DEVELOPING STUDENTS' SPATIAL ABILITY IN UNDERSTANDING THREE-DIMENSIONAL REPRESENTATIONS

MASTER THESIS



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UNIVERSITAS NEGERI SURABAYA PROGRAM PASCASARJANA PROGRAM STUDI PENDIDIKAN MATEMATIKA 2015

DEVELOPING STUDENTS' SPATIAL ABILITY IN UNDERSTANDING THREE-DIMENSIONAL REPRESENTATIONS

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DEDICATION

I dedicate this thesis to my family and those who support me endlessly: My mother, Roisah

Who teaches me to balance my life and pursue my dreams. My father, Joyo Suprapto

Who shows me how to not give up this life.

My sisters, <u>fli</u> Frnawati and Iin Dwi Susanti

Who inspire me in many ways.

I WILL ALWAYS LOVE MY FAMILY

ABSTRACT

Hendroanto, Aan. 2015. *Developing Students' Spatial Ability in Understanding Three-Dimensional Representations*. Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisors: (I) Prof. I Ketut Budayasa, Ph.D. and (II) Dr. Abadi, M.Sc.

Keywords: Spatial Ability, Spatial Visualization, Spatial Orientation, Understanding 3D representations, Realistic Mathematics Education (RME), Design Research.

Spatial ability is known as the main key to develop students' ability in understanding three-dimensional (Abbreviated as 3D) representations that becomes the anchor of the development of 3D geometry thinking. Therefore, to develop students' ability in understanding 3D representations, we must develop their spatial ability. The present study aims to design a sequence of activities to help young learners developing their ability in understanding 3D representations. To develop such activities, spatial visualization task and spatial orientation task are combined with the aspects of understanding 3D representations. The sequence is also designed based on the characteristics of Realistic Mathematics Education (RME) and students' learning style. In addition, the present study also targets to contribute to the local instruction theory of developing students' spatial ability such as how the design works and how students' learning goes. Accordingly, design research is chosen as the research approach in order to produce both the instructional materials and also the theory. We conducted two implementations of teaching experiment involving 30 third-grader students of SD Laboratorium UNESA, Surabaya. The result of the first implementation showed that the activities need some improvements to have better support for the students. After the second trial, the activities indicated a better support to the development of students' spatial ability in understanding 3D representations. These activities are identifying pictures in photography activity and also drawing and constructing objects of building blocks in reporting temples activity. During the implementation, these activities gave students chance and guided them to explore the views of 3D objects and their representations. Students' learning progress is also in line with the hypothetical learning trajectory.

ABSTRAK

Hendroanto, Aan. 2015. *Developing Students' Spatial Ability in Understanding Three-Dimensional Representations*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembmbing: (I) Prof. I Ketut Budayasa, Ph.D. and (II) Dr. Abadi, M.Sc.

Kata Kunci: Kemampuan Spasial, Spasial Visualisasi, Spasial Orientasi, Memahami Representasi 3D, Pendidikan Matematika Realistik, Design Research.

Kemampuan spasial dikenal sebagain kunci utama untuk mengembangkan kemampuan siswa dalam memahami representasi tiga dimensi (disingkat 3D) yang merupakan dasar dalam perkembangan berpikir geometry 3D. Oleh karena itu, untuk mengembangkan kemampuan siswa dalam memahami representasi 3D, kita harus mengembangkan kemampuan spasial siswa. Penelitian kali ini bertujuan untuk mendesain serangkaian kegiatan untuk membantu siswa mengembangkan kemampuan mereka dalam memahami representasi 3D. Untuk mendesain kegiatan ini, tugas atau kegiatan yang berkaitan dengan spasial orientasi dan spasial visualisasi dikombinasikan dengan aspek-aspek dalam memahami representasi 3D. Serangkaian kegiatan ini juga dikembangkan berdasarkan karakteristik dari Pendidikan Matematika Realistic (RME) dan gaya belajar siswa. Sebagai tambahan, penelitian kali ini juga bertujuan untuk berkontribusi dalam mengembangkan lokal teori untuk mengembang kemampuan spasial siswa seperti bagaimana kegiatan berjalan and bagaiman respon siswa serta perkembangannya berjalan. Sesuai dengan tujuan ini, design research dipilih sebagai pendekatan dalam penilitian untuk menghasilkan baik kegiatan dan teorinya. Penelitian ini melaksanakan dua kali implementasi yang melibatkan 30 siswa kelas 3 dari SD Laboratrium UNESA, Surabaya. Hasil dari implementasi pertama menunjukan bahwa kegiatan yang didesain memerlukan beberapa perbaikan untuk lebih meningkatkan kualitas kegiatan bagi siswa. Setelah implementasi kedua, kegiatan yang didesain mengindikasikan hasil yang lebih baik dalam membantu siswa untuk mengembangkan kemampuan spasial untuk memahami representasi 3D. Kegiatan yang didesain ini terdiri dari mengidentifikasi foto-foto dalam kegiatan fotografi dan kegiatan menggambar dan mengknstruksi objek-objek dari blok-blok kayu dalam kegiatan melaporkan candi. Selama implementasi, kegiatan-kegiatan yang didesain memberikan siswa kesempatan dan menuntun mereka untuk mengeksplore objek-objek 3D dan representasinya. Progres belajar siswa juga sesuai dengan trajek belajar yang telah diprediksikan (HLT).

PREFACE

Assalamu'alaikum Wr.Wb.

Alhamdulillah to The Almighty, Allah SWT, so that I can finally finish this thesis without much difficulties. Blessing and greetings to the prophet Muhammad SAW who inspires people in the world to be wise and merciful among the others.

This thesis is one of my great works that I have ever achieved in my short life. Of course, I would not be able to complete it without assistance, guidance, motivation and support from the people involved in this study. Therefore, I would like to thank and express my gratefulness to:

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I do hope this study will contribute to the development of mathematics education, especially in Indonesia. Finally, I fully realize that the present study still has so many weakness in many aspects. Therefore, any critics, comments, and suggestions are really welcomed.

> Surabaya, June 2015 Author,

Aan Hendroanto

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CHAPTER I

INTRODUCTION

A. Research Background

Spatial ability is a very important skill not only for the future career of the students, but also for their daily life. For instance, an architect need good spatial visualization ability to do the job and predict the final product of constructions or assignments (Schmidt, 2001). In daily life, students may work with photographs and they need to understand the perspective of the image. Another example is that in traveling, people need to visualize scenic travel locations and find the orientation of their position on the map. Many studies have been conducted on the field of spatial ability and the results suggested the importance of spatial ability for learning mathematics (Guay, 1977; Batista, 1982; Revina et al., 2011; Risma et al., 2013). Moreover, some researches showed that there is a significant positive correlation between spatial ability and student's achievements (Guay, 1977; Batista, 1982; Tatre, 1990). In 1988, Lohman explained that there are three major factors of spatial ability that supports students' learning on mathematics. They are 1) spatial orientation, 2) spatial visualization, and 3) spatial relation. However, it is not yet specifically known which fields of mathematics is influenced by spatial ability.

In 2010, Pittalis and Christou investigated the relation between students' spatial ability and their three-dimensional (later we abbreviate it into 3D) geometry thinking. The results showed that spatial ability supports the

development of students' 3D geometry thinking which consists of 4 different types: 1) *understanding 3D representations*, 2) *spatial structuring*, 3) *conceptualization of mathematical properties*, and 4) *measurement*. Pittalis and Christou (2010) concluded that the improvement of students' 3D geometry thinking follows the development of students' spatial ability. Additionally, they also found that spatial orientation and spatial visualization contribute to the development of students' ability in understanding 3D representations, spatial structuring, and measurement. Meanwhile, spatial relation only supports one type of 3D geometry thinking, conceptualization of mathematical properties. Although spatial ability has an important role in the development of students' 3D geometry thinking, it still does not yet receive much attention in the Indonesian curriculum (Revina et al., 2011; Risma et al., 2013).

In Indonesian mathematics classroom in primary education, there are activities on spatial ability such as constructing net, determining directions, finding the volume of space figures and map reading. However, these activities do not fully support students' spatial ability in 3D geometry thinking. These activities merely focus on three types of 3D geometry thinking: spatial structuring, measurement, and conceptualization. The activities do not sufficiently support students' ability in understanding 3D representations. A lack of spatial ability in understanding 3D representations. A lack of spatial ability in understanding 3D representation can impede students' learning and can be a problem in their further education especially in the field of geometry. For instance, Ben Haim (1985) indicated that students will had difficulties in reading two-dimensional representations of solid objects (3D) which lead to the deficiency of understanding of volume measurement. Students in senior high school also often encounter difficulties in 3D geometry reasoning like understanding the relation between elements in space. Parzysz (1988) showed that students in primary education often have problems in differentiating between knowing versus seeing. Unable to distinguish knowing versus seeing means that they struggle to distinguish the properties of the object on the real situation and the properties of the object on its representation. They often forget to consider the unseen parts of objects if the representations are in 2D drawings. They get confused to draw or understand and mentally manipulate 3D representations (van Den Brink, 1993). On the other hand, the development of students' ability in understanding 3D representations can support the development of the other three types of 3D geometry thinking (Pittalis & Christou, 2010; Duval, 1998). Thus, understanding in 3D representations is like a foundation for students' 3D geometry thinking. Considering the importance of understanding 3D representations, we shall spend more time to study how to develop it. As we mentioned before, to develop students' ability in understanding 3D representations, we need to support students' spatial orientation and spatial visualization (Pittalis & Christou, 2010; Duval, 1998). Therefore, we need more activities related to spatial orientation and spatial visualization to support students' spatial ability in understanding 3D representations.

On the other hand, most of studies only investigate the correlation between spatial ability and students' achievements or mathematical abilities. There are only few studies that investigate how to develop students' spatial ability, especially in Indonesia. There is still limited knowledge of how to support students' spatial ability for young learners, especially in understanding 3D representations (Revina et al., 2011). The recent study by Revina et al. (2011), designed instructional activities involving spatial visualization task to support students' spatial structuring to promote the understanding of volume measurement. In her study, she concluded that spatial visualization tasks indeed improved students spatial structuring on volume measurement. However, her study only focused on students' spatial structuring and measurement. It was still lack of supporting students' ability in understanding 3D representations. Another study, Risma et al. (2013), showed how to design activities to supports the development of student's spatial abilities. She designed spatial visualization and spatial orientation task to help 3rd grade students developing their spatial ability. The study revealed that spatial orientation task and spatial visualization task can improve students' spatial ability. However, those activities were still too general on developing students' spatial ability and still lack of bridging the students to understand 3D representations. Therefore, we argue that more studies are needed on this area.

Based on those reviews, this study aims to design a sequence of learning activities for 3rd grade students to support their spatial ability in understanding 3D representations. We know that 3rd grade students have already understood planar figures and its representations. However, they do not yet learn further about space figures. Furthermore, students also are not yet familiar with mental-

moved object such as imagining a 3D object rotated or cut and moved. They usually work with models or manipulatives to help them imagining or thinking. Therefore, the idea is to design learning activities of spatial orientation task and spatial visualization task to bridge students' understanding and spatial ability to work on 3D representations. In the end of the activities, the students are expected to have better spatial ability in understanding 3D representations.

B. Research Question

Based on the aforementioned research background, the research question is formulated as follows "*How can spatial orientation task and spatial visualization task support the development of students' spatial ability in understanding 3D representations?*"

C. Aims of the Research

The aim of the research is to produce instructional materials and to contribute to the local instruction theory of developing students' spatial ability in understanding 3D representations. The contribution is an innovation in designing the series of learning activities to support students' spatial ability. In addition, it also includes the learning trajectory of the activities and how it supports students.

D. Definition of Key Terms

In this study, there are some key terms that need to be defined to avoid miss-interpretation. These key terms are:

1. Spatial orientation

Spatial orientation is operating on relationships between different positions in space with respect to your own position. It is the ability to remain unconfused if the perceptual perspective of the person viewing the object is changed or moved.

2. Spatial visualization

Spatial visualization is performing mental movements of two- and threedimensional objects. It is also the ability to create mental images and manipulate them.

3. Task

Task in this study means context-based problems to engage students to do investigations and observations to answer and solve the problems.

4. Spatial orientation task

Spatial orientation task is tasks related to the use of spatial orientation. In this study, we only employ spatial orientation task of finding the position of stand point of different views of 3D objects represented in the form of either models or photos.

5. Spatial visualization task

Spatial visualization task is tasks related to the use of spatial visualization. In this study, we only employ spatial visualization task of determining the standard views of 3D objects represented in the form of either models or photos.

6. To support

"To support" means to facilitate, engage, and stimulate the students to do investigations, observations, and discussions to achieve certain purposes.

7. Ability

Ability is the capacity of an individual to understand, remember, and do certain performances.

8. Spatial ability

Spatial ability is an ability related to the use of space. It consists of three major types: 1) spatial visualization, 2) spatial orientation, and 3) spatial relation. In this study, the focus is only on developing students' spatial visualization and spatial orientation.

9. The development of students' spatial ability

Development of students' spatial ability means the progress of how students solved spatial ability problems during the teaching and learning process.

10. Understanding

Understanding is connecting schemes available in mind with new schemes of certain concepts.

11. Three-dimensional representations

Three-dimensional representations abbreviated as 3D representations are a set of representations to replace the existence of certain 3D objects. These

representations have three levels: 1) Original Objects, 2) Models/Close representations, and 3) Drawings/Distant representations.

12. Understanding three-dimensional representations

Understanding 3D representations means the process of recognizing and manipulating the representations of 3D objects. It is integrated into the activity of coding and decoding representations of 3D objects.

E. Criteria of the Study

This study aims to develop instructional materials and a sequence of activities to support the development of students' spatial ability in understanding 3 dimensional representations. Therefore, we set up criteria to what extent the learning material and the activities have supported the development of the students' spatial ability in understanding 3 dimensional representations.

- Students show a development of their spatial ability during the activities. The development is indicated by students' progress in solving the tasks and the problems in each activity related to the spatial ability. Students' strategies and answers will be analyzed to explain the progress.
- 2. Students show a development of their ability in understanding 3D representations during the activities. This is indicated by students' progress in coding and decoding process of the 3D representations. Coding means students drawing or representing objects while decoding means reading or interpreting representations.

3. The actual learning trajectory (ALT) is in line with the hypothetical learning trajectory (HLT). This is indicated by comparing students' actions as a respond to the problems or activities to the predicted students' actions in the hypothetical learning trajectory. The comparison is in the form of matrix (Dierdrop's matrix analysis). The result of the analysis will be used to revise and improve the activities and the HLT.

F. Significance of the Research

The significances of the research can be divided into four kinds:

- For mathematics education in general, the present study means to contribute to the development of local instruction theory in the domain of spatial ability. The contributions are the product of a sequence of lessons and also the description of the learning process during the implementation.
- 2. For the researcher, the present study is a master thesis as part of the requirements to achieve a master degree in mathematics education. It is also to gain more knowledge for the future career of the researcher.
- 3. For the teacher who was involved in the research, the product becomes innovations in mathematics teaching and learning. This study also can add teacher's knowledge about realistic mathematics education and students' spatial ability.
- 4. For the students who were involved in this study, the sequence gives an opportunity to develop their spatial ability and also their ability in understanding 3D representations.

CHAPTER II

THEORETICAL FRAMEWORK

A. Spatial Ability

In the online dictionary, ability is a capacity of a person to do something physically or mentally (dictionary.com, accessed in 17th May 2015). In addition, Oxford dictionary added that ability means a level of skill or a person's capability. Based on those definitions, we tend to define ability as the capacity of an individual to understand, remember, and do certain performances physically or mentally. There are beliefs that person's abilities are fixed when he/she was born (Dweck & Leggett, 1988). That is why people commonly say that a person was born with a gift or ability to learn mathematics or to learn certain subject such as languages, music or arts. However, numerous studies show that abilities grow when students learn. For instance, Newcombe (2010) emphasized that students' abilities and achievements follows their consistent effort and hard work. According to many scientists, some abilities influence student's performances in some school subjects (Newcombe, 2010). One of these abilities is spatial ability which influences student's performances especially in mathematics (Tartre, 1990; Casey et al., 1992; Hoffler, 2010; Newcombe, 2010).

The issue of spatial ability arose in the early 20th century because of its relatedness to students' achievements in some school subjects, especially mathematics. Many scientists and psychologist have tried to define what spatial

ability is and to categorize it in sub-abilities (McGee, 1979; Tartre, 1990; Clements, 2003). Tartre (1990) defined spatial ability as a mental skill concerned with understanding, manipulating, reorganizing, or interpreting relationships visually. In the other words, spatial ability is the ability to formulate mental images and manipulate them in the mind (McGee, 1979). Refer to the use of the words "spatial", Olkun (2003) defined spatial ability as the ability associated to the use of space. Based on these definitions of spatial ability, we define spatial ability as the ability related to the use of space concerned with understanding, manipulating, reorganizing, or interpreting relationships visually.

In the literature, spatial ability is distinguished into two major abilities: spatial visualization and spatial orientation (McGee 1979). There are also some scientists who determine three major factors of spatial ability: spatial visualization, spatial orientation, and spatial relation (Lohman, 1988). Spatial visualization is the ability to create mental images and manipulate them (McGee, 1979). Clements (2003) added that spatial visualization is about performing mental movements of two- and three-dimensional objects. It is different from spatial orientation with respect to what is to be moved. Spatial orientation is operating on relationships between different positions in space with respect to your own position (McGee, 1979). Tartre (1990) explained it as the ability to remain unconfused if the perceptual perspective of the person viewing the object is changed or moved. Unlike spatial visualization, spatial orientation does not require mentally moving the objects. For instance, given a picture of the front side of a house and imagining how it looks like if you see it from the left side is spatial visualization. In contrast, determining the position or angle of where the photo of the house is taken is spatial orientation. Overall, spatial visualization is more related to mental visualization and mental transformation while spatial orientation is related to how people conceptualize space such as map reading, navigation, and drawing perspective.

Many studies have been conducted to analyze the importance of spatial ability. Most of them suggest that there is a positive correlation between spatial ability and student's achievement, especially mathematics (Guay & McDaniel, 1977; Batista et al., 1982; Tartre, 1990; Newcombe, 2010; Pittalis & Christou, 2010; Cheng & Mix, 2014). Furthermore, Guay and McDaniel (1977) found that this positive correlation between student's spatial ability and student's achievement in mathematics does not only occur at secondary school and at college level but also among elementary students. The mean scores of spatial test between low and high mathematics achievers among elementary students are statistically significant. This evidence is supported by more recent studies by Cheng and Mix (2014). In their research involving 6- and 8- year old students, they established that the students' mathematics ability follows their spatial ability. Hegarty and Waller (2005) also found that spatial ability is required to develop mathematical thinking which is in line with the findings by Cheng and Mix (2014). In addition, Kell et al. (2013) revealed that spatial ability has a unique role in the development of students' creativity and technical innovation in the future. In their longitudinal 30-year study, they found that spatial ability plays an important role not only in the process of assimilating and utilizing students' prior knowledge but also in developing new knowledge. Additionally, spatial ability supports students' creative thought and innovative productions. In conclusion, spatial ability is very important to support students' mathematical thinking because it supports students' creativity and also has significant positive correlation with students' achievements in mathematics.

In a sense, spatial ability also can be beneficial in many areas (Schmidt, 2001; Newcombe, 2010; Revina et al., 2011; Risma et al., 2013; Kell et al., 2013). For instance, an architect will really need good spatial ability to do a good job, to create and predict the final form of unfinished constructions (Schmidt, 2001). In daily life, students may work with pictorial representations and they need to understand and sometimes, mentally manipulate these images. Another example is reading, understanding and making map as students need to visualize scenic travel location and find the orientation of their position (Schmidt, 2001; Kurniadi et al., 2013). Newcombe (2010) added that many people from different fields, mostly science, rely on their spatial ability such as geoscientists, engineers and surgeons. People also need spatial ability in simple daily activities such as understanding photographs, arranging furniture, imagining locations, finding positions and understanding directions or instructions. On the other hand, a low spatial ability will lead to several problems for the students. One of those problems is the ability to understand pictorial representations, especially three-dimensional geometry (3D geometry). For instance, Ben Haim et al. (1985) indicated that students have

difficulties in reading two-dimensional representations of solid objects (3D shapes) which lead to a lack of understanding of volume measurement and a low understanding in spatial structuring. Revina et al. (2011) added the students often forget to count the blind spots or hidden parts of the object in the representations since the representations are in 2D drawings. All in all, spatial ability gives many benefits for the students both for their development of mathematical thinking and for their daily life activities. On the contrary, low spatial ability will impede their understanding and their performance in mathematics.

Considering the importance of spatial ability above, we surely have to support and facilitate the development of students' spatial ability. Uttal et al. (2012) conducted a meta-analysis regarding the potential to develop students' spatial ability. The study covered 217 research studies on spatial training from 1984 until 2009. The result showed a positive size effect .47. This means that spatial training can indeed enhance students' spatial ability. Furthermore, the study also confirmed that the training effect was stable and was not interfered by the delay between pre- and posttest. Therefore, this finding suggests that spatial ability obviously can be developed by training.

In conclusion, spatial ability is indeed important for the students not only to support their understanding of space and their performance in mathematics but also for their practical and future life. Spatial ability is not a fixed skill but it can be developed through spatial training.

B. Spatial Ability in Understanding Three-Dimensional Representations

In the previous section, we described how spatial ability determines students' learning and achievement especially in mathematics. A recent research by Pittalis and Christou (2010) also revealed and emphasized that spatial ability supports the development of students' 3D geometry thinking. They conducted a research involving students from grade 5, 6, 7, 8, and 9 in Cyprus to examine the relation between students' spatial ability and their 3D geometry thinking. They found that spatial ability is a very important contributor to the development of students' 3D geometry thinking. There is a direct effect between the spatial ability and 3D geometry thinking. That direct effect suggests that an improvement of spatial ability results in an enhancement of 3D geometry thinking. Therefore, to support students' 3D geometry thinking, we must support their spatial ability.



Figure 2.1 Four types of 3D geometry thinking

According to Pittalis and Christou (2010), 3D geometry thinking consists of 4 different types of reasoning. The types are understanding 3D representations, spatial structuring, conceptualization of mathematical properties, and measurement. These four types of geometry thinking are in line with Duval's theory of 3D geometry thinking. Duval (1998) added that understanding 3D representations is the foundation of the other types of reasoning. Figure 2.1 illustrates how the types of 3D geometry thinking supports each other. Therefore, supporting the development of students' ability in understanding 3D representation is important since it supports the development of the other types of 3D geometry thinking.

On the other hand, low understanding on 3D representations can lead to some problems such as difficulties in reading two-dimensional representations and determining the blind spot of the objects in the representations since the representations are in 2D drawings (Ben Haim et al., 1985; Revina et al., 2011). Moreover, students also have difficulties in drawing, representing, and reasoning about 3D objects such as parallel or perpendicular lines in space. Unfortunately, most of the textbooks and teaching activity use 2D drawings to represent 3D objects such as cubes, boxes, organs, and maps. Therefore, students often confuse to understand the representations (Revina et al., 2011; Risma et al., 2013). Importantly, low ability in understanding 3D representations can impede the development of their reasoning in 3D geometry thinking.

According to the online dictionary, understanding means "to perceive the meaning" or "to grasp the idea" (dictionary.com, accessed in 17th May 2015). Understanding is the knowledge about something that somebody have. In mathematics, understanding means connecting between schemes available in mind with new schemes of certain concepts (Sierpinska, 1994). Meanwhile, 3D

representations is a set of representations of 3D objects. Understanding 3D representation is not easy, particularly for young learners. It consists of two types of reasoning: 1) recognizing and 2) manipulating. Recognizing is dealing with mental movement or construction such as determining the shape of a 3D object based on its net. The second type, manipulating, means students are able to identify, interpret, and determine the shape of 3D objects based on the representations (Pittalis & Christou, 2010). In the present study, we focus more on understanding 3D representation in the aspect of manipulation of 3D representations than recognizing them.

Since the ancient time, people always have difficulties in understanding the representations of spatial objects. This due to the difference of the dimension of the objects and its representations (Parzysz, 1988). The only way to represent 3D objects is by making their model or two-dimensional drawings (2D drawings). For instance, a box is originally a 3D object but the drawing is on 2D layout or paper. As a result, some of the shapes changed into another shapes because of the effect of perspective like a rectangle can be a parallelogram or a rhombus. Thus, people have to imagine and create mental images of these shapes of the box based on the drawing. Different from 3D objects, 2D shapes' representations are easier to recognize since the representations have the same dimension as long as it does not have perspective views. Parzysz (1988) categorized object representations into three different levels. Table 2.1 displays the levels.

n

Table 2.1 Levels of 3D representations

		Geometry Objects		
	Levels	2D	3D	
	Level 0	Shapes on objects	Objects	
Close representation	Level 1	Drawing	Model	
Distant representation	Level 2		Drawing	•

Level 0 means the object itself while level one is called close representations. At level 1, the dimension of the objects and its representations are the same. Thus, the properties of the representations are close to the properties of the real one. The highest level of representations is a drawing of 3D objects or 2D representations of 3D objects which are called distant representations. For instance, if the object is a container, then the close representation of it is a model of the container such as toys, wooden box, or something similar to the container. Meanwhile, the distant representation of the object is a drawing of the container.

Distant representations are the most difficult representations to understand, especially for young learners (Parzysz, 1988). The reason is because there is a lot of information missing when we move from a lower level to a higher level of representations. For instance, a right angle in a cube sometimes will be not equal to 90° in its drawing. Parallel lines sometimes are not parallel in their representations. Drawings of 3D objects also have hidden parts or unseen parts which are usually called as the blind spots of the representations. As a result, most people will have difficulties to understand the object or even to draw it (Parzysz, 1988). Unfortunately, many subjects involve 3D drawings such as mathematics. Lack of ability in understanding 3D representations will impede student's performance in learning mathematics, particularly in 3D geometry.

Considering the importance of understanding 3D representations for the students, we obviously must pay more attention to it (Clements, 2003; Olkun, 2003; Revina et al., 2011; Risma et al., 2013). Previously, we know that an improvement of students' spatial ability will lead to an improvement in students' 3D geometry thinking. Now, the problem is how to support the development of students' spatial ability to assist their 3D geometry thinking. Unfortunately, most researches only focus on why spatial ability is related to students' performance and only few studies examine how to develop it and how to apply it in classroom activities, especially for primary students. The recent research was done by Revina et al. (2011). She designed instructional activities involving spatial visualization tasks with the purpose to support students' spatial structuring and understanding of volume measurement. Another study, Risma et al. (2013) showed how instructional design supports the development of student's spatial abilitiy. She created spatial visualization and spatial orientation tasks to help 3rd grade students developing their spatial ability. These tasks are based on the book of Mathematics in Context (MIC). Nevertheless, these activities need to be focused on students' ability in understanding 3D representations. Therefore, we argued that more research are needed in this area.
Hence, the present study aims to design instructional activities or sequence of lessons to support students' spatial ability in understanding 3D representation.

In the present study, the designed activities consist of spatial ability tasks which related to the aspects of understanding 3D representations. Pittalis and Christou (2010) explained that some spatial ability tasks have connection to the aspects of understanding 3D representations. Based on their research, only spatial orientation and spatial visualization contribute to the development of students' ability in understanding 3D representations, spatial structuring, and measurement. Meanwhile, spatial relation only contributes to the development of students' conceptualization of mathematical properties. Therefore, supporting students' spatial ability in understanding 3D representations should focus on the aspect of spatial orientation and spatial visualization. Spatial orientation and spatial visualization are related to object perspectives and image perspectives which are about sketches of a solid in different representational modes, manipulation of images of 3D objects, and isometric views of a cube (bird eye view).

In conclusion, understanding 3D representations consists of two types of reasoning which are recognizing and manipulating. The development of students' ability in understanding 3D representations follows the development of students' spatial ability, particularly spatial orientation and spatial visualization. Therefore, to support students' spatial ability in understanding 3D representations, we should support their spatial orientation and spatial visualization.

C. Spatial Ability in Indonesian Curriculum

In Indonesia, spatial ability does not yet receive much attention especially for young learners in primary education (Revina et al., 2011; Risma et al., 2013). Based on the curriculum, there are activities on reading map and on the sense of cardinal direction for 5th grader. However, these activities only focus on developing students' spatial orientation. Furthermore, these activities also only focus on navigation skills. The only activity on spatial visualization is the construction of nets of 3D objects like cube, box, and pyramid. However, this subject aims to introduce the concept of net to the students. It is also more related to the recognizing process in understanding 3D representations. This means there is no specific competence for developing students' spatial ability. Furthermore, the activities also did not completely support students' ability in understanding 3D representations. Lack of activities that support students' spatial ability and develop their ability in understanding 3D representations, will affect their further learning. Recently, in the mathematics textbooks released by PMRI foundation, there are some tasks about side seeing, looking 3D objects from different perspectives. These tasks are related to spatial orientation and spatial visualization. However, the book is not yet widely used in Indonesia.

D. Realistic Mathematics Education

In the previous section, we explained that this study will design a sequence of lessons to support students' spatial ability in understanding 3D

representations. These lessons are developed based on the principals and the characteristic of Realistic Mathematics Education (RME) from the Netherlands. This approach is based on the belief that "mathematics is a human activity" (Freudenthal, 1991). This means mathematics is the product of students' activity. Instead of teaching mathematics as a ready-made subject, teachers allow the students to construct their own ideas and concepts of mathematics (Freudenthal, 1991). There are 3 principles in designing activities on RME which are called design heuristics (Gravemeijer et al., 2003). They are didactical phenomenology, guided reinvention, and emergent modelling. The activities in the present study are developed based on the first principle, didactical phenomenology (Fredenthal, 1991). As a result, the activities in the learning process shall begin with meaningful contexts. According to Gravemeijer et al. (2003), the meaningful problems should be experientially real for the students such that they can imagine the situation. Thus, the problems do not require the students to have experienced it in real life as long as students can imagine it. To design such activities and problems, conducting a didactical phenomenological analysis is recommended to organize phenomena within the mathematical concepts to be developed by students (Gravemeijer et al., 2003). Since this study is for Indonesian elementary students, the activities are based on the culture of the region which is familiar enough to the students.

RME has five tenets as the combination of Van Hiele's three levels and Freudenthal's didactical phenomenology. They are 1) the use of context, 2) the use of model, 3) the use of students' contribution, 4) students' interactivity, and 5) intertwinement (Treffers, 1987). The present study does not implement all these five tenets because the design aims to develop spatial ability in geometry field. How these tenets will be integrated in the design of the present study will be discussed below.

1. The use of context

According to van den Heuvel-Panhuizen (1998), contexts are used as the starting points of the lesson so that the instruction does not start from formal level. The aim of the contexts is to activate students' prior knowledge and to engage them in meaningful activities. Context is also the source of concept construction and the zone of application (Leen Streefland, 1991). Simon and Tzur (2004) emphasized that the use of contexts in the lesson will give students' activity goals which at the same time, bring them toward the goals of the lesson or mathematical goals. In the present study, each activity begins with a context that familiar to the students. For instance, to develop students' awareness of spatial visualization and spatial orientation, we use the story of a photographer. In this story, students will be asked to help the photographer to figure out the best position for the camera to capture the objects.

2. The use of model

Gravemeijer et al. (2003) described the use of model in RME is student-generated ways of organizing their strategy or activity mathematically. The advantage of the model is to bridge the students from informal understanding to more formal understanding. However, in this study, the use of model is not applied in the Activities because in geometry field, we define the model used in the activities as tools or learning media to help students visualize object or imagine and create visual imagery of objects. This kind of model has a different role from RME model.

3. The use of students' construction and production

Treffers (1987) emphasized that students' construction and production have a key role in progressive mathematizing. He added that students' construction and production in the learning process can be interpreted as differentiated production procedures that are used to solve problems. Streefland (1991) stated that students will have greater initiative if they construct and produce their own procedure to find the solution. In this study, to engage the students to use their own constructions and production, we employs open problems for the discussions and asked students to solve the problem using any strategy. Therefore, the students are free to use their strategy and their terms. Students also will be encouraged to construct their own object (problem) that will be used in another activity.

4. Interactivity

Interactivity in RME teaching and learning means that learning mathematics is both an individual activity and a social activity (Gravemeijer et al., 2003). Social activity such as group work, sharing, and comparing is important for the students to get and share ideas and to improve their strategies. In the present study, we emphasized the interactivity between teacher-students and student-students in each activity. Therefore, there are whole class discussions and small groups' discussions in each activity to support the interactivity in the class. The whole class discussions are in the form of informal presentation where the students have to share and discuss their finding to the class and also teacher-led discussion among groups. The teacher will guide both the discussion and the presentation to make sure everyone has the same understanding. The students also have chance to discuss their strategy with their peers in small group discussion. Unlike the whole class discussion, teacher will move to every group to guide, scaffold, and help them if they find difficulties.

5. Intertwinement

Gravemeijer et al. (2003) underlined that mathematical domains are integrated to each other. Therefore, in the design, the activities should put mathematical domains in a close connection. In the present study, we employ some basic activities on spatial structuring and symmetry. These activities can be a basic understanding to develop the concept of volume measurement or surface area. One of the activity also involves the sense of vision line of angles and photography.

E. The Role of Students' Learning Style in the Design

Learning style is said to be the key to unlock student's full potential in the learning process (Dunn & Dunn, 1979; Deporter & Hernacki, 1992). There are

three basic types of student's learning styles according to Deporter and Hernacki (1992): 1) visual learners, 2) auditory learners, and 3) kinesthetic/tactile learners. Visual learners easily process information on pictures, diagram, charts, and etc. They prefer to learn through seeing by using representations, diagrams, charts and etc. On contrary, auditory learners feel more comfortable if they learn by listening or hearing. Usually, they use music, songs to learn and remember something. Meanwhile, kinesthetic learners enjoy learning by experiencing like doing or touching things. This type of learners likes to learn something physically by using models or tools to directly observe and investigate the concepts they want to learn. Dunn and Dunn (1979) emphasized that if learning approach match student's learning style then student's performance can be optimized.

Considering the influence of students' learning style, we tried to combine the students' learning style as one of the foundation to develop the sequence of activities. For this purpose, we firstly need to know the preference of elementary students in learning mathematics. According to Park (2000), most of the elementary students tend to have kinesthetic and visual learning style. This finding suggests that elementary students easily learn something by experiencing, touching, and observing things visually. Therefore, the use of tools, and physical models or media are highly recommended for kinesthetic and visual learners.

Based on the study of Newcombe (2010), investigating and observing physical objects are also the best way to support the development of students'

spatial ability, especially for young learners. Supporting Newcombe's finding, Tracy (1987) and Uttal et al. (2013) also suggests the use of physical objects like building blocks, puzzle play, and construction models. Piaget and Inhelder (Cited in Clements, 2003) argued that children ideas develop from intuitions grounded in actions such as building, drawing, moving, and perceiving. In the other words, children ideas about space do not come from passively looking, but from actively investigating. Supporting Piaget's and Inhelder's statement, Tracy (1987) claimed that students' spatial ability are supported by extracurricular activities involving toys such as building blocks, erector sets, and many more. These recommendations are in line with the need of students' learning style where they prefer to learn by experiencing physically and visually.

Based on the analysis above, we decide to involve the use of real objects such as building blocks, photos and model of buildings in the design. We design the activities by integrating the use of physical objects in the activities of spatial orientation task and spatial visualization task.

F. The Role of Teachers in RME Classroom

Teachers play an important role during the sequence of activities in the present study. This role is important and can determine the success of the sequence during the implementation. Unlike the conventional approach, RME requires teachers who actively guide and facilitate students' discussion (Zulkardi, 2002). Gravemeijer (cited in Zulkardi, 2002) explained that RME

teachers have 4 roles: *a facilitator, an organizer, a guide*, and *an evaluator*. Based on these 4 roles, in the present study, we determine five teachers' roles during the learning process.

1. Guiding the learning activities

Teachers organize and guide the learning process. Organizing and guiding the learning process means teachers conduct the activity and make sure that it flows on the right direction towards the goal of the lesson. In addition, teachers also make sure and prepare everything needed during the learning process.

2. Presenting the contexts

Presenting the contexts is also one of the important teachers' roles during the lesson. It is important that the students understand the contexts and the circumstances of the problem. Otherwise, students will be confuse about what to do or mislead to different direction of the activity. Therefore, teachers have to be able to present the contexts and make sure that everyone understands them all.

3. Leading discussion

In Indonesian mathematics classroom, students are mostly passive and afraid to express their opinion. Therefore, during the discussion including the presentation, teachers must help them by leading the discussion and posing question related to the problem. Leading discussion also enables teachers to drive the students to the demanded goals. In this study, there will be small group discussion, class discussion, and also presentation. In a group discussion, teachers move around from one group to another to help them investigating the problems. During the class discussion and presentation, teachers will pose more questions to the students related to what they have done and help them sharing the result of their discussion.

4. Encouraging students to actively participate

There is a possibility that some students are passively participate during the process. In this case, teachers have to be active to encourage the students to actively take part in the discussion and the learning process. In RME design, students' participation is very important and becomes one of the characteristic of RME, interactivity. Therefore, teachers' assistance is really needed in the present design.

5. Bad advisor

Bad advisor means teachers sometimes need to propose an answer that is neither right nor wrong. In some circumstances, teachers even need to propose wrong solution. For instance, when students have identify and put the photograph in the right position, teachers can ask "why didn't you pick this one?". Another example is when students have to construct a building block similar to a given photo, teachers can propose another construction which is similar to the photo but it is different from students' work. Applying bad advisor has some benefits in the discussion such as guiding students to notice something they do not aware of, giving students the clues, or checking students' consistency. In some cases, bad advisor also gives teachers opportunity to further analysis how students strategy and thinking goes. However, teachers must be careful when proposing answers since there may be a possibility students will assume that teachers' answer is the right one.

These five teachers' roles are not easy to implement, especially Indonesian teachers who are not familiar with RME. Wubbels et al. (1997) explained that as a consequence of RME, teachers have to be prepared by the researcher beforehand to assist them fulfilling their role in RME classroom. This preparation is required to further support teachers, especially those who are used to be teacher-centered in their class. For this reason, the present study provides teachers all the materials needed during the learning process. Moreover, teachers' guide and lesson plan that consist of contexts, goals, possible students' answers and also possible teachers' actions are provided. These learning materials can be found in Appendix 1 of this document.

G. Design Research

This study is meant to be an innovation and an improvement for mathematics education, particularly on developing students' spatial ability and their ability in understanding 3D representations. In addition, the aim of this study is also to contribute to the local instruction theory of how mathematical instructions can support the development of student's spatial ability, specifically understanding 3D representations. Therefore, we design a sequence of activities for 3rd grade students to support their spatial ability. Meanwhile,

we also develop the theory of how the lesson is designed and implemented. For that reason, we need a research approach that enables us to design the lessons and so at the same time, it also let us develop the theory. One of the research approaches which support that aims is design research. According to Gravemeijer and Cobb (2006), the main aim of the design research is to develop both instruction theory and educational materials. In addition, design research have powerful link between theory and its practice in which most of educational researches do not have (van de Akker et al., 2006). Design research also offers opportunity to deeply investigate and understanding students' thinking and learning (van Erde, 2013). In the same way, the aim of the present study is also to examine both the development and the process of students' learning, and the role of the instructions in their development. Moreover, design research also gives not only products such as educational materials but also gives insight of how to implement and use it in mathematics classroom (Van den Akker et al., 2006). It bridges the gap between educational practice, e.g. implementing, and its theory, e.g. designing and analyzing.

Design research as a research approach has all advisory aims (Bakker & van Erde, 2013). Therefore, it is good to assist studies that aim to develop innovation on teaching and learning to improve education. This is in line with the aims of this study which investigates how to design activities to improve mathematics education. In general, according to Cobb et al. (2003) design research has characteristics that support the present study:

- Its aim and purposes: Design research aims to develop theories about learning and becomes bridge between educational theory and practice. It also has predictive and advisory aim including descriptive, comparative, and evaluative.
- Interventionist nature: it is different from experiment approach. Design research has better ecological validity. Thus the finding of this study will be more applicable in reality.
- 3. Prospective and reflective components: in design research, reflection is done after each lesson. That is why it possible there will be changes to the original design for the next lessons. This gives flexibility during the designing process to further improve and adjust the activities.
- Cyclic nature: Design research forms an iterative process consisting 3 phases: 1) preparation and designing, 2) teaching experiment, 3) retrospective analysis. Figure 2.2 shows the cyclic nature of design research.



Figure 2.2 The cyclic nature of design research (van Eerde, 2013)

In the beginning, "K" in Figure 2.2 means researchers do thought experiment based on the literatures. Afterwards, this is followed by "D", designing the activities. The process is then continued by "E", implementing the design in the teaching experiment. In the end of the cycle, researchers do retrospective analysis, "R". The result of analysis becomes the starting points or consideration to begin the new cycle. The details of the three phases in design research is explained below:

1. Preparation and designing

The earliest step in this phase is conducting literature reviews and formulating the research aims and the general research aims (van Eerde, 2013). According to Gravemeijer and Cobb (2006), this phase is then followed by defining general mathematical goal of the study. The result of literature reviews and generating mathematical goals becomes the anchor of the design of activities. In this phase, researchers also develop conjectured local instruction theory of the design (Gravemeijer & Cobb, 2006). It consists of both provisional instructional activities and conjectured of learning process. Later, this conjectured local instruction theory is called Hypothetical Learning Trajectory (HLT) (Simon, 1995).

