FROM AREA TO AREA MODEL:

DEVELOPING A LOCAL INSTRUCTION THEORY FOR LEARNING THE MULTIPLICATION OF BINOMIALS BY USING AN AREA MODEL

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of

Master of Science (M.Sc)

In

International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University

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FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
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STATEMENT PAGE

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1. All the data, information, analyses and the statements in analyses and conclusions that presented in this thesis, except from the reference sources are the result of my observations, researches, analyses and views with guidance of my supervisors.
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This statement is truly made. If any fouls in my above statement is found in the future, I am ready to get academic sanctions, such as cancelation of my degree that I have got through this thesis.

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ABSTRACT

Algebra is often seen as a gatekeeper to success in higher education. In contrast to its importance, studies show that students often find difficulties in learning algebra. One of the difficulties is the transition from arithmetic to algebraic thinking. This paper reports a design research study aiming to contribute to local instruction theory on supporting students' learning processes of binomials multiplication. The soul of this study is the emergent modeling from area to area model, representing Pendidikan Matematika Realistik Indonesia (an Indonesian version of Realistic Mathematics Education), to bridge the gap between arithmetic and algebraic thinking. Using area and numbers as the main context, we design mathematical activities and a Hypothetical Learning Trajectory (HLT) to guide the seventh grade students (12-13 years old) to understand the multiplication of two binomials. Two cycles of teaching experiment were conducted and recorded. Students' written works were collected and interviews were held. Those data were analyzed around the potential of area model to support students' learning processes and to improve the HLT. The findings of this study suggest that the context of area supports students' understanding of binomials multiplication, and it is a potential tool to bridge the arithmetic and algebraic thinking. Further, area model is a potential tool to solve binomials multiplication problems.

Keywords: *algebra, binomials multiplication, secondary school, area model, local instruction theory, RME, PMRI, HLT*

ABSTRAK

Aljabar dikenal sebagai pintu gerbang menuju sukses di tingkat pendidikan lanjut. Berlawanan dengan pentingnya aljabar, penelitian menunjukkan bahwa siswa seringkali mengalami kesulitan dalam mempelajari aljabar. Salah satu kesulitan tersebut adalah selama proses transisi dari proses berpikir aritmatik ke aljabar. Penelitian ini merupakan sebuah penelitian "design research" yang bermaksud untuk berkontribusi di "local instruction theory" dalam membantu siswa mempelajari perkalian dua faktor. Inti dari penelitian ini adalah "emergent modelling" yang merujuk pada Pendidikan Matematika Realistik Indonesia (versi Indonesia dari "Realistic Mathematics Education"), untuk menjembatani antara proses berpikir aritmatik dengan proses berpikir aljabar. Menggunakan luas dan bilangan sebagai konteks utamanya, kami membuat kegiatan-kegiatan matematika dan "Hypothetical Learning Trajectory" (HLT) sebagai rujukan untuk siswa-siswa kelas tujuh (12-13 tahun) untuk memahami perkalian dua faktor. Penelitian ini terdiri dari dua siklus pembelajaran yang semuanya direkam. Selain itu, semua hasil pekerjaan siswa juga dikumpulkan dan juga dilaksanakan wawancara kepada guru maupun siswa. Data-data hasil rekaman, pekerjaan tertulis dan wawancara dianalisis seputar kemungkinan "area model" dapat digunakan untuk membantu proses belajar siswa serta untuk memperbaiki HLT. Hasil penelitian ini menunjukkan bahwa konteks luas dapat membantu pemahaman siswa terhadap perkalian dua faktor dan mampu dijadikan sebagai alat untuk menjembatani proses berpikir aritmatik ke proses berpikir aljabar. Lebih lanjut, "area model" menunjukkan potensinya untuk digunakan dalam penyelesaian perkalian dua faktor.

Kata kunci: *aljabar, perkalian dua faktor, SMP, area model, local instruction theory, RME, PMRI, HLT*

SUMMARY

Algebra is seen as one of the most important subjects in mathematics for lower secondary school for its' ability to support students' success in higher education and future real life. However, many students face difficulties when they are working with algebra. One of these difficulties is the transition from arithmetic to algebraic thinking. Most students see algebra as a brand new thing instead of a generalized arithmetic. Hence, they merely focus on algorithm without knowing the relation between each part in algebra. With this background, this study is to bridge the arithmetic and the algebraic thinking in multiplication by using area model, which has been proved as a useful tool to make sense multiplication in arithmetic. In this manner, we use numbers and area as the main contexts.

Design research is chosen as the appropriate research approach where we develop a sequence of mathematical activities to support students' learning processes in binomials multiplication. Design research emerges three phases: preparation, experiment and retrospective analysis. During the preparation phase, we study literatures to determine the goal of this study and as a basic to make the initial HLT. The HLT is a learning line consists of the mathematical activities, its description and goals, and the conjectures of students' thinking. During the second phase, the initial HLT is tested in a pilot experiment and analyzed to improve the mathematical activities and the HLT. The improved HLT is implemented in a natural classroom and analyzed based on students' learning process to see whether area model success of fail to support students' learning processes in the multiplication of two binomials. In both teaching experiments, we collect the students' written work, observe and video taped the learning process. In each before and after the teaching experiments, we conduct pre- and post-test to enhance the quality of our findings based on the learning processes. Retrospective analysis is done by the end of both teaching experiments, means that the third phase is between the first and second teaching experiment and after the second teaching experiment.

Based on the retrospective analysis, we find that the context of area support students' understanding of binomials multiplication and area model is a useful tool to solve binomials multiplication. Further, area model shows a potential to be used not merely in binomials multiplication, but also in linear algebraic multiplication with one or more variables and in factorization.

RINGKASAN

Aljabar seringkali dipandang sebagai salah satu cabang ilmu terpenting dalam matematika untuk SMP. Hal ini dikarenakan aljabar sangat menentukan keberhasilan siswa di tingkat pendidikan yang lebih tinggi dan di kehidupan nyata. Akan tetapi, banyak siswa yang mengalami kesulitan selama mempelajarai aljabar. Salah satu kesulitan yang sering dijumpai adalah adanya masa transisi dari proses berpikir aritmatik ke proses berpikir aljabar. Banyak siswa yang tidak menyadari bahwa aljabar merupakan suatu hal baru, bukan peningkatan dari aritmatik. Dalam hal ini, mereka hanya fokus pada algoritma yang diberikan tanpa mengaitkan hubungan antara unsur dalam aljabar dan operasinya dengan aritmatik. Dengan latar belakang ini, penelitian ini bertujuan untuk menjembatani proses berpikir aritmatik dengan proses berpikir aljabar dalam perkalian dengan menggunakan *area model,* yang mana sudah dibuktikan sebagai media yang ampuh untuk memahami perkalian dalam aritmatik. Dalam penelitian ini, kami menggunakan bilangan dan luas sebagai konteks utamanya.

Design research dipilih sebagai pendekatan penelitian yang cocok dalam penelitian ini, dimana kami mengembangkan serangkaian kegiatan pembelajaran matematika yang mendukung proses belajar siswa dalam perkalian dua faktor. Dalam *design research,* terdapat tiga fase utama, yakni: persiapan, eksperimen dan analisis. Selama fase persiapan, kami mempelajari literatur dan penelitian yang sudah ada untuk merumuskan tujuan penelitian ini dan membuat HLT awal. HLT berisi aktivitas matematika, deskripsi dan tujuan dari aktivitas tersebut, serta perkiraan tentang proses berpikir siswa dalam melakukan aktivitas matematika tersebut. Selama fase ke dua, HLT awal diimplementasikan di siklus 1 pada sebuah kelas kecil untuk menyempurnakan HLT awal tersebut. HLT yang telah disempurnakan diimplementasikan pada siklus kedua dimana subjek yang digunakan adalah satu kelas. Selama proses pengimplementasian dari HLT, kami mengumpulkan hasil pekerjaan siswa, merekam proses pembelajaran dan melakukan observasi selama pembelajaran berlangsung. Di tiap akhir siklus, fase ke tiga, yaitu analisis dilaksanakan. Selain itu, pre-test dan post-test juga dilaksanakan pada setiap sebelum dan sesudah implementasi guna memperkuat kualitas kesimpulan dari penelitian ini.

Berdasarkan analisis dari data yang dikumpulkan, kami menemukan bahwa konteks luas sangat membantu siswa untuk memahami makan dari perkalian dua faktor. Selain itu, *area model* juga merupakan alat yang bisa digunakan siswa untuk mempermudah melakukan perkalian dua faktor. Lebih jauh lagi, *area model* juga berpotensi untuk digunakan sebagai alat pada topik lain di aljabar, seperti perkalian aljabar linear dengan satu atau lebih variable dan pada topic faktorisasi.

PREFACE

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I dedicated this thesis for my parents (Suwarso M. Damami and Siti Marfungah) and sisters (Soliah and Mustanginah), who gave me full supports and trusts. Also for my niece, mba Nala, whose birthday I always missed due to completing my study. Further, I want to thank to all people who supported me and were there for me since the beginning of my study: Karsten MacDonald, Pak Gunarjo and all teachers and staffs in BCU, PMRI team, lectures from LB UNSRI, Daria anggawati for always be in my side, Farih and Fera for always give me place to stay for a little escape; and people who supported me when I was in the Netherlands: Bunda for all the breakfasts service when I was busy, Qori Uci for the friendship, sleep over and never ending travelling and photo session, Ari Susanti and Raymond Matthijs for the feeling like home, Michael Lutam and Chysanti Arumsari for being so handy in helping me during my thesis. Last but not least, kak Mimi for being a crazy roommate.

I consciously understand that this thesis is far away from perfect. Thus, any insightful critics and constructive ideas will be gladly accepted.

Palembang, June 2015

Nur Chasanah

CURRICULUM VITAE

Nur Chasanah, born in Kebumen, December 10, 1986, is a student of International Master Program on Mathematics Education (IMPoME) of Sriwijaya University – Utrecht University 2013. Being the youngest child in the family, she is raised with freedom to pursue her dreams, having different passion with her two older sisters, and she is grateful to get the permit to sometimes leave study and travelling a lot during her undergraduate study. She spent most of her childhood and received her education in Kebumen (TK PGRI Jatijajar, SDN 3 Jatijajar, SMPN 1 Ayah, SMAN 1 Gombong) and received her bachelor degree in mathematics education from Yogyakarta State University in 2011.

Further, she had been working for two years in Bina Cita Utama National Plus School in Palangka Raya, Central Borneo where she taught mathematics for high school. During her job as a teacher, working with foreigners, she started to learn to be an active English speaker and joined in some workshop and training program for anti-bullying held by the school foundation, Sejiwa Foundation, Yayasan Usaha Mulai Foundation, and Barclays Capital, as a participant, trainee for becoming a trainer and finally as a trainer herself.

She was awarded a scholarship in 2013 to continue his master degree called International Master Program on Mathematics Education (IMPoME) focusing on Realistic Mathematics Education and design research. He got an English training from Dikti for 1.5 months at Sriwijaya University and continued studying her first semester in the mathematics education department for graduate students in Sriwijaya University. During the year of 2014, she completed her study in Utrecht University, the Netherlands.

Lastly, since she has no permanent resident and loves to travel, she can be reached through phone: 085747974987 or email: nur.chasanah.1012@gmail.com.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF BOXES

LIST OF APPENDICES

CHAPTER 1. INTRODUCTION

Algebra, as a gatekeeper to success in higher education, college preparatory as well as many career paths, benefits all students. Many studies (Capraro & Joffrion, 2006; Edwards, 2000; Erbas, 2005, Gamoran & Hannigan, 2000; Stephen, 2005) have shown the superiority of secondary school algebra. Hence, Gamoran and Hannigan (2000) stated that "algebra for everyone" has been a popular slogan of the reformation of school algebra. Algebraic skills serve as a basis for advance mathematics as well as other subjects like physic and economics. Besides they also provide advantages in students' real life (Usiskin, 1988).

The National Council of Teachers of Mathematics (2008) has defined algebra as "a way of thinking and a set of concepts and skills that enable students to generalize, model, and analyse mathematical situations." Moreover, algebra is seen as the first domain in school mathematics, which encourages students' abstract reasoning through making a transition from the concrete arithmetic to the more advanced algebra, which includes symbols. However, Kieran (2004) found that students often encounter difficulties during this transition (Susac, et al., 2014).

With regard to students' difficulties, the term transition mainly refers to students' way of thinking, viewing and expressing, in terms of notation as well as arithmetic processes from pre-algebraic or early algebraic concepts (without variables) to secondary school algebra. There is generally less time for this transition than there should be, due to time limitation in the curriculum. School algebra for secondary school level usually starts at a formal level and gives less chance for students to reason. Hence, many students are lost in understanding, which results in a gap between high- and low-achiever students.

Besides this transition, Al Jupri et al (2014) stated that there are common problems that students encounter during their phase of learning school algebra. The common problems are mostly related to how algebra is being taught in school. The formal level of teaching leads to students' difficulties in the students' understanding of variables, the arithmetic processes, and the strategies to solve the algebraic problems.

When the concept of variable has been introduced, students may still experience one of the most common difficulties, namely understanding the concept of variable (Kieran et al., 1991; Booth, 1988). Kieran et al (1991) agreed with Booth's (1988) claims that in most countries, school algebra is more taught and seen as an extension of arithmetic. Apparently, arithmetic has also letters in it, but they have a different meaning from the ones in algebra. In arithmetic, letters are used to represent meter, centimetre, etc. In algebra, students need to understand that a letter represents a variable, which refers to a range of values of unknown quantity. Confusion over this change as well as a formal way of teaching algebra causes students lack a numerical referent to interpret the meaning of letters in algebra.

Regarding the arithmetic processes, Usiskin (1988) noted that algebra is also seen as generalized arithmetic. Hence, arithmetic becomes the soul of algebra and when students have difficulties in solving arithmetic problems, they will also have difficulties in solving algebraic problems. In fact, many students still have misunderstandings about arithmetical conventions. Usiskin (1988) and Booth (1988) argued that some of the difficulties of students to learn algebra are related to the arithmetic processes during the understanding and solving of algebraic problems.

Usiskin (1988) noted that one of the fundamental issues in school algebra instruction is the limited amount of strategy that the students learned. In fact, students are given algorithms and only need to follow these. Usiskin (1988) also marked that students should be required to be able to do various manipulative skills in solving algebra problems. These skills are useful for students as preparation for higher levels of education, and as such these students will be able to manipulate algebraic problem solving more creatively. These skills are also useful when students who cannot solve problems with one specific method will now be able to find a way to solve the problems.

Many studies have been conducted concerning students' difficulties towards school algebra as well as finding a good instructional design to teach the materials. Yet, most of them focus on the concept of variables and finding solutions, and a topic like multiplication of two algebraic factors is likely still being taught in a formal way. No studies have been conducted to find out the best way of teaching this topic or to investigate whether some models will work on supporting students' understanding in this topic. These formal teaching and learning processes provide less opportunities for low-achiever students to understand the concept and wider the possibility for them to fail. Hence, this research is to design a sequence of learning on finding the product of two algebraic factors and the research question is "how does area model promote students' understanding of multiplication of two binomials?"

CHAPTER 2. THEORETICAL FRAMEWORK

This theoretical framework employs some literature reviews to address the structure of thinking for designing materials that help students with their difficulties in school algebra. The studies we discuss provide insight in the reasons why it is very important for early high school students to succeed in studying algebra and why there should be a reform of how algebra is taught in school. In this study, Realistic Mathematics Education (RME) is chosen as the domain specific theory for the reformation. The use of models is then derived from RME to cover students' difficulties toward the materials. Meanwhile, an overview of the Indonesian curriculum is provided to give insight into the mathematical goal that should be achieved by the students.

2.1 School Algebra

2.1.1 The Concept of School Algebra

Algebra comes from an Arabic word "al-jabar" introduced by Al-Khuwarizmi in his book *Kitab al-mukhtasar fi hisab al-jabar wal-muqabala.* Al-jabar means 'restoration' or 'completion', and at the same time also means equation (Grandz, 1937; Wilson, 1995). The book contains three fundamental types of quadratic equation and became the first source for learning algebra for Europeans (Wilson, 1995).

School algebra is algebra taught in school. Usiskin (1988) noted that school algebra is quite different from the algebra taught to mathematics majors. Usiskin (1988) claimed that school algebra is closely related to understanding variables and their operations. The operations themselves represent the generalisation of arithmetic operations.

Battista (1998) stated that learning about numerical procedures in arithmetic should begin in primary school. This should continue until students are able to manipulate algebraic symbols based on their reflection on the numerical procedures. This idea is crucial for students' learning and conceptual development of algebra since it will support the transition from arithmetic to algebra (Kriegler, 2004).

In school algebra, there is usually an uncertain line between informal and formal algebra. Not many people know that algebra can be seen not merely as generalized arithmetic, but also as a language as well as a tool for functions and mathematical modelling (Kriegler, 2004).

Algebra as a language is closely related to the existence of variables and their expressions. Students have to understand that variables in algebra hold different meaning based on the contexts. As a language, algebra employs students' abilities to read, write and manipulate numbers and variables in formulas, expressions, equations and inequalities. Moreover, students are also required to be able to interpret each solution in algebra (Kriegler, 2004).

As a tool for functions and mathematical modelling, Herbert and Brown (1997) relate algebra with context where algebraic ideas can be applied, especially in real life and relevance. Hence, Kriegler (2004) added some mathematical activities that develop algebraic skills in the context of functions and mathematical modelling. These activities include seeking, expressing and generalizing patterns and rules in real world context, developing coordinate graphing techniques, representing mathematical ideas using equations, tables and graphs, and working with input and output patterns.

Furthermore, Usiskin (1988) noted two others conceptions of school algebra beside generalized arithmetic and as a tool to solve certain problems. School algebra is also seen as the study of relationships among quantities. Lastly, school algebra is seen as a study of structures.

2.1.2 Students' Difficulties towards School Algebra

A study by Booth (1988) showed that students faced many difficulties in learning school algebra. Traced by the errors made by the students, Booth (1988) categorized these difficulties into four aspects: (i) the focus of algebraic activity and the nature of the "answer", (ii) the use of notations and conventions in algebra, (iii) the kinds of relationships and methods used in arithmetic, and (iv) the meaning of letters as variables.

In arithmetic as well as other domains of mathematics, which have been learned for years by students before starting high school algebra, the focus of the activities is on finding the exact numerical answer. However, this is not the case in school algebra. The focus in school algebra is on the derivations of the procedures, relationships between expressions and manipulations using variables. Hence, students will learn about expressions and simplification of forms. This situation leads to confusion for students who are still focused on finding numerical answers (Booth, 1988).

Students' difficulties with the use of notations and convention in algebra seems to be closely related with their understanding of arithmetic. Warren (2003) stated that in traditional school mathematics, students are assumed to have required the following prior knowledge: (i) understanding relationships between quantities, such as a comparison to decide which quantity is less than or greater; (ii) understanding properties and relations between operations, including the associative, commutative, and distributive property; (iii) understanding relationships across the quantities, such as understanding equations and inequalities. The traditional approach on teaching early secondary school algebra implicitly assumes that all students have understood all this prior knowledge based on their experiences with arithmetic in the primary grades. However, in fact, students face many difficulties related to the numerical context itself. Hence, as a solution, sense of the operation among the three previously-mentioned areas of knowledge should be developed before starting school algebra (Warren, 2003).

Battista et al (1991) noted that another major difficulty in learning school algebra is understanding the letters as variables. The concept of variable itself represents different meanings. However, most often variables are used to represent a range of values or unknowns. Students' difficulties in understanding the concept of variable is based on their prior knowledge in arithmetic where they also have letters representing different things, such as units of length.

Warren (2003) also claimed that studies have conveyed students' struggles in understanding the concept of variable, solving algebraic equations and translating word problems into algebraic symbols.

Al Jupri et al. (2014) noted some difficulties of students in early school algebra learning in Indonesia. Most of them are similar to common difficulties of students. The difficulties mentioned by Al Jupri et al. (2014) are: (i) applying arithmetic operation, (ii) understanding the notion of variable, (iii) understanding algebraic expressions, (iv) understanding the different meanings of the equal sign, and (v) mathematization, which is related to translating real world phenomena problem into mathematical language with symbols.

2.2 Secondary School Algebra

2.2.1 Why is Secondary School Algebra Important

Silva, et al. (2006) argued that studying algebra for the seventh and eighth grades is a reasonable standard. Furthermore, Capraro and Joffrion (2006) stated that in early high school, students are expected to be able to use symbolic algebra. Understanding equations and algebraic relationships are the fundamental preparation for using advanced algebraic concepts. Hence, early high school grades are seen as the critical point to gain success for higher education.

Based on their study comparing eighth and tenth graders who did or did not take algebra in their school as college preparation, Gamoran and Hannigan (2000) concluded that all students benefit from taking algebra. The study also showed that most of the students who took algebra only in eight grade gained more achievement.

Silva et al. (2006) said that when a teacher expects that all children can learn algebra, students who fail in completing school algebra would be seen as "low-ability" students. However, students in fact are facing many difficulties in learning school algebra and an innovation is needed in teaching algebra in order to support students' understanding and to support them to succeed in school algebra.

2.1.3 The Transition from Arithmetic to Algebra

To be successful in the transition from arithmetic to algebra, students should have mastered the understanding of the associative, distributive and commutative laws, as well as addition and division, which they have learned in primary school. To begin with, early algebra is taught by reflecting on the operations in arithmetic and representing the quantities and numbers with letters.

Warren (2003) argued that the attainment of early secondary school algebra learning really depends on students' experience with arithmetic. The transition from arithmetic to algebra takes place in a relatively short period as such the line between arithmetic and early algebra is often vague (Warren, 2003).

Learning numerical relations of a situation is considered the start of the transition from arithmetic thinking to algebraic thinking. Discussing the transition explicitly and trying to represent unknowns with letters is the next step. One thing is retained from the first to the later step, which is the ability to operate, from operating numbers to operating numbers with unknowns or variables. Hence, students can use the same model to learn arithmetic and algebra (Warren, 2003).

However, there is a gap between arithmetic and algebraic thinking where students often experience difficulties (Kieran, 2004; Warren, 2003). To bridge the gap, Warren (2003) mentioned a recent mathematical reform, which involves generalizing patterns, visual patterns, and tables of values. Meanwhile, concrete materials are proposed to develop the understanding of variables. More recently, people start to use spreadsheets and computers in teaching initial algebra (Warren, 2003).

2.1.4 Binomials as a Part of School Algebra in Indonesian Curriculum

Considering the benefit to students who learn algebra in eighth grade, the target of this study is algebra for eighth graders. In Indonesia, the concept of formal algebra, which explicitly involves variables, is first introduced in the seventh grade of lower secondary school. Meanwhile, binomials, as the more advance level of algebra, are being introduced in eighth grade under the topic of quadratic equations. Table 1 describes how quadratic equations are being introduced for the first time in grade eight of lower secondary school in Indonesia.

Table 1 Binomials in the Indonesian Curriculum

The second semester of the 8th grade of Indonesian lower secondary school	
3. Understanding and applying knowledge (factual, conceptual and procedural) based on curiosity toward science, technology,	3.3 Determining the value of a quadratic equation with one unknown variable

art and culture related to phenomena and real life occurrence	

The curriculum seems to pay attention more to how to solve quadratic equations. Therefore, since many students experience difficulties in the transition from arithmetic to algebra, it is important to strengthen the basis for binomials before moving on to solving quadratic equation problems. Hence, this study aims to design the fundamental for quadratic equations in a sequence of instructional activities to support students' understanding of multiplication of two binomials, which will result in a quadratic expression.

2.3 Realistic Mathematics Education (RME) and the Use of Models

2.3.1 Realistic Mathematics Education (RME)

Freudenthal (1973) viewed mathematics as a human activity. Based on Freudenthal's idea, Realistic Mathematics Education (RME) was developed as a domain specific theory instruction (Treffers, 1987; De Lange, 1987). The design in this study is focused on the following tenets of RME:

1. Intertwining

   Learning mathematics does not merely mean learning mathematics. In the design, the instructional activities are meant to enhance students' abilities and awareness of the relation between algebra and arithmetic. The relation between algebra and arithmetic is learned through the context of the area of a rectangle. Hence, the students need to have a prior understanding about the concept of area in geometry. Therefore, students do not merely learn the connection between not merely arithmetic and algebra, but also at the same time, that between those two branches of mathematics and geometry.

2. Phenomenological exploration

   The mathematical activities are started within a concrete context. The design mainly employs numerical numbers together with a realistic phenomenon as the context. The context offers an exploration to structure a model as a bridge to help the transition from arithmetic to algebra.

3. Interactivity

Discussion among students will open much new knowledge when listening to others' point of view. In the design of this study, the teacher encourages whole class as well as group discussions. In this study, students will be free to explore and share their understanding about the concept of binomial multiplication as well as the strategies in solving the problems through the discussions. Students, then, will be supposed to construct their own understandings about the material based on their own prior understanding and their conclusions from the discussions.

2.3.2 Rectangular Area Model to Model Binomials Multiplications

Inline with the concept of RME, models are developed to promote students' learning and understanding on mathematics. Several models have proven useful to help students to understand multiplication. Two of recently promoted models to encourage understanding of multiplication are the ratio table and the area model.

The ratio table has been used to promote the initial understanding in not merely multiplication, but also division (Widjaya et al, 2010; Dole, 2008). More over, the ratio table has proven to be a useful model when the learning process is extended from multiplication to proportional reasoning. This model helps students to develop mental strategies for solving problems related to proportional, such as fraction problems. Middleton and van den Heuvel-Panhuizen (1995) noted the usefulness of the ratio table in understanding the concept of cross multiplication. Means that this model is very useful as a computational tool as well as a conceptual tool for making connection between the concepts of ratio, fraction, and percent. However, according to Dole (2008), there are several issues with the use of the ratio table, such as (i) it is time-consuming to construct, (ii) since ratio tables can be extended infinitely, students may feel the need to continuously putting numbers in empty cells and not knowing when to stop, (iii) students sometimes forget that the ratio table is meant to show the sequence of their calculations, not the order from the smallest to the largest, and (iv) students' progressive calculations sometimes do not match with the given ratio.

