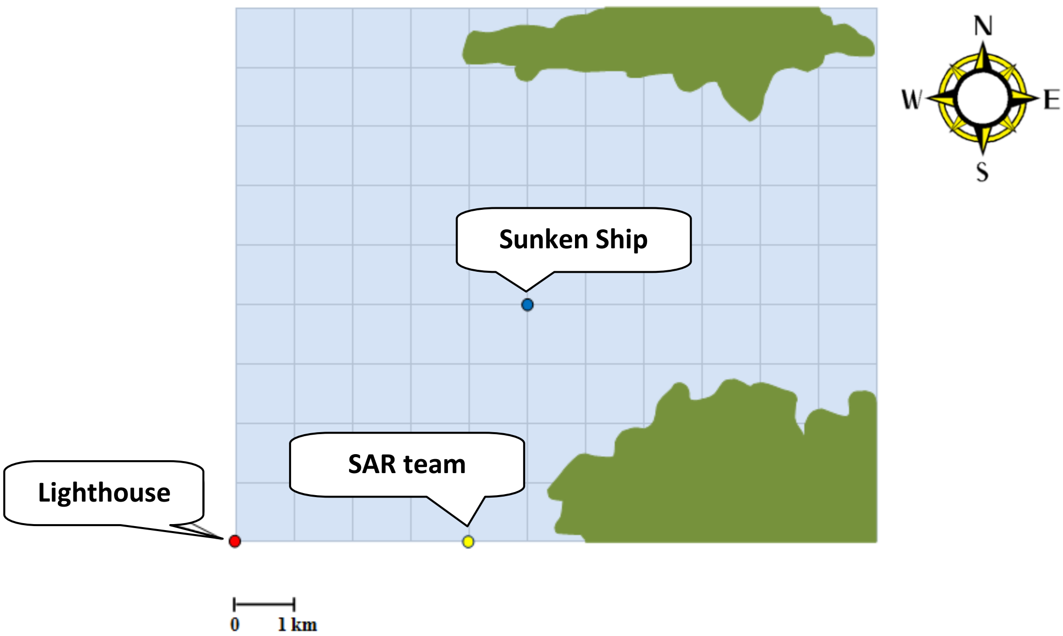
Answer:

2. Look at the figure of the chessboard below!

Where is the position of the "*Black Queen*" and the *"White King"*?

Jawaban:

3. Look at the sea map below!



a. Based on the sea map above, where is the coordinate of the sunken ship and SAR team if they are viewed from the lighthouse?

Answer:

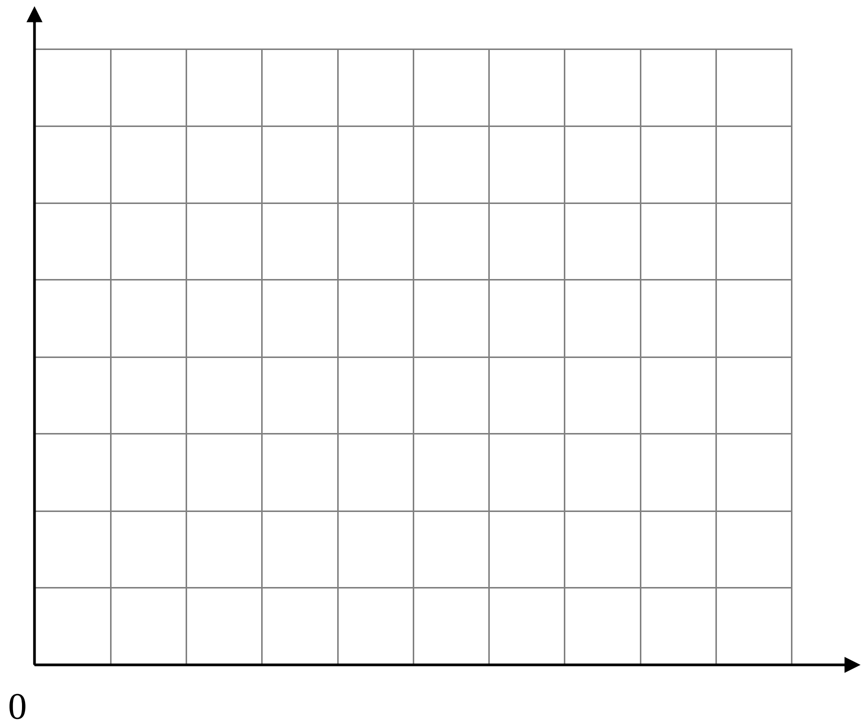
b. Based on the sea map above, where is the location of the lighthouse if it is viewed from the lighthouse itself?

Answer:

4. Look at the conversation below!

Bima : There are three coordinates, namely A (0, 5); B (0, 1); dan C ($8\frac{1}{2}$, 1). From those three coordinates, we can make rectangle ABCD if the coordinate of point D can be determined

Nabila : Let's find the coordinate of point D!



Plot point D on the diagram above and what is the coordinate of point D!

Answer:

APPENDIX E
STUDENT WORKSHEET

Good morning,

Recently, the local government of South Sumatera announces that they will develop a satellite city named Jakabaring. The city will be different with the other cities in Indonesia because it will adopt a system of ***City Blocks*** like in Barcelona, Spain.

Photograph of City Blocks in Barcelona

Sumber: http://www.shutterstock.com/s/residential-district/search.html?page=1&inline=213033817

Street Map of City Blocks in Barcelona

Sumber:http://w20.bcn.cat/Guiamap/Default_en.aspx#x=30982&y=82638&z=2&w=973&h=647&base=GuiaMartorell

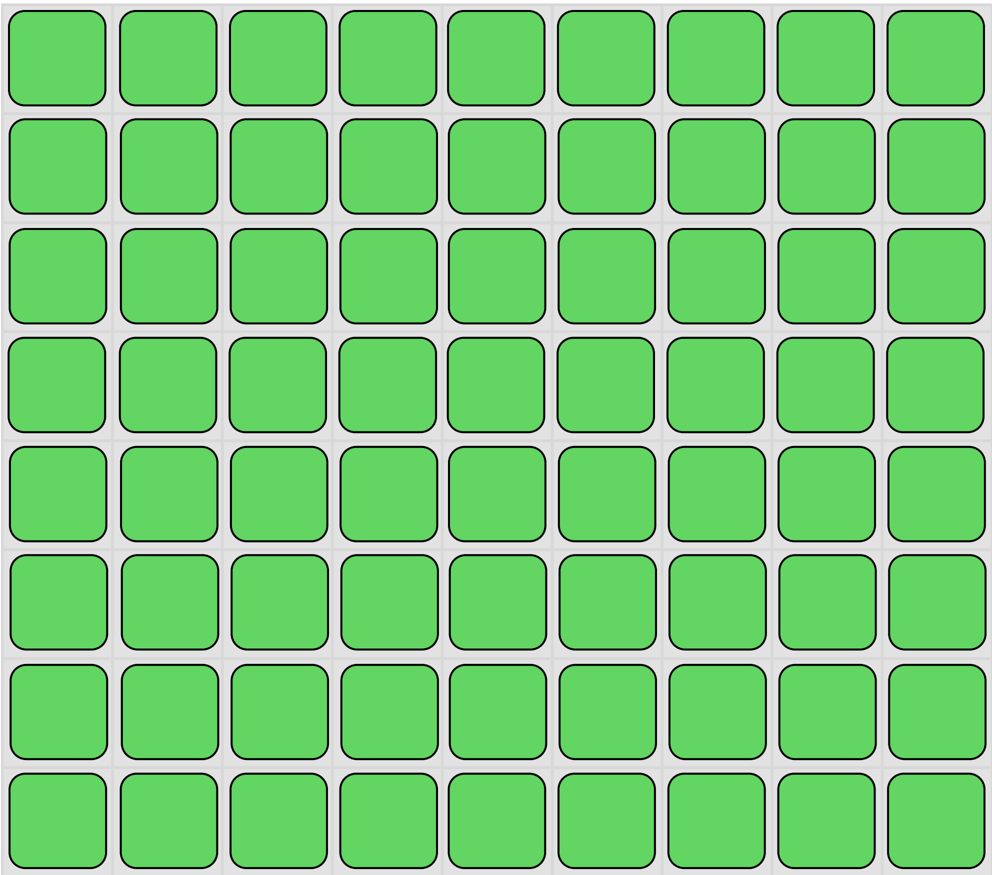
Worksheet 1

Groups : 1) …….. **3) ……….** **5) ………**

2) …….. **4) ……….**

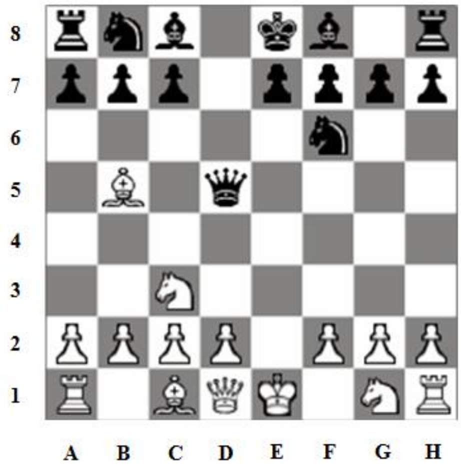
At the beginning of the project, the government does not give the names for the streets, yet they will label the blocks. Make a system of labeling blocks that can help people, who are getting around the city, find the location of a certain block quickly and easily!

City Blocks Jakabaring



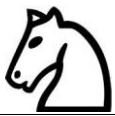
Write your system on the given paper poster and present your poster in front of the class!

Where are the locations of "these" chess pawns?