The conjectured local instruction theory is then elaborated based on observations, interviews and prior knowledge of the students. As a result, the elaboration product is named the elaborated hypothetical learning trajectory. The elaborated HLT consists of the descriptions of learning goals, learning activities and conjectured students' thinking and actions (Simon, 1995). HLT is the bridge between instruction theory and its concrete teaching experiment. It consists of learning goals, learning activities, hypothetical learning process or conjectures of how students thinking and understanding. It will be tested in a small group of students and analyzed in the next phase. The result of the analysis will be used to revise and improve the HLT.

2. Teaching experiment

When the first phase have produced the provisional activities and the elaborated HLT, the second phase can start (Gravemeijer & Cobb, 2006). Although the term used in this phase is "experiment", the teaching experiment is not the same as experimental or quasi-experimental researches. There is no control group, comparison between groups and etc. like the experimental researches. The main aim of the teaching experiment is to try or demonstrate but both to test and improve the activities and the HLT (Gravemeijer & Cobb, 2006).

Usually there are two teaching experiments which are preliminary teaching experiment (pre TE) and teaching experiment (TE). In the pre TE, the sequence of learning activities and the elaborated HLT as the product of designing process, are implemented in a small group of students, around 5-6 students. In addition, the researcher will be the teacher and at the same time also becomes an observer of the learning process. The purpose of this pre TE is to test the activities and the HLT so that the HLT can be improved before the teaching experiment. Consequently, the new HLT is called the improved HLT. In the teaching experiment, the improved HLT from the preliminary teaching experiment will be implemented. Different from the pre TE, the teaching experiment involves all the students in the regular class. The experiment also employs the regular teacher in the school. Before conducting the pre TE and the TE, researchers have to decide what kind of data to be collected and how these data will be analyzed (van Eerde, 2013).

3. Retrospective analysis.

The goal of the retrospective analysis may vary based on the theoretical intent of the design experiment (Gravemeijer & Cobb, 2006). However, the main aim of the analysis is to contribute to the local instruction theory. All the collected data will be analyzed to describe and explain the activities, the learning processes and the HLT during the experiment. According to Gravemeijer and Cobb (2006), the analysis based on the retrospective, systematic and thorough analysis of the collected data. The conclusion can be drawn by backtracking through the whole analysis. Therefore, in the retrospective analysis, researchers shall go back and forth between the HLT, the activities, and the empirical observations or data (van Eerde, 2013).

Importantly, retrospective analysis also investigates how the instruction support students' learning and their progressive development. If the analysis indicates that there are holes in the designed activities, then the sequence can be changed, revised and improved. If the changes are radically made then another teaching experiment can be conducted (Gravemeijer &

Cobb, 2006). HLT can also become a guidance in doing the retrospective analysis by comparing it with the actual learning trajectory (ALT) that happened in the experiment (Dierdrop et al., 2011). Later, this comparison is called Dierdrop's matrix analysis. The result of the analysis will become the answers to the research questions and contribute to the local instruction theory.

H. The Present Study and The Outline of Hypothetical Learning Trajectory

The present study aims to design a sequence of lessons to support the development of students' spatial ability in understanding 3D representations. We design these activities for young learners, 3rd grade elementary students. According to Clement (2003), 8-9 years old children begin to develop spatial orientation and spatial visualization. Therefore, the design will be implemented for 3rd grade students to assist their early development on spatial ability. Based on the previous studies on spatial ability, some recommend the use of physical objects such as building blocks, photos, puzzles and etc. The use of physical objects are not only the best media to develop students' spatial ability but also suitable for students' learning style. As we already know, most of elementary students tend to be visual and kinesthetic learners. Therefore, physical objects will be integrated in the design to engage students during the activities.

Based on the study of Pittalis and Christou (2010), to support students' understanding in 3D representations, the design shall focus on developing students' spatial orientation and spatial visualization. Therefore, the designed

activities consist of spatial orientation task and spatial visualization task combined with the use of physical objects as the media for students' investigations. In addition, the investigations are also based on kinesthetic and visual learning style. The following Table 2.2 summarizes the recommended activities to support the students based on the previous researches. The table also becomes the outline of the hypothetical learning trajectory.

Task	Activity	Hypothesized Results
	Observing physical	- Students will develop prior
	objects	knowledge of spatial orientation
		and spatial visualization of 3D
		objects.
		- Students also will be used to work
Spatial orientation		with 3D objects.
task	Finding the position	- Students will develop their spatial
	of given standard	orientation by finding the position
	views of 3D objects	of standard views.
		- Students have prior knowledge of
		3D objects and its distant
		representations (standard views).
	Drawing standard	- Students will develop their spatial
	views of 3D objects	visualization by drawing the
Spatial		standard views of 3D objects.
visualization task		- Students can understand distant
		representations of 3D objects
		(Pictures).
	Building 3D objects	- Using their spatial ability, students
Spatial orientation	based on its standard	can build 3D objects based on its
task combined	views	standard views.
with spatial		- Students understand distant
visualization task		representations (standard views) of
~		3D objects (Pictures).
Spatial orientation	Determining	- Using their spatial ability, students
task combined	standard views of 3D	can discover the properties of
with spatial	objects based on its	standard views of 3D objects.
visualization task	standard views.	

Table 2.2 The outline of hypothetical learning trajectory

The lessons will be developed based on the characteristics of Realistic Mathematics Education (RME). The general aim of this study is to develop the activities and to contribute to the local instruction theory of how to support students' spatial ability in understanding 3D representations. Therefore, the study is guided by the research question: *how can spatial orientation task and spatial visualization task support the development of students' spatial ability in understanding 3D representation?*

CHAPTER III

RESEARCH METHOD

A. Research Approach and Timeline of the Study

This study uses design research as the research approach to both design the activities and develop the instructional theory. In design research, there are three phases to be done. The description of activities in each phase is discussed below.

1. Preparation and designing

This phase begins with studying literature and formulating general research question. We also define general aims of the study which is to contribute to the local instruction theory of how to support the development of students' spatial ability in understanding 3D representations. This is continued by again studying literature and relevant researches. The results become the anchor to design the activities. The outline of general hypothetical learning trajectory is also defined as a guidance in the designing process. This step will be ended with the elaboration of the general hypothetical learning trajectory based on the result of observations, and interviews with the teacher and the subject of the study.

Since the present study uses PMRI as the approach in the learning activities, we also conduct didactical phenomenological analysis. It aims to find phenomena or events in Indonesia that relates to the topic. These phenomena are then integrated to the lesson as the context so that the problems and the activities will be sufficiently real for the students. Based on the result, we further design the sequence of learning activities to support the development of students' spatial ability in understanding 3D representations for subject' particular region.

As explained in the first paragraph, we elaborate the *Hypothetical Learning Trajectory* (HLT) of the activities in detail based on the result of the observations and interviews. This HLT consists of learning goals, learning activities, and hypothetical learning process or conjectures of how students thinking and understanding. In addition, we added proposed teachers' actions to anticipate students' conjectures during the learning process. In this phase, we produce the first version of elaborated HLT. It will be tested and implemented in the classroom. This process is called preliminary teaching experiment and teaching experiment that will be discussed in the next phase.

2. Teaching experiment

In this phase, there are two teaching experiments which are preliminary teaching experiment (pre TE) and teaching experiment (TE). In the pre TE, the sequence of lessons and the elaborated HLT as the product of the designing process, are implemented in a small group of students, around 6 students. In addition, the researcher will be the teacher and at the same time also becomes an observer of the learning process. The purpose of this pre TE is to test the activities and the elaborated HLT so that they can be improved before the teaching experiment starts. During the preliminary teaching experiment, the researcher helped by one other observer will collect data such as classroom observations, students' written works, and semi-structured interviews. Those data will be analyzed qualitatively to improve or revise the elaborated HLT and the activities. Consequently, the new HLT is called the improved HLT.

In the teaching experiment, the improved HLT and the improved activities from the preliminary teaching experiment will be tested and implemented. Different from the pre TE, the teaching experiment involves all the students in the regular class, in this case 24 students of grade 3 in SD Laboratorium UNESA Surabaya. The experiment also employs the regular teacher in the school. During the activities, the researcher will only be an observer and collect data like classroom observations, students' written works, small interviews with the students and interviews with the teacher after the lesson. Classroom observations will be registered as videos and field notes. After each lessons or during the lessons, researcher may reflect on the design and the HLT. It is possible that the HLT will be adjusted a little bit based on the result of the previous lesson, for instance, because of the teacher runs out time. Therefore, an analysis of the lesson is also conducted both at the end of the sequence and also at the end of each lesson. The idea is to have an insight of what students have learned and whether the goals are reached. In this phase, we also do member checking with the teacher. In the end of the teaching experiment, a posttest is done to support all the collected data and as the complementary data for the analysis. It will be analyzed qualitatively to support the findings during the sequence.

3. Retrospective analysis.

After conducting the teaching experiment and collecting the data, we move to the next phase which is a retrospective analysis. The analysis involves all the process from the designing process until the implementation of the design. Importantly, retrospective analysis also investigates how the instructional materials support students' learning and their progressive development. The improved HLT becomes a guidance in doing the retrospective analysis. The actual learning trajectory (ALT) that happened in the teaching experiment will be compared to the HLT. The result of the analysis can be a guidance to revise and improve the activities and become the answers to the research question.

The following table 3.1 shows the schedule from the designing phase until the teaching experiment and the retrospective analysis.

No.	Descriptions of activity	Date		
Preliminary Design				
1	Studying literature and designing the 1 st	15 August 2014 – 10		
	version HLT	January 2015		
2	Observation	17 February 2015		
3	Interview with the regular teacher	17 February 2015		
4	Pretest for pre TE	18 February 2015		
Preliminary Teaching Experiment (pre TE)				
5	Lesson 1 Playing with the camera 1	20 February 2015		

 Table 3.1 Timeline of the research.

No.	Descriptions of activity	Date
6	Lesson 2 Playing with the camera 2	23 February 2015
7	Lesson 3 Reporting new temples	24 February 2015
8	Lesson 4 Building temples	26 February 2015
9	Lesson 5 Making gift boxes	2 March 2015
10	Posttest after pre TE	3 March 2015
11	Analyzing the data from preliminary	4 – 25 March 2015
	teaching experiment	
11	Pretest for TE	30 March 2015
Teaching Experiment (TE)		
12	Discussion with teacher regarding the	1 March – 7 April 2015
	lessons	
13	Preparation for teaching experiment	25 – 30 March 2015
14	Lesson 1 Playing with the camera 1	31 March 2015
15	Lesson 2 Playing with the camera 2	1 April 2015
16	Lesson 3 Reporting new temples	2 April 2015
17	Lesson 4 Building temples	6 April 2015
18	Lesson 5 Making gift boxes	7 April 2015
19	Posttest after TE	7 April 2015
20	Analyzing the data from teaching experiment	7 April – 20 Mei 2015
21	Publishing the result of the pre TE in SEA	18 – 19 April 2015
	DR 2015 in Palembang	

B. Subject of the Study

The subject of the present study are 30 students of grade 3 and a regular teacher of SD Laboratorium UNESA in Surabaya. The students are around 8-9 years old with composition of 12 females and 12 males. SD Laboratorium UNESA is a general elementary school located in the central city of Surabaya. Each class consists of 25-30 students which are vary in social class. As we have explained before, the teaching experiment of this study consists of two phase. The first one is a preliminary teaching experiment which involves only six students of grade 3B of the school. In this pre TE, the activities and the HLT are tested to check the learning line and the conjecture of students' thinking in the HLT. The result will be used to improve and revise the HLT. The second phase is the teaching experiment which involves 24 students of grade 3C and a regular teacher of the class. During the teaching experiment, the students will be grouped into small groups of 4-5 students.

C. Data Collection

This study focuses on the qualitative aspects of the teaching experiments. Therefore, most of the data collected in the teaching experiment is qualitative data. Moreover, we use different kind of data to assist the analysis of the teaching experiments. There are four types of data in this study: 1. Classroom observations, 2. Students' written works, 3. Interviews 4. Pretest and posttest. These data are collected in the first two phases: preparation phase, preliminary teaching experiment, and teaching experiment.

1. Preparation phase

In this phase, we conduct a classroom observation, interviews with the teacher and the students, and a pretest. The observation is also to gather data of students' experiences and facility in the school. Furthermore, it also aims to know the culture of the class in the regular mathematics teaching and learning process. This teaching and learning culture is called social norms

and socio-mathematical norm (Yackel & Cobb, 1996). To do this, some parts of the regular lesson will be videotaped and combined with the field notes from the observer. The topic for the observation can be seen in Appendix 1. To further confirm how the students usually learn mathematics, we also did an interview with the teacher. This interview is to identify students' experiences, prior knowledge and struggles regarding the topic. In addition, during the interview, we also examine teacher beliefs and her/his educational background to determine what kind of supports the teacher have provided and need to be conduct the teaching experiment. The list of topics for the interview with the teacher can be seen in Appendix 1.

Lastly, in this phase, we conduct a pretest for the students. The aim is to have an insight of their prior knowledge regarding the topic and how far their spatial ability on understanding 3D representation. The pretest consists of 3 problems with different goals and purposes. The first problem is about finding the position of objects in the pictures with respect to the position of the other object. The goal is to investigate students' sense of direction. The second one is about determining the photo of the objects if the position of the camera is given. The purpose is to examine students' awareness of spatial orientation. The last problem is about determining the shape of a building blocks if they are viewed from the side. This problem is to further investigate students' ability to recognize the shape of an objects based on its distant representation. The pretest form can be found in Appendix 1 of this document. 2. Preliminary teaching experiment

As mentioned in the previous section, a preliminary teaching experiment only employed six students of class 3B. The researcher will the teacher and the observer. In addition, one more observer is employed to help the researcher documenting the process. The aim is to test the elaborated HLT as part of the effort to improve the HLT before the teaching experiment. During this phase, we did classroom observations by making videotapes and field notes during the lessons. In addition, students' written works are also collected to be examined. Students' written works is collected from student worksheet for each lesson. The worksheet can be found in Appendix 1 of this document. The result will be used to improve the HLT and the activities before the activities are conducted in the teaching experiment.

3. Teaching experiment

Teaching experiment involves 24 3rd-grader students and a regular teacher of the class. Students worked in small groups of 4-5 students. In this teaching experiment, the researcher in this occasion will be the observer to see the activity of the students in the focus group. Another observer is also employed to record the activity of the other groups during the teaching experiment. All the activities of the focus group are videotaped and analyzed as part of the classroom observation. All the students' written works are also gathered to be examined. In addition, the observer also has

small interviews with the students in between the observation to clarify what the students did.

Finally, after all the lessons are done, we conduct a posttest as a complementary to all the data we have gathered. This posttest will be held in the same day after the teaching experiment ended. The content of the posttest is similar to the pretest but the questions are increased. In addition, the first problem is not included since the goal of the first question is to investigate students' prior knowledge. As the replacement for this first problem, we add one more problem about building an object based on given standard views. The posttest form can be found in Appendix 1.

4. Validity and reliability of data collection

To improve the validity of the data collection, all the teaching material such as problems, worksheets, and the sequence, are consulted to some experts and have been used in another similar studies (Revine et al., 2010; Risma et al., 2013). Before conducting the teaching experiment, there is also preliminary teaching experiment to check and improve the HLT and the activities. This study also employs many types of data to confirm and support the findings. The use of many kinds of data contributes to the validity of the data collection. There are three types of data: 1) Classroom observations, 2) Students' written works, and 3) Interviews. The classroom observations and the interviews are registered by videotapes and audiotapes to increase the reliability of the data. This is because compared to the field notes, videotaping and audiotaping illustrates the original learning

activities. Field notes sometimes can carry observer tendency that lead to miss interpretation of the situation.

D. Data Analysis

All the data collected during the phases will be analyzed and examined qualitatively. The data analysis is categorized into 4 phases based on the purpose of the data. They are pretest and preparation phase, preliminary teaching experiment, teaching experiment, and posttest.

1. Pretest and data on the preparation phase

Pretest is conducted in the preparation phase after one day before teaching experiment starts. Qualitatively, the result of the test is partly examined to check and give insight of students' prior knowledge. The analysis of the result of the pretest refer to the goal of each question. The description of each question in the pretest can be seen in Appendix 1 of this document. Similarly, the data from the classroom observation and the interview with the teacher will be analyzed qualitatively to have pictures of social norm and sociomathematical norm in the class. Social norm is very important since there is possibility that the social norm in the class is far from the desired social norm in the design. In this case, the observation result can be a guidance how to create the desired social norm during the teaching experiment. The result of the analysis also will be used as a guideline to improve and adapt activities. The result of the analysis also becomes guidance to anticipate what strategy or answer that students may come up during the teaching experiment.

2. Preliminary teaching experiment

There are two types of data in this teaching experiment: classroom observations and students' written works. Classroom observation, the video and field notes, will be watched as a whole to find the moment where students have interesting discussion, work, or idea related to the problems. Interesting discussion means a moment when the students discuss the problems like the HLT have predicted. The fragments have a discussion related to big ideas of the activity. These fragments will be transcribed and analyzed to improve the HLT. Students' written works also will be examined to enrich the conjectures of the HLT. To further check the HLT, the actual learning and the conjecture will be paired in the form of a matrix/table. In the matrix, if the actual learning meet the HLT, then we will put sign "+" in the table. If the actual learning only match half of the HLT or just some part of the HLT, then we put sign "±" in the matrix. The overview of matrix will be used to improve both the activities and the HLT.

3. Teaching experiment

In the teaching experiment, we conducted a bigger teaching and learning activity with 24 students. The collected data are classroom observations, students' written works and small interview with the students. Classroom observations will be in the form of videotaping and field notes. The videotaping also will be more focus on the focus group. Similar to the videotaping in the preliminary experiment, the video in this cycle also will be watched as a whole and will be fragmented based on the important findings of the students during the discussion. To select these fragments, students' discussions will be overviewed and matched with the HLT. The moments where the students have similar discussion like in the HLT will be selected. Furthermore, if students had another answers or strategies which is not in the HLT will also be selected to be examined. These fragments are transcribed to be embedded in the analysis. The field notes during the lessons also will be collected and deeply studied to support the analysis and the findings of the students.

The HLT becomes guidance to do the analysis. Besides, the HLT is used to select the fragments, it is also used to determine students learning during the discussion. HLT helps us indicating how far students' learning and how the teacher responds to the students. For this reason, the actual learning and the HLT are compared to each other in the form of matrix or table. If the actual learning meet the HLT, then we will put sign "+" in the matrix. If the actual learning only match half of the HLT or just some part of the HLT, then we put sign "±" in the matrix. From this matrix, how students' learning can be clearly explained and compared to the HLT.

Small interviews are done during the teaching experiment. The aim is to check and confirm students' activities and understanding. These data can be a support for the other collected data during the analysis. Finally, students' written works are used to support the findings and to check students thinking in the video. It also served as the evidence of the students learning and thinking. Overall, the result of the analysis will be used as a guideline to derive conclusion and answer the research questions. In addition, this analysis will be used to determine the implication of the study, suggestion for future research, and to revise the HLT.

4. Posttest

The posttest is conducted after the sequence of lessons is end, but it is in different days or right after the last lesson finish. The result is analyzed in qualitative way. The data will not be analyzed statistically or quantitatively. Qualitative way means the posttest will be analyzed partly and thoroughly according to the need to support the findings during the teaching experiment. The posttest also can be the evidence of the result of students' learning. Each question in the posttest will analyzed separately according to the goal of each question. The description of the goal of each question can be seen in the description of posttest in Appendix 1.

5. Validity and reliability of data analysis

In this analysis, we employs different kind of data such as classroom observation, interview, and students' works. The use of various data aims to improve the internal validity of the data analysis. To further support the validity of data analysis, we also did member checking with the teacher after each lesson to avoid main problems that may arise. In this study, we employed the regular teacher of the class in the school to expand the ecological validity of this study. The composition of the classroom also is not changed. During the analysis, the researcher also did inter-subjectivity analysis with the expert and the supervisors of this study to support the trackability of the analysis.

CHAPTER IV

RESULT AND DISCUSSION

A. Preliminary Teaching Experiment

1. Preparation and Designing Phase

In this phase we conduct a classroom observation, an interview with the regular teacher, a didactical phenomenological analysis, and the elaboration of the general HLT. The aim is to get more information and additional data such as students' experiences, social norms, socio-mathematical norms, and students' prior knowledge. The result becomes a reference to design the activities and also to elaborate the general hypothetical learning trajectory. The details of the result of each activity in this phase are described below:

a. Observation

Classroom observation was conducted in February 17, 2015. The class had 14 males and 12 females. In total, there were 26 students from various backgrounds and economy classes. The teacher started the lesson by reexplaining the topic in the previous meeting: *Comparing fraction*. At first, the teacher gave problems written in whiteboard and asked the class to find the answer. If there were two or more answers, the teacher discussed them by drawing figures that represent the value of the fraction such as pie, chocolate bar and etc. These drawings were used to help the students finding the right answer. Afterwards, the teacher wrote 5 questions of proofing that the fractions were equal, and instructed the students to solve it individually in their book. During the process, the teacher only sat on his table but if the students had difficulties, they bring their work to the teacher. The teacher then helped them one by one until the end of the lesson. The lesson took approximately 3x35 minutes but in the last 35 minutes, the teacher left the class since he had to accompany his daughter to the hospital.

During the lesson, the teacher actively involved students' opinions or answers. For example, to explain why 1/2 is more than 1/3, teacher asked students' opinion before explaining the proof. Thus, the teacher did not merely explain the proof while the students were just listening. But, they have good two-way communication between the teacher and the students. However, the lesson was lack of students' discussion since they worked individually and there were no group discussion or class discussion. During the exercise, only some of the students came to the front to consult their work to the teacher. Mostly, they wanted to confirm whether their answer is right or wrong. After the teacher confirmed that their answer was right, they closed their book and wait for the class to end.

Regarding the social norm during the lesson, the classroom was quite crowded especially during exercises. The students who had finished the work began to play and yelled each other. The teacher sometimes warned the students who do not pay attention by asking questions or directly telling them to be quiet. The teacher always asked students' opinion or answer before he presented the solution. If students had question, they directly went to the front of the class while yelled that they have questions. Furthermore, students sat in pairs forming rows and columns (Figure 4.1). However, although they were in pairs, they often worked individually and even complained to the teacher if their partner looked their answer. Students rarely work in small groups and this is confirmed by the teacher during the interview. Class discussion after the exercise was also rarely conducted. After the students solve the problems, they always asked teacher to confirm whether their solution is right or wrong.



Figure 4.1 Mathematics teaching and learning in Class 3C

Socio-mathematical norms can be easily observed during the lesson. For each problems, the teacher asked the students for different strategy to prove the fraction. There were 4 different strategies used during the lesson: number line, bar, pie, and multiplication but the teacher only suggested the students to use either the bar or the number line. The students then had to decide for themselves which one they like to used.

In conclusion, the students in this class do not yet familiar with discussion, especially in small groups since they rarely do it during the regular teaching and learning activities. In addition, the class is crowded when the students have finished their work in the class. This is also due to teacher method of having private consultation in front of the class. The students usually ask the teacher's confirmation of their answers whether they are right or wrong. But,
the students have familiar with employing many strategies to solve a problem. The teacher also actively guide the students and try to engage all the students during the lesson.

b. Teacher Interview

We interviewed the teacher after conducting the classroom observation. The interview was to study further the background of the teacher and how the teacher usually works with the students. The regular teacher of the class was Mr. Daluri, S.Pd. He had been teaching for approximately 15 years. Originally, he was a mathematics teacher and became an elementary teacher who not only taught mathematics but also the other subjects such as bahasa Indonesia, sains and etc. He also participated in some workshops and projects of PMRI in UNESA. However, he did not fully agree with the philosophy of RME in mathematics teaching and learning. For instance, he did not like open questions because in his opinion, too many possible answers can make students confuse. Even so, he admitted the importance of teaching mathematics that relates to students' life. In his teaching, the teacher also sometimes use realistic strategies to explain the solution.

In teaching mathematics, Daluri explained that he likes to have short stories to motivate the students. However, mostly the stories are rarely related to the students' activity. Afterwards, he explained the concept that he want to teach while demonstrating the strategy to understand the concept or solve the problems related to the concept. During his explanation or demonstration, he often involves students by asking students' opinion. In the end, he gives students problems and asks them to solve that problems. Students. Nevertheless, the teacher do not like to have discussion for the whole class because it takes too much time. He prefer to have a private discussion with each of the students during the exercises. Mostly, private discussions are happened if and only if: 1) students have difficulties to solve the problems, or 2) students wants to clarify their answers (correct or wrong answers).

Regarding the social norm in the class, if students want to ask question or show opinion, the teacher instructs the students directly come to him. However, he further explained that the rule are changing over time depending on the situation. If during the learning activity, there are students who do not pay attention or disturb the learning process, the teacher will directly warn them and if necessary, he will get angry. This is because students often take lightly teacher's warning and continue to not participate in the class. Regarding the classroom management, Daluri likes to have a row-column formation but he said it can be changed depending on the types of learning activity. According to him, 1 up to 10 students actively participate in the class while 11 up to 20 students are moderately active during the classroom activity. Meanwhile, 1 up to 5 students passively participate in the teaching and learning mathematics.

Theoretically, Daluri has prepared the learning activity for the students in the document of "Rencana Pelaksanaan Pembelajaran" (RPP) or lesson plans. However, he stated that he is not typical teacher who follow strictly the lesson plan. He said that during the learning, there are many possibilities to change the plan depending on the situation. He also added that personally, he is open to new things, innovations in the teaching and learning especially teaching mathematics.

In conclusion, the regular teacher had a good background of mathematics education and had been teaching mathematics for many years. He also experienced and learned RME on some PMRI projects (Pendidikan Matematika Realistik Indonesia). He was quite open with new things. However, he did not prefer to use lesson plan to organize his teachings. In addition, he also was not used to have small discussions among students and students' presentations.

c. Didactical Phenomenological Analysis

The process of didactical phenomenological analysis have been conducted since the literature review was done. However, after conducting the observation and interview with the teacher, we get some additional data such as the culture in the region, students' experiences and etc. In designing the activities, we also have to select and choose the context such that the problem will be sufficiently real for the students. Therefore, to find a good context for the activities we involve the data of students' experiences and culture in the region.

Based on our interview with the teacher, students in the SD Laboratorium UNESA are familiar with the use of camera digital such as smart phone or digital camera. When confirmed by the researcher, students admitted that they were able to use the camera digital or smart phone. In addition, students also are able to bring a camera digital to school with the permission of their parent and teachers. Therefore, we plan to utilize students' ability of using the camera in the activities. As we know, spatial orientation and spatial visualization task involves the use of many pictures of 3D objects. The camera digital can be used to gather pictures from many angles or to observe the objects in the activities. Therefore, we decide that the context of the activities is about photographers who like to capture objects by using their camera. The use of camera also can be a bridge for students' understanding from working with the physical objects to working with its pictures (representations). Furthermore, this context supports students' learning style since photography activity enables them to directly see, observe, touch, and experience the objects to develop their spatial ability.

The photography activity will be combined with spatial orientation task and visualization task to as grounded activity for students to develop their spatial ability. For building objects activity, we consider the use of building blocks under the context of temple in Java. The context of temple is chosen since the students are quite familiar with this kind building. In Java, people in the past built the temple by using blocks of stones arranged like a pyramid or a gate. Therefore, building 3D objects will be meaningful for the students in the context of temple. In the activity, students are allowed to build their own temple so that they will be more engaged and motivated to be creative. The context of photography and temple is also combined into drawing activity. Hence, by using the camera, students have to also draw the views of the temple in the camera into a layout to be reported to the government.

In conclusion, photography activity is chosen as the context of the spatial orientation and spatial visualization task because it allows students to work with objects and pictures. They have been familiar with the use of camera in daily life. The context of temple is also selected as it allows students to explore more building blocks activities in a meaningful way.

d. Elaboration of the Hypothetical Learning Trajectory

This study aims to contribute to the local instruction theory of spatial ability. Therefore, we design a sequence of activities to support the development of students' spatial ability. In this section, we elaborate the Hypothetical Learning Trajectory (HLT) of each lesson that will be used in this study. This elaborated HLT consists of mathematical goals, mathematical activities, and conjectures of students' thinking. In addition, teacher's actions are proposed for particular conjectures of students may do during the learning process. The elaborated HLT describes the flow of the students learning and becomes guidance during the teaching experiment. The elaboration of the HLT is based on experiences, students' prior knowledge, and students' learning styles. It is also developed based on the results of class observations, didactical phenomenological analysis, and interview with the teacher.

The sequence consists of 5 lessons of 2x35 minutes in approximately 1 week to reach the goal. These five lessons can be seen in the table 4.1. The

lessons require a teacher who has 5 roles described in the theoretical framework. These teacher's roles are important to create an active learning atmosphere for the students. Therefore, to assist the teacher, we also design a teacher guide. This teacher guide consists of lesson plan and teaching notes for each lesson. Lesson plan explains the flow of the activities and what teachers have to do in each lesson. It includes technical matters during the teaching process such as learning material/media, required tools, and also time estimation for the activities. Furthermore, lesson plan gives briefly overviews of the goals and mathematical concept/skills in the activities. Teaching notes for teachers. It has an overview of students' activities and some suggestions for teachers. It has an overview of students' activities and examples of students' answer. Teaching notes are elaborated together with the student worksheets. All these documents can be found in Appendix 2.

At the beginning, the students already have prior knowledge and abilities on some aspects of spatial ability. First of all, students have sense of direction. For instance, students can distinguish where their right and their left is. Students also know the difference between 3D objects and 2D shapes. For example, they are able to show some examples of 3D objects such as box, drum, and etc. or objects that has particular 2D shapes on its sides such as the surface of paper, book, pizza and etc. It is not necessary for them to know the formal terms "3D" and "2D" as long as they can distinguish the shape of those objects. In grade 2, they already learn about 2D shapes "Bangun datar" such as rectangle, triangle, square, and circle. Thus, they also know the names for these 2D shapes.

After participating the sequence, we expect the students to have better spatial ability in understanding 3D representations. They are able to identify, determine and draw standard views of 3D objects based on their models or their distant representations. For instance, students can determine the shape of a house in a photograph if it is viewed from the side or from the back. They are also able to interpret, recognize, manipulate and communicate the representations of 3D objects. For example, students can distinguish the photos of two different objects. They can identify, predict and read the blind spots of the distant representations of 3D objects in the photograph based on the given information. They also can build 3D objects based on its photographs or its standard views.

In the aforementioned paragraph, the elaboration of HLT consists of three main parts in which one of them is conjectures of students' thinking. This part originally consists of possible students' thinking as a response to the mathematical activities. However, the thinking process is a cognitive activity that only can be observed by studying students' behavior during the learning. In addition, spatial ability in this study is indicated by students' actions when dealing with spatial task. Therefore, the conjectures of students' thinking in the elaborated HLT later will be more students' actions (behavior) than their thinking to reflect their thinking processes registered by their answers, actions, and strategies. The details of these parts in each lesson are discussed below.

Meetings	Learning Goals	Spatial ability task	3D representation	Students' Activities
Meeting 1 Playing with Camera 1	 Students experience spatial orientation and spatial visualization of an object. Students understand that 3D objects have different views. 	 Preliminary spatial orientation task and visualization task. Developing students awareness of spatial orientation and 	1. Preliminary activities to work with models and representations.	 Teachers give the students an arrangement of 4 objects (Apartment, two Houses, and Tower) and asks students to determine where the students shall put the camera to have a picture of all the object, two objects, and one object. Students also have to find the best position to take photos of the objects. Students will work with two camera models and a digital camera.
Meeting 2 Playing with the camera 2	 Students are able to identify the stand point of different standard views of an objects. Students are able to distinguish between standard views of an object and standard views of another similar objects. 	 Spatial orientation task. Developing students' spatial orientation through identifying photographs. 	 Developing students' ability to recognizing the position of different views of an object with the help of the models. Bridging students' ability to work with distant representation of 3D objects. 	 The students analyze and identify the photos of 3D objects. They have to put them in the right position of where it was taken. There are 12 photos from 8 positions. However, among these photos there are 4 photos that do not belong to the object. Students will move around the objects to find the right position of the camera in the given pictures. Students will work with two camera models and a digital camera.
Meeting 3 Reporting new temples	 Students are able to draw standard views of 3D object from different stand points based on its model. Students are able to draw standard views of an object based on its distant representations. 	 Preliminary spatial visualization task. Spatial visualization task Developing students' spatial visualization through perspective task. 	 Developing students' ability to determine and draw different views of 3D objects from different stand points based on either its model or its representation. 	 Students create their own temple by using exactly 8 cubes and then sketch or draw the standards views of the temple by using the camera models and a digital camera. Students also have to draw the standard views of a new temple in a bird eye photo. They draw the view in the layout. Students also are allowed to use building blocks to help them thinking.
Meeting 4 Fixing the reports	 Students are able to identify the properties of standard views of 3D objects. Students are able to draw the standard views of an object based on its opposite standard view. 	 Spatial visualization task combined with spatial orientation task. Developing students' spatial ability to support their understanding of 3D representation 	 Investigating basic relations among standard views of 3D objects. Recognizing the views of an object from different stand points based on photograph and another standard views. 	 Teachers give 5 photos (front, sides, and top) which belong to an object of building blocks. Student have to guess how these photos are originally arranged in the layout. Teachers will not provide them the object and camera. Teachers give three cases of different 3D objects. Each case consists of 3 standard views. Students have to predict and draw the shape of the object on the other stand points.
Meeting 5 Building a temple	 Students are able to identify, predict, and read the blind spot of distant representations of 3D objects. Students are able to interpret, recognize, and communicate distant representations of 3D objects. Students are able to construct an object based on its standard views. 	 Spatial visualization task combined with spatial orientation task. Developing students' spatial ability to support their understanding of 3D representation and spatial structuring 	 Developing students' ability to read, interpret and reason distant representation of 3D objects. Building an object based on its photograph and its standard views. 	 Given photos of a new temple (bird eye photos). Students have to build the temple by using cubes/building blocks. Afterwards, teachers give three standard views of the temple (top, back, side) to equip the bird eye photo. Students also have to build a temple based only on its standard views in a template/layout without its bird eye photo by using building blocks.

Table 4.1 Learning line for the preliminary teaching experiment

1) Lesson 1: Playing with Camera 1

Learning goals:

- a) Students experience and investigate the orientation and visualization of an object.
- b) Students understand that 3D objects have different views from different stand points.

Mathematical activity and conjecture of students' thinking:

In this first lesson, the student will act as a photographer. Before, students are divided into small groups of 3-4 students. The context is that the student as a photographer have to find a position for their camera in the layout to have a good picture of the buildings in a city. There are three problems in total for the first activity in this lesson: 1) investigating what building they will see in the photo if they capture the buildings from different positions, 2) Determining the position of a building with respect to the other building in the photo taken from different positions, and 3) finding positions to capture either all the buildings, two buildings, or only one building and also choose the best position to take photos.

For this first activity, teachers provide each group a worksheet, two camera models, a digital camera and a miniature of the city. The worksheet can be found in the appendix 2 and the teacher guide for this lesson is in appendix 3. The following Figure 4.2 shows the camera models to help students' investigation and the arrangement of the objects.





Camera ModelMiniature of the cityFigure 4.2 A set of the city and camera model for lesson 1

Conjectures of students' answers, actions and strategies for the first problem of investigating what building students will see in the photos from different positions:

- a) Some students may prefer to use the camera digital and move around the object since they can directly see the photo. But some may argue that the camera model is more advantageous since it is simpler to use and they can see the building through it. In this case, teachers leave the students choose the camera for themselves. Probably, teachers can ask why they prefer to use the camera.
- b) Students probably still confuse how to use the camera model. Therefore, teachers can give a demonstration to the students to give an example how to use it and how to fill the table in the worksheet.
- c) Students may take the camera out of position into bird eye angle. They think that the views of the object from bird aye angle will be the same as the views from the ground. In this case, teachers argue to help students that the photographer must stay on the ground. Thus they cannot take the camera from the layout and ask the students to make comparison so that they realized that the views are different.

- d) Students have no difficulties at all and can determine what building they will see from different position. In this case, teachers can ask further question to confirm students' strategy like "how can you figure that out? Tell me!", "now, if you do not have the camera, can you imagine what buildings you will see from this position?"
- e) Students may do not need to use camera to determine what buildings they will see. In this case, teachers ask to confirm their strategy and why they did not use the camera.
- f) Students may argue that they can still see a little part of the buildings and confuse whether it count as visible or not. In this case, teachers argue that it is okay to just see parts of the building as long as they can determine what kind of building that is, like a house, a tower, or an apartment. But, if they cannot determine the building, then it is counted as unseen.

After attending this activity, students will think that the objects have different views from different positions. The views of the objects from bird eye angle are different from the views of the objects from the ground. Thus, they slowly realize that to obtain the images of the objects, they need to either directly looking the objects from the position on the ground or looking the objects from bird eye angle and then imagine the building they will see.

In the second problem, students have to determine the position of the certain building with respect to the other buildings. Figure 4.3 is the questions for this problem in the student worksheet. The worksheet can be found in the appendix of this document.

- 2. If you take a photo from position 3, where is the position of the red house and the tower with respect to the apartment?
- 3. If you take a picture from position 7, where is the position of the red house and the tower with respect to the apartment?
- 4. If you take a photo from position 1, where is the position of the red house and the tower with respect to the apartment?
- 5. If you take a picture from position 5, where is the position of the red house and the tower with respect to the apartment?

Figure 4.3 Second problem of the activity in lesson 1

Conjectures of students' answers, actions, and strategies:

- a) Students mostly will use the terms "right or left" and also "front or back".
 But it is also possible that they will combine both such as "it is in the front-left side of the apartment"
- b) Students probably will be confuse whether the left and the right is with respect to the building or to themselves. For instance, a photo from position 3 shows that the red house is on the left side of the apartment. But some students may say the house is on the right side because their right arm is on that side too. In this case, teachers have to asks the students to confirm their orientation like "the house is on the left side of what?" or teachers can asks two students to stand and face each other and have to show their right hand on the same time. This way enables teachers to explain that the direction is relative with respect to something.
- c) Students maybe also use cardinal directions such as north, south, and etc.to determine the location. This is okay as long as it is true. However, if the layout is moved then the cardinal direction will change.

After this activity, students will think that the position of building are changing if they look from different positions as the views are also changed. The last problem is to determine in which location students can capture all the buildings, three buildings, or two buildings. Afterwards, they have to decide which best position to take a picture.

Conjectures of students' answers, actions, and strategies:

- a) Students easily figure out the position of the camera by moving around to see the objects from the camera.
- b) Students may have difficulties to find the position where they only see two objects or one object. In this case, teachers suggest the students to use the camera carefully by closing the other eye. It is also possible to demonstrate how to use the camera to the students. Afterwards, teachers ask them what they see from the camera and how many objects they can see.
- c) In case of finding the position to capture 1 or 2 objects, students may argue that they can still see a little part of the objects. In this case, teachers argue that they have to see the whole objects not just a little part of it.

After all groups completing the worksheet, teachers ask one group to explain their findings in front of the class. Teachers start and led the discussion by posing the following questions:

- a) Have you completed question number one?
- b) If you take pictures from position 1, what buildings will you see in the photos?
- c) What about position 2?

d) And etc.

During the discussion, teachers also give chances to react to the group's finding or to express their opinion or questions. For questions 2 until 5, teachers invite another group to explain their answer. Similarly, teachers lead the discussion by asking the group questions in the worksheet. For the last questions, teachers ask another group again to explain their answer. Again, teachers ask students' answers for the rest of questions in the worksheet. After the presentation, teachers close the lesson by guiding the students to make a conclusion of what they have done.

During the process of the investigation in this lesson, we conjecture students will have insight of the orientation of an object. In the first activity, students will gain the idea of stand points. The appearances of the object from many stand point are different. They began to be able to imagine the views of the objects in their mind.

2) Lesson 2: Playing with Camera 2

Learning goals:

- a) Students are able to identify the stand point of different standard views of an object.
- b) Students are able to distinguish between the standard views of an object and the standard views of another similar objects.

Mathematical activity and conjecture of students' thinking:

The activity begins with a story of a cameraman who took pictures of the stuff on the table (building blocks). He made some shoots and printed the

photos together with all the photos from different session. However, when he tried to select the photos, he confused to distinguish the photos of the object and the position of where they were taken. Therefore, students have to help him identify which photos belong to the objects and the position of the camera when the photos were taken.

In this session, students work in small groups of 3-4 students (the same group as the previous lesson). Each group is given a set of building blocks on the table, the photos (12 photos) and two camera models. They can use the cameras to help them identify the photographs. During the investigation, students have to answer questions in the worksheet. The worksheet is available in the appendix. Firstly, students have to cut the photos. Afterwards they have to investigate to which photos are of the object and at the same time finding the position of the camera when the photos were taken. Students have to be careful since there are 4 photos that are not of the objects. These photos were from a different photo session. Students have to identify these photos and make sure they do not put them in the worksheet. Figure 4.4 shows the photos and the layout which are attached in Appendix 1 of this document.