Outhred and Mitchelmore (2004) stated that the rectangular array model is the basis to model multiplication, to represent fractions and as the basis for area formula. The latter might be the reason why others refer to this rectangular array model as area model since it is closely related to area. Meanwhile, Ball et al. (2005) claimed that the area model, in respect to the algorithm of multiplication, is an effective way to represent its *meaning*. Ball et al. (2001) added that when using the area model as a representation for multiplication, one has to pay attention to the units, including the differences between linear (side lengths) and area measurements.

Paying more attention to the units, this model is more suitable to model binomials multiplication since the model can show how a variable represent unknown as a unit. The area model can also connect the multiplication of numerical numbers using the area of a rectangle with algebraic multiplication. Furthermore, Rathouz (2011) found in her pilot project that the area model could show how the distributive property works in the binomial multiplication. The recognition of the distributive property is similar to the formal way of teaching binomial multiplication: FOIL (first, outside, inside, last), which has been used globally.

Nevertheless, Fosnot and Jacob (2010) also mentioned that array model, which refers to area model as well, is a concrete tool that can be used by the students to explore the commutative, distributive and associative properties. In their book, they used array model to model binomials multiplication. In this regard, they described the array model as a total area of a rectangle with factors as the measures of the sides.

2.3.3 Socio-mathematical Norms and The Role of the Teacher

Nevertheless, the successful of the design is closely related to the social norms, socio-mathematical norms and how the teacher carries the lesson to make the students actively construct the new knowledge. General classroom norms mainly refer to understandings about the acts that the students are expected to do. Whereas, socio-mathematical norms refer to normative aspects of mathematical discussion that specific to teacher and students mathematical activities in the classroom (Cobb & Yackel, 1996). These socio-mathematical norms are different with general classroom norms since these are specific to mathematical aspects of students' activities. Cobb and Yackel (1996) listed the normative understandings in socio-mathematical norms, including understandings of what count as mathematically different, mathematically sophisticated, mathematically efficient, mathematically elegant, and what count as an acceptable mathematical explanation and justification. However, the role of a teacher is needed as a facilitator to build good socio-mathematical norms.

As Simon (1995) noted, leaving the students alone so, they will construct mathematical understanding or putting them in groups so they can communicate while solving the problems is not very helpful. Hence, to assist the teacher, this study designs a series of teacher guides as well as a hypothetical learning trajectory to give a view to the teacher of how she should pose a problem or question, stimulate the students to share their ideas and strategies, conduct and lead a discussion and how she should react to students' answers. The role of the teacher is even more crucial since she is the one who directly interact with the students; as such, she is the one who selects topics for discussion or reacts directly to unexpected things during the lesson.

Based on the aforementioned studies, we aim in this study to design instructional activities using the area model to support students' understanding of binomials multiplication. Then the research question of this study is **How does area model promote students' understanding of the multiplication of two binomials?** To answer this research question, some sub-research question is proposed, which are:

1. How do the students use and understand the area model?
2. How does students' ability in translating from area model into algebraic expression?
3. Do the students able to connect area model with binomials multiplication?
4. How does the improvement of the students' ability (by using area model as a tool) in solving binomials multiplication?

CHAPTER 3. METHODOLOGY

3.1. Research Approach

Innovation on learning algebra in Indonesia is lacking and students face many difficulties during the learning process of understanding the concept of binomials multiplication. Therefore an innovation and improvement are needed and we need to design new instructions in the teaching and learning processes of algebra. Hence, design plays a critical role in the development of mathematics education. The design is made to implement a theory as such it can be evaluated, or even refined.

Aiming to give a contribution to improving mathematics education and contribute to the local instruction theory by designing a sequence of instructions, this study investigates how area model support students' understanding in the concept of binomial multiplication. This study is meant not merely to generalize whether the area model support students' understanding on algebraic multiplication, but also to describe in what way it supports. In the end, retrospective analysis on the study leads to a conclusion, which answer the research question. Combining design and research at the same time is one of the characteristics of design research. Therefore, design research is chosen as the appropriate means to develop a sequence of instructional theory for learning the concept of binomial multiplication.

Design research is occupied as the means to achieve the goal of this study and to answer the research question. In conducting a design research, Gravemeijer and Cobb (in van den Akker et al, 2006) categorize three phases, which are: (i) preparing for the experiment, (ii) experimenting in the classroom, and (iii) conducting retrospective analysis.

1. Preparing for the experiment

   In this phase, the researcher begins to analyze problems and identifying goals. By studying literatures, common problems related to students' difficulties or learning issues could be pointed out. Hence, mathematical learning goals can be derived from there and contribute to the design of a learning trajectory. The literature study will also benefit for supporting the designing process. It gives insight about how the instructions should be developed. Consequently, instructional activities are designed during this phase. Since the aim of this design study is to contribute to a local instructional theory, all aspects on designing the instruction, including conjectures of students' thinking toward the activities as well as the role of the teacher based on the classroom culture and history, should be well thought out. Hence, discussion with the teacher is done to know the prior knowledge as well as the characteristics of the students and the teacher itself. Social and socio-mathematical norms are also pointed out. In other words, a Hypothetical Learning trajectory (HLT) is made during this phase as well. However, the conjectures in the HLT are dynamic, in which they can be adjusted or changed due to students' actual learning process during the pilot project.

In this study, literature review is focused on students' difficulties toward school algebra in early secondary school. A study in the current Indonesian curriculum for the eighth grader is also done to choose and get insight about the material, which is binomial multiplication, and how it has been being taught at school. The next focus on the literature review is on finding the most suitable model to help students understanding the concept of binomial multiplication. While designing the activities, the model is combined with realistic context in order to engage students' interest. A first Hypothetical Learning Trajectory is made based on the researcher's predictions based on literature reviews and interview with the teacher, students and the result of the pre-test for the first cycle.

2. Experimenting in the classroom

   In this phase, design experiments are conducted. Gravemeijer and Cobb (in van den Akker et al, 2006) stated that the aim of this phase is to test as well as to improve the conjectured local instruction theory that has been made during the first phase. Moreover, understanding about how it works is crucial in regards to be able to answer the research question.

To be more specific, the design in this study is implemented and analyzed to gather data in order to answer the research question, which is how area model supports students' understanding on binomial multiplication. The design is implemented and tested in the classroom within two cycles. The first cycle is a pilot experiment where only a limited group of students join as the participants and the researcher plays the role of teacher. In this pilot experiment, HLT is used as the guideline for the researcher to deliver the materials, the focus of each activity as well as the focus of the data collection and discussion. The aim of this pilot experiment is to adjust the design, including the activities, the sequence, and the conjectures. Some changes may be done to improve the design. The second cycle is an actual teaching experiment where it involves a whole class with the initial mathematics teacher in that class. In this cycle, the improved design with a refined HLT is implemented. The HLT functions as the guidelines for the teacher as well as researcher and observer.

In the second cycle, a discussion with the teacher is held after each lesson. During the discussion, reflection on some important things is pointed out, including the strong and weak points of the lesson, what goes right and what goes wrong during the lesson, and what needs to be changed and what need to be kept for the next cycle. Furthermore, the researcher and the teacher also share ideas and opinions about activities for the next lesson, discuss whether it should be refined or adjust based on the previous lesson. The redesigning process during the cycle where researcher together with the teacher adjusts or refines the design is inline with the theory of a cumulative cyclic process in design research.

3. Conducting a retrospective analysis

   The retrospective analysis is done after each cycle. The collected data from the teaching experiment is analyzed using the HLT as the guideline. In this manner, the conjectures about students' thinking and learning in the HLT are compared with the observed students' learning processes in the classroom. However, there is a need for attention to mention not merely on what happens in the classroom that support the HLT, but also observations that contradict the HLT. The analyses of these learning processes which support and contradict the conjectures on HLT leads to a conclusion which is used to answer the research question.

3.2. Data Collection

This study is conducted in the SMP 1 Palembang and involves the eight grades students from two different classes. Data collection is in two phases, during the preparation for the experiment and during the classroom experiment phases.

3.2.1 Preparation Phase

In this phase, the data collection involves the teacher and all students who participate in both cycles. The researcher collects data about students' prior knowledge as well as the social and socio-mathematics norms in the classroom. Hence, interview with the teacher as well as some random students is the first step. The interview is a semi-structured interview, which means that there are some guidelines for the main questions, but more questions are elaborated during the interview based on the answers or responses by the interviewees. The scheme for the interview can be seen in the appendix 1. Video recording during the interview is important as such the result can be analyzed more. During the interview, it is also important to get to know the teacher's way of teaching, classroom norms, students' characteristics, students' abilities in algebra as well as multiplication, and estimation about how many in the classroom who are high and low achiever in mathematics, especially in algebra and arithmetic.

The next step is giving pre-test for two classes, which students will participate in the teaching experiments. This means that even though not all of the students from the first class participate during the pilot project, the whole class get the pre-test. Besides, the researcher also does a classroom observation in the natural classroom to see how the students interact to each other as well as to the teacher, whether the students are common to give opinions, discuss or work in groups. The observation also gives insight about the classroom norms. In the end, the result of the pre-test as well as the interview and observation is then used as the starting points of the HLT.

3.2.2 First Teaching Experiment (Cycle 1)

The first teaching experiment is conducted as the pilot experiment (first cycle). The participants are not all students, but merely about 3 to 4 students who are able to represent the whole class, means that they are not the highest and lowest achievers in the class. The researcher plays role as a teacher and the initial teacher observes and make field notes during the teaching experiment.

Besides investigating the students' learning and reasoning processes, the aim of this cycle is to test the hypothesis and conjectures in the HLT. The HLT in this manner is the initial HLT that has been developed during the preparation phase. The following data area collected: students' written work, video recording of all activities and field notes by the teacher of observations during the lesson. The scheme for the observation can be seen in the appendix 2. The data is focused around students' learning. In the end, the analysis of the data from this cycle is used to revise and improve HLT for the second teaching experiment.

3.2.3 Second Teaching Experiment (Cycle 2)

The revised HLT from the previous teaching experiment is used in this cycle. However, the participants are different. This cycle involves a whole class with the classroom mathematics teacher as the teacher. However, there is merely one focus group to be observed by the researcher and her partners, other master students who are also doing design research for their thesis. The focus group is chosen based on the results on pre-test and on interview with the teacher, where the member of the focus group are students with middle mathematics abilities. The researcher collects the same data as in the previous cycle by making video recordings of the whole class teaching and learning processes as well as the discussion among the students in the focus group. Hence, there are two video recorders used in this study, one with the focus of whole class environment, including how the teacher delivers the context and how she conducts the mini lesson and the whole class discussion, and one camera focusing on the focus group. Moreover, the researcher and her partners make field notes.

3.2.4 Pre- and Post-test

The design in this study aims to introduce binomial as well as binomial multiplication. Hence, the students have not learnt yet about binomial. In this regards, the pre-test is design to be different with the post-test. The content of the pre-test is mainly to check the required students' prior knowledge to learn the topic. In this sense, the pre-test mainly consists of problems related to arithmetic multiplication, operations in linear algebra as well as the concept of variable. In this case, the students are also required to give their strategies to get the solutions for the problems. However, there is also a problem related to binomial multiplication in the pre-test. The students' answers from this problem mean to be compared with the students' answer from the post-test.

In the post-test, the problems are designed to check students' understanding, strategies and the use of the area model. Hence, the post-test consist of problems, which are served with the models as well as realistic problems where the students are given opportunity to explore and use the model. Moreover, more formal problems and problems with different contexts with the one in the activities are also given to check students' understanding and abilities to apply their understanding. The pre-test and post-test can be seen in the appendix 3 and 4.

As have been mentioned before, a pre-test is done before the teaching experiment. This test is aimed to check students' prior knowledge about arithmetic and algebra, especially on their abilities on multiplication, their understanding about variables and multiplication on algebra. The pre-test is given for students from the first and second cycles. In this manner, researcher collects written works of all students. The results from the pre-test are used to choose students for cycle one. Meanwhile, the results from the pre-test of the students for the second cycle are used for choosing the focus group on the second cycle. The group in the first cycle and the focus group in the second cycle are chosen among the middle ability students with assumption that they will be able to work together in one group and they are also able to represent the whole class achievements.

The post-test is given at the end of each cycle. This test is aimed to assess students' understanding about the concept of algebraic multiplication using area model that they have learnt during the teaching experiment. After the post-test, focus group from the second cycle is interviewed about their work to know how they solve the problems, their reasoning, strategies and understanding. The interview is also recorded on video. Hence, different from the pre-test, the researcher not only collects students' written works but also video recorded observations.

3.2.5 Validity and Reliability

In regard to the data collection, it is important to consider the aspects of validity and reliability. Validity refers to whether the data collection really measures what is intended to be measured based on the research questions. Meanwhile, reliability refers to the independency of the researchers.

Since this study involves collecting different types of data, including students' written work, interviews, video observations and field notes, it enables to do data-triangulation which can improve the internal validity of the data. Furthermore, this study is held in a natural classroom while the focus group is chosen among middle mathematics ability students and the teacher is the initial teacher. These contribute to the ecological validity of this study. On the other hand, the use of video recordings to collect data contributes to the internal reliability of the research because it increases the consistency between the field notes from the observation from the real facts recorded on the videos.

3.3. Data Analysis

Researcher needs to analyze all collected data and draw conclusion based on that. This conclusion is meant to answer the research question. To analyze the data, we conduct a qualitative analysis focusing on students' learning processes and how the area model supports students' understanding on binomials multiplication.

3.3.1 Pre-test

The result of the pre-test is analyzed to investigate students' prior knowledge relevant with the topic of algebraic multiplication. There are some aspects on students' prior knowledge needs to be revealed in the analysis of the pre-test, including students' knowledge about multiplication of whole numbers, the area of rectangle, variables and linear algebraic addition, subtraction and multiplication. Furthermore, students' informal knowledge and their misconceptions about the prior knowledge are also gathered.

3.3.2 First Teaching Experiment (Cycle 1)

The collected data including the students' written work, video observation and field notes for all the activities during this cycle are analyzed in regard to investigate the learning process of the students. Fragments of the recording, which show the crucial moments of how the students find the big ideas, strategies, share, explain and confront each other about their ideas or even struggle in doing the problems, are chosen.

The analysis will focus on improving the HLT and the mathematical activities. The initial HLT is used as the guideline whether the actual learning process of the students meet the conjectures on the HLT. Furthermore, the HLT is revised and improved based on the analysis of the first cycle.

3.3.3 Second Teaching Experiment (Cycle 2)

The improved HLT is used as the guideline for the analysis of the second teaching experiment to investigate the learning process of the students. Fragments from the video recording is chosen and transcribed to show and interpret students' reasoning. The selection of the fragments is based on the same reason with the previous cycle, that show students' learning and reasoning processes, when they find big ideas, strategies, sharing, explaining and confronting ideas with their friends as well as their struggles. The analyses of the fragments, the students' written work and the whole activities then compared with the conjectures in the HLT in regards to the students' learning processes to draw conclusion in order to answer the research question. Some unexpected things may also be discussed on the analysis.

Inline with the goal of this study, the analysis will be focus on (i) whether it is inline with the conjectures on HLT, means that all data that support and contradict the HLT as well as students' struggles need to be collected, (ii) whether the area model support students' understanding toward binomial multiplication or not and (iii) how the area model supports or fails to support students' understanding on binomial multiplication. These analyses are also used as the basic to redesigning the HLT and improve the design to contribute to the local instructional theory.

3.3.4 Post-test

The qualitative analysis is employed to analyze the students' written work of the post-test. The analysis is not merely comparing the quantitative result on this test with the result on the pre-test, but also looking deeply on students' work, including their strategies, the use of area model, and their misconceptions and errors. The post-test is aimed to investigate the development of students' understanding toward the topic as well as their strategies, misconceptions and the use of area model. The result of this analysis contributes to the analysis of the teaching experiment and used to draw conclusion as well.

3.3.5 Validity and Reliability

Enhancing the quality of this study can be done by improving the validity and the reliability in terms of data analysis. Validity is categorized into internal, external and ecological validities. Whereas, reliability is merely categorized into internal and external reliabilities.

Internal validity can be improved by the data triangulation between all methods of analyzing the data, including analysis of the students' written work, video recorded observations, the interviews and field notes. Meanwhile, external validity refers to the generalization of the conclusion of the study. This means that whether the products in this context of study, including the HLT, instructional theory and the sequence of activities, can be implemented in different context. By making explicit the crucial elements of the learning process and how it can influence the whole learning process, those crucial elements can contribute to the external validity.

Discussion with partners during the retrospective analysis about the crucial elements, such as students' written work and fragments of video observation and interview, enhance the internal reliability of this study. Meanwhile, giving a clear and complete explanation and description in a good order about the learning process enhance the traceability which contributes to the external reliability of this study.

CHAPTER 4. HYPOTHETICAL LEARNING TRAJECTORY

A hypothetical learning trajectory (HLT) is developed based on the understanding of students' prior knowledge. It is used as a vehicle for learning plan, which consists of the goal of the students' learning, mathematical activities and conjectures about students' learning processes (Simon & Tzur, 2009).

4.1 Overview of the Design

Since the aim of this study is to investigate how area model supports students' understanding on multiplication of two binomials, the instructional activities are design to promote the use of area model for multiplication. Furthermore, the activities are designed to lead the students to use the area model to model binomial multiplications and to find the product of the multiplications.

In this study, there are six lessons, which are implemented in eighth graders of SMP N 1 Palembang. The design is implemented within two cycles. The first cycle is a pilot experiment, which takes only 3 to 4 students from a class, whereas the second cycle involves a whole different class.

Furthermore, the general overview and mathematical ideas of the activities are described in the table 3.

Table 2 Overview of the activities

Lesson	Activity	Description	Learning Goal(s)
1 and 2	"Hutan Rakyat"	The students multiply numbers using area model in the context of "Hutan Rakyat". By the end of the second meeting, the students are reminded (and discuss) about the concept of variable.	1. To lead the students to multiply using area model 2. To introduce the notion of *rectangle formula* and *pieces formula.* 3. To emerge variables in the multiplication.

3	From area to area model	In the beginning of the lesson, the students match drawings with algebraic expressions. Furthermore, the activities continue with the same context about "Hutan Rakyat" but the measurements of the sides of the lands involve unknown lengths represented by variables.	To lead the students to use the area model to do multiplications of two binomials. However, formerly it is important to make sure that all students understand the translation from drawings to algebraic expressions and vice versa. This checking is done during the first session of the lesson, which is the matching activity.
4	The rose garden plan	The students are challenged by a problem to compare the area of a square rose garden plan and rectangular rose garden plan. In this case, the students are given examples of their seniors' work and have to order the work based on specific reason. After the rose garden problem, the students are asked to solve three problems about finding the area of coconut plantations where not all parts of the lands are used as the coconut plantation. The idea of this activity is that students can apply the strategies they learnt during the previous session of this meeting.	To create a situation where the students investigate how to find the product of multiplication of binomials, which involves subtraction.
5	"The puzzle game" and "How to explain?"	During the first activity, the students need to fill in the blanks and complete the rectangle and pieces formulas. Some of the problems are actually related to factorize quadratic	1. To lead the students to use their understanding about area model and some strategies derived from it to factorize quadratic expressions

		expressions. The next activity is mainly to check students' understanding by confronting them with formal algebraic problems where the students are asked to use their understanding during the experiment of the design to solve the problems.	2. To check how the students solve formal mathematics problems using their understanding about area model and other strategies they have learnt during the experiment of this design

4.2 Lesson 1: "Hutan Rakyat" Project

4.2.1 Mathematical Goal

The goal of this lesson is to lead the students to multiply using area model and to introduce the notion of *rectangle formula* and *pieces formula.* These two notions are the basic of the design in this study.

4.2.2 Starting Points

Since the students are secondary school students, they have learnt how to multiply two one-digit numbers and how to find the area of a rectangle. Meanwhile, related to algebra, the students have also learnt about linear equation systems, which means that they have learnt the concept of variable as well.

4.2.3 Mathematical Activities

Preliminary activity

To start with, the teacher poses two problems. The second problem is given after the students have finished the first problem. The problems are: (i) explaining to 4^{th} grade students how to multiply 5 and 17. (The teacher writes down on the blackboard 5x17 and the teacher emphasizes that the 4^{th} graders are having difficulties in understanding that the *saving* needs to be added up to the final result), (ii) The teacher has a rectangular piece of land, that is 17 meters long and 5 meters wide (draw the rectangle on the board while telling the story). The teacher is planning to divide the piece of the land into two parts, 10 meters of its length to plant vegetables and 7 meters a lawn. The students need to find the area of the whole land and the area for the vegetable plantation as well as the grass lawn. The partition can be seen in the following picture (the teacher draws the picture on the blackboard).

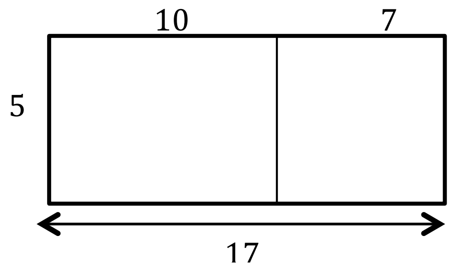


Figure 1 Representation (drawing) for the problem

To solve the problems, the students are given some time to think and discuss with their neighbor. Afterward, the teacher asks some students to give opinions, ideas or strategies to solve the problems. To make sure that all students understand their friends' explanation, the teacher asks the whole class whether they understand it or not. If some of them do not understand, the teacher asks another students to explain using their own words.

The discussion is then started especially to compare students' strategies in solving both problems. In this discussion, the main concepts of this design are introduced. They are a rectangle formula and a pieces formula. In the case of the second problem (finding the area of a rectangle), the formulas are:

- The **rectangle formula** (of the whole rectangle) is 17x5
- The **pieces** formula is 50 + 35

Both formulas will be the core concepts of this design.

Group work

There are two sessions of the group work. In the first session, the teacher starts the discussion by engaging the students' interest by asking the following question: if the students have a piece of land, what do they want to do with it. Furthermore, the problem that the students need to work with is depicted in box 1.

Your uncle has a piece of land with a length of 15 meters and a width of 11 meters. He plans to build a house on the land with and area of about 100 m^2. He also plans to have a garage with 3 meters wide. In front of the garage is a paved drive. For the rest of the land, he plans to have a nice garden.

1. Draw the house plan of your uncle's house
2. Is the area of the garage and paving together larger than the area of the garden? Explain your answer.
3. What is the total area of the land?
4. Find the *rectangle formula* and the *pieces formula* of the house plan

Box 1 The house plan problem

Half of the groups makes poster for their answer and the teacher chooses one group to present their answer. After the presentation, the teacher starts a class discussion by asking whether are other groups who have different house plan. The teacher also gives opportunity for those who want to ask, comment or give remarks. The teacher leads the students to reason and compare answers.

The second session is started by a story about the government project namely "Hutan Rakyat". The teacher asks whether some of the students know about the project or not, and then tells about the aims, the advantages and the regulations of the project. The teacher mainly focuses on the partition of the lands to fulfil the regulation. After that, the teacher hands out worksheet where the students need to fill in the blanks in area models and complete the rectangle and the pieces formula. By the end of the worksheet, the problems changed into using the area model to find the products of two multiplications.

4.2.4 Conjectures of Students' Answers and Suggestion for Teacher's Reaction

Preliminary activity

In answering the first question, most students will propose to use formal calculation at first. However, encouraged by the teacher that the 4^{th} graders have difficulties in understanding that the *saving* needs to be added up to the final result, they will try to find other ways. Therefore, the students may propose to differ the number based on its place value. As such they will get more easy number on the counting process (divide 17 into 10 and 7). Students then multiply 5 and 10 as well as 5 and 7, and add up the products to get the final answer. It is also possible that the students use repeated addition. In this manner, the teacher can stimulate the whole class to respond to this strategy and compare all strategies they have found to find the most effective strategy.

For the second problem, the students may say that the strategy they use for the previous method is similar or identical with how they solve this problem. Some others may say that the first and second problems are mainly about the same multiplication. In this case, the teacher encourages the students to compare and make conclusion of both problems. The students may conclude that doing multiplication is like finding area of a rectangle. They may also conclude that the strategy, making partitions of the rectangle as well as to split the big number based on the place value, is easier and they can do that even without scratch paper. The most crucial conclusion is that the pieces formula is the product of the multiplication in the rectangle formula. Hence, is the students do not arrive in that conclusion, the teacher scaffold them by asking the relation between those two formulas.

***Group* work**

During the introduction of the group work, the students give their ideas about what they want to build when they have a piece of land. In this case, some students will say that they want to build a house. Therefore, the teacher starts discussing the context, which is about making house plan. The following is the hypothesis for each sub-question:

1. For the first sub-question, the groups will probably have different size of the house, but the house plan will be the same.
2. For the second sub-question, the groups may come up with different answer based on their preferable of the measurement of the house. The students, however, may face difficulties in explaining their answer.

3. The students will not face any difficulty in solving this problem. However, some of them may still use formal calculation and some others may use area model and/or use the house plan they made as the model of the multiplication.
4. The students find the *rectangle formula* and the *pieces formula* by employing their understanding and information about the *rectangle formula* and the *pieces formula* from the previous problems and the information in the worksheet. Moreover, they will not have any difficulty.

A story about "Hutan Rakyat" project starts the second session of the discussion. The students give their opinions and ask questions about this project. During the group work afterward, for the first three numbers, some students may find difficulties in solving these problems. The difficulties could be in filling the missing value or finding the *rectangle formula* and the *pieces formula.* Moreover, the students will use their knowledge about the *area model* to solve them. The teacher, however, pays more attention to groups or students who have difficulties in solving these problems since this is a crucial phase for students to be able to understand the future materials. When the students do not use their knowledge about the *area model*, encourage them to think about using it. For the last problem, some students may find difficulties on the multiplication with the fractions. Others may feel unsure about their answer. The rest may face no difficulty at all. Hence, they probably check their answer with formal calculation or vice versa. When the students face difficulties, the teacher may help them by giving examples of simpler cases or numbers.

By the end of the discussion, the students realize that their difficulty with the formal calculation, including fractions multiplication, can be solved using rectangular area model.

4.3 Lesson 2: "Hutan Rakyat" Project

4.3.1 Mathematical Goals

The goals of this lesson are to continue to lead the students to multiply using area model and, by the end of the lesson, to emerge variables in the multiplication.