Sumber:http://commons.wikimedia.org/wiki/File:AAA_SVG_Chessboard_and_chess_pieces_03.svg

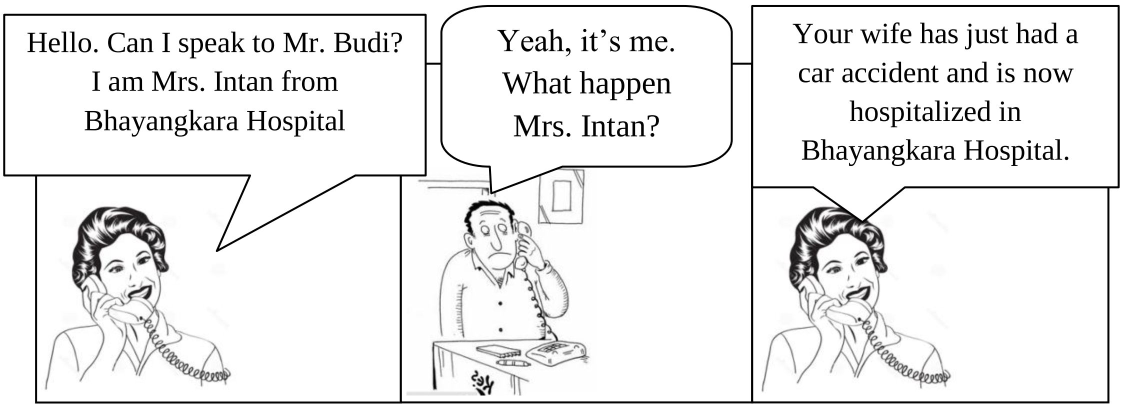
Complete the table below!

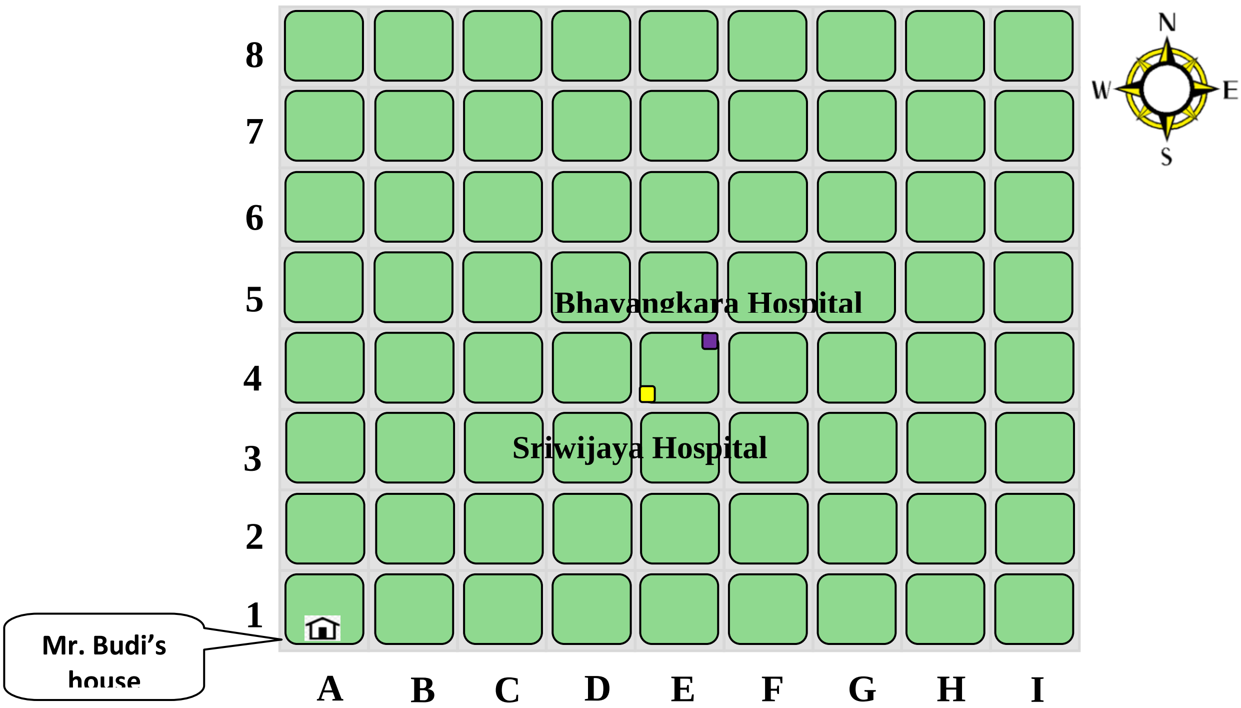
No.	Chess Pawns	Location
1.	White Knight 	
2.	Black Knight 	
3.	Black Queen 	
4.	White Bishop 	
5.	Black Rook 	

Worksheet 2

A Ten years later, Jakabring City has been expanded into a big city and there are many public places in the city, one of which is hospitals.

Perhatikan petikan percakapan di bawah ini!



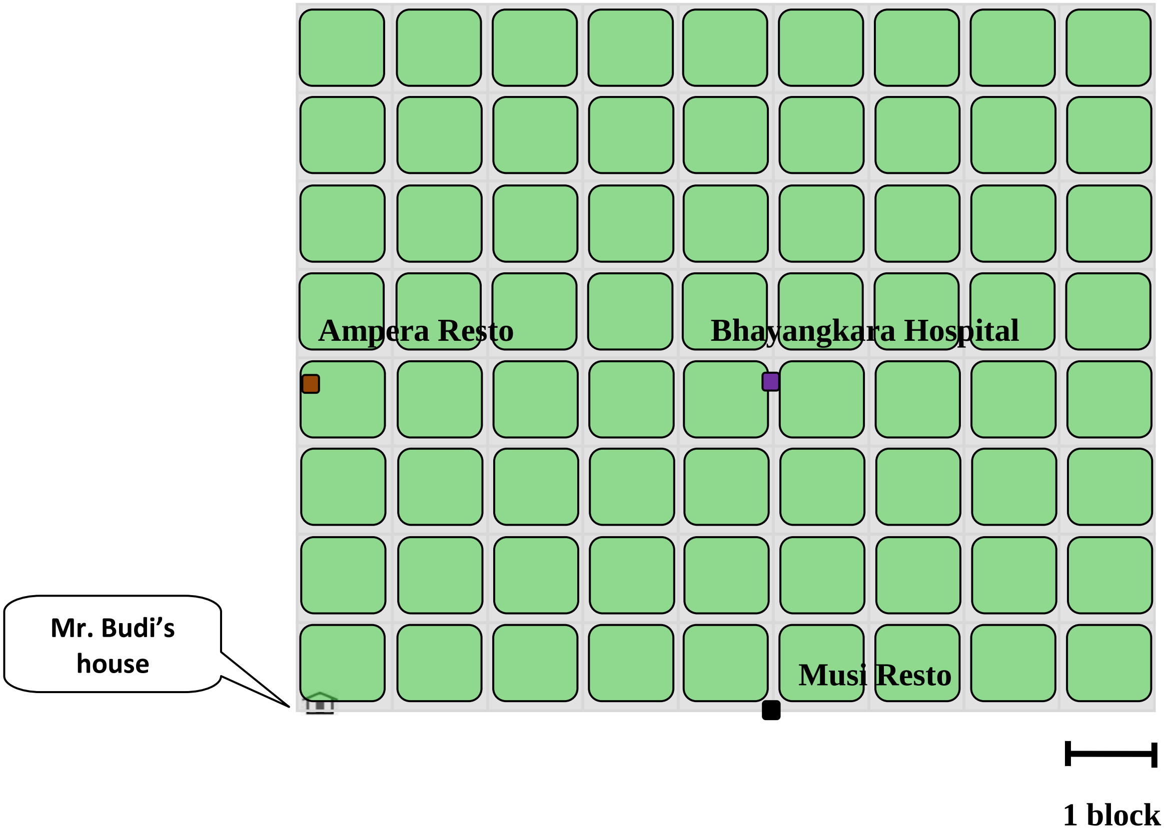


Suppose you are Mrs. Intan, how will you tell the precise location of Bhayangkara hospital to Mr. Budi?

Location

B After receiving the call, Mr. Budi went immediately to Bhayangkara hospital.

Based on the street map below, make a shortest route that Mr. Budi probably takes from his house to Bhayangkara hospital!



1 Based on the route above, what is distance (in blocks) that Mr. Budi's passed to arrive at Bhayangkara hospital?

Answer:

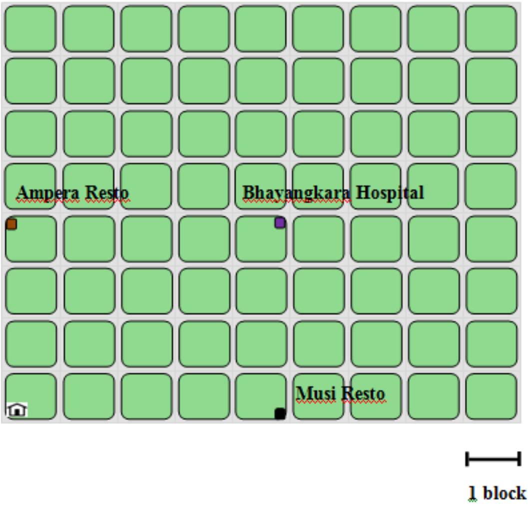
2 Present your work in front of the class with details:

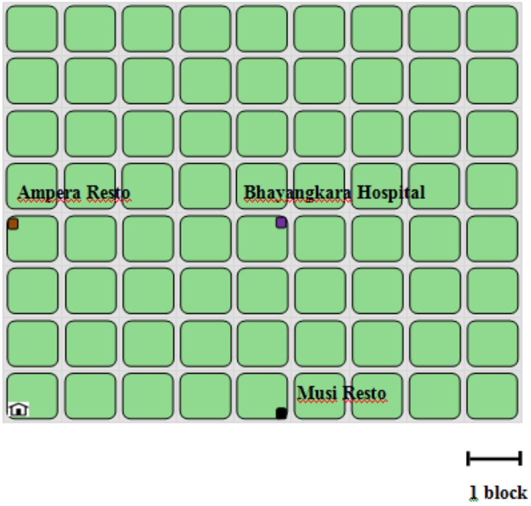
- Draw the shortest route that you have been chosen
- Show how did you determine the distance (in blocks) between Mr. Budi's house and Bhayangkara hospital

3 What can you conclude about the distance covered by those routes that have been presented by your fellow students?

Answer:

4 Draw two shortest routes from Mr. Budi's house to Bhayangkara hospital that merely need minimal turn!





5 For each route above, what is the horizontal and the vertical distance? What the results are the same for both routes above?

Answer:

6 What can you conclude about the relationship between the total distance (the answer of your first question) and its horizontal and vertical distance (the answer of your fifth question)?