Figure 4.4 Photos and layouts for the activity in lesson 2

In this activity, teachers instruct the students to only use the camera models. The digital camera will be used in the discussion.

Conjecture of students' answers, actions, and strategies:

- a) Students may put all the picture into its position correctly without much difficulties. They find the position by moving around the object and matching the view they see in the camera model and in the photo. If they see a match view, they put the photo to the position. In this case, teachers may ask further questions to confirm student's strategy and understanding like "how did you figure it out?" "Why was this photo put in this position?" The purpose is to gather more information about students' strategy. Teachers also have to dig up more information of how the students choose the right photos and why the other 4 photos are not taken from the object.
- b) Students may unaware of the camera and put the photos orderly based on the number on it. In this case, teachers choose one of the students' answer and show it that the photo is different from the real picture in the camera model. Afterwards, teachers may demonstrate how to get the position of that photo to its right position. This situation can happen if the students do not know how to use the camera or do not understand the problem or just unaware of the problem.
- c) Students may falsely put the picture on its opposite number. For instance, the photo from the camera 1 is put in the camera 5. In this case, teachers evoke the students to use the camera and check whether the picture in the

photos and real camera are the same. Teachers then can asked the students to find the position of an object with respect to the other object in the photo. Afterwards, students have to compare the position of the object in the photo and in the camera. By doing this, students may realize the difference. This is different from the previous case since the students may know how to use the camera but they falsely put the photos.

d) Students can wrongly choose pictures of the object. For instance, for the following photos, teachers can ask the students to compare the photos and the real camera to scaffold the students. Teachers invite the students to see the difference of the picture such as asking "Where is the position of this block in the photo? what about in the camera? Do you see it also in the camera?" "where is this cube facing to?" "is it the same as the photo?"



e) Look at the teacher guide for the example of students' work

After attending this activity, some students will be able to imagine the images of the objects without the use of camera. They will think that they can obtain the views of the object from bird eye angle by imagining look the objects from the ground. Students also will think 3D objects looked flat from certain positions that will lead them to the standard views.

In the beginning, teacher's role is to deliver the context and make sure everyone is understand the problem. Teachers guide the students and act as a facilitator during small group discussion and also during classroom discussion. After all groups completing worksheet, teachers ask one group to explain how they figuring out the photos and discus it with the other students. Teachers start and led the discussion by posing the following question questions:

- a) Where did you put photo 1? Where is the position of the camera when photo 1 was taken?
- b) Does every group put photo 1 in the same position?

c) Now use your digital camera does the result shows the same picture?Teachers then continue this question for the other cameras. Afterwards, teachers ask the following questions to confirm their strategy:

- a) How did you figure out the position of the photos? Can you explain it to us? For example, how did you know the position of the photo?
- b) Why did you not choose this photo [photos that does not belong to the object?
- c) So, which photos did you not choose? Why?

In case of the group has wrong photo and debated by the other groups. This question can be asked to the group and do not necessary to be posed in the end. In the end of the activity, teachers guide the students to conclude what they have been done and what they learn in this lesson.

3) Lesson 3: Reporting New Temples

Learning goals:

- a) Students are able to draw standard views of 3D objects from different stand points based on its close representation.
- b) Students are able to draw standard views of 3D objects from different stand points based on its distant representation.

Mathematical activity and conjecture of students' thinking:

The activities are based on the context of temples in Indonesia. It begins with the story of an archeologist, Reza. He wants to share his experience as an archeologist. If he finds a new historical temple in Indonesia then he has to report his discovery to the government so that he has more assistance such as getting funds and tools for further exploration. In the report, he has to attach the standard views/drawings of the temple which consist of how the temple looks like from the front, the left, the right, and the top. Based on this context, students are given problems of reporting the discovery of a temple in small group of 3 or 4 students. Each group gets the building blocks, a digital camera and two camera models to help them determining and drawing the views.

The first problem in this lesson is that the students have to create their own temple by using exactly 8 cubes. Afterwards, they have to report their temple by drawing the standard views of the temple in the given layout. First, students will only use the camera model without the digital camera. Figure 4.5 shows the layout for this activity.



Figure 4.5 Layout for lesson 3

Conjecture of students' answers, actions, and strategies:

 a) Students may create a simple temple like a vertical six blocks or horizontal six blocks or another simpler cases. In this case, teachers have to challenge the students to have more unique temple.



b) Students may draw bird eye drawing. In this case, teachers must refer to the last problem in the lesson 1 and give an example of standard drawing by using the camera. For instance, for the following temple, students may draw this way:



- c) The drawings are maybe rough and different from the real object. In this case, teachers suggest the students to use ruler to make the drawing better.
- d) Students are able to create proper drawings but they miscount the cube or make small mistake like misplaced the cube in the drawing. To help the students, teachers evoke the students to carefully investigate the number of cubes and their position.
- e) Students can draw the standard views of the temple without big difficulties. In this case, teachers can further investigate how they did it and asks them to check their drawing.
- f) For the top view, some students maybe have difficulties. Therefore, teachers shall put the temple in the ground and demonstrate if the camera is exactly on the upper side of the temple.

After the groups finish the drawings, teachers instruct the students to check their drawings by using their digital camera.

In the last problem, students have to draw the standard view of a new temple only from the left (2). In addition, the drawing is done not in a layout but in the worksheet. Thus, there is a move from the use of layout which help the students to imagine the temple to not use the layout which is maybe a little bit confusing for them (see the worksheet in Appendix 1). Similar to the second problem, the temple is only in photograph. Teacher will not give the blocks. The temple is shown in Figure 4.6 below.



Figure 4.6 A new temple for the second problem of lesson 3

Conjectures of students' answers, actions, and strategies:

- a) Students will imagine the shape of the temple like the previous activity and imagine the shape to draw.
- b) Probably, students count the cube as a reference to draw the shape.
- c) Probably students will draw a little bit sloppy since they do not work in the layout like the previous problems. For instance:



In this case, teachers scaffold the students by asking the height of the temple by counting the cubes in students' drawing compared to the photograph. Teachers also can refer to the previous drawing as an example or to give demonstration.

After all the group finish the task, teachers ask some groups to share their strategy and their drawings. After all the students finish the drawing, teachers choose one group to present their work in front of the class. Teachers lead the discussion by posing questions like:

a) Can you show us your temple?

- b) Show the drawing to the others. How you did it?
- c) Why there is only three cubes (for example) in this drawing?
- d) Etc

Sometimes, teachers also give chances to the other students to react or if they have question. Teachers then choose another group to present the second problem. Teachers help the discussion by giving questions to the group.

a) Can you show us your drawing?

- b) What is this drawing? Where was it taken?
- c) Does everyone has the same drawing from the same position?
- d) How did you determine that the drawing would be like that?
- e) Why were there only 5 square/rectangle/cubes in its base?Teachers then check all the drawings (front, left/right, Top)
- f) Why are all the drawings the same?

Teachers also give chances to the other students to react or give opinions to the presentation, especially if they have different strategy or opinion. For the last problem, teachers ask all the groups to show each other the drawings and ask some students to determine how they did it. Afterwards, teachers bring the class to an end by guiding the students to conclude what they have been learned in this session.

After attending this activity, students will think that to get the views of the objects in photos, they have to imagine the object like the building blocks and then imagine moving the objects or looking the objects in mind to determine the shape.

4) Lesson 4: Fixing the report

Learning goals:

- a) Students are able to identify the properties of standard views of 3D objects.
- b) Students are able to draw the standard views of an object based on its opposite standard view.

Mathematical activity and conjecture of students' thinking:

Teachers begin the activity by instructing students to work in the same group as the previous lessons. The context in this meeting is about Reza who messed up the old report of a new temple last night. Some report lost its photos. Therefore, students are asks to help him to rearrange the photos in the report or draw a new one. In the first problem, students are asked to guess how to put the photos of a report into the right position like how they were. Teachers give each group 5 photos and a layout. The photos and the layout are shown in Figure 4.7.



Figure 4.7 The layout and the photos for the first problem of lesson 4 Conjectures of students' answers, actions, and strategies:

a) Students firstly put the top view of the temple since the photo obviously belongs to the top view. The reason maybe because it has no 'similar'

picture like the others. However, some students may takes time to realize which photo the top view is. In this case, teachers take an example of students work from previous lesson and asks them to investigate it to find the pattern or how the photo relate to each other.



- b) Students may do not have idea how to put the photo and saying that it can be anything. In this case, teachers can asks the students about their experience in the previous lesson. For instance, teacher can begin by assuming one photo is belong to certain position. Afterwards, teachers ask how it will look like from the opposite.
- c) Students nicely choose 'similar' photo for the opposite position. However, they falsely put the photos with respect to the top view. In this case, teachers scaffold the students by imagining how many cube the students will see from the front or side based on the top view. For example:



d) Students correctly put the photos by relating the photos to its opposite and the top view. In this case, maybe the students will conclude that there are two possibilities of how the photos are arranged based on the top view. In this case, teachers can ask further question like what makes them different, and whether it influences the shape of the new temple.





After all the groups finish their work, teachers select one group to present their work in front of the class. Teachers lead the discussion by posing questions to the presenter like

- a) How did you put those photos in the report? Can you show us?
- b) Why did you put them this way? Please explain to us
- c) How did you figure out the photo from the top?

Why this photo is in the opposite of this one?

d) How about the other group? are there groups that have different arrangements?

If there are groups that have different answer, teacher can invite the group to discuss it. In the end of the lesson, teachers help the students to conclude what they have been done or learned.

Afterwards, teachers ask the students to move to the second problem (see the worksheet and the teacher guide in the appendix). In this second activity, teachers give each groups 3 layouts of different cases. Each layout contains 3 different standard drawings. In the first case, the photos are from the top, the left, and the front. The photos in second case are from the top, from the right and from the left. The last one contains only two drawings from the top, and the back. The task is that students have to complete the standard views in the each report based on the remaining photos attached. Teachers must warn the students that maybe only some reports can be repaired. In this activity, teachers do not provide the students neither building blocks nor the camera. Students have to draw to completely fill the layout. The following Figure 4.8 presents the three different cases in this problem.



Figure 4.8 Three different cases for the second problem of lesson 4 Conjectures of students' answers, actions, and strategies:

- a) To draw the photo from the opposite direction of the given photo, students may use the photo in the opposite as the reference. Based on the previous investigation, they may imitate the opposite views to draw the views in the layout.
- b) Students understand that the shape will be 'similar' to its opposite, but they are wrongly draw the orientation. The following pictures show that possibilities.



In this case, teachers make sure why the students did that because there is possibilities that what students imagine or students' orientation is correct. If the drawings stand then they are correct drawings. If students did it wrong (they do not have reason why they did that), teachers scaffold the students by making the photo stands and ask the students to imagine what they will see from the opposite. Teachers also can refer to the previous investigation of the two reports.

- c) Some groups may claim that they cannot draw the empty photo for the 2nd and 3rd cases. In this case, teacher should ask more why they cannot draw it. Probably, students are aware that they need the opposite photos of the views.
- d) Some groups may predict the empty photos in the 2nd and the 3rd cases this way:



In this case, teachers ask the students such as "How could you know/ figure it out?" "Why do you have three blocks there?". Probably, students just imagine or assume that the temple will be like that. In this case, teacher can refer to the previous investigation. Teacher may use blocks to illustrate the possibilities of the temple. After all the groups finish their work, teachers select one group to present their work in front of the class. Teachers lead the discussion by posing questions to the presenter like:

- a) How many reports can you fix? Which one is that?
- b) How did you fix it? Why did the view become like this?
- c) How many reports left? Why did not you fix it?
- d) How about the other group?

After discussing students' answers, teachers can close the class by guiding students to conclude what they have done today.

5) Lesson 5: Building the Temples

Learning goals:

- a) Students are able to identify, predict, and read the blind spot of distant representations of 3D objects.
- b) Students are able to interpret, recognize, and communicate distant representation of 3D objects.
- c) Students are able to construct a 3D object based on its standard views.

Mathematical activity and conjecture of students' thinking:

The activities are still based on the story of discovering temples. Reza visited the government office and got new reports of a new temple. He was asked by the government to build a miniature of the new temple. Since Reza is busy, he asks students to help him. The students will work in the same small

groups of 3-4 people like the previous lesson. They will deal with two problems of building a miniature of the new temple.

The first problem tells the students that they receive a report of a new temple from archeologists. However, unlike the other reports, they only attach one bird eye photograph of the temple. Therefore, they have to make the model of the temple based on only its photo. This problem aims to make the students aware of the blind spot of the representation. The photograph have weakness in representing 3D object since it has blind spot of the representation. Figure 4.9 show the photos of the new temple (see also student worksheet and teaching notes in the appendix).



Figure 4.9 Photograph of the new temple in the first problem of lesson 5

Conjectures of students' answers, actions, and strategies:

a) Students probably cannot construct properly the object similar to the photo. For example, the following construction has error in the side. In this case, teachers help the students by evoke the students to determine the position of the cubes in the photo like "do you see this column? Where does this lie in this block?"



b) At first, students may just consider the visible cubes and construct it the way it looks like in the photo. In this case, teachers shall ask "Do you know how it look like in the back of the temple?" "Are there another cubes in there?" "How do you decide it?" the purpose is to bring the students to aware that the blind spot cannot be predicted unless they have clue. Here is some of the possible answer for this conjecture:



- c) Students probably aware of the unseen part and they assume the back view of the temple. They may have construction similar to the second conjecture. In this case, teachers argue by asking "What if everyone has different assumption?" "Will they have similar temple in the end?". Student may think that without further information they cannot build the temple.
- d) There is a possibility the students will claim they cannot make the model and debate each other of how the blind spot look like since we cannot

determine clearly what is in the back. In this case, teachers guide the students to arrive at the conclusion themselves that the blind spot cannot be determined unless we have more information about it. This information will be given to them later.

Teachers give 15 minutes for the students to think and construct their temple. Afterwards, teachers invite all students' attention into class discussion. Teachers begin the discussion by asking one of group's construction and the compare to the other. If there are differences, teachers shall discuss it, especially if it is about the blind spot. If there is no differences, teachers shall propose a construction which is similar to the picture but have more cubes in the blind spot. The aims is to drive the students to be aware of the blind spot and we cannot determine how it looks like in the back.

After all the students agree that the blind spot/ the back side of the temple cannot be determined, teachers give each group the standard views of the temple. The discussion is then continued by asking the students whether now they can make the model or not. The expected conclusion of this first problem is that they cannot make the model based only on one picture. Teachers really play crucial role in this discussion by guiding the students, propose question to make them aware of the blind spot. Here is the standard views of the temple.



Figure 4.10 Standard views of the temple for the first problem of lesson 5

After getting the standard view, we predict students will be able to build the temple. The temple will be like:



After concluding the first problem, the students will get another report of a new temple. This time, the report includes the standard views of the temple. Similarly, students have to build the temple by using 9 building blocks (see student worksheet in the appendix). Figure 4.11 shows students worksheet and the standard views of the temple.



Figure 4.11 Standard views in the layout for the second problem of lesson 5

Conjectures of students' answer:

 a) Based on their experience on the first problem, students put the blocks by following the top view first and then they adjust the object by looking at the standard view of each side.



b) Students also might start with the side views first and then began to adjust with the other view.



- c) Students may directly construct the object based on their feeling/sense.Afterwards, they adjust it based on the standard view.
- d) Some students may propose to flip the paper such that the side view will be vertical to fit it to the real object and then construct it. This was quite helpful to imagine the form of the object. Look at the following illustration:



e) Students can mistakenly flip the paper to the inside so that the top part will be at the bottom. In this case, teachers can ask them to remember how they

draw the views in the lesson 2. Then, teachers ask them "Where is the top part of the temple in your drawing?" "If you folding it, where will it be?"



f) There is possibilities that students will mistakenly put the blocks since the view from stand point 1 and 4 have huge differences. In this case, teachers invite the students to check again their construction. Question like "Why did you put this block in here? Because I did not see it in the views" can be used to begin the investigation.



g) Some groups probably construct two different temple regarding the number of cube to be used. The following constructions show these possible temples with different number of cubes. In this case, teachers ask students to read again the instruction given in the students' worksheet.



In the end of the activities, teachers select one groups to present their work and explain how they get it. It is important for the teachers to guide the discussion and make sure all the students understand the explanation. We expect all the groups have the same constructions and the number of the cubes like the instructions given in the worksheet. The following Figure 4.12 is the final construction of the object.



Figure 4.12 The correct construction of the temple in activity 2 lesson 5

2. Preliminary Teaching Experiment Phase

The preliminary teaching experiment was conducted at 20 February until 2 March 2015. There were five lessons in which each lesson was around 2x35minutes. Before the experiment, at 18 February 2015, students had a pretest to examine their prior knowledge while in the end of the sequence, they did a posttest to further provide us additional data of their learning.

The teaching was attended by six students of grade 3 from class 3B of SD Laboratorium UNESA Surabaya. These 6 students were then divided into two small groups of three students during the lessons. Each group was vary in terms of students' ability in mathematics. Two students were also
categorized as passive students in the class by their teacher. Two students were always actively participate in the regular mathematics class. In the second meeting, one of the students were absent because of sick. As a result, one students were added temporarily to the group to replace the absence. In the next meeting, the regular student came back to the group until the end of the sequence.

The rooms for conducting the preliminary teaching experiment were mostly in multimedia room of the school. Only in meeting 2, students had to study in the teachers' office of the school. In this occasion, the researcher became the teacher and at the same also, also became an observer. In addition, one more observer was employed to help the researcher documented the process. All the data were collected at the end of each lesson such as videos, and students' written works. Overall, the preliminary teaching experiment did not face so many obstacles until the end of the sequence.

3. Retrospective Analysis Phase

a. Pretest

Pretest was given two days before the teaching experiment at 18th February 2015. There were 6 students attended the test. The aims is to get insight of students' prior knowledge and their spatial ability. The test consists of three parts. The first one asked students to determine the position of an objects with respect to the other objects. The second part is about choosing the photo of the object if the camera is put in certain position from the object. The last one asked students to determine the shape of the objects in a photograph if the object is viewed from one of its sides. For more details of the problem, see the pretest form in Appendix 1.

In the first part (Figure 4.13), students were asked to mention the name of their friends who stands either in the right side of the left side of the students in the photo. Most of the students answered Rudi and Soni who are standing in the right side, while Mia and Roni are on the left side. This finding suggests that the students had difficulties to imagine or orientate themselves in the photograph. Only one students, Aydin, who was able to determine who stands in the right or in the left based on his position in the photo. In conclusion, the first question reveals that the students mostly could not orientate themselves into the photograph.



Figure 4.13 Kanaya's answer for the first problem in the pretest

In the second part, students have to choose the correct photos taken from the position B and C (Look the pretest form). Most students answered the question by randomly choosing the photos. Only one student, Kanaya, chose the correct photos (Figure 4.14). This finding suggests that most of the students do not have yet good spatial orientation to understand and answer the problem. They cannot imagine the situation in the photograph and identify the buildings in the photo from position B and C.



Figure 4.14 Kanaya's answer for part B of the pretest

In the last part, students also struggled to answer the question (Figure 4.15). Most of the students just tried to copy the object in the photo. Kanaya claimed that she could determine the shape but she struggled to draw it. Only Aydin could determine the correct shape. However, we were not sure whether he did know it or just randomly guessed. In conclusion, most of the students have difficulties to imagine the problems in part B and C. This suggests that they cannot yet understand distant representation of 3D object. Furthermore, students do not have enough spatial visualization ability to imagine the shape of the object.



Figure 4.15 Ratu's answer for part C of the pretest

In conclusion, based on the result of the pretest, we can conclude that students have sense of direction but they were not able to imagine the object in the photo as the center of orientation to determine the direction. Students' spatial ability, particularly spatial orientation and spatial visualization, is lacking which influence their inability to understand the objects in the photo.

b. Lesson 1

The lesson took approximately 2x35 minutes. At the beginning, the teacher opened the lesson by explaining that for today and the next 4 meetings, the students were going to work with pictures and 3D objects. For that purpose, students were divided into 2 groups of 3 students. To begin the activity, the teacher told the context of the activity, Toni "the photographer". Afterwards, students worked in small groups to solve the problems in the worksheet. There were 4 different problems (Look the students' worksheet in Appendix 1). The first problem was determining what buildings they will see in the photo if they take pictures from each position. The second problem asked students to describe the position of the red house and the tower with respect to the apartment from several different camera positions. The last problem asked students to choose the best position to take pictures of the buildings. After students had completed the worksheet, the teacher chose one group to present their answers for question 1, 2, and 3 in the worksheet.

Based on our observation, students did not have much difficulties for the first problem. They could determine what buildings they will see from each position. By using the camera model or digital camera, they moved around the table to investigate each position (Figure 4.16). Some students took pictures from bird eye angle and struggled to decide what building they will see. However, after the teacher argued that Toni cannot fly and he takes picture from the ground, the students agreed that the camera should be in the ground. There was also a group who confused since there were buildings that can be slightly seen in the photo. The teacher argued that if only small part of the building is seen, then the building is categorized as unseen.



Figure 4.16 Students move around the object to take pictures in activity 1 of lesson 1

Group 2 claimed that using camera model was easier to investigate the buildings. On the other hand, group 1 liked to use the digital camera because they can see the photo directly. In this occasion, both of the group needed the camera to help them visualize the object. This suggests that they could not yet visualize the object without using the camera. When they did not use the camera, some buildings were visible from their point of view. However, when they looked through the camera, the buildings were actually hidden from their sight. This finding also confirms that the camera indeed help the students visualize the object and creating the images of the objects in their mind. The following Figure 4.17 was the answer of the first group for the first problem in this lesson. The table indicates that students were able to pose themselves in the desired stand points. Furthermore, they were also able to observe the surrounding of the object by determining what they saw. This suggests that students had directly experienced early spatial orientation and spatial visualization of the object.

Posisi	Rumah Merah	Rumah pink	Menara	Apertemen
1	V	-	V	17
2	an Jone V- molech	ann Trumin	01-10-5	V
3			dolon se vicin	1 V
4 00			1	
5			\checkmark	1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 -
6	-	\checkmark	lut-state is	V
7	V		م م الجب ال	V
8		- 2	V	4

Figure 4.17 The answer of group 1 for the first activity of lesson 1

In the second problem (question 2 until 5), students found it confusing. Some students described the position of the red house and the tower based on their right and left (self-orientation). Meanwhile, some used the orientation of the building to determine the left and the right. After discussing it with the teacher, all the students agreed to use the orientation of the apartment as the reference to determine where the left and the right were. Students also used the term "depan" (front) or "belakang" (back) to determine the positions. Group 2 even combined the words become "samping depan kiri" (front-left side) or "samping belakang kanan" (backright side). Furthermore, none of them used cardinal direction to explain the positions. Overall, students did not have much difficulties in this activity after they had the agreement. This finding proves that the result of the pretest that most of the students is more capable of using themselves as the center of orientation than using the other object apart from their body as the center. To improve this activity, the teacher suggested to add details of the building like windows and doors so that students can also use the terms "front" or "back" with respect to the position of the door of the building.

In the third problem, students checked again each position by using the camera to determine which positions can capture two, three, or all the buildings. The students correctly answered this question by again moving around the objects and check every position. The only problem was when they saw slightly part of the building and confused whether it was counted as "seen" or "unseen". Interestingly, none of the group used the result from table 1 (question 1) to determine the position of camera to capture some or all the buildings.

For the final question, all the groups suggested Toni to take pictures from position 3, 4, or 5. The reason was because they could capture all the buildings from that positions. This was in line with our hypothetical earning trajectory. The following Table 4.2 compares further students' actual learning trajectory and our hypothetical learning trajectory.

Problem	HLT	ALT	Note
1. Determining the	Students use the camera	- By moving around the	+
buildings they	model or the digital camera	object, students saw through	
will see in the	and move around the	the camera to determine	
photograph	building to check what they	what they would see. Some	
from different	will see in the photo.	used the camera model and	
stand points.	Probably, some students	some used their digital	
	look the building from bird	camera.	
	eye positions.	- Some students struggled to	
		use the camera in the correct	

 Table 4.2 Matrix analysis for lesson 1

Problem	HLT	ALT	Note
		stand point. They placed the camera in the bird eye's stand point.	
2. Describing the position of an object based on the other object in the photograph.	Students will use their orientation of right and left to determine the position of the building. Probably, they will also use the orientation of the apartment or the other buildings. For instance, they use the terms "front", "back", "left of" and etc. Cardinal direction are also probably will be used too by the students.	 Some students used their orientation to describe the position of the house and the tower with respect to the apartment. Some used the orientation of the apartment to determine the position of the house and the tower. 	+
3. Determining stand points where only two, three, or all the buildings can be captured.	Students identify each position by moving around and look again the object with or without the camera. Probably, some students will refer to the table in the first activity to choose the positions.	 Students figured the stand points by using the camera and again moving around the object. Students thought that the position where someone can capture all the building is better. 	+
4. Suggesting Toni the best position to take photos of the buildings.	Students will choose the position based on the number of buildings that they can see in each position like position 3, 4, 5 since all the building can be seen. Some students may prefer their personal choices like focusing their photo to one spot or choose only some of the buildings.	- Students chose position 3, 4, and 5 as the best positions to capture the buildings. It is because they could see all the buildings from that positions.	+

Based on these explanations, we can conclude that in this first lesson, students had directly experienced early spatial orientation and spatial visualization by observing the object from different stand points using the camera. They were also able to describe the position of the building based on both the apartment as the center of orientation and their own body as the center of orientation. During the activities, students were able to identify the buildings only with the use of the camera. Without the camera, they struggle to determine the buildings they see since they cannot visualize the objects from their angle/point of view. In this case, the camera did help and support them creating the images of the objects which they could not do independently yet.

c. Lesson 2

In the beginning of the lesson, students directly sat in the same small groups as the previous lesson. However, one of the students from group 2, Hafid, was not able to come. Therefore, to replace him, the teacher invited Fadhil to attend the lesson. Before starting the lesson, the teacher reminded the students about Toni "the photographer". After telling the context, the teacher distributed the material including the worksheet for each groups. In this lesson, students were asked to find the position of the camera where the printed photos were taken by Toni. However, among the photos, there were photos that do not belong to the object. Therefore, students had to distinguish these photo while trying to find the position of the correct photos. At first, students were only allowed to use the camera model. Digital camera were forbidden and would be used only in the class discussion later on. After figuring the position of the photos, the teacher instructed the students to answer the questions in the worksheet regarding how they solved the problem. Afterwards, the teacher asked one group to present their work in front of the class. During the discussion, the teacher evoked the students to use their digital camera to check their answer by directly making the photo of the objects. Approximately, the lesson took 2x35 minutes for students' investigation and presentation.

The main problem in this lesson was finding the position of the camera where the photos were taken. To find the position, group 1 took randomly one photo based on their guessing and moved around the table while seeing through the camera model. If they found a match between the view in the camera and the photo, they quickly glued it to the position. If they did not find the match, they claimed that the photos did not belong to the object. Differently, the second group investigated orderly from photo 1, photo 2 and so on. Similarly, if they found a match in the camera model, they glued the photo to its position. However, they struggled to find the position of photo 5 and 6. They keep repeating moving around the table to find the position of these photo 5 and 6 but they could not find it. It seemed that they forget that there were photos that do not belong to the object. After the teacher reminded them that there were wrong photos, they realized that photo 5 and 6 did not belong to the object. Some students such as Kanaya and Aydin began to not use the camera model to determine the position. They just saw the object from the position directly without the camera. The following Figure 4.18 shows how students investigates the photos. In the first photo, Aydin did not use the camera model and in another photo, Aurum used the camera to help her investigation.



Figure 4.18 Students investigate the position of the photo in lesson 2

The use of the camera indicates that students still need a tool to bridge them visualizing the object to acquire its images or its view. Differently, Aydin and Kanaya began to work without the camera which suggests they started to be able to visualize the object in their mind.

In the beginning, group 2 falsely chose photo 5 and put it in the layout. However, after the teacher asked them to look at where the "L" object was facing, they realized that the object in the photo had different position from the real object. Therefore, they categorized the photo into unselected photo. None of the groups falsely put the photos into wrong positions. The only struggle they had was identifying the photos that did not belong to the object. The following Fragment 4.1 illustrates how students figured out the wrong photos and distinguished them from the other photos.

1	Teacher	: Have you done finding all the photos and glued them? Remember!
		Some photo are not of the objects. Afterwards, answer the questions.
2	Kanaya	: Try this position! [Looking for position of photo 5] ahh not this one
3	Ratu	: Try there!
4	Teacher	: If you do not find the position, maybe it is not from that position.
5	Kanaya	: Not this one [try another position for photo 5]
6	Teacher	: So, is this photo of the object? [photo 5]
7	Kanaya	: No, it is not
8	Teacher	: Okay, then you have to put it here.
		Group 2 and friends continue to investigate photo 6.
9	Kanaya	: What if it is reversed? This one is reversed [photo 6]
10	Teacher	: So, is the photo taken from the objects?
11	Kanaya	: No, it is not
12	Teacher	: Okay, then find another photos. But, you need to be sure that this photo
		is out of the object. Why this photo is not taken from the object.

13	Kanaya	: This part is not the same [pointing to part of the object in photo 6]
14	Ratu	: This part faces towards different direction [pointing the real object to
		compare it to photo 6]
15	Kanaya	: Yeah, it faces to the side. It should face this way [the right side]

Fragment 4.1 Students' discussion in lesson 2

From that fragment, students firstly looked for the position of the wrong photo and then identify the difference of the object in the photo and the real object. In the worksheet, most students also explained that they distinguished the unselected photos because the object in the photos faced to the opposite direction of the real object, "Kebalik" or "reversed". This means the students compared the view in the photo to the view of the object. The following Figure 4.19 were students' answer of how they distinguished the photos. This suggests that students had been able to connect and compare the information they had in the photo and the information they got from the real object. This is a basic ability in spatial orientation where they operating on relationships of different information from different position in space.



Figure 4.19 Students' explanation of determining the wrong photos

In the end of the lesson, the teacher asked both group to check their photos by using their digital camera to confirm their answers. The following Table 4.3 shows the comparison between students' actual learning trajectory and hypothetical learning trajectory that we had designed.

Problem	HLT	ALT	Note
 Determining the stand points where the photos were taken. 	- Students move around the objects while comparing the view in the photo to the view they see in the camera model. Some students may do it without the camera model.	 Some students used the camera model to find the position of the camera. They compared the view of the object in the photo to the view they saw in the camera. Some students did not need camera model to help them comparing the view in the photos and the view of the object they will see from the stand points 	+
2. Distinguishing the photos of the object from the wrong photos	 Students firstly try to find the position by moving around the objects. After they cannot find the position, they will consider that the photo is not of the object. Students probably will refer to one of the stand point where they can see a similar view like the wrong photo has. Afterwards, they explain that the objects has differences in some parts like facing the opposite direction or the cube is misplaced. 	 Students firstly compared the photos to the view of the object from all possible stand points. If they did not find a match then the photos are not of the object. Students realized the photos were not of the object since from a certain stand point, the object in the photo had different direction from the real one. 	+
3. Determining the position of the top view photo	- They will directly find out that the view is from the top of the object since the photo is different from the other.	 Students knew directly the photo was taken from the top of the object. 	+

 Table 4.3 Matrix analysis for lesson 2

Based on the observation, students were able to identify the position of each photo correctly by moving around the object and matching the view of the object with the photo. The camera model becomes a helpful tool for those who had difficulties to observe the object. Some students even began to not use the camera which means they could visualize the object. Therefore, we claimed that students' were able to relate the information they had in the photo with the information they saw from the real object. This was indicated by the fact that students could determine the position of the photos and identify the wrong photos of the object. In addition, Dierdrop's matrix analysis also shows that the learning went as we have predicted in the HLT. Therefore, we conclude that the activities have supported the development of students' spatial orientation and do not need to be improved.

d. Lesson 3

This lesson aims to develop students' spatial visualization ability by determining and drawing the standard views of the temple from different stand points. There are two goals in this lesson. First, students are able to draw standard views of 3D objects from different stand points. Different from the first goal, the second goal insists students to draw the standard views based only on the distant representations of the object. These goals are embedded into two parts of problems in this lesson. In the first part, students draw the standard views of a temple made by the students while the second part asked students to draw the standard views of a given temple presented in its distant representation (photo). The groups were still the same as the previous lesson. Group 1 had Aura, Aydin, and Hafid while in

group 2, there are Kanaya, Ratu, and Youvriel. The lesson took around 2x35 minutes for students' investigation and presentation.

In the first part, each group created a temple by using exactly 6 building blocks. As we have expected, group 1 tried to make a simple temple by forming a vertical line and a block. However, after the teacher challenged them to construct a unique temple, they recreated the temple into a temple in Figure 4.20. Differently, group 2 directly construct a temple showed in the Figure 4.20. At first, students have difficulties how to draw it, before the teacher demonstrated the strategy by using the camera. Group 2 determined the views by putting the camera into the position and then they looked through the camera. While seeing the object, they drew the shape by using their pencil and ruler. Similarly, group 1 also did the same. In addition, Aydin directly drew the shape without looking to the camera. However, even though the camera model was not used, the students still need to kneel down and put their eye at the same level as the object. This finding confirms that the students were difficult to determine and imagine the shape without the camera. The process of visualizing and drawing is difficult for them even the object is concrete. However, they succeed to draw all the standard views with the use of camera model.



Group 1 Group 2 **Figure 4.20** The temples constructed by the students

Group 2 found a unique strategy to determine the position of the cube in their drawing. In Figure 4.21, they shaded the cube which had position one level behind the unshaded cube. This made them easier to imagine that actually there were another cube behind the unshaded cube. This action of the students were not predicted in the HLT. The following Fragment 4.2 draws how group 2 explained the use of shading in their drawing.

1	Teacher	: Is this kanaya's drawing?
2	Ratu	: Yeah
3	Teacher	: where is the position of this cube (the cube placed in the front side of
		the object) in this drawing?
4	Kanaya	: there (pointing to the cube in her drawing)
5	Teacher	: Oh, this one?
6	Kanaya	: Yes, this cube is in the front and this cubes (cubes with shading) are
		behind the first cube.
7	Teacher	: Oh, did you put shading there?
8	Kanaya	: Yeah
9	Teacher	: okay, then you have to write here (beside the drawing) to explain that the shading indicates the position of the cubes.

Fragment 4.2 Students' discussion in lesson 3

From the fragment, the teacher did not notice the shading in the beginning. He asked the position of the cube in the drawing in order to investigate students' understanding. However, Kanaya and her friends explained that there is shading in their drawing to indicate the position. This finding reveals that students understood that the cubes in their drawing were not at the same level of position. Therefore, as the solution to differentiate the positions, they put sign to show that the cube is in front of the other cubes or behind the other cubes.



Figure 4.21 Group 2 shaded the cube to indicate its position

In this activity, the only problem was that the students were quite crowded, especially group 1. This was due to Hafid who disturbed the other students and tried to provoke his group by destroying the temple. The teacher tried to warn him but he was still crawling. As a result, the first group's work was not as good as group 2. The drawings can be seen in Figure 4.22. In general, all groups did not have much difficulties and could draw the views correctly. In the end of the investigation, the teacher asked the both group to check the work of the other group by using their camera digital. To do this, students took pictures from their digital camera and then compare it to the drawings.



Figure 4.22 The drawings of group 2 in activity 1 of lesson 3

In the next part of this lesson, students had to draw the standard views of the temple in a photo. The photo can be seen in Figure 4.23. Both group looked struggle to Figure out how the shape or the view will look like. In group 2, Kanaya told her friend to see the photo from the edge of the right side of the photo. Kanaya added that this could help her imagine the shape. Figure 4.23 shows the drawings of group 2 for this activity. Group 1 also did something the same. They look the photograph from the edge. In addition, Aydin flipped the photograph and trying to look from different angle. He thought that if he flipped the photo then he could imagine the back view and maybe it could help him imagine the side view. The only difference between the two groups was the shading. Like activity 1, group 2 put another shading in their drawing.



Figure 4.23 The drawings of group 2 in activity 2 of lesson 3

In the end, the teacher discussed both problems from the first and the second part with all the students and asked them which one was the easier. Most of them agreed that the second one was harder since they do not have the object. They added it was difficult to imagine the shape without the object. Although they succeed in drawing the object, some students still confused and struggle to solve it. In this case, we indicate that the students need more activities to bridge their understanding from working with the concrete object into working with its distant representation. Judging from

the outcome of the first activity, we know that students quite struggled just to determine the shape of the concrete object. Therefore, to further support the students, we can add an activity between the first and the second problems by giving them the temple in a photo and let students construct the object first before drawing the shape. With this activity, students can experience and feel how the object in the photo will be in reality.

Overall, the lesson went as we have predicted. The following Table 4.4 shows that the actual learning trajectory of the students is in line with our hypothetical learning trajectory. However, we did not predicted that students would look from the edge of the photo and tried to imagine the shape of the object.

Problem	HLT	ALT	Note
1. Drawing the	- Students draw the	- At first, some students	+
standard views of	shape while looking	confused to draw the	
the temple with	through the camera	shape since they thought	
the help of	model and tried to	that the drawing will be	
camera model	imitate it. Some	bird eye picture. After	
and the miniature	students maybe do not	they look to the camera,	
of the object.	need the camera model	they realized that the	
	to determine the shape.	shape was not 3-	
	Some students may	dimensional.	
	falsely draw the	- Students used camera	
	temple from bird eye's	model to see the shape	
	angle.	and then draw it to the	
		layout.	
2. Drawing the	- Students will imagine	- Students tried to look the	±
standard views of	the shape of the temple	temple from edge of the	
the temple only	like the first activity	photos and then	
based on its bird	and draw the shape.	imagined the shape.	
eye photograph.	- They maybe count the	- Some of them also flip	
	cubes they see as a	the photos to help them	
	reference to draw.	imagine the shape	

 Table 4.4 Matrix analysis for lesson 3

During the learning process, we observed that students had develop prior spatial visualization by determining the standard views of the temple. In the second activity, students needed more time to determine the shape. This indicated that they struggled to imagine and identify the shape since they did not have the object. Some students tried to imagine the shape by putting their eye in the edge of the photo. Clearly, they tried to imagine doing the same strategy as they worked in activity 1. It shows that activity 1 successfully assisted them to develop their thinking when solving the problem in activity 2. However, they were quite struggle to solve the next activity. Therefore, we conclude that the activities in lesson 3 helped the students develop their spatial visualization although more activities are probably needed to further support them to shift from understanding the model to understanding its distant representation (drawings).

e. Lesson 4

In this lesson, the context was about fixing reports. The lesson has two goals embedded into 2 different type of problems. The first problem asked students to figure out how to put 5 loose photos into its position in the report. The second problem gave students situation where two photos in each three different report were lost. Therefore, students have to fix the report by guessing how the view of the temple in the lost photo. They have to draw the view based on the remaining photos they had in the given report.

This activity seemed to be very difficult for the students. Both groups did not know how to figure out the photos or at least guess which photo was from which stand points. To help the students, the teacher provided each group the report of a temple from the previous meeting. The teacher instructed them to investigate the report and observed to see if they noticed something to help them putting the photos. However, after several minutes, students still confused and had no idea. As a result, the teacher guide each group by referring to the previous report. The teacher also directly asked students to see what happened between a photo and its opposite photo in the report. The aims is to make the students notice that a photo and its opposite photo in the report always looks "similar". Some of the students understood the properties, but most of them did not. In this case, the teacher tried to guide both groups by making the photo become standing in the report like in the proposed action in the HLT. Students then looked this photo from opposite direction while imagined they had the temple in the middle of the layout. By doing this, some students like Aydin understood it but the others, like Ratu and Aura, did not. In the end, students agreed that if the position of two photos oppositely face each other, then the view will be "similar" (like a mirror). Nevertheless, they still could not imagine and understood completely why the photos were "similar". The following Figure 4.24 is the answers of both groups.



Figure 4.24 Students' arrangement of the report in activity 1 of lesson 4

We indicated several causes of why students had difficulties to grasp the idea of the activities. First, students cannot imagine the object since they do not have yet the experience to build the object based on its standard views. As we know, that building activities would be conducted in the fifth meeting. That was why the students did not have the idea of the object from the given standard views. Second, students' learning style in elementary school did not yet align with the activities where they easily learning by touching, experiencing things. But, in this activity, they did not have something to perceive like nor the experience to imagine perceiving the object. We figured that students did not have the feeling of the existence of the object. That was why they cannot reason and fully accept the properties. In this case, to further improve the activities for the next teaching experiment, we suggest to postpone lesson 4 until the students have experienced building activities in the lesson 5. By doing this, students will have enough experiences to perceive the object or imagine the object of the given standard views in this lesson.

In the last problem of fixing three reports, students easily completed the first report. For example, group 2 knew directly that the view in the lost photo would be "similar" to its opposite. Similarly, group 1 also did the same. However, for the other two reports, both groups found difficulties. The other two reports were supposed to be unable to be fixed. However, both groups could draw the view by guessing randomly or imitating the top view. Both group 1 and 2 fixed the second report by assuming that the temple must be like Figure 4.25. Therefore, the view on each stand points would be the same. This finding once again proves that the students still could not understand the properties of standard views completely.