4.3.2 Starting Points

The starting points of this lesson are based on what have been learnt by the students in the previous meeting. The starting points are that the students have been able to multiply numbers using the area model and they are able to find the rectangle formula and pieces formula of an area model.

4.3.3 Mathematical Activities

Group Work

To start with, the teacher reminds the students about the previous meeting, which is about making partition of pieces of lands for "Hutan Rakyat" project as well as about the rectangle formula and the pieces formula. The teacher then explains that the students will tackle similar problems to the ones in the previous meeting. However, this time the known values are not always the length of the rectangular sides.

The students are given a worksheet, which contains similar problems to the ones in the previous meeting. However, the missing values are more varying. Moreover, there are also some problems if the lands are in a square shape (see students' worksheet day 2). There are also problems where the students need to find the product of multiplication by first draw the area model that represent the multiplication.

The students work with their group to tackle problems in the worksheet. During the group work, the teacher walks around in the class and checks each group to make sure all groups are on the right path. The teacher pays attention on some students or groups with difficulties since this is a crucial phase for students to be able to learn the future materials. In this case, the teacher may help by giving scaffolding.

After the group discussion, the teacher starts a short class discussion, especially about problems with lands in a square shape. Some students share their ideas and strategies to solve the problems; other students give comments, remarks and questions. The teacher leads the discussion and gives each student an equal opportunity to express their ideas, and to accept, accommodate and compare students' ideas and strategies.

Mini Lesson – emerging variables

The teacher tells a story about her friend's house. The story is:

> *"My friend has a house that is 10 x 10 meters in size. In front and on the right sides of the house, he has a lawn of the same dimension. However, I do not know the exact measurement of the grass garden. It could be 3 or 5 meters in length."*

The teacher asks two students to draw a representation picture in front of the class. One student draws the lawn with a length of 3 meters and another student draws the lawn with a length of 5 meters. After that, the teacher asks the whole class to find the rectangle formula and the pieces formula for the whole land for both 3 and 5 meters length lawn.

The teacher asks follow up question: **"the length of the garden is unknown. How can you express that in the drawing?"** The students are given some time to discuss in pairs to solve the problem. After the discussion, the teacher asks some students to voluntarily share their ideas. Some of them may say they can use a letter or variable to represent the unknown length. The notion of a variable may appear since the students have learnt variables in their previous grade. If students do not come up with the notion of a variable, scaffold them by reminding them of simple problems where a specific value is represented by a variable.

A student draws the new rectangle with the variable in front of the class. To make sure all students get the idea of a variable to represent the unknown length, the teacher leads a short discussion about what things can be represented by a variable in algebra. The teacher accepts, accommodates and compares students' ideas. At the end of the lesson, the teacher tells the students that the *rectangle formula* of the land is $(10 + x)(10 + x)$ and the *pieces formula* of the land is $100 + 20x + x^2$.

4.3.4 Conjectures of Students' Answers and Suggestion for Teacher's Reaction

Group Work

Even though the students do similar problems with the previous meeting, a few students may still have difficulties on filling in the missing values or finding the rectangle and the pieces formulas. In this case, it is important for the teacher to pay more attention to the students who still have difficulties. The teacher needs also to check whether the students have correct understanding.

To solve the four first problems, the students use their knowledge about the area of a rectangle to solve the problems. By filling in the blanks, the students will make relation between multiplication and division. For the 3rd and 4th problems, where the areas are in square shape, the students will employ their understanding about the characteristics of rectangle as well as square to decide that a square is actually a special rectangle. If the students struggle with this understanding, the teacher helps them by asking the characteristics of both figures and asks the students to compare and make conclusion. Here the students will grasp that a square is a special rectangle.

In solving the last part of the worksheet, which is about drawing area model and use it to solve some multiplications, the students will not face any difficulties since they have been working with the area model for two lessons. Few students may get confuse and the teacher can scaffold them by reminding them about the "Hutan Rakyat" problems where they need to make partition for the lands. Furthermore, the students will make partition in the area models based on place value of the numbers, and few probably will make partition based on easy numbers.

Mini Lesson

Recalling the story from the teacher: *"My friend has a house. The size is 10 x 10 meters. In front and on the right side of the house, he has a lawn* with *the same size. However, I do not know the exact measurement of the lawn. It could be 3 or 5 meters in length"* there are several conjectures for students' answers and the sequence of the discussion. Bellow is the sequence of the mini lesson:

1. Some students are voluntarily drawing the rectangle in front of the class. If there is no student who volunteers to draw the *area model*, the teacher asks two of them.
2. The correct drawing will look like this:

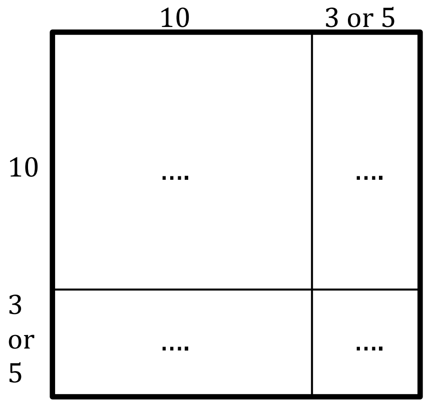


Figure 2 Representation for the correct drawing of the house plan

If the drawing is incorrect, the teacher asks other students' opinion about the drawing and to re-drawing until they have the correct drawings.

3. Since the teacher asks the students to find the rectangle and pieces formulas afterward, the students recall their understanding about those two formulas and most of them will be able to find the formulas for the situation. If some students forget about the formulas, ask other students to explain it. This also means to check students' understanding of the concept of the rectangle formula and the pieces formula.
4. After finding the formulas, the discussion is continued by a question by the teacher: **"the length of the grass garden is unknown. How can you express that in the drawing?"** After that, the teacher lets the students to discuss about how to answer the problem in pairs. After short discussion, some students may say that they can use a question mark (?) or a letter to represent the unknown length. Some may directly propose to use a variable. In this case, the role of the teacher is very crucial. The teacher leads the discussion and give some questions to lead the students to an agreement that the students can write a letter as a representation of the unknown length (since Indonesian students are mostly familiar with the letter x in algebra, most of them may propose to use x).
5. The next is to ask some students to draw the new rectangle in front of the class and to explain their drawing. In this case, some students may get confused where to put the letter (variable). Hence, the teacher may ask

another student who knows about the position of the letter to explain. The correct drawing will look like this:

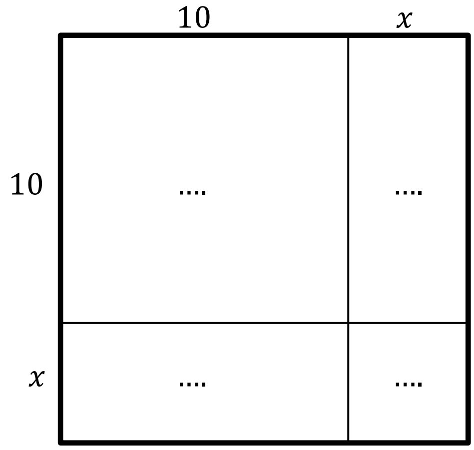


Figure 3 Representative drawing for the problem

6. If in the previous step the students do not come up with the word variable, ask them what they usually call the letter in mathematics. Since the students have learnt algebra and variable from the previous grade and event previous semester, some of them may state the notion of "variable" and others will agree.
7. Next, the teacher asks the students what can be represented by a variable in algebra. Students' answer may vary, but one of them is variable as a representation of unknown length. The teacher needs to accept, accommodate and suggest the students to compare their answers. If none of the students come up with variable as a representation of unknown length, the teacher helps them by reminding about the problem on this mini lesson.
8. The teacher explains that the $(10 + x)$ is called a factor. The teacher then asks the students to share their ideas to define what is factor.
9. By the end of the mini lesson, the teacher tells students that: (1) the rectangle formula of the area is $(10 + x)(10 + x)$ and the pieces formula is $100 + 20x + x^2$. Some students may start to give their opinions. However, it is important for the teacher not to make any conclusion about the students' opinions and ask them to re-give their opinions in the next meeting.

4.4 Lesson 3: From Area to Area Model

4.4.1 Mathematical Goals

The goal of this lesson is to lead the students to use the area model, derived from the area problems, to do a multiplication of two binomials.

4.4.2 Starting Points

The starting points for this lesson are derived from what the students learnt in the previous meeting. Therefore, the starting points are that the students can make area model to solve multiplication as well as can find the rectangle formula and the pieces formula of the model. Moreover, the students have also been reminded about the concept of variable as a representation of an unknown value.

4.4.3 Mathematical Activities

To start with, the teacher reminds the students about the concept of variable which has been discussed in the precious meeting. Furthermore, this lesson is divided into four sessions of group work. Before starting the group work, the teacher hands out worksheet to all groups. There is no restriction for the time allocation for each session, each group can move on to do the next session problems after completing the previous session problems. Moreover, all activities in this lesson are in the context of group work.

The matching games

In this session, the students work to match drawings with same value expressions. The aim of this activity is to make sure all students understand correctly and able to translate from drawings to their expressions and vice versa.

"Hutan Rakyat" problems

During this session, the students do similar problems in the same context, which is "Hutan Rakyat". However, this time the partition of the lands will involve unknown lengths represented by variables. Moreover, in this meeting the students need to work with variables as well.

During the group work, the teacher walks around in the class and checks each group to make sure all groups are on the right path and have correct understanding. The teacher pays attention to some students or groups with difficulties since this is a crucial phase for students to be able to learn the future materials. In this case, the teacher may help by giving scaffolding or reminding and asking the students to compare the material in this meeting with the one in the previous meeting as well as in the first session of this meeting. During the discussion process, the students reason that the product of two equal variables is the square of the variable itself.

House plan problem

In this session, the students work in groups to find the rectangle formula and the pieces formula of a house plan. Moreover, the students also need to find and state the relationships between both formulas. It is very crucial for the students to discuss and share their ideas about the relationships between the rectangle formula and the pieces formula. Hence, the students are asked to make a poster for this problem to be discussed during the math congress.

Making up a story

In this session, each group has to make up a story or situation that can represent the multiplication of $(x + 6)$ and $(x + 2)$. Furthermore, the students are asked to draw an area model to represent the situation and use it to find the product of the multiplication. In the end, the students need to write their story, area model and steps of using the area model to find the product of multiplication in poster paper.

Math Congress

The teacher asks two groups to represent problems from the 3rd and the 4th sessions. In the first discussion, the discussion focus on how the students apply their understanding about area model and use it to solve the house plan problem. More important, the teacher leads the discussion, as such all students will understand the relationships between the rectangle formula and the pieces formula. The teacher accepts and accommodates students' ideas and encourages the students to compare their ideas, strategies and answers.

For the second discussion, the discussion would be about the area model and how the students use it to find the product of two binomials $[(x + 6)(x + 2)]$. After the presentation, the teacher lets other groups to share their stories as well as give questions, answers for the questions, comments or remarks. The teacher leads the discussion by giving each student an equal opportunity to express their ideas, and the teacher will accept, accommodate and compare students' ideas and strategies.

To end the lesson, the teacher encourages the students to formulate a conclusion or share their opinions about the use of area model to find a product of two algebraic factors. Afterward, the teacher gives homework, which is to prove that the rectangle formula and the pieces formula given by the teacher in the previous meeting during the mini lesson are correct.

4.4.4 Conjectures of Students' Answers and Suggestion for Teacher's Reaction

The matching game

For the first problems where the drawing is a line with $(10 + x)$ long, some students may think to match that drawing with $10x$ instead of $(10 + x)$. This is because in the previous activities, for similar problems but without variable, the students think that they need only to arrange the value, not add. Hence, the teacher scaffolds the students by asking some additional lengths in number line.

For the rectangular drawings, the students will not face any meaningful difficulties. This is because: (1) the students have known how to translate the length of the sides of the rectangle based on their understanding in the previous problem this meeting, (2) the students have learnt about rectangle and pieces formulas from the previous meeting and already have understanding that they can find the area of the rectangle by multiplying the lengths of the sides.

"Hutan Rakyat" problems

There are several conjectures for students' answers in this session, which are:

1. Since the students have learnt algebra with one variable for the first and second numbers, including all the operations, some of them will face no meaningful difficulties in doing these problems. Since this part is a crucial moment during this learning, the teacher pays more attention to the students who have difficulties in solving these problems.
2. For the third and fourth numbers, the students understand that when a variable is multiplied by itself, the product will be the square of the variable ($x \times x = x^2$). If students hardly understand the multiplication of two identical variables,

the teacher scaffolds them by asking the result of some whole numbers multiplied by itself and leads them to square numbers.

3. The students check the answer using their understanding and strategy from the previous problems. In this case, it is important for the teacher to make sure that all students have correct understanding.

House plan problem

In this session, the students use their understanding from the previous problems to solve this problem. The students will use the house plan as an area model to model the multiplication and find the rectangle and pieces formulas. Some other students may draw new area model which looks exactly like the area model from the previous problems. Few students will get confuse because this is the first time they have to multiply not only with x but with $2x$. However, other students will be able to explain to their friends who have that struggle. More than that, the students will not face any difficulties.

For the part where the students need to find the relationships between rectangle formula and pieces formula, the students understand that the pieces formula is the product of the multiplication expressed by the rectangle formula. If some students do not come up with this conclusion, remind them about the area formula and the pieces formula on rectangle with whole numbers from the previous activities. In this case, the *rectangle formula* represents the multiplication and the sum of the *pieces formula* represents the product of the multiplication. Some students will also connect the rectangle formula and the pieces formula with the general (geometric) area formula for rectangle.

Making up a story

Each group will make different story in this session. However, most of them will take area of lands or places as their context since they have been solving problems in this context. Few other groups may use different context based on their preference, including games. Few others may use different unit of context, such as time, price, etc.

To draw the area model of the situation, the students employ their understanding from the previous problems and draw an *area model*. If some groups have difficulties about how to draw it, the teacher reminds them about the area models in the previous activities. However, the students may find difficulty in translating information from their story to the area model. In this case, the teacher may come up with questions about the story to lead the students to refine their story or to complete their area model. Furthermore, the students use their understanding about how to find the rectangle formula and the pieces formula as well their relationships to find the product of the multiplication of $(x + 6)$ and $(x + 2)$ represent the story.

Math Congress

There are two groups presenting their answer, first group present the house plan problem and the second group presenting their story which represent the multiplication of$(x + 6)$ and $(x + 2)$. The conjectures for the discussion are described bellow:

1. During the discussion for the house plan problem, the students will argue each other that they will need to draw a new area model or they can simply use the house plan as the area model to solve the problem. The teacher leads the discussion and suggests the students to compare their ideas. By the end of the discussion, the teacher leads the students to make conclusion that it is both fine to make new area model or use the house plan.
2. Some students may still confuse about multiplying $2x$ with both x and 3. However, other students will be able to explain their ideas and strategies and the reason of their strategies. These strategies are basically from their knowledge in their previous grade (multiplying $2x$ and 3) and previous activity (multiplying $2x$ and x).
3. The students will discuss and share their ideas about the relationships between the rectangle formula and the pieces formula. The conjectures of their ideas have been given in the "house plan problem" part (see previous page).
4. After the second presentation, the students will ask the presenter to explain in more detail about their context and how they translate that into the area model. Some may propose to enhance the context. Others may give comments or remarks.
5. Other groups share their contexts and compare each other.

In using the area model to find the product of the multiplication, the discussion will relate this the strategy to solve this problem with their conclusion on the previous problem.

4.5 Lesson 4: The Rose Garden Plan

4.5.1 Mathematical Goals

The goal of this lesson is to create a situation where the students investigate how to find the product of $(x+a)(x-b)$ where a and b are constants. In other words, this lesson aims to find the product of multiplication of binomials, which involves subtraction.

4.5.2 Starting Points

The starting points for this lesson are derived from what the students learnt in the previous meeting, which are: (i) the students are able to do operations with numbers and variables and (ii) the students are able to use area model to do multiplication of two binomials. However, the binomials include only additional signs.

4.5.3 Mathematical Activities

The *Rose* Garden Plan

To start with, the teacher tells a story about a high school student who wants to build a square rose garden. In this case, the teacher draws the square rose garden plan in front o the class as follow:

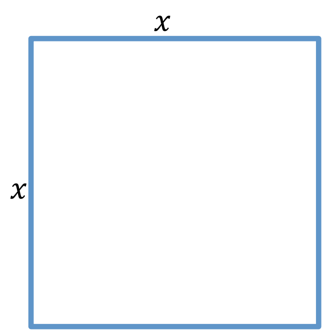


Figure 4 The square rose garden plan

The teacher then continues the story. The story is depicted in the box 2.

Ayu wants to build a square rose garden. In her plan, she measures the length of her land for the rose garden is x. However, Ayu's sister wants to have a rectangular rose garden. Hence, she suggests adding and reducing the same length in the right and front sides of the garden. She argues that they are going to need the same amount of rose plants since the area will be the same. Do you agree with Ayu's sister?

Box 2 The rose garden story

Afterward, the teacher asks the students how long they will cut and add to the measurement of the square rose garden to get the rectangular rose garden. Some students may propose to cut and add 1 meter and others will propose different length size. The teacher makes the agreement with the students to use, for example, 1 meter since it is an easy number.

The next step is to ask the students to draw the rectangular rose garden plan next to the square rose garden plan in front of the class. Ask the other students whether the drawing is correct or incorrect. Give opportunity to other students as well to share their opinions about the correct drawing. If some students do not understand with ones' explanation, ask another students to re-explain. If the drawing is incorrect, the teacher asks other students to give comments or opinions and to re-draw until they get the correct drawing as follow:

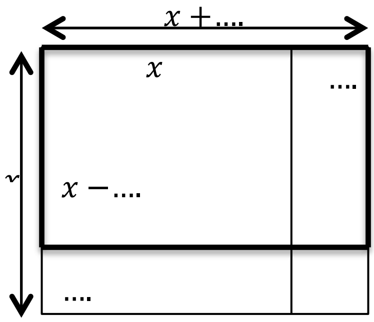


Figure 5 The rectangular rose garden plan

Next, the teacher continues the discussion by asking the students whether they agree with Ayu's sister or not. The teacher gives some time for the students to discuss with their neighbour. After the discussion, the teacher gives opportunity for students to share their ideas and opinions. In this case, the teacher accepts and accommodates all students' ideas and opinions. Moreover, the teacher encourages the students to compare the ideas and opinions. However, it is important that the teacher does not lead to any conclusion.

By the end of the discussion, the teacher tells the students that she has been giving the problem last year to their seniors and is going to give them some of their seniors' answers. In this case, the students are asked to order the answers. They can make order based on any criterion. After that, the students are given the examples of answers and they work in groups to order the answers.

After the group discussion, the teacher leads a whole class discussion and gives opportunity for two groups that have different orders to share their orders and the reasons why they make those orders. In this case, all students are given opportunity to ask, comment, argue or give remarks. The teacher suggests the students to compare each strategies of the answers, and discuss about its' effectiveness, advantages and disadvantages. By the end of the lesson, the teacher suggests the students that they can choose and use their favourite strategy to solve problems.

Coconut Problems

After the whole class discussion, the teacher delivers the next context, which is about the coconut plantations. The context is: "Some villagers in Sekayu want to make coconut plantations in some parts of their lands. The plans of how they make the coconut plantation can be seen in the worksheet. Now, your task is to find the area of the coconut plantation in each plan." Each group gets a worksheet, which contains three coconut problems. One of the problems in the worksheet is in figure 6.

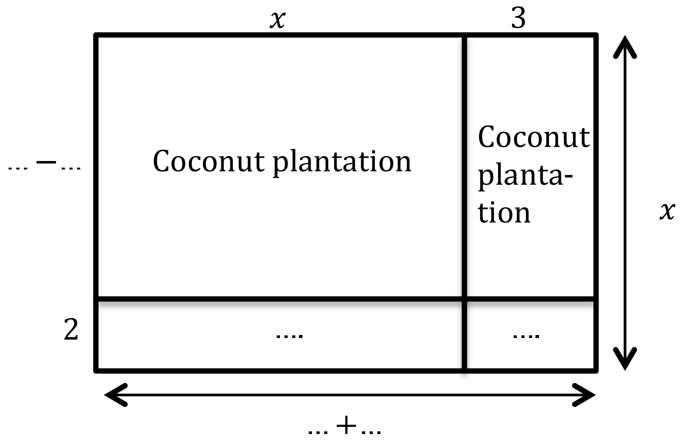


Figure 6 Example of the coconut plantation problem

Besides finding the area of the coconut plantations, the students also need to find the rectangle formula as well as the pieces formula. In this case, the students are freely to choose any strategy they think is most efficient and helpful. By the end of the group work, the teacher draws a table in front of the class and to put students' answers for all three problems. However, the table plots answers based on the strategies used by the students. In this case, the teacher leads the students to make conclusions related to the strategies and the results of all strategies.

4.5.4 Conjectures of Students' Answers and Suggestion for Teacher's Reaction

The rose garden plan

In drawing the rectangular rose garden plan, some students may find difficulties in regard to their initial drawing or spatial abilities. However, most students will be able to draw it easily and can explain to others by showing it step by step. This means, the students may first cut off 1 meter from 1 side, and then add 1 meter to other (non-parallel) side.

During the first whole class discussion about whether the students agree or not with Ayu's sister's opinion, some students may, in the first place, reason that they agree with Ayu's sister that the area will not change. Some others reason that the area will be different. In this case, the role of the teacher is to encourage the students to investigate and compare what their seniors have been done with this problem and make their final conclusions after the investigation.

During the group investigation, in making order of some students' answers, most groups will make an order based on whether the answer is correct or incorrect. Most groups will order from what they think the worst to the one they think is the best answer. Some others may do in the reverse order. The students' definition decide which one is better than the other one may based on the level of understandable of the answer, the appearance, the effectiveness of the strategy. Moreover, during the discussion, the students will come to an understanding that all the strategies, including the one using multiplication table, are just like generalization of the area model to a simpler model. However, when some students do not come to that understanding, the teacher will leads the students to understand that during the whole class discussion afterward.

During the whole class discussion, the teacher plays a role as a facilitator to help the students compare their orders, how they make the orders and their reasons. In this case, the teacher needs to be neutral and explain that none of the order is wrong. The teacher also encourages the students to compare, give remarks, questions or opinions to each other. The efficiency and accuracy of the strategies are also main topic of this discussion. Furthermore, the whole class discussion will lead to a conclusion that the areas of both rose garden plans are different. By the end of the discussion, the teacher leads the students to conclude that all strategies are correct and tells the students that they can use any strategy the feel most comfortable with.

The coconut problems

The first conjecture is that the students will not face any meaningful difficulty in finding the rectangle formula and the pieces formula. If some students still find difficulties or forget how to find them, the teacher asks other students who understand and still remember about them to explain. Second, the students will use one of the strategies, including using multiplication table, from the previous activity about rose garden plan to solve these problems. However, some other students may still use original area model.

4.6 Lesson 5: The Puzzle Game and the Challenge

4.6.1 Mathematical Goals

There are two main goals of this lesson, which are: (i) the students are able to use their understanding about area model, rectangle formula and pieces formula as well as some strategies they have learnt to factorize quadratic expressions and (ii) use area models as well as other strategies to solve formal algebraic problems related to finding the product of binomials multiplication and factorize quadratic expressions.

4.6.2 Starting Points

The starting points of this lesson are all the materials that have been learnt by the students in the previous meetings, including the use of area model to multiply two binomials, rectangle formula, pieces formula, strategies and other derivation models from the area model.

4.6.3 Mathematical Activities

In line with the previous lessons, this lesson is carried out in a group work atmosphere. There are mainly two sessions, which have different goals. The first is a puzzle game, which carries out the first goal and the second is basically to check students' understanding toward the whole lessons.

The puzzle game

To begin with, the teacher tells a story about puzzle game. To finish the game, each participant has to fill in all blanks correctly. Moreover, each participant has also to complete the formulas. The game was so interesting that they will be able not merely to find the product of two binomials, but also find the binomials itself. Hence, the teacher brings the game to the group work.

During the group work, the students are given worksheets, which contain two kinds of problems. Moreover, in both kinds of problems the students need to fill in the blanks and complete the formulas. The first problems related to multiplication table and the second problems related to area model-looks like. Examples of both kinds of problems are represented in figure 7.

×	x	7
x	…	…
-2	…	…

Rectangle formula = $(x + 7)(\dots\dots\dots\dots)$

Pieces formula=

	$4x$	…
$2x$	…	…
2	…	**6**

Rectangle formula =

Pieces formula= $x^2 + 3x - 1$

Figure 7 Examples of the types of problems

The main aim of this activity is that the students can relate and use their understandings about how to do multiplication binomials with the reverse of it, which is factorize quadratic expressions into two binomials.

The Challenge: How to explain?

After finishing the first worksheet, the teacher tells the students that there are other students from different school who have difficulties with this material. Hence, the teacher wishes that the students could help them. Before meeting them, the students need to practice in order to be able to explain well. Hence, the teacher has collected some problems, which are going to be asked by students from the different school. In this case, the teacher hands out each group worksheet consist of seven formal problems. The problems are:

1. $x(15 - x) = \ldots\ldots$
2. $(x + 4)(x + 6) = \ldots\ldots$
3. $(x + 7)(x - 7) = \ldots\ldots$
4. $(x - 3)(x - 5) = \ldots\ldots$
5. $(\ldots\ldots\ldots)(\ldots\ldots\ldots) = x^2 + 8x + 16$
6. $(\ldots\ldots\ldots)(\ldots\ldots\ldots) = x^2 + 5x - 14$
7. $(\ldots\ldots\ldots)(\ldots\ldots\ldots) = x^2 - 7x + 6$

To solve the problems, the students are allowed to use any strategies. Furthermore, each group needs to make poster of their answer, strategies and the way they explain their answer and strategies. Each group are inquired to make poster of their answer for one of the fourth to the seventh problems. The main aim of this activity is to check students understanding about the whole materials they learnt during the implementation of this design.