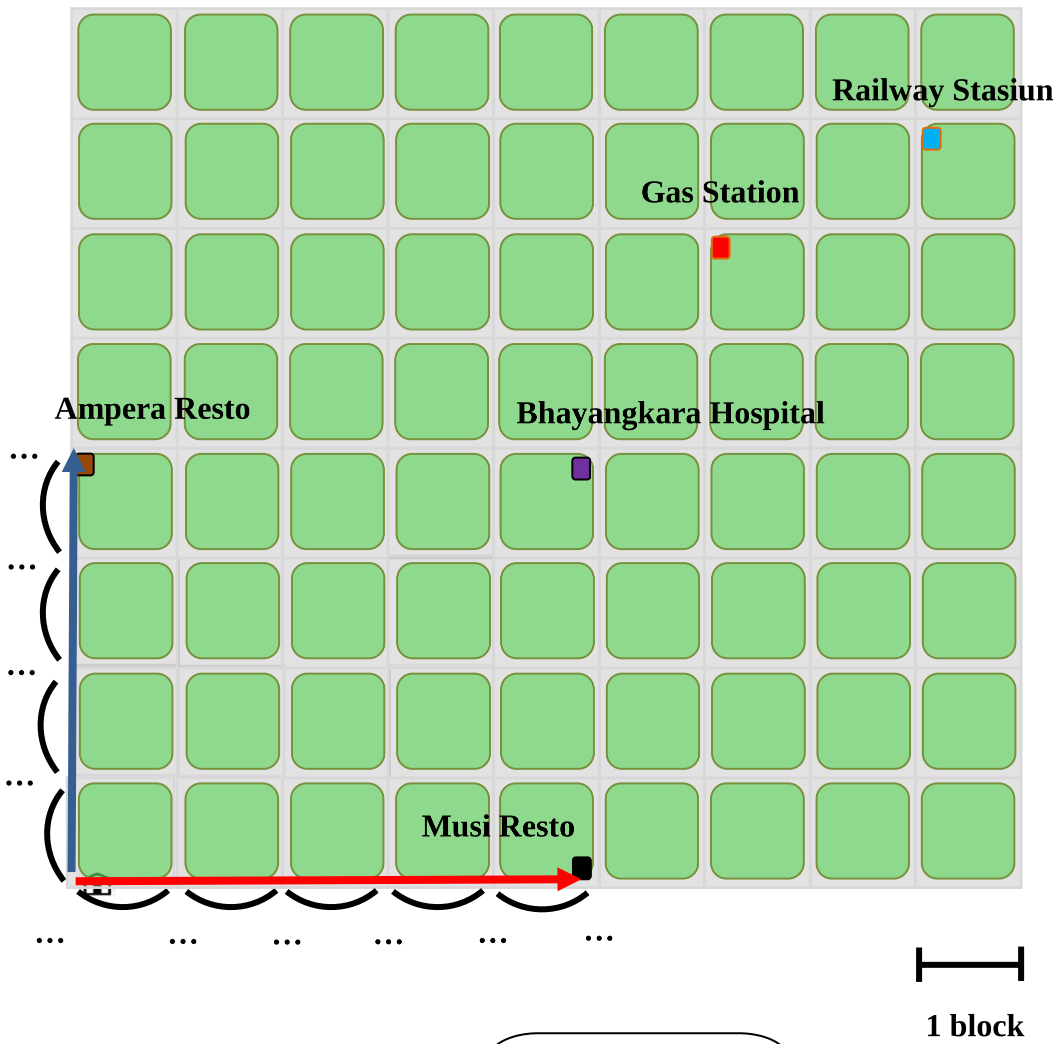
Answer:

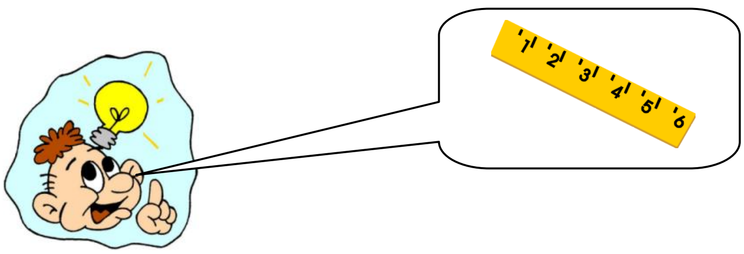
Discussion

Making a system that can be used to determine the shortest distance between Mr. Budi's house to **any places** (Musi Resto, Ampera Resto, Bhayangkara hospital, Gas Station, Railway Station) easily!

Hint

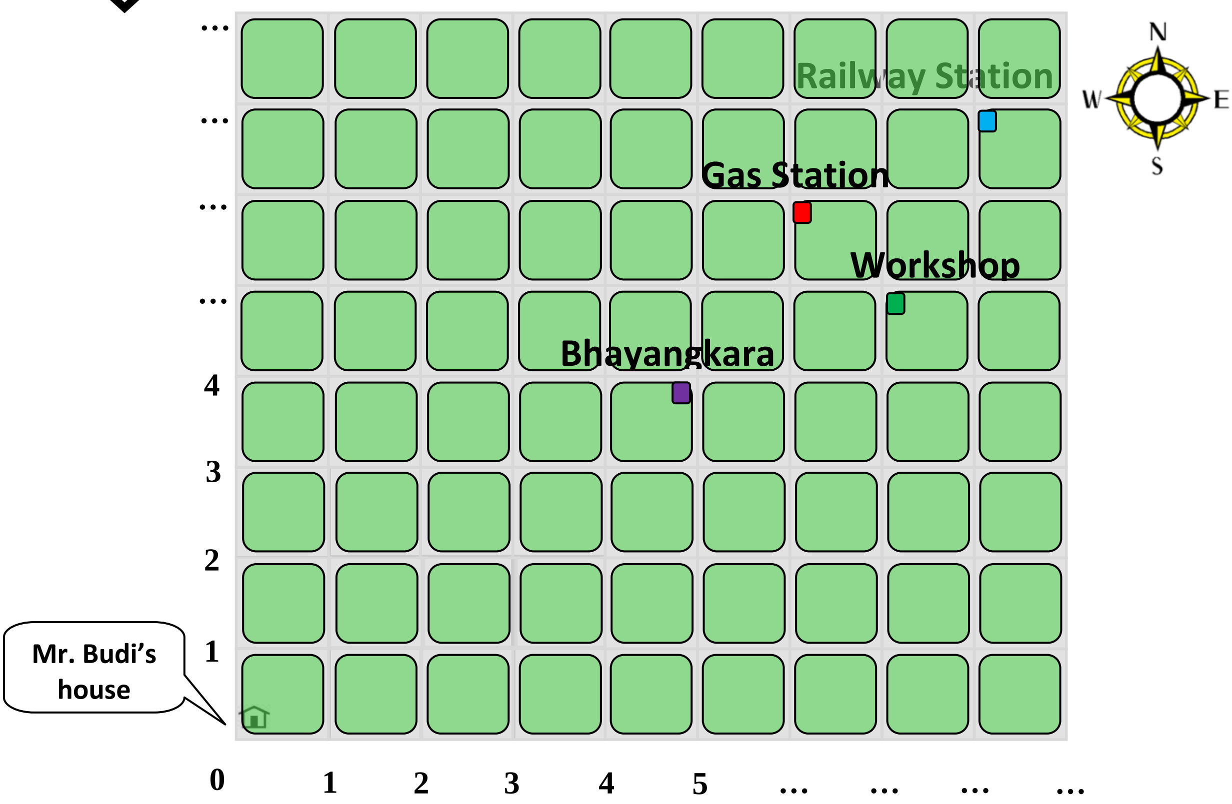
Use the idea of the relationship between the total distance and its horizontal and the vertical distance!





Worksheet 3

A Look at the street map of Jakabaring city blocks below!



Mr. Budi and family will go to Lampung to have vacation by Sriwijaya Train Express. Before arriving at Kertapati Station, their car machine has trouble such that they need to go to a nearest workshop.

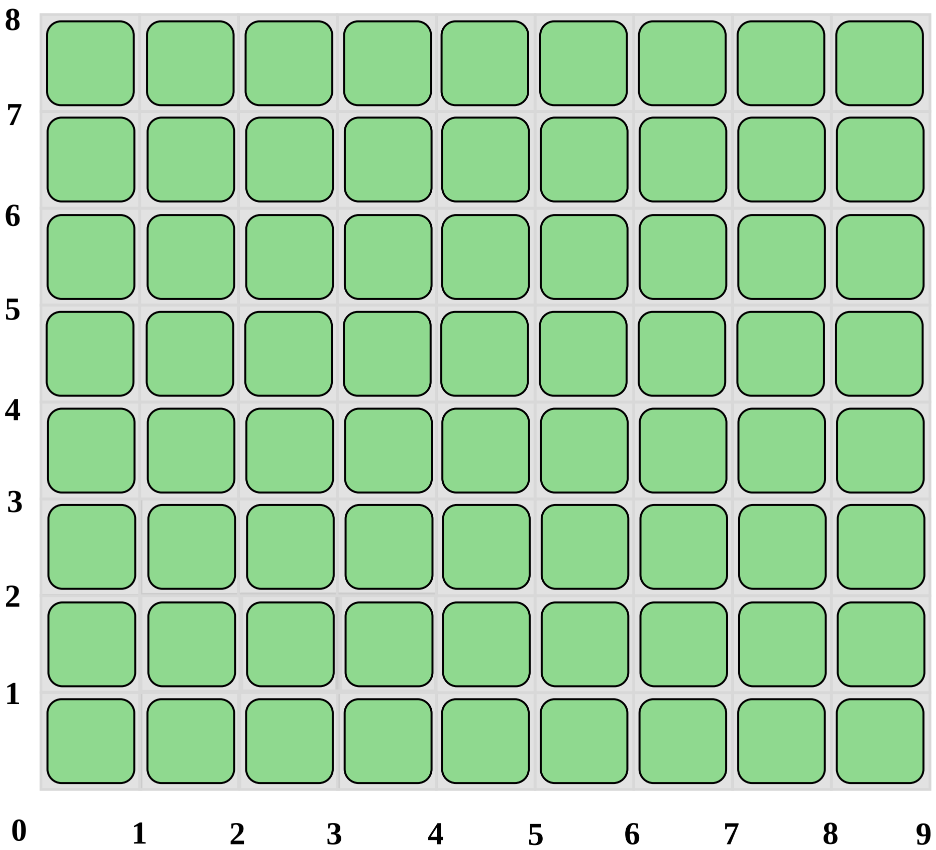
Based on the street map above, where is the location of the workshop and Kertapati railway station from Mr. Budi's house?

Answer:

B

A fire accident happened in some blocks in Jakabaring city due to short-circuit. The electricity powers are turned off and police line is installed surrounding the burned blocks.