Figure 4.25 The temple imagined by the students for report 3 in activity 2 of lesson 4

After all the groups competed their work, the teacher invited all the students to discuss the problem and asked them to compare the fixed report from both groups. By doing this, students realized that the fixed reports from both group were different and concluded that they could not fix the report. The following dialogue in Fragment 4.3 illustrates the discussion.

1	Teacher	: Look the second report! Are your answers the same?
2	Kanaya	: They are different
3	Teacher	: Why are they different?
Stude	ents were this	nking to answer the question but could not figure out the reason
5	Teacher	: What if the right photo is not known in here. You could draw it by
		making "similar" shape to its opposite but it is reversed. Right?
2	Students	: Yeah
3	Teacher	: Now you do not have photos remained either in the back or in the
		front. How could you determine the drawing?
4	Kanaya	: Because there is a hole there. So, I assume it. [Pointing to the top view]
6	Teacher	: But, do the shape actually can be drawn?
5	Aurum	: Yes if we have the blocks.
7	Teacher	: Yeah, the problem is we do not have the block. So can you draw it?

8 Aydin : Yes, you can imagine and assume it

Fragment 4.3 Students' discussion in lesson 4

From the fragment, we can conclude that students imagined the temple and then tried to determine the shape. However, in the end of the discussion, students realized that if they assumed the temple then the reports would be different which means they could not restore the original report. From this discussion, we find that students could not apply the properties of standard views in this activity. From the statement that students imagined the temple, we also get information that students' strategy is that they build the temple in mind to determine the shape. This is why students struggled correctly fixing the report since building the temple from standard views is the activity for lesson 5.

In general, the goals of the lesson were not completely achieved. This was indicated by two things. First, students cannot make sense of the properties of the standard view although they could accept it. Second, students cannot apply the properties of the standard views in the activity 2 and imagined building the temple instead. In addition, the comparison between the ALT of the student and our HLT in Table 4.5 presents further information that the lesson did not happen fully as we have predicted. Student had more difficulties than we had expected.

Problem	HLT	ALT	Note
1. Rearranging the	 Students firstly will 	- In the beginning, students	±
photos in a	put the top view of the	confused how to rearrange the	
report of a new	temple. They can	photos. It needs teacher direct	
temple	identify the top view	guidance for them to realize	

 Table 4.5 Matrix analysis for lesson 4

Problem	HLT	ALT	Note
	since the other photos	some position of the photos.	
	do not have similar	They first put the top view	
	shape.	and then put the other photos	
	- They put the other	around the top.	
	photo by adapting the	- Students struggled to imagine	
	number of cube in side	the object and could not	
	photos to the top view.	accept just by learning the	
		properties of the standard	
		views	
2. Drawing the	- By using the properties	- For the first report, students	±
shape of the	that the opposite	drew the shape by using the	
temple from	photos are similar,	properties of the opposite side	
some stand	students draw the	of standard views.	
points based on	shape of the temple	- For the other reports, students	
the remaining	based on its opposite	assumed the shape of the	
standard photos	photo.	temple and then drew the	
	- For the reports that	shape of the temple.	
	does not have enough	- Some students just randomly	
	photo, students will	imitated the shape of the	
	assume the shape first	temple in the other given	
	before realizing that	standard views in the report.	
	the shape cannot be		
	determined.		

f. Lesson 5

The last lesson was about building temples by using building blocks. Students were asked to construct a miniature of the temple in the given photos. There were two sessions of activities in this lesson. The temple in the first session was presented in a bird eye photograph like in Figure 4.26. Afterwards, the teacher also provided the students the standard views of the temple (front, side, and top view) to help students finish the construction. Differently, the second session only gave each group photos of standard views of a temple placed in the report. During 2x35 minutes, students were engaged in investigating how to build the temple based on its distant representation. There were two goals to achieve in this lesson: 1) Students are able to identify, predict, and read the blind spot of distant representations of 3D objects, 2) Students are able to interpret, recognize, and communicate distant representation of 3D objects, and 3) Students are able to construct a 3D object based on its standard views.



Figure 4.26 The photo of the object in the first activity in lesson 5

In the first session, both group constructed the same temple similar to the photograph and has the same construction in the back side (Figure 4.27). They did not realized that they had assumed how the unseen part looked like. Therefore, to stimulate the students about the unseen part of the temple, the teacher proposed another temples. The temple was similar to the students' temple but it had different form in the unseen part (Figure 4.27). The teacher then asked why the temple had similar shape like in the photo but it had different form in the back. Some students began to realize that they did not know the form of the temple in the back. Aydin added that on that part, the temple could have more cubes or even no cubes at all. Thus, all the students agreed that they could not see the back part and also could not completely build the temple in the photograph. The following Fragment 4.4 illustrates the discussion of how students realized the unseen part of the photo.

1	Teacher	: So, the temple of group 1 and group 2 are the same. Has the temple
		been similar to the photo?
2	Both group	: Yes
3	Teacher	: What if I take these two cubes and this cube. Is the temple still similar
		to the photo?
4	Aydin	: Yes, it is
5	Hafid	: No
6	Aura and A	ydin: Yes, it is still similar to the photo, Fid!
7	Kanaya	: If you look from this point, yes it is still similar
8	Teacher	: So, which is the correct one? Does it have these cubes
		(removed cubes) or not?
9	Kanaya	: Yes, it has
10	Teacher	: How did you know it Kanaya?
11	Kanaya and	l Aydin: hehehe
12	Aura	: guess it sir.
13	Teacher	: Oh by guessing, if you are not allowed to guess it, then how?
14	Kanaya	: then you could not figure it out
15	Aura	: we have to ask Reza first or the government
16	Teacher	: what do you want to ask?
17	Aura	: Sir, is there another cubes in the back of the temple?
18	Teacher	: okay, so if you just have this photo, can you figure out how the back
		part is looked like?
19	Students	No, we cannot
20	Teacher	: So, can you build the miniature of the temple?
21	Ratu	: No, you cannot
22	Kanaya and	Aura: You can build it, but the back part is incomplete
	2	

Fragment 4.4 Students' discussion in lesson 5

Afterwards, the teacher asked both group what they need to

completely build the temple. Some students answered they have to ask Reza

for more photo in the back side.



Teacher

Figure 4.27 Students' construction and teacher's proposed model

In response to the students' request, teacher then gave both group another photos. These photos were the standard views of the temple. Group 2 began to adjust their construction from the back view and moved to the side view. Similarly, the other group also did the same. In the end, they put the top view above the temple and finished the construction. However, group 1 and group 2 has different result (Look Figure 4.28). Therefore, the teacher guide group 1 by asking them to look again the bird eye photo of the temple. By doing this, they realized their mistake and corrected the temple.



Figure 4.28 Students' final constructions in activity 1 of lesson 5

In the second session, the teacher gave each group standard views of a temple (front, side, and top view) set in a report. Students had to build the temple based only on the standard views. As we had predicted, both group begin the construction from the top view and then adjust the temple based on its side views. During the investigation and construction, both group did not have much problem. However, the final product between the two groups were different. Figure 4.29 shows the difference. To discuss this, the teacher then invited both group to check the other group construction. After checking the construction of the other group, students concluded that there were two temples could be build based on the photos. In the end, the teacher told the students that the temple consisted of exactly 10 cubes. With this additional information, both groups finally had the same temple.



Figure 4.29 Students' final constructions in activity 2 of lesson 5

In these activity, students built the temple without having much problem or difficulties. The learning process flowed as we had planned. Students' actions, strategy, and solution were also in line with our HLT. This can be seen in the following Dierdrop's matrix analysis in Table 4.6. This finding indicates that the activities help students to notice the blind spot of the representations and help them decoding the distant representations of the object by building it. Unlike the 4th lesson, the last lesson seemed to be easier for the students. Students explained that unlike the lesson 4, they had the blocks in this lesson. When students constructed the temple from one of the standard views and then adjusted it to the other view. There is a process of combining spatial orientation and spatial visualization to interpret the drawing and build the temple which fit to each standard views. Therefore, we can detect that students' strategy was the combination of their strategies in lesson 2 and 3. On the other hand, we did not see the effect of lesson 4 to the student's strategy.

Problem	HLT	ALT	Note
1. Constructing a	- Students construct the	- Students constructed the	+
temple based on	temple by assuming the	temple such that it looked	
its bird eye	unseen part of the temple.	similar to the photograph	
photograph		without realizing the	

 Table 4.6 Matrix analysis for lesson 5

Problem	HLT	ALT	Note
	 Some students probably aware of the unseen part but they assume it to build the temple. Some may directly state they cannot build the temple since the photo is not enough. Students will conclude that they do not know how the unseen part looked like and need more photos of the temple. 	 unseen part of the object in the photos. Students then decided they could not build the temple and they needed to ask Reza the view of the temple from the back. 	
2. Constructing a temple based on its bird eye photograph and its standard views	 Students continue their construction in activity 1 by adjusting the shape with the photos. Probably students will work from the back view and then the side view or top view. In the end they compare it again with the bird eye photo. 	 Students firstly constructed the temple based on the back view and then the side view. In the end, they finished the temple based on its top view. Group 1 did a mistake by putting another cube like Figure 5.16. After teacher asked them to compare it to the bird eye photograph, they fixed it correctly. 	+
 Constructing a temple based on its standard views 	 Students construct the temple from the top view and then adjust the construction to the side views. They put the cubes in the top view and then they look the temple from the side to add another cubes. Some probably begin the construction from the sides and then adjust it to the other side views and the top vies. 	 Students began constructing the temple from its top view. Afterwards, they adjusted the temple by putting another cube based on the side views. 	+

Based on the explanation, we can conclude that students are able to identify the blind spot of the object. Furthermore, students are also able to connect the standard views of the temple to construct and build the temple by using the cubes. Therefore, the activities in this lesson successfully helped students to develop their spatial ability to decode the information in the distant representations of the object. Thus, the activities do not need further improvement. However, to follow up the finding in the lesson 4, lesson 5 is recommended to be done before the lesson 4.

g. Posttest

Posttest was held one day after the fifth lesson. All the six students attended the test for approximately 40 minutes. After the test, examiner interviewed the students one by one to confirm their answer and their strategy to solve the problems in the test. The test itself consists of three problems. Different from the pretest, the first problem was about spatial orientation. Students were asked to find the position of the camera for each given photo. The second problem was the same as the third problem in the pretest. It was about drawing the view of the object from the side. The last problem was constructing an object based on the standard views of the object. The aims was to investigate students understanding of 3D representation. The posttest form can be found in the appendix.

Based on the result, most of the students were able to answer the first question. During the interview, they also understood how they chose the photo for each position. For example, Aura indicated the photo by imagining if she stands in a position then she will see some buildings. This building can be reference to find the photo. Among the students, Youvriel was the only students who could not answer the first question. Whenever, he was asked why he chose the photo for a particular position, he answered that he could not imagine the photo. He just knew one photo from position C.

The second problem about determining the view of an object from the front seemed to be difficult for the students. Previously in the pretest, only Aydin could draw the correct view while Kanaya had correct view but she could not draw it. In the posttest, Hafid and Kanaya showed that he could determine the shape. On the other hand, Aydin confused with the view and could not draw it. The rest of the students were the same as Aydin either confused or cannot draw it. If we go back to the learning process in the lesson 3, both the groups were able to determine the shape of the same temple/object viewed from the side. This finding reveals that the learning process was not optimal. The bridging from the concrete object to the representation need to be conducted more in grounded way.

In the last problem, most of the students did not find much difficulties. Most of them correctly construct the object by using building block. When they were asked why the object becomes like they had, they explained it clearly. Only Youvriel that could not constructed the object and confused to explain why the object looked like picture 4.

h. Discussion and Conclusion of the Preliminary Teaching Experiment

During the preliminary teaching experiment, we noticed things that are in line with our hypothetical learning trajectory and also things that we did not expected before. Based on the analysis of each lesson, we know that in general, the activities indeed supported the development of students' spatial ability. This was indicated by the progress of how students solved the problems in each lesson. In lesson 1, students developed the sense of the orientation and visualization of an object by observing the object from various angles or stand points. They were able to indicate the visible buildings and its position from different positions. At this level, they still do not yet able to visualize the objects without the camera. Students then developed further their spatial orientation ability by participating in the activities of lesson 2. They were able to identify the position of camera where the given photos were taken. This shows that students were able to make connections between the information they had in the photo with the real situation to find the correct position of where the camera stand. In addition, they also could identify and distinguish the photos that were not of the object. Some students also did not to use the camera since they began to be able to visualize the objects and create their images by looking from bird eye angle.

In the next activity, students showed a positive progress of their spatial visualization. Given an object of building blocks, they were able to identify and draw the standard views of the object. Their strategies are developed from using the camera model until directly seeing the object without the use of camera to identify the standard view. Students' drawings of the standard views of the objects also showed an improvement. In the pretest, most of the

students drew the view in bird eye drawing (imitating the photo of the object) and could not imagine the shape. During the activities, they were able to identify the shape correctly by using the camera model. In the end of the activity, they were even able to identify the standard views of a given object in a photograph. Therefore, we conclude that the activities had supported students' spatial ability.

Regarding students understanding of 3D representations, students also showed a positive progress during the activities. In the pretest, most of the students could not encode or interpret the drawing of an object in a photograph. They hardly imagined the shape and hardly put their imagined shape into drawing. However, during the third lesson, students understanding were bridged from working with the model (building blocks) until working with the distant representations (photograph). Their drawings were the proof that they imagined the shape and put it into drawing although the shift was not optimal since some students still struggle to do the process. With more activities to support this shifting process, probably can help students having better performance. In the lesson 5, students also demonstrated that they were able not only to build an object based on its bird eye picture, but also construct an object based on its standard views. There were a process of encoding and decoding in students' answers during the activities of drawing and building the object. In the other words, students interpreted and processed the information to draw or build the object. These progress are an indications of the development of students' ability in

understanding 3D representations. Generally, students' responses, strategy, and thinking during the activity are in line with the HLT.

During the preliminary teaching experiment, we also found some facts that we did not expected. First, students' idea of shading their drawing to indicate location of the cubes. In the third activities, Group 2 had a unique idea to shade their drawing so that they remember that there is another cube in front of the shaded cube. Second, the fourth activity was not happened as we expected. Students could not discover the properties of standard views of an object in the activity 1 of lesson 4. In the process, the teacher had to directly guide the students to derive the properties from the first activity. In addition, students did not have sense of the properties. The fourth finding was that students hardly imagined the object in the activity 2 of lesson 4. This was because they did not have yet building activity that would be in the next lesson. Therefore, when the teacher tried to evoke students to imagine the object, students struggled and confused. Fifth, although students could imagine and draw the standard views of the object in lesson 3, during the posttest, some of the students struggled to determine the shape of the object given in a photograph.

Based on these findings and analysis, we consider the following improvements to have better activities that support students during the learning. The following Table 4.7 summarize the recommendations for further improvements of the activities for the teaching experiment.

Lesson	Activities	Improvement
Lesson 1	 Observing objects using camera model to determine the visible buildings from different stand points Describing position of buildings from different stand points Finding Position to capture one, two, or all buildings and also finding best position to capture the buildings 	 Improving the miniature of the city by adding some details to the buildings like doors and windows. Adding one more question in the last problem to find the best position to capture one building e.g. the pink house. Some minor revisions shall be done like rephrasing the sentence.
Lesson 2	 Investigating printed photos to find the position of the camera where the photos were taken At the same time, students also do selecting the printed photos to distinguish the photos of the objects and the photos out of the objects 	There were no major revision for the activities in lesson 2. There were only some minor revisions like rephrasing the sentences and improve the quality of the photos in the worksheet.
Lesson 3	 Determining and drawing the standard views of a temple constructed by the students using 6 cubes. Determining and drawing a standard view, from certain stand point, of a temple presented in a bird eye photo. 	 Increasing the number of the cubes used to construct the temple into 8 cubes to enable more creative and complex temples. Adding one more activities to further bridge students' understanding from the use of model to the use of distant representation. Minor revision like rephrasing the sentences and improve the quality of the pictures used in the students' worksheet.
Lesson 4	 Rearranging the photos of standard views of a temple such that they fit perfectly into the layout as they were. Fixing reports by determining and drawing the lost photos of standard views of temple. 	 Conduct the activities in this lesson after the students have experienced the building temple activities. Adding an activity as the preliminary investigation to discover the properties of standard views.
Lesson 5	1. Building a temple presented in a bird eye photo by using building blocks. Afterwards, students also have to adjust	1. Conduct this building activity before students explore the properties of standard views of an object

Table 4.7 Recommendations to improve the activities
Lesson	Activities	Improvement
	the constructed temple based on its standard views (Back, Left, and top).Building a temple based on its standard views (Front, back, left, right, and top).	2. Hide the clue of using 9 cubes in the second activity such that it has two possible answers to evoke students' reasoning and creativity to construct the temple.

B. Teaching Experiment

1. Preparation Phase

In this phase, we did the follow up of the retrospective analysis of the preliminary teaching experiment by revising and improving both the activities and the HLT. In addition, we also conducted a pretest for the students who participated in the teaching experiment. The result of the pretest will be used to indicate students' prior knowledge regarding the topic and to anticipate another conjectures of students' thinking during the lessons.

Based on the facts and the findings during the preliminary teaching experiment, we improved the HLT and also the design of the activities in order to have better support for the students as follow:

a. We added some problems in the pretest and posttest to better have insight of students' thinking and ability. In the pretest, we added a problem to investigate students' sense of direction with two centers of orientation. In the previous pretest, we only investigated students' ability to determine the direction based on the people in picture as the center of orientation. Meanwhile, the new posttest had an additional problem in part B where students also have to draw the shape of the object from position B. In both pretest and posttest, we include a question to count the number of cubes used in the photo. Some sentences are also rephrased to hinder students' confusion. The improved pretest and posttest can be found in Appendix 1.

b. Based on the regular teacher's suggestion, we redesign the buildings in the miniature of the city. All the buildings now has doors and windows on its sides. This is to enable more possibilities for the students to determine and describe the position of buildings in the photo in lesson 2. For instance, since the building has the door, students can identify which front side or back side of the buildings. In the students' worksheet, we also added a new question in the end of the investigation. Before, students only have to suggest Toni the position to capture all the building. Now, there is another task to suggest Toni to find the best position to capture the pink house. This aims to check students' spatial orientation and preference of choosing position to capture the buildings. The following Figure 4.30 shows the buildings and the additional question.



Figure 4.30 The photo of the new buildings and the additional problem in lesson 2.

c. In the third activity, we added another activity of determining the standard views of an object in a photograph. This activity aims to bridge students' understanding to move from models (concrete objects) to distant representations (pictures). In this part, students are allowed to use building blocks to help them imagine the shape. However, they have to build the object in the photo themselves. In addition, students have to count the number of cubes used in the photo before starting to draw the shape of the object in activity 2 and 3 of lesson 3. The number of cubes to build a temple for the first activity is also increased up to 8 cubes to enable more creative constructions and level of difficulty. The distant representation of the objects is also improved by adding colors and table to approach the real situation. The new activity can be seen in the students' worksheet in the appendix. The following Figure 4.31 shows the new activity in lesson 3.



Figure 4.31 The photo of the new activities and the new photo of objects in lesson 3.

A new conjecture of students thinking is also added to the HLT. This is based on the drawing of group 2 the pre TE. They put sign to indicate the position of the cubes. The new conjecture is as follow:

"Students probably put sign like shadow or colors on their drawing to indicate the position of the cubes. For example, they put shadow in the following drawing to indicate that there is another cube in the front. The sign also can be used to show that the cube is one step behind unsigned cubes. Probably, students also will use these signs to further indicate the level of the position of the cube. For instance, unsigned cube means the nearest cube to the camera and then all the cubes placed one step behind are blue while for those placed two step behind are red and so on."



d. After considering the flow of the activities in the design, the old lesson 4 are moved to the new fifth lesson while the old lesson 5 becomes the new lesson 4. In other words, we swapped lesson 4 and lesson 5. This is done since the fourth activity is more abstract than the activity in lesson 5. Furthermore, students need to have sense of building the object based on its standard views before attending the activities in the old lesson 4. Activities in the old lesson 4 ask students to imagine the

object from the given standard views. Therefore, the old lesson 5 needs to be conducted before the old lesson 4. The new sequence of activities is presented in the Table 4.8.

e. In the new lesson 4 (The old lesson 5) of building temple, we change the second problems such that it has two possible answers. This aims to investigate students' creativity and students' ability to interpret the standard views. The following Figure 4.32 shows the two possible answers:



Figure 4.32 The two possible answer for the second problem in lesson 4.

f. The new fifth lesson (The old lesson 4) has a new activity to give students preliminary investigation to the properties of standard views. This is to anticipate students' confusion and students' lack of sense of the properties during the preliminary teaching experiment. The old first activity of lesson 4 (Arranging the standard views in a report) is moved to the least problem after students fixing the report by drawing the lost standard views in activity 2 of the new lesson 5. The following Figure 4.33 is the new preliminary investigation in the beginning of lesson 5. It also can be found in the appendix of students' worksheet (Appendix 1).

Hali,, kita bertemu lagi. Kemaren aku lihat beberapa kaparan yang kalian buat. Semuanya keren dan unik. Pada saat melihat laporan itu pula aku menemukan hal yang menazi Coba kalian pertahtikan laporan kalionyang diberikan gurumu	Bandingkan pula gambar dari bagian kanan dan kiri candi. 1. Apakah ada kemiripan dari kedua gambar tersebut? Jelaskan!
kemudian jawab pertanyaan berikut.	2. Apakah ada perbedaan pada kedua gambar tersebut? Jelaskan!
andinakan aambar dari baajan deban dan belakana candi.	
 Apakah ada kemiripan dari kedua gambar tersebut? Jelaskan! 	Kesimpulan apa yang kalian dapat dari pertanyaan di atas?
2. Apakah ada perbedaan pada kedua gambar tersebut? Jelaskani	

Figure 4.33 The new investigation problem in lesson 5.

All the revision are based on the findings that we encountered during the preliminary teaching experiment. The improved HLT can be found in the appendix. The following Table 4.8 generally describes the improved design of the activities after the preliminary teaching experiment. These activities are then implemented in the teaching experiment.

Table 4.0 Learning fine for the teaching experiment	Table 4.8	Learning	line for	the teaching	experiment
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Meetings	Learning Goals	Spatial ability tasks	3D representation	Students' Activities
Meeting 1 Playing with Camera 1	 Students experience spatial orientation and spatial visualization of an object. Students understand that 3D objects have different views. 	1. Preliminary spatial orientation and spatial visualization task	1. Preliminary activities to work with models and representation.	1. Teachers give the students an arrangement of 3 objects (Apartement, Houses, and tower) and asks students to determine where the students shall put the camera to have a picture of all the object, two objects, and one object. Students also have to find the best position to take photos of the object.Students will work with two camera models and a digital camera
Meeting 2 Playing with the camera 2	 Students are able to identify the stand point of different standard views of an objects. Students are able to distinguish between standard views of an object and standard views of another similar objects. 	 Spatial orientation task Developing students' spatial orientation through identifying photographs. 	 Developing students' ability to recognizing the position of different views of an object with the help of the models. Bridging students to work on distant representation of 3D objects. 	 The students analyze and identify the photos of 3D objects. They have to put them in the right position of where it was taken. There are 12 photos from 8 position. However, among these photos there are 4 photos that do not belong to the object. Students will move around the objects to find the right position of the camera in the given pictures. Students will work with two camera models and a digital camera.
Meeting 3 Reporting new temples	 Students are able to draw standard views of 3D object from different stand points based on its model. Students are able to draw standard views of an object based on its distant representations. 	 Preliminary spatial visualization task Spatial visualization task Developing students' spatial visualization through perspective task 	1. Developing students' ability to determine and draw different views of 3D objects from different stand points based on either its model or its representation.	 Students create their own temple by using exactly 8 cubes and then sketch or draw the standards views of the temple by using the camera model and digital camera. Students also have to draw the standard views of a new temple in a bird eye photo. They draw the view in the layout. Students also are allowed to use building blocks to help them thinking. Afterwards, students have to draw one standard view of a temple presented in a photo. Different from the previous task, the drawing is on different layout.
Meeting 4 Building a temple	 Students are able to identify, predict, and read the blind spot of distant representations of 3D objects. Students are able to interpret, recognize, and communicate distant representation of 3D objects. Students are able to construct an object based on its standard views. 	 Spatial visualization task combined with spatial orientation task Developing students' spatial ability to support their ability in understanding 3D representation and spatial structuring 	 Developing students' ability to read, interpret and reason distant representation of 3D objects. Building an object based on its photograph and its standard views. 	 Given photos of a new temple (bird eye photos). Students have to build the temple by using cubes/building blocks. Afterwards, teachers give three standard views of the temple (top, back, side) to equip the bird eye photo. Students also have to build a temple based only on its standard views in a template/layout without its bird eye photo by using building blocks.
Meeting 5 Fixing the reports	 Students are able to identify the properties of standard views of 3D objects. Students are able to draw the standard views of an object based on its opposite standard view. 	 Spatial visualization task combined with spatial orientation task Developing students' spatial ability to support their ability in understanding 3D representation 	 Investigating basic relations among standard views of 3D objects. Recognizing the views of an object from different stand points based on photograph and another standard views. 	 At the beginning, students will investigate the previous reports from the previous lessons. They compare the back view and the front view of the temple to find the properties of the standard views. Similarly, they also check the top, the left and the right view in the reports. Teachers give three cases of different 3D objects. Each case consists of 3 standard views. Students have to predict and draw the shape of the object on the other stand points. Afterwards, teachers give 5 photos (front, sides, and top) which belong to an object of building blocks. Student have to guess how these photos are originally arranged in the layout. Teachers will not provide them the object and camera.

2. Teaching Experiment Phase

Teaching experiment was conducted at 31 March until 7 April 2015. There were five lessons during that week involving 24 students and a regular teacher of the class. Actually, the class consisted of 25 students but one student was absent for the whole week. Each lesson took 2x35 minutes with two observers, including the researcher, recording students' activities. The teaching experiment itself was held in the aula of the school so that students had more space during the activity. But, Lesson 3 and 5 were conducted in different place, in the multimedia room, since the aula was used by another event.

During the sequence, students were divided into six small groups of 4-5 students. These group stayed the same until the end of the sequence. To get more specific data, we choose one group as the focus group to be observed by the researcher. This group was group 4. Meanwhile, the other observer recorded the other groups' activities. The observers sometimes interviewed the students to confirm their understanding or to dig further what students really did during the activities. After the lesson ended, all the data including the field notes and students' written works were collected or documented. The researcher also did small interview with the teacher after each lesson to analysis the findings and prepare for the next meeting.

Actually, students got a pretest before the sequence began and at the end of the sequence, students also got a posttest to provide us additional data during the experiment. However, in the posttest, two students could not come and attend the test because of sick. In general, the teaching experiment went well without much obstacles encountered. The details of students' activity in each lesson will be discussed and analyzed in the retrospective analysis.

3. Retrospective Analysis Phase

a. Pretest

Pretest was conducted one day before the teaching experiment began. There were 24 students attending the test for around 30 minutes. Similar to the pretest in the preliminary teaching experiment, the test consists of three parts. The first part is to investigate students' sense of direction in a relative system of "Right or Left". The second part was to check how students solve spatial orientation problems while the third part was about spatial visualization problems. The aim is to have insight of students' spatial ability and their understanding of 3D representations. After the pretest, 10 of the students were chosen to be interviewed by the researcher to check and confirm their works.



Figure 4.34 The photo for the first and second question in part A of the pretest.

The first part of the pretest consists of two questions. The first one asked students to determine what kind of fruit placed in the right or left side of the photo. On contrary, the second question required students to imagine themselves in the photo and determine who stands on the right side or on the left side. The photo for these questions can be seen in the Figure 4.34. For the first question, almost all the students could mention which fruits are on the right or on the left. Among 24 students, only one students falsely mentioned the fruit. In contrast, only 7 students answered correctly the second question. During the interview of some students, mostly they could not imagine themselves to become the person in the photo and determine the left side or the right side. This shows us that students' prior orientation is themselves. They were not common yet referring to things as the reference for orientation.

The second question asked students to figure out how the photo will look like if they take pictures of things on a table from certain position. The photo of the question is in Figure 4.35 in which a miniature of a city sat on the table. Students are instructed to imagine themselves walking around the table and stop in position B and C. Afterwards, they have to choose two photos among 4 given photos that represent the view of the object from position B and C.



Figure 4.35 The photo of the objects for the problem in part B of the pretest.

The result of the pretest shows that only 7 students could correctly answer this problem. Based on the data of the students from the teacher, some of these 7 students were high achievers in mathematics. The other 17 students mostly said that the question was too difficult and they could not imagine it. Even among the 7 students, one student that was randomly involved to be interviewed said that she actually randomly choose and did not know which the photos will be. This finding gives us a picture of how students spatial orientation at their age. Mostly, they cannot imagine themselves to be in the photo and recognize the shape. During the interview with some of the students, they showed that actually they could follow the instruction and imagine walking around the table. However, they did not have idea how to imagine the situation. In conclusion, most of the students have poor spatial ability. They were not able yet to process the information in the given situation to determine the view of the photo.



Figure 4.36 The photo of an object for the problem in part C of the pretest.

In the third part, the problem was about drawing the view of an object in a photo from a certain stand point. The photo of the object is in Figure 4.36. The students generally could not solve this problem. There were only 7 students who drew the view close to the right one. Figure 4.37a and 4.37b gives examples of the correct drawing by the students while the other examples in Figure 4.37 are wrong. During the interview with some of the students, they said that the drawing was difficult and also could not imagine the shape of the object. Mostly, they just copy the object in the photo like Figure 4.37d. They admit it during the interview because they did not want to let the question blank. That is why the draw something on it. This result confirms that students' ability to identify the shape of the object was lack. Probably, some of them were able to imagine the shape but they hardly drew it on the paper. However, when they were interviewed, none of them showed that they were able to imagine the object.



Figure 4.37 Students' answers for the problem in part C of the pretest.

In conclusion, the pretest result shows that students' spatial ability mostly were poor. They cannot imagine the object in the photo and identify its view. Furthermore, problem 2 also indicates that students could not yet connect different information of an object from different position in space. However, all the students already had sense of direction to determine left and right based on their body as the center of orientation.

b. Lesson 1

As we have explained in chapter IV, students became a photographer with the tasks to take pictures in a city. Figure 4.38 shows the miniature of the city and the stand points for students to look the city. There were three tasks to do in small groups of 4-5 students. Firstly, students have to determine what buildings they will see from each stand point. Secondly, they also have to describe the position of the other buildings in the photo with respect to the apartment. Lastly, each group have to find the best position to capture the buildings. The aim of these problems is to engage students to observe and experience the orientation of objects.



Figure 4.38 The miniature of the city on the layout for lesson 1.

In the first problem, students in the focus group (Group 4) used both the camera model and the digital camera to observe the city. To determine what buildings they saw, they moved around the table and stopped at each stand points. They looked through the camera model while closing the other eye. This students' strategy indicates that the camera helped the students to visualize the objects. To confirm their answer, they also captured the city from each stand point by using the digital camera. Based on our observation, students really experienced to observe the views of the object from many angles. In the other group, there was a students who saw the city from a bird eye angle. That is why the students stated that all the buildings were visible in each stand point. However, after the teacher argued that the photographer stand on the ground and cannot fly, this students accepted the reason and changed the way she looked the city. The latter finding confirms that the students still could not yet imagine the images of the object from bird eye angle. They think that the bird eye angle and the horizon produce the same views. Without the camera, they struggled to determine the visible buildings which indicates they could not yet create images from bird eye angle. Overall, all groups did not have much difficulties to solve the first problem.

The following Figure 4.39 shows the focus group written work of first problem.

Posisi	Rumah Merah	Rumah Pink	Menara	Apertemer
1	V	-	V	V
2	V	-1	-	V
3	V	4	1	~
4	4	1	1	-
5	V	~	1	~
6		2		V
7	1	\checkmark	-	V
8	~	-	1	\neg

Figure 4.39 The Focus group's answer for problem 1 in lesson 1.

In the Figure 4.39 above, students correctly put checks to indicate the buildings they saw. During the class discussion, some groups mistakenly identify the visible buildings from stand point 2 and 6. This was due to the angle of vision was very narrow like Figure 4.40 shows us. This caused some groups count a slightly seen pink house as visible while the other groups counted it as unseen. The focus group also decided to count the pink house for stand point 2. However, teacher asked the students to not put a check for the pink house.



Figure 4.40 The vision line for stand point 6 and 2 of the problem in lesson 1.

In the second problem of describing the position of the buildings, all the group used their body as the center of orientation to determine which side is right or left. None of them used cardinal direction or the apartment as the center of orientation. Figure 4.41 is one of the answers of the students in the focus group. As we can see in their answer, they wrote the position of the red house was in the right side of the apartment and the pink house was in the front of the apartment. They did refer to the apartment but to determine the side, they used their orientation of right and left. We also noticed that students were not influenced by the position of the door of each building. For example, the focus group in Figure 4.41 said that the pink house is in the front of the apartment. In fact, the pink house is behind the apartment because the door or the front side of the apartment faced to stand point 1. All the group did the same thing by ignoring the real condition and describing the position of the building based on what they saw in the camera.

5. Jika kamu mengambil gambar dari pe merah, rumah pink dan menara terhada	osisi 5, dimana posisi rumah p apertemen pada foto?
Romah merch. di barran Aparlemen Romah pink. Di depun Aparlemen	
Menara . sebelah kahan Apartemen	Translation:
	Red house: In the right side of the apartment
	Pink house: In front of the apartment
	Tower: In the right side of the apartment

ł.

Figure 4.41 The Focus group's answer for problem 2 in lesson 1.

In the third problem of finding the best position, students mostly chose stand point 3 and 5. Mostly the reason was because they could see all the buildings. On the other hand, to capture the pink house, students' answers were vary. The reason was personal justification that the pink house looked more beautiful in photo. Three groups picked stand point 5 to capture the pink house since in the photo, the pink house will be in the front side.

During the investigation, students really depended on the use of camera. For instance, Aurum, one of the students in the focus group, guessed the buildings she saw from a stand point. But she wrongly answered that she could saw it since she did it from bird eye angle. After using the camera, she realized that there was a building that she could not see. This finding suggests that students could not imagined the standard views by looking from bird eye angle. The camera helps them picturing the view of the object in their mind. This indicates that the camera and the activities have supported the students to experience spatial objects and spatial images. In conclusion, students experienced directly and observed spatial objects and spatial images. The camera really did help students picturing the views of the buildings they see. Furthermore, they also could describe the position of the buildings by using relative system of right and left.

c. Lesson 2

In this lesson, students worked in small groups to investigate the printed photos that the teacher gave them. There were 12 photos where 8 photos were of the object while the rest was not. Each group had a set of object in the middle of table (Look Figure 4.42) and they have two tasks to do. First, students have to figure out the position of each photo where it was taken. Second, they also have to distinguish the wrong photos and describe why they are not from the object. The aim of the activities is to help students making connections between the photos and the object as a bridge to develop their spatial orientation.



Figure 4.42 The drawings of group 2 in activity 1 of lesson 2

During the investigation, students moved around the table like they did in the previous lesson. They used the camera model to identify the view of the object. To find the position of the photos, students in the focus group picked the photo and then moved around to check the view of the object. If they found a match between the view of the object in the photo and the view of the real object, then they claimed that the photo was taken from there. Similarly, to distinguish the wrong photo, they also checked each position and compared them with the real object. If they could not find a match, then they put aside the photo. The following scheme in Figure 4.43 illustrates students' strategy to investigate the position of the photo.



Found a match Do not have a match **Figure 4.43** The scheme of the students' investigation in lesson 2

As the discussion went further, students started not to use the camera model to help them picturing the view. This is a sign of development of students' spatial ability. Before, they hardly imagined the view from bird eye angle without the camera. This indicates students began to be able to create mental images based on what they saw from bird eye angle. This was different from the lesson 1 when students still need the camera and look directly from the horizon. The way students differentiate the wrong photos was also growing. Adit, a students from the focus group, straightly compared the view in the photo and the view from the real object from a stand point. He indicated that there was a part of the object that face towards different direction from the object in the photo. Hence, he concluded that the photo was not of the object. The following Fragment 4.5 of discussion illustrates how Adit argued the wrong photo.

1 Adit : [Photo 7] this should be here, but this part should be there [Pointing the L object in the photo and the real object].



2 Observer : What should be there?



- 3 Adit : The part is here [In the photo]. It is actually there [In the real object]
- 4 Observer : Is it reversed?
- 5 Adit : Yes, it is reversed

Fragment 4.5 Students' discussion in lesson 2 of teaching experiment

Figure 4.44 shows how students reasoned why the photo was not from the object. They circled parts of the object and wrote "kebalik" which means "reversed". This indicates that students did making connection between the information in the photo with the information they got from the real object. By comparing the photos to find a match, they had developed preliminary spatial orientation from a given small situation of an object.





During the class discussion, there was one group, group 5 that had wrong answers. Figure 4.45 shows the answers and circles the misplaced photos. From students' answers, we noticed that only photos in position 1, 4, 5 and 6 were correct. We conjecture a possibility of students' thinking that they placed the four photos randomly. In the students' written work, they wrote that the number of photos were exceed the number of positions. It seems they referred to the previous layout where the number of stand point was 8. In addition, they thought that only photo 1 until 8 were used. This possibility is in line with students' explanation in their written work that the numbers on the photos are not the same. Unfortunately, we did not record this group's investigation during the observation. Therefore, we cannot clearly show what the students really intended to do. Afterwards, in the class discussion, the group realized their mistake and wanted to revise their answers. It seems the discussion among the students did not go very well in that group. This case actually have been predicted in the HLT. However, the teacher did not give further guidance for this group in small group discussion.



Figure 4.45 The answers of group 5 for the problem in lesson 2

Based on the analysis above, we conclude that the activity in this lesson have help students developing spatial orientation. During the activity,

students are engaged to process different information of the same object. They connect all the information from different position in space by figuring out the position of the photos. In this case, they compared the view of the object with the view of the real object on the table. During the activities, students also began to be able to create mental images of the object without the use of camera model. Furthermore, they were able to do this from bird eye angle.

d. Lesson 3

In small groups, students became an archeologist with the task to report their temple by drawing its standard views from 5 different stand points: front, back, left, right, and top. There were three problems in this lesson. Firstly, students created their own temple by using exactly 8 building blocks and then drew its standard views. Second problem was asking students to draw the standard views of the temple in a photograph given by the teacher. Lastly, they also have to determine and draw the standard views of the temple in a photo from a certain stand point. The aim of these problem is to develop students' spatial visualization by investigating and drawing the views of the object. Furthermore, the activities also aims to assist the students to shift from working with the model into working with its distant representation.

In the first problem, students in the focus group constructed a temple shown in Figure 4.46. To draw its standard views, some of them used the

camera model to help them imagine the shape of the object. Differently, Erdi did not need the camera model. He could identify the shape and drew the shape easily from bird eye angle. In fact, most of the students in the other group still need the camera model to help them drawing the shape. Some students from the other group even draw the shape in bird eye style. They hardly imagined the shape until they carefully look the shape from the camera. This was due to the task that asked not only to determine or imagine but also to draw it in the layout. In the previous lesson, students were able to determine the view of an object without the camera model. However, since now they had to put the images in the drawing, they struggled and needed the camera. In this step, the camera really helped the students to determine the view and guided them in the drawing process. The following Figure 4.46 is the answer of group 4 (the focus group). From their drawing, students showed that they can identify the shape and draw it. To do this, students need a good visualization of the shape. Even for drawings from stand points 2 and 4, they drew it perfectly.



Figure 4.46 Focus group's temple and the drawings of its standard views.

On the other hand, some students found difficulties to identify the shape. Debi, a student from group 5, found herself difficult to identify the shape of her temple. Figure 4.47 shows her initial drawing as the view of the temple from the direction of the arrow. When she was interviewed by the observer to confirm her understanding of her drawing, she confused and smiled as a sign that she was in doubt. In the Figure 4.47, Debi confused to point the position of the cube in her drawing (Circled in red). Even though her teammates tried to argue and show her that the cube should not be there, she was still in doubt. After the observer asked her to use the camera again and asked Debi and her friends to check the drawing, she admitted that the shape of the temple in her drawing was different from the photos or from what she looked from the camera. The process of identifying and imagining the shape needs higher level of spatial visualization and understanding 3D representations. It is different from the previous tasks when students only need to recognize and match the information. Some students need more time to do this kind of tasks. The camera model once again played an important role to help those who struggled to visualize the objects. Like Debi's case, students realized their mistake and used the camera once again to get the view of the object.