After the group discussion, there are two groups present their answer in context of pretending to explain their strategies to solve the problems to other students. The teacher, however, tells a rule to the students that she does no longer lead the discussion, but plays a role as a student from the different school. In this case, other students who do not present pretend as the students from other school as well and ask questions, give remarks, or even help the presenter to answer the question. After the discussion ends, the teacher takes role as a leader of a whole class discussion to ask the students to make conclusions, remarks, and share their ideas about what they have been learnt during the implementation of this design.

4.6.4 Conjectures of Students' Answers and Suggestion for Teacher's Reaction

The puzzle game

At first, most students will struggle in filling the blanks when the pieces formula is given but the rectangle formula is asked. The blanks they are struggling with especially the terms, which consist of multiplication between a number and a variable. In this case, the teacher may rescue the students by encouraging the students to compare that kind of problems with the previous problems on the same worksheet where the rectangle formula is given and the pieces formula is asked. The teacher may suggest the students to try working backwards for the previous problems, which has been solved. This means, the students ignore the known rectangle formula first and use the pieces formula they have gotten. After that, they do the same steps with what they have done for that problem but in reverse order.

The Challenge: How to explain?

There are several conjectures for students' answer in solving the problems. Those conjectures are:

1. For the first four problems where the students are asked to find the product of binomials multiplications, most groups will use original area models since they have used that model in the earlier meetings to solve this kind of problems.
2. Some groups may change their way of solving those four problems after two or three problems since they think the original area model is not efficient enough and takes more time. Hence, they use the multiplication table or generated area model instead.
3. For solving the following three problems about factorizing quadratic expressions, the students will refer to how they solve the similar problems in the previous session of this meeting. The difficulties will remain the same, which is deciding the terms, which represent the multiplication of a number and a variable, and also the sign of them, whether they are positive or negative.

During the presentation, the students will ask questions and give remarks, especially about how to predict the values of the terms, which consist of a number and a variable. The students may have different strategies or some students may still struggle. Hence, the students are asked by other students to explain how they get those values and how are their strategies. In the end, the teacher encourages the students to conclude what they have learnt.

CHAPTER 5. RETROSPECTIVE ANALYSIS CYCLE 1

In the previous chapter, we have described the hypothetical learning trajectory (HLT) in learning binomials multiplication. This HLT is to be tested and implemented in the first cycle. The retrospective analysis in this chapter is focused on how to improve the HLT as well as the mathematical activities based on the implementation of the initial HLT. Furthermore, the discussion is focused on the mathematical activities and instructions, which are improved after the preliminary teaching in cycle 1.

In Indonesian curriculum, binomials multiplication is a topic in grade eight. However, since the material has been taught in the grade eight, this study involves seven graders to prevent bias. Before conducting a preliminary teaching, students from grade 7.3 of secondary school 1 Palembang did a pre-test lasting for 20 minutes. Based on this pre-test, seven students were selected as the candidates to participate in the preliminary teaching. Those students were students with average scores in the pre-test. Moreover, based on the interview with the teacher, she chose four out of the seven students as the medium achievers in the class to participate during the preliminary teaching. The four students consist of two male and two female students. During the preliminary teaching the instructional activities as well as the initial HLT were tested to the four students in four meetings. However, since there was a mid semester test, the preliminary teaching was done not in a sequence. Once per week for the first two meetings, one week break for the test, and the next two meetings in the same week after the test. Post-test and students' interview were done in the day after the last meeting.

5.1 Pre-Test Cycle 1

The pre-test aims to know students' prior abilities about the prerequisite knowledge and to make sure that most students have no understanding about binomials multiplication (the topic in this study) to prevent bias. An interview with selected students who participated in the preliminary teaching and few other students who showed high and low scores during the pre-test about how the students solve the problems in the pre-test was done. This interview aims to get the insight to students' understanding, strategies, struggles, ways of thinking and misconceptions toward the prerequisite knowledge and students' current understanding and strategies to solve binomials multiplication. The information from the interview is used to adjust the instructional activities and the HLT for the preliminary teaching.

The four students who participated in the preliminary teaching were those who have average understanding about the prerequisite knowledge, showing common understanding about the concept of variable in the class and show no ability to solve binomials multiplication. They are Diana, Alifia, Ghifary and Aldi. Students' understanding and knowledge about the concept of variable is also important to adjust the instructional activity in case of the transition process from *area model* for integers multiplication to *area model* for binomials multiplication. The prerequisite knowledge in this study consists of the students' abilities to find the rectangle area as well as knowing its formula, ability to do addition, multiplication of integers and ability to simplify linear equation. Over all, the following are the prior knowledge, which are used to adjust the instructional activities and the initial HLT.

a. Students' ability to find the area of a rectangle and the formula for rectangle area

   To be able to use *area model* as a tool to solve binomials multiplication, the students must first have an understanding about the area of a rectangle itself. The formula to find the area of a rectangle is then used in this study as a new concept, which is *rectangle formula.*

   In this pre-test, 2 out of 27 students did not answer the problem correctly. Instead of finding the area of the given rectangle, one student tried to find the perimeter of the rectangle and the other student made mistake on calculating the multiplication. Based on the interview, it can be concluded that the first student was confused and forgot the area and perimeter formula. Meanwhile, all interviewed students claimed that all of them use the formal formula of rectangle area to find the area of rectangle. None of them use different strategy. Taking the average mathematical ability, the four students who participated in the preliminary teaching were able to find the area of rectangle correctly in the pre-test.

b. Students ability to do addition and multiplication of integers

One other crucial ability to be able to use *area model* as a tool to solve binomials multiplication is the students' ability to do arithmetical operation with integers, especially addition and multiplication. The students did not have much issue with this ability. Based on the result of the pre-test, two students, indeed, seem to have issue with negative numbers in addition (but not in multiplication). One of the two students, ghifary, was chosen to participate in the preliminary teaching. Based on the interview with ghifary, he did not do the problem correctly because he did not pay attention carefully. The rest of the students seem to have no issue with addition and multiplication of integers.

c. Students knowledge and ability to simplify linear equation with one variable

The previous algebraic material that has been learnt by the students was linear equation system with one variable. In this matter, the students have learnt how to find a solution of linear equation system with one variable. There are two things revealed in the result of this pre-test: (1) almost half of the students did not do the problems correctly and (2) most of the students have understood that they can sum up the terms with same variable.

For the first case, there are two kinds of mistakes done by the students. The most common mistake is that the students tried to find the value of the variable. The problem, however, demands the students to merely simplify the equation, which means that the students need only to sum up terms with same variable. They actually have done the problem correctly, but confusion made them decided to find the exact numerical answer. The other students made mistake because of miscalculation or by adding all numbers in the problems. The first mistake, strengthen by the interview, shows how the students are oriented to find the exact numerical answer. This is inline with the list of common mistakes by Booth (1998).

About three quarters of the total students have understood that terms with same variable can be sum up to simplify the algebraic expression. These students include the students who decided to find the exact numerical answer for the problems. At first, they have answered the problem in a correct way, but then they tried to find the exact numerical answer. Hence, they are also categorized as those who have understood that terms with same variable can be sum up.

For the students who participate in the preliminary teaching, two of them (Diana and Aldi) were selected from those answer the problems correctly. Meanwhile, Alifia and Ghifary made mistake in simplifying linear algebraic expression. Alifia's mistake was merely in her counting, whereas Ghifary's mistake was that he tried to find the exact numerical value for the variables.

d. Students ability to solve binomials multiplication

The students were expected to have no understanding and ability about how to solve binomials multiplication. Since the instructional activities in this study is designed to promote the use of *area model* to help the students solving binomials multiplication. Any knowledge about this ability can make bias in this study. Hence, all students who participated in the preliminary teaching did not solve this problem correctly and have incorrect understanding about how to find the product of binomials multiplication.

The problem was to find the product of $(x+1)$ and $(x+2)$. Students' answers for this problem vary. Merely one student in the class answered the problem correctly by using FOIL (first, outer, inner, last) strategy. He knew the strategy based on his private lesson. This student was excluded from the list of students who participate in the preliminary teaching.

Some of the students answers are: $(2x+3)$, $(x+2x)=3x$, $x+x=-1-2$, $2x+3x=5x$, $1x.2x=2x$, $(x+x)(1+2)=2x.3=6x$. Based on the students' answers and the interview, it can be concluded that the students have no understanding about the use of distributive property that they have been used in arithmetic to solve this algebraic problem.

What the students did was mainly manipulating the numbers and the variables to get the answer. The interview revealed that the students themselves did not fully understand about what they did to solve the problem, whether their strategies were correct or incorrect. Some students, however, was very confidence that they solved the problem in a correct way. This moment is the crucial moment for the students as they start to make sense the transition from arithmetic thinking to algebraic thinking.

e. Students understanding about the concept of variable

As a consequence of have been learning about linear equation system with one variable, the students have been introduced and learnt about the concept of variable. However, almost all students agreed that variable is a letter after a number (coefficient). Some students gave explanation by giving the examples of variables in some terms. This is a result of the lessons where the teacher (in the class and their private mathematics teachers) emphasized that a variable is the letter in algebraic expressions. The students who participated in the teaching experiment hold the same understanding with this common understanding about variable.

Conclusion based on the Pre-test Cycle 1

Based on the pre-test, almost students know the area formula of a rectangle and are able to find the area of a rectangle. Moreover, they also do not have meaningful issue with arithmetic operations (especially addition and multiplication) with integers. These two abilities are sufficient for the students to learn the mathematical activities designed in this study.

About three quarters of the students have the ability to solve linear equation problems. They, indeed, are able to simplify linear algebraic expressions. This means that they know that terms with same variable can be sum up to simplify the expression. However, one third of those students were still oriented to find the exact numerical answer. Hence, even though they have the ability to simplify the algebraic expressions, they thought they were doing incorrectly and were determined to find an exact numerical value for the variable.

Furthermore, the students also show their shallow understanding about the concept of a variable. Some of them know that it is something related to algebra, the rest merely relate a variable with general mathematics. Meanwhile, all students agreed that a variable is a letter in mathematics (whether it is in algebra or just general mathematics), which comes after a number. In conclusion, the students have used variables in mathematics without knowing exactly the meaning and the use of it.

Moreover, the most of students know how to multiply a number with a binomial. Therefore, they are able to use their understanding about distributive property in doing this multiplication. However, when they have to face binomials multiplication, none of them used distributive property. Based on the interview, some of the students knew that there must be something about the use of distributive property or that they have to do some cross operations between the two given binomials.

To sum up, the result of the pre-test match with the prediction and all students are qualified to join the lesson with instructional and mathematical activities designed in this study. The problems in the pre-test itself are qualified to test all the required abilities of the students to join in this study. Furthermore, the four students participated in the preliminary teaching were students with average mathematical abilities based on the result of the pre-test as well as acknowledge from the teacher. The four students' abilities represent typical class abilities, which are: (1) knowing how to find the area of a rectangle, (2) knowing how to do addition and multiplication of integers, (3) 3 students were able to simplify a linear algebraic expression and 1 student tried to find the exact numerical answer for the variable, (4) do not know how to do binomials multiplication and (5) have the same understanding about variable as a letter in algebra.

5.2 Preliminary Teaching Cycle 1

The preliminary teaching involves the researcher as the teacher, four students as the participants and an observer for each meeting. The four students divided themselves into two groups. Group 1 consists of Diana and Alifia; and group 2 consists of Aldi and Ghifary. The observer made a field note and discussed the learning process afterward with the researcher. The discussion went along with the analysis of the video recording of the preliminary teaching. The preliminary teaching lasts for four meetings; with 70 to 80 minutes long per meeting. The remarks and feedbacks both from the discussion and interview with the students are analysed to improve the instructional activities as well as the HLT. The analysis of each meeting is described as follow:

5.2.1 Meeting 1

This meeting did not go so well due to the condition of the room. The researcher who acted as the teacher, the four students and an observer sat in the floor and just a couple meters far, there were some other students practicing for English speech competition. Many times, those other students distracted the students. Furthermore, there were three main activities in this meeting, which described as follow.

Preliminary Discussion

This activity aims at introducing the notions of *rectangle formula* and *pieces formula.* Rectangle formula is the multiplication of the total length and width of the area model, whereas pieces formula is the summation of the areas of all pieces inside the area model. More information about the rectangle formula and pieces formula can be seen in figure 1.

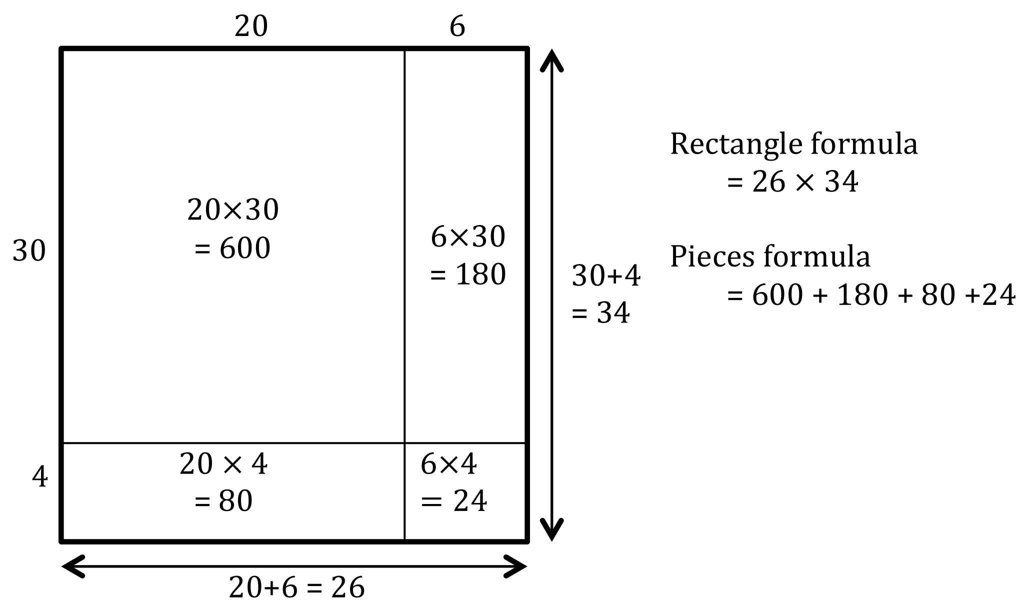


Figure 8. Rectangle formula and pieces formula in area model

In this discussion, the students were given two problems, which actually have same content: multiplication of 5 and 17. Beforehand, the researcher engaged the students' attention by asking whether they have younger brother(s) or sister(s) and whether the younger brother(s) or sister(s) have entered school or not. They all have younger brother or sister and three of them were in primary school grade 2, 4 and 5. The question successfully engaged the students' attentions.

The first problem was to find a way to explain how to do multiplication of 5 and 17 to their younger brother or sister who were in grade 2 and 4. Alifia came up with answer that her brother had to know by heart the multiplication. It is common in Indonesia that primary school students are asked to memorize multiplication until 100. The researcher then confronted the students with situation that their younger brothers could not memorize the multiplication. The four students finally agreed that they used the formal way to calculate multiplication (in line with the HLT) and they all claimed that their younger brother or sister understood that formal strategy. Since none of the students had any more-understandable strategy when the researcher asked, the lesson continued with the second problem.

The second problem was about a piece of land which measurement was 5 m wide and 17 m long. The owner of the land divided the land into two parts, 10 m long for vegetables plantation and 7 m long for lawn. The researcher asked how the students find the area of each and Aldi suggested counting one by one per each area. None of the students found meaningful difficulties since they had known the area formula for a rectangle. They all found the same correct answer for the area of land for plantation, lawn and the total area of the land.

The researcher started a discussion by asking the students to compare both problems. All students stated that the first and the second problems actually were the same problems. Further, they stated that instead of memorizing, they could use the "land-division" method to teach multiplication to their younger sister and brother. The following fragment shows that the students concluded that they prefer to use the strategy using *area model* from the second problem to solve the first problem.

The researcher tried to explore more by questioning the reason why the students chose that model instead of memorizing and formal method. Ghifary, Alifia and Aldi stated respectively that the "land-division" method was more effective because it was simpler and they could avoid "saving". However, none of the students gave further explanation. Moreover, the researcher also asked whether it is fine to choose different pair of numbers to replace 10 and 7 for the 17. Ghifary said it was not fine while Alifia said that different numbers would give different result. The researcher tried to gained more about the students' reasoning, but it was impossible. None of the students were focus on the discussion and all of them were very passive.

By the end of the discussion, the researcher introduced the notions of *rectangle formula* and *pieces formula, which* will be used during the preliminary teaching.

House Plan Problem

In this activity, the students were expected to learn how to draw area model. The problem was to draw a house plan where given some information as follow:

- The measurement of the land was 15 m long and 11 m wide,
- The area of the house was about 100 m^2,
- Beside the house, there was a garage with a paved drive in front of it. In front of the house was a garden,
- The width of the garage was 3 m.

The students were able to pick up useful information from the problem to draw the house plan. In this case, the shape of the land and the measurements of the house as well as the garage. At first, both group agreed that they needed to draw a rectangle as a representative of the land. Further, to investigate how the house plan should be, group 1 decided to put the house first in the house plan and group 2 chose to draw the garage first. Figure 2 shows the progress of both groups in drawing the house plan.

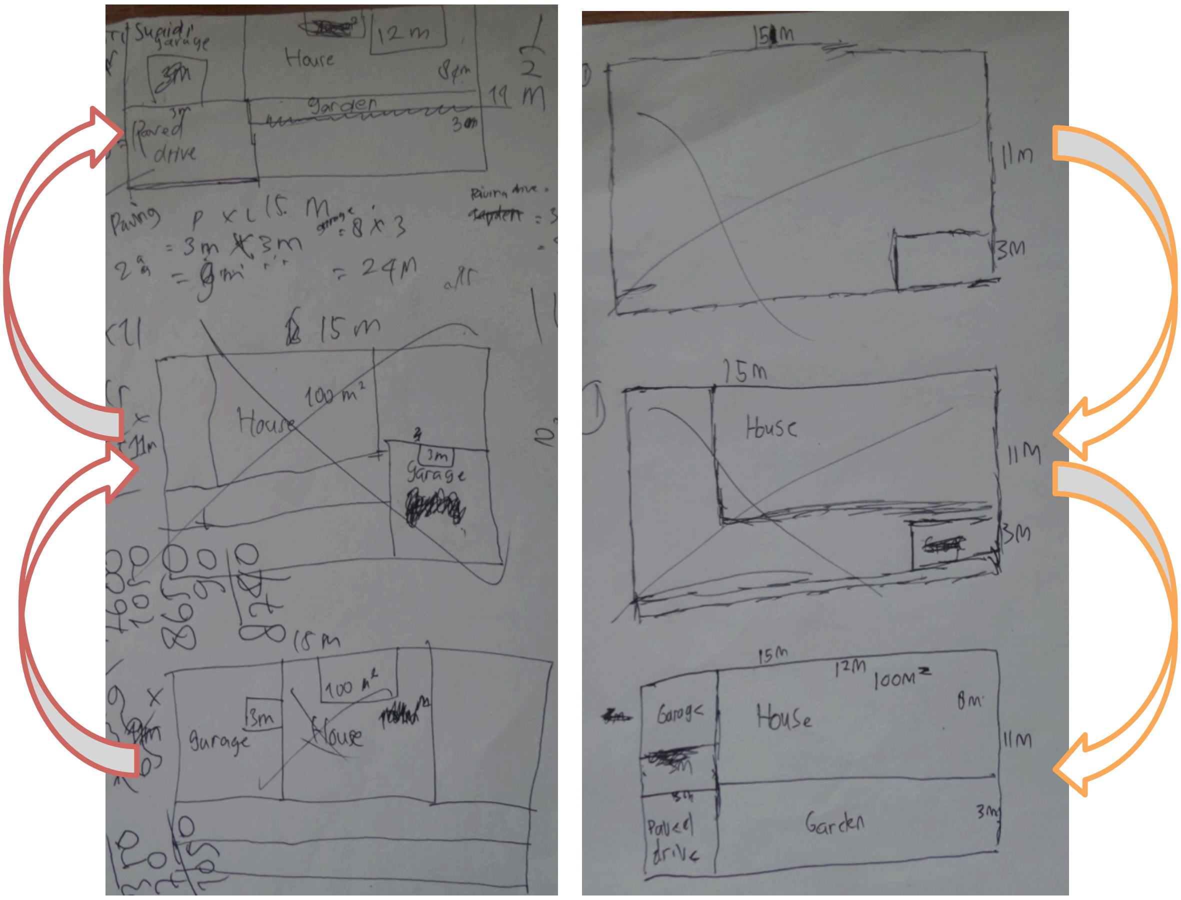


Figure 9. The progress in drawing the house plan by group 1 (left) and group 2 (right)

In the students' point of view, there were possibly some spaces in the land which were not used to build the four components of the house plan, the house, garage, garden and paving. In case of the first group who decided to draw the house first, they drew the house and wrote down 100 m^2 as the estimation of the area of the house. Further, they drew a garage in the left side of the house. From the figure 2, it can be seen that group 1 still had some empty spaces. After some internal discussion among Diana and Alifia, members of group 1, they noticed that the area of the empty spaces should be smaller when they counted the total area of the land and compare it with the area of the house. In confusion, they tried to find better place in the land to put the garage (see the transition of group 1's work from the bottom to the middle house plan).

There was quite a discussion in group 1 until they finally came up with the final house plan drawing. In the discussion, Alifia and Diana counted the total area of the land and compare it with the area of the house and predicted the area of the garden and the paving. This discussion led to a conclusion that there was impossible to have spare space in the land according to the total area of the land, area of the house and the measurement of the garage. Hence, group 1 came up with their final house plan drawing in figure 2. However, in respect to the fact that there were some students practicing for English speech competition and that the discussion was in low voice, the discussion couldn't be heard clearly in the video recording.

The next step was figuring out the exact measurements of the house plan. Line 8 from the following fragment shows how the students determined the length of the house.

1 Researcher : how is the measurement of the garage?
2 Alifia : the width is 3 m
3 Researcher : which one is the 3 m?
4 Alifia : this (pointing at the width of the garage which has been marked 3 m in her drawing)
5 Researcher : nah, so how about the measurement of the house?
6 Diana : 12 m (pointing to the length of the house)
7 Researcher : 12 m. why 12 m?
8 Diana : because the length of the land is 15 m and taken 3 m for the garage.

Meanwhile, starting by drawing the garage, group 2 developed an understanding that the house should be exactly next to the garage. Considering the area of the house and the total area of the land, they estimated that the house should be big enough in the drawing. Therefore, they ended up with the second drawing (see the middle drawing of group 2 in figure 2). Feeling weird with the position as well as the area of the garage, they tried to moved its' location to adjust the area, considering the common real area of a garage. In the end, they drew exactly the same house plan with group 1.

Group 2 employed the same strategy with group 1 in determining the length of the house. While group 1 still confused about the width of the house, group 2 has determined that the width was 8 meter. Line 6 in the following fragment shows how group 2 determined the width of the house by employing the given information, which was the area of the house was around 100 meter square.

1 *Researcher : (asking group 2) And then what is the length of the width?*
2 *Ghifary : 8 m*
3 *Researcher : (asking to Diana and Alifia) Do you know why he (Ghifary) chose 8 m?*
4 *[Silent]*
5 *Researcher : Ghifary, could you explain to your friends??*
6 *Ghifary : Because (the area of the house is) around 100; 12 x 8 = 96; (which is) almost 100.*

Moreover, the students stated that the measurements could be different. It was because the students arrived on realization moment that around 100 for the area means that the area could be more than or less than 100.

However, poster session was impossible to be done in this lesson due to the time and the condition of the room. The students took longer time finishing this activity and there were many other students in the room practicing for a competition. Hence, after finishing the house plan drawing, both groups were asked to present and explain their house plan to the other group. Since both house plans were the same, there were not many discussions regarding the house plan making.

Private Forest Problem

This activity was started by a story about *private forest project* in which South Sumatera is joining the project. The researcher firstly introduced the project and asked whether the students knew about the project. Luckily, one of the students, Alifia, knew a little about the project. This led to a nice discussion where alifia explained to the other students about what she knew about this project. The researcher continued the discussion by asking do the students know about the regulation in *Private forest project.* Since none of the students answered, the researcher continued by telling them the regulation, which is each forest need to have more than one plant. The students talked about the advantages of having various plants in one forest, especially regarding to its ecosystem.

After the discussion, the researcher told the students that they were going to help people in Lahat in their plans for the project. In this case, the students need to find the total area for the land as well as for each plantation, using *rectangle* *formula* and *pieces formula.* The students did this problem in two groups, the male and female groups. During the group work, the students faced no meaningful difficulties. This inline with the prediction in the HLT and information gathered from the pre-test that most students do not have difficulties with standard arithmetic operations, like multiplication and addition. Furthermore, when the students were asked to implement their understanding in using *rectangle formula* and *pieces formula* at solving the private forest problem in the multiplication, the students faced no meaningful difficulties. Figure 3 is a sample of students' work.

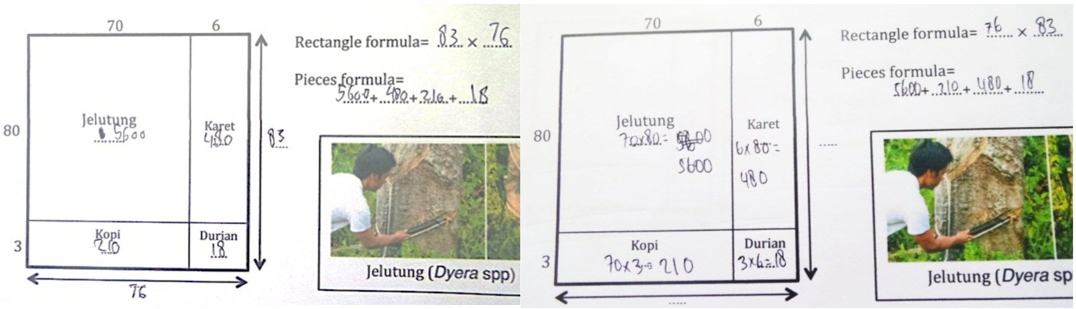


Figure 10. Example of Group 1 (right) and group 2 (left) work in private forest problems

The two groups had their own preference in solving the problems. Group 2 preferred to put their final answer in the blanks, showed in the left figure. Whereas, group 1 did not want to loose track and wrote how they get to their final answer for each piece of land, which was the multiplication. Furthermore, the students did not find any meaningful difficulties and were very enjoy and confident in solving the problems.