The police line is installed on four street lamp towers located at the coordinate (1,1); (7,1); (7,5) and (1,5). Plot the location of those four lamp towers including its police line on the street map of Jakabaring!



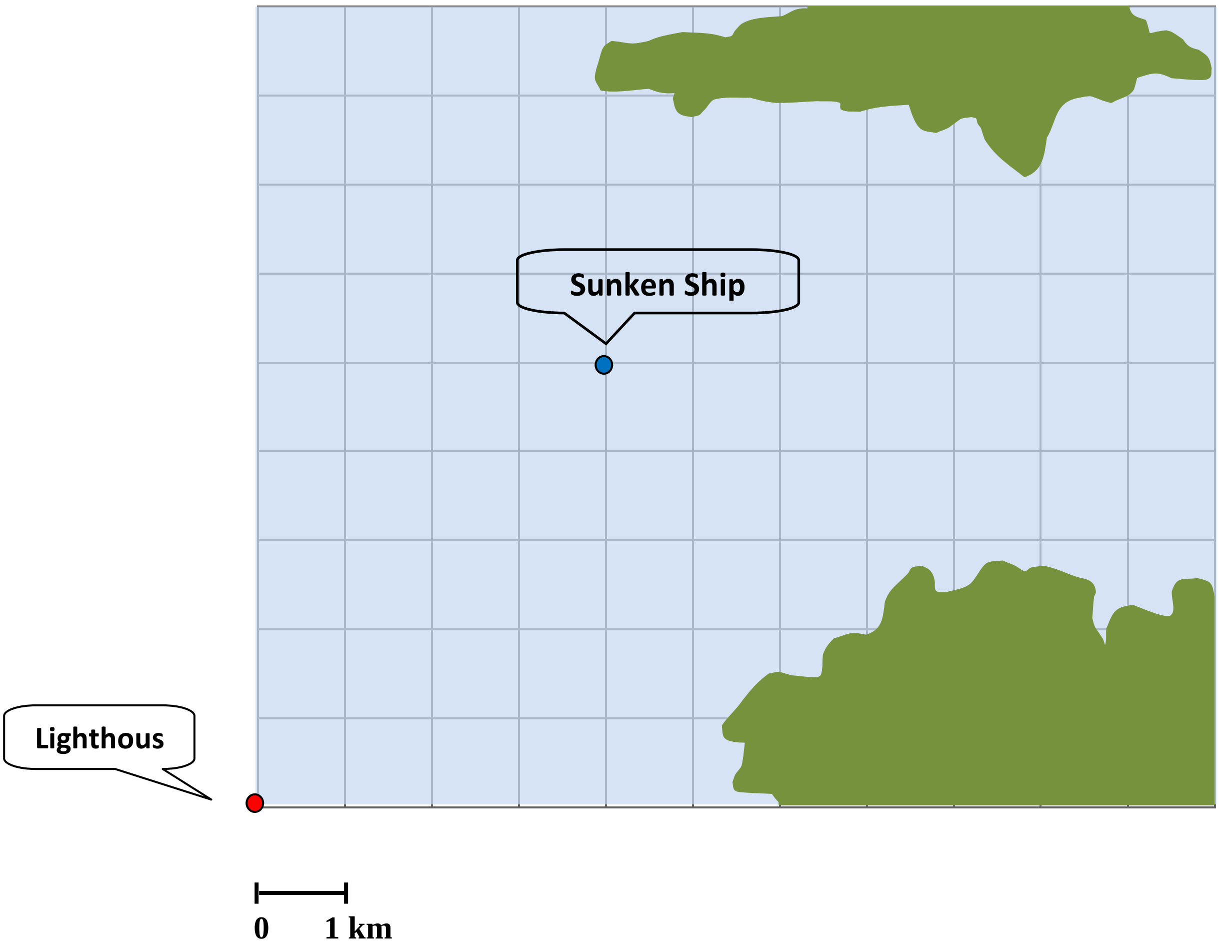
Based on the result above, give the shaded area for the burned blocks!

Worksheet 4

Srwijaya Ship has voyaged in the Java Sea and suddenly has trouble with the machine. Shortly, the backside of the ship began to sink such that the captain instructs his crew to ask for a help from SAR team in the lighthouse by sending the coordinate of their ship.

Illustration

1. Suppose you are the crew of the ship. Based on the sea map below, what is the coordinate of your ship if it is viewed from the lighthouse?

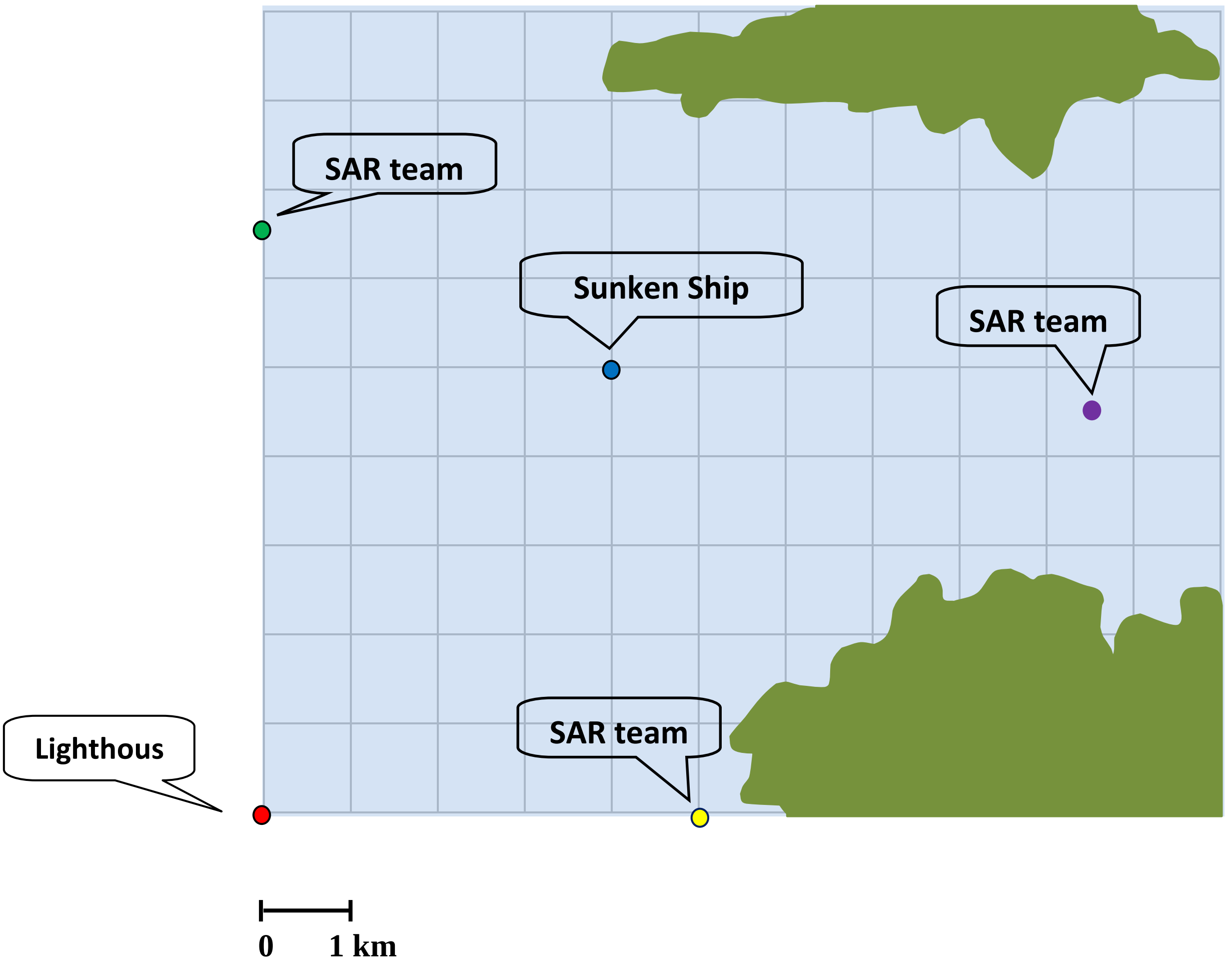


Answer:

After getting the information from the crew of Sriwijaya Ship, the lighthouse officer sends three SAR team (team A, B, and C) to do evacuation. Before conducting the evacuation, those three teams report the coordinate of their each position.

Illustration

Based on the sea map below, what is the coordinate of those three SAR teams?



Answer:

Worksheet 5

A

Students in class 5A of Pusri Primary School are playing "secret quadrilateral" game with the following rules.

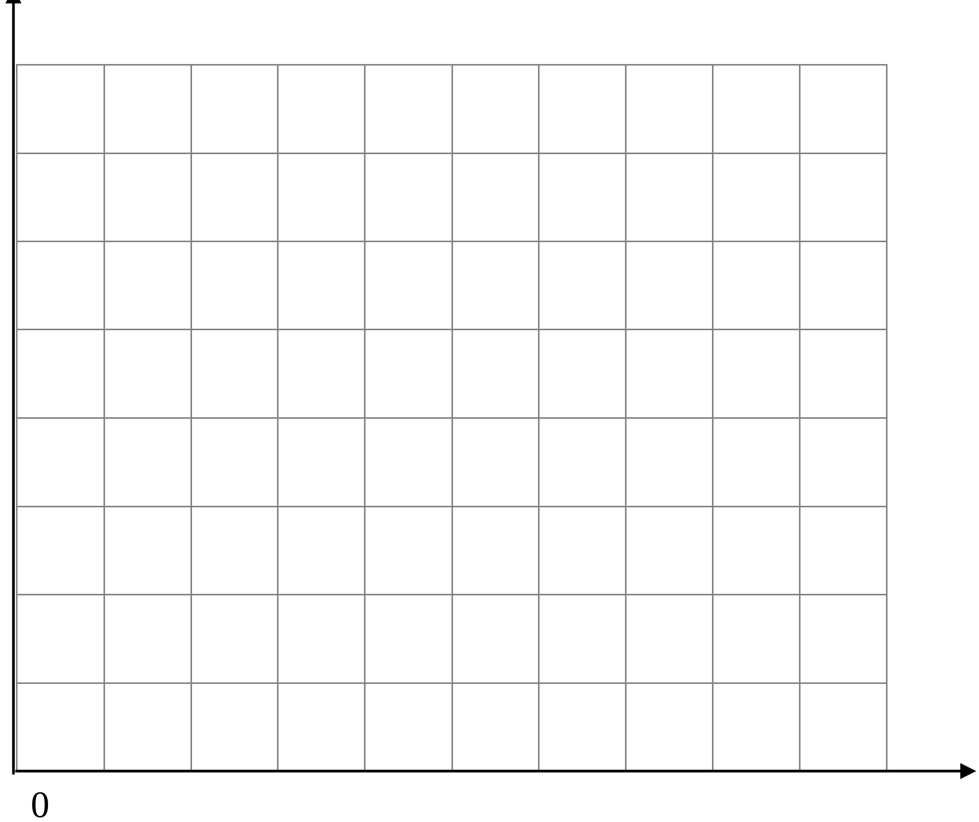
"SECRET QUADRILATERAL"

1. The first player determine four coordinates of a secret quadrilateral
2. The second player plot those four coordinates on the given diagram and determine the name of the formed quadrilateral

Hafiz and Sari are playing "secret quadrilateral" game as shown in the following conversation.