Figure 4.47 The drawings of Debi from group 1 in activity 1 of lesson 3

In the next problem, students were given a photo of a temple (Look Figure 4.48). Firstly, Aurum counted the number of the cubes and she got 10. However, Erdi and Adit argued that the cubes should be 11 because she missed to count the cube under the other cube in the photo. This finding suggests that Aurum's understanding of the representation was not completely clear. In fact, some students in the other groups did the same as what Aurum did. After some of their friends argued that the cube was actually existed, they realized it and could imagined it. To convince Aurum, Erdi even constructed the object by using building blocks to show that actually the cubes were 11.



Figure 4.48 The drawings of the focus group in activity 2 of lesson 3

During the discussion, the use of the cubes really helped students to imagine the object. As we have predicted, they were not able to directly create mental images of the object to be manipulated. For instance, by using the cubes in the second activity, students could figure out the correct number of the cubes to build the object which was 11. Some students were able to notice the cube underneath the other cube, but for most of the students, that was difficult. What Erdi did by demonstrating the object to explain the number of cubes truly became a trigger for Aurum and Rere to imagine the existence of the last cube.

To identify the shape, all the group had to construct first the object to help them imagining it. This fact shows that the students could not work directly without the model of the object. That was why as we had predicted they created the model to help them drawing. For the third grade manipulating and recognizing the view of the object in a photo was very difficult. In this lesson, students could not completely work without the blocks. In the third problem in Figure 4.49, students also used the block to help them identifying the shape. However, since the drawing layout was in the other paper, it forced them to memorize shape before drawing it in the layout.



Figure 4.49 The drawings of the focus group in activity 3 of lesson 3

During the process of drawing, there were two groups that drew the view of the object in bird eye mode. It seems they tried to imitate or copy the view of the object in the photo (Look Figure 4.50). In this case, the students in the groups could not yet imagine to look the object from the instructed stand point. This suggests that the shift to manipulating 3D representation with distant representations was difficult for the students in

grade 3 if they did not have the model. After the teacher guided them by using the cube to help them think and imagine the view, they revised the drawing and corrected them. When the teacher gave them the cubes, students could imagine the shape of the object viewed from the stand point. As we have conjectured, students who found difficulties in this problem needed more assistance of the use of model of the object to help them understanding the photo.



Figure 4.50 Examples of drawings in activity 3 of lesson 3

Again, the use of cubes in this problem really helped the students to realize and identify the shape. Some maybe could do this without the cube after experiencing the activity 2 in this lesson. However, students mostly still need it. For instance, students could easily imagine the view of the object when the teacher asked the students to look at the real object and compare it with the photo. When the object was taken away, students had already mental images in their mind regarding its view from the desired stand point.

In summary, the use of the cubes really becomes the key to help students' understanding the object in its distant representation (photo). Most of the students were able to create mental images or visualization by using the cubes as the guidance. The cubes also helped students shift from working with the concrete objects into working with its distant representation. Furthermore, the camera model also played an important role by helping students visualizing the object and guiding the drawing process.

e. Lesson 4

In lesson 4, students worked to build a temple based on its photographs. There were 2 problems employed in this occasion. The first problem asked students to construct a temple presented in a bird eye photograph (Figure 4.51). The aim of this first problem is to give students preliminary activity before they decode the standard views of an object. The problem was also to make students realize the blind spot of distant representations. On the other hand, the next problem was constructing a temple based only on its standard views. Unlike the first problem, students did not have the bird eye photos of the object. The aim is to develop further students' ability in understanding 3D representation. These problems were also to investigate how students' spatial ability assists them in the discussion and investigation.



Figure 4.51 The bird eye photo of the object in activity 1 of lesson 4

In the first problem, most groups took more time to finish the object in the bird eye photo longer than the students in the preliminary teaching experiments. Students mostly created the miniature by counting the number of the cubes used in the photo while constructing the object. All the group agreed that they need 15 cubes without realizing that they had guessed how the back part of the object looks like. After several minutes, all the group finished constructing the object without much difficulties. The object is shown in Figure 4.52. At this stage, all the group constructed the same object with the same back view. Therefore, to begin the discussion about the blind spot, the teacher proposed another temple which was similar to the photo but it had different view from the back (Look Figure 4.52). After discussing it with the students, all the groups agreed that the back part cannot be built unless they have another photos from the back. Students figured out the blind spot by comparing their construction with the teacher's proposed construction using the camera model. They viewed the constructed object from a stand point where the photo was taken. By doing this, they saw both objects have similar view from that stand point.





Figure 4.52 Students' construction and teacher's proposed model in lesson 4

After the teacher gave each group another photos of the temple from the back, the right, and the top, students began adjusting their construction. They started from the top view and then compared the result with the view from its side. The following objects in Figure 4.53 are students' final constructions. Most of the group built the object like Figure 4.53a, but there were others that build the object like Figure 4.53b. The teacher then asked them to compare the temple with the bird eye photo. Those who had the object like Figure 4.53a changed it into Figure 4.53b by removing one cube. This was because in the bird eye photo that cube was not exist.

In the beginning students did not realize the back part of the objects. Although they had created mental images of the objects like Figure 4.52, they did not aware that they had assumed the unseen part of the temple without supporting information. In this case, students' spatial orientation and spatial visualization played an important role to figure out why the proposed temple by the teacher was correct. For instance, in the focus group, Erdi tried to find the stand point of the given bird eye photo in the temple he constructed. His spatial orientation had enabled him to find directly the stand point and then moved the camera to the same stand point in the temple constructed by the teacher. By doing this, he and his group found out that both construction were the same and the unseen part could not be determined. In order to build the object, students' spatial visualization ability also helped them build the correct temple. Firstly, they need to create mental images of the objects in mind based on the photo and turn it into reality by using cubes. Then, they also needed to adjust the temple based on the given standard views. Sometimes, they did mental movement in the process to find the correct construction.



Figure 4.53 Students' final constructions of activity 1 in lesson 4

The second problem asked students to build a temple based on its standard views. The standard views were placed in the layout of a report like Figure 4.54. This problem is more difficult than the first one since the students do not have the bird eye picture of the object. Thus, they did not have clear picture of how the object will look like. To build this temple, some groups begin the construction from the side view. Then, they made the object satisfy the other view. Based on our observation, students really struggled to adjust the object. On the other hand, most of the groups started the construction from top view and later, they revised it based on the side views. Although these groups also struggled to build the object, they were quicker than some groups who started building the object from the side.



Figure 4.54 The drawings of group 2 in activity 1 of lesson 3

The following scheme describes the process students in the focus group went through to create the miniature of the temple. They started by putting cubes on the top view and then adjusted the object based on the side view. Afterwards, they checked the object from another stand points to revise the object again. This process continued until the object satisfied all the standard views. During the process, the students in the focus group had to repeat to adjust the object from the same stand point. This happened because when they tried to revise the object from one stand point, the view of the object from the previous stand point was broken. Thus, they had to repeat revising the object again. In the end of the activity, most of groups including the focus group constructed the object like Figure 4.55a in the scheme. However, there was one group that had final temple like Figure 4.55b in the scheme. The teacher then added an information that the number of the cubes used to build the object was only 9. Therefore, students revised their temple by removing one cube to make it satisfy the information.



Figure 4.55 The scheme of how students construct the object in lesson 4

From students' strategy, we can see that students employed spatial ability to understand the standard views and build the object. First, to recognize the shape of the temple, they combined spatial orientation and spatial visualization at the same time to build and adjust the object from each stand point. Secondly, they manipulate the images of the object in mind and the actual object such that it can be fit to the standard views. The manipulation aspect did not happen as much as the first problem since this problem did not involve bird eye picture. However, the manipulation and the recognition process happened at the same time in order to build the correct object. This process can be see when students tried to perceive the shape of the object from one stand point and at the same time have to imagine the form of the object from the other stand point.

Based on the analysis above, we can conclude that students are able to identify the blind spot of the object through this activity. Furthermore, students are also able to connect the standard views of the temple to construct and build the temple by using the cubes. Therefore, the activities in this lesson successfully helped students to develop their ability to decode the information in the distant representations of the object. In the process of solving the problems, students used their spatial ability to understand the standard views/photographs and construct the temples.

f. Lesson 5

The last lesson was about investigating the properties of standard views of an object. There were three problems to do in small groups. The first problem was a preliminary investigation to find the properties of standard views of an object. The second problem asked students to fix three reports of a temple. These reports lose some of its standard views. Therefore, each group have to draw the shape of the lost standard views in the reports without knowing how the object look like. The last problem in this meeting was about rearranging the photos of standard views of a temple in the report. Similarly, students did not know how the temple look like and they also do

not have the building blocks of the object. These problems aim to develop further students' understanding of 3D representation and to investigate how students' spatial ability assist them in the discussion and investigation.

In the first problem of investigating the properties of standard views, students did not find much difficulties. Mostly, students identified that the shape and its opposite had the same number of cubes. The shapes were also "similar". During the discussion, group 5 added that the shape and its opposite faced different direction. In the conclusion, all the groups agree that the front and back views were similar. The left and the right view were also similar. On contrary, the top view did not have similarities with the other standard views. To discover the properties, we noticed that students did not use much their spatial ability in the process. Although, they successfully understood the properties based on the previous report, they did not experience further spatial ability and understanding 3D representation. Probably, in the future, the investigation should be more difficult and employ the use of blocks so that they can explore more and develop more than they did in this lesson.



Figure 4.56 The drawings of group 4 in activity 2 of lesson 5

In the second problem, students did really well. They concluded that only report 1 could be repaired while report 2 and 3 were impossible to fix (Look Figure 4.56). At the beginning, some group confused whether report 2 and 3 could repaired or not. The teacher guided them by referring them to their previous investigation about the properties of the standard views. Afterwards, they used the properties of standard views of an object to determine the lost shapes in each report. To draw the shape, they looked to the opposite view and imitated it. The following Fragment 4.6 of discussion illustrates how students explained and fixed the report in the second problem.

1 : Why did the drawing become like this? [Pointing the red circle] Observer



2 Adit

3

4

5

- : Because of this [Pointing the green circle] Observer : Oh, this one [red circle] is similar to this?
- : Yes Adit
- : What about this? [Pointing the black circle] Observer
- 6 Aurum : This [Pointing the blue circle]
- 7 Observer : So, can this report be repaired?
- Students in the group: Yes 8
- 9 : What about this? [Report 2] If this photo is lost, can you draw this Observer view? [Red circle in the picture below]



- 10 Erdi
- : Yes, you can. Using this [green circle]
- : Okay, now what about this stand point? Can you draw it? [blue 11 Observer circle]
| 12 | Erdi | : I think you can. The object is two level high so there will be two |
|----|----------|--|
| | | squares. |
| 13 | Observer | : How many level? How do you know this? |
| 14 | Erdi | : I guess. Umm [thinking] |
| 15 | Observer | : If you have this photo [Black circle], can you find its view from |
| | | here? [Blue circle] |
| 16 | Erdi | : yes, you can |
| 17 | Observer | : What if you do not have it? |
| 18 | Erdi | : Then you cannot drew it |

Fragment 4.6 Students' discussion in lesson 5 of teaching experiment

From the fragment, students clearly referred to the properties of standard views that they found in the first investigation. Students also tried to imagine building the object in their mind. This can be seen at line 12 in the fragment. However, students struggled to imagine the building because they did not have the side view. As a result, they guessed or assumed the object as we have predicted. When the observer brought back the properties of the standard drawing, students realized that actually they could not do that.

In the process of reasoning for report 2 and report 3, students did imagined the object in the middle of the layout. As what Erdi did in the discussion above, they tried to create mental images of the object based on the given standard views. This clearly needs good spatial ability of the student. In the process, students struggled to complete the images of the object in mind since they did not have the remaining standard views. Students' spatial ability played the same role as the previous activity in lesson 4 as students imagined building the objects. We noticed only some students could reason using imagined building activity. Some students did not do this mentally building object and directly refer to the properties of the standard views. This indicates that understanding 3D representation in mental level is difficult for their current level. During the discussion, some groups wrongly imitated the opposite view without changing its orientation. To guide these students, the teacher asked them to look the previous report and once again search for the similarities of a standard view and its opposite. By doing this, they realized that they had made mistake and the view should be not only similar but also reversed.

In the last problem, students firstly confused how to rearrange the photo. Some of them guessed by putting the photos randomly. However, they realized that the object cannot be built and the arrangement did not satisfy the properties of standard views of an object. To guide the students, the teacher asked them to imagine the object and count the number of cubes on the top view. By doing this, they realized to put the "L" view into stand point 1 and 3 because the shape had the same number of cubes in a row. Meanwhile the rest was placed oppositely on the stand point 2 and 4 like Figure 4.57a. During the discussion in the focus group, the observer asked why they did not put the L on the stand point 3 vertically or reversely. The students responded that the shape would not fit the properties of standard views, if they put them reversely from the current position like Figure 4.58b and 4.57c. Besides, the number of the cubes in a row must be the same as the row on the top view.



Figure 4.57 The arrangement of standard views by students in lesson 5

From students' discussion, we can see that students struggled to put the photos such that they fit altogether. To do it, they imagined the object on the top view to be observed. By doing this, they obtained the front and back views by referring to the number of cubes in the row. Imagining the object in the middle involves the use of spatial ability. In order to recognize the form of the object, students probably did the building process like lesson 4 of on the top view. For the future research, probably asking students to build the object to check whether their arrangement are fit or not, is good addition so that students could directly see their imagination of the temple..

In summary, in this lesson, students were able to identify the properties of standard views by investigating the previous reports. During the investigation of drawing views based on the other standard views, some students imagined building the object to help them reasoning the drawing. However, they did not success build it and referred to the properties of the views to repair the report. Students' spatial ability did not play a big role in the preliminary investigation of the properties. However, during the second and third activity, students' spatial ability help students to imagine the object based on the standard views.

g. Posttest

Posttest was done after the last lesson. It involved only 22 students because 2 students were absent because of sick. The posttest consists of three parts similar to the previous posttest used for the preliminary teaching experiment. The first two parts are the same problem in the second and the third part of the pretest. However, we develop a little bit more by adding more tasks to it. The last part of the posttest was about building objects based on its standard views. In addition, the researcher assessed directly by interviewing the students one by one to check their understanding when building the object. After the students finished the test, 10 of the students were chosen to be interviewed by the researcher regarding their answers in each part of the posttest.



Figure 4.58 The photo for the questions in part A of the posttest.

The first problem asked students to match the given 4 photos into the position of where they were taken. Figure 4.58 shows the object and how they were set up on the table. In the pretest, only 7 students could answer correctly the problem while the others could not imagine the situation. Among the 7 students, there were also those who just randomly answered and fortunately it was correct. Different from the result of the pretest, 15

students were able to solve this problem in the posttest. Meanwhile, 2 students only solved half of the photos while the other half was incorrect. We interviewed some of the students, especially those in the focus group, regarding how they solve the problem. One of them, Aurum admitted that she could imagined the situation on the table easily. The strategy she used to solve it was by imagining what building she would see. She added these buildings must occur in the photo. During the pretest, Aurum hardly imagined the object and could not solved it. She said that the problem was too difficult for her. Therefore, when she was asked by the interviewer regarding her statement during the pretest, she just smiled and said 'I do not know but it is easy'. She figured out the photos by imagining the object. The key word here is "imagine". This suggests that Aurum created mental images of the object situated on the table. She imagined herself looking the object from each position to determine which building she would saw. Before, she admitted in the pretest that she cannot imagine the problem and gave up. The way students solved the problem and reasoned it indicated that they have developed their spatial ability and their understanding of 3D representation.



Figure 4.59 The photo of the object for the problem in part B of the posttest.

In the second problem, students drawing two views of the object in Figure 4.59 from stand point A and B. There were 15 students who drew correct view of the object. The other 4 students drew the views correctly but they falsely put the drawing on the layout. For instance, Aurum drew correctly the view of the object from stand point A, but when she was interviewed, she realized that she put the drawing on stand point B instead. Different from the posttest result, we knew only 7 students could solve this problem in the pretest. This emphasizes that the activities have supported students to identify the view of the object and help them understanding the representations. However, we also found some students who still could not solve the problem. There were four students who could not imagine the view of the object in the photo. We also found interesting drawing by some students who draw the object in bird eye style. These drawing was not usual bird eye view since the students did not copy it from the photo. They imagined the object from the stand points and then drew it in bird eye style. This kind of skill need considerably good spatial ability. The following Figure 4.60 shows some examples of students' drawings in the second problem of the posttest.



Figure 4.60 Examples of students' drawing in the second part of the posttest.

In the third part of the posttest, students were given cubes to build the object based on its standard views (Look Figure 4.61). During the lesson 4, students looked really struggle to build the object. They started from its top view and adjusted it based on its side view. Surprisingly, most of the students easily solved this last problem. Probably, the object was simpler than the one they got previously. There were 17 students could both built the object and explained it during the interview. They showed good understanding of the object and related the standard views to the real object. Figure 4.61 presents some of the example students' constructions. There were only 3 students who could build the object while the other 2 students built the object but they could not explain it and did not show good understanding of the object.



Figure 4.61 The photo of an object for the problem in part C of the pretest.

Based on the result of the posttest, students were able to solve the problem without having much difficulties. This result confirms what we found during the learning activities that students developed their spatial ability in understanding 3D representations.

h. Discussion and the Conclusion of Teaching Experiment

1) Students' spatial ability and understanding of 3D representations

Based on the description and the analysis of each lesson, we found several points to discuss regarding the activities and how they support the students. First of all, the activities did help students to develop their spatial ability. This was indicated by what students did during the activities especially in lesson 1, 2, and 3. In lesson 1, students observed the object by using the camera model and moving around the table to identify the buildings they saw. This process truly helped the students as a bridge to develop spatial orientation and spatial visualization. This is because to have good spatial orientation, students have to be able to process and connect many information of an object from different positions [McGee, 1979]. This was done by observing directly the object. The activity in lesson 1 also introduced students to the visualization process of objects from certain stand points. In this lesson, students still needed the camera to do the visualization. This process later became the preliminary activity for students to create mental images of the object. During the investigation, some students claimed that all the buildings can be seen from each stand points. This proves that the students actually have not been able to imagine and visualize the view and concluded based on how they saw the objects from bird eye angle. However, by using the camera, they finally understand how they should view the object. The camera helped them to do the visualization of the objects from different positions. As the results of lesson 1, students had been able to gather and process different information in space as a preliminary ability to spatial orientation and visualization. In addition, students also had been able to perform preliminary mental movements registered by physical movements and create mental images with the help of camera model.

In lesson 2, students forced to compare the photos with the real object. The strategy and the reasoning performed by the students were also develop. First, they need the camera to help them view the object, but then they left it out. This indicates that students had been able to visualize and create images (mental images) of the object in their mind from bird eye angle. To figure out the position of the photo, the strategy grew from checking all the position one by one into directly choosing a possible stand point for the photo. This indicates that there is a process of students viewing the object in their mind from those possible stand points. That is why they can guess the possible positions. Furthermore, the comparison activity suggests that students making connections between views from different positions. In this case, the views in the photos are compared to the views of the real object. According to (McGee, 1979), this the basic ability of spatial orientation. Therefore, we conclude that in lesson 2, students had developed spatial orientation ability by making connections among different information in space. Furthermore, they also were able to create mental images without the help of the camera.

In lesson 3, students further develop their spatial ability, especially their spatial visualization. From their activity, we can see that students firstly struggled to determine and draw the standard views of the object. Some still drew the shape incorrectly both during the pretest and also during the beginning of the investigation. In the process, the use of the camera played an important role to help students visualize the shape. Those who difficult to imagine the shape or drew the shape incorrectly used the camera to look the object so that the view of the object can be obtained. The camera offered a guidance for the students both to determine shape and also to draw the views. After experiencing this process, some students began not to use the camera model to identify and draw the view. At this stage, students have develop the ability to create mental images of the object and manipulate it in mind. Furthermore, in the end of the activity in lesson 3, students could determine the shape of the object represented in a photo. For this problem, only some of the students could imagine the shape without the use of the model. According to McGee (1979), determining the side views of a 3D object presented in a photo shows the ability of spatial visualization. In this lesson, all the students could build the model of the object in the photo and chose appropriate stand point to correctly solve the problem. It indicates that the students were able to understand the representation. Therefore, we conclude that students in lesson 3 had developed spatial visualization ability. They were able to do mental movement and create mental images from distant representations.

Regarding students' ability in understanding 3D representation, we claimed that it developed during the activities. In addition, students' spatial ability really supported students to understand the representation of the objects. In the first lesson, students observed the object by using the camera. Previously they could not understand the problems in the pretest and said that they could not imagine the situation. However, during the activities, they got chances to both observe the objects directly and compare the object with its photo. Observing objects helped students to create mental images and imagine or visualize the views. This also gave them idea of how the objects have so many different views from different positions. Furthermore, observing objects or models also become the preliminary mental movements for students to manipulate the representations. As a result, students were able to imagine the situation given in the photograph like Aurum did in the posttest. This is also proven when students struggled to interpret the photo in lesson 3. At that time, students used the blocks to construct the object and then observe it. They imagined the model as the object in the photo. Afterwards, they looked for the view of the object from the desired stand point.

Regarding the aspects of understanding 3D representation, students were able to do recognizing and manipulating process during the activities and also during the posttest. During the investigation, students' ability in manipulating representation developed mostly in lesson 3, lesson 4, and lesson 5. Lesson 1 and lesson 2 gave students preliminary ability to do the manipulation like creating mental images and mental movements. In lesson 3, students succeed to determine the side view of 3D objects in the photo. This indicates students had developed ability to manipulate representation. This ability then improved further during the lesson 4 and lesson 5. In these lessons, students did connecting many information, creating images and manipulate them to build the object. However, students only developed basic ability in recognizing representations since the activities focus more on manipulating ability. This is also because recognizing needs more advance spatial ability and it is more suitable for higher grade students. In fact, students were already able to recognize 3D representations before attending the activities in a grounded level like determining the picture of a box and etc. In lesson 3 and lesson 4, students developed the basic concept of recognizing ability by building the objects based on their standard views.

During the building process, students had to recognize the representation like the position of the cubes without necessarily performing mental movement. Students also became capable of realizing the blind spot of representations. Sometimes, the two aspects of understanding 3D representations went together during the activities.

In understanding the representations, students' spatial ability played important role as we have predicted. For instance in lesson 4, students really used their spatial ability to build the objects in the photos. In the first activity, students did not realize the blind spot of the representation. When the new model proposed, some students claimed that the object was different. However, some of them realize that their problem was how they view photo. They could find the stand point where the photo was taken to explain that the back part cannot be seen. Therefore, they conclude that it was pointless to build the temple if they did not have another photo from the back.

In the second activity of lesson 4, students were also supported by their spatial ability. To completely build the object, they adjusted the object based on its view from different stand points. Unconsciously, students had done connecting the information from different positions and visualize the object to revise it until they got the correct object. Otherwise, they will never finished the object and keep revising without an end. At the beginning, some groups struggled to build the object and could not imagine it. However, they slowly started over and began constructing the object from the top view. Then, they looked for the other view of the object. They imagined how they made the object such that it has similar view.

Generally, students' activities during the lessons were in line with our HLT. We have predicted how students would react to the problems and activities. Compared to the preliminary teaching experiment, students' reactions to the activities in the teaching experiment were less surprising. The following Table 4.9 of Dierdrop's matrix analysis presents the comparison between the HLT and the ALT of the teaching experiment. The plus sign means that students' reactions were in line with the HLT. On the other hand, the minus indicates that students' responses were not predicted in the HLT. Meanwhile, \pm means that students' reactions were in the HLT, but there were also reactions that out of the HLT. Based on the observation, all students reactions during the learning activities have been predicted in the HLT. This indicates that the learning process went as we have conjectured. Therefore, the HLT does not need to be revised any further.

Lesson	Problem	HLT	ALT	Note
Lesson 1	1. Determining the buildings they will see in the photograph from different stand points.	Students use the camera model or the digital camera and move around the building to check what they will see in the photo. Probably, some students look the building from bird eye positions.	 By moving around the object, students saw through the camera to determine what they would see. Some used the camera model and some used their digital camera. Some students struggled to use the camera in the correct stand point. They placed the camera in the bird eye's stand point. 	+
	2. Describing the position of an object based on the other object in the photograph.	Students will use their orientation of right and left to determine the position of the building. Probably, they will also use the orientation of the apartment or the other	- Students used their orientation of right and left to describe the position of the house and the tower with respect to the apartment.	+

Table 4.9 Matrix analysis of the teaching experiment

Lesson	Problem	HLT	ALT	Note
		buildings. For instance, they use the terms "front", "back", "left of" and etc. Cardinal direction are also probably will be used too by the students.		
	3. Determining stand points where only two, three, or all the buildings can be captured.	Students identify each position by moving around and look again the object with or without the camera. Probably, some students will refer to the table in the first activity to choose the positions.	 Students figured the stand points by using the camera and again moving around the object. Students thought that the position where someone can capture all the building is better. 	+
	 Suggesting Toni the best position to take photos of the buildings. 	Students will choose the position based on the number of buildings that they can see in each position like position 3, 4, 5 since all the building can be seen. Some students may prefer their personal choices like focusing their photo to one spot or choose only some of the buildings.	- Students chose position 3, 4, and 5 as the best positions to capture the buildings. It is because they could see all the buildings from that positions.	+
	 Determining the stand points where the photos were taken. 	 Students move around the objects while comparing the view in the photo to the view they see in the camera model. Some students may do it without the camera model. 	 Some of the students used the camera model to find the position of the camera. They compared the view of the object in the photo to the view they saw in the camera. Some of the students did not need camera model to help them comparing the view in the photos and the view of the object they will see from the stand points 	+
Lesson 2	2. Distinguishing the photos of the object from the wrong photos	 Students firstly try to find the position by moving around the objects. After they cannot find the position, they will consider that the photo is not of the object. Students probably will refer to one of the stand point where they can see a similar view like the wrong photo has. Afterwards, they explain that the objects has differences in some parts like facing the opposite 	 Students firstly compared the photos to the view of the object from all possible stand points. If they did not find a match then the photos are not of the object. Students realized the photos were not of the object since from a certain stand point, the object in the photo had different direction from the real one. 	+

Lesson	Problem	HLT	ALT	Note
		direction or the cube is		
		misplaced.		
	3. Determining the	- They will directly find out	- Students knew directly the	+
	position of the	that the view is from the top	photo was taken from the top of	
	top view photo	of the object since the photo	the object.	
		is different from the other.		
	1. Drawing the	- Students draw the shape	- Students used camera model to	+
	standard views of	while looking through the	see the shape and then draw it	
	the temple with	camera model and tried to	to the layout.	
	the help of camera	imitate it. Some students	- Some students confused to	
	model and the	maybe do not need the	thought that the drawing will be	
	object	the shape. Some students	bird eve picture. After they look	
	00ject.	may falsely draw the temple	to the camera, they realized that	
		from hird eye's angle	the shape was not 3-	
		nom ond eye s angle.	dimensional	
	2. Drawing the	- Students build the model of	- Students firstly built the object	
	standard views of	the object and then identify	using cubes. Afterwards they.	
	a temple presented	its standard views.	drew its standard views.	
T O	in a photograph	- Students will imagine the		
Lesson 3		shape of the temple like the		
		first activity and draw the		
		shape.		
	3. Drawing a	- Students will imagine the	- Students tried to look the	+
	standard view of	shape of the temple like the	temple from edge of the photos	
	a temple only	first activity and draw the	and then imagined the shape.	
	based on its bird	shape.	- Most of the students built the	
	eye photograph.	- They maybe count the cubes	object on the table and then	
		they see as a reference to	look the object from desired	
		draw.	stand point in the photo.	
		- Probably, they also		
		constructed the object		
		imagining its view		
	1 Constructing a	- Students construct the	- Students constructed the temple	
	temple based on	temple by assuming the	such that it looked similar to the	I
	its bird eve	unseen part of the temple.	photograph without realizing	
	photograph	- Some students probably	the unseen part of the object in	
		aware of the unseen part but	the photos.	
		they assume it to build the	- Students then decided they	
Lesson 4		temple. Some may directly	could not build the temple and	
		state they cannot build the	they needed more views of the	
		temple since the photo is not	temple from the back.	
		enough.		
		- Students will conclude that		
		they do not know how the		
		unseen part looked like and		

Lesson Problem HLT		HLT	ALT	Note
		need more photos of the		
		temple.		
	2. Constructing a	- Students continue their	- Students firstly constructed the	+
	temple based on	construction in activity 1 by	temple based on the top view	
	its bird eye	adjusting the shape with the	and then the side view.	
	photograph and	photos.	- Some groups did a mistake by	
	its standard views	- Probably students will work	putting another cube. After the	
		from the back view and then	teacher asked them to compare	
		the side view or top view. In	it to the bird eye photograph,	
		the end they compare it	they fixed it correctly.	
		again with the bird eye		
		photo.		
	3. Constructing a	- Students construct the	- Students began constructing the	+
	temple based on	temple from the top view	temple from its top view.	
	its standard views	and then adjust the	Afterwards, they adjusted the	
		construction to the side	temple by putting another cube	
		views. They put the cubes in	based on the side views.	
		the top view and then they	- Some started the construction	
		look the temple from the	from the side and struggled to	
		side to add another cubes.	finish the temple. Then, they	
		- Some probably begin the	started over the temple from its	
		construction from the sides	top view like the others.	
		and then adjust it to the		
		other side views and the top		
	1 Investigating the	Vies.	Students compared the number	
	nevious reports	of the cubes in each view to	of the cubes between the two	+
	of temples to	compare	views and concluded that both	
	determine the	- They also compare its	views had the same number of	
	properties of	"similar" shape but the	cubes	
	standard views	orientation is reversed	- Students also found that the	
	Standard Views	- They conclude that the top	shape of the two views were	
		view does not have similar	similar but they were oppositely	
		view like the others.	reversed.	
	2. Drawing the	- By using the properties that	- For the first report, students	+
	shape of the	the opposite photos are	drew the shape by using the	
	temple from	similar, students draw the	properties of the opposite side	
	some stand points	shape of the temple based	of standard views.	
	based on the	on its opposite photo.	- Some students firstly assumed	
T	remaining	- For the reports that does not	the lost views of the temple.	
Lesson 5	standard photos	have enough photo, students	But then, they realized that the	
		will assume the shape first	lost views cannot be determined	
		before realizing that the		
		shape cannot be determined.		
	3. Rearranging the	- Students firstly will put the	- In the beginning, students	+
	photos in a report	top view of the temple.	firstly put the top view and then	
	of a new temple	They can identify the top		

Lesson	Problem	HLT	ALT	Note
		view since the other photos	put the other photos around the	
		do not have similar shape.	top views by guessing.	
		- They put the other photo by	- They then used the properties	
		adapting the number of cube	of the standard views to help	
		in side photos to the top	them put the photos. They also	
		view.	reasoned the number of the	
			cubes in the row of each views	
			to compare it to the top view	

2) Classroom norm/Social norm and Socio-mathematical norm

Regarding the social norm in the class, students were still quite crowded during the lessons. Beside they were used to it, there were two reasons why they were crowded in the class. First, the activities asked students move around and observing objects while discussing it. These activities kept students active and moving during the learning process. Second, the teacher organized the class by only providing each group a table without chairs for the students. Therefore, when the learning have passed 40-50 minutes, students felt tired of standing and sat on the floor. During the lessons, students always worked in small groups. Even though, they were not common with it, they looked used to it and actively discussed the problems with their teammates. Sometimes, students still tried to ask confirmations from the teacher or the observer like they usually did in the regular class.

Regarding the socio-mathematical norm, students were not common with multiple answers. Hence, when they faced problems which has two possible answers, they always asked which one was correct. In addition, when the observer questioned what and how students answered the problem, they always thought that the answers were incorrect. This indicates that students were not used to explain their answers. However, after several lessons, they get used to describe their answers and how they found them.

3) Teacher's Role

Based on the observation, the teacher did not play all the roles the design required. There were several indicators of why the teacher did not perform well enough. First, in the beginning of the lesson 1, the teacher did explain the context of the activities to the students. However, he did not do it for the next lessons. He explained directly the activities and asked students to read the story in the worksheet. We had warned the teacher but he keep forgetting to tell briefly the context. Since students could read the context in the worksheet, this problem can be anticipated. Second, the teacher did not facilitate the discussion of the whole class well enough. He sometimes forget what to discuss and at one moment, he judged students' answer by asking the other groups to judge whether the answer was right or wrong. Although the judge was from the students, we did not suggested this in during the preparation with the teacher. Sometimes, the teacher also forgot what to do after an activity and asked the observer what he should do. Actually, the lesson have been discussed with the teacher a week before the teaching experiment. In addition, one day before the lesson began, we also discussed the activities to help the teacher preparing himself. However, since the teacher only asked the observer when the students still discussed the problem in small group, it did not interrupt the learning activities.

We also noticed the positive effect of the teacher during the learning activities. He could actively guide and facilitate students' discussion in small group. He also could manage the crowd by motivating the students and engaging them to the activities.

CHAPTER V

CONCLUSION AND SUGGESTION

A. Conclusion of the Study

In this section, we draw a conclusion to answer the research question that becomes the anchor this study. The research question is "*How can spatial orientation task and spatial visualization task support the development of students' spatial ability in understanding three-dimensional representations?*". Based on the retrospective analysis, spatial orientation tasks and spatial visualization tasks can support the development of students' spatial ability in understanding 3D representations. How the activities support students can be looked into the progress of students' learning that happened during the teaching experiment. The following findings indicate the learning process that happened in each activity and how they contribute to the development of students' spatial ability in understanding 3D representations. These findings are:

- 1. Observing objects from different positions in lesson 1 and finding the positions of photos in lesson 2 helped students to develop their spatial orientation ability. Furthermore, these activities also gave students preliminary experiences to create mental images and manipulate them in mind. In addition, the activities became a preliminary entry to develop students' spatial visualization.
- 2. Drawing standard views of an object in lesson 3 supported students to develop their spatial visualization ability. The activities helped students

visualize the standard views and shift from working with concrete objects into working with the objects' distant representation. Furthermore, the activity also developed students' ability in understanding 3D representations such as recognizing and manipulating information of objects presented in distant representations like pictures and photos.

- 3. Building objects based on their photos in lesson 4 developed students' ability in understanding 3D representations. The activity helped students identifying the blind spot of the representations and manipulating the representations to create images of the objects. This was indicated by students' capability of turning the images of the object in mind into the concrete object by using the building blocks. In order to build the objects, students had to recognize and manipulate the mental images of the object based on its photos.
- 4. Students' spatial ability played an important role in the development of students' ability in understanding 3D representations. During the lesson 4 and lesson 5, students showed and demonstrated strategy related to spatial orientation and spatial visualization in order to reason and understanding the representations of the temple.

The aim of this study is not only to produce the instructional materials to develop students' spatial ability but also to contribute to the local instruction theory (LIT) of how to develop students' spatial ability. The following Table 5.1 is the learning trajectory as the final result of the sequence of activities in this study. This learning trajectory becomes the local instruction theory of the present study about students' spatial ability in understanding 3D representations.

Context	Tools	Activity	Ability
Photography activity / Playing with the camera	Observing objects Finding the position of the photos	Finding the best position to capture the city by identifying the building students see from different stand points Finding the position of where the given photos were taken from the object of building blocks	 Preliminary experiences of spatial orientation and spatial visualization Preliminary experiences of mental movement and creating mental images Supporting students spatial orientation Preliminary support for spatial visualization Preliminary experiences of mental movement and creating mental images
Reporting temples	Drawing standard views of an object	Identifying and drawing the standard views of the temples represented in a model or in a photo	 Supporting students spatial visualization Mental movement and mental images Supporting students' ability in understanding 3D representation such as working with distant representation
Building temples	Building objects based on its photos	Building temple by using building blocks such that the temple was similar to the photos	 Spatial ability supports students' ability in understanding 3D representation Creating mental images, recognize and manipulate them.
Fixing reports of a temple	Observing the properties of standard views	Investigating the properties of standard views of an object and rearranging the photos of standard views of an unknown objects in the layout	 Spatial ability supports students' ability in understanding 3D representation Creating mental images and manipulate them.

Table 5.1 Learning trajectory in local instruction theory of the present study

B. Weaknesses of the study

We know that this study has so many weaknesses and limitations due to the time, condition, and situation from the beginning until the end of the data analysis. One of the weaknesses we noticed is data analysis. The analysis of the data focuses only on the result of the focus group. Although we also included some of the findings from the other students in the class, it is still not cover all the participants. However, we discussed the result of the pretest and posttest for all students in the focus group and in the whole class.

C. Suggestions

Based on the conclusion and the weakness of the study, we suggest to consider:

- For further investigation, one can combine the basic understanding on spatial structuring into the problems. For instance like counting the cubes, guessing the number of the cubes needed to construct an object in the photo. This kind of investigations is not maximally embedded in the sequence.
- Conducting the activities before the students learn further about 3D object. This is because the activities in this sequence will develop students' basic ability that support their development in 3D geometry thinking.
- 3. The analysis of the data cannot cover the whole subject. We suggest the use of data triangulation like having two or three focus group in the teaching experiment. In the analysis, the data of the three focus groups can be compared to have stronger findings, conclusion and theory. However, this

will take more time to analyze data, especially classroom observation and also need many observer in the teaching experiment.

D. Recommendations for future research

This study only focuses on developing students' spatial ability in understanding 3D representation. In the hierarchy of 3D geometry thinking, understanding 3D representation is the basic understanding among the others: 1) understanding 3D representation, 2) spatial structuring, 3) conceptualization of mathematical properties, and 4) measurement. We recommend the future researches to design activities to support students in developing the other types of thinking. As we know, spatial structuring can lead to the understanding of volume measurement while conceptualization of mathematical properties can be a bridge to measurement.

The other recommendation is the consideration of auditory learning style. As we already know, the sequence of lessons in this study was developed based on the kinesthetic and visual learning style. This due to most of the students in elementary school have visual and kinesthetic learning style. In addition, visual and kinesthetic approach are also the most appropriate style to develop students' spatial ability. Therefore, we did not accommodate those who prefer to have auditory learning style. Future researches can consider it to be integrated into the activities.

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Appendix 1

- **1. List of topics for classroom observation**
- 2. List of topic for interview with the teacher
- 3. Pretest
- 4. Posttest
- **5. Student Worksheet**

List of Topics for Classroom Observation

(Pre-experiment)

Topics	Questions
Practical setting	- How many students are there?
	- How does teacher arrange or organize the students?
	- Availability of learning media in the class/school
	- If there is a group discussion, how teacher organize/divide
	the group?
	- Does teacher use learning media for the activity?
	- Does the teacher use worksheet or textbook or any
	instructional material during the learning activity?
Students participation	- Does the students have discussion with their friends and teacher?
	- How active is the students during the learning process?
	- How does students respond teacher's
	explanation/instructions?
	- Who are the active learners?
	- Who are the passive learners?
Teacher role	- How does teacher conduct the activities?
	- How does teacher respond to students' questions?
	- How does teacher respond if there is student who has
	difficulties in the learning?
	- What the teacher do when the students have discussion
	among their fellow?
	- How does teacher react to a right solution proposed by the students?
	- How does teacher react to a wrong solution proposed by
	the students?
Social norm and	- What do student do if they want to ask questions?
classroom culture	- Is there discussion among students? Among students and
	teacher?
	- Do students solve the problem by themselves? Or
Mathematical activity	- What type of question does the teacher propose? Open
	question?
	- Do students use different strategy to solve the problems?
	- Is there discussion about how the students solve the
	problem?
	- Is the solution/strategy produced by the students? Or
	given by the teacher?
	- How does teacher respond to wrong answers/strategies?

List of Topics for Interview with the Teacher

(Pre-experiment)

Topics	Questions
Background of	- How long does the teacher become a teacher?
teacher's education	- What is the motivation of the teacher to be an elementary
	teacher?
	- How are teacher's opinions about mathematics education?
	- How difficult is teaching mathematics?
	- Does teacher likes to teach mathematics compared to
	another subject?
	- Does the teacher know about RME/PMRI?
	- Does the teacher have experiences in this kind of research
	(design research)?
Classroom	- How does teacher usually arrange students'
management	position/formation during the learning process?
	- How does teacher divide the students when they have
	group discussion?
	- How does teacher do to create a good learning
	environment for the students?
	- Do you have certain agreement with the students if they
	want to do something like asking question, propose
	solution, asking permission?
Learning techniques	- What kind of approach does teacher usually use to teach
	mathematics?
	- Does teacher have preference of teaching method?
	- How does teacher build classroom discussion?
Planning skills	- Does the teacher use lesson plan/teacher guide?
	- How does teacher design the learning material? Or just
	following textbook?
	- How does teacher do if a lesson do not go well as he/she
	has planned?
Class Composition	- How many students is in your class?
	- How active is the students in your class in your opinion?
	- How many percents of the students have high ability,
	medium/average ability, and low ability on mathematics?
	- Who is the most active and passive students in the class?
	(active means participating in the activities)
	- How does teacher divide the students if he/she has group
	discussion?



Nama:

Bagian A

1.



Gambar di atas merupakan foto berbagai macam buah-buahan mulai dari jeruk, apel, anggur, dan pisang.

Tahukah kamu buah apa yang terletak di paling ujung kiri foto?_____ Tahukah kamu buah apa yang terletak di paling ujung kanan foto?_____

2.