The students did not have difficulties with fraction operation or division. Hence, none of them experienced meaningful difficulties during the next step of this kind of problem, which were using *area model* to do the multiplication of $2\frac{1}{2}$ and $2\frac{1}{2}$ and finding the blanks when the blanks were not merely in the pieces area of the rectangle.

Conclusion of preliminary teaching meeting 1

During the preliminary teaching, none of the students fully paid attention and focused to the discussion. This was mainly because there were some students in the same room who practiced for an English speech competition and distracted them. Moreover, the students were very passive during the lesson. it was hard to start a discussion, and even when each group had a discussion, they spoke very low. Hence, some important discussions were not done effectively. Those discussions include the relation between the numbers of rectangle and pieces formulas, as well as the relation between the two formulas. As such, the students hardly conclude and reason about those relations.

In the second activity, drawing a house plan, the students took much longer then expected. The students took a very long time to decide how the house plan would look like or where to put the house, garage etc. At the beginning they were about to draw their imaginary house plan, and matched it with the given information. However, all students agreed without any meaningful difficulty to draw a rectangle as a representation of the land at the first place.

The last activity, doing the private forest problems, was the activity where the students felt most confidence and fully engage in solving the problems. It is because they knew what they needed to do and they did not find meaningful difficulties. Hence, no change is needed in this activity.

5.2.2 Meeting 2

Preliminary Discussion

Before the lesson started, the researcher asked the students about the concept of *rectangle formula* and *pieces formula* to remind them. After making sure that all students remember and have correct understanding about those two concepts, the researcher started a discussion about the house plan the students have made in the previous meeting, especially the division of the land and the position of the house, garage, paving and the garden.

The researcher then started a story about unique-shape buildings. As an example, the researcher talked about *cube house* in Rotterdam, the Netherlands and showed the pictures. The students were excited and discussed some stuff like how it feels to be inside the house. Furthermore, the researcher started to talk about the context, which was a unique square house (the house plan was exactly the same with the students' house plan from the previous meeting) but, the wide of the garage was the same with the wide of the garden.

The students were able to draw the house plan, but they were confused about the measurement of the house plan, then the researcher scaffold by supposing the wide of the garden and the garage was 3 m. After that, the researcher brought back the problem in this activity, which was the unknown length for the wide of the garage and garden. The following fragment shows how *variable* emerged as a solution in this problem to represent unknown length.

1	*Researcher*	*: Now the problem is that the measurement of the house is known, 10 m x 10 m. but the wide of the garage and the garden are still unknown. Since it is unknown, how to make the house plan?*
2	*[silent]*	
3	*Researcher*	*: What can we use when it is unknown?*
4	*Alifia*	*: 3x, 3y (pointing at part with unknown length of the house plan)*
5	*Researcher*	*: We can use x and y. why? What are x and y?*
6	*Alifia and Diana*	*: Variable*
7	*Researcher*	*: What variable is for?*
8	*Diana*	*: (to represent) the same area*
9	*Researcher*	*: hmm,, are the areas the same?*
10	*Alifia*	*: To replace*
11	*Researcher*	*: To replace what?*
12	*Aldi*	*: The unknown number (unknown length)*
13	*Researcher*	*: Then what should be here (pointing at the wide of the garage)? 3x or what?*
14	*Aldi*	*: Only x*
15	*Diana*	*: Anything*
16	*Researcher*	*: What anything? What do you want?*
17	*All students*	*: x*
18	*Researcher*	*: Ok then now I'm going to ask. If you want to replace it with variable, is it 3x or x?*
19	*All students*	*: 3x*
20	*Researcher*	*: 3x or x?*
21	*All students*	*: 3x*
22	*Researcher*	*: Why?*
23	*[Silent]*	
24	*Researcher*	*: what was the function of variable?*
25	*Diana and alifia*	*: to replace the unknown number*
26	*Diana*	*: so it is x*

The fragment shows that the context successfully led the students to an understanding that a variable can be used to represent unknown. Students' understanding about variable shifted from defining variable solely as a letter come after a number in algebra into meaningful definition of variable, which is a representation of unknown lengths. The students then completing the house plan drawing by adding variable x as the wide of the garden and the garage. After that, the researcher started to talk about what the students would learn in this meeting, which was finding the *rectangle formula* and the *pieces formula* of this kind of shape (the house plan with some unknowns). However, the students needed to solve problems in *the matching games* first.

The Matching Games

This mathematical problems in this activity aims at making sure all students had correct understanding and able to formulating geometric drawings into algebraic expressions. This aim is related to the ability in relating *area model* drawing with formal algebraic expressions. This ability is one of the compulsory to join the next mathematical activity. Hence, it is very important that all students succeed and build correct understanding in this activity.

In this activity, the students matched the geometric drawings with its' suitable expressions by using arrows. There were no meaningful difficulties. The discussions were merely around two things. First, between Alifia and Diana in doing the first problem, whether the drawing in figure 4 (left) was $10x$ or $10 + x$. The researcher scaffold Alifia and Diana by taking number line as an example closely to the problem and replace the variable with numbers. Accordingly, both students agreed that the drawing expressed $10 + x$.

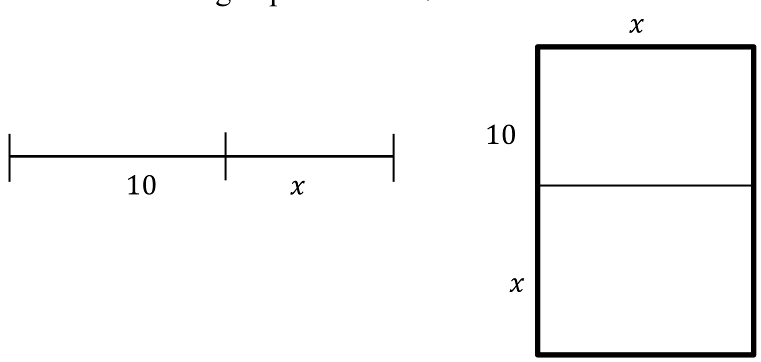


Figure 11. the first problem (left) and the third problem (right) in the matching games

The second discussion was about the different answers between group 1 and group 2. The drawing of the problem can be seen the figure 4 (right). Some students hold an algorithm that the measurement of the left-side should come first in a multiplication, followed by the measurement of the top-side. Group 1 matched the drawing with answer f, which is $(10 + x)x$. Meanwhile, group 2 matched the drawing with answer b, which is $x(10 + x)$. Both group argued that their answer was correct and other group's answer was incorrect. The following fragment shows how the discussion goes.

1 *Researcher : (asking to group 1) hmm… Can you explain why f?*
2 *Diana : Because from the previous meeting, the first (that come up in the expressions) is the 10 add up with* x *(the left side of the rectangle)*
3 *Researcher : hmm… this one first? (pointing at the side representing* $10 + x$*) and then? This (10) plus this (*x*) multiply by this (*x*)?*
4 *Group 1 : ya*
5 *Researcher : (asking group 2) and then how about you? Why b?*
6 *Male students : [smiling and silent] (since they think that group 1's answer is correct, they assume that their answer is incorrect and do not want to explain their answer)*
7 *Researcher : (to all students) now I have a question. The multiplication on b and f, are they actually the same?*
8 *Diana : (asking Alifia, Ghifary and Aldi) same right?*
9 *Alifia, Ghifary, Aldi: same (nodding)*
10 *Researcher : so if the answer is f, is it correct?*
11 *All students : correct*
12 *Researcher : answer b?*
13 *Diana : wrong*
14 *Alifia, Aldi, Ghifary: laugh*
15 *Aldi : correct*
16 *Alifia, Ghifary : (nodding)*
17 *Researcher : is the result (for both expressions) are the same?*
18 *All students : same*
19 *Researcher : can we use both?*
20 *All students : yes*

Line 7 to 20 shows that the students finally agreed that $x(10 + x)$ and $(10 + x)x$ are actually the same. This means the students had a moment of realization that commutative property works in algebraic multiplication. Moreover, they realized that they could manipulate the expressions to get the same value. The algorithm

The Private Forest Problems

The students needed to do the same thing like in the previous private forest problems, which were filling the blanks and finding rectangle and pieces formulas. The private forest problems in this lesson emerged variables to represent unknown measurements of the private forests. Since the students have understood how to solve the private forest problems and able to represent geometric drawings into algebraic expressions that they learnt from the matching problems, there were no meaningful difficulties.

During the group work, the students got confused to representing some algebraic expression. In this case, they came back to their paper on matching games, made their own examples using number lines, and deepening their understanding. As predicted, other discussion was about the product of x times x. The following fragment shows how Diana found that x times x equals x^2.

1 *Diana* : x *times* x *equals ....*

2 *Alifia* : *Nah… (agreeing that she also has a problem in finding* x *times* x*)*

3 *Ghifary* : *[laugh] (means that he also has the same problem)*

4 *Diana* : x *to the power of 2!*

5 *Researcher* : x *to the power of 2. Why is it* x *to the power of 2?*

6 *Diana* : *Because there are two* x*s*

7 *Researcher* : *Why is it "power"?*

8 *Ghifary* : *Why?*

9 *Diana* : *Because the* x*s are the same.* x *is a representation of unknown number. If we take an example* x *is 5, 5 "what" is 5 times 5 ((what is a number in 5 form represents 5 times 5)?*

10 *Ghifary, Alifia, Aldi: 25*

11 *Diana* : *25. Can you reform 25 into 5?*

12 *[Silent, confused face]*

13 *Researcher* : *25 equals 5…*

14 *All students* : *times 5*

15 *Researcher* : *5 times 5 equals*

16 *Diana* : 5^2

17 *Researcher* : *5 times 5 times 5 equals*

18 *Diana* : 5^3

19 *Researcher : so that's why unknown multiply by itself, become*
20 *Alifia : oh… x^2*
21 *Researcher : Yes, the square of the unknown*

Line 9 shows how Diana tried to determine the result of x times x by supposing x with a number. This example by Diana is useful to help other students understand the multiplication of same variable. This is exactly inline with the prediction in the HLT. There is no discussion, which is not like what expected in the HLT. The problems foster students' understanding for square of a variable. Therefore, there is no change needed in this activity.

The House plan problem

In this activity, the students were given one problem about rectangular house plan like seen in figure 5. In this problem, the students need to find the rectangle and pieces formula. Moreover, they were asked whether there was a relation between the two formulas and to explain the relation.

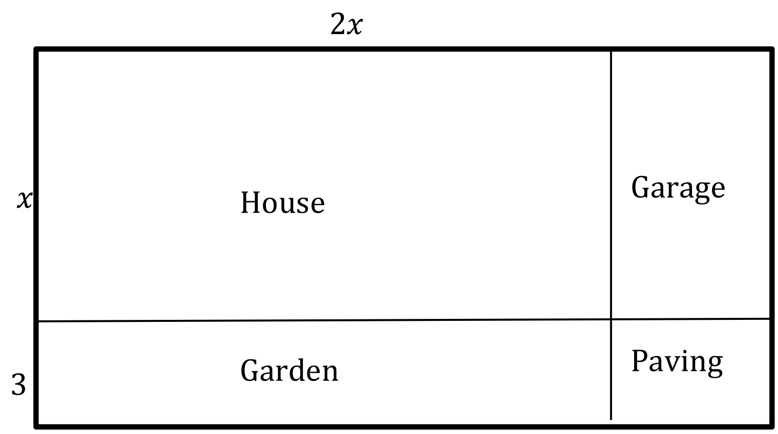


Figure 12. The house plan problem

The students did not have problem in finding the rectangle and the pieces formula. In accordance with the prediction, the students had also already aware of the relation between rectangle and pieces formulas. The following fragment shows how Ghifary and Aldi explain the relation based on their point of view.

Diana: is there any relation between rectangle formula and pieces formula?

1 *R: nah, is there any relation?*
2 *Ghifary: yes there is. If we sum up this (pointing at pieces formula) we get rectangle formula*
3 *R: yes you can write that way*

4 *R: is there any relation between rectangle formula and pieces formula? Aldi?*
5 *Aldi: the results are the same*
6 *R: so, the sum in pieces formula and the product in rectangle formula are the same. Others, how do you think?*
7 *All: same*

Making up a story

In this activity, the students were asked to make up a story which represent the multiplication of $(x + 6)$ and $(x + 2)$. In this case, the students need to make the context for the multiplication themselves. The stories made by the two groups were similar, which were about a rectangular plantation and its area. The additional represent the partition of the land. In finding the product of the multiplication, the students were confused about which one is the product. The researcher scaffolds them by reminding them about multiplication of integers in the first meeting.

Conclusion of preliminary teaching meeting 2

In this meeting, the students started to freely discuss the problems. The activities supported students' understanding and became the base or prerequisite knowledge before the next activities. Over all, this meeting went smooth and all discussion matches with the prediction. Hence, there is no need to change the activities, content or order in this meeting.

5.2.3 Meeting 3

The Rose Garden Problem

This activity was started with a story about Ayu who wanted to build a small square rose garden. However, her sister suggested to build a rectangular rose garden since it would be more beautiful. The students were engaged to the story as they commented and speak out their preferable, which was the rectangular rose garden. The story continued by Ayu's sister, who claimed that the total area of both square and rectangular rose garden would be the same when they reduce 1 m from one side and add 1 m to another (not parallel) side. All students agreed with Ayu's sister. In this case, the students' first understanding matches the prediction in HLT.

Finally, the researcher showed some work examples and asked the students to try to understand the strategy used in the answers. Furthermore, the students were also asked to order the answers, based on anything they like, such as the correctness and the clearness of the answers, or the sophisticated strategies. This was the first time the students encountered this kind of task. Hence, it took a while for them to really understand what they need to do. At first, the students were simply ordering the answers without trying to understand the strategies on each answer. In this case, the students agreed that the answer in figure 8 was the best answer; inline with their previous believes that the area would not change. The researcher then re-explained the task, and the students started to try to understand each answer.

Both groups ordered the answers based on the clearness of the strategies and the correctness of the answers. Means that their considerations included how easy or difficult to understand the strategy. They ended up with the same order. While the students engaged in group discussion, the researcher asked several questions to make sure all students knew what they need to do and to gain more information about how the students understand the answers and the strategies.

Both group ended up with same order. The first one can be seen in figure 6 (left). In understanding the strategy in this answer, the students did not have meaningful difficulty. They associate the strategy with the area model they have been using in the previous meetings. However, it was really different with the second answer in figure 6 (right).

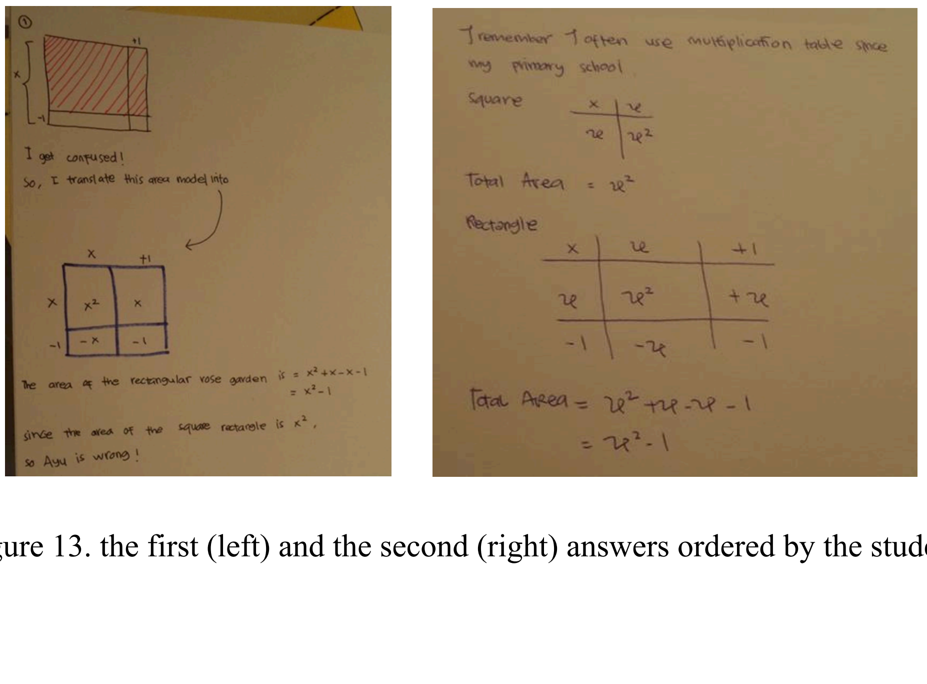


Figure 13. the first (left) and the second (right) answers ordered by the student

When the researcher asked about the strategy in the left answer, the students related the strategy with the concept of *rectangle formula* and *pieces formula.* Meanwhile, when the researcher asked what the difference between the left and the right answers in figure 13, Ghifary said it was the strategy that made them different. Furthermore, Diana added that the right answer used crossed multiplication. It shows that the students did not really understand the meaning of crossed multiplication.

Starting with Diana's claim that the right answer employs crossed multiplication, the researcher asked more about it and Alifia stated that that was crossed multiplication on a table. After some discussion, the students agreed to call that strategy as multiplication table. To gain more, the researcher asked them to explain how a multiplication table works. In this case, Alifia, helped by the other students, use integers, instead of the given example answer, to explain it. On her explanation, she said that they needed to take one number in the left column and one number in the top line of the multiplication table, took a horizontal line from the left number and a vertical line from the top number. The meeting point of the two lines represented the product of the multiplication.

Even though they have discussed about the multiplication of one variable to itself, the students said that they felt more comfortable and sure when they just used numbers in their effort to understand how the multiplication table works. Their preference in using numbers in trying to understand some algorithms was seen from the previous meeting, when they assuming variable as a constant number. When the researcher brought the students back to the context, Dianna explained to others how the multiplication table in the answer worked. It showed that Diana's explanation was clear and all students did not have many difficulties understanding how the multiplication table was used to solve the problem.

Figure 14 shows students' third order. The students knew this answer was incorrect since the final answer for the total area was different with the previous two answers, which they claimed as correct answer. However, they were first struggle in finding the incorrect things in the strategy. They thought the strategy was correct. After some discussion, the students realized that the incorrect thing in this answer was the product of the multiplication of x and $x - 1$ (see the red sign in the picture).

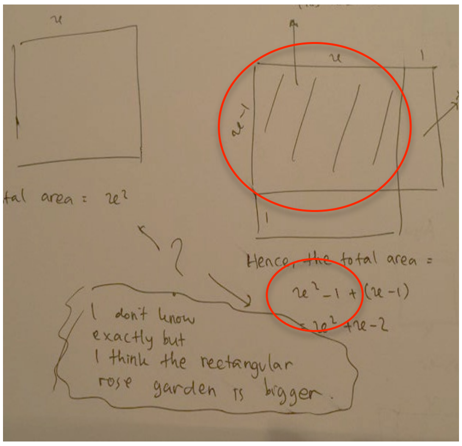


Figure 14. Students' third order

Previously in the beginning of the lesson, all students agreed that the answer in figure 15 (represented Ayu's sister's opinion) was correct. However, after the discussion and agreement that the first two answers were correct, the students re-examined this answer. Employing numbers (see figure 15 bottom), the students realized that the answer was incorrect and put it in the last order.

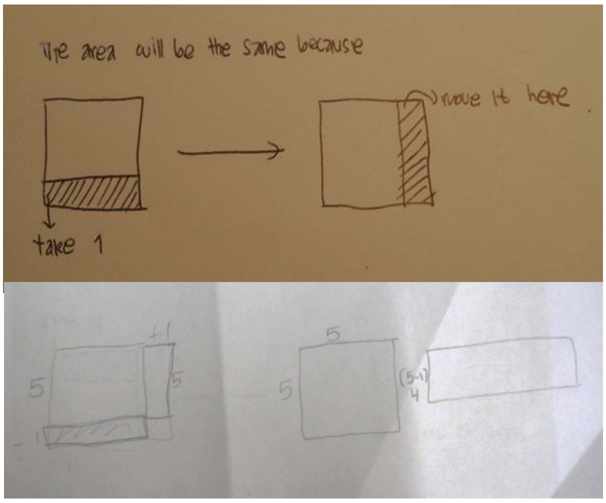


Figure 15. Students' last order (top) and students' strategy to find out that the answer was incorrect (bottom)

The Palm Oil Plantation

The palm oil plantation problems were actually the same with private forest problems, which was about land division. The difference was that in this problem, not all of the partitions of the land were used. In other words, the problems involved subtractions. The aim of this activity was to enhance students understanding and made more sense of area model or multiplication table in solving binomials multiplication. When the students were asked about their preferable to use which strategy, all of them chose the left strategy, which is more similar to the *area model* they learnt since the first meeting of the preliminary teaching. Therefore, all of them used area model to solve these problems.

Exercises

In this activity, the students were asked to do problems in improved area model and multiplication table. The aim of this activity was to have the students used to using area model or multiplication table to solve binomials multiplication. The students were not handy yet with multiplication table, hence they often looked back to the work example from the rose garden problem.

Conclusion of preliminary teaching meeting 3

The students built up understanding that they could use area model not as precisely the same with what they have learnt before. Further, they could use it as a tool to do binomials multiplication. How they draw the model was also shifted. They were no longer paying attention to the proportion and measurement of the length of the sides. This showed a shift of the area model from a representation to fully a tool. Based on the theory of RME, this shift represents emergent modelling of the area model.

5.2.4 Meeting 4

Exercise

In this meeting, the students were given formal mathematics where they need to do binomials multiplication with whatever tool they wanted. Moreover, there were also given inverse problems. Figure 16 shows the written work when the second group encountered factorization problem for the first time. This problem was basically a factorization problem, but not in formal or straight form.

The known was the *pieces formula* or the product of the multiplication and the students needed to find the *rectangle formula*, which was the multiplication.

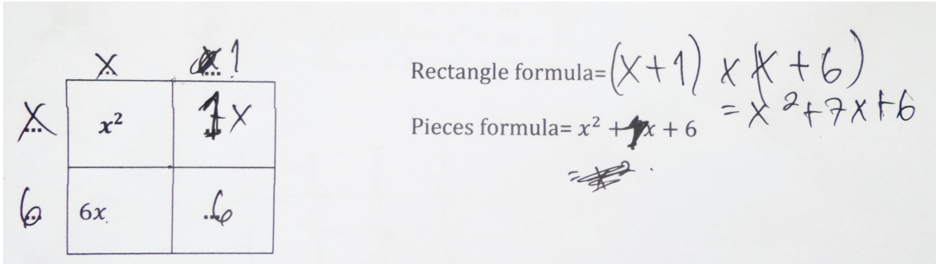


Figure 16. Students' written work on the first factorization problem

Alifia and Diana realized that this problem was similar to the previous problems where they need to do binomials multiplication. Hence, they sensed that this problem hold the same principles. As the consequence, in solving this problem they related with the previous problems and faced no meaningful difficulty in making sense and finding that x^2 was made of x times x. Furthermore, both students agreed to put 6 in the bottom right of the inside of the area model, and got 6 and 1 in the sides of the area model. However, both of them agreed to put $7x$, taken from the *pieces formula,* in the rest spot, the right upper in the area model. In this case, since both students had found the 1 and 6, the researcher confronted their answer ($7x$) with the question of finding the piece area for that spot in regard to the multiplication of 1 and x. Both students answered that the piece formula for that was $1x$, which was different from their initial answer. However, when the researcher tried to ask them to further think about that, both students found difficulty. Hence, since the first group turned out to be able to understand and make sense this problem themselves, the researcher conducted a whole class discussion asking the first group, consists of Aldi and Ghifary, to explain to the second group.

1	Researcher	: (asking to Aldi and Ghifary) Diana and Alifia were confused whether it should be $1x$ or $7x$. How do you think?
2	Aldi	: $1x$.
3	Researcher	: $1x$?
4	Ghifary	: Because this (pointing at $6x$ and $1x$ inside the area model) should be sum up. $6x$ and $1x$ equals $+7x$.
5	Diana	: Repeat please.

6	Researcher	: How, what's needed to be sum up?
7	Ghifary	: $6x$ (pointing at $6x$ in the table) plus $1x$ (pointing at $1x$ in the table) equals $7x$ (pointing at $7x$ in the *pieces formula*)
8	Diana	: Why it is "sum up"?
9	Ghifary	: This is "plus"
10	Alifia + Diana	: Hah? Where is the "+"?
11	Aldi	: no. *pieces formula* is the sum of all of these (pointing at the $x^2, 6x, 1x$ and 6), right?
12	Alifia + Diana	: (clapping hands)
13	Researcher	: Aldi said *pieces formula* means you need to sum up all of these (pointing at the $x^2, 6x, 1x$ and 6). So what's the effect here, to decide whether it should be $1x$ or $7x$?
14	Aldi + Ghifary	: if it is $6x$ plus $7x$, the result is more than the expected answer.
15	Ghifary	: The result is .... (counting with fingers)
16	Alifia	: $13x$.
17	Reseacrher	: $13x$. And here (pointing in the *pieces formula*) is…?
18	Ghifary	: that's the answer. $7x$ (pointing at $7x$ in *pieces formula*)

The line 8 to 11 shows that Aldi and Ghifary were able to relate this problem with their previous understaning about area model and the notions of *rectangle formula* and *pieces formula*. This ability led to decision to choose $1x$ instead of $7x$. The resoning of Aldi and Ghifary about their choice was shown in the line 4 to 7 as well as line 14 to 18.

Conclusion of preliminary teaching meeting 3

In this meeting, we found out that the students were really confident in doing not merely binomials multiplication, but also factorization. This proves the potential of area model as a tool in teaching factorization as well.

5.3 Post-Test Cycle 1

The result of the post-test enhances the findings on the preliminary teaching. Based on the comparison of the results on the first and second problems as well as the interview with the students, the students were mostly confidence and able to do multiplication of two binomials using improved area model. One of them was still confused of how to use multiplication table. Further, they prefer to use improved area model than multiplication table to solve given formal problems on binomials multiplication and factorization. All of them answered the factorization correctly even though they took a quite long time to solve it. The last problem revealed that two students were able to emerge area model to solve the problem. One was still confuse but she has already the idea of how to solve it but ran out of time, whereas one other student was lost.