Hafiz : There are four coordinates written on the secret message, namely A (0, 2); B (7, 2); C (10, 6); D (3, 6)

Sari : Let's see what quadrilateral figure that is formed if those four coordinates are connected!



Based on the diagram above, what is the name of the quadrilateral figure?

B

The activity in class 5A is continued with "secret coordinate" game with the following rules.

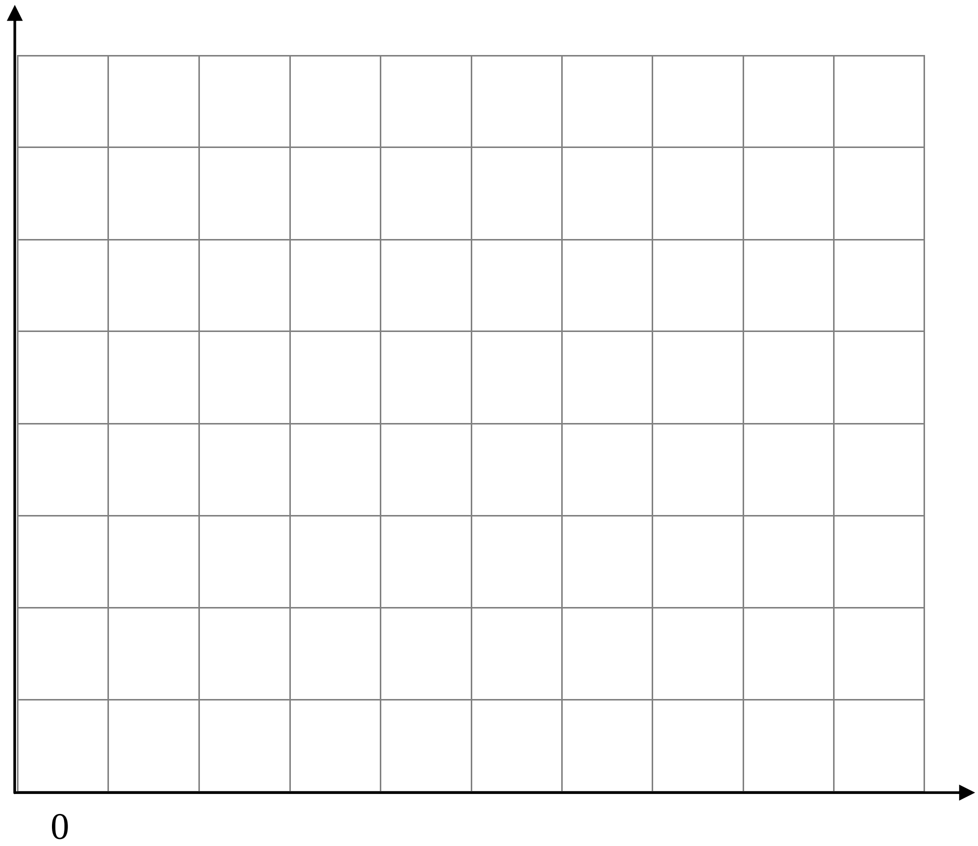
"SECRET COORDINATE"

1. The first player determine three out of four coordinates of a certain quadrilateral
2. The second player plot those three coordinates on the given diagram and determine the forth coordinate

Rif'at and Nadya are playing "secret coordinate" game as shown in the following conversation.

Rif'at : There are three coordinates, K (0, 3); L ($2\frac{1}{2}$, 0); dan M (8, 3). From those three coordinates, we can make a kite KLMN if we can determine the coordinate of point N

Nadya : Let's find the coordinate of point N!



Based on the diagram above, what is the coordinate of point N?

APPENDIX F

Duration of Lesson

2 × 35 minutes

Learning Goals (1st Session)

1. Students are able to make a system of organizing things, a system of labeling blocks (city blocks), using a grid system involving rows and columns
2. Students are able to understand that locating an object in the grid system by only using one parameter (rows or columns) is less efficient
3. Students are able to understand that to locate an object in the grid system at least needs two parameters (rows and columns)
4. Students are able to understand that the grid system is used to identify locations in the form of region (cell)

Learning Goals (2nd Session)

1. Students aware that the alphanumeric grid system is typically used in the conventional chess notation
2. Students are able to identify the location of particular pawns on the chessboard using the alphanumeric grid system
3. Students are aware that the alphanumeric grid system can be used to label the blocks

Materials

Student's worksheet, picture of city blocks, a street map of city blocks, and picture of chessboard form of region

Learning Activities of 1st Session *(45 minutes)*

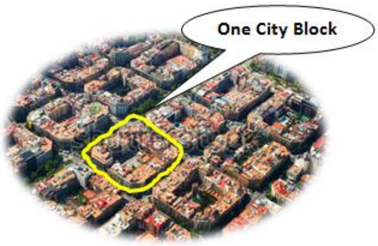
Overview of the activity:
Students in a group of 4-5 people are asked to make a system of labeling blocks (in city blocks) that allows them to perceive the city blocks as a collection of cells (looks like a rectangular grid) consisting of rows and columns. The system they make is called a grid system that is used to identify

Orientation *(5 minutes)*

1. Tell a story about planning of adopting the system of city block for a new satellite city in South Sumatera of Indonesia as described below.

Recently, the local government of South Sumatera announces that they will develop a satellite city named Jakabaring. The city will be different with the other cities in Indonesia because it will adopt a system of City Blocks like in Barcelona, Spain.

2. Show the photograph and the street map of the city blocks in Barcelona.



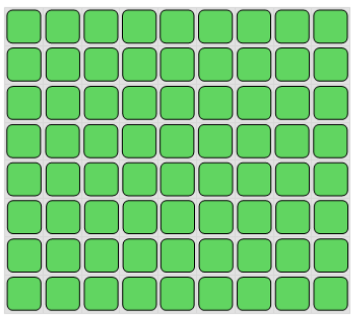
3. Describe the meaning of city blocks, namely a rectangular area in a city surrounded by streets and usually containing several buildings such as house, mall, apartment, etc.
4. To have students get a clearer depiction of city blocks, ask question: *have you ever heard about city blocks in a game?*

Predictions of Students' Response	The Proposed Actions of the Teacher
Some of students refer to *Simcity* or *City Blocks* or the other games	Ask students to tell how the city blocks on that game look like
None of them know about the game related to city blocks	Give the example: *Simcity* or *City Blocks* game and describe that each block contain several buildings such as house, mall, apartment, etc

Working Group and Classroom Discussion *(30 minutes)*

5. Distribute the student's worksheet and ask students in group of 4-5 people to solve the given problem and to make a poster of their work for 20 minutes.

At the beginning of the project, the government does not have the names for the streets yet. Instead, they will label the blocks to start their project. Discuss with your group to make a system of labeling the blocks that can help people, who are getting around the city, find the location of a certain block quickly and easily!



Informal Assessment

This problem assesses students' ability to identify the city blocks as rows and columns and to indicate whether they can make a grid system involving rows and columns.

6. Assist the working group by supporting students to find or reason about a better system for labeling the blocks.

Example of Students' Solution	The Proposed Actions of the Teacher
Label the blocks using ordinal number starting from 1 to 72 horizontally or vertically	Asking: • *Do the system help people, who are getting around the city, find a certain block (such as block 25) without passing through all blocks?* • *Do people not confuse find block 25 using that system?*
Perceive the city blocks as rows and columns, which are labeled with letters and numbers or the other way around	• Asking: *Do the system help people, who are getting around the city, find a certain block without passing through all blocks?* • Ask students to point blocks 7C or C7 (conditionally) quickly
Perceive the city blocks as row and column, which are labeled with two different numbers	• Asking: *Do the system help people, who are getting around the city, find a certain block without passing through all blocks?* • Ask students to point blocks 7.3 or 3.7 (conditionally) quickly

7. Ask two representational groups to present their poster for and ask other groups to give remarks or questions related to the presentation (20 minutes)

Student's Work	The Proposed Actions of the Teacher
The group use ordinal number to label the blocks (either horizontally or vertically)	Asking: • *Do the system help people, who are getting around the city, find a certain block (such as block 36) without passing through all blocks?* • *Do people not confuse find block 25 using that system?* Do not judge that the labeling system only using the ordinal number is less efficient
The group use other systems involving rows and columns to label the blocks	Asking: *Do the system help people, who are getting around the city, find a certain block (such as block 8F or 8.6 depends on students' solution) easily and quickly without passing through all blocks?*

8. Ask students which system that they like the most and its reason

Learning Activities of 2nd Session *(25 minutes)*

Overview of the activity:

The alphanumeric grid system is a system with an grid with assorted numbers and letters employed to identify regions in the form cell. Students in groups are asked to tell the location of some pawns on the chessboard using the chess notation, which is the example of the use of the alphanumeric grid system. In the end of the lesson, students are asked to reason whether this system is applicable for labeling the blocks

Orientation *(5 minutes)*

9. Talk about students' experience of playing chess and ask: do you know about the chess notation, namely a system used to tell the position of pawns on a chessboard?