Bayangkan gambar di atas adalah gambar kamu dan teman-temanmu. Kamu berdiri di tengah memakai baju warna hijau sementara Temantemanmu berdiri disampingmu. Lihat gambar di atas dan jawab pertanyaan berikut:

Siapa saja yang berdiri di sebelah kananmu? ______ Siapa saja yang berdiri di sebelah kirimu? _____



Gambar di atas menunjukkan beberapa miniatur bangunan di atas meja. Bangunan tersebut adalah dua rumah, satu apartemen, dan satu tower. Bayangkan kamu berjalan mengelilingi meja tersebut dan berhenti pada posisi B dan C. Pada dua posisi tersebut, kamu memfoto bangunan di meja tersebut.

Tentukan manakah diantara foto berikut yang merupakan hasil foto dari posisi B? Tentukan manakah diantara foto berikut yang merupakan hasil foto dari posisi C?













Perhatikan objek di atas! Objek tersebut berupa kubus-kubus yang ditumpuk membentuk suatu bangunan di atas meja. Bayangkan kamu memfoto objek di atas dengan kamera dari posisi A. Gambarlah bagaimana bentuk objek pada foto yang dihasilkan!

Ada berapa kubus yang ditumpuk pada gambar di atas? Kubus kayu

Gambar foto dari posisi A disini!

DESCRIPTION OF THE PRETEST

Time Allocation: 30 Minutes

Question	Problem	Purpose	Analysis
Part A	Determining the position of an objects with respect to the other objects.	The question aims to investigate students' prior knowledge, particularly students' sense of direction.	The analysis of this question will be used to adapt the activities and also to improve the HLT.
Part B	Identifying the photo of the object if the camera is put in certain position	The purpose is to have an insight of students' awareness of spatial ability and their ability in manipulating representations in mind.	The result will be used to improve the HLT, especially to predict possible students' answers.
Part C	Determining the shape of an objects if the object is viewed from the side. The object is presented in a photograph	The purpose is to investigate students' spatial ability in understanding representations especially to recognize and manipulate the object based on distant representation of the object.	The analysis of this question is used to determine students' ability to read 3D representation.


Gambar di atas menunjukkan beberapa miniatur bangunan di atas meja. Bangunan tersebut adalah dua rumah, satu apartemen, dan satu tower. Bayangkan kamu berjalan mengelilingi meja tersebut dan memfoto bangunan tersebut dari setiap posisi A, B, C, dan D. Foto-foto berikut adalah hasilnya. Tentukan pada setiap foto berikut posisi dimana foto tersebut diambil!



Foto 1 diambil dari posisi____



Foto 2 diambil dari posisi____



Foto 3 diambil dari posisi_____



Foto 4 diambil dari posisi____





Perhatikan objek di atas! Objek tersebut berupa kubus-kubus kayu yang ditumpuk membentuk suatu bangunan di atas meja. Bayangkan kamu memfoto objek di atas dengan kamera dari posisi A dan B. Gambarlah bagaimana bentuk objek pada foto yang dihasilkan!

Ada berapa kubus yang ditumpuk pada gambar di atas? Kubus kayu

Gambar foto dari posisi A disini!	Gambar foto dari posisi B disini!	

Bagian C

Bangun candi berikut dengan menggunakan kubus kayu yang diberikan oleh gurumu! Berilah namamu pada candi tersebut dengan menggukan potongan kertas yang diberikan gurumu.



DESCRIPTION OF POSTTEST

Time Allocation: 30 Minutes

Question	Problem	Purpose	Analysis
Part A		The purpose is to investigate	
	Identifying the photo of the object if the camera is put in certain position	students' spatial ability and	
		also their ability in	
		manipulating representation	
		after attending the sequence	
		of lesson.	
Part B		This question aims to	
	Determining the shape of an	examine students' spatial	The analysis will be
	objects if the object is viewed	ability in understanding 3D	used as complementary
	from the side. The object is	representations, especially to	analysis to support the
	presented in a photograph	recognize and manipulate	findings' during the
		them to determine the shape	teaching experiment.
		of objects.	
Part C		The aim is to examine	
	Building an object based on the	students' spatial ability to	
	standard views of the object.	recognize and manipulate	
	Students use building blocks to	3D representations,	
	construct the objects.	particularly its standard	
		views.	

Students Worksheet

Lesson 1

- Lesson 2
- Lesson 3
- Lesson 4
- Lesson 5

PEMBELAJARAN 1 BERMAIN DENGAN KAMERA Bagian 1

NO KELOMPOK:

NAMA:

Halo, namaku Toni. Aku seorang fotografer. Kemanapun aku pergi, aku selalu membawa kamera untuk memfoto berbagai hal. Hari ini, aku pergi ke suatu kota untuk mengambil gambar bangunan di sana.

Toni pergi ke suatu kota dimana ia menemukan 4 bangunan. Keempat bangunan itu adalah sebuah apertemen, dua rumah, dan sebuah menara. Ia ingin mencari posisi untuk mengambil foto semua bangunan dalam satu kali potret. Bantulah Toni mencari posisi yang bagus untuk mengambil foto bangunan di kota itu!

Berikut susunan bangunan tersebut. (Lihat layout)

Toni meminta pendapatmu untuk menentukan posisi terbaik untuk mengambil gambar bangunan tersebut. Pertama-tama, jawablah pertanyaan di halaman selanjutnya! Kemudian kamu dapat menentukan dimanakah posisi terbaik untuk mengambil gambar berdasarkan diskusi kelompokmu.

Jawablah pertanyaan di bawah ini!

 Fotolah bangunan tersebut dari posisi 1 sampai posisi 8 dengan kameramu atau kamera buatan yang disediakan. Berilah tanda centang "√" pada table berikut bila kamu dapat melihat bangunan tersebut pada kamera. Jika kamu tidak melihat bangunan tersebut berilah tanda strip "-" pada tabel. Misalnya, pada posisi 1, Toni dapat melihat rumah merah, menara dan apertemen, maka ia memberi tanda centang pada ketiga bangunan tersebut dan tanda strip pada rumah pink. Bagaimana dengan posisi yang lain?

Posisi	Rumah Merah	Rumah Pink	Menara	Apertemen
1	١	-	١	١
2				
3				
4				
5				
6				
7				
8				
			•	

- 2. Jika kamu mengambil gambar dari posisi 3, dimanakah posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?
- 3. Jika kamu mengambil gambar dari posisi 7, dimana posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?

Jawablah pertanyaan di bawah!

4. Jika kamu mengambil gambar dari posisi 1, dimana posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?

5. Jika kamu mengambil gambar dari posisi 5, dimana posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?

Jawablah pertanyaan di bawah!

Posisi mana sajakah yang memungkinkan untuk mengambil gambar yang memuat semua bangunan?

Posisi mana sajakah yang memungkinkan untuk mengambil gambar yang hanya memuat 3 bangunan saja?

Posisi mana sajakah yang memungkinkan untuk mengambil gambar yang hanya memuat 2 bangunan saja?



atas, menurutmu posisi manakah yang terbaik untuk mengambil gambar bangunan-bangunan itu? Jelaskan jawabanmu agar Toni dapat mengerti alasanmu.

Jelaskan alasanmu di sini!



Jika Toni hanya ingin memfoto rumah pink saja, posisi manakah yang menurut kalian paling baik?Setelah melakukan investigasi dengan menjawab pertanyaan di





Layout for lesson 1. The original size is A3.

PEMBELAJARAN 2 BERMAIN DENGAN KAMERA Bagian2

No Kelompok: _

Nama:

Hallo, kita bertemu lagi, Toni "si fotografer". Kemarin, aku mengambil banyak sekali foto dari benda yang dapat kamu lihat di meja sekarang. Aku telah mencetak semuanya, akan tetapi foto-foto itu bercampur dengan foto-foto lain yang kuambil pada waktu yang berbeda. Aku bingung untuk menemukan manakah foto dari benda yang kuambil kemarin. Dapatkah kamu membantuku seperti kemarin?



Kamu harus membantu Toni untuk menemukan manakah foto yang benar yang diambil dari berbagai posisi. Toni mengirimimu foto di balik halaman ini. Kamu bisa menggunting dan menempelkannya pada posisi dimana Toni mengambilnya. Akan tetapi, berhati-hatilah karena di antara foto-foto itu, terdapat beberapa foto yang diambil pada waktu yang berbeda dengan benda yang mirip. Jangan sampai salah memilih foto ya!





Aku sangat berterima kasih. Kamu telah menolongku dua kali. Sekarang, aku penasaran bagaimana kamu memilih foto tersebut. Tolong jelaskan padaku mengapa kamu tidak memilih foto yang lain?

Jawablah pertanyaan berikut ini!

- 1. Berapa banyak foto yang kamu tempel?
- 2. Foto nomor berapa saja yang kamu pilih dan tempel?

- 3. Berapa banyak foto yang tidak kamu pilih dan tidak kamu tempel?
- 4. Foto nomor berapa saja yang tidak kamu pilih dan tidak kamu tempel?
- 5. Tempelkan foto yang tidak terpilih di balik halaman ini! Kemudian tuliskan alasan disamping foto tersebut kenapa kamu tidak memilihnya!

<u>Ini adalah foto yang diambil Toni!</u> Kamu dapat menggunting dan menempelkan foto berikut pada layout







Layout for lesson 2. The original size is A3.



NO KELOMPOK: ____

NAMA:_

Hallo, namaku Reza. Aku adalah seorang arkeolog dan seorang ahli yang suka menjelajahi situs sejarah seperti candi. Di Indonesia, jika kamu menemukan sebuah candi baru, maka kamu harus melaporkannya pada pemerintah. Dalam laporan itu, kamu harus melampirkan foto/gambar candi dari bagian depan, belakang, samping kiri, samping kanan, dan bagian atas dari candi tersebut.

Aku ingin berbagi pengalaman denganmu sebagai seorang arkeolog. Sekarang, ikuti petunjukku:

- 1. Andaikan, kamu adalah seorang arkeolog.
- 2. Sekarang, bangunlah satu candi yang menarik dengan menggunakan tepat 8 kubus.
- 3. Laporkan candimu pada pemerintah dengan melukis gambar candimu dari bagian depan, belakang, kiri, kanan dan atas candi menggunakan kamera yang disediakan.

Aku tunggu laporan candi kalian!!!

Berikut ini contoh gambar dalam laporan candi milik reza. Lihat!



Candi milik Reza



Gambar candi milik Reza tampak dari depan





Hai kawan, kamu membuat candi yang menarik dan juga laporan yang bagus. Aku ingin memberitahumu sesuatu. Kemarin, aku menemukan sebuah candi baru. Aku sudah kirimkan gambarnya untukmu.

Aku memiliki tugas untukmu. "Tolong buatkan laporan dari candi tersebut!". Aku benar-benar sibuk saat ini untuk mengivestigasi lebih jauh candi itu. Aku sudah kirimkan foto candinya. Aku harap foto itu cukup untuk panduan membuat laporannya.





Foto candi baru

Candi baru di atas memiliki alas berbentuk persegi.

Tahukah kamu berapa bongkah batu yang ada pada candi di bawah ini?_____

Gambarlah candi pada lembar kerja yang telah dibagikan!

Hai kawan, Kemarin, aku menemukan candi baru lagi. Aku telah menggambar candi itu untuk membuat laporannya, tetapi belum selesai. Tolong bantu aku untuk menyelesaikannya. Terimakasih banyak, kawan!!! ©

Tahukah kamu berapa bongkah batu yang ada pada candi di bawah ini?_____

Gambar bentuk candi berikut difoto dari posisi tanda panah!





Layout for Lesson 3. The original size is A3.



No Kelompok: _

Nama:_

Hallo ^-^

Kemarin, aku pergi ke kantor pemerintah. Mereka menerima laporan tentang candi baru. Masalahnya, pengirim laporan hanya mengirimkan satu foto dari candi tersebut. Mereka tidak melampirkan semua foto seperti yang kamu biasa lakukan sebelumnya. Padahal pemerintah harus membuat miniatur candi tersebut. Sekarang mereka bingung karena mereka hanya memiliki satu foto dari candi tersebut.





Tolong bantu pemerintah membuat miniatur dari candi dalam foto tersebut dengan menggunakan kubus. Foto candi dapat dilihat di bawah.

Aku harus pergi dulu, beritahu gurumu jika kamu dapat menemukan bagaimana untuk membangun miniatur. Terima kasih, kawan!



Foto Candi Baru



Tampak Belakang Candi



Tampak Samping Candi



Tampak Atas Candi



No Kelompok: _____

Nama: _____

Hai,, aku memiliki kabar gembira. Pemerintah mengirimiku laporan lain. Kali ini, semua foto disertakan. Tolong bantu aku untuk membangun miniatur candi yang dilaporkan!

Aku tahu kamu dapat melakukannya, terima kasih!

Ini adalah foto candi yang dilaporkan:

Kamu dapat menggunakan layout yang diberikan oleh gurumu.





Tampak Depan Candi





Tampak Atas Candi



Layout for activity 2 in lesson 4. The original size is A3.

PEMBELAJARAN 5

KEGIATAN 1 – Menyelidiki Laporan

No Kelompok:

Nama:

Haii,, kita bertemu lagi. Kemaren aku lihat beberapa laporan yang kalian buat. Semuanya keren dan unik. Pada saat melihat laporan itu pula aku menemukan hal yang menarik. Coba kalian perhatikan laporan kalianyang diberikan gurumu kemudian jawab pertanyaan berikut.



Lihat laporan-laporan tersebut!

Bandingkan gambar dari bagian depan dan belakang candi.

- 1. Apakah ada kemiripan dari kedua gambar tersebut? Jelaskan!
- 2. Apakah ada perbedaan pada kedua gambar tersebut? Jelaskan!

Bandingkan pula gambar dari bagian kanan dan kiri candi.

- 1. Apakah ada kemiripan dari kedua gambar tersebut? Jelaskan!
- 2. Apakah ada perbedaan pada kedua gambar tersebut? Jelaskan!

Bagaimana dengan gambar dari bagian atas? Adakah gambar dari bagian lain yang mirip dengan gambar dari bagian atas?

Kesimpulan apa yang kalian dapat dari jawaban pertanyaan-pertanyaan di atas?

Previous reports for activity 1 in lesson 5. The original size is A4.







PEMBELAJARAN 5 KEGIATAN 2 - Memperbaiki Laporan

No kelompok: _

Nama:

Hallo, kamu telah menolongku berkali-kali. Aku sangat beruntung memiliki teman sepertimu, kawan. Aku mempunyai beberapa laporan lama dari penemuan berbagai candi. Karena sudah sangat lama, beberapa foto hilang. Aku penasaran apakah kamu dapat memperbaikinya. Tolong bantu aku lagi untuk menentukan bagaimana bentuk laporan aslinya berdasarkan foto yang kita miliki di sini.





Ada tiga laporan utnuk diperbaiki. Kamu tidak harus memperbaiki semuanya. Mungkin saja, sebagian laporan tidak bisa diperbaiki.

Jika ada laporan yang tidak dapat diperbaiki, tolong beritahu aku "kenapa" dengan menjawab pertanyaan di bawah.

🟓 Jawablah pertanyaan di bawah!

Apakah ada laporan yang bisa diperbaiki? Jika ada sebutkan laporan berapa!

Jelaskan bagaimana kalian memperbaikinya?

Apakah ada laporan yang tidak bisa diperbaiki? Jika ada sebutkan dan jelaskan kenapa tidak bisa diperbaiki!

Three reports for activity 2 in lesson 5. The original size is A4.





Haii,, Tadi malam, aku mendapat sebuah laporan tentang candi baru di Jawa Tengah. Akan tetapi, aku tidak sengaja menjatuhkan laporan itu dan semua fotonya lepas. Aku tidak tahu bagaimana untuk menempel foto tersebut di tempat semula.



Tolooong T.T., bantu aku memperbaiki laporan ini!

Reza tidak sengaja merusak laporan itu. Awalnya, foto-foto itu disusun dalam laporan seperti biasanya. Lihat pada gambar!

Ada 5 foto pada laporan. Reza tidak ingat letak foto-foto tersebut. Bantu Reza untuk menemukannya.

Dapatkah kalian tebak bagaimana susunan foto-foto tersebut pada laporan.



Jelaskan bagaimana kalian memperbaiki laporan di atas?



Photos for activity 3 in lesson 5. The layout below is originally as big as A4.

Appendix 2

Teacher Guide:

Lesson 1

Lesson 2

Lesson 3

Lesson 4

Lesson 5

PEDOMAN GURU

Mengembangkan Kemampuan Spasial Anak



Untuk Siswa Kelas 3 SD

PEDOMAN GURU

Dokumen ini merupakan pedoman bagi guru untuk menjalankan pembelajaran yang telah didesain menggunakan pendekatan matematika realistic. Terdapat 5 pertemuan yang terdiri dari berbagain macam kegiatan dan masalah bagi siswa. Kegiatan ini bertujuan untuk membantu siswa mengembangkan kemampuan spasial mereka untuk memahami representasi bangun tiga dimensi. Setiap pertemuan memerlukan sekurang-kurangnya 2x35 menit pembelajaran. Guru dapat menggunakan informasi pada dokumen ini untuk mengetahui bagaimana pembelajaran dilakukan.

Setiap pertemuan terdiri dari dua dokumen yang berbeda. Yang pertama, dokumen yang berisi tentang berbagai hal yang harus guru lakukan selama pembelajaran. Dokumen ini disebut Rencana Pelaksanaan Pembelajran (RPP). Dokumen yang kedua berisi tentang hal-hal yang mungkin akan siswa lakukan sebagai respon terhadap kegiatan yang diberikan seperti jawaban siswa, strategy, dan kesalahan yang mungkin dilakukan. Pada lampiran RPP terdapat pula material atau media yang diperlukan selama pembelajaran seperti miniatur rumah atau kamera buatan. Guru harus mempersiapkan terlebih dahulu media ini dengan cara memperbanyak dan membentuknya sebelum pembelajaran dilakukan.

Agar pembelajaran berlangsung sesuai dengan yang diinginkan maka guru harus mempertimbangkan hal-hal berikut yaitu:

Peran Guru:

Selama pembelajaran, guru berperan aktif membantu dan memfasilitasi dalam melakukan investigasi. Dalam hal ini guru hanya lah membantu siswa memecahkan masalah dengan tidak memberikan jawaban. Bila diperlukan, guru bisa memberikan demonstrasi sebatas contoh agar siswa mengerti apa yang harus mereka lakukan. Penting bagi guru untuk memastikan bahwa semua kelompok memahami konteks pembelajaran. Oleh karena itu, guru harus berkeliling memantau setiap kelompok dan membantu mereka jika mengalami kesulitan. Guru dapat memberikan sedikit gambaran dari konteks/cerita yang mendasari kegiatan agar siswa mempunyai gambaran sebelum membaca LKS. Selain itu, guru juga berperan sebagai organisator pembelajaran yang mengontrol alur pembelajaran serta waktu yang diperlukan sehingga siswa tahu apa yang harus dikerjakan dan lakukan selama

berpartisipasi aktif dalam pembelajaran. Sebagai contoh, jika salah seorang siswa terlihat hanya diam atau bermain, guru memperingatkan dengan memberi pertanyaan atau perintah dan semacamnya.

Norma Sosial:

Selama pembelajaran, guru menekankan pada siswa pentingnya kerjasama dan diskusi dalam kelompok. Oleh karena itu, siswa harus saling membantu dalam menginvestigatsi setiap masalah. Apabila siswa ingin menyampaikan pendapat atau pertanyaan maka terlebih dahulu harus mengangkat tangan. Penting pula untuk mengingatkan siswa bahwa selama pembelajaran guru tidak akan memberikan jawaban mana yang benar atau salah. Siswa harus menemukan dan membuktikan sendiri mana jawaban yang benar dan mana yang salah. Beberapa norma di atas bertujuan untuk mempermudah dan mendorong siswa aktif bekerja menyelesaikan setiap masalah yang diberikan.

Pada akhirnya guru dapat melakukan sedikit perubahan pada pembelajaran jika situasi atau kondisi tidak memungkinkan untuk melaksanakannya sesuai dengan rencana. Pedoman guru ini dibuat dengan tujuan untuk membantu guru dalam mengaplikasikan pembelajaran yang telah direncanakan.



Pembelajaran 1

Nama sekolah	: SD Laboratorium Unesa, Surabaya
Mata Pelajaran	: Matematika
Kelas/Semester	: III/2
Indikator	: Siswa menyelidiki tampilan obyek tiga dimensi dari berbagai sudut
	pandang.
Aloksi Waktu	: 2 x 35 menit

A. Tujuan Pembelajaran

- 1. Siswa dapat melakukan investigasi secara langsung orientasi dan visualisasi dari suatu obyek tiga dimensi.
- 2. Siswa memahami bahwa obyek tiga dimensi memiliki tampilan yang berbeda-beda jika dilihat dari sudut pandang yang berbeda.

B. Kemampuan Matematika yang diajarkan

Tidak seperti figur dua dimensi, obyek tiga dimensi memiliki tampilan yang berbeda dari berbagai arah. Arah dimana objek tersebut dilihat disebut sudut pandang. Dalam pembelajaran kali ini, siswa akan dikenalkan dengan tampilan yang berbeda dari obyek tiga dimensi dari berbagai sudut pandang. Sebagai tambahan, kegiatan ini juga menstimulasi siswa untuk menemukan konsep dari garis penglihatan dan daerah yang tak terlihat dari obyek tiga dimensi. Sebagai contoh, benda-benda di bawah dilihat dari berbagai arah. Hasilnya memperlihatkan bentuk yang berbeda dari obyek. Posisi mug juga berubah dari kiri ke kanan. Kegiatan ini penting bagi siswa sebagai aktifitas pendahuluan untuk mengembangkan spasial orientasi dan visualisasi siswa.



Obyek tiga dimensi

Tampilan dari sudut pandang yang berbeda

C. Metode dan Pendekatan Pembelajaran

- 1. Pendekatan pembelajaran: Pendidikan Matematika Realistik Indonesia (PMRI)
- 2. Metode: diksusi kelompok dan diskusi kelas

D. Sumber dan Media Pembelajaran

- 1. Sumber: Lembar kerja siswa (LKS)
- 2. Media: dua kamera buatan, satu kamera digital, dan satu set miniatur kota untuk setiap kelompok.

Keterangan:

- LKS terlampir pada dokumen ini
- Jaring-jaring kamera buatan dan miniatur kota terlampir pada bagian akhir dokumen ini. Guru dapat memperbanyak jaring-jaring tersebut sesuai kebutuhan. Guru harus menggunting jaring-jaring tersebut dan membentuknya dengan lem sebelum digunakan dalam pembelajaran. Berikut ini adalah gambar miniatur kota dan kamera buatan yang telah selesai di buat.



Kamera Buatan



Miniatur Kota pada soal

E. Skenario Pembelajaran

1. Pembukaan (10 menit)

 Guru menjelaskan bahwa hari ini dan empat pertemuan mendatang, siswa akan mempelajari tentang benda-benda ruang yaitu menganalisis foto dan menggambar benda-benda dengan bantuan kamera. Karena itu, siswa akan dibagi menjadi kelompok kecil yang terdiri dari 3-4 siswa dan diberi nama kelompok 1, kelompok 2, dst. Guru juga memberitahu bahwa kelompok ini tidak akan berubah sampai 3 pertemuan mendatang. Untuk itu, siswa harus menghafal anggota kelompoknya.
- Guru meminta siswa untuk duduk melingkar dengan kelompoknya dalam satu meja. Guru juga menyampaikan norma yang berlaku selama pembelajaran. Setelah semua kelompok siap, Guru dapat memulai kegiatan.
- 2. Kegiatan inti (55 menit)
 - Untuk memulai kegiatan, guru bercerita tentang konteks kegiatan yaitu "Toni si fotografer".

"Hari ini, kamu akan bertemu dengan Toni. Dia seorang fotografer dan suka memfoto banyak hal. Kemarin, ia berjalanjalan ke suatu kota dan melihat banyak bangunan seperti bangunan yang ada dimeja kalian. Dia meminta bantuan kalian untuk memberikan saran dari posisi mana sebaiknya dia memfoto bangunan tersebut agar hasilnya bagus. Bisakah kalian membantunya?"

- Guru kemudian membagikan LKS, miniatur kota, dan kamera buatan. Setiap kelompok mendapat dua kamera buatan dan satu kamera digital (dibawa oleh siswa dari rumah).
- Setelah semua kelompok mendapatkan media pembelajaran di atas, guru menginstruksikan siswa untuk membaca cerita terlebih dahulu dan jka ada hal yang tidak dimengerti, siswa bisa menanyakannya pada guru. Guru juga menjelaskan bahwa siswa dapat memakai model kamera ataupun kamera digital untuk melihat dan memfoto bangunan tersebut. Selama penyelidikan, siswa harus menjawab pertanyaan yang ada pada LKS. (Untuk lebih detailnya silahkan baca LKS dan catatan mengajar pada dokumen ini)
- Selama diskusi siswa, guru berkeliling dan membimbing siswa mengerjakan LKS dan membantu siswa jika siswa merasa kesulitan. (40 menit)
- Setelah semua siswa menyelesaikan LKS, guru meminta salah satu kelompok untuk menjelaskan penemuan mereka di depan kelas (15 menit) Guru memulai dan memimpin jalannya diskusi dengan mengajukan pertanyaan di bawah:
 - Apakah kamu sudah menjawab pertanyaan nomor 1?
 - Jika kamu mengambil foto dari posisi 1, bangunan apa saja yang dapat kamu lihat dalam foto itu?
 - Bagaimana dengan posisi 2? Bangunan apa saja yang kamu lihat?

- dst.

Di sela-sela diskusi, guru juga memberi kesempatan pada kelompok lain untuk memberi tanggapan atau menyampaikan pendapat atau pertanyaan pada presenter.

Untuk pertanyaan berikutnya, 2 sampai 5, guru menunjuk kelompok lain untuk menjelaskan jawaban mereka. Sama seperti sebelumnya, guru memimpin jalannya diskusi dengan menanyakan pertanyaan dalam LKS pada kelompok. Guru juga menanyakan jawaban siswa untuk sisa pertanyaan yang ada dalam LKS. (silahkan lihat catatan mengajar untuk beberapa contoh jawaban siswa)

3. Penutupan (5 menit)

- Setelah diskusi selesai, guru mengingatkan siswa untuk memberi nama mereka dan nama kelompoknya pada LKS. Lembar kerja siswa itu kemudian dikumpulkan.
- Setelah itu, guru membimbing siswa untuk membuat kesimpulan tentang apa yang mereka lakukan hari ini.
- Guru juga mengingatan siswa bahwa pertemuan selanjutnya mereka akan menerima foto-foto yang dibuat oleh Toni. Untuk itu, setiap kelompok paling sedikit membawa satu gunting dan lem.
- Guru kemudian menutup pembelajaran.

F. Penilaian

Sumber penilaian : Observasi diskusi kelompok, LKS, dan performa selama pembelajaran

Ada tiga aspek penilaian yaitu penilaian sikap, penilaian kompetensi pengetahuan, dan penilaian keterampilan. Penilaian kompetensi pengetahuan didasarkan pada LKS kelompok. Sedangkan penilaian sikap dan keterampilan ditentukan berdasarkan observasi guru selama pembelajaran berlangsung. Jika siswa menunjukkan perilaku sesuai dengan indikator maka guru memberikan tanda centang " $\sqrt{}$ " pada kolom keterangan "Ya". Sebaliknya, jika siswa tidak menunjukan perilaku yang disebutkan dalam indikator, maka guru mencentang kolom keterangan "Tidak". Total poin dilihat dari banyaknya jawaban "Ya". Satu keterangan "Ya" bernilai 1 poin, sedangkan keterangan "Tidak" bernilai 0 poin.

Aspek	Penilaian	Ket		Total
Penilaian	Indikator	Ya	Tidak	Poin
Sikap	 Siswa mengikuti pembelajaran dari awal sampai akhir kegiatan Siswa ikut berpartisipasi dalam diskusi kelompok dan investigasi Siswa berani menyampaikan pendapat atau mengajukan pertanyaan pada saat diskusi kelas Siswa ikut berperan dalam menciptakan susasana yang kondusfi untuk belajar seperti membantu teman yang kesulitan atau mengajak teman yang kurang berpartisipasi selama kegiatan 			
Kompetensi Pengetahuan	 Siswa memahami permasalahan dan kegiatan yang dilakukan. Contoh siswa mengerti bahwa mereka harus mencari posisi terbaik untuk memfoto bangunan. Siswa dapat mengidentifikasi bangunan yang terlihat serta posisinya dengan atau tanpa bantuan kamera. Siswa dapat menentukan posisi dimana mereka mampu memotret beberapa atau semua bangunan. Siswa dapat memberikan saran atau pendapat tentang posisi terbaik untuk memotret semua bangunan atau memotret salah satu bangunan beserta dengan alasan lengkap kenapa posisi tersebut dipilih. 			
Keterampilan	 Siswa dapat menggunakan kamera buatan serta kamera digital selama diskusi dan investigasi dengan benar. Siswa dapat menjelaskan, menunjukkan dan mendemonstrasikan hasil investigasi kelompok jika ditanya oleh guru atau kelompok lain. 			
Total Poin:				

CATATAN MENGAJAR - Pembelajaran 1 Image: Description of the temperature of t

Gambaran: Pembelajaran ini bertujuan untuk memberikan siswa kegiatan pendahuluan untuk mengembangkan kemampuan spasial orientasi pada benda tiga dimensi. Pada pembelajaran ini, siswa menggunakan kamera buatan dan kamera digital utnuk melihat miniatur bangunan. Mereka berjalan mengelilingi meja untuk menemukan posisi yang benar dimana mereka dapat melihat semua obyek atau hanya sebagian obyek. Siswa akan bekerja dalam kelompok kecil yang terdiri dari 3-4 siswa. Selama penyelidikan, siswa harus menjawab pertanyaan dalam LKS untuk menunjukkan apa yang mereka temukan dan diskusikan. Guru berkeliling untuk membantu jika siswa menemukan kesulitan

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Tentang halaman ini: Halaman ini mengenalkan konteks masalah pada siswa. Guru mungkin menjelaskan gambaran cerita secara singkat sebelum membagiakan LKS pada setiap kelompok.



Toni meminta pendapatmu untuk menentukan posisi terbaik untuk mengambil gambar bangunan <u>tersebut</u>. Pertama-tama, jawablah pertanyaan di h<u>alaman selanjutnya</u>! Kemudian kamu dapat menentukan dimanakah posisi terbaik untuk mengambil gambar berdasarkan diskusi kelompokmu.

Halo, namaku Toni. Aku seorang fotografer. Kemanapun aku perai, aku selalu membawa kamera untuk memfoto

berbagai hal. Hari ini, aku pergi ke suatu kota untuk

mengambil gambar bangunan di sana.

Toni pergi ke suatu kota dimana ia menemukan 4 bangunan. Keempat bangunan itu adalah sebuah apertemen, dua rumah, dan sebuah menara. Ia

ingin mencari posisi untuk mengambil foto semua bangunan dalam satu kali

potret. Bantulah Toni mencari posisi yang bagus untuk mengambil foto

Berikut susunan bangunan tersebut. (Lihat layout)

bangunan di kota itul

Jawablah pertanyaan di bawah inil

 Fotolah bangunan tersebut dari posisi 1 sampai posisi 8 dengan kameramu atau kamera buatan yang disediakan. Berilah tanda centang "\" pada table berikut bila kamu dapat melihat bangunan tersebut pada kamera. Jika kamu tidak melihat bangunan tersebut berilah tanda strip "-" pada tabel. Misalnya, pada posisi 1, Toni dapat melihat rumah merah, menara dan apertemen, maka ia memberi tanda centang pada ketiga bangunan tersebut dan tanda strip pada rumah pink. Bagaimana dengan posisi yang lain?

Posisi	Rumah Merah	Rumah Pink	Menara	Apertemen
1	N	-	N	N
2				
3				
4				
5				
6				
7				
8				

- Jika kamu mengambil gambar dari posisi 3, dimanakah posisi rumah merah, <u>rumah</u> pink dan menara terhadap apertemen pada foto?
- Jika kamu mengambil gambar dari posisi 7, dimana posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?

Alat dan Bahan: Guru memerlukan miniatur kota dan dua kamera buatan untuk setiap kelompok. Miniatur kota dan kamera buatan diberikan pada lampiran RPP. Guru harus menggunting dan mengelem jarring-jaring yang disediakan di dokumen ini untuk mengkonstruksi kota pada LKS.

Konsep Matematika: Pada sesi ini, siswa dikenalkan dengan konsep gambar standar. Konsep ini penting untuk mengerti konstruksi obyek tiga dimensi dari gambar dua dimensi dan juga untuk membantu siswa mengembangkan kemampuan spasial orientasi dan visualisasi. Lebih lanjut, kegiatan ini juga membantu siswa mengembangkan konsep garis penglihatan (*Vision Line*).

Tentang halaman ini: pada halaman ini, siswa akan menemukan pertanyaan yang harus mereka jawab selama penyelidikan. Pada meja siswa memberi tanda check jika mereka melihat obyek dan memberi tanda strip jika mereka tidak melihat obyek. Di bawah ini contoh jawaban siswa.

Posisi	Rumah Merah	Rumah pink	Menara	Apertemen
1	N	-	N	N
2	N	-	-	١
3	N	١	١	١
4	N	N	N	N
5	N	N	Ν	N
6	-	١	-	١
7	N	١	-	٧
8	N	-	N	N

Jawablah pertanyaan di bawahl

4. Jika kamu mengambil gambar dari posisi 1, dimana posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?

5. Jika kamu mengambil gambar dari posisi 5, dimana posisi rumah merah, rumah pink dan menara terhadap apertemen pada foto?

Jawablah pertanyaan di bawah!

Posisi <u>mana sajakah</u> yang memungkinkan untuk mengambil gambar yang memuat semua bangunan?

Posisi <u>mana sajakah</u> yang memungkinkan untuk mengambil gambar yang hanya memuat 3 bangunan <u>saja</u>?

Posisi mana sajakah yang memungkinkan untuk mengambil gambar yang hanya memuat 2 bangunan <u>saja</u>?

- 2. Rumah merah berasa di sebelah kiri apertemen Menara berada di depan apertemen
 - 3. Rumah merah berada di sebelah kanan apertemen
 - Menara berada di depan apertemen
 - 4. Rumah merah berada di sebelah kanan apertemen Menara berada di tengah antara rumah merah dan apertemen
- 5. Rumah merah berada di sebelah kiri apertemen Menara juga berada di sebelah kiri apertemen

Komentar:

- Siswa mungkin menggunakan gambar untuk menunjukkan posisi rumah dan menara
- Siswa juga mungkin bingung dengan orientasi mereka untuk menentukan posisi kiri atau kanan terhadap bangunan atau terhadap diri mereka sendiri.
- Mungkin siswa juga menggunakan arah mata angin untuk mendeskripsikan posisi misal utara, selatan, dst.

Tentang halaman ini: halaman ini terdiri dari pertanyaan lanjutan untuk penyelidikan siswa. Siswa diminta untuk membuktikan, di posisi manakah, mereka bisa memotret semua bangunan atau hanya tiga atau dua bangunan.

Contoh jawaban siswa :

Untuk memotret semua bangunan: kamu dapat melakukannya pada posisi 3,4, dan 5.

Untuk memotret hanya tiga bangunan: kamu dapat melakukannya pada posisi 1, 7, dan 8.

Untuk memotret hanya dua bangunan: kamu dapat melakukannya pada posisi 2 dan 6.



atas, menurutmu posisi manakah yang terbaik untuk mengambil gambar bangunan-bangunan itu? Jelaskan jawabanmu agar Toni dapat mengerti alasanmu.

💯 Jelaskan alasanmu di sini!





Tentang halaman ini: Halaman ini berhubungan dengan pertanyaan terakhir untuk investigasi siswa. Pertanyaan ini tidak memiliki jawaban tetap. Untuk itu, siswa dapat memiliki pendapatnya masing-masing selama mereka menjelaskan alasannya dengan benar.

Contoh jawaban siswa:

Jawaban siswa mungkin berbeda. Ada kemungkinan siswa memiliki pendapatnya masing-masing. Di bawah ini contoh jawaban siswa:

- Posisi terbaik adalah antara posisi 3, 4 atau 5 karena kami dapat memotret semua bangunan. Tapi, kami lebih menyarankan posisi 4 karena kami dapat melihat sisi dari rumah dan juga apertemen.
- Aku pikir posisi 3 adalah terbaik dikarenakan kamu dapat memotret semua bangunan dan rumah. Sebagai tambahan, posisi 3 berada tepat di depan bangunan jadi foto yang dihasilkan pasti bagus.
- Menurut pendapatku, posisi 5 atau 6 adalah lokasi terbaik untuk memotret rumah pink. Foto akan lebih fokus pada rumah dibandingkan bangunan yang lain. Orang-orang akan bingung jika terlalu banyak bangunan di dalam foto.

Catatan: Guru menerima semua jawaban dan tetap menanyakan mengapa mereka memilih posisi itu. Dalam berdiskusi, penting untuk semua kelompok menyampaikan ide mereka kepada kelompok yang lain. Guru harus memfasilitasi siswa melakukannya.

		Tower
Apartemen		
		Tower











Pembelajaran 2

Nama Sekolah	: SD Laboratorium Unesa, Surabaya
Mata Pelajaran	: Mathematics
Kelas/Semester	: III/2
Indikator	: Siswa mengidentifikasi posisi kamera dimana suatu foto diambil.
Alokasi Waktu	: 2 x 35 menit

A. Tujuan Pembelajaran

- 1. Siswa dapat mengidentifikasi sudut pandang dari gambar standar suatu obyek tiga dimensi.
- 2. Siswa dapat membedakan gambar standar dari suatu obyek dengan gambar standar dari obyek lain.

B. Kemampuan Matematika yang diajarkan

Obyek tiga dimensi memiliki tampilan yang berbeda dari arah yang berbeda yang disebut dengan sudut pandang. Benda ruang selalu memiliki gambar standar dari sudut pandang yang berbeda yaitu dari depan, dari belakang, dari kanan, dari kiri, dan dari atas. Sebagai cntoh, foto bangunan di bawah dilihat dari 5 sudut pandang yang berbeda. Dalam pembelajaran kali ini, siswa harus mampu mengidentifikasi posisi sudut pandang dari gambar standar suatu obyek. Tujuannya untuk mengembangkan kemampuan spasial orientasi siswa yang sangat penting untuk belajar matematika lebih jauh dikemudian hari seperti geometri, sistem koordinat, membaca peta, dst.



C. Pendekatan dan Metode Pembelajaran

- a. Pendekatan pembelajaran: Pendidikan Matematika Realistik Indonesia (PMRI)
- b. Metode: Diskusi kelas dan kelompok

D. Sumber dan Media Pembelajaran

- a. Sumber : Lembar Kerja Siswa (LKS)
- Media: Gunting, lem, dua kamera buatan dan kamera digital, satu set *building blocks*, dan lembar kerja untuk setiap kelompok

Keterangan:

- Kamera buatan terdapat pada lampiran dokumen pembelajaran 1. Guru harus memperbanyak dan membentuk kamera tersebut dengan lem sebelum digunakan dalam pembelajaran.
- Bangun *building block* pada layout harus dibentuk dan disesuaikan dengan tanda yang terdapat pada layout. Guru dianjurkan untuk membentuk objek dengan memasang double tip pada setiap kubus sehingga benda tidak mudah hancur apabila layout secara sengaja atau tidak sengaja tertarik oleh siswa.
- Gambar berikut adalah contoh benda yang sudah tertempel pada layout dengan menggunakan double tip. Bagian yang berwarna lebih gelap pada layout merupakan bagian yang paling tinggi dari obyek.



E. Skenario Pembelajaran

- 1. Pembukaan (5 menit)
 - Guru menjelaskan bahwa hari ini siswa akan menyelidiki foto milik Toni yang sudah di cetak. Untuk itu, guru meminta siswa untuk duduk melingkar dengan kelompoknya pada satu meja.
 - Guru juga memeriksa kesiapan setiap kelompok seperti kamera digital, gunting dan lem. Jika diperlukan, guru dapat mengingatkan kembali norma sosial selama pembelajaran berlangsung seperti: mengangkat tangan jika bertanya, kerja sama

dan diskusi adalah yang terpenting, dan siswa harus menemukan dan menentukan apakah jawaban mereka benar atau salah.