5.4 Improvement of the HLT

The Hypothetical Learning Trajectory (HLT) described in the previous chapter is still valid. However, based on the preliminary teaching in cycle 1, there are some parts of the HLT need to be improved.

5.3.1 Meeting 1

In this meeting, the discussion did not run very smooth. Hence, some important discussions were missing. To prevent the missing discussion, there are three questions added in the worksheet. The three questions are:

1. What is the relation between rectangle formula, pieces formula, a multiplication and a product of multiplication?
2. How do you make the partition of the area model (how you divide the numbers)?
3. Is there any interesting thing that you find related to the use of area model?

In answering these questions, students are expected to have group discussion and reason about the relation between the formulas and a multiplication. Answers for these questions are expected to be the conclusion of students' learning in this meeting.

The second improvement is to prevent passive whole class discussion in case the students do not know anything about private forest, a poster will be spread among the students one day before the lessons. The poster consists information about the project and its' regulation, which is there should be more than one type of plant in one piece of land. The students are expected to read the poster and raise interest and awareness about the useful of this project. This will engage them in the whole class discussion about the private forest.

The third improvement for the mathematical activities is that since (1) all students came to drawing a rectangle as a representation of the land before trying to draw the house plan, and that (2) the students took much longer in doing the second activity; the drawing of the rectangle represents the land will be given in the worksheet. When the time allocated for doing this activity is shortens, there will be time for students to present their work and hold a deeper discussion.

The students have no problem with the private forest activity where they needed to work with the *area model.* In this case, this activity made the students felt most confidence because they knew what they needed to do and they were able to do all the problems easily. Hence, there is no change needed to improve this activity. However, in the preliminary teaching for the introduction of the context here, the researcher was lucky because one of the students knew about the project. Hence, a nice discussion could be held. To prevent from a failure in engaging the students with a nice discussion because none of the students know about this project, a poster about this project will be spread in the classroom one day before the meeting. Hence, the students will have insight about the project, or browse additional information about it and bring them up during the discussion.

5.3.2 Meeting 2

This meeting ran smoothly, the goals were achieved and all students' activities matched the predictions. Hence, there is no change for this meeting.

5.3.3 Meeting 3

In the first task about the rose garden, it took a while for the students to really understand the task and what they needed to do. This is because they never encounter similar tasks. Hence, the teacher in the next cycle needs to pay attention to give the instruction clearly. Moreover, after the discussion about all the strategies used in the samples, the students were not ready to solve the palm oil problems. The students definitely needed some more practices to fully understand the strategies. As such, there will be some practices after the rose garden problems where the students need to solve the given problems using advanced area model or multiplication table. In this case, the palm oil problems will be put in the last meeting.

5.3.4 Meeting 4

This meeting will be start with some refreshment about the problems from the previous meeting to make sure all students have understood how to use area model for binomials multiplication. Next, the palm oil problems will be given and followed by more problems. There are additional problems, which related to factorization. This is because in the last meeting of the preliminary teaching as well as the post-test, most students were very confidence and enthusiast in solving factorization problems.

CHAPTER 6. RETROSPECTIVE ANALYSIS CYCLE 2

After the HLT and the mathematical activities have been improved, they were being implemented in a natural classroom in cycle 2. In this stage, the teacher was no longer the researcher, but the initial mathematics teacher in that classroom. Before conducting the teaching experiment, the researcher first conducted twice classroom observations during mathematics lessons and interview with the teacher to get insight into her perspectives.

The retrospective analysis in this chapter focus is on answering the research question. As such, the analysis will be around how the area model supports students' learning process. Before the teaching experiment, a pre-test was done to choose the focus group. There is no change in all items in the pre-test from the first to the second cycle.

6.1 Profile of the Class and the Teacher

There were two classroom observations done in the class for the teaching experiment. In the first observation, the students were doing normal class where they need to do some mathematical problems. In the second observation, the students were doing presentation.

Based on the observation and the interview with the teacher, it is known that there were 27 students in the class, consists of 8 male students and 19 female students. The students sit in pairs in rows, just like common Indonesian classroom. There were also groups in the class consisted of 3 to 4 students in a group, with 1 to 2 male students in each group. In this case, one student had one chair and one table. During the second observation where the students sat in groups, each group just chose wherever they wanted to sit. This is ineffective during the presentation because the groups sat in crowd and there were many students who did not pay attention since they were hidden by other groups or students. To avoid this issue during the discussion in the teaching experiment, the researcher proposed a classroom plan. The proposed classroom plan used during the teaching experiment can be seen in the figure 17. The blue circles represent the position of the group.

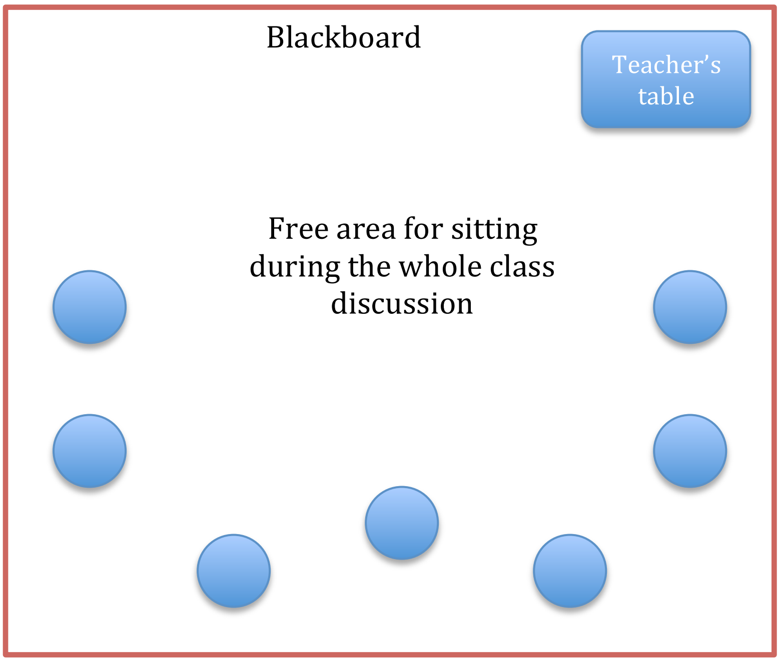


Figure 17. Proposed classroom plan

During the group presentation, the presenters spoke very low and the other students barely pay attention. There was no interaction and discussion between the presenters and the other students. Moreover, the students seemed to hold back. They were afraid of asking or giving the incorrect answers. The teacher seemed to be the talkative one and most students were passive because they were afraid of saying something incorrectly. In this case, the teacher was sometimes friendly and sometimes not. However, the teacher indeed gave opportunity to express their ideas, but the students rarely took that opportunity. The mathematics content that the students learnt was the formal mathematics, with formulas and everything. However, the teacher stated that some times they used manipulative in her teaching.

Based on the classroom observation to the class that will be used for the teaching experiment cycle 2, there was one big issue that the researcher was concerned about, the time management. In both observations, the lesson was started 40 minutes and 30 minutes late. This was of course a big issue since in one period of teaching; the time allocation was 40 minutes. 40 minutes late meant that the students lost one period of teaching. In this case, the researcher talked with the teacher and made agreement with the students and other teachers who taught in the previous period as such the students would not be late in starting the learning processes during the teaching experiment.

The teacher, named Ibu Apriya, is a very cooperative teacher. She has been teaching mathematics for 5 years. She has been teaching grade 7, 9 and senior high school as well. However, since she never teach grade 8, she never teach binomials multiplication in particular. She usually explains, presents or demonstrates the topics in the classroom. Sometimes she promotes a discussion as well, but the students are quite passive. She also usually starts the lesson with reminding the students about the homework or the previous topics, not quite engaging for high school students.

However, the beliefs she holds are that it is better not to really implement Pendidikan Matenatika Realistik Indonesia (PMRI), an adoption from Ducth's Realistic Mathematics Education (RME) in the actual or daily base classroom, due to the amount of material and the limited time allocation. It is a typical way of thinking of teachers in Indonesia. It is not that she against or contradicts the principle of PMRI, but she finds some common issues in teaching mathematics in Indonesia as an excuse to not implementing PMRI.

Further, she has some misunderstanding about some concepts in PMRI. Being familiar with PMRI since her undergraduate degree does not mean that she has master all concepts in PMRI. When asked to explain what does she know about PMRI, she mentioned about how a teacher is demanded to use real model or manipulative, such as real net of three dimensional shapes, fruits, etc. Her understanding about PMRI is narrow on the use of model. Even, the definition of model she holds is limited to manipulative, not such tools the students may use to solve mathematics problems. As such, she stated that using models in mathematics lessons takes time. The model should be real. Her definition of real is those real objects that often seen or touched by the students. She did not mention about real as those who are on students' imagination, cartoon, etc.

Inline with many Indonesian teachers, as she claimed, she argued that to fully implement PMRI in daily base is very hard. Due to the lot numbers of materials in the curriculum and limited time allocation for teaching them all. Thus, she implements PMRI in some, but not all, of her classes.

6.2 Pre-Test Cycle 2

Aims at knowing students' prior abilities about the prerequisite knowledge and to make sure that most students have no understanding about binomials multiplication (the topic in this study) to prevent bias. In this cycle 2, the pre-test was not done in while mathematics class as like in the cycle 1. Instead, half of this was in a mathematics class and another half was in break time. Thus, the initial teacher was not there during the second half of the pre-test. Further, different with the preliminary teaching, the interview was short and informal in the classroom during the break time due to the time constrain. This interview aims to get the insight to students' understanding, strategies, struggles, ways of thinking and misconceptions toward the prerequisite knowledge and students' current understanding and strategies to solve binomials multiplication.

The result of the pre-test for both cycle 1 and cycle 2 were similar. The pre-test was to measure five aspects of students' understanding and abilities. Those aspects are explained as follow.

a. Students' ability to find the area of a rectangle and the formula for rectangle area

   To be able to use *area model* as a tool to solve binomials multiplication, the students must first have an understanding about the area of a rectangle itself. The formula to find the area of a rectangle is then used in this study as a new concept, which is *rectangle formula.* In this pre-test, all students were able to answer the first problem to find the area of a rectangle. None of them showed miscalculation as well.

b. Students ability to do addition and multiplication of integers

   One other crucial ability to be able to use *area model* as a tool to solve binomials multiplication is the students' ability to do arithmetical operation with integers, especially addition and multiplication. Based on the result of the pre-test, merely three out of 27 students, indeed, seem to have issue with negative numbers in addition (but not in multiplication). Hence, it can be concluded that this class is good enough as subject of this study.

c. Students knowledge and ability to simplify linear equation with one variable

The previous algebraic material that has been learnt by the students was linear equation system with one variable. In this matter, the students have learnt how to find a solution of linear equation system with one variable. There are two things revealed in the result of this pre-test: (1) about three quarters of the students did not do the problems correctly and (2) almost of the students have understood that they can sum up the terms with same variable. For the first case, there is one common mistake done by the students. They tended to find the value of the variable. The problem, however, demands the students to merely simplify the equation, which means that the students need only to sum up terms with same variable. They actually have done the problem correctly, but confusion made them decided to find the exact numerical answer. In other words, the students were still number-oriented.

d. Students ability to solve binomials multiplication

The students were expected to have no understanding and ability about how to solve binomials multiplication. Since the instructional activities in this study is designed to promote the use of *area model* to help the students solving binomials multiplication. Any knowledge about this ability can make bias in this study. Hence, all students who participated in the preliminary teaching did not solve this problem correctly and have incorrect understanding about how to find the product of binomials multiplication.

The problem was to find the product of $(x + 1)$ and $(x + 2)$. Students' answers for this problem vary. Types of students' answer were similar to the pre-test cycle 1, such as $(2x + 3)$, $(4x + 5x)$, $(x + 3)$, etc. Based on the students' answers and the interview, it can be concluded that the students have no understanding about the use of distributive property that they have been used in arithmetic to solve this algebraic problem. What the students did was mainly manipulating the numbers and the variables to get the answer. The interview revealed that the students themselves did not fully understand about what they did to solve the problem, whether their strategies were correct or incorrect. One student in the class answered the problem correctly by using FOIL (first, outer, inner, last) strategy. This student was excluded from the option for the focus group.

e. Students understanding about the concept of variable

   As a consequence of have been learning about linear equation system with one variable, the students have been introduced and learnt about the concept of variable. However, almost all students agreed that variable is a letter after a number (coefficient). This shows students' shallow understanding about the concept of variable.

Conclusion based on the Pre-test Cycle 1

Based on the pre-test, all students know the area formula of a rectangle and are able to find the area of a rectangle. Moreover, they also do not have meaningful issue with arithmetic operations (especially addition and multiplication) with integers. These two abilities are sufficient for the students to learn the mathematical activities designed in this study. To sum up, the result of the pre-test shows that most students in this class hold typical abilities, which are: (1) knowing how to find the area of a rectangle, (2) knowing how to do addition and multiplication of integers, (3) 3 students were able to simplify a linear algebraic expression and 1 student tried to find the exact numerical answer for the variable, (4) do not know how to do binomials multiplication and (5) have the same understanding about variable as a letter in algebra. These conditions are the prerequisite condition for the students to join the mathematical activities design in this study. Thus, this class is eligible as the subject of this study.

6.3 The Focus Group

At the beginning, the researcher had made agreement with the teacher to remake the groups and discuss the students in the focus group. Those students should be chosen based on their result of the pre-test and interview with the teacher. Those students should be the average mathematical abilities students and are able to work in group. However, due to the time constraint, the researcher merely had one weekend to discuss with the teacher. Unfortunately, the teacher was not able to come to the discussion appointment about the new group formation due to family issue. Hence, this study uses the existing groups. This is also increase the ecological validity of this study. The focus group was chosen based on the result of the pre-test. In this case, since all groups consist of various mathematical abilities students, the researcher chose the group with no highest or lowest mathematical ability students. In other words, the focus group can represent the whole groups in the class.

6.4 Teaching Experiment Cycle 2

One or few days before each meeting, the researcher handed out the lesson plan and the students' worksheet to the teacher. More than that, a discussion about the lesson was held every time before the lesson to make sure the teacher fully understood and no misconception about the flow and the content of the lesson plan. In this cycle, the researcher played a role as an observer, together with one permanent observer. The researcher decided to keep the same and not many observers to increase the validity and the reliability of the data. By doing this, the observer could see each development of the students and could judge fairly. The lesson plan was also given to the other observer before the teaching experiments were conducted. In this manner, the observers would study the lesson plan together. After each teaching experiment, the researcher and the other observer discussed about the lesson and the video recording.

The researcher was first afraid of the students who might not pay attention or cooperate during the teaching experiment. However, the changes position for the whole class discussion, group work position and how the teacher delivered the context, were very helpful in this teaching experiment. Previously, based on the observations, the researcher suggested some changes in the classroom positions and classroom norms. As a result, the students were engaged and paid attention. Moreover, they were willing to share their ideas and the discussion ran very smooth then expected. However, time was an issue in this cycle. Some conditional changes were made in order to adjust the situation.

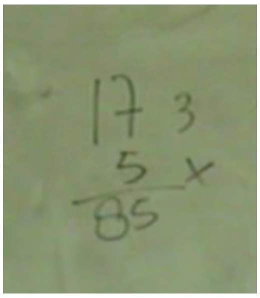
6.4.1 Meeting 1

The main aim of this meeting was to introduce the area model and let the students used to using it. The whole lesson ran smoothly. There were three main activities in this meeting: preliminary discussion, house plan problem and private forest problem, and lasted for 3 periods, or 120 minutes. Two days before the lesson, the researcher had spread posters consist of information about the private forest (see appendix). Unfortunately, due to family issue of the teacher, the lesson started 40 minutes late, which means there were merely 80 minutes left from 120 minutes planned. Therefore, since it was really hard to get extra lesson from the school, the researcher decided to omit the second activity, which was the house plan problem. However, after the lesson had finished, there was news coming that the next teacher still has not arrived yet. Hence, the teaching experiment might continue with the second activity.

The preliminary discussion

The aims of this discussion are to introduce *area model* and the two concepts in are model, which are *rectangle formula* and *pieces formula.* The teacher followed the lesson plan by posing questions about students' younger sisters/brothers and challenge them to find a strategy to explain the multiplication of 17 and 5 to their younger sisters/brothers. Teacher's way of telling the story was engaging and none of the students did not pay attention or engage to the discussion. The students sat in the floor forming U letter and the teacher was in the middle to tell the story. When the teacher asked the students to discuss with their neighbour, providentially the students did that.

Figure 18 the situation in the classroom



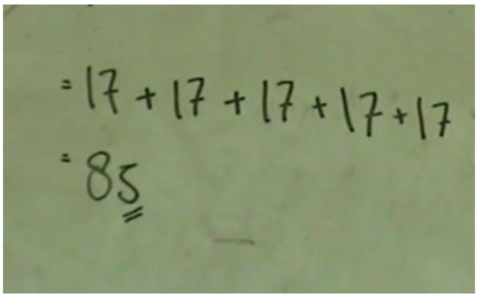


Figure 19. first answer (left) and second answer (right) by students

The teacher posed a problem about how to teach their younger brothers or sisters who are still in early primary school how to do multiplication of 17 and 5. As predicted, the first answer came from the students was to do formal calculation as they used to do (see figure 19 left). The teacher asked other students whether it is possible for their younger brothers or sisters to easily understand that strategy and answer. All students said no. By doing this, the teacher had opened a neighboured-discussion about new strategies to do it. After short period of discussion, one students came up with the second answer, which was repeated addition of 17 (see figure 19 right).

Figure 20. the third answer by a student

Both the students and the teacher were not content by the options of the answer. Hence, the students still tried to find new strategies. One student came up with counting manually one by one (see figure 20). Moreover, there was also one student who came up with a strategy, which was similar to the principle in the use of area model in this research. This principle is to separate the numbers based on its' place value (see figure 21).

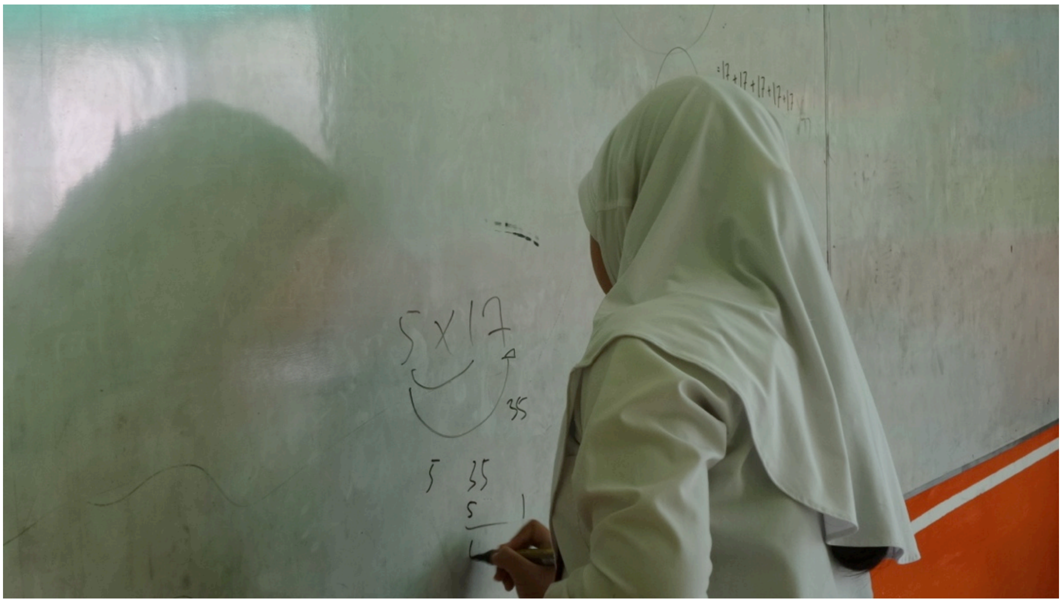


Figure 21. The strategy that similar to the principle of the use of area model

For the last answer given by the student, most students in the class claimed that the strategy was way too complicated for students. Whereas, some of the students stated that the strategy was simpler and easier. The teacher did not lead the students to make a conclusion at this point.

The teacher then posed the second question, which was actually the same question (the multiplication of 17 and 5) but in different context. In this case, the teacher said that she has a piece of land where she was planning to plant chilli and some vegetables. In this case, the measurement of the land was 17 m and 5 m, and she planned to divide the land into two parts, for the chilli and the vegetables. The division could be seen in figure 11. In this case, all students ended up with the same strategy, finding the area of each partition before finding the total area (see figure 22)

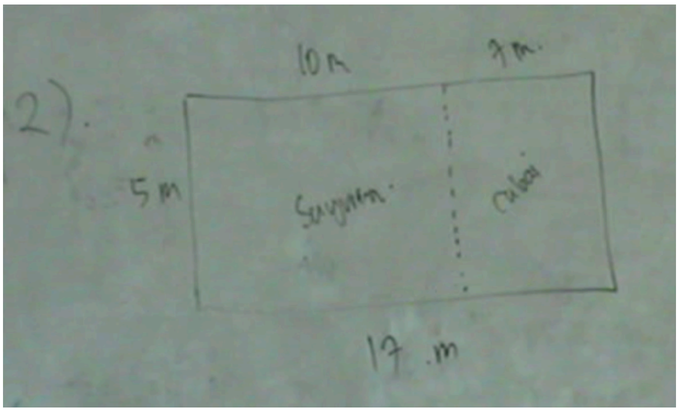


Figure 22. The second problem

After one student wrote the answer on the board, the discussion continued. This time, the teacher asked the students what they could say from the two problems. The first thing said by the students was that the two problems were actually the same, which was multiplying 17 and 5. The other students stated that they could use the strategy from the second problem to solve the first problem. Moreover, some students claimed that the second problem explained the fourth answer given by the student, which was not easily understandable at first.

To end the discussion, the teacher introduced the name, area model, and two formulas, rectangle formula and pieces formula, which were going to be used in the lesson.

The private forest problems

The teacher started by reminding the students about the poster they were given two days before. The poster consisted information about private forest. After small talks about this project, the teacher delivered the problems, which was finding the rectangle formula and the pieces formula of some pieces of lands for the forest project. There were partitions in each land since people were not allowed to plant one same type of plant. After that, the students worked in group.

Since the students did not have problem with multiplication of numbers, none of the group found meaningful difficulties. They easily understand how the area model worked. As a consequence, the spent less time in doing this activity. However, since the time was almost over, the researcher and the teacher decided to do the group presentation in the next lesson.

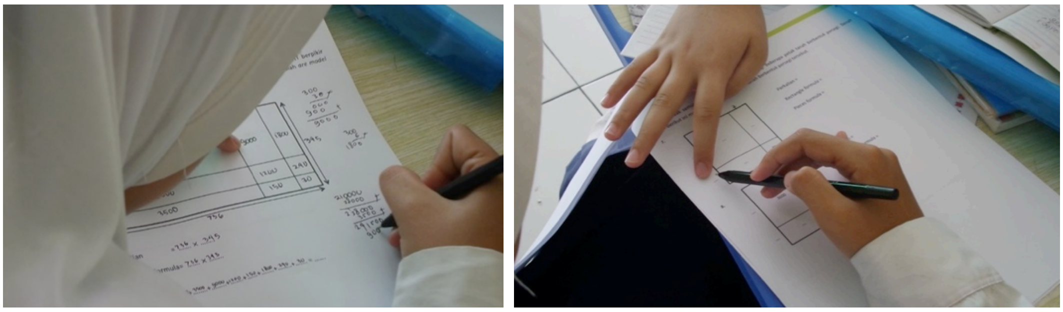


Figure 23. The group work

By the end of the problems, the students were asked to answer some question related to their understanding of area model and the two formulas. The students were asked to find the relation between rectangle formula, pieces formula and multiplication and its' product. All of the group were able to relate between the area model and multiplication. Moreover, the students were asked to explain how they would suggest to divide the numbers as such it would be easier to use area model. In this case, there were vary answers. Most students stated that they should use place value, whereas some others suggested to simply divide the numbers into two, or anything as such the sum of those partition numbers will be the initial number. By answering these questions, the students discussed with their group about it. Their answers were their own conclusion for their learning progress.

The house plan problem

This activity was firstly omitted from the lesson because the lesson started 40 minutes late. However, 10 minutes after the lesson had been ended, the next teacher did not show up. Hence, the lesson was continued with the house plan activity. The aim of this activity shifted because of the fact that they have learnt how to use area model in the previous activity. However, this activity was able to support the progress from the previous meeting. If previously they have concluded how to divide the numbers as such it would be easier to use area model, this activity showed the students that they could use area model without any fix regulation on dividing the numbers.

In this activity, the students were asked to draw house plan with some given information: the land was a rectangular land with 15 m length and 11 m width, the area of the house was about 100 m^2, there was a garage beside the house, a garden in front of the house and a paving in front of the garage. Last, the width of the garage was 3 m.

Figure 24. The students drew the house plan

The focus group agreed to draw the garage at first, since its' measurement was the one they were exactly sure (see the erased-right middle line of the house plan in figure 25). They drew it rectangle, 3 m wide and 5 m long. The next step was to determine the position and the exact measurement of the house. Hence, the students tried to find a pair of numbers which multiplication would produce a number around 100 (see the blue circle in the outside the house plan in figure 25).

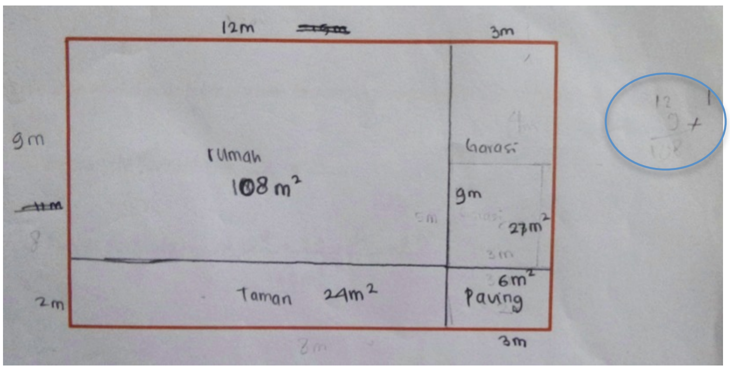
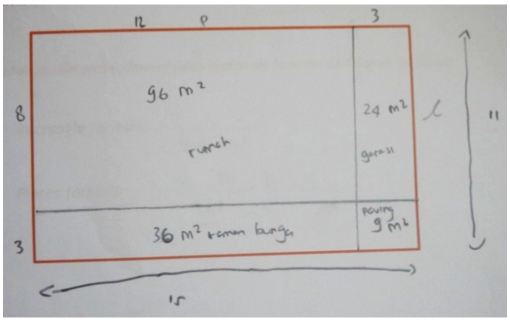


Figure 25. Focus group's house plan drawing

After one trial, the focus group found that the length of the house could be 12 m and the width could be 9 m. In this case, the following fragment shows how the researcher came to make sure that the students realized the possibility of other pairs for the house measurement. Line 4 of the fragment shows students' awareness of other possibilities.