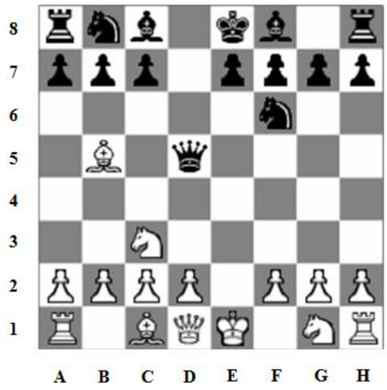
<table>
<tr><th>Predictions of Students' Explanation</th><th>The Role of the Teacher</th></tr>
<tr><td>The vertical columns (from the white's left or from the white's right) are labeled A through H, while the horizontal rows (from the white's or from the black's side) are numbered 1 to 8</td><td rowspan="3">• Accept all of possible explanation and clarify the conventional chess notation, which has been agreed among all chess organization
• Explain the system for the conventional chess notation, but does not tell how the system works on describing the location of a certain pawn</td></tr>
<tr><td>The vertical columns of squares (either from the white's left or from the white's right) are numbered 1 to 8, while the horizontal rows (either from the white's side or from the black's side) are labeled A through H</td></tr>
<tr><td>None of students know about the system for chess notation</td></tr>
</table>

10. Explain the system for the conventional chess notation, but not telling how the system works on locating a certain pawn

To describe the position of paws, we need to identify each square of the chessboard by a unique coordinate pair of a letter and a number. The vertical columns of squares from the white's left are labeled A through H, while the horizontal rows from the white's side are numbered 1 to 8

Working Group and Classroom Discussion *(15 minutes)*

11. Distribute the student's worksheet (see Worksheet 1, Appendix E) and ask students in pairs to do the worksheet for about 10 minutes



No.	Pawns	Locations
1.	White Knight	
2.	Black Knight	
3.	Black Queen	
4.	White Bishop	
5.	Black Rook	

About the Assessment

This problem assesses students' ability to identify the location of particular pawns on the chessboard using the alphanumeric grid system.

12. Let students to work with their own pairs and accept all possible solution, which will be discussed deeply in the section of class discussion
13. Ask two pairs present their work on telling the location of a certain pawns and ask other pairs to give remarks or question related to the presentation (5 minutes)

Students' Answer	The Proposed Actions of the Teacher
A coordinate pair of a number and a letter respectively describes the location of a certain pawn. For instance, the white knights are located at squares 3C and 1G	Accept all of possible explanation • If student A move the white queen to F5, ask a student to appoint the location of F5. Similarly, If student B move the black queen to 5F, ask a student to appoint the location 5F • Ask question whether F5 and 5F position the pawns in the different squares or not
A coordinate pair of a letter and a number respectively describes the location of a certain pawn. For instance, the white knights are located at squares C3 and G1	
For each two identical pawns such as white knight, black knight, white bishop, or black rook that have two locations, some students only tell one location	Remind students that there are some identical pawns

Closing *(5 minutes)*

14. Back to labeling city blocks problem and ask students whether the system used for the chess notation can be applied for the blocks or not (in the case

that none of students come up with of the use of the alphanumeric grid system for labeling the blocks)

Students' Response	Actions of the Teacher
Agree that the developed system for the chess can be applied for the blocks	Ask students to show how the system works for labeling the blocks
Do not agree that the developed system for the chess can be applied for the blocks	• Ask students to show why the system does not work for the blocks • Ask another student to argue by showing how the system works for labeling the blocks

About the Assessment

This question assesses students' reasoning whether they aware that the alphanumeric grid system can be used to label the blocks.

10. Lead students to make a conclusion about the use of the alphanumeric grid system and invite them to reflect on what they already learned during the lesson.

Duration of Lesson

2 × 35 minutes

Learning Goals

Students are able to make a system for determining the *taxicab* distance between an origin position and a certain object. This main goal can be specified into sub-goals as follow.

1. Students are able to realize that the alphanumeric grid system needs refinement to locate two objects or more that are laid on the same cell
2. Students are able to find the *taxicab* distance between two objects in the city blocks
3. Students are able to look for different shortest paths on the map of city blocks with the same starting and ending points
4. Students are able to understand that any paths with the same starting and ending points cover the same *taxicab* distance
5. Students are able to find a shortest path that only involves one turn
6. Students are able to interpret the grid lines in the rectangular grid as number lines (rulers) only consisting of whole numbers

Materials

Student's worksheet, the street map of Jakabaring city blocks, and pencil colours

Learning Activities of 1st Session *(25 minutes)*

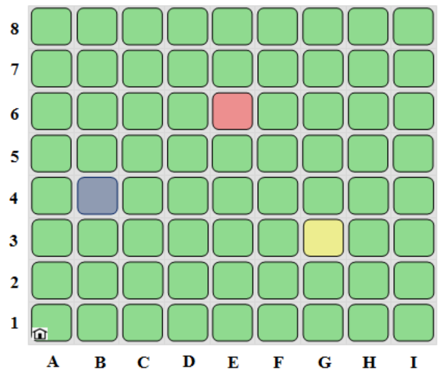
Overview of the activity:

Since the grid system identifies cell rather than points, the precise location cannot be described. Therefore, in this activity, students are shown to the fact that the label of the blocks is less accurate to locate two or more objects laid on the same blocks. Accordingly, they need to put additional information such as the compass directions if they want to tell the location of a certain object by using the grid system.

Orientation *(5 minutes)*

1. Remind students about the city blocks and ask whether the alphanumeric grid system used in the chess notation system is applicable for the labeling the blocks
2. Provide the street map of the city blocks and ask a student to explain how the alphanumeric grid system works for the blocks.

   (Here the student is expected to come up with the following system)



3. Ask students to determine the label of blue, red, and yellow blocks

Predictions of Students' Reaction	The Proposed Actions of the Teacher
The blue, red, and yellow blocks are located on B4, E6, and G3 respectively	• Accept all of different answer • Ask question whether B4 and 4B appoint the different block or not
The blue, red, and yellow blocks are located on 4B, 6E, and 3G respectively	

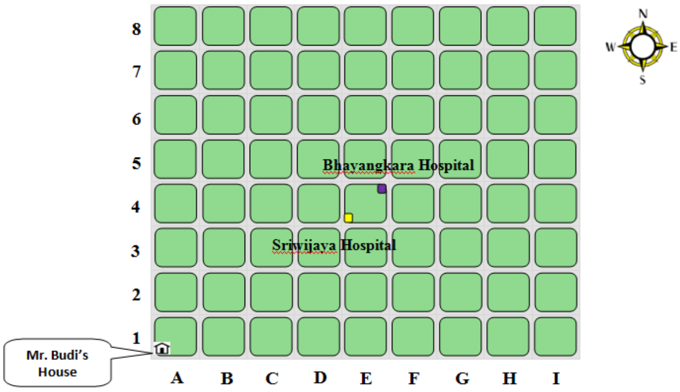
About the Assessment

This question assesses students' ability to identify the location of a certain blocks in the city blocks using the alphanumeric grid system.

Working Group and Classroom Discussion *(20 minutes)*

4. Distribute the student's worksheet and ask students to do part A (see Worksheet 2, Appendix E) in a group of 4 -5 people for 10 minutes

   Determine the precise location of Bhayangkara hospital based on the figure below!



About the Assessment

This problem assesses students' reasoning that the alphanumeric grid system needs refinement to locate two objects or more that are laid on the same cell.

5. Assist the working group by supporting students to be able to tell the location of the hospital precisely

Students' Answer	The Proposed Actions of the Teacher
Similar to the chess notation problem, Bhayangkara hospital is located in block E4 or block 4E.	Ask questions: • *How is about the location of Sriwijaya Hospital?* • *Is there any chance that Mr. Budi will come to the wrong hospital (Sriwijaya)?* • *Will Mr. Budi find Bhayangkara Hospital quickly and precisely based your information?*
Put additional information such as the compass direction to the label of the blocks. Therefore, the hospital is located on the northeast corner or the north part of block E4 or 4E	Ask questions: • *Why do you mean by northeast?* • *Why do you put this information? (refer to northeast)* • *How is about the location of Sriwijaya Hospital?* • *How can you be sure that Mr. Budi will arrive at the right hospital based on your information?*

6. Ask two representational groups to present their the solution of the given problem and ask other groups to give remarks or questions related to the presentation (10 minutes)

Student's Presentation	The Proposed Actions of the Teacher
The group only uses a coordinate pair of a letter and a number to locate the objects	Asking: *Is there any chance Mr. Budi will arrive at the wrong hospital (Sriwijaya Hospital) if you only tell the label of the blocks*?

The group puts additional information such as the compass directions to the coordinate pair	• *How can you be sure that Mr. Budi arrive at the right hospital based on your information?* • Asking another groups to give remarks and question about the presented solution

7. Ask question: *Is it enough to tell the location of a certain object precisely only using the label of the block?*

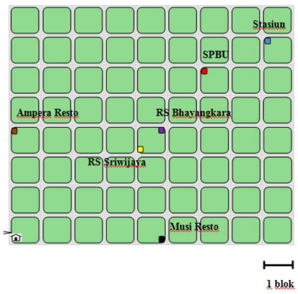
Students' Response	The Proposed Actions of the Teacher
It is not enough because we need put additional information, such as related to the compass direction	*What is the advantage if we put such additional information to tell the location of a certain object?*
It is enough because we just need to get around the certain block to find the intended object.	*In your opinion, is it not time consuming if we want to find a certain location using that kind of information?*

Learning Activities of 2nd Session *(45 minutes)*

Overview of the activity:
Students are guided to reinvent the first quadrant of the Cartesian coordinate system by making a system of locating an object from a origin position using the *taxicab* distance. To start with, students are asked to investigate some shortest paths between two object that leads to the conclusion that any paths with the same starting and ending points cover the same *taxicab* distance, which involve the horizontal and the vertical distance. In the end, students can interpret the grid lines as rulers

Working Group and Classroom Discussion *(40 minutes)*

8. Tell that Mr. Budi looks think about the possible shortest distance
9. Show a PPT about determining the distance between 2 objects
10. Give worksheet 2 part B and ask them to solve the problem for 15 minutes



1. *Based on your shortest route, what is the distance (in blocks) that must be passedby Mr. Budi?*
2. *Make a poster for your shortest route including its corresponding distance*

About the Assessment
This problem assesses students' ability in determining the taxicab distance between two objects in the city blocks

11. Assist the working group by supporting students to clarify their answer so that can minimize the misunderstanding or misinterpretation about the given problems.