2. Kegiatan Inti (60 menit)

- Untuk memulai kegiatan, guru bercerita tentang Toni si fotografer.
 - "Toni "si fotografer". Kemarin, ia mendatangiku dan meminta bantuan lagi. Ia mengambil gambar sebuah benda tetapi setelah ia mencetaknya, foto tersebut tercampur dengan foto lain. Mungkin kamu dapat membantunya, ia akan menceritakan ceritanya dalam LKS. Kalian bisa membacanya"
- Guru kemudian membagikan LKS. Setiap kelompok juga mendapatkan dua kamera buatan, satu kamera digital, 12 foto, dan *building blocks* dalam lembar kerja. Untuk mengatur *building blocks* guru hanya perlu mengikuti tanda pada layout dan mengikuti foto pada LKS. (silahkan lihat lembar kerja dan LKS pada dokumen ini)
- Setelah itu, guru menginstruksi siswa untuk memotong dan menempelkan foto pada posisi dimana foto itu diambil. Guru juga memperingatkan siswa bahwa diantara foto-foto itu, ada beberapa foto yang tidak sesuai dengan obyek dikarenakan foto tersebut diambil pada waktu yang berbeda. Siswa hanya boleh menggunakan model kamera untuk menentukan posisi foto. Kamera akan digunakan ketika diskusi kelas (untuk lebih detailnya, silahkan lihat foto dan catatan mengajar dalam dokumen ini)
- Guru juga memberitahukan pada siswa bahwa ada pertanyaan yang harus siswa jawab di LKS. Selama diskusi kelompok, guru berkeliling dan membimbing mereka mengerjakan LKS dan membantu mereka jika ada yang kesulitan. (40 menit)
- Setelah semua kelompok menyelesaikan LKS, guru meminta salah satu kelompok untuk menjelaskan bagaimana mereka memilih dan menempatkan foto-foto tersebut serta mendiskusikannya dengan siswa yang lain. (15 menit) Guru terlebih dahulu menanyakan jawaban siswa pada LKS. Guru memulai dan memimpin jalannya diskusi dengan mengajukan pertanyaan di bawah:
 - Dimana kamu meletakkan foto 1? Dimana posisi kamera pada saat foto satu diambil?

- Apakah semua menempatkan foto 1 di tempat yang sama? Adakah yang berbeda?

- Coba kalian sekarang cek dengan kamera digital! Apakah hasilnya sama? Guru kemudian melanjutkan pertanyaan ini untuk foto yang lain. Setelah itu, guru menanyakan pertanyaan di bawah untuk memastika strategi siswa:

- Bagaimana kamu menemukan posisi foto? Dapatkah kamu menjelaskannya? Misalnya, bagaimana kamu mengetahui bahwa foto ini posisinya disini?
- Mengapa kamu tidak memilih foto ini (foto yang tidak sesuai dengan obyek)?
- Jadi, foto mana yang tidak kamu pilih? Mengapa kamu tidak memilih foto itu?

Seandainya kelompok itu memilih foto yang salah dan diperdebatkan oleh kelompok lain, pertanyaan ini dapat ditanyakan pada kelompok tersebut dan tidak perlu untuk diajukan di akhir.

• Pada sesi akhir diskusi, guru menyuruh siswa untuk mengecek foto masingmasing dengan menggunakan kamera digital. Hal ini untuk mengevaluasi apakah foto yang dipilih benar dan berada pada posisi yang tepat.

3. Diskusi kelas dan penutupan (5 menit)

- Setelah diskusi selesai, guru memerintahkan siswa untuk meninggalkan pekerjaan mereka di kelas karena pekerjaan itu akan dikirimkan pada Toni (hanya pura-pura).
- Akhir pembelajaran, guru membimbing siswa untuk membuat kesimpulan tentang apa yang mereka lakukan hari ini.
- Guru juga mengingatkan siswa bahwa pertemuan selanjutnya mereka akan membuat banyak gambar jadi mereka harus membawa peralatan seperti penggaris, pensil, dsb.
- Guru kemudian menuup pembelajaran

F. Penilaian

Sumber penilaian : Observasi diskusi kelompok, LKS, dan performa selama pembelajaran

Ada tiga aspek penilaian yaitu penilaian sikap, penilaian kompetensi pengetahuan, dan penilaian keterampilan. Penilaian kompetensi pengetahuan didasarkan pada LKS kelompok. Sedangkan penilaian sikap dan keterampilan ditentukan berdasarkan observasi guru selama pembelajaran berlangsung. Jika siswa menunjukkan perilaku sesuai dengan indikator maka guru memberikan tanda centang " $\sqrt{}$ " pada kolom keterangan "Ya". Sebaliknya, jika siswa tidak menunjukan perilaku yang disebutkan dalam indikator, maka guru mencentang kolom keterangan "Tidak". Total poin dilihat dari banyaknya jawaban "Ya". Satu keterangan "Ya" bernilai 1 poin, sedangkan keterangan "Tidak" bernilai 0 poin.

Aspek	Penilaian	Ket		Total
Penilaian	Indikator	Ya	Tidak	Poin
Sikap	 Siswa mengikuti pembelajaran dari awal sampai akhir kegiatan Siswa ikut berpartisipasi dalam diskusi kelompok dan investigasi Siswa berani menyampaikan pendapat atau mengajukan pertanyaan pada saat diskusi kelas Siswa ikut berperan dalam menciptakan susasana yang kondusfi untuk belajar seperti membantu teman yang kesulitan atau mengajak teman yang kurang berpartisipasi selama kegiatan 			
Kompetensi Pengetahuan	 Siswa memahami permasalahan dan kegiatan yang dilakukan. Contoh siswa mengerti bahwa mereka harus menseleksi foto yang benar dan mencari posisi dimana foto tersebut diambil. Siswa dapat mengidentifikasi posisi dimana beberapa foto dengan atau tanpa bantuan kamera buatan. Siswa dapat membedakan beberapa foto yang besaral dari objek dengan foto dari objek lain. Siswa dapat menentukan semua posisi foto dengan benar dan menjelaskan kenapa beberapa foto bukan berasal dari objek yang diberikan. 			
Keterampilan	 Siswa dapat menggunakan kamera buatan serta kamera digital selama diskusi dan investigasi dengan benar. Siswa dapat menjelaskan, menunjukkan dan mendemonstrasikan hasil investigasi kelompok jika ditanya oleh guru atau kelompok lain. 			
Total Poin:				

CATATAN MENGAJAR - Pembelajaran 2



Nama

No Kelompok: _____

Hallo, kita bertemu lagi, Toni "si fotografer". Kemarin, aku mengambil banyak sekali foto dari benda yang dapat kamu lihat di meja sekarang. Aku telah mencetak semuanya, akan tetapi foto-foto itu bercampur dengan foto-foto lain yang kuambil pada waktu yang berbeda. Aku bingung untuk menemukan manakah foto dari benda yang kuambil kemarin. Dapatkah kamu membantuku seperti kemarin?

Kamu harus membantu Toni untuk menemukan manakah foto yang benar yang diambil dari berbagai posisi. Toni mengirimimu foto di balik halaman ini. Kamu bisa menggunting dan menempelkannya pada posisi dimana Toni mengambilnya. Akan tetapi, berhati-hatilah karena di antara foto-foto itu, terdapat beberapa foto yang diambil pada waktu yang berbeda dengan benda yang mirip. Jangan sampai salah memilih foto yal



Tujuan:- Siswa dapat mengidentifikasi sudut pandang dari gambar standar suatu obyek tiga dimensi.

- Siswa dapat membedakan gambar standar dari suatu obyek dengan gambar standar dari obyek lain.

Alokasi waktu: 2x35 menit

Gambaran: Pembelajaran ini bertujuan untuk mengembangkan kemampuan spasial orientasi siswa dengan memberikan masalah tentang orientasi suatu obyek. Pada pembelajaran ini, siswa akan bekerja dalam kelompok kecil yang terdiri dari 3-4 siswa untuk melakukan penyelidikan bersama. Mereka menggunakan kamera buatan dan kamera digital untuk melihat obyek pada lembar kerja. Mereka berjalan mengelilingi meja untuk menemukan posisi dimana foto itu diambil. Selama penyelidikan, siswa harus menjawab pertanyaan pada LKS untuk menunjukkan apa yang mereka temukan. Siswa membutuhkan gunting untuk menggunting foto dan lem untuk menempelkan foto pada lembar kerja. Untuk itu, guru harus memberitahu siswa untuk membawa gunting dan lem pada akhir pertemuan sebelumnya.

Tentang halaman ini: Halaman ini bertujuan mengenalkan konteks masalah pada siswa. Guru bisa menyampaikan gambaran cerita terlebih dahulu sebelum membagikan LKS pada setiap kelompok.





Alat dan Bahan: Setiap kelompok membutuhkan setidaknya satu gunting, lem, dan satu set *building blocks*, dua kamera buatan, dan satu kamera digital. Terdapat area yang ditandai pada lembar kerja sebagai petunjuk untuk guru dimana *building blocks* ditempelkan atau ditempatkan.

Konsep Matematika: kegiatan pada pembelajaran ini membantu siswa untuk mengembangkan kemampuan spasial orientasi pada obyek tiga dimensi. Kegiatan ini juga memiliki peran penting dalam mengembangkan pemahaman pada geometri tiga dimensi seperti menentukan bentuk benda, manipulasi secara mental dan perspektif.



Komentar:

- Siswa mungkin mengalami kebingungan ketika mereka mencoba mengidentifikasi posisi foto. Dalam hal ini, guru harus membantu mereka dengan mendemonstrasikan bagaimana mencari posisi salah satu foto sebagai contoh kepada siswa.
- Siswa juga mungkin mengalami kesulitan untuk membedakan foto mana yang bukan dari obyek. Dalam hal ini, guru bias memancing mereka dengan menanyakan "berapa banyak kotak yang dapat kamu lihat pada kamera?""menghadap kemanakah bentuk "L" ini?""apa bedanya antara dua foto ini? Mereka sama tapi ada sesuatu yang aneh dan beda bukan?"

Tentang halaman ini: setelah menginvestigasi posisi foto, siswa harus menjawab pertanyaan pada halaman ini. Hal ini untuk menunjukkan jawaban mereka dan untuk mengetahui strategi mereka untuk menyelesaikan masalah.

Contoh jawaban siswa:

- Berapa banyak foto yang kamu tempel? 8 foto
- 2. Foto nomor berapa saja yang kamu pilih dan temple? Foto 1, 2, 3, 4, 7, 8, 9, dan 11.
- Berapa banyak foto yang tidak kamu tempel?
 4 foto
- 4. Foto nomor berapa saja yang tidak kamu pilih dan tempel? Foto 5, 6, 10, dan 12.
- 5. Jelaskan mengapa foto tersebut tidak dipilih? Beri tanda pada foto bagian yang berbeda dari benda aslinya!

Karena foto itu berbeda dengan obyek. Bentuk L menghadap sisi yang lain. Tidak sama dengan aslinya. Terbalik.



💽 Jawablah pertanyaan berikut ini!

1. Berapa banyak foto yang kamu tempel?

2. Foto nomor berapa saja yang kamu pilih dan tempel?

3. Berapa banyak foto yang tidak kamu pilih dan tidak kamu tempel?

- 4. Foto nomor berapa saja yang tidak kamu pilih dan tidak kamu tempel?
- Tempelkan foto yang tidak terpilih di balik halaman ini! Kemudian tuliskan alasan disamping foto tersebut kenapa kamu tidak memilihnya!

Melaporkan Penemuan Candi



Pembelajaran 3

Nama Sekolah	: SD Laboratorium Unesa, Surabaya
Mata pelajaran	: Matematika
Kelas/Semester	: III/2
Indikator	: Siswa dapat menggambar tampilan standar dari suatu obyek 3D
	berdasarkan model dan fotonya.
Alokasi waktu	: 2 x 35 menit

A. Tujuan Pembelajaran

- 1. Siswa dapat menggambar tampilan standar dari suatu obyek 3D dari sudut pandang yang berbeda berdasarkan modelnya.
- 2. Siswa dapat menggambar tampilan standar dari suatu obyek 3D dari sudut pandang yang berbeda hanya berdasarkan foto perspektifnya (*Distant Representation*).

B. Kemampuan Matematika yang diajarkan

Suatu obyek 3D memiliki tampilan yang berbeda dari sudut pandang yang berbeda. Sebuah obyek selalu memiliki 5 tampilan standar dari 5 sudut pandnag yang berbeda: depan, belakang, kiri, kanan dan atas. Sebagai contohnya, kotak di bawah dilihat dari 5 sudut pandang yang berbeda.



Pembelajaran hari ini bertujuan untuk mengembangkan spasial visualisasi siswa dengan meminta siswa untuk mengidentifikasi dan mengambar tampilan standar dari suatu obyek 3D.

C. Pendekatan dan Metode Pembelajaran

- a. Pendekatan pembelajaran: Pendidikan Matematika Realistik Indonesia (PMRI)
- b. Metode: diskusi kelas dan kelompok

D. Sumber dan Media Pembelajaran

- a. Sumber : Lembar Kerja Siswa (LKS)
- b. Media: *building blocks*, dua kamera buatan, kamera digital dan satu lembar kerja untuk setiap kelompok.

Keterangan:

- Kamera buatan terdapat pada lampiran dokumen pembelajaran 1. Guru dapat memperbanyak dan digunakannya pada pembelajaran.
- Untuk meningkatkan efisiensi penggunaan *building blocks*, setiap kelompok pada sebaiknya hanya diberi tepat 11 kubus. Pada kegiatan satu, siswa hanya memerlukan 8 kubus, sedangkan kegiatan kedua memerlukan 11 kubus.

E. Skenario Pembelajaran

- 1. Pembukaan (5 menit)
 - Guru menjelaskan bahwa hari ini mereka akan bermain dengan kamera lagi dan menggambar.
 - Untuk itu, guru meminta siswa duduk melingkar dengan kelompoknya dalam satu meja. Guru juga menginstruksi siswa untuk menyiapkan peralatan menggambar seperti pensil, penghapus, dan penggaris.

2. Kegiatan Inti (60 menit)

• Untuk memulai kegiatan, guru memberitahu siswa tentang konteks pembelajaran kali ini.

"Aku memiliki teman, namanya Reza. Dia adalah seorang arkeolog yang sering meneliti dan menemukan banyak candi di Indonesia. Hari ini, dia ingin membagikan pengalamannya selama bekerja menjadi arkeolog. Kamu akan tahu apa yang harus kamu lakukan dalam LKS."

• Guru kemudian membagikan LKS, 11 kubus dan lembar kerja untuk setiap kelompok. Setiap kelmpok juga mendapatkan dua kamera buatan. (silahkan lihat lembar kerja dan LKS yang terdapat pada dokumen ini)

- Guru meninstruksi siswa untuk membuat candi mereka sendiri dengan menggunakan 8 kubus (tidak kurang dan tidak lebih) di tengah atau di luar layout/lembar kerja yang telah dibagikan. Guru juga harus memastikan bahwa candi tidak akan terlalu susah dan terlalu mudah. Contohnya jika siswa hanya meletakkan kubus membentuk bangun horisontal atau vertikal saja seperti garis, maka guru harus menantang mereka untu membuatnya lebih menarik. (5 menit) (Lihat tugas pada LKS)
- Setelah semua kelompok memiliki candinya masing-masing, guru meminta mereka untuk melaporkannya pada pemerintah dengan membuat gambar standar dari candi dalam lembar kerja yang telah dibagikan. Siswa dapat menggunakan kamera buatan untuk membantu dalam menggambar. Kamera digital tidak boleh digunakan dulu karena kamera tersebut akan dipakai untuk mengecek hasil gambar setelah diskusi (Lihat tugas pada LKS)
- Selama diskusi kelompok, guru berkeliling dan membimbing siswa dalam mengerjakan LKS dan membantu mereka jika ada yang mengalami kesulitan. (15 menit)
- Setelah semua siswa menyelesaikan gambar, guru menginstruksikan setiap kelompok untuk mengecek hasil gambar kelompok sebelahnya dengan menggunakan kamera digital masing-masing. Jika ada yang salah/berbeda maka guru bisa mendiskusikan di tempat kelompok tersebut dan membantu siswa memecahkan masalah tersebut. (5 menit)
- Setelah proses pengecekan selesai, guru menginstruksikan siswa untuk melanjutkan pada kegiatan berikutnya yaitu menggambar candi lain yang hanya bisa dilihat pada foto yang dikirm reza. Guru menekankan siswa boleh menggunakan strategi apapun termasuk dengan menggunakan kubus. Tetapi untuk kegiatan 3, kubus tidak boleh digunakan. (Lihat catatan mengajar dan LKS) (15 menit)
- Guru memilih satu kelompok untuk mempresentasikan pekerjaan mereka pada kegiatan kedua di depan kelas. Guru memimpin jalannya diskusi dengan mengajukan pertanyaan seperti: (10 menit)
 - Dapatkah kamu menggambar candinya?
 - Perlihatkan gambarnya pada siswa lain. Bagaimana kamu menggambarnya?
 - Mengapa hanya 3 kubus (misalnya) pada gambar ini?

- Apakah kelompok lain membuat gambar yang sama? Adakah yang berbeda?
- dst

Terkadang, guru juga memberikan kesempatan pada siswa lain untuk menanggapi atau jika mereka memiliki pertanyaan.

- Guru kemudian memilih kelompok lainnya untuk mempresentasikan permasalahan ketiga. Guru membantu siswa berdiskusi dengan mengajukan pertanyaan pada kelompok. (10 menit)
 - Dapatkah kamu memperlihatkan gambar milikmu?
 - Darimana kamu mengambilnya?
 - Bagaimana kamu membutikan bahwa gambarnya akan menjadi seperti itu?
 - Mengapa hanya ada 3 peregi/persegipanjang/kubus disini?

Guru juga memberikan kesempatan pada siswa lain untuk menanggapi atau memberi pendapat pada presentasi, terutama jika mereka memiliki strategi atau pendapat yang berbeda.

3. Penutupan (5 menit)

- Setelah diskusi selesai, guru menginstruksikan setiap kelompok untuk menggantung atau menempelkan gambar mereka pada dinding jadi kelompok atau orang lain dapat melihatnya.
- Guru kemudian membimbing siswa untuk membuat kesimpulan tentang apa yang mereka lakukan hari ini.
- Guru menutup pembelajaran dengan mengingatkan siswa bahwa pertemuan selanjutnya siswa akan tetap membuat gambar. Untuk itu, mereka harus membawa peralatan menggambar lagi.

F. Penilaian

Sumber penilaian : Observasi diskusi kelompok, LKS, dan performa selama pembelajaran

Ada tiga aspek penilaian yaitu penilaian sikap, penilaian kompetensi pengetahuan, dan penilaian keterampilan. Penilaian kompetensi pengetahuan didasarkan pada LKS kelompok. Sedangkan penilaian sikap dan keterampilan ditentukan berdasarkan observasi guru selama pembelajaran berlangsung. Jika siswa menunjukkan perilaku sesuai dengan indikator maka guru memberikan tanda centang " $\sqrt{}$ " pada kolom keterangan "Ya". Sebaliknya, jika siswa tidak menunjukan perilaku yang disebutkan dalam indikator, maka guru mencentang kolom keterangan "Tidak". Total poin dilihat dari banyaknya jawaban "Ya". Satu keterangan "Ya" bernilai 1 poin, sedangkan keterangan "Tidak" bernilai 0 poin.

Aspek	Penilaian	Ket		Total
Penilaian	Indikator	Ya	Tidak	Poin
Sikap	 Siswa mengikuti pembelajaran dari awal sampai akhir kegiatan Siswa ikut berpartisipasi dalam diskusi kelompok dan investigasi Siswa berani menyampaikan pendapat atau mengajukan pertanyaan pada saat diskusi kelas Siswa ikut berperan dalam menciptakan susasana yang kondusfi untuk belajar seperti membantu teman yang kesulitan atau mengajak teman yang kurang berpartisipasi selama kegiatan 			
Kompetensi Pengetahuan	 Siswa memahami permasalahan dan kegiatan yang dilakukan. Contoh siswa mengerti bahwa mereka harus membentuk dan menggambar candi dari berbagai sisi. Siswa dapat menggambar tampilan candi minimal dari 2 sudut pandang dengan baik. Siswa dapat mengidentifikasi dan menggambar tampilan candi berdasarkan foto pada kegiatan kedua dengan atau tanpa bantuan kubus kayu. Minimal siswa dapat menggambar tampilan candi dari satu sudut pandang. Siswa dapat mengidentifikasi dan menggambar tampilan candi berdasarkan foto pada kegiatan kedua dengan atau tanpa bantuan kubus kayu. Minimal siswa dapat menggambar tampilan candi dari satu sudut pandang. Siswa dapat mengidentifikasi dan menggambar candi hanya berdasarkan fotonya pada kegiatan ketiga tanpa bantuan kubus kayu. 			
Keterampilan	 Siswa dapat menggunakan kamera buatan serta kamera digital selama diskusi dan investigasi dengan baik dan benar. Siswa dapat menjelaskan, menunjukkan dan mendemonstrasikan hasil investigasi kelompok jika ditanya oleh guru atau kelompok lain. 			
		Т	otal Poin:	

CATATAN MENGAJAR - Pembelajaran 3



NAMA:



Aku ingin berbagi pengalaman denganmu sebagai seorang arkeolog. Sekarang, ikuti petunjukku: 1. Andaikan, kamu adalah seorang arkeolog.

2. Sekarang, bangunlah satu candi yang menarik dengan menggunakan tepat 8 kubus.



- 3. Laporkan candimu pada pemerintah dengan melukis gambar candimu dari bagian depan, belakang, kiri, kanan dan atas candi menggunakan kamera yang disediakan.
- Aku tunggu laporan candi kalian!!!

Berikut ini contoh gambar dalam laporan candi milik reza. Lihat!



Candi milik Reza

Gambar candi milik Reza tampak dari depan

Tujuan: - Siswa dapat menggambar tampilan standar dari suatu obyek 3D

- dari sudut pandang yang berbeda berdasarkan modelnya.
- Siswa dapat menggambar tampilan standar dari suatu obyek 3D dari sudut pandang yang berbeda hanya berdasarkan foto perspektifnya (Distant Representation).

Alokasi waktu: 2x35 menit

Gambaran: Tujuan pembelajaran ini adalah untuk mengembangkan spasial visualisasi siswa dengan membayangkan bentuk/tamilan obyek dari berbagai sisi yang berbeda. Pada pembelajaran ini, siswa akan bekerja dalam kelompok kecil yang terdiri dari 3-4 siswa untuk melakukan penyelidikan. Pada kegiatan pertama, siswa menggambar tampilan standar candi yang mereka buat dari 5 sisi (depan, belakang, kiri, kanan dan atas). Untuk melakukannya, siswa akan dibagikan model (building blocks) dan juga dua kamera buatan serta kamera digital.

Tantang halaman ini: Guru disarankan untuk menyampaikan gambaran cerita pada halaman ini sebelum membagikan LKS pada tiap kelompok. Di halaman ini terdapat petunjuk tentang apa yang harus siswa lakukan untuk memulai penyelidikan.

Komentar:

- Siswa harus mengkonstruksi candi mereka tepat dengan 6 kubus.
- Guru harus menantang siswa untuk membuat candi yang unik, Hal ini dilakukan agar siswa tidak membuat candi yang terlalu mudah dikonstruksi seperti garis, menara, atau hanya sebuah kotak.
- Guru dapat memberi siswa saran untuk membuat candi tertentu jika mereka tidak memiliki ide.
- Obyek dapat diletakkan di tengah lembar kerja. Tapi ketika mereka menggambar dari sisi atas, mereka harus memindahkannya.

Alat dan Bahan: siswa membutuhkan peralatan menggambar seperti pensil, penghapus, dan penggaris. Untuk itu, guru harus mengingatkan siswa untuk membawa peralatan itu pada pembelajaran sebelumnya.

Konsep Matematika: Kegiatan ini merupakan pendahuluan untuk mengembangkan *3D geometry thinking* siswa. Selain itu, ada banyak konsep matematika yang menggunakan kemampuan visualisasi siswa dalam proses penalaran seperti konstruksi jaring-jaring suatu bangun ruang, mencari volume, perspektif, dan masih banyak lagi.

Di bawah ini merupakan contoh dari jawaban siswa:





Hai kawan, kamu membuat candi yang menarik dan juga laporan yang bagus. Aku ingin memberitahumu sesuatu. Kemarin, aku menemukan sebuah candi baru. Aku sudah kirimkan gambarnya untukmu.

Aku memiliki tugas untukmu. "Tolong buatkan laporan dari candi tersebut!". Aku benar-benar sibuk saat ini untuk mengivestigasi lebih jauh candi itu. Aku sudah kirimkan foto candinya. Aku harap foto itu cukup untuk panduan membuat laporannya.



Foto candi baru

Candi baru di <u>atas</u> memiliki alas berbentuk persegi. Tahukah kamu berapa bongkah batu yang ada pada candi di bawah ini? __ Gambarlah candi pada lembar kerja yang telah dibagikan! **Tentang halaman ini:** Ini adalah konteks dari kegiatan kedua. Guru harus memastikan bahwa siswa mengerti konteks dan permasalahan. Khususnya, siswa harus mengerti bahwa dasarnya adalah persegi sehingga tidak ada kubus lain di bagian yang tak terlihat.

Contoh jawaban siswa:



Komentar:

- Guru harus berkeliling karena ada besar kemungkinan siswa mengalami kesulitan untuk membayangkan bentuknya. Jika demikian guru bisa menyarankan siswa untuk menggunakan kubus kayu sebagain bantuan.
- Jika siswa tidak dapat membayangkan bentuknya, guru dapat pula membantu mereka dengan menggunakan gambar yang menyerupai seperti kubus dan meminta siswa untuk menggambar sisinya. Setelah itu, guru membawa siswa untuk melihat candi seperti konstruksi kubus.



Tentang halam ini:

Halaman ini memuat tugas terakhir pada pembelajaran kali ini. Siswa harus mengerti konteks sebelum memulai penyelidikan. Di sisni, siswa diminta untuk menggambar bentuk candi dari sudut pandnag 2 (kiri). Berbeda dengan tugas sebelumnya. Siswa menggambar bentuk pada LKS, tidak pada layout.

Contoh jawaban siswa:

Komentar:

- Guru sebisa mungkin melarang siswa untuk tidak menggunakan bantuan kubus lagi. Namun jika terpaksa maka guru bisa menggunakannya.
- Jika siswa tidak dapat membayangkan bentuknya, guru dapat membantu mereka menggunakan gambar yang menyerupai seperti kubus dan meminta siswa untuk menggambar sisinya. Setelah itu, guru membawa siswa untuk melihat candi seperti konstruksi kubus
 Kemungkinan siswa akan menggambar dengan meniru bentuk obyek pada foto. karena mereka tidak mengerjakan pada layout seperti permasalah sebelumnya. Contohnya:



Membangun Miniatur Candi



Pedoman Guru untuk Pembelajaran 4

- RPP (Rencana Pelaksanaan Pembelajaran)
- Catatan mengajar

-

Pembelajaran 4

Nama Sekolah	: SD Laboratorium Unesa, Surabaya
Mata Pelajaran	: Matematika
Kelas/Semester	: III/2
Indikator	: - Siswa dapat mengkonstruksi suatu obyek 3D berdasarkan fotonya
	(Bird eye photograph).
	- Siswa dapat mengkonstruksi suatu obyek 3D berdasarkan gambar
	standar.
Alokasi waktu	: 2 x 35 menit

A. Tujuan Pembelajaran

- 1. Siswa dapat mengidentifikasi, memprediksi dan membaca bagian yang tidak terlihat dari *distance representation* dari suatu obyek 3 dimensi.
- 2. Siswa dapat menafsirkan, mengidentifikasi dan mengkomunikasikan *distance representation* dari suatu obyek 3 dimensi.
- 3. Siswa dapat mengkonstruksi suatu obyek 3 dimensi berdasarkan gambar standarnya.

B. Kemampuan Matematika yang diajarkan

Membaca representasi obyek 3D tidak mudah bagi siswa kelas 3 SD. Membaca berarti mengerti dan hal itu membutuhkan kemampuan visualisasi dan orientasi yang baik. Selama proses membaca dan memahami representasi obyek 3D, siswa harus membayangkan menggerakan, memutar dan memanipulasi obyek tersebut. Memahami representasi 3D yang paling sederhana adalah membuat model obyek 3D tersebut berdasarkan fotonya (*Bird eye photograph*). Namun, hal ini sulit dan tidak mungkin dilakukan karena kita tidak pernah tahu bagaimana tampilan obyek jika dilihat dari belakang. Bagian yang tidak terlihat dari obyek ini lah yang disebut sisi tak terlihat. Pembelajaran kali ini bertujuan membuat siswa sadar dengan bagian yang tak terlihat ini dan memberi pengalaman awal untuk mengkonstruksi obyek berdasarkan tampilan standarnya.

Pada pertemuan kali ini, siswa diminta untuk mengkonstruksi obyek berdasarkan foto dan juga berdasarkan tampilan standarnya.





Gambar standar suatu candi

C. Pendekatan dan Metode Pembelajaran

- a. Pendekatan pembelajaran: Pendidikan Matematika Realistik Indonesia (PMRI)
- b. Metode: diskusi kelompok dan kelas

D. Sumber dan Media Pembelajaran

- a. Sumber: Lembar Kerja Siswa (LKS)
- b. Media: foto, *building blocks*, dan laporan/lembar kerja untuk setiap kelompok.

Keterangan:

 Untuk meningkatkan efisiensi penggunaan kubus, guru sebaiknya hanya memberi setiap kelompok tepat 20 kubus. Pada kegiatan satu, setiap grup hanya memerlukan 15 sampai 20 kubus. Sedangkan pada kegiatan kedua, setiap grup memerlukan tepat 9 kubus.

E. Skenario Pembelajaran

- 1. Pendahuluan (1 menit)
 - Guru menjelaskan bahwa hari ini mereka akan membuat candi dengan menggunakan balok (building blocks). Untuk itu, siswa diminta duduk melingkar dengan kelompoknya dalam stau meja.

2. Kegiatan inti 1 (35 menit)

• Untuk memulai kegiatan, guru menyampaikan pada siswa cerita dibalik kegiatan kali ini.

"Kemarin, Reza pergi ke kantor pemerintah dan ia mendapat laporan tentang candi yang baru ditemukan. Ia diminta oleh pemerintah untuk membangun miniatur candi tersebut. Namun, ia sedang sedikit sibuk jadi ia meminta kamu untuk membantunya lagi."

(Baca pada LKS)

- Guru kemudian membagikan LKS dan *building blocks* pada setiap kelompok (Lihat LKS)
- Guru menyampaikan pada siswa bahwa mereka memiliki 10 menit untuk mendiskusikan dengan kelompoknya dan mengkonstruksi candi. Guru kemudian berkeliling dan membimbing siswa mengerjakan LKS. (10 menit) (Lihat catatan mengajar utnuk melihat contoh jawaban siswa)

Diskusi (10-15 menit)

- Guru meminta perhatian siswa untuk diskusi kelas. Guru memulai diskusi dengan menanyakan hasil konstruksi dari salah satu kelompok dan membandingkannya dengan konstruksi kelompok lain. Jika ada perbedaan, guru harus mendiskusikannya, terutama jika terkait dengan sisi tak terlihat.
- Jika ada kelompok yang menyadari bahwa bagian belakang candi tak terdefinisi atau tak terbaca, guru bisa menanyakan pertanyaan berikut pada siswa seperti:
 - Mengapa kamu tidak dapat mengkonstruksi candi itu?
 - Apakah ada bagian dari candi pada foto yang tidak kamu ketahui?
 - Bagaimana dengan kelompok lain, apakah kelompok yang lain juga memiliki kesulitan yang sama?
- Jika tidak ada perbedaan dan semua kelompok dapat membuat candi (tidak menyadari bagian yang tidak terlihat), guru harus menunjukkan konstruksi yang mirip dengan foto namun memiliki lebih banyak balok di sisi tak terlihat (di belakang candi yang terdapat bagian yang tidak terlihat pada foto)(Lihat catatan mengajar). Kemudian, guru dapat menanyakan pertanyaan di bawah untuk membuat siswa menyadari bagian yang tak terlihat.
 - Apakah candi ini mirip dengan foto?
 - Mengapa kita memiliki foto yang sama namun candi yang kita konstruksi berbeda?

- Ada yang tahu alasannya?
- Bagaimana tampilan candi jika dilihat dari belakang?
- Dapatkan kita benar-benar membuat candi yang sama persis?
- Setelah semua siswa menyetujui bahwa sisi tak terlihat/bagian belakang candi tidak dapat dibuktikan, guru menyimpulkan bahwa satu foto saja tidak cukup. Kemudian guru menanyakan pada siswa foto seperti apa yang mereka butuhkan untuk mengkonstruksi candi. Setelah itu, guru memberi setiap kelompok tampilan standar dari candi. Guru menanyakan kembali apakah sekarang mereka dapat mengkonstruksi candi atau tidak. (10 menit)
- Setelah semua kelompok selesai mengkosntruksi candi, guru beralih ke kegiatan selanjutnya.

3. Kegiatan inti 2 – Tampilan standar (33 menit)

- Mula-mula, guru membagikan LKS dan layout/laporan baru dengan gambar standar. (Lihat LKS dan catatan mengajar)
- Guru kemudian membagikan LKS, foto dan *building blocks* untuk setiap kelompok.
- Guru berkeliling utnuk melihat dan membantu diskusi siswa (15 menit)
- Setelah semua kelompok menyelesaikan pekerjaannya, guru menunjuk satu kelompok untuk mempresentasikan model mereka. Guru memimpin diskusi dengan mengajukan pertanyaan. Pertama, kelompok tersebut harus menunjukkan hasil konstruksi pada kelompok yang lain. Jika ada perbedaan antar kelompok, maka guru harus mendiskusikan strategi mereka dan alasan mereka mengapa mereka memiliki model seperti itu. Jika perbedaan dikarenakan banyak kubus kayu yang digunakan maka guru harus menyuruh siswa untuk mengecek kembali dan meyakinkan siswa bahwa ternyata ada dua candi yang mungkin dibuat dari laporan (Lihat catatan mengajar). Setelah siswa setuju dan menyimpulkan bahwa ada dua candi yang mungkin, guru kemudian memberikan tambahan informasi bahwa candi tersebut ternyata hanya memiliki 9 bongkah batu. Dengan tambahan informasi ini, setiap kelompok diharapkan kini memiliki konstruksi yang sama. (18 menit)

4. Penutup (1 menit)

• Guru membimbing siswa untuk membuat kesimpulan tentang apa yang mereka lakukan hari ini.

• Guru juga mengingatkan siswa untuk tidak merusak miniatur karena miniatur itu akan didokumentasikan.

F. Penilaian

Sumber penilaian : Observasi diskusi kelompok, LKS, dan performa selama pembelajaran

Ada tiga aspek penilaian yaitu penilaian sikap, penilaian kompetensi pengetahuan, dan penilaian keterampilan. Penilaian kompetensi pengetahuan didasarkan pada LKS kelompok. Sedangkan penilaian sikap dan keterampilan ditentukan berdasarkan observasi guru selama pembelajaran berlangsung. Jika siswa menunjukkan perilaku sesuai dengan indikator maka guru memberikan tanda centang " $\sqrt{}$ " pada kolom keterangan "Ya". Sebaliknya, jika siswa tidak menunjukan perilaku yang disebutkan dalam indikator, maka guru mencentang kolom keterangan "Tidak". Total poin dilihat dari banyaknya jawaban "Ya". Satu keterangan "Ya" bernilai 1 poin, sedangkan keterangan "Tidak" bernilai 0 poin.

Aspek	Penilaian Ket		et	Total
Penilaian	Indikator	Ya	Tidak	Poin
Sikap	 Siswa mengikuti pembelajaran dari awal sampai akhir kegiatan Siswa ikut berpartisipasi dalam diskusi kelompok dan investigasi Siswa berani menyampaikan pendapat atau mengajukan pertanyaan pada saat diskusi kelas Siswa ikut berperan dalam menciptakan susasana yang kondusfi untuk belajar seperti membantu teman yang kesulitan atau mengajak teman yang kurang berpartisipasi selama kegiatan 			
Kompetensi Pengetahuan	 Siswa memahami permasalahan dan kegiatan yang dilakukan. Contoh siswa mengerti bahwa pada kegiatan ini mereka harus membangun candi yang sesuai denga foto atau laporan yang diberikan. Siswa dapat membangun candi yang mirip dengan <i>bird eye</i> foto meskipun tidak sesuai dengan gambar standar. Siswa dapat membangun candi yang sesuai <i>bird eye</i> foto dan sesuai dengan gambar standar. Siswa dapat membangun candi hanya berdasarkan gambar standard an informasi tambahannya pada kegiatan kedua. 			
Keterampilan	 Siswa dapat menjelaskan, menunjukkan dan mendemonstrasikan hasil investigasi kelompok jika ditanya oleh guru atau kelompok lain. 			
Total Poin:				
CATATAN MENGAJAR - Pembelajaran 4



Nama

No Kelompok: _

.

Hallo ^-^ Kemarin, aku pergi ke kantor pemerintah. Mereka menerima laporan tentang candi baru. Masalahnya, pengirim <u>laporan</u> hanya mengirimkan satu foto dari candi <u>tersebut</u>. Mereka tidak melampirkan semua foto seperti yang kamu biasa lakukan sebelumnya. Padahal pemerintah harus membuat miniatur candi tersebut. Sekarang mereka bingung karena mereka hanya memiliki satu foto dari candi tersebut.

to dari ua foto Padahal rsebut. hanya



Tolong bantu pemerintah membuat miniatur dari candi dalam foto <u>tersebut</u> dengan menggunakan kubus. Foto candi dapat dilihat di bawah. Aku harus pergi dulu, beritahu gurumu jika kamu dapat

menemukan bagaimana untuk membangun miniatur. Terima kasih, kawan!



Foto Candi Baru

- **Tujuan:** Siswa dapat mengidentifikasi, memprediksi dan membaca sisi yang tidak terlihat dari *distance representation* dari suatu obyek 3D.
 - Siswa dapat menafsirkan, mengidentifikasi dan mengkomunikasikan *distance representation* dari suatu obyek 3D.
 - Siswa dapat mengkonstruksi suatu obyek 3D berdasarkan gambar standarnya.

Alokasi waktu:2x35 menit

Gambaran: Pada kegiatan pertama, siswa harus mengkonstruksi miniatur candi pada foto. Tujuannya untuk membuat siswa menyadari sisi tak terlihat dari gambar. Kegiatan ini juga menstimulasi penalaran siswa tentang tampilan standar suatu obyek. Guru menyediakan *building blocks* untuk siswa. Setiap kelompok memerlukan paling sedikit 20 balok.seperti biasa setiap kelompok akan bekerja dengan kelompok kecil yang terdiri dari 3-4 siswa.

Tentang halaman ini: halaman ini menjelaskan konteks dibalik kegiatan dan tugas untuk siswa. Di bagian bawah ada foto candi yang harus siswa bangun.

Konsep Matematika: Tugas ini meliputi tidak hanya terkait dengan representasi 3D tetapi juga *spatial structuring*. Siswa menggunakan pengetahuan sebelumnya spasial orientasi dan visualisasi untuk menyelesaikan masalah ini. Jenis penalaran seperti ini sangat penting terutama pada bidang geometri.

Alat dan Bahan: foto terlampir pada dokumen ini. Guru juga membutuhkan banyak *building block*.



Untuk kasus ini, guru membantu siswa dengan cara menggunakan strategy pada pertemuan sebelumnya dengan membayangkan melihat obyek dari samping. Selanjutnya, guru bias bertanya pertanyaan seperti "dimana letak tiga kubus ini terhadap balok yang ini?" (kolom kubus)





Komentar:

- Siswa mungkin tidak menyadari bagian yang tidak terlihat dari candi pada foto. Dalam hal ini, guru bias mengajukan candi yang mirip dengan foto tapi ada perbedaan pada bagian belakang. Setelah itu, guru dapat menanyakan siswa seperti apakah tampilan candi dari belakang.
- Setelah siswa menyimpulkan bahwa mereka memerlukan foto lebih banyak, guru kemudian memberikan setiap kelompok tiga foto standar candi tersebut. Foto ini terlampir pada dokumen ini.

Jawaban siswa:

Gambar di samping merupakan contoh jawaban siswa. Gambar-gambar itu mirip pada bagian depan dan samping namun berbeda pada bagian belakang. Beberapa siswa/kelompok mungkin menyatakan bahwa mereka tidak bisa membangun candi itu. Sebagian lagi mungkin mengatakan bahwa mereka bisa membangun candi dengan menebak bagaimana bentuk belakang candi.

Pada kasus disamping, guru mengajukan konstruksi yang lain tetapi mirip dengan candi yang ada di dalam foto, tetapi terdapat perbedaan pada bagian belakang candi. Dari sini, guru kemudian menggiring siswa menuju kesimpulan bahwa bagian belakang candi tidak bias dikonstruksi. Dioerlukan foto yang lebih banyak



Gambaran: kegiatan kedua pada pembelajaran ini masih tentang menkonstruksi candi. Kali ini siswa harus mengkonstruksi candi berdasarkan tampilan standarnya menggunakan *building blocks*. Tampilan standar ini diberikan/disusun pada layout/laporan.

Tentang halaman ini: Halaman ini memuat konteks untuk kegiatan dan juga foto candi. Foto tidak dapat digunting karena guru akan menyediakan foto lain untuk setiap kelompok dan sebuah layout/lembar kerja.

Komentar:

- Guru dapat menyarankan siswa untuk mendirikan foto. Sehingga mereka bisa membayangkan candi dengan mudah.
- Siswa sebaiknya memulai konstruksi dari tampilan atas terlebih dahulu. Hal ini lebih mudah daripada mengkonstruksi candi dari samping.





Memperbaiki Laporan Candi



Pedoman Guru untuk Pembelajaran 5

- RPP (Rencana Pelaksanaan Pembelajaran)
- Catatan mengajar

1

PEMBELAJARAN 5

Nama Sekolah	: SD Laboratorium Unesa, Surabaya	
Mata pelajaran	: Mathematics	
Kelas/Semester	: III/2	
Indikator	: Siswa dapat mengidentifikasi dan menggambar tampilan standar dari	
	suatu obyek berdasarkan sifat dan hubungannya dengan tampilan standar	
	yang lain dari obyek tersebut.	
Alokasi waktu	: 2 x 35 menit	

A. Tujuan Pembelajaran

- 1. Siswa dapat mengidentifikasi sifat pencerminan dari tampilan standar dari suatu obyek 3D.
- 2. Siswa dapat menggambar tampilan standar dari obyek berdasarkan tampilan standar yang ada posisi sebaliknya.

B. Konsep Matematika

Siswa telah belajar tentang bagaimana mengidentifikasi dan menggambar tampilan standar dari suatu obyek. Namun, siswa belum belajar tentang sifat dari tampilan standar. Tampilan standar dari suatu obyek memiliki sifat yang unik. Contohnya, tampilan depan dan tampilan belakang obyek di bawah hampir serupa. Perbedaannya hanya pada orienatasinya. Tampilan belakang menampilkan kebalikan dari tampilan depan. Tampilan itu seperti cermin karena sisi kiri menjadi sisi kanan.



C. Pendekatan dan metode pembelajaran

- a. Pendekatan pembelajaran: Pendidikan Matematika Realistik Indonesia (PMRI)
- b. Metode: diskusi kelompok dan diskusi kelas

D. Sumber dan media

- a. Sumber: Lembar Kerja Siswa (LKS)
- Media: Laporan candi pada dua kegiatan sebelumnya, satu set foto, satu lembar kerja dan tiga laporan untuk setiap kelompok.

Keterangan:

- Foto gambar standar pada kegiatan pertama dapat diperoleh di LKS. Guru harus memisahkan foto ini terlebih dahulu dari LKS. Setelah siswa mengkonstruksi candi pada kegiatan pertama dan menyimpulkan bahwa mereka butuh lebih banyak foto, guru dapat memberikan foto ini pada siswa.
- Guru harus memfoto dua laporan siswa pada kegiatan sebelumnya dan mencetaknya untuk pembelajaran kali ini.

E. Skenario pembelajaran

1. Pedahuluan (5 menit)

- Guru menjelaskan bahwa mereka akan bekerja dengan foto atau gambargambar candi lagi. untuk itu, siswa harus duduk dengan kelompoknya dan menyiapkan alat menggambar seperti pensil, penggaris, dan dsb.
- Bila diperlukan guru kembali mengingatkan norma sosial yang berlaku pada pembelajaran kali ini.

2. Kegiatan Inti (60 menit)

• Untuk memulai kegiatan, guru terlebih dahulu menyampaikan gambaran kegiatan.

"Reza kemarin menemukan banyak laporan candi yang sudah lama. Beberapa foto pada laporan itu lepas dan hilang. Ia memintaku untuk menolongnya lagi memperbaiki laporan tersebut. Jadi, sekarang saya minta bantuan kamu untuk menyelidiki bagaimana memperbaiki laporan ini." (Bacal cerita pada LKS)

- Guru membagikan LKS dan foto untuk setiap kelompok dan menginstruksikan mereka untuk mengerjakan kegiatan pertama.. Setiap kelompok juga mendapatkan dua laporan candi yang mereka buat pada pertemuan sebelumnya. Laporan ini berguna untuk membantu siswa menyelidiki laporan yang rusak.
- Dalam kegiatan pertama ini, siswa harus melakukan investigasi pada dua laporan yang mereka buat sebelumnya. Investigasi dilakukan dengan cara menjawab pertanyaan pada LKS. (10 menit)
- siswa akan diberi tiga layout/laporan yang berbeda. Mereka harus menggambar gambar/tampilan standar yang lain pada layout dengan hanya berdasarkan tampilan standar yang lain dari obyek tersebut. (15 menit) (lihat LKS dan catatan mengajar)
- Selama diskusi kelompok, guru berkeliling dan membimbing siswa untuk mengerjakan LKS dan membantu mereka jika mengalami kesulitan. (Lihat catatan mengajar untuk melihat kemungkinan apa yang akan dilakukan siswa dan jawabannya)
- Jika kelompok telah menyelesaikan penyelidikan, guru memilih satu kelompok untuk mempresentasikan kegiatan pertama. Guru memimpin diskusi dengan mengajukan pertanyaan di bawah: (15 menit)
 - Apa yang kalian simpulkan dari perbandingan gambar depan dan belakang?
 - Bagaimana dengan gambar dari kiri dan kanan? Jadi kesimpulan kalian seperti apa?
 - Bagaimana dengan kelompok lain?
 - Pada kegiatan berikutnya, berapa banyak laporan yang dapat kamu perbaiki? Bagaimana dengan kelompok yang lain? Laporan yang manakah itu?
 - Bagaimana kamu memperbaikinya? Mengapa gambar dari sudut pandang ini menjadi seperti ini? Coba jelaskan!
 - Mengapa kamu tidak dapat memperbaiki laporan yang lain? Apa yang membuat laporan itu tidak mungkin untuk diperbaiki?

- Guru juga memberikan kesempatan pada kelompok lain untuk menyempaikan pendapat jika mereka memiliki jawaban atau strategi yang berbeda.
- Setelah semua kelompok menyelesaikan kegiatan satu, siswa beralih pada kegiatan selanjutnya. Pada kegiatan kedua siswa harus menyusun foto laporan yang lepas sehingga terbentuk candi yang diinginkan. (Lihat LKS dan catatan mengajar) (10 menit)
- Setelah siswa selesai, guru memilih kelompok lain untuk mempresentasikan permasalahan selanjutnya. Guru memulai dan memimpin jalannya diskusi dengan mengajukan pertanyaan pada kelompok tersebut seperti: (10 menit)
 - Bagaimana kamu meletakkan foto yang lepas pada laporan? Coba tunjukkan pada teman-temanmu!
 - Mengapa kamu meletakkan foto-foto tersebut seperti itu? Tolong jelaskan!
 - Bagaimana kamu menentukan foto dari bagian atas?
 - Adakah yang memiliki jawaban/susunan yang berbeda?

3. Penutup (5 menit)

- Setelah semua presentasi selesai, guru dapat mengakhiri pembelajaran. Guru membimbing siswa untuk membuat kesimpulan tentang apa yang mereka lakukan hari ini.
- Guru juga meminta siswa untuk mengumpulkan LKS mereka karena akan diberikan pada Reza.

F. Penilaian

Sumber penilaian : Observasi diskusi kelompok, LKS, dan performa selama pembelajaran

Ada tiga aspek penilaian yaitu penilaian sikap, penilaian kompetensi pengetahuan, dan penilaian keterampilan. Penilaian kompetensi pengetahuan didasarkan pada LKS kelompok. Sedangkan penilaian sikap dan keterampilan ditentukan berdasarkan observasi guru selama pembelajaran berlangsung. Jika siswa menunjukkan perilaku sesuai dengan indikator maka guru memberikan tanda centang " $\sqrt{}$ " pada kolom keterangan "Ya". Sebaliknya, jika siswa tidak menunjukan perilaku yang disebutkan dalam indikator, maka guru mencentang kolom keterangan "Tidak". Total poin dilihat dari banyaknya jawaban "Ya". Satu keterangan "Ya" bernilai 1 poin, sedangkan keterangan "Tidak" bernilai 0 poin.

Aspek	Penilaian	Ket		Total
Penilaian	Indikator	Ya	Tidak	Poin
Sikap	 Siswa mengikuti pembelajaran dari awal sampai akhir kegiatan Siswa ikut berpartisipasi dalam diskusi kelompok dan investigasi Siswa berani menyampaikan pendapat atau mengajukan pertanyaan pada saat diskusi kelas Siswa ikut berperan dalam menciptakan susasana yang kondusfi untuk belajar seperti membantu teman yang kesulitan atau mengajak teman yang kurang berpartisipasi selama kegiatan 			
Kompetensi Pengetahuan Keterampilan	 Siswa memahami permasalahan dan kegiatan yang dilakukan. Contoh siswa mengerti bahwa pada kegiatan ini mereka harus menggambar tampilan candi berdasarkan foto yang dimiliki. Siswa dapat menentukan sifat gambar standar dari suatu candi misalnya siswa menyimpulkan bahwa tampilan depan dengan tampilan belakang memiliki bentuk yang mirip tetapi terbalik. Siswa dapat menggambar gambar standar candi berdasarkan gambar yang berada di posisi sebaliknya. Siswa bisa menyimpulkan laporan 2 dan laporan 3 tidak dapat diperbaiki karena foto yang diketahui tidak cukup Siswa dapat menemukan salah satu kemungkinan susunan foto pada kegiatan ketiga. Siswa dapat menjelaskan, menunjukkan dan mendemonstrasikan hasil investigasi kelompok jika ditanya oleh guru atau kelompok 			
Total Poin:				

CATATAN MENGAJAR - Pembelajaran 5



- **Tujuan:** Siswa dapat mengidentifikasi sifat tampilan standar dari suatu obyek 3D.
 - Siswa dapat menggambar tampilan standar dari obyek berdasarkan tampilan standar yang ada posisi sebaliknya.

Alokasi waktu: 2x35 menit

Gambaran: pada kegiatan ini, siswa diminta untuk menyelidiki dua laporan yang mereka buat pada pertemuan sebelumnya. Siswa harus membandingkan foto dari bagian depan dan belakanng candi serta foto dari bagian kanan dan kiri candi. Diharapkan siswa mampu menemukan sifat percerminan dari gambar standar tersebut dengan bahasa informal mereka sendiri.

Tantang halaman ini: halaman ini dimulai dengan konteks dari kegiatan/masalah diikuti dengan pertanyaan lanjutan tentang bagaimana siswa harus menyelidiki dua laporan yang diberikan. Lembar kerja dan laporan untuk kegiatan ini terlampir di bagian belakang dokumen ini. Guru membagikan dua laporan dari pertemuan sebelumnya.

Contoh jawaban siswa:

Gambar belakang dan depan.

- 1. Ya, bentuk dan banyak kubusnya sama.
- 2. Ya, ada kubus yang letaknya terbalik

Gambar kanan dan kiri.

- 1. Sama bentuknya, banyak kubusnya juga sama
- 2. Ya berbeda, bentuknya terbalik

Kesimpulan:

- Gambar dari bagian depan dan belakang bentuknya mirip tapi terbalik.
- Bentuknya berbeda tetapi memiliki bentuk yang mirip dan banyak kubus yang sama.



Hallo, kamu telah menolongku berkali-kali. Aku sangat beruntung memiliki teman sepertimu, kawan. Aku mempunyai beberapa laporan lama dari penemuan berbagai candi. Karena sudah sangat lama, beberapa foto hilang. Aku penasaran apakah kamu dapat memperbaikinya. Tolong bantu aku lagi untuk menentukan bagaimana bertuk laporan aslinya berdasarkan foto yang kita miliki di sini.



Ada tiga laporan utnuk diperbaiki. Kamu tidak harus memperbaiki semuanya. Mungkin saja, sebagian laporan tidak bisa diperbaiki. Jika ada laporan yang tidak dapat diperbaiki, tolong beritahu aku "kenapa" dengan menjawab pertanyaan di bawah.

🔎 Jawablah pertanyaan di bawah!

<u>Apakah ada laporan yang bisa diperbaiki? Jika ada sebutkan laporan berapal</u>

Jelaskan bagaimana kalian memperbaikinya?

<u>Apakah ada laporan</u> yang <u>tidak bisa diperbaiki</u>? Jika ada sebutkan dan jelaskan kenapa tidak bisa diperbaiki!

Gambaran: pada kegiatan ini, siswa diminta untuk menggambar tampilan standar berdasarkan foto yang diberikan pada laporan. Ada tiga laporan. Pertama, laporan yang memuat 3 foto dari bagian depan, kiri dan kanan. Kedua, laporan yang berisi foto dari bagian kiri, kanan dan atas. Yang ketiga memuat tiga foto dari bagian depan, atas dan belakang.

Tantang halaman ini: halaman ini dimulai dengan konteks dari kegiatan/masalah diikuti dengan pertanyaan lanjutan tentang bagaimana siswa menggambar gambar pada laoran. Lembar kerja dan laporan untuk kegiatan ini terlampir di bagian belakang dokumen ini. Guru membagikan tiga laporan di atas setelah semua kelompok menyelesaikan kegiatan satu dalam pembelajaran ini.

Komentar pada permasalahan:

- Guru mengingatkan siswa bahwa mungkin ada laporan yang tidak bisa diperbaiki. Artinya, beberapa gambar tidak dapat tentukan hanya berdasarkan foto yang diketahui/diberikan. Siswa harus menyelidiki "mengapa"
- Sebenarnya, hanya satu laporan yang dapat dilengkapi/diperbaiki yaitu laporan yang pertama.



Di bawah ini contoh pekerjaan siswa:





Contoh dari jawaban siswa:

Apakah ada laporan yang bisa diperbaiki?

Jawab: ya laporan satu bisa karena bentuknya mirip dengan foto yang lainnya Apakah ada laporan yang tidak bisa diperbaiki??

Jawab: ya ada, laporan 2 dan laporan 3 tidak bisa diperbaiki karena fotonya kurang





Haii,, Tadi malam, aku mendapat sebuah laporan tentang candi baru di Jawa Tengah. Akan tetapi, aku tidak sengaja menjatuhkan laporan itu dan semua fotonya lepas. Akutidak tahu bagaimana untuk menempel foto tersebut di tempat semula. Tolooong T.T., bantu aku memperbaiki laporan ini!

Reza <u>tidak sengaja merusak</u> laporan <u>itu</u>. Awalnya, foto-foto itu disusun dalam laporan seperti biasanya. Lihat pada gambar!

Ada 5 foto pada laporan. Reza tidak ingat letak foto-foto tersebut. Bantu Reza untuk menemukannya.

Dapatkah kalian tebak bagaimana susunan foto-foto tersebut pada laporan.



Jelaskan bagaimana kalian memperbaiki laporan di atas?

Gambaran: pada kegiatan pertama, siswa harus menemukan posisi foto pada laporan (layout). Mereka harus menebak urutan atau posisi karena mereka tidak diketahui obyeknya. Siswa hanya akan bekerja dengan foto tampilan standar dari 5 sudut pandang (kanan, kiri, atas, depan, dan belakang). Seperti biasanya, siswa akan bekerja dalam kelompok yang terdiri dari 3-4 siswa.

Tentang halaman ini: halaman ini memuat konteks dibalik kegiatan dan tugas untuk bahan investigasi siswa. Ada juga foto pada halaman ini tapi foto ini hanya untuk ilustrasi. Siswa tidak bisa mengguntingnya seperti pemebelajaran sebelumnya. Guru memberikan foto yang akan digunakan secara terpisah bersamaan dengan LKS ini.

Komentar:

- Jika siswa bingung maksud dari pertanyaan, guru bias meminta siswa untuk menjelaskan apa saja langkah yang mereka lakukan untuk menyusun foto.
- Ada dua susunan foto yang mungkin, yaitu pada contoh jawaban siswa di halaman sebelumnya. Semua jawaban dapat diterima diterima asalkan susunannya sesuai dengan tampilan dari atas



Gambar di samping adalah contoh jawaban siswa.

Komentar:

- 1. Jika siswa bingung dalam menentukan foto mana yang dari tampak atau bagian atas, guru bisa merujuk pada pekerjaan siswa pada pertemuan sebelumnya dan menanyakan pada siswa apa yang mereka dapat simpulkan mengenain gambar dari kiri dan kanan atau gambar dari depan dan belakang.
- 2. Guru dapat menyarankan siswa untuk meletakkan foto sedemikian sehingga foto bias berdiri pada posisinya. Kemudian, siswa diminta membayangkan obyeknya dan menebak bagaimana tampilan obyek dari posisi sebaliknya. Dengan melakukan hal ini, siswa mungkin lebih mudah membayangkan gambar pada sisi yang lainnya.
- 3. Ada juga kemungkinan siswa memahami sifat 'kemiripan' pada tampilan standar tapi mereka tidak menyesuaikannya dengan tampilan atas. Contohnya:





4. Pada kasus ini, guru membantu siswa dengan meminta mereka menghitung banyakanya kubus jika dilihat dari sisi atas dan membandingkannya dengan tampilan dari sisi samping.

Appendix 3

- 1. Examples of students' works in Pre TE
- 2. Examples of students' works in TE
- 3. The Improved HLT

Examples of Students' Written Works in Preliminary Teaching Experiment

1. Students' written works in the pretest







TERIMA KASIH ATAS SARANNYA

Kim. Tawablah pertanyaan di bawah inil Sekarang, kamu berdiri dan ambil foto dari posisi I sampai posisi B. Beriah tanda check "\" pada table berikut bila kamu dapat melilat bangunan tersebut pada kamera. Jika kamu tidak melihat bangunan tersebut barilah tanda strip "-" pada tabel. Misalnya, pada posisi 1, Tari dapat melihat rumah merah, menara dan apertenen, maka ia memberi tanda check pada ketiga bangunan tersebut dan tanda strip mada munich turk Bangungan danam badit uana laita.
 Jawablah pertanyaan di bawahi

 4. Jika kamu mengambil gambar dari pasisi 1, dimana posisi rumah merah dan menara terhadap apertemen?
 Rumah merah samping kanan apar temen Menara samping bir apartemen pada rumah pink. Bagaimana dengan posisi yang lain? Posisi Rumah Merah Rumah pink Menara Apertemen 5. Jika kamu mengambil gambar dari posisi 5, dimana pasisi rumah merah dan menara tertindap apertemen? Kiri Samping Kangun apartier neen : Menaro dan 2 y 7 3 V runab merah 5 V 6 8 2 6 Jawablah pertanyaan di bawahl Jika kamu mengambil gambar dari posisi 3, dimanakah posisi rumah merah dan menara terhadap apertemen? Pasisi mana sajakah yang memungkinkan untuk mengambil gambar yang menuat semua bangunan? 5,3,4, menara didepan apartemen rumah meran disan piny depan anartema kiri Posisi mana sajakah yang memungkinkan untuk mengambil gombar yang hanya memuat 3 bangunan saja? 3. Jiko kanu menganbil gambar dari posisi 7, dimana posisi rumah menghalan menana terhadap apertemen? Sladalang apartarkan = media fa fumah merah disampin bebladang Apartarkan in kanan 7, 8, 1 8 Pasisi mana sajakah yang memungkinkan untuk mengambil gan hanya memuat 2 bangunan saja? G $_1 \to$ nbar yang -Setelah melakukan investigasi dengan menjawab pertanyaan di atas, menurutmu posisi manakah yang terbaik untuk mengambil gambar bangunan-bangunan inu? Jelaskan jawabanmu agar Toni dapat mengerti alasanmu. Jelaskan alasanmu di sinil 3-4- dan 5 Karena semua banguran terlihat dengan jelas atau semuanya kelihatan

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 Tahukah kamu perbedaan foto dari bagian kiri candi (2) dan foto dari bagian kanan candi (4)? Jelaskan!
 4 Jailar to balik Ah... Aku tahu idemu. Tapi, aku bingung mengikuti strategi dan pemikiranmu. Jawablah pertanyaan di bawah agar aku bisa memahami bagaimana kamu meperbaiki laporan tersebut. San ping Aya Jawablah pertanyaan di bawahl 1 allowant per remain a contrain 1 Bagaiman kamin kanih foto untuk bagian atas, depan, belakara, samping kiri, dan samping kanan Jelankant di pilih yang sanca dulu yang tidak sa ma berarti ntu 1 alas 2. Tahukah kamu perbedaan antara foto dari bagian depan candi (1) dan foto dari bagian belakang candi (3)? Jelaskan! $\frac{1}{2}$ atasnyn Kelompok Z Niya, Patu, Youuriel bela hang samping samping Abas. devan





/ Bagian B POSTTEST A Hama: Canay 3 Bagian A ₩S^B с A Bayangkan kamu memfoto objek di atas dengan kamera dari posisi A. Gambarlah bagaimana bentuk objek pada foto yang dihasilkan! D 0 Gambar Disini! A Ada miniature bangunan di atas meja seperti pada foto di atas. Bangunan tersebut adalah dua rumah, satu apartemen, dan satu tawer. Bayangkan kanu berjalan mengelilingi meja tersebut dan memfoto bangunan tersebut dari setiap posisi A, B, C, dan D. Foto-foto berikut adalah hasilnya. Tentukan pada setiap foto berikut posisi dimana foto tersebut mbil deri posisi Foto 2 diambli dari p -# đ E A Foto 4 diambil dari posisi_____ Foto 3 diambil dari posisi_____

6. Students' written works in the posttest

Examples of Students' Written Works in Teaching Experiment

1. Students' written works in the pretest





Gambar di atas menunjukkan beberapa miniature bangunan di atas meja. Bangunan tersebut dalah dua rumah, satu apartemen, dan satu tawer. Bayangkan kamu berjalan mengelilingi meja tersebut dan berhenti pada posisi B dan C. Pada dua posisi tersebut, kamu umenfoto bangunan di meja tersebut.

Tentukan manakah diantara foto berikut yang merupakan hasil foto dari pasisi B2 $\emptyset~4$ Tentukan manakah diantara foto berikut yang merupakan hasil foto dari pasisi C?A







Perhatikan objek di atasl Objek tersebut berupa kubus-kubus yang ditumpuk membentuk suatu bangunan di atas meja. Bayangkan kamu memfoto objek di atas dengan kamera dari posisi A. Gambarlah bagaimana bentuk objek pada foto yang dihasilkan!



Jawablah pertanyaan di bawah inil 1. Fotolah bangunan tersebut dari posisi 1 sampai posisi 8 dengan kameramu atau kamera buatan yang disediakan. Berilah tanda centang "V" pada table berikut bila kamu dapat melihat bangunan tersebut pada kamera. Jika kamu tidak melihat bangunan tersebut berilah tanda strip "-" pada tabel. Misalnya, pada posisi 1, Toni dapat melihat rumah merah, menara dan apertemen, maka ia memberi tanda centang pada ketiga bangunan tersebut dan tanda strip pada rumah pink. Bagaimana dengan posisi yang lain? Posisi Rumah Merah Rumah Pink Menara Apertemen 1 V V V ٧ 7 -v 5 -1 22 2 6 ~ V 8 7 -2. Jika kamu mengambil gambar dari posisi 3, dimanakah posisi rumah Boisi 3, Posisi 4. V Posisi 5 merah, rumah pink dan menara terhadap apertemen pada foto? Rumah Marah . Di kanan Apartemen Rumah Pink = Di Eiri Apartanan Manam Di depan Apartanan 3. Jika kamu mengambil gambar dari posisi 7, dimana posisi rumah menan, rumah pink dan menara terhadap apertemen pada foto? Rumah smaRah = Di kilai agalawan kumah Dinkz di Sabelak kaman aDarlanen mana da di Kalakang apar ken en





Posisi mana sajakah yang memungkinkan untuk mengambil gambar yang

Posisi mana sajakah yang memungkinkan untuk mengambil gambar yang hanya memuat 2 bangunan saja?6 2

Aku sangat berterima kasih. Kamu telah menolongku dua kali. Sekarang, aku penasaran bagaimana kan memilih foto tersebut. Tolong jelaskan pada mengapa kamu tidak memilih foto yang lain? Jawablah pertanyaan berikut ini 1. Berapa banyak toto yang kamu tempel? 8 Feto Foto 7 9 oto 2. Fota namor berapa saja yang kamu pilih dan tempel? Foto 4.9,3,8,2,6,01 3. Berapa banyak foto yang tidak kamu pilih dan tidak kamu tempel? \mathcal{U} Fot o and and learns lectaknys terbalis 4. Foto nomor berapa saja yang tidak kamu pilih dan tidak kamu tempel? 7.10.12.5 5. Tempelkan foto yang tidak terpilih di balik halaman inil Kemudian tuliskan alasan disamping foto tersebut kenapa kamu tidak Foto 12 memilihnyal Foto 5 Farma Potonya tidak ada yang Cacok dungan bangu.

















POSTTEST	Begian B
Image: Product of the second secon	Arritation abjek di atasi Objek tersebut berupa kubus-kubus kayu yang di tumpuk membertuk suatu banguan di atas meja. Bayangkan kanu menfertu objek di atasi denga kumera dari pasisi A da B. Gambaridat bagiamana bertuk objek pada fatu yang dihasilkani Ada berapa kubus yang ditumpuk pada gambar di atasi Kubus kayu Mathematikati di disili Gember fata denga di disili
Foto 2 diambil dari posist	

6. Students' written works in the posttest

The Improved Hypothetical Learning Trajectory

The following elaboration of HLT have been improved based on the findings and the result of retrospective analysis in the preliminary teaching experiment. We only present the elaboration of lessons that had major revision for the next teaching experiment. The lessons with only minor revisions are not displayed in this section since they can be found in the previous elaborated HLT. Therefore, lesson 1 and lesson 2 are not elaborated again in this version. The details of the other lessons with major revisions are presented below.

1) Lesson 3: Reporting New Temples

Learning goals:

- a) Students are able to draw standard views of 3D objects from different stand points based on its close representation.
- b) Students are able to draw standard views of 3D objects from different stand points based on its distant representation.

Mathematical activity and conjecture of students' answer:

The activities are based on the context of temples in Indonesia. It begins with the story of an archeologist, Reza. He wants to share his experience as an archeologist. If he finds a new historical temple in Indonesia then he has to report his discovery to the government so that he has more assistance such as getting funds and tools for further exploration. In the report, he has to attach the standard views/drawings of the temple which consist of how the temple looks like from the front, the left, the right, and the top. Based on this context, students are given problems of reporting the discovery of a temple in small group of 3 or 4 students. Each group gets the building blocks, a digital camera and two camera models to help them determining and drawing the views.

The first problem in this lesson is that the students have to create their own temple by using exactly 8 cubes. Afterwards, they have to report their temple by drawing the standard views of the temple in the given layout. First, students will only use the camera model without the digital camera. Figure 4 shows the layout for this activity.



Figure 4. Layout for lesson 3

Conjecture of students' answers, actions, and strategies:

 a) Students may create a simple temple like a vertical six blocks or horizontal six blocks or another simpler cases. In this case, teachers have to challenge the students to have more unique temple.



b) Students may draw bird eye drawing. In this case, teachers must refer to the last problem in the lesson 1 and give an example of standard drawing by using the camera. For instance, for the following temple, students may draw this way:



c) Students probably put sign like shadow or colors on their drawing to indicate the position of the cubes. For example, they put shadow in the following drawing to indicate that there is another cube in the front. The sign also can be used to show that the cube is one step behind unsigned cubes. Probably, students also will use these signs to further indicate the level of the position of the cube. For instance, unsigned cube means the nearest cube to the camera and then all the cubes placed one step behind are blue while for those placed two step behind are red and so on.



d) The drawings are maybe rough and different from the real object. In this case, teachers suggest the students to use ruler to make the drawing better.

- e) Students are able to create proper drawings but they miscount the cube or make small mistake like misplaced the cube in the drawing. To help the students, teachers evoke the students to carefully investigate the number of cubes and their position.
- f) Students can draw the standard views of the temple without big difficulties.In this case, teachers can further investigate how they did it and asks them to check their drawing.
- g) For the top view, some students maybe have difficulties. Therefore, teachers shall put the temple in the ground and demonstrate if the camera is exactly on the upper side of the temple.

After the groups finish the drawings, teachers instruct the students to check their drawings by using their digital camera.

The second problem gives the students a new temple in a photograph (Figure 5). Students have to report this discovery but they only have a photograph of the new temple. Teachers do not provide them the building blocks and the camera models like the previous one. In the other words, they have to imagine the standard views based on the photo and the context of the problem.



Figure 5. A new temple for the second problem of lesson 3

Conjecture of students' answers, actions, and strategies:

- a) There is possibility the students will draw based on what they see and tried to make bird eye drawing since they only have the bird eye photo of the temple. In this case, teachers can give them a simple example like a cube, a container or a tent or something familiar to the students such that they can imagine it. If they still difficult teachers can use building blocks to make the object and ask students to compare it to the photo.
- b) Students may do not draw squares but rectangle and the drawing is also a little bit rough. This is not a big problem since it is just a sketch. However, it is better for teachers to suggest the students to use a ruler.
- c) Students can mistakenly count or unaware of the number of squares. For instance,



In this case, teachers invite the students to confirm the number of cube and how they are placed in the photo.

 d) Students draw the standard views of the temple without any difficulties by counting the cube and using ruler.

In the last problem, students have to draw the standard view of a new temple only from the left (2). In addition, the drawing is done not in a layout but in the worksheet. Thus, there is a move from the use of layout which help the students to imagine the temple to not use the layout which is maybe a little bit confusing for them (see the worksheet). Similar to the second problem, the temple is only in photograph. Teacher will not give the blocks. The temple is shown in Figure 6 below.



Figure 6. A new temple for the third problem of lesson 3

Conjectures of students' answers, actions, and strategies:

- a) Students will imagine the shape of the temple like the previous activity and imagine the shape to draw.
- b) Probably, students count the cube as a reference to draw the shape.
- c) Probably students will draw a little bit sloppy since they do not work in the layout like the previous problems. For instance:



In this case, teachers scaffold the students by asking the height of the temple by counting the cubes in students' drawing compared to the photograph. Teachers also can refer to the previous drawing as an example or to give demonstration.

After all the group finish the task, teachers ask some groups to share their strategy and their drawings. After all the students finish the drawing, teachers
choose one group to present their work in front of the class. Teachers lead the discussion by posing questions like:

- a) Can you show us your temple?
- b) Show the drawing to the others. How you did it?
- c) Why there is only three cubes (for example) in this drawing?
- d) Etc

Sometimes, teachers also give chances to the other students to react or if they have question. Teachers then choose another group to present the second problem. Teachers help the discussion by giving questions to the group.

- a) Can you show us your drawing?
- b) What is this drawing? Where was it taken?
- c) Does everyone has the same drawing from the same position?
- d) How did you determine that the drawing would be like that?
- e) Why were there only 5 square/rectangle/cubes in its base?Teachers then check all the drawings (front, left/right, Top)
- f) Why are all the drawings the same?

Teachers also give chances to the other students to react or give opinions to the presentation, especially if they have different strategy or opinion. For the last problem, teachers ask all the groups to show each other the drawings and ask some students to determine how they did it. Afterwards, teachers bring the class to an end by guiding the students to conclude what they have been learned in this session.

2) Lesson 4: Building the Temples

Learning goals:

- a) Students are able to identify, predict, and read the blind spot of distant representations of 3D objects.
- b) Students are able to interpret, recognize, and communicate distant representation of 3D objects.
- c) Students are able to construct a 3D object based on its standard views.

Mathematical activity and conjecture of students' answer:

The activities are still based on the story of discovering temples. Reza visited the government office and got new reports of a new temple. He was asked by the government to build a miniature of the new temple. Since Reza is busy, he asks students to help him. The students will work in the same small groups of 3-4 people like the previous lesson. They will deal with two problems of building a miniature of the new temple.

The first problem tells the students that they receive a report of a new temple from archeologists. However, unlike the other reports, they only attach one bird eye photograph of the temple. Therefore, they have to make the model of the temple based on only its photo. This problem aims to make the students aware of the blind spot of the representation. The photograph have weakness in representing 3D object since it has blind spot of the representation. Figure 7 shows the photos of the new temple (see also student worksheet and teaching notes in the appendix).



Figure 7. Photograph of the new temple in the first problem of lesson 5 Conjectures of students' answers, actions, and strategies:

a) Students probably cannot construct properly the object similar to the photo. For example, the following construction has error in the side. In this case, teachers help the students by evoke the students to determine the position of the cubes in the photo like "do you see this column? Where does this lie in this block?"



b) At first, students may just consider the visible cubes and construct it the way it looks like in the photo. In this case, teachers shall ask "Do you know how it look like in the back of the temple?" "Are there another cubes in there?" "How do you decide it?" the purpose is to bring the students to aware that the blind spot cannot be predicted unless they have clue. Here is some of the possible answer for this conjecture:



- c) Students probably aware of the unseen part and they assume the back view of the temple. They may have construction similar to the second conjecture. In this case, teachers argue by asking "What if everyone has different assumption?" "Will they have similar temple in the end?". Student may think that without further information they cannot build the temple.
- d) There is a possibility the students will claim they cannot make the model and debate each other of how the blind spot look like since we cannot determine clearly what is in the back. In this case, teachers guide the students to arrive at the conclusion themselves that the blind spot cannot be determined unless we have more information about it. This information will be given to them later.

Teachers give 15 minutes for the students to think and construct their temple. Afterwards, teachers invite all students' attention into class discussion. Teachers begin the discussion by asking one of group's construction and the compare to the other. If there are differences, teachers shall discuss it, especially if it is about the blind spot. If there is no differences, teachers shall propose a construction which is similar to the picture but have more cubes in the blind spot. The aims is to drive the students to be aware of the blind spot and we cannot determine how it looks like in the back. After all the students agree that the blind spot/ the back side of the temple cannot be determined, teachers give each group the standard views of the temple. The discussion is then continued by asking the students whether now they can make the model or not. The expected conclusion of this first problem is that they cannot make the model based only on one picture. Teachers really play crucial role in this discussion by guiding the students, propose question to make them aware of the blind spot. Here is the standard views of the temple.



Figure 8. Standard views of the temple for the first problem of lesson 5 After getting the standard view, we predict students will be able to build the temple. The temple will be like:



After concluding the first problem, the students will get another report of a new temple. This time, the report includes the standard views of the temple. Similarly, students have to build the temple by using building blocks (see student worksheet in the appendix). Figure 9 shows students worksheet and the standard views of the temple.



Figure 9. Standard views in the layout for the second problem of lesson 5 Conjectures of students' answers, actions, and strategies:

 a) Based on their experience on the first problem, students put the blocks by following the top view first and then they adjust the object by looking at the standard view of each side.



b) Students also might start with the side views first and then began to adjust with the other view.



c) Students may directly construct the object based on their feeling/sense.
 Afterwards, they adjust it based on the standard view.

d) Some students may propose to flip the paper such that the side view will be vertical to fit it to the real object and then construct it. This was quite helpful to imagine the form of the object. Look at the following illustration:



e) Students can mistakenly flip the paper to the inside so that the top part will be at the bottom. In this case, teachers can ask them to remember how they draw the views in the lesson 2. Then, teachers ask them "Where is the top part of the temple in your drawing?" "If you folding it, where will it be?"



f) There is possibilities that students will mistakenly put the blocks since the view from stand point 1 and 4 have huge differences. In this case, teachers invite the students to check again their construction. Question like "Why did you put this block in here? Coz I did not see it in the views" can be used to begin the investigation.



g) Some groups probably construct two different temple regarding the number of cube to be used. The following constructions show these possible temples with different number of cubes.



In the end of the activities, teachers select one groups to present their work and explain how they get it. It is important for the teachers to guide the discussion and make sure all the students understand the explanation. We expect all there are groups that have different constructions since the number of cubes can be 9 or 10. If there are no group having this case then teachers must propose themselves to make students aware of it. Afterwards, teachers tell the students that the temple only need 9 cubes. We expect after getting this information, students will have the same construction of the temple like Figure 10.



Figure 10. The correct construction of the temple in activity 2 lesson 4

3) Lesson 5: Fixing the report

Learning goals:

- a) Students are able to identify the properties of standard views of 3D objects.
- b) Students are able to draw the standard views of an object based on its opposite standard view.

Mathematical activity and conjecture of students' answer:

Teachers begin the activity by instructing students to work in the same group as the previous lessons. The context in this meeting is about Reza who messed up the old report of a new temple last night. Some report lost its photos. Therefore, students are asks to help him to rearrange the photos in the report or draw a new one. At beginning, teachers give students two report from the previous lesson and instruct the students to investigate the reports by answering questions in the worksheet. The following Figure 11 is the question in the worksheet.

 Apakah ada perbedaan pada kedua gambar tersebut? andingkan pula gambar dari bagian kanan dan kiri candi. Apakah ada kemiripan dari kedua gambar tersebut? Apakah ada perbedaan pada kedua gambar tersebut? 	
ndingkan pula gambar dari bagian kanan dan kiri candi. 1. <u>Apakah ada kemiripan dari kedua gambar tersebut?</u> 2. Apakah ada perbedaan pada kedua gambar tersebut	Jelaskan!
 <u>Apakah ada kemiripan dari kedua gambar tersebut</u>? Apakah ada perbedaan pada kedua gambar tersebut 	
2. Apakah ada perbedaan pada kedua gambar tersebut	Jelaskan!
	? Jelaskar

Figure 11. The questions to investigate students' reports in lesson 5

Conjectures of students' answers, actions, and strategies:

- a) Students probably answer that the views are different since there is part of the view placed in opposite position. However, they note that the shapes are almost similar and have the same number of cubes.
- b) Some students may say there is a similarity between the two views but the views are a little bit different since the position are not the same.
- c) Students probably note that the number of cubes the similarity between the two views is the number of the cubes. The difference between the two views is the shape or the direction or position of some cubes

Afterwards, teachers ask the students to move to the second problem (see the worksheet and the teacher guide). In this second activity, teachers give each groups 3 layouts of different cases. Each layout contains 3 different standard drawings. In the first case, the photos are from the top, the left, and the front. The photos in second case are from the top, from the right and from the left. The last one contains only two drawings from the top, and the back. The task is that students have to complete the standard views in the each report based on the remaining photos attached. Teachers must warn the students that maybe only some reports can be repaired. In this activity, teachers do not provide the students neither building blocks nor the camera. Students have to draw to completely fill the layout. The following Figure 12 presents the three different cases in this problem.



Figure 12 Three different cases for the second problem of lesson 4 Conjectures of students' answers, actions, and strategies:

- a) To draw the photo from the opposite direction of the given photo, students may use the photo in the opposite as the reference. Based on the previous investigation, they may imitate the opposite views to draw the views in the layout.
- b) Students understand that the shape will be 'similar' to its opposite, but they are wrongly draw the orientation. The following pictures show that possibilities.



In this case, teachers make sure why the students did that because there is possibilities that what students imagine or students' orientation is correct. If the drawings stand then they are correct drawings. If students did it wrong (they do not have reason why they did that), teachers scaffold the students by making the photo stands and ask the students to imagine what they will see from the opposite. Teachers also can refer to the previous investigation of the two reports.

- c) Some groups may claim that they cannot draw the empty photo for the 2nd and 3rd cases. In this case, teacher should asks more why they cannot draw it.
 Probably, students are aware that they need the opposite photos of the views.
- d) Some groups may predict the empty photos in the 2nd and the 3rd cases this way:



In this case, teachers ask the students such as "How could you know/ figure it out?" "Why there is three block here?". Probably, students just imagine or assume that the temle will be like that. In this case, teacher can refer to the previous investigation. Teacher may use blocks to illustrate the possibilities of the temple.

After all the groups finish their work, teachers select one group to present their work in front of the class. Teachers lead the discussion by posing questions to the presenter like:

First problem:

- a) Did you find the similarities between the front and the back view of the temple?
- b) What about the difference?
- c) What is your conclusion after investigating the views? What about the other group?

Second problem

- a) How many reports can you fix? Which one is that?
- b) How did you fix it? Why did the view become like this?
- c) How many reports left? Why did not you fix it?
- d) How about the other group?

After the discussion is finished, teachers move to the last problem. In this last problem, students are asked to guess how to put the photos of a report into the right position like how they were. Teachers give each group 5 photos and a layout. The photos and the layout are shown in Figure 13.



Figure 13. The layout and the photos for the first problem of lesson 5

Conjectures of students' answers, actions, and strategies:

a) Students firstly put the top view of the temple since the photo obviously belongs to the top view. The reason maybe because it has no 'similar' picture like the others. However, some students may takes time to realize which photo the top view is. In this case, teachers take an example of students work from previous lesson and asks them to investigate it to find the pattern or how the photo relate to each other.



- b) Students may do not have idea how to put the photo and saying that it can be anything. In this case, teachers can asks the students about their experience in the previous lesson. For instance, teacher can begin by assuming one photo is belong to certain position. Afterwards, teachers ask how it will look like from the opposite.
- c) Students nicely choose 'similar' photo for the opposite position. However, they falsely put the photos with respect to the top view. In this case, teachers scaffold the students by imagining how many cube the students will see from the front or side based on the top view. For



d) Students correctly put the photos by relating the photos to its opposite and the top view. In this case, maybe the students will conclude that there are two possibilities of how the photos are arranged based on the top view. In this case, teachers can ask further question like what makes them different, and whether it influences the shape of the new temple.



After all the groups finish their work, teachers select one group, different from the previous group, to present their work in front of the class. Teachers lead the discussion by posing questions to the presenter like

- a) How did you put those photos in the report? Can you show us?
- b) Why did you put them this way? Please explain to us
- c) How did you figure out the photo from the top?Why this photo is in the opposite of this one?
- d) How about the other group? are there groups that have different arrangements?

If there are groups that have different answer, teacher can invite the group to discuss it. In the end of the lesson, teachers help the students to conclude what they have been done or learned.