1	*Researcher*	*: Does the area have to be exactly 100 m^2?*
2	*Students*	*: No.*

3	*Researcher*	*: Could it be more?*
4	*Students*	*: Yes. And it could also be less.*



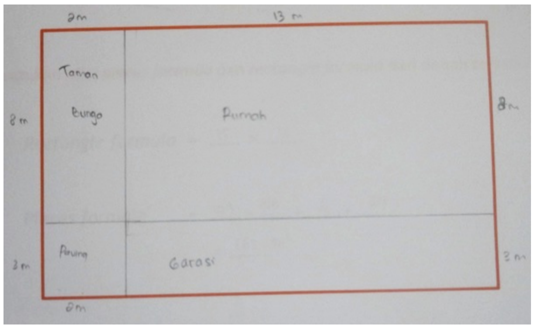


Figure 26. Example of other house plan drawings

Meanwhile, as predicted in the HLT, there were some other difference house plans (see figure 26). The left drawing in figure 26 shows group's drawing that decided to have the area of the house less than 100 m^2. The right drawing in figure 26 shows a group who determined the house plan very different with other groups. The garage was not in the right side of the house, but in the bottom side of the house. These differences led to different correct answer for the next questions, which was to compare the area of the garden and the garage. Each group then drew their house plan in a poster paper. However, presentation could not be done due to the time limitation. Hence, the presentation would be done as the preliminary activity in meeting 2.

Conclusion of meeting 1

Based on the teaching and learning activities in this lesson, it can be concluded that the students were very welcome to use the area model in solving multiplication table. Even though there were two new concepts, the rectangle formula and the pieces formula, the implementation of the design was not complicated. The students easily understood about how the area model worked. Based on the interview with some students, the area model helped them in doing multiplication easier. In conclusion, the mathematical activities in this lesson supported students' ability in doing multiplication. Further, the successful of this meeting was also a good base for the students to join the mathematical activities in the next meeting.

6.4.2 Meeting 2

This meeting lasted for 80 minutes. To start with, a presentation from the previous meeting was done.

The presentation

The first presentation was about how to use an area model to find the product of 115 times 76. Three students came in the front representing their group to show others about their answer. One student used place value to make partition. Interestingly, two students did not employ place value. Instead, they used easy numbers, such as dividing 76 not into 70 and 6, but 50, 20 and 6 (see figure 27).

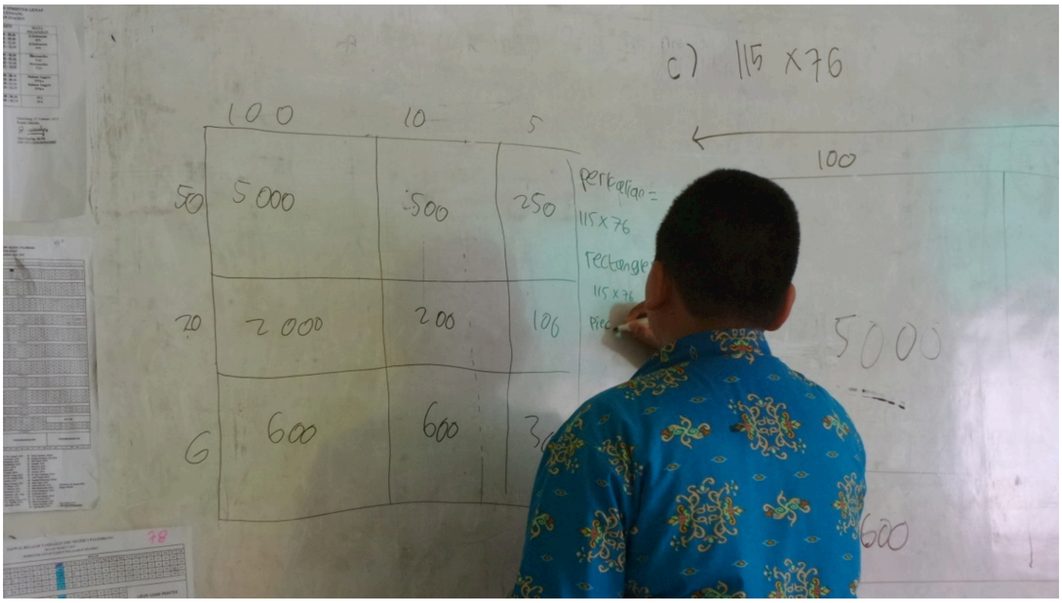


Figure 27. Example of how a student used area model for multiplication

Based on the answers, the teacher and the students made conclusion about how to make the division of the numbers. The conclusion was that area model was an open tool where students could use it to solve multiplication problems and put any number on it. However, the teacher gave suggestion by the end of the discussion to use place value for the numbers' division. This, of course, contradicted with the plan and affected the students. The students, who were enthusiastic and excited during the first meeting and the previous discussion, became more afraid in speaking or giving answer. This due to their anxious in giving the so-called "wrong" answer.

As a consequence, the second discussion was more passive. In this discussion, three students were stood in the front to show their group's posters. These three posters were different. Other students were to give remarks, comments and questions. The students were mostly passive. The teacher encouraged the students by asking what did they think about the differences in the three house plans.

Figure 28. Three students held their house plan posters in front of other students

The students commented on the different measurement and different shape of the house plans. The students got more passive. However, they agreed that all of the house plans were correct since they fulfilled the given information on the problem.

Emerging variables

The discussion was continued using the house plan. In this case, the teacher told a story about her friend's house. The house was unique. Not only the measurement of the land was square, but also the house was a square. Because the area was the same, around 100 m^2, the students and the teacher agreed that the measurement of the sides of the house was 10 m. The teacher drew the new house plan and asked them what could be the measurement of the width of the garage and the garden. The teacher gave example if the width of the garage was 3 m. All students agreed that the width should be the same. Hence, the width of the garden was also 3 m. The teacher then asked how the students determine the rectangle and the pieces formulas.

After a discussion about how to find the rectangle and the pieces formula, the teacher gave further story. She said, she did not know exactly what was the width of the garage and garden. In this case, some students proposed to use x, followed by a discussion about what x was. All students claimed that x was a variable. However, when the teacher followed the lesson plan questioning students' understanding about what was variable, the students stated something like a letter, which came after a number (see line 2 to 8 of the following fragment). This was inline with their answer in the pre-test. The teacher scaffold the students by brought them back in the context. Finally, some students claimed that variable was something to represent unknown length (see line 11 to 17 of the following fragment). It proved that the context had helped the students to make sense one of the definitions of a variable.

1	*Teacher*	*: What is variable?*
2	*Student1*	*: (a) Letter*
3	*Teacher*	*: Is there anything else?*
4	*Student2*	*: (an) Alphabet*
5	*Teacher*	*: Is there anything else?*
6	*[silent]*	
7	*Teacher*	*: What is actually a variable?*
8	*Student3*	*: (a) Letter that comes after (a) number*
9	*Student4*	*: "peubah" (the Bahasa translation for variable)*
10	*Students*	*: (inaudible) [talking about numbers, consonant, etc)*
11	*Teacher*	*: Look, is the value known?*
12	*Student5*	*: Yes*
13	*Students*	*: Not yet (correcting student 5)*
14	*Teacher*	*: Is the value known already?*
15	*Students*	*: Not yet*
16	*Teacher*	*: Nah, so, what is variable?*
17	*Students*	*: Representation of the unknown*

However, the discussion went much further then expected. The teacher asked the students how to find the rectangle and the pieces formula of the new house plan. One student went to write the answer on the board, as can be seen in figure 29. The discussion about how to multiply two same variables, in this case x and x, was carried out by the teacher. All students seemed to understand.

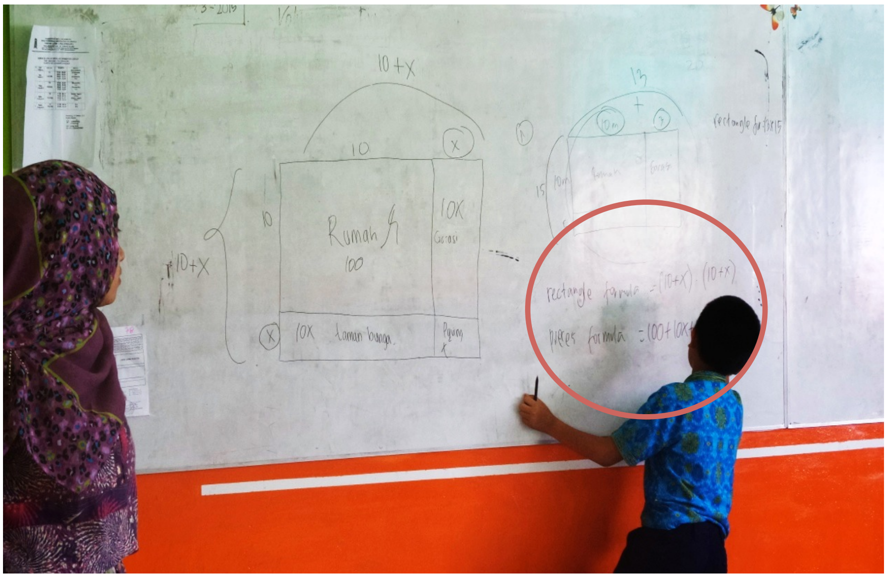


Figure 29. The students determined the rectangle and pieces formula

After the discussion, the students started to work in group. There were two types of problems. The first type was matching games. In this task, the students needed to match between geometrical shapes and algebraic expressions. The second was the same private forest project. However, in this meeting, some lengths of the lands were unknown. Variables were used to represent the unknown lengths. Figure 30 shows the sequence of the tasks.

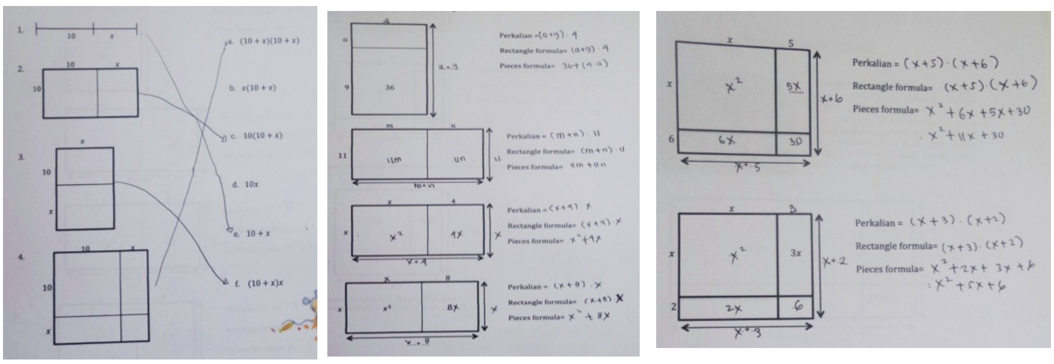


Figure 30. Mathematical problems in meeting 2

As a consequent of the too-far whole class discussion, the group work finished very fast. During this group discussion, as like in the preliminary teaching, the students were expected to make sense that the multiplication of two same variables equals the square of that variable. There was neither meaningful discussion nor difficulties. Students' understanding about how to use area model supported them as such they knew what they needed to do. The too-far discussion helped them to understand and to find the answers.

Making up a story

The last task was to make students' own context to represent and solve a binomials multiplication. The focus group chose to continue using house plan whereas some other groups chose different context, such as land division, and room division. Being able to correctly apply their understanding about using area model for multiplication and making a correct story which represent the multiplication, the students were fully understand the mathematics content and able to use the area model as a tool to solve binomials multiplication.

The conclusion of meeting 2

Continuing the first meeting activities, the presentation confirmed students' opinion that the use of the area model does not have to be precisely in one rule. Instead, the students could use any way of dividing the numbers as like their preference. The numbers' division does not have to be based on the place value of the numbers. This gave freedom for the students to use the area model based on their needs. This is also a base for the next mathematical content, binomials multiplication, which actually the same multiplication but included variables to represent unknown lengths. In this case, the area model did no longer consider place value as a divisor, but the known or exact numbers and variables.

Regardless the too-far discussion during the preliminary discussion, the students easily followed the mathematical tasks. They did not find any meaningful difficulties. This proved that the previous meeting about the introduction of the area model using numbers to the students was successfulness and able to support students' learning process in this meeting.

6.4.3 Meeting 3

This meeting lasted for 80 minutes. The aim of this meeting was to let the students realize that the area model they had been using could be improved. The improved area model in this case was that the area model was no longer unite with the context, but used as a tool to help them solve mathematics problems. In this case, emergent modelling was the soul of this shift. The new area model was no longer representing the measurement of the problems. It is used as like table which length of the sides did not depending on the measurements.

The rose garden problem

This meeting started with a story about one of the student, Andi, who wanted to build a square rose garden. In this case, his sister wanted to build a rectangular rose garden instead. Thus, the teacher asked a student to draw a square as a representation of the garden plan. In this case, the students wrote x in each side to emphasize that the drawing was a square. After that, the teacher asked the students could they make a rectangular rose garden plan with the same area of the square rose garden plan. Some students said they could make it by dividing the square rose garden into two rectangular parts and put one rectangular part beside the other part as such it would build up a rectangular rose garden with same area. However, the teacher said that the sister had her own way to make it rectangle, which can be shown in the following fragment.

1 *Teacher : (telling the strategy of the Andi's sister in building her rectangular rose garden plan) So we cut this (pointing at the top rectangular area of the square rose garden plan) and put it here (pointing at the right rectangular area of the square rose garden plan) (see figure 27).*
2 *Teacher : If the length of this is 1 (pointing at the width of the cut area), this one becomes (pointing at the width of the right rectangle drawing, next to the initial garden plan drawing; see figure 27)?*
3 *Students :* $x - 1$
4 *Teacher: So this one is (pointing at the length of the rectangle)?*
5 *Students :* $x + 1$
6 *Teacher: Do you think the area will be the same?*
7 *Students : Yes, the same.*
8 *Teacher : Can you find the area of the two garden plans?*
9 *Student1 : Area of the square is side times side; and area of the rectangle is length times width*

10 Teacher : Do you think both area are the same?
11 Students : Yes

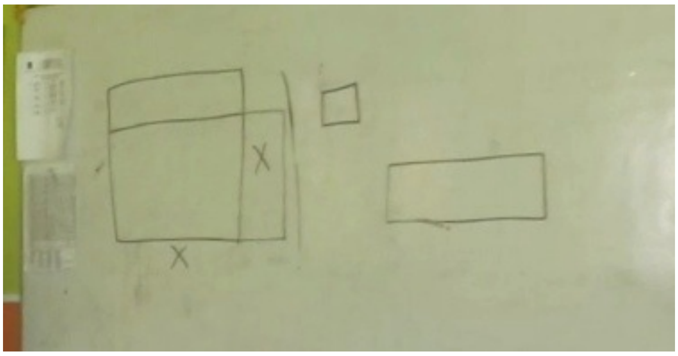


Figure 31. The students' drawings to represent the problem

Line 2 to 5 shows students' ability in translating geometric drawing into algebraic expression, which proved the successful of the previous meeting. In other word, the previous meeting had been proven to support students' ability in transforming situation or drawing into algebraic expression. Further, line 7 and 11 shows students' prior understanding towards the material. This is a good base before the students experiencing a cognitive conflict. The cognitive conflict confront their prior understanding that the area would be the same, with their understanding of area model to do binomials multiplication, where the multiplication resulted on different correct answer. To make sure and show the students their changing understanding, the teacher asked the students who claimed that the area would be the same to raise their hand. Automatically, all students raised their hand, and none of the students raised their hand when the teacher asked who did think that the area would change (see figure 32).

Figure 32. Students who raised their hands for same area of both rose garden plans (left) and did not raise their hands for changes area

Afterward, the students worked in group where they were given some examples of the answer for the rose garden problems. In this case, the students were asked to try to understand each answer in order to be able to order those answers. In ordering the answers, the students were given freedom in which thing they based their order on. The focus group automatically stated that the answer in figure 33 (left) was correct.

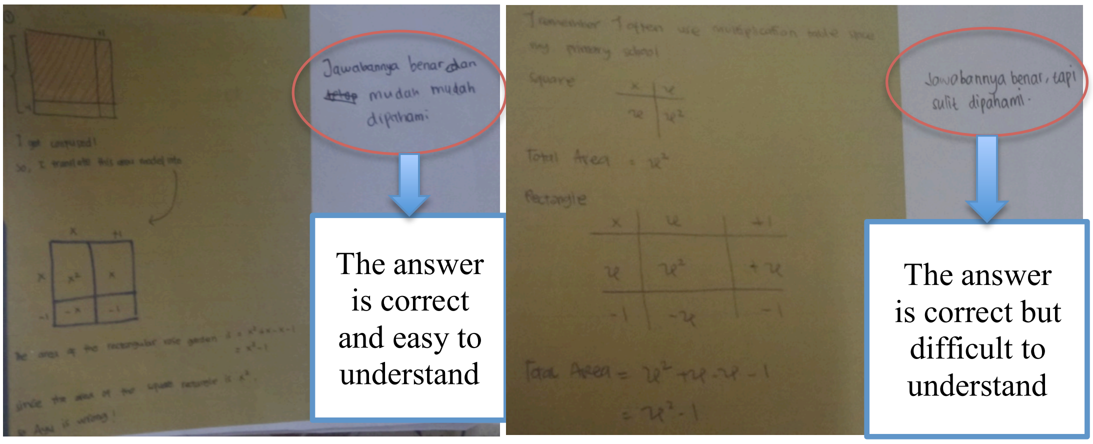


Figure 33. The first (left) and the second (right) orders by the focus group

The focus group ordered the left answer on figure 29 in the first place. Their reason was that because this answer was correct and easy to understand. They employed their understanding about area model to understand this answer. Moreover, understanding this answer and realizing that the answer was correct by different with the correct answer of their prior understanding (see figure 30), let the students to re-examine the answer in figure 34.

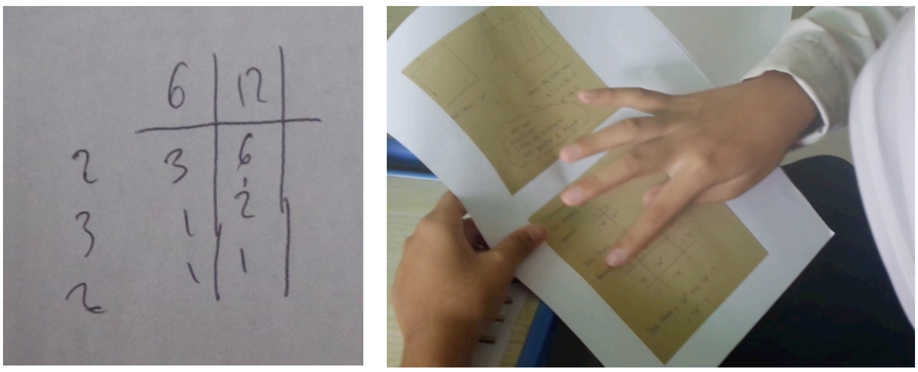


Figure 34. Trial and error for understanding multiplication table (left) and a student who was explaining how the multiplication table worked to her group (right)

The second answer based on the group's order was the right answer in figure 29. The students found a huge difficulty in understanding the strategy and model used in this answer, which was a multiplication table. The students in focus group were first tried to use numbers to help them understand the answer. In this case, they thought that the final row should be filled with 1, just like what in the answer. Finally, they ended up with using ratio table for division, instead of multiplication table (see the left picture in figure 30). After unsuccessfully understanding the answer, they tried to see the answer from different perspective. Finally, one student found that in this table, they needed to multiply those on the left side with those on the top side to get the inside numbers.

Finally, the third position in the order came into the answer, which was thought to be the correct answer. The students tried to find in which part of this answer was incorrect. After some trial and error, they found that the cut piece was too long to put in the right side of the rest garden plan. In this case, to make sure, they tried to put numbers. Their strategy can be seen in the right picture of figure 35.

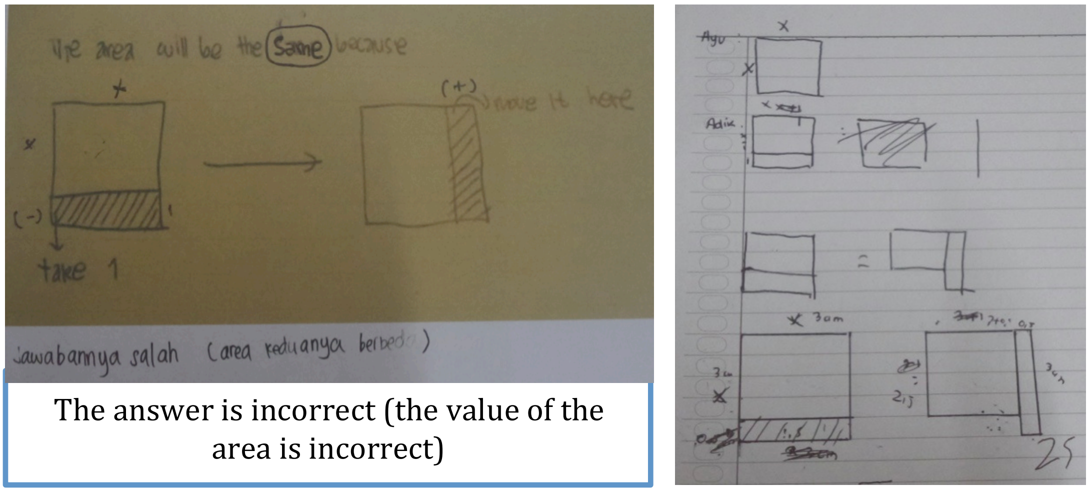


Figure 35. Students' strategy to re-examine their prior undersstanding

Due to the time limitation, the students did not spend a lot of time in trying to understand the answer, which they put in the last order. However, they had been sure that the answer was incorrect. This answer, which they ordered the last, can be seen in figure 36. In conclusion, the focus group order the answers based on the correctness of the answer and the easiness of the strategy and answer to be understood.

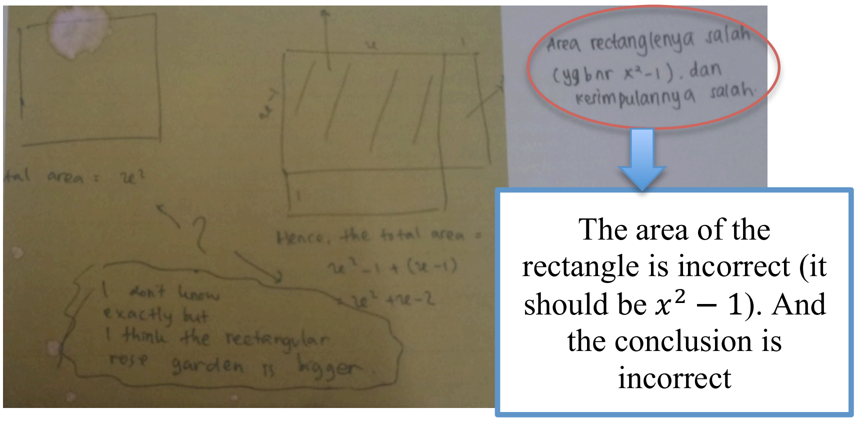


Figure 36. The last answer in the order

Lastly, the teacher started a whole class discussion where she asked two groups to represent their order and the reason behind their order. In this discussion, it was revealed that there were still groups which thought that the answer in figure 29 (left) was correct, and there were difference orders and reason to order. However, all of them came to a conclusion that for answers in figure 29, the left answer was way more easy to understand and the right answer was too complicated to understand. In other words, the students chose the improved area model to be more easy to use than the multiplication table. The reason why the teacher and students concluded that the model used in the left answer in figure 29 was that this was really similar with the area model they had used in the previous meetings. Moreover, in this model, the students did not need to express the model in similar drawing to the real measurement.

Applying what you have known

To get the students used to using the advanced-area model or multiplication table (as an option, in regards that the multiplication table holds the same multiplication algorithm and principles with area model), the students were given problems to fill in the blanks in the model and table and complete the formulas. The students found difficulties especially in doing the multiplication table-problems. They often looked back to the example from the rose garden problem. Further, the students were firstly struggling to do the last problem related to factorization (see the right picture in figure 37). However, after some discussion and looking back to the principle in using area model, few of the students uncovered how to use area model to do factorization.