No	Students' Answer	Actions of the Teacher
1	Make any shortest routes between the house and Bhayangkara hospital with the total distance 9 blocks (it can be less or more depending on students' answer)	Ask questions: • *What is the total distance?* • *How did you calculate the distance?* • *Please, give arrow as the direction for your routes*
2	Make a poster with the given poster paper containing the street map of Jakabaring city blocks	

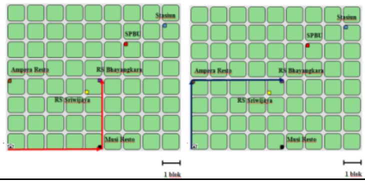
12. Ask each group to put their poster in front of the class and to investigate two other posters that is used to answer the third question
13. Ask students to continue the task no 3 – 5 as shown below.
    3. *What can you conclude about the distance covered by those routes that have been presented by your fellow students?*
    4. *Draw two shortest routes from Mr. Budi's house to Bhayangkara hospital that merely need minimal turn!*
    5. *For each route above, what is the horizontal and the vertical distance? What the results are the same for both routes above?*

About the Assessment

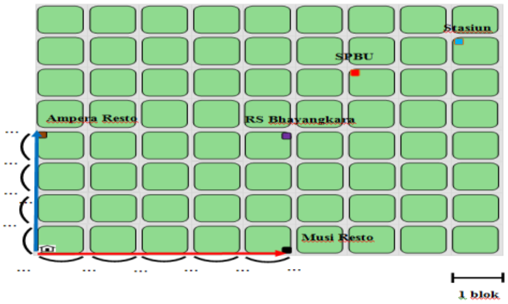
This problem assesses students' ability in arguing that any taxicab routes with the same starting and ending points cover the same distance. In further, it also assess students' capability of determining taxicab routes

14. Assist the group discussion

No	Students' Answer	Actions of the Teacher
3	All of the shortest routes between the house and Bhayangkara hospital covers the same distance	Ask students to show how they calculate distance for three different paths
	The routes do not cover the same distance because they make mistake on drawing the shortest path	Ask students to examine the possibility that there exist non-shortest path
	The routes do not cover the same distance because the routes with	Suggest students to calculate the distance for each shortest

	more turns are longer than the routes with fewer turn	path that they make and see the results of their calculation
4	The shortest routes with one turn are shown below 	Ask questions: • *What is the total distance for each route?* • *What can you say about the distance of both routes?*
5	The horizontal and the vertical distance of both routes are the same, which are 5 blocks and 4 blocks respectively	Ask: *what can you say about the relationship between the total distance and both the horizontal and the vertical distance?*

15. Ask a representational group to present their work in front of the class and conduct the class discussion with the guidance as mentioned in the table
16. Ask students to look at worksheet 2 part C and show the street map of Jakabring city blocks as shown below.



17. Ask students to determine the distance between the house and Ampera Resto/ Musi Resto by simultaneously labeling the gridlines with positive integers
18. Ask: what is the distance between Ampera Resto and Bhayangkara hospital same with the distance between the house and Musi Resto?
19. Ask: what is the distance between Musi Resto and Bhayangkara hospital same with the distance between the house and Ampera Resto?
20. Ask: what can you conclude about the calculation of distance between the house and Bhyangkara hospital?
21. Ask students to determine the horizontal and the vertical distance between the house and any places in the Jakabaring city blocks
22. Lead students to make a conclusion about the use of the alphanumeric grid system and invite them to reflect on what they already learned during the lesson.

Lesson 3 Teacher Guide

Duration of Lesson

2 × 35 minutes

Learning Goals

1. Students are able to locate an object on the rectangular grid by using the horizontal and the vertical distance.
2. Students are able to make agreement to locate an object on the rectangular grid
3. Students are able to understand about an ordered pair
4. Students are able to locate any objects on the rectangular grid using the ordered pair

Materials

Student's worksheet and the street map of Jakabaring city blocks

Learning Activities of 1st Session *(25 minutes)*

Overview of the activity:

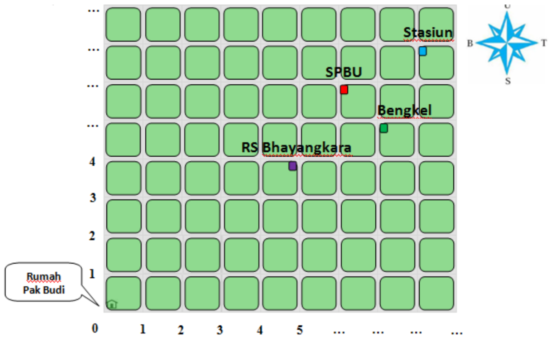
Students in groups of 4-5 people are challenged to describe the location of a certain object in the city blocks by using the horizontal and the vertical distance. To some extent, students may put additional information to tell the location precisely such as using the cardinal direction or the relative system. This will be preliminary activity before students are introduced to the use of an ordered pair.

Orientation *(5 minutes)*

1. Tell a story that Mr. Budi and family will go to Lampung to have vacation by Sriwijaya Train Express. Before arriving at Kertapati Station, their car machine has trouble such that they need to go to a nearest workshop.

Working Group and Classroom Discussion *(25 minutes)*

2. Distribute the student's worksheet and ask students in group of 4 -5 people to determine the location of the railway station and the workshop as shown below.



3. Ask: *what are the meaning of numbers notated on the given street map? (Here, students are expected to clarify that that the numbers notated on the given system represent the distance)*

About the Assessment

This problem assesses students' ability locate an object in the city blocks as the representation of the rectangular grid by using the horizontal and the vertical distance. *(Note: the information related to the horizontal and the vertical distance are embedded in the system as shown in figure 8 above)*

4. Assist the group discussion by supporting students to clarify their own answer

Prediction of Student's Answer	The Role of the Teacher
Use the combination of the distance and the cardinal directions such as the station is located 8 blocks to the east and turn 7 blocks to the north from the house and similarly for the workshop	*Why do you add the information about the cardinal direction such as "north" and "south"?*
Use a coordinate of the horizontal and the vertical distance such as the station is located 8 blocks horizontally and 7 blocks vertically or the other way around and similarly for the workshop	Asking: • *In which direction you start to move?* • *Why do add the information like "horizontally" and "vertically"?*
Only use the distance without giving the direction to locate the objects such as the station is located 8 blocks and 7 blocks and similarly for the workshop	Asking: • *Which direction you go first?* • *If we start with the different direction, does it arrive at the same place?*

5. Ask two representational groups present their the solution of the given problem and ask other groups to give remarks or questions related to the presentation (10 minutes)

Student's Presentation	The Proposed Actions of the Teacher
The group only uses of distance to locate the objects without giving the direction	• Ask other groups to give remarks or questions related to the presentation • Asking: *if we start with the different direction, does it arrive at the same place?*
The group considers both the distance and the certain direction to locate the objects	*Why do you put additional information about the direction, such as "north" or "east" depending on student's answer?*

5. Ask question: Is it enough to tell the location of a object only using the distance? *(Here, students are expected to realize that they need information about the direction)*

Learning Activities of 2nd Session *(45 minutes)*

Overview of the activity:

To understand how the ordered pair works, students are confronted with a problem about which distance (the horizontal or the vertical distance) that should be considered as the first and the second coordinate. This activity will lead them to an agreement about which distance that should be notated as the first and the second coordinate.

Orientation *(5 minutes)*

6. Tell a story about TV news reporting a fire accident in the city blocks
7. Ask students about their experience about witnessing a fire disaster

Working Group and Classroom Discussion *(35 minutes)*

8. Distribute the student's worksheet and ask students in groups to solve the following problem for 15 minutes (see also Worksheet 3, Appendix E).

A fire accident happened last night in a certain neighborhood of Jakabaring city blocks due to short-circuit. The electricity powers from four electrical towers are turned off and the police line is installed around the danger zone. The four electrical powers are located on (1,1); (7,1); (7,5) and (1,5). Plot the location of those four lamp towers including its police line on the street map of Jakabaring!

About the Assessment

This problem assesses students' reasoning about the need of an agreement related to the issue *for which direction (the horizontal or the vertical distance)* they should notate the first coordinate of an ordered pair

9. Assist the group discussion by supporting students to clarify their own answer

Predictions of Students' Answer	The Proposed Actions of the Teacher
Interpret the first coordinate of the ordered pair as the horizontal distance	Ask students the interpret the vertical distance as the first coordinate
Interpret the first coordinate of the ordered pair as the vertical distance	Ask students the interpret the horizontal distance as the first coordinate
Interpret the first coordinate of the ordered pair as two possibilities, either the horizontal and the vertical distance	Ask questions: • *Can we use the notation (refer to the ordered pair) to locate an object?* • *What can we do if we want to use the notation as the conventional way of locating a certain object?*

10. Ask two representational groups present their the solution of the given problem ask other groups to give remarks or questions related to the presentation (20 minutes)

Student's Presentation	The Proposed Actions of the Teacher
Students interpret the first coordinate of the ordered pair as either the horizontal or the vertical distance	• Ask other groups to give remarks or questions related to the presentation • Ask them to plot the objects that is contrast with their interpretation.
Students interpret the first coordinate of the ordered pair as two possibilities, either the horizontal and the vertical distance	Asking: *what can we do if we want to use the notation as the conventional way of locating a certain object?*

11. Make agreement with the students that the horizontal distance is notated as the first coordinate of an ordered pair
12. Introduce that the given notation is called an ordered pair

Closing *(5 minutes)*

13. Lead students to make a conclusion in their group about the use of an ordered pair to locate a certain object and invite them to reflect on what they already learned during the lesson.

Lesson 4

Teacher Guide

Duration of Lesson

2 × 35 minutes

Learning Goals

Students are able to locate a certain object, represented as a point, in the form of the ordered pair without providing the grid lines. The sub-goals are specified as follow.

1. Students are able to locate a certain object laid on the horizontal or the vertical axis
2. Students are able to locate a certain object involving "half" coordinate
3. Students are able to locate the origin in the form of the ordered pair (0,0)

Materials

Student's worksheet and the sea map

Learning Activities of the 1st Session *(25 minutes)*

Overview of the activity:

Students in groups of 4-5 people are challenged to locate object in the sea map using the ordered pair without providing the grid lines. students are expected to produce the grid lines and label the axes by themselves to be able to locate the object using the ordered pair. Students are also challenged to tell the location a certain object, which is not located in the intersection of the grid lines. It will be the enrichment for students to locate an object by using fraction, one of which is involving "half" as its coordinate

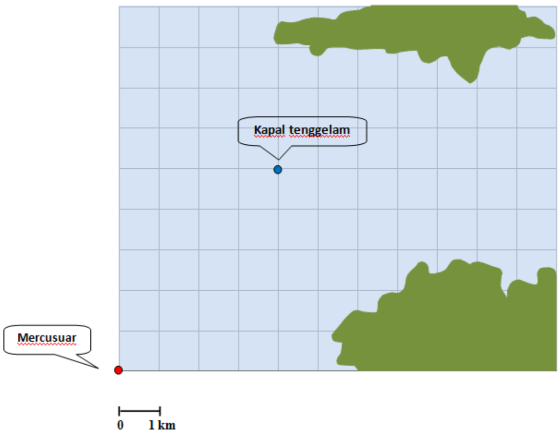
Orientation *(10 minutes)*

1. Talk about the experience of having trip by using a ship.
2. Tell a story about the tragedy of the sunken ship in the Java sea as described below.

One day, Srwijaya Ship has voyaged in the Java Sea and suddenly has trouble with the machine. Shortly, the backside of the ship began to sink such that the captain instructs his crew to do emergency call. The crews open their map and call the officer of the lighthouse to tell the coordinate of their ship

Working Group and Classroom Discussion *(20 minutes)*

3. Distribute the worksheet and ask students to determine the coordinate of the sunken ship if it is seen from the lighthouse
4. Give additional information that the crew and the lighthouse officer have the same sea map. It was conducted to make the context makes sense for the students and they can start to work with the given sea map.



About the Assessment
This problem assess students' ability to locate an object on the rectangular grid using an ordered pair as the agreement they made before

5. Assist the group discussion

Students' Answer	Response of the Teacher
The location of the sunken ship is 5 km to the north and 4 km to the east or the other way around	Asking: ➢ *What do you mean by 1 km her?* ➢ *Can you write down the coordinate of the sunken ship*
Coordinate of the sunken ship is (4, 5)	Asking: *For which direction do you notate the first coordinate?*
Coordinate of the sunken ship is (5, 4)	Asking: • *For which direction do you notate the first coordinate?* • *Let's to remember the agreement that we made before about the ordered pair*

6. Ask a representational group to present their work and ask other students to give comments, remarks, or questions related to the presentation

Learning Activity of the 2nd Session *(45 minutes)*

Gambaran umum dari aktivitas:

Siswa dalam kelompok (4 – 5 orang) ditantang untuk melokasikan objek-objek yang lain (kapal-kapal timSAR) yang terletak pada sumbu-x atau sumbu-y serta objek yang yang salah satu koordinatnya mengandung pecahan "setengah" dengan menggunakan notasi pasangan berurutan

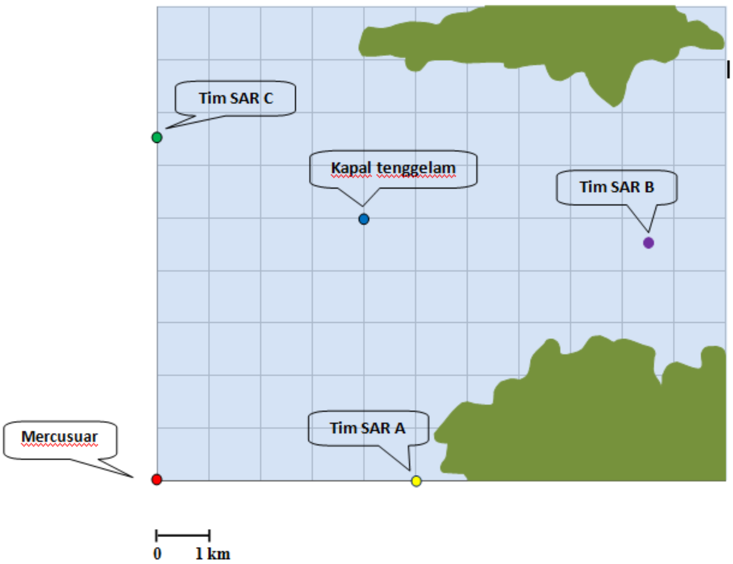
Introduction (5 minutes)

7. Continue the story about the sunken ship as written below.

After getting the information from the crew of Sriwijaya Ship, the lighthouse officer sends three SAR team (team A, B, and C) to do evacuation. Before conducting the evacuation, those three teams report the coordinate of their each position.

Working Group and Classroom Discussion *(35 minutes)*

8. Distribute the student's worksheet and ask students in group to determine the coordinate of the rescue teams for about 25 minutes.



About the Assessment

This problem assesses students' ability to make the coordinate grid (optional) and to locate a certain object in the form of an ordered pair that has zero and "half" as its coordinate

9. Assist the group discussion by supporting students to clarify student answer

Students' Answer	Responses of the Teacher
Coordinate of SAR team A is notated as (*x*, 0)	Asking: *How did you determine its horizontal and vertical distance?*
Coordinate of SAR team A is notated as (*x*, 0)	Asking: • *How did you determine its horizontal and vertical distance?* • *Let's to remember about the agreement about the use of the ordered pair that we made before*
Coordinate of SAR team B that has value "a half" is represented using fractions	Asking: *Why do you use fractions here?*
Coordinate of SAR team B that has value "a half" is represented using fractions	Asking: *Why do you use fractions here?*
Coordinate of SAR team A is notated as (0, *y*)	Asking: *How did you determine its horizontal and vertical distance?*
Coordinate of SAR team A is notated as (*y*, 0)	Asking: • *How did you determine its horizontal and vertical distance?* • *Let's to remember about the agreement about the use of the ordered pair that we made before*

10. Ask student to determine the tell the location of the lighthouse

Students' Answer	The Role of the Teacher
It is located at 0	Giving probing questions: • *How many number lines (rulers) that you can see in the map?* • If the notated numbers actually label the axes, so *how many zero that actually you have*?
It is located at (0, 0)	Asking for clarification: *Why do you put two numbers "zero" on that?*

About the Assessment
This question assesses students' ability to locate the origin in the form of the ordered pair (0, 0)

Closing *(5 minutes)*

10. Lead students to make a conclusion in their group about locating an object in the form of an ordered pair that involves "half" coordinate and invite them to reflect on what they already learned during the lesson

Lesson 5 Teacher Guide

Duration of Lesson

2 × 35 minutes

Learning Goals

1. Students are able to plot and locate any points on the Cartesian diagram from the origin using an ordered pair
2. Students are able to identify a special quadrilateral figure formed by four points represented in the form of ordered pair

Materials

Student's worksheet and the blank sheet

Learning Activities *(70 minutes)*

Overview of the activity:

Students in this activity are expected to be able to plot and locate any points on the Cartesian diagram in the form of ordered pairs. The understanding of the Cartesian coordinate system plays an important role to be able to do graph work, one of which is making graph. Here, by making a special quadrilateral figure through locating and plotting points on the plane, students start to learn about graph making, but in the simple shape (not curve). In addition, the main issue mostly encountered by students when plotting points in Cartesian diagram is from which position students plot the points, whether from the origin or from a certain points.

Orientation *(5 minutes)*

1. Introduce a game named "Secret Quadrilateral" in which someone needs to guess a certain quadrilateral shape formed by four points (in the form of an ordered pair) with the certain rules (see Worksheet 5, Appendix E).
2. Ask students to mention the example of the special quadrilateral they knew (rectangle, square, parallelogram, trapezoid, rhombus, and kite)
3. Tell a story about two children play "Guess the Shape" game

Working in Pairs *(15 minutes)*

4. Distribute the student's worksheet and ask students in pairs to solve the following problem (see also Worksheet 5, Appendix E).

   Hafiz and Sari are playing "secret quadrilateral" game as shown in the following conversation.

Hafiz : There are four coordinates written on the secret message, namely A (0, 2); B (7, 2); C (10, 6); D (3, 6)
Sari : Let's see what quadrilateral figure that is formed if those four coordinates are connected!

5. Assist the group work and notice from which position students plot the points, either from the origin or from a certain points.

Predictions of Students' Answer	The Roles of the Teacher
Plot each four given points from the origin and interpret the ordered pair as (*x*, *y*) with the geometrical shape is a rectangle with the horizontal side as the longest side	In the case that students interpret the vertical coordinate first, the teacher can recall them to think about the agreement they make during working with "the danger zone" problem in the context of city blocks
Plot each four given points from the origin and interpret the ordered pair as (*x*, *y*) with the geometrical shape is a rectangle with the horizontal side as the longest side	
Plot a certain point not from the origin, but from the other certain point. For example, if they had plotted a point at (2, 2) and the next point was at (7, 2), they would place the points 7 units to the right and 2 units to the above of (2, 2), at (9, 4)	Asking for clarification: • *Why you plot the point from the previous point?* • *Think about the vertical and the horizontal distance represented by the coordinate pair!*

Orientation *(5 minutes)*

6. Introduce a game named "Secret Coordinate" in which someone needs to guess the coordinate of the forth point if given three points and its geometrical shape

Working in Pairs and Classroom Discussion *(30 minutes)*

7. Distribute the student's worksheet and ask students in a group of 4 -5 people solve the problem as shown below for about 15 minutes.

Rif'at and Nadya are playing "secret coordinate" game as shown in the following conversation.

Rif'at : There are three coordinates, K (0, 3); L ($2\frac{1}{2}$, 0); dan M (8, 3). From those three coordinates, we can make a kite KLMN if we can determine the coordinate of point N
Nadya : Let's find the coordinate of point N!

About the Assessment
This question assesses students' ability to plot any points in the form of an ordered pair and to identify a special quadrilateral figure formed by four points *(Note: it needs to be noticed whether students plot a new point from the origin or from the previous point)*

8. Assist the group work and notice from which position students plot the points, either from the origin or from a certain points

Students' Answer	The Role of the Teacher
Read the given coordinate as (x, y) and the secret coordinate is at $(0, 6\frac{1}{2})$	*Show me how did you plot the given coordinate!*
Read the given coordinate as (y, x) and the secret coordinate is at $(6\frac{1}{2}, 0)$	• *Show me how did you plot the given coordinate!* • *Let's to remember about the agreement about the use of the ordered pair that we made before"*

9. Ask some representational groups to present their solution of the given problem for about 15 minutes.

Working in Pairs *(15 minutes)*

10. Ask students in pairs to play the games "Secret quadrilateral" and "Secret coordinate" with the mentioned rules before

Closing *(5 minutes)*

11. Lead students to make a conclusion that plotting points on the Cartesian plane start from the origin and invite them to reflect on what they already learned.