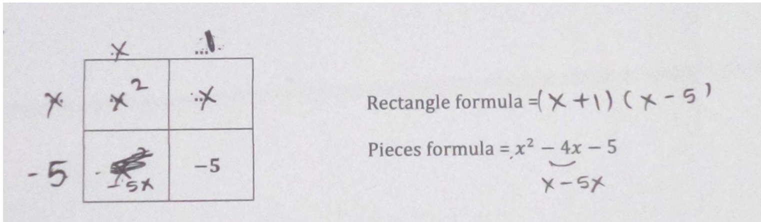


Figure 37. Example of the problems and answers of the focus group

1 *Student 1 : This is x, this is x (pointing at the x on the left and top sides of the area model: see figure 33)*
2 *Researcher : How is the answer for no. 8 (the problem in figure 33)? Why do you think that's x?*
3 *Student 1 & 2: Because of this* x^2 *(pointing at* x^2 *in the pieces formula)*
4 *Researcher : Ok. And then?*
5 *Student 1 : Means that this is....??*
6 *Student 3 : This one is -2 and this one -2 (pointing at the top right and left bottom out sides of the area model)*
7 *Student 1 : Oh no, no, no wrong! the result is different.*
8 *Student 1 : If this one 2 and this one 2, and then we sum up. the result is 4.*
9 *Student 2 : 2 is multiplied with what number to get -5?*
10 *Student 3 : Oia....*
11 *Researcher : hmmm, how do you think? If here is 2 and here is 2, does it suit the -5 (the last number in the pieces formula)?*
12 *Student 1 & 2 : No.*
13 *Researcher : How do you think? -5 is a multiplication of?*
14 *Student 2 : 1 and 2*
15 *Researcher : Please try.*
16 *Student 2 : eh, 1 and …… 5.*
17 *Student 1 : 1 and 5 (repeating)*
18 *Student 3 : 1 and 5 will not equal to the product (-5)*
19 *Researcher : 1 and 5. Can (combination of) 1 and 5 be (-)4 (the middle number in the pieces formula)?*
20 *Student 2 & 3 : No.*
21 *Researcher : So, can it be 1 and 5?*

22 *Student 2* *: Eh, it can, it can.*
23 *Researcher* *: Ok.. So which one is 1 and which one is 5?*
24 *Student 2* *: This one 1….. (pointing at the top right out side of the area model)*
25 *Student 4* *: Furthermore, this is -4x right? (pointing at the middle term on the pieces formula)*
26 *Student 1* *: Yes.*
27 *Student 4* *: It means 1 (pointing at the top right out side the area model) plus ….*
28 *Student 1* *: This one is 3 (the pair of 1, pointing at the left bottom outside the area model)… eh,,,*
29 *Student 2* *: -4*
30 *Researcher* *: 1 and …?*
31 *Student 1* *: This one is 5*
32 *Student 4* *: Isn't it -5?*
33 *Student 2 & 3* *: 1 and -5*
34 *[writing 1 and -5]*
35 *Student 1* *: It means …. (writing* $(x-1)(x-5)$ *on the rectangle formula)*

The fragment shows the focus group's discussion when they first encountered factorization problem. This was the second factorization problem, but the group decided to do it first. Line 8 and 9 shows that the students had realized that the two numbers, 1 and -5, should result on -4 (the middle number of the pieces formula) when added and -5 (the last number of the pieces formula) when multiplied. They employed trial and error to find the suitable numbers.

Conclusion of meeting 3

In this meeting, choices were given to the students. There were improved area model and multiplication table. The improved area model was introduced as a tool to solve more complicated binomials multiplication problems. Multiplication table was given as an option for the students. However, the main principle of how the improved area model and multiplication table worked was the same. In practical, the students faced difficulties in understanding how the multiplication table worked, but easily understand how the improved area model worked. Obviously, the students prefer to use the improved area model over the multiplication table. This was because they were used to using it since the previous meeting. Multiplication table might give the same result as the improved area model if the students were used to using it. In conclusion, the students showed

6.4.4 Meeting 4

This meeting was done in the same day with the previous meeting, but was separated by other subject. This meeting lasted for about 50 minutes due to incidental stop.

The palm oil plantation: the appearance of FOIL strategy

In this activity, the students were working on palm oil plantation problems. In this problem, the students need to implement their understanding from the previous meeting, whether about improved area model or multiplication table to solve the problems. The problems were basically binomials multiplication, which involved subtraction. The aim of this activity was to enhance students' ability in using improved area model or multiplication table to solve problems.

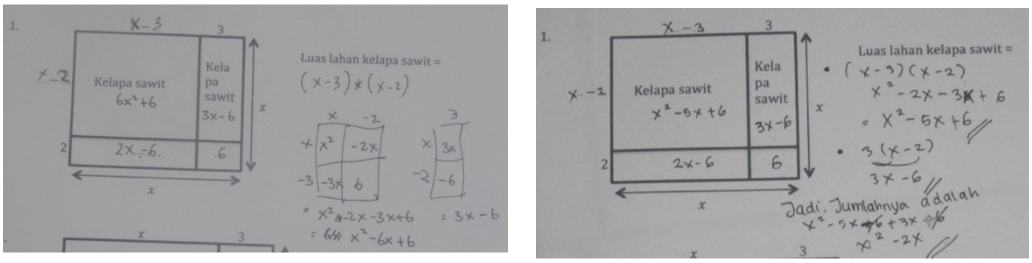
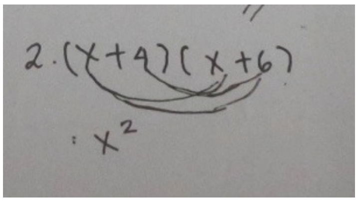


Figure 38. Example of focus group's work (left) and example of other group's work which employed FOIL strategy

Since the students were still struggling on the previous meeting, they carried their struggles into this meeting. Hence, some group frustrated and, unpredictably, used FOIL (First, Outer, Inner, Last) strategy. It was started with merely few groups who used that, but then most of the groups started using that as well. In this case, the researcher asked few groups and they said they have been taught recently about the FOIL strategy. However, they also showed struggle on applying FOIL strategy as well. They knew that there should be connection between the variable and number in specific order as can be seen in figure 39. However, many of them did not really know what to do with those connections.

To prevent from further things, which contradict the design of this study, the lesson was stopped and a new lesson was design to recover this condition.



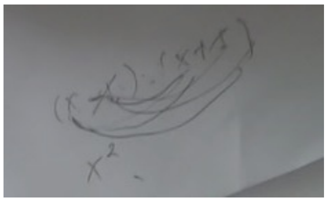


Figure 39. Prove of the appearance of FOIL strategy as well as students' confusion over this strategy

Conclusion of meeting 4

The intervention of the FOIL strategy was obviously an issue in implementing the design in this study. The one who got the most disadvantages from this situation were the students themselves. The students were really confused. They were on the path to understanding the use of improved area model or multiplication table. But they were afraid of giving the wrong answer and decided to use FOIL strategy as had been taught by their teacher outside the implementation of this design. However, they were still confused about the FOIL strategy as well. This distracted students' learning processes. Hence, other lesson was really needed to bring back the students to the initial path.

6.4.5 Meeting 5

To cover the effect of the appearance of FOIL strategy, meeting 5 was designed. This meeting lasted for about 60 minutes and was started by some problems using area model and multiplication table in order to lead the students to again intuitively use them to do binomials multiplication. The name of this activity was "*How further can you implement?*" and lasted for about 60 minutes.

How further can you implement?

This activity was divided into three parts. In the first part, the students were asked to fill in the blanks and complete the multiplication and its' product. This aimed at bringing the students back to the track and support students' learning processes using area model. This part was mainly the same with the problems from the second activity in the third meeting.

The second part of this activity involved merely algebraic expressions. The students were asked to either find the product of a multiplication or to factorize a quadratic expression. This is the crucial meeting where the students finally got the feeling of using area model to do binomials multiplication and fully made sense on how to do factorization. Examples of problems in this part can be seen in figure 40 (left).

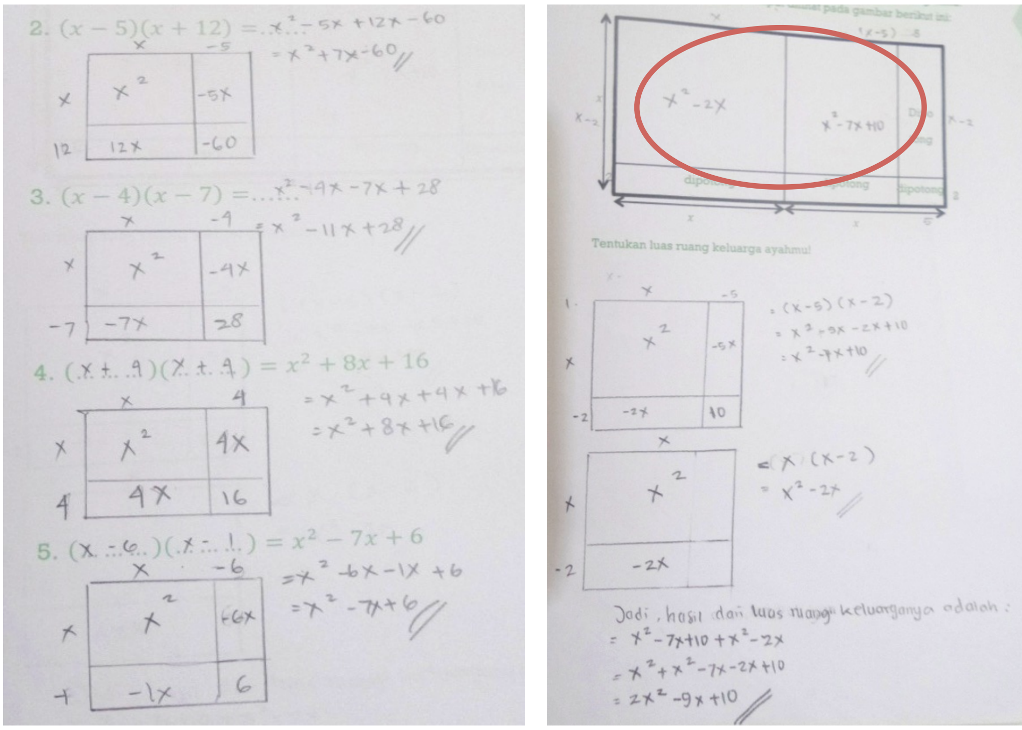


Figure 40. Work by the focus group

The last problem was a carpet problem. Given information about two square carpets with x long sides. To cover the living room (the two areas inside the red circle), they need to cut the carpet, as shown in the figure 36 (right). Based on the answer, we can see that the focus group divided the area into two. This division was based on the carpet. At first, they counted the right carpet by using area model. The following fragment shows how the focus group thought about the left carpet.

1 *Researcher* : *Ok. How about the other (the left living room) area?*

2 *Student 2* : *This is a combination right? (Means that the* $x^2 - 7x + 10$ *is for both living room areas)*

3 *Student 1* : *No! This is only for this area (pointing at the right living room area).*
4 *Student 2 & 3* : *So we make one (area model) more (for this)*
5 *Student 1* : *Just draw for both (drawing area model)*
6 *Student 2* : *Don't you think the result (the area model) will be different?*
7 *Student 1* : *Yes and we need only to sum up (the final result, not the area model)*
8 *Student 2* : *No, I mean this one and this one ... (pointing at the right and then the left living room area)*
9 *Student 1* : *This one has -5, and this one is only x.*

The fragment clearly tells about students understanding and steps on solving this problem, the reason why they drew the second area model. In drawing the second area model, they at first thought that there were four pieces areas. However, they then realized that the left carpet was cut merely in the bottom area. Thus, they needed to just draw area model with two pieces areas.

Besides the focus group, there were other groups, which answered the problem in similar way. Most of them combined the two areas first and then used area model to find the total area. Examples of this common strategy were given in figure 41.

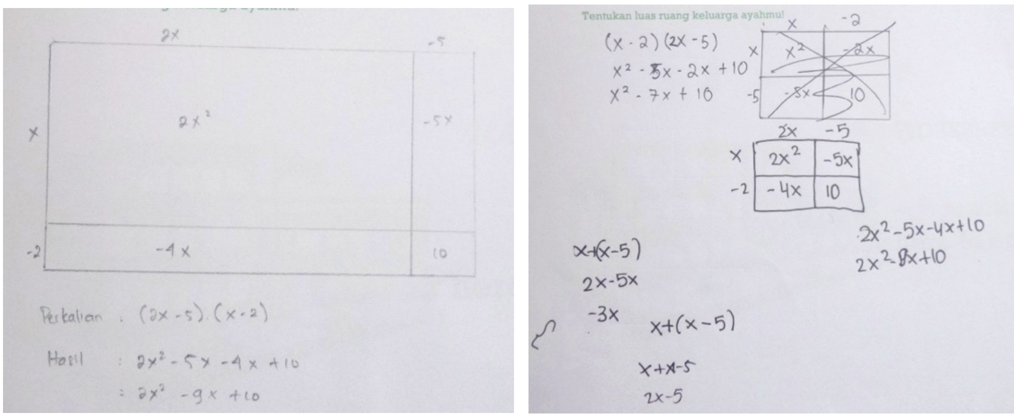


Figure 41. Common strategies by other groups

6.5 Checklist of the conjectures in the HLT

The HLT consists of activities, its aims and descriptions, and conjectures of students thinking. During the teaching experiments, many of the conjectures match the actual learning processes, but some did not. Table 3 shows the comparison between the conjectures in the HLT with the actual learning processes during the teaching experiment.

Table 3. Checklist of the conjectures of the HLT

Meeting	Activity	Conjectures	Actual learning processes
1	Two problems on multiplication of 17 and 5	• The students will come up with some strategies to do the multiplication, such as: formal calculation, repeated addition, counting one by one • The students will conclude that the area context of the second problem can be used as a strategy to teach multiplication for young children	• Same • Same
	The private forest project	• The students will not find any meaningful difficulties • The students will conclude that in employing area model to do multiplication, it is easier to divide the number based on its' place value	• Same • Some groups concluded that it is easier to divide number based on its' place value, while other groups use friendly number or simply half the number
	The house plan	• The students come up with vary house plans drawing and measurement	• Same
2	Emerging	• The students	• Same

	variable	propose to use variable to represent unknown length	
	The matching games	• Some students confuse in translating geometry into algebraic expressions, but soon they will fully understand and build up same understanding	• Same
	The private forest problems	• No meaningful difficulties	• Same
	Making up a story	• No meaningful difficulties, the students will make stories about land division and house plan	• The students make stories about land division for plantation, house plan and room plan
3	The rose garden plan	• The students first think that the area will not change • Students' understanding of improved area model and multiplication table is derived from their understanding of area model. Hence, they will easily understand how improved area model and multiplication table work • The students change their first thought	• Same • The students easily understand the improved area model based on their understanding of area model, but they hardly understand the multiplication table • Same

		that the area will not change	
	Exercises	• The students deepen their understanding of improved area model and multiplication table	• Same
4	The palm oil plantation	• The students use their understanding of improved area model or multiplication table to solve problems with some subtractions on its' factor	• Most of the students used FOIL strategy
5	Exercises	• The students go back in using improved area model or multiplication table to solve problems • The students make sense and find strategy to factorize	• The students used improved area model to solve problems • Same
	How further you can go? – carpet problem	• The students use improved area model to solve the problem	• Same

6.6 Post-Test Cycle 2

There were no changes in the items of the post-test. All 27 students in the class joined the post-test. An interview was held with the focus students to get insight into their answers in the post-test and their remarks during the teaching experiment. There were six problems on the post-test, which were divided into three parts. The first part consisted of the first and second problems. In the first problem, the students were given an area model where they needed to fill in the blanks and wrote down the multiplication and the product based on the area model. The second problem was basically the similar problem, but it used multiplication table instead of area model. With same level of difficulties, 23 out of 27 students answered correctly for the first number and merely 16 out of 27 students answered correctly for the second problem. This shows how the students felt more comfortable using area model, and how area model support students' ability and learning process in solving binomial multiplication.

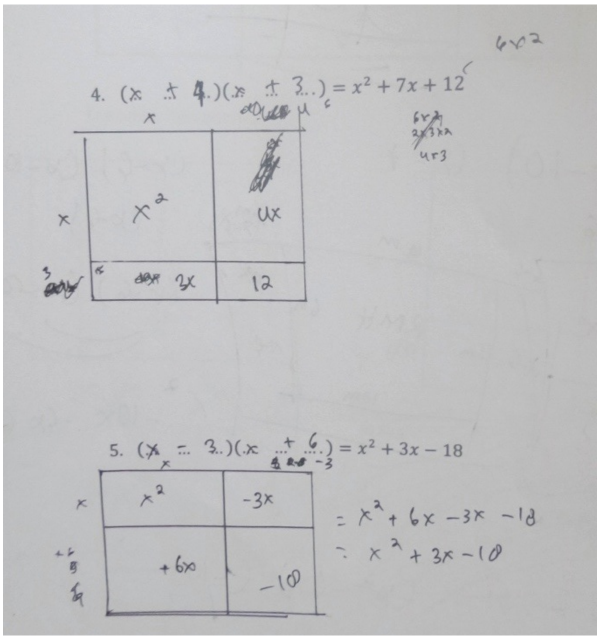


Figure 42. Answer from a focus student for the two factorization problems

The second part of the post-test consisted of three problems. One problem where the students needed to do binomials multiplication, and two problems where the students need to do factorization. In this case, all problems came in formal level and there was no suggestion or order to use area model or multiplication table. As predicted, all students used area model to solve all of the three problems. This strengthens the previous conclusion that the students prefer to use area model instead of multiplication table. 25 out of 27 students answered correctly for doing the binomials multiplication problem. However, 23 and 19 out of 27 students respectively answered the two factorization problems correctly. In this case, most students seemed to spend their time longer in the factorization problems and did trial and error. However, many of their effort resulted in correct answer. Figure 36 shows one example of students' trial and error strategy in factorized the quadratic expression.

The last part consisted of one problem where the students had the chance to implement their knowledge and understanding, which had been learnt during the teaching experiment, or even improved it. In this problem, the students were given information that there was a house surrounded (in the front, right and left sides) by a grass lawn. In this case, the students needed to find the area of the grass lawn.

However, most students seemed to have difficulties in solving this problem. Only 9 out of 27 were able to finally answer the problem correctly. Further, there were many different strategy used by the students in solving this problem. Most common mistake done by the students was that they simply substitute 6 and 10 (the width and the length of the house) from x and multiplied it. Hence, they multiplied $x - 6$ and $x - 10$ using area model.

Another common strategy in solving this problem was finding the total area of the land and subtracting the area of the house. In this case, there were two kinds of mistake in solving the problem using this strategy. First was that the students forgot that the length of the land was $2x + 10$. They forgot to add one x and resulted in $x + 6$. In the end, they multiplied $2x + 10$ with $x + 6$, the width of the land. The second kind of the mistake was that the students forgot to subtract the area of the house from the total area.

Few students, in their confusion, tried to solve the problem by changing the variable into a certain number. As such, they ended up giving exact numbers, without variable, as their final answer. Two students who answered the problem correctly was finding the area for each partition, and added them all and subtracted the area of the house.

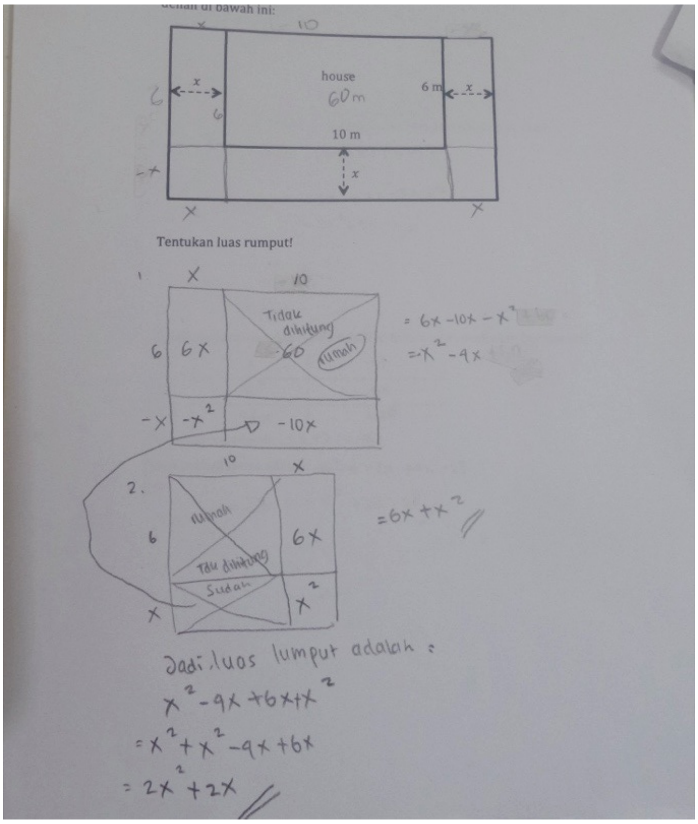


Figure 43. Answer for the last problem by a focus student

Interestingly, some students made division of the drawing. One of those students was a focus student. Figure 20 shows her strategy, of making division, in solving this problem. The crossed area means that those areas did not need to be counted. Those areas included the area of the house, and the area of grass lawn in front of the house because this area had been counted in the other drawing. However, a minor mistake she did. During the interview, it was revealed that since she was thinking that she drew two front-grass lawn areas (in which she had crossed one of it), so she needed to subtract somewhere. Hence, in a hurry because the time was running out, she simply subtracted (see figure 37 where she made the $10x$ as a negative) the $10x$, or the area of one front-grass lawn. Lastly, she added up all the area she got, including the $-10x$.

Conclusion of the post-test

Over all, the result of the post-test shows an improvement in students' abilities in solving binomials multiplication and, as a bonus, factorization. However, students' abilities in doing binomials multiplication are higher then their abilities in doing factorization. The result shows how area model, which had been being learnt during the teaching experiment, supports the students and becomes a tool to solve the problems. In this case, area model was overpowering the multiplication table, showed by the number of students who answered the first part of the post-test correctly. The first problem in area model resulted on much more correct answers then the second problem which related to multiplication table. In the second part of the post-test when the students were given formal problems, all of them chose to use area model. This overpowering of the area model shows potential of this as a tool in quadratic algebra. However, multiplication table might show the same potential if it had been taught from the beginning and the students had used to it.

CHAPTER 7. CONCLUSION AND SUGGESTION

The main aim of this study is to design the fundamental material to support students' understanding of binomials multiplication in a sequence of instructional activities. In this case, a research question is proposed, which is how does area model promote students' understanding of binomials multiplication. In order to answer this research question, some sub-research questions need to be answered. From those answers, we can draw a conclusion, which answer the research question. Those sub-research questions are:

1. How do the students use and understand the area model?
2. How does students' ability in translating from area model into algebraic expression?
3. Do the students able to connect area model with binomials multiplication?
4. How does the improvement of the students' ability (by using area model as a tool) in solving binomials multiplication?

7.1 Conclusion: Answering the research question

The first sub-research question is how do the students use and understand the area model. This sub-research question can be answered by looking back at the firs meeting of the teaching experiment. Starting with land division and using numbers, the students did not find any difficulties. It was very easy for the students to make sense the context and to know what they need to do. Further, they claimed that during the preliminary discussion and interview that area model is a useful tool for doing multiplication.

The second research question is about the students' ability in translating from area model into algebraic expression. This sub-research question was covered in the second meeting of the teaching experiment. In the matching activity, the students were able to make sense, some with support of line numbers, which algebraic expression matched the drawing. Further, the drawings in the next activity were all area models, and none of the students found difficulties in representing the drawing into algebraic expressions. This is a chain understanding with the previous meeting. Since the students had understood the area model, and how to use it for multiplication, they had had ability to translate the numbers in the model into a multiplication and its' product. Consequently, the students used that ability to translate the drawings into algebraic expressions. Their understanding about line numbers had supported them as well.

So far, based on the first and second meeting, the students have successfully associated area model with multiplication; with the help of rectangle and pieces formulas concept. With the emergent of variables in the second meeting, the students had successfully connected area model with binomials multiplication. The students who were all successfully used area model for binomials multiplication prove this. Even more, they were able to find a context for the multiplication. In this case, the students had started to use area model as a tool to solve binomials multiplication, no longer as a representation of problems. Furthermore, during the shift and improvement of students' understanding, they revealed how to do the inverse algorithm, which is doing factorization. This shows a potential use of area model to further teach factorization as well. Additionally, the area model might be also potential for a series of multiplication algebra in early secondary school. It can be a choice to be used as a tool in teaching a series of algebraic multiplication lesson, start from multiplication in linear algebra until binomials multiplication and factorization.

Last, the improvement of students' abilities in solving binomials multiplication problem can be clearly seen in the pre-test and post-test. There is a huge difference in the students' abilities in solving binomials multiplication problem. If previously in the pre-test almost all of the students could find the product of binomials multiplication, the post-test result revealed that almost all of the students were able to solve binomials multiplication.

Based on the aforementioned answers for all the sub-research questions, it can be concluded that a sequence of lessons that employs area model as a tool does support students' understanding in binomials multiplication. The result of this study also suggests that area model has a potential for teaching multiplication in linear algebra or for teaching factorization. Further, the strength of this design is the simplicity of the mathematical materials. The basic context is numbers. This design can be apply in any region. It is also easy to find the story for the division of the area model, such as land or house division. When the students have no problem with multiplication of numbers, this design will be easy yet challenging for them.

Thus, the contribution of this study to the local instruction theory is the series of learning processes using area model to support students' understanding in binomials multiplication. The students have to first learn the area model. When they feel comfortable enough, variables can be emerged. In this case, the lesson shifted from numbers to algebra. As the time flies, the area division, which is first stand for representation of area of something, emerge into area model, a tool to solve algebraic multiplication, including linear and binomials. We define this shift as the *emergent modelling*. This shift is the crucial moment of this series of lessons. After students have been able to use area model as a tool to solve algebraic multiplication, let the students explore by giving some problems. In this case, the students might find the principle in doing the inverse of binomials multiplication, which is factorization.

7.2 Suggestion

The suggestions in this part are derived into two different areas: the teaching practices in schools who want to implement this design and further research.

For the teaching practices

To be noted, the classroom norms and socio-mathematical norms are very important aspects in determining the success of the implementation of this design. For those who want to implement this design, this study also gives suggestion on the position of the students during the whole class discussion and group discussion. For the teacher, be remember to create a situation where the students are freely express their thoughts and mathematical ideas.

The crucial point in this design is during the shift of area model, from a representation of land division into a tool to solve binomials multiplication. Hence, put more exercise in this area if needed. However, students' first encounter of area model is also important as the base of further lessons. Thus, it is important to put another one meeting to get the students used to area model if necessary.

For further research

Quadratic algebra is one of the issues in Indonesia, especially in factorization. Students are confused about how to factorize quadratic expressions. This difficulty may leads to a bigger failure in students' future when they have to face more complicated algebra. We believe that when students are able to reason and make sense about binomials multiplication, which will result on quadratic expression, it will ease them to make sense factorization. Thus, developing students' understanding and abilities in doing binomials multiplication is crucial as the base for the students. This study has proved the advantages of the use of area model as a tool to teach binomials multiplication. Yet, a potential use of area model for factorization problems is also revealed, but in shallow analysis since this is not the main focus of this study. Hence, further research may focus on how to implement and analysis more deeply in the potential of area model to do factorization.